PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE

ESCUELA DE INGENIERIA

A FACTOR BASED DYNAMIC RISK MODEL FOR FIXED INCOME PORTFOLIOS: AN APPLICATION IN AN EMERGING MARKET

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Thesis submitted to the Office of Research and Graduate Studies in partial fulfillment of the requirements for the Degree of Master of Science in Engineering

Advisor:
GONZALO CORTÁZAR

Santiago de Chile, April, 2014

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A mi madre, por todo su apoyo en este proceso.

A mi novia, por su comprensión y paciencia.

A mis amigos más cercanos, por sus consejos y ayuda.

AGRADECIMIENTOS

Quisiera agradecer al profesor Gonzalo Cortázar, por su constante direccionamiento siempre enfocado hacia el aporte de valor de esta tesis, junto con su paciencia y fuerte compromiso con este trabajo, lo que facilitó enormemente el cumplimiento de los objetivos de este proceso.

Además, quiero destacar la importante participación de Héctor Ortega, la cual fue indispensable para garantizar la calidad de lo logrado.

Por último, quisiera agradecer a la Pontificia Universidad Católica de Chile, a CONICYT, y a FONDECYT (a través del proyecto 1130352), instituciones que me apoyaron de manera importante durante el proceso.

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RESUMEN

Una tendencia reciente relacionada con la gestión de riesgo se ha vuelto muy popular entre académicos y ejecutores, incluyendo las mayores empresas financieras. Consiste en expresar el riesgo de un portafolio en términos de factores de riesgo subyacentes, los cuales reflejan exposición a variaciones de mercado que se encuentran "detrás de escena" y revelan el "mito de la diversificación", entregando a los gerentes de portafolios un gran conocimiento sobre el riesgo de sus carteras de inversión, en momentos de mayor complejidad. Adicionalmente, muchos estudios relacionados han concluido que, para capturar los cambios de volatilidad y en la correlación entre factores, se deben considerar coeficientes que cambien en el tiempo al modelar el riesgo.

En esta Tesis, proponemos un modelo de riesgo que incluye ambas tendencias y se enfoca en el mercado de renta fija, el cual es un importante mercado a nivel mundial en términos de volumen y no ha sido estudiado de manera profunda en cuanto a estos métodos. Además, presentamos una metodología para validar la robustez del modelo a través de Value-at-Risk Back-Testing. Adicionalmente, probamos nuestro modelo en un mercado emergente con baja liquidez, dado que los coeficientes dependientes del tiempo permiten capturar cambios en la volatilidad y rápidos movimientos en la correlación de factores, características comúnmente encontradas en estos mercados.

Palabras Claves: Portfolio Risk, Factor Model, Fixed-Income, Dynamic Model, Kalman Filter, Value-at-Risk.

ABSTRACT

A recent trend regarding risk management has become very popular between academics and practitioners, including the biggest financial firms. It consists on expressing portfolio risk in terms of underlying risk factors, which reflects exposures to market variations behind the scenes and unveils the "myth of diversification", giving managers a greater insight about their portfolio risk in moments of distress. In addition, many studies related to this have concluded that, in order to capture volatility shifts and changes in the co-movement between factors, time-varying sensitivities need to be considered when modeling risk.

In this Thesis, we propose a risk model that includes both trends and focuses on the fixed-income market, which is an important market worldwide in terms of volume and has not been highly studied regarding these methods; along with a methodology for validating the robustness of the model through Value-at-Risk Back-Testing. Additionally, we test our model in an emerging market with low liquidity, as time-varying coefficients allow for capturing shifting volatility and rapid co-movements change among factors, features commonly found on these environments.

Keywords: Portfolio Risk, Factor Model, Fixed-Income, Dynamic Model, Kalman Filter, Value-at-Risk.

1. INTRODUCTION

Portfolio diversification among asset classes or single securities, along with the measurement of their marginal contribution to risk, have been portfolio manager's main concerns when regulating portfolio risk, since the development of current risk metrics like Value-at-Risk and Tracking Error (Litterman 1996, Lee and Lam 2001, Figelman 2004, Berkelaar et al. 2006). Chow and Kritzman (2001) are one of the first to propose the term Risk Budgeting as a way to allocate a risk measure into various categories, being these: asset classes, portfolio managers or any other segmentation. Blitz and Hottinga (2001) and Molenkamp (2004) extend this notion and state a model where an optimum Tracking Error allocation can be made on different assets or families, giving no more than a certain amount of risk to each one.

Other authors have focused on decomposing risk into some underlying financial factors. Sharpe (2002) states that by expressing the return of individual assets as a linear function of factors, the problem of estimating risk and correlations on an asset-by-asset basis is reduced in a significant way. More recent studies have also used this approach (Menchero and Poduri 2008, Balduzzi and Robotti 2010, Bender et al. 2010, Schaefer and Behrens 2011) but they are based on stocks portfolios and hedge funds, leaving aside the fixed-income market, which is one of the biggest worldwide in terms of volume.

Leading financial firms have also explored this approach for different kinds of portfolios. Page (2010) suggests that "risk factor correlations tend to be more robust against regime shifts than asset classes correlations", Werley (2011) proposes a factor model for global portfolios arguing that "factors reflect exposure behind the scenes and attempt to separate background investment forces from genuine investment skills", Junod (2012) stands that diversification by risk factors is likely to replace the conventional asset classes approach because "allocating capital across assets represents superficial diversification if payoffs are exposed to the same set of risk factors"[1].

Current studies of underlying factors on the fixed-income market have focused mainly on reducing pricing errors, forecasting future bond returns or yield spreads and reporting factor sensitivities for different countries (Collin Dufresne et al. 2001, Chen et al. 2005, Cochrane and Piazzesi 2005, Ammann et al. 2007, Dunis and Morrison 2007). Although they have not focused on risk decomposition, they are helpful for highlighting the main trends regarding which factors to use.

The first approach for defining relevant risk factors was proposed by Litterman and Scheinkman (1991) and Litterman et al. (1991), who used PCA[2] to define a set of yield factors for bonds. Although they are non-observable by construction, subsequent studies discovered important correlations between these factors and observed changes in yield curves[3], which allows their estimation from market observations (Ang and Piazzesi 2003, Diebold and Li 2006). Another important step in this matter was made by Kahn (1991) and Ferson and Harvey (1993), who indicate that if the objective is to explain bond returns, the first difference of these observable factors should be used.

[1]Other relevant corporate and institutional articles regarding this are Cheyette (2002) and Shaefer and Behrens (2011).

[2]PCA stands for Principal Components Analysis.

[3]Mainly shifts on the level, slope and curvature of the yield curve.

The second approach was introduced by Fama and French (1993). These authors proposed explaining bond returns with a factor that captures unexpected changes in bond returns[4], a firm's default probability factor[5], another one accounting for stocks market movements[6] and two factors related to firms' fundamentals[7].

The third most influencing trend uses indices from stocks and bonds as factors[8] and was proposed by Blake et al. (1993) and Elton et al. (1995).

When using a factor model, the use of a constant CAPM and other time-invariant models for stocks has been criticized because they lack the ability to capture changes in market volatility and factors co-movements (Wang 2003, Andersen et al. 2006), which can result in an inaccurate estimation of risk or market premium (Shah and Moonis 2003, Fama and French 2006). In the bond sector, factor approaches for studying the movements of the yield curve have also found that time varying coefficients allow for a better return and spread predictability, but their focus is only on reporting the dynamics of these coefficients (Brandt and Diebold 2006, Pozzi and Wolswijk 2008, Aβmann and Boysein-Hogrefe 2012). In order to model these time variations, one of the preferred approaches

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[4]The TERM factor is defined as a long-term bond return from the current month minus the one-month treasury bill rate from the previous month.

[5]The DEFAULT factor is defined as the return of a long-term corporate bonds portfolio minus a long-term government bonds portfolio, both representing the market.

[6]The MARKET factor is built as the cap-weighted return of every stock in the market, minus the risk-free rate.

[7]The first is the Small-Minus-Big factor (SMB), defined as the return of companies with low market capitalization, minus the return of companies with high market capitalization. This captures the performance of small companies compared with big firms. The second is the High-minus-Low factor (HML), defined as the return of companies with high Book-to-Market ratio (undervalued firms), minus the return of the ones with low ratio (overvalued).

[8]These authors use a Global Bond Index, which is a value-weighted index of corporate and government bonds; a Mortgage backed Securities Index, a High-Yield Index which helps to explain corporate bonds and Short-Term, Mid-Term and Long-Term Indices for corporate and government bonds.

is the Kalman Filter[9], as it mixes historic information with a specified dynamic and has proven to perform better than other methods (Faff et al. 2000, Jostova and Philipov 2005, Megner and Bulla 2008, Adrian and Franzoni 2009, Huang and Hueng 2009, Das and Ghoshal 2010).

In this Thesis, we present a dynamic model that represents fixed-income portfolio risk in terms of underlying risk factors, which can be successfully applied under challenging market conditions, commonly found in emerging economies. We define a step-by-step methodology that includes how to define the relevant risk factors as well as how to back-test its accuracy through Value-at-Risk estimations.

Our approach uses a set of observable variables that correlate with principal components (Litterman and Scheinkman 1991, Litterman et al. 1991) and another set that aims for explaining corporate bonds returns (Fama and French 1993).

Risk sensitivities to these factors are defined as time-varying and calibrated using an extended Kalman Filter method. The dynamic chosen for the betas is the Random Walk (Mergner and Bulla 2008) and as a benchmark, we estimated the static version of the model, in which betas are not time-varying.

We implement the model in a challenging emerging market like the Chilean economy, since its lack of liquidity, strong dependence on foreign demand, high volume growth rate and other singularities, expose it to large volatility and factor sensitivities changes, all of which are events suitable for using dynamic coefficients.

[9] In these models, the state variables are the betas or coefficients relating the returns with factors, which are time-varying.

This Thesis is organized as follows. Section "Model and Methodology" describes both in detail. Section "Application in the Chilean Market" presents an application of the methodology including data and results. The last section concludes.

2. MODEL AND METHODOLOGY

In this section, we present the complete methodology including the model for decomposing risk into factors, the necessary steps for defining a set of factors, the application of the Kalman Filter for obtaining the beta time series and the validation of the model through Value-at-Risk Back-Testing.

2.1 Portfolio Representation through Indices

Having chosen a bond portfolio as object of study, we start by selecting fixed-income market indices representing the main asset families in this portfolio[10]. Every instrument in which a position is hold must be mapped into an index of similar characteristics, constructed with similar securities and with available historical information. This step is very common among portfolio managers, as it is also the first step of risk decomposition into asset families. It is usually performed by using a heuristic or econometric analysis.

To illustrate how this mapping is done, assume a manager has a portfolio of N' instruments like the following:

[10]These indices are commonly available in most markets.

$$\mathrm{P}' = (p_1, p_2, \dots, p_{N'}) \tag{2.1}$$

Where p_i represents instrument i and the position on the instruments on time t are $w_{1t}, w_{2t}, \dots, w_{N't}$. Thus, the portfolio return r'_{pt}, is represented as a linear function of the return r'_{it} of every instrument on time t as:

$$r'_{pt} = \sum_{i=1}^{N'} w_{it} r'_{it} \tag{2.2}$$

Portfolio mapping into indices consists on assuming that the portfolio is formed by N market indices I, where N is less or equal to N':

$$P = (I_1, I_2, \dots, I_N) \tag{2.3}$$

Where every instrument p_i has similar characteristics to the instruments considered in I_{it}. In this way, the proxy for the portfolio return r_{pt} is constructed using the return r_{it} of every index, in such a way that r_{pt} is similar to r'_{pt}:

$$r_{pt} = \sum_{i=1}^{N} w_{it} r_{it} \tag{2.4}$$

2.2 Risk Decomposition Model

Once a portfolio representation through indices is achieved, the next step is to define a model that decomposes risk in underlying factors.

Following Sharpe (2002), a linear relation between asset classes and factors can be assumed. As a result, the decomposition of each index in M factors is represented by:

$$r_{it} - r_{ft} = \alpha_i + \sum_{k=1}^{M} \beta_{ik} f_{kt} + \epsilon_{it} \tag{2.5}$$

Where r_{it} is the index return from equation (2.4), β_{ik} is the sensitivity of the index i to the factor k and f_{kt} is the value of the time series of factor k on time t. Generally, a factor approach is used to explain the excess return over the risk free rate, represented here by r_{ft}. Replacing equation (2.5) on equation (2.4) we obtain the factor decomposition of the portfolio:

$$r_{pt} = \sum_{i=1}^{N} w_{it} \left(\alpha_i + \sum_{k=1}^{M} \beta_{ik} f_{kt} + \epsilon_{it} + r_{ft} \right) \tag{2.6}$$

Rearranging this and using the fact that $\sum_{i=1}^{N} w_{it} = 1$, the resulting equation is:

$$r_{pt} = \sum_{i=1}^{N} w_{it} \alpha_i + \sum_{k=1}^{M} \left(\sum_{i=1}^{N} w_{it} \beta_{ik} \right) f_{kt} + \sum_{i=1}^{N} w_{it} \epsilon_{it} + r_{ft} \tag{2.7}$$

In this last equation, $\beta_{Pkt} = \sum_{i=1}^{N} w_{it} \beta_{ik}$ can be seen as the sensitivity of the portfolio to factor k. By construction, it depends both on investment decisions w_{it} made by the portfolio manager and the sensitivities of the indices to the factors β_{ik}.

This notion is central for understanding how risk is separated between those two influences.

Defining $\alpha_{Pt} = \sum_{i=1}^{N} w_{it}\alpha_i$ and $\epsilon_{Pt} = \sum_{i=1}^{N} w_{it}\epsilon_{it}$ the portfolio decomposition can be written as:

$$r_{pt} = \alpha_{Pi} + \sum_{k=1}^{M} \beta_{Pkt} f_{kt} + \epsilon_{Pt} + r_{ft} \tag{2.8}$$

which is analogue to equation (2.4) but incorporating the factor decomposition.

Once the portfolio is decomposed and the important variables are expressed, the next step is to decompose a risk measure of the portfolio into the contribution to risk of every factor. In our case, the risk measure utilized is the portfolio Value-at-Risk, which, following Litterman (1996), can be decomposed in the following way:

$$\begin{aligned} VaR(\beta, w) = \sum_{k=1}^{M} \frac{\delta VaR(\beta, w)}{\delta \beta_{Pkt}} \beta_{Pkt} + \frac{\delta VaR(\beta, w)}{\delta \alpha_{Pt}} \alpha_{Pt} \\ + \frac{\delta VaR(\beta, w)}{\delta \epsilon_{Pt}} \epsilon_{Pt} + \frac{\delta VaR(\beta, w)}{\delta r_{ft}} r_{ft} \end{aligned} \tag{2.9}$$

Where:

$$MCR = \frac{\delta VaR(\beta, w)}{\delta \beta_{Pkt}} \tag{2.10}$$

is defined as the Marginal Contribution to Risk of factor k, which represents the sensitivity of the VaR to a small change in β_{Pkt}, and

$$CR = \frac{\delta VaR(\beta, w)}{\delta \beta_{Pkt}} \beta_{Pkt} \tag{2.11}$$

is the current Contribution to Risk of factor k to the portfolio.

This decomposition is what allows portfolio managers to understand the risk of their portfolios in a certain period. As it can be seen, the MCR and the CR depend now on investment decisions and also in market movements. As a consequence, when deciding the proportions to invest, the exact change of the portfolio VaR will be known.

2.3 Determining the Factor Set

Once the model for decomposing risk is set, the next step is to define a core group of factors explaining most of the volatility of the selected indices. For this, we first define certain variables proposed in the literature and then test them empirically using a Backward Elimination procedure.

To set the candidate variables, we start by using an observable version of the Litterman-Scheinkman factors as the core drivers of the model, since they reflect movements in the yield curve of interest rates, which are a proxy of the state of the market and the macroeconomic environment of an economy.

To define them, we follow the literature that uses principal components as factors[11]. First, a set of market indices that differ from the ones used for portfolio mapping shall be chosen, each one including government bonds. We call these indices

[11]The most relevant studies proposing this methodology for financial purposes include Litterman and Scheinkman (1991), Litterman et al. (199), Singh (1997) and De Jong et al. (2001).

"Duration Indices", as each one of them is obtained using bonds with the same duration. Second, we perform PCA on these indices and calculate their loadings[12]. As reported in Litterman and Scheinkman (1991), the first three principal components are enough for explaining most of the return series variances (on average, the first one explains 89.5%, the second one accounts for 8.5% and the third, 2.0%), however, a verification of this is important for validating the results up to this phase.

To achieve the observable version of these factors, once the principal component factors are obtained, we must relate them with observable time series constructed from rate movements. Following Ang and Piazzesi (2003) and Diebold and Li (200), who study the relation of principal components with observable time series, a close correlation between two series must be found. This last step is critical as factors must be observable and have economic interpretation in order to be used by practitioners.

To construct observable time series from the yield curve, we follow Kahn (1991), Ferson and Harvey (1993), Singleton (2000) and Diebold and Li (2006), and obtain the three factors as:

$$Level_t = y_{mid\ t} \tag{2.12}$$

$$Slope_t = y_{long\ t} - y_{short\ t} \tag{2.13}$$

[12]For a detailed explanation on principal components analysis, please refer to Anderson (1984) and Pearson (2002).

$$Curvature_t = y_{long\ t} - 2y_{mid\ t} + y_{short\ t} \tag{2.14}$$

where $y_{short\ t}$, $y_{mid\ t}$ and $y_{long\ t}$ are a short-term, a mid-term and a long-term maturity yield time series respectively, obtained from the local yield curve. It is important to highlight that, depending on the market, the maturities of these series can vary significantly and every factor may be constructed with independent series[13].

In addition, if the objective is to study bond returns as it is in this Thesis, the first difference of these factors should be computed using the following definitions:

$$\Delta\, Level_t = y_{mid\ t} - y_{mid\ (t-1)} \tag{2.15}$$

$$\Delta\, Slope_t = \left(y_{long\ t} - y_{short\ t}\right) - \left(y_{long\ (t-1)} - y_{short\ (t-1)}\right) \tag{2.16}$$

$$\begin{aligned} \Delta Curvature_t &= \left(y_{long\ t} - 2y_{mid\ t} + y_{short\ t}\right) \\ &\quad - \left(y_{long\ (t-1)} - 2y_{mid\ (t-1)} + y_{short\ (t-1)}\right) \end{aligned} \tag{2.17}$$

The next step is to study the correlation of these series with the principal components factors, because only if there is a high correlation between them, they can be assumed to be observable versions of the Litterman-Scheinkman Factors[14].

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[13]For example, y_{midt} used to compute $Level_t$ does not need to be the same as the one used to define $Curvature_t$.

[14]As a comparison, Diebold and Li (2006) find correlations greater than 0.97 for these series.

Once the core factors are set, regression models for different asset families may be proposed. As an example, we define one model for government bonds and another one for corporate bonds. The government bond's model is:

$$r_{it} = \alpha_i + \sum_{k=1}^{3} \beta_{ik} PFC_{kt} + r_{ft} + \epsilon_{it} \qquad (2.18)$$

Where $PCF's$[15] are the observable time series highly correlated with principal components factors.

The corporate bond's model is:

$$r_{it} = \alpha_i + \sum_{k=1}^{3} \beta_{ik} PFC_{kt} + \sum_{l=1}^{4} \beta_{il} FF_{lt} + r_{ft} + \epsilon_{it} \qquad (2.19)$$

Where $FF's$ are some of the Fama-French Factors (Fama and French 1993), which aim to explain corporate bond's returns. These factors are called $Default$, $Market$, SMB and HML, since they are related to firms default probability and stock market movements, elements not incorporated in $PCF's$ Factors.

As every market may differ on the important factors affecting their instrument returns, a factor filtering based on Backward Elimination must be performed previous to stating the final model for every index.

As a consequence, the model can be summarized as follows:

[15] PCF stands for Principal Components Factors.

$$r_{it} = \alpha_i + \sum_{k=1}^{K_i} \beta_{ik} f_{kt} + r_{ft} + \epsilon_{it} \tag{2.20}$$

Where f_{kt} are the factors remaining after the Backward Elimination and K_i is the number of factors remaining for index i.

2.4 Dynamic Model through Time-Varying Coefficients

Once the factors have been specified for every index, the last regression of the Backward Elimination defines a set of static coefficients α_i's and β_{ik}'s, which relate each index with a factor. This set of coefficients conform the static model, which serves as a benchmark when measuring the performance of the dynamic version.

The time variation of the coefficients is achieved by setting a stochastic model estimated using the Kalman Filter. In this method, the state variables represented in this Thesis by the α_i's and β_{ik}'s, are allowed to follow a specific random process over time and the parameters of these process are estimated using historical information.

We chose this method because it allows for a direct estimation of the time-varying variables through a numerical method, updating the variables as new information appears, which gives an important advantage in comparison to techniques that estimate coefficients through a volatility calculation.

The Kalman Filter method has two important parts. The first one is called Signal or Measurement equation and relates the independent variables with the dependent one, in a very similar way than a Linear regression does. The second part can be a group of equations which are known as State or Transition equations. They describe how the state variables evolve over time, specifying the dynamic and setting the parameters to estimate.

Estimations with this method result in a constant update of the state variables over time, where on each step, only the information previous to that step is considered. This implies that at initial time periods the variables are unstable as there is scarce information available, gaining accuracy in subsequent iterations[16].

In a general way, the method can be summarized as follows:

$$y_t = Z_t x_t + d_t + \epsilon_t \tag{2.21}$$

$$x_t = T x_{(t-1)} + c_t + \eta_t \tag{2.22}$$

where equation (2.21) and (2.22) are the Signal and State equations respectively, y_t is the portfolio return, x_t are the state variables (in our case, the factor sensitivities), Z_t is the matrix of time-dependent coefficients (the factors in our approach), matrix T is the transition matrix that contains the parameters defining the dynamic of state variables, d_t and c_t are auxiliary vectors that can be known or not and ϵ_t and η_t are Gaussian errors, with no correlation at any lag.

The variables to be estimated are the transition matrix, the elements of the auxiliary vectors[17], the coefficients x_t and the variance of the errors.

In our approach, we use the Kalman Filter estimation for every index separately. This is because each one has its own sensitivities to different factors, which will

[16] For a detailed explanation on this methodology, please refer to Harvey [1989].

[17] The estimation of these vectors will depend on the dynamic specified, as is shown later on this section.

depend on the individual characteristics of the index. The Signal equation corresponds to the linear relation between the index and the factors as in equation (2.20), but in this case all the α's and β's are time-varying. Thus this equation is:

$$r_{it} = \alpha_{it} + \sum_{k=1}^{K_i} \beta_{ikt} f_{kt} + r_{ft} + \epsilon_{it} \tag{2.23}$$

As a consequence, a time series for each α_i and β_{ik} will be the result of this estimation. These series are the output that explains how the factor sensitivity of every index changes to account for movements in market volatility and factor correlations.

Although several different dynamics are proposed for using as random processes for the state variables, we decided to include a widely used dynamic in related literature, the Random Walk (Bhar 2000, Megner and Bulla 2008, Das and Ghoshal 2010).

In this approach, the new value of variables is a combination of its previous value plus a Gaussian error η_{ikt}. This gives certain freedom of movement to the variables, depending on the estimated volatility of the errors in the equation. The State equation for this dynamic is the following:

$$\beta_{ik(t)} = \beta_{ik(t-1)} + \eta_{ik(t)} \tag{2.24}$$

This specification implicitly means that the coefficients are estimated as unobservable variables.

2.5 Validation through Back-Testing for Indices

The first step of the validation is to confirm that the risk of every index is correctly accounted for by the factor set. In this Thesis, we propose using a Value-at-Risk Back-Testing procedure for every index independently, as it provides several advantages. First, a Value-at-Risk approach verifies that the total risk of the index is correctly included in the model and not just an active risk relative to a benchmark, as is the case when using other measures like Tracking Error. Second, studying every index separately gives much more transparency than analyzing the whole portfolio at once, since it eliminates the influence that the proportion invested on every index has on the initial portfolio risk and any zero-sum effect that could arise between returns of different indices. Third, the Value-at-Risk is a widely known methodology validated and used currently by the U.S. banking system, which gives a solid empirical background for its selection.

Only after executing a Back-Test for every index, we continue to test the whole portfolio, to verify that the total risk is also correctly considered.

The mathematical details and step-by-step procedure of the indices and portfolio validation is presented in Appendix A.

3. APPLICATION IN THE CHILEAN MARKET

In this section, we perform an application of the methodology proposed for the Chilean bond market.

3.1 Data

As explained before in this Thesis, the Chilean market was chosen to test the proposed model as it is an emerging economy and, as such, it has a series of features that make it suitable for understanding portfolio risk from a factor perspective, along with capturing shifting sensitivities using time-varying coefficients.

In this market, the main financial institution is the Chilean Central Bank, which is in charge of the monetary policy, controlling inflation and debt issuance. This institution is independent from the government.

Fixed-payment securities are classified into two categories. The financial intermediation market, which includes instruments with maturity of less than a year[18], and the fixed-income market, compounded by all other instruments, being these mainly central bank and government bonds[19], corporate bonds[20], pension bonds and mortgage securities. In this application, we focus on studying the factors affecting the risk of bonds, including the ones issued by the central bank, the government and private firms, since they represent the majority of the market in terms of volume.

[18]This includes central bank short-maturity bonds, banks deposits and corporate commercial effects.
[19]Although the central bank and government are independent institutions, their bonds have usually the same rate and are modeled together.
[20]These bonds belong to three categories: bonds issued by the banking system, firms debt and securitized bonds.

The portfolio we chose contains every government bond available and corporate bonds with a risk classification of A- or upper, reaching a 95% of total bonds volume. As bonds represent in turn a 93% of the Fixed-Income market (Figure 3-1), we are considering a portfolio that accounts for the 88.3% of total Fixed-Income

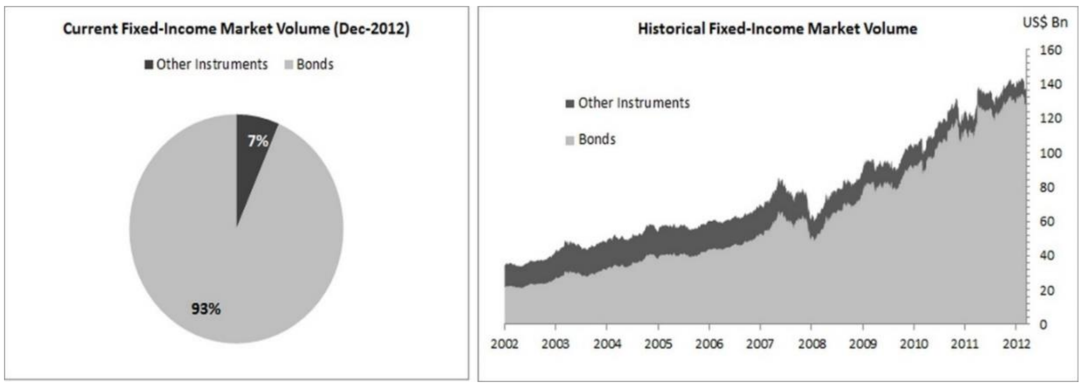

Figure 3-1: Current and historical fixed-income volume distribution in the Chilean market

volume.

This construction has two advantages. First, although any other portfolio can be selected, choosing a portfolio with an important variety of instruments allows for testing the risk model on a greater set of asset families. Second, as the portfolio accounts for most of the market, the set of factors defined will be relevant for a great part of the debt sector and not just a subset.

Specifically, we chose a set of 24 mutually exclusive market indices from Riskamerica, which is the local institution in charge of providing the market value for fixed-income securities and also provides other services like indices, being many of them published in Bloomberg.

Government and central bank bonds[21] are divided in 12 of these indices, according to their duration, currency and type. The other 12 indices represent corporate bonds, being differentiated according to their duration and risk classification. These Indices and their construction process are specified in Appendix B.

In order to obtain the Litterman-Scheinkman factors, we used a series of market bond indices of different durations, along with empirical observations of the local yield curve. The indices were obtained from Riskamerica and their description can be found in Appendix C. The yield curve's empirical observations were obtained from Riskamerica's fixed-income valuations database, which is the official system used in the Chilean market. Their description can also be found in Appendix C[22].

To define the Fama-French Factors, we used a series of indices from Bloomberg and from Riskamerica, along with observations of the local three-month treasury bill from the Riskamerica's valuation system as the risk-free rate. The details of this information is presented in Appendix D.

As the validation of the application in the Chilean market was achieved using a Value-at-Risk Back-Testing approach, we took in consideration the common practices on this matter, which are to use an Out of Sample range with daily data and at least one year of observations. Thus, we decided to use daily returns and rates observations and to set an Out-of-Sample range of three years.

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[21]As government and central bank bonds are modeled jointly, indices representing these bonds include both types. As a consequence, we decided to refer to them simply as government bonds.

[22]The process explaining how the factors are obtained from this information is described in the next section.

Since the 2008-09 financial crisis was between the dates under study, in order to define the exact dates for the In-Sample and the Out-of-Sample periods, we considered what is concluded in many post-crisis studies regarding Value-at-Risk measures, including papers published after the Asian and the Sub Prime crises. Pownall and Koedijk (1999) state that in periods of turmoil, the additional downside risk is not captured in Value-at-Risk methods and thus proposes other ways to manage risk during crises. Bao et al. (2006) study how Value-at-Risk behaves in five emerging markets during the Asian crisis and concludes that it performs reasonably well before and after the crisis, but not during it. Norling and Selling (2010) add that including the Sub Prime crisis in an In-Sample period, results in a model that is exceeded less than the optimal percentage of times, and that including it in the Out-of-Sample range leads to a model that is exceeded more than that percentage. Moreover, McAleer et al. (2010) state that calm periods and crises must be studied using different Value-at-Risk models for each moment.

Following these notions and with the objective of using a single Value-at-Risk model[23], we decided to leave the crisis aside, without including it in the In-Sample nor in the Out-of-Sample periods.

Figure 3-2 shows the daily return of the Chilean bond market and its 2-months volatility. As shown in this Figure, there are specific moments in which the

[23]Although we encourage further development ofour methodology in the direction of analyzing what happens during periods of turmoil, it is not the scope of this thesis.

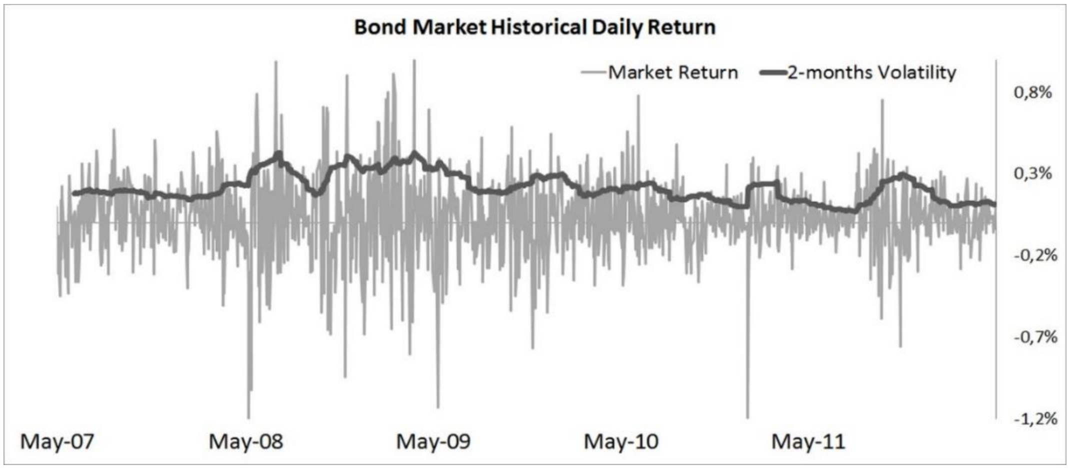

Figure 3-2: Bond market historical daily return and its 2-month volatility

volatility jumps and stays in a higher level than the rest of the period. With this, we defined a 1-year crisis period that goes from May the 1st, 2008 to May the 1st, 2009. Since the oldest observation available is from January the 2nd, 2004[24], we left a 6-month period from that date so the Kalman Filter could gain stability. Thus, the In-Sample range was set from July the 1st, 2004 to April the 30th, 2008 (959 business day observations).

As we aimed to have three years in the Out-of-Sample period, it was defined from May the 4th, 2009 to April the 30th, 2012 (753 business day observations).

3.2 Portfolio Mapping

The first step of our methodology is to map the portfolio into the set of selected indices. In order to do this, the percentage of the portfolio invested on every index is needed. As the portfolio we chose mimics most of the bond sector, to define the

[24]That is the first business day of that year.

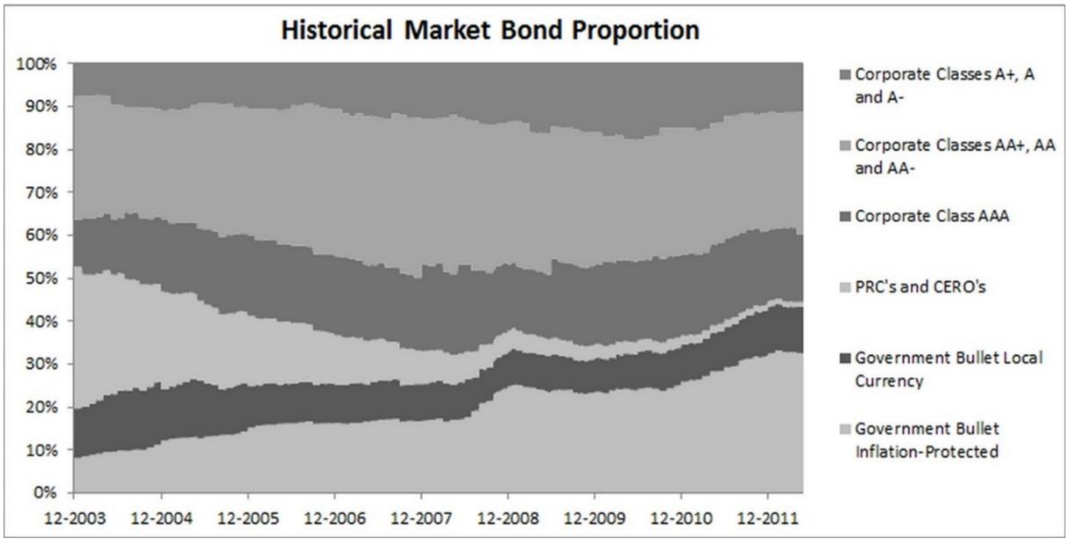

Figure 3-3: Historical market proportion of main bonds classification

percentage of our portfolio invested on every index we simply used the market proportions of these indices[25].

As the proportion of different kinds of bonds changes through time, each w_{it} from equation (2.4) is in fact a time series accounting for the proportion of every index i in the market, on time t. Figure 3-3 shows the historical proportions of these indices. This means that the returns of our portfolio are affected over time by the return of every index being part of it, as well as for the shifting proportions invested on every index.

3.3 Factor Selection and Construction

As the majority of the Chilean bonds are issued in an inflation-protected currency called “Unidad de Fomento” or “UF”, we decided to define two sets of Litterman-

[25]As an example, suppose that 23% of the bonds we account for can be classified in “Index A”. Then, the percentage of our portfolio in “Index A” is 23%.

Scheinkman Factors, one for inflation-protected bonds and the other for local currency bonds. To compute these factors, we conducted principal components analysis for the indices shown in Appendix C to obtain the inflation-protected factors and the local currency factors.

In line with related literature, we report in Table 3-1 that the first three components are enough to explain most of bonds volatility. We also find that, for inflation-protected and local currency bonds, these three components affect the returns as level, slope and curvature shifts respectively.

As shown in Figure 3-4, the first component affects bonds from different durations in a similar way. The second one affects short bonds with a different direction than

Table 3-1: Variance contribution of inflation protected and local currency principal components

Component Type	Component Order	Variance Contribution	Cumulative Variance Contribution
Inflation-Protected	**1st**	**71.22%**	**71.22%**
Inflation-Protected	**2nd**	**17.29%**	**88.51%**
Inflation-Protected	**3rd**	**3.88%**	**92.40%**
Inflation-Protected	4th	2.30%	94.70%
Inflation-Protected	5th	1.71%	96.41%
Inflation-Protected	6th	1.03%	97.44%
Inflation-Protected	7th	0.95%	98.39%
Inflation-Protected	8th	0.61%	99.00%
Inflation-Protected	9th	0.50%	99.50%
Inflation-Protected	10th	0.34%	99.84%
Inflation-Protected	11th	0.16%	100%
Local Currency	**1st**	**74.98%**	**74.98%**
Local Currency	**2nd**	**13.75%**	**88.73%**
Local Currency	**3rd**	**5.75%**	**94.48%**
Local Currency	4th	3.47%	97.95%
Local Currency	5th	0.66%	98.61%
Local Currency	6th	0.58%	99.19%
Local Currency	7th	0.52%	99.72%
Local Currency	8th	0.28%	100%

long bonds, as a slope shift does. The third component has a positive influence on short and long bonds but a negative one on mid-duration bonds, resembling curvature shifts.

Once the time series of the principal components were obtained, we used historical information from the yield curve also presented in Appendix C, to define several time series representing shifts in the level, slope and curvature of the interest rate using equations (2.15), (2.16) and (2.17). After this, we looked for high correlations between any of these series and the previously obtained principal components.

For the inflation-protected bonds, we found a level and a slope series highly correlated with the first two principal components. The same happened with the local currency bonds (Table 3-2). As a contrast, no strong correlation between any of the series representing curvature shifts and the third principal components was found for any type of bond. Thus, we decided to include in the model only the level and slope variables (these results are detailed in Appendix E).

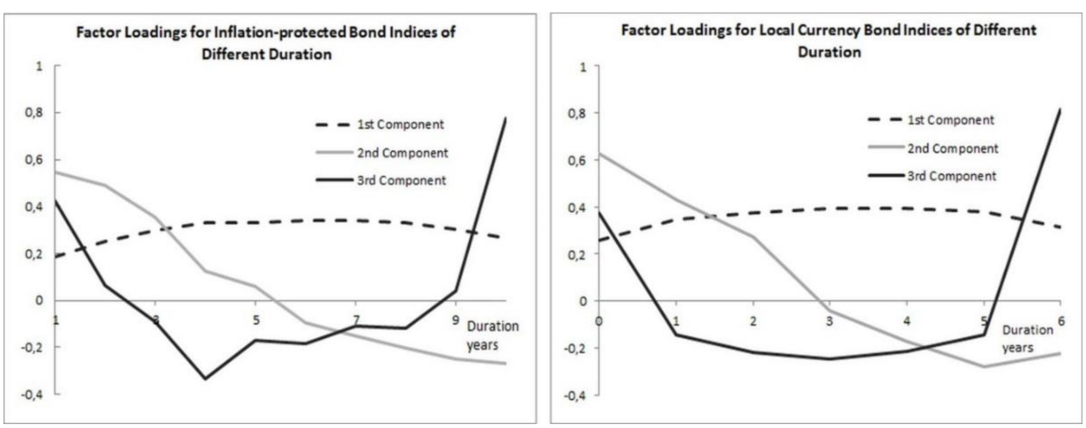

Figure 3-4: Effects of principal components on bond indices of different durations.

For the inflation-protected bonds, we found a level factor represented by shifts in the 7-year real yield curve and a slope factor formed by shifts in the 10-year yield minus the 2-year yield. In the case of local currency bonds, the level factor corresponds to shifts in the 4-year nominal yield curve and the slope factor is formed by changes in the difference of the 10-year and the 1-year yields. Main results are summarized in Table 3-2 and further details are in Appendix E.

To explain movements in the returns of corporate bonds, we defined local Fama-French Factors using data detailed in Appendix D. The $default$ factor was defined as the daily return of corporate bonds of a 10 or more years duration, minus the

Table 3-2: Observable Variables from yield curves' movements and their correlation with principal components.

Variable Name	Definition	Principal Component	Correlation with PC
Real 7 years	$\Delta(7y)_t$	1st Real PC	−0.92
Real 10 years - 2 years	$\Delta(10y-2y)_t$	2nd Real PC	0.79
Nominal 4 years	$\Delta(4y)_t$	1st Nominal PC	−0.97
Nominal 10 years - 1 year	$\Delta(10y-1y)_t$	2nd Nominal PC	0.85

return of government bonds of the same duration interval, which are considered long-term bonds for the local market. The $market$ factor was defined as the daily return of the IGPA Index, which is a cap-weighted index including all available stocks, minus the risk-free rate, computed as the nominal three-month rate from the previous month[26].

[26]This rate is originally annualized so it has to be converted into daily units for its proper use.

To define the SMB and the HML factors, we followed the steps introduced in Fama and French (1993). Every December in the time series, we separate the available stocks into two groups (Big and Small) using their market capitalization information and selecting the median market-cap of the market as a threshold. In that month, we also classify the stocks according to their Book-to-Market ratio into three groups: Low for the lowest 30%, Medium for the Next 40% and High for the highest 30%. In this way, a stocks are classified into 6 portfolios, Small and Low, Small and Medium, Small and High, Big and Low, Big and Medium and Big and High. Then, for the whole following year, the SMB factor is computed as the daily average return of all stocks classified as Small, minus the average daily return of all the Big stocks. In the same way, the HML factor is computed as the average daily return of the High stocks minus the Low stocks.

One last aspect to consider is that the returns of the bond indices from Appendix B are all expressed in local currency. As consequence, in the case of the inflation-protected indices, it is not enough to use the previously defined factors to explain their returns. This is because part of the return is by construction related to inflation and these factors, being generated from real yield curves and market movements, do not account for that portion. Taking this into consideration, we decided to incorporate an inflation factor only for the inflation-protected indices, represented by the daily return of the Bloomberg CLF Index[27]. Once the initial model is set for every type of index (they are summarized in Table 3-3), the next step is to carry out a Backward Elimination procedure for each one of them, in order to discard non-relevant factors empirically. In addition, this procedure also outputs the factor β's or coefficients that define the static benchmark against which the time-varying

Table 3-3: Factors of the initial regression models.

Index Type	Level	Slope	Fama-French	Others
Gov. Inflation-Protected	$\Delta(7y)_t$	$\Delta(10y-2y)_t$	-	$Inflation_t$
Gov. Local Currency	$\Delta(4y)_t$	$\Delta(10y-1y)_t$	-	-
Corp. Inflation-Protected	$\Delta(7y)_t$	$\Delta(10y-2y)_t$	$Default_t$ $Market_t$ SMB_t HML_t	$Inflation_t$

model is tested. Table 3-4 summarizes the results once the Backward Elimination was achieved. Further details are presented in Appendix F.

As shown in the Table, the level factor has always a negative influence in bond returns (in other words every β is negative for the level factor). This is in line with the fact that an upper shift in the yield curve brings down the price of bonds and, therefore, affects their return negatively. Another effect that validates the obtained sensitivities is that the effect of this factor is greater in longer duration bonds. This is explained by considering that the duration is the sensitivity of bonds to yield changes, thus, longer durations mean greater β's.

[27]This index represents the value of the Chilean inflation-protected currency (UF) in terms of the local currency, that is, CLP/UF. The daily return of this index is a proxy for the daily inflation.

Table 3-4: Linear regression results after backward elimination procedure.

Index Name	Alpha	Level	Slope	Default	Market	SMB	HML	Inflation	R^2 adj.
Gov. Bullet	2.22×10^{-3}	-510.52	327.92	-	-	-	-	0.99	0.69
UF 0-3	*-2.21	*-12.60	*13.62					*13.13	
Gov. Bullet	-4.47×10^{-3}	-1216.90	113.48	-	-	-	-	0.89	0.65
UF 4-6	*-1.17	*-31.39	*3.57					*6.64	
Gov. Bullet	5.25×10^{-3}	-1753.29	-	-	-	-	-	0.88	0.55
UF 7-9	*-0.08	*-20.59						*3.65	
Gov. Bullet	1.71×10^{-3}	-2700.63	-352.34	-	-	-	-	1.40	0.52
UF 10+	*-0.19	*-18.02	*-3.37					*3.42	
Gov. PRC	-4.95×10^{-3}	-113.68	385.65	-	-	-	-	0.91	0.52
0-1	*-1.71	*-4.28	*13.79					*12.16	
Gov. PRC	-6.11×10^{-3}	-555.65	383.79	-	-	-	-	0.98	0.81
2-3	*-3.23	*-33.05	*26.39					*15.78	
Gov. PRC	-4.65×10^{-3}	-1077.48	198.28	-	-	-	-	0.87	0.81
4-5	*-1.90	*-32.12	*9.28					*9.11	
Gov. ZERO	1.01×10^{-2}	-373.05	35.15	-	-	-	-	-	0.69
0-3	*3.45	*-11.98	*18.09						
Gov. ZERO	-1.77×10^{-3}	-1295.96	164.50	-	-	-	-	0.93	0.66
4-6	*-0.37	*-29.91	*6.35					*6.62	
Gov. ZERO	-5.15×10^{-3}	-1947.35	-94.35	-	-	-	-	1.11	0.66
7-9	*-1.12	*-33.65	*-2.73					*5.21	
Gov. Bullet	2.72×10^{-3}	-279.07	31.03	-	-	-	-	-	0.54
CLP 0-3	*2.98	*-15.55	*2.71						
Gov. Bullet	2.52×10^{-3}	-1282.45	-509.46	-	-	-	-	-	0.78
CLP 4+	*1.50	*-34.36	*-21.84						
Corp. AAA	-5.06×10^{-3}	-394.66	383.22	0.02	-	-	-	1.07	0.64
UF 0-3	*-1.76	*-13.35	*21.19	*2.42				*11.10	
Corp. AAA	2.99×10^{-3}	-1287.45	119.96	-	-	-	-	1.16	0.39
UF 4-6	*0.42	*-15.08	*2.58						
Corp. AAA	1.31×10^{-2}	-1729.47	-	-	-	-	-	-	0.52
UF 7-9	*1.84	*-24.91							
Corp. AAA	2.06×10^{-3}	-2746.30	-464.42	0.07	-	-	-	0.99	0.78
UF 10+	*0.27	*-28.43	*-6.29	*2.34				*3.64	
Corp. AAC	-3.17×10^{-3}	-446.91	350.89	-	-	-	-	1.11	0.83
UF 0-3	*-1.20	*-15.99	*29.31					*21.02	
Corp. AAC	1.08×10^{-3}	-1069.92	192.56	-	-	-	-	1.04	0.88
UF 4-6	*0.48	*-41.72	*16.71					*17.54	
Corp. AAC	0.02	-1834.19	-139.49	-	-	-	-	-	0.86
UF 7-9	*5.88	*-49.39	*-5.53						
Corp. AAC	4.90×10^{-3}	-2475.30	-390.37	0.09	-	-	-	1.28	0.78
UF 10+	*0.74	*-31.03	*-6.27	*3.23				*5.42	
Corp. AC	-4.42×10^{-3}	-376.04	319.44	-	-	-	-	1.07	0.65
UF 0-3	*-1.53	*-13.15	*20.34					*16.49	
Corp. AC	4.77×10^{-3}	-1019.13	139.91	-	-	-	-	0.85	0.66
UF 4-6	*1.24	*-33.85	*7.29					*5.71	
Corp. AC	9.22×10^{-3}	-1749.55	-77.99	-	0.01	-	-	0.99	0.75
UF 7-9	*1.90	*-41.53	*-2.72		*2.53			*6.32	
Corp. AC	0.03	-2062.86	-	-	-	-	-	-	0.50
UF 10+	*0.43	*-7.11							

* values with this sign represent the t-value of the coefficient above.

The slope factor affects short bonds positively and long bonds negatively. This is explained by the fact that a movement in the slope is usually compounded by two movements. If the slope moves upwards, then the long yield moved upwards and/or the short yield moved downwards. In most cases, this causes a drop in prices for longer bonds but raises prices for shorter bonds, implying a negative return for the first and a positive return for the latter.

The bond market in general seems to be minimally influenced by shifts in firm's default probability and stock market moves, since factors $default$ and $market$ are statistically significant in only 4 of the 12 corporate indices. Furthermore, factors SMB and HML resulted to be non-significant for all corporate indices under study. This means that Chilean corporate bonds are not influenced by the relative performance of small companies compared with big firms (movements captured by the SMB factor), nor the investment opportunities observable in the difference between undervalued and overvalued firms (captured by the HML factor).

As the effect of inflation is completely considered in the return of inflation-protected[28] bonds, the sensitivity to the inflation factor should be close to 1 in all cases. Even though this is the case, there are certain variations in some indices. This could happen because, as the inflation factor is constructed as a linear projection between the UF value of a certain month and the next one, there can be intra-month

[28] It is important to recall that the return of all indices is expressed in local currency, thus, part of the return of all inflation-protected Indices is explained by inflation.

Table 3-5: Descriptive statistics of the final factors set.

Factor Name	Definition	Average (bp)	Std. Dev. (bp)	Min. (bp)	Max. (bp)
Level (Real)	$\Delta(7y)_t$	2.35×10^{-4}	0.014	-0.061	0.108
Slope (Real)	$\Delta(10y-2y)_t$	-5.26×10^{-4}	0.025	-0.135	0.225
Level (Nominal)	$\Delta(4y)_t$	6.62×10^{-4}	0.016	-0.111	0.086
Slope (Nominal)	$\Delta(10y-1y)_t$	-1.42×10^{-3}	0.029	-0.179	0.197
Default	$Rcorp_t - Rgov_t$	0.117	37.130	-268.453	189.093
Market	$Rmkt_t - r_{f_t}$	5.326	77.565	-606.838	482.276
Inflation	$Inflation_t$	1.273	2.278	-8.077	21.906

Table 3-6: Correlation coefficients of the final factors set

Factor Name	Level (Real)	Slope (Real)	Level (Nominal)	Slope (Nominal)	Default	Market	Inflation
Level (Real)	1	0.075	0.280	0.085	0.218	0.076	0.050
Slope (Real)	0.075	1	0.045	0.058	0.147	-0.001	-0.052
Level (Nominal)	0.280	0.045	1	-0.168	0.095	0.065	-0.039
Slope (Nominal)	0.085	0.058	-0.168	1	0.054	0.024	0.019
Default	0.218	0.147	0.095	0.054	1	0.062	0.001
Market	0.076	-0.001	0.065	0.024	0.062	1	-0.065
Inflation	0.050	-0.052	-0.039	0.019	0.001	-0.065	1

speculation related to the UF value that may affect daily bond returns, generating a mismatch with the projected daily inflation.

Considering these results for the Backward Elimination procedure, only the SMB and the HML factors are discarded from the study, not being considered for the dynamic version of the model.

3.4 Factor Statistics

Tables 3-5 and 3-6 summarize the main descriptive statistics and the correlation coefficients respectively, for the final set of selected factors. The factors representing movements in fixed-income yield curves, that is, the level and slope factors have averages that are very close to zero base points, as would be expected in daily time series representing fixed-income instruments. However, the minimum and the maximum values differ considerably from the average. The $default$ factor has a positive but small average, which shows that in general, corporate bonds were just slightly more profitable than government bonds during the period under study. Something similar occurs with the $market$ factor, evidencing that the market stocks portfolio was just slightly more profitable than the risk-free rate, on average. In the case of the $inflation$ factor, the standard deviation is very low compared with the average, which is expected as it is a daily measure of a variable that usually does not vary in a considerable way in the Chilean market, as the central bank is particularly focused on maintaining it under control.

Having a look at the correlation between the factors, it can be seen that no important relation exists between them, which is a key point in validating the robustness of the regression models used on this Thesis. The fact that they can be considered as independent factors means that a high number of different risk influences are being considered and that the R^2 measures are more reliable. For further visibility on the factors, their histograms are presented in Appendix G.

3.5 Dynamic Risk Sensitivities

Once the final set of factors for every index was achieved, the dynamic version of the model was estimated using the Kalman Filter method. As input to the model, we set the static value of the β's to be the initial value of the estimation which allows to compare if there are any structural changes being detected by the dynamic model that are not accounted for in the static version. Table 3-7 summarizes the final values of the sensitivities for every index and compares them with the initial (static) values. To illustrate examples of the time movement of the β's, Figures 3-5 and 3-6 illustrate the time series of the In-Sample and Out-of-Sample ranges of the indices Government Bullet UF 10+ and Corporate AAA UF 10+, respectively. The time series of the sensitivities for every index are presented in Appendix H.

Table 3-7 shows that the final values of the dynamic sensitivities of the level factor are close to the static value for most government bonds. Figure 3-5 and the exhibits in Appendix H support this observation for the final values in the In-Sample as well as in the Out-of-Sample ranges.

A more notorious difference for government bonds occur in the slope and inflation factors, where the time variation of the β's capture significant differences with the static model for both the In-Sample and the Out-of-Sample ranges. In the case of corporate bonds, their sensitivities to the level, slope and default factors are very distant from the static value. In Appendix H, a significant shift on this factor is appreciated in the Out-of-Sample range for most of the indices. This would support the idea that, as a result from the financial crisis, there were shifts in general

Table 3-7: Dynamic risk sensitivities (final values)

Index Name	Model	Alpha	Level	Slope	Default	Market	Inflation
Gov. Bullet	Dynamic	-1.70×10^{-2}	-534.45	286.17	-	-	1.12
UF 0-3	Static	2.22×10^{-3}	-510.52	327.92	-	-	0.99
Gov. Bullet	Dynamic	-1.72×10^{-2}	-1142.91	103.89	-	-	0.98
UF 4-6	Static	-4.47×10^{-3}	-1216.90	113.48	-	-	0.89
Gov. Bullet	Dynamic	6.84×10^{-2}	-1590.03	37.16	-	-	0.78
UF 7-9	Static	5.25×10^{-3}	-1753.29	-	-	-	0.88
Gov. Bullet	Dynamic	-7.68×10^{-2}	-2461.35	-233.04	-	-	0.46
UF 10+	Static	1.71×10^{-3}	-2700.63	-352.34	-	-	1.40
Gov. PRC	Dynamic	8.37×10^{-2}	-236.33	307.31	-	-	1.26
0-1	Static	-4.95×10^{-3}	-113.68	385.65	-	-	0.91
Gov. PRC	Dynamic	1.91×10^{-2}	-597.10	317.08	-	-	1.24
2-3	Static	-6.11×10^{-3}	-555.65	383.79	-	-	0.98
Gov. PRC	Dynamic	-5.96×10^{-3}	-1077.23	157.47	-	-	1.05
4-5	Static	-4.65×10^{-3}	-1077.48	198.28	-	-	0.87
Gov. ZERO	Dynamic	6.01×10^{-2}	-390.16	344.26	-	-	1.30
0-3	Static	1.01×10^{-2}	-373.05	35.15	-	-	-
Gov. ZERO	Dynamic	-2.67×10^{-2}	-1238.67	191.79	-	-	1.48
4-6	Static	-1.77×10^{-3}	-1295.96	164.50	-	-	0.93
Gov. ZERO	Dynamic	-6.36×10^{-2}	-1881.23	-46.19	-	-	1.51
7-9	Static	-5.15×10^{-3}	-1947.35	-94.35	-	-	1.11
Gov. Bullet	Dynamic	2.47×10^{-2}	-323.41	64.22	-	-	-
CLP 0-3	Static	2.72×10^{-3}	-279.07	31.03	-	-	-
Gov. Bullet	Dynamic	2.52×10^{-3}	-1282.45	-509.46	-	-	-
CLP 4+	Static	2.87×10^{-2}	-1173.84	-344.79	-	-	-
Corp. AAA	Dynamic	8.77×10^{-2}	-466.37	266.59	0.15	-4.17×10^{-4}	1.15
UF 0-3	Static	-5.06×10^{-3}	-394.66	383.22	0.02	-	1.07
Corp. AAA	Dynamic	5.68×10^{-2}	-1077.11	178.61	0.12	-3.94×10^{-3}	0.39
UF 4-6	Static	2.99×10^{-3}	-1287.45	119.96	-	-	1.16
Corp. AAA	Dynamic	-2.29×10^{-2}	-1438.25	30.68	0.21	4.48×10^{-3}	0.96
UF 7-9	Static	1.31×10^{-2}	-1729.47	-	-	-	-
Corp. AAA	Dynamic	-7.71×10^{-2}	-2142.06	-202.10	0.41	4.94×10^{-3}	1.29
UF 10+	Static	2.06×10^{-3}	-2746.30	-464.42	0.07	-	0.99
Corp. AAC	Dynamic	7.97×10^{-2}	-438.891	279.78	2.85×10^{-2}	-5.66×10^{-4}	1.15
UF 0-3	Static	-3.17×10^{-3}	-446.91	350.89	-	-	1.11
Corp. AAC	Dynamic	7.40×10^{-2}	-917.58	185.49	0.11	4.79×10^{-3}	1.14
UF 4-6	Static	1.08×10^{-3}	-1069.92	192.56	-	-	1.04
Corp. AAC	Dynamic	-1.89×10^{-2}	-1391.66	29.03	0.20	7.67×10^{-3}	1.20
UF 7-9	Static	0.02	-1834.19	-139.49	-	-	-
Corp. AAC	Dynamic	-1.23×10^{-2}	-1958.83	-125.76	0.35	7.62×10^{-4}	1.23
UF 10+	Static	4.90×10^{-3}	-2475.30	-390.37	0.09	-	1.28
Corp. AC	Dynamic	0.13	-443.17	243.37	5.21×10^{-2}	-6.27×10^{-4}	1.05
UF 0-3	Static	-4.42×10^{-3}	-376.04	319.44	-	-	1.07
Corp. AC	Dynamic	0.15	-887.98	149.55	0.14	9.73×10^{-3}	0.91
UF 4-6	Static	4.77×10^{-3}	-1019.13	139.91	-	-	0.85
Corp. AC	Dynamic	6.22×10^{-2}	-1437.92	14.07	0.20	7.62×10^{-3}	1.06
UF 7-9	Static	9.22×10^{-3}	-1749.55	-77.99	-	0.01	0.99
Corp. AC	Dynamic	1.77×10^{-2}	-1893.14	-55.11	0.32	8.13×10^{-3}	1.17
UF 10+	Static	0.03	-2062.86	-	-	-	-

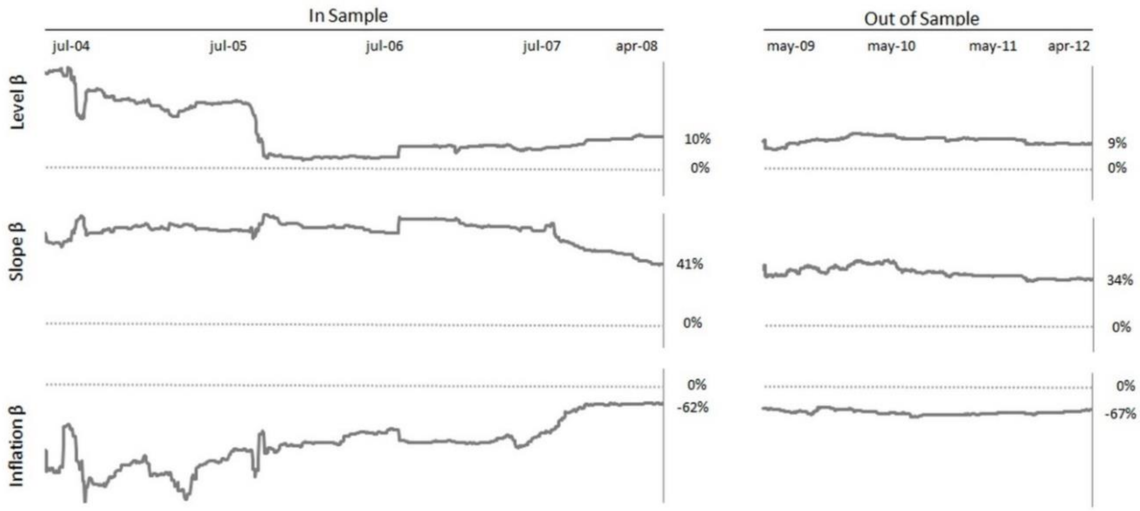

Figure 3-5: Time series of factors sensitivities for the index “Government Bullet UF 10+”.

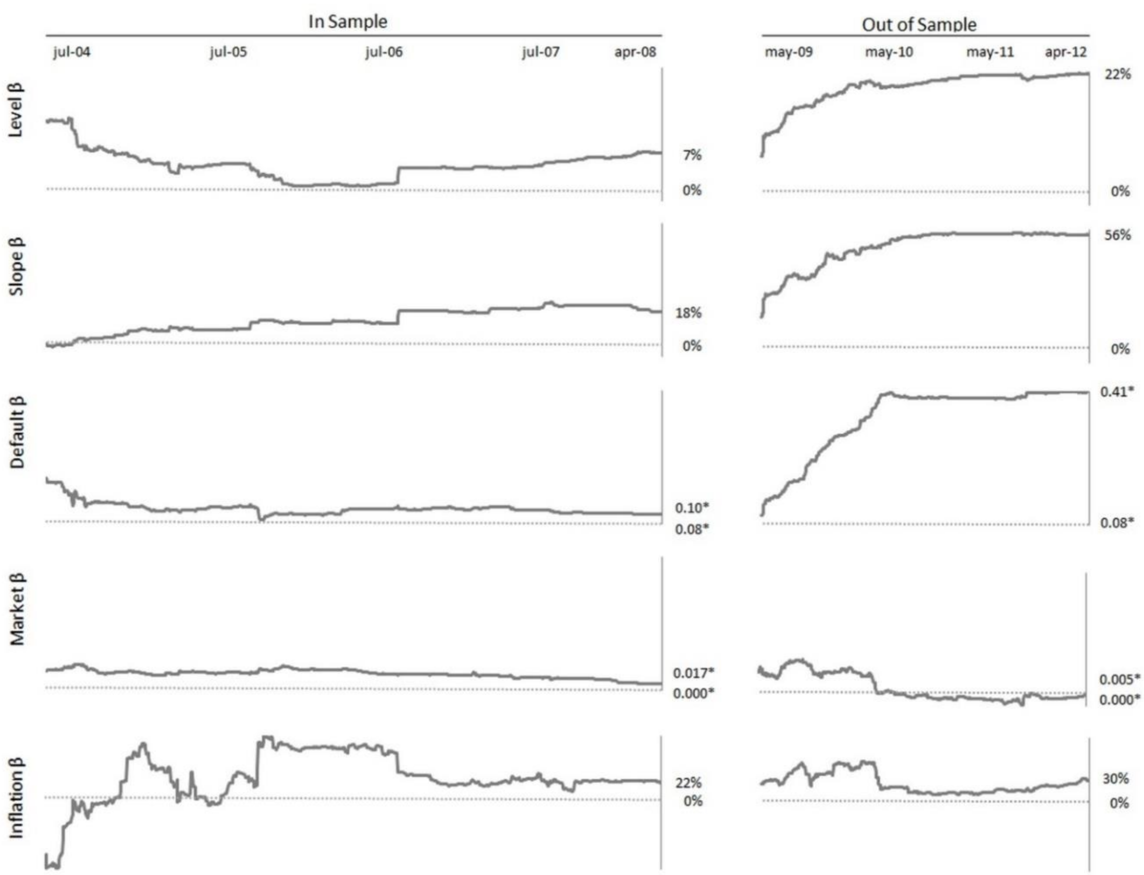

Figure 3-6: Time series of factors sensitivities for the index “Corporate AAA UF 10+”.

economic conditions, which were captured by the dynamic model through adaptations in the sensitivities.

These adaptations are particularly important from an economic perspective for the $default$ factor. After the crisis, the significant shift of the $default$ sensitivities affects all corporate indices (details can be found in the exhibits of Appendix H). This indicates that firms' default probability started to be considered as a relevant macroeconomic factor only after the crisis, influencing corporate bonds prices and returns from that moment. The importance of the factor did not vanish after a while and continues until nowadays, which means that the market learned from the crisis. Something similar but at a smaller scale happens for the $market$ sensitivities. Figure 3-6 shows that before the crisis, the β's were very similar to the static value (zero in most cases). In the Out-of-Sample range though, a small adaptation takes place and in the case of the index illustrated as example, this factor's influence in the returns changes from positive to negative. A similar small adaptation can also be distinguished for the rest of corporate indices.

Regarding the $inflation$ factor, comparing the exhibits of this factor in Appendix H, two observations can be highlighted. The first one is that for government bonds, there is a more significant variation of the dynamic β's relative to the static ones, compared to the case of corporate bonds. The second one is that, for government bonds, the sensitivities for shorter bonds tend to be positive, while the sensitivities for bonds with longer durations is negative[29].

Despite of their differences, both the dynamic and the static models have some characteristics in common, which also confirm their correctness from an interpretative perspective. First, level β's are negative for all indices and greater in absolute terms as the indices get longer in duration, confirming that bond prices and returns decrease as yields increase. Second, slope sensitivities for short bonds are positive and for long bond, negative.

3.6 Validation and Back-Testing

The following step after obtaining the dynamic specification of the sensitivities is to validate the model from a risk perspective. As was mentioned in previous sections, we decided to implement a Value-at-Risk Back-Testing procedure which is explained in detail in Appendix A.

For every index, the procedure works by calculating N returns, where N is the number of In-Sample days, using only the right section of Equation (2.20), this is, considering only the factor and β's information for every index. With those returns, the Value-at-Risk for the first Out-of-Sample day is computed. This is repeated until the Value-at-Risk is calculated for every Out-of-Sample day. On each iteration, the In-Sample window is moved one day forward, maintaining N days. Thus, the procedure is a Back-Test with historical Value-at-Risk calculation and a rolling

[29]This can be seen clearly in the case of government bullet UF indices.

window, with the variation of using factors' and β's information instead of index returns.

We performed the Back-Testing for the static and the dynamic versions of the model. The only difference in specification between them is that in the static case the β's used are all the same and in the dynamic version every day has a specific sensitivity for each factor.

To validate the Back-Testing, the Value-at-Risk calculations are compared with the realized returns of the indices in the Out-of-Sample period. As a consequence, this procedure can be seen as an assessment of whether the representation of indices through factors and sensitivities implies a significant loss of information or not.

The criteria used for determining if the indices pass or not the Back-Test is the Kupiec Test (Kupiec 1995), which consists on determining the percentage of days that the realized returns violate the computed Value-at-Risk, relative to the total Out-of-Sample days. If this percentage is statistically equal to the Value-at-Risk confidence level[30], then the index is correctly expressed by the factor decomposition.

The statistic calculated in the Kupiec Test is called the Proportion of Failure (POF) and it distributes x^2 with one degree of freedom. This means that if the Value-at-Risk is calculated with a 5% confidence, the POF must be smaller than 3.84.

[30]Tipically 5% or 1%.

Table 3-8: Results of the Back-Test for every Index.

Index Name	Model	Num. of Failures	Failure %	POF Statistic	Kupiec Test Status
Gov. Bullet	Dynamic	42	5.57%	0.51	Passed
UF 0-3	Static	42	5.57%	0.51	Passed
Gov. Bullet	Dynamic	29	3.85%	2.26	Passed
UF 4-6	Static	45	5.97%	1.42	Passed
Gov. Bullet	Dynamic	29	3.85%	2.26	Passed
UF 7-9	Static	62	8.23%	13.98	**Failed**
Gov. Bullet	Dynamic	39	5.17%	0.05	Passed
UF 10+	Static	60	7.96%	11.92	**Failed**
Gov. PRC	Dynamic	44	5.84%	1.07	Passed
0-1	Static	52	6.90%	5.17	**Failed**
Gov. PRC	Dynamic	44	5.84%	1.07	Passed
2-3	Static	54	7.17%	6.62	**Failed**
Gov. PRC	Dynamic	45	5.97%	1.42	Passed
4-5	Static	54	7.17%	6.62	**Failed**
Gov. ZERO	Dynamic	42	5.57%	1.42	Passed
0-3	Static	76	10.09%	32.15	**Failed**
Gov. ZERO	Dynamic	40	5.31%	0.15	Passed
4-6	Static	54	7.17%	6.62	**Failed**
Gov. ZERO	Dynamic	41	5.44%	0.30	Passed
7-9	Static	47	6.24%	2.27	**Failed**
Gov. Bullet	Dynamic	86	11.42%	48.71	**Failed**
CLP 0-3	Static	127	16.86%	141.78	**Failed**
Gov. Bullet	Dynamic	43	5.71%	0.76	Passed
CLP 4+	Static	33	4.38%	0.62	Passed
Corp. AAA	Dynamic	44	5.84%	1.07	Passed
UF 0-3	Static	74	9.82%	29.18	**Failed**
Corp. AAA	Dynamic	36	4.78%	0.07	Passed
UF 4-6	Static	51	6.77%	4.50	**Failed**
Corp. AAA	Dynamic	35	4.64%	0.20	Passed
UF 7-9	Static	47	6.24%	2.27	Passed
Corp. AAA	Dynamic	46	6.10%	1.82	Passed
UF 10+	Static	51	6.77%	4.50	**Failed**
Corp. AAC	Dynamic	45	5.97%	1.42	Passed
UF 0-3	Static	62	8.23%	13.98	**Failed**
Corp. AAC	Dynamic	45	5.97%	1.42	Passed
UF 4-6	Static	62	8.23%	13.98	**Failed**
Corp. AAC	Dynamic	40	5.31%	0.15	Passed
UF 7-9	Static	41	5.44%	0.30	Passed
Corp. AAC	Dynamic	40	5.31%	0.15	Passed
UF 10+	Static	39	5.17%	0.05	Passed
Corp. AC	Dynamic	57	7.56%	9.10	**Failed**
UF 0-3	Static	100	13.28%	76.26	**Failed**
Corp. AC	Dynamic	52	6.90%	5.17	**Failed**
UF 4-6	Static	88	11.68%	52.35	**Failed**
Corp. AC	Dynamic	42	5.57%	0.51	Passed
UF 7-9	Static	60	7.96%	11.92	**Failed**
Corp. AC	Dynamic	39	5.17%	0.05	Passed
UF 10+	Static	55	7.30%	7.41	**Failed**

Table 3-8 summarizes the results of the Back-Tests for the dynamic and static versions of the model.

As can be seen in the Table, the static model has failures for most indices, while the dynamic model is successful in allowing risk management in almost every index. This means that the time-variation of the sensitivities is a very relevant issue in managing risk in a factor approach.

There are two kinds of failures in a Back-Testing procedure. The one where there is a lack of violations (less than the expected % level) and the one where there is an excess of them, each having different implications. The first one is the less expensive for the investor, as it results in a sub-optimal allocation of his assets, since it indicated that there is a greater amount of money under risk that what actually is. The second one is the most dangerous and does not allow for a proper risk management. It indicated that there is a smaller amount of assets under

risk than what there really is, being significant losses the most frequent consequence of using it.

In the static model, all the failures are due to an excess of Value-at-Risk violations (failures of the second kind). Thus, that model as it is, cannot be used to perform risk management through factors.

The dynamic model, on the contrary, allows for 21 of the 24 indices under study to pass the Kupiec test, which represents a 90.9% of success in terms of volume, relative to the total studied. As the β's change over time, the model adapts in two situations. In times where there is a great quantity of violations, it expands the Value-at-Risk estimation faster than the static model to stay close to the permitted failure percentage. On the contrary, when there are fewer violations, the estimation is contracted.

These results are considered to be positive as they allow for a decomposition of portfolio risk into relevant macroeconomic factors through the time-varying sensitivities obtained, while maintaining the ability of conducting a proper portfolio risk management with one of the most commonly used methodologies, the Value-at-Risk. This means that there is no loss of robustness in risk management when turning to decompose it into factors instead of asset classes.

The indices that fail represent the government bullet bonds in local currency (CLP), with a duration interval of 0 to 3 years, and the corporate inflation-protected bonds with durations of 0 to 3 years and 4 to 6 years. These indices have some characteristics in common, which allow for an explanation of their behavior.

The first one is a short to medium duration of bonds. This implies greater dependence on short rates which are generally more volatile, especially in emerging markets where there are few transactions and each one affects prices and returns considerably. The second one applies to corporate bonds and is related to their risk rank. As the corporate bonds indices failing are ranked as “A Consolidated”, that is, including classes A+, A and A-, there is an important volatility of their yield spreads, thus, there is greater volatility in their prices[31].

These considerations are important because, even though we would have preferred to account every index as a success for the dynamic model, it is not a surprise that these particular indices are the ones to fail.

To test our methodology in a real portfolio, we chose the market portfolio of government and corporate bonds. This portfolio was mapped into the indices used for the study by keeping the market proportions of these indices as their proportion in the portfolio.

The Value-at-Risk was calculated under the dynamic and the static models in the same way as for the indices, with the consideration of calculating the returns using Equation (2.8) instead of Equation (2.20), as the computation is performed on a portfolio composed by various indices. We also computed a Historical Value-at-Risk as a second benchmark, which is calculated using the historical realized returns of the indices, that is, using Equation (2.4) and not considering the decomposition of the indices into factors. This calculation is performed with a rolling window of the historical data, keeping the same amount of days into this window.

This benchmark is particularly important as it is a commonly used methodology when performing risk decomposition into asset classes. Thus, if the dynamic Value-

[31]Even though there are bonds with lower risk rank, they represent less than 5% of the market in terms of volume. This means that in practice, the categories A+, A and A- are the riskiest on the market.

Table 3-9: Results of the Back-Test for the market portfolio.

Model	POF Statistic	Kupiec Test Status	Extended Christoffersen Statistic	Extended Christoffersen Test Status
VaR from Dynamic Betas	0.077	Passed	5.053	Passed
VaR from Historic Returns	0.200	Passed	5.623	Passed

at-Risk is similar to the historical calculation, little information would have been lost from a risk perspective when decomposing indices returns into factors.

Table 3-9 reports the Kupiec Test statistics for the Dynamic and the Historic models. It also reports the Christoffersen Extended Test (Christoffersen 1998).

This test consists on determining in an integrated way, if a VaR estimation is correct, that is, if it has the expected proportion of failures, but also if VaR violations are independent from one another[32], which means they are not clustered in short time periods. It is based on the calculation of the Extended Christoffersen Statistic, which distributes x^2 with two degrees of freedom. This means that if for a certain portfolio the Value-at-Risk is performed with a 5% confidence, the test statistic calculated for that portfolio must be smaller than 5.99 in order to say that the VaR succeeded in the test.

As can be seen in the previous Table, both models pass the Kupiec Test, which means they are correct measures of the portfolio Value-at-Risk, even though, as we reported before, there were some indices not passing the test when studied alone. Both methodologies also pass the Christoffersen Extended Test, meaning that there

[32] Independence for this test means that the fact of having a VaR violation on time t does not affect the probability of having another violation on day $t + 1$.

are no violations clusters and that the model adapts correctly in high volatility periods.

This result implies that, by using the set of factors introduced in this study, risk management can be performed for the Chilean bond market when studying their

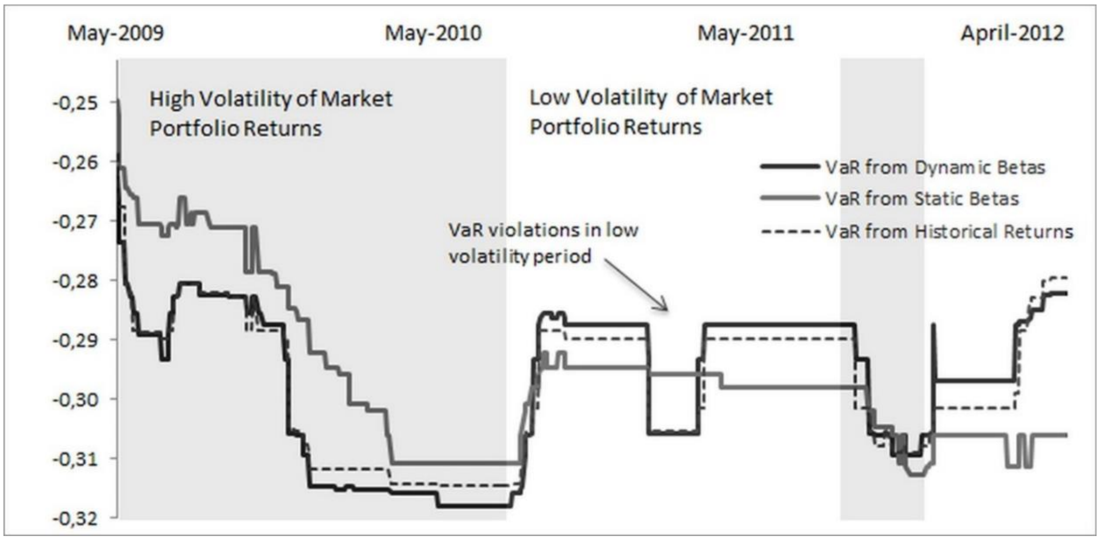

Figure 3-7: Value-at-Risk time series for the market portfolio.

behavior from a factor perspective, maintaining the possibility of decomposing risk into the marginal contribution of every factor.

Figure 3-7 shows the time series of the Value-at-Risk calculations from dynamic β's and the VaR from historical returns. The correlation between these two time series is 0.97, which contrasts with the correlation of the VaR from the static β's and the historical, which is only 0.65.

From May 2009 to May 2010, there is a high volatility period in the return of the market portfolio. In this period, both the dynamic and the historical VaR adapt and shift towards a higher estimation, while the static VaR is much slower to perform this change. During low volatility periods, the dynamic and the historical VaR are again faster in changing to a lower estimation than the static VaR. This is even more notorious in the last low volatility period, where the static VaR does not react at all. Between February and April 2010, although there is low volatility in portfolio returns, there are a few especially low returns which resulted into VaR violations for all models. Here, there was a rapid adaptation of the VaR estimation for the historical and the dynamic models, which is a particularly important example of the performance of these models against the static version.

4. CONCLUDING REMARKS

This thesis shows how to implement a factor-based risk model for fixed-income portfolios following recent trends now common for equity funds.

We propose decomposing fixed-income portfolio risk into a set of underlying factors and setting a dynamic model with time-varying factor loadings to be able to capture market shifts.

We define a five-step procedure to decompose risk into factors that include mapping the portfolio into indices, choosing an initial set of underlying factors, constructing a static and a dynamic factor model, and a back-testing procedure to perform model validation.

Results of applying this procedure in difficult market conditions like those of the Chilean low-liquidity market are very encouraging showing that a factor-based risk model for fixed-income portfolios may be a useful way to model risk.

APPENDICES

APPENDIX A. METHODOLOGY VALIDATION THROUGH VALUE-AT-RISK BACK-TESTING

Index Validation

In order to validate the factors for every index from a risk perspective, we propose using a Back-Testing approach for each one of them and verifying that, through these models, a correct measure of Value-at-Risk can be calculated. For this, we first choose a specific time period to represent the In-Sample range for the Back-Testing, which must include three or more years of daily observations for the factors and indices. This In-Sample interval is represented by:

$$T_{in\ sample} = (t-M, t-M+1, \dots, t-1) \tag{A.1}$$

Where M is the number of observations, $t-M$ is the first observation and $t-1$ is the last observation.

Then, we select an Out-of-Sample range, which should have a length of one or more years and is represented by:

$$T_{out\ of\ sample} = (t, t+1, \dots, t+S) \tag{A.2}$$

Where there are $S+1$ observations, being t the first one and $t+S$ the last one.

After this, we continue by Calculating a historic Value-at-Risk for every index i for day t of the Out-of-Sample range, using all the observations from the In-Sample range. That is, for day t and index i, we compute:

$$\Phi_{it} = \left(r_{i(t-M)}, r_{i(t-M+1)}, \dots, r_{i(t-m)}, \dots, \quad r_{i(t-2)}, r_{i(t-1)}\right) \tag{A.3}$$

Where M is the number of In-Sample observations and r_{t-m} is the return of the m^{th} previous day to day t.

The key aspect of this model is that every return is calculated with equation (2.23), and not with historical information from the index, incorporating in this way the time-varying coefficients to the risk measure. The returns are computed with contemporaneous information from the coefficients and the factors as:

$$r_{i(t-m)} = \alpha_{i(t-m)} + \sum_{k=1}^{K_i} \beta_{ik(t-m)} f_{k(t-m)} + r_{f(t-m)} \tag{A.4}$$

After Φ_{it} is computed, according to certain error percentage α^{*}[33], we calculate the VaR for index i and day t as:

$$VaR_{it} = \Phi|_{\alpha^{*}\%} \tag{A.5}$$

where $\Phi|_{\alpha^{*}\%}$ is the return in Φ_{it} that represents the α^{*} percentile.

After the VaR for day t is computed, the In-Sample window is moved one day to include the information for β's and factors from day $(t - M + 1)$ up to day t and the VaR for day $(t + 1)$ is computed. If the process continues, a time series of the VaR for every index is achieved. This time series include the VaR of every Out-of-Sample day:

$$VaR_i = \left(VaR_{i(t)}, VaR_{i(t+1)}, \dots, VaR_{i(t+S-1)}, VaR_{i(t+S)}\right) \tag{A.6}$$

[33] Typically, Back-Tests are performed with an error percentage of 5% or 1%.

Where t, $t+1$,…, $t+S-1$, $t+S$ are the days of the Out-of-Sample range.

Since the time series of VaR_i are risk measures obtained from the model we are proposing, by assessing the correctness of these VaR_i's, an evaluation of the entire methodology is achieved. Thus, the problem of validating our proposal is reduced to testing these risk measures.

For this, we follow Kupiek (1995) and Campbell (2005) and define a Hit Vector for every index as:

$$I_i = \left(I_{i(t)}, I_{i(t+1)}, \dots, I_{i(t+S-1)}, I_{i(t+S)}\right) \tag{A.7}$$

This vector informs whether a VaR violation has occurred for the Out-of-Sample range for index i according to:

$$I_{it} \begin{cases} 0 \ if \ VaR_{it} < r_{it} \\ 1 \ if \ VaR_{it} \geq r_{it} \end{cases} \tag{A.8}$$

Where r_{it} are the known historical returns of the index in the Out-of-Sample interval. This means that the computed VaR_{it}'s are benchmarked with the actual information of what happened with the return of the index for the Out-of-Sample days. If the documented return for certain day is a loss and it is greater than the computed VaR[34], then a violation has occurred and $I_{it} = 1$.

Once this vector is constructed for every index, the next step is to test if the percentage of VaR violations is statistically equal to the α^* defined. The notion behind

[34]Note that a loss greater than the VaRis represented by $VaR_{it} \geq r_{it}$ as these are both negative measures.

this is that the VaR calculations would be correct only if it fails the amount of times it was supposed to fail. So, if there is a greater or smaller percentage of VaR violations, the model will be rejected.

Kupiek [1995] defines a test statistic to evaluate the correctness of a VaR model, used nowadays by most of the banking systems worldwide, including the U.S. system (Campbell 2005). It consists of the following maximum likelihood statistic which is called the Proportion of Failures:

$$\Pi_i = 2Ln\left[\left(\frac{1-\frac{N_i}{S}}{1-\alpha^*}\right)^{S-N_i}\left(\frac{N_i}{S}\right)^{N_i}\right] \tag{A.9}$$

Where $N_i = \sum_{t=1}^{S} I_{it}$ is the number of VaR violations that occurred in the Out-of-Sample range for index i, S is the number of observations Out-of-Sample and α^* is the error percentage assumed.

The null hypothesis of this test statistic is that the VaR measure is correct. As a consequence, since $\Pi_i \sim X^2$ with one degree of freedom, for $\alpha^* = 5\%$ the null hypothesis is rejected if $\Pi_i > 3.84$ and for $\alpha^* = 1\%$, it is rejected if $\Pi_i > 5.99$.

By evaluating every test statistic Π_i, an evaluation of the model for every index from a risk perspective can be accomplished.

Portfolio Validation

In addition to the verification of the methodology for every index, we also incorporate a Back-Testing for the whole portfolio under study. For this, we follow a similar methodology, but we calculate measures for the portfolio instead of the indices.

We calculate the VaR for day t by first computing:

$$\Phi_{Pt} = \left(r_{P(t-M)},\ r_{P(t-M+1)}, \dots,\ r_{P(t-m)}, \dots,\quad r_{P(t-2)}, r_{P(t-1)}\right) \tag{A.10}$$

Where $r_{P(t-m)}$ is the return of the whole portfolio for day $t-m$, calculated based on a variation of Equations (2.7) and (2.8), which represent the portfolio return in terms of the factors and their sensitivities. These variations consist on incorporating the time dependence of the β's and α's and the resulting equations are:

$$r_{P(t-m)} = \sum_{i=1}^{N} w_{i(t-m)}\alpha_{i(t-m)} + \sum_{k-1}^{M}\left(\sum_{i=1}^{N} w_{i(t-m)}\beta_{ik(t-m)}\right) f_{k(t-m)} + \sum_{i=1}^{N} w_{i(t-m)} + r_{f(t-m)} \tag{A.11}$$

and:

$$r_{P(t-m)} = \alpha_{P(t-m)} + \sum_{k-1}^{M} \beta_{Pk(t-m)} f_{k(t-m)} + r_{f(t-m)} \quad \text{(A.12)}$$

Which are the variations of equations (2.7) and (2.8) respectively.

Once Φ_{Pt}'s are calculated for every Out-of-Sample day, the VaR time series for the whole portfolio can be computed in the same way as VaR_i in Equation (A.6) and it results in:

$$VaR_P = \left(VaR_{P(t)}, VaR_{P(t+1)}, \dots, VaR_{P(t+s)}, \quad VaR_{P(t+S-1)}, VaR_{P(t+S)}\right) \quad \text{(A.13)}$$

Where $VaR_{P(t+s)}$ is the portfolio VaR for day $t+s$.

The calculation of the Hit Vector and the test statistic for the portfolio, I_P and Π_P respectively, is achieved by re-defining Equations (A.8) and (A.9) as:

$$I_{Pt} \begin{cases} 0 \; if \; VaR_{Pt} < r_{Pt} \\ 1 \; if \; VaR_{Pt} \geq \; r_{Pt} \end{cases} \quad \text{(A.14)}$$

and

$$\Pi_P = 2Ln\left[\left(\frac{1-\frac{N_P}{S}}{1-\alpha^*}\right)^{S-N_P} \left(\frac{N_P}{S}\right)^{N_P}\right] \quad \text{(A.15)}$$

Where r_{Pt} is the reported return of the portfolio Out-of-Sample day t, $N_P = \sum_{t=1}^{S} I_{Pt}$ is the number of VaR violations occurred in the Out-of-Sample range and S is the number of Out-of-Sample observations.

Since Π_P also distributes X^2, the verification of the portfolio VaR is the same as the one for indices.

APPENDIX B. SELECTED CHILEAN BOND MARKET INDICES

The indices chosen are constructed, updated and managed by Riskamerica, a firm that provides financial services, market indices and official bond valuation to Chilean institutions.

Name	Ticker	Type	Issuer	Currency	Duration	Risk Class
Government Bullet UF 0-3	RACLGOB-B-UF-0-3	Bullet	Government Central Bank	Inflation Protected	0 to 3 years	-
Government Bullet UF 4-6	RACLGOB-B-UF-4-6	Bullet	Government Central Bank	Inflation Protected	4 to 6 years	-
Government Bullet UF 7-9	RACLGOB-B-UF-7-9	Bullet	Government Central Bank	Inflation Protected	7 to 9 years	-
Government Bullet UF 10+	RACLGOB-B-UF-10P	Bullet	Government Central Bank	Inflation Protected	10 years or more	-
Government Bullet CLP 0-3	RACLGOB-B-UF-0-3	Bullet	Government Central Bank	Chilean Peso	0 to 3 years	-
Government Bullet CLP 4+	RACLGOB-B-UF-4P	Bullet	Government Central Bank	Chilean Peso	4 years or more	-
Government PRC 0-1	RACLGOB-PRC-0-1	Amortized	Government Central Bank	Inflation Protected	0 to 1 years	-
Government PRC 2-3	RACLGOB-PRC-2-3	Amortized	Government Central Bank	Inflation Protected	2 to 3 years	-
Government PRC 4-6	RACLGOB-PRC-4-6	Amortized	Government Central Bank	Inflation Protected	4 to 6 years	-
Government ZERO 0-3	RACLGOB-ZERO-0-3	Zero-Coupon	Government Central Bank	Inflation Protected	0 to 3 years	-
Government ZERO 4-6	RACLGOB-ZERO-4-6	Zero-Coupon	Government Central Bank	Inflation Protected	4 to 6 years	-
Government ZERO 7-9	RACLGOB-ZERO-7-9	Zero-Coupon	Government Central Bank	Inflation Protected	7 to 9 years	-
Corporate AAA UF 0-3	RACLCO-AAA-UF-0-3	Mixed	Private Firms	Inflation Protected	0 to 3 years	AAA
Corporate AAA UF 4-6	RACLCO-AAA-UF-4-6	Mixed	Private Firms	Inflation Protected	4 to 6 years	AAA
Corporate AAA UF 7-9	RACLCO-AAA-UF-7-9	Mixed	Private Firms	Inflation Protected	7 to 9 years	AAA
Corporate AAA UF 10+	RACLCO-AAA-UF-10P	Mixed	Private Firms	Inflation Protected	10 years or more	AAA
Corporate AAC UF 0-3	RACLCO-AAC-UF-0-3	Mixed	Private Firms	Inflation Protected	0 to 3 years	AA-, AA, AA+
Corporate AAC UF 4-6	RACLCO-AAC-UF-4-6	Mixed	Private Firms	Inflation Protected	4 to 6 years	AA-, AA, AA+
Corporate AAC UF 7-9	RACLCO-AAC-UF-7-9	Mixed	Private Firms	Inflation Protected	7 to 9 years	AA-, AA, AA+
Corporate AAC UF 10+	RACLCO-AAC-UF-10P	Mixed	Private Firms	Inflation Protected	10 years or more	AA-, AA, AA+
Corporate AC UF 0-3	RACLCO-AC-UF-0-3	Mixed	Private Firms	Inflation Protected	0 to 3 years	A-, A, A+
Corporate AC UF 4-6	RACLCO-AC-UF-4-6	Mixed	Private Firms	Inflation Protected	4 to 6 years	A-, A, A+
Corporate AC UF 7-9	RACLCO-AC-UF-7-9	Mixed	Private Firms	Inflation Protected	7 to 9 years	A-, A, A+
Corporate AC UF 10+	RACLCO-AC-UF-10P	Mixed	Private Firms	Inflation Protected	10 years or more	A-, A, A+

Exhibit B.1: Chilean bond market indices.

These indices are built by evaluating, every month, which bonds should be kept, incorporated or eliminated from each index according to their Duration and Risk Classification. To obtain the price of the index every day, the daily price of every instrument is weighted using their proportion in the Central Value Deposit, which is an institution that keeps track of 98% of the instruments issued in the Chilean Market. This proportion is computed at the beginning of every month and is maintained until the next one. The Duration used to classify bonds corresponds to the Macaulay Duration and the Risk Classification is the lowest between the information published by the classification firms Fellet-Rate, FitchRatings and Humphreys.

APPENDIX C. BOND INDICES AND YIELD CURVE OBSERVATIONS FOR LITTERMAN-SCHEINKMAN FACTORS

The following indices are constructed including government Bonds' information from the Chilean market and are used to obtain principal components for the bond sector, in the process of obtaining the Litterman-Scheinkman factors. Indices in Exhibits C.1 are used to compute the principal components for inflation-protected bonds and the ones in Exhibit C.2; for bonds in local currency. Yield curves in Exhibit C.3 are used to obtain empirical definitions for level, slope and curvature shifts of the yield curve. Their correlation with the Principal Components is studied order to discover which empirical observation can be used as a factor.

Name	Ticker	Duration
Government CLP Duration 0	RACLGOB-CLP-D0	0 to 0.5 years
Government CLP Duration 1	RACLGOB-CLP-D1	0.5 to 1.5 years
Government CLP Duration 2	RACLGOB-CLP-D2	1.5 to 2.5 years
Government CLP Duration 3	RACLGOB-CLP-D3	2.5 to 3.5 years
Government CLP Duration 4	RACLGOB-CLP-D4	3.5 to 4.5 years
Government CLP Duration 5	RACLGOB-CLP-D5	4.5 to 5.5 years
Government CLP Duration 6	RACLGOB-CLP-D6	5.5 to 6.5 years
Government CLP Duration 7+	RACLGOB-CLP-D7P	6.5 years and more

Exhibit C.1: Inflation-protected government indices.

Name	Ticker	Duration
Government UF Duration 1	RACLGOB-UF-D1	0.5 to 1.5 years
Government UF Duration 2	RACLGOB-UF-D2	1.5 to 2.5 years
Government UF Duration 3	RACLGOB-UF-D3	2.5 to 3.5 years
Government UF Duration 4	RACLGOB-UF-D4	3.5 to 4.5 years
Government UF Duration 5	RACLGOB-UF-D5	4.5 to 5.5 years
Government UF Duration 6	RACLGOB-UF-D6	5.5 to 6.5 years
Government UF Duration 7	RACLGOB-UF-D7	6.5 to 7.5 years
Government UF Duration 8	RACLGOB-UF-D8	7.5 to 8.5 years
Government UF Duration 9	RACLGOB-UF-D9	8.5 to 9.5 years
Government UF Duration 10+	RACLGOB-UF-D10P	9.5 years and more

Exhibit C.2: Local currency government indices.

Curve Name	Type	Time Horizon
Zero Real 1 month	Inflation-Protected	1 month
Zero Real 3 months	Inflation-Protected	3 months
Zero Real 6 months	Inflation-Protected	6 months
Zero Real 1 year	Inflation-Protected	1 year
Zero Real 2 years	Inflation-Protected	2 years
Zero Real 3 years	Inflation-Protected	3 years
Zero Real 4 years	Inflation-Protected	4 years
Zero Real 5 years	Inflation-Protected	5 years
Zero Real 6 years	Inflation-Protected	6 years
Zero Real 7 years	Inflation-Protected	7 years
Zero Real 8 years	Inflation-Protected	8 years
Zero Real 9 years	Inflation-Protected	9 years
Zero Real 10 years	Inflation-Protected	10 years
Zero Real 15 years	Inflation-Protected	15 years
Zero Real 20 years	Inflation-Protected	20 years
Zero Nominal 1 month	Nominal	1 month
Zero Nominal 3 months	Nominal	3 months
Zero Nominal 6 months	Nominal	6 months
Zero Nominal 1 year	Nominal	1 year
Zero Nominal 2 years	Nominal	2 years
Zero Nominal 3 years	Nominal	3 years
Zero Nominal 4 years	Nominal	4 years
Zero Nominal 5 years	Nominal	5 years
Zero Nominal 6 years	Nominal	6 years
Zero Nominal 7 years	Nominal	7 years
Zero Nominal 8 years	Nominal	8 years
Zero Nominal 9 years	Nominal	9 years
Zero Nominal 10 years	Nominal	10 years
Zero Nominal 15 years	Nominal	15 years
Zero Nominal 20 years	Nominal	20 years

Exhibit C.3: Yield curve observations.

APPENDIX D. INDICES AND YIELD CURVE OBSERVATIONS FOR FAMA-FRENCH FACTORS

The following information was used to define empirical Fama-French Factors for the Chilean market.

Name	Type	Source	Factor
Government UF Duration 10+	Index	Riskamerica	Default
Corporate UF Duration 10+	Index	Riskamerica	Default
IGPA	Index	Bloomberg	Market
Zero Nominal 3 months	Yield Curve	Riskamerica	Market
Return of every Chilean stock	Indices	Riskamerica and Bloomberg	SMB and HML

Exhibit D.1: Time series for Fama-French factors.

APPENDIX E. CORRELATION COEFFICIENTS BETWEEN PRINCIPAL COMPONENTS AND VARIABLES CONSTRUCTED FROM YIELD CURVE OBSERVATIONS

The following Exhibits summarize the correlation coefficients between different observable variables defined from yield curve observations and the Principal Component aimed to be matched. Variables with maximum correlation are highlighted.

Variable Name	Definition	Source Curve Type	Correlation with 1st PC
Real 1 month	$\Delta(1month)_t$	Real	-0.12
Real 3 months	$\Delta(3months)_t$	Real	-0.14
Real 6 months	$\Delta(6months)_t$	Real	-0.25
Real 1 year	$\Delta(1y)_t$	Real	-0.49
Real 2 years	$\Delta(2y)_t$	Real	-0.62
Real 3 years	$\Delta(3y)_t$	Real	-0.75
Real 4 years	$\Delta(4y)_t$	Real	-0.83
Real 5 years	$\Delta(5y)_t$	Real	-0.88
Real 6 years	$\Delta(6y)_t$	Real	-0.91
Real 7 years	$\boldsymbol{\Delta(7y)_t}$	**Real**	**-0.92**
Real 8 years	$\Delta(8y)_t$	Real	-0.91
Real 9 years	$\Delta(9y)_t$	Real	-0.89
Real 10 years	$\Delta(10y)_t$	Real	-0.87
Real 15 years	$\Delta(15y)_t$	Real	-0.79
Real 20 years	$\Delta(20y)_t$	Real	-0.74

Exhibit E.1: Correlations of observable level shifts from real yield curves with the first principal component.

Variable Name	Definition	Source Curve Type	Correlation with 1st PC
Nominal 1 month	$\Delta(1month)_t$	Nominal	-0.46
Nominal 3 months	$\Delta(3months)_t$	Nominal	-0.64
Nominal 6 months	$\Delta(6months)_t$	Nominal	-0.75
Nominal 1 year	$\Delta(1y)_t$	Nominal	-0.83
Nominal 2 years	$\Delta(2y)_t$	Nominal	-0.91
Nominal 3 years	$\Delta(3y)_t$	Nominal	-0.96
Nominal 4 years	$\boldsymbol{\Delta(4y)_t}$	**Nominal**	**-0.97**
Nominal 5 years	$\Delta(5y)_t$	Nominal	-0.95
Nominal 6 years	$\Delta(6y)_t$	Nominal	-0.92
Nominal 7 years	$\Delta(7y)_t$	Nominal	-0.87
Nominal 8 years	$\Delta(8y)_t$	Nominal	-0.83
Nominal 9 years	$\Delta(9y)_t$	Nominal	-0.79
Nominal 10 years	$\Delta(10y)_t$	Nominal	-0.76
Nominal 15 years	$\Delta(15y)_t$	Nominal	-0.64
Nominal 20 years	$\Delta(20y)_t$	Nominal	-0.57

Exhibit E.2: Correlations of observable level shifts from nominal yield curves with the first principal component.

Variable Name	Definition	Source Curve Type	Correlation with 2nd PC
Real 20 years - 6 months	$\Delta(20y - 6months)_t$	Real	0.57
Real 20 years - 1 year	$\Delta(20y - 1y)_t$	Real	0.75
Real 20 years - 2 years	$\Delta(20y - 2y)_t$	Real	0.77
Real 20 years - 3 years	$\Delta(20y - 3y)_t$	Real	0.72
Real 15 years - 6 months	$\Delta(15y - 6months)_t$	Real	0.56
Real 15 years - 1 year	$\Delta(15y - 1y)_t$	Real	0.75
Real 15 years - 2 years	$\Delta(15y - 2y)_t$	Real	0.78
Real 15 years - 3 years	$\Delta(15y - 3y)_t$	Real	0.74
Real 10 years - 6 months	$\Delta(10y - 6months)_t$	Real	0.55
Real 10 years - 1 year	$\Delta(10y - 1y)_t$	Real	0.74
Real 10 years - 2 years	$\boldsymbol{\Delta(10y - 2y)_t}$	**Real**	**0.79**
Real 10 years - 3 years	$\Delta(10y - 3y)_t$	Real	0.75
Real 9 years - 6 months	$\Delta(9y - 6months)_t$	Real	0.54
Real 9 years - 1 year	$\Delta(9y - 1y)_t$	Real	0.73
Real 9 years - 2 years	$\Delta(9y - 2y)_t$	Real	0.78
Real 9 years - 3 years	$\Delta(9y - 3y)_t$	Real	0.75
Real 8 years - 6 months	$\Delta(8y - 6months)_t$	Real	0.53
Real 8 years - 1 year	$\Delta(8y - 1y)_t$	Real	0.73
Real 8 years - 2 years	$\Delta(8y - 2y)_t$	Real	0.78
Real 8 years - 3 years	$\Delta(8y - 3y)_t$	Real	0.75
Real 7 years - 6 months	$\Delta(7y - 6months)_t$	Real	0.52
Real 7 years - 1 year	$\Delta(7y - 1y)_t$	Real	0.72
Real 7 years - 2 years	$\Delta(7y - 2y)_t$	Real	0.78
Real 7 years - 3 years	$\Delta(7y - 3y)_t$	Real	0.75
Real 6 years - 6 months	$\Delta(6y - 6months)_t$	Real	0.50
Real 6 years - 1 year	$\Delta(6y - 1y)_t$	Real	0.70
Real 6 years - 2 years	$\Delta(6y - 2y)_t$	Real	0.77
Real 6 years - 3 years	$\Delta(6y - 3y)_t$	Real	0.76
Real 5 years - 6 months	$\Delta(5y - 6months)_t$	Real	0.48
Real 5 years - 1 year	$\Delta(5y - 1y)_t$	Real	0.68
Real 5 years - 2 years	$\Delta(5y - 2y)_t$	Real	0.76
Real 5 years - 3 years	$\Delta(5y - 3y)_t$	Real	0.76

Exhibit E.3: Correlations of observable slope shifts from real yield curves with the second principal component.

Variable Name	Definition	Source Curve Type	Correlation with 2nd PC
Nominal 20 years - 6 months	$\Delta(20y - 6months)_t$	Nominal	0.85
Nominal 20 years - 1 year	$\Delta(20y - 1y)_t$	Nominal	0.82
Nominal 20 years - 2 years	$\Delta(20y - 2y)_t$	Nominal	0.78
Nominal 20 years - 3 years	$\Delta(20y - 3y)_t$	Nominal	0.75
Nominal 15 years - 6 months	$\Delta(15y - 6months)_t$	Nominal	0.84
Nominal 15 years - 1 year	$\Delta(15y - 1y)_t$	Nominal	0.83
Nominal 15 years - 2 years	$\Delta(15y - 2y)_t$	Nominal	0.80
Nominal 15 years - 3 years	$\Delta(15y - 3y)_t$	Nominal	0.78
Nominal 10 years - 6 months	$\Delta(10y - 6months)_t$	Nominal	0.84
Nominal 10 years - 1 year	$\mathbf{\Delta(10y - 1y)_t}$	**Nominal**	**0.85**
Nominal 10 years - 2 years	$\Delta(10y - 2y)_t$	Nominal	0.81
Nominal 10 years - 3 years	$\Delta(10y - 3y)_t$	Nominal	0.80
Nominal 9 years - 6 months	$\Delta(9y - 6months)_t$	Nominal	0.84
Nominal 9 years - 1 year	$\Delta(9y - 1y)_t$	Nominal	0.85
Nominal 9 years - 2 years	$\Delta(9y - 2y)_t$	Nominal	0.81
Nominal 9 years - 3 years	$\Delta(9y - 3y)_t$	Nominal	0.80
Nominal 8 years - 6 months	$\Delta(8y - 6months)_t$	Nominal	0.83
Nominal 8 years - 1 year	$\Delta(8y - 1y)_t$	Nominal	0.84
Nominal 8 years - 2 years	$\Delta(8y - 2y)_t$	Nominal	0.82
Nominal 8 years - 3 years	$\Delta(8y - 3y)_t$	Nominal	0.81
Nominal 7 years - 6 months	$\Delta(7y - 6months)_t$	Nominal	0.84
Nominal 7 years - 1 year	$\Delta(7y - 1y)_t$	Nominal	0.85
Nominal 7 years - 2 years	$\Delta(7y - 2y)_t$	Nominal	0.82
Nominal 7 years - 3 years	$\Delta(7y - 3y)_t$	Nominal	0.81
Nominal 6 years - 6 months	$\Delta(6y - 6months)_t$	Nominal	0.85
Nominal 6 years - 1 year	$\Delta(6y - 1y)_t$	Nominal	0.84
Nominal 6 years - 2 years	$\Delta(6y - 2y)_t$	Nominal	0.82
Nominal 6 years - 3 years	$\Delta(6y - 3y)_t$	Nominal	0.81
Nominal 5 years - 6 months	$\Delta(5y - 6months)_t$	Nominal	0.83
Nominal 5 years - 1 year	$\Delta(5y - 1y)_t$	Nominal	0.84
Nominal 5 years - 2 years	$\Delta(5y - 2y)_t$	Nominal	0.82
Nominal 5 years - 3 years	$\Delta(5y - 3y)_t$	Nominal	0.82

Exhibit E.4: Correlations of observable slope shifts from nominal yield curves with the second principal component.

Variable Name	Definition	Source Curve Type	Correlation with 3rd PC
Real +20 years -2(15 years) + 10 years	$\Delta\left(+20y - 2*15y + 10y\right)_t$	Real	0.01
Real +15 years -2(10 years) + 5 years	$\Delta\left(+15y - 2*10y + 5y\right)_t$	Real	0.09
Real +20 years -2(10 years) + 1 year	$\Delta\left(+20y - 2*10y + 1y\right)_t$	Real	-0.18
Real +15 years -2(9 years) + 3 years	$\Delta\left(+15y - 2*9y + 3y\right)_t$	Real	0.03
Real +15 years -2(8 years) + 1 year	$\Delta\left(+15y - 2*8y + 1y\right)_t$	Real	-0.20
Real +10 years -2(7 years) + 4 year	$\Delta\left(+10y - 2*7y + 4y\right)_t$	Real	0.07
Real +9 years -2(7 years) + 5 year	$\Delta\left(+9y - 2*7y + 5y\right)_t$	Real	0.11
Real +10 years -2(6 years) + 2 years	$\Delta\left(+10y - 2*6y + 2y\right)_t$	Real	-0.16
Real +9 years -2(6 years) + 3 year	$\Delta\left(+9y - 2*6y + 3y\right)_t$	Real	-0.02
Real +8 years -2(6 years) + 4 years	$\Delta\left(+8y - 2*6y + 4y\right)_t$	Real	0.01
Real +9 years -2(5 years) + 1 year	$\Delta\left(+9y - 2*5y + 1y\right)_t$	Real	-0.27
Real +8 years -2(5 years) + 2 years	$\Delta\left(+8y - 2*5y + 2y\right)_t$	Real	-0.27
Real +7 years -2(5 years) + 3 years	$\Delta\left(+7y - 2*5y + 3y\right)_t$	Real	-0.05
Real +7 years -2(4 years) + 1 year	$\Delta\left(+7y - 2*4y + 1y\right)_t$	Real	-0.31
Real +5 years -2(3 years) + 1 year	$\Delta\left(+5y - 2*3y + 1y\right)_t$	Real	-0.32

Exhibit E.5: Correlations of observable curvature shifts from real yield curves with the third principal component.

Variable Name	Definition	Source Curve Type	Correlation with 3rd PC
Nominal +20 years -2(15 years) + 10 years	$\Delta\left(+20y - 2*15y + 10y\right)_t$	Nominal	0.23
Nominal +15 years -2(10 years) + 5 years	$\Delta\left(+15y - 2*10y + 5y\right)_t$	Nominal	0.25
Nominal +20 years -2(10 years) + 1 year	$\Delta\left(+20y - 2*10y + 1y\right)_t$	Nominal	0.21
Nominal +15 years -2(9 years) + 3 years	$\Delta\left(+15y - 2*9y + 3y\right)_t$	Nominal	0.25
Nominal +15 years -2(8 years) + 1 year	$\Delta\left(+15y - 2*8y + 1y\right)_t$	Nominal	0.21
Nominal +10 years -2(7 years) + 4 year	$\Delta\left(+10y - 2*7y + 4y\right)_t$	Nominal	0.26
Nominal +9 years -2(7 years) + 5 year	$\Delta\left(+9y - 2*7y + 5y\right)_t$	Nominal	0.27
Nominal +10 years -2(6 years) + 2 years	$\Delta\left(+10y - 2*6y + 2y\right)_t$	Nominal	0.23
Nominal +9 years -2(6 years) + 3 year	$\Delta\left(+9y - 2*6y + 3y\right)_t$	Nominal	0.24
Nominal +8 years -2(6 years) + 4 years	$\Delta\left(+8y - 2*6y + 4y\right)_t$	Nominal	0.25
Nominal +9 years -2(5 years) + 1 year	$\Delta\left(+9y - 2*5y + 1y\right)_t$	Nominal	0.17
Nominal +8 years -2(5 years) + 2 years	$\Delta\left(+8y - 2*5y + 2y\right)_t$	Nominal	0.22
Nominal +7 years -2(5 years) + 3 years	$\Delta\left(+7y - 2*5y + 3y\right)_t$	Nominal	0.21
Nominal +7 years -2(4 years) + 1 year	$\Delta\left(+7y - 2*4y + 1y\right)_t$	Nominal	0.13
Nominal +5 years -2(3 years) + 1 year	$\Delta\left(+5y - 2*3y + 1y\right)_t$	Nominal	0.05

Exhibit E.6: Correlations of observable curvature shifts from nominal yield curves with the third principal component.

APPENDIX F. LINEAR REGRESSION RESULTS BEFORE AND AFTER THE BACKWARD ELIMINATION PROCEDURE

Index Name	Procedure	Alpha	Level	Slope	Default	Market	SMB	HML	Inflation	R^2 adj.
Government Bullet	before/after	2.22×10^{-3}	-510.52	327.92	-	-	-	-	0.99	0.69
UF 0-3		*-2.21	*-12.60	*13.62					*13.13	
Government Bullet	before/after	-4.47×10^{-3}	-1216.90	113.48	-	-	-	-	0.89	0.65
UF 4-6		*-1.17	*-31.39	*3.57					*6.64	
Government Bullet	before	-5.08×10^{-4}	-1812.64	-93.75	-	-	-	-	0.83	0.55
UF 7-9		*-0.10	*-20.59	*-1.85					*3.41	
Government Bullet	after	5.25×10^{-3}	-1753.29	-	-	-	-	-	0.88	0.55
UF 7-9		*-0.08	*-20.59						*3.65	
Government Bullet	before/after	1.71×10^{-3}	-2700.63	-352.34	-	-	-	-	1.40	0.52
UF 10+		*-0.19	*-18.02	*-3.37					*3.42	
Government	before/after	-4.95×10^{-3}	-113.68	385.65	-	-	-	-	0.91	0.52
PRC 0-1		*-1.71	*-4.28	*13.79					*12.16	
Government	before/after	-6.11×10^{-3}	-555.65	383.79	-	-	-	-	0.98	0.81
PRC 2-3		*-3.23	*-33.05	*26.39					*15.78	
Government	before/after	-4.65×10^{-3}	-1077.48	198.28	-	-	-	-	0.87	0.81
PRC 4-5		*-1.90	*-32.12	*9.28					*9.11	
Government	before	-9.62×10^{-2}	-373.11	363.80	-	-	-	-	0.99	0.73
ZERO 0-3		*-0.06	*-1.05	*1.56					*0.37	
Government	after	1.01×10^{-2}	-373.05	35.15	-	-	-	-	-	0.69
ZERO 0-3		*3.45	*-11.98	*18.09						
Government	before/after	-1.77×10^{-3}	-1295.96	164.50	-	-	-	-	0.93	0.66
ZERO 4-6		*-0.37	*-29.91	*6.35					*6.62	
Government	before/after	-5.15×10^{-3}	-1947.35	-94.35	-	-	-	-	1.11	0.66
ZERO 7-9		*-1.12	*-33.65	*-2.73					*5.21	
Government Bullet	before/after	2.72×10^{-3}	-279.07	31.03	-	-	-	-	-	0.54
CLP 0-3		*2.98	*-15.55	*2.71						
Government Bullet	before/after	2.52×10^{-3}	-1282.45	-509.46	-	-	-	-	-	0.78
CLP 4+		*1.50	*-34.36	*-21.84						

* values with this sign represent the t-value of the coefficient above.

Exhibit F.1: Linear regression results for government bonds.

Index Name	Procedure	Alpha	Level	Slope	Default	Market	SMB	HML	Inflation	R^2 adj.
Corporate AAA	before	-5.35×10^{-3}	-393.80	384.31	0.02	1.95×10^{-3}	0.01	-3.39×10^{-3}	-	0.64
UF 0-3		*-0.05	*-1.10	*1.63	*0.21	*0.03	*0.03	*-8.32×10^{-3}	-	
Corporate AAA	after	-5.06×10^{-3}	-394.66	383.22	0.02	-	-	-	1.07	0.64
UF 0-3		*-1.76	*-13.35	*21.19	*2.42				*11.10	
Corporate AAA	before	2.51×10^{-3}	-1286.88	121.72	3.69×10^{-3}	3.96×10^{-3}	-0.16	0.13	1.18	0.39
UF 4-6		*0.34	*-14.69	*2.54	*0.08	*0.49	*-2.11	*2.51	*3.80	
Corporate AAA	after	2.99×10^{-3}	-1287.45	119.96	-	-	-	-	1.16	0.39
UF 4-6		*0.42	*-15.08	*2.58					*3.93	
Corporate AAA	before	8.00×10^{-4}	-1777.86	-56.89	0.05	3.36×10^{-3}	-0.08	0.04	0.71	0.52
UF 7-9		*8.27×10^{-4}	*-4.99	*-0.24	*0.61	*0.06	*-0.16	*0.10	*0.26	
Corporate AAA	after	1.31×10^{-2}	-1729.47	-	-	-	-	-	-	0.52
UF 7-9		*1.84	*-24.91							
Corporate AAA	before	5.23×10^{-4}	-2743.63	-459.23	0.07	0.01	-0.01	0.02	1.05	0.78
UF 10+		*6.85×10^{-2}	*-28.10	*-6.20	*2.31	*1.88	*-0.24	*0.42	*3.73	
Corporate AAA	after	2.06×10^{-3}	-2746.30	-464.42	0.07	-	-	-	0.99	0.78
UF 10+		*0.27	*-28.43	*-6.29	*2.34				*3.64	
Corporate AAC	before	-3.07×10^{-3}	-446.86	350.40	-3.29×10^{-3}	-1.74×10^{-3}	0.02	-0.01	1.10	0.83
UF 0-3		*-0.03	*-1.25	*1.49	*-0.03	*-0.03	*0.04	*-0.03	*0.41	
Corporate AAC	after	-3.17×10^{-3}	-446.91	350.89	-	-	-	-	1.11	0.83
UF 0-3		*-1.20	*-15.99	*29.31					*21.02	
Corporate AAC	before	9.11×10^{-4}	-1070.71	194.43	0.01	-2.32×10^{-4}	0.01	-4.69×10^{-3}	1.05	0.88
UF 4-6	before	*0.42	*-42.51	*16.70	*1.99	*-0.11	*0.97	*-0.34	*17.76	
Corporate AAC	after	1.08×10^{-3}	-1069.92	192.56	-	-	-	-	1.04	0.88
UF 4-6		*0.48	*-41.72	*16.71					*17.54	
Corporate AAC	before	3.41×10^{-3}	-1833.53	-124.43	0.01	5.44×10^{-3}	-0.02	0.03	1.09	0.87
UF 7-9		*0.03	*-5.15	*-0.53	*0.2	*0.10	*-0.03	*0.08	*0.41	
Corporate AAC	after	0.02	-1834.19	-139.49	-	-	-	-	-	0.86
UF 7-9		*5.88	*-49.39	*-5.53						
Corporate AAC	before	3.89×10^{-3}	-2471.98	-386.48	0.09	8.22×10^{-3}	0.02	9.31×10^{-3}	1.32	0.78
UF 10+		*0.04	*-6.95	*-1.64	*0.98	*0.15	*0.05	*0.02	*0.49	
Corporate AAC	after	4.90×10^{-3}	-2475.30	-390.37	0.09	-	-	-	1.28	0.78
UF 10+		*0.74	*-31.03	*-6.27	*3.23				*5.42	
Corporate AC	before	-4.38×10^{-3}	-374.43	319.65	5.04×10^{-3}	-1.63×10^{-4}	-0.01	8.53×10^{-3}	1.07	0.65
UF 0-3		*-0.04	*-1.05	*1.36	*0.05	*-3.03×10^{-3}	*-0.02	*0.02	*0.40	
Corporate AC	after	-4.42×10^{-3}	-376.04	319.44	-	-	-	-	1.07	0.65
UF 0-3		*-1.53	*-13.15	*20.34					*16.49	
Corporate AC	before	4.56×10^{-3}	-1019.83	140.90	8.14×10^{-3}	2.43×10^{-3}	-0.02	9.41×10^{-3}	0.86	0.66
UF 4-6		*1.14	*-32.24	*7.16	*0.71	*0.60	*-0.79	*0.30	*5.86	
Corporate AC	after	4.77×10^{-3}	-1019.13	139.91	-	-	-	-	0.85	0.66
UF 4-6		*1.24	*-33.85	*7.29					*5.71	
Corporate AC	before	9.84×10^{-3}	-1746.39	-82.12	-0.03	0.01	-0.05	0.05	0.97	0.75
UF 7-9		*2.00	*-42.08	*-2.92	*-1.63	*2.52	*-1.03	*1.31	*6.16	
Corporate AC	after	9.22×10^{-3}	-1749.55	-77.99	-	0.01	-	-	0.99	0.75
UF 7-9		*1.90	*-41.53	*-2.72		*2.53			*6.32	0.75
Corporate AC	before	0.01	-2245.57	-269.57	0.01	0.01	0.09	-0.09	1.02	0.52
UF 10+		*0.16	*-6.31	*-1.14	*0.20	*0.25	*0.19	*-0.23	*0.38	
Corporate AC	after	0.03	-2062.86	-	-	-	-	-	-	0.50
UF 10+		*0.43	*-7.11							

* values with this sign represent the t-value of the coefficient above.

Exhibit F.2: Linear regression results for corporate bonds.

APPENDIX G. FACTORS DESCRIPTIVE STATISTICS

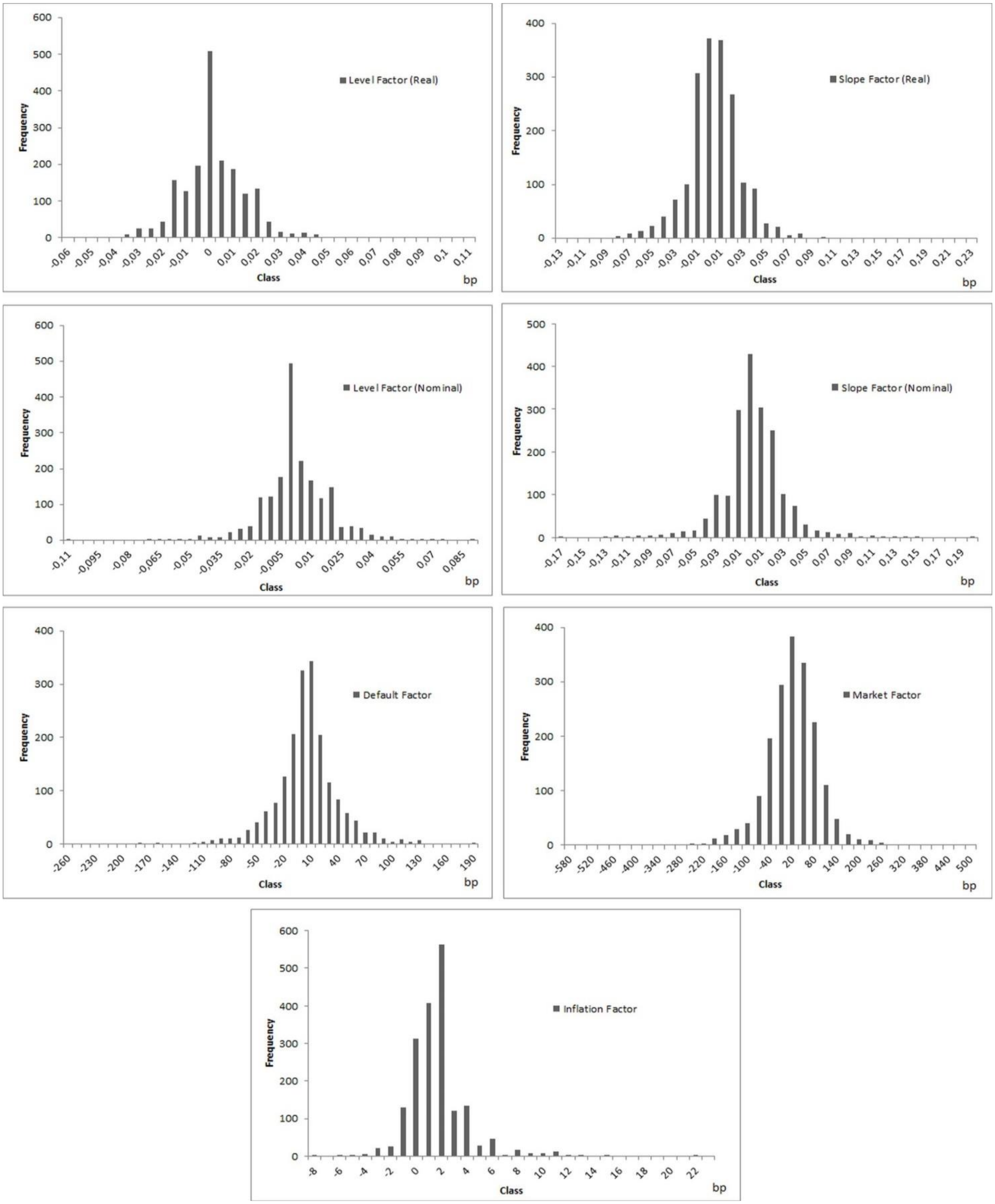

Exhibit G.1: Factors' histograms.

APPENDIX H. DYNAMIC FACTORS SENSITIVITIES

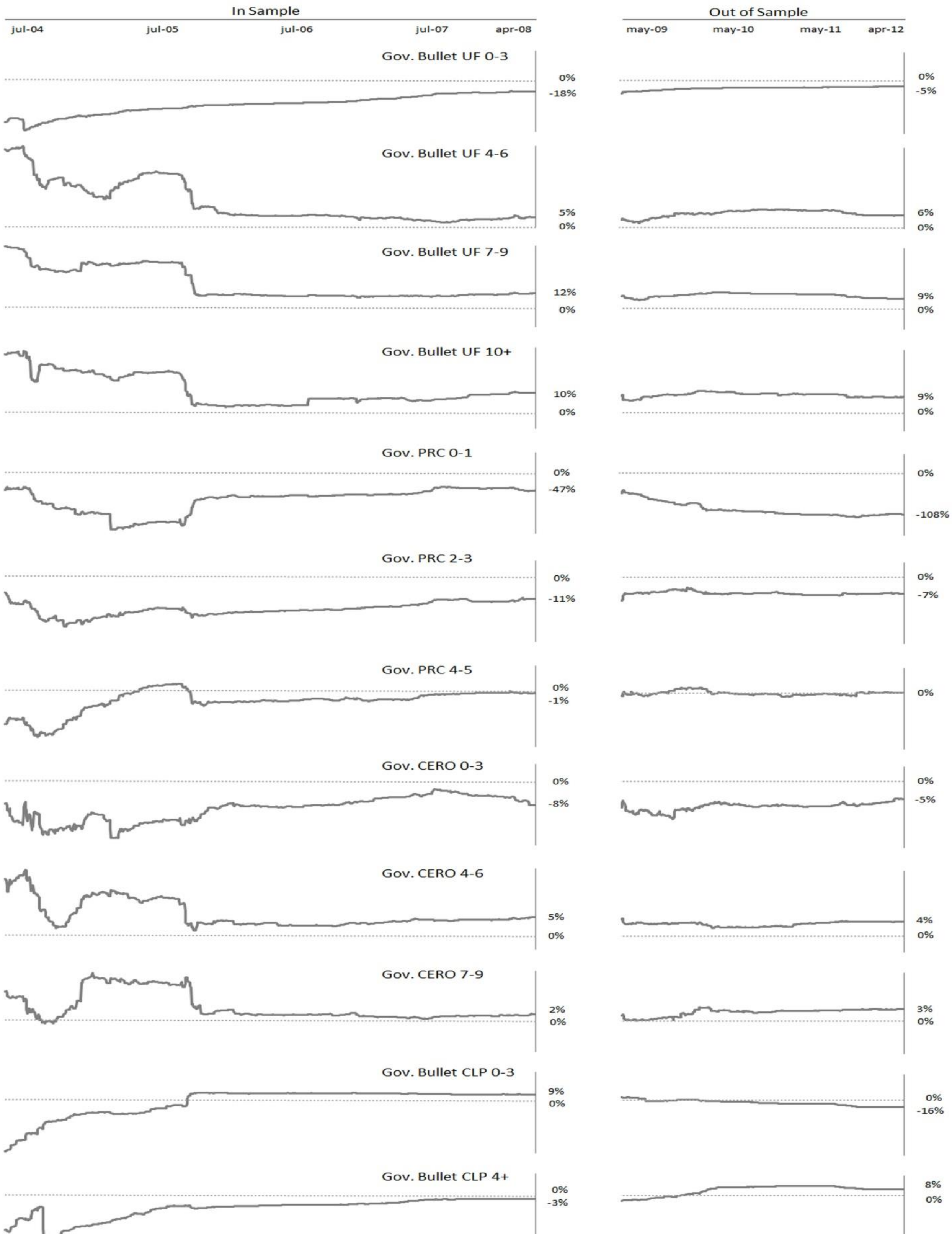

Exhibit H.1: Level factor sensitivities for government bonds.

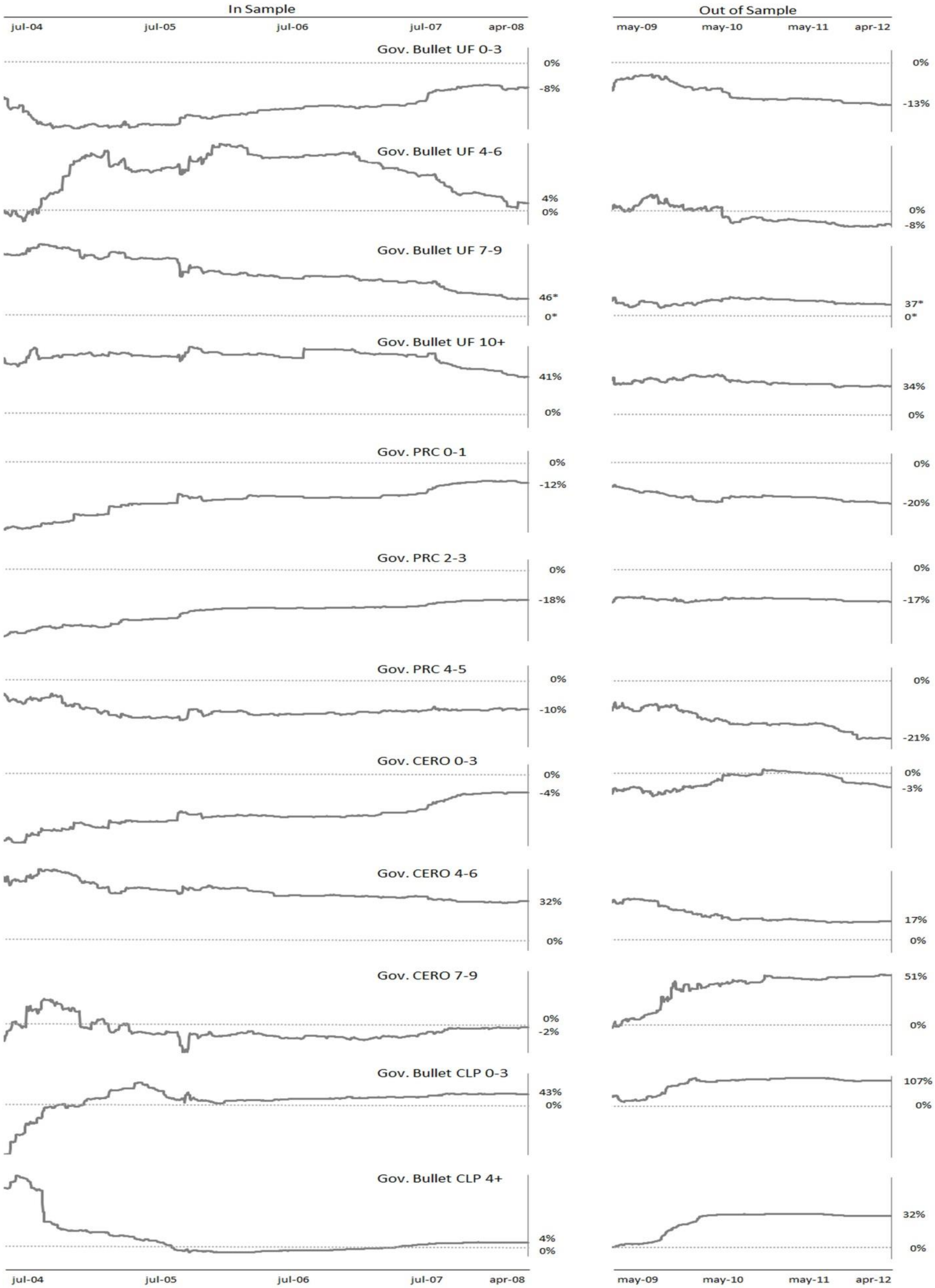

Exhibit H.2: Slope factor sensitivities for government bonds.

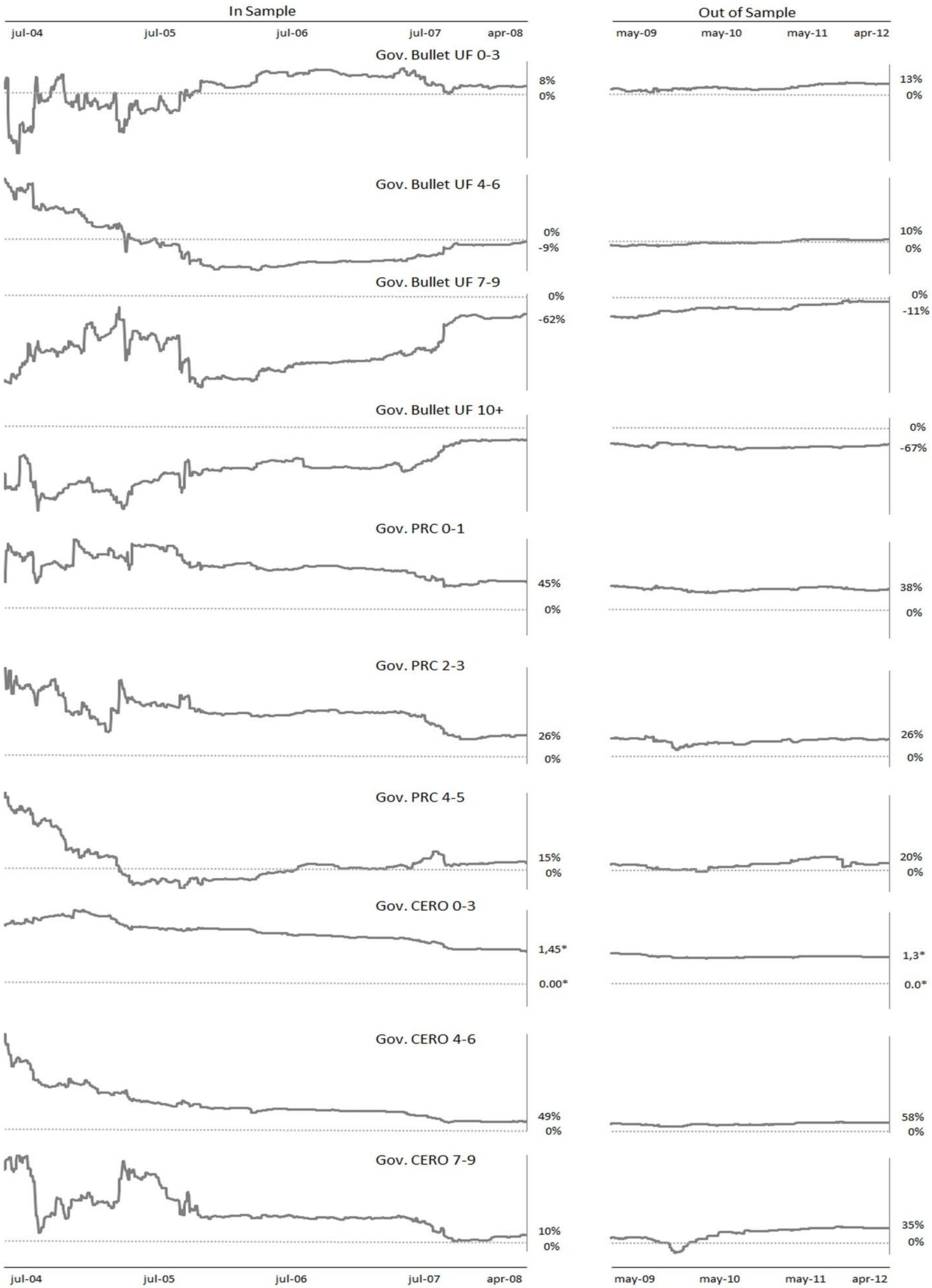

Exhibit H.3: Inflation factor sensitivities for government bonds.

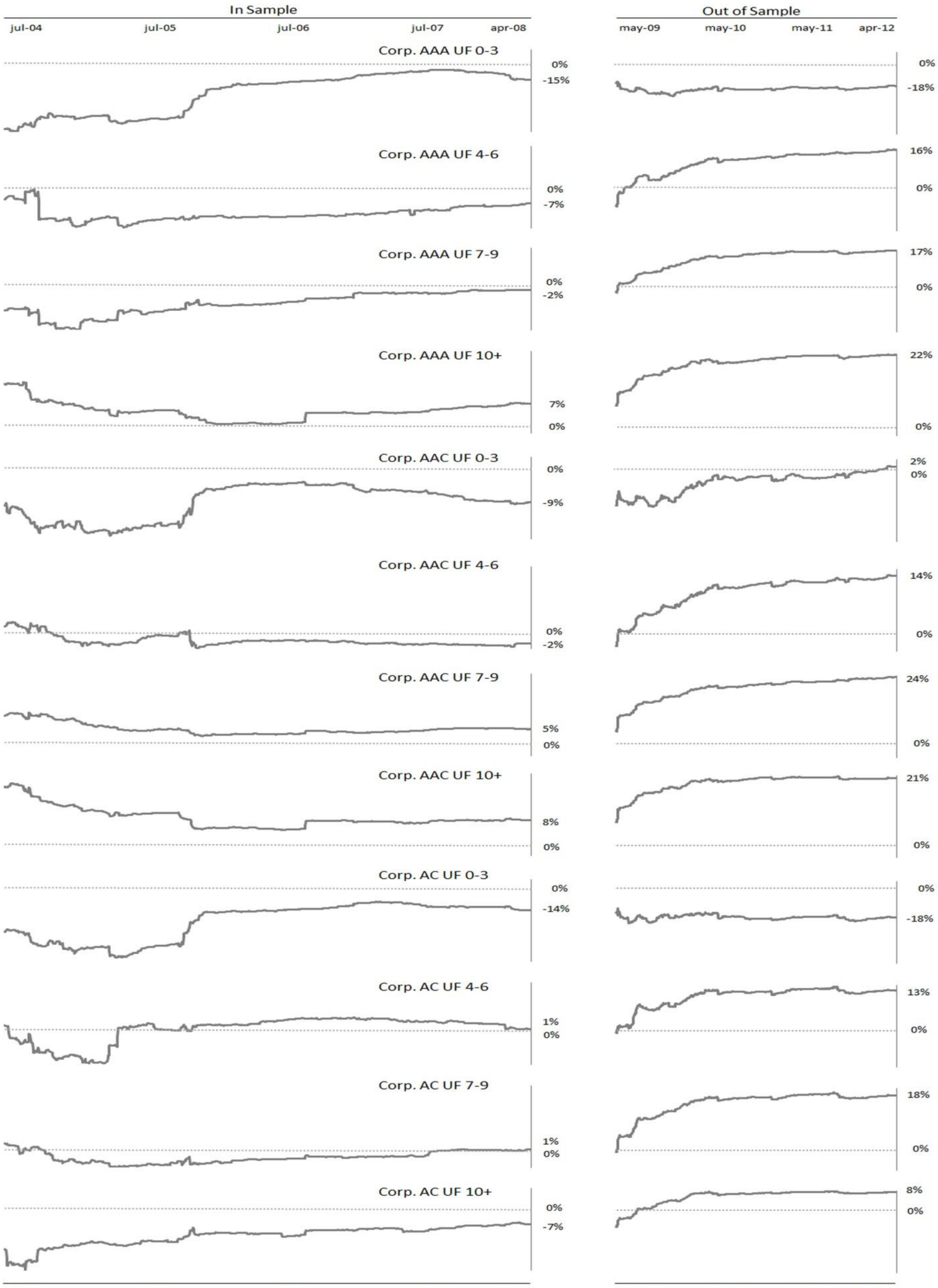

Exhibit H.4: Level factor sensitivities for corporate bonds.

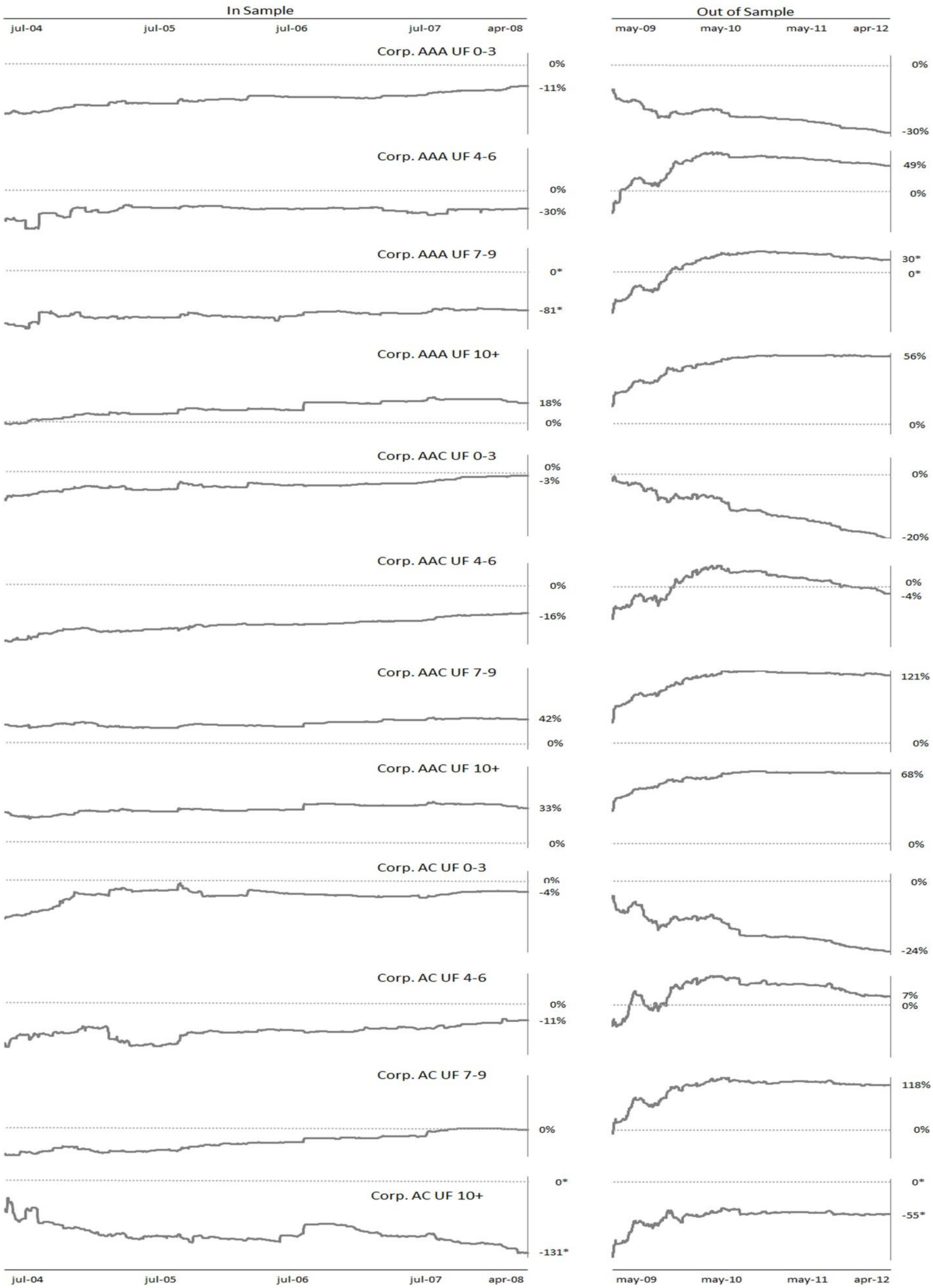

Exhibit H.5: Slope factor sensitivities for corporate bonds.

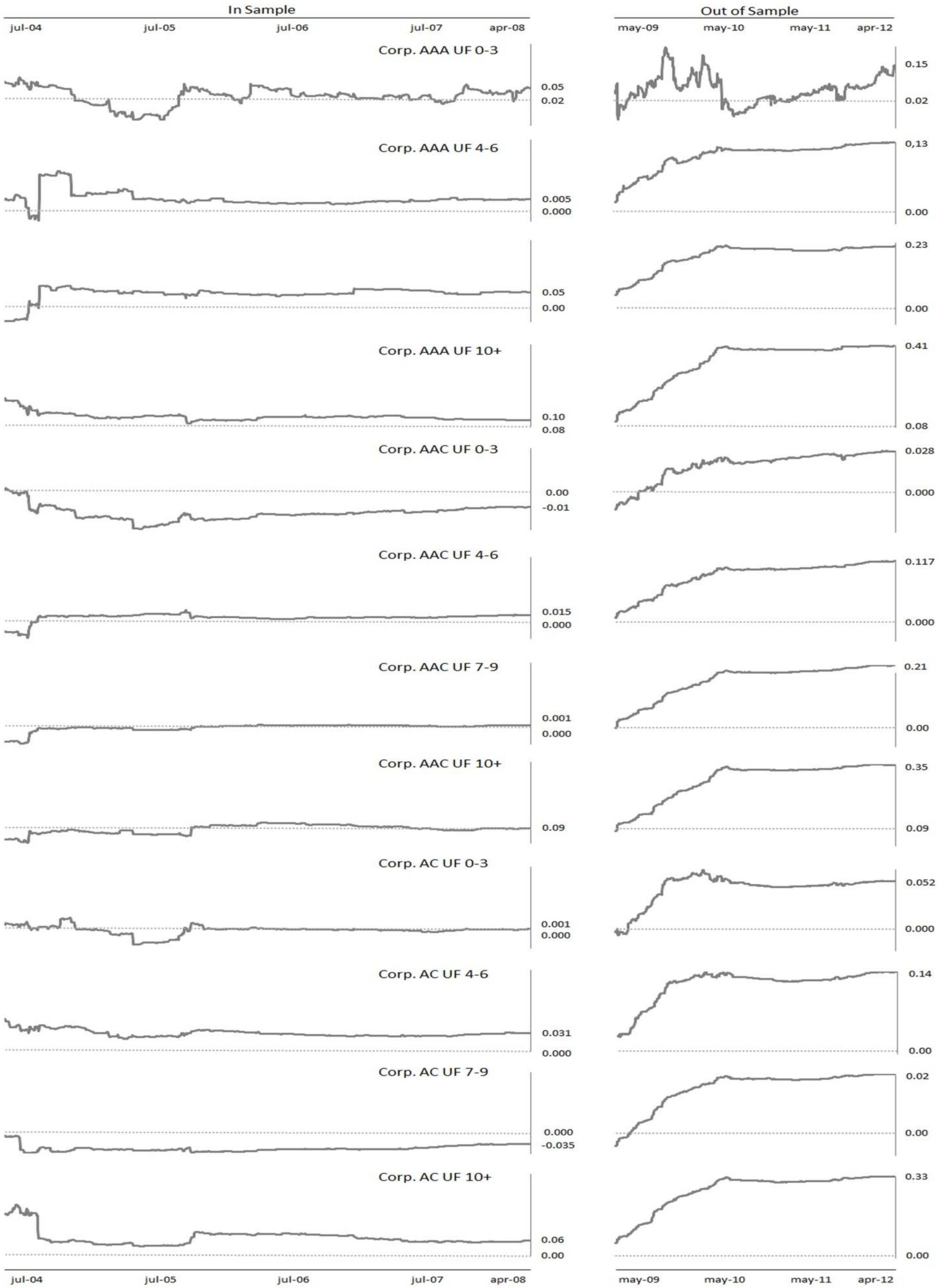

Exhibit H.6: Default factor sensitivities for corporate bonds.

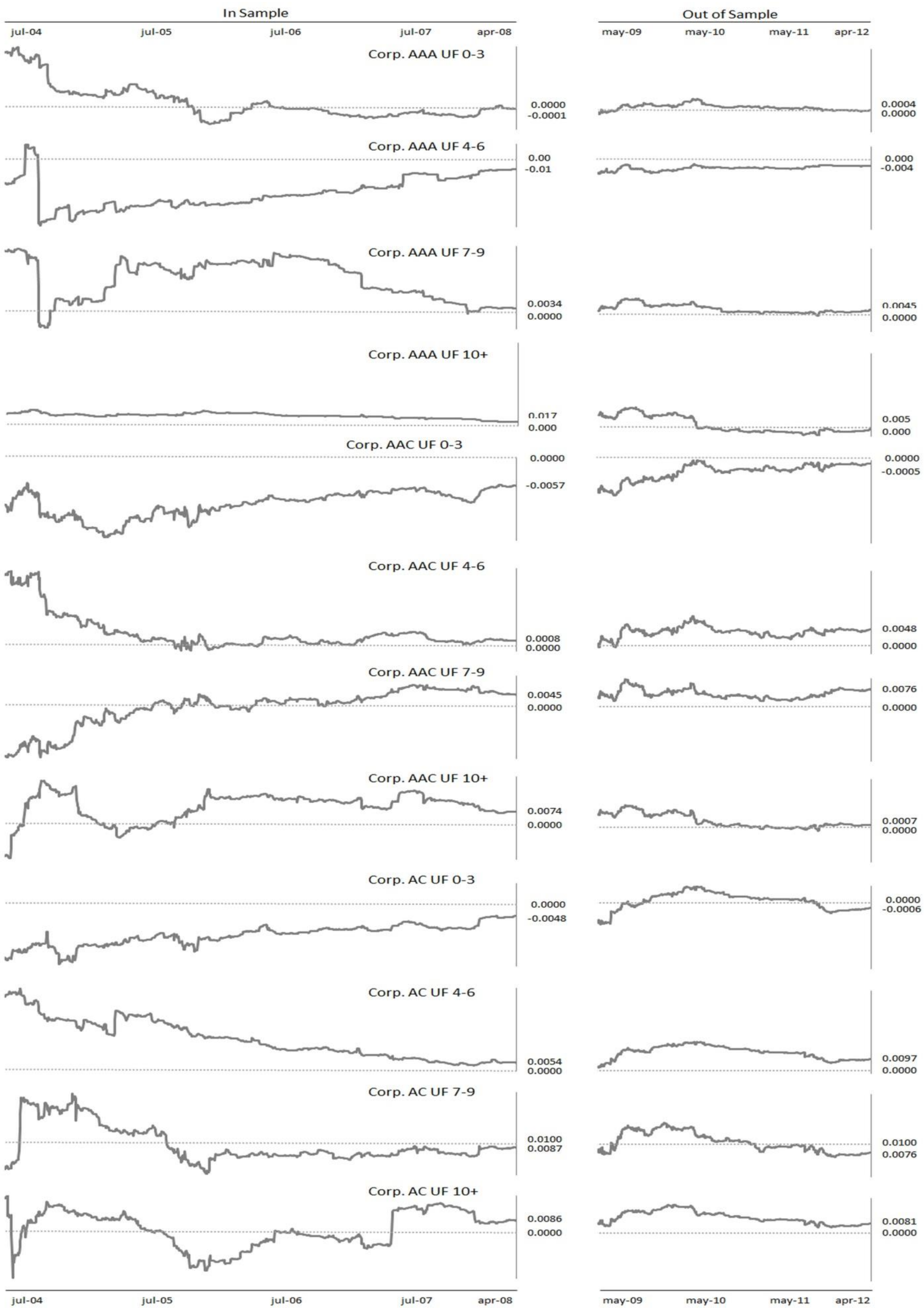

Exhibit H.7: Market factor sensitivities for corporate bonds.

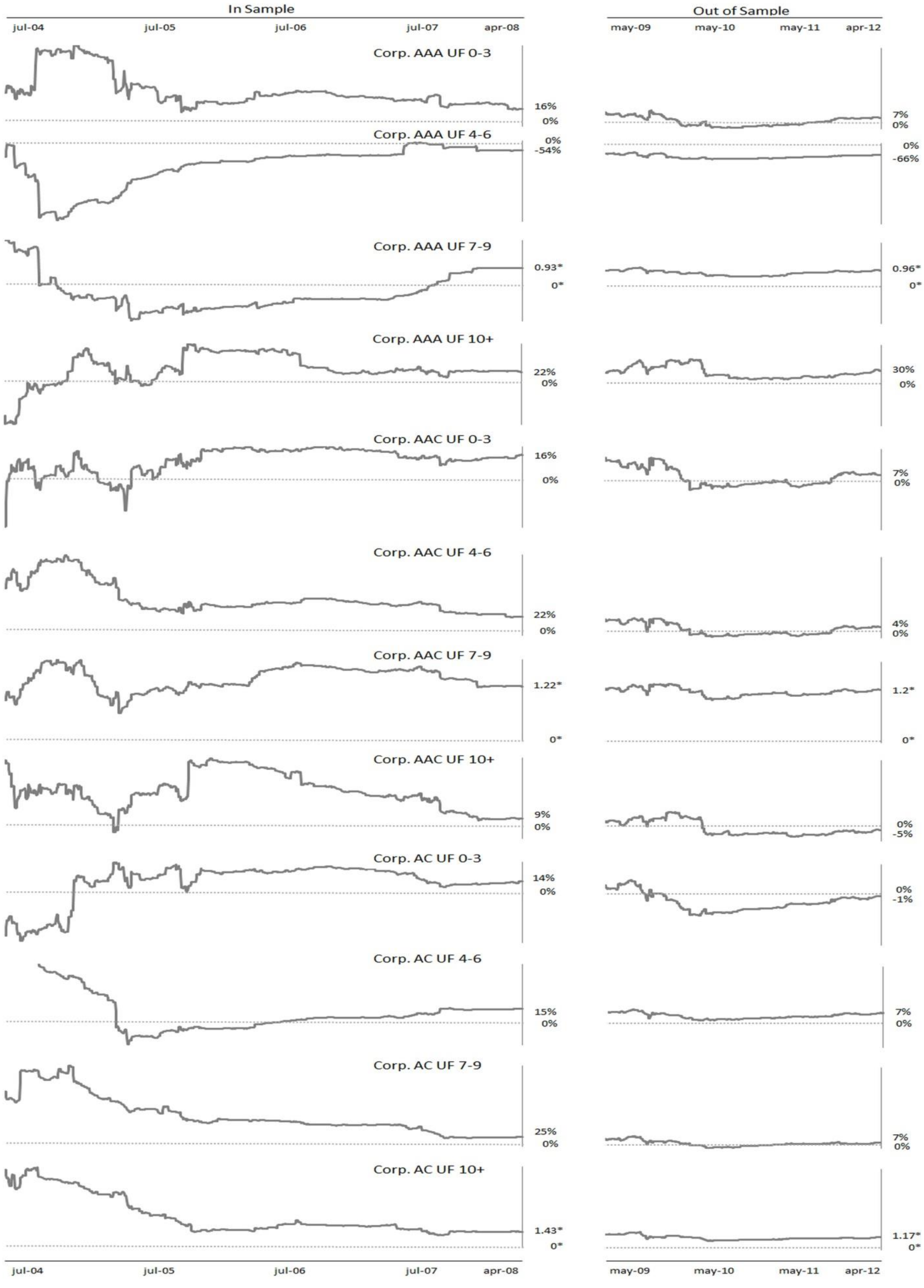

Exhibit H.8: Inflation factor sensitivities for corporate bonds.

REFERENCES

Adrian, T., F. Franzoni. “Learning About Beta: Time-Varying Factor Loadings, Expected Returns and the Conditional CAPM”. Journal of Empirical Finance, Vol. 16, No. 4 (2009), pp. 537-556.

Ammann, M., A.H. Kind, R. Seiz. “What Drives the Performance of Convertible-Bond Funds?”.Working Paper, University of St. Gallen, 2007.

Andersen, T.G., T. Bollerslev, F.X. Diebold, G. Wu. “Realized Beta: Persistence and Predictability”. Advances in Econometrics, Vol. 20 (2006), pp. 1-39.

Anderson, K.W. An Introduction to Multivariate Statistical Analysis.New York: John Wiley & Sons, 1984.

Ang, A., M. Piazzesi. “A No Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables”.Journal of Monetary Economics, Vol. 50 (200), pp. 745-787.

Aßmann, C., J. Boysen-Hogrefe. “Determinants of Government Bond Spreads in the Euro Area: In Good Times as in Bad”. Empirica,Vol. 39 (2012), pp. 341-356.

Balduzzi, P., C. Robotti. “Asset Pricing Models and Economic Risk Premia: A Decomposition”. Journal of Empirical Finance, Vol. 17(2010), pp. 54-80.

Bao, Y., T. Lee, B. Saltoglu. “Evaluating Predictive Performance of Value-at-Risk Models in Emerging Markets: A Reality Check”.Journal of Forecasting, Vol. 2 (2006), pp. 101-128.

Bender, J., R. Briand, F. Nielsen, D. Stefek. “Portfolio of Risk Premia: A New Approach to Diversification”. Journal of Portfolio Management, Vol. 36 (2010), pp. 17-25.

Berkelaar, A.B., A. Kobor, M. Tsumagari. “The Sense and Nonsense of Risk Budgeting”. Financial Analysts Journal, Vol. 62 (2006),pp. 63-75.

Bhar, R. “Modelling Australian Bank Bill Rates: A Kalman Filter Approach”. Applied Financial Economics, Vol. 10 (2000), pp. 49-58.

Blake, C.R., E.J. Elton, M.J. Gruber. “The Performance of Bond Mutual Funds”. The Journal of Business, Vol. 66 (1993), pp. 371-403.

Blitz, D.C., J. Hottinga.“Tracking Error Allocation”. Journal of Portfolio Management, Vol. 27 (2001), pp. 19-25.

Brandt, M.W., F.X. Diebold. “A No-Arbtirage Approach to Range- Based Estimation of Return Covariances and Correlations”. TheJournal of Business, Vol. 79 (2006), pp. 61-74.

Campbell, S.D. “A Review of Backtesting and Backtesting Procedures”.Finance and Economics Discussion Series, Board of Governors of the Federal Reserve System (U.S.), 2005.

Chen, Y., W. Ferson, H. Peters. “The Timing Ability of Fixed Income Mutual Funds”.Working Paper, EFA Moscow Meetings, 2005.

Cheyette, O. “The Barra U.S. Fixed Income Risk Model”. Barra Fixed Income Research, 2002.

Chow, G., M. Kritzman. “Risk Budgets”. Journal of Portfolio Management,Vol. 27 (2001), pp. 56-60.

Christoffersen, P.F. “Evaluating Interval Forecasts”. International Economic Review, Vol. 39, No. 4 (1998), pp. 841-862.

Collin-Dufresne, P., R.S. Goldstein, J.S. Martin.“The Determinants of Credit Spread Changes”. The Journal of Finance, Vol. 56 (2001),pp. 2177-2207.

Das, A., T.K. Ghoshal. “Market Risk Beta Estimation Using Adaptive Kalman Filter”. International Journal of Engineering, Science and Technology, Vol. 2 (2010), pp. 1923-1934.

De Jong, F., J. Driessen, A. Pelsser. “Libor Market Models versus Swap Market Models for Pricing Interest Rate Derivatives: AnEmpirical Analysis”. European Finance Review, Vol. 5 (2001), pp. 201-237.

Diebold, F.X., C. Li. “Forecasting the Term Structure of Government Bond Yields”. Journal of Econometrics, Vol. 130 (2006), pp. 337-364.

Dunis, C.L., V. Morrison. “The Economic Value of Advanced Time Series Methods for Modelling and Trading 10-year GovernmentBonds”. The European Journal of Finance, Vol. 13 (2007), pp. 333 - 352.

Elton, E.J., M.J. Gruber, C.R. Blake. “Fundamental Economic Variables, Expected Returns and Bond Fund Performance”. The Journal of Finance, Vol. 50 (1995), pp. 1229-1256.

Faff, R.W., D. Hillier, R. Brooks. “Australian Industry Beta Risk, the Choice of Market Index and Business Cycles”. Applied FinancialEconomics, Vol. 10 (2000), pp. 49-58.

Fama, E.F., K.R. French. “Common Risk Factors in the Return of Bonds and Stocks”. Journal of Financial Economics, Vol. 33(1993), pp. 3-56.

Fama, E.F., K.R. French. “The Value Premium and the CAPM”. Journal of Finance, Vol. 61 (2006), pp. 2163-2185.

Ferson, W.E., C.R. Harvey. “Explaining the Predictability of Asset Returns”. Research in Finance, Vol. 11 (1993), pp. 65-106.

Figelman, I. “Optimal Active Risk Budgeting Model”. Journal of Portfolio Management, Vol. 30 (2004), pp. 22-35.

Harvey, C.R. “Time-Varying Conditional Covariances in Tests of Asset Pricing Models”. Journal of Financial Economics, Vol. 24(1989), pp. 289-317.

Huang, P., C.J. Hueng. “Interest Rate Risk Factor and Stock Returns: A Time-Varying Factor-Loadings Model”. Applied Financial Economics, Vol. 19 (2009), pp. 1813-1824.

Jostova, G., A. Philipov. “Bayesian Analysis of Stochastic Betas”.Journal of Financial and Quantitative Analysis, Vol. 40 (2005), pp.747-778.

Junod, S. “Towards the 3rd Generation of Asset Allocation”. Deutsche Bank Private Wealth Management, 2012.

Kahn, R.N. “Bond Performance Analysis: A Multifactor Approach”. Journal of Portfolio Management, Vol. 18 (1991), pp. 40-47.

Kupiec, P.H. “Techniques for Verifying the Accuracy of Risk MeasurementModels”. The Journal of Derivatives, Vol. 3 (1995), pp. 73-84.

Lee,W., D.Y. Lam. “Implementing Optimal Risk Budgeting”. Journal of Portfolio Management, Vol. 28 (2001), pp. 73-80.

Litterman, R. “Hot Spots and Hedges”. Risk Management Series,Goldman Sachs & Co, 1996.

Litterman, R., J. Scheinkman. “Common Factors Affecting Bond Returns”. The Journal of Fixed Income, Vol. 1 (1991), pp. 54-61.

Litterman, R., J. Scheinkman, L. Weiss. “Volatility and the Yield Curve”. The Journal of Fixed Income, Vol. 1 (1991), pp. 49-53.

McAleer, M., J. Jimenez-Martin, T. Perez. “Optimal Risk Management Before, During and After the 2008-09 Financial Crisis”.Medium for Econometric Application, Vol. 18, No. 1 (2010), pp. 20-28.

Menchero, J., V. Poduri. “Custom Factor Attribution”. Financial Analysts Journal, Vol. 64 (2008), pp. 81-92.

Mergner, S., J. Bulla. “Time-Varying Beta Risk of Pan-European Industry Portfolios: A Comparison of Alternative Modelling Techniques”. European Journal of Finance, Vol. 8 (2008), pp. 771-802.

Molenkamp, J.B. “Risk Allocation Under Shortfall Constraints”. Journal of Portfolio Management, Vol. 30 (2004), pp. 46-52.

Norling, N., D. Selling. ”An Empirical Evaluation of Value-at-Risk During the Financial Crisis”. Working Paper, Lund University, 2010.

Pearson, N.D. Risk Budgeting: Portfolio Problem Solving with VaR.New York: John Wiley & Sons, 2002.

Page, S. “The Myth of Diversification: Risk Factors vs. Asset Classes”. PIMCO Viewpoints Articles, 2010.

Pownall, R., K.G. Koedijk. “Capturing Downside Risk in Financial Markets: The Case of the Asian Crisis”. Journal of InternationalMoney and Finance, Vol. 18 (1999), pp. 853-870.

Pozzi, L., G.Wolswijk. “Have Euro Area Government Bond Risk PremiaConverged to their Common State?”.Working Paper, Erasmus University, 2008.

Schaefer, S., J. Behrens.“Fixed Income Review - Final Report”.Norwegian Ministry of Finance, 2011.

Shah, A., S.A. Moonis.“Testing for Time Variation in Beta in India”.Journal of Emerging Markets Finance, Vol. 2 (2003), pp. 163-180.

Sharpe, W.F. “Budgeting and Monitoring Pension Fund Risk”. Financial Analysts Journal, Vol. 58 (2002), pp. 74-86.

Singh, M.K. “Value-at-Risk Using Principal Components Analysis”. Journal of Portfolio Management, Vol. 24 (1997), pp. 101-112.

Singleton, K.J. “Yield Curve Risk in Japanese Government Bond Markets”.International Review of Finance.Vol 1 (2000), pp. 97-121.

Wang, K.Q. “Asset Pricing with Conditioning Information”. The Journal of Finance, Vol. 58 (2003), pp. 161-196.

Werley, A. “Factor Risk Management: A Generalized Methodology for Multi-Asset Class Portfolios”. J.P. Morgan Endowments &Foundations Group, 2011.