PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE

ESCUELA DE INGENIERIA

A COMPOUND REAL OPTION APPROACH FOR DETERMINING THE OPTIMAL INVESTMENT PATH FOR RESIDENTIAL PV-STORAGE SYSTEMS

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Thesis submitted to the Office of Research and Graduate Studies in partial fulfillment of the requirements for the Degree of Master of Science in Engineering

Advisor:
TOMÁS REYES

Santiago de Chile, December 2019

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To my family and schoolfellows

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ABSTRACT

The use of residential Photovoltaic-Storage systems may produce large economic benefits to owners and has expanded rapidly in recent years. Nonetheless, large uncertainties regarding the profitability of these systems make it necessary to incorporate flexibilities in their economic evaluations. This paper offers a new method to evaluate the compound flexibility of both the option of delaying investments and the option of further expanding the capacity of solar photovoltaic modules and batteries during the investment horizon. Flexibility is modeled as a compound real option, whose value is computed using a novel method that we call Compound Least Square Monte Carlo (CLSM). We applied our model to the investment decisions associated to a residential Photovoltaic-Storage system in Chile. Our results show that a household should invest in Photovoltaic-Storage systems in 60% of possible future scenarios. Additionally, in 36% of future scenarios, it is recommended to break the investment down into two steps or more. The value of the compound flexibility suggests that investors should use the proposed CLSM method in the economic valuation of multi-stage projects, since considering only the single flexibility of postponing the investment could promote sub-optimal decisions.

Keywords: Batteries; Least Square Monte Carlo; Optimal investment path; Real options; Residential PV-Storage systems; Solar power.

RESUMEN

El uso de sistemas residenciales de almacenamiento fotovoltaico puede producir beneficios económicos para los propietarios y se ha expandido rápidamente en los últimos años. Las grandes incertidumbres con respecto a la rentabilidad de estos sistemas hacen necesario incorporar flexibilidades en sus evaluaciones económicas. Este documento ofrece un nuevo método para evaluar la flexibilidad de tener la opción compuesta de retrasar y ampliar progresivamente la capacidad de módulos fotovoltaicos y baterías durante un tiempo de inversión. La flexibilidad se modela como una opción real compuesta, cuyo valor se calcula utilizando un nuevo método que denominamos *Compound Least Square Monte Carlo* (CLSM). Aplicamos el modelo a un sistema residencial de almacenamiento fotovoltaico en Chile. Nuestros resultados muestran que un hogar debe invertir en sistemas de almacenamiento fotovoltaico en el 60% de los posibles escenarios futuros. Además, en el 36% de los escenarios futuros, se recomienda dividir la inversión en dos o más etapas. El valor de la flexibilidad compuesta sugiere que los inversionistas deben usar el método CLSM propuesto en la valoración económica de proyectos de múltiples etapas, ya que considerar únicamente la flexibilidad de posponer la inversión podría promover decisiones subóptimas.

Palabras Claves: Baterías; Energía solar; Least Square Monte Carlo; Opciones reales; Ruta de inversión óptima; Sistemas de almacenamiento fotovoltaico residencial.

I. ARTICLE BACKGROUND

Several countries are promoting renewable energy sources and discouraging fossil-fuel-based energy generation. On the one hand, these initiatives have resulted in a large integration of non-dispatchable energy sources, such as solar power, which, in turn, is demanding more flexibility in order to balance power supply and demand. On the other hand, the adoption of energy storage systems has expanded rapidly in recent years, mainly due to the observed decrease in the cost of batteries. In this context, the implementation of systems combining solar photovoltaic (PV) modules and batteries (PV-Storage systems) has significantly increased at the residential level.

The use of residential PV-Storage systems may produce large economic and operational benefits to the owner (Cucchiella et al., 2017; Hesse et al., 2017; Shaw-Williams et al., 2018; Tervo et al., 2018; Truong et al., 2016). In addition, an increase in the PV-Storage systems' penetration rate should translate into social and environmental benefits for the entire society. However, the willingness of households to privately invest in a PV-Storage system depends mainly on its economic valuation. Roughly speaking, for households to be willing to invest, the savings in electricity bill costs plus the potential benefits from selling energy to the grid should be larger than the capital investment costs of the PV-Storage system.

A large body of studies on the economic valuation of residential PV-Storage systems have been performed (Barbour & González, 2018; Cucchiella et al., 2017; Flatley et al., 2016; Hoppmann et al., 2014; Ramteen Sioshansi, 2010; Shaw-Williams et al., 2018; Tervo et al., 2018; Truong et al., 2016; Uddin et al., 2017). Most of these studies focus on the cost-effectiveness of adding a storage system to reduce the mismatch between the PV supply and the house's energy demand, enabling the household to use the energy that is not consumed when the generation exceeds the demand. Their results suggest that: (i) investments in small residential PV systems are already profitable under certain conditions (e.g., high electricity prices), (ii) policies promoting investments in batteries will not be necessary in the long run, (iii) PV-Storage systems are likely to promote the ongoing trend toward distributed electricity generation, and (iv) more investment in technical infrastructure will be required to support this trend.

Most of the studies do not incorporate flexibility in the economic valuation of projects, and only compute the net present value (NPV) of the rigid projected cash flows generated by a certain combination of PV modules and storages devices. This rigid valuation of discounted cash flows (DCF), assumes that the investor takes a passive attitude once the initial investment is executed. However, in these types of projects the household has usually the flexibility to react to uncertain scenarios that differ from what was originally expected. The most frequent flexibilities are to postpone, expand, and abandon a project. These flexibilities can be incorporated into the valuation by using a Real Options Analysis (ROA).

Renewable energy investments usually imply high cost and benefits, are partially irreversible, and are subject to high uncertainty (Henao, et al., 2017)[1]. As a consequence, researchers usually incorporate flexibilities in the valuation of this kind of projects and they do so with a real option approach. However, only a few papers apply ROA to valuate investment decisions in residential PV-Storage systems (Gahrooei et al., 2016; Martinez-Cesena et al., 2013; Moon & Baran, 2018). In particular, Moon and Baran (2018) considered the flexibility of postponing the initial investment of residential PV modules and concluded that the NPV method may underestimate economic profitability, and thus lead to earlier-than-optimal investments in PV modules. Additionally, Gahrooei et al. (2016) incorporated the flexibility of expanding and delaying investments in PV modules, using a dynamic optimization problem and showed that having these two flexibilities increases the net present value of the investment. However, to the best of our knowledge, none of these studies have jointly considered the flexibility of postponing the initial investment and the option to invest in a compounded way; for instance, considering the option to first invest in PV modules and then to add batteries, as it is done in this study.

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[1] Refer to Blanco et al. (2011), Zhang et al. (2016), Loncar et al. (2017), and Rios et al. (2019) to see some examples.

Based on the Least Square Monte Carlo (LSM) algorithm proposed by Longstaff and Schwartz (2001) to valuate American options, this paper proposes a new method using a real option approach to valuate both the flexibility of delaying the investments and the option of expanding the capacity of both PV modules and batteries during the evaluation time in a compounded way, where the householder can invest multiple times. Real options are valued using similar models to those used to price American stock options and represent the option and not the obligation of the investor to execute an action related to the project (e.g., to postpone, to expand, to abandon).

There are multiple methodologies to value real options (Boyle, 1977; Cohen, et al, 1972; Cox, et al, 1979). Monte Carlo simulation is one of these methodologies and presents several advantages (Boyle, 1977; Fatone, et al, 2015). Least Square Monte Carlo (LSM) is a particular use of the Monte Carlo simulation to value real options and provides robustness to the results. Intuitively, LSM computes the optimal investment time comparing at every period the NPV of investing in that period with the value of having the option and not the obligation to invest in the future, which is called the continuation value. Therefore, the investor executes to option to invest, expand or abandon the project only if the payoff from immediate implementation is greater than the continuation value.

This paper proposes a new valuation algorithm that estimates the compound optimal investment path for a PV-Storage system. We called this method Compound Least Square Monte Carlo (CLSM). CLSM expands the use of the LSM algorithm, including all its benefits, to compound options in the context of multi-stage project. Therefore, CLSM enables the investor to take into account the expected cash flows, and also the flexibility of reacting different under favorable or unfavorable scenarios. In other words, the economic valuation of the flexible project using this approach adds to the traditional rigid net present value the flexibility to modify the original path of the project.

CLSM method enables us to identify the effect of having the option to invest in a compounded way during the investment time for different combinations of PV modules and batteries. As a consequence, it allows us to compute the optimal investment strategy depending on the given scenario.

We use a high-income house in Santiago de Chile as a base case to analyze the economic viability and the optimal investment path for a residential PV-Storage system. The house owner behaves as a private rational investor interested in reducing the cost of her/his electricity bill through the implementation of a residential PV-Storage system. We consider that the house owner has five different investment possibilities as the result of some combinations of two levels of power capacity from PV modules (P_{min} and P_{max}) and two levels of storage capacity from batteries (B_{min} and B_{max}).

The household has the option to invest directly in multiple combinations of PV-Storage (i.e., only P_{min}, only P_{max}, P_{min} + B_{min}, P_{max} + B_{min}, and P_{max} + B_{max}) and remain in that state for the rest of the evaluation time, or she/he can make multiple investments, upscaling to states with higher solar power production or larger battery capacity (e.g., investing P_{min} first and then moving to P_{min} + B_{min} in a later period). Therefore, during the investment time, the household has multiple possible investment paths, composed of one or more transitions. Additionally, each path has a terminal state, that is the state with the highest capacity of PV modules and batteries of the path. A certain combination of PV modules and batteries (e.g., P_{max} + B_{max}) can be the terminal state of more than one path (e.g., to invest directly in P_{max} + B_{max} or to invest first in P_{max} and then add B_{max}).

We simulate multiple scenarios to take into account the large uncertainties regarding the profitability of the PV-Storage system. In each of these scenarios, the price of electricity, and the cost of solar modules and batteries are modeled as independent Geometric Brownian Motion processes (GBM).

Our results show that households invest different quantities of PV modules and batteries when they have the option to invest in a compounded way. In particular, the flexibility given by the CLSM method accounts for 20% of the net present value of the flexible project, and it makes the project 26% more profitable than the traditional rigid valuation. Using this method, the household invests in a PV-Storage system in 60% of the future scenarios.

The proposed model outperforms the traditional rigid valuation and LSM method as it considers each of these two approaches as one of the multiple possible paths. Particularly, in 36% of the time it is recommendable to invest in multiple steps, taking advantage of the compound option to postpone part of the project until more favorable future scenarios. Specifically, when the household invests in batteries it does so in a compounded way, because it invests first in PV modules and then adds the storage devices. Finally, consistently with real options theory, high levels of uncertainties increase the value of the flexibility and, as a consequence, increase the profitability of the flexible project.

Future studies related to residential PV-Storage systems should implement the CLSM method to other combinations of residential PV-Storage systems and also to dynamic economies and investment behaviors. Additionally, further research should consider different combinations of basis functions to determine the conditional continuation value for the CLSM algorithm.

Finally, the value of using a compound real options approach in the economic valuation of PV-Storage projects demonstrated in this work suggests that the CLSM methodology can be extended to value other types of multi-stage projects, especially if they are exposed to high uncertainties. Considering the traditional rigid valuation or only the flexibility to postpone the investment could promote sub-optimal investments decisions. The above limitations will be a viable avenue for further research and model improvement.

II. Hypothesis and Objectives

This paper has two main hypotheses:

1. Adding the flexibility to invest in a compounded way increases the expected NPV
2. CLSM changes the investor behavior and incentivize the household to invest in multiple steps

The objectives of this study are the following:

1. Recapitulate the state of the art in relation to PV Storage systems
2. Explain how batteries can be modeled as an option
3. Show how the CLSM increases the expected NPV of the projects enabling the investor to invest in multiple steps
4. Illustrates how to implement and expand the use of the CLSM

III. A Compound Real Option Approach for Determining the Optimal Investment Path for Residential PV-Storage Systems

NOMENCLATURE

T_{inv}	Latest time where household can invest
T	Time Horizon
S_i	State with a certain combination *i* of PV modules and Batteries
E^t	Electricity price at t
M^t	PV modules Cost at t
B^{t}	Batteries Cost at t
$RB_{S_i \to S_j}^t$	Rigid Benefit of moving from S_i to S_j at t
$BS_{S_i}^t$	Benefit of having a combination *i* of PV modules and Batteries installed at t
Q^t	House demand for electricity at t
$BCS_{S_i}^t$	Percentage of bill cost savings that state S_i generates at t when compared to not having PV modules installed (S_0)
$SC_{S_i \to S_j}^t$	Initial setup costs to move from S_i to S_j at *t*
$REC_{S_i \to S_j}^t$	Discounted value of the renovation costs incurred between t and T to replace the PV modules and/or batteries (necessaries to move from S_i to S_j) for new ones after their lifespans
$SV_{S_i \to S_j}^t$	Salvage value at T of the investment to move from S_i to S_j at t
$RC_{S_i \to S_j}^t$	Rigid cost to move from S_i to S_j at time t
r	Discount rate
$RNPV_{S_i \to S_j}^t$	Rigid (Traditional) NPV at *t* of investing to move from S_i to S_j
$FNPV_{S_i \to S_j}$	Discounted NPV of investing to move from S_i to S_j at the optimal investment time
${CV^{t,p}}_{S_i \to S_j}$	Continuation value represents the expected net present value generated by moving from S_j to any higher state in path *p* after *t*
$CNPV_{S_i \to S_j}^{t,p}$	Compound NPV represents the sum between the Rigid NPV of investing to move from S_i to S_j at *t* and the value of having the option to continue expanding from S_j in a given path p
α	Drift of a Wiener process
σ	Volatility of a Wiener process
dZ	Increment of a Wiener process

1. INTRODUCTION

The implementation of systems combining photovoltaic (PV) modules and batteries (PV-Storage systems) has significantly increased at the residential level in line with the expansion of renewable energy generation and the use of storage systems. The willingness of households to privately invest in a PV-Storage system depends mainly on its economic valuation.

Several studies on the economic valuation of residential PV-Storage systems have been performed with focus on the cost-effectiveness of adding residential storage systems (Barbour & González, 2018; Cucchiella et al., 2017; Flatley et al., 2016; Hoppmann et al., 2014; Ramteen Sioshansi, 2010; Shaw-Williams et al., 2018; Tervo et al., 2018; Truong et al., 2016; Uddin et al., 2017). Some of these studies have computed the profitability of investing in future periods. However, to the best of our knowledge, none of these studies have jointly considered the flexibility of postponing the initial investment and the option to invest in a compounded way; for instance, considering the option to first invest in PV modules and then to add batteries, as it is done in this work. More specifically, this paper proposes a new approach for approximating the value of compound real options in the context of multi-stage projects based on the Least Square Monte Carlo (LSM) algorithm proposed by Longstaff and Schwartz (2001) to valuate American options.

The household can invest in different PV modules and battery capacities over a period of time and then add more PV modules and/or batteries. Accordingly, the proposed model allows us to identify the effect of having the option to invest in a compounded manner during the investment decision process, and the benefit-cost thresholds for different combinations of PV modules and batteries. We implemented the model to analyze the economic viability and the optimal investment path of a residential PV-Storage system in Chile.

The results in our case study show that the household should invest in a residential PV-Storage system in 60% of possible future scenarios. Additionally, our results suggest that on average in 36% of future scenarios it is optimal to invest in two steps or more, taking advantage of the option to postpone part of the investment until more favorable future scenarios occur. And even more importantly, the analysis of the value of the compound flexibility shown in this work suggests that investors should use the proposed Compound Least Square Monte Carlo (CLSM) method in the economic valuation of multi-stage projects, especially if they are exposed to large uncertainties, since considering only the single flexibility to postpone an investment could promote sub-optimal decisions.

Sensitivity analyses illustrate how more favorable future scenarios encourage the household to invest in states with higher capacity PV modules and batteries. For instance, this would be the case if the economic benefits are high because of an increase in electricity price and the costs are low because of a decrease in the costs of the PV modules and/or batteries. As a consequence, this may promote owners to invest in a compounded manner, because they would first invest in PV modules and then add batteries. All these results help us to show that large levels of uncertainties increase the value of a flexible project.

The rest of the paper is outlined as follow. Section 2 contains a review of the literature. Section 3 explains the valuation framework and the proposed valuation model. Section 4 presents a case study and the numerical results. Section 5 shows some sensitivity analyses. Section 6 concludes the paper.

2. LITERATURE REVIEW

In this section, we present a literature review of three important topics related to this research work. First, we review the current state of research about residential PV-Storage systems. Then, we focus on the real options analysis (ROA) as a way of adding flexibility to these types of projects. Finally, we review different approaches to valuate real options.

2.1 Economic Valuation of Residential PV-Storage Systems.

Previous research on the economic valuation of residential PV-Storage systems is not yet decisive in the profitability and social/private welfare generated by these systems. Hoppmann et al. (2014) reviewed several studies about the economic viability of PV-Storage systems. Their results suggest that investments in small residential PV systems are already profitable under certain conditions (e.g., high electricity prices). However, more investment in technical infrastructure will be required to support this trend.

More recently, studies are still discrepant on the benefits of the implementation of PV-Storage systems, mainly due to the uncertainty about the cost evolution of batteries. On the one hand, some authors confirm that batteries are unprofitable to install with current tariffs for most consumers (Uddin et al., 2017), and that widespread battery adoption will not occur unless retail electricity prices rise in addition to other conditions such as an increase in the reward for exported electricity (Barbour & González, 2018).

On the other hand, other works indicate that pairing lithium-ion battery storage systems with residential PV modules is profitable under current conditions in the United States (Tervo et al., 2018), Germany (Hesse et al., 2017; Truong et al., 2016), Italy (Cucchiella et al., 2017), and Australia (Shaw-Williams et al., 2018). However, subsidies, Feed-in Tariffs, self-consumption, considerations of battery degradation and costs, and the control of the ratio between PV module and battery capacities are some of the current conditions existing in these countries that are crucial for profitability to be achieved.

2.2 Real Options Analysis to Add Flexibility to PV-Storage Projects

ROA has been used widely on studies related to renewable energy investment, because this kind of investments implies high cost and benefits, is partially irreversible, and are subject to high uncertainty (Henao, et al., 2017)[2]. However, only a few papers apply ROA to valuate investment decisions in residential PV-Storage systems (Gahrooei et al., 2016; Martinez-Cesena et al., 2013; Moon & Baran, 2018).

[2] Refer to Zhang et al. (2016), Loncar et al. (2017), and Rios et al. (2019) to see some examples.

Differently, in this paper, we use ROA to value residential PV-Storage systems, considering the option of investing in a compounded way by expanding both the PV module and battery capacities during the evaluation horizon.

2.3 Real Option Valuation and Least Square Monte Carlo Algorithm

Real options are valued using similar models to those used to price American stock options. An American stock option is a financial security giving the right, but not the obligation, to the holder to buy or sell an asset at a certain price within a specified period of time (Black & Scholes, 1973). There are three main methodologies to value these options: Binomial trees (Cox et al., 1979), closed algebraic solutions (Cohen et al., 1972), and Monte Carlo simulations (Boyle, 1977). Monte Carlo simulations present multiple advantages. One is the flexibility regarding the distribution used to generate returns on the underlying risky asset (i.e., the investment project). Additionally, contrary to closed algebraic solutions, this method does not need strong assumptions (Fatone et al., 2015). Finally, the simulation method has also clear advantages in cases where the underlying risky asset returns are generated by a mixture of stochastic processes, such as the case presented in this paper (Boyle, 1977).

Least Square Monte Carlo (LSM) is a particular use of the Monte Carlo simulation to value real options. The LSM method provides robustness to the results and contains two important features: i) the initial value of the project is uncertain and ii) the volatility of the parameters over the evaluation time is non-constant (Mariscal et al., 2018). The LSM method estimates the conditional expected payoff from continuation (i.e., to wait until future periods to execute the option) from cross-sectional information in the simulation by using an ordinary least squares regression model (Longstaff & Schwartz, 2001). This continuation value is compared at any period, with the payoff that comes from immediately exercising the option. The option is only executed if the payoff from immediate implementation is greater than the continuation value. In this context, Gamba (2007) applied the LSM method on embedded real options adding to the continuation value of each option the value of the flexibility to invest afterwards. In this case, the investor was forced to invest in a determined unique path.

We proposes a new algorithm called Compound Least Square Monte Carlo (CLSM) that expands the use of the LSM algorithm including all its benefits to compound options. In our case, at any period, the household has the option to invest in multiple combinations of PV-Storage systems, remaining in that state for the rest of the evaluation horizon, or she/he can make multiple investments, upscaling to states with higher solar power production or larger battery capacity.

3. METHODOLOGY

We assume a rational household interested in reducing electricity bill cost through the implementation of a residential PV-Storage system. The household can invest multiple times in a limited period of time, always upscaling to states with higher solar power production and/or larger battery capacity. During the investment horizon (T_{inv}) the household can make multiple investments, while during the remaining valuation time ($T - T_{inv}$) the household cannot invest, and she/he will remain with the PV-Storage system she/he had at the end of T_{inv}.

As an example, Figure 1 shows five investment possibilities that result from some combinations of different levels of power capacity from solar PV modules and different levels of storage capacity from batteries. S_0 is the base case, where no PV modules or batteries are installed. For every pair of states i, j ($j > i$), S_j has larger or equal PV-Storage capacity than S_i. The household has the option to invest once, moving directly to any state and remain in that state for the rest of the valuation time, or she/he can make multiple investments during the investment horizon (e.g., moving to S_1 in the first period, moving to S_2 in the third period and finally, moving to S_5 in the ninth period).

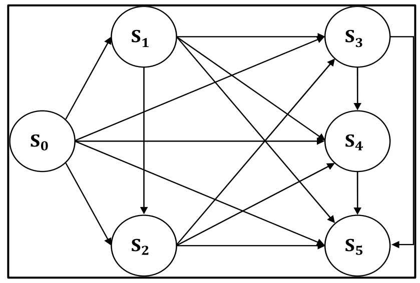


Figure 1: Possible States and Transitions.

That is, during the investment time, the household has multiple possible investment paths, composed of one or more transitions. Single Transition Paths are those with only one transition (e.g., moving from S_0 to S_1 in the third period, and staying there for the remaining valuation horizon), while Multi Transitions Paths are those with two or more transitions (e.g., moving from S_0 to S_1 in the fifth period, then moving from S_1 to S_4 in the sixth period, and finally moving to S_5 in the ninth period). Each path has a terminal state, which is the state with the highest capacity of PV modules and batteries of the path. There are 27 possible investment paths in Figure 1.

The price of electricity, and the unitary cost of solar modules and batteries are modeled as independent Geometric Brownian Motion (GBM) processes:[3]

$$dE(t) = E(t) \cdot \alpha_e \cdot dt + E(t) \cdot \sigma_e \cdot dZ_e, \tag{1}$$

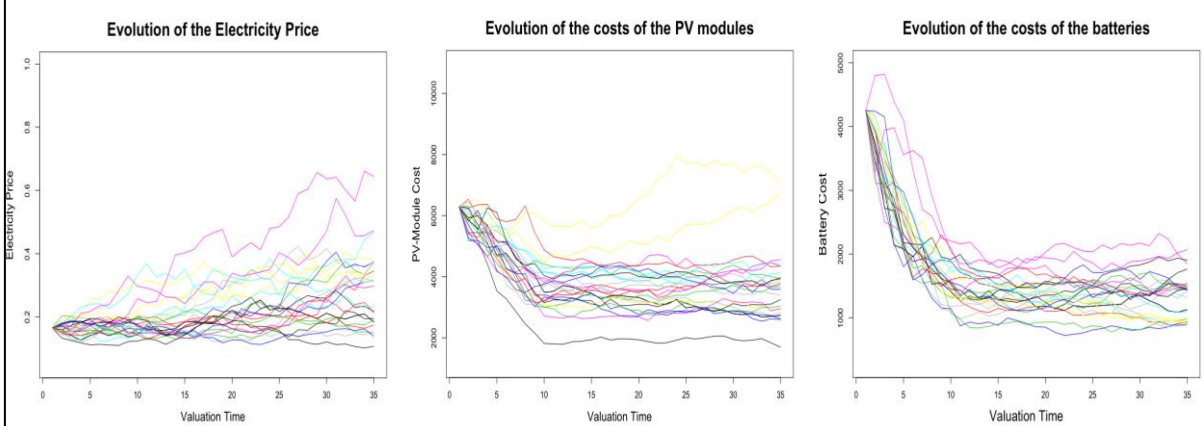
$$dM(t) = M(t) \cdot \alpha_m \cdot dt + M(t) \cdot \sigma_m \cdot dZ_m, \tag{2}$$

$$dB(t) = B(t) \cdot \alpha_b \cdot dt + B(t) \cdot \sigma_b \, dZ_b, \tag{3}$$

where $E(t)$, $M(t)$ and $B(t)$ are the price of electricity, PV module cost and battery cost at time t, respectively. Additionally, α, σ, and dZ represent the drift, volatility and the increment of a Wiener process, respectively (Hull, 2006). In every period, the price and costs can increase or decrease depending on the drift and the volatility of each GBM. For

[3] Others studies that have used GBM to model uncertainty in real option valuations are: Moon & Baran (2018); Pindyck (1999); and Tang et al. (2014).

example, Figure 2 shows some possible evolutions over time of the electricity price and the costs of PV modules and batteries.



(i) (ii) (iii)

Figure 2: **(i) Electricity price GBM, (ii) PV modules costs GBM, (iii) batteries costs GBM.**[4]

To describe the methodology in a logic sequence, the rest of this section is outlined as follows: Section 3.1 explains the benefits and costs of moving between two states of PV-Storage systems, considering a rigid project. Section 3.2 first presents the conventional valuation of discounted cash flows for a rigid project (Section 3.2.1); and then, it shows how the traditional LSM method accounts for the single flexibility to postpone the investment (Section 3.2.2). Finally, this section shows how CLSM method account for the compound flexibility to postpone the initial investment and to expand the project (Section 3.2.3).

[4] For illustrative purposes, we only show 25 future scenarios in Figure 2.

3.1 Rigid Benefits and Cost of Moving between States

For a given future scenario, moving from any state, S_i, to another state with higher capacity, S_j, has rigid incremental benefits and costs. Rigid benefits are computed as the difference in the electricity bill cost paid by the household at each state. Rigid costs are all the costs necessary to move to state S_j. All benefits and costs are measured in USD.

3.1.1 Rigid Benefits

As mentioned above, moving to a higher state in a certain future scenario generates an incremental benefit to the household. In particular, moving from S_i to S_j in period $\hat{t}$ generates benefits equivalent to the difference between the electricity bill cost of states S_i and S_j for the remaining valuation horizon $(T - \hat{t}\ )$. The total benefit of a given transition is called rigid benefit ($RB^{\hat{t}}_{S_i \to S_j}$) and is computed as the net present value of the incremental annual cash flows of moving from S_i to S_j for the remaining years of the valuation horizon. Assuming a discount rate of r, rigid benefits are computed as follows:

$$RB^{\hat{t}}_{S_i \to S_j} = \sum_{t=\hat{t}}^{T} (BS^t_{S_j} - BS^t_{S_i}) \cdot e^{-r(t-\hat{t}\ )}, \tag{4}$$

where, for each period t, benefits per state ($BS^t_{S_i}$) are computed as the multiplication of the house demand for electricity (Q^t), the price of electricity (E^t), and the percentage of bill cost savings that a certain state generates ($BCS^t_{S_i}$) when compared to S_0 (e.g., a PV-Storage combination of P_{min} + B_{min} could decrease the bill cost by 60% with respect to not having any PV module and battery). Therefore, the benefit of being in state i in period t is expressed as follows:

$$BS^t_{S_i} = Q^t \cdot E^t \cdot BCS^t_{S_i} \tag{5}$$

3.1.2 Rigid Costs

Investment costs of moving from S_i to S_j in period $\hat{t}$ are divided in three components: (i) initial setup cost, (ii) renovation cost, and (iii) salvage value. Initial setup costs ($SC^{\hat{t}}_{S_i \to S_j}$) are incurred at period $\hat{t}$ ($\hat{t} \in [0, T_{inv}]$) when the household invests to move to a higher state. These setup costs are computed using the GBMs' value of the module and battery costs for a certain future scenario and period.

Since the lifespan of any component of the PV-Storage system could be shorter than the remaining valuation time after the initial setup $(T - \hat{t})$, the household has to reinvest in the components of the PV-Storage system at one or multiple times during the valuation time. Thus, renovation costs ($REC^{t}_{S_i \to S_j}$) are incurred between $\hat{t}$ and T when the household has to replace the PV modules and/or batteries for new ones after their lifespans.

Finally, at the end of the valuation horizon, the household recovers the salvage value ($SV^{\hat{t}}_{S_i \to S_j}$) of the PV-Storage system. The $SV^{\hat{t}}_{S_i \to S_j}$ is computed as the multiplication between the $SC^{t}_{S_i \to S_j}$ in period T and the remaining fraction of the lifespan of the PV-Storage system. Therefore, the rigid cost ($RC^{\hat{t}}_{S_i \to S_j}$) from moving from S_i to a higher state S_j at time $\hat{t}$ is computed as follows:

$$RC^{\hat{t}}_{S_i \to S_j} = SC^{\hat{t}}_{S_i \to S_j} + \sum_{t=\hat{t}}^{T} \left(REC^{t}_{S_i \to S_j}\right) \cdot e^{-r(t-\hat{t})} - SV^{\hat{t}}_{S_i \to S_j} \cdot e^{-r(T-\hat{t})}, \tag{6}$$

where the renovation cost is zero for the years in which no PV modules or batteries are replaced and it is equal to $SC^{t}_{S_i \to S_j}$ in the years that PV modules or batteries are replaced.

3.2 PV-Storage System Valuation

The best way of understanding the proposed CLSM methodology is comparing the traditional valuation methods with our compound approach. In this section, we first review the conventional valuation of a rigid project; and then, we recall how the traditional LSM method accounts for the single flexibility of postponing an investment. Finally, we show

how the CLSM method accounts for the compound flexibility of postponing the initial investment and then expanding a project. All flexibilities are measured in USD.

3.2.1 Rigid Valuation

Let us take a subset of two states of Figure 1, as presented in Figure 3. For each future scenario, in this case, the household invests immediately (i.e., in $t = 0$) moving from S_i to S_j.

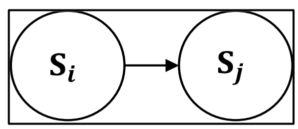


Figure 3: Representation of a transition in the system represented in Figure 1 (*j* > *i*).

Rigid valuation calculates the rigid benefits, costs, and NPV of investing immediately, moving from S_i to S_j. We compute a matrix of rigid benefits ($RB_{S_i \to S_j}$) and costs ($RC_{S_i \to S_j}$). Each of these matrices has N future scenarios and only one column since the household only has one period to invest. Therefore, the element $(n, 1)$ of matrix $RB_{S_i \to S_j}$ has the present value of the rigid benefit of upscaling from S_i to S_j in scenario n in time $t = 0$ and is computed as explained in (4) with $t = 0$. Analogously, the element $(n, 1)$ of matrix $RC_{S_i \to S_j}$ has the present value of the cost of upscaling from S_i to S_j computed as explained in (6) with $t = 0$. Then, the matrix $RNPV_{S_i \to S_j}$ is computed as the difference between the present values of the rigid benefits ($RB_{S_i \to S_j}$) and costs ($RC_{S_i \to S_j}$), and it contains the rigid NPV of investing in $t = 0$ for each of the N future scenarios. Thus:

$$RB_{S_i \to S_j} = \begin{bmatrix} \vdots \end{bmatrix}_{Nx1} \tag{7}$$

$$RC_{S_i \to S_j} = \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_{Nx1} \tag{8}$$

$$RNPV_{S_i \to S_j} = RB_{S_i \to S_j} - RC_{S_i \to S_j} \tag{9}$$

The expected rigid NPV of moving from S_i to S_j is the average $RNPV_{S_i \to S_j}$ of all future scenarios. Therefore, the household decides to invest if the expected NPV is positive.

3.2.2 Single Flexibility Valuation

In this case, for each future scenario, the household has the flexibility of choosing to postpone the investment. Thus, she/he is not forced to invest in $t = 0$, but has the flexibility of investing at any time between 0 and T_{inv}.

Single flexibility valuation first calculates the rigids benefits, costs, and NPV of investing and moving from S_i to S_j (see Figure 3) at any time between 0 and T_{inv}. Therefore, we compute a matrix of rigid benefits ($RB_{S_i \to S_j}$) and costs ($RC_{S_i \to S_j}$) of NxT_{inv}, where N is the number of future scenarios, and T_{inv} is the number of periods in which the household can invest. The values in these matrices present the value of the projected benefits and costs during the remaining valuation time after investing in a certain year. For example, element (n, t) in matrix $RB_{S_i \to S_j}$ has the present value of the incremental benefit of moving from S_i to S_j in scenario n in period t. Analogously, element (n, t) of matrix $RC_{S_i \to S_j}$ has the present value of the cost of upscaling from S_i to S_j in scenario n in period t. Then, the matrix $RNPV_{S_i \to S_j}$ is computed as the difference between the present values of the rigid benefits ($RB_{S_i \to S_j}$) and costs ($RC_{S_i \to S_j}$), and it shows the NPV of investing at any time between 0 and T_{inv} for each of the N future scenarios. Accordingly:

$$RB_{S_i \to S_j} = \begin{bmatrix} \cdot & \cdots & \cdot \\ \vdots & \ddots & \vdots \\ \cdot & \cdots & \cdot \end{bmatrix}_{NxT_{inv}} \tag{10}$$

$$RC_{S_i \to S_j} = \begin{bmatrix} \cdot & \cdots & \cdot \\ \vdots & \ddots & \vdots \\ \cdot & \cdots & \cdot \end{bmatrix}_{NxT_{inv}} \tag{11}$$

$$RNPV_{S_i \to S_j} = RB_{S_i \to S_j} - RC_{S_i \to S_j} \tag{12}$$

For each future scenario n, the LSM algorithm computes the optimal investment time of moving from S_i to S_j (instead of investing immediately as in the rigid valuation presented in Section 3.2.1). Intuitively, LSM computes the optimal investment time t^* comparing at every period t (between 0 and T_{inv}) the NPV of investing in period t with the value of having the option and not the obligation to invest in the future, which is called the continuation value ($CV_{\mathrm{S_j} \to \mathrm{S_k}}$) in the LSM method (Longstaff and Schwartz, 2001). Then, the flexible NPV ($FNPV_{S_i \to S_j}$) in the future scenario n is simply the discounted value of the n-th element of the rigid NPV ($RNPV_{S_i \to S_j}$) in the optimal investment time:

$$FNPV_{S_i \to S_j}(n, 1) = \ RNPV_{S_i \to S_j}(n, t^*) \cdot \mathrm{e}^{(-t^* \cdot r)} \tag{13}$$

Finally, the expected value of having the option and not the obligation to move from S_i to S_j at any time between 0 and T_{inv} is the average $FNPV_{S_i \to S_j}$ of all future scenarios.

3.2.3 Compound Flexibility Valuation

The CLSM method enables the household to valuate multiple investments in a compounded way. For example, let us take a subset of three states of the system represented in Figure 1, as presented in Figure 4. Table 1 shows all three possible paths and their transitions in this selected subset of states. In this case, paths I and II have a single transition, while path III is compound and has two transitions. The valuation of paths I and II is the same used in the single flexibility valuation explained in Section 3.2.2. On the other hand, the compound valuation of path III is only possible with the CLSM algorithm, as explained next.

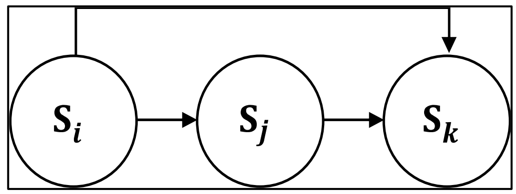


Figure 4: Possible transitions in a subset of three states of the system represented in Figure 1 (k > j > i).

Table 1: Possible paths of the system represented in Figure 4.

Path (1)	Transition 1 (2)	Transition 2 (3)	Terminal State (4)
I	$S_i \rightarrow S_j$	-	S_j
II	$S_i \rightarrow S_k$	-	S_k
III	$S_i \rightarrow S_j$	$S_j \rightarrow S_k$	S_k

For path III, the CLSM calculates first the rigids benefits, costs, and NPV of investing and moving from S_i to S_j and from S_j to S_k at any time between 0 and T_{inv}. The following matrices of rigid benefits, costs, and NPV are calculated in the same way as calculating the single flexibility valuation for each future scenario and transition.

From S_i to S_j:

$$RB_{S_i \rightarrow S_j} = \begin{bmatrix} . & \cdots & . \\ \vdots & \ddots & \vdots \\ . & \cdots & . \end{bmatrix}_{NxT_{inv}} \quad (14)$$

$$RC_{S_i \rightarrow S_j} = \begin{bmatrix} . & \cdots & . \\ \vdots & \ddots & \vdots \\ . & \cdots & . \end{bmatrix}_{NxT_{inv}} \quad (15)$$

$$RNPV_{S_i \rightarrow S_j} = RB_{S_i \rightarrow S_j} - RC_{S_i \rightarrow S_j} \quad (16)$$

From $\mathrm{S_j}$ to S_k :

$$RB_{S_j \rightarrow S_k} = \begin{bmatrix} \cdot & \cdots & \cdot \\ \vdots & \ddots & \vdots \\ \cdot & \cdots & \cdot \end{bmatrix}_{NxT_{inv}} \tag{17}$$

$$RC_{S_j \rightarrow S_k} = \begin{bmatrix} \cdot & \cdots & \cdot \\ \vdots & \ddots & \vdots \\ \cdot & \cdots & \cdot \end{bmatrix}_{NxT_{inv}} \tag{18}$$

$$RNPV_{S_j \rightarrow S_k} = RB_{S_j \rightarrow S_k} - RC_{S_j \rightarrow S_k} \tag{19}$$

Then, the CLSM method computes the optimal investment times and the NPV of the path considering two steps:

Step I:

In this step, we calculate the optimal investment time of the multiple transitions in path III, for each future scenario, using a compound NPV ($CNPV_{S_i \rightarrow S_j}$) matrix. For a certain future scenario n and period t, the $CNPV_{S_i \rightarrow S_j}$ represents the value of investing and moving from S_i to S_j in path III, and it is computed as the sum of the rigid NPV ($RNPV_{S_i \rightarrow S_j}$) –as explained in section 3.2.2– and the continuation value ($CV_{\mathrm{S_j} \rightarrow \mathrm{S_k}}$). Thus,

$$CNPV_{S_i \rightarrow S_j}(n,t) = RNPV_{S_i \rightarrow S_j}(n,t) + CV_{\mathrm{S_j} \rightarrow \mathrm{S_k}}(n,t) \tag{20}$$

The continuation value ($CV_{\mathrm{S_j} \rightarrow \mathrm{S_k}}$) represents the flexibility to expand afterwards and is quantified as the expected net present value generated by moving from a given state to any higher state in the same path during the future. In other words, moving to a particular PV-Storage system confers the household the right and not the obligation to continue investing afterwards to move to higher capacity levels of PV modules and batteries under favorable future conditions. As explained before, the continuation value ($CV_{\mathrm{S_j} \rightarrow \mathrm{S_k}}$) is computed using the LSM method.[5]

[5] In financial terms, the CLSM method fills the null values of the CV matrix used in LSM for the out-of-money options, using the same conditional expectation function used for the CV of the in-the-money options. Thus, out-of-the-money options are not used in the determination of this conditional expectation function.

It is also important to notice that the $CNPV_{S_i \rightarrow S_j}$ matrix depends on the path, because the $CV_{\mathrm{S_j} \rightarrow \mathrm{S_k}}$ values all possible future expansions in this path. For instance, for a path that has two transitions, as path III in Table 1 ($S_i \rightarrow S_j$ and $S_j \rightarrow S_k$), the continuation value from moving from S_i to S_j (first transition), only considers the option to expand from S_j to S_k (second transition). However, if a path has three transitions (e.g., $S_i \rightarrow S_j, S_j \rightarrow S_k$, and $S_k \rightarrow S_l$), the continuation value of moving from S_i to S_j (first transition), will consider the value of the option to expand from S_j to S_k (second transition), and also the value of the option to continue expanding from S_k to S_l (third transition). When the transition is to the terminal state of the path (e.g., S_k for path III), there are no future expansion possibilities and, therefore, the continuation value is zero.

Step II:

The CLSM calculates the optimal investment time for every transition applying the LSM method with the $CNPV$ matrix (as the expected payoff from immediate exercise of the option) instead of applying the method with the $RNPV$ matrix, as in the case that there is no flexibility to expand. If a path is composed by multiple transitions (e.g., path III: $S_i \rightarrow S_j$ and $S_j \rightarrow S_k$), there will be one optimal investment time per transition.[6] Then, the flexible NPV vector of an investment path is simply the sum of the discounted values of the elements of the rigid NPV matrices in the optimal investment times of each of the transitions within this path. Thus, for example, the flexible NPV of path III ($FNPV_{S_i \rightarrow S_j \rightarrow S_k}$) in a certain future scenario n where the optimal investment times are $t^*_{S_i \rightarrow S_j}$ and $t^*_{S_j \rightarrow S_k}$ is:

$$FNPV_{S_i \rightarrow S_j \rightarrow S_k}(n) = RNPV_{\mathrm{S_i} \rightarrow \mathrm{S_j}}\left(n,\, t^*_{S_i \rightarrow S_j}\right) \cdot \mathrm{e}^{\left(-r \cdot t^*_{S_i \rightarrow S_j}\right)} + RNPV_{\mathrm{S_j} \rightarrow \mathrm{S_k}}(n,\, t^*_{S_j \rightarrow S_k}) \cdot \mathrm{e}^{(-r \cdot t^*_{S_j \rightarrow S_k})} \tag{21}$$

[6] For example, for path III that has two transitions ($S_i \rightarrow S_j$ and $S_j \rightarrow S_k$), the CLSM method computes the optimal investment time of the second transition on the condition that it has to be done after the optimal investment time of the first investment.

Finally, after computing the optimal investment times and NPVs of all possible paths, the CLSM selects, in each future scenario, the best path by maximizing the NPV. Therefore, the CLSM selects the optimal path in each future scenario n as:

$$OPNPV(n,1) = Max\left[FNPV(n,1)_{S_i\rightarrow S_j\rightarrow S_k};\ FNPV(n,1)_{S_i\rightarrow S_j};\ FNPV(n,1)_{S_i\rightarrow S_k}; 0\right] \tag{22}$$

where the element $(n,1)$ of the optimal path NPV matrix ($OPNPV$) is the value of the maximum flexible NPV of all paths in scenario n. The expected value of having the option and not the obligation to move from S_i to S_j or from S_i to S_k, in a direct or compound way, at any time between 0 and T_{inv} is the average NPV of the $OPNPV$ vector.

4. CASE STUDY: RESIDENTIAL INVESTMENTS IN THE CHILEAN MARKET

We use the Chilean electricity market as a case study. We assume a rational household interested in reducing the cost of her/his electricity bill through the implementation of a residential PV-Storage system. The household has five different investment possibilities as the result of some combinations of two levels of power capacity from solar PV modules (P_{min} and P_{max}) and two levels of storage capacity from batteries (B_{min} and B_{max}); see Figure 5. The household energy consumption is considered, on average, 577 kWh per month (i.e., $Q = 577\ kWh$) based on the study of Shaw-Williams et al. (2018).

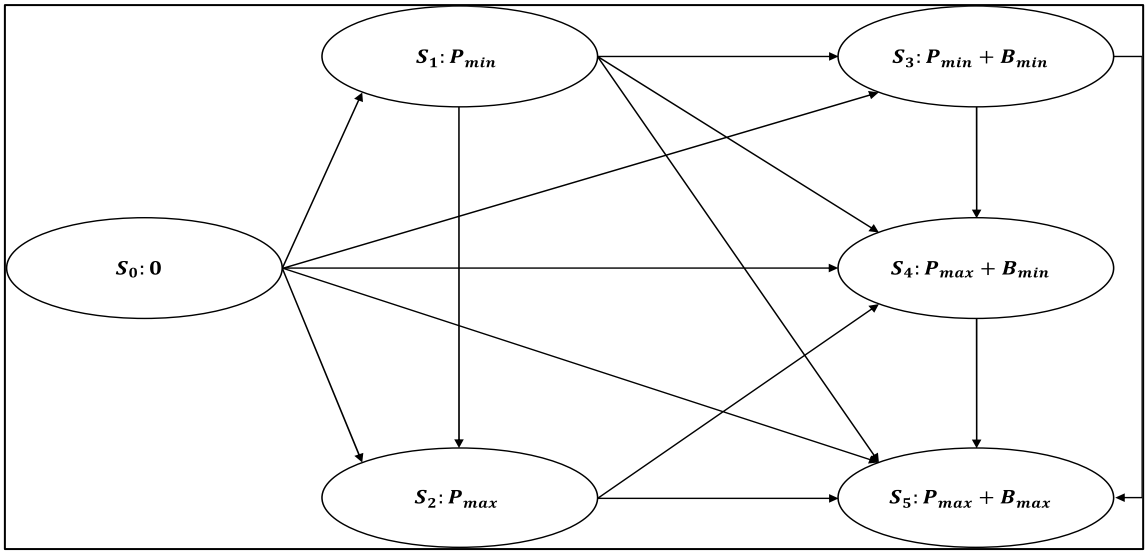


Figure 5: Map of the transitions and states in the case study

The household has the option to invest directly in the following combinations of PV modules and batteries: (i) P_{min}; (ii) P_{max}; (iii) P_{min} + B_{min}; (iv) P_{max} + B_{min}; and (v) P_{max} + B_{max} and remain in that state,[7] or she/he can invest multiple times, always upscaling to states with higher solar PV power production and/or larger battery capacity during the valuation horizon (e.g., invest P_{min} first, then move to P_{min}+ B_{min}, then to P_{max}+ B_{min}, and finally move to P_{max}+ B_{max})[8]. Table 2 shows the 23 possible investment paths of the investment options shown in Figure 5.

[7] For simplicity, the state $P_{min} + B_{max}$ was excluded, because the capacity of B_{max} is designed for P_{max}.
[8] Analogously, the option to abandon was excluded, because the maintenance cost is negligible.

Table 2: Possible investment paths in the case study.

<table>
<tr><th>Path
(1)</th><th>1st Trans.
(2)</th><th>2nd Trans.
(3)</th><th>3rd Trans.
(4)</th><th>4th Trans.
(5)</th><th>Terminal State
(6)</th></tr>
<tr><td>1</td><td>P_{min}</td><td></td><td></td><td></td><td>P_{min}</td></tr>
<tr><td>2</td><td>P_{max}</td><td></td><td></td><td></td><td rowspan="2">P_{max}</td></tr>
<tr><td>3</td><td>P_{min}</td><td>P_{max}</td><td></td><td></td></tr>
<tr><td>4</td><td>$P_{min} + B_{min}$</td><td></td><td></td><td></td><td rowspan="2">$P_{min} + B_{min}$</td></tr>
<tr><td>5</td><td>P_{min}</td><td>$P_{min} + B_{min}$</td><td></td><td></td></tr>
<tr><td>6</td><td>$P_{max} + B_{min}$</td><td></td><td></td><td></td><td rowspan="6">$P_{max} + B_{min}$</td></tr>
<tr><td>7</td><td>P_{min}</td><td>$P_{max} + B_{min}$</td><td></td><td></td></tr>
<tr><td>8</td><td>P_{min}</td><td>P_{max}</td><td>$P_{max} + B_{min}$</td><td></td></tr>
<tr><td>9</td><td>P_{min}</td><td>$P_{min} + B_{min}$</td><td>$P_{max} + B_{min}$</td><td></td></tr>
<tr><td>10</td><td>P_{max}</td><td>$P_{max} + B_{min}$</td><td></td><td></td></tr>
<tr><td>11</td><td>$P_{min} + B_{min}$</td><td>$P_{max} + B_{min}$</td><td></td><td></td></tr>
<tr><td>12</td><td>$P_{max} + B_{max}$</td><td></td><td></td><td></td><td rowspan="12">$P_{max} + B_{max}$</td></tr>
<tr><td>13</td><td>P_{min}</td><td>$P_{max} + B_{max}$</td><td></td><td></td></tr>
<tr><td>14</td><td>P_{min}</td><td>$P_{min} + B_{min}$</td><td>$P_{max} + B_{max}$</td><td></td></tr>
<tr><td>15</td><td>P_{min}</td><td>$P_{min} + B_{min}$</td><td>$P_{max} + B_{min}$</td><td>$P_{max} + B_{max}$</td></tr>
<tr><td>16</td><td>P_{min}</td><td>$P_{max} + B_{min}$</td><td>$P_{max} + B_{max}$</td><td></td></tr>
<tr><td>17</td><td>P_{min}</td><td>P_{max}</td><td>$P_{max} + B_{max}$</td><td></td></tr>
<tr><td>18</td><td>P_{min}</td><td>P_{max}</td><td>$P_{max} + B_{min}$</td><td>$P_{max} + B_{max}$</td></tr>
<tr><td>19</td><td>P_{max}</td><td>$P_{max} + B_{max}$</td><td></td><td></td></tr>
<tr><td>20</td><td>P_{max}</td><td>$P_{max} + B_{min}$</td><td>$P_{max} + B_{max}$</td><td></td></tr>
<tr><td>21</td><td>$P_{min} + B_{min}$</td><td>$P_{max} + B_{max}$</td><td></td><td></td></tr>
<tr><td>22</td><td>$P_{min} + B_{min}$</td><td>$P_{max} + B_{min}$</td><td>$P_{max} + B_{max}$</td><td></td></tr>
<tr><td>23</td><td>$P_{max} + B_{min}$</td><td>$P_{max} + B_{max}$</td><td></td><td></td></tr>
</table>

We used an investment horizon of 10 years (i.e., $T_{inv} = 10$). Taking into consideration the lifespan of the PV modules and batteries –i.e., 25 and 10 years, respectively (Shaw-Williams et al., 2018)– and that the household can invest in a PV module up to the last year of the investment horizon (i.e., $t = 10$), we considered a valuation horizon of 35 years (i.e., $T = 10 + 25 = 35$).[9] A 5% nominal discount rate is used to discount the future cash flows, as the average value presented in the literature for projects with similar risk (Moon & Baran, 2018; Shaw-Williams et al., 2018; Tervo et al., 2018; Truong et al., 2016). Finally, to represent the uncertainties about the future, we simulated 50,000 future scenarios for each of the GBMs.

Next, in Section 4.1, we describe the parameters used in the computation of the benefits and costs of the multiple paths that the household can invest during the investment horizon. Then, Section 4.2 presents the main results of the case study.

4.1 Parameters Associated to Benefit and Cost Calculations

To compute the benefits, we need the initial value of the electricity price, and the drift and volatility of the electricity price for the GBM, see (1). The initial value of the electricity price for Santiago de Chile is US $0.165 kWh based on statistical data from the main electricity company in Santiago (ENEL, 2019). We used an annual drift of 0.023 equivalent to the projected inflation rate in Chile (Chilean Government, 2019), and a volatility of 0.082 based on the volatility of historical Chilean electricity prices (ENEL, 2019). Additionally, we need the bill cost savings' percentages for each state to compute the benefits generated by the installation of PV modules and batteries, see (4). We used PV-Storage combinations and bill cost savings consistent with those used by Shaw-Williams et at. (2018).[10] Table 3 summarizes the values used in the calculation of benefits.

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[9] For example, if for certain future scenario the last investment is in the ninth year, the household will remain in that PV-Storage system for 26 more years.

[10] We assumed a constant household demand. Then, the bill saving percentages remain constant during the valuation horizon.

Table 3: Parameters used for computing the benefits (BCS means Bill Cost Savings).

Parameter (1)	Value (2)	Parameter (1)	Value (2)
Initial Elect. Price	$0.165 USD kWh	B_{max}	13.5 Kwh
α_e	0.023 for $t\in[0,10]$	$BCS(P_{min})$	45%
α_e	0 for $t\in[10,T]$	$BCS(P_{max})$	65%
σ_e	0.082	$BCS(P_{min}+B_{min})$	60%
P_{min}	3 Kw	$BCS(P_{max}+B_{min})$	78%
P_{max}	5 Kw	$BCS(P_{max}+B_{max})$	83%
B_{min}	4 Kwh		

To compute the costs, we need the initial setup cost of the PV modules and batteries, and the drifts and volatilities of their corresponding GBMs, see (2) and (3). Additionally, we need the lifespans of PV modules and batteries to determine the Renovation Cost and Salvage Values, see (6).

PV modules set up costs are the ones offered by the main solar company in the Chilean market (ENEL X, 2019). The initial investment cost is $6,290 and $9,440 for P_{min} and P_{max}, respectively, and it costs $5,090 to expand from P_{min} to P_{max}. Based on the study of Moon and Baran (2018) in the US for panels of 3 Kw and 5Kw, we used a drift of -0.061, and a volatility of 0.0659 for the corresponding GBMs. Finally, we used a lifespan of 25 years for all PV modules based on (Moon and Baran, 2018).

Battery costs are based on Tesla's Powerwall battery (Tesla, 2019). We follow Tervo et al. (2018) to compute the investment costs for different capacities, considering a capacity cost of $392.86 /kWh, plus $700 for the balance of systems and a residential installation cost of $1,000. Additionally, we add import fees of 26% to import the batteries to Chile (Servicio Nacional de Aduanas, 2019). Therefore, the initial investment costs are $4,120 and $8,820 for B_{min} and B_{max}, respectively, and it costs $6,840 to expand from B_{min} to B_{max}.

For the drift costs of the batteries, we used data from Bloomberg (Bloomberg NEF, 2019), where the expected decrease in battery prices (i.e., investment costs) for 2024 and 2030 is 46% and 65% of its initial value, respectively. Therefore, considering the exponential trajectory that the battery costs have followed historically (Bloomberg NEF, 2019), we use a drift of -0.12 for the first 5 years, -0.07 from the fifth to the tenth year, and zero for the remaining valuation horizon.[11] Analogously, battery cost volatilities are 0.102 based on historical costs showed by Bloomberg (Bloomberg NEF, 2019). Finally, we used a lifespan of 10 years based on the Tesla Powerwall battery (Tesla, 2019). We applied a 5% discount if the household expands the entire system in the same year (i.e., adds PV modules and batteries at the same time), based on multiple quotes in the Chilean market. Table 4 summarizes the information about the costs of the PV modules and batteries.

Table 4: Parameters used for computing the costs.

Parameter (1)	Value (2)	Parameter (1)	Value (2)
Lifespan (Batteries)	10 years	*Lifespan (PV*	25 years
B_{min} *cost*	\$4,120 USD for t=0	P_{min} *cost*	\$6,290 USD for t=0
B_{max} *cost*	\$8,820 USD for t=0	P_{max} *cost*	\$9,440 USD for t=0
$B_{min} \rightarrow B_{max}$ *expansion cost*	\$6,840 USD for t=0	$P_{min} \rightarrow P_{max}$	\$5,090 USD for t=0
α_b	-0.12 for $t \in [0,5]$	α_m	-0.061 for $t \in [0,10]$
α_b	-0.07 for $t \in [5,10]$	α_m	0 for $t \in [10, T]$
α_b	0 for $t \in [10, T]$	σ_m	0.0659 for $t \in [10, T]$
σ_b	0.102		

4.2 Numerical Results

This section presents the main results of the case study. First, it shows the NPV of the flexible project, which is the average NPV of the optimal path selected in each future scenario. Then, it shows the frequency that each path was selected as optimal. Finally, it shows the flexibility value of having the option to postpone and expand the project when compared to a rigid project, in which the household is forced to invest directly in $t = 0$.

[11] Therefore, the expected cost for 2024 is $(1 - 0.12)^5 = 0.53$ and $(0.88)^5(0.93)^6 = 0.35$ for 2030.

The average NPV of the flexible project is \$7,851, which is 746% more profitable than investing in P_{max} + B_{max} in $t = 0$ and 26% more profitable than investing in the most profitable rigid project (i.e., P_{max}) in $t = 0$. Additionally, the first investment or transition is, on average, during the fifth year, and it usually consists of investing in P_{max}. Therefore, our results recommend postponing the initial investment regardless of the preferred PV-Storage system. More importantly, our results suggest that, on average, in 36% of future scenarios it is optimal to invest in two steps or more, taking advantage of the option to postpone part of the investments until more favorable future scenarios occur. For instance, in 32% of the future scenarios, our results suggest investing first in P_{max} and only after certain number of years move to $P_{max} + B_{min}$.

4.2.1 Path Selection

Table 5 shows the paths more frequently recommended to be taken. Column 2 shows the frequency that the path was recommended (i.e., it was optimal) and column 3 shows the expected NPV when a certain path is selected as optimal. Additionally, columns 4 to 6 show the final state of each available transition in each path and their median investments times in parenthesis. For example, the last row of the table shows that the household invests in path 19 in 3.6% of the future scenarios, and she/he moves, on average, from S_0 to P_{max} in period four and from P_{max} to $P_{max} + B_{max}$ in period nine. Finally, column 7 shows the frequency that a given terminal state was recommended as optimal in all future scenarios.

Table 5: Most frequent optimal paths selected by the CLSM method.

Path (1)	Freq. (%) (2)	E(NPV) [$] (3)	1st Transition (4)	2nd Transition (5)	3rd Transition (6)	Terminal State Freq. (7)
1	0.3	426	P_{min} (10)			P_{min} (0.3 %)
2	39.9	5,009	P_{max} (5)			P_{max} (39.9%)
4	0.2	1,183	$P_{min} + B_{min}$ (10)			$P_{min} + B_{min}$ (0.2%)
6	23.4	8,380	$P_{max} + B_{min}$ (7)			$P_{max} + B_{min}$ (55.5%)
7	0.2	8,304	P_{min} (1)	$P_{max} + B_{min}$ (10)		
8	0.1	7,157	P_{min} (2)	P_{max} (9)	$P_{max} + B_{min}$ (10)	
10	31.6	10,365	P_{max} (3)	$P_{max} + B_{min}$ (10)		
11	0.21	9,363	$P_{min} + B_{min}$ (6)	$P_{max} + B_{min}$ (8)		
12	0.1	15,482	$P_{max} + B_{max}$ (7)			$P_{max} + B_{max}$ (3.9%)
17	0.1	20,328	P_{min} (1)	P_{max} (7)	$P_{max} + B_{max}$ (8)	
19	3.6	14,109	P_{max} (4)	$P_{max} + B_{max}$ (9)		

Our results suggest that the household should invest in PV modules during the next ten years in 99.8% of the future scenarios (i.e., S_0 is selected as optimal only 0.2% of the time). Moreover, our results recommend implementing only PV modules in 40.2% of future scenarios and PV modules with batteries in 59.6% of future scenarios (see column 7). The two most common terminal states are: $P_{max} + B_{min}$, which is optimal in 55.5% of the future scenarios, and P_{max}, which is recommended 39.9% of the time.

Regarding the paths, the three most recommended paths are paths 2, 6, and 10. Path 2 (i.e., to invest only in P_{max}) is optimal in 39.9% of the future scenarios, investing on average during the fifth year with an expected NPV of $5,009. Path 6 (i.e., to invest directly in $P_{max} + B_{min}$) is recommended in 23.4% of the future scenarios, investing on average during the seventh year with an expected NPV of \$8,380. Finally, path 10 (i.e., to invest first in P_{max} and then add B_{min}) is recommended in 31.6% of the future scenarios, investing on average during the third and tenth year, respectively, with an expected NPV of \$10,365.

Non-tabulated results show that during favorable future scenarios, when the benefits are high because of an increase in the electricity price and the costs are low because of a decrease in the costs of the modules and batteries, the household invests in states with a large capacity of PV modules and batteries. This explains why the household usually invests in $P_{max} + B_{min}$ instead of $P_{max} + B_{max}$, because the benefits generated by $P_{max} + B_{min}$ and $P_{max} + B_{max}$ differ by only 5% (see Table 3), but the cost of B_{max} is on average more than twice the cost of B_{min} (see Table 4). Therefore, $P_{max} + B_{max}$ is only recommended in extremely favorable scenarios when the extra 5% of benefits justifies the much larger extra investment.

On the other hand, during unfavorable scenarios, when the electricity price decreases in combination with an increase in the costs of PV modules and/or batteries, the household usually invests in P_{max}, because it generates a 65% bill cost savings compared to the base case and without needing a storage device. P_{min} is only recommended in extremely unfavorable scenarios because it is the state with the highest relative profitability, but it generates few absolute benefits due to its low capacity.

4.2.2 Transitions

Figure 6 shows the most recommended transitions between states. The width of the arrows between states is proportional to the accumulated frequency that the household decides to move between them. As Figure 6 shows, most of the time, our results recommend investing directly into P_{max} or $P_{max} + B_{min}$. Additionally, once the household moves to P_{max}, it usually waits until the battery costs drop and then move to $P_{max} + B_{min}$.

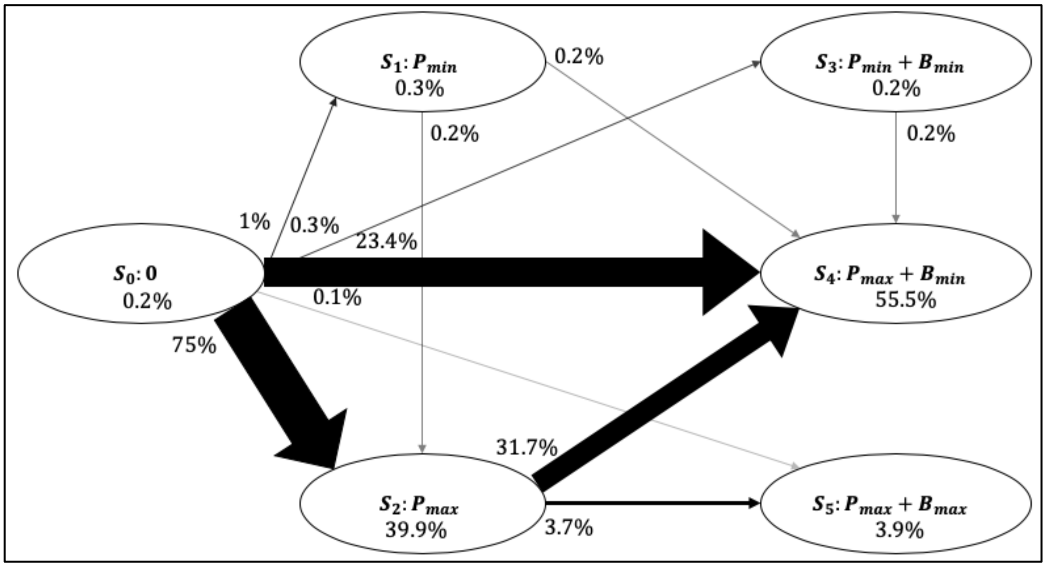

Figure 6: Most recommended transitions (The width of the arrows between states is proportional to the frequency that the household decides to move between them).

4.2.3 Value of Single and Compound Flexibility

We considered single and compound flexibility to compare the results generated by the rigid analysis with the results generated by the LSM, and CLSM methods, respectively. Table 6 shows the rigid, single flexibility and compound flexibility NPVs (columns 2 to 4), the value of the single and compound flexibilities (columns 5 and 6), and the number of paths existing for different PV-Storage systems (column 7). The rigid NPV of a certain state, S_i, is the expected NPV of moving from S_0 to S_i in $t = 0$ (column 2). Additionally, the single flexibility NPV of S_i is the expected NPV of moving from S_0 to S_i in the optimal investment time (column 3). Finally, the compound flexibility NPV of S_i is the expected NPV of investing in any investment path with terminal state S_i or any other state with a lower capacity of PV modules and/or batteries during the investment time (column 4). The single flexibility value of a certain state, S_i, is the difference between the single flexibility NPV (column 3), and the rigid NPV (column 2). The compound flexibility value is the difference between the compound flexibility NPV (column 4) and the rigid NPV (column 2). The number of paths for each PV-storage system S_i is the number of investment paths with terminal state S_i or any other state with a lower capacity of PV modules and/or batteries (column 7). Therefore, the value of the compound flexibility is always larger or equal to the single flexibility value because the compound flexibility includes the option to postpone (i.e., single flexibility).

Table 6: Value of the expected NPVs and flexibilities for each terminal state

	NPV			Flexibilities		
State (1)	Rigid NPV (2)	Single Flex NPV (3)	Compound Flex NPV (4)	Single Flexibility Value (5)	Compound Flexibility Value (6)	Number of Paths (7)
P_{min}	4,589	5,212	5,212	623	623	1
P_{max}	6,250	7,341	7,342	1,091	1,092	3
$P_{min} + B_{min}$	2,753	5,464	5,827	2,711	3,075	3
$P_{max} + B_{min}$	4,108	7,337	7,840	3,229	3,732	11
$P_{max} + B_{max}$	-1,216	5,905	7,851	7,121	9,067	24

As Table 6 shows, the value of the single flexibility of P_{min} is \$623, and it is the amount of money that the household would be willing to pay for having the option to wait rather than investing today in P_{min}. Additionally, the compound flexibility value of P_{min} is also \$623, because there is no extra flexibility since there is only one possible path from S_0 to P_{min} (see column 7). In the same way, the single flexibility value of $P_{min} + B_{min}$ is \$2,711. However, in this case, the compound flexibility is \$3,075, since the household has the option to postpone the direct investment, but it also has the option to invest in any of the 3 paths with terminal state $P_{min} + B_{min}$ (i.e., Path 4 and 5) or P_{min} (i.e., Path 1) during T_{inv} instead of directly moving from S_0 to $P_{min} + B_{min}$ in $t = 0$.

As expected, the value of compound flexibility increases with the number of possible investment paths. For example, the compound flexibility values of $P_{max} + B_{max}$ and $P_{max} + B_{min}$ consider 24 and 11 possible investment paths, respectively, and the difference between the value of the compound and single flexibilities of $P_{max} + B_{max}$ (i.e., $ 1,946) is 3.9 times the same difference in the case of $P_{max} + B_{min}$ (i.e., $ 503).

5. SENSITIVITY ANALYSIS

In this section, we conduct some sensitivity analyses to assess the impacts of variations in certain parameters on the NPV of the PV-Storage project, the distribution of the terminal states, and the value of flexibility. Specifically, we analyze the impacts of varying by plus and minus 25% the following parameters: (i) Drift of the PV modules cost; (ii) Drift of the batteries cost; (iii) Drift of the electricity price; (iv) Bill cost savings for all states; and (v) Volatility of the PV modules costs, batteries costs, and electricity price.

Table 7 shows the main results of these analyses. Column 2 shows the variations in the parameters and column 3 illustrates the expected flexible NPV with each variation. Then, columns 4 to 9 show the distribution of the terminal states. Additionally, columns 10 and 11 show the initial investment time and the frequency of compound optimal paths, respectively. Finally, columns 12 and 13 show the values of the single and compound flexibilities, respectively.

Table 7: Results of the sensitivity analyses.

			Frequencies [%]								Flexibilities	
(1)	Variation (2)	NPV [$] (3)	S_0 (4)	P_{min} (5)	P_{max} (6)	$P_{min}+B_{min}$ (7)	$P_{max}+B_{min}$ (8)	$P_{max}+B_{max}$ (9)	Initial Investment Time (10)	Compound Paths (11)	Single Flex Value (12)	Compound Flex Value (13)
Base Case	0%	7,851	0.2	0.3	39.9	0.2	55.5	3.9	6.4	36.0%	7,121	9,067
PV modules cost drift	-25% (α_m=-0.05)	7,389	0.5	0.8	37.8	0.4	56.5	4.1	6.1	35.6%	6,835	8,720
	+25% (α_m=-0.08)	8,296	0.1	0.1	41.6	0.1	54.1	4.1	6.6	37.1%	7,386	9,401
Batteries cost drift	-25% (α_b =-0.09, -0.05)	7,632	0.2	0.4	61.0	0.1	37.4	1.0	6.2	25.9%	7,193	9,856
	+25% (α_b=-0.15, -0.09)	8,129	0.2	0.3	21.0	0.3	68.0	10.3	6.5	40.3%	7,137	8,530
Electricity price drift	-25% ($\alpha_e = 0.02$)	6,386	0.5	0.8	48.8	0.2	47.6	2.0	6.1	29.8%	7,467	9,400
	+25% ($\alpha_e = 0.03$)	9,523	0.1	0.2	30.9	0.1	61.4	7.3	6.6	41.8%	6,807	8,722
Volatility of the drifts	-25% (σ_b=0.08 ; σ_m=0.05; $\sigma_e = 0.06$)	7,714	0.0	0.0	38.0	0.0	61.1	0.9	6.6	36.8%	7,120	8,925
	+25% (σ_b=0.13 ; σ_m=0.08; $\sigma_e = 0.10$)	8,037	1.0	1.0	39.6	0.5	49.5	8.3	6.2	36.1%	7,276	9,247
Bill cost Savings	-25%	4,600	1.6	1.8	58.3	0.3	37.3	0.6	8.1	18.5%	9,460	11,133
	+25%	11,495	0.0	0.1	28.6	0.2	56.0	15.1	3.9	48.2%	5,180	7,457

5.1 PV Module Drift Costs

Increasing and reducing the drifts by 25%, change the cost of the modules by 8% and -9%, respectively, and affect the NPV of the flexible project on average by 6% and -6%, respectively (see column 3). Additionally, changing the drift of the PV modules costs does not significantly affect the distribution of the terminal states, because all states have PV modules and, therefore, all states increase or decrease their NPV accordingly (see columns 4-9).

Regarding the investment time, an increase in the drift of the costs of the PV modules slightly postpones the first investment since the household decides to defer the investment until costs decrease (see column 10). Additionally, variations in the drift do not significantly affect the average number of transitions of the optimal paths (see column 11).

Finally, increasing the drift of the cost of the PV modules increases the value of single and compound flexibilities, since the household prefers to postpone the initial investment waiting for lower future costs (see column 12 and 13).

5.2 Battery Drift Costs

Increasing and reducing the drifts by a 25%, changes the costs of the batteries by 11% and -9%, respectively, and affect the NPV of the flexible project on average by 3% and -2%, respectively (see column 3). Additionally, a higher drop in battery costs (i.e., an increase in the drift of the battery cost) generates more future favorable scenarios for the states with storage and, therefore, it increases the frequency of the terminal states with batteries from 60% to 79% when compared to the base case (see columns 7-9). In contrast, when the household expects a smaller drift, it invests in batteries only in 39% of the future scenarios.

Regarding the investment time, varying the drift in the cost of the batteries does not affect the initial investment time significantly, because the first investment usually consists of only PV modules (see column 10). Additionally, increasing the drift in battery costs increases the frequency of compound paths selected as optimal, since the household is encouraged to invest in batteries during the investment time, and these investments are usually made as expansions from other states with only PV modules (see column 11).

Finally, varying the drift in the battery cost does not considerably affect the value of the single flexibility because it increases both the rigid and the single flexibility NPVs approximately by the same amount (see column 12). In contrast, by increasing the drift of the battery cost, the value of the compound flexibility decreases, because, on the one hand, the NPV of investing in t=0 increases considerably due to the decrease in renovation costs, and on the other hand, the optimal flexible NPV only increases in 79% of future scenarios, when the household decides to invest in PV modules and batteries (see column 13).

5.3 Electricity Drift Price

Increasing and reducing the drift of the electricity price by 25% affects the benefits of each state directly –see (5)– and affects the NPV of the flexible project on average by 21% and -19%, respectively (see column 3). Additionally, increasing the drift in the electricity price (i.e., benefits) justifies larger investments in PV-Storage systems and, therefore, it increases the frequency of terminal states with PV modules and batteries from 60% to 69% when compared to the base case (see columns 7-9). In contrast, when the household expects lower prices, it invests in PV modules and batteries in only 50% of future scenarios.

Regarding the investment time, an increase in the drift in the price of electricity postpones the first investment slightly since the household decides to defer the investment until costs decrease (see column 10). Additionally, an increase in the drift promotes the household to invest in higher capacities of PV modules and batteries and, therefore, it increases the average number of transitions of the flexible project, since the household usually invests in a compound way towards the highest states (see column 11).

Finally, increasing the drift in the price of electricity decreases the value of single and compound flexibilities, since it increases the expected NPV of investing in any state in $t = 0$ (i.e., rigid project). Consequently, the amount of money that the household would be willing to pay for having the option to wait and invest in a compounded way decreases (see columns 12 and 13).

5.4 Volatility

Increasing and decreasing the volatility does not change the expected values of the PV modules costs, batteries costs, and electricity price. However, being consistent with real options theory, where the NPV of a project increases with higher levels of uncertainty, increasing and decreasing the volatility affect the NPV of the flexible project on average by 2% and -2%, respectively (column 3), since volatility increases the frequency of extremely favorable and unfavorable future scenarios (i.e., with high or low NPV, respectively). Therefore, higher volatility also increases the frequency of extreme states (i.e., $P_{max} + B_{max}$ and S_0) (see columns 4 and 9).

Regarding the investment time, volatility does not significantly affect the initial investment time (column 10), nor the frequency of compound paths (column 11), since it does not change the expected values of the PV modules costs, batteries costs, and electricity price.

Finally, consistently with real options theory, increasing the volatility increases the value of single and compound flexibilities (see columns 12 and 13).

5.5 Bill Cost Savings

Increasing and reducing the bill cost savings per state by 25% affects the benefits of each state directly –see (5)– and the NPV of the flexible project on average by 46% and -42%, respectively (see column 3). Additionally, increasing the bill cost savings per state justifies larger investments in PV-Storage systems and, therefore, it increases the frequency of terminal states with batteries from 60% to 71% when compared to the base case. In contrast, when the household expects fewer bill costs savings per state, it invests in batteries only in 38% of the future scenarios (see columns 4-9).

Regarding the investment time, high and low levels of bill cost savings accelerate the initial investment on average by 2.5 and -1.7 years, respectively, compared to the base case (see column 10). Additionally, an increase in the bill cost savings promotes the household to invest in higher capacities of PV modules and batteries and, therefore, it increases the average number of transitions of the flexible project, since the household usually invests in a compound way towards the highest states (see column 11).

Finally, analogously to varying the drift in the electricity price, increasing the bill cost savings per state decreases the value of single and compound flexibilities (see columns 12 and 13).

5.6 Comparative Analysis

In summary, the two most sensitive parameters regarding the NPV are the bill cost savings and the electricity price because they directly affect the benefits of all PV-Storage systems. In contrast, the expected NPV is less sensitive to variations in the drifts of the costs of the PV modules or batteries, because they are only part of the total cost of the PV-Storage system. Finally, variations in the volatility slightly affect the NPV, since they do not change the expected values of the costs and benefits per state. Figure 7 illustrates the impact of each parameter on the expected NPV of the flexible project.

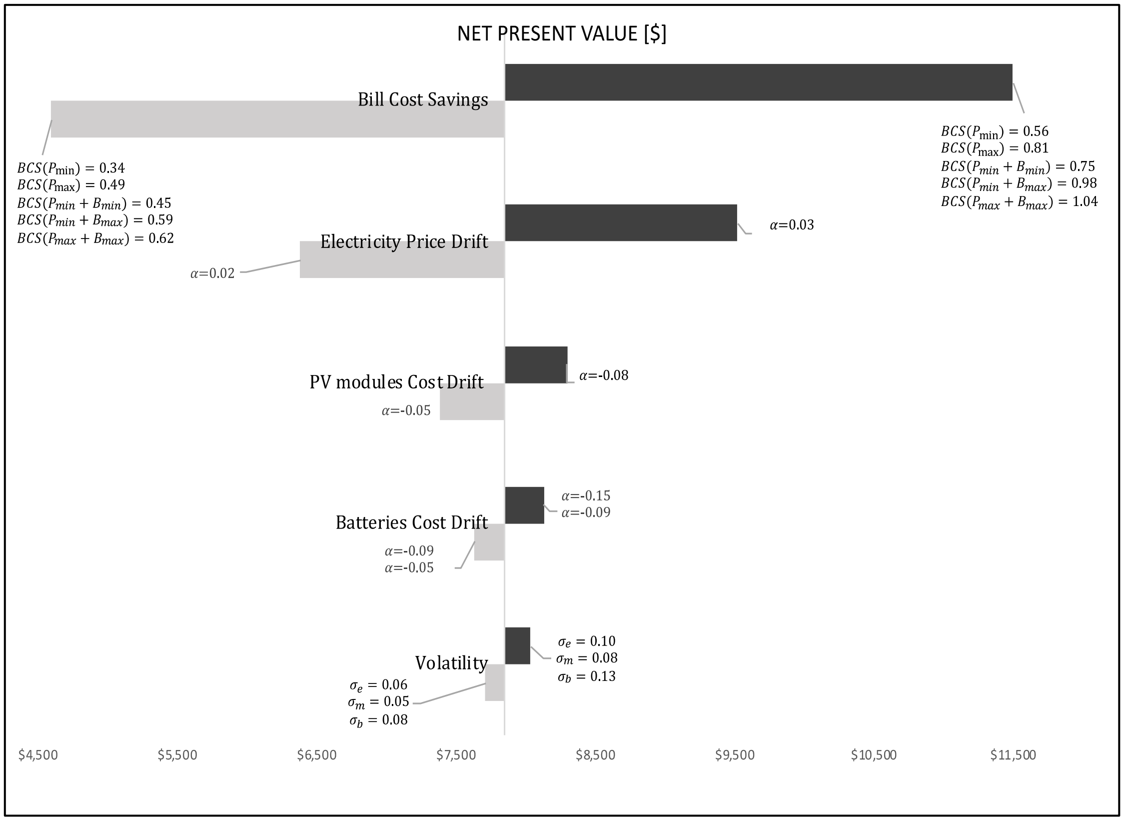

Figure 7: NPV of the flexible project under different values of the parameters.

6. DISCUSSION AND CONCLUSIONS

Residential PV-Storage systems may produce economic and operational benefits to the owner. The willingness of households to privately invest in a PV-Storage system depends primarily on their economic valuation. Recent studies are still discrepant on the profitability of residential PV-Storage projects, mainly due to large uncertainties regarding the future of energy prices and the cost of PV modules and batteries. Therefore, it is necessary to incorporate in the economic valuation of residential PV-Storage systems the flexibility of postponing and investing in a compounded way, so the household has the option to react differently under various possible future scenarios. We valued these flexibilities using a Compound Real Options Analysis.

This paper estimates the value of flexibility and optimal investment path using a novel method called Compound Least Square Monte Carlo (CLSM), which takes into account that the household has the flexibility of investing in multiple stages. We consider that the household has different investment possibilities as the result of some combinations of the possible levels of power capacity from PV modules and the possible levels of storage capacity from batteries. During the investment horizon, the household has multiple possible investment paths, made by one or more transitions between PV-Storage systems. At any time, the household has the option to invest, and with each new investment, a different range of future possible investments becomes available. We applied the proposed methodology to analyze the economic viability and the optimal investment path of residential PV-Storage systems in Chile.

The application of the CLSM method to residential PV-Storage systems in Chile offers interesting new insights compared to other antecedents. Our results show that the expected NPV of the compound flexible project is 26% more profitable than investing in the most profitable rigid project in $t = 0$. Additionally, the household invests in PV-modules and batteries in 60% of possible future scenarios, investing most of the time in the maximum power capacity of PV modules (P_{max}), and the minimum storage capacity of batteries (B_{min}). Regarding the flexibilities considered by our method, our results suggest that, on average, in 36% of the future scenarios it is optimal to invest in multiple steps, taking advantage of the compound option to postpone part of the project until more favorable future scenarios occur. Our sensitivity analyses show that when the household invests in states with higher capacity PV modules and batteries it does so in a compounded way, because it invests first in PV modules and then adds the batteries. Finally, consistently with real options theory, high levels of uncertainties increase the value of the compound flexible project.

In future studies we propose to expand the implementation of the CLSM method to other combinations of residential PV-Storage systems and also to dynamic economies and investment behaviors. Additionally, we propose to use other stochastics models, instead of GBM, to capture different types of learning curves when modeling the costs of

PV modules and Batteries. Alternatively, the value of the compound flexibility demonstrated in this work suggests that the methodology can be extended to value other types of multi-stage projects, especially if they are exposed to high uncertainties, since considering only the option to postpone could promote sub-optimal investments decisions.

7. ACKNOWLEDGMENTS

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