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A technique for natural gauge boson masses

by

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Dedicada a mis padres

Abstract

In this work, a novel mechanism for spontaneous symmetry breaking is presented. This mechanism allows to avoid quadratic divergencies and is thus capable of addressing the hierarchy problem in gauge theories. Using the scale-dependent effective action Γ_k minimally coupled to a gravitational sector, variational parameter setting provides a mass and vacuum expectation value as a function of the constants arising in the low scale expansion of Newtons' and cosmological couplings. A comparison with experimental data, such as the Higgs mass, allows putting restrictions on these constants. This generic approach allows comparing with explicit candidates for an effective field theory of gravity. As an example, we use the asymptotic safety scenario, where we find restrictions on the matter content of the theory.

Ackowledgments

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Chapter 1

Acronyms used

- GR: General Relativity
- SM: Standard Model
- EWSB: Electroweak Symmetry Breaking
- SSB: Spontaneous Symmetry Breaking
- VEV: Vacuum Expectation Value
- QFT: Quantum Field Theory
- RG: Renormalization Group
- AS: Asymptotic Safety
- FP: Fixed Point
- VPS: Variational Parameter Setting
- PMS: Principle of Minimal Sensitivity
- FRGE: Functional Renormalization Group Equation
- QED: Quantum Electrodynamics
- EEA: Effective Average Action.

Contents

Chapter 2

Introduction

The Standard Model describes the elementary particles and their interactions in a successful way. However, there are good reasons for looking for physics beyond the Standard Model. One such motivation is the subject of this paper; the so-called hierarchy problem in theories with Spontaneous Symmetry Breaking (SSB) (1; 2; 3; 4; 5; 6).

2.1 SSB, quadratic divergencies, and the hierarchy problem in a nutshell

The measurements of the Higgs boson at the Large Hadron Collider (7; 8) confirms that its existence and properties are consistent with the Standard Model (SM). Unlike all other particles of the SM the Higgs is a fundamental scalar, which gives rise to the question, whether the SSB mechanism, which is induced by the Higgs field, is natural (9; 10; 11; 12; 13; 14; 15; 16; 17; 18; 19; 20; 21; 22; 23). The central issue is the strong sensitivity of masses of scalar particles against radiative corrections, leading to the so-called hierarchy problem and the failure of the notion of naturalness. It is, of course, possible that naturalness is not always a good guiding principle for the understanding of nature (24), but if one would have the choice, a natural description is certainly preferable.

The hierarchy problem affects only scalar particles since Dirac and gauge fields are technically natural. Just as for scalar fields, the mass term associated with spin-1/2 or spin-1 fields is invariant under a global symmetry, but there is further an enhanced symmetry when the mass parameter goes to zero. Since, in this case, quantum corrections respect the enhanced symmetry, the associated corrections will be proportional to the symmetry-breaking term. As a consequence, the loop corrections to masses of Dirac and gauge fields will be suppressed by the smallness of the tree-level parameter, and no fine-tuning is needed when further corrections are incorporated.

Unfortunately, the same does not hold for the mass term for scalar particles. In the SM, the term $m^2 H^\dagger H$, with H being the $SU(2)_L$ Higgs field doublet, is invariant under any gauge or global symmetry acting on it. Further, no additional symmetry is enhanced when $m \to 0$, and therefore the mass parameter is exposed to any contribution coming from the UV sector. The fact that at the quantum level, it has the sensitivity to the physics in the UV reflects the lack of arguments to justify the stability of the Higgs mass parameter against radiative corrections. Quite generally, in a theory with multiple mass-scales one finds that the Higgs mass m accumulates quantum corrections from all (coupled) particles at all energy scales. Thus, m is affected by heavy particles through the appearance of quadratic divergences (unlike the technically natural spin-$\frac{1}{2}$ or spin-1 fields). As we will see in the next chapter, in an effective field theory approach of the SM, where the momenta of virtual particles are involved, the radiative corrections are cut-off at the scale Π. The running of m from Π up to some experimental-scale μ is given in (3.37), and grows as (25),

$$m_H^2\left(\mu\right) = m_0^2\left(\Pi\right) + \delta m^2, \tag{2.1a}$$

$$\delta m^2 = \frac{\Pi^2}{16\pi}\left(-4y^2 + 4e^2 + 4g^2 C_2(\Box) + \lambda\right), \tag{2.1b}$$

where λ, y, e and g_2 are the Higgs quartic, Yukawa, Abelian, and non-Abelian gauge couplings, respectively. It is possible to provide a physical meaning for the cutoff Π. For example, from a Wilsonian perspective, Π would represent the space-time lattice spacing. Moreover, the quadratic divergences can be seen (at least approximately) as a placeholder for a physical threshold, identifying the regulator with the heavy particles coupled to the Higgs. Following (2.1a), an explanation of why the observed Higgs mass remains small requires a large fine-tuning such that the tree-level parameter exactly cancels the huge correction in (2.1b). If one starts with a bare action, the quadratic divergences for scalar fields always arise when the quantum corrections are incorporated. Accordingly, a light Higgs scalar cannot survive in a natural way if the theory is expected to hold up to large energy scales, such as the Planck scale. This is referred to the fine-tuning, hierarchy, or naturalness problem and turns out to be independent of the scheme one uses to renormalize the theory.

Historically speaking, there are three traditional ways of addressing the problem of quadratic divergences. The first one is embedding the SM into a new kind of symmetry, which acts in such a way that the Higgs mass is protected by this symmetry, turning it into a technically natural parameter. One possible option to implement this idea is based on a new fermion-boson symmetry called supersymmetry (26; 27; 28; 29; 30), where a cancellation between loops of different statistics takes place.

The second option invokes the possibility of bringing down the cutoff of the SM through an electroweak symmetry breaking spawned by a dynamically generated vacuum condensate of a strongly coupled group, known as technicolor or Higgsless models (31; 4; 32; 33; 34). Nevertheless, the discovery of a light Higgs mass (7; 8) ruled most of these models out. However, various extensions (35; 36; 37; 38; 39) suggest alternative ways to preserve this idea.

The third option establishes a set of vacua of the SM, over which the Higgs mass varies according to some statistical distribution (40; 41; 42). The anthropic principle provides a guide for explaining the observed light Higgs boson mass and the closeness to the QCD and weak scales without resorting to additional symmetries or a lower cutoff.

An incomplete list of more recent alternatives include:
4) NNaturalness (43; 44), which relies on multiples copies of the SM in the same universe, each with a different vacuum expectation value.

5) Twin Higgs (45; 46; 47) models are based on the incorporation of discrete symmetries that allow different SM quantum numbers, and the representation of two Higgs doublets into a fundamental $SU(4)$.
6) Noncommutative perturbative dynamics (48) assumes the separation of UV and IR physics, using a noncommutative theory. Here, non-trivial mixtures of UV and IR phenomena can explain different hierarchies in nature.
7) Asymptotic Safety (49) works with a non-trivial UV fixed point for gravity. The observed Higgs mass is predicted from supposing that the Standard Model plus gravity are valid up to Planck scale energies and assuming that there are no new fundamental degrees of freedom at intermediate scales.
8) Cosmological relaxation (50) is a model, where the cosmological evolution of the Universe is diving the Higgs boson mass to a much smaller value than the Planck scale.

2.2 SSB without quadratic divergencies

The goal of this work is to point out a novel way of inducing spontaneous symmetry breaking. It allows to generate masses of gauge bosons without quadratic divergences. This mechanism thus avoids the corresponding hierarchy problem mentioned in the previous subsection 2.1.

For this purpose, the starting point will be a bare action without "dangerous" interactions like the quartic Higgs coupling. Quantum corrections to this classical bare action yield a scale dependence at the level of the gauge couplings contained in an effective action Γ_k. The arbitrary renormalization scale cannot be part of physical observables. It will be set following the Variational Parameter Setting (VPS) (51) prescription, which can be understood as the principle of minimal sensitivity (52), applied to quantum field theory background calculations. The VPS prescription allows to minimize the scale-dependent effective action with respect to variations of its field- and parameter content, giving a set of non-linear differential equations, frequently referred to as gap equations (53). These gap equations are different from the equations one would obtain from the initial bare action.

It is possible to choose a bare action $S(\Phi)$ such that no quadratic divergences arise in the effective action $\Gamma_k(\phi)$, where ϕ is the quantum expectation value of the field Φ. This means for constant values of the renormalization scale k, there will be no terms, which are usually necessary to generate SBB. However, in every quantum field theory calculation the scale k has to be set in order to arrive at a testable prediction. This necessity is a consequence of the incomplete nature of any perturbative or effective quantum field theory approach. It depends on the observable one is interested in, whether one choses k as a function of kinematic variables, renormalized parameters, or something else. Thus, assuming the scale to be an independent constant was actually inconsistent. For example, when one is interested in background configurations one way to proceed is the scale setting of the "improving solutions" procedure, leading to Uehling-type potentials (56; 57; 58; 59; 60; 61; 62; 63; 64; 65). However, the scale setting in "improving solutions procedures" leads to an anomalous violation of the underlying gauge symmetries. Fortunately, such a breaking of gauge symmetries is not necessary. As shown in (51), the afore mentioned VPS prescription allows to derive an optimal scale setting $k \to k_{opt}$, which preserves the underlying gauge symmetries of the effective action Γ_k. After the replacement $k \to k_{opt}$ the effective action Γ_k becomes an optimal effective action Γ_{opt}.

The main massage of this paper is that for SSB to occur, it is sufficient that it occurs at the level of this optimal effective action Γ_{opt}, as opposed to the non-optimal effective action Γ_k, or the bare action S. The advantage of this is that quadratic divergences arising from quantum corrections of the bare action S in the standard SSB are absent when SSB only occurs at the level of Γ_{opt} since the optimal effective action has all quantum corrections already incorporated (67; 66). No additional quantum corrections have to be incorporated into Γ_{opt} and thus no quadratic divergencies occur.

This idea is conceptually appealing. In section 5.1 it is implemented for scalar Quantum Electro Dynamics (QED). It turns out that the resulting optimal effective action Γ_{opt} only allows for SSB if the gauge fields form a condensate. Even though, this might be an interesting possibility it deviates from our original intention. One realizes that for the program to work, one needs a scale dependent vacuum contribution to the effective action. This is the reason why we proceed with an effective description of quantum gravity, where this vacuum contribution is given in terms of a cosmological constant. As example we study the Asymptotic Safety (AS) approach. The AS (68) conjecture provides a consistent description of gravity as a non-perturbatively renormalizable quantum field theory (69; 70; 71) and a scenario for testing the results of this work. Moreover, the scale dependence of the gravitational and cosmological constant has been extensively studied, in (72; 73; 74; 75; 76; 77; 78) as well as properties and consequences of the scale-dependent Einstein Hilbert action (79; 80; 81; 82; 83; 84) and Gaussian massless matter fields minimally coupled to an external metric (85; 86; 87; 88; 89; 90; 91; 92).

The thesis is organized as follows: in chapter 3, the nature of the electromagnetic and weak forces will be examined, providing a conceptual explanation of the standard SSB mechanism. In chapter 4, the techniques of the functional renormalization group (FRG) are studied, giving a derivation of the Wetterich equation and then proceed to introduce the AS scenario for quantum gravity. With particular emphasis on the application of the FRG to the Einstein-Hilbert action, the compatibility of minimally coupled matter degrees of freedom with the AS program is explored. chapter 5 provides the mechanism where the electromagnetic sector is minimally coupled to a metric field, conducing to a symmetry-breaking potential without the necessity of a quartic self-interaction. Expressions for the mass and VEV of the Higgs boson, as well as a benchmark between the gravitational parameters coming from an infrared expansion, are worked out. Our results are compared with those obtained in the context of the functional renormalization group. Finally, a summary, some comments, and ideas for future work are given in chapter 6.

Chapter 3

Basics on Quantum Field Theory and Hierarchy Problem

3.1 Spontaneous Symmetry Breaking in $U(1)$ gauge theory

The essential idea underlying the Higgs mechanism can be illustrated considering the classical Abelian Yang-Mills theory. The $U(1)$ gauge-invariant kinetic term for the gauge field is,

$$\mathcal{L}_{kin} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \tag{3.1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. (3.8) is invariant under the local gauge transformation $A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \partial_\mu \eta(x)$, for any η and x. Please note that the insertion of a mass term for the gauge field in the Lagrangian breaks the local $U(1)$ gauge symmetry. The addition of a complex scalar field with charge $-e$ that couples both, to itself and to the photon to the Lagrangian (3.1) gives,

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2, \tag{3.2}$$

where $D_\mu = \partial_\mu - ieA_\mu$ and $\lambda > 0$ for the scalar potential to be bounded from below. It is easily seen that this Lagrangian is invariant under the gauge transformation,

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \partial_\mu \eta(x) \ ; \ \phi(x) \to e^{i\eta(x)} \phi(x). \tag{3.3}$$

At this point, there are two possibilities for the theory. If $\mu^2 < 0$, the state of minimum energy is unique, located at the vacuum state $\phi\phi^\dagger = 0$, and the potential preserve all the symmetries of the Lagrangian, which describes massless electrodynamics with a massless photon and a charged scalar field ϕ with mass μ.

The other scenario takes place when $\mu^2 > 0$, and the potential develops a non-trivial degeneracy of minimums, acquiring a vacuum expectation value different to zero,

$$\langle\phi\rangle = \sqrt{\frac{\mu^2}{2\lambda}} \equiv \frac{v}{2}. \tag{3.4}$$

Now, the global $U(1)$ gauge symmetry is spontaneously broken. One of the main features of this breakdown in symmetry lies in the Lagrangian (3.2) can describe the physics of a massive gauge boson. To see this, let's pick the vacuum along the direction of the real part of ϕ,

$$\phi = \frac{v+h}{\sqrt{2}} e^{\frac{i\chi}{v}}, \tag{3.5}$$

where h and χ, which are the fields related to the Higgs and Goldstone bosons, respectively, corresponds to scalar fields without VEV. Replacing (3.5) into (3.2),

$$\begin{aligned}\mathcal{L} = &\left[(\partial_\mu - ieA_\mu(x))\left(\frac{v+h}{\sqrt{2}}e^{i\frac{\chi}{v}}\right)\right]^\dagger \left[(\partial^\mu - ieA^\mu(x))\left(\frac{v+h}{\sqrt{2}}e^{i\frac{\chi}{v}}\right)\right] \\ &- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mu^2\left(\frac{v+h}{\sqrt{2}}\right)^2 - \lambda\left(\frac{v+h}{\sqrt{2}}\right)^4 \end{aligned} \tag{3.6}$$

$$= \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^2v^2}{2}A_\mu A^\mu}_{\text{massive gauge boson}} + \underbrace{\frac{1}{2}\partial_\mu h\partial^\mu h + \mu^2 h^2}_{\text{massive scalar field}} + \underbrace{\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + evA_\mu\partial^\mu\chi}_{\text{Goldstone boson}} + ..., \tag{3.7}$$

where the dots represents interactions terms between h and χ. The Lagrangian (3.6) describes the physics of a massive gauge field A_μ with mass $m_A = ev$, a massive scalar field h with mass $m_h = \sqrt{2\lambda}v$ and a massless Goldstone boson χ. The gauge symmetry allows removing the confuse $A_\mu - \chi$ mixing by working in the unitary gauge. It is important to note that the total counting of degrees of freedom (DOF) before and after the SSB is the same. One starts with a massless vector field (two DOF) and one complex scalar field (two DOF) and ends with a massive vector field (three DOF) and one real scalar field (one DOF). This phenomenon is sometimes described as the vector field has eaten up the Goldstone boson χ in the unitary gauge, becoming massive. In the next section, this toy model will be expanded to a more realistic theory.

3.2 Electroweak Symmetry Breaking

In this section, the nature of weak forces is analyzed. At high energies, the standard model is invariant under a $SU(2) \otimes U(1)$ gauge group. However, the $U(1)$ gauge group at such energies is not the electromagnetic $U(1)_{em}$. To make the distinction, at high energies, the gauge group $U(1)$ turns out to be the Abelian group $U(1)_Y$ hypercharge. As one lowers the energy, this symmetry breaks into,

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em}. \tag{3.8}$$

One calls the $SU(2)_L \otimes U(1)_Y$ the electroweak symmetry, and the process of breaking symmetry at low energy is called electroweak symmetry breaking (EWSM).

One of the great achievements of the SM is the explanation of the EWSM process. EWSB decodes how to go from $SU(2)_L \otimes U(1)_Y$ to $U(1)_{em}$ at low energies through the Higgs mechanism. The central problem associated with EWSB can be summarized as the masses of the mediators of the Weak force, the gauge bosons. Experimentally, weak bosons have masses. The only way, however, to introduce masses for the gauge bosons without spoiling unitarity and renormalizability. Consequently, it is not possible to write these mass terms directly in the SM Lagrangian without violating gauge invariance. The simplest way to solve this mass problem is through the Higgs mechanism.

To see how the Higgs mechanism provides mass to gauge bosons, consider the SM Lagrangian.

$$\mathcal{L}_{SM} = \mathcal{L}_{kin} + \mathcal{L}_{int} \tag{3.9}$$

Typically, the interaction terms contain,

$$\mathcal{L}_{int} = \mathcal{L}_{Yukawa} - V(\phi), \tag{3.10}$$

where the potential $V(\phi)$ contains only scalar terms related to the Higgs field ϕ, whose shape is responsible for the EWSB,

$$V(\phi) = \mu^2\Phi^\dagger\Phi + \lambda\left(\Phi^\dagger\Phi\right)^2, \tag{3.11}$$

with Φ being a scalar doublet field. For $\mu^2, \lambda > 0$, the potential has a minimum different to zero which occurs at the vacuum expectation value (VEV),

$$|\Phi|^2 = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}. \tag{3.12}$$

All these minimum configurations are connected by gauge transformations which change the phase of the complex field Φ without affecting its modulus. It is important to note that the Lagrangian (3.9) is still invariant under gauge transformations and all its characteristic properties, as current conservation, still stand in place.

Expanding the Higgs doublet around the minimum v,

$$\Phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix} = \frac{1}{\sqrt{2}}\exp\left[\frac{i\sigma_i\theta^i(x)}{v}\right]\begin{pmatrix}0 \\ h(x)+v\end{pmatrix}, \tag{3.13}$$

where $h(x)$ represents fluctuations around v, and the θ_i's shift $\phi(x)$ along the flat minimum of the Higgs' potential. These θ_i's are known as Nambu-Goldstone, massless particles associated with $v \neq 0$ symmetry breaking. Rotating the fields $\theta_i(x)$ by a $SU(2)_L$ gauge transformation so that $\theta_i(x) = 0$ (the so-called unitary gauge), the kinetic term for the

Higgs field will be,

$$\mathcal{L}_{H,kin} = \frac{1}{2}\begin{pmatrix}0 & h+v\end{pmatrix}\left(\partial^\mu - igT_aW^{\mu,a} - ig^{'}B^\mu Y\right)^\dagger \left(\partial_\mu - igT_aW^a_\mu - ig^{'}B_\mu Y\right)\begin{pmatrix}0\\ h+v\end{pmatrix}, \tag{3.14}$$

where $T^a = \frac{1}{2}\sigma^a$ and Y are the generators of the $SU(2)_L$ and $U(1)_Y$ gauge group, respectively. The covariant derivative contains the gauge field of $U(1)_Y$ (B_μ) and $SU(2)_L$ (W_μ). Collecting the terms containing only the interaction among the Higgs and gauge fields,

$$\mathcal{L}_{H,kin} \ni \frac{g^2v^2}{8}(W_1^\mu W_{\mu,1} + W_2^\mu W_{\mu,2}) + \frac{v^2}{8}\begin{pmatrix}W_3^\mu & B_\mu\end{pmatrix}\begin{pmatrix}g^2 & -gg^{'}\\ -gg^{'} & g^{'2}\end{pmatrix}\begin{pmatrix}W_{\mu,3}\\ B_\mu\end{pmatrix}. \tag{3.15}$$

Defining the charged W^-_μ, and its complex conjugate as,

$$W^-_\mu = \frac{1}{\sqrt{2}}(W^1_\mu + iW^2_\mu) \tag{3.16a}$$
$$W^+_\mu = \frac{1}{\sqrt{2}}(W^1_\mu - iW^2_\mu). \tag{3.16b}$$

Inserting (3.16) into (3.15),

$$\mathcal{L}_{H,kin} \ni \frac{g^2v^2}{4}(W^{+\mu}W^-_\mu + \frac{v^2}{8}\begin{pmatrix}W_3^\mu & B_\mu\end{pmatrix}\begin{pmatrix}g^2 & -gg^{'}\\ -gg^{'} & g^{'2}\end{pmatrix}\begin{pmatrix}W_{\mu,3}\\ B_\mu\end{pmatrix}, \tag{3.17}$$

thereby, the weak bosons acquire a mass $\frac{gv}{2}$ through this mechanism. The remaining task is to calculate the mass of B_μ and W_3 fields, which are mixed through the coupling's matrix in (3.17). The two remaining neutral gauge bosons Z and A are defined in terms of the weak mixing angle $\tan\theta_w = \frac{g^{'}}{g}$,

$$\begin{aligned}Z_\mu &\equiv \cos\theta_w W_{\mu,3} - \sin\theta_w B_\mu\\ &= \frac{1}{\sqrt{g^2+g^{'2}}}(gW^3_\mu - g^{'}B_\mu)\end{aligned} \tag{3.18a}$$
$$\begin{aligned}A_\mu &\equiv \sin\theta_w W_{\mu,3} + \cos\theta_w B_\mu\\ &= \frac{1}{\sqrt{g^2+g^{'2}}}(g^{'}W^3_\mu + g^{'}B_\mu).\end{aligned} \tag{3.18b}$$

After the identification of (3.18) in (3.17), one obtains,

$$\mathcal{L}_{H,kin} \ni \frac{g^2v^2}{4}W^{+\mu}W^-_\mu + \frac{v^2}{8}(g^2+g^{'2})Z^\mu Z_\mu. \tag{3.19}$$

The kinetic part of the Lagrangian includes a mass for the Z boson $\frac{v}{2}\frac{g}{\cos\theta_w}$ while the photon's mass remains vanish. The gauge bosons acquire a mass through their interaction with the VEV. The fact that three goldstone bosons have been obtained mass is closely related to the election of the unitary gauge to absorb the Nambu-Goldstone bosons. Three $\theta_i(x)$ have been "eaten up" by three gauge bosons.

Although the Higgs mechanism is embedding the SM an explanation for how EWSB can provide a mass for the gauge bosons, the discovery of light Higgs bosons raises new questions about its origin. Before the discovery of this particle, two kinds of fundamental fields were known: fermions with half-integer spin and bosons with spin-1. The Higgs, however, is the only fundamental spin-0 scalar particle observed, and this leads to a problem with the quantum corrections and the corresponding divergences associated with scalar fields. The next section is devoted to calculating these divergences explicitly.

3.3 On Veltman's condition and hierarchy problem

In this section, we will briefly discuss what the hierarchy problem in the light of a computation of the self-energy for fermions, gauge, and scalar fields.

3.3.1 Fermions

The physical mass for the fermion field is given by,

$$m_\psi^2 = m_{\psi,0}^2 + \delta m_\psi^2, \tag{3.20}$$

where $m_{\psi,0}$ is the bare mass parameter appearing in the Lagrangian, and δm_ψ represents the quantum contribution due to loop corrections. At one loop, the main contribution is,

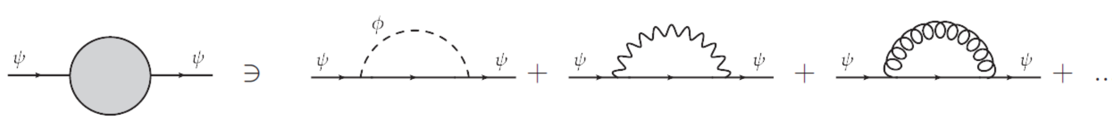
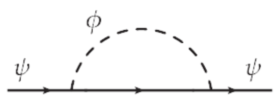


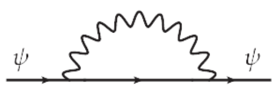
Figure 3.1: One loop contribution to the fermion self energy

The second diagram of Figure 3.1 gives the following contribution,



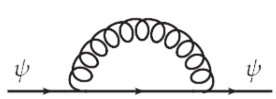
$$\begin{aligned}
&= \int \frac{d^4k}{(2\pi)^4}(-iy)\frac{i(\not{k}+m_\psi)}{k^2-m_\psi^2}(-iy)\frac{i}{k^2-m_\phi^2} \\
&= y^2\int_0^1 dx \int \frac{d^4k}{(2\pi)^4}\frac{\not{k}+m_\psi}{\left[k^2-xm_\psi^2-(1-x)m_\phi^2\right]^2} \\
&= \frac{iy^2}{(4\pi)^2}\int_0^1 dx \int_0^{\Pi^2} dk_E^2 \frac{m_\psi k_E^2}{\left[k_E^2+xm_\psi^2+(1-x)m_\phi^2\right]^2} \\
&= \frac{iy^2}{(4\pi)^2}\frac{m_\psi}{m_\psi^2-m_\phi^2}\left(m_\psi^2\log\frac{\Pi^2+m_\psi^2}{m_\psi^2}-m_\phi^2\log\frac{\Pi^2+m_\phi^2}{m_\phi^2}\right)
\end{aligned} \tag{3.21}$$

In writing down (3.21) we made use of a Wick rotation. The third diagram of Figure 3.1 gives,



$$\begin{aligned}
&= \int \frac{d^4k}{(2\pi)^4}(ie\gamma^\nu)\frac{i(\not{k}+m_\psi)}{k^2-m_\psi^2}\frac{-ig_{\mu\nu}}{k^2-\mu^2}(ie\gamma^\mu) \\
&= -e^2\int_0^1 dx \int \frac{d^4k}{(2\pi)^4}\frac{-2\not{k}+4m_\psi}{\left[k^2-xm_\psi^2-(1-x)\mu^2\right]^2} \\
&= \frac{-ie^2}{(4\pi)^2}\int_0^1 dx \int_0^{\Pi^2} dk_E^2 \frac{4m_\psi k_E^2}{\left[k_E^2+xm_\psi^2+(1-x)\mu^2\right]^2} \\
&= \frac{-ie^2}{(4\pi)^2} m_\psi \log\frac{\Pi^2+m_\psi^2}{m_\psi^2},
\end{aligned} \tag{3.22}$$

where to get to the last line, it has been applied the limit $\mu \to 0$. The non-Abelian gauge boson contribution to the fermion self-energy is identical to the contribution from the Abelian gauge boson, except for the quadratic Casimir factor $C_2(\Box)$ coming from the $SU(N)$ non-Abelian gauge group theory,



$$= \frac{-ig^2}{(4\pi)^2}C_2(\Box)m_\psi\log\frac{\Pi^2+m_\psi^2}{m_\psi^2}, \tag{3.23}$$

3.3.2 Gauge Bosons

The quantum corrections to the Abelian and non-Abelian gauge boson masses are shown in Figure 3.2

Fortunately, evaluating all these diagrams is unnecessary because the gauge symmetry protects gauge boson masses. To see how the mechanism works, the process will be em-

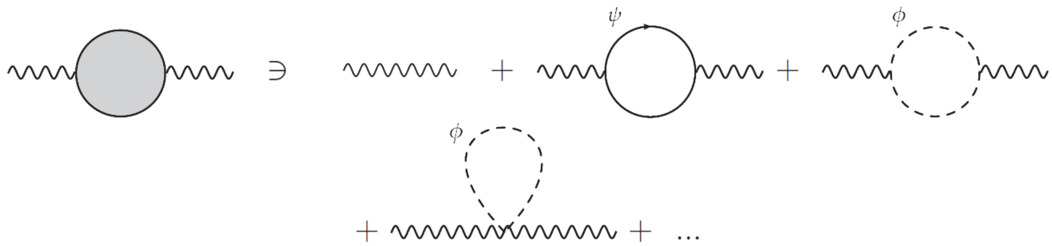


Figure 3.2: One loop contribution to the Abelian gauge bosons

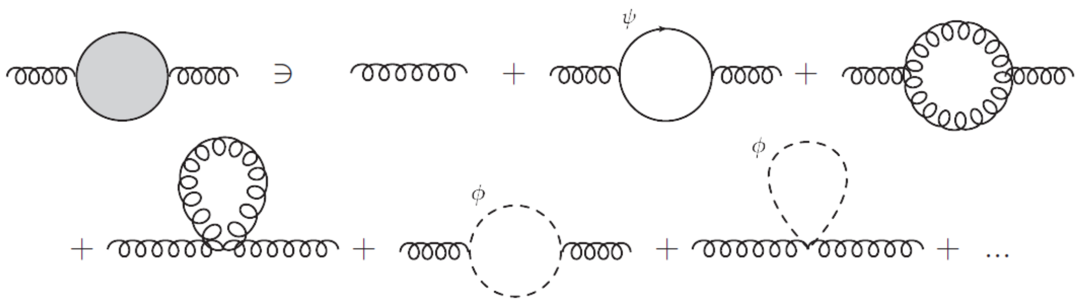


Figure 3.3: One loop contribution to the non-Abelian gauge bosons

ployed on Abelian gauge bosons, but a similar arguments protect non-Abelian gauge boson mass. The Abelian gauge transformation,

$$\phi(x) \to e^{i\alpha(x)}\phi(x), \tag{3.24}$$

implies that terms like $\phi\phi^*$ are invariant. The covariant derivative is constructed such that both, the mass and kinematic terms are invariant under gauge transformations.

$$\begin{aligned} D_\mu\phi &\equiv (\partial_\mu - ieA_\mu)\,\phi \\ &\to i\partial_\mu\alpha e^{i\alpha}\phi + e^{i\alpha}\partial_\mu\phi - ieA_\mu^{'}\phi \\ &= e^{i\alpha}\left(\partial_\mu - ie\left[A_\mu^{'} + \frac{1}{e}\partial_\mu\alpha\right]\right)\phi. \end{aligned} \tag{3.25}$$

If A_μ transform under the local gauge transformation as,

$$A_\mu \to A_\mu^{'} = A_\mu - \frac{1}{e}\partial_\mu\alpha, \tag{3.26}$$

then the kinetic term (3.25) is,

$$\begin{aligned} D_\mu\phi &\to e^{i\alpha}\,(\partial_\mu - ieA_\mu)\,\phi \\ &= e^{i\alpha}D_\mu\phi, \end{aligned} \tag{3.27}$$

and $D_\mu\phi D^\mu\phi^\dagger$ is invariant under (3.24) and (3.26). With this gauge transformation in mind, the bare gauge boson mass in the Lagrangian (3.9) must respect the symmetry (3.24). For the mass term of the gauge boson,

$$\mathcal{L} \ni M_\gamma A_\mu A^\mu, \tag{3.28}$$

the gauge transformation (3.26) yields,

$$\begin{aligned} m_\gamma A^\mu A_\mu \to m_\gamma \left(A^\mu - \frac{1}{e}\partial^\mu\alpha\right)\left(A_\mu - \frac{1}{e}\partial_\mu\alpha\right) \\ = m_\gamma A^\mu A_\mu - \frac{2}{e}m_\gamma A^\mu\partial_\mu\alpha + \frac{1}{e^2}m_\gamma\partial^\mu\alpha\partial_\mu\alpha. \end{aligned} \tag{3.29}$$

The problem with (3.29) is that it depends explicitly on an unphysical gauge parameter α. The only possibility to keep the gauge invariance is by setting $M_\gamma = 0$. Even though this analysis was carried out to tree-level, the quantum corrections respect the original symmetries of the theory, so the one-loop contribution to the self-energy of the gauge boson mass cannot have huge contributions in its mass term.

3.3.3 Scalars

The physical mass for the scalar field is given by,

$$m_\phi^2 = m_{\phi,0}^2 + \delta m_\phi^2. \tag{3.30}$$

At one loop, the main contribution will be,

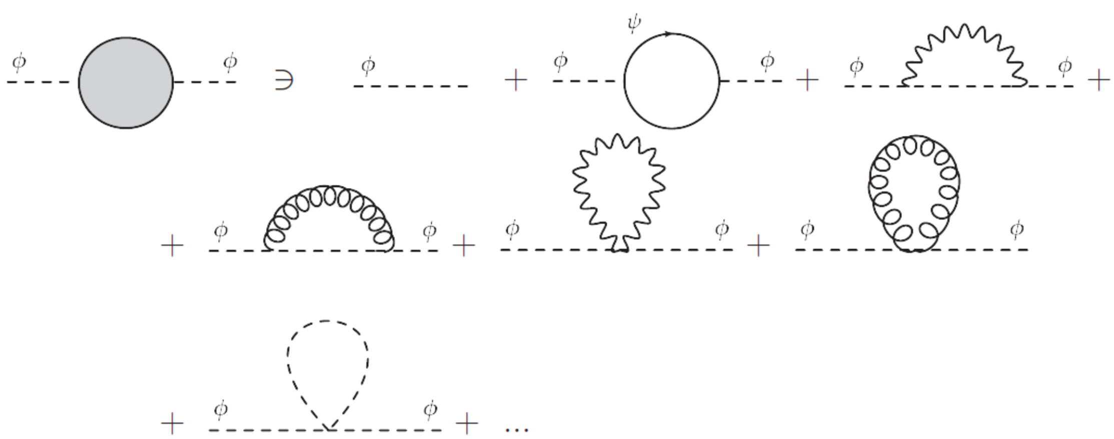
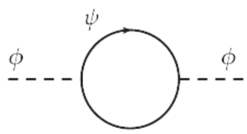


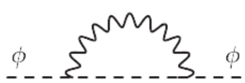
Figure 3.4: One loop contribution to the scalar self energy

The third diagram of Figure 3.4 gives the fermion contribution to the self-energy,



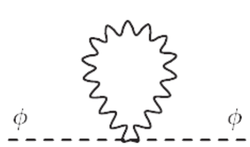
$$
\begin{aligned}
&= \int \frac{d^4k}{(2\pi)^4}(-1)\mathrm{Tr}\left[(-iy)\frac{i(\not{k}+m_\phi)}{k^2-m_\phi^2}(-iy)\frac{i\not{k}}{k^2-m_\phi^2}\right]\\
&=-4y^2\int\frac{d^4k}{(2\pi)^4}\frac{k^2+m_\psi^2}{\left(k^2-m_\psi^2\right)^2}\\
&=-\frac{4iy^2}{(4\pi)^2}\int_0^{\Pi^2}dk_E^2\frac{k_E^2(-k_E^2+m_\psi^2)}{\left(k_E^2+m_\psi^2\right)^2}\\
&=\frac{iy^2}{(4\pi)^2}\left(\Pi^2-3m_\psi^2\log\frac{\Pi^2-m_\psi^2}{m_\psi^2}+\text{finite terms}\right).
\end{aligned}
\tag{3.31}
$$

The fourth diagram of Figure 3.4 gives,



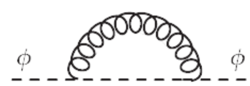
$$
\begin{aligned}
&=\int\frac{d^4k}{(2\pi)^4}(ie\gamma^\nu)\frac{i}{k^2-m_\phi^2}(ie\gamma^\mu)\frac{-ig_{\mu\nu}}{k^2-\mu^2}\\
&=-4e^2\int_0^1dx\int\frac{d^4k}{(2\pi)^4}\frac{1}{\left[x(k^2-m_\phi^2)+(1-x)\mu^2\right]^2}\\
&=-\frac{4ie^2}{(4\pi)^2}\int_0^1dx\int_0^{\Pi^2}dk_E^2\frac{k_E^2}{\left[k_E^2+xm_\phi^2+(1-x)\mu^2\right]^2}\\
&=-\frac{ie^2}{4\pi^2}\frac{1}{m_\phi^2-\mu^2}m_\phi^2\log\frac{\Pi^2+m_\phi^2}{m_\phi^2}.
\end{aligned}
\tag{3.32}
$$

To arrive to the last line in (3.32) the $\mu\to 0$ limit has been applied. The evaluation of the sixth diagram of Figure 3.4 gives,

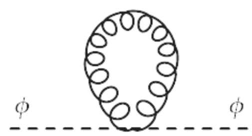


$$
\begin{aligned}
&=\int\frac{d^4k}{(2\pi)^4}(-2ie^2\gamma_{\mu\nu})\frac{-ig^{\mu\nu}}{k^2-\mu^2}\\
&=-4e^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-\mu^2}\\
&=\frac{4ie^2}{(4\pi)^2}\int_0^{\Pi^2}dk_E^2\frac{k_E^2}{\left[k_E^2+\mu^2\right]^2}\\
&=\frac{ie^2}{4\pi^2}\Pi^2.
\end{aligned}
\tag{3.33}
$$

The non-Abelian contribution coincides with the Abelian one expect of group theory factors,

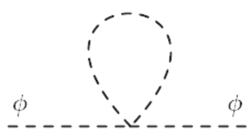


$$= \frac{-ig^2}{4\pi^2} C_2(\square) \log \frac{\Pi^2 + m_\phi^2}{m_\phi^2} \tag{3.34}$$



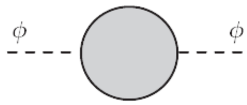
$$= \frac{-ig^2}{4\pi^2} C_2(\square) \Pi^2. \tag{3.35}$$

The quartic scalar self coupling is,



$$\begin{aligned} &= \int \frac{d^4k}{(2\pi)^4} (-i\lambda) \frac{i}{k^2 - m_\phi^2} \\ &= - \frac{i\lambda}{(4\pi)^2} \int dk_E^2 \frac{k_E^2}{k_E^2 + m_\phi^2} \\ &= \frac{i\lambda}{(4\pi)^2} \left(-\Pi^2 + m_\phi^2 \log \frac{\Pi^2 + m_\phi^2}{m_\phi^2} + \text{finite terms} \right). \end{aligned} \tag{3.36}$$

The total one-loop contribution to the scalar self-energy depends quadratically on the cut-off Π,



$$\begin{aligned} &= - \frac{i}{(4\pi)^2} \left(-4y^2 + 4e^2 + 4g^2 C_2(\square) + \lambda \right) \Pi^2 + \mathcal{O}(\log \Pi^2) + \text{finite terms} \\ &\sim \ \Pi^2. \end{aligned} \tag{3.37}$$

The last formula is known as "Veltman condition" in his attempt to solve the hierarchy problem. Now, suppose the standard model is viewed as an effective field theory (EFT) up to the Planck scale $\Pi = M_{Pl} \sim 10^{19}$, then the quantum corrections are $\mathcal{O}(M_{Pl}^2)$. For the case of scalar particles with masses much smaller than the Planck scale, (3.30) implies a significant and incredibly precise value of the bare mass parameter to cancel the contribution of quantum corrections and then reproducing a light-type particle. Tuning non-physical parameters to reproduce certain physical phenomena seems to be forced a theory to fit nature, and, at the same time, seems contrary to the scientific's goal of using theoretical models for naturally describing physical phenomena. The problem of this quadratic divergence in the quantum corrections of scalar particles is referred to as "*hierarchy, fine-tuning problem* or *quadratic divergences*." The problem can be reformulated in terms of the enormous difference between the weak and the Planck scale.

Two questions naturally arise from this analysis. The first question is whether the SM needs to be treated as an EFT. Suppose that the SM could run up to arbitrarily high energies. If this were the case, it would hit the Landau pole in the hypercharge coupling. One of the main consequences of the divergent behavior of the coupling in the SM lies in the formation of non-zero vacuum condensates of fermions in the UV (around 10^{41} GeV), which is inconsistent with the long-range degrees of freedom in the infrared. The SM needs to be an EFT at some cutoff Π to avoid that kind of inconsistency whether or not one is concerned about the implication of quantum gravity.

The second question is what to think about the quadratic divergence and if a different renormalization scheme might avoid (or, at least, alleviate) the hierarchy problem. In QFT, one deals with divergences by introducing counterterms in the Lagrangian and fix their coefficients according to some renormalization scheme, and then use this scheme to perform finite predictions for observables at other scales. Following this criterion, one might not be worried by the quadratic divergence. But even if one does not ascribe physical significance to the quadratic divergence alone, it gives signals for the existence of sensitivity to UV physics, understanding the quadratic divergence as a placeholder for physical threshold. In fact, from a Wilsonian perspective, the fundamental theory is finite, and divergences in the effective field theory are physical (for example, the cutoff is equivalent to the lattice spacing, or could be represented mass scale of particles rendering the Higgs mass finite). The counterterms added to the original Lagrangian just manifest fine-tuning processes. Formulated in this way, it is clear that it does not matter what scheme one is using to do the calculation. For instance, dimensional regularization hides the explicit dependence on Π^2 in its intermediate results, but the dependence on the high-energy parameters is still there.

Chapter 4

Functional Renormalization Group and its Applications

The standard model of particle physics, together with gravity, describes a set of interactions capable of expressing a vast number of physical phenomena from those occurring in particle colliders at LHC to everyday physics as the projectile motion thrown near the Earth's surface. Nevertheless, it would be absurd trying to explain how the train machinery works in terms of gluons and photons, as well as an astronomer, wouldn't want to understand the laws of a macroscopic universe in terms of every single star in the cosmos. Instead, a description of a physical system suitable for a length scale we are interested in is highly desirable. Thus, the aim lies in covering an effective description on different scales, going from physics at short distances scales to a characterization of the vast array of phenomena observed at larger distances scales. The connection is from the microscopic to macroscopic dynamics since different microscopic theories can lead to the same effective description of lower energy scales. The goal of this chapter is to derive a systematic approach to work with this question by deducing a well-suited formalism that allows dealing with both the perturbative (QED) and the nonperturbative (QCD and gravity) scenarios.

4.1 Derivation of the scale dependent effective action

Consider a scalar theory with fields $\psi^a, a = 1, ..., N$, and d Euclidean dimensions. Starting with the generating functional of the n-point correlation functions in the path integral representation,

$$Z[J] = \int \mathcal{D}\psi \text{exp} \left(\int_x J\psi - S[\psi] \right), \tag{4.1}$$

one defines a scale dependent generating functional by inserting artificially a cutoff term $\Delta S_k[\psi]$,

$$Z_k[J] = \int \psi \exp\left(-S[\psi] + \int_x J\psi - \Delta S_k[\psi]\right). \tag{4.2}$$

The target of the new term consists in suppressing the low momenta modes $p^2 \ll k^2$. For this purpose, the required form of the cutoff term has to be at most quadratic in the field ψ,

$$\Delta S_k[\psi] = \frac{1}{2}\int_q \psi^*(q) R_k(q) \psi(q), \tag{4.3}$$

where $R_k[\psi]$ is called cutoff function and can be viewed as a momentum-dependent mass term. A cutoff term quadratic in the fields ensures that a one-loop equation can be exact (93). In general, there is a freedom in the choice of the specific shape of R_k except for a few basic requirements,

i In the deep infrared,

$$\lim_{\frac{k^2}{q^2}\to 0} R_k(q) \to 0,$$

which implies that the regulator $\Delta S_k[\psi]$ vanishes for $k \to 0$, for all q. As a consequence, one ensures the standard generating functional as well as the full quantum effective action.

ii R_k must be continuous and monotonically decreasing in q and monotonically increasing in k.

iii For $q^2 \ll k^2$

$$\lim_{\frac{q^2}{k^2}\to 0} R_k(q) > 0.$$

It implies that $R_k(q)$ must be an infrared regulator that suppress dynamics of low momentum modes by a soft mass-like infrared cutoff.

iv For $k \to \Lambda$ (k $\to \infty$ when $\Lambda \to \infty$) no modes integrated out yet and

$$\lim_{k^2 \to \Lambda \to \infty} R_k(q) \to \infty,$$

which induces that the functional integral is dominated by the stationary point of the action in this limit.

For practical calculations, it is useful to give some examples of common used shapes of cutoff profiles. The exponential cutoff (94) is given by,

$$R_k(q) = \frac{q^2}{\exp \frac{q^2}{k^2} - 1}, \tag{4.4}$$

while the optimized cutoff (95),

$$R_k(q) = \left(k^2 - q^2\right)\Theta\left(1 - \frac{q^2}{k^2}\right), \tag{4.5}$$

allows to do almost all the integral involved in the rest of the next sections analytically, which is why we will adopt it in the rest of this thesis. Even though the fixed points obtained in 4.6 and 4.7 have a dependence of the shape of R_k, variations on the position in theory space are too small, and regardless of the form of R_k, the value always remains in the first quadrant. Return to the generating functional, the scale-dependent $W_k[J]$ for the connected Green functions is defined by,

$$Z_k[J] = \exp\left(W_k[J]\right). \tag{4.6}$$

Employing a modified Legendre transformation,

$$\Gamma_k[\phi] = -W_k[J] + \int_x J(x)\phi(x) - \Delta S_k[\phi], \tag{4.7}$$

the variation condition on the action over the fields gives the equation of motion for $\phi(x)$,

$$\begin{aligned}
\frac{\delta\Gamma_k[\phi]}{\delta\phi(x)} &= -\int_y \frac{\delta W_k[J]}{\delta J(y)}\frac{\delta J(y)}{\delta\phi(x)} + \int_y \frac{\delta J(y)}{\delta\phi(x)}\phi(y) + J(x) - \frac{\delta\Delta S_k[\phi]}{\delta\phi(x)} \\
&= -\int_y \phi(y)\frac{\delta J(y)}{\delta\phi(x)} + \int_y \frac{\delta J(y)}{\delta\phi(x)}\phi(y) + J(x) - \frac{\delta\Delta S_k[\phi]}{\delta\phi(x)} \\
&= J(x) - \frac{\delta\Delta S_k[\phi]}{\delta\phi(x)} \\
&= J(x) - R_k(x)\phi(x).
\end{aligned} \tag{4.8}$$

Now we turn to the derivation of the flow equation. It describes the change of the scale-dependent effective with a change of the renormalization group scale k, and thus how the effective action on different scales are connected. Defining the renormalization group time t as,

$$t = \log\left(\frac{k}{\Lambda}\right) \Rightarrow \partial_t = \frac{\partial}{\partial t} = k\frac{\partial}{\partial k} = k\partial_k. \tag{4.9}$$

To get the flow equation, the first step consists in taking the derivative of the Average Effective Action (EAA) with respect to t, with constant field ϕ and variable current J,

$$
\begin{aligned}
\partial_t \Gamma_k[\phi] &= -\partial_t W_k[J] - \int_x \underbrace{\frac{\delta W_k[J]}{\delta J(x)}}_{\phi(q)} \partial_t J + \int_x \phi(x) \partial_t J(x) - \partial_t \Delta S_k[\phi] \\
&= -\partial_t W_k[J] - \partial_t \Delta S_k[\phi].
\end{aligned}
\tag{4.10}
$$

.

In order to find the explicit flow equation, the derivative of the cutoff term is required,

$$
\begin{aligned}
\partial_t \Delta S_k[\phi] =& \partial_t \frac{1}{2} \int_q \phi^*(q) R_k(q) \phi(q) \\
=& \frac{1}{2} \int_q \phi^*(q) \left(\partial_t R_k(q)\right) \phi(q).
\end{aligned}
\tag{4.11}
$$

Performing the derivatives of the modified generator of the connected Green's functions explicitly, one gets,

$$
\begin{aligned}
\partial_t W_k[J] &= 1 \; \partial_t W_k[J] \\
&= \exp\left(-W_k[J]\right) \exp\left(W_k[J]\right) \left(\partial_t W_k[J]\right) \\
&= \exp\left(-W_k[J]\right) \partial_t \mathrm{exp}\left(W_k[J]\right),
\end{aligned}
\tag{4.12}
$$

the properties of the previous part of the flow are analyzed in a better way going back to the path integral representation since the scale dependence appears only in the cutoff term,

$$
\begin{aligned}
\partial_t W_k[J] &= \exp\left(-W_k[J]\right) \left(\partial_t \mathrm{exp}\left(W_k[J]\right)\right) \\
&= \exp\left(-W_k[J]\right) \partial_t Z_k[J] \\
&= \exp\left(-W_k[J]\right) \partial_t \int \mathcal{D}\psi e^{\mathcal{I}} \\
&= \exp\left(-W_k[J]\right) \int \mathcal{D}\psi \left(-\partial_t \Delta S_k[\psi]\right) e^{\mathcal{I}} \\
&= \exp\left(-W_k[J]\right) \int \mathcal{D}\psi \left(-\frac{1}{2} \int_q \psi^*(q) \left(\partial_t R_k(q)\right) \psi(q)\right) e^{\mathcal{I}} \\
&= -\frac{1}{2} \int_q \left(\partial_t R_k(q)\right) \exp\left(-W_k[J]\right) \int \mathcal{D}\psi \psi^*(q) \psi(q) e^{\mathcal{I}},
\end{aligned}
\tag{4.13}
$$

where it has been defined,

$$
\mathcal{I} \equiv -S[\psi] + \int_x J(x)\psi(x) - \Delta S_k[\psi]. \tag{4.14}
$$

The term $\psi^*(q)\psi(q)$ appearing in (4.13) suggests expressing the integral over the scalar fields together with the generating functional in terms of the connected Green's functions,

$$\begin{aligned}
\exp\left(-W_k[J]\right)\frac{\delta^2}{\delta J(q)\delta J^*(q)}\exp\left(W_k[J]\right) &= \exp\left(-W_k[J]\right)\frac{\delta}{\delta J(q)}\left(\exp\left(W_k[J]\right)\frac{\delta W_k[J]}{\delta J^*(q)}\right)\\
&= \underbrace{\frac{\delta W_k[J]}{\delta J(q)}}_{\phi(q)}\underbrace{\frac{\delta W_k[J]}{\delta J^*(q)}}_{\phi^*(q)} + \underbrace{\frac{\delta^2 W_k[J]}{\delta J(q)\delta J^*(q)}}_{<\psi^*(q)\psi(q)>}\\
&= \phi(q)\phi^*(q) + <\psi^*(q)\psi(q)>_{k,\text{connected}}\\
&= \phi(q)\phi^*(q) + G_k(q,q),
\end{aligned} \tag{4.15}$$

where $G_k(q,q)$ are the Green functions of connected k. Inserting (4.15) into (4.13),

$$\begin{aligned}
\partial_t W_k[J] &= -\frac{1}{2}\int_q \left(\partial_t R_k(q)\right)\left(G_k(q,q) + \phi^*(q)\phi(q)\right)\\
&= -\frac{1}{2}\int_q \left(\partial_t R_k(q)\right) G_k(q,q) - \underbrace{\frac{1}{2}\int_q \phi^*(q)\left(\partial_t R_k(q)\right)\phi(q)}_{\partial_t \Delta S_k[\phi]}\\
&= -\frac{1}{2}\int_q \left(\partial_t R_k(q)\right) G_k(q,q) - \partial_t \Delta S_k[\phi].
\end{aligned} \tag{4.16}$$

Collecting all ingredients, the flow equation for the scale-dependent effective action can be obtained in terms of the Green function for fixed ϕ and variable source,

$$\begin{aligned}
\partial_t \Gamma_k[\phi] &= -\partial_t W_k[J] - \partial_t \Delta S_k[\phi]\\
&= \frac{1}{2}\int_q \left(\partial_t R_k(q)\right) G_k(q,q) + \partial_t \Delta S_k[\phi] - \partial_t \Delta S_k[\phi]\\
&= \frac{1}{2}\int_q \left(\partial_t R_k(q)\right) G_k(q,q).
\end{aligned} \tag{4.17}$$

What remains to do in order to obtain a closed functional differential equation such that it only involves Γ_k is to express $G(q,q)$ in terms of the effective action and relate the flow equation with the two-point function effective action. The green function is defined by,

$$G(p,q) = \frac{\delta^2 W_k[J]}{\delta J^*(p)\delta J(q)}. \tag{4.18}$$

From the modified Legendre transformation (4.7), the second functional derivative is applied to find the variation of the source term with respect to the field,

$$\frac{\delta J(q)}{\delta \phi^*(q^{'})} = \frac{\delta^2 \Gamma_k[\phi]}{\delta \phi^*(q^{'})\delta \phi(q)} + R_k(q)\delta(q-q^{'}). \tag{4.19}$$

equation (4.18) implies the following identity for the delta function,

$$\begin{aligned}
\delta(q^{'}-q) &= \frac{\delta \phi^*(q)}{\delta \phi^*(q^{'})} \\
&= \frac{\delta}{\delta \phi^*(q^{'})}\frac{\delta W_k[J]}{\delta J(q)} \\
&= \int_{q''} \frac{\delta}{\delta \phi^*(q^{'})}\frac{\delta W_k[J]}{\delta J(q)}\frac{\delta J(q^{''})}{\delta J(q'')} \\
&= \int_{q''} \frac{\delta^2 W_k[J]}{\delta J(q'')\delta J(q)}\left[\frac{\delta^2 \Gamma_k[\phi]}{\delta \phi^*(q')\delta \phi(q'')} + R_k(q^{''})\delta\left(q^{''}-q^{'}\right)\right].
\end{aligned} \tag{4.20}$$

Thus, the scale-dependent inverse propagator has a cutoff function and a second functional derivative. Using (4.20), the functional renormalization group equation (FRGE), *a.k.a.* Wetterich equation (57), is dictated by,

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left(\frac{\partial_t R_k}{\Gamma_k^{(2)}[\phi] + R_k}\right), \tag{4.21}$$

where the trace is extended to every index, so it includes integration over continuous indices as well as different kind of fields (taking bosons with a plus and fermions with a minus sign). At this point, several comments are in order.

- Even though the FRGE (4.21) has a structure of an one-loop equation, it corresponds to an exact equation. In fact, it is possible to demonstrate that the action at scale k is related to the full quantum effective action $\Gamma_{k\to 0}[\phi] = \Gamma[\phi]$ and the classical action $\Gamma_{k\to\infty}[\phi] = S[\phi]$.
- The trace of (4.21) is UV-finite due to the introduction of the cutoff term in the EAA. The derivative $\partial_t R_k$ ensures the UV regularization since its predominant support lies on a smeared momentum shell near $q^2 \sim k^2$.
- The solution of (4.21) corresponds to an RG trajectory in the so-called *theory space* or the space of all functionals of ϕ parametrized by the dimensionless couplings of the theory. The two ends of the path (if any) are the full quantum effective action and the bare action. Possible trajectories concerning the last point will be analyzed in the next chapter.

4.2 Grafical representation, higher n-point functions and truncation

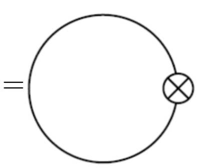
Before going on with the derivation and implications of the correlation functions, it is useful to give a graphical representation for $\partial_t \Gamma_k$ and understand the FRGE in the sense of Feynman diagrams. To do that, for each modified propagator and RG-time derivative cutoff insertion, one can assign a straight line and crossed circle, respectively, showed in Figure 4.1, where we have defined G_k through the following identity,

Figure 4.1: Graphical represetation of the modified propagator G_k (left figure) and the RG time derivative of the cutoff function (right figure).

$$\mathbb{1} = \left(\Gamma_k^{(2)}[\phi] + R_k\right) G_k. \tag{4.22}$$

The previous prescription allows representing the right-hand side (r.h.s.) of (4.21) by a loop with a cutoff insertion,

$$\partial_t \Gamma_k =$$

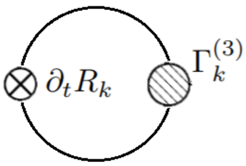


. (4.23)

This diagrammatic representation emphasizes that the flow equation does not contain any functional integrals. As discussed above, the FRGE is an exact one-loop equation; however, from a practical point of view is, in general, impossible to get exact solutions of (4.22). The reason behind this idea can be observed from a geometrical point of view. The flow equation generates a vector field with non-vanishing components in (possible infinitely) directions in theory space. Solving this kind of system is, in general impossible, since the FRGE constitutes a system of an infinite number of coupled differential equations. More concretely, the n-point vertex function provides an example that shows the impossibility previously discussed. The *1*-point functions,

$$\frac{\delta}{\delta\phi}\partial_t \Gamma_k[\phi] = -\frac{1}{2}\mathrm{Tr}\left\{(\partial_t R_k)\left[\Gamma_k^{(2)} + R_k\right]^{-1}\Gamma_k^{(3)}\left[\Gamma_k^{(2)} + R_k\right]^{-1}\right\}, \tag{4.24}$$

has a graphical representation given by,

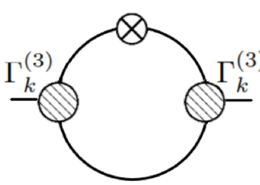
$$\partial_t \Gamma_k^{(1)} = -\tfrac{1}{2}$$ 

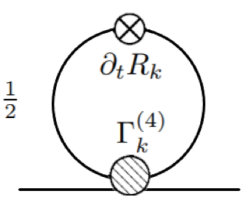
$$. \tag{4.25}$$

while the two-point function is obtained applying one more functional derivative,

$$\begin{aligned}\frac{\delta^2}{\delta\phi\delta\phi}\partial_t\Gamma_k^{(2)} =& \mathrm{Tr}\left\{(\partial_t R_k)\left[\Gamma_k^{(2)}+R_k\right]^{-1}\Gamma_k^{(3)}\left[\Gamma_k^{(2)}+R_k\right]^{-1}\Gamma_k^{(3)}\left[\Gamma_k^{(2)}+R_k\right]^{-1}\right\} \\ &-\frac{1}{2}\mathrm{Tr}\left\{(\partial_t R_k)\left[\Gamma_k^{(2)}+R_k\right]^{-1}\Gamma_k^{(4)}\left[\Gamma_k^{(2)}+R_k\right]^{-1}\right\}. \end{aligned}\tag{4.26}$$

The graphical representation is splitted into two diagrams, one for each line in the r.h.s. of (4.26),

$$\partial_t \Gamma_k^{(2)} =$$ 

$$-\tfrac{1}{2}$$ 

$$. \tag{4.27}$$

What does this imply? The main implication of both, diagrammatic and analytical representation, is the so-called hierarchy of the flow equation: the flow of the n-point function involves the knowledge of $\Gamma_k^{(n+1)}[\phi]$ and $\Gamma_k^{(n+2)}[\phi]$, i.e., it is always necessary to know the structure of the higher-order terms. To deal with this kind of infinite tower of differential equations is, in general impossible.

—Usually, a solution to the lack of exact solutions consists of truncating the theory space by choosing an ansatz and setting the remaining directions (or couplings) to zero. Therefore, the flow equation in this truncation. Generally, the selection of a particular truncation and the following justification of why the chosen truncation encapsulates the physics successfully behind the problem we are interested in turns out to be a non-trivial problem. To judge the quality of the truncation, one can study the stability of the results under an extension of the truncation and then analyzing the effects of the neglected operators. The limitations of this check become evident. The results obtained may be very stable under several successive enlargements of the truncation, concluding that these results are close enough to the exact one. However, there must always be a possibility of ignoring an operator, which has the potential to spoil this picture dramatically.

Just as the perturbative approach neglects operators corresponding to higher powers of the coupling, a suitable guiding principle for a non-perturbative regime resides in doing a derivative expansion sort operator by the number of derivatives they posses. In the next three sections, a particular truncation using up to two derivatives shall be adopted to explore the gravitational sector with matter minimally coupled to a metric field.

4.3 Asymptotic Safety and Non-perturbative Renormalizability

What happens with the terms that help to describe the different forces of nature? It is well known that the couplings involved in theories like quantum electrodynamics grow out of control as energy increases, hitting a Landau pole in the far-ultraviolet (UV). However, if one stays away from such energies, perturbation theory has bestowed upon the best predictions for experimental measurements that physics has ever known. Similarly, general relativity may be seen as an effective quantum field theory (96; 97; 98). Although the fact that gravity is a non-renormalizable theory (99), it is possible to compute quantum effects due to graviton loops while keeping the momenta of particles in the loops up to some reference cut-off scale. As an example, it has been possible to compute quantum corrections to the Newtonian potential (100) unambiguously. The problems arise when one pushes the cut-off to some energy regimes like the Planck scales, or beyond. For a generic theory not necessarily restricted to gravity, a UV completion faces two kinds of problems: the finiteness of the couplings at arbitrary energy-scales and the predictivity if the theory under consideration is non-perturbative.

For answering these questions, the starting point will be an effective action which is constituting by an arbitrary number of monomials, each made up of a combination of fields compatible with the symmetries of the theory and multiplying by the running couplings,

$$\Gamma_k\left[\phi\right] = \int d^d x \sum_{n=0}^{\infty} \sum_i g_i^{(n)}\left(k\right) \mathcal{O}_i^{(n)}\left(\phi\right), \tag{4.28}$$

where ϕ denotes a generic field for carrying the discussion as general as possible, $g_i^{(n)}\left(k\right)$ are the running couplings defined at energy scale k, and $\mathcal{O}_i^{(n)}$ are all possible operators constructed with the field ϕ and its derivatives compatible with the symmetries of the theory. If d is the canonical dimension of $g_i^{(n)}\left(k\right)$, the dimensionless couplings have the following scaling structure,

$$\tilde{g}_i^{(n)}\left(k\right) = k^{-d} g_i^{(n)}\left(k\right). \tag{4.29}$$

where $\tilde{g}$ represents the dimensionless version of the running coupling. Let $\mathcal{Q}$ be an abstract notation for some physical quantity (cross-section, decay rates, or anything else). For any shape or physical meaning associated with $\mathcal{Q}$, it will be a function of the renormalization scale k (appearing in the renormalization procedure of the theory), the kinematical parameters of the process (momentas, angles, etc.) and the running couplings. Suppose the structure of $\mathcal{Q}$ is not so intricate such that one can extract suitable powers of k. The most general expression of an on-shell physical quantity can be expressed in its dimensionless form as,

$$\tilde{\mathcal{Q}} = k^D \mathcal{Q}\left(K, \tilde{g}_i^{(n)}(k)\right), \tag{4.30}$$

where K stands for any dimensionless kinematical quantities and their ratios, for a canonical dimension of $\mathcal{Q}$ equal to D. From (4.30) is clear that the UV behavior of the physical quantity is governed by the dimensionless running couplings $\tilde{g}_i^{(n)}$. A healthy UV limit of Q implies the $\tilde{g}_i^{(n)}$'s renormalization group trajectories reach finite values, or fixed points (FP) at infinite energy scale,

$$\lim_{k\to\infty} \tilde{g}_i(k) = \tilde{g}_i^* \tag{4.31}$$

measured in units of k. If a particular theory meets the conditions (4.31), the dimensionless physical quantities will not blow up when the cutoff approaches to arbitrarily high energies.

To answer the second question, one needs to investigate the various RG trajectories which present finite UV-limit. At this stage, it is useful to introduce the theory space, as space spanned by the essential dimensionless couplings. Essential couplings mean all the couplings which cannot be removed by a field redefinition. The set of asymptotically safe trajectories and the corresponding existence of an FP give a criterion for selecting a QFT description at any energy scale. However, in this scenario, two kinds of problems associated with the predictivity of the asymptotic safe theory may occur. Suppose a vast number of essential couplings posses RG trajectories that hit a finite value when $k \to \infty$. For seeking the proper RG flow of the real world, an infinite number of initial conditions have to be determined by resorting to experiments (as many measures as essential couplings posses the theory), so the theory would lose predictivity. On the other hand, the theory can have a maximal predictive power if there is a single trajectory that arrives at the FP in the UV regime. An acceptable intermediate stage arises when those RG trajectories ending at finite values correspond to a reasonably limited number of essential couplings. Those conditions (finite and predictive) define the so-called "*asymptotic safety*" scenario or the non-perturbative renormalizability. Understanding this second condition implies a more in-depth knowledge of the characteristics of the FP.

4.4 Predictivity and Fixed Points

What is the relation between predictivity and the study of the FP's? As in the case of perturbative renormalizability, the theory under consideration must have a finite number of parameters that can be measured experimentally.

Consider N essential couplings and an FP's of their respective flows. These FP's come in two types: Gaussian FP, which forms a $\vec{0}$-vector ($\tilde{g}_1 = 0, \tilde{g}_2 = 0, ..., \tilde{g}_N = 0$) and Non-Gaussian FP, where at least one of them is different to zero, $\tilde{g}_i \neq 0$. The properties of both types of points will be discussed at the end of this section. In theory space, it would be possible to identify a subspace consisting of all the trajectories that reach an attractive FP in the UV, called the "*critical surface*" $\mathcal{C}$. By definition, no matter where is the location of the starting point of the flow, it will always find the FP. Otherwise, if the locus of the starting point is outside of $\mathcal{C}$, the trajectory will, in general, exhibit divergences for $k \to \infty$. Then, using the finiteness in the far UV allows discarding RG trajectories not belonging to $\mathcal{C}$. The numbers of parameters that can be chosen to approach the FP must be equal to the dimensionality of $\mathcal{C}$.

In general, the simplest way to investigate the dimension of $\mathcal{C}$ is by determining its tangent space at the FP after having linearized the flow of the coordinates. Suppose the variation of $\tilde{g}_i$'s concerning to the RG time are known for all the relevant couplings,

$$\beta_i\left(\tilde{g}_j\right) = \partial_t \tilde{g}_i. \tag{4.32}$$

Condition (4.31) requires that all beta functions vanishing at the FP,

$$\beta_i\left(\tilde{g}_j^*\right) = 0. \tag{4.33}$$

Linearizing the beta functions in the vicinity of the FP,

$$\beta_i\left(\tilde{g}_j\right) = \beta_i\left(\tilde{g}_j^*\right) + \sum_j \mathcal{M}_{ij}\left(\tilde{g}_j^*\right)\left(\tilde{g}_j - \tilde{g}_j^*\right) \tag{4.34}$$

where,

$$\mathcal{M}_{ij} = \frac{\partial \beta_i}{\partial \tilde{g}_j} \tag{4.35}$$

is the stability matrix. The first term of (4.34) is zero by construction, and the new coordinate center $y = \left(\tilde{g}_j - \tilde{g}_j^*\right)$ is small enough at the neighbourhood of the FP for neglecting second order and higher corrections. By diagonalizing (4.35), an eigenvector V_i associated to $\mathcal{M}$ and their corresponding eigenvalues θ_i give a solution of the system of first order differential equation (4.34). Integrating (4.34), the general solution written as a function of V and θ is,

$$\tilde{g}_i(k) = \tilde{g}_i^* + \sum_n c_n V_i^n k^{\theta_n}, \tag{4.36}$$

where the c_n are free integration parameters. Due to the stability matrix $\mathcal{M}$ is not manifestly symmetric, its eigenvalues may come out as complex conjugate pairs. In order to get physical information out of these equations, one has to note three cases. If $\mathrm{Re}(\theta < 0)$ the sum (4.36) goes to zero for $k \to \infty$. Directions that come with a negative sign are called *relevant* directions, and they allow to keep unconstrained the constant c_n (corresponding to the free parameters of the theory). Provided that the numbers of relevant directions (or the dimensionality of our subspace $\mathcal{C}$) with $\theta_n < 0$ remains finite leads to a predictive theory in the sense of the previous discussion. To have a well-defined UV limit, the *irrelevant* directions, $\theta_n > 0$, yield $c_n = 0$. Those conditions define the UV critical surface unequivocally for the gravitational case. Finally, the third option with eigenvalues $\theta_n = 0$ implies going beyond the linearized analysis for establishing whether it is marginally relevant or irrelevant.

We conclude this chapter by mentioning the physical difference between GFP and NGFP. General features known in perturbative renormalizability are valid in a neighborhood of the GFP, and the relevant critical exponents are equal to the canonical mass dimension of the couplings providing the flow studied. Following (4.29), by dimensional analysis one can write the beta function of dimensionful couplings as,

$$\beta_i\left(\tilde{g}_j\right) = -d_i\tilde{g}_j + \alpha_i(\tilde{g}_j), \tag{4.37}$$

where the first term contains the classical scaling while the second one incorporates the quantum fluctuations coming from the loop corrections, with $\alpha_i(\tilde{g}_j) = \beta_i(g_i k^{-d_j})$. Since the region of interest is nearing the FP, a Taylor expand of α_i in (4.37) is carried out,

$$\alpha_i\left(\tilde{g}_j\right) = \alpha_{ij}\tilde{g}_j + \alpha_{ijk}\tilde{g}_j\tilde{g}_k + \mathcal{O}(\tilde{g}^3). \tag{4.38}$$

Combining (4.37) and (4.38) into (4.35), the stability matrix elements,

$$\mathcal{M}_{ij} = -d_i\delta_{ij} + \alpha_{ij}, \tag{4.39}$$

evaluated at the FP will be diagonal with minus the canonical dimension as eigenvalues. As a consequence, $\mathcal{C}$ is spanned by the relevant couplings that are power-counting renormalizable. The AS scenario at the GFP recovers the perturbative renormalizable and asymptotically free characteristic of most of the quantum field theories. The most exciting feature appears when one is facing an NGFP since the scaling exponents, in this case, may receive large contributions of loop corrections. This scenario turns out especially interesting in the case of gravity, which is non-perturbative renormalizable.

4.5 The background FRGE

The main aim of this and the next section is to derive the one-loop beta function of the gravitational couplings contained in the so-called Einstein-Hilbert truncation using the FRGE in the context of the AS program. To define the functional RG for the single metric flow equation to dynamical gravity, we shall employ the background field method, splitting the metric field into a fixed background and a quantum fluctuation,

$$g_{\mu\nu} = \overline{g}_{\mu\nu} + h_{\mu\nu}. \tag{4.40}$$

Even when the form of the average effective action is not specified, one assumes that the complete action is a function of the full metric $S(\overline{g}_{\mu\nu}, h_{\mu\nu})$ and the gauge fixing and ghost action break the quantum gauge invariant but preserve background gauge transformations. The later means that the cutoff R_k in (4.3) can no longer be a function of q but must be defined through a covariant differential operator. The choice is not unique, and it will be discussed in the next subsection.

As shown in chapter 4, to obtain a well-defined EEA, the cutoff action must be quadratic in the quantum field. Adding to the Euclidean partition function,

$$e^{W(j,\bar{\sigma},\sigma)} = \int (dhd\bar{c}dc)\, e^{-\left\{S(h;\bar{g})+S_{GF}(h;\bar{g})+S_{gh}(\bar{c},c;\bar{g})+\int dx\sqrt{\bar{g}}(j^{\mu\nu}h_{\mu\nu}+\sigma^{\mu}c_{\mu}+\bar{\sigma}_{\nu}\bar{c}^{\nu})\right\}}, \tag{4.41}$$

where all quantum fields have been linearly coupled to source terms, the quadratic cutoff action given by,

$$\Delta S_k = \frac{1}{2}\int d^dx\sqrt{\bar{g}}h_{\mu\nu}R_k^{\mu\nu\rho\gamma}(\bar{g})h_{\rho\gamma} - \int d^dx\sqrt{\bar{g}}\bar{c}_{\nu}g^{\mu\nu}R_k^{gh}(\bar{g})c_{\mu}. \tag{4.42}$$

The EEA corresponds to the Legendre transformation minus the cutoff ΔS_k,

$$\Gamma_k\left(h,\bar{c},c;\bar{g}\right) = -W_k(j,\bar{\sigma},\sigma;\bar{g}) + \int d^dx\sqrt{\bar{g}}\left(j^{\mu\nu}h_{\mu\nu}+\sigma^{\mu}c_{\mu}+\bar{\sigma}_{\nu}\bar{c}^{\nu}\right) - \Delta S_k(h,\bar{c},c;\bar{g}). \tag{4.43}$$

This functional obeys the FRGE (4.21),

$$\frac{d\Gamma_k}{dt} = \frac{1}{2}\mathrm{Tr}\left(\frac{1}{\sqrt{\bar{g}}}\frac{\delta^2\left(\Gamma_k+\Delta S_k\right)}{\delta\varphi\delta\varphi}\right)^{-1}\frac{d}{dt}\frac{\Delta S_k}{\delta\varphi\delta\varphi}, \tag{4.44}$$

where $\varphi = h_{\mu\nu}, \bar{c}_{\nu}, c_{\nu}$ and $\sqrt{\bar{g}}$ is needed to cancel out tensorial objects. Until now, (4.44) is a closed equation for Γ_k, but it's still intractable since the modified inverse propagator and the cutoff term are unknown. Since ΔS_k does not contain mixed $h-c$ nor $h-\bar{c}$ terms, the trace can be written as the sum of two parts,

$$\frac{d\Gamma_k}{dt} = \frac{1}{2}\mathrm{Tr}\left(\frac{1}{\sqrt{\bar{g}}}\frac{\delta^2\left(\Gamma_k+\Delta S_k\right)}{\delta\varphi\delta\varphi}\right)^{-1}_{hh}\frac{dR_k}{dt} - \mathrm{Tr}\left(\frac{1}{\sqrt{\bar{g}}}\frac{\delta^2\left(\Gamma_k+\Delta S_k\right)}{\delta\varphi\delta\varphi}\right)^{-1}_{\bar{c}c}\frac{dR_k^{gh}}{dt}. \tag{4.45}$$

In order to study the hessian of Γ_k in the FRGE, let $\overline{\Gamma}_k(\bar{g}) \equiv \Gamma_k(0,0,0;\bar{g})$ be the gauge invariant functional obtained by putting all the quantum expectation values to zero. Hence, the full EAA can be splitted into a gauge invariant part and the remainder,

$$\Gamma_k(h,\bar{c},c;\bar{g}) = \overline{\Gamma}_k(\bar{g}+h) + \hat{\Gamma}_k(h,\bar{c},c;\bar{g}). \tag{4.46}$$

The next aim is writing down the flow equation for the gauge-invariant part of the action. We start by deriving the general form of the hessian for Γ_k. Although the ghost action is bilinear in the ghost field, the covariant derivative appearing in the Faddeev-Popov operator carried the full metric field $g_{\mu\nu}$, introducing mixed $h-$ghost terms. Hence the hessian takes the form,

$$\frac{\delta^2\Gamma_k}{\delta\varphi\delta\varphi} = \begin{pmatrix} \frac{\delta^2\Gamma_k}{\delta h\delta h} & \frac{\delta^2\Gamma_k}{\delta h\delta\bar{c}} & \frac{\delta^2\Gamma_k}{\delta h\delta c} \\ \frac{\delta^2\Gamma_k}{\delta\bar{c}\delta h} & 0 & \frac{\delta^2\Gamma_k}{\delta\bar{c}\delta c} \\ \frac{\delta^2\Gamma_k}{\delta c\delta h} & \frac{\delta^2\Gamma_k}{\delta c\delta\bar{c}} & 0 \end{pmatrix}. \tag{4.47}$$

Since we are interested in the flow equation of the functional when all quantum expectation values are set to zero, $\nabla_\mu \to \overline{\nabla}_\mu$ in the Faddeev-Popov operator and the interactions between the metric fluctuation and the ghost fields go to zero, leaving (4.47) as,

$$\frac{\delta^2\Gamma_k}{\delta\varphi\delta\varphi} = \begin{pmatrix} \frac{\delta^2\Gamma_k}{\delta h\delta h} & 0 & 0 \\ 0 & 0 & \frac{\delta^2\Gamma_k}{\delta\bar{c}\delta c} \\ 0 & \frac{\delta^2\Gamma_k}{\delta c\delta\bar{c}} & 0 \end{pmatrix}. \tag{4.48}$$

The structure (4.48) implies that (4.44) simplifies to,

$$\frac{d\overline{\Gamma}_k}{dt} = \frac{1}{2}\mathrm{Tr}\left(\frac{1}{\sqrt{\bar{g}}}\frac{\delta^2\Gamma_k}{\delta h\delta h} + R_k\right)^{-1}\frac{dR_k}{dt} - \mathrm{Tr}\left(\frac{1}{\sqrt{\bar{g}}}\frac{\delta^2\Gamma_k}{\delta\bar{c}c} + R_k\right)^{-1}\frac{dR_k^{gh}}{dt}. \tag{4.49}$$

The main trouble with (4.49) consists in we are dealing with different functionals. On the left-hand side (l.h.s.), one is varying the gauge-invariant part $\overline{\Gamma}_k$, while the traces on the right-hand side (r.h.s.) involve the full action Γ_k, and thus (4.49) is not a closed equation. To solve the r.h.s. of the Wetterich equation, the information about the hessian of $\hat{\Gamma}_k$ is required. To get some insights on $\hat{\Gamma}_k$, consider the one-loop EEA at vanishing fluctuation,

$$\begin{aligned}\overline{\Gamma}_k^{(1)}(\bar{g}) &= S(\bar{g}) + \frac{1}{2}\mathrm{Tr}\log\left(\frac{1}{\sqrt{\bar{g}}}\frac{\delta^2\left(S+S_{GF}\right)}{\delta h\delta h} + R_k\right)_{h=0} - \mathrm{Tr}\log\left(\frac{1}{\sqrt{\bar{g}}}\frac{\delta^2 S_{gh}}{\delta\bar{c}\delta c} + R_k^{gh}\right) \\ &= S(\bar{g}) + \frac{1}{2}\mathrm{Tr}\log\left(\frac{1}{\sqrt{\bar{g}}}\left(\frac{\delta^2 S}{\delta\bar{g}\delta\bar{g}} + \frac{\delta^2 S_{GF}}{\delta h\delta h}\right) + R_k\right) - \mathrm{Tr}\log\left(\frac{1}{\sqrt{\bar{g}}}\frac{\delta^2 S_{gh}}{\delta\bar{c}\delta c} + R_k^{gh}\right),\end{aligned} \tag{4.50}$$

where we have applied that S is a quadratic functional of the full metric field, and the gauge fixing condition is linear in h, thus its action is quadratic in h. Performing an "RG-improvement" by replacing the scale-dependent action $S \to \overline{\Gamma}_k$ in the r.h.s. of (4.50) and taking the RG-time derivative one gets,

$$\frac{d\overline{\Gamma}_k}{dt} = \frac{1}{2}\mathrm{Tr}\left(\frac{1}{\sqrt{\bar{g}}}\left(\frac{\delta^2\overline{\Gamma}_k}{\delta\bar{g}\delta\bar{g}} + \frac{\delta^2 S_{GF}}{\delta h\delta h}\right) + R_k\right)^{-1}\frac{dR_k}{dt} - \mathrm{Tr}\left(\frac{1}{\sqrt{\bar{g}}}\frac{\delta^2 S_{gh}}{\delta\bar{c}\delta c} + R_k^{gh}\right)^{-1}\frac{dR_k^{gh}}{dt}. \tag{4.51}$$

A comparison between (4.49) and (4.51) suggests,

$$\hat{\Gamma}_k(h,\bar{c},c;\bar{g}) = S_{GF}(h;\bar{g}) + S_{gh}(\bar{c},c;\bar{g}), \tag{4.52}$$

which is also consistent with (4.46) because when all the quantum expectation values go to zero it follows,

$$\hat{\Gamma}_k(0,0,0;\bar{g}) = 0. \tag{4.53}$$

4.6 The Einstein-Hilbert truncation

As a first step towards the inclusion of quantum gravitational effects using the definition of non-perturbative renormalizability, the RG flow for a particularly simple truncation will be discussed. In the Einstein-Hilbert truncation, the gauge-invariant part of the action,

$$\overline{\Gamma}_k = \frac{1}{2\kappa^2}\int d^dx\sqrt{g}\left(2\Lambda_k - R\right), \quad \kappa = \sqrt{8\pi G_k}, \tag{4.54}$$

together with the ghost and gauge fixing action (4.52). Before proceeding to compute the beta functions for the scale-dependent gravitational couplings, it is necessary to define the different types of cutoffs. For concrete calculations, the shape of R_k in (4.42) needs to be specified. In general, one has some freedom in this choice (as long as it meets the requirements given in section 4.1). Traditionally, the argument of the cutoff function R_k is a differential operator of the form,

$$\Delta = -\nabla^2 + \mathbf{E}, \tag{4.55}$$

where Δ is the Levi-Civita connection of g, ∇ a covariant derivative acting on both the gravitational field and other gauge connections coupled to the internal degrees of freedom of the field, and $\mathbf{E}$ a linear map acting on the quantum fields. Let us split the endomorphism into $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$, where $\mathbf{E}_1$ does not include any coupling and $\mathbf{E}_2$ only contains terms associated with couplings. One can distinguish three different types of cutoffs,

- *Type I*, if the operator is the Bochner Laplacian -∇^2
- *Type II*, if the operator is $-\nabla^2 + \mathbf{E}_1$. Note that this choice depends on the fields we are working with.
- *Type III*, if the operator is the full connection $-\nabla^2 + \mathbf{E}_1 + \mathbf{E}_2$.

In this thesis, all the following calculations will be performed using the type II cutoff since it is technically simpler than type III, whereas type I gives rise to difficulties with fermion fields (see Appendix B).

The first step to compute the RG flow is to define the bilinear action of the graviton. The quadratic part of the Einstein-Hilbert action is greatly simplified in the Feynman gauge due to the cancellation between the non-minimal terms in the Hessian and gauge fixing. Expanding up to second order in the quantum fluctuation, the terms in (4.46) become (139),

$$\begin{aligned}\overline{\Gamma}_k + S_{GF} &= \frac{1}{64\pi G_k}\int d^dx\sqrt{\overline{g}}h_{\mu\nu}K^{\mu\nu\alpha\beta}\Delta^{\rho\sigma}_{(h)\alpha\beta}h_{\rho\sigma}, \\ S_{gh} &= -\int d^dx\sqrt{\overline{g}}\bar{c}^{\mu}\left(-\overline{\nabla}^2\delta^{\mu}_{\nu} - \overline{R}^{\mu}_{\nu}\right)c^{\nu}.\end{aligned} \tag{4.56}$$

Here $\Delta^{\rho\sigma}_{(h)\alpha\beta} = -\overline{\nabla}^2\mathbf{1}^{\mu\nu}_{\rho\sigma} + W^{\mu\nu}_{\rho\sigma}$, where W is a tensor structure linear in curvature, and

K is a tensor involving the background metric,

$$K^{\mu\nu\rho\sigma} = \frac{1}{4}\left(\overline{g}^{\mu\alpha}\overline{g}^{\nu\beta} + \overline{g}^{\mu\beta}\overline{g}^{\nu\alpha} - \overline{g}^{\mu\nu}\overline{g}^{\alpha\beta}\right). \tag{4.57}$$

This definition coincides with (72) except that the cosmological constant present in the operator $\Delta^{\rho\sigma}_{(h)\alpha\beta}$ has been removed. The operators $R_k^{\mu\nu\rho\sigma}$ and $R^{\mu}_{\nu,k(gh)}$ used for characterizing the evolution between low and high energy modes are necessary to compute the trace,

$$R_k^{\mu\nu\rho\sigma} = Z_N K^{\mu\nu\rho\sigma} R_k\left(\Delta_{(h)}\right) \tag{4.58a}$$

$$R_{k\nu}^{(gh)\mu} = \delta^{\mu}_{\nu} R_k\left(\Delta_{(gh)}\right), \tag{4.58b}$$

for gravitons and ghosts, respectively. The wave function renormalization of the graviton is defined by the Einstein's constant, $Z_N = (16\pi G_k)^{-1}$. Modified inverse propagators $\left(\Gamma^{(2)} + R_k\right)^{-1}$ can be expressed as,

$$\frac{1}{\sqrt{\overline{g}}}\frac{\delta^2\Gamma_k}{\delta h_{\mu\nu}\delta h_{\rho\sigma}} + R_k^{\mu\nu\rho\sigma} = Z_N K^{\mu\nu\rho\sigma}\left(\Delta^{\rho\sigma}_{(h)\alpha\beta} + R_k(\Delta_{(h)})\right) \tag{4.59a}$$

$$\frac{1}{\sqrt{\overline{g}}}\frac{\delta^2\Gamma_k}{\delta\overline{c}_\mu\delta c^\mu} + R_{k,\nu}^{(gh)\mu} = \delta^{\nu}_{\mu}\Delta_{(gh)} + \delta^{\nu}_{\mu}R_k\left(\Delta_{(gh)}\right). \tag{4.59b}$$

Once defined the modified inverse propagator in the trace of the Wetterich equation, all that remains to do is to calculate the derivative of the cutoff function with respect to the RG time,

$$\frac{dR_k^{\mu\nu\rho\sigma}}{dt} = Z_N K^{\mu\nu\rho\sigma}\left(\partial_t R_k - \eta_N R_k\right) \tag{4.60a}$$

$$\frac{dR_{k\nu}^{(gh)\mu}}{dt} = \partial_t R_k \delta^{\mu}_{\nu}, \tag{4.60b}$$

where $\eta_N = \frac{1}{Z_N}\frac{dZ}{dt}$ is the anomalous dimension of the field G_k. Only results up to one-loop, i.e., neglecting the scale dependence of the gravitational coupling contained within the trace, are taken into account in the derivation of cutoff with respect to the RG time. In the local behavior, an asymptotic expansion of the heat kernel is valid for an arbitrary function ζ of some operator Δ (see Appendix A),

$$\mathrm{Tr}\zeta(\Delta) = \frac{1}{(4\pi)^{\frac{d}{2}}}\sum_{n=0}^{\infty} Q_{\frac{d}{2}-n}(\zeta)B_{2n}(\Delta), \tag{4.61}$$

where Q is a functional that depends on the gamma function, and B is a function of scalars composed from curvatures and its respective covariant derivatives. In Appendix A, we present a detailed calculation of the traces and Q-functionals for the optimized cutoff. Evaluating the Q-functional and extracting the heat kernel coefficients, the beta functions for dimensionless Newton and cosmological constants in d dimensions are,

$$\beta_g = (d-2)g + B_1 g^2 \tag{4.62a}$$

$$\beta_\lambda = -2\lambda + \frac{A_1}{2} g + B_1 g \lambda, \tag{4.62b}$$

with,

$$A_1 = \frac{16\pi(d-3+8\lambda)}{(4\pi)^{\frac{d}{2}}\Gamma\left[\frac{d}{2}\right](1-2\lambda)} \tag{4.63a}$$

$$B_1 = -\frac{4\pi(5d^5-3d+24-8\lambda(d+6))}{3(4\pi)^{\frac{d}{2}}\Gamma\left[\frac{d}{2}\right](1-2\lambda)}. \tag{4.63b}$$

To leading order in the small cosmological constant, A_1 and B_1 are just numbers. After solving the differential equations in four dimensions, one gets,

$$\beta_g = 2g - \frac{23}{3\pi} g^2 \tag{4.64a}$$

$$\beta_\lambda = -2\lambda + \frac{1}{2\pi} g + \frac{8}{3\pi} g\lambda. \tag{4.64b}$$

which have a non-trivial fixed point at,

$$g^* = \frac{6\pi}{23} \ , \ \lambda^* = \frac{3}{62}. \tag{4.65}$$

The flow is shown in Figure 4.2. All the RG trajectories in the plane $g > 0$ are being attracted to this NGFP, from which one concludes that a nontrivial UV-attractive FP in the $\lambda - g$ plane appears already at the lowest level in perturbation theory. It is easy to see that the eigenvalues of the stability matrix defined in (4.35) are $\theta_1 = -4, \theta_2 = -2$ (two relevant directions), while their eigenvectors are $V_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$,$V_2 = \begin{pmatrix} \frac{-23}{124\pi} \\ 1 \end{pmatrix}$. An important observation is pointed out: when different types of cutoffs (or different covariant differential operators adopted as an argument of the infrared regulator function R_k) are used to compute the beta functions, only the B_1 coefficient will change.

4.7 Gaussian matter fields minimally coupled to an external metric

Once calculated the flow equation for the dimensionless gravitational couplings, we proceed to analyze the contribution of the matter fields to these beta functions. The matter degrees of freedom induce divergences related to the external metric. When the metric becomes dynamic, they will also contribute to the RG flow for G_k and Λ_k. Consider N_S scalar fields, N_D Dirac fields, and N_V Abelian vector fields, all of them minimally

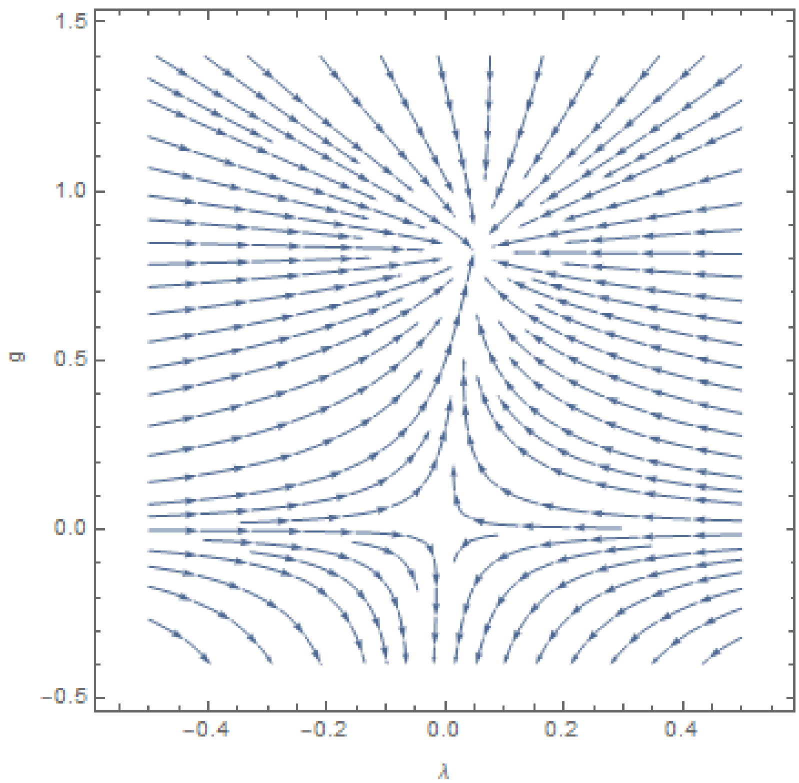


Figure 4.2: The Einstein-Hilbert flow in the $\lambda - g$ plane in four dimensions using the Feynman-de Donder gauge with type II cutoff

coupled to a dynamical metric field. The action is given by,

$$\Gamma_{matter} = S_S(\phi, g) + S_D(\psi, \bar{\psi}, g) + S_V(A, c, \bar{c}, g), \tag{4.66}$$

where,

$$\begin{aligned}
S_S(\phi, g) =& \frac{1}{2}\int d^dx\sqrt{g}g^{\mu\nu}\sum_{i}^{N_S}\partial_\mu\phi^i\partial_\nu\phi^i \\
S_D(A, c, \bar{c}, g) =& i\int d^dx\sqrt{g}\sum_{i}^{N_D}\bar{\psi}_i \not{D}\psi^i \\
S_V(A, c, \bar{c}, g) =& \frac{1}{4}\int d^dx\sqrt{g}\sum_{i=1}^{N_V} g^{\mu\nu}g^{\kappa\lambda}F^i_{\mu\kappa}F^i_{\nu\lambda} + \frac{1}{2\xi}\int d^dx\sqrt{g}\sum_{i=1}^{N_V}\left(g^{\mu\nu}\nabla_\mu A^i_\nu\right)^2 \\
&+ \int d^dx\sqrt{g}\sum_{i=1}^{N_V}\bar{c}_i(-\nabla^2)c_{i,1}
\end{aligned}$$

Since the action consists of kinetic terms of massless field, the modified inverse propagator will involve kinetic operators for each type of field,

$$P_k(\nabla^2) = -\nabla^2 + \mathbf{E}_1 + R_k(\nabla^2). \tag{4.68}$$

The endomorphism $\mathbf{E}_1$ for both scalar and ghost fields is $\mathbf{E}_1 = 0$. For Dirac fields, $\mathbf{E}_2 = \frac{R}{4}\mathbf{1}$, and for Maxwell fields in the Feynman gauge is given by the Ricci tensor acting on vectors. The FRGE (4.21) with the truncation (4.64) plus gauge fixing and ghost terms is,

$$\begin{aligned}\frac{d\Gamma_k}{dt} =& \frac{N_S}{2}\mathrm{Tr}\left(\frac{\partial_t R_k\left(\nabla_S^2\right)}{P_k\left(\nabla_S^2\right)}\right) - \frac{N_D}{2}\mathrm{Tr}\left(\frac{\partial_t R_k\left(\nabla_D^2\right)}{P_k\left(\nabla_D^2\right)}\right)\\ &+\frac{N_V}{2}\mathrm{Tr}\left(\frac{\partial_t R_k\left(\nabla_V^2\right)}{P_k\left(\nabla_V^2\right)}\right) - N_V\,\mathrm{Tr}\left(\frac{\partial_t R_k\left(\nabla_{gh}^2\right)}{P_k\left(\nabla_{gh}^2\right)}\right),\end{aligned} \tag{4.69}$$

where the vectorial part has been split into a part with the ghost fields and the remainder. Applying (4.61), the trace corresponding to the scalar kinetic operator in four dimensions reads,

$$\mathrm{Tr}\left(\frac{\partial_t R_k\left(\nabla_S^2\right)}{P_k\left(\nabla_S^2\right)}\right) = \int\frac{\sqrt{g}d^4x}{(4\pi)^2}\left\{Q_2\left(\frac{\partial_t R_k\left(\nabla_S^2\right)}{P_k\left(\nabla_S^2\right)}\right) + \frac{R}{6}Q_1\left(\frac{\partial_t R_k\left(\nabla_S^2\right)}{P_k\left(\nabla_S^2\right)}\right) + \mathcal{O}(R^2)\right\}. \tag{4.70}$$

The trace involving the Dirac field is,

$$\mathrm{Tr}\left(\frac{\partial_t R_k\left(\nabla_D^2\right)}{P_k\left(\nabla_D^2\right)}\right) = \int\frac{\sqrt{g}d^4x}{(4\pi)^2}\left\{4Q_2\left(\frac{\partial_t R_k\left(\nabla_D^2\right)}{P_k\left(\nabla_D^2\right)}\right) - \frac{R}{3}Q_1\left(\frac{\partial_t R_k\left(\nabla_D^2\right)}{P_k\left(\nabla_D^2\right)}\right) + \mathcal{O}(R^2)\right\}, \tag{4.71}$$

while the trace associated with Maxwell and scalar ghost fields are given by,

$$\mathrm{Tr}\left(\frac{\partial_t R_k\left(\nabla_V^2\right)}{P_k\left(\nabla_V^2\right)}\right) = \int\frac{\sqrt{g}d^4x}{(4\pi)^2}\left\{4Q_2\left(\frac{\partial_t R_k\left(\nabla_V^2\right)}{P_k\left(\nabla_V^2\right)}\right) - \frac{R}{3}Q_1\left(\frac{\partial_t R_k\left(\nabla_V^2\right)}{P_k\left(\nabla_V^2\right)}\right) + \mathcal{O}(R^2)\right\} \tag{4.72a}$$

$$\mathrm{Tr}\left(\frac{\partial_t R_k\left(\nabla_{gh}^2\right)}{P_k\left(\nabla_{gh}^2\right)}\right) = \int\frac{\sqrt{g}d^4x}{(4\pi)^2}\left\{2Q_2\left(\frac{\partial_t R_k\left(\nabla_{gh}^2\right)}{P_k\left(\nabla_{gh}^2\right)}\right) + \frac{R}{3}Q_1\left(\frac{\partial_t R_k\left(\nabla_{gh}^2\right)}{P_k\left(\nabla_{gh}^2\right)}\right) + \mathcal{O}(R^2)\right\}. \tag{4.72b}$$

Inserting (4.70), (4.71) and (4.72) into (4.69) together the evaluation of the Q-functional obtained from Appendix A, the projection of the flow equation in the first power of the

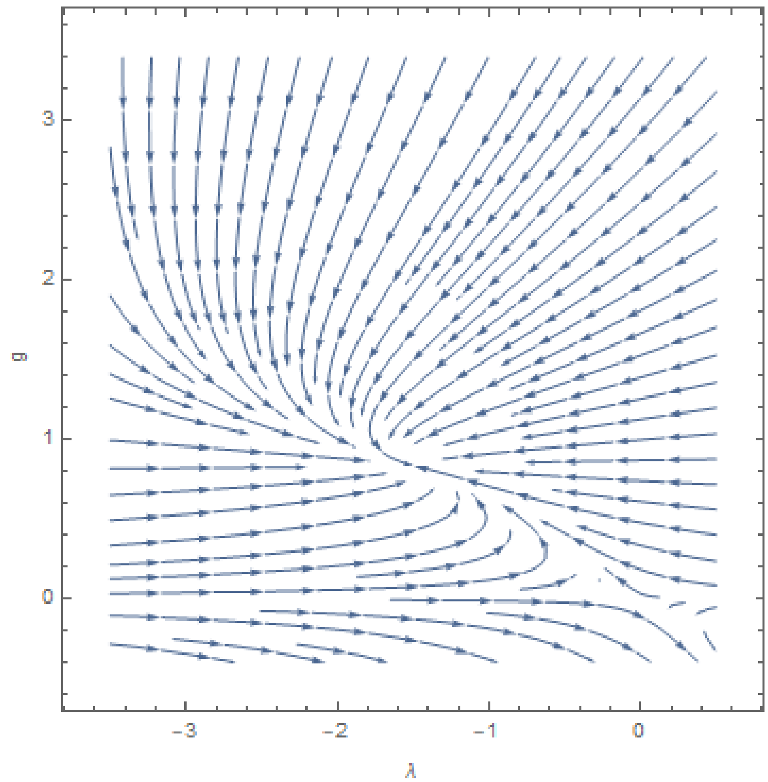


Figure 4.3: The flow corresponding to an Einstein Hilbert term with massless matter degrees of freedom minimally coupled to the metric field in the $\lambda - g$ plane in four dimensions using the Feynman-de Donder gauge with type II cutoff

Ricci scalar reduces to,

$$\frac{d\Gamma_k}{dt} = \frac{1}{2(4\pi)^2} \int \sqrt{g} d^4x \left\{ (N_S - 4N_D + 2N_V)k^4 + \frac{1}{3}k^2 R(N_S + 2N_D - 4N_V) \right\} \quad (4.73)$$

Comparing the l.h.s. of the FRGE, written for the action (4.54) with the r.h.s. given in (4.73) together with (4.64), the beta function for the dimensionless cosmological and Newton's constant are,

$$\beta_g = 2g + \frac{1}{6\pi} g^2 \left(N_S + 2N_D - 4N_V - 46\right), \quad (4.74a)$$

$$\beta_\lambda = -2\lambda + \frac{1}{4\pi} g \left(N_S - 4N_D + 2N_V + 2\right) + \frac{1}{6\pi} \lambda g \left(N_S + 2N_D - 4N_V - 16\right), \quad (4.74b)$$

which have a non-trivial fixed point at,

$$g^* = -\frac{12\pi}{N_S + 2N_D - 4N_V - 46} \quad , \quad \lambda^* = -\frac{3}{4} \frac{N_S - 4N_D + 2N_V + 2}{N_S + 2N_D - 4N_V - 31}, \quad (4.75)$$

Please note that in absent of matter degrees of freedom, the fixed point in the $\lambda - g$ plane reduces to (4.65). For the matter content of the SM ($N_S = 4, N_D = \frac{45}{2}$ and $N_V = 12$), the flow is shown in Figure 4.3.

The following comments are in order. First, the calculation has been done without specifying the external metric. Hence, the result of the beta function is background-independent. Second, the sum over the heat kernel coefficient is closely related to the optimized cutoff. In particular, in this framework, the sum in (4.62) terminates in the third term to $d = 4$. For more general cutoffs, a calculation of beta function for curvature scalars of cubic and higher-order would require the knowledge of a higher heat kernel coefficient. Third, the beta functions for both the metric and matter sector were calculated in the so-called type II cutoff. The only difference with the type I is in the gravitational and the Abelian gauge fields contribution. The 46 is replaced by 22 in (4.74a) and -16 by 8 in (4.74b) for the graviton's fluctuation, while $4N_V$ is replaced by N_V.

The set (4.74) will be used in the next chapter to provide a comparison of the result obtained in this thesis with those found in this chapter.

Chapter 5

Results

5.1 Scalar QED without quartic interaction

As first example one can consider a theory that contains charged spin-zero particles that interact with photons. The bare action is given by,

$$\mathcal{S}(A_\mu, \Phi) = \int d^4x \left(\frac{a_b}{2}(D_\mu\Phi)^*(D^\mu\Phi) - \frac{m_b^2}{2}\Phi^*\Phi - \frac{1}{4e_b^2}F_{\mu\nu}F^{\mu\nu} \right), \tag{5.1}$$

where $D_\mu = \partial_\mu - iA_\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and Φ is a complex scalar field.

5.1.1 Optimized effective action

Following Wilson's idea (101), one can define an average effective action Γ_k as the functional obtained after integrating out the quantum fluctuations, which contain momenta $q^2 > k^2$. By changing k, this scale-dependent effective action can be seen as a smooth interpolation between the microscopic ultraviolet action $\Gamma_{k\to\infty}$ and the full quantum effective action in the infrared limit $\Gamma_{k\to 0}$. The effective action for (5.1) reads,

$$\Gamma_k = \int d^4x \left(\frac{a_k}{2}(D_\mu\phi)^*(D^\mu\phi) - \frac{m_k}{2}\phi^*\phi - \frac{1}{4e_k^2}F_{\mu\nu}F^{\mu\nu} \right). \tag{5.2}$$

This effective action has no hierarchy problem, but also no standard SSB. The couplings (a_k, m_k, e_k) are now scale-dependent quantities. To avoid the logarithmic divergences appearing in the QED couplings due to deep infrared scale $k \to 0$, the Renormalizaton Group (RG) scale is split into its reference fixed part $k_0 = m_0$ and its variable part $k^{'}$ as $k = m_0 + k^{'}$. Identifying the reference scale as m_0 and defining the gravitational couplings in the infrared limit at this scale as shown in Figure 5.1, the couplings can be expanded in the vicinity of m_0 using the dimensionless quantity $\frac{k}{k_0}$ as the expansion parameter. By using the dimensionless quantity $\frac{k}{k_0}$ as the expansion parameter, the couplings can be expanded, in the vicinity of the infrared scale k_0

$$a_k = a_0 + \xi_{a,1}\frac{k^{'}}{m_0} + \xi_{a,2}\frac{k^{'2}}{m_0^2} + \mathcal{O}\left(\frac{k^{'}}{m_0}\right)^3 \tag{5.3a}$$

$$m_k^2 = m_0^2 + \xi_{m,1}\frac{k^{'}}{m_0} + \xi_{m,2}\frac{k^{'2}}{m_0^2} + \mathcal{O}\left(\frac{k^{'}}{m_0}\right)^3 \tag{5.3b}$$

$$\frac{1}{e_k^2} = \frac{1}{e_0^2} + \xi_{e,1}\frac{k^{'}}{m_0} + \xi_{e,2}\frac{k^{'2}}{m_0^2} + \mathcal{O}\left(\frac{k^{'}}{m_0}\right)^3. \tag{5.3c}$$

The set of coefficients (ξ_{ij}) with $i = (a, m, e)$ and $j = (1, 2)$ is obtained from the beta functions of (5.1). Those beta functions further depend on the renormalization scheme.

It is desirable that physical observables are independent of the particular renormalization scheme used to renormalize the theory and the corresponding unphysical parameters involved in this process. However, if the prediction, calculated by a series of approximations, depends on unphysical parameters, then the parameters should be chosen such that variations will minimize the sensitivity of the observable on those parameters. Following this criterion, one looks for a scale setting of the renormalization scale as a function of physical variables $k = k(\phi, \xi_i, F_{\mu\nu}, ...)$. This identification results from applying the variational principle to k by promoting the scale k to a field (51) at the level of the effective action. As shown in (62; 102; 51), this minimization can be written as

$$\begin{aligned} &\frac{\delta\Gamma\left(A_\mu, \phi(x), k(x), a_k, m_k\right)}{\delta k} = 0 \\ \Rightarrow \quad &\frac{\mathrm{d}\mathcal{L}\left(A_\mu, \phi(x), k(x), a_k, m_k\right)}{\mathrm{d}k}\Big|_{k=k_{opt}} = 0. \end{aligned} \tag{5.4}$$

In contrast to most other scale settings, the above procedure allows maintaining the original gauge symmetries, such as gauge and/or diffeomorphism invariance (51). The philosophy underlying this procedure has been developed in (62; 102; 51) and it has been successfully applied in different contexts (103; 104; 105; 106; 107; 108). For consistency, with the expansion (5.3), also the effective action has been expanded up to k^2. The condition (5.4) allows resolving the renormalization-point ambiguity by selecting a single scale and fix it as a function of dynamical field variables,

$$k_{opt} \to \frac{-\xi_{e,1}F_{\mu\nu}F^{\mu\nu} + 2\xi_{a,1}\left|D_\mu\phi\right|^2 - 2\xi_{m,1}\phi^2}{2\left(\xi_{e,2}F_{\mu\nu}F^{\mu\nu} - 2\xi_{a,2}\left|D_\mu\phi\right|^2 + 2\xi_{m,2}\phi^2\right)}. \tag{5.5}$$

Since there are no new undetermined integration constants in (5.5), one can insert the solution $k = k_{opt}$ back into (5.2). This gives an optimal effective action independent of the arbitrary scale k,

$$\Gamma_{opt} = \int d^4x \left\{ \frac{2\xi_{e,1}\xi_{e,2}\xi_{m,1} - 4m_0^2\xi_{e,2}^2 - \xi_{e,1}^2\xi_{m,2}}{8\xi_{e,2}^2}\phi^2 + \right. \tag{5.6}$$
$$\left. \frac{(\xi_{e,2}\xi_{m,1} - \xi_{e,1}\xi_{m,2})^2}{4F^{\mu\nu}F_{\mu\nu}\xi_{e,2}^3}\phi^4 + \mathcal{L}_{kin} + \mathcal{L}_{const} + \mathcal{O}(\phi^5) \right\},$$

where the potential has been expanded to order ϕ^4 in a weak field approximation. Kinetic factors of ϕ and A_μ are contained in $\mathcal{L}_{kin}$ and quantities without any field factor ϕ are assembled into $\mathcal{L}_{const}$. One notes that (5.6) has a quadratic and a quartic term in ϕ, which is necessary for the standard SSB mechanism to take place. However, one also notes that the quartic coupling is only well behaved for a finite electromagnetic background field, $< F_{\mu\nu}F^{\mu\nu} >\neq 0$. Even though this is an interesting feature, it is not the type of SSB we are interested in.

5.1.2 Values of parameter expansion from QED sector

It is instructive to calculate the explicit values of the parameters $\xi_{e,j}, \xi_{m,j}$. Those ξ's have some scheme dependence and can be obtained by applying perturbative methods. When the integral for obtaining an explicit expression of the coupling as a function of the scale is carried out, the lower limit (unlike the case of gravity) cannot be zero, which explains the expansion around k_0 instead of 0 in (5.3). Expressing $k\frac{\partial\alpha}{\partial k}$, with $\alpha \equiv \frac{e^2}{4\pi}$, in terms of the known scalar QED β-function up to one loop in the minimal subtraction scheme (109) gives the RG equation.

$$k\frac{\mathrm{d}\alpha}{\mathrm{d}k} = \frac{\alpha^2}{6\pi}. \tag{5.7}$$

Integrating this equation between the initial and an intermediate scale, the running coupling takes the form,

$$\alpha(k^2) = \frac{\alpha(k_0)}{1 - \frac{1}{3\pi}\alpha(k_0)\ln\left(\frac{k}{k_0}\right)}. \tag{5.8}$$

The expansion for the running coupling e_k around $k = k_0$ by imposing $e_k \to e_{k_0} \equiv e_0$ for the first term of the series (see Figure 5.1) and rearranging (5.8) gives,

$$\frac{1}{e_k^2} = \frac{1}{e_0^2} - \frac{1}{24\pi^2}\left(\frac{k'}{m_0}\right) + \frac{1}{48\pi^2}\left(\frac{k'^2}{m_0^2}\right) + \mathcal{O}\left(\frac{k'}{m_0}\right)^3. \tag{5.9}$$

The one-loop contribution to the anomalous mass dimension in Lorentz gauge (109), $\gamma_m = -\frac{3e^2}{8\pi^2}$, follows the same treatment. Integrating γ_m with the initial condition in k_0, an expansion (like (5.9)) for the running coupling m_k is obtained,

$$m_k^2 = m_0^2 - \frac{3e_0^2 m_0^2}{8\pi^2}\left(\frac{k'}{m_0}\right) + m_0^2 e_0^2\left(\frac{24\pi^2 + e_0^2(9\pi - 1)}{128\pi^3}\right)\left(\frac{k'}{m_0}\right)^2 + \mathcal{O}\left(\frac{k'}{m_0}\right)^3. \quad (5.10)$$

Thus, the set of parameters ($\xi_{i,j}$) from the scalar sector can be identified from a comparison between (5.3), (5.9) and (5.10) as,

$$\xi_{e,1} = -\frac{1}{24\pi^2} \quad (5.11a)$$

$$\xi_{e,2} = \frac{1}{48\pi^2} \quad (5.11b)$$

$$\xi_{m,1} = -\frac{3e_0^2 m_0^2}{8\pi^2} \quad (5.11c)$$

$$\xi_{m,2} = m_0^2 e_0^2\left(\frac{24\pi^2 + e_0^2(9\pi - 1)}{128\pi^3}\right). \quad (5.11d)$$

The set of equations (5.11) is valid only for scalar QED up to one loop in perturbation theory in the Lorentz gauge, in the minimal subtraction scheme. Possible contributions on these U(1) coefficients coming from the gravitational sector will be discussed in section 5.3.

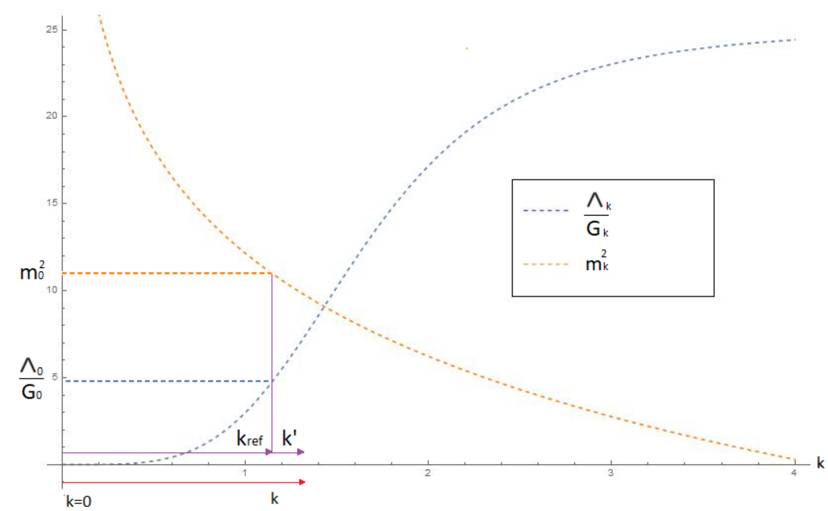


Figure 5.1: Graphic representation of reference scale used in this work.

5.2 Gravitational sector minimally coupled to a charged scalar

As explained in the introduction, a cosmological term is important for the implementation of our ideas. Thus, let's consider a gravitational sector coupled to matter. In the leading order truncation (meaning that higher-order dipheomorphism invariant operators like R^2, $R_{\mu\nu}R^{\mu\nu}$,... are neglected), the simplest effective action of gravity coupled to a charged scalar reads,

$$\Gamma_k = \int d^4x\sqrt{-g}\left\{\kappa(2\Lambda_k - R) + \frac{m_k^2}{2}\phi^*\phi + \frac{a_k}{2}(D_\mu\phi)^*(D^\mu\phi) - \frac{1}{4e_k^2}F^{\mu\nu}F_{\mu\nu} + c_f\mathcal{L}_{kin,\psi}\right\}. \tag{5.12}$$

One notes that all couplings (κ_k, Λ_k, m_k, a_k, e_k), except the one of the sterile fermions c_f (fermions which only interact with the metric field), are scale dependent quantities. In this action, Λ_k stands for the scale-dependent cosmological constant, R is the Ricci scalar, and $\kappa = (16\pi G_k)^{-1}$, where G_k is the running counterpart of the gravitational coupling. In addition to (5.3), an expansion around the infrared zone $k \to k_0$ in the gravitational coupling is needed to get an optimal scale. In order to maintain consistency with the expansion (5.3), the effective action (5.12) and the gravitational couplings also are expanded to the same order. We examine the solutions of the gravitational couplings with an Einstein-Hilbert truncation in the deep infrared. In this limit, one finds, independent of the implementation of the Wilsonian renormalization procedure, an RG running of Newton's and the cosmological constant of the form,

$$G(k) = G_0\left(1 + C_1G_0k^2 + C_2G_0^2k^4\right) + \mathcal{O}(k^6) \tag{5.13}$$
$$\Lambda(k) = \Lambda_0 + C_3\Lambda_0G_0k^2 + C_4\zeta\left(G_0\Lambda_0\right)k^4 + \mathcal{O}(k^6), \tag{5.14}$$

with $C_{1,2,3,4}$ being real numbers. Depending on the sign of C_1, (5.13) shows screening or anti-screening property of gravity. When an expansion is made around k_0 instead of zero, one can redefine the constants C_i giving

$$\tilde{\Lambda}_k = \tilde{\Lambda}_0 + \xi_{\tilde{\Lambda},1}\left(\frac{k'}{m_0}\right) + \xi_{\tilde{\Lambda},2}\left(\frac{k'}{m_0}\right)^2 + \mathcal{O}(k')^3, \tag{5.15}$$

where a change of the cosmological variable $\tilde{\Lambda}_k = \frac{\Lambda_k}{G_k}$ has been applied. Please note that if one is interested in the effective Higgs potential, (5.15) is enough to get an overview of the gravitational's contribution to this model. In particular, G_k from (5.13) is not needed, because this part of the action is proportional to R, which does not take part in the SSB process. Based on a study on field-parametrization dependence of the renormalization group flow in the vicinity of non-Gaussian fixed points in quantum gravity, a beta function derived from EH action can be used to fix the parameters ξ_i in (5.15). One can further look at how the massless-matter fields affect asymptotically safe quantum gravity. In this case, the parameters ξ_i would have a dependence on the number and the nature of matter fields. For now, the gravitational parameter set $\xi_{\tilde{\Lambda},j}$ is kept arbitrary. In the next section, a physical benchmark for this set $\xi_{\tilde{\Lambda},j}$ will be worked out.

As in (5.5), the scale setting is performed by demanding (5.12) to be insensitive under infinitesimal changes of k, giving

$$k_{opt} = \frac{\mathcal{H}_{R,1} + \mathcal{H}_{F,1} + \mathcal{C}_1 - 2\xi_{m,1}\phi^2 + 2|D\phi|^2\xi_{a,1}}{\mathcal{H}_{R,2} + \mathcal{H}_{F,2} + \mathcal{C}_2 + 4G_0^3\left(\xi_{m,2}\phi^2 - \xi_{a,2}|D\phi|^2\right)}. \tag{5.16}$$

Herein, the functions $\mathcal{H}_R$ and $\mathcal{H}_F$ have as arguments the Ricci tensor R and electromagnetic tensor $F_{\mu\nu}F^{\mu\nu}$, respectively, in addition to the infrared value of the running G_k. The constants $\mathcal{C}_{1,2}$ only contain infrared couplings and electromagnetic constants $\xi_{e,m,\ldots}$. When the optimal scale is inserted back into the effective action (5.12) (that includes gravitational effects), one gets an optimal effective action independent of k,

$$\begin{aligned}\Gamma_{opt} = \int d^4x\sqrt{-g}\,\Big\{&+\frac{\mu^2}{2}|\phi|^2 - \frac{\lambda}{4}|\phi|^4 - \frac{1}{4\tilde{e}}F_{\mu\nu}F^{\mu\nu} \\ &+\mathcal{L}_{kin} + \mathcal{L}_{coup} + \mathcal{L}_{const} + \mathcal{O}(\phi^6)\Big\}\,,\end{aligned} \tag{5.17}$$

where $\mathcal{L}_{kin}$ contains kinetic terms for the real scalar and gauge fields. Couplings with higher-order factors and Ricci scalar quantities are collected into $\mathcal{L}_{coup}$, and the Lagrangian part independent of Ricci scalar, electromagnetic strength and scalar field are named $\mathcal{L}_{const}$. The effective potential has again been expanded up to order ϕ^4. The optimal effective action (5.17) is written following the usual notation with μ and λ appearing in the Abelian Higgs mechanism,

$$\frac{\mu^2}{2} = \frac{m_0^2}{2} + \frac{\xi_{\tilde{\Lambda},1}\left(\xi_{m,2}\xi_{\tilde{\Lambda},1} - 2\xi_{m,1}\xi_{\tilde{\Lambda},2}\right)}{8\xi_{\tilde{\Lambda},2}^2} \tag{5.18a}$$

$$\frac{\lambda}{4} = \frac{\left(\xi_{m,2}\xi_{\tilde{\Lambda},1} - \xi_{m,1}\xi_{\tilde{\Lambda},2}\right)^2}{32\xi_{\tilde{\Lambda},2}^3}. \tag{5.18b}$$

If those parameters have the correct values, then the field ϕ acquires a VEV, and the U(1) global symmetry will be spontaneously broken. Thus, (5.17) shows that, even if one starts with a model like (5.12), which has not SSB, it is possible to get this feature for the optimal effective action Γ_{opt}. This is particularly true if $\mu^2, \lambda > 0$.

5.3 Gauge boson masses

In this section, restrictions on the RG parameters ξ_i will be studied.

5.3.1 Mass and vacuum expectation value of scalar and gauge fields

The parameters $\xi_{m,1}$ and $\xi_{m,2}$ appearing in (5.18) can be obtained studying the changes of the anomalous mass dimension. Since the action (5.12) considers the Einstein-Hilbert contribution, the gravitational sector may, in principle, have an impact on the behavior of electromagnetic couplings. Gravitational corrections to the beta function in quantum field theories have been analyzed in (110; 111; 112; 113; 114; 115; 116; 117; 118; 119). Non-Abelian gauge fields coupled to gravity in (3+1) dimensions give rise to an additional term in the one-loop beta function proportional to the inverse square of Planck scale, improving the asymptotic freedom of N= 4 Super Yang-Mills theory (120; 121). In (122) it is pointed out that quadratic divergences coming from the gravitational sector are responsible for asymptotic freedom of QED beta function in a gauge-independent context with energy scale near the Planck scale. For the case where a complex scalar field is minimally coupling to perturbative quantized Einstein gravity with an explicit gauge dependence in the photon and graviton propagator, the total vacuum polarization tensor depends on the gauge parameters, surface terms, a dimensionless constant, and the ultraviolet momentum cutoff, as explained in (123; 124). In the last case, there are several reasons for neglecting the gravitational contribution to the usual beta function of the U(1) gauge coupling,

- Choosing the gauge parameter, ξ appearing in the graviton propagator in (124), equal to $\frac{5}{13}$, a cancellation of a gravitational contribution to be takes place.
- Using dimensional regularization instead of a momentum cutoff, such that arbitrary parameter contained in the gravitational term is set to 0 (123).
- Some studies (125; 126; 127) have shown the beta function of scalar electrodynamics possesses no contribution coming from gravitational interactions.
- In the infrared $k \approx k_0$ all gravitational contributions to the beta functions of matter will be strongly suppressed by the Planck-scale. This is the reason why the standard model without gravity is a successful quantum field theory in the first place.

Given the arguments expressed above, the gravitational contribution to the electromagnetic beta functions will be neglected. One condition on the effective potential in (5.17) for producing positive Higgs parameters and then SSB was that $\lambda > 0$. This determines the sign of $\xi_{\tilde{\Lambda},2}$. This can be seen by replacing (5.11c) and (5.11d) in (5.18b) and demanding $\lambda > 0$. For a negative value of $\xi_{\tilde{\Lambda},2}$ one has to solve the inequality

$$\left[e_0^4 \left(9\pi - 1\right) \xi_{\tilde{\Lambda},1} + 24 e_0^2 \pi^2 \left(\xi_{\tilde{\Lambda},1} + 2 m_0^2 \xi_{\tilde{\Lambda},2}\right)\right]^2 < 0, \tag{5.19}$$

which has no solution for $e_0, \xi_{\tilde{\Lambda},1} \in \mathbb{R}$. Thus, the requirement for the field ϕ to acquire a VEV is $\xi_{\tilde{\Lambda},2} > 0$. From (5.17) and (5.18), the vacuum expectation value (VEV) of the scalar field is,

$$\phi_{VEV}^2 = \frac{2\left(4 m_0^2 \xi_{\tilde{\Lambda},2}^2 + \xi_{\tilde{\Lambda},1}\left(\xi_{m,2}\xi_{\tilde{\Lambda},1} - 2\xi_{m,1}\xi_{\tilde{\Lambda},2}\right)\right)\xi_{\tilde{\Lambda},2}}{\left(\xi_{m,2}\xi_{\tilde{\Lambda},1} - \xi_{m,1}\xi_{\tilde{\Lambda},2}\right)^2}. \tag{5.20}$$

Suppose that the scalar potential in (5.17) is near one of the minima (say the positive one), then it is convenient to define a fluctuation of the scalar field $\phi(x) = \phi_{VEV} + \eta(x)$. The squared mass of the complex scalar field $\eta(x)$ is then

$$m_\eta^2 = 2\left(m_0^2 + \frac{\xi_{\tilde{\Lambda},1}\left(\xi_{m,2}\xi_{\tilde{\Lambda},1} - 2\xi_{m,1}\xi_{\tilde{\Lambda},2}\right)}{4\xi_{\tilde{\Lambda},2}^2}\right). \tag{5.21}$$

Further, the U(1) coupling is no longer constant because the scale setting condition (5.4) implies a dependence on the field ϕ,

$$\frac{1}{\tilde{e}^2} = \frac{1}{e_0^2} + \frac{\left(4\xi_{\tilde{\Lambda},1} + \xi_{m,1}\phi_{VEV}^2\right)\left(4\xi_{e,2}\xi_{\tilde{\Lambda},1} - 8\xi_{e,1}\xi_{\tilde{\Lambda},2} + h\phi^2\right)}{4\left(4\xi_{\tilde{\Lambda},2} + \xi_{m,2}\phi^2\right)^2}, \tag{5.22}$$

where $h = \xi_{e,2}\xi_{m,1} - 2\xi_{e,1}\xi_{m,2}$. The mass term for the gauge bosons is obtained through the product between the inverse of (5.22) and the VEV of the scalar field (5.20),

$$m_A^2 = \frac{64\xi_{\tilde{\Lambda},2}^3\left(\frac{\mathcal{F}_1}{4\xi_{\tilde{\Lambda}}^2} - m_0^2\right)}{\left(\xi_{m,2}\xi_{\tilde{\Lambda},1} - \xi_{m,1}\xi_{\tilde{\Lambda,2}}\right)^2}\left(\frac{4}{e_0^2} + \frac{\mathcal{F}_3\left(\xi_{e,2}\mathcal{F}_3 + 6\xi_{e,1}\xi_{m,2}\xi_{\tilde{\Lambda},2}\mathcal{F}_1 - 4\xi_{e,1}\mathcal{F}_2\right)}{\left(3\xi_{m,2}^2\xi_{\tilde{\Lambda},1}^2\xi_{\tilde{\Lambda},2} - 6\xi_{m,1}\xi_{m,2}\xi_{\tilde{\Lambda},1}\xi_{\tilde{\Lambda},2}^2 + 2\mathcal{F}_2\right)^2}\right)^{-1}, \tag{5.23}$$

with,

$$\mathcal{F}_1 = \xi_{\tilde{\Lambda},1}\left(2\xi_{m,1}\xi_{\tilde{\Lambda},2} - \xi_{m,2}\xi_{\tilde{\Lambda},1}\right) \tag{5.24a}$$

$$\mathcal{F}_2 = \xi_{\tilde{\Lambda},2}^3\left(\xi_{m,1}^2 + 2m_0^2\xi_{m,2}\right) \tag{5.24b}$$

$$\mathcal{F}_3 = 2\xi_{m,2}^2\xi_{\tilde{\Lambda},1}^3 - 3\xi_{m,1}\xi_{m,2}\xi_{\tilde{\Lambda},1}^2\xi_{\tilde{\Lambda},2} + 4m_0^2\xi_{m,1}\xi_{\tilde{\Lambda},3}^3. \tag{5.24c}$$

After insertion of the set of U(1) parameters (5.11) into (5.20) and (5.21), the mass and VEV of Higgs field are determined as a function of gravitational parameters appearing in the infrared expansion of $\frac{\Lambda_k}{G_k}$ and m_0,

$$v^2 = 256\pi^3\xi_{\tilde{\Lambda},2}\left(\frac{e_0^4\left(9\pi - 1\right)\xi_{\tilde{\Lambda},1}^2 + 512m_0^2\pi^3\xi_{\tilde{\Lambda},2}^2 + 24e_0^2\pi^2\xi_{\tilde{\Lambda},1}\zeta_1}{e_0^4\left(e_0^2\left(9\pi - 1\right)\xi_{\tilde{\Lambda},1} + 24\pi^2\zeta_2\right)^2}\right), \tag{5.25a}$$

$$m_\eta^2 = \frac{1}{256\pi^3\xi_{\tilde{\Lambda},2}^2}\left(e_0^4\left(9\pi - 1\right)\xi_{\tilde{\Lambda},1}^2 + 512\pi^3m_0^2\xi_{\tilde{\Lambda},2}^2 + 24e_0^2\pi^2\xi_{\tilde{\Lambda},1}\zeta_1\right), \tag{5.25b}$$

with $\zeta_1 = \left(\xi_{\tilde{\Lambda},1} + 4m_0\xi_{\tilde{\Lambda},2}\right)$ and $\zeta_2 = \left(\xi_{\tilde{\Lambda},1} + 2m_0\xi_{\tilde{\Lambda},2}\right)$. At this point, it is important to mark a few comments about the result obtained in (5.25). The initial action (5.17) has no elements to produce mass for the U(1) gauge boson since the quartic interaction is missing. After applying the VPS procedure, to get an optimal effective action, a symmetry breaking effective potential appears when the gravitational sector is taken into account. As a result, masses are driven by gravitational parameters. However, these parameters must also meet certain requirements. The hermicity of the optimal scale Lagrangian implies that the parameters in (5.18) must be real, and thus this Lagrangian respects charge, parity, and time-reversal symmetries. Mimicking the usual Higgs mechanism for SSB, the sign of the mass term is chosen negative. Moreover, the effective coupling λ must be positive as a requirement for the scalar potential to be bounded from below. As shown above, this implies that $\xi_{\tilde{\Lambda},2} > 0$.

5.3.2 Benchmark of gravitational parameters

Due to the dependence on infrared coefficients ξ_i provided in the expansion of the couplings involved in the theory, one can expect to get restrictions from physically observed gauge boson masses. This exercise is done despite the fact that the model presented in (5.17) is more a conceptual case study rather than a competitive phenomenological model since it has neither electro-weak nor Yukawa couplings implemented. Observed experimental values of the Higgs mass and VEV will be employed to get a better idea about the distribution of the allowed parameters $\xi_{\tilde{\Lambda},1}$, $\xi_{\tilde{\Lambda},2}$ in agreement with the observations. The quantities m_η and v in (5.25) have four free parameters, e_0, m_0, $\xi_{\tilde{\Lambda},1}$ and $\xi_{\tilde{\Lambda},2}$. We impose that,

- $e_0 = \sqrt{4\pi\alpha} \approx 0.3028$ (128). The U(1) coupling e_0 in (5.12) takes the value of the vertex function in spinor electrodynamics when all three particles (one incoming fermion, incoming photon and outgoing fermion) are on shell, i.e. the elementary charge e. In the deep infrared, this choice $k' \to 0$ is justified due to the long-range character of QED. One confirms numerically that the difference between $\tilde{e}$ and e_0 is negligible.
- $v = \left(\sqrt{2}G_F\right)^{-\frac{1}{2}} = 246.2197\text{GeV}$ (129), because the experimental uncertainty on the Higgs mass m_H is much larger than the uncertainty on the VEV of the Higgs field measured in the muon decay v_H, only the best fit value $v = 246.2197$ will be considered to fix m_0 as a function of the gravitational parameters $\xi_{\tilde{\Lambda},1}$ and $\xi_{\tilde{\Lambda},2}$.
- $\xi_{\tilde{\Lambda},2} > 0$. Although the parameters $\xi_{\tilde{\Lambda},1}$ and $\xi_{\tilde{\Lambda},2}$ are arbitrary, when the two preceding points are applied to (5.25b), the bound on $\xi_{\tilde{\Lambda},2}$ arises from imposing real values for m_η.

The boundaries for the gravitational parameters $\xi_{\tilde{\Lambda},1}$ and $\xi_{\tilde{\Lambda},2}$ can be obtained by associating the limits of the Higgs boson mass with the limits of m_η in (5.25b), and the result is shown in Figure 5.2. Parameters enclosed in the shadow region ensure completion of experimental requirements previously discussed.

5.4 Comparison with the functional renormalization group

Up to now, our calculations never made use of a specific shape of gravitational beta functions. Now, it would be useful to perform a comparison with explicit realizations of those beta functions and exploring the constraints imposed such that the proposed model takes place. The evolution of the scale-dependent dimensionless couplings is dictated by the functional renormalization group equation (*a.k.a.* Wetterich equation),

$$\frac{\mathrm{d}\Gamma_k}{\mathrm{d}t} = \frac{1}{2}\mathrm{STr}\left(\frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)}[\phi] + \mathcal{R}_k}\right). \tag{5.26}$$

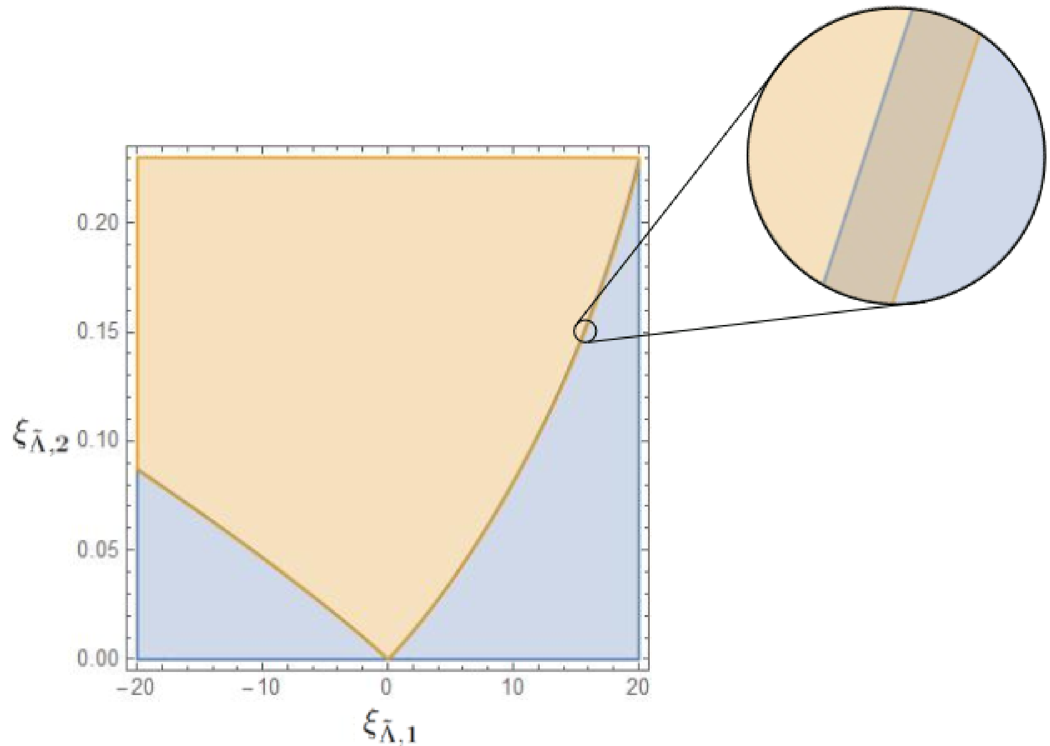


Figure 5.2: Allowed parameter range in the gravitational parameter $\xi_{\tilde{\Lambda},1}$ and $\xi_{\tilde{\Lambda},2}$. The orange region represents gravitational parameters which give a mass for the U(1) gauge boson lower than $m_\eta = 125.33$ GeV while the blue region represents parameters greater than $m_\eta = 124.85$ (lower and upper experimental limit of the mass of the Higgs boson).

The Wetterich equation is formulated such that it depends on renormalization group time $t = \ln \frac{k}{k_0}$, the modified inverse propagator $\Gamma_k^{(2)}[\phi] + \mathcal{R}_k$ involving a second functional derivative of the scale-dependent effective action with respect to the fields and the momentum cutoff $\mathcal{R}_k$ chosen in such a way to suppress the contributions of field modes smaller than the cutoff scale k^2 (57; 130; 131; 132). The notation STr represents a generalization where the trace is performed over momenta as well as particle species and spacetime or internal indices, including a factor (-1) for fermionic fields. Despite the fact that the Wetterich equation (5.26) is an exact one-loop equation, for practical computations, one deals with the necessity to truncate the theory space to avoid recursive solutions coming from an infinite tower of coupled differential equations.

To perform the comparison between (5.13), (5.18) and the functional renormalization group, equation (5.26), a matter sector presented in section 4.7 is required in addition to the usual Einstein-Hilbert truncation derived in section 4.6. In the following analysis, the results from section 5.2 will be compared with the evolution of gravitational couplings using the FRGE. To find suitable conditions for the relevant variables involved in the comparison, one shall vary the numbers of fields N_D, N_S, and N_V such that the conditions encountered in the AS scenario are met. The flow equations (4.74a) and (4.74b) can be integrated analytically, and the dimensionful running version of the Newton coupling turns out to be,

$$G_N = \frac{G_k}{1+\frac{1}{2}\mathcal{C}_1 k^2 G_k}, \tag{5.27}$$

where G_N is the Newton constant measured in the deep infrared $k \to 0$. The dimensionful running version of the cosmological constant gives

$$\begin{aligned}\Lambda_k =& -\frac{\mathcal{C}_2}{\mathcal{C}_3\left(\mathcal{C}_1+\mathcal{C}_3\right)}\left(\frac{2+\left(\mathcal{C}_1+\mathcal{C}_3\right)k^2G_k}{G_k}\right)+\frac{\Lambda_0 G_N\left(2+\mathcal{C}_1 k^2 G_k\right)^{1+\frac{\mathcal{C}_1}{\mathcal{C}_3}}}{2^{1+\frac{\mathcal{C}_3}{\mathcal{C}_1}}G_k}\\ &+\frac{2^{-\frac{\mathcal{C}_3}{\mathcal{C}_1}}\mathcal{C}_2\left(2+\mathcal{C}_1 k^2 G_k\right)^{1+\frac{\mathcal{C}_3}{\mathcal{C}_1}}}{\mathcal{C}_3\left(\mathcal{C}_1+\mathcal{C}_3\right)G_k}.\end{aligned} \tag{5.28}$$

In (5.27) and (5.28) we have defined

$$\mathcal{C}_1 = \frac{1}{6\pi}\left(N_S+2N_D-4N_V-46\right), \tag{5.29a}$$

$$\mathcal{C}_2 = \frac{1}{4\pi}\left(N_S-4N_D+2N_V+2\right), \tag{5.29b}$$

$$\mathcal{C}_3 = \frac{1}{6\pi}\left(N_S+2N_D-4N_V-16\right). \tag{5.29c}$$

Consider the expansion around the quantity $k = (k^{'}+m_0)/m_0$ (see Figure 5.1). By working out at the infrared the system of differential equations (4.74a) and (4.74b) according to 5.1, the modified running $\tilde{\Lambda}$ up to order $k^{'2}$ reads,

$$\tilde{\Lambda}_k = \tilde{\Lambda}_0 + m_0^4\mathcal{C}_2\cdot\left(\frac{k^{'}}{m_0}\right)+3\cdot 2^{-\mathcal{C}_1}m_0^4\mathcal{C}_2\cdot\left(\frac{k^{'}}{m_0}\right)^2+\mathcal{O}\left(G_0,\Lambda_0\right). \tag{5.30}$$

Since the crucial requirement of the method described in the two previous sections on the gravitational parameters generated in (5.15) is $\xi_{\tilde{\Lambda},2} < 0$, the compatibility will be dictated by the sign of $\mathcal{C}_2$. To compare our result with those found in the AS program, the requirements that the gravitational parameters $(\xi_{\tilde{\Lambda},1}, \xi_{\tilde{\Lambda},2})$ need to meet can be summarized as follows,

1. *Positive Newton's fixed point* $g^* > 0$: The low value ($k \lesssim M_{pl},$) of Newton's gravitational coupling, is restricted by observations based on laboratory experiments at the scale $k_{lab} \backsimeq 10^{-5}eV$.

2. *Relevant directions*: Insofar as the corresponding fixed points for the gravitational couplings of a pure gravity-theory have two relevant directions (89), one expects that the addition of a small number of matter degrees of freedom does not change this behavior and the subsequent parametrization in theory space.

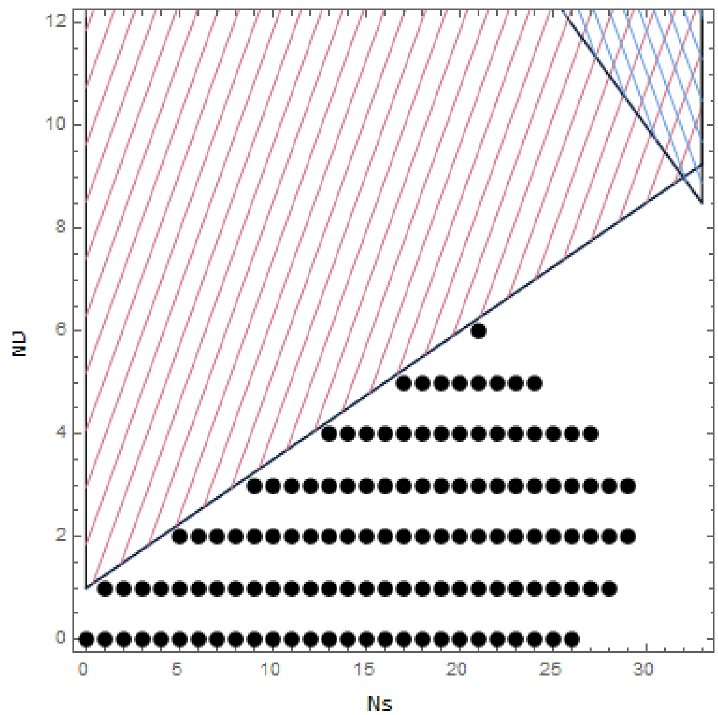


Figure 5.3: Dynamical matter degrees of freedom compatible with a gravitational fixed point with two relevant directions for $N_V = 1$ (explicit values are listed in Appendix C), represented by black bullets. The shaded blue region represents a zone where a negative Newton's fixed point takes place while the shaded red area contains points associated with $\xi_{\tilde{\Lambda},2} < 0$.

3. *Positive value of* $\xi_{\tilde{\Lambda},2} > 0$: As discussed before, the requirement to ensure that our model guarantees SSB at the level of the effective action needs $\lambda > 0$ in (5.18b), or equivalently, $\mathcal{C}_2 > 0$ in (5.30).

The first two criteria were already pointed out in (89), and they are shown in Appendix C for different matter field configurations, while the third selection criterium is necessary for the validity in (5.18b). These conditions determine how many fields N_S, $N_D, \ldots$ may be incorporated such that the proposed mechanism for SSB is in agreement with the requests of AS. Figure 5.3 shows the region in the N_S-N_D-plane where a fixed point with the demands previously enumerated exist for $N_V = 1$ at one-loop approximation in the anomalous dimension using a type II cutoff. The addition of sterile fermionic degrees of freedom carries a single prediction $N_D = 1$ for numbers of Dirac fermions can be inferred from Figure 5.3 while the numbers of scalar fields keep unchanged ($N_S = 2$), maintaining the same sign $\xi_{\tilde{\Lambda},2} > 0$.

Chapter 6

Conclusions And Remarks

In the present work, a novel mechanism for spontaneous symmetry breaking is suggested that circumvents the appearance of quadratic divergences by avoiding the breaking to take place at the classical level. It is shown that SSB still can occur at the quantum level, namely after setting the renormalization scale k of the effective quantum action Γ_k. As scale setting procedure, the VPS method is used. This allows arriving at an optimized effective quantum action Γ_{opt}, which, under certain conditions, produces SSB.

Despite the fact that the toy models of our study do not contain Yukawa, weak, or strong couplings, the underlying mechanism can be expected to work also for models containing these features. It is shown that within this type of model, one can impose phenomenological conditions on Γ_{opt}. These conditions (in particular, the requirement $\xi_{\tilde{\Lambda},2} > 0$) do then allow to put restrictions on the free parameters and the number of scalar, vector, and Dirac fields. For the example of asymptotically safe quantum gravity coupled to matter, it is shown that, for a given number of scalar fields, these conditions impose an upper and a lower bound on the number of Dirac fields, as shown in Figure 5.3.

We further analyze to which extent the results depend on the gauge choice, the truncation, and the shape of the cut-off function in Appendix B. The inclusion of graviton and ghost anomalous dimension, as well as the anomalous dimension of the matter fields, derived in (89), does not affect the findings discussed above. Note further, that the results for the masses of the Abelian and scalar fields obtained in section 4 are entirely independent of the existence of UV fixed points and the respective UV completion proposed in the AS scenario.

In a future study, we plan to perform an implementation with all couplings necessary to arrive at the Glashow-Weinberg-Salam model (133; 134; 135) coupled to gravity.

Appendix A

Evaluation of traces and Q-functionals

This appendix is devoted to showing how to evaluate the traces appearing in the r.h.s. of (4.45). The trace is typically a general function of a differential operator, where in general, the spectrum of the operator is unknown. Here we present a series of tools to deal with this complicate situation using a derivative expansion.

Consider the trace of some function W of the covariant Laplace Δ,

$$\mathrm{Tr}W(\Delta) = \sum_n W(\lambda_n), \tag{A.1}$$

where λ_n are the eigenvalues. Introducing the Laplace anti-transform $\tilde{W}$,

$$W(z) = \int_0^\infty ds e^{-zs}\tilde{W}(s). \tag{A.2}$$

Replacing (A.2) into (A.1),

$$\begin{aligned}\mathrm{Tr}W(z) &= \sum_n \int_0^\infty ds e^{-\lambda_n s}\tilde{W}(s) \\ &= \int_0^\infty ds \mathrm{Tr}K(s)\tilde{W}(s),\end{aligned} \tag{A.3}$$

where λ_n are the eigenvalues. Introducing the Laplace anti-transform $\tilde{W}$, where $\mathrm{Tr}K(s)$ corresponds to the trace of the heat kernel of Δ. In order to find a manageable expression for $\mathrm{Tr}K(s)$, one first thinks in a flat d-dimensional space where the heat kernel can be calculated using Fourier analysis. The Fourier transform of $K(\vec{x},\vec{y};t)$ on the first coordinate is a function $\tilde{K}(\vec{q},\vec{y};t)$ satisfying the equation,

$$\frac{d}{dt}\tilde{K}(\vec{q},\vec{y};t) + q^2\tilde{K}(\vec{q},\vec{y};t) = 0, \tag{A.4}$$

with the initial condition,

$$\tilde{K}(\vec{q},\vec{y};0) = e^{-\vec{q}\cdot\vec{y}}. \tag{A.5}$$

The solution of (A.4) is $\tilde{K}(\vec{q},\vec{y};t) = e^{-i\vec{q}\cdot\vec{y}-q^2 t}$. The inverse Fourier transform gives back the heat kernel K,

$$\begin{aligned} K(\vec{x},\vec{y};t) &= \int \frac{d\vec{q}}{(2\pi)^d} e^{-q^2 t + i\vec{q}\cdot(\vec{x}-\vec{y})} \\ &= \frac{1}{(4\pi t)^{\frac{d}{2}}} e^{-\frac{|\vec{x}-\vec{y}|^2}{4t}}, \end{aligned} \tag{A.6}$$

where we have used the Gaussian integral to get the second line of (A.5). In flat space, if V denotes the volume,

$$\mathrm{Tr}K(t) = \frac{V}{(4\pi t)^{\frac{d}{2}}}. \tag{A.7}$$

In the case of a curved manifold, they look locally like Euclidean space. Hence, in the limit $t \to 0$, the trace of the heat kernel K must reduce to (A.7) in flat space. The deviations from this form must be proportional to the curvature invariants since they take into account the variations of the metric from a flat background. The trace of the heat kernel has an asymptotic expansion around $t \to 0$ of the form,

$$\mathrm{Tr}K(t) \approx \frac{1}{(4\pi t)^{\frac{d}{2}}} \left[B_0(\Delta) + tB_2(\Delta) + t^2 B_4(\Delta) + ... \right], \tag{A.8}$$

where,

$$B_n(\Delta) = \int d^4x \sqrt{g}\, \mathrm{Tr} b_n(\Delta), \tag{A.9}$$

and $b_n(\Delta)$ are scalars constructed from the curvature and their covariant derivatives. Each b_n contains n-derivatives of the metric, such that $b_0 \propto R^0$, $b_1 \propto R$ and so on. Using the expansion (A.8), one can rewrite (A.3) as,

$$\begin{aligned} \mathrm{Tr}W(\Delta) &= \int_0^\infty ds \tilde{W}(s) \frac{1}{(4\pi s)^{\frac{d}{2}}} \left[B_0(\Delta) + sB_2(\Delta) + s^2 B_4(\Delta) + ... \right] \\ &= \frac{1}{(4\pi)^{\frac{d}{2}}} \int_0^\infty ds \tilde{W}(s) \left[B_0(\Delta) s^{-\frac{d}{2}} + B_2(\Delta) s^{-\frac{d}{s}+1} + B_4(\Delta) s^{-\frac{d}{2}+2} + ... \right] \\ &= \frac{1}{(4\pi)^{\frac{d}{2}}} \left[Q_{\frac{d}{2}}(\tilde{W}) B_0(\Delta) + Q_{\frac{d}{2}-1}(\tilde{W}) B_2(\Delta) + ... + Q_0(\tilde{W}) B_d(\Delta) + ... \right], \end{aligned} \tag{A.10}$$

where the Q-functionals are defined as,

$$Q_n(\tilde{W}) = \int_0^\infty ds s^{-n} \tilde{W}(s). \tag{A.11}$$

To get an explicit relation between the Q-functional and W, consider the $i-th$ derivative of W with respect to z,

$$W^{(i)}(z) = (-1)^i \int_0^\infty ds s^i e^{-sz} \tilde{W}(s). \tag{A.12}$$

The application to the Q-functional to (A.12) gives the following relation,

$$Q_n(W^{(i)}) = (-1)^i Q_{n-i}(\tilde{W}). \tag{A.13}$$

By other hand, the gamma function is defined as,

$$1 = \frac{1}{\Gamma(n)} \int_0^\infty dz z^{n-1} e^{-z}. \tag{A.14}$$

The definition of $\Gamma(n)$ allows relating Q and W through a Mellin transformation,

$$\begin{aligned} Q_n(W) &= \frac{1}{\Gamma(n)} \int_0^\infty \int_0^\infty ds dz s^{-n} z^{n-1} e^{-z} \tilde{W}(s) \\ &= \frac{1}{\Gamma(n)} \int_0^\infty dz z^{n-1} \int_0^\infty ds s^{-n} \tilde{W}(s) e^{-z} \\ &= \frac{1}{\Gamma(n)} \int_0^\infty dz z^{n-1} \underbrace{\int_0^\infty ds e^{-zs} \tilde{W}(s)}_{W(z)} \\ &= \frac{1}{\Gamma(n)} \int_0^\infty dz z^{n-1} W(z). \end{aligned} \tag{A.15}$$

In writing down the third line of (A.15), we have performed the change of variable $z \to zs$. For $m \geq 0$,

$$Q_{-m}(W) = (-1)^m W^{(m)}(0). \tag{A.16}$$

The equations (A.15) and (A.16) can be emerged from a general formula. If $n \geq 0$, (A.15) works, while if $n < 0$ one can choose a positive integer r such that $n + r > 0$, then,

$$Q_n(W) = \frac{(-1)^r}{\Gamma(n+r)} \int_0^\infty dz z^{n+r-1} W^{(r)}(z). \tag{A.17}$$

Now we can calculate the integrals appearing in chapter 4. We restrict ourselves to the type II cutoff ($\Delta = -\nabla^2 + q^2 \mathbb{1}$) and the optimized cutoff function (4.5). It is convenient to measure the cutoff and the modified inverse propagator in units of k^2. Let us define the dimensionless variable y by $z = k^2 y$. The cutoff function R_k and the modified inverse propagator $P_k(z)$ are,

$$R_k(z) = k^2 r(y) \ \ , \ \ P_k(z) = k^2(y + r(y)), \tag{A.18}$$

for some function r. In the particular case of the optimized cutoff, $r(y)$ takes the form,

$$r(y) = (1-y)\Theta(1-y). \tag{A.19}$$

The Q-functional are,

$$\begin{aligned}
Q_n\left(\frac{\partial_t R_k}{(P_k+q)}\right) &= \frac{1}{\Gamma(n)}\int_0^\infty dz z^{n-1}\left(\frac{\partial_t R_k}{(P_k+q)}\right) \\
&= \frac{1}{\Gamma(n)}\int_0^\infty k^{2(n-l+1)} dy y^{n-1}\left(\frac{r(y) - yr^{'}(y)}{(y+r(y)+\tilde{q})^l}\right) \\
&= \frac{2k^{2(n-l+1)}}{\Gamma(n)}\left\{\int_0^\infty dy \frac{y^{n-1}r(y)}{(y+r(y)+\tilde{q})^l} - \int_0^\infty dy \frac{y^n r^{'}(y)}{(y+r(y)+\tilde{q})^l}\right\} \\
&= \frac{2k^{2(n-l+1)}}{\Gamma(n)}\left\{\frac{(1+\tilde{q})^{-l}}{n(n+1)} + \frac{(1+\tilde{q})^{-l}}{1+n}\right\} \\
&= \frac{2k^{2(n-l+1)}}{n!(1+\tilde{q})^l},
\end{aligned} \tag{A.20}$$

where $\tilde{q} = qk^{-2}$. For the Einstein-Hilbert term, the Q-functionals of the ghost term are,

$$Q_1\left(\frac{\partial_t R_k(\Delta_{gh})}{P_k(\Delta_{gh})}\right) = 2k^2 \ \ , \ \ Q_2\left(\frac{\partial_t R_k(\Delta_{gh})}{P_k(\Delta_{gh})}\right) = k^4, \tag{A.21}$$

while in the general case for the metric fluctuations, the cosmological constant Λ and the anomalous dimension η_n in the arguments of the inverse propagator and cutoff function, respectively, are given by,

$$Q_1\left(\frac{\partial_t R_k(\Delta_h) - \eta_N R_k(\Delta_h)}{P_k(\Delta_h) - 2\Lambda}\right) = \frac{k^2}{(1-2\lambda)}\left(2 - \frac{\eta_N}{2}\right), \tag{A.22a}$$

$$Q_2\left(\frac{\partial_t R_k(\Delta_h) - \eta_N R_k(\Delta_h)}{P_k(\Delta_h) - 2\Lambda}\right) = \frac{k^4}{(1-2\lambda)}\left(1 - \frac{\eta_N}{6}\right), \tag{A.22b}$$

with λ being the dimensionless version of the cosmological constant. The traces of the scalars b_n of (A.10) are taken from (139).

Appendix B

Spectral sum for the Dirac operator

To decide which class of cutoff gives the correct contribution to the Newton running coupling in four dimensions, the r.h.s. of the FRG equation is evaluated with an independent method. To make the discussion more concrete, the contribution of fermionic fields to the relevant terms of β_g in $d = 4$ with the optimized cutoff are,

$$-\frac{N_D}{96\pi^2}\int d^4\sqrt{g}R\,, \quad \text{for the type I cutoff} \tag{B.1a}$$

$$\frac{N_D}{48\pi^2}\int d^4\sqrt{g}R\,, \quad \text{for the type II cutoff.} \tag{B.1b}$$

These values differ not only in the number but even in the sign. The one-loop EEA can be defined in terms of the Dirac operator,

$$\Gamma_k = -\mathrm{Tr}\,\log\left(|\not{D}| + R_k^D(|\not{D}|)\right). \tag{B.2}$$

The argument of R_k^D has to be the modulus of the Dirac operator since one wants to suppress the modes depending on the wavelength of the corresponding eigenfunctions. For convenience, we choose,

$$R_k^D(z) = (k-z)\Theta(k-z). \tag{B.3}$$

The heat kernel calculation employed in chapter 4 does not put any restriction on the background. In the particular case of spherical background, however, the calculations are greatly simplified due to the spectrum of the Dirac operator is known, and the r.h.s. of the FRG equation reads,

$$\mathrm{Tr}\left[\frac{\partial_t R_k^D(|\not{D}|)}{P_k^D(|\not{D}|)}\right] = \sum_n m_n \frac{\partial_t R_k^D(|\not{D}|)}{P_k^D(|\not{D}|)} = \sum_n m_n \Theta(k - |\lambda_n|), \tag{B.4}$$

with λ_n and m_n the eigenvalues and the multiplicities of the Dirac operator in S^d, respectively,

$$\lambda_n^{\pm} = \pm\sqrt{\frac{R}{d(d-1)}}\left(\frac{d}{2}+n\right) \quad , \quad m_n = 2^{\left[\frac{d}{2}\right]}\binom{n+d-1}{n} \ , \ n = 0, 1, 2, ... \tag{B.5}$$

where the square parenthesis denotes the integer part. The sum in (B.4) can be computed using the Euler-Maclaurin formula,

$$\sum_{i=0}^{n} f(i) = \int_0^n f(x)dx - B_1\left(f(n) - f(0)\right) + \sum_{k=1}^{p} \frac{B_{2k}}{(2k)!}\left(f^{(2k-1)}(n) - f^{(2k-1)}(0)\right) + \mathcal{C}, \tag{B.6}$$

where B_i are the Bernoulli's numbers, and $\mathcal{C}$ is a remainder. After collecting the volume factor, only terms involving the zeroth and the first powers of the curvature scalar R are needed for computing Newton's constant flow. In (B.6), only the integral contains terms proportional to R; thus it is enough to compute the integral,

$$\begin{aligned}\sum_{i=0}^{n} m_n\Theta(k - |\lambda_n|) &= \int_0^n m_n\Theta\left(k - \sqrt{\frac{R}{d(d-1)}}\left(\frac{d}{2}+n\right)\right) dn \\ &= 2^{\left[\frac{d}{2}+1\right]}\int^{k\sqrt{\frac{d(d-1)}{R}}-\frac{d}{2}} \binom{n+d-1}{n} dn. \end{aligned}\tag{B.7}$$

Changing variables to $n \to n^{'} - \frac{d}{2}$, (B.7) can be written as,

$$\begin{aligned}\sum_{i=0}^{n} \Theta(k - |\lambda_n|) &= 2^{\left[\frac{d}{2}\right]+1}\int_0^{k\sqrt{\frac{d(d-1)}{R}}} \binom{n^{'}+\frac{d}{2}-1}{n^{'}-\frac{d}{2}} dn \\ &= 2^{\left[\frac{d}{2}\right]+1}\int_0^{k\sqrt{\frac{d(d-1)}{R}}} \frac{(n^{'}+\frac{d}{2}-1)!}{(n^{'}-\frac{d}{2})!(d-1)!} dn^{'} \\ &= \frac{2^{\left[\frac{d}{2}\right]+1}}{(d-1)!}\int_0^{k\sqrt{\frac{d(d-1)}{R}}} \left(n^{'}+\frac{d}{2}-1\right)...\left(n^{'}-\frac{d}{2}+1\right) dn^{'}. \end{aligned}\tag{B.8}$$

As will become clear later, the terms we are interested in coming from the integral of the two highest powers of $n^{'}$,

$$\underbrace{\left(n^{'}+\frac{d}{2}-1\right)...\left(n^{'}-\frac{d}{2}+1\right)}_{d-1\text{times}} = n'^{d-1} - n'^{d-3}\sum_{k=1}^{\left[\frac{d-1}{2}\right]}\left(\frac{d}{2}-k\right)^2 + \mathcal{O}(n'^{d-5}). \tag{B.9}$$

Rewriting the sum as $\sum_{k=1}^{\left[\frac{d-1}{2}\right]}\left(\frac{d}{2}-k\right)^2=\frac{1}{24}d(d-1)(d-2)$, the integral in (B.8) becomes,

$$\begin{aligned}\sum_{0}^{n} m_n\Theta\left(k-|\lambda_n|\right) &= \frac{2^{\left[\frac{d}{2}\right]+1}}{(d-1)!}\int_0^{k\sqrt{\frac{d(d-1)}{R}}}\left(n'^{d-1}-\frac{n'^{d-3}}{24}d(d-1)(d-2)\right)dn' \\ &= \frac{2^{\left[\frac{d}{2}\right]+1}}{d!}\left(k\sqrt{\frac{d(d-1)}{R}}\right)^d-\frac{2^{\left[\frac{d}{2}\right]+1}}{24(d-2)!}\frac{d}{24}\left(k\sqrt{\frac{d(d-1)}{R}}\right)^{d-2} \\ &= \frac{2^{\left[\frac{d}{2}\right]+1}d^{\frac{d}{2}}(d-1)^{\frac{d}{2}}}{d!}\left(\frac{k^d}{R^{\frac{d}{2}}}-\frac{d}{24}\frac{k^{d-2}}{R^{\frac{d}{2}-1}}\right). \qquad \text{(B.10)}\end{aligned}$$

By other hand, the volume of a n-dimensional Euclidean sphere with radius r is given by,

$$V_d=(4\pi)^{\frac{d}{2}}\frac{\Gamma(\frac{d}{2})}{\Gamma(d)}r^d. \qquad \text{(B.11)}$$

The scalar curvature R is related with the radius of the sphere r by $R=\pm\frac{d(d-1)}{r}$, thus (B.11) is,

$$V_d=\frac{2\pi^{\frac{d+1}{2}}}{\Gamma(\frac{d+1}{2})}\left(\frac{d(d-1)}{R}\right)^{\frac{d}{2}}. \qquad \text{(B.12)}$$

Collecting a volume factor, the trace in (B.4) is,

$$\sum_{i=0}^{n} m_n\Theta(k-|\lambda_n|)=\frac{2^{\left[\frac{d}{2}\right]}}{\Gamma\left(\frac{d}{2}+1\right)(4\pi)^{\frac{d}{2}}}V_d\left(k^d-\frac{d}{24}k^{d-2}R\right). \qquad \text{(B.13)}$$

In four dimensions, the corresponding contribution to the fermionic fields exactly agrees (except for V_d) with (B.1b). The agreement of the spectral sum with the type II cutoff heat kernel calculation represents a useful consistency check. It suggests that the later gives the correct result, whereas the type I cutoff does not.

Appendix C

Fixed points and relevant directions

N_s	N_D	g^*	λ^*	θ_1	θ_2
0	1	0.7891	-0.0355	3.4679	1.8295
4	1	0.7874	0.0474	3.3244	1.9828
8	1	0.7712	0.1323	3.3392	2.1429
12	1	0.7985	0.2119	4.7225	1.6221
16	1	1.984	0.236	16.0381	4.2369
20	1	5.8534	0.1827	59.1008	15.8484
25	1	27.9783	0.0831	468.104	56.6011
30	1	-7.4924	-3.0458	4.3406	3.0051
35	1	-4.3416	-2.2382	4.2789	2.7502
0	2	0.9061	-0.1388	3.6279	1.6733
4	2	0.9255	-0.0513	3.5503	1.7274
8	2	0.9161	0.0457	3.5004	1.8615
12	2	0.8959	0.1407	3.8755	1.9107
16	2	1.0522	0.2183	7.004	1.1397
20	2	2.9317	0.2021	23.4523	5.4593
25	2	13.2672	0.0905	139.134	16.5161
30	2	-6.5262	-2.1898	4.6413	2.9661
35	2	-3.951	-1.7535	4.4352	2.7362
0	4	1.2332	-0.4412	3.8211	1.4862
4	4	1.3634	-0.3722	3.8152	1.4125
8	4	1.4801	-0.2736	3.8299	1.3405
12	4	1.5275	-0.1386	3.9264	1.3165
16	4	1.4747	0.0126	4.1321	1.6424
20	4	1.5462	0.1276	6.4885	1.4076
25	4	4.5202	0.0938	24.9725	2.0416
0	6	1.7796	-0.9772	3.919	1.3963
4	6	2.2679	-1.073	3.9343	1.2565
8	6	3.2689	-1.3068	3.9642	1.0238
12	6	10.5941	-3.5853	3.9829	0.3888
0	8	2.8677	-2.096	3.9688	1.3603
4	8	5.0095	-3.304	3.984	1.1922
8	8	63.0118	-112.51	2.1179	-1.0674
0	9	3.9567	-3.2412	3.9837	1.3547
2	9	5.6446	-4.423	3.99	1.2799
4	9	10.5569	-7.9074	3.9964	1.1821
6	9	$3.72\text{x}10^7$	-1087.061	$1.25\text{x}10^9$	$6.22\text{x}10^8$
0	10	6.1542	-5.5762	3.9945	1.3547
2	10	11.888	-10.3525	4	1.2801
4	10	712554.8395	490.2162	$3.487\text{x}10^6$	$1.74\text{x}10^6$
0	11	12.9664	-12.8612	4.0024	1.3589

Table C.1: Selected gravitational fixed points and relevant directions for $N_V = 1$ for type II cutoff, Feynman-de Donder gauge and one loop approximation. The first and second column indicate the matter content. The third and fourth column are the fixed points for the Newton's and cosmological constant. The fifth and sixth column represents the minus of the critical exponents.

Appendix D

Consistency of flow equations

Formulations of the FRGE require the inclusion of an IR regulator to ensure the integration of all degrees of freedom of fields possessing fluctuations of momenta higher than k. The choice of the arguments of the cut-off function $\mathcal{R}_k$ gives rise to diversity in the shape of the couplings' flow involved in the theory. However, physical results must remain independent of the selection of the shape and the corresponding endomorphism used in the cut-off function. The last sentence shall be used to check the result obtained in this work. Since the criteria for discriminating the result of the presented mechanism has to do with the sign of $\mathcal{C}2$, different truncations with various types of cut-off and expansion of the cosmological constant were investigating setting $N_S = 2$, $N_V = 1$ and the number of Dirac fields being a number between 2 and 9, where we know the model works while fulfilling the conditions required by the AS. The results are presented in Table D.1, confirm that this analysis is robust under changes in the truncation procedure.

Furthermore, the characterization of $\mathcal{C}_2$ number is quite general because in an ultra-local parametrization described in (138) is independent of the gauge choice. Due to the structure of the $\mathcal{C}_2$ term in (5.29b), the sign of $\xi_{\tilde{\Lambda},2}$ is also independent of how many scalar fields are incorporated while keeping N_D at some fixed number.

Most of the findings presented in this appendix might seem evident, but some issues appear when one gets solutions of the FRGE with one or the other of the kinetic operator. In particular, (139; 140; 137) shows that the spectrum of ∇^2 (type I cut-off) and $\nabla^2 - \frac{R}{4}$ (type II cut-off) may turn out in ambiguities in the sign of the fermionic contribution to the running of Newton's constant. In other words, the background-field dependence of $\mathcal{R}_k$ can alter results in the background approximation of physical observables. Since only type II cut-off gives the sign according to the infrared observation of G, the beta functions eq. 5.25a. This result has been corroborated by employing a completely independent method to evaluate the r.h.s. of (5.26) (140). However, the SSB through PMS takes place whatever the inverse propagator or gauge specifies employed in obtaining the flow equation for the gravitational running couplings.

Ref	Truncation	Gauge	Specifics	$\mathrm{sgn}(\xi_{\tilde{\Lambda},2})$
(89)	EH with SM matter	$\alpha = \beta = 1$	type Ia cutoff lowest order in Λ	Positive
(89)	EH with SM matter	$\alpha = \beta = 1$	type Ia cutoff first order in Λ	Positive
(89)	EH with SM matter	$\alpha = \beta = 1$	type Ib cutoff lowest order in Λ	Positive
(89)	EH with SM matter	$\alpha = \beta = 1$	type Ib cutoff first order in Λ	Positive
(89)	EH with SM matter	$\alpha = \beta = 1$	type II cutoff lowest order in Λ	Positive
(89)	EH with SM matter	$\alpha = \beta = 1$	type II cutoff first order in Λ	Positive
(136)	EH with SM matter	$\alpha = 0, \beta = 1$	type II cutoff lowest order in Λ	Positive*
(136)	EH with SM matter	$\alpha = 0, \beta = 1$	type II cutoff first order in Λ	Positive*
(137)	$f(R)$ to R^9 with SM matter	$\alpha = 0, \beta = -\infty$	type I cutoff lower order in Λ	Positive
(137)	$f(R)$ to R^9 with SM matter	$\alpha = 0, \beta = -\infty$	type I cutoff first order in Λ	Positive
(137)	$f(R)$ to R^9 with SM matter	$\alpha = 0, \beta = -\infty$	type II cutoff lower order in Λ	Positive
(137)	$f(R)$ to R^9 with SM matter	$\alpha = 0, \beta = -\infty$	type II cutoff first order in Λ	Positive

Table D.1: The compatibility of the result obtained in (5.17) is investigated for various studies of the gravitational RG flow in the presence of SM massless matter fields minimally coupled to an external metric. blueIn the third column, different gauge (labeled by the gauge parameters α and β) are explored the second The information exhibited in the column "specifies" contains the choice of the covariant differential operator used as the argument of the cutoff function and the expansion of the cosmological constant, explained in the references of the first column. The fifth column gives the sign of the relevant parameter involved in the process of SSB in (5.18b)

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