PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
SCHOOL OF ENGINEERING

MANAGING UNCERTAINTY IN AGROFORESTRY PROBLEMS: APPLICATIONS OF OPERATION RESEARCH MODELS AND METHODOLOGIES IN THE WINE AND FORESTRY INDUSTRIES

MAURICIO ANDRÉS VARAS VALDÉS

Thesis submitted to the Office of Graduate Studies
in partial fulfillment of the requirements for the degree of
Doctor in Engineering Sciences

Advisor:
SERGIO MATURANA VALDERRAMA

Santiago de Chile, January 2016.

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Tesis enviada a la Dirección de Investigación y Postgrado en cumplimiento parcial de los requisitos para el grado de Doctor en Ciencias de la Ingeniería.

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RESUMEN

Esta investigación se centra principalmente en la aplicación de modelos y metodologías de investigación operativa a problemas que surgen en la industria forestal y de la agricultura. A pesar que los modelos desarrollados difieren tanto en su naturaleza como en el ámbito de aplicación, éstos comparten el mismo objetivo: incrementar la efectividad de los procesos productivos modelados al incorporar la incertidumbre que afecta su desempeño.

En el segundo capítulo de esta tesis se considera el problema de programación de la producción en aserraderos sujeto a incertidumbre. En particular, el modelo propuesto por Maturana et al. (2010) es reformulado con el fin de hacer frente a la incertidumbre tanto en el abastecimiento de materias primas como en las órdenes de los productos. A través de la aplicación del enfoque de optimización robusta propuesto por Bertsimas & Sim (2004), se desarrollan tres modelos robustos: en el primero, sólo se toma en cuenta la incertidumbre en la demanda; en el segundo, sólo incertidumbre en el abastecimiento; y en el tercero, ambas fuentes de incertidumbre son consideradas. En cada uno de ellos, varios experimentos son realizados con el fin de evaluar la robustez de las soluciones, el efecto del nivel de conservatismo en la función de costos y en impacto de la incertidumbre en la programación del aserrado. Además, y mediante simulación de Monte Carlo, se plantean varias estrategias para seleccionar un adecuado nivel de conservatismo con implicancias en la toma de decisiones.

En el tercer capítulo se considera la problemática asociada a la gestión de vinos premium y se desarrolla una heurística de dos etapas con el fin de establecer niveles adecuados de inventarios cuando una política (*s-1, s*) se emplea para la gestión de *stock*. Se asume que las órdenes por productos llegan de acuerdo a procesos de Poisson independientes; que los inventarios de vinos etiquetados son revisados en forma continua; que hay sólo una máquina que etiqueta; y que el proceso

de etiquetado se realiza sólo si existen órdenes por productos. Considerando la minimización de la suma de los costos estacionarios de productos en proceso, en inventario y faltantes por unidad de tiempo, la heurística opera como sigue: en la primera etapa, el proceso de etiquetado es modelado como un sistema *polling* asimétrico que opera bajo una política de servicio exahustiva y se emplea el enfoque de Winands et al. (2006) para obtener los tiempos medios de espera por producto. Luego, en la segunda etapa se aplica el teorema de Palm (Palm, 1938) que reduce el problema global a la resolución de un problema del vendedor de diarios por cada tipo de etiqueta. La heurística se expone en forma sucinta mediante ejemplos numéricos y también se evalúa su precisión a través de un modelo de simulación.

En el cuarto capítulo se analiza la estrategia de *postponement* como herramienta para hacer frente a la incertidumbre en las órdeners por productos en las viñas. El objetivo de este trabajo, que se basa en el desarrollado por Cholette (2009), es evaluar bajo qué condiciones operacionales un modelo de planificación de la producción (táctico) que opera desacoplando las líneas de etiquetado y embotellado, entrega mejores planes de producción que otro, el cual sólo considera inventario de productos etiquetados. Ambos modelos de programación entera mixta son analizados bajo un marco de horizonte rodante considerando diversas condiciones de holgura en la capacidad productiva, longitud del horizonte y grados de incertidumbre por los productos. Con significancia estadística se muestra que el nivel de beneficios derivados del uso de *postponement* son significantes, especialmente, cuando la holgura productiva es baja, el horizonte de planificación es extenso y el grado de incertidumbre es alto.

Finalmente, en el quinto capítulo se presentan las conclusiones de esta tesis donde se describen los hallazgos, las debilidades de las formulaciones y propuestas para trabajo futuro.

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ABSTRACT

This thesis was centered fundamentally on the application of operation research models and methodologies to problems that arise in the forestry and agricultural industries. Although those models differ in their nature and scope, they share a common objective: to increase the effectiveness of the production process modeled through the handling of the environmental (and system) uncertainty that affects its performance.

In the second chapter of this thesis, we considered the problem of scheduling production under uncertainty in a sawmill, where the deterministic model proposed by Maturana et al. (2010) was extended to account for uncertainties in product demand and availability of raw materials. The robust optimization methodology of Bertsimas & Sim (2004) was applied to develop three robust models: the first considers only demand uncertainty; the second considers only supply uncertainty; and the third considers both types of uncertainties. For each of these models, we carried out experiments to evaluate both the robustness of the solutions and the effect of the conservatism level on the solutions. This last allowed us to determine the impact of both sources of uncertainty, and their interaction on the production schedules. We analyzed the behavior of the robust solutions using Monte Carlo simulation. This analysis allowed us to provide several managerial insights that could help schedulers choose the appropriate level of conservatism with respect to each source of uncertainty.

In the third chapter of this thesis, we focused on the management of premium wines and we developed a two-stage heuristic to set the stock levels when an (*s-1,s*) policy is followed for finished goods. Bearing in mind the operations of a small export-focused Chilean winery that we had worked with, we assumed that orders arrive according to independent Poisson processes, inventories of bottled and labeled wines are reviewed continuously in time, a *postponement* strategy is employed for the labeling process, and there is only one labeling machine. Considering that the wine decision

maker is concerned with minimizing the sum of the steady-state expected values of the *WIP*, overage and underage costs per unit time, the heuristic operates as follows: in the first stage, we model the labeling process as a polling system under exhaustive service, and we apply the results of Winands et al. (2006) in order to obtain the mean delays for each end product. Then, in the second stage, we apply Palm's theorem (Palm, 1938), which reduces the stock levels, setting the problem to solving a newsvendor-type problem for each end product. We provided some numerical examples, and we also addressed the accuracy of the proposed heuristic.

In the fourth chapter of this thesis, we focused on the use of labeling postponement as a tool to cope with order uncertainty at wineries. The main objective of our work, which builds off Cholette (2009), was to gain insights about under which conditions a tactical production planning model that allows decoupled bottling and labeling would provide better plans than a benchmark model that only allows holding inventory of finished goods. We tested both MILP models in a rolling horizon framework under different conditions of capacity tightness, horizon length, and demand uncertainty. Our results show that there is little or no benefit in using postponement when there is both a significant slack in resources capacity, and the actual orders are not very different from the forecasted ones. However, when the capacity becomes tight due to large-sized orders, and the order forecast is less accurate, we found that postponement performs better in terms of both inventory and shortage levels, but at the expense of an increase in the setup quantity performed.

Finally, in the fifth chapter we present the conclusions of this thesis summarizing the major findings and shortcomings as well as suggesting future work on a chapter-by-chapter basis.

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1. INTRODUCTION

Operations research models have increasingly been used for helping decision makers with the management of natural resource operations in several areas, such as agriculture, fisheries, forestry and mining. For a recent review of applications in all four sectors, see the work of Bjørndal et al. (2012).

Given both the hard task of managing those natural resource sectors and organizational needs for efficiency, operations research models found, almost sixty years ago, two particularly fruitful research areas where they have proven to be an effective tool for supporting decision makers.

As A. Weintraub & Romero (2006) state, forestry has been an active user of decision support systems based on this kind of models, whereas agriculture is one of the fields in which operations research models were first used and have been most widely applied.

Although forestry and agriculture share common problems such as scarcity, concern for the environmental effects of production, and the need for efficient production processes, they differ in the nature of the resources and the way they are handled, the time horizons considered, the planning and operational processes, and the environmental impacts. Correspondingly, how the production is managed in each sector, and the objectives that decision makers are primarily concerned about, can be quite different.

For example, as Bjørndal et al. (2012) argue, decisions in forestry are centered on the management of plantations and their production activities in order to meet their demands while adhering to supply restrictions. On the other hand, in agriculture, farmers are primarily concerned with how to plant crops and/or raise animals more efficiently in order to increase their profits.

This last, of course, defines the operations research models features and how they will be used to support decision makers.

1.1. Operation research models in forestry

The forest industry plays an important role in the Chilean economy, being the second largest sector after mining. Indeed, at the year 2014 it generated around 2.7% of the GDP, with forest products comprising 8.1% of Chilean total exports and having a valuation of just over 6 billion US dollars.

The Chilean forestry sector is completely private, based mainly on large firms that own pine plantations (some eucalyptus too), and are vertically integrated with pulp plants, sawmills, and paper markets (Epstein et al., 1999). In fact, only two large companies dominate this industry in Chile: Holding Arauco and Forestal Mininco, both having forest plantations and plants for producing cellulose, paper, lumber, and other related products (Maturana et al., 2010).

In forestry, problems addressed by operations research models are often divided according to their strategic, tactical, or operational decision level of application.

At the strategic level, for example, operations research models address two or three tree rotations of 80 years each, within which silvicultural policies, aggregate harvesting, sustainability of timber production, and environmental concerns are considered (Richards & Gunn, 2003). Typically, those models aggregate time, space and tree species to maintain a moderate size, and their use is considered a standard industry practice (Bjørndal et al., 2012).

A successful application in the Chilean forest industry of a long-range strategic planning tool, called MEDFOR, can be found in Epstein et al. (1999). In their work, an optimization model is designed to support several strategic decisions, such as producing timber through a planning horizon of 50 years, maintaining a consistent supply for the industrial plants, estimating the volume of log exports, and selecting silviculture regimes for plantations, among others.

On the other hand, tactical planning in forestry serves as an interface between strategic planning and detailed operational decisions (Church, 2007). As Bjørndal et al. (2012) argue, at this decision level, both tree species and timber products are aggregated, but spatial aspects are considered in detail. Typical horizons encompass decisions related to harvesting already planted trees, which can range from many years for pine or eucalyptus plantations to several decades for slower growing species. Also, decisions involving harvesting relate to location in order to address environmental concerns and how to build infrastructure.

A successful application of a tactical planning model in a Chilean forest firm can be found in Andalaft et al. (2003). In their work, a mixed integer programming model was developed to address decisions about how to integrate road building and harvesting; and the optimization problem is solved through model strengthening and Lagrangian Relaxation methods. Readers interested in the Lagrangian Relaxation Method for solving integer programming problems are referred to Fisher (2004).

Finally, operational decisions in forestry are made with planning horizons ranging from one day to several months, and include harvesting, machine location, and transportation scheduling. The most relevant decisions at the operational level include: which areas should be harvested within the planning horizon; how to cut (or buck) trees into logs so as to efficiently satisfy the required demand in length, diameter, and quality; how to allocate harvesting equipment; and how to haul timber (Bjørndal et al., 2012).

A successful application of operations research models to support operational decisions in forestry can be found in A. Weintraub et al. (1996). Timber transport plays an important part in the overall operational costs of forest industries (at least 40%). Basically, daily transport is based on trucks hauling logs of various dimensions from different forest harvesting locations to destinations such as plants or ports. Given the latter, the authors developed an operative and computerized system, ASICAM, based on a simulation process with heuristic rules, to support daily truck scheduling decisions. The model quickly develops daily schedules for fleets of up to several hundred trucks, and its use has significantly reduced vehicle queuing and transportation costs.

1.1.1. The forest production chain and the role of sawmills

As Bjørndal et al. (2012) state, the forest production chain extends from trees in forests to primary and secondary transformation plants, and then to markets as processed products like paper, panels or simply lumber. Fig. 1.1 shows the basic links of this chain corresponding to harvesting, plants, mills, stockyards, and ports, through which the products above flow.

Being strongly dependent on exports, the forest industry has to ensure that its profits remain competitive so, as Carlsson & Rönnqvist (2005) argue, supply chain management and operations research models have proven to be successful in order to increase the efficiency in several value-added activities that take place in the forest supply chain (or the wood flow chain as well). There have been studies on this last. To cite just a few, the Chilean forest supply chain and models applied on it are described in A. Weintraub & Epstein (2005), whereas the Swedish one is described in Carlsson & Rönnqvist (2005).

Now, as Carlsson & Rönnqvist (2005) state, customer orientation is the focus of the forest industry, which implies that the right kind of products should be delivered to the (internal or external) customer in the right quantity, at the right time, and in the right place. Undoubtedly, in order to meet their product demands at minimum costs, forestry firms face the need to coordinate, especially at the

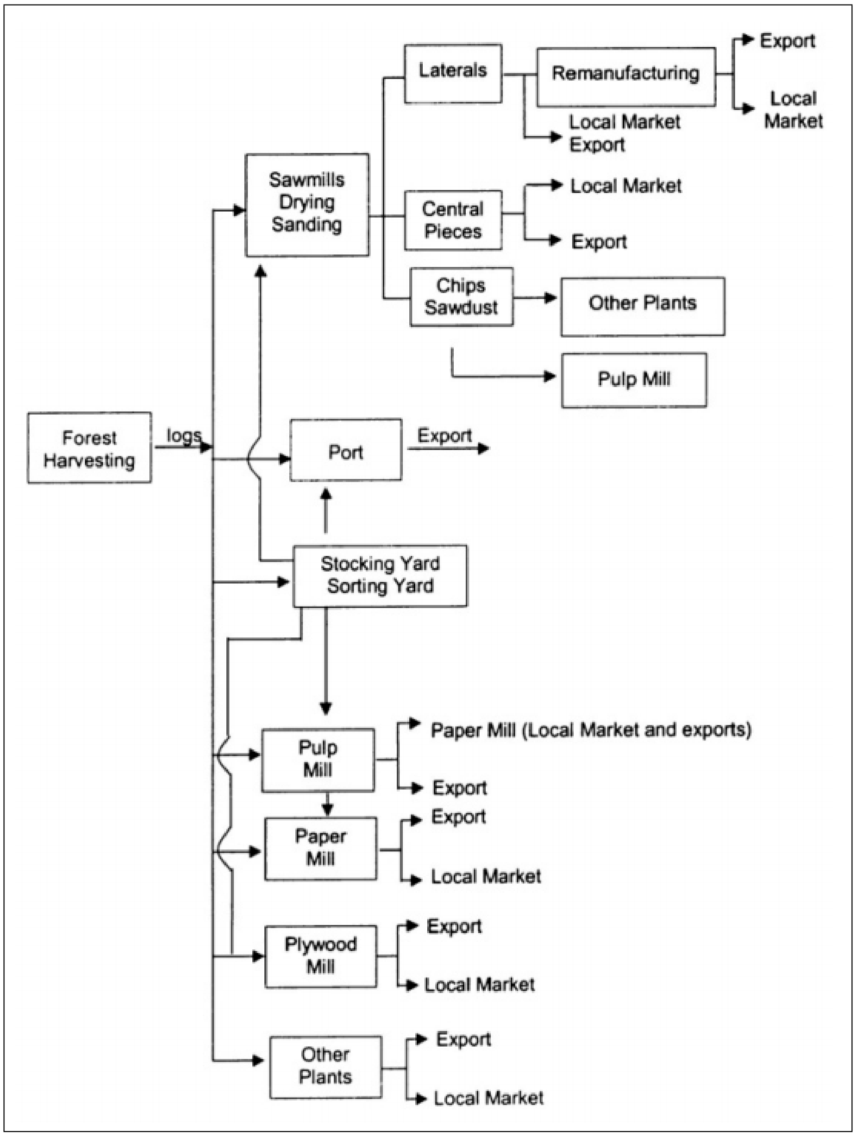

FIGURE 1.1. The physical forest supply chain.
Source: A. Weintraub & Epstein (2005).

operational decision level, the sales contracted with their harvesting, transportation, and production capabilities.

In the last few years, Chilean forestry firms (especially the two large companies that dominate this industry) have significantly increased their exports to many different markets around the world. In particular, lumber has gained more importance and has become more profitable as these companies have become more efficient in their operations. On this last, one of the keys to increase

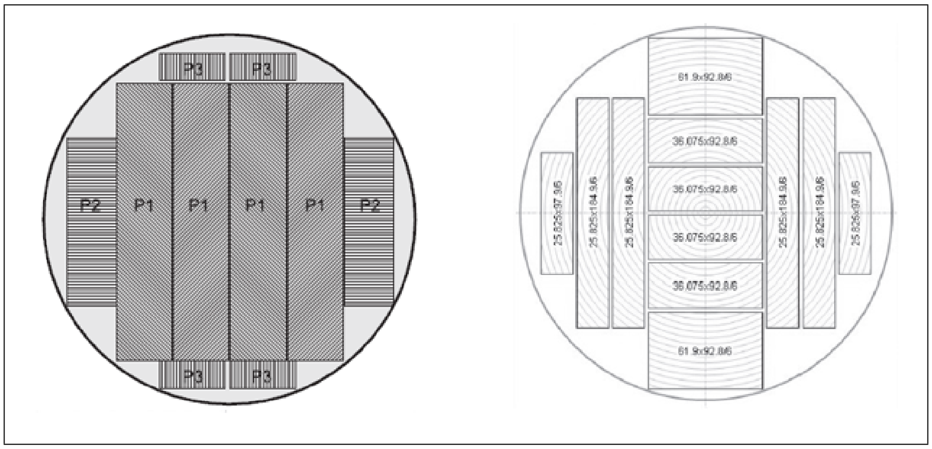

FIGURE 1.2. Examples of cutting patterns.
Source: Maturana et al. (2010).

the efficiency of lumber production and, at the same time, the reliability of the wood flow chain, is improving production scheduling at the sawmills (Maturana et al., 2010).

Planning production at sawmills consists of programming the log pieces to be processed, taking into account, on one hand, the diameter, length, and grade of each log piece and, on the other, the finished goods that are needed to fulfill the orders,. The length of the log piece and the cutting pattern (see Fig. 1.2, which is how the log is cut, determine the products that are generated and also the yield, which is how much of the volume of the log ends as finished goods.

1.1.2. The sawmill production scheduling problem

The problem of scheduling production at sawmills has been studied by different researchers. For example, Singer & Donoso (2007) propose a model for optimizing both production and inventory decisions within a system of plants, where the sawmill production process is modeled in terms of two transformation stages and two inventory stages. Other authors like Todoroki & Rönnqvist (1999), or Winn et al. (2004) have studied the combinatorial problem of how to select optimal cutting patterns for different types of logs. On the other hand, Carnieri & Mendoza (2000) proposed a model for optimizing the problem of cutting boards for dimensioned parts. Todoroki & Rönnqvist (2002) proposed a log sawing optimization system (implemented in a log sawing simulator), which, taking into account the demand for timber products of different qualities, can optimize either the yield or value generated from a given log. Another approach, proposed by A. J. Weintraub et al. (1997), uses simulation to find a feasible schedule for a sawmill, which is used as a complement to an infinite capacity MRP system. More recently, Maturana et al. (2010) have proposed the use of a mathematical programming model to support the development of feasible production schedules, and its performance is tested against a heuristic that was used by a sawmill programmer in a southern Chilean forestry firm. The mathematical model showed superior performance in almost all test instances except two, were both found the optimal solution.

1.1.3. The impact of uncertainty on the development of feasible production schedules

On this last, it is important to point out that the development of feasible production schedules could be a difficult task to support, not only for the inherent complexities of the production process, but also because sawmills schedulers have to deal with many different sources of uncertainty.

For example, as A. Weintraub & Epstein (2005) argue, sales and production departments communicate directly; therefore, demand uncertainties that arise from spot sales are transmitted immediately to the plants. On the other hand, as Epstein et al. (2007) argue, to secure a continuous supply to the mills, forest companies usually have safety stocks of raw materials. Nevertheless, the delivery of logs to particular locations is organized by priorities (A. Weintraub & Epstein, 2005), which implies that if some disruptive events occur, like a change in weather conditions or queues in transports, then the inflow of logs to the sawmills can be severely affected. Finally, there is also variability in the yield coefficients associated with the cutting patterns used (Alvarez & Vera, 2014), which implies that the volume of the logs that ends as finished goods is also uncertain.

Overall, the above can have a significant impact on the production activities since, for example, a decision maker trying to meet an order might not have enough raw material to process; the product might not meet the quality specifications required by the market; or the order might exceed production capacity.

1.1.4. Research on the sawmill production problem under uncertainty

As Mula et al. (2006) argue, models for production planning that do not recognize uncertainty can be expected to generate inferior or even infeasible planning decisions compared to models that explicitly account for it.

In general, the literature presents different approaches for handling uncertainty in optimization models, such as Sensitivity Analysis (SA), Stochastic Programming (SP), and Robust Optimization (RO); and several researchers have applied such approaches in order to cope with the drawbacks of uncertainty in sawmills. Readers interested in a deep discussion about SP and RO methodologies are referred to Birge & Louveaux (2011) and Beyer & Sendhoff (2007) respectively.

For example, Zanjani et al. (2010) proposed a robust production planning approach for the sawmill planning problem that considers uncertainty in the quality of raw materials. Zanjani et al. (2013) proposed a stochastic mixed-integer programming model in order to cope with both randomness in yield and in demand. Finally, Alvarez & Vera (2014) applied the RO methodology to the sawmill planning problem, where variability is assumed in the yield coefficients associated to the cutting patterns used.

1.1.5. The scope of scheduling production for a sawmill: a robust optimization approach

Although there were many sources of uncertainties, the first part of this thesis focused on the development of feasible production schedules, through a mathematical programming model, which takes into account the two main sources that the Chilean forest company that Maturana et al. (2010) worked with faced: demand uncertainty and raw material uncertainty.

Demand uncertainty was important to consider because the Chilean sawmill exported most of its production to different markets all over the world. These markets demanded, in general, different products, and the orders had relatively short lead times since they had to be sent by ship to different ports, most of which were far away. On the other hand, raw material uncertainty was also important since the type of raw material received by the company, most often than not, was not the one requested. The reason is that the raw material came from another company that managed the forest. When the sawmill company requested a certain amount of raw material, the forest company would decide which part of the forest to cut, trying to satisfy the request of the sawmill, but also taking into account the distance the logs would have to travel from the forest to the sawmill. Also, the part of the forest that was cut usually had trees of different diameter, classes, and qualities that did not exactly match those requested by the sawmill. So, although the raw material received was not usually the one the sawmill required, they had to do the best they could with it.

In order to take into account both sources of uncertainty in the optimization model, we chose to use the RO approach instead of SP mainly for one reason: the problem was that the sawmill managers did not have very good data on both sources of uncertainties to allow us to infer probability distributions, which led us to use the RO approach proposed by Bertsimas & Sim (2004) since this method does not require knowledge of the probability distributions. Furthermore, the robust problem remains linear, making it easier to solve using standard optimization tools.

However, although a probability distribution is not needed, we did require that the decision maker provide a value for the budget of uncertainty that will be used, which determines the level of conservatism of the solutions. Since the appropriate budget of uncertainty to use depends on the characteristics of the decision maker and on the characteristics of the problem, we needed to give the decision maker some guidance on how to decide on the uncertainty budget to use. This last led us to carry out a detailed analysis of the effect of the uncertainty budget on the robust solutions, which allowed us to understand how the degree of conservatism affects the solutions of the RO approach in this particular case.

1.2. Operation research models in agriculture

As stated in the introduction, agriculture is one of the fields where operations research was first used and has been widely applied (A. Weintraub & Romero, 2006). Beginning with the work of Waugh (1951), who established least-cost combinations of feeding stuff and livestock rations through linear programming; in the last sixty years, many other agricultural problems have been addressed as well as several OR models and methodologies have been applied.

Particularly, both A. Weintraub & Romero (2006) and Bjørndal et al. (2012) review several operations research models and approaches employed to solve them, whereas Plà-Aragonés (2015), provides an overview of models and methodologies applied recently and successfully in this field.

Operations research models in agriculture can be characterized by several dimensions. For a relevant taxonomy see Plà-Aragonés (2015). On this, the decision problem addressed, the methodology employed, and the level of application of the proposed model are the ones that stand out.

In terms of the decision problem, operations research models can be divided depending on if they address production planning, decision analysis to assess either risk situations or management alternatives, efficiency, sustainability, location, diet issues, and replacement problems. On the other hand, several methodologies have been employed to tackle the respective proposed model; for example, linear programming (LP), stochastic programming (SP), simulation (SIM), metaheuristics (MHEU), multiobjective programming (MO), risk analysis (RA), forecasting (FOR), data envelopment analysis (DEA), multicriteria Analytic hierarchy process (AHP), Markov decision process

TABLE 1.1. Taxonomy for cited papers.

Research	Decision Problem	Methodology	Level
Albornoz & Canales (2006)	Planning	SP	Farm
Moreira et al. (2011)	Efficiency	SPF	National
Toledo et al. (2011)	Risk	AHP	Regional
Albornoz et al. (2015)	Planning	MO	Farm
González-Araya et al. (2015)	Planning	LP	Farm

(MDP), stochastic production frontier (SPF), and so on. Finally, the scope could be at the farm, regional, national or supply chain level of application.

Readers interested in a further discussion about the above methodologies are referred to: Dantzig (1998) for LP, Birge & Louveaux (2011) for SP, Banks et al. (1998) for SIM, Glover & Kochenberger (2003) for MHEU, Ehrgott (2006) for MO, Makridakis et al. (2008) for FOR, W. W. Cooper et al. (2011) for DEA, Vaidya & Kumar (2006) for AHP, Puterman (2014) for MDP, and Aigner et al. (1977) for SPF.

Some recent examples of successful applications of OR models for problems arising or inspired in the Chilean agricultural industry can be found in Albornoz & Canales (2006); Ferrer et al. (2008); Moreira et al. (2011); Toledo et al. (2011); Albornoz et al. (2015); González-Araya et al. (2015). Table 1.1 characterize these research according to the three dimensions defined above.

The authors' work on lobster fishery in Chile is summarized in Albornoz & Canales (2006). The paper presents the formulation and algorithmic resolution of a two-stage stochastic nonlinear programming model with recourse. The proposed model considers a long-term planning horizon and specifically allows an optimal total allowable catch quota to be obtained for the first planning period. This model takes into account biomass dynamics, the conditions guaranteeing sustained species management, and uncertain parameters such as growth rate and species carrying capacity. These parameters are explicitly incorporated via a discrete random variable (scenarios). The proposed model is solved by Lagrangian decomposition using AMPL, in combination with solver MINOS. Also, the article presents the results obtained with this methodology and the conclusions drawn from their seminal work.

In Moreira et al. (2011) the authors estimate and analyze the technical efficiency (TE) component of productivity for a sample of wine grape producers in Chile. The data includes 38 farms with specific input-output information for individual blocks yielding a total of 263 observations. They use a Cobb-Douglas model to estimate a stochastic production frontier and to obtain TE scores both at the individual block and at the farm level. The results suggest that the average farm level TE is 77.2%, while the block level TE ranges from 23.4 to 95.0%.

The objective of Toledo et al. (2011) work was to prioritize risk factors that are highly relevant for farmers in Central South Chile. The multicriteria Analytical Hierarchical Process methodology was used to define a decision structure with four risk factors or criteria: climate, price and direct cost variability, human factor, and commercialization. The results obtained showed that there are no important imbalances in the weightings of different risk factors. Price and cost variability was the most important factor, whereas climate was the least important. Their research also confirmed that there are spatial differences in the weightings obtained for the distinct risk factors that determine distinct risk levels for the respective agricultural activities according to geographic region.

Considering that the process for agriculture planning starts by delineating the field into site-specific rectangular management zones to face within-field variability, in Albornoz et al. (2015), the authors propose a bi-objective model that minimizes the number of these zones and maximizes their homogeneity with respect to a soil property. Then they use a method to assign the crops to the different plots to obtain the best profit at the end of the production cycle subject to water forecasts for the period, humidity sensors, and the chemical and physical properties of the zones within the plot. With this crop planning model, they can identify the best management zones of the previous bi-objective model. Finally, they show a real-time irrigation method to decide the amount of water for each plot, at each irrigation turn, in order to maximize the total final yield. This is a critical decision in countries like Chile where nowadays water shortages are frequent. In their study, they integrate these stages in a hierarchical process for the agriculture planning, and empirically prove its efficiency.

To help planning agricultural work in apple orchards, in González-Araya et al. (2015), the authors propose an optimization model, working through harvest time windows, which incorporates fruit ripeness. The model seeks to minimize labor costs, equipment use, and loss of fruit quality in order to meet the demand of packing plants, considering their processing capacities as well as the production in orchards. The author shows that the application of the proposed model on three apple orchards in the Maule Region, Chile, led to a 16% decrease in both labor costs and loss of income for harvesting fruits with poor quality.

1.2.1. The agri-food supply chain and the case of wine

Although OR models in agriculture have not shown the same particular success that the forestry sector exhibits (which was driven mainly by the fact that forest lands and firms are quite large in both area and financial resources), it is important to point out that agriculture, and particularly the agri-food industry which deals with agricultural products obtained from crops and destined for human consumption- are becoming more capital intensive, and also a more complex business than it traditionally was (Plà-Aragonés, 2015).

Indeed, as Ahumada & Villalobos (2009) state, the emergence of better informed customers that want to have precise information about the farming, marketing, and distribution practices used to bring the agricultural products into the shelves of their supermarkets, has been translated into additional regulations and market-driven standards, which affect both the design and operation of several value-added processes that take place in the agri-food industry. Moreover, this complexity is also increased when the activities from production to distribution encompass more than one country.

Therefore, new customer behaviors in conjunction with the opening to international markets have resulted gradually in shifting the managerial focus from a single echelon, such as the farmer, to the management of the overall agri-food supply chain.

The term agri-food supply chain has been coined to describe the activities from production to distribution that bring agricultural or horticultural products from the farm to the table (Aramyan et al., 2006). Thus, these supply chains are composed by the organizations responsible for the production (farmers), distribution, processing and the marketing of these products to the final customers. For an excellent review of many of the planning models that have been developed in the agri-food supply chain see Ahumada & Villalobos (2009). More recently, Shukla & Jharkharia (2013) also provide a review on what they call the agri-fresh produce supply chain, which focuses on food products that are perishable and have a short shelf life.

One Chilean agri-food product, perishable but with a long shelf life, which exposes a remarkable story (Knowles & Sharples, 2002); immersed in a complex industry that has undergone numerous and profound transformations over the past 30 years (Visser, 2004; Felzensztein, 2014); with a supply chain that is particularly complex given its relationship with the importers (Sánchez Loyola et al., 2008); and which has become the major ambassador of Chile in the minds of foreign customers, is wine.

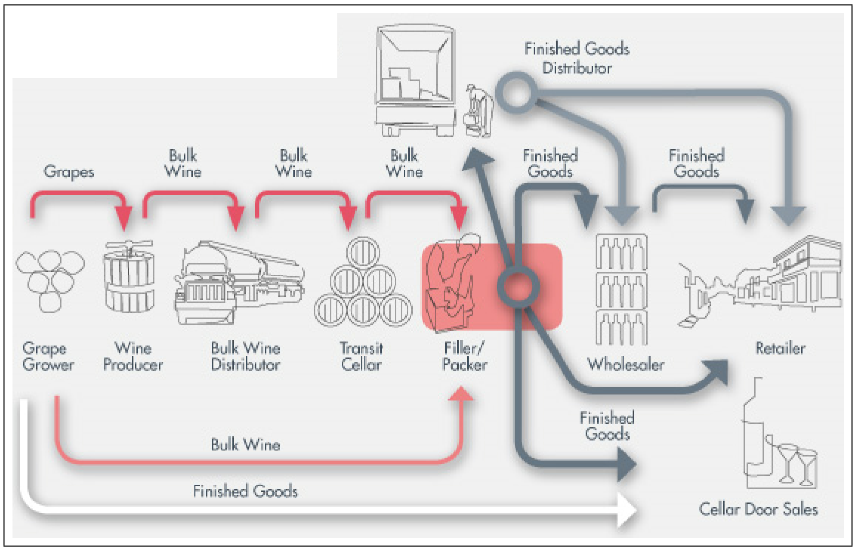

FIGURE 1.3. A representative scenario of the wine supply chain.
Source: http://www.fla.net.au/.

The wine supply chain is depicted in Fig. 1.3. For a discussion about the activities performed by the main actors of this chain see Saglietto et al. (2014, pp. 5). According to Petti et al. (2006), the main stages of this chain, which are described below, comprise: grape production, wine production, packaging, and distribution.

The grape production stage consists of the agricultural operations, such as pruning, tillage pest control activities, harvest, equipment maintenance, etc. Furthermore, transport that occurs within the field operations for workers and products is taken into account in this phase. The wine production stage includes operations such as stemming and crushing, fermentation, and storage. Here, the grape is transformed into wine, by first becoming must, and then –through the fermentation process– wine. Processes included in the clarification of wine comprising racking, fining, filtration, and refrigeration, are also an important part of this stage in order to purify the output, i.e. wine. Finally, storage takes place. In this phase, wine is stored in order to be aged. In the packaging stage, processes such as bottle filling, corking, capsuling, labeling, box filling, and placement on pallets, can be taken into consideration. And the distribution phase is a mainly transport-related one, and can be considered at a local, regional, national, or international level, depending on the strategy and production capacity of the firm.

Research on the wine supply chain has been done before. For example, Adamo (2004) analyzes how current trends in Supply Chain Management are affecting the global wine supply chain, and builds on the specific case of Argentinean wineries that sell their products in the U.S. market. To do so, the author's research starts by analyzing each tier of the supply chain using Porter's Five Forces model in order to understand the characteristics of each tier; how these forces impact the supply chain as a whole; and how companies interact between tiers. Finally, the author describes possible changes in the supply chain configuration due to the adoption of these new trends by organizations along the chain, and describes some of the major aspects that Argentinean wineries should take into account in order to gain a better competitive advantage along the chain.

In Garcia et al. (2012) a comprehensive framework for measuring logistics performance in the wine industry is presented. This framework was created following well-established approaches, including the SCOR model, and includes three abstraction levels that simplify the logistics performance analysis of wine supply chains. The performance measurement model was validated through its application in a benchmarking study of the logistics operations efficiency of six wineries from Mendoza (Argentina), which is presented as a case study.

Finally, Mac Cawley (2014) argues that wine companies face two important challenges in their supply chain: the international shipping temperatures and their effect on the perceived quality of the wine, and the optimization of the bottling schedule. The wine maker takes special care in producing the best quality product, which is then shipped to the importer/distributor or customer, generally in non-refrigerated containers at the mercy of the prevailing environmental conditions. The contribution of the author's work is that it is the first to measure, for a significant period of time, the temperatures along the international wine supply chain and link them to the specific supply chain processes. Also, this is the first research that analyzes the effect of shipping temperature on the perceived quality of the product by those who make the purchase decision for importers, restaurants, and supermarkets. Results indicate that wine is very likely to have been exposed to extreme temperatures during shipping. For white wines, tasters are able to detect differences in wines that have been exposed to shipping temperatures and show a preference towards them. For red wines, they are unable to detect differences.

The wine supply chains all over the world are not exempt from both public scrutiny and the internalization of new regulations and practices that Ahumada & Villalobos (2009) refer to. For example, Cimino & Marcelloni (2012) argue that, in the last decade, several factors have determined an increasing demand for wine supply chain transparency. Indeed, amalgamation, fraud, counterfeiting, use of hazardous treatment products and pollution are affecting the trust of customers, who are more and more oriented to consider the so-called *credence attributes* rather than price. Thus, customers have started to demand a more detailed information on the overall process from the grape to the bottle. Moreover, as Sánchez Loyola et al. (2008) state, nowadays the worldwide wine industry is facing a more competitive scenario as a result of the high number of wine companies trying to access new markets. And in this context, wine importers are the key actors to settle down in those markets. Undoubtedly, their primary need is to establish a relationship with firms that offer a reliable service with appropriate lead times and fill rate. Thus, a proper coordination of the overall wine supply chain that enables both traceability and reliability is a *sine qua non* condition to compete not only in the international market, but also to enhance their positions as incumbents in their national markets.

On the other hand, and according to the Wine Supply Chain Council[1] (WSCC), there are great opportunities to make wine supply chains lean. Although the concept is associated with the world of manufacturing, some of the same issues apply to wine supply chains. Particularly, there are several issues that one can address in order to enhance the wine supply chain competitiveness. These issues include: elimination of waste, management of variability, synchronization and alignment, and continuous improvement.

Elimination of Waste refers to double-handling, such as moving wine among tanks, repacking bottles and repalletizing cartons.

The Management of Variability is an issue of concern because, for example, process time variability means downstream customers must endure occasional stock-outs, or else protect themselves with extra inventory, which represents an additional expense. On the other hand, temperature variability damages the wine by creating piston-like movements of the cork, which admits oxygen into the bottle, which might change the chemical and sensory profile of wines.

Synchronization and Alignment means that all participants in the supply chain must coordinate to match production with consumption. Nevertheless, wine production requires long lead times and depends on unforeseeable factors such as the weather. Furthermore, many of the markets for alcohol are regulated in complex and arbitrary ways. Such factors create dilemmas all along the supply

[1] http://wscc.scl.gatech.edu/

chain. For example, should the winery make-to-stock or make-to-order? Or should it produce unlabeled bottles for later customization?

Finally, Continuous Improvement implies that there must be processes in place to constantly review and evaluate supply chain performance as a whole, and not just the independent enterprises along the chain. Moreover, the supply chain must look ahead to new challenges, such as, in terms of sustainability strategies, the carbon footprint profile of the entire life cycle of wine-making linked with market interactions, or the support, in terms of commercial and logistic strategies, of additional sales channels, which generally have different labeling and packaging requirements.

Several of these issues can be tackled by operations research models and, in the recent years, many applications have been reported either in production (viticulture, harvesting, and winery processes) as well as in post-production activities (Moccia, 2013). In the next subsection, we review some relevant studies.

1.2.2. OR models applied in the wine industry

A couple of examples of relevant applications of operations research models and techniques to the activities performed along the wine supply chains are Cakici et al. (2006); Kolympiris et al. (2006); Berruto et al. (2006); Cholette (2007); Ferrer et al. (2008); Ertugrul & Isik (2009); Hernandez et al. (2009); Mac Cawley (2014).

Regarding viticulture applications, Hernandez et al. (2009) present a routing model to support new viticulture practices aimed at minimizing pesticide spraying in viticulture. Minimization of pesticide spraying induces a complex routing problem for spraying units. Thus, the authors motivate a Multi-Trip Vehicle Routing Problem with Time Windows where the customers are the vineyard parcels to be visited by the spraying units, in given time windows, under capacity constraints, and with a maximal number of simultaneous routes.

In terms of harvesting applications, Ferrer et al. (2008) present a practical tool for optimally scheduling wine grape harvesting operations taking into account both operational costs and grape quality. They solve a mixed-integer linear programming model to support harvest scheduling, labor allocation, and routing decisions. A novel quality loss function is used to represent wine quality reduction at each vineyard block due to premature or deferred harvest with respect to an optimal date. They present computational results that show that the proposed tool could be used to support grape harvest planning in a large vineyard, at both a tactical and operational level.

In terms of winery processes applications, Cakici et al. (2006); Kolympiris et al. (2006); Berruto et al. (2006); Ertugrul & Isik (2009); Mac Cawley (2014) provide relevant operations research models.

Cakici et al. (2006) model and analyze the cellar tank piping network at E. & J. Gallo Winery. The objective is to determine the routing path through the cellar tank piping network that minimizes wine damage and optimizes the resources used. The proposed MILP cannot be solved by a state-of-the-art commercial solver for more than six flows. Hence, the authors propose a heuristic that iteratively solves single-flow problems, and determines an incremental network by a greedy approach. The solution found by this heuristic is then fed into a scheduling model that, with the objective of minimizing the makespan, determines the starting and ending times of flows considering pump availability and other side constraints.

Kolympiris et al. (2006) provide an optimization model which focusing on the capacity utilization of tanks sequence the wine flow through the production process. The model is formulated as a mixed integer program which accounts for winemaking specifications, market conditions, grape availability, and tank capacity. An empirical example is provided to demonstrate results and uses of the model.

Berruto et al. (2006) present a new scheduling method for planning bottling activities in modern wineries. The bottling plant has been assumed as a single-machine job shop where wine orders of different amounts and due dates must be processed. To limit the complexity introduced by the large number of variables and constraints, the sequencing is obtained by means of a two-step procedure based on mixed-integer linear programming algorithms. The optimization takes into account, for each wine type, data on production rates, storage levels, minimum batch size, labor and storage costs, and the risk for not having the minimum storage, which causes lost sales. The method operates on a finite time horizon, typically of four weeks, with a recursive rescheduling at each week. The effectiveness of the proposed method is shown by an example using data collected in a large winery in the Piedmont region of Italy.

Ertugrul & Isik (2009) present a mixed-integer linear programming model for the production planning of a winery in Denizli, Turkey. The winery must decide its production mix considering minimum lot sizes, setup costs of the packaging lines, and other side constraints. The main decision variables are binary and determine whether a type of wine is produced or not. Constraints enforce winery capacities and bounds on the produced quantities. The objective function to be maximized accounts for the net revenue. The model is easily solved by a commercial solver.

In Mac Cawley (2014), the author develops a model that produces solutions for the wine bottling lot-sizing and scheduling problem with sequence dependent setup times, in an adequate time frame, which can be implemented by large wineries. The author also develops a model and algorithm that produces fast, good, and robust solutions for the winery lot-sizing and scheduling problem with sequence dependent setup times. Also, the author implements an effective decomposition algorithm that uses the structure of the problem, which can be applied to other families of sequence dependent scheduling and lot-sizing problem. Results indicate that the model achieves reductions of 30% in the total plan costs.

And finally, in terms of post-production activities (specifically in marketing), Cholette (2007) points out that wineries must find distributors to represent their wines, whereas distributors seek interesting wines to sell to clients. Thus, at trade shows, wineries and distributors often form partnerships. Given the latter, the author creates a system to qualify potential partnerships for one of such trade shows, the World Wine Market, by formulating a mixed-integer program that is an embellished transportation model. The author bases the parameters on answers to a questionnaire obtained from a subset of trade-show attendees who were seeking partners. Of 675 possible matches between wineries and distributors, the program recommended 31 as the optimal set of matches, which resulted in an allocation of just over 50,000 cases of wine.

1.2.3. The impact of uncertainties in wine production activities

Galbraith (1973) defines uncertainty as the difference between the amount of information required to perform a task and the amount of information already possessed. As Mula et al. (2006) argue, in the real world there are many forms of uncertainty that affect production processes.HO (1989) categorizes them into two groups: (i) environmental uncertainty and (ii) system uncertainty. Environmental uncertainty includes uncertainties beyond the production process, such as demand uncertainty and supply uncertainty. System uncertainty is related to uncertainties within the production process, such as operation yield uncertainty, production lead time uncertainty, quality uncertainty, failure of the production system and changes to product structure, to name a few.

Both types of uncertainty could degrade severely the performance of the overall wine supply chain, and some researchers have begun to make contributions in order to mitigate their effects. Research on system uncertainties in wine production activities can be found in Carrillo Higueras et al. (2008); Bohle et al. (2010); Cuadrado Labra et al. (2013); whereas research on environmental uncertainties in wine production activities can be found in Sánchez Loyola et al. (2008); Cholette (2009).

Considering that many uncertain events are present inside production systems, and these events interrupt and delay the order sequencing at production lines, a robust and flexible heuristic for wine orders scheduling is developed in Carrillo Higueras et al. (2008), and tested in a case of study related to a Chilean export-oriented wine producer. The objective is to find a way to have the highest number of on-time jobs, and thus minimize the rescheduling necessity. Hence, the authors introduce idle times inside the original scheduling in order to have slacks that could absorb the uncertain times derived from, for example, machine breakdowns, power cuts, defectives items, uncertain setup times, and so on.

Bohle et al. (2010) present an extension of the model of Ferrer et al. (2008), considering uncertainty in the actual productivity that can be achieved when harvesting. In order to cope with this, the authors applied the robust methodology proposed by Bertsimas & Sim (2004) to an aggregate redundant constraint. The objective is to study how effective robust optimization is solving this problem in practice. Also, they develop alternative robust models and show results for some test problems obtained from actual wine industry problems.

In Cuadrado Labra et al. (2013), the author aims to generate a robust planning over the process of harvest assuming that the parameters under uncertainty are the ones related to grape fermentation times. It is argued, because of the complex relationship between the parameters under uncertainty, that it is impossible to apply directly any of the existing approaches for robust optimization, but using a variation of the adversary problem approach proposed by Bienstock & ÖZbay (2008), it is possible to decrease the effects of the natural variation of these parameters.

Sánchez Loyola et al. (2008) study the impact produced by the utilization of (make-to-order/make-to-stock) MTO/MTS hybrid strategies in a wine industry firm subject to demand size uncertainty. To solve this, a simulation model is utilized to replicate a wine processing facility. This model considers analyzing and processing stages, that is to say the cycle since the order arrives to the company until it gets completed and delivered to importers. Besides, it is considered the creation of different kinds of products and client types determined by their lead times. The experiments measure the impact by producing some products under an MTO strategy, and others with an MTS strategy, having as a base case a purely MTO strategy. Also, it is studied the effect caused by changing the lead times of the different clients combined with hybrid strategies.

Cholette (2009) argues that wineries must allocate production across multiple sales channels before demand is known, thus misallocation derived from uncertainty may result in undesirable surpluses in some channels and lost sales opportunities in others. Given the latter, the author investigates this problem by constructing a mathematical model for postponing channel differentiation. The author provides a process overview for a prototype winery and presents a two-stage stochastic linear program with fixed recourse that maximizes the expected profit over a distribution of demand scenarios. In the first stage, the winery allocates production to finished goods by channel and to intermediate inventory points. Once demand is known, recourse variables include the transformation of intermediate inventories. Results from solving this model using a mix of data derived from interviews and literature review, suggest that a considerable portion of production should be held at both the labeling and packaging level, and can lead to significant improvement in product profitability.

1.2.4. The management of high-value goods in the wine industry under uncertainty

Taking into account both environmental and system uncertainties, but also the synchronization and alignment issues discussed by WSCC, in the third chapter of this thesis we focus on premium wines, which are the most profitable for small export-focused wineries, and we develop a simple two stage heuristic in order to support wine managers with the stocking of finished goods decisions.

The objective was to develop a practical tool that could be easily implementable in order to manage these goods.

Bearing in mind the operations of a small export-focused Chilean winery that we worked with, we consider the case where orders for premium wines arrive according to independent Poisson processes; inventories of bottled and labeled wines are reviewed continuously in time; a *postponement* strategy is employed for the labeling process; a $(s\text{-}1,s)$ policy is followed for the management of finished goods; and there is only one labeling machine, which exhibits random processing and setup times.

Considering that the winery planner is concerned in minimizing the sum of the steady-state expected values of the *WIP*, overage and underage costs per unit time; the heuristic operates as follows: in the first stage, we model the labeling process as a polling system under exhaustive service, and we apply the results of Winands et al. (2006) in order to obtain the mean delays for each end product. Then, in the second stage, we apply Palm's theorem (Palm, 1938) , which reduces the stock levels setting problem to the solving of a newsvendor-type problem for each end product.

We provide some numerical examples, and also, we address the accuracy of the proposed heuristic (by means of a simulation model) showing that this last becomes more accurate as long as the demand rates for each label type are low relative to the total demand.

When developing our analysis about how the heuristic solutions are related in our approach, we needed to fully characterize the newsvendor problem under Poisson demand in order to prove some statements. Particularly, we needed to show how both the optimal order quantity and its expected cost behave as a function of the mean product demand, which, at the same time, defines the level of uncertainty that a decision maker faces in this context. Thus, as a by-product of this chapter, we formalize those statements in the Appendix of this thesis. We provide some numerical examples, and we expect that our analysis could be used as supporting material for courses on inventory management.

1.2.5. Lot-sizing in an export-focused winery and the impact of postponement practices under uncertainty

Finally, in the fourth chapter of this thesis, both the environmental uncertainty and the synchronization and alignment issues are taken into account, and we analyze, by comparing the outputs of two mathematical programming models that differ in what kind of inventory is held, the impact of postponement practices on the performance of tactical production decisions.

In order to diminish the mismatch between demand and supply, one way to cope with product misallocation due to both demand variety and variability is by postponing product differentiation, which shifts the focus to the use of mixed MTS/MTO strategies and, for an export-focused winery, a critical boundary point is the labeling process.

On this, although the results of Cholette (2009) show the potential value of adopting postponement practices in the wine industry, the authors' strategic modeling framework does not capture some important operational aspects, such as batch bottling, changeover times, production capacity in the lines, etc., which a decision maker must consider when planning the winery operations. Thus, it is an open research question if the benefits of using postponement can be achieved when several operational constraints are taken into account.

Based on the operations of a large export-oriented Chilean winery we worked with, we present an overview of the operational complexities that a wine decision maker must consider, and we assess the performance in terms of inventory, lost sales, and setup quantities of delayed product differentiation under different planning environments.

To assess these performance measures, we develop two different MILP planning models that differ in the kind of inventory they can keep. Both models can be reduced to a capacitated lot-sizing with setup times problem, which is known to be NP-hard, so the relative performance of their optimal solutions is analyzed only in instances that could be solved in a reasonable computing time.

Moreover, since decisions must be made periodically, our analyses are circumscribed to a rolling horizon framework (a very used industrial practice), and on how capacity tightness, horizon length, and forecast inaccuracy affect the performance of both models, and thus the attractiveness of postponement.

1.3. Objective and main hypothesis of this thesis

The objective of this thesis was to improve the decision making process for planning problems that arise in the natural resource industry, particularly in forestry and agriculture, through the application of OR methodologies and the development and resolution of mathematical models.

The main hypothesis of this work is that through the application of OR methodologies it is possible to increase both the efficiency and effectiveness of the production process that is modeled through the handling of the environmental and/or system uncertainty that affects its performance.

1.4. The remainder of this thesis

The remainder of this thesis is organized as follows. Chapter 2 presents the paper*Scheduling production for a sawmill: a robust optimization approach* published in International Journal of Production Economics, April 2014, vol. 150, pp. 37-51. Chapter 3 presents the paper *On the* *management of high-value goods in the wine industry under uncertainty: a two-stage heuristic to setting stock levels under (s-1,s) inventory control policies*, which will be submitted soon to evaluation in European Journal of Operational Research. Chapter 4 introduces the paper *Lot-Sizing in an export-focused winery: the impact of postponement practices on the performance of tactical production decisions*, which is under its last revisions. Chapter 5 presents the conclusions of this thesis, whereas, in the Appendix, we present the paper *A note on the newsvendor under Poisson demand*, which is a by-product of chapter 3 insights and will be submitted soon to Operational Research Letters.

2. UNCERTAINTY IN SAWMILLS: A ROBUST OPTIMIZATION APPROACH TO COPE WITH BOTH SUPPLY AND DEMAND INACCURACIES.

2.1. Introduction

Optimization models are increasingly being used for planning natural resource operations. See Bjørndal et al. (2012) for a recent review. In particular, forestry has been an active user of decision support system based on these kind of models; see A. Weintraub & Romero (2006) for a review. There has also been work on planning models in the agri-food supply chain; see Ahumada & Villalobos (2009) for a review. Nevertheless, planning natural resource operations is difficult to support since decision makers have to deal with many different sources of uncertainty including, of course, demand uncertainty. This can have a significant impact on production activities since, for example, a decision maker trying to meet an order might not have enough raw material to process; the product might not meet the quality specifications required by the market; or the order might exceed production capacity. Some recent applications of optimization to planning natural resources, taking into account uncertainty, have been reported in the literature. Vlajic et al. (2012) propose an integrated framework to support the analysis and design of robust food supply chains. They classify supply chain disturbances, sources of food supply chain vulnerability, and suggest redesign principles and strategies to achieve robust supply chain performances. Verderame et al. (2010) review the contributions within the planning and scheduling communities related to uncertainty analysis. They analyze several independent sectors in order to find commonalities and potential avenues for future interdisciplinary collaborations. They discovered that many different approaches, such as two-stage stochastic programming, parametric programming, fuzzy programming, chance constraint programming, robust optimization techniques, conditional value-at-risk, and other risk mitigation procedures, have been applied in the analyzed sectors. Tan & Çömden (2012) present a planning methodology to match the random supply of annual fruits and vegetables from a number of contracted farms and the random demand from the retailers during the planning period. They use an extension of the newsvendor problem to determine the farm areas and the seeding times for annual plants that survive for only one growing season that maximizes expected total profit, considering uncertain demand.

In general the literature presents different approaches for handling uncertainty in optimization models, such as, *sensitivity analysis* (SA), *stochastic programming* (SP), and *robust optimization* (RO).

SA is an *ex-post* analysis approach that studies how the solution changes when certain parameters or data change. This approach is not very helpful for planning since it can only provide information on how sensitive the solution is to changes in input parameters. Furthermore, SA usually only does *ceteris paribus* analysis, since joint sensitivity analysis tends to be impractical.

SP is the most rigorous of the three approaches and its use has been widely reported in the literature. For example, Alonso-Ayuso et al. (2011) developed a multistage Stochastic Integer Programming model to help make more robust decisions based on a range of timber price scenarios over time, maximizing the expected net profit. The SP approach assumes that the probability distributions of the uncertain parameters are known or can be accurately estimated. In order to find a solution that is feasible for all (or almost all) instances of the uncertain data, the expected value of some function of the decision and random variables is maximized. This approach, however, is the most complex and hard to understand for most managers. It also requires much data, some of which is very difficult to obtain, and the problems frequently are intractable or require too much time to be solved to optimality. See Birge & Louveaux (2011) for a detailed discussion of SP.

Finally, the RO approach has been proposed as an alternative modeling approach to develop models which solutions are less sensitive to data uncertainty without considering a specific distribution of uncertain parameters. Beyer & Sendhoff (2007) give a detailed review on the different ways that RO can be implemented. For example, Leung et al. (2007) developed a RO to solve production planning problems for perishable products in an uncertain environment in which the setup costs, production costs, labor costs, inventory costs, and workforce changing costs are minimized. They used the approach proposed by Mulvey et al. (1995), which assumes a set of scenarios may occur, each with a known probability. The model was used to solve the case of a Hong Kong plush toy company.

Soyster (1973) had previously proposed a deterministic linear optimization model that tries to find solutions that always remain feasible for a *columnwise uncertainty* case. However, the resulting model produces solutions that tend to be too conservative. A step forward was taken independently by Ben-Tal & Nemirovski (1998, 1999, 2000), El Ghaoui & Lebret (1997), and El Ghaoui et al.

(1998). In order to find less conservative solutions, these authors proposed models in which the uncertain data are assumed to belong to a set with some specific geometric structure. Nevertheless, a practical drawback is that, depending on the uncertain set, this may lead to nonlinear models that are harder to solve than Soyster's approach, although interior point methods could be used to solve them (Nesterov et al., 1994).

More recently Bertsimas & Sim (2004) proposed an approach for robust linear optimization that retains the advantages of Soyster's linear framework and is at least as flexible as the one proposed by Ben-Tal and Nemirovsky and El-Ghaoui and Lebret. This methodology has been applied successfully to different types of problems and of uncertainties. For example, Erera et al. (2009) applied RO to a dynamic repositioning problem, where the uncertainty was the forecasts for future supplies and demands for assets at different time epochs. Bohle et al. (2010) applied RO to a wine grape harvesting scheduling optimization problem, where the uncertainty was the actual harvesting productivity. Wei et al. (2011) applied RO to an inventory and production planning problem with uncertain demands and number of returns. Alem & Morabito (2012) applied RO to the production planning problem of a furniture factory, with uncertain product demand and uncertain cost coefficients. Finally Aouam & Brahimi (2013) applied RO to an integrated production planning and order acceptance problem, with uncertain demand. Although the general approach is the same, the resulting models are quite different, depending on the underlying problem and the uncertainties that were taken into account.

One of the features of the RO approach is that it requires an additional input, which are the so-called uncertainty budgets. These budgets should be based on the decision makers' risk aversion, since they determine the conservatism of the solutions. Unfortunately, how the solution behaves when the budget of uncertainty changes is not obvious. Most of the applications of the RO approach mentioned above carried out computational experiments to analyze the behavior of the solutions for different uncertainty budgets. However, the results of the analysis tend to be complicated and would be difficult to convey to most decision makers who have to face the real problems.

In this chapter we will use the sawmill scheduling production problem described in Maturana et al. (2010) to analyze the effect of using the RO approach to handle uncertainty. Maturana et al. (2010) developed a deterministic optimization model to solve the production scheduling problem of a Chilean sawmill and compared its performance with a heuristic. The comparison was carried out using various scenarios during a six week planning horizon. The results showed that the mathematical model outperformed the heuristic on almost all instances, except two, in which the heuristic also found the optimal solution. This demonstrated the feasibility of using a mathematical model to help schedule the operations of the sawmill. However the model assumes that all the data is known with certainty, which is not true for the real world problem.

Although there were many sources of uncertainties, we focus in this chapter on the two main ones the Chilean company we worked with faced: demand uncertainty and raw material uncertainty. Demand uncertainty was important because the Chilean sawmill exported most of its production to different markets all over the world. These markets demanded, in general, different products, and the orders had relatively short lead times since they had to be sent by ship to different ports, most of which were faraway.

Raw material uncertainty was also important since the type of raw material received by the company, most often than not, was not the one requested. The reason is that the raw material came from another company that managed the forest. When the sawmill company requested a certain amount of raw material, the forest company would decide which part of the forest to cut, trying to satisfy the request of the sawmill, but also taking into account the distance the logs would have to travel from the forest to the sawmill. Also, the part of the forest that was cut usually had trees of different diameter classes and qualities, that didn't exactly match those requested by the sawmill. So although the raw material it received usually was not the one the sawmill required, they had to do the best they could with this raw material.

The sawmill planner handled both types of uncertainty by using a rolling planning horizon. However, we thought it might be better to use a mathematical model that explicitly handled these uncertainties. The problem was that the sawmill managers didn't have very good data on both sources of uncertainties to allow us to infer probability distributions, which led us to use the Bertsimas and Sim RO approach, since this method does not require knowledge of the probability distributions. Furthermore, the robust problem remains linear making it easier to solve using standard optimization tools.

However, although a probability distribution is not needed, we did require that the decision maker give us a value for the budget of uncertainty that will be used. Since, as mentioned earlier, the appropriate budget of uncertainty to use depends on the characteristics of the decision maker and on the characteristics of the problem, we needed to give the decision maker some guidance on how to decide on the uncertainty budget to use. This led us to carry out a detailed analysis of the effect of the uncertainty budget on the robust solutions, which allowed us to understand how the degree of conservatism affects the solutions of the RO approach in this particular case.

The rest of this chapter is organized as follows: in section 2.2 we review the RO approach proposed by Bertsimas and Sim. In section 2.3 we present a description of the sawmill scheduling production planning problem. The robust reformulation of this problem is described in section 2.4. In section 2.5 we present the computational experiments and the analysis of their results. Finally, some concluding remarks are given in section 2.6.

2.2. Robust optimization methodology

In this section we describe the robust optimization approach proposed by Bertsimas & Sim (2004) based on polyhedral uncertainty sets. Let us consider a general linear programming problem:

$$\begin{aligned} \text{minimize} \quad & c^T x \\ \text{subject to} \quad & \alpha_i^T x \le b_i, \quad i = 1, \dots, m \\ & x \ge 0 \end{aligned} \tag{2.1}$$

We denotes by A the constraint matrix formed with the row vectors α_i^T, $i = 1, \dots, m$. Without any loss of generality, we assume that the uncertainties affect only the elements in matrix A. If there is uncertainty on b_i a new variable x_{n+1} can be introduced into the model, and the constraint associated with the uncertain parameter can be rewritten as $\alpha_i^T x - x_{n+1} b_i \le 0$, with $x_{n+1} = 1$, which includes b_i into the matrix A (Bertsimas & Sim, 2004). Similarly, if the objective function is also subject to uncertainties, the model can be rewritten to minimize z, and the constraint $c^T x - z \le 0$ is added to the set of constraints becoming part of the matrix A (Bertsimas & Thiele, 2006). We define $\mathcal{J}_i$ as the set of coefficients in row i of matrix A that are subject to uncertainty. Each coefficient a_{ij}, $j \in \mathcal{J}_i$, is modeled as a symmetric, independent, and bounded random variable $\tilde{a}_{ij}$ that takes values on the interval $[\bar{a}_{ij} - \hat{a}_{ij}, \bar{a}_{ij} + \hat{a}_{ij}]$. We define the scaled deviation of parameter a_{ij} from its nominal value $\bar{a}_{ij}$ as $z_{ij} = (\tilde{a}_{ij} - \bar{a}_{ij})/\hat{a}_{ij}$, where $|z_{ij}| \le 1$.

One of the main ideas behind Bertsimas and Sim's approach is the introduction of a parameter that can be used to control the level of conservatism of the robust solution. For every constraint i, a not necessarily integer threshold Γ_i is introduced in order to bound the total (scaled) variation of the uncertain parameters as follows: $\sum_{j\in\mathcal{J}_i}|z_{ij}| \leq \Gamma_i,\ \forall i$. The choice of a particular *budget of uncertainty* Γ_i, allows decision-makers to evaluate the trade-offs between robustness and performance of the solution according to their risk aversion. Also, as stated in Bertsimas & Sim (2004), having a parameter that might vary between 0 (the nominal case) and $|\mathcal{J}_i|$ (the worst case) allows greater flexibility to build a robust model without excessively affecting the optimal cost.

In the robust optimization framework the notions of uncertainty sets and a robust counterpart problem are very important. Uncertainty in the data is described through deterministic sets (typically bounded and convex) each of which contains most or all possible values that may be realized for the uncertain parameters. In particular, the uncertainty set in Bertsimas and Sim's approach is given by:

$$U = \{A \in \mathbb{R}^{m\times n} : a_{ij} = \bar{a}_{ij} + z_{ij}\hat{a}_{ij},\ \forall i,j; \quad \sum_{j\in\mathcal{J}_i}|z_{ij}| \leq \Gamma_i, \forall i; \quad |z_{ij}| \leq 1 \quad \forall i,j\} \tag{2.2}$$

so the problem we consider in this chapter is given by:

$$\begin{aligned} \text{minimize} \quad & c^T x \\ \text{subject to} \quad & Ax \leq b, \quad \forall A \in U \\ & x \geq 0 \end{aligned} \tag{2.3}$$

The robust counterpart is the deterministic problem associated with the uncertain problem, as in (2.3). Depending on the uncertainty set used, different robust counterparts with different levels of tractability may be obtained. In the case that concerns us, the robust counterpart is constructed by maximizing the left-hand side of the constraints over the set of admissible scaled deviations that can be represented by a protection function for each uncertain constraint. This leads to the following problem:

$$\begin{aligned} \text{minimize} \quad & c^T x \\ \text{subject to} \quad & \bar{\alpha}_i^T x + \beta_i(x,\Gamma_i) \leq b_i, \quad i = 1,\ldots,m \\ & x \geq 0 \end{aligned} \tag{2.4}$$

where $\bar{\alpha}_i^T$ denotes the nominal data for row i and the protection function for each constraint $i = 1,\ldots,m$ is:

$$\begin{aligned}\beta_i(x,\Gamma_i) = \quad & \text{maximize} && \sum_{j\in\mathcal{J}_i} |x_j|\hat{a}_{ij} z_{ij} \\ & \text{subject to} && \sum_{j\in\mathcal{J}_i} z_{ij} \le \Gamma_i \\ & && 0 \le z_{ij} \le 1, \quad \forall j \in \mathcal{J}_i.\end{aligned} \tag{2.5}$$

Through the application of strong duality on (2.5), Theorem 1 of Bertsimas & Sim (2004) proves that (2.4) is equivalent to the following linear formulation:

$$\begin{aligned} \text{minimize} \quad & c^T x && \\ \text{subject to} \quad & \bar{\alpha_i}^T x + z_i\Gamma_i + \sum_{j\in\mathcal{J}_i} p_{ij} \le b_i, && \forall i \\ & z_i + p_{ij} \ge \hat{a}_{ij} y_j, && \forall i, j \in \mathcal{J}_i \\ & -y_j \le x_j \le y_j, && \forall j \\ & p_{ij} \ge 0, && \forall i, j \in \mathcal{J}_i \\ & y_j \ge 0, && \forall j \\ & z_i \ge 0, && \forall i \\ & x_j \ge 0, && \forall j \end{aligned} \tag{2.6}$$

where the variables z_i and p_{ij} are the dual variables of the constraints in (2.5). The main advantage of this formulation is that the problem remains as difficult to solve as the original problem, since it preserves its linearity, even allowing for integer variables.

So by construction, if up to $\lfloor \Gamma_i \rfloor$ of the coefficients a_{ij} change within their bounds, and at most one coefficient a_{it_i} changes by $(\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{a}_{it_i}$, the solution of the robust formulation (2.6) will remain deterministically feasible. Moreover, by the uncertainty set U, even if the above statement is not fulfilled, the robust solution will be feasible with very high probability (Bertsimas & Sim, 2003).

2.3. The scheduling production model

In this section we present the sawmill production scheduling model that will be used to analyze the effect of using RO to handle uncertainty. A deterministic version of this model is described in Maturana et al. (2010), which is related to the models presented in Clark (2005); Alvarez & Vera (2014); de Araujo et al. (2007).

TABLE 2.1. Sets used in the Scheduling Production Model

$\mathcal{I}$	:	Set of raw materials. (Indexed by i)
$\mathcal{P}$	:	Set of products. (Indexed by p)
$\mathcal{T}$	:	Set of periods. (Indexed by t)
$\mathcal{N}$	:	Set of subperiods. (Indexed by n)

TABLE 2.2. Variables used in the Scheduling Production Model

x_{pn}	:	Quantity of product p to be produced in subperiod n.
s_{pt}	:	Inventory (or stock) of product p at the end of period t.
r_{in}	:	Inventory of raw material i at the end of subperiod n.
m_{in}	:	Quantity of raw material i to be processed in subperiod n.

2.3.1. Problem description

The model considers raw materials (logs) and products (lumber). The decision horizon is divided into periods (e.g., weeks); each of which is divided into subperiods (e.g., days within the week). Processing of raw materials involves the application of a manufacturing process (cutting pattern) that generates a given set of products. We assume that each raw material is associated with a specific manufacturing process, which takes a fixed and known time to be carried out. Supply of raw materials occurs at the beginning of each subperiod and the demand for products takes place at the end of each period. Thereby, the main decision in this problem is how much raw material to process of each type in each subperiod in order to fulfill the orders for products, while trying to maximize the yield or the profit. The yield is the percentage of the log volume that becomes a finished good, as opposed to waste.

Note that we are maintaining the assumption made in Maturana et al. (2010) regarding the relation between cutting patterns and raw material types. This assumption is that for any log diameter class, the optimal cutting pattern will be used. We also assume that these optimal cutting patterns have been previously determined. In the case of the Chilean company we worked with, this was done using log sawing simulators.

2.3.2. Model formulation

Tables 2.1, 2.2, and 2.3 show the notation used in the mathematical model. Table 2.4 shows the notation from the General Lot-Sizing and Scheduling Problem (GLSP) of de Araujo et al. (2007).

TABLE 2.3. Parameters used in the Scheduling Production Model

C_n	:	Time available to process in subperiod n.
A	:	Time required to process one unit of raw material.
D_{pt}	:	Demand of product p in period t.
R_{in}	:	Supply of raw material type i in subperiod n.
Y_{ip}	:	Yield of product p obtained when raw material type i is processed.
S	:	Inventory penalty (or cost) for holding a unit of product.
B	:	Backlog penalty (or cost) for delaying delivery of a unit of product.
M	:	Inventory penalty (or cost) for holding a unit of raw material.

TABLE 2.4. General Lot-Sizing and Scheduling Problem Notation

N_t	:	Number of subperiods in period t, $t \in T$.
$F_t = 1 + \sum_{\tau}^{t-1} N_\tau$	:	The index of the first subperiod in period t, with F_1 = 1.
$L_t = F_t + N_t - 1$	:	The index of the last subperiod in period t, with L_1 = N_1.

In order to facilitate the robust reformulation we will describe in the next section, we had to make some changes to the model described in Maturana et al. (2010). Basically, the cost associated with having either excess inventory or unmet demand was replaced by a convex, piecewise linear holding/shortage cost with an unrestricted product inventory variable. We also assume that some demand may not be met and that raw materials are non-perishable. Also note that we are not replacing r_{in} in the objective function. The modified model is the following:

$$
\begin{aligned}
\text{minimize} \quad & \sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}} \max\{S s_{pt}, -B s_{pt}\} + \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}} M r_{in} \\
\text{subject to} \quad & \sum_{i\in\mathcal{I}} Y_{ip} m_{in} = x_{pn} && \forall p \in \mathcal{P}, n \in \mathcal{N} \\
& s_{pt} = s_{p0} + \sum_{k=1}^{L_t} x_{pk} - \sum_{k=1}^{t} D_{pk} && \forall p \in \mathcal{P}, t \in \mathcal{T} \\
& \sum_{i\in\mathcal{I}} A m_{in} \le C_n && \forall n \in \mathcal{N} \\
& r_{in} = r_{i0} + \sum_{l=1}^{n} R_{il} - \sum_{l=1}^{n} m_{il} && \forall i \in \mathcal{I}, n \in \mathcal{N} \\
& m_{in}, r_{in} \ge 0 && \forall i \in \mathcal{I}, n \in \mathcal{N}.
\end{aligned}
\tag{2.7}
$$

The objective function of this model minimizes the sum of the holding/shortage cost associated with the products and the holding cost associated with raw materials. These costs are computed at different times. While the former is calculated at the end of a period, the latter is calculated at the end of a subperiod. Note that this allows simultaneously taking into account cost components that are relevant in different time horizons. Raw material costs, for example, are very important in the short term, while final product costs follow a longer cycle.

The first set of constraints determines the total amount of product p produced in the subperiod n as a result of processing all different raw materials of indices i in subperiod n. The yield of product p for a specific raw material i is given by Y_{ip}, so x_{pn} will accumulate the total obtained. The second set of constraints relates the inventories of products with the amounts of product produced and demanded. The third set of constraints ensure that the production capacity is not exceeded in each subperiod. The fourth set of constraints relates the inventories of raw materials with the amounts of raw material processed and supplied. Finally, there are the non-negativity constraints.

2.4. Robust reformulation

To the model presented in (2.7) we added two sources of uncertainty: the demand of products and the supply of raw materials. We did not consider uncertainty in the yield coefficients associated with cutting patterns as is considered in Alvarez & Vera (2014). In the next two subsections we apply the robust optimization framework in order to construct the robust counterpart of the uncertain scheduling production model. We closely follow Bertsimas & Thiele (2006), since the inventory problem of that paper has a similar structure to the problem we are addressing. Also, for the sake of clarity, we address the sources of uncertainty separately.

2.4.1. Uncertainty in product demand

We model the product demand D_{pt} ($\forall\ p,\ t$) as an uncertain parameter that takes values in the interval $[\bar{D}_{pt} - \hat{D}_{pt}, \bar{D}_{pt} + \hat{D}_{pt}]$. We define the scaled deviation of D_{pt} from its nominal value as $z_{pt} = (D_{pt} - \bar{D}_{pt})/\hat{D}_{pt}$, where $|z_{pt}| \leq 1$. For every product p, we impose *budgets of uncertainty* Γ_{pt} at each period t to restrict the cumulatively scaled deviation up to time t as $\sum_{k=1}^{t} |z_{pk}| \leq \Gamma_{pt}$. The uncertainty budgets take values in the interval $[0, t]$, and are assumed to be nondecreasing with time. Furthermore, the budget increase cannot exceed the number of new parameters added at each time period. That is, for each product p, the following conditions must hold:

(i) $\Gamma_{pt} \leq \Gamma_{pk}, \forall k \geq t.$

(ii) $\Gamma_{p,t+1} - \Gamma_{pt} \leq 1, \forall t.$

Considering the previous definitions, the uncertainty set is given by:

$$U = \{D \in \mathbb{R}^{|\mathcal{P}|\times|\mathcal{T}|} : D_{pt} = \bar{D}_{pt} + z_{pt}\hat{D}_{pt} \ \forall p,t; \quad \sum_{k=1}^{t} |z_{pk}| \leq \Gamma_{pt} \ \forall p,t; \quad |z_{pt}| \leq 1 \ \forall p,t\} \tag{2.8}$$

In order to construct the robust counterpart we consider the following differentiable model equivalent to (2.7), where the piecewise linear holding/shortage cost has been added to the constraints and the expression for the inventory of products has been replaced to take into account that $z \in U$:

$$\begin{aligned}
\text{minimize} \quad & \sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}} \delta_{pt} + \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}} M r_{in} && && (2.9)\\
\text{subject to} \quad & \sum_{i\in\mathcal{I}} Y_{ip} m_{in} = x_{pn} && \forall p,n && (2.10)\\
& \delta_{pt} \geq S\left(s_{p0} + \sum_{k=1}^{L_t} x_{pk} - \sum_{k=1}^{t} \bar{D}_{pk} - \sum_{k=1}^{t} z_{pk}\hat{D}_{pk}\right) && \forall p,t && (2.11)\\
& \delta_{pt} \geq -B\left(s_{p0} + \sum_{k=1}^{L_t} x_{pk} - \sum_{k=1}^{t} \bar{D}_{pk} - \sum_{k=1}^{t} z_{pk}\hat{D}_{pk}\right) && \forall p,t && (2.12)\\
& \sum_{i\in\mathcal{I}} A m_{in} \leq C_n && \forall n && (2.13)\\
& r_{in} = r_{i0} + \sum_{l=1}^{n} R_{il} - \sum_{l=1}^{n} m_{il} && \forall i,n && (2.14)\\
& m_{in}, r_{in} \geq 0 && \forall i,n. && (2.15)
\end{aligned}$$

Note that z is not a variable in this model and that the protection function $\beta_{pt}(\hat{D}_{pt}, \Gamma_{pt})$ related to the pair of holding/shortage constraints (2.11) and (2.12) is given by:

$$\begin{aligned}
\beta_{pt}(\hat{D}_{pt}, \Gamma_{pt}) = \quad \text{maximize} \quad & \sum_{k=1}^{t} z_{pk}\hat{D}_{pk}\\
\text{subject to} \quad & \sum_{k=1}^{t} z_{pk} \leq \Gamma_{pt}\\
& 0 \leq z_{pk} \leq 1 \qquad \forall k \leq t.
\end{aligned} \tag{2.16}$$

where the dual of this problem is:

$$\begin{aligned} \text{minimize} \quad & q_{pt}\Gamma_{pt} + \sum_{k=1}^{t} u_{kpt} \\ \text{subject to} \quad & q_{pt} + u_{kpt} \geq \hat{D}_{pk} \quad \forall k \leq t \\ & q_{pt}, u_{kpt} \geq 0 \qquad\quad \forall k \leq t. \end{aligned} \tag{2.17}$$

The protection function arises from minimizing $\sum_{k=1}^{t} z_{pk}\hat{D}_{pk}$ in constraint (2.11) and maximizing $\sum_{k=1}^{t} z_{pk}\hat{D}_{pk}$ in constraint (2.12), over the set of admissible scaled deviations (2.8) (Bertsimas & Thiele, 2006). Considering that (2.16) is feasible and bounded, by strong duality, both primal and dual problem have the same optimal value. Replacing (2.17) in (2.11) and (2.12), we obtain the following robust counterpart for the scheduling production problem with uncertain demand:

$$\begin{aligned} \text{minimize} \quad & \sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}} \delta_{pt} + \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}} M r_{in} \\ \text{subject to} \quad & \sum_{i\in\mathcal{I}} Y_{ip} m_{in} = x_{pn} && \forall p, n \\ & \delta_{pt} \geq S\left(s_{p0} + \sum_{k=1}^{t}\sum_{n=F_k}^{L_k} x_{pk} - \sum_{k=1}^{t}\bar{D}_{pk} + q_{pt}\Gamma_{pt} + \sum_{k=1}^{t} u_{kpt}\right) && \forall p, t \\ & \delta_{pt} \geq -B\left(s_{p0} + \sum_{k=1}^{L_t} x_{pk} - \sum_{k=1}^{t}\bar{D}_{pk} - q_{pt}\Gamma_{pt} - \sum_{k=1}^{t} u_{kpt}\right) && \forall p, t \\ & q_{pt} + u_{kpt} \geq \hat{D}_{pk} && \forall p, t, k \leq t \\ & \sum_{i\in\mathcal{I}} A m_{in} \leq C_n && \forall n \\ & r_{in} = r_{i0} + \sum_{l=1}^{n} R_{il} - \sum_{l=1}^{n} m_{il} && \forall i, n \\ & m_{in}, r_{in} \geq 0 && \forall i, n \\ & q_{pt}, u_{kpt} \geq 0 && \forall p, t, k \leq t \end{aligned} \tag{2.18}$$

2.4.2. Uncertainty in supply of raw materials

Similarly to the product demand, we model the supply of raw materials R_{in} $(\forall i, n)$ as an uncertain parameter that takes values in the interval $[\bar{R}_{in} - \hat{R}_{in}, \bar{R}_{in} + \hat{R}_{in}]$. The scaled deviation of R_{in} from its nominal value is defined as $z'_{in} = (R_{in} - \bar{R}_{in})/\hat{R}_{in}$, where $|z'_{in}| \leq 1$. For every raw material i, we give *budgets of uncertainty* Γ'_{in} at each subperiod n to restrict the cumulatively scaled deviation up to n as $\sum_{l=1}^{n} |z'_{il}| \leq \Gamma'_{in}$. The uncertainty budgets takes values on the interval

$[0, n]$, and are characterized in the same way as Γ_{pt}. In this case, the uncertainty set is given by:

$$U = \{R \in \mathbb{R}^{|\mathcal{I}|\times|\mathcal{N}|} : R_{in} = \bar{R}_{in} + z'_{in}\hat{R}_{in},\ \forall i, n; \quad \sum_{l=1}^{n} |z'_{il}| \leq \Gamma'_{in},\ \forall i, n; \quad |z'_{in}| \leq 1\ \forall i, n\} \tag{2.19}$$

To construct the robust counterpart, we consider the following differentiable model, which is based on (2.18), where $z' \in U$ and the expression for the inventory of raw materials r_{in} has been replaced both by the objective function and the non-negativity constraint:

$$\begin{aligned}
\text{minimize} \quad & \sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}} \delta_{pt} + \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}} M(r_{i0} + \sum_{l=1}^{n}\bar{R}_{il} - \sum_{l=1}^{n} m_{il}) + \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{l=1}^{n} M z'_{il}\hat{R}_{il} & \\
\text{subject to} \quad & \sum_{i\in\mathcal{I}} Y_{ip} m_{in} = x_{pn} & \forall p, n \\
& \delta_{pt} \geq S\left(s_{p0} + \sum_{k=1}^{L_t} x_{pk} - \sum_{k=1}^{t}\bar{D}_{pk} + q_{pt}\Gamma_{pt} + \sum_{k=1}^{t} u_{kpt}\right), & \forall p, t \\
& \delta_{pt} \geq -B\left(s_{p0} + \sum_{k=1}^{L_t} x_{pk} - \sum_{k=1}^{t}\bar{D}_{pk} - q_{pt}\Gamma_{pt} - \sum_{k=1}^{t} u_{kpt}\right), & \forall p, t \\
& q_{pt} + u_{kpt} \geq \hat{D}_{pk}, & \forall p, t, k \leq t \\
& \sum_{i\in\mathcal{I}} A m_{in} \leq C_n, & \forall n \\
& r_{i0} + \sum_{l=1}^{n}\bar{R}_{il} + \sum_{l=1}^{n} z'_{il}\hat{R}_{il} - \sum_{l=1}^{n} m_{il} \geq 0, & \forall i, n \\
& m_{in} \geq 0, & \forall i, n \\
& q_{pt}, u_{kpt} \geq 0, & \forall p, t, k \leq t
\end{aligned} \tag{2.20}$$

Since the uncertainty affects part of the objective function, we applied the approach described in section 2: a new variable α_{in} was created to replace the affected part of the objective function and the following constraint was added to (2.18):

$$\alpha_{in} \geq \sum_{l=1}^{n} z'_{il}\hat{R}_{il},\ \forall i, n. \tag{2.21}$$

The protection function in this case arises from maximizing $\sum_{l=1}^{n} z'_{il}\hat{R}_{il}$ in the new constraint (2.21) and minimizing $\sum_{l=1}^{n} z'_{il}\hat{R}_{il}$ in the non-negativity of raw materials constraint over the set of admissible scaled deviations (2.19). Thus, the protection function is given by:

$$\begin{aligned} \beta_{in}(\hat{R}_{in}, \Gamma'_{in}) = \quad & \text{maximize} && \sum_{l=1}^{n} z'_{il} \hat{R}_{il} \\ & \text{subject to} && \sum_{l=1}^{n} z'_{il} \leq \Gamma'_{il} \\ & && 0 \leq z'_{il} \leq 1, \quad \forall l \leq n. \end{aligned} \tag{2.22}$$

where the dual of this problem is:

$$\begin{aligned} & \text{minimize} && v_{in}\Gamma'_{in} + \sum_{l=1}^{n} w_{lin} \\ & \text{subject to} && v_{in} + w_{lin} \geq \hat{R}_{il} && \forall l \leq n \\ & && v_{in}, w_{lin} \geq 0 && \forall l \leq n. \end{aligned} \tag{2.23}$$

Considering that (2.22) is feasible and bounded, by strong duality, both primal and dual problem have the same optimal value. Replacing (2.23) in (2.20), we obtain the following robust formulation for the uncertain scheduling production problem:

$$\begin{aligned} & \text{minimize} && \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} \delta_{pt} + \sum_{i\in\mathcal{I}} \sum_{n\in\mathcal{N}} M(r_{i0} + \sum_{l=1}^{n} \bar{R}_{il} - \sum_{l=1}^{n} m_{il}) + \sum_{i\in\mathcal{I}} \sum_{n\in\mathcal{N}} M\alpha_{in} \\ & \text{subject to} && \alpha_{in} \geq v_{in}\Gamma'_{in} + \sum_{l=1}^{n} w_{lin} && \forall i, n \\ & && \delta_{pt} \geq S\left(s_{p0} + \sum_{k=1}^{L_t} x_{pk} - \sum_{k=1}^{t} \bar{D}_{pk} + q_{pt}\Gamma_{pt} + \sum_{k=1}^{t} u_{kpt}\right) && \forall p, t \\ & && \delta_{pt} \geq -B\left(s_{p0} + \sum_{k=1}^{L_t} x_{pk} - \sum_{k=1}^{t} \bar{D}_{pk} - q_{pt}\Gamma_{pt} - \sum_{k=1}^{t} u_{kpt}\right) && \forall p, t \\ & && q_{pt} + u_{kpt} \geq \hat{D}_{pk} && \forall p, t, k \leq t \\ & && v_{in} + w_{lin} \geq \hat{R}_{il} && \forall i, n, l \leq n \\ & && \sum_{i\in\mathcal{I}} Y_{ip} m_{in} = x_{pn} && \forall p, n \\ & && \sum_{i\in\mathcal{I}} A m_{in} \leq C_n && \forall n \\ & && r_{i0} + \sum_{l=1}^{n} \bar{R}_{il} - v_{in}\Gamma'_{in} - \sum_{l=1}^{n} w_{lin} - \sum_{l=1}^{n} m_{il} \geq 0 && \forall i, n \\ & && m_{in}, v_{in}, w_{lin} \geq 0 && \forall i, n, l \leq n \\ & && q_{pt}, u_{kpt} \geq 0 && \forall p, t, k \leq t \end{aligned} \tag{2.24}$$

2.5. Computational analysis of the model

The main motivation of using Robust Optimization is to achieve a good trade-off between the level of conservatism and the loss of optimality. In fact, a robust solution is immune to changes in parameters, within a certain range, but there might be a loss of optimality. Decision makers need to know how the model behaves under various uncertainty and variability assumptions and also how the level of conservatism with respect to each source of uncertainty affects the robust solution. We have addressed these issues with the computational experiences we describe in this section.

The model was tested on the *Ideal Forest Case* described in detail in Maturana et al. (2010), which consists of 7 products, 6 raw materials, a planning horizon of 6 periods, and 7 subperiods per period. The data is summarized in appendix 2.A. The mathematical model was coded in AMPL and solved using CPLEX 11 on a PC with an Intel® Core™ 2 Duo CPU processor and 4 GB of RAM. In this case, the robust scheduling planning model described in (2.24) generates a problem with 7,581 variables and 2,099 constraints, which is solved to optimality in less than 1 second (through dual simplex iterations) for every instance that we run.

We designed three experiments to evaluate the performance of the robust solutions provided by (2.24). The first considers only uncertainty in product demand, the second considers only uncertainty in raw material supply, and the third considers both uncertainties simultaneously. This allowed us to determine the impact of both sources of uncertainty separately and also the combined effect. In each experiment we define several levels of variability for the uncertain parameters, and for each of them we manipulate the budgets of uncertainty in order to vary the level of conservatism. We evaluated the behavior of robust solutions when we vary the level of conservatism, and we also analyzed their average performance with respect to uncertainty using Monte Carlo simulation.

2.5.1. Uncertainty in demand parameters

We assume that for each product p the uncertain demand D_{pt} is independent and identically distributed, with a nominal value $\bar{D}_{pt}$, and a maximum variation expressed as a fraction of the nominal value. Therefore, $\hat{D}_{pt}$ can be replaced by $\gamma_p \bar{D}_{pt}$ in (2.24), where γ_p can be understood as the level of variability of the uncertain demand for product p. Five levels of variability were considered for each product: 5%, 10%, 20%, 40%, and 80%, that is, $\gamma_p \in \{0.05, 0.1, 0.2, 0.4, 0.8\}$.

Different approaches have been proposed to determine the budgets of uncertainty in order to represent different levels of conservatism. For example, Bienstock & ÖZbay (2008) use random generation, Bertsimas & Thiele (2006) use an algorithm that scales the budgets as $\sqrt{t+1}$, while Adida & Perakis (2006) use time-dependent linear functions. This last approach is appealing since it allows varying the level of conservatism using only one parameter. Thus, we consider in this experiment the following expression for the budgets of uncertainty: $\Gamma_{pt} = \phi t$ where $\phi \in [0, 1]$. This fulfills the conditions explained in 2.4.1 and allows us to consider the non-protection case ($\phi = 0$), the full protection case ($\phi = 1$), and many intermediate cases.

Note that since the budget of uncertainty is the same for all products, we can specify each instance of (2.18) using only one parameter, which avoids the combinatorial problem of instance generation. It also facilitates the analysis of the results.

We generated 101 instances of the problem by varying ϕ, which we call the *demand conservatism* parameter, from 0 to 1, in steps of 0.01. Each of these instances was solved for 5 levels of variability of the demand (5%, 10%, 20%, 40%, and 80%), resulting in 505 robust solutions of (2.18). Fig. 2.1 shows the deterioration of these solutions as the demand conservatism parameter increases from $\phi = 0$, which is the nominal problem, to $\phi = 1$, for the different levels of variability of the demand. Although the solutions deteriorate as conservatism increases for all the variabilities considered, the effect is stronger for higher variabilities.

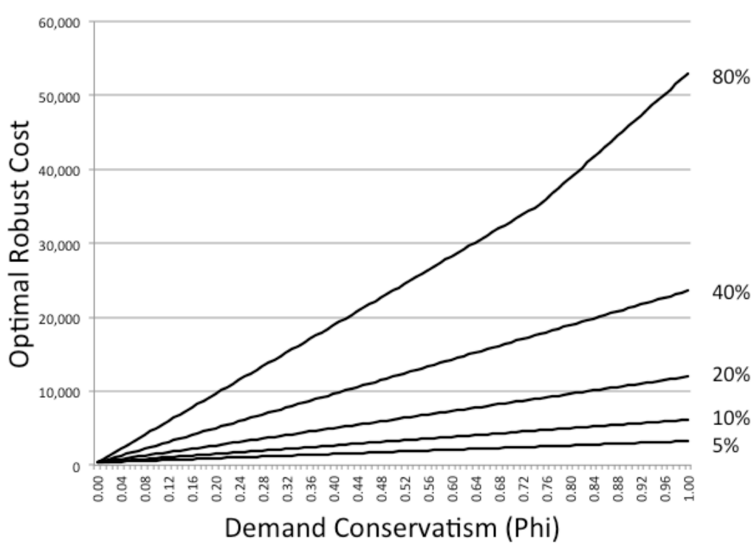

FIGURE 2.1. Optimal robust operational cost for several levels of γ_p.

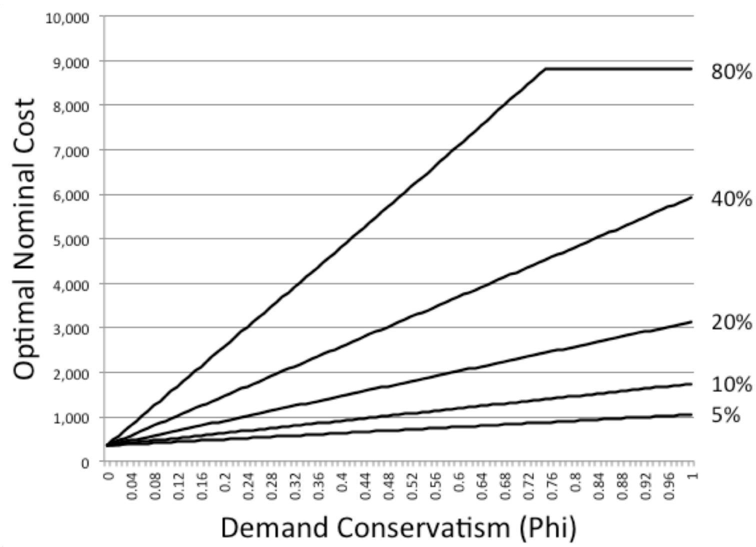

FIGURE 2.2. Optimal nominal operational cost for several values of γ_p.

Since the objective function of (2.18) includes the term $q_{pt}\Gamma_{pt} + \sum_{k=1}^{t} u_{kpt}$, which is not included in (2.7), the optimal robust solutions of (2.18) were evaluated in (2.7) in order to facilitate the comparison of using different levels of conservatism. Fig. 2.2, which presents these results, shows how the optimal operational cost increases as the level of conservatism rises. However, when demand variability is 80%, the optimal operational cost becomes constant for levels of demand conservatism greater than 0.75. To understand the reason for this behavior it is important to consider how the different cost components vary when the conservatism level changes.

In the nominal problem, the optimal value of 355.8 is completely determined by the inventory cost of raw material since the optimal production schedule completely satisfies the nominal product demand in each period. Given that the backlog cost B is greater than the product holding cost S, robust solutions attempt to avoid backlogged demand by producing more when the orders for products are greater than expected. Thus, as conservatism increases, finished goods holding/shortage costs increase and raw material holding costs decrease, as raw material is increasingly converted into finished goods. In Fig. 2.3, which shows how raw material inventory costs vary with demand conservatism, we can see that for a demand variability of 80% raw material inventory cost reaches zero when demand conservatism reaches 0.75. This means that raw materials were totally consumed so no more finished goods can be produced. Thus, although the model attempts to produce more as

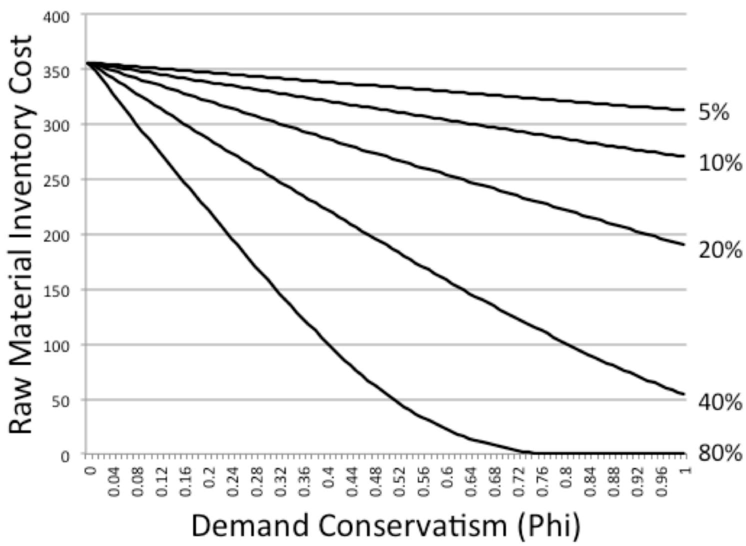

FIGURE 2.3. Raw material inventory cost for several values of γ_p.

the demand conservatism increases, when it reaches 0.75 it cannot do it any more, since it runs out of raw material. Therefore the optimal nominal cost stops increasing.

The managerial implication of this analysis is that if the variability of the demand is very high, being too conservative might lead to a solution where raw materials are exhausted. This might not be what the decision maker intended.

To better understand how robust solutions perform under uncertainty, we used Monte Carlo simulation by randomly generating 1,000 scenarios for the demand parameters D_{pt}, according to a normal distribution with a 6σ coverage, for each level of variability. The average nominal operational costs obtained by evaluating the robust solutions in (2.7) for each scenario are presented in Fig. 2.4.

As can be seen in Fig. 2.4, the nominal solution ($\phi = 0$) performs in average better than the most conservative solution ($\phi = 1$) for every level of variability. The figure also shows the existence of an optimal level of demand conservatism, which results from the trade-off between the holding/shortage cost and the raw material inventory cost. The average holding/shortage cost increases when the level of conservatism is very high, since the inventory of products tends to grow. However, for low levels of conservatism, the cost also increases due to backlogged demand. Thus, there is a level of conservatism, which depends on the magnitudes of both B and S, in which the holding/shortage product cost is at its minimum. The inventory of raw material cost, on the other

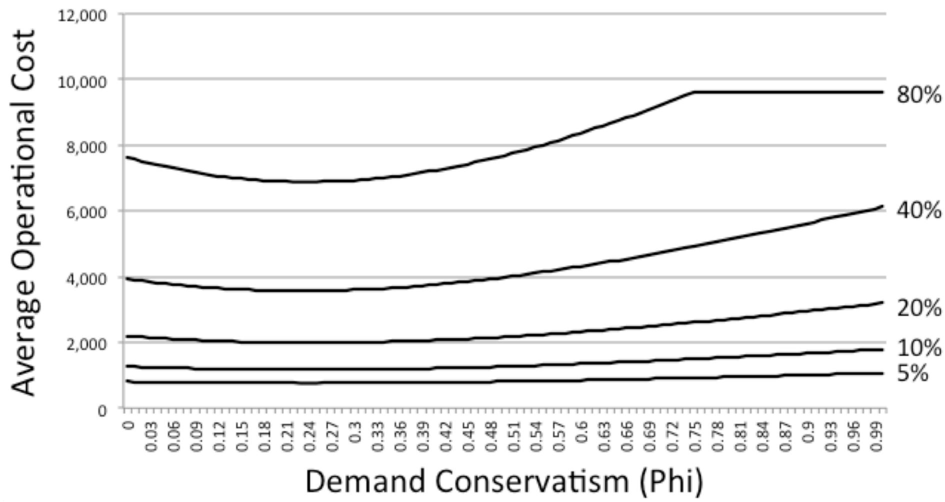

FIGURE 2.4. Average nominal operational cost for several values of γ_p.

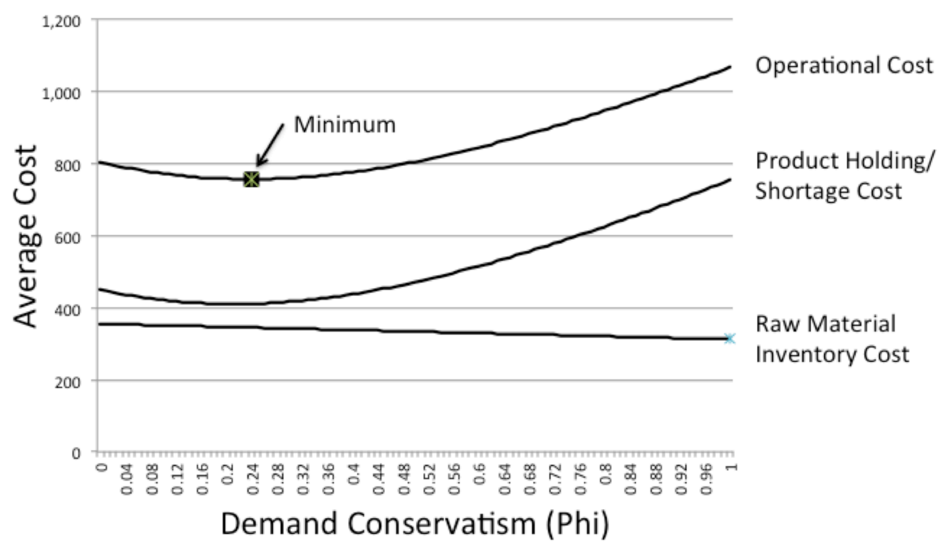

FIGURE 2.5. Optimal level of conservatism for γ_p = 5%.

hand, diminishes when demand conservatism increases reaching a minimum at the full protection case. Thereby, the optimal conservatism parameter is displaced to the right. Fig. 2.5 shows this behavior and the optimal level of conservatism for a variability of 5%. However, it is important to note that this optimal level, $\phi = 0.24$, was the same for all the levels of variability considered.

Since average behavior sometimes hides important aspects, we also looked at the standard deviations of the nominal operational costs generated by the Monte Carlo Simulation. The average

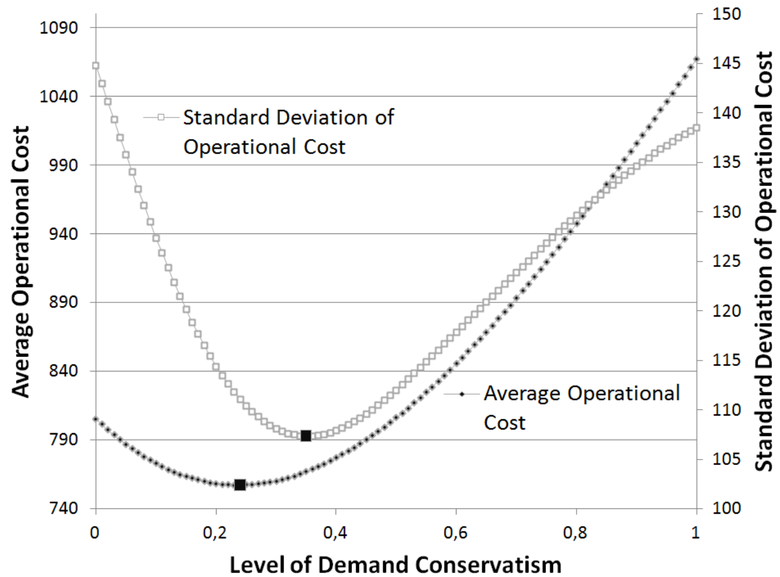

FIGURE 2.6. Average and standard deviations of robust solutions for γ_p = 5%.

and standard deviation of these nominal operational costs, for a 5% variability level, and for different levels of demand conservatism, are presented in Fig. 2.6. This figure shows that there is a level of conservatism ($\phi = 0.35$) that minimizes the variability of the nominal operational cost, which is different from the one that minimizes the average nominal operational costs ($\phi = 0.24$).

From a managerial point of view, this last result is very useful since it constrains the choice of a particular level of conservatism to a subset of the interval $[0, 1]$. Consider Fig. 2.7, which shows the average and standard deviation of the nominal operational costs generated by the Monte Carlo Simulation when we vary the level of demand conservatism from 0 to 1, for a 5% demand variability. We highlighted in this figure an efficient frontier given by the set of Pareto-optimal points considering both the average and standard deviation of the nominal operational costs. From Fig. 2.6, we can see that this Pareto-optimal trade-off takes place between the levels of conservatism that minimize each of the curves. Therefore, a rational decision maker needs only to focus on the interval $[0.24, 0.35]$ when selecting a level of demand conservatism according to the decision maker risk aversion. Any other choice is dominated by either a higher average operational cost, for a given standard deviation, or a lower standard deviation for a given average operational cost.

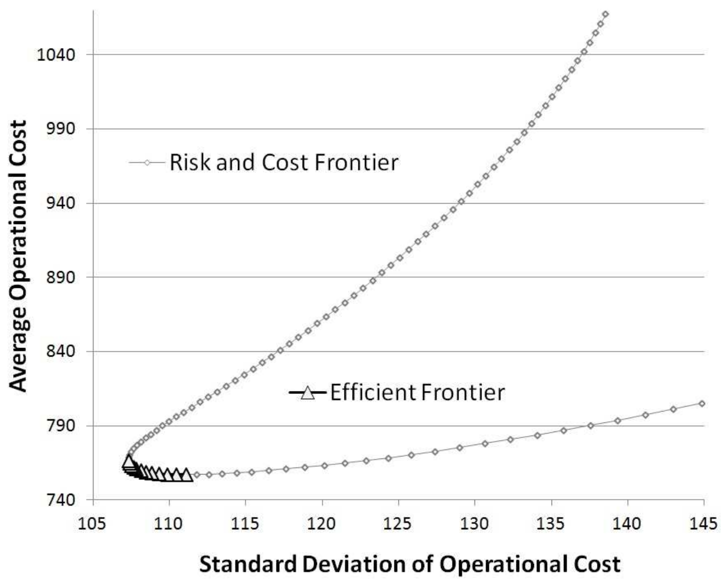

FIGURE 2.7. Possible combinations of average and standard deviation of operational costs for for γ_p = 5%.

Finally, we analyzed how well robust solutions satisfy the uncertain demand for products. For each scenario, we computed the following service level index (called Type I service level or ready rate) that determines how often demand was not backlogged during the planning horizon for a given robust solution:

$$SL = 1 - \frac{\sum_{t,p} b_{pt}}{|\mathcal{P}||\mathcal{T}|} \tag{2.25}$$

where

$$b_{pt} = \begin{cases} 1 & \text{if } s_{pt} < 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.26}$$

Note that *SL* does not discriminate by the magnitude of the backlog. The results of the Monte Carlo simulation, shown in Fig. 2.8, clearly indicate that the service level increases with the level of conservatism, regardless of the variability level. The only exception is for the 80% variability case, when the raw material inventory is exhausted. In that case, increasing the level of conservatism over 0.75 does not increase the service level any further.

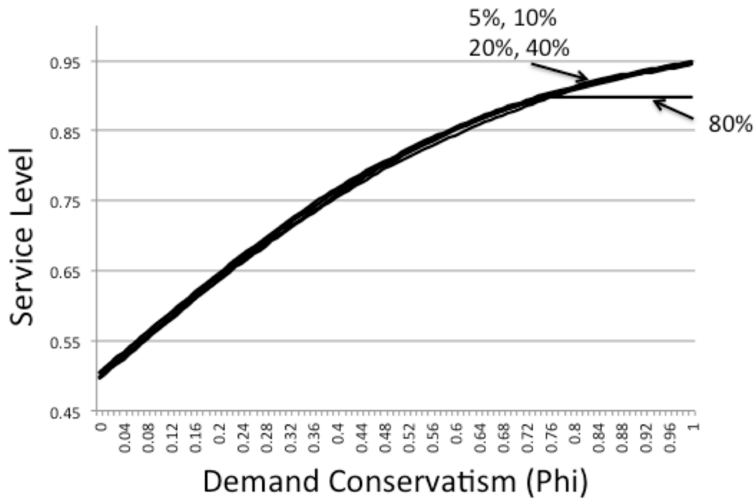

FIGURE 2.8. Average service level (*SL*) for different levels of demand conservatism and γ_p.

From a managerial perspective, the results shown in Fig. 2.8 add another dimension to the problem of selecting the proper demand conservatism level, which is the service level that will be offered to the clients. For instance, a risk-averse decision maker may be willing to accept a higher average operational cost and a higher standard deviation than those in the efficient frontier, in order to achieve a higher service level than those that can be attained with the conservatism levels within this frontier. However, even with this added dimension, choosing a demand conservatism level lower than 0.24 is not rational, since it would be dominated by another demand conservatism level with a higher service level and either a lower average cost for the same standard deviation or a lower standard deviation for the same average cost.

Finally, it is worth noting the convexity of the average operational cost and the concavity of the average service level as a function of demand conservatism. This implies that as we increase the level of demand conservatism, in the relevant subset of $[0, 1]$, the improvements in the service level diminish while the degrade in the average operational cost become larger.

2.5.2. Uncertainty in supply parameters

Similarly to the uncertainty in demand parameters, we assume that for each raw material i the uncertain supply R_{in} is independent and identically distributed, with a nominal value $\bar{R}_{in}$, and a maximum variation given by $\delta\bar{R}_{in}$. We assume the following levels of variability δ for the uncertain supply: 5%, 10%, 20%, 40%, 80%, and, as before, we determine the uncertainty budgets as $\Gamma'_{in} =$

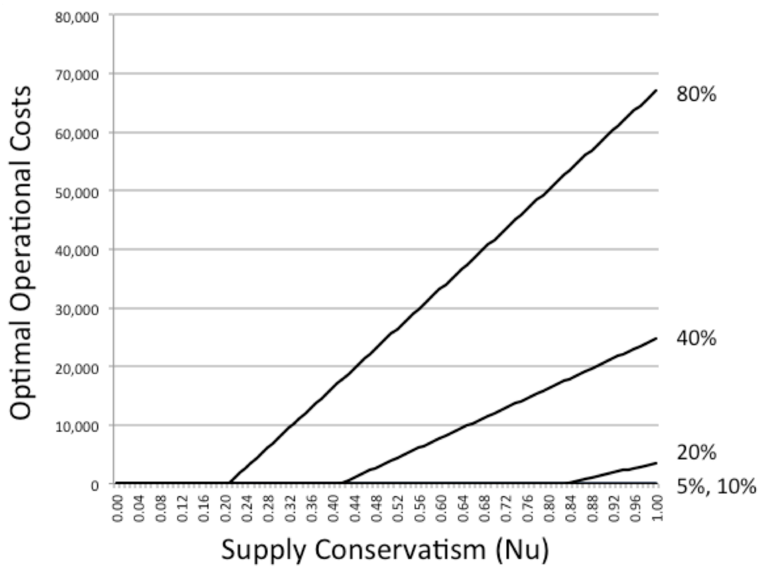

FIGURE 2.9. Optimal nominal operational costs for several levels of δ.

$\nu n \; \forall i, n$ with $\nu \in [0, 1]$. Since in this case we are interested solely on supply uncertainty, we set $\Gamma_{pt} = 0 \; \forall p, t$ in (2.24).

For each variability level we generated and solved an instance of (2.24) for values of the supply conservatism parameter ν from 0 to 1 in steps of 0.01. As the term $v_{in}\Gamma'_{in} + \sum_{l=1}^{n} w_{lin}$ is included in the objective function of (2.24) through the effect on α_{in}, we evaluated each robust solution in (2.7) in order to facilitate the analysis of the effect of the conservatism parameter on the nominal cost function. The results, which are presented in Fig. 2.9, show that the nominal operational cost increases with the supply conservatism level for all the variabilities considered. Also, the higher the variability, the higher is the increase in the nominal costs.

Although this behavior is expected, to a certain degree, it is worth noting that the stronger deterioration that takes place for variability levels of δ = $\{20\%, 40\%, 80\%\}$ is mostly due to the increase in product holding/shortage costs. For variability levels of 10% or less, there is no significant increase in cost when conservatism is increased, and the full protection case ($\nu = 1$), only represents an increase of 4% in the nominal operational cost for δ = 5% and of 13% for δ = 10%. Moreover, and for every instance of the supply conservatism parameter, the robust solutions do not incur in product holding/shortage costs. Therefore, despite the increase in conservatism in raw material supply, the orders for products are met in each period by postponing processing some raw material to later subperiods. However, for higher levels of variability, a threshold appears for the supply conservatism level beyond which, in order to achieve a solution that remains either deterministically feasible or feasible with a high probability, the fulfillment of all product orders is no longer possible. The higher the variability, the lower is this threshold level. In our case, these thresholds are ν = {0.83, 0.41, 0.2} for δ = {20%, 40%, 80%}, respectively. Beyond these conservatism levels, less raw material is processed and, consequently, the backlog cost gradually increases.

As before, we also conducted a feasibility study to analyze how well the robust solutions would perform in practice. For each level of variability we randomly generated 1,000 scenarios for the raw material supply parameters R_{in} according to a normal distribution with a 6σ coverage. For each scenario, we evaluated the robust solutions and we computed a feasibility index, defined below, which indicates how often the raw material inventory constraint was not violated during the planning horizon:

$$FL = 1 - \frac{\sum_{i,n} c_{in}}{|\mathcal{I}||\mathcal{N}|} \tag{2.27}$$

where

$$c_{in} = \begin{cases} 1 & \text{if } r_{in} < 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.28}$$

The results of the Monte Carlo simulation presented in Fig. 2.10 show that as conservatism increases, the feasibility level also increases. For levels of ν greater than 0.4, the feasibility is very high, at least 99.5%, regardless of the variability. For lower levels of conservatism the feasibility level varies with the variability level. For the nominal case ($\nu = 0$) the feasibility levels are *FL* = {85%, 85%, 84%, 83%, 79%} for δ = {5%, 10%, 20%, 40%, 80%} respectively. This is due to the infeasibilities that may occur in the first subperiods because of the large deviations in R_{in}.

How could the results shown in Figures 2.9 and 2.10 help a decision maker choose the appropriate level of conservatism? Consider Fig. 2.11, which shows the average optimal nominal costs and the average feasibility index values for different levels of supply conservatism in the case of a 20% variability level. Note that for a conservatism level higher than the threshold 0.83, the nominal cost increases very quickly since it becomes impossible to fulfill all product orders and backlog costs appear. Therefore it seems reasonable to consider only values of $\nu \in [0, 0.83]$. Within this

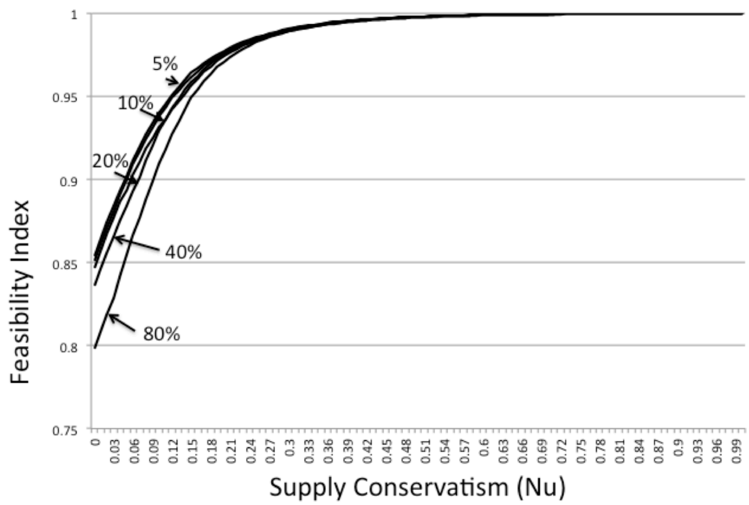

FIGURE 2.10. Average feasibility index (FL) for different levels of supply conservatism and δ.

range the decision maker should select a value that depends on the trade-offs the planner is willing to make between the increased cost and the increased feasibility index. However, as the conservatism increases, the optimal nominal cost grows at an increasing rate, while the feasibility level rises at a diminishing rate. Therefore, most decision makers would probably choose a conservatism level between 0.2 and 0.5. A value lower than that would mean a sharp fall in the feasibility index and only a moderate decrease in the optimal nominal cost, while a larger value would mean a small increase in the feasibility index at the expense of a significant increase in the optimal nominal cost. This can be more clearly seen in Fig. 2.12, which shows the combinations of the optimal nominal cost and the feasibility index for a 20% variability. In this figure the values for a conservatism level between 0.2 and 0.5 are shown in a darker color. Note that a 4% increase in the optimal nominal cost allows achieving a 98.5% feasibility level. However, to increase the feasibility level further, the optimal nominal costs must rise very fast.

2.5.3. Uncertainty in both demand and supply parameters

Finally we analyze the behavior of robust solutions when both product demand and raw material supply are uncertain. From the previous two subsections we have seen that robust solutions deal with both sources of uncertainty in a different way. When product demand is uncertain, robust solutions attempt to avoid backlogged demand by processing more raw material so that the performance of the solution is not severely affected when uncertain demands deviate from their nominal values

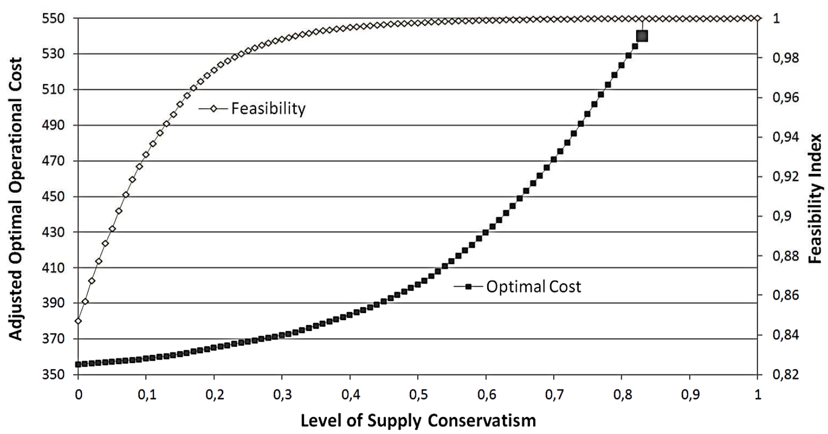

FIGURE 2.11. Average optimal nominal costs and average feasibility levels for δ = 20%.

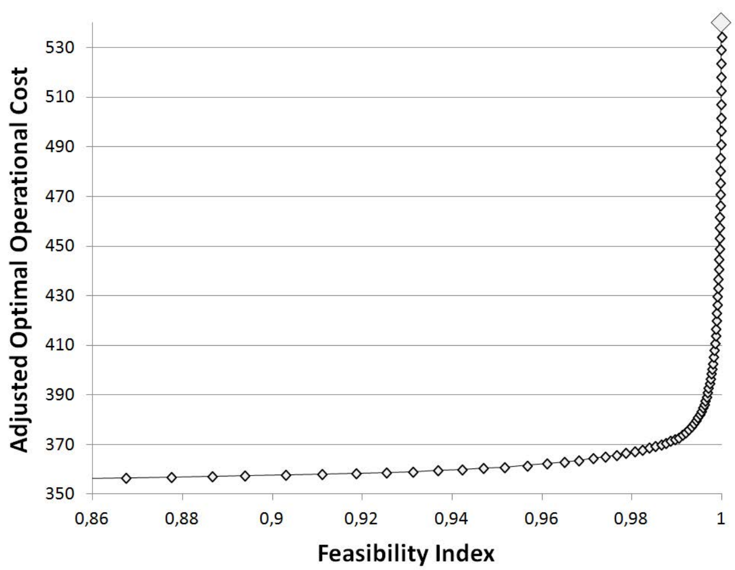

FIGURE 2.12. Possible combinations of both average optimal nominal costs and average feasibility index values for δ = 20%.

to their lower bounds. When raw material supply is uncertain, robust solutions gradually decrease the amount of raw material processed when the level of conservatism exceeds the threshold level, which depends on the variability level, in order to achieve feasibility.

Note, however, that the worst case for raw material inflow imposes bounds to the worst case for product demand. From the robust formulation for the uncertain scheduling production problem (2.24), we can see that as the budget of uncertainty on the raw material inventory constraint increases, the maximum quantity that can be processed decreases. This reduces the set of feasible robust solutions and in turn limits the overproduction that may be required by demand conservatism.

Thus, in order to analyze how the robust approach for supply uncertainty constrains the robust approach for demand uncertainty, several scenarios were generated considering pairs of variability given by the Cartesian product of $\{5\%, 10\%, 20\%, 40\%\}$ with itself. For each pair (γ, δ) we vary both levels of conservatism, ϕ and ν, from 0 to 1, in steps of 0.01. Each model instance of (2.24) was solved and evaluated in (2.7). Due to the similarity of results, we only present those for the worst pair of variabilities considered, γ=40% and δ=40%.

In Fig. 2.13, which shows the optimal nominal operational costs as the level of supply conservatism ν and the level of demand conservatism ϕ increase, we can see that the level of supply conservatism dominates the behavior of robust solutions, when it is larger than 0.42. For levels of supply conservatism lower than 0.13, it is possible to achieve the overproduction required by demand conservatism. In fact, for each $\phi \in [0, 1]$ the corresponding robust solution for any $\nu \in [0, 0.13]$ processes the same quantity of raw material, but with a different schedule. This explains the smooth increase in the optimal nominal operational cost for $\phi = 1$ as ν increases in this interval.

Fig. 2.13 also shows that for a given $\phi > 0$ there exists a $\hat{\nu}(\phi) \in [0.14, 0.41]$ such that for $\nu \in [\hat{\nu}(\phi), 0.41]$ the optimal nominal operational cost is decreasing. This means that, as ν increases from $\hat{\nu}(\phi)$ to 0.41, the scarcity of raw material supply makes it impossible to achieve the overproduction required by ϕ, so production must be reduced in order to assure a feasible raw material processing plan. Furthermore, this implies that for each $\nu \in [0.14, 0.41]$ there exists a $\hat{\phi}(\nu) \in [0, 1]$ that generates the maximum level of production, as shown in Fig. 2.14. For all $\phi > \hat{\phi}(\nu)$, the same raw material processing plan is obtained. This explains why for a given $\nu \in [0.14, 0.41]$ there is a threshold level for demand conservatism beyond which the operational cost does not change. Overall, the existence of threshold levels in demand conservatism allow us to assess how supply conservatism constrains the robust approach on demand uncertainty. In that sense, the subset of demand conservatism parameters $D(\nu) = \{\phi : \phi \geq \hat{\phi}(\nu)\}$ are dominated by supply conservatism ν since it is not possible to achieve the production level that is required.

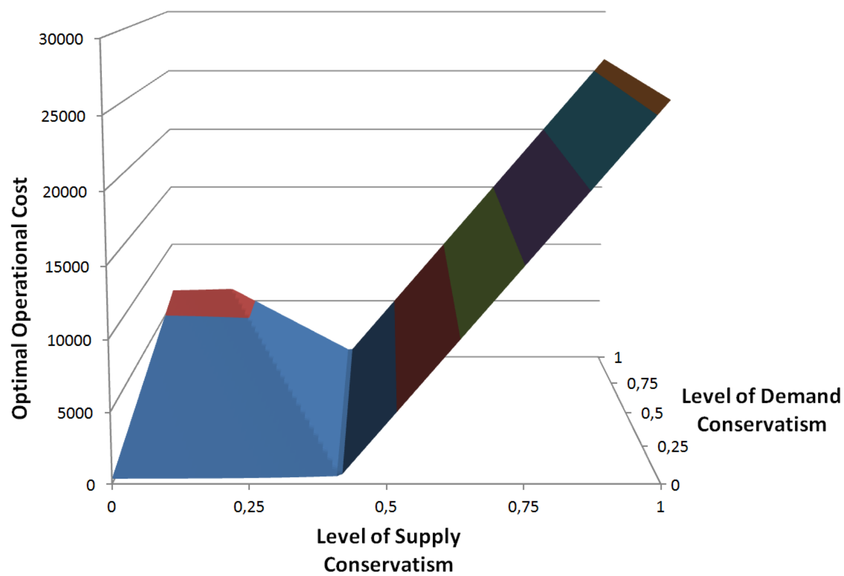

FIGURE 2.13. Optimal nominal operational costs for γ=40% and δ=40%.

When the supply conservatism level is greater than 0.41, the operational cost depends solely on supply conservatism, as can be seen in Fig. 2.13. Moreover Fig. 2.14 shows that for $\nu = 0.42$ the maximum level of production is achieved at $\phi = 0$, i.e., $\hat{\phi}(0.42) = 0$, so higher levels of demand conservatism do not change the raw material processing plan. Therefore, for values of $\nu > 0.42$, robust solutions behave as if there were only uncertainty in the supply, that is, demand conservatism is fully dominated by supply conservatism. Recall from 2.5.2 that $\nu = 0.41$ was the threshold level for supply conservatism under 40% of variability. This explains the steep increase in operational costs when supply conservatism is greater than 0.41, which is the same that can be appreciated in Fig. 2.9.

To gain more insight on the performance of the robust solutions, we carried out a Monte Carlo simulation to analyze how different levels of conservatism impact service and feasibility levels, and the average operational costs. As before, we randomly generated 1,000 scenarios for each pair (γ, δ) considering variability in both raw materials parameters R_{in} and demand parameters D_{pt} according to a normal distributions with a 6σ coverage. We then evaluated the robust solutions in each scenario. For the sake of brevity, we focus only on the worst pair of variabilities considered, γ=40% and δ=40%.

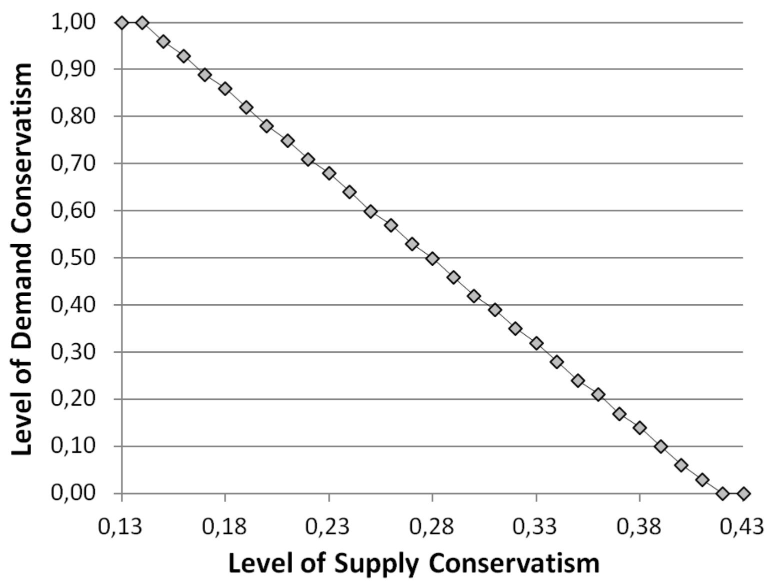

FIGURE 2.14. Demand conservatism levels which generate the maximum level of production for each $\nu \in [0.13, 0.43]$ under γ=40% and δ=40%.

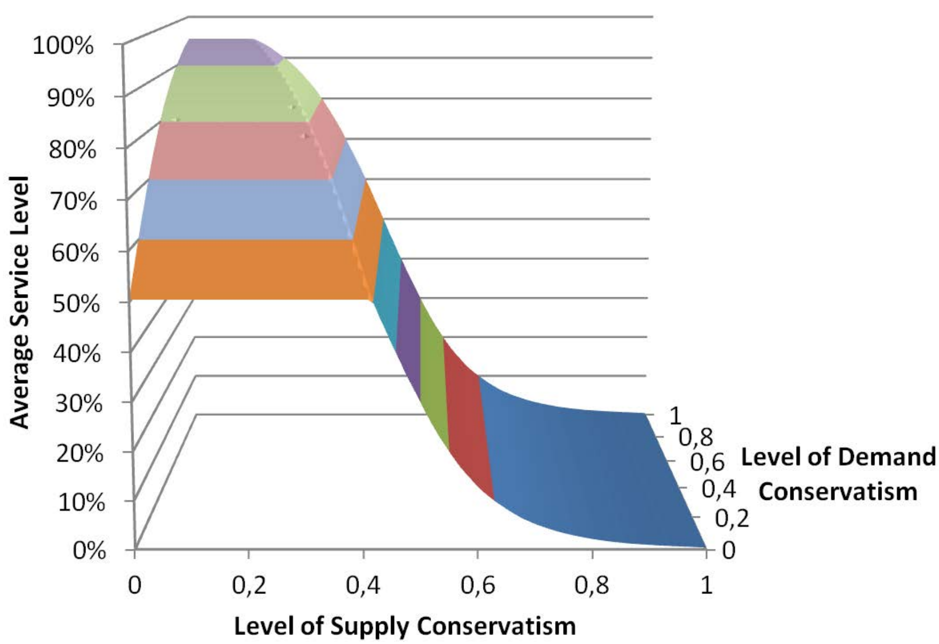

FIGURE 2.15. Average service levels for γ=40% and δ=40%.

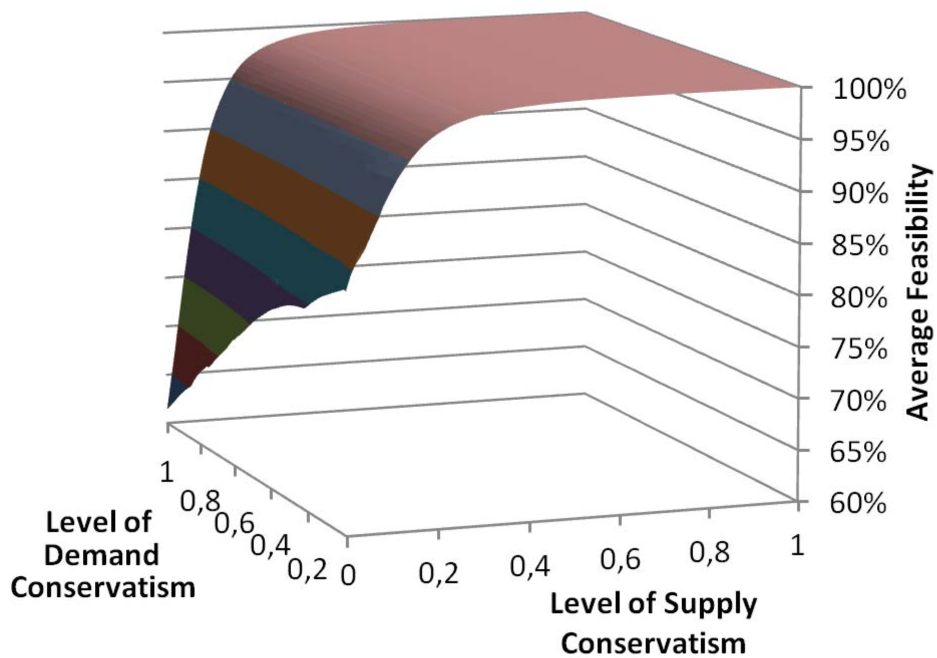

FIGURE 2.16. Average feasibility for γ=40% and δ=40%.

Fig. 2.15 shows the average service level. With respect to demand conservatism, the service level exhibits the same behavior as that of Fig. 2.8 for levels of supply conservatism of 0.13 or less. However, as ν increases beyond 0.13, the maximum average service levels begins to decrease, and the threshold levels for demand conservatism shown in Fig. 2.14 define the dominated region in which average service levels cease to increase with demand conservatism. Beyond the supply conservatism threshold level ($\nu > 0.41$) demand conservatism is fully dominated and the average service level decreases steeply.

Fig. 2.16 shows the average feasibility level. Recall that the average feasibility level is increasing with respect to the supply conservatism parameter, as shown in Fig. 2.10. However, as the demand conservatism increases, the resulting overproduction may result in shortages of raw material. Thus, for lower levels of supply conservatism, the average feasibility diminishes as demand conservatism increases. Again the dominance of supply conservatism manifests itself through a set of thresholds for the demand conservatism beyond which the average feasibility ceases to decrease. As before, beyond $\nu = 0.41$ demand conservatism is fully dominated by supply conservatism and the average feasibility level function behaves exactly as shown in Fig. 2.10.

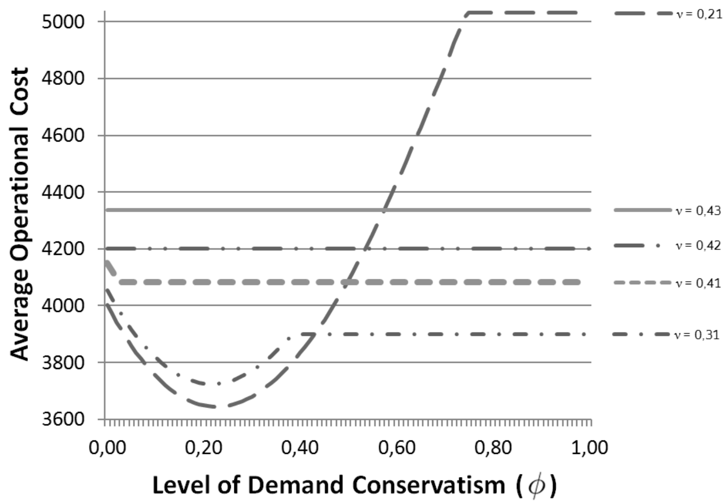

FIGURE 2.17. Average cost curves for several levels of supply conservatism.

The results of the Monte Carlo simulation for the average operational cost are harder to visualize. Fig. 2.17 shows the behavior of the average operational cost as a function of the demand conservatism parameter for selected levels of supply conservatism. To facilitate the analysis, let $z(\phi, \nu)$ be the average operational cost function and $z^*(\nu)$ be the minimum average operational cost for a given level of supply conservatism. Also let $\phi^*(\nu)$ be the level of demand conservatism such that $z^*(\nu) = z(\phi^*(\nu), \nu)$ and $\hat{\phi}(\nu)$ be the threshold level for demand conservatism from which $z(\phi, \nu)$ does not change, i.e., $z(\hat{\phi}(\nu), \nu) = z(\phi, \nu) \; \forall \; \phi \geq \hat{\phi}(\nu)$. Table 2.5 shows $\phi^*(\nu)$, $z^*(\nu)$, *SL*($\phi^*(\nu)$), *FL*($\phi^*(\nu)$), $\hat{\phi}(\nu)$, $z(\hat{\phi}(\nu), \nu)$, *SL*($\hat{\phi}(\nu)$) and *FL*($\hat{\phi}(\nu)$) for selected values of ν. In this table we can see that $\phi^*(\nu)$ is non-increasing with ν and that $z^*(\nu)$ increases with ν. Furthermore, there is a threshold level $\hat{\nu}$ for the supply conservatism parameter such that $\phi^*(\nu) = \hat{\phi}(\nu) \; \forall \; \nu \geq \hat{\nu}$. This implies that supply conservatism can dominate not only the level of production but also the trade-off between average product holding-shortage costs. In the table the threshold level is 0.36.

From a managerial point of view, the results shown in Table 2.5 and Fig. 2.17 could help a decision maker choose the appropriate levels of both supply and demand conservatism. Since for values of $\nu \geq 0.14$ the levels of feasibility are very high, a possible strategy for selecting the conservatism parameters is doing it in two stages. First, the decision maker should choose a level of supply conservatism ν' that reflects the decision maker preferences in regard to the feasibility level.

TABLE 2.5. Average operational costs for selected values of raw material supply and product demand conservatism parameters.

ν	$\phi^*(\nu)$	$z^*(\nu)$	$SL(\phi^*(\nu))$	$FL(\phi^*(\nu))$	$\hat{\phi}(\nu)$	$z(\hat{\phi}(\nu), \nu)$	$SL(\hat{\phi}(\nu))$	$FL(\hat{\phi}(\nu))$
0.14	0.23	3616	66.5%	94.8%	1.00	6202	94.6%	93.7%
0.18	0.23	3629	66.5%	96.7%	0.86	5509	92.1%	96.4%
0.22	0.23	3650	66.5%	97.9%	0.71	4883	88.7%	97.8%
0.26	0.23	3679	66.5%	98.6%	0.57	4354	83.9%	98.5%
0.30	0.22	3714	65.9%	99.0%	0.42	3968	77.5%	99.0%
0.34	0.22	3757	65.9%	99.3%	0.28	3781	69.4%	99.3%
0.36	0.21	3780	64.8%	99.4%	0.21	3780	64.8%	99.4%
0.38	0.14	3847	60.0%	99.5%	0.14	3847	60.0%	99.5%
0.40	0.06	3985	54.8%	99.5%	0.06	3985	54.8%	99.5%
0.42	0.00	4200	49.6%	99.5%	0.00	4200	49.6%	99.5%

Then asses the trade-off between cost and service level on the subset $A(\nu') = \{\phi : \phi^*(\nu') \leq \phi \leq \hat{\phi}(\nu')\}$. The level of supply conservatism that makes $\phi^*(\nu) = \hat{\phi}(\nu)$, which in this case is $\nu = 0.36$, seems to be a natural upper bound in the first stage. For higher values the service level starts to decrease steeply, the improvements in feasibility levels are negligible, and the minimum average operational costs start to increase very fast. In the second stage, for example, if the decision maker chose $\nu = 0.30'$ in the first stage, then the value of the demand conservatism should be chosen from $A = \{\phi : 0.22 \leq \phi \leq 0.42\}$, depending on the decision maker's risk aversion.

2.6. Concluding remarks and future research

We have shown that the robust optimization approach can be used to help schedule production for a sawmill plant subject to final product demand and raw material supply uncertainty. In particular, we showed that the Bertsimas and Sim approach could be adapted to this problem to properly protect against uncertainty in both final product demand and raw material supply. The robust optimization approach has at least three practical advantages over other approaches. The first is that it does not require having probabilistic distributions of the uncertain parameters. In practice, it is very hard to obtain the required data to estimate these distributions since the environment is constantly changing. Second, it preserves the linearity of the model, which allows solving larger problems than other approaches. The third is that it allows decision makers to choose the budget of uncertainty, which is a critical parameter to obtain a robust solution. Making this choice, based on their experience and risk aversion, could help give decision makers more confidence on the results obtained by the model.

However, allowing the decision maker to choose the budget of uncertainty can also be a problem, since there are no general guidelines for choosing an appropriate value. For this reason we decided to study the behavior of the optimal solutions under different levels of conservatism and variability of the uncertainty. This allowed us to observe that under certain conditions there might be a level of conservatism that achieves better results, in average, than either the nominal model, or the fully protected model. We believe it would be very helpful for a decision maker to know how the different levels of conservatism affect the feasibility and service levels and the trade-off in terms of loss of optimality that is necessary to incur in order to improve these levels. For example, knowing that one source of uncertainty tends to dominate under certain conditions, as occurred in our experiments, also would be helpful for decision makers. Without this knowledge, the applicability of the robust optimization approach is greatly diminished. Although some of the published applications of robust optimization, like Bohle et al. (2010); Wei et al. (2011); Alem & Morabito (2012); Aouam & Brahimi (2013), analyze the solution behavior for different values of the uncertainty budget, the complexity of the analysis would not be helpful for most decision makers. The analysis presented in this chapter we believe is simple enough to be of help to most decision makers.

One of the limitations of the approach we used is that it is static. All decisions must be made at the beginning of the planning horizon, with no possibility of adapting them to the realization of uncertain data later on, as is done in Chapter 4 with rolling horizons. We might use in the future an online algorithm where some data are released through time, or dynamic stochastic programming to improve this. Another alternative is to use the Affinely Adjustable Robust Counterpart approach proposed by Ben-Tal et al. (2004). This approach allows defining some of the uncertain parameters as "non-adjustable variables", while the other variables, the "adjustable variables", can be chosen after the realization. Nevertheless, the current approach of using a rolling horizon has the advantage of preserving the linearity of the problem structure, which greatly facilitates the solution process.

A final conclusion is that the Bertsimas and Sim robust optimization approach is a good option for dealing with uncertainty to support production scheduling problems in real-world problems, mainly since it preserves the lineal structure of the deterministic model as opposed to other approaches, which are theoretically attractive, but difficult to apply. However, we believe that more study on how to determine the uncertainty budget, under different conditions, is required in order to help decision makers choose the appropriate one.

Appendix 2.A: the data

The following tables show the data used in our models.

Set	Cardinality
$\mathcal{I}$	6
$\mathcal{P}$	7
$\mathcal{T}$	6
$\mathcal{N}$	42

Table 1: Set Cardinality

Parameters	Value
C_n	24 $\forall n$
A	0.04
R_{in}	60 $\forall i, n$
S	1
B	2
M	0.01

Table 2: Parameter Values

Product	Demand
1	612.5 $\forall t$
2	37.8 $\forall t$
3	105 $\forall t$
4	525 $\forall t$
5	454.3 $\forall t$
6	280 $\forall t$
7	85.4 $\forall t$

Table 3: Product Demands

	Product1	Product2	Product3	Product4	Product5	Product6	Product7
Log1	0.235	0.067	0	0.469	0.229	0	0
Log2	0.346	0.03	0	0.377	0.247	0	0
Log3	0.1	0	0.15	0.25	0.09	0.4	0.01
Log4	0.1	0	0.15	0.25	0.09	0.4	0.01
Log5	0.509	0	0	0	0.275	0	0.216
Log6	0.46	0.011	0	0.154	0.367	0	0.008

Table 4: Yield Matrix of Optimal Cutting Patterns

3. ON THE MANAGEMENT OF HIGH-VALUE GOODS IN THE WINE INDUSTRY UNDER UNCERTAINTY: A TWO-STAGE HEURISTIC TO SETTING STOCK LEVELS UNDER (*S-1,S*) INVENTORY CONTROL POLICIES.

3.1. Introduction

Operations research models and methodologies have increasingly being used for helping decision makers in the management of natural resource operations in several areas like agriculture, fisheries, forestry, and mining. For a recent review of applications in all four sectors see the work of Bjørndal et al. (2012). Particularly, as A. Weintraub & Romero (2006) state, agriculture is one of the fields in which operations research models were first used, almost fifty years ago, and have been most widely applied. On this sector highlight the management of products destined for human consumption, and planning models for the agri-food supply chain have been the subject of an active research (Ahumada & Villalobos, 2009). Regarding with this kind of products lies the wine, and in the recent years, researchers have start to make contributions along the wine supply chain (Moccia, 2013).

3.1.1. Managing high-value goods in the wine industry under uncertainty: preliminary considerations

Taking into account both environmental and system uncertainties, but also the synchronization and alignment issues discussed by Wine Supply Chain Council[1] (WSCC), in this chapter we focused on premium wines, and we developed a simple two-stage heuristic to support wine managers with the stocking of finished goods decisions.

As Cholette (2009) argues, the global growth of both export programs and private label brands lead wineries to allocate their often limited production across an increasing variety of sales channels which, more often than not, demand different packaging or labeling requirements. For simplicity, we consider the case where the only distinctive attribute among the finished goods is the label. In this case, the number of different labels determines the number of premium SKUs that the winery handle, which is not a strong assumption.

[1] http://wscc.scl.gatech.edu/

Moreover, we assume that the winery manage these wines under *postponement* in the labeling process, which refers to delaying the labeling until more information about customer orders is available. For a review on postponement practices see Van Hoek (2001). This last is a common practice for this kind of goods mainly due to manufacturing constraints. In fact, Garcia et al. (2012) establish two reasons to postpone the labeling process for these non-highly rotating products. First, premium wines need an aging process in the bottle to increase their quality, so bottling must be performed before the orders for these products arrives. And second, when exporting, labels need to be customized to incorporate importers information and some specific data depending on the country of destination. Note that finished goods across different sales channels (or international markets) may contain the same wine but they cannot be treated as substitutes . So, as shown by Cholette (2009), labeling postponement can also be employed to reduce the product misallocation risk' due to order uncertainty.

Furthermore, we assume that the arrival of orders for bottled and labeled wines follows independent Poisson process. This last is a common assumption in the literature to modeling the demand process of products which exhibit both high manufacturing costs and low inventory turnover (like spare parts, for example). At the expense of generality, we assume that the product orders are for one case of wine, which is not a too strong assumption considering that, for example, the price of a single bottle of Clos Apalta -a premium wine of the Chilean winery Lapostolle- rise above US$150.

Now recall the synchronization and alignment issues established by the WSCC (see subsection 1.2.1). Due to both the commonality of a bottled wine and the lack of substitutability of a labeled one, a pure bottle-to-stock and label-to-order inventory control policy for premium wines diminishes the misallocation risk entirely, but this last is only one of several dimension that a wine decision maker must handle. Another one is the backorders (and its costs) which could rise sharply in this context. In fact, as Soman et al. (2004) argue, the (large) number of setups required by a high-diversity product portfolio could degrade severely the order performance measures, e.g. average response time or average order delay, which are the focus of pure *make-to-order* strategy. Moreover, even if a *group scheduling policy* (GS) is employed -for which Benjaafar & Sheikhzadeh (1997) shown that could accommodate a manufacturing system in presence of setups to better handle a higher product mix variety under uncertainty through capacity system preservation- the average response time (i.e. average part flow time) could be excessive if the workload is high. Thus, by Little's law (Little, 1961), a *poor* average part flow time implies a high backorder level. There must be a trade-off, and given that premium wines usually have both high manufacturing cost and low demand rates, we assume that a (*s-1,s*) inventory control policy with $s \geq 1$ is more suitable for managing the stocks of bottled and labeled products (see the comments of Feeney & Sherbrooke (1966, pp. 1)). For an appealing introduction for this kind of policy see Muckstadt & Sapra (2010, Ch. 7).

As Muckstadt & Sapra (2010, pp. 185) argue, the amount in *resupply* (which in our case correspond to the number of bottles waiting for or being labeled) at a random point in time is the key random variable in the study of the behavior of systems managed using an (*s-1,s*) policy. Once its stationary distribution is known, we can easily determine the stationary distribution for on-hand and backordered inventory. On this, we assume that the wine decision maker is concerned with minimizing the sum of the steady-state expected values of the *WIP*, overage and underage costs per unit time. Obviously, the amount in *resupply* at a random point depends on how manufacturing is managed, and there are a couple of situation when these distributions turn out to be simple expressions. On this, relevant examples are when the manufacturing system is modeled as a M/M/∞ or M/G/∞ queuing system, for which the amount in *resupply* at a random point follows a Poisson distribution (see Scarf (1958)).

In this chapter, we consider the case where there is only one labeling machine (i.e. only one resource handle the customization of premium wines), and as in Benjaafar & Sheikhzadeh (1997), a GS policy for the labeling process is employed. This last means that, where once the labeling machine is setup for a particular label type, it continues processing orders from that type until all orders are exhausted. Then, the labeling machine is setup for the next label type, and the switching is made from one label to another in a cyclic order. As long as setup times are sequence independent (which is quite common in labeling), and there is no priority among orders (which is a reasonable assumption), we think that this kind of policy is simple enough to be easily implementable by wine managers. Besides, we consider that for each label type the arrival of orders, processing times and the setup time, are i.i.d random variables and those could differ among labels types. This last, in the context of polling systems, means that we model the manufacturing process as a asymmetric cyclic queue system in which the labeling machine operates under an exhaustive service policy. For an excellent review of polling models see Takagi (1988). Given the latter, Figure 3.1 represent the dynamic of the manufacturing system/labeling process.

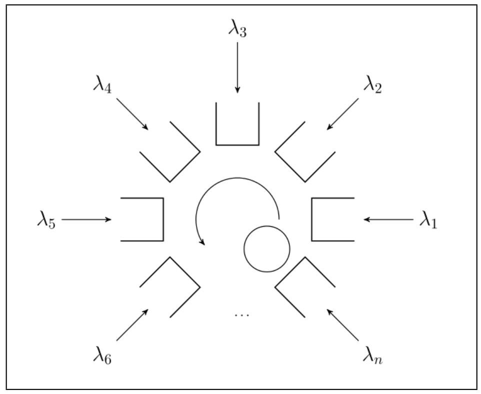

FIGURE 3.1. Dynamic of the labeling process.
Source: Gareth Jones. Own work.

As Boon et al. (2011) argue, the key performance measures of a polling system like the previously described are the queue lengths and the waiting times. On this, one of the most remarkable results regarding the marginal queue length distribution has been obtained by Fuhrmann & Cooper (1985). They observed that the marginal queue length distribution can be decomposed as the sum of two independent random variables: the queue length of an M/G/1 queue with only type i customers in isolation, and the queue length at an arbitrary moment during the intervisit period, i.e. the time between the visit completion of the queue for type i customers and the next visit beginning to this queue. This last observation results in the following probability generating function (PGF) of the marginal queue length distribution:

$$\tilde{L}_i(z) = \frac{(1-\rho_i)(1-z)\tilde{B}_i(\lambda_i(1-z))}{\tilde{B}_i(\lambda_i(1-z))-z} \cdot \frac{\tilde{L}_i^{V_{c_i}}(z) - \tilde{L}_i^{V_{b_i}}(z)}{\lambda_i E[I_i](1-z)} \tag{3.1}$$

where: λ_i is the arrival rate for queue i; ρ_i is the load at queue i; $\tilde{B}_i(\cdot)$ is the Laplace-Stieltjes transform (LST) of the service time distribution at queue i; $E[I_i]$ is the mean intervisit time for queue i, and; $\tilde{L}_i^{V_{c_i}}$ and $\tilde{L}_i^{V_{b_i}}$ denote the LST of the number of customers in queue i at the beginning and completion of a visit to this queue, respectively.

Note that this transform expression could be used to calculate the stationary distributions of the number of orders in resupply that Muckstadt & Sapra (2010) refers (and thus the stationary distribution for on-hand and backordered inventory for each label type). Nevertheless, as Choudhury & Whitt (1996) argue, this kind of transform expressions have been successfully exploited to derive means and sometimes second moments for the corresponding performance measures, but seldom used to calculate the distributions themselves. As the authors state, one possibly reason for this last is that methods for transform inversion are not especially well known. Another reason is that usually the transforms are not available directly; i.e. the transform at one polling instant is expressed in terms of the transform at another polling instant.

Although Choudhury & Whitt (1996) show that these polling transforms often can be quite easily computed and inverted numerically to calculate the required distributions, we think that less computationally intensive and simpler approaches could be more suitable to support decision making (at least initially) in the wine industry, especially, considering that operations research models and methodologies are recently beginning to spread in production and post-production activities. Note that simplicity and restrictive assumptions usually go together, but simpler approaches could have high chances to be validated and applied by wine managers as long as the context of applicability does not differ too much of the restrictive assumptions of the approach.

In order to support the managing of N high-value goods in the wine industry under uncertainty, we develop a simple two-stage heuristic which requires, in the first stage, solving a system of N^2 linear equations, and in the second, solving N straightforward nonlinear integer programing problems.

The proposed heuristic is developed taking into account the seminal work of Sherbrooke (1968), and thus, it make use of the remarkable theorem of Palm (Palm, 1938), which as Ewing (2002) argues, forms the basis of modern (*s-1,s*) theory for multi-echelon recoverable part inventory systems. Palm's Theorem can be restate as follows:

Theorem 3.1. *Suppose s is the stock level for an item which demands are generated by a Poisson process with rate λ. Assume that the resupply time random variables have density functions $g(\tau)$ with mean $\bar{\tau}$, and cumulative distribution functions $G(\tau)$. Suppose further that the resupply times are i.i.d. from customer order to customer order. Then the steady state probability that x units are in resupply is given by:*

$$\pi(x) = e^{-\lambda\bar{\tau}}\frac{(\lambda\bar{\tau})^x}{x!}, \;\; \forall x \in \mathbb{Z}^+. \tag{3.2}$$

Proof: see Palm (1938) or Muckstadt & Sapra (2010, pp. 188). □

What makes remarkable Palm's Theorem is the fact that the steady-state probabilities, which enable us to compute the stationary distribution for on-hand and backordered inventory for each label type, depends only on the first moment of the resupply time distribution: that means, in our context, that the stationary distribution of the amount in *resupply* for each label type could be obtained through the knowledge of the mean delay of an order, plus the expected labeling time, which can be computed efficiently for the kind of polling system discussed in this paper.

Literature exhibits some relevant approaches that could be used to determine exactly the mean delay of a customer. For example, one of such approach is the *buffer occupancy method* developed by R. Cooper & Murray (1969); R. Cooper (1970); Eisenberg (1972). This method is based on the buffer occupancy variables X_{ij}, which denote the queue length at queue j at a polling instant of queue i, with $i, j = 1, 2, \ldots, N$. The buffer occupancy method requires the solution of N^3 linear equations with unknowns $E[X_{ij}X_{jk}]$ (recall how to compute covariances) to compute the mean delays in all N stations simultaneously. Another approach, is the *station time method* by Ferguson & Aminetzah (1985). In the station time approach, all mean delays are obtained simultaneously starting from the station time variables $U_i, i = 1, 2, \ldots, N$, where U_i, is composed of the time the server spends servicing customers at queue i plus the preceding setup time in the case of exhaustive service. The station time technique induces a set of N^2 linear equations with unknowns $E[U_iU_j]$. More recently, Winands et al. (2006) developed a quite novel approach to compute the mean delays for exhaustive type polling systems in a pure probabilistic manner. The authors derive a set of N^2 linear equations for these delay figures with the help of the following two basic queueing results: (i) the PASTA property, i.e., Poisson arrivals see time averages and (ii) Little's Law. The unknowns in these equations are $E[L_{ij}]$, the mean queue length at queue i at an arbitrary epoch within a station time, also called a visit time, of queue j. This last approach, based on the Mean Value Analysis, is quite appealing and is the approach that we applied in this paper.

Overall, in order to support wine managers with the stocking of finished goods decisions we propose the use of the following greedy heuristic: first, compute the mean delay of the orders through Winands et al.'s equations, and then, compute the optimal stock level for each label type through the stationary distribution for on-hand and backordered inventory derived from Palm's theorem. This last, in turn, is the main contribution of this research.

The rest of this chapter is organized as follows: in Section 3.2 we review the approach of Winands et al. (2006) and we show how to compute the mean delay of a customer by means of a numerical example. In Section 3.3 we show how to set the wine stock levels using an (*s-1,s*) inventory control policy. Particularly, the properties of the related optimization problems and some straightforward bounds for the optimum of those are discussed. In Section 3.4 we test the accuracy of the proposed heuristic, whereas in Section 3.5 concluding remarks are established.

3.2. Computing the mean delays of customer orders for an asymmetric cyclic queue system under an exhaustive service policy

As state in subsection 3.1.5, there are several approaches that can be employed in order to determine the mean delay of a customer in polling systems. In this paper, we considered the novel approach of Winands et al. (2006) mainly for one reason: the derivation of the system of equations is based on standard queueing results and has a pure probabilistic interpretation. For the sake of the explanation, we review their approach next, taking into account the context of our problem.

3.2.1. Winands et al. approach

Let's start considering the following set of assumptions:

(i) We consider a system with one single server for $N \geq 1$ queues, in which there is infinite buffer capacity for each queue.

In our context, this means that there is one labeling machine for premium wines dedicated to serving N label types, and no orders for a given case of wine are rejected.

(ii) The server visits and serves the queues in a fixed cyclic order under both a *First Come First Serve* (FCFS) discipline and a *Exhaustive* policy. The queues are indexed by i, with $i = 1, 2, \ldots, N$, in the order of the server movement. For compactness of presentation, all references to queue indices greater than N or less than 1 are implicitly assumed to be modulo N, e.g., queue $N + 1$ actually refers to queue 1.

For our problem, this means that once the labeling machine is setup for a particular label type, it continues processing orders from that type, according to its arrival position, until all orders are exhausted. Then, the labeling machine is setup for the next label type, and the switching is made from one label to another in a cyclic order. Note that as long as the setups are sequence-independent and orders are not prioritized, the order within the cycle is irrelevant.

(iii) Customers arrive at all queues according to independent Poisson processes with rates λ_i. The service times at queue i are independent, identically distributed random variables with mean $E[B_i]$ and second moment $E[B_i^2]$. When the server starts service at queue i, a setup time is incurred of which the first and second moment are denoted by $E[S_i]$ and $E[S_i^2]$, respectively. These setup times are identically distributed random variables, independent of any other event involved, and in particular, they are independent of the service times.

Recall that we assumed that the orders are for one case of bottled and labeled wines. Therefore, for each label type the arrival rates, service times and setup times, are measured in cases/unit of time, units of time/case and units of time, respectively. On the other hand, although both processing times and setup times are generally product-independent at the labeling machine, there are situations where the labeling process depends on the bottle shape (especially the processing times at the machine). This latter justifies to consider the product-dependent assumptions of Winands et al. (2006). On the other hand, the Poisson arrival assumption was made in order to simplify the subsequent analysis.

Now, consider the following definitions:

- Let $E[S]$ be the mean total setup time in a cycle, which is given by:

$$E[S] = \sum_{i=1}^{N} E[S_i] \tag{3.3}$$

- Let $E[R_{B_i}]$ the mean residual service time, and $E[R_{S_i}]$ the mean residual setup time for queue i, which can be expressed as follows[2], respectively:

$$E[R_{B_i}] = \frac{E[B_i^2]}{2E[B_i]},\ i = 1, 2, \ldots, N. \tag{3.4}$$

[2] Recall the *random incidence problem*. For further information, see Ibe (2014, Chapter 12, pp. 389).

$$E[R_{S_i}] = \frac{E[S_i^2]}{2E[S_i]},\ i = 1, 2, \ldots, N. \tag{3.5}$$

- Let the occupation rate (excluding setups) at queue i defined as $\rho_i = \lambda_i E[B_i]$. Thus, total occupation rate ρ is given by[3]:

$$\rho = \sum_{i=1}^{N} \rho_i \tag{3.6}$$

- Let the cycle length of queue i be defined as the time between two successive arrivals of the server at this queue. It is well-known that the mean cycle length is independent of the queue involved and is given by[4] (see Kuehn (1979)):

$$E[C] = \frac{E[S]}{1-\rho} \tag{3.7}$$

- Let θ_i be the visit time of queue i, which is composed of the service period of queue i, i.e. the time the server spends servicing customers at queue i, plus the preceding setup time. Since the server is working a fraction ρ_i of the time on queue i, the mean of a visit period of queue i under an exhaustive service is:

$$E[\theta_i] = \rho_i E[C] + E[S_i] \tag{3.8}$$

- Let an (i, j)-period $\theta_{i,j}$ be defined as the sum of j consecutive visit times starting in queue i. The corresponding mean length is given by:

$$E[\theta_{i,j}] = \sum_{n=i}^{i+j-1} E[\theta_n] \tag{3.9}$$

- On this, the fraction of the time $q_{i,j}$ the system is in an (i, j)-period equals:

$$q_{i,j} = \frac{E[\theta_{i,j}]}{E[C]} \tag{3.10}$$

- Moreover, the mean of the residual (i, j)-period is given by:

$$E[R_{\theta_{i,j}}] = \frac{E[\theta_{i,j}^2]}{2E[\theta_{i,j}]} \tag{3.11}$$

---

[3] A necessary and sufficient condition for the stability of this polling system is simply $\rho < 1$. See Takagi (1988, pp. 11).

[4] Note the nice parallel between $E[C]$ and the optimal cycle length of the *Economic Lot Scheduling Problem* when the sum of the setup times exceeds the idle time in the manufacturing system. See Pinedo (2005, Chapter 7).

Recall that, in order to apply Palm's theorem, we are interested in to obtain the mean delay of an order for a given case of wine, $E[W_i]$, which is defined as the time in steady state from an order's arrival for a given label type i until the start of its service. By Little's Law, these mean delays are related to the mean queue lengths (excluding the customer possibly in service), which are denoted by $E[L_i]$, for $i = 1, 2, \ldots, N$. The analysis of Winands et al. (2006) is oriented towards the determination of $E[L_{i,n}]$, which is defined as the mean queue length at queue i at an arbitrary epoch within a visit time of queue n, with $i, n = 1, 2, \ldots, N$. The mean queue length $E[L_i]$ can be expressed in terms of $E[L_{i,n}]$ as follows:

$$E[L_i] = \sum_{n=1}^{N} q_{n,1} E[L_{i,n}] \tag{3.12}$$

Now, consider a tagged order at the moment it arrives at queue i, $i = 1, 2, \ldots, N$. Based on PASTA property, it is know that the state distribution seen by this tagged order is identical to the equilibrium distribution. So, this order has to wait for the servicing of all order L_i, who were already waiting in this queue on its arrival. Further, with probability ρ_i the server is working at queue i on its arrival and the tagged order has to wait for the residual service time of the order in service as well. On the other hand, with probability $E[S_i]/E[C]$ the server is in a setup phase for queue i and the delay of the customer is increased by a residual setup time. Finally, with probability $1 - q_{i,1}$ the server is at one of the other queues and the service of the tagged customer is delayed until the server starts service again at queue i. The latter time period is equal to the a residual $(i+1, N-1)$-period plus a setup time for queue i.

Through the above appealing reasoning, Winands et al. (2006) develop the following arrival relation for the mean delay $E[W_i]$ of a type-i order:

$$E[W_i] = E[L_i]E[B_i] + \rho_i E[R_{B_i}] + \frac{E[S_i]}{E[C]} E[R_{S_i}] + (1 - q_{i,1})(E[R_{\theta_{i+1,N-1}}] + E[S_i]) \tag{3.13}$$

and through the application of Little's Law,

$$E[L_i] = \lambda_i E[W_i] \tag{3.14}$$

yields to the derivation of the mean queue lengths expressions for exhaustive polling systems (by using the *Mean Value Analysis* (MVA) approach), which the authors state as follows:

$$E[L_i] = \frac{\lambda_i}{1-\rho_i}(\rho_i E[R_{B_i}] + \frac{E[S_i]}{E[C]}E[R_{S_i}] + (1-q_{i,1})(E[R_{\theta_{i+1,N-1}}] + E[S_i]),\ i = 1,\dots,N. \tag{3.15}$$

As Winands et al. (2006) argue, the unknown $E[R_{\theta_{i+1,N-1}}]$ form the stumbling block to the straightforward computation of the mean queue lengths (and thus the mean delay of an order) via this expression. To obtain those unknowns, the authors relate them to $E[L_{i,n}]$ and derive a set of equations for these quantities.

First, as the authors state, note that under an exhaustive service policy no type-i orders are left at the end of a visit time of queue i. Thus the following property can be obtained: the number of type-i orders present at an arbitrary moment within an $(i+1,j)$-period equals the number of Poisson arrivals during the age of this $(i+1,j)$-period. Since the age is in distribution equal to the residual time, the following set of $N(N-1)$ of equations holds:

$$\sum_{n=i+1}^{i+j} \frac{q_{n,1}}{q_{i+1,j}} E[L_{i,n}] = \lambda_i E[R_{\theta_{i+1,j}}], i = 1,\dots,N; j = 1,\dots,N-1. \tag{3.16}$$

And second, by substitution of Identity (3.12) into expression (3.15), it yields the following set of N equations:

$$\sum_{n=1}^{N} q_{n,1} E[L_{i,n}] = \frac{\lambda_i}{1-\rho_i}(\rho_i E[R_{B_i}] + \frac{E[S_i]}{E[C]}E[R_{S_i}] + (1-q_{i,1})(E[R_{\theta_{i+1,N-1}}] + E[S_i])),\ i = 1,\dots,N. \tag{3.17}$$

Equations (3.16) and (3.17) represent a set of N^2 linear equations for $2N^2$ unknowns, $E[L_{i,n}]$ and $E[R_{\theta_{i,j}}]$. Therefore, it is necessary to derive additional equations by expressing $E[R_{\theta_{i,j}}]$ in terms of $E[L_{i,n}]$. In an appealing and pure probabilistic manner, the authors derive the following set of extra equations based on the initiation of a busy period (for further details, see Winands et al. (2006, Subsection 3.1)):

$$E[R_{\theta_{i,1}}] = \frac{1}{1-\rho_i}(E[L_{i,i}]E[B_i] + \frac{\rho_i E[C]}{E[\theta_{i,1}]}E[R_{B_i}] + \frac{E[S_i]}{E[\theta_{i,1}]}E[R_{S_i}]),\ i = 1,\dots,N. \tag{3.18}$$

and for $i = 1, \ldots, N$ and $j = 2, \ldots, N$:

$$E[R_{\theta_{i,j}}] = \frac{q_{i,1}}{q_{i,j}} \left(\frac{E[R_{\theta_{i,1}}]}{\prod_{n=1}^{j-1}(1-\rho_{i+n})} + \sum_{n=1}^{j-1} \frac{E[S_{i+n}] + E[L_{i+n,i}]E[B_{i+n}]}{\prod_{m=n}^{j-1}(1-\rho_{i+m})} \right) + (1 - \frac{q_{i,1}}{q_{i,j}}) E[R_{\theta_{i+1,j-1}}] \tag{3.19}$$

Finally, as the authors argue, eliminating $E[R_{\theta_{i,j}}]$ from Equations (3.16) and (3.17) with the help of Equations (3.18) and (3.19), renders a set of N^2 linear equations for equally many unknowns $E[L_{i,n}]$. After solving these equations, the unconditional mean queue lengths and mean delays can be computed via Identity (3.12) and Little's Law (3.14).

3.2.2. A numerical example

Consider a two-queue polling system with exhaustive service, i.e. $N = 2$. Suppose that the service and setup times follow exponential distributions with means equal to 1 for both customer types. Furthermore, assume that λ_1 = 0.6 and λ_2 = 0.2. Then,

The first and second moment of both service and setup times are,

$$E[B_1] = E[B_2] = 1$$

$$E[B_1^2] = E[B_2^2] = 2$$

$$E[S_1] = E[S_2] = 1$$

$$E[S_1^2] = E[S_2^2] = 2$$

The mean total setup time in a cycle is,

$$E[S] = \sum_{i=1}^{2} E[S_i] = 2$$

The mean residual service time and the mean residual setup time for each queue are,

$$E[R_{B_1}] = \frac{E[B_1^2]}{2E[B_1]} = \frac{2}{2 \cdot 1} = 1$$

$$E[R_{B_2}] = \frac{E[B_2^2]}{2E[B_2]} = \frac{2}{2 \cdot 1} = 1$$

$$E[R_{S_1}] = \frac{E[S_1^2]}{2E[S_1]} = \frac{2}{2 \cdot 1} = 1$$

$$E[R_{S_2}] = \frac{E[S_2^2]}{2E[S_2]} = \frac{2}{2 \cdot 1} = 1$$

The occupation rate (excluding setups) at the queues are,

$$\rho_1 = \lambda_1 E[B_1] = \frac{6}{10} \cdot 1 = \frac{6}{10}$$

$$\rho_2 = \lambda_2 E[B_2] = \frac{2}{10} \cdot 1 = \frac{2}{10}$$

The total occupation rate ρ is therefore,

$$\rho = \sum_{i=1}^{2} \rho_i = \rho_1 + \rho_2 = \frac{8}{10}$$

The mean cycle length is,

$$E[C] = \frac{E[S]}{1-\rho} = \frac{2}{1-\frac{8}{10}} = 10$$

The visit time for each queue is,

$$E[\theta_1] = \rho_1 E[C] + E[S_1] = \frac{6}{10} \cdot 10 + 1 = 7$$

$$E[\theta_2] = \rho_2 E[C] + E[S_2] = \frac{2}{10} \cdot 10 + 1 = 3$$

The (i, j)-period is:

$$E[\theta_{1,1}] = \sum_{n=1}^{1} E[\theta_n] = E[\theta_1] = 7$$

$$E[\theta_{1,2}] = \sum_{n=1}^{2} E[\theta_n] = E[\theta_1] + E[\theta_2] = 7 + 3 = 10$$

$$E[\theta_{2,1}] = \sum_{n=2}^{2} E[\theta_n] = E[\theta_2] = 3$$

$$E[\theta_{2,2}] = \sum_{n=2}^{3} E[\theta_n] = E[\theta_2] + E[\theta_{3 \ mod \ 2}] = 3 + 7 = 10$$

The fraction of the time the system is in an (i, j)-period equals:

$$q_{1,1} = \frac{E[\theta_{1,1}]}{E[C]} = \frac{7}{10}$$

$$q_{1,2} = \frac{E[\theta_{1,2}]}{E[C]} = \frac{10}{10}$$

$$q_{2,1} = \frac{E[\theta_{2,1}]}{E[C]} = \frac{3}{10}$$

$$q_{2,2} = \frac{E[\theta_{2,2}]}{E[C]} = \frac{10}{10}$$

The first $N^2 = 4$ equations are given by (3.16) (the first two) and (3.17) (the last two) as follows,

$$(1)\ \frac{q_{2,1}}{q_{2,1}} E[L_{1,n}] = \lambda_1 E[R_{\theta_{2,1}}]$$

$$\therefore E[L_{1,2}] = \frac{6}{10} E[R_{\theta_{2,1}}]$$

$$(2)\ \frac{q_{1,1}}{q_{1,1}} E[L_{2,1}] = \lambda_2 E[R_{\theta_{1,1}}]$$

$$\therefore E[L_{2,1}] = \frac{2}{10} E[R_{\theta_{1,1}}]$$

$$(3)\ \sum_{n=1}^{2} q_{n,1} E[L_{1,n}] = \frac{\lambda_1}{1-\rho_1}(\rho_1 E[R_{B_1}] + \frac{E[S_1]}{E[C]} E[R_{S_1}] + (1 - q_{1,1})(E[R_{\theta_{2,1}}] + E[S_1]))$$

$$\frac{7}{10} E[L_{1,1}] + \frac{3}{10} E[L_{1,2}] = \frac{\frac{6}{10}}{1 - \frac{6}{10}} (\frac{6}{10} \cdot 1 + \frac{1}{10} \cdot 1 + (1 - \frac{7}{10})(E[R_{\theta_{2,1}}] + 1))$$

$$\therefore \frac{7}{10} E[L_{1,1}] + \frac{3}{10} E[L_{1,2}] = \frac{9}{20} E[R_{\theta_{2,1}}] + \frac{6}{4}$$

$$(4)\ \sum_{n=1}^{2} q_{n,1} E[L_{2,n}] = \frac{\lambda_2}{1-\rho_2}(\rho_2 E[R_{B_2}] + \frac{E[S_2]}{E[C]} E[R_{S_2}] + (1 - q_{2,1})(E[R_{\theta_{1,1}}] + E[S_2]))$$

$$\frac{7}{10} E[L_{2,1}] + \frac{3}{10} E[L_{2,2}] = \frac{\frac{2}{10}}{1 - \frac{2}{10}} (\frac{2}{10} \cdot 1 + \frac{1}{10} \cdot 1 + (1 - \frac{3}{10})(E[R_{\theta_{1,1}}] + 1))$$

$$\therefore \frac{7}{10} E[L_{2,1}] + \frac{3}{10} E[L_{2,2}] = \frac{1}{4} + \frac{7}{40} E[R_{\theta_{1,1}}]$$

At this stage, the unknowns are $E[L_{1,1}]$, $E[L_{1,2}]$, $E[L_{2,1}]$, $E[L_{2,2}]$, $E[R_{\theta_{2,1}}]$ and $E[R_{\theta_{1,1}}]$. The two extra linear equations required can be determined by (3.18) as follows,

$$(5)\ E[R_{\theta_{1,1}}] = \frac{1}{1-\rho_1}(E[L_{1,1}]E[B_1] + \frac{\rho_1 E[C]}{E[\theta_{1,1}]} E[R_{B_1}] + \frac{E[S_1]}{E[\theta_{1,1}]} E[R_{S_1}])$$

$$E[R_{\theta_{1,1}}] = \frac{1}{1 - \frac{6}{10}} (E[L_{1,1}] \cdot 1 + \frac{\frac{6}{10} \cdot 10}{7} \cdot 1 + \frac{1}{7} \cdot 1)$$

$$\therefore E[R_{\theta_{1,1}}] = \frac{10}{4}E[L_{1,1}] + \frac{10}{4}$$

$$(6)\ E[R_{\theta_{2,1}}] = \frac{1}{1-\rho_2}(E[L_{2,2}]E[B_2] + \frac{\rho_2 E[C]}{E[\theta_{2,1}]}E[R_{B_2}] + \frac{E[S_2]}{E[\theta_{2,1}]}E[R_{S_2}])$$

$$E[R_{\theta_{2,1}}] = \frac{1}{1-\frac{2}{10}}(E[L_{2,2}]\cdot 1 + \frac{\frac{2}{10}\cdot 10}{3}\cdot 1 + \frac{1}{3}\cdot 1)$$

$$\therefore E[R_{\theta_{2,1}}] = \frac{5}{4}E[L_{2,2}] + \frac{5}{4}$$

The set of equations can be written in matrix form as,

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & -\frac{6}{10} \\ 0 & 0 & 1 & 0 & -\frac{2}{10} & 0 \\ \frac{7}{10} & \frac{3}{10} & 0 & 0 & 0 & -\frac{9}{20} \\ 0 & 0 & \frac{7}{10} & \frac{3}{10} & -\frac{7}{40} & 0 \\ -\frac{10}{4} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{5}{4} & 0 & 1 \end{pmatrix} \begin{pmatrix} E[L_{1,1}] \\ E[L_{1,2}] \\ E[L_{2,1}] \\ E[L_{2,2}] \\ E[R_{\theta_{1,1}}] \\ E[R_{\theta_{2,1}}] \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{6}{4} \\ \frac{1}{4} \\ \frac{10}{4} \\ \frac{5}{4} \end{pmatrix}$$

And the solution of the system is given by,

$$\begin{pmatrix} E[L_{1,1}] \\ E[L_{1,2}] \\ E[L_{2,1}] \\ E[L_{2,2}] \\ E[R_{\theta_{1,1}}] \\ E[R_{\theta_{2,1}}] \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & -\frac{6}{10} \\ 0 & 0 & 1 & 0 & -\frac{2}{10} & 0 \\ \frac{7}{10} & \frac{3}{10} & 0 & 0 & 0 & -\frac{9}{20} \\ 0 & 0 & \frac{7}{10} & \frac{3}{10} & -\frac{7}{40} & 0 \\ -\frac{10}{4} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{5}{4} & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ \frac{6}{4} \\ \frac{1}{4} \\ \frac{10}{4} \\ \frac{5}{4} \end{pmatrix} = \begin{pmatrix} \frac{129}{35} \\ \frac{12}{5} \\ \frac{82}{35} \\ \frac{11}{5} \\ \frac{82}{7} \\ 4 \end{pmatrix}$$

Now, through identity (3.12), we obtain the mean queue lengths as follows,

$$E[L_1] = \sum_{n=1}^{2} q_{n,1}E[L_{1,n}] = q_{1,1}E[L_{1,1}] + q_{2,1}E[L_{1,2}] = \frac{7}{10}\cdot\frac{129}{35} + \frac{3}{10}\cdot\frac{12}{5} = \frac{33}{10}$$

$$E[L_2] = \sum_{n=1}^{2} q_{n,1}E[L_{2,n}] = q_{1,1}E[L_{2,1}] + q_{2,1}E[L_{2,2}] = \frac{7}{10}\cdot\frac{82}{35} + \frac{3}{10}\cdot\frac{11}{5} = \frac{23}{10}$$

And, by Little's law (3.14),

$$E[W_1] = \frac{E[L_1]}{\lambda_1} = \frac{33}{10} \cdot \frac{10}{6} = \frac{33}{6}$$

$$E[W_2] = \frac{E[L_2]}{\lambda_2} = \frac{23}{10} \cdot \frac{10}{2} = \frac{23}{2}$$

Finally, the average time that the orders spends in the system are:

$$E[W_1^s] = E[W_1] + E[B_1] = \frac{33}{6} + 1 = \frac{39}{6}$$

$$E[W_2^s] = E[W_2] + E[B_2] = \frac{23}{2} + 1 = \frac{25}{2}$$

And those are the *sine qua non* input for the approach that we develop in the next section.

3.3. Setting heuristically wine stock levels using an (*s-1,s*) inventory control policy

3.3.1. The optimization problem

As Muckstadt & Sapra (2010, pp. 202) argue, setting stock levels for items managed using an (*s-1,s*) policy will depend on the objectives and constraints that are stipulated. On this, we follow the work of Gross & Harris (1971), and we consider the case when the wine decision maker is concerned on minimizing the sum of the steady-state expected values of the *WIP*, overage and underage costs per unit time. For the sake of the explanation, consider the notation of Table 3.1.

For each label type i, with $i = 1 \ldots N$, if x cases are outstanding at a random point in time, then, assuming that $s_i \geq x$, the on-hand inventory level is given by $s_i - x$, the average on-hand inventory level is $\sum_{x=0}^{s_i}(s_i - x)p_i(x)$, and the expected inventory carrying cost per unit time is $C_I \sum_{x=0}^{s_i}(s_i - x)p_i(x)$. Moreover, if $s_i < x$, then the backorder level is $x - s_i$, the average backorder level is given by $\sum_{x=s_i}^{\infty}(x - s_i)p_i(x)$, and the expected cost of backorder per unit of time is $C_B \sum_{x=s_i}^{\infty}(x - s_i)p_i(x)$. Finally, the *WIP* cost is given simply by $C_W \sum_{x=0}^{\infty} xp(x)$.

Therefore, the expected cost function per unit time when there are N label types for premium wines is given by:

$$C(\mathbf{s}) = C(s_1, \ldots, s_N) = C_W \sum_{x=0}^{\infty} xp(x) + C_I \sum_{i=1}^{N} \sum_{x=0}^{s_i}(s_i - x)p_i(x) + C_B \sum_{i=1}^{N} \sum_{x=s_i}^{\infty}(x - s_i)p_i(x) \tag{3.20}$$

TABLE 3.1. Notation

$s_i \in \mathbb{Z}^+$	:	stock level for label type i (cases).
$N \in \mathbb{Z}^+$	:	number of labels that the winery handle for premium wines (indexed by i).
$C_I \in \mathbb{R}^+$	:	overage penalty cost (US$ /case/unit time held on shelf).
$C_B \in \mathbb{R}^+$	:	underage penalty cost (US$/case/unit time in backorder).
$C_W \in \mathbb{R}^+$	:	work-in-process holding costs (US$/case/unit time on order).
$C(\mathbf{s}) \in \mathbb{R}^+$	:	expected cost function per unit time.
$X_i \in \mathbb{Z}^+$	:	number of cases on-order for label type i when there are N different label types (random variable).
$p_i(x) \in [0,1]$	:	steady state probability that x cases are on-order for label type i when there are N different label types.
$p(x) \in [0,1]$	:	steady state probability that x cases of any label type are on-order when N label types are handle.
$E[W_i^s] \in \mathbb{R}^+$	:	average response time (i.e. mean delay plus the service time) for an order of label type i, when there are N label types, in an asymmetric cyclic queue system under an exhaustive service policy.

As stated before, we assume that the orders for labeled wines arrives according to an independent Poisson process with average demand rate of λ_i. Now, due to Palm's Theorem, the steady state probability that x cases are on-order for label type i is simply:

$$p_i(x) = e^{-\lambda_i E[W_i^s]} \frac{(\lambda_i E[W_i^s])^x}{x!}, \; i = 1, \ldots, N. \tag{3.21}$$

Moreover, considering that the sum of independent Poisson processes is also a Poisson process, the expected number of cases that are on-order in steady state is:

$$\sum_{x=0}^{\infty} x p(x) = \sum_{i=1}^{N} \lambda_i E[W_i^s] \tag{3.22}$$

Hence, in order to setting the optimal stock levels for labeled wines, given a number of labels N, the following optimization problem P_N) must be solved:

$$\text{minimize} \quad C_W \sum_{i=1}^{N} \lambda_i E[W_i^s] + C_I \sum_{i=1}^{N} \sum_{x=0}^{s_i} (s_i - x) p_i(x) + C_B \sum_{i=1}^{N} \sum_{x=s_i}^{\infty} (x - s_i) p_i(x) \tag{3.23}$$

$$\text{s.t.} \quad p_i(x) = e^{-\lambda_i E[W_i^s]} \frac{(\lambda_i E[W_i^s])^x}{x!} \qquad \forall i = 1..N \tag{3.24}$$

$$s_i \in \mathbb{Z}^+ \qquad \forall i = 1..N \tag{3.25}$$

3.3.2. Solving the wine stock setting problem

First, we analyze some properties of problem $P_N)$, and then we show how to solve it. Consider the following propositions.

PROPOSITION 3.1. *Problem $P_N)$ can be expressed as an equivalent nonlinear separable programming problem.*

Proof: Given the average response time for each label type as an input, the *WIP* costs expression $C_W \sum_{i=1}^{N} \lambda_i E[W_i^s]$ of $C(\mathbf{s})$ is a constant. Now, let $f_i(s_i)$ be defined as:

$$f_i(s_i) = C_I \sum_{x=0}^{s_i} (s_i - x) p_i(x) + C_B \sum_{x=s_i}^{\infty} (x - s_i) p_i(x) \tag{3.26}$$

which is nonnegative $\forall s_i \geq 0$. So the following optimization problem is equivalent to $P_N)$:

$$P_N^*) : \text{minimize} \qquad g(\mathbf{s}) = \sum_{i=1}^{N} f_i(s_i) \tag{3.27}$$

$$\text{s.t.} \qquad p_i(x) = e^{-\lambda_i E[W_i^s]} \frac{(\lambda_i E[W_i^s])^x}{x!} \qquad \forall i = 1..N \tag{3.28}$$

$$s_i \in \mathbb{Z}^+ \qquad \forall i = 1..N \tag{3.29}$$

which is a separable programming problem in the stocking variables s_i. $\square$

This last result, although simple, is very useful since we can now focus on each label type in order to define the optimum of problem $P_N)$. The following one, establish a nice property of functions $f_i(s_i)$ and $g(\mathbf{s})$.

PROPOSITION 3.2. *For each label type i, the function $f_i(s_i)$ is discretely convex on the stock level s_i. Moreover, $g(\mathbf{s})$ is also discretely convex.*

Proof: for this case, it suffices to show that the first forward differences of $f_i(s_i)$ are increasing (see Yüceer (2002), Theorem 1 and Corollary 1). Let $\Delta f_i(s_i)$ be defined as $f_i(s_i + 1) - f_i(s_i)$. After some algebra (see Appendix 3.A), it can be shown that:

$$\Delta f_i(s_i) = (C_I + C_B) \sum_{x=0}^{s_i} p_i(x) - C_B \tag{3.30}$$

and

$$\Delta^2 f_i(s_i) = \Delta f_i(s_i+1) - \Delta f_i(s_i) = (C_I + C_B) p_i(s_i+1) > 0 \tag{3.31}$$

thus

$$\Delta f_i(s_i+1) > \Delta f_i(s_i),\ \forall s_i \in \mathbb{Z}^+ \quad \square \tag{3.32}$$

We now focus our analysis in the characterization of the optimum of $f_i(s_i)$.

Theorem 3.2. *s_i^* is a global optimum of $f_i(s_i)$ if satisfies:,*

$$\begin{cases} \Delta f_i(s_i^* - 1) \leq 0 \leq \Delta f_i(s_i^*) & \textit{if } s_i^* > 0 \\ 0 \leq \Delta f_i(s_i^*) & \textit{if } s_i^* = 0 \end{cases}$$

Proof: Local optimality conditions are shown by Gross & Harris (1971, Chapter 1). Thus, due to both Proposition 3.2 and the statements of Ui (2006), s_i^* is also a global optimum of $f_i(s_i)$. $\square$

Now, we are ready to establish how to set the optimum of P_N), which by Proposition 3.1, can be reduced to the solving of N newsvendor problems under Poisson demand. For a recent review of the newsvendor problem see Qin et al. (2011), and also the excellent review of Khouja (1999).

Defining $\sum_{x=0}^{-1} p_i(x) = 0$,

PROPOSITION 3.3. *s_i^*, the optimum of $f_i(s_i)$, is the smallest non-negative integer which satisfies,*

$$\sum_{x=0}^{s_i^*-1} p_i(x) \leq \epsilon \leq \sum_{x=0}^{s_i^*} p_i(x) \tag{3.33}$$

where $\epsilon = \frac{C_B}{C_I + C_B}$.

Proof: Replace the expression (3.30) in the conditions of Theorem 3.2. $\square$

For each label type, it is easy to find s_i^*, the optimum of $f_i(s_i)$, which correspond to a heuristic solution for the wine setting stock problem. As Gross & Harris (1971) argue, to find s_i^* in this case one could evaluate $\Delta f_i(s_i)$ for each value of s_i starting with $s_i = 0$ and stopping when

Theorem 3.2 condition is satisfied, that is, when $\Delta f_i(s_i)$ becomes positive for the first time. Nevertheless, note that this process could be inefficient if $\lambda_i E[W_i^s]$ is large. So in order to enhance the search procedure, the following Proposition provides simple bounds for the optimum of $f_i(s_i)$ that can be employed in more efficient numerical search procedures such as, for example, the bisection algorithm.

PROPOSITION 3.4. *The following bounds are valid for s_i^*, with $i = 1, \ldots, N$:*

(i) *If $0 < \epsilon < \frac{1}{2}$, then $0 \leq s_i^* \leq \left\lfloor \min\{\frac{\lambda_i E[W_i^s]}{1-\epsilon}, \lambda_i E[W_i^s] + \frac{4}{3}\} \right\rfloor$.*

(ii) *If $\frac{1}{2} \leq \epsilon < 1$, then $\lceil \max\{\lambda_i E[W_i^s] - \ln(2), 0\} \rceil \leq s_i^* \leq \left\lfloor \frac{\lambda_i E[W_i^s]}{1-\epsilon} \right\rfloor$.*

Proof: By Proposition 3.3, s_i^* must satisfy,

$$1 - \epsilon \leq \sum_{x=s_i^*}^{\infty} p_i(x) \tag{3.34}$$

and by *Markov's Inequality*,

$$\sum_{x=s_i^*}^{\infty} p_i(x) \leq \frac{\lambda_i E[W_i^s]}{s_i^*} \tag{3.35}$$

thus,

$$s_i^* \leq \frac{\lambda_i E[W_i^s]}{1 - \epsilon},\ \forall\, 0 \leq \epsilon < 1. \tag{3.36}$$

On the other hand, if $\epsilon \geq \frac{1}{2}$, then by Proposition 3.3,

$$\frac{1}{2} \leq \epsilon \leq \sum_{x=0}^{s_i^*} p_i(x) \tag{3.37}$$

which means that s_i^* is greater or equal to the median of the random demand process.

Conversely, if $\epsilon < \frac{1}{2}$, then

$$\sum_{x=0}^{s_i^*-1} p_i(x) \leq \epsilon < \frac{1}{2} \tag{3.38}$$

which means that $s_i^* - 1$ is lower than the median.

As the number of cases that are on-order for label type i follows a Poisson distribution with parameter $\lambda_i E[W_i^s]$, the following inequalities hold according to Choi (1994),

$$\lambda_i E[W_i^s] - \ln(2) \leq \text{median}(X_i) < \lambda_i E[W_i^s] + \frac{1}{3} \tag{3.39}$$

hence,

$$\begin{cases} \lambda_i E[W_i^s] - \ln(2) \leq s_i^*, & \text{if } \epsilon \geq \frac{1}{2} \\ s_i^* - 1 < \lambda_i E[W_i^s] + \frac{1}{3}, & \text{if } \epsilon < \frac{1}{2} \end{cases}$$

and given that $s \in \mathbb{Z}^+$,

$$\begin{cases} 0 \leq s_i^* \leq \left\lfloor \min\{\frac{\lambda_i E[W_i^s]}{1-\epsilon}, \lambda_i E[W_i^s] + \frac{4}{3}\} \right\rfloor, & \text{if } 0 \leq \epsilon < \frac{1}{2} \\ \lceil \max\{\lambda_i E[W_i^s] - \ln(2), 0\} \rceil \leq s_i^* \leq \left\lfloor \frac{\lambda_i E[W_i^s]}{1-\epsilon} \right\rfloor, & \text{if } \frac{1}{2} \leq \epsilon < 1 \end{cases} \quad \square$$

Finally, we state how the optimal solutions of the newsvendor problems are related in our approach, which indeed is quite intuitive. Consider Proposition 3.5.

PROPOSITION 3.5. *Assume that C_B and C_I are the same for each label type. If $\lambda_i E[W_i^s] < \lambda_j E[W_j^s]$, with $i, j = 1, \ldots, N$ and $i \neq j$, then:*

(i) *$s_i^* \leq s_j^*$, and*

(ii) *$f_i(s_i^*) < f_j(s_j^*)$.*

Proof of 1. : A succinct proof of the first part of this proposition could be develop based on the concept of *stochastic ordering* (see Ross et al. (1996, Chapter 9)). It is said that a random variable X is stochastically larger than a random variable Y, denoted by $X \geq_{st} Y$, if:

$$P\{X \geq x\} \geq P\{Y \geq x\}, \quad \forall x \in \text{Support}\,(X) \cap \text{Support}\,(Y) \tag{3.40}$$

In the newsvendor problem context, Song (1994) proved that a stochastically larger demand implies a non-lower optimal stock level (see Proposition 3.2 and Lemma 3.3 of Song (1994)). Moreover, Ross et al. (1996) showed that a Poisson random variable is stochastically increasing in its mean (see example 9.2(a) of Ross et al. (1996), p.411). Given the later, if $\lambda_i E[W_i^s] < \lambda_j E[W_j^s]$, then $X_j \geq_{st} X_i$, and thus $s_i^* \leq s_j^*$. $\square$

Proof of 2. : In order proof the second part of this proposition we need to state some properties of the newsvendor problem under Poisson demand. Let $h^*(\lambda)$: $\mathbb{R}^+ \to \mathbb{R}^+$ be defined as,

$$h^*(\lambda) = \min_{s\in\mathbb{Z}^+} f(s,\lambda) = \min_{s\in\mathbb{Z}^+} C_I \sum_{x=0}^{s}(s-x)p(x,\lambda) + C_B \sum_{x=s}^{\infty}(x-s)p(x,\lambda) \tag{3.41}$$

where $p(x,\lambda) = e^{-\lambda}\frac{\lambda^x}{x!}$.

Hence, $h^*(\lambda)$ is the minimum expected cost, in the newsvendor problem context, if the demand follows a Poisson distribution with parameter λ. We first restate the following lemma by Jain et al. (2012).

Lemma 3.1. *Let $s^*(\lambda) = \arg\min_{s\in\mathbb{Z}^+} f(s,\lambda)$. Then, for each compact and not empty subset of $\mathbb{R}^+$ of the form $\mathbb{S}(\bar{\lambda}) = \{x \in \mathbb{R}^+ : 0 \leq x \leq \bar{\lambda}\}$, with $\bar{\lambda} > -\ln(\epsilon)$, there exists a partition of $\mathbb{S}\left(\bar{\lambda}\right) = \cup_{l=0}^{\bar{s}-1}[\lambda_{l-1},\lambda_l[\cup [\lambda_{\bar{s}-1},\lambda_{\bar{s}}]$, with $\lambda_{-1} = 0$ and $\lambda_{\bar{s}} = \bar{\lambda}$, such that, $s^*(\lambda) = l$ on $[\lambda_{l-1},\lambda_l[$, $\forall\, 0 \leq l < \bar{s}$, $s^*(\lambda) = \bar{s}$ on $[\lambda_{\bar{s}-1},\lambda_{\bar{s}}]$, where λ_l is the unique root of the transcendental equation,*

$$t_l(\lambda_l) = \sum_{i=0}^{l} e^{-\lambda_l}\frac{\lambda_l^i}{i!} - \epsilon = 0 \tag{3.42}$$

over $[0,\bar{\lambda}]$, with $0 < \epsilon < 1$.

Proof of Lemma 3.1: The existence of the parameters $\{\lambda_l\}_{l=0}^{\bar{s}-1}$, and thus the partition of $\mathbb{S}\left(\bar{\lambda}\right)$, was proven by Jain et al. (2012, Lemma1). In their proof, replace λ by 1, and suppose $\bar{\lambda} = T$. On the other hand, uniqueness of these parameters can be proved by contradiction. For $l = \{0..\bar{s}-1\}$, suppose that ξ_1^l and ξ_2^l are two roots of $t_l(\xi)$ in $[0,\bar{\lambda}]$, with $\xi_1^l \neq \xi_2^l$, and by construction, both are greater than 0. In this context, note that $t_l(\xi)$ is differentiable and hence, continuous on $[\xi_1^l,\xi_2^l]$. Thus, by the *Mean Value Theorem*, we can find a number ν in (ξ_1^l,ξ_2^l) such that:

$$\frac{t_l(\xi_2^l) - t_l(\xi_1^l)}{\xi_2^l - \xi_1^l} = t_l'(\nu) \tag{3.43}$$

Since $t_l(\xi_2^l) = t_l(\xi_1^l) = 0$, because they are roots of $t_l(\xi)$, we get,

$$0 = t_l'(\nu) \tag{3.44}$$

Nevertheless, recall that $t_l'(\xi) = -e^{-\xi}\frac{\xi^l}{l!}$ is negative for all $\xi > 0$. So, there is not exists such ν in (ξ_1^l,ξ_2^l) for which $0 = t_l'(\nu)$, implying that $\xi_1^l = \xi_2^l$, and thus the roots of $t_l\left(\lambda\right)$ are unique in λ for each valid value of l. $\square$

Due to Lemma 3.1, $s^*(\lambda)$ is an increasing step function in λ. Besides, close form expressions are available only for $\lambda_0 = -\ln(\epsilon)$ and $\lambda_1 = -1 - W_{-1}(-\frac{\epsilon}{e})$, where $W_{-1}(x)$ is the lower branch of the Lambert W function.

Now consider the following Lemma.

Lemma 3.2. *For any interval of the partition of* $\mathbb{S}(\bar{\lambda})$, $h^*(\lambda)$ *is continuous, convex and increasing in* λ.

Proof of Lemma 3.2: Convexity was proved by Rossi et al. (2014), see their Appendix B. The increasing behavior was proved by Jain et al. (2012), see their Lemma 2. For the sake of the explanation, we briefly describe both proofs.

First, note that $p(x, \lambda)$ is a continuous function of λ. Thus, $f(s, \lambda)$ is also continuous in λ.

Second, note that $f(s, \lambda)$ can be wrote as (see Appendix 3.B),

$$f(s, \lambda) = (C_I + C_B) \sum_{j=0}^{s-1} F(j, \lambda) + C_B(\lambda - s) \tag{3.45}$$

now,

$$\frac{\partial}{\partial \lambda} f(s, \lambda) = -(C_I + C_B)F(s-1, \lambda) + C_B \tag{3.46}$$

whereas,

$$\frac{\partial^2}{\partial \lambda^2} f(s, \lambda) = (C_I + C_B)p(s-1, \lambda) \tag{3.47}$$

Note that $\frac{\partial^2 f(s,\lambda)}{\partial \lambda^2} \geq 0, \ \forall \lambda > 0$, and given $s \in \mathbb{Z}^+$, so $f(s, \lambda)$ is convex in λ.

And third, let $\lambda^* = \arg\min_{\lambda > 0} f(s, \lambda)$ be the unique root of $\sum_{x=0}^{s-1} e^{-\lambda^*} \frac{(\lambda^*)^x}{x!} = \epsilon$. Hence, $f(s, \lambda) \geq f(s, \lambda^*), \ \forall \lambda \geq \lambda^*$. Now consider the interval $[\lambda_{l-1}, \lambda_l[$. If $\lambda \in [\lambda_{l-1}, \lambda_l[$, then by Lemma 3.1, $s^*(\lambda)$=argmin$_{s \in \mathbb{Z}^+} f(s, \lambda) = l$. But recall that if $s^*(\lambda) = l$, $f(l, \lambda)$ reach its minimum at λ_{l-1}. Thus, $f(l, \lambda) \geq f(l, \lambda_{l-1})$.

Therefore, $h^*(\lambda) = f(l, \lambda)$ is continuous, convex and increasing in $[\lambda_{l-1}, \lambda_l[, \ \forall l = 0..\bar{s} - 1$, and also for $[\lambda_{\bar{s}-1}, \lambda_{\bar{s}}]$. $\square$

Finally consider the last Lemma,

Lemma 3.3. *For any interval* $[\lambda_{l-1}, \lambda_l[$, $0 \leq l < \bar{s}$, *it holds that* $h^*(\lambda_l) = f(l, \lambda_l) = f(l + 1, \lambda_l)$, *which implies* $s^*(\lambda_l)$=*argmin*$_{s \in \mathbb{Z}^+} f(s, \lambda_l)$={*l, l+1*}.

Proof of Lemma 3.3: We need to prove that,

$$f(l, \lambda_l) = f(l + 1, \lambda_l) \tag{3.48}$$

now consider (A.22),

$$(C_I + C_B) \sum_{j=0}^{l-1} F(j, \lambda_l) + C_B(\lambda_l - l) = (C_I + C_B) \sum_{j=0}^{l} F(j, \lambda_l) + C_B(\lambda_l - l - 1) \tag{3.49}$$

rearranging terms,

$$C_B = (C_I + C_B) F(l, \lambda_l) \tag{3.50}$$

that is equivalent to,

$$\epsilon = F(l, \lambda_l) \tag{3.51}$$

and this last equality holds due to Lemma 3.1. □

Due to Lemmas 3.2 and 3.3, $h^*(\lambda)$ is both continuous and increasing in $\mathbb{S}(\bar{\lambda})$. Now, suppose that $0 < \lambda_1 < \lambda_2 \leq \bar{\lambda}$. Then $h^*(\lambda_1) < h^*(\lambda_2)$. Let $\lambda_1 = \lambda_i E[W_i^s]$ and $\lambda_2 = \lambda_j E[W_j^s]$. Thus the following holds: $h^*(\lambda_i E[W_i^s]) = f_i(s_i^*) < f_j(s_j^*) = h^*(\lambda_j E[W_j^s])$, which proves the second part of Proposition 3.5. □

3.3.3. A numerical example (cont.)

Consider again the two-queue polling system with exhaustive service of Subsection 2.2. Recall that for each label type the average time that the orders spends in the system are $E[W_1^s] = \frac{39}{6}$ and $E[W_2^s] = \frac{25}{2}$ units of time, respectively. Now, suppose further that $C_I = 100$ (\$/case/units of time), $C_B = 500$ (\$/case/units of time) and $C_W = 50\% \cdot C_I = 50$ (\$/case/units of time). For this case, the machine load is $\rho = 80\%$, and the target service level for each label type is at least $\epsilon = 83.3\%$.

The newsvendor problem for label type 1 is given by:

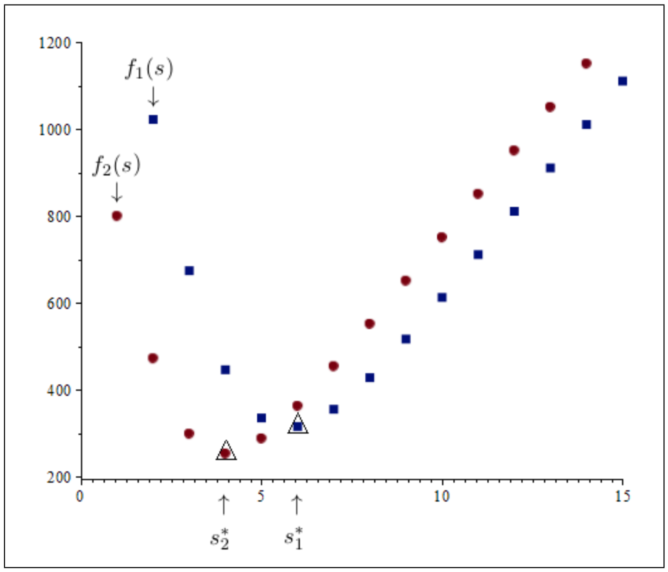

FIGURE 3.2. Optimal solutions for both newsvendor problems.

$$P^*_{L1}) : \text{minimize} \qquad f_1(s_1) = C_I \sum_{x=0}^{s_1}(s_1 - x)p_1(x) + C_B \sum_{x=s_1}^{\infty}(x - s_1)p_1(x) \tag{3.52}$$

$$\text{s.t.} \qquad p_1(x) = e^{-\frac{39}{10}} \frac{(\frac{39}{10})^x}{x!} \tag{3.53}$$

$$s_1 \in \mathbb{Z}^+ \tag{3.54}$$

Whereas the newsvendor problem for label type 2 is given by:

$$P^*_{L2}) : \text{minimize} \qquad f_2(s_2) = C_I \sum_{x=0}^{s_2}(s_2 - x)p_2(x) + C_B \sum_{x=s_2}^{\infty}(x - s_2)p_2(x) \tag{3.55}$$

$$\text{s.t.} \qquad p_2(x) = e^{-\frac{25}{10}} \frac{(\frac{25}{10})^x}{x!} \tag{3.56}$$

$$s_2 \in \mathbb{Z}^+ \tag{3.57}$$

Figure 3.2 show the optimal solutions for both problems.

On the other hand, Table 3.2 shows the values for a given set of feasible solutions, and how the conditions of Theorem 2.1 and Proposition 3.5 are fulfilled at the optimums.

TABLE 3.2. Optimal solutions and values.

s	0	1	2	3	4	5	6	7	8	9	10
$f_1(s)$	1,950	1,462.14	1,021.65	673.53	445.47	334.49	**314.83**	354.52	427.28	516.16	612.02
$\Delta f_1(s)$	-487.85	-440.48	-348.12	-228.05	-110.98	-19.66	**39.68**	72.75	88.88	95.86	98.59
$f_2(s)$	1,250	799.25	471.62	297.91	**252.46**	287.16	361.95	453.44	550.89	650.21	750.04
$\Delta f_2(s)$	-450.74	-327.62	-173.71	-45.45	**34.70**	74.78	91.48	97.45	99.31	99.83	99.96

TABLE 3.3. Stationary distribution estimation of the number of order waiting or being processed.

		Number of orders											
	Frecuency	0	1	2	3	4	5	6	7	8	9	10	≥ 11
Label 1	Relative	0.126	0.160	0.154	0.131	0.104	0.081	0.062	0.046	0.034	0.025	0.019	0.058
	Cumulative	0.126	0.286	0.440	0.571	0.674	0.755	0.817	**0.863**	0.898	0.923	0.942	1.000
Label 2	Relative	0.287	0.226	0.149	0.097	0.067	0.046	0.033	0.024	0.018	0.013	0.010	0.030
	Cumulative	0.287	0.513	0.662	0.759	0.826	**0.872**	0.905	0.929	0.947	0.960	0.970	1.000

Thus, a heuristic solution for label type 1 is to hold an inventory position of 6 cases of premium wines, whereas for label type 2, the heuristic solution implies to keep an inventory position of 4 cases.

3.4. The heuristic accuracy

How accurate is the solution provided by the heuristic in our working example? To answer this question we develop a simulation model, which was implemented using Arena software (Rockwell Softwares Corporation), in order to estimate the stationary distribution of the queue lengths at the labeling process.

3.4.1. The heuristic accuracy in our working example

After a replication length of 10,000,000 hours, Table 3.3 shows both the relative and cumulative frequencies of the number of cases that are on-order for each label type.

Considering that the critical ratio is equal to $\epsilon = 83.3\%$, the optimal solution for label type 1 is to hold an inventory position of 7 cases of premium wines (recall that we are solving a simple newsvendor problem), whereas for label type 2, the optimal solution implies to hold an inventory position of 5 cases. As expected (recall Palm's Theorem statements), our approach underestimate the optimal inventory positions with relative errors of 14.2% and 20% for label type 1 and 2, respectively.

3.4.2. The symmetrical case

To assess under which conditions our heuristic could be more accurate, we consider again the input data of our working example, but under a simpler manufacturing environment which avoids the combinatorial problem of instance generation. For the sake of the exposition, we consider the case where the labeling process is modeled as symmetric cyclic queue system in which the labeling machine operates under an exhaustive service policy.

This system can be described as follows. According to Bertsimas et al. (1989, pp. 7): consider a queueing system that consists of k queues $Q_1, Q_2, \ldots, Q_k$ each one with infinite capacity. Customers arrive at each queue according to independent Poisson processes with the same arrival intensity λ/k. The queues are served by a single server that visits the queues in a fixed cyclic order $Q_1, Q_2, \ldots, Q_k, Q_1, Q_2, \ldots$ The setup time from one queue to another are independent identically distributed random variables with mean $\bar{s}$ and second moment $\overline{s^2}$. The service times at every queue are independent identically distributed random variables with mean $\bar{b}$ and second moment $\overline{b^2}$. The traffic intensity is $\rho = \lambda\bar{b}$. Finally, the server servicing each queue i, until the queue is empty before proceeding.

By mean of the so-called pseudoconservation law (see Boxma (1989)), a close form expression for the mean waiting time in each queue is available, and given by:

$$E[W_q(k)] = \frac{\lambda\overline{b^2}}{2(1-\lambda\bar{b})} + \frac{\overline{s^2}-\bar{s}^2}{2\bar{s}} + \frac{\bar{s}(k-\lambda\bar{b})}{2(1-\lambda\bar{b})} \tag{3.58}$$

then, the mean waiting time in the system for each order is:

$$E[W(k)] = \frac{\lambda\overline{b^2}}{2(1-\lambda\bar{b})} + \frac{\overline{s^2}-\bar{s}^2}{2\bar{s}} + \frac{\bar{s}(k-\lambda\bar{b})}{2(1-\lambda\bar{b})} + \bar{b} \tag{3.59}$$

and by Little's Law, the average number of orders waiting or being processed in each queue is simply:

$$E[L(k)] = \frac{\lambda}{k}\left(\frac{\lambda\overline{b^2}}{2(1-\lambda\bar{b})} + \frac{\overline{s^2}-\bar{s}^2}{2\bar{s}} + \frac{\bar{s}(k-\lambda\bar{b})}{2(1-\lambda\bar{b})} + \bar{b}\right) \tag{3.60}$$

Supposing that the service (b) and setup times (s) follow exponential distributions with means equal to 1 for each label type, the arrival intensity (λ) is 0.8 and the critical ratio (ϵ) is 83.3%, Table 3.4 shows $E[L(k)]$, for $k = 1, \ldots, 10$, the heuristic inventory position for each queue k according to Proposition 3.3, which is denoted by s_k^h, and the aggregate inventory position. On the

TABLE 3.4. Heuristic inventory position for each queue in the symmetric case.

k	2	3	4	5	6	7	8	9	10
$E[L(k)]$	3.400	2.933	2.700	2.560	2.467	2.400	2.350	2.311	2.280
s_k^h	5	5	4	4	4	4	4	4	4
$\sum_k s_k^h$	10	15	16	20	24	28	32	36	40

TABLE 3.5. Estimation of the stationary distribution at the symmetrical case after a replication length of 10,000,000 hours.

		Number of orders										
k	Frecuency	0	1	2	3	4	5	6	7	8	9	≥ 10
2	Relative	0.176	0.190	0.158	0.121	0.090	0.066	0.049	0.036	0.027	0.020	0.068
	Cumulative	0.176	0.366	0.524	0.645	0.734	0.800	**0.849**	0.885	0.912	0.932	1.000
3	Relative	0.202	0.203	0.162	0.121	0.088	0.063	0.045	0.033	0.023	0.017	0.043
	Cumulative	0.202	0.405	0.568	0.689	0.776	**0.840**	0.885	0.918	0.941	0.957	1.000
4	Relative	0.213	0.210	0.166	0.123	0.088	0.062	0.043	0.030	0.021	0.014	0.030
	Cumulative	0.213	0.423	0.589	0.711	0.799	**0.861**	0.905	0.935	0.955	0.970	1.000
5	Relative	0.222	0.214	0.168	0.124	0.088	0.061	0.042	0.028	0.019	0.012	0.023
	Cumulative	0.222	0.435	0.604	0.728	0.815	**0.876**	0.918	0.946	0.965	0.977	1.000
6	Relative	0.227	0.216	0.171	0.126	0.089	0.060	0.040	0.026	0.017	0.011	0.018
	Cumulative	0.227	0.442	0.613	0.738	0.827	**0.887**	0.927	0.954	0.971	0.982	1.000
7	Relative	0.229	0.217	0.172	0.127	0.089	0.060	0.040	0.025	0.016	0.010	0.015
	Cumulative	0.229	0.446	0.618	0.745	**0.834**	0.894	0.934	0.959	0.975	0.985	1.000

TABLE 3.6. Comparison between the heuristic and the optimal solution.

k	2	3	4	5	6	7	8	9	10
s_k^h	5	5	4	4	4	4	4	4	4
s_k^*	6	5	5	5	5	4	4	4	4
Absolute error	1	0	1	1	1	0	0	0	0
Relative error	16.7%	0.0%	20.0%	20.0%	20.0%	0.0%	0.0%	0.0%	0.0%

other hand, Table 3.5 shows both the relative and cumulative frequencies of the number of cases that are on-order as function of the numbers of labels, whereas Table 3.6 resumes both the heuristic and the optimal inventory positions.

As expected, the output of this greedy heuristic becomes more accurate as long as the demand rates for each label type are low relative to the total demand, and there is a slack in production resources. Recall that in the statement of Palm's Theorem, resupply times are assumed to be i.i.d. from customer order to customer order, condition which is fulfilled in a M/M/∞ or M/G/∞ queuing system, but not here, where there is correlation between customer orders for each label type.

3.5. Concluding Remarks

In this chapter, we focused on the management of premium wines and we develop a two stage heuristic in order to setting the stock levels when an (*s-1,s*) policy is followed for finished goods. Bearing in mind the operations of a small export-focused Chilean winery that we had worked with, we assumed that orders arrive according to independent Poisson processes, inventories of bottled and labeled wines are reviewed continuously in time, a *postponement* strategy is employed for the labeling process, and there is only one labeling machine. Considering that the wine decision maker is concerned with minimizing the sum of the steady-state expected values of the *WIP*, overage and underage costs per unit time, the heuristic operates as follows: in the first stage, we modeled the labeling process as a polling system under exhaustive service, and we apply the results of Winands et al. in order to obtain the mean delays for each end product. Then, in the second stage, we applied Palm's theorem which reduces the stock levels setting problem to the solving of a newsvendor-type problem for each end product. We provided some numerical examples, and also, we addressed the accuracy of the proposed heuristic.

We intend this research to be a starting point for developing a practical tool for wine managers to support their decision making on the management of premium wines. The developed approach should be sufficiently simple to be validated and applied by wine managers through a Decision Support System (DSS), as long as the context of applicability does not differ too much of the restrictive assumptions of the approach. Simplicity and restrictive assumptions usually go together. However, the assumption about order size could be relaxed to support real-world business needs.

Appendix 3.A: Proposition 3.2

Here we show that $\Delta f_i(s_i) = (C_I + C_B)\sum_{x=0}^{s_i} p_i(x) - C_B$. By definition:

$$\Delta f_i(s_i) = f_i(s_i+1) - f_i(s_i)$$

then,

$$\Delta f_i(s_i) = C_I \sum_{x=0}^{s_i+1}(s_i+1-x)p_i(x) + C_B \sum_{x=s_i+1}^{\infty}(x-s_i-1)p_i(x) - C_I \sum_{x=0}^{s_i}(s_i-x)p_i(x) - C_B \sum_{x=s_i+1}^{\infty}(x-s_i)p_i(x)$$

Now,

$$C_I \sum_{x=0}^{s_i+1}(s_i+1-x)p_i(x) = C_I \sum_{x=0}^{s_i}(s_i-x)p_i(x) + C_I \sum_{x=0}^{s_i} p_i(x)$$

On the other hand,

$$C_B \sum_{x=s_i+1}^{\infty}(x-s_i-1)p_i(x) = C_B \sum_{x=s_i+1}^{\infty}(x-s_i)p_i(x) - C_B(1-\sum_{x=0}^{s_i} p_i(x))$$

Thus,

$$\Delta f_i(s_i) = C_I \sum_{x=0}^{s_i} p_i(x) - C_B(1-\sum_{x=0}^{s_i} p_i(x)) = (C_I+C_B)\sum_{x=0}^{s_i} p_i(x) - C_B \quad \square$$

Appendix 3.B: Lemma 3.2

Here we show that $f(s,\lambda) = (C_I + C_B)\sum_{j=0}^{s-1} F(j,\lambda) + C_B(\lambda - s)$, where $F(j,\lambda) = \sum_{x=0}^{j} p(x,\lambda)$. Let $x^+ = max\{x, 0\}$, and consider $X \sim$ Poisson distribution with mean λ.

Now,

$$f(s,\lambda) = C_I E(s-X)^+ + C_B E(X-s)^+$$

Gallego & Moon (1993) showed that:

$$(X-s)^+ = (s-X)^+ + (X-s)$$

thus,

$$f(s,\lambda) = (C_I+C_B)E(s-X)^+ + C_B E(X-s)$$

or

$$f(s,\lambda) = (C_I+C_B)E(s-X)^+ + C_B(\lambda-s)$$

And given that $E(s-X)^+$ can be written as $\sum_{j=0}^{s-1} P(X \leq j)$, then

$$f(s,\lambda) = (C_I+C_B)\sum_{j=0}^{s-1} F(j,\lambda) + C_B(\lambda-s) \quad \square$$

4. LOT-SIZING IN AN EXPORT-FOCUSED WINERY UNDER UNCERTAINTY: THE IMPACT OF POSTPONEMENT PRACTICES ON THE PERFORMANCE OF TACTICAL PRODUCTION DECISIONS.

4.1. Introduction

Operations research models are increasingly being used to help decision makers to manage natural resource operations in agriculture, fisheries, forestry, and mining. For a recent review of applications in all four sectors, seeBjørndal et al. (2012). As A. Weintraub & Romero (2006) state, agriculture is one of the fields in which operations research models were first used, almost fifty years ago, and have been most widely applied.

On this last, an area that has received increasing attention is that of the agri-food supply chain, which focuses on the particularities of the production of food. For an excellent review of many of the planning models that have been developed see Ahumada & Villalobos (2009). More recently, Shukla & Jharkharia (2013) also provide a review on what they call the agri-fresh produce supply chain, which focuses on food products that are perishable and have a short shelf life. Other food products, such as wine, which is the product that we will focus on this chapter, are perishable but have a long shelf life.

In recent years, several applications of operations research have been reported in production –viticulture, harvesting, and winery processes– as well as in post-production activities. See Moccia (2013) for a recent review. A couple of examples of relevant applications of operations research models and techniques to the wine grape harvesting are Ferrer et al. (2008) and Bohle et al. (2010).

For wineries, planning their operations to meet product demand has become more challenging than before, in part, due to the global growth of both export programs and private label brands, which means an increasing variety of sales channels typically with different packaging or labeling requirements (Cholette, 2009). This last greatly complicates the wineries production planning process. Note that product proliferation increases inventory levels and setup frequencies, which means higher costs.

Furthermore, and unlike many other industries where essential raw materials can be sourced year-round, wine production follows a yearly cycle, which begins with harvesting operations, and forces wineries to plan their production based on sales forecast. Moreover, large wineries, such as

Concha y Toro in Chile, typically have several vineyards in different locations; however, they also tend to centralize their manufacturing and distribution facilities due to the economies of scale they can obtain. Careful planning for transporting bulk wine allows bottling plants to operate efficiently.

Given an increasing number of SKUs with customer specific features, on the one hand, and the complexity of the wine supply, on the other; neither pure make-to-order (MTO) nor make-to-stock (MTS) strategies may be the most efficient or effective production options. As Soman et al. (2004) note, the large number of setups required by a highly diversified product portfolio could severely reduce the order performance measures, such as average response time and average order delay, which are the focus of a pure MTO strategy. On the other hand, uncertainty in product orders could reduce the product performance measures, such as item fill rate and average inventory levels, which are the focus of a pure MTS strategy. In fact, the likelihood of shortages in some products and overstock in others could be, in practice, very high, especially for export-focused wineries. As Cholette (2009) pointed, products across different sales channels or international markets may contain the same wine but they cannot be treated as substitutes. If we focus solely on bottled wine, the lack of substitutability can be attributed simply to the label, which, for example, may be written in different languages or could include country-specific regulations.

In the past, several researchers, such as Van Donk (2001) and Soman et al. (2004) have explored combining MTO and MTS strategies in the food processing industry. In the wine industry, Garcia et al. (2012) show empirically the potential value of adopting mixed strategies based on a benchmark study conducted on a sample of wineries from Mendoza (Argentina). The authors show that the winery with the shortest production cycle time (i.e. the one with the shortest time since the order is received until it is ready to be transported to the port), used an MTS strategy for its highly rotating products and a label-to-order strategy for more slow moving products. They give two main reasons for using this last strategy in this context. Firstly, premium wines need an aging process in bottle to increase their quality, so bottling must be performed before the orders for these products arrive. Secondly, when exporting, labels need to be customized in order to incorporate data that depends on the country of destination. Note that in the label-to-order strategy the bottling and labeling lines need to be decoupled, so part of the combined strategy can be specified as a bottle-to-stock and a label-to-order policy. Now, since labeling a bottle increases its opportunity cost, delaying the product differentiation seems to be an effective tool, not only for reducing the misallocation problem, but also for increasing profits. In fact, considering the assignment of wine production to multiple sales channels in an uncertainty environment, Cholette (2009) developed a strategic model that analyzes, using a two-stage stochastic programming model with recourse (see Birge & Louveaux (2011)), how to differentiate the production under several demand scenarios. The authors results suggest that a significant portion of wine production should remain as work-in-process (*WIP*) inventory, either unlabeled or in bulk, in order to maximize the expected profits when uncertainty is taken into account.

Behind the idea of holding *WIP* inventory instead of bottled and labeled wines, two broader managerial concepts lie: *Postponement*, which is delaying the development of differentiating processes or activities until more information about customer orders is available (Van Hoek, 2001), and the *Customer Order Decoupling Point* (CODP), which separates the part of the supply chain oriented towards activities for customer orders from the part of the supply chain whose processes are based on forecasting and planning (Hoekstra & Romme, 1992). According to Cheng et al. (2010), the decoupling point can be determined by the customer expected waiting time and availability/accuracy of demand information in the system, while the differentiation point is determined by operational factors, such as product characteristics and manufacturability. Both are related concepts, and in order to reduce the product misallocation risk through postponement, the decoupling point should be set before the differentiation processes. In fact, postponement strategies have shown to be an effective tool for mitigating uncertainties in various industries (Yang et al., 2004).

Although the results of Cholette (2009) show the benefits of adopting postponement practices in the wine industry, the authors strategic modeling framework does not capture some important operational aspects, such as batch bottling, setup times, production capacity in the lines, etc., that a decision maker must consider when planning the winery operations. Thus, it is an open research question if the benefits of using postponement can be achieved when the operational constraints are taken into account. Assuming the only difference among wines exported to different countries is how they are labeled, it seems natural to place the CODP between the bottling and labeling processes at export-focused wineries. A wine labeled for one export market cannot be sent to another. Then, the use of postponed manufacturing would tend to reduce inventory carrying costs and lost sales, but at the expense of increases in the processing cost due to the loss in economies of scales (Zinn & Bowersox, 1988).

Based on the operations of a large export-oriented Chilean winery we worked with, we present an overview of the operational complexities that a wine decision maker must consider, and we assess the performance in terms of inventory, lost sales, and setup quantities of delayed product differentiation under different planning environments. To assess the performance, we use two different MILP planning models that differ in the kind of inventory they can keep. Both models can be reduced to a capacitated lot-sizing with setup time problem, which is known to be NP-hard, so the relative performance of their optimal solutions is analyzed only in instances that could be solved in a reasonable computing time. As decisions occur periodically, our models employ a rolling horizon framework. The main contribution of this chapter is that we demonstrate how capacity tightness, horizon length, and forecast inaccuracy affect the performance of both models, and thus the attractiveness of postponement.

Finally, it is important to point out that since our analysis considers only the bottling and labeling processes and treats those as exogenous; we considered the wine in the storage tanks as raw material and we refer to work in progress (*WIP*) when the wine that has been bottled but not yet labeled. This is a different approach than the one Cholette (2009) has taken, where tanked wine is also considered *WIP*.

The rest of this chapter is organized as follows: in Section 4.2 we present an overview of winery production operations. The wine production planning models are described in Section 4.3. In Section 4.4, we present both the computational experiments and the statistical analysis, whereas some concluding remarks are discussed in Section 4.5.

4.2. Overview of winery production operations

Although wine production is a long process that starts with the wine grape and continues with the harvest, fermentation, and other activities, most of which are under control of the oenologists, we will concern ourselves only with the last part of the process that the winery manages, which is shown in Fig. 4.1. Since most of the Chilean wineries sell their production FOB, or Free on Board, which means that the buyer is in charge of the transportation from the port of shipment to its final destination, the main concern of the wineries is to have the wine ready to ship at the port. Transporting the bottled wine from the winery to the port is relatively easy since the ports of Valparaíso and San Antonio are very close to the wine producing regions. Therefore, the main problem the winery faces, which is not under the control of the oenologists, is being able to bottle and label the wines that were ordered by the clients and having them ready to be transported to the port in time to be loaded on board of the ship specified by the buyer. In other words, the wine production planner must coordinate the operations shown in Fig 4.1 in order to meet the stack date, which is the last date the container that carries the wine shipment has to be at the port. If this date is not met, it means that the buyer will not receive his shipment on time, which is very bad in the highly competitive international wine industry. Being able to meet the stack date for all the products ordered by a client when the winery exports many different products to many different markets, is not easy.

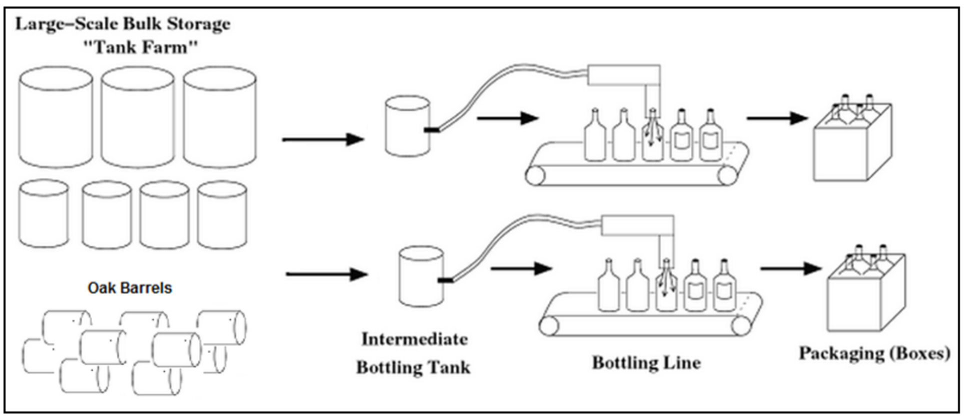

FIGURE 4.1. Wine production process.
Source: Mohais et al. (2012)

Wineries can also sell bulk wine, which can be bottled elsewhere. Some larger Chilean wineries such as Concha y Toro ship some of their more inexpensive wines in bulk. However, bottled wine commands higher prices. In fact, smaller wineries typically export almost only bottled wines. Therefore, large wineries must have large bottling facilities, which typically have several bottling lines. As shown in Fig. 4.1, each bottling line has an intermediate bottling tank that holds the wine that will be bottled and labeled. Although Fig. 4.1 shows the bottling and labeling line as one unit; in practice, they are separate units that can be set up to operate as one. However, when needed, they can be decoupled. Bottling and labeling lines are usually highly automated and can run at a very high speed. Nevertheless, they require long setups when changing the product format or the wine being bottled.

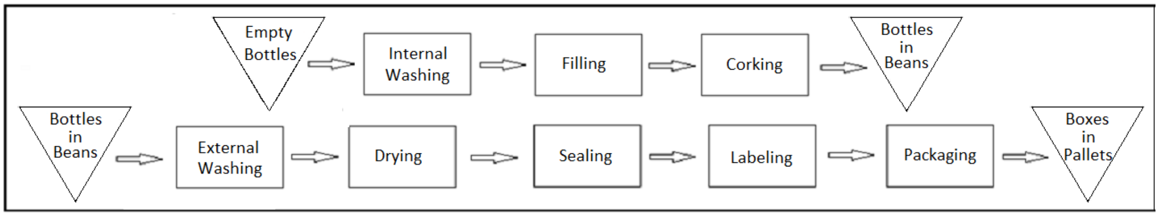

FIGURE 4.2. Bottling and labeling activities.
Source: Arcos (2004).

The storage capacity of the intermediate tanks that feed the bottling line determines the maximum number of bottles that can be filled in one batch. It also determines the minimum amount since, for technical reasons, they must be filled at least to half their capacity.

Since vineyards or wine cellars are not always located near to the bottling and labeling centers, the wine must be transported to the processing centers, in advance, in large tank trucks. Also, the problem of transferring the wine from the tank farm shown in Fig. 4.1 is not trivial since when part of the wine of a large volume tank is transferred to an intermediate tank, the remaining wine has to be reallocated to other large volume tanks in the tank farm. If a large volume tank is half empty, the wine will oxidize very quickly

We will now focus on the bottling and labeling activities that are shown in Fig. 4.2. The critical activities, which usually determine the capacity of the line, are the bottle filling and labeling activities. Processing speeds of bottling lines tend to be greater than the processing speed of labeling, so when the lines work coupled the throughput of the line is usually determined by the labeling activity.

One of the main problems of the bottling line is that it has to go through lengthy setups each time that either the wine being bottled, or the type of bottle being used changes. When the wine changes, both the intermediate bottling tank and the filling device on the bottling line must go through a cleaning process that removes all traces of the wine that was being filled. The length of the setup is sequence dependent on the type and quality of the wine. The time required to switch from a red wine, such as a Cabernet Sauvignon, Malbec, Merlot, Syrah, etc., to a white wine, such as a Chardonnay, Sauvignon Blanc, Riesling, etc., is much longer than switching from a white wine to a red wine. In the first case, the cleaning has to be much more thorough to avoid getting a pink wine, instead of a white wine. Also, switching from a regular wine to a premium wine takes longer than the converse since mixing regular wine with a premium wine is much worse than getting some premium wine in a regular wine. Typical changeover times are almost two hours when going from a red to a white wine, while going from a white wine to a red one takes about one hour. The labeling process, on the other hand, is more agile than the bottling one since the setup times are much shorter and they are not sequence dependent. For example, changing the label roll takes at most half an hour.

Although some wineries might prefer to operate the bottling line and the labeling line coupled as a single line, there are instances in which it is necessary to separate them. For example, the best quality wines must be aged for a long time in their bottle. Storing them with the label on them is not convenient since the bottles get dirty during the time they are stored, which can be two or more years, and they must be washed before they are sold. This would damage the labeling making it necessary to remove it and re-label it. Another reason is to postpone product differentiation, as discussed earlier. This reduces *WIP* inventory, but introduces some inefficiencies, such as extra handling of the unlabeled bottles.

Note that a decoupling operation enables the generation of buffers that allow mitigating variations in production rates of the production processes. This may be used to develop a more robust plan against possible disruptions in the lines although the main advantage of postponing the labeling process is to reduce product misallocation, as a result of batch bottling, while keeping lower levels of *WIP* inventory due to the pooling effect of buffers.

Is it always better to hold *WIP* inventory rather than bottled and labeled wine, which is ready to ship? Is it better to use a label-to-order policy or a label-to-stock policy? According to the literature, the answer is that it depends on several aspects. A greater diversity of sales channels and higher levels of uncertainty in product orders would favor a label-to-order policy. On the contrary, few sales channels and low levels of demand uncertainty tend to favor the label-to-stock policy. Other aspects may influence which policy is better, such as the length of the planning horizon the winery uses, and whether the winery has a shortage of production capacity. The remainder of this chapter will explore these issues.

4.3. Models for wine production planning

In this section, we present two lot-sizing models for wine production planning that only differ in what kind of inventory they may keep. Although it is possible to hold both *WIP* (unlabeled bottles) and finished goods inventory (labeled bottles), our objective is to isolate the effect of postponing labeling on several performance measures that are meaningful for production planning. In the next section, we specify these performance measures and how we will compute them.

In terms of complexity, both problems can be reduced to a capacitated lot-sizing with setup time problem, which means that both are NP-hard (Maes et al., 1991). So, as Clark & Clark (2000) point out, they are difficult problems to solve to optimality, except for small instances.

On the other hand, the models are intended to help wine production planners to determine lot sizes, not with scheduling the line. Although it is possible to develop a model that includes scheduling, in the spirit of the *General Lotsizing and Scheduling Problem* (Fleischmann & Meyr, 1997), solving this model would be even more difficult than just the lot-sizing one given the inclusion of the sequencing decisions that have to be made due to the sharing of common resources. In practice, scheduling the line is not too difficult, once the lot sizes have been decided, since the number of jobs that can be carried out during a shift is relatively small, diminishing the combinatorial problem of finding a feasible schedule of sequencing.

Finally, it is important to point out that when we refer to a given type of wine, it could be not only by varietal, but also by size.

4.3.1. Model with postponement in the labeling process

We will first describe the model that permits keeping unlabeled bottles in inventory, that is, that allows postponed labeling.

4.3.1.1. Notation

The model requires the following four sets of index:

- $\mathcal{K}$: Set of production lines (indexed by k).
- $\mathcal{I}$: Set of all types of wines produced in the winery (indexed by i).
- $\mathcal{J}_i$: Set of labels that can be assigned to a wine i (indexed by j).
- $\mathcal{T}$: Set of periods to be considered (indexed by t).

The model considers the following decision variables:

- $x_{kijt} \in \mathbb{R}^+$: Bottles to be processed in k, filled with i, labeled with j, and sold in period t.
- $w_{kit} \in \mathbb{R}^+$: Bottles to be processed in k, filled with i, and left unlabeled in period t.
- $p_{kijt} \in \mathbb{R}^+$: Bottles to be processed in k, previously filled with i, labeled with j, and sold in period t.
- $s_{it} \in \mathbb{R}^+$: Stock of bottles filled with i, but left unlabeled at the end of period t.
- $y_{kit} \in \mathbb{Z}^+$: Number of times that the intermediate wine tank, which feeds k, is filled with i in period t ($t \geq 2$).
- $u_{kit} \in \mathbb{R}^+$: Underutilization of a single intermediate wine tank ($t \geq 2$).
- $zbl_{kijt} \in \{0, 1\}$: : Indicates if, in k, there are bottles to be both filled and labeled, with i and j respectively in period t.
- $zb_{kit} \in \{0, 1\}$: : Indicates if, in k, there are bottles to be filled with i, but left unlabeled in period t.
- $zl_{kijt} \in \{0, 1\}$: Indicates if, in k, there are bottles previously filled with i to be labeled with j in period t.

Finally, the model requires the following parameters that must be input as data:

- P_{ki} : Wine allocation decisions for the tanks in the first period (liters).
- C_i : Capacity of the bottle which will be filled with wine i.
- B_{kt} : Capacity of the bottling line k in period t (hours).
- L_{kt} : Capacity of the labeling line k in period t (hours).
- TA_k : Capacity of the intermediate wine tank that feeds line k (liters)
- D_{ijt} : Demand for wine i labeled with j in period t (bottles).
- $S_i = s_{i0}$: Initial stocks of unlabeled bottled wine i (bottles).
- STB_k : Setup time needed to adjust the bottling line k (hours).
- STL_k : Setup time needed to adjust the labeling line k (hours, $< STB_k$).
- $STBL_k$: Setup time needed to adjust the production line k (hours, $max\{STB_k, STL_k\}$).
- TB_k : Time required for filling a bottle with wine in line k (hours).
- TL_k : Time required for labeling a bottled wine in line k (hours, $> TB_k$).
- TBL_k : Time required to completely process a bottle in line k (hours, $max\{TB_k, TL_k\}$).
- λ : Ratio of shortage cost vs. holding cost.

- δ : Ratio of setup cost vs. holding cost.

4.3.1.2. The objective function

The objective function is given by the following expression:

$$\min \sum_{i,t} s_{it} + \lambda \sum_{i,j,t} (D_{ijt} - \sum_k (x_{kijt} + p_{kijt})) + \delta \sum_{k,i,t} (zb_{kit} + \sum_j (zl_{kijt} + zbl_{kijt}))$$

The objective function minimizes the sum of the stock of undifferentiated bottles and the weighted sum of the shortages, and the number of setups. It can be viewed as a scalarization of a multi-objective minimization problem. Parameters λ and δ reflect the relative undesirability of both shortage and number of setups with respect the overall inventory level.

4.3.1.3. The constraints

The constraints of the model are the following:

$$TA_k \cdot P_{ki} = C_i(w_{ki1} + \sum_j x_{kij1}), \forall\, (k,i). \tag{4.1}$$

$$TA_k(y_{kit} - u_{kit}) = C_i(w_{kit} + \sum_j x_{kijt}), \forall\, (k,i,t \geq 2). \tag{4.2}$$

$$\sum_k (x_{kijt} + p_{kijt}) \leq D_{ijt}, \forall\, (i,j,t). \tag{4.3}$$

$$s_{it} = s_{i(t-1)} + \sum_k (w_{kit} - \sum_j p_{kijt}), \forall\, (i,t). \tag{4.4}$$

$$\sum_{i,j} (TBL_k x_{kijt} + STBL_k zbl_{kijt}) + \sum_i (TB_k w_{kit} + STB_k zb_{kit}) \leq B_{kt}, \forall\, (k,t). \tag{4.5}$$

$$\sum_{i,j} (TBL_k x_{kijt} + STBL_k zbl_{kijt} + TL_k p_{kijt} + STL_k tzl_{kijt}) \leq L_{kt}, \forall\, (k,t). \tag{4.6}$$

$$x_{kijt} \leq \left(\frac{min(B_{kt}, L_{kt})}{TBL_k} \right) zbl_{kijt}, \forall\, (k,i,j,t) \tag{4.7}$$

$$w_{kit} \leq \left(\frac{B_{kt}}{TB_k} \right) zb_{kit}, \forall\, (k,i,t). \tag{4.8}$$

$$p_{kijt} \leq \left(\frac{L_{kt}}{TL_k} \right) zl_{kijt}, \forall\, (k,i,j,t). \tag{4.9}$$

$$0 \leq u_{kit} \leq 0.5, \forall\, (k,i,t \geq 2). \tag{4.10}$$

$$p_{kijt}, s_{it}, w_{kit}, x_{kijt} \geq 0, \forall\, (k,i,j,t). \tag{4.11}$$

$$y_{kit} \in \mathbb{Z}^+, \forall\, (k, i, t \geq 2). \tag{4.12}$$

$$zbl_{kijt}, zb_{kit}, zl_{kijt} \in \{0, 1\}, \forall\, (k, i, j, t). \tag{4.13}$$

Constraints (4.1) and (4.2) ensure that the wine must be either an unlabeled and stored one, or a bottled and labeled wine. Besides, to represent the complexity of wine inflows and large-scale tank management at processing centers, it is assumed that the allocation-to-tanks decisions have to be made at least with one period of anticipation. As stated before, the bottling process is a batch one and, for technical reasons, the intermediate tanks have to be filled at least to half of their capacity, as enforced by constraint (4.10). Constraint (4.3) establishes that orders will be fulfilled though some combination of tanked wine (raw materials) or work in progress (unlabeled, bottled wine). In the first case, bottling and labeling lines operate coupled, and in the latter, only the labeling lines are used. Constraint (4.3) also prevents the overproduction of finished goods, which is necessary to prohibit non-negativity of lost sales. Constraint (4.4) updates the *WIP* stock, and by (4.11) backlog is forbidden. Constraints (4.5) and (4.6) guarantee that the capacity of the bottling and labeling line are not exceeded, respectively. Note that due to the possibility of decoupling the lines, both the postponement process and the differentiation process just undermine the resources that they use. Besides, the triangle inequality that exposes the setup costs in the objective function implies that, in order to satisfy a product demand in a given line and period, either direct production and postponement process or direct production and differentiation process can be performed, but cannot do both pairs. Finally, constraints (4.7), (4.8), and (4.9) ensure that there must be a setup if any process, i.e. direct production, postponement, and differentiation, is performed, respectively. Last, but not least, (4.11), (4.12), and (4.13) establish the nature of the variables.

This *MIP* model could have at most $\mathcal{I} \cdot (\mathcal{T} - 2 \cdot \mathcal{K} + 4 \cdot \mathcal{K} \cdot \mathcal{T} + 4 \cdot \mathcal{J} \cdot \mathcal{K} \cdot \mathcal{T})$ variables (where $\mathcal{J}$ is the set among $\mathcal{J}_i$ with the largest cardinality) from which $\mathcal{I} \cdot (\mathcal{T} - \mathcal{K} + 2 \cdot \mathcal{K} \cdot \mathcal{T} + 2 \cdot \mathcal{J} \cdot \mathcal{K} \cdot \mathcal{T})$ are continuous, $\mathcal{I} \cdot \mathcal{K} \cdot \mathcal{T} \cdot (2 \cdot \mathcal{J} + 1)$ are binary, and $\mathcal{K} \cdot \mathcal{I} \cdot (\mathcal{T} - 1)$ are integer. For the other hand, the model could have at most $\mathcal{I} \cdot (\mathcal{T} - \mathcal{K}) + \mathcal{T} \cdot (2 \cdot \mathcal{K} + \mathcal{I} \cdot \mathcal{J} + 3 \cdot \mathcal{I} \cdot \mathcal{K} + 2 \cdot \mathcal{I} \cdot \mathcal{J} \cdot \mathcal{K})$ regular constraints, excluding those related to the nature of the variables.

4.3.2. A benchmark model without postponement in the labeling process

In this subsection, we describe the benchmark model. Only finished goods are now stored, and labeling postponement is forbidden.

4.3.2.1. Notation

The benchmark model considers the following new decision variables. Although we use the same letters, those variables have changed either their decision scope, or their set dimensionality.

- $w_{kijt} \in \mathbb{R}^+$: Bottles to be processed in k, filled with i, labeled with j, and stored in period t.
- $p_{ijt} \in \mathbb{R}^+$: Bottles filled with i and labeled with j, taken from stock and sold in t.
- $s_{ijt} \in \mathbb{R}^+$: Stock of bottles filled with i and labeled with j at the end of period t.
- $z_{ijt} \in \{0, 1\}$: Dummy variable that establishes a logical behavior.

Also, the benchmark model requires the following new parameter that replaces S_i:

- $S_{ij} = s_{ij0}$: Initial stocks of labeled and bottled wine (bottles).

4.3.2.2. The objective function

The objective function in the benchmark model is given by the following expression:

$$\min \sum_{i,j,t} s_{ijt} + \lambda \sum_{i,j,t} (D_{ijt} - \sum_{k} x_{kijt} - p_{ijt}) + \delta \sum_{k,i,j,t} zbl_{kijt}$$

As before, the objective function minimizes the weighted sum of the holding cost, the shortage cost and the setup cost. Note that the expression for the setup costs reflects the fact that the lines always work coupled.

On the other hand, note that we have made the strong assumption that the holding cost of a bottle ready to be sold is the same as of an unlabeled bottle. Although this is not fulfilled in practice, the assumption was made in order to reflect in both models the same level of undesirability about holding any kind inventory.

4.3.2.3. The constraints

The constraints of the benchmark model are the following:

$$TA_k \cdot P_{ki} = C_i \sum_{j} (w_{kij1} + x_{kij1}), \forall \, (k, i). \tag{4.14}$$

$$TA_k(y_{kit} - u_{kit}) = C_i \sum_j (w_{kijt} + x_{kijt}), \forall\, (k, i, t \geq 2). \tag{4.15}$$

$$\sum_k x_{kijt} + p_{ijt} \leq D_{ijt}, \forall\, (i, j, t). \tag{4.16}$$

$$s_{ijt} = s_{ij(t-1)} + \sum_k w_{kijt} - p_{ijt}, \forall\, (i, j, t). \tag{4.17}$$

$$\sum_k w_{kijt} \leq \sum_{l=t+1}^{T} D_{ijl} \cdot z_{ijt}, \forall\, (i, j, t). \tag{4.18}$$

$$p_{ijt} \leq D_{ijt} \cdot (1 - z_{ijt}), \forall\, (i, j, t) \tag{4.19}$$

$$TBL_k \cdot \sum_{i,j} (x_{kijt} + w_{kijt}) + STBL_k \cdot \sum_{i,j} zbl_{k,i,j,t} \leq min(B_{kt}, L_{kt}), \forall\, (k, t). \tag{4.20}$$

$$w_{kijt} + x_{kijt} \leq \left(\frac{min(B_{kt}, L_{kt})}{TBL_k} \right) \cdot zbl_{kijt}, \forall\, (k, i, j, t) \tag{4.21}$$

$$0 \leq u_{kit} \leq 0.5, \forall\, (k, i, t) \tag{4.22}$$

$$p_{ijt}, s_{ijt}, w_{kijt}, x_{kijt} \geq 0, \forall\, (k, i, j, t) \tag{4.23}$$

$$y_{kit} \in \mathbb{Z}^+, \forall\, (k, i, t \geq 2) \tag{4.24}$$

$$zbl_{kijt}, z_{ijt} \in \{0, 1\}, \forall\, (k, i, j, t). \tag{4.25}$$

4.3.2.4. Discussion of the benchmark model

As stated at the beginning of this section, the main difference between this model and the last relies on the fact that only finished goods inventory is held, which means that the lines always work coupled. This changes how demand is fulfilled, with constraint (4.16) establishing that in each period the orders must be satisfied either through direct production or the stock of finished goods. Likewise, constraints (4.14) and (4.15) reflect that wine within the intermediate tanks is destined to be sold or processed, then stored as finished goods. Accordingly, constraint (4.17) updates the stock of every type of bottled and labeled wine. On the other hand, constraints (4.18) and (4.19) impose a logical solution behavior, which resembles, in part, the well-known Wagner-Whitin optimality conditions (Wagner & Whitin, 1958). In fact, those constraints state that in each time period the finished inventory of a given wine can either increase or decrease, but cannot do both. Moreover, if a particular finished inventory increases, then the quantity of bottled and labeled wines to be processed for storage can only be used to meet future demands. As explained before, the bottling and labeling lines operate coupled, so constraint (4.20) assures that the capacity of production lines is not exceeded. Recall that the setups of the bottling lines are greater than those of the labeling lines, and conversely, the processing times of the bottling lines are shorter than those of the labeling lines. Hence, the time needed both to clean the lines and to process the wine is dominated by the bottling setup and the labeling processing times, respectively. Constraint (4.21) establishes that there must be a setup if a batch is performed. Lastly, (4.23), (4.24), and (4.25) establish the nature of the used variables.

The *MIP* benchmark model could have at most $\mathcal{I} \cdot (3 \cdot \mathcal{J} \cdot \mathcal{T} - 2 \cdot \mathcal{K} + 2 \cdot \mathcal{K} \cdot \mathcal{T} + 3 \cdot \mathcal{J} \cdot \mathcal{K} \cdot \mathcal{T})$ variables (where $\mathcal{J}$ is the set with the largest cardinality among $\mathcal{J}_i$) from which $\mathcal{I} \cdot (2 \cdot \mathcal{J} \cdot \mathcal{T} - \mathcal{K} + \mathcal{K} \cdot \mathcal{T} + 2 \cdot \mathcal{J} \cdot \mathcal{K} \cdot \mathcal{T})$ are continuous, $\mathcal{I} \cdot \mathcal{J} \cdot \mathcal{T} \cdot (\mathcal{K} + 1)$ are binary, and $\mathcal{K} \cdot \mathcal{I} \cdot (\mathcal{T} - 1)$ are integer. Moreover, this model could have at most $\mathcal{K} \cdot \mathcal{T} - \mathcal{I} \cdot \mathcal{K} + 4 \cdot \mathcal{I} \cdot \mathcal{J} \cdot \mathcal{T} + 2 \cdot \mathcal{I} \cdot \mathcal{K} \cdot \mathcal{T} + \mathcal{I} \cdot \mathcal{J} \cdot \mathcal{K} \cdot \mathcal{T}$ regular constraints, excluding those related to the nature of the variables.

Compared with the previous model, the benchmark model will always have the same quantity of integer variables, but fewer binary (at least $\mathcal{I} \cdot \mathcal{K} \cdot \mathcal{T} + \mathcal{I} \cdot \mathcal{J} \cdot \mathcal{T} \cdot (\mathcal{K} - 1)$). The relationship between the quantity production lines and labels will determine which model has more continuous variables and constraints. In fact, the difference between the continuous variables from the previous model and this model is given by $\mathcal{I} \cdot \mathcal{T} \cdot (2 \cdot \mathcal{J} - \mathcal{K} - 1)$, whereas the difference between constraints is defined by $\mathcal{T} \cdot (\mathcal{I} + \mathcal{K} + \mathcal{I} \cdot \mathcal{K} + \mathcal{I} \cdot \mathcal{J} \cdot (\mathcal{K} - 3))$.

4.4. Experimental analysis

4.4.1. A framework for decision making and the experimental scope

Considering that with demand forecasts the availability of both resources and inventories are also frequently updated, Chand et al. (2002) argue that a widespread industry practice to deal with uncertainties relies on a decision making under a rolling horizon framework. This approach, by the way, is followed by the winery we worked with. Indeed, it was commonly seen that due to rush orders or cancelations from importers, the demand forecasts need to be periodically updated. Furthermore, wine production quantities planned for the weeks ahead are used mostly for estimating both wine inflows to the production center, and dry material supply requirements, which include the order placing for labels, bottles, corks, and seals among others.

In a rolling horizon framework, two parameters need to be set: the replanning periodicity (RP) and the horizon length (HL). The first one defines the frozen portion of the horizon, whereas the second establishes the scope of the planning. The length of both parameters must be carefully assessed due to the trade-offs involved, as these trade-offs have practical implications in the wine industry. Note that a low replanning periodicity reduces the planning *nervousness*, thus the dry material supply and inflow decisions become easier at a cost of manufacturing flexibility, such as the ability to react to order changes. On the other hand, a long planning horizon favors the planning of tactical resources, as manpower through shift management, but it also adds more uncertainty to the manufacturing process due to error-prone forecast. The subject winery typically makes the production planning decisions on a weekly basis (i.e. RP = 1). Horizon lengths, on the other hand, could change according to the order acceptance policy of its sales department. Nevertheless, the winery usually considers, at least, three weeks ahead.

Moreover, orders for cases of wine exhibit a pronounced seasonality over the calendar year. Thus, the capacity tightness of the manufacturing process also varies potentially resulting in lost sales and systematic overtime when the available processing time in the lines is exceeded by the time needed to process all product orders. Recall that the investment on production machinery is a strategic decision, so in the short term, overtime and outsourcing are the only options to cope with high demand levels.

Considering what was explained before, the objective of this chapter is to establish insights about under which conditions of capacity tightness (CP), horizon length (HL) and forecast inaccuracy (as a measure of order uncertainty degree, FI hereafter), a tactical production planning model for which the bottling and labeling stage could operate decoupled provide, on a rolling horizon framework, better solutions than a benchmark one that only holds inventories of finished goods (so the lines always work coupled).

We think that these three production planning parameters define the smallest set of relevant settings that allow a fair comparison of the performance of both models, and are also in line with the test framework of Clark (2005). In the next subsections, we expose how to modify these parameters for experimental issues, and we define the performance measures that we use to compare the models outputs.

TABLE 4.1. Sets and parameter instance values

Sets	Cardinality	Parameters	Value
		C_i	0.75 liters $\forall i$.
$\mathcal{K}$	2	B_{kt}	54 hours $\forall k, t$.
		L_{kt}	54 hours $\forall k, t$.
		TA_k	10,000 liters $\forall k$.
		STB_k	1.5 hours $\forall k$.
$\mathcal{I}$	2	STL_k	0.5 hours $\forall k$.
		$STBL_k$	1.5 hours $\forall k$.
		TB_k	0.00014 hours $\forall k$.
		TL_k	0.00028 hours $\forall k$.
$\mathcal{J}_i$ ($\forall i$)	3	TBL_k	0.00028 hours $\forall k$.
		λ	1,000
		δ	0.001

4.4.2. Sets and instance parameters, rolling horizon setup and performance measures

As stated before, both models are $\mathcal{NP} - hard$. So, given that for each level of CP, HL, and FI, it is needed to solve both models several times (not only due to the approach followed, but also in order to develop a statistical analysis on the outputs); in this chapter, we focused solely on very small instances that can be solved to optimality in a reduced computing time.

In the last section of this chapter, we established guidelines to cope with large-sized instances, and we left to future research the development of heuristics for this problem (see the work of Buschkühl et al. (2010) for an excellent classification and review of solution approaches for capacitated lot-sizing problems).

According to the data of the winery we worked with, but modified for experimental purposes, Table 4.1 shows the values that were used in the experiments. Also, all the tests were carried out with an RP of one period.

In a rolling horizon framework, the focus lies on the set of implemented production decisions, which refer to the lot-sizing and setup structure at the frozen portion of the horizon. As shown in Fig. 4.3, these decisions take into account the actual or *certain* demand (D_{ijt}) and the error-prone forecasts made in that period for future periods ($F^t_{ijt'}$). In our computational experiments, we chose three levels of HL = $\{3, 4, 5\}$ (so $t' = t + 1 \ldots t + HL + 1$) to test their impact on the implemented decisions.

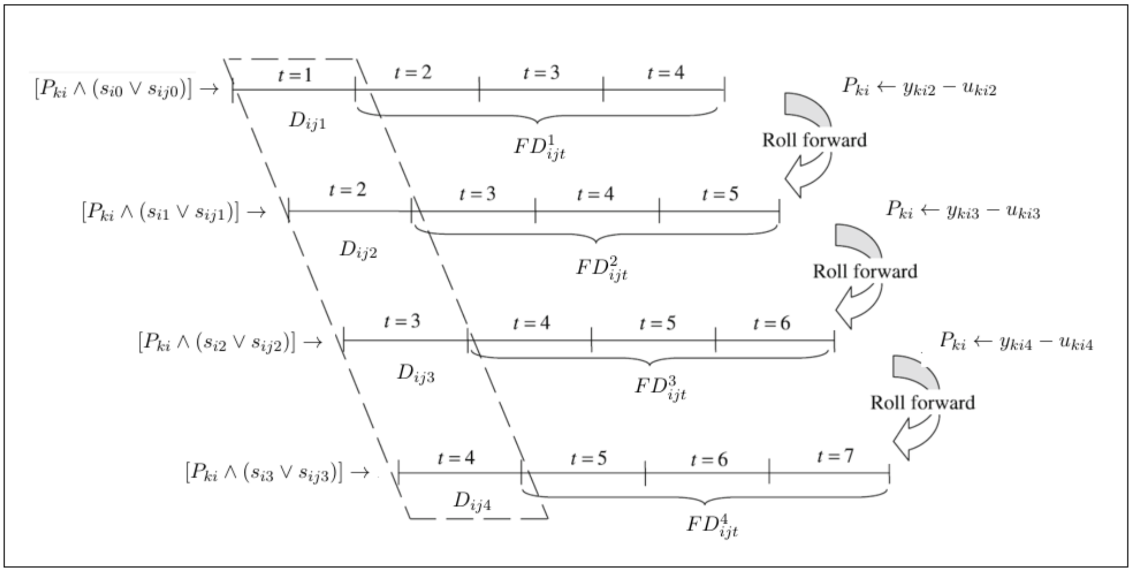

FIGURE 4.3. Rolling Horizon Framework with a RP of one period and an HL of four periods. D_{ijt} refers to the actual demand in period t, whereas $F^t_{ijt'}$ refers to the forecast made in t for the certain demand in period t', with $t < t'$.
Source: Asad & Demirli (2010).

Moreover, whenever the planning horizon is rolled forward, those decisions define both the wine-to-tanks allocation and the initial stocks of wines for the next planning periods, which represent the boundary conditions for the next decision making. On this last, the first period needs special treatment, and three sets of boundary conditions could be chosen for it. The first one (worst case scenario) implies setting all these parameters to 0, and recording only the performance measures of the subsequently implemented actions. The second one (best case scenario) implies setting all these parameters as variables, to solve the models for some periods ahead, and then to use their optimal values as inputs. And last, the third case implies to set these values randomly. To *stress* both models behavior, and due to its simplicity, we choose the first approach for each instance that we run.

Finally, let τ be the number of times that the horizon is rolled forward and η the number of replications. As performance measures (PM) of the quality of the implemented decisions, we chose to use the average per period levels (in the whole system) of inventory (IL), shortages (SL), and setups (STL); and we set $\tau = 20$ and $\eta = 100$ for each level of FI, HL and CP, in order to reduce the dispersion of those values. Table 4.2 and 4.3 show how the performance measures values were computed where, for example, $s^{\nu}_{i\theta}$ and $D^{\nu}_{ij\theta}$ refer respectively to the end stock of wine i, and the

TABLE 4.2. Computation of the performance measures (average per period) for the model which allows postponement.

Performance Measure	Computation
Inventory Level (IL)	$\frac{1}{\eta}\sum_{\nu=1}^{\eta}\frac{1}{\tau-1}\sum_{\theta=2}^{\tau}\sum_{i} s_{i\theta}^{\nu}$
Shortage Level (SL)	$\frac{1}{\eta}\sum_{\nu=1}^{\eta}\frac{1}{\tau-1}\sum_{\theta=2}^{\tau}\sum_{i,j}(D_{ij\theta}^{\nu}-\sum_{k}(x_{kij\theta}^{\nu}+p_{kij\theta}^{\nu}))$
Setup Level (STL)	$\frac{1}{\eta}\sum_{\nu=1}^{\eta}\frac{1}{\tau-1}\sum_{\theta=2}^{\tau}(\sum_{k,i}(zb_{ki\theta}^{\nu}+\sum_{j} zl_{kij\theta}^{\nu})+\sum_{k,i,j} zbl_{kij\theta}^{\nu})$

TABLE 4.3. Computation of the performance measures (average per period) for the model without postponement.

Performance Measure	Computation
Inventory Level (IL)	$\frac{1}{\eta}\sum_{\nu=1}^{\eta}\frac{1}{\tau-1}\sum_{\theta=2}^{\tau}\sum_{i,j} s_{ij\theta}^{\nu}$
Shortage Level (SL)	$\frac{1}{\eta}\sum_{\nu=1}^{\eta}\frac{1}{\tau-1}\sum_{\theta=2}^{\tau}\sum_{i,j}(D_{ij\theta}^{\nu}-\sum_{k} x_{kij\theta}^{\nu}-p_{ij\theta}^{\nu})$
Setup Level (STL)	$\frac{1}{\eta}\sum_{\nu=1}^{\eta}\frac{1}{\tau-1}\sum_{\theta=2}^{\tau}\sum_{k,i,j} zbl_{kij\theta}^{\nu}$

actual (or certain) demand at the ν-th rolling horizon run when the beginning of the horizon is period θ.

4.4.3. Capacity tightness and *certain* demand data generation

To test how both models make use of scarce resources, we considered three levels of capacity tightness following the approach explained in Clark & Clark (2000), but extended for our purposes. Roughly speaking, Clark & Clark's approach assumes that, under a condition of tight capacity, in each period, the overall available processing time in the lines is equal to the expected overall production time needed for processing the product orders on a lot-for-lot basis. Note that not only does the latter encourage both models to reduce the shortages efficiently, but it also allows scaling the demand values to the quantities of wines types, labels, and production lines for experimental analysis. Under their approach, the product's mean demand per period is given by:

$$\bar{D}_t = \gamma \cdot \frac{\sum_k min(B_{kt}, L_{kt}) - \mathcal{I} \cdot \mathcal{J} \cdot (\sum_k STBL_k/\mathcal{K})}{\mathcal{I} \cdot \mathcal{J} \cdot (\sum_k TBL_k/\mathcal{K})} \tag{4.26}$$

where γ is the scale parameter. We chose three values for $\gamma = \{1, 0.5, 0.1\}$, and for each of these, we generated 100 certain demand sets (according to the number of replications η) by distributing uniformly the certain demand for a wine type i and a label type j in period θ for the ν-th replication ($D^{\nu}_{ij\theta} : \nu = 1..\eta$) in the interval [0.75$\cdot\bar{D}_\theta$, 1.25$\cdot\bar{D}_\theta$].

Note that as the horizon is rolled forward τ times, the whole planning horizon for data generation is $\tau + max_l\{HL_l\}$ for each capacity tightness level considered. It is interesting to point out that we fixed the number of finished goods ($|\mathcal{I}| \cdot |\mathcal{J}_i| = 6$) increasing capacity tightness through the scale parameter, thus tending to force larger batch sizes; therefore, efficiency losses due to the economies of scale in production tend to diminish.

4.4.4. Error-prone demand forecasting

Finally, one dimension of the experiments is about how both models cope with demand uncertainty due to error-prone demand forecasting. According to our experimental framework, this means to define for each γ and HL the forecasted values $F^{\nu,\beta}_{ij\theta}$ ($\beta = \theta - 1 - HL..\theta - 1$) made for the certain values $D^{\nu}_{ij\theta}$ (see Fig 4.3). One comprehensive approach to incorporate uncertainty on demand parameters in a rolling horizon framework is developed by Clark (2005), and for the sake of the explanation, we expose this approach next.

Clark's method assumes that as the lead-time from the forecast to the actual or certain demand decreases, accuracy tends to improve with the incorporation of new market and customer information into the forecast. The latter can be appreciated in Fig. 4.4. As shown in this figure, a non-negative parameter α is employed to control the degree of the forecast inaccuracy, where $\alpha = 0$ implies a perfect forecast. The approach considers a base value V_T for the first forecast F_T of a particular product demand, T periods ahead in the horizon, which is simulated as the eventual demand value –or certain demand value– V_0, multiplied by unity plus a random element that is proportional to T, α and the standardized normally distributed random variable r: $V_T = max\;\{0, V_0 \cdot (1 + T \cdot \alpha \cdot r)\}$. Note that the larger the value of α, the further the first forecast base value V_T is likely to be from the certain demand value V_0, whereas V_T and V_0 take the same value when $\alpha = 0$. Roughly speaking, the convex combination between V_T and V_0 behaves like a sort of path from which the forecast accuracy randomly *walks* over, but improving. Formally, as the planning horizon rolls forward the updated forecast F_t of a given demand in period t (with $t = T, T-1, ..., 1, 0$), it is simulated as the interpolated base value $V_t = V_0 + (t/T) \cdot (V_T - V_0)$

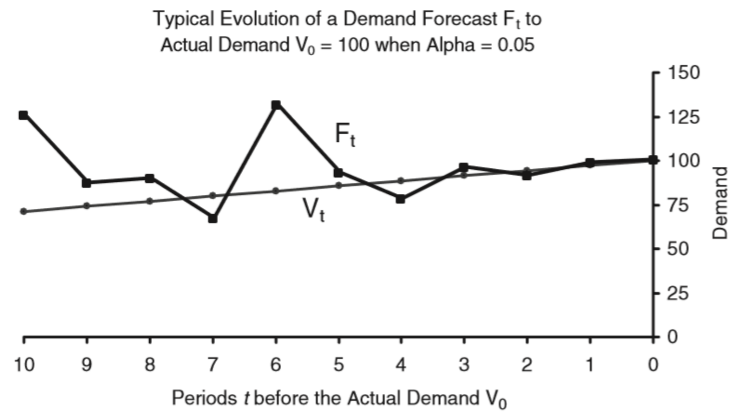

FIGURE 4.4. Clark's forecasting simulation approach.
Source: Clark (2005).

multiplied by unity plus a perturbation, which is proportional to the period, α, and an instance of a standard normal variable r: $F_t = max\ \{0, V_t \cdot (1 + t \cdot \alpha \cdot r)\}\ \forall\ t = T, T-1, ..., 1, 0$. Note that as V_t converges to the eventual demand value, so does F_t.

We applied this approach for each of the three hundred demand sets, which represent the certain values for their corresponding capacity levels, considering 60 levels of uncertainty ($\alpha = 0.005..0.3$ in increments of 0.005) and the three values for HL (which is defined as $T - 1$ in this framework).

4.4.5. Experimental results

Overall, the followed framework required that for each level of capacity tightness (γ), uncertainty degree (α) and horizon length (HL), both models were solved $\eta \cdot \tau = 2,000$ times in order to compute the associated performance measures (i.e. $IL(HL, \gamma, \alpha)$, $SL(HL, \gamma, \alpha)$ and $STL(HL, \gamma, \alpha)$, see Table 4.2 and 4.3).

The models and the rolling horizon approach were coded in AMPL (Fourer et al., 1993), and each of the 2,196,000 instances were solved using CPLEX 12.6.0.0 (64-bits) on a PC with an Intel Core i7 CPU processor and 16 GB of RAM.

For the sake of both analysis and explanation, we arbitrarily classified the uncertainty levels into three groups of the same cardinality: α_L = $\{$0.5%..10%$\}$, α_M = $\{$10.5%..20%$\}$ and α_H = $\{$20.5%..30%$\}$; and we refer to them as low, medium and high demand uncertainty levels.

TABLE 4.4. Average per period performance of implemented decisions for γ = 0.1

Loose capacity (γ = 0.1)										
Horizon Length (HL)	Level of uncertainty (LU)	Inventory Level (IL)			Shortage Level (SL)			Setups Level (STL)		
		PP	DP	Diff	PP	DP	Diff	PP	DP	Diff
HL = 3	α_L = {0.5%..10%}	644	650	-6	606	602	3	9.46	7.69	1.77
	α_M = {10.5%..20%}	1,811	1,865	-54	1,754	1,737	17	9.07	7.26	1.81
	α_H = {20.5%..30%}	3,030	3,404	-374	2,805	2,674	131	8.59	6.75	1.84
HL = 4	α_L = {0.5%..10%}	637	637	0	603	603	1	9.57	7.63	1.94
	α_M = {10.5%..20%}	1,819	1,819	-1	1,714	1,714	-1	9.17	7.25	1.92
	α_H = {20.5%..30%}	3,106	3,129	-23	2,601	2,618	-16	8.72	6.79	1.93
HL = 5	α_L = {0.5%..10%}	630	633	-3	604	602	2	9.45	7.72	1.73
	α_M = {10.5%..20%}	1,803	1,806	-3	1,680	1,678	2	9.08	7.38	1.70
	α_H = {20.5%..30%}	3,182	3,205	-23	2,403	2,418	-15	8.68	6.92	1.76

TABLE 4.5. Statistical analysis for the differences with γ = 0.1

Loose capacity (γ = 0.1)													
HL	LU	Inventory Level (IL)				Shortage Level (SL)				Setup Level (STL)			
		S-W	p-value	T or W	p-value	S-W	p-value	T or W	p-value	S-W	p-value	T or W	p-value
3	α_L	0.956	47.0%	-4.167	0.1%	0.914	7.6%	5.680	0.0%	0.957	48.6%	362.173	0.0%
	α_M	0.832	0.3%	-3.808 ”-”	0.0%	0.819	0.2%	-3.920 ”+”	0.0%	0.871	1.2%	-3.923 ”+”	0.0%
	α_H	0.935	19.1%	-14.707	0.0%	0.951	38.2%	11.923	0.0%	0.968	70.7%	344.115	0.0%
4	α_L	0.915	7.9%	0.201	84.3%	0.942	25.8%	1.559	13.6%	0.958	50.5%	413.365	0.0%
	α_M	0.974	84.4%	-0.436	66.8%	0.930	15.7%	-0.639	53.1%	0.974	83.4%	408.637	0.0%
	α_H	0.940	24.3%	-6.016	0.0%	0.899	3.9%	-3.920 ”-”	0.0%	0.934	18.0%	380.310	0.0%
5	α_L	0.962	57.8%	-1.947	6.7%	0.927	13.4%	5.242	0.0%	0.952	39.4%	340.871	0.0%
	α_M	0.929	14.6%	-3.264	0.4%	0.973	82.4%	8.331	0.0%	0.954	42.9%	452.181	0.0%
	α_H	0.913	7.2%	-9.026	0.0%	0.958	50.4%	-7.088	0.0%	0.966	67.6%	224.295	0.0%

Given this layering, for each combination of demand uncertainty level, horizon length, and capacity tightness, we perform a pair-wise analysis in order to establish if the mean (or median) of the performance measure values of both models differs with statistical significance.

Particularly, for each performance measure we applied first the Shapiro-Wilk test (Shapiro & Wilk, 1965) –S-W hereafter– to test if the difference between the values follows a normal distribution (null hypothesis), and then, accordingly, we applied either the Student's t-test T hereafter or the Wilcoxon signed-rank test (Wilcoxon, 1945), –W hereafter– to establish if the mean or median of those values statistically differs (alternative hypothesis). Tables 4.4 to 4.9 expose the main results of this research, where all the statistical analyses were carried out considering a significance level of 5%.

4.4.5.1. *Loose* capacity case

Consider Tables 4.4 and 4.5. For each horizon length, as the forecast inaccuracy increases, on average, both models exhibit an increase in their inventory and shortage levels, but conversely, the number of setups that they carried out diminishes. This last means that due to order uncertainty

TABLE 4.6. Average per period performance of implemented decisions for γ = 0.5

Intermediate capacity (γ = 0.5)										
Horizon Length (HL)	Level of uncertainty (LU)	Inventory Level (IL)			Shortage Level (SL)			Setup Level (SL)		
		PP	DP	Diff	PP	DP	Diff	PP	DP	Diff
HL = 3	α_L = {0.5%..10%}	3,106	3,181	-75	3,031	3,017	14	9.17	7.18	1.99
	α_M = {10.5%..20%}	9,010	9,108	-98	8,787	8,790	-4	8.99	7.02	1.97
	α_H = {20.5%..30%}	14,932	15,267	-335	14,410	14,489	-79	8.76	6.79	1.97
HL = 4	α_L = {0.5%..10%}	3,110	3,135	-24	3,017	3,016	1	9.13	7.19	1.94
	α_M = {10.5%..20%}	9,068	9,137	-69	8,569	8,584	-14	8.96	7.06	1.90
	α_H = {20.5%..30%}	15,394	15,784	-390	13,346	13,408	-62	8.80	6.85	1.95
HL = 5	α_L = {0.5%..10%}	3,116	3,127	-11	3,013	3,015	-2	9.13	7.20	1.93
	α_M = {10.5%..20%}	8,999	9,047	-47	8,409	8,422	-13	8.98	7.07	1.91
	α_H = {20.5%..30%}	15,804	16,676	-872	12,243	12,285	-42	8.85	6.87	1.98

TABLE 4.7. Statistical analysis for the differences at γ = 0.5

Intermediate capacity (γ = 0.5)														
HL	LU	Inventory Level (IL)				Shortage Level (SL)				Setup Level (STL)				
		S-W	p-value	T or W	p-value	S-W	p-value	T or W	p-value	S-W	p-value	T or W	p-value
3	α_L	0.971	77.3%	-16.751	0.0%	0.966	67.2%	7.381	0.0%	0.960	54.5%	390.183	0.0%
	α_M	0.944	28.3%	-13.571	0.0%	0.957	48.7%	-0.794	43.7%	0.974	82.9%	315.191	0.0%
	α_H	0.902	4.5%	-3.920 ”-”	0.0%	0.948	33.9%	-6.521	0.0%	0.963	60.4%	284.988	0.0%
4	α_L	0.920	10.1%	-5.581	0.0%	0.945	29.2%	0.839	41.2%	0.972	79.1%	294.333	0.0%
	α_M	0.945	30.3%	-7.465	0.0%	0.949	35.9%	-2.795	1.2%	0.965	64.5%	322.665	0.0%
	α_H	0.858	0.7%	-3.920 ”-”	0.0%	0.963	59.8%	-7.755	0.0%	0.946	30.5%	250.745	0.0%
5	α_L	0.951	38.7%	-4.334	0.0%	0.949	35.9%	-1.418	17.2%	0.931	15.8%	329.295	0.0%
	α_M	0.983	96.3%	-8.905	0.0%	0.905	96.3%	-2.706	1.4%	0.968	70.2%	350.476	0.0%
	α_H	0.808	0.1%	-3.920 ”-”	0.0%	0.809	0.1%	-3.248 ”-”	0.1%	0.965	64.5%	187.324	0.0%

TABLE 4.8. Average per period performance of implemented decisions for γ = 1.0

Tight capacity (γ = 1.0)										
Horizon Length (HL)	Level of uncertainty (LU)	Inventory Level (IL)			Shortage Level (SL)			Setup Level (SL)		
		PP	DP	Diff	PP	DP	Diff	PP	DP	Diff
HL = 3	α_L = {0.5%..10%}	5,128	12,641	-7,513	7,561	10,815	-3,255	9.45	6.42	3.03
	α_M = {10.5%..20%}	13,815	35,187	-21,372	20,598	25,815	-5,217	9.20	6.30	2.90
	α_H = {20.5%..30%}	21,682	59,670	-37,988	34,641	41,486	-6,845	8.85	5.88	2.97
HL = 4	α_L = {0.5%..10%}	5,121	22,034	-16,913	7,521	11,600	-4,078	9.48	6.29	3.19
	α_M = {10.5%..20%}	13,845	58,587	-44,741	20,190	29,792	-9,602	9.22	5.95	3.27
	α_H = {20.5%..30%}	22,328	101,292	-78,965	32,651	44,594	-11,944	8.90	5.33	3.57
HL = 5	α_L = {0.5%..10%}	5,128	31,351	-26,223	7,522	12,832	-5,309	9.5	6.16	3.34
	α_M = {10.5%..20%}	13,811	84,946	-71,135	19,712	34,243	-14,531	9.25	5.59	3.66
	α_H = {20.5%..30%}	22,557	155,528	-132,971	30,645	47,859	-17,213	8.97	4.73	4.24

TABLE 4.9. Statistical analysis for the differences at γ = 1.0

Tight capacity (γ = 1.0)													
HL	LU	Inventory Level (IL)				Shortage Level (SL)				Setup Level (STL)			
		S-W	p-value	T or W	p-value	S-W	p-value	T or W	p-value	S-W	p-value	T or W	p-value
3	α_L	0.886	2.2%	-3.920 ”-”	0.0%	0.941	24.5%	-28.295	0.0%	0.911	6.7%	112.796	0.0%
	α_M	0.967	68.7%	-20.471	0.0%	0.953	40.7%	-28.926	0.0%	0.948	33.2%	374.935	0.0%
	α_H	0.962	58.2%	-32.578	0.0%	0.963	61.1%	-60.302	0.0%	0.949	34.8%	227.853	0.0%
4	α_L	0.935	19.3%	-11.380	0.0%	0.898	3.9%	-3.920 ”-”	0.0%	0.878	1.6%	-3.920 ”+”	0.0%
	α_M	0.918	9.3%	-21.726	0.0%	0.901	4.2%	-3.920 ”-”	0.0%	0.932	17.2%	157.576	0.0%
	α_H	0.936	20.4%	-29.760	0.0%	0.961	56.0%	-79.746	0.0%	0.982	96.0%	169.224	0.0%
5	α_L	0.927	13.3%	-11.876	0.0%	0.920	10.1%	-9.438	0.0%	0.961	57.3%	238.482	0.0%
	α_M	0.958	49.9%	-20.400	0.0%	0.934	18.2%	-28.869	0.0%	0.972	87.8%	95.941	0.0%
	α_H	0.937	21.1%	-28.720	0.0%	0.980	93.3%	-89.894	0.0%	0.939	23.3%	131.192	0.0%

both models employed larger batch sizes, which increase the efficiency in the lines, but, at the same time, this worsens the wine misallocation problem.

In comparative terms, and for each combination of horizon length and uncertainty degree, the model with postponement in the labeling process performs, on average, more setups than the benchmark one. Nevertheless, only in two cases ($\{HL_4, \alpha_H\}$ and $\{HL_5, \alpha_H\}$, respectively) it achieves with statistical significance lower per period levels of both inventory and shortage, thus being an attractive alternative to direct production.

The rest of the cases, on the other hand, show dissimilar results. Particularly, for the shortest horizon length (HL_3), the postponement strategy kept in average less inventory per period than the direct production approach, but this last could handle better the shortages. Also, the higher the forecast inaccuracy degree, the greater the differences between the performance of both models. Indeed, for this horizon length, the highest level of uncertainty (α_H) provides the worst relative performance regarding lost sales for the postponement model, which degrades its possible use. Note that this last supports the pooling effect of labeling postponement, but not its capacity to manage the differentiation of the less *WIP* inventory efficiently.

Another interesting result comes from the analysis of HL_4, where this last statement does not hold. For the low and medium level of forecast inaccuracy (α_L and α_M), the average performance of both models, in terms of inventory and lost sales period levels, does not differ statistically, which means that the direct production approach must be employed due to its lower use of setups.

Finally, for the HL_5 case, again, the lowest level of uncertainty favors the use of direct production because the average per period level of inventory for both models does not differ statistically, whereas for the medium level of uncertainty, the pooling effect of postponement is statistically supported but, as the HL_3 case, the direct production approach could handle shortages better.

In summary, neither model consistently outperforms the other in the environment with loose capacity tightness. However, longer planning horizons and higher levels of order uncertainty favor keeping inventory as *WIP* instead of as finished goods.

4.4.5.2. *Intermediate* capacity case

Consider Tables4.6 and 4.7. As before, the model with postponement in the labeling process performed, on average, more setups than the benchmark one; nevertheless, the first one increased its competitiveness against the latter. In fact, for each combination of horizon length and forecast inaccuracy, the pooling effect of postponement is statistically supported, which implies that for all cases the benchmark model could not provide average Pareto-superior solutions.

Moreover, in five of nine cases ($\{HL_3, \alpha_H\}$, $\{HL_4, \alpha_M\}$, $\{HL_4, \alpha_H\}$, $\{HL_5, \alpha_M\}$ and $\{HL_5, \alpha_H\}$), the postponement model achieved with statistical significance both lower average per period levels of inventory and shortages, which favors its use.

For this capacity environment, the worst relative performance of the model with postponement is achieved at the lowest level of both horizon length and forecast inaccuracy (HL_3, α_L), where it kept, on average, less inventory per period than the benchmark one, but this last experienced a lower level of lost sales with statistical significance.

For the rest of the cases ($\{HL_3, \alpha_M\}$, $\{HL_4, \alpha_L\}$ and $\{HL_5, \alpha_L\}$), the comparison on the attractiveness of both models is reduced to the default production planning trade-off. Indeed, for these cases, the shortages do not differ statistically, so both the reduction on the average per period inventory levels, and the increase on the quantity of setup carried out need to be assessed to define the attractiveness of both models.

4.4.5.3. *Tight* capacity case

Last, consider Tables 4.8 and 4.9. As the *Loose* and *Intermediate* capacity cases the model with postponement performed, on average, more setups than the benchmark one, but in this case, these differences are significantly greater and tend to increase markedly for both higher horizon length and forecast inaccuracy levels. In fact, for the (HL_5, α_H) case, the difference of 4.24 is almost of the same magnitude as the average setups quantity performed by the benchmark model.

For this capacity tightness environment, not only is the pooling effect of labeling postponement statistically supported for each case, but also the order of magnitude of the differences stands out. For example, the shift from holding bottled and labeled wines to keeping *WIP* inventory implies, in the worst case (HL_3, α_L), a reduction of 59.4% (7,513/12,641) in the average per period inventory levels; whereas, in the best case (HL_5, α_H), the reduction reaches 85.5% (132,971/155,528).

Moreover, for the nine cases considered, delaying the labeling process exhibits a strong and statistically significant ability to manage efficiently the differentiation of the lower *WIP* inventory. For the two previous cases, which also represent respectively the worst and best scenarios in absolute terms, switching the model of production from the benchmark implies, for (HL_3, α_L), a reduction of 3,255 in the average quantity of lost sales; whereas, for the (HL_5, α_H) case, the reduction reaches 17,213. In relative terms, on the other hand, the (HL_3, α_H) case exhibits the smallest reduction with 16.5% (6,845/41,486), whereas the greatest reduction is achieved by (HL_5, α_M), where it reaches 42.4% (14+,531/34,243).

4.5. Concluding remarks

This chapter addressed the production planning problem faced by wineries by two mixed integer programs that differ in which technological constraints associated with the manufacturing process they incorporate. Considering that for export-focused wineries or wineries that use several sales channels in the same territorial market, their finished goods exhibit a lack of substitutability; we analyzed the benefits of delaying product differentiation for satisfying properly future orders using as proxy a model that can only maintain, as inventory, bottled but not labeled wines.

Assuming that the planning decisions are made on a rolling horizon basis, we found that decoupling the lines has minimal value if neither production capacities are tightly constrained nor orders differ significantly from forecasts. However, if the capacity becomes tight due to large-sized orders and the forecast of those turns more inaccurate, then we found that postponement practices become a very attractive option for decision makers. Particularly, they allow to provide a higher service level to the customers (in terms of fewer shortages), with fewer semi-finished goods, but at the expense of an increase in the setup costs.

From a managerial point of view, we expect that the insights of this paper could establish guidelines to help decision makers in the wine industry choose the right model of production according to their planning environment. But also, we think that the results of this paper could encourage wine managers to adopt the postponement concept when it applies in an industrial sector (i.e. agribusiness) that, as Van Hoek et al. (1998) emphasize, is quite behind in the adoption of postponement methodologies compared to others such as automotive or high-tech.

As stated in section 4.3, both models are NP-hard, and thus difficult to solve to optimality except for small problem instances. To cope with real-sized ones, and thus make the use of the lot size models attractive to wine production planners, we expect to develop two kinds of heuristics to obtain both close to optimal production plans, and non-dominated solutions regarding quality and computing time compared to the CPLEX branch-and-cut incumbents.

First, and based on some preliminary experiments, we have found that the Lagrangean Relaxation bound (see the seminal paper of Geoffrion (2010), and the excellent although not new work of Fisher (2004) for a review) obtained through dualizing constraints (4.5) and (4.6) in the model with postponement in the labeling process (stated in 4.3.1.2) is very close to the best lower bound found by CPLEX after about one and a half hour of computing time. We expect that the integrality gap is small, so we plan to develop at least one Lagrangian Relaxation-based heuristics to solve larger instances of this problem.

On the other hand, since the order forecasts are unreliable and will most likely change, solving the models to optimality provides a limited benefit considering the what-if analysis that decision makers commonly need to develop. We believe that the latter justifies the use of approximate models that could provide good solutions in a reduced amount of computing time. Given its versatility and ease of implementation on AMPL, we expect to implement Fix-and-Relax heuristics (see the work of Mercé & Fontan (2003) for a very comprehensive overview), which combines an MIP-based algorithm with a rolling horizon framework. A key element of this heuristics is the treatment of the *Approximation Window*, which corresponds to the portion of the horizon that is relaxed to reduce the computational burden. We plan to develop three simplification strategies for it: the first, or the benchmark one, implies replacing the binary variables with their linear relaxation; the second involves a linear approximation for the setups variables, which means removing them and compensating this by the increase in the values of the processing times (see the work of Clark & Clark (2000)); and finally, the third approach involves dualizing the capacity constraints for this portion of the horizon, and penalizing capacity infeasibilities with values of the Lagrangian multipliers that have been set up by the default subgradient method at the first iteration of the Fix-and-Relax framework.

5. CONCLUSIONS AND FUTURE RESEARCH

This thesis was centered fundamentally on the application of operations research models and methodologies to problems that arise in forestry and agricultural industries. Although those models differ in their nature and scope, they share a common objective: to increase the effectiveness of the production process modeled through the handling of the environmental and/or system uncertainty that affects its performance.

Particularly, in the second chapter of this thesis, we considered the problem of scheduling production under environmental uncertainty in a sawmill, where the deterministic model proposed by Maturana et al. (2010) was extended to account for uncertainties in product demand and availability of raw materials. The robust optimization methodology of Bertsimas & Sim (2004) was applied to develop three robust models: the first considers only demand uncertainty, the second considers only supply uncertainty, and third considers both types of uncertainties. For each of these models, we carried out an experiment to evaluate the robustness of the solutions and the effect of the conservatism level on these last. This allowed us to determine the impact of both sources of uncertainty, and their interaction on the production schedules. We analyzed the behavior of the robust solutions using Monte Carlo simulation. This analysis provides several managerial insights that could help schedulers choose the appropriate level of conservatism with respect to each source of uncertainty. But one of the limitations of the approach we used is that it is static. All decisions must be made at the beginning of the planning horizon, with no possibility of adapting them to the realization of uncertain data later on, as it is done in the fourth chapter with rolling horizons. We might use, in the future, an online algorithm where some data are released through time, or dynamic stochastic programming to improve this. Another alternative is to use the *Affinely Adjustable Robust Counterpart* approach proposed by Ben-Tal et al. (2004). This approach allows to define some of the uncertain parameters as *non-adjustable variables*, while the other variables, the *adjustable variables*, can be chosen after the realization.

On the other hand, in the third chapter of this thesis, we focused on the management of premium wines, bearing in mind the operations of a small export-focused Chilean winery that we had worked with. Unlike what was done in the previous chapter, we focused on the underlying inventory control problem rather than the production planning of finished goods. Nevertheless, this last issue is analyzed in chapter four. Moreover, system uncertainty was also taken into account due to both processing times and setup times variability is part of the modeling framework. For this problem, we considered the case where orders arrive according to independent Poisson processes, inventories of bottled and labeled wines are reviewed continuously in time, a *postponement* strategy is employed for the labeling process (which is subject to randomness), and there is only one labeling machine. Each of these assumptions was made based on a trade-off between complexity and reality. Moreover, we assumed that an (*s-1,s* policy is followed for each of those, which is a common assumption for goods with both high manufacturing cost and low inventory turnover. If the wine decision maker is concerned about minimizing the sum of the steady-state expected values of the *WIP*, overage and underage costs per unit time; then, the problem of setting the stock levels for finished goods can be addressed by means of a nonlinear stochastic programming problem, but with an objective function that is not easily evaluable. To make what-if type analysis easier, we developed a simple two-stage heuristic that operates as follows: in the first stage, we modeled the labeling process as a polling system under exhaustive service, and we applied the results of Winands et al. (2006) to obtain the mean delays for each end product. Then, in the second stage, we applied Palm's theorem (Palm, 1938) reducing the stock levels setting problem, which is an integer stochastic programming problem, to the solving of a set of straightforward newsvendor problems for each end product. We provided some numerical examples, and we showed that the heuristic becomes more accurate as long as the demand rates for each label type are low, relative to the total demand, and there is slack in production resources. The objective of this research was to develop an approach that could be easily implementable in order to support wine managers with the stocking of finished goods decisions. Nevertheless, the main limitation of the approach is how the arrival process of the orders is modeled. Recall that we assumed that the product orders are for one case of wine. To cope with other sales policies, we expect next to consider the case when the arrival of orders is modeled as a compound Poisson process. For this case, the MVA approach of Winands et al. (2006) can be applied again –without seriously complicating the analysis– to obtain the mean delays in the system, but using the generalization of Palm's theorem (see Muckstadt & Sapra (2010, pp. 190)) will require to compute efficiently the cumulative distribution function of a compound Poisson process in order to set easily the wine stock levels.

Furthermore, in the fourth chapter of this thesis we analyzed the use of postponement practices as a tool to cope with order uncertainty at wineries. Unlike the previous two chapters where environmental and/or system uncertainties were explicitly taken into account by the modeling framework;

for this problem, the focus was to test how two different production models can cope with order uncertainty when production decisions are taken based on the forecast on a rolling horizon basis. This last is a widespread practice to deal with environmental uncertainty, and the winery we worked with followed this approach. Therefore, taking into account both the insights of Cholette (2009) and the test framework of Clark (2005), our objective was to provide guidelines about under which conditions of capacity tightness, horizon length, and demand uncertainty, allowing a decoupled bottling and labeling process, which can be understood as a labeling postponement practice, better tactical production plans could be provided, compared to a manufacturing policy where this degree of freedom is forbidden. For both situations, a mixed integer programming problem was developed, and we tested the quality of both models regarding inventory levels, shortages, and setups, in a rolling horizon framework. Our results showed that there is little or no benefit in using labeling postponement when there is a significant slack in resources capacity, and the actual orders do not differ too much from the forecasted ones. However, when the capacity becomes tight due to large-sized orders, and the order forecast is less accurate, we found that labeling postponement performs better regarding fill rates, inventory, and shortage levels, but at the expense of an increase in the number of setups employed. One of the limitations of our analysis is that we considered a manufacturing policy where if postponement is allowed, then it is employed for all products, and the average demand of all of these is the same. In the future, it would be interesting to analyze mixed policies where, for example, labeling postponement is forbidden for highly-rotating products and allowed for low-rotating products. Moreover, one could develop a mixed integer programming model with hierarchical decision levels, where one of the decisions variables would be if a given product in a certain period is produced under postponement in the labeling process or not. However, with larger instances for this kind of NP-hard model, commercial solvers such as CPLEX are unlikely to provide a good solution in a reasonable computing time. In order to circumvent this difficulty, it would be necessary to develop heuristics (like Relax-and-Fix, see for example Mercé & Fontan (2003)), metaheuristics (like Variable Neighborhood search, see Mladenović & Hansen (1997)), or a combination of both (like MIP-based Neighborhood Search Heuristics, see James & Almada-Lobo (2011)), in order to obtain both close to optimal production plans and non-dominated solutions in terms of quality and computing time compared to the CPLEX branch-and-cut incumbents.

Finally, it is important to point out that although the chapters in this work do not use the same methodological approach to address their respective problems, they serve to illustrate that operations research provides a full tool kit to address real world issues in the agricultural and forestry sectors.

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APPENDIX A. THE NEWSVENDOR PROBLEM UNDER POISSON DEMAND

The newsvendor problem is fundamental in inventory management. It concerns the order placing of a single product, in a single period, under stochastic demand, considering both underage and overage costs. In this appendix, we analyze the newsvendor problem supposing that the product demand follows a Poisson distribution. Particularly, we show how both the optimal order quantity and its expected cost behave as a function of the mean product demand, which also defines the level of uncertainty which a decision maker faces in this context. We expect that this short note could be used as supporting material for courses on inventory management.

Keywords: Optimization; Newsvendor problem; Uncertainty.

A.1. Introduction

The newsvendor problem is the stochastic demand, single product and static inventory management problem. It consider both overage and underage costs. Literature reviews has been done before. For example, see the excellent work by Khouja (1999) and the more recent review by Qin et al. (2011). In this context, one line of research deals with the study of the effects of increased variability of demand. For example see Gerchak & Mossman (1992); Ridder et al. (1998).

In this appendix we study the newsvendor problem with Poisson-distributed demand. Specifically, we describe the behavior of the optimal order quantity and the optimal expected costs, as functions of the mean product demand, denoted λ, which also define the level of variability that the decision maker faces.

Let $D = D(\lambda)$ be a Poisson-distributed demand. Its mean is $E[D] = \lambda$ and its variance is $V[D] = \lambda$. The unit overage and underage costs are denoted by C_o and C_u, respectively. For notational convenience, let $x^+ = \max(0, x)$. If s units are ordered, the expected overage costs are $C_o \cdot E[s - D(\lambda)]^+$, whereas the expected underage costs are $C_u \cdot E[D(\lambda) - s]^+$. Let $f(s, \lambda) = C_o \cdot E[s - D(\lambda)]^+ + C_u \cdot E[D(\lambda) - s]^+$ be the total cost function, considering linear underage and overage costs, when s units are ordered and the random demand follows a Poisson distribution with mean product demand λ. Finally, let denote $\epsilon = \frac{C_u}{C_o+C_u}$ the critical ratio. Thus, the problem can be stated simply as,

$$\min_{s\in\mathbb{Z}^+} f(s,\lambda) = C_o \cdot E\left[s - D(\lambda)\right]^+ + C_u \cdot E\left[D(\lambda) - s\right]^+ \tag{A.1}$$

The contribution of this appendix is twofold. First, we bound the optimal order quantity for (A.1), and state optimality conditions. And second, we develop results on the behavior of the optimal solution $s^*(\lambda) = \arg\min_{s\in\mathbb{Z}^+} f(s,\lambda)$, and the optimal value of this problem, $h^*(\lambda) = \min\{f(s,\lambda) : s \in \mathbb{Z}^+\}$, as functions of λ.

The rest of this appendix is organized as follows. In section A.2 we characterize and bound the optimal order quantity. In section A.3 we analyze the behavior of the optimal solution, $s^*(\lambda)$. In section A.4, we analyze the behavior of the optimal value, $h^*(\lambda)$. Finally some concluding remarks are given in section A.5.

A.2. Setting the optimal order quantity and bounds

Given $\lambda \in \mathbb{R}^+$, the following integer nonlinear programming problem equivalent to (A.1), must be solved,

$$P(\lambda)\Big) \quad \text{minimize} \quad f(s,\lambda) = C_o \sum_{x=0}^{s} (s-x)p(x,\lambda) + C_u \sum_{x=s}^{\infty} (x-s)p(x,\lambda) \tag{A.2}$$

$$\text{s.t.} \quad p(x,\lambda) = e^{-\lambda}\frac{\lambda^x}{x!} \tag{A.3}$$

$$s \in \mathbb{Z}^+ \tag{A.4}$$

The following propositions can be stated.

PROPOSITION A.1. *Function $f(s,\lambda)$ is discretely convex on the order quantity s, with λ fixed.*

Proof: It suffices to show that given λ, the first forward differences of $f(s,\lambda)$ are increasing (Yüceer, 2002, Theorem 1). Let $\Delta f(s,\lambda) = f(s+1,\lambda) - f(s,\lambda)$. After some algebra we obtain,

$$\Delta f(s,\lambda) = (C_o + C_u)\sum_{x=0}^{s} p(x,\lambda) - C_u \tag{A.5}$$

and,

$$\Delta^2 f(s,\lambda) = \Delta f(s+1,\lambda) - \Delta f(s,\lambda) = (C_o + C_u)p(s+1,\lambda) > 0 \tag{A.6}$$

thus,

$$\Delta f(s+1,\lambda) > \Delta f(s,\lambda),\ \forall\ s \in \mathbb{Z}^+ \tag{A.7}$$

$\square$

We now focus our analysis in the characterization of the optimum.

Theorem A.1. *$s^*(\lambda)$ is a global optimum of $P(\lambda)$ if it satisfies,*

$$\begin{cases} \Delta f(s^*(\lambda)-1,\lambda) \leq 0 \leq \Delta f(s^*(\lambda),\lambda) & \textit{if } s^*(\lambda) > 0 \\ 0 \leq \Delta f(s^*(\lambda),\lambda) & \textit{if } s^*(\lambda) = 0 \end{cases}$$

Proof: Local optimality is shown by Gross & Harris (1971, Chapter 1). And due to both Proposition A.1 and the statements of Ui (2006), $s^*(\lambda)$ is also a global optimum of $P(\lambda)$. $\square$

Now, we are ready to establish how to find the optimal order quantity.

PROPOSITION A.2. *$s^*(\lambda)$, the optimum of $P(\lambda)$, is the smallest non-negative integer which satisfies,*

$$\sum_{x=0}^{s^*(\lambda)-1} p(x,\lambda) \leq \epsilon \leq \sum_{x=0}^{s^*(\lambda)} p(x,\lambda) \tag{A.8}$$

Proof: Replace the expression (A.5) in the conditions of Theorem A.1. $\square$

Note that for given λ, it is easy to find the optimal order quantity. As Gross & Harris (1971) argue, $s^*(\lambda)$ can be solved by evaluation of $\Delta f(s,\lambda)$ for each value of s starting with $s=0$ and stopping when Theorem A.1 is satisfied, that is, when $\Delta f(s,\lambda)$ becomes positive for the first time. Nevertheless, this process could be slow. The following proposition provides simple bounds for the optimum of $P(\lambda)$ that can be employed in more efficient numerical search procedures such as, for example, the bisection algorithm.

PROPOSITION A.3. *The following bounds are valid for $P(\lambda)$,*

(i) *If $0 < \epsilon < \frac{1}{2}$, then $0 \leq s^*(\lambda) \leq \left\lfloor \min\{\frac{\lambda}{1-\epsilon}, \lambda + \frac{4}{3}\} \right\rfloor$.*
(ii) *If $\frac{1}{2} \leq \epsilon < 1$, then $\lceil \max\{\lambda - \ln(2), 0\} \rceil \leq s^*(\lambda) \leq \left\lfloor \frac{\lambda}{1-\epsilon} \right\rfloor$.*

Proof: By Proposition A.2, $s^*(\lambda)$ must satisfy,

$$1 - \epsilon \leq \sum_{x=s^*(\lambda)}^{\infty} p(x, \lambda) \tag{A.9}$$

by *Markov's Inequality*

$$\sum_{x=s^*(\lambda)}^{\infty} p(x, \lambda) \leq \frac{\lambda}{s^*(\lambda)} \tag{A.10}$$

thus

$$s^*(\lambda) \leq \frac{\lambda}{1 - \epsilon}, \forall\, 0 \leq \epsilon < 1 \tag{A.11}$$

On the other hand, if $\epsilon \geq \frac{1}{2}$ by Proposition A.2,

$$\frac{1}{2} \leq \epsilon \leq \sum_{x=0}^{s^*(\lambda)} p(x, \lambda) \tag{A.12}$$

which means that $s^*(\lambda)$ is greater or equal to the median of demand process. Conversely, if $\epsilon < \frac{1}{2}$ then

$$\sum_{x=0}^{s^*(\lambda)-1} p(x, \lambda) \leq \epsilon < \frac{1}{2} \tag{A.13}$$

which means that $s^*(\lambda) - 1$ is lower than the median. As random demand D follows a Poisson distribution with parameter λ, the following inequalities hold according to Choi (1994),

$$\lambda - \ln(2) \leq \text{median}(D) < \lambda + \frac{1}{3} \tag{A.14}$$

hence,

$$\begin{cases} \lambda - \ln(2) \leq s^*(\lambda), & \text{if } \epsilon \geq \frac{1}{2} \\ s^*(\lambda) - 1 < \lambda + \frac{1}{3}, & \text{if } \epsilon < \frac{1}{2} \end{cases}$$

and given that $s \in \mathbb{Z}^+$,

$$\begin{cases} 0 \leq s^*(\lambda) \leq \left\lfloor \min\{\frac{\lambda}{1-\epsilon}, \lambda + \frac{4}{3}\} \right\rfloor, & \text{if } 0 \leq \epsilon < \frac{1}{2} \\ \lceil \max\{\lambda - \ln(2), 0\} \rceil \leq s^*(\lambda) \leq \left\lfloor \frac{\lambda}{1-\epsilon} \right\rfloor, & \text{if } \frac{1}{2} \leq \epsilon < 1 \end{cases}$$

$\square$

Summarizing, the optimal solution $s^*(\lambda)$ can be computed faster using Proposition A.3. It is particularly important if the parameter λ is large.

A.3. Behavior of the optimal solution $s^*(\lambda)$

We now turn our attention to the optimal order quantity, s^*, as a function of the parameter λ.

PROPOSITION A.4. *$s^*(\lambda)$, the optimum of $P(\lambda)$, is non-decreasing in λ.*

Proof 1: Let $F(s, \lambda)$ be the cumulative distribution function of a Poisson random variable with parameter λ up to s, i.e.

$$F(s,\lambda) = \sum_{x=0}^{s} p(x,\lambda) = \sum_{x=0}^{s} e^{-\lambda}\frac{\lambda^x}{x!} \tag{A.15}$$

The first partial derivative of $F(s, \lambda)$ with respect λ is given by,

$$\frac{\partial F(s,\lambda)}{\partial \lambda} = -\frac{e^{-\lambda}\cdot\lambda^s}{s!} = -p(s,\lambda) \tag{A.16}$$

which is negative for all $\lambda > 0$. It means that $F(s, \lambda)$ is decreasing in λ, for fixed s. From Proposition A.2, $F(s, \lambda) \geq \epsilon$ in order to $s = s^*(\lambda)$ be the optimum of $P(\lambda)$.

If parameter λ is changed to $\lambda' > \lambda$, then $F(s^*(\lambda), \lambda')$ could be no longer greater or equal to ϵ.

Thus, if $F(s^*(\lambda), \lambda') < \epsilon$, then $s^*(\lambda)$ must be increased in order to be optimum [1]. □

Proof 2: Another way to prove this proposition is based on *stochastic ordering* (Ross et al., 1996, Chapter 9). A random variable X is stochastically larger than a random variable Y, denoted by $X \geq_{st} Y$ if,

$$P\{X \geq x\} \geq P\{Y \geq x\}, \quad \forall x \in \text{Support}\,(X) \cap \text{Support}\,(Y) \tag{A.17}$$

In the newsvendor problem context, Song (1994) proved that a stochastically larger demand implies a non lower optimal order quantity (see the author's Proposition 3.2 and Lemma 3.3). Moreover, Ross et al. (1996) showed that a Poisson random variable is stochastically increasing in its parameter (see example 9.2(a) of the authors' book, p. 411). Therefore, if $\lambda' \geq \lambda$, then

[1] Note that $F(s, \lambda')$ is a increasing function in the order quantity s for fixed λ', because it is a cumulative distribution function.

$D(\lambda') \geq_{st} D(\lambda)$, and thus $s^*(\lambda) \leq s^*(\lambda')$. $\square$

Let $h^*(\lambda)$: $\mathbb{R}^+ \to \mathbb{R}^+$ be defined as,

$$h^*(\lambda) = \min_{s \in \mathbb{Z}^+} f(s, \lambda) \tag{A.18}$$

Hence, $h^*(\lambda)$ is the minimum expected cost, if the demand follows a Poisson distribution with parameter λ. We first restate the following lemma by Jain et al. (2012).

Lemma A.1. *Let* $s^*(\lambda) = \arg\min_{s \in \mathbb{Z}^+} f(s, \lambda)$. *Then, for each compact and not empty subset of* $\mathbb{R}^+$ *of the form* $\mathbb{S}(\bar{\lambda}) = \{x \in \mathbb{R}^+ : 0 \leq x \leq \bar{\lambda}\}$, *with* $\bar{\lambda} > -\ln(\epsilon)$, *there exists a partition of* $\mathbb{S}\left(\bar{\lambda}\right) = \cup_{l=0}^{\bar{s}-1}[\lambda_{l-1}, \lambda_l[\cup [\lambda_{\bar{s}-1}, \lambda_{\bar{s}}]$, *with* $\lambda_{-1} = 0$ *and* $\lambda_{\bar{s}} = \bar{\lambda}$, *such that,* $s^*(\lambda) = l$ *on* $[\lambda_{l-1}, \lambda_l[$, $\forall\, 0 \leq l < \bar{s}$, $s^*(\lambda) = \bar{s}$ *on* $[\lambda_{\bar{s}-1}, \lambda_{\bar{s}}]$, *where* λ_l *is the unique root of the transcendental equation,*

$$t_l(\lambda) = \sum_{i=0}^{l} e^{-\lambda} \frac{\lambda^i}{i!} - \epsilon = 0 \tag{A.19}$$

over $[0, \bar{\lambda}]$, *with* $0 < \epsilon < 1$.

Proof: The existence of the parameters $\{\lambda_l\}_{l=0}^{\bar{s}-1}$, and thus the partition of $\mathbb{S}\left(\bar{\lambda}\right)$, was proven by Jain et al. (2012, Lemma 1). In their proof, replace λ by 1, and suppose $\bar{\lambda} = T$.

On the other hand, uniqueness of these parameters can be proved by contradiction. For $l = \{0..\bar{s}-1\}$, suppose that ξ_1^l and ξ_2^l are two roots of $t_l(\xi)$ in $[0, \bar{\lambda}]$, with $\xi_1^l \neq \xi_2^l$, and by construction, both are greater than 0. In this context, note that $t_l(\xi)$ is differentiable and hence, continuous on $[\xi_1^l, \xi_2^l]$. Thus, by the *Mean Value Theorem*, we can find a number ν in (ξ_1^l, ξ_2^l) such that:

$$\frac{t_l(\xi_2^l) - t_l(\xi_1^l)}{\xi_2^l - \xi_1^l} = t_l'(\nu) \tag{A.20}$$

Since $t_l(\xi_2^l) = t_l(\xi_1^l) = 0$, because they are roots of $t_l(\xi)$, we get,

$$0 = t_l'(\nu) \tag{A.21}$$

Nevertheless, recall that $t_l'(\xi) = -e^{-\xi}\frac{\xi^l}{l!}$ is negative for all $\xi > 0$. So, there is not exists such ν in (ξ_1^l, ξ_2^l) for which $0 = t_l'(\nu)$, implying that $\xi_1^l = \xi_2^l$, and thus the roots of $t_l(\lambda)$ are unique in λ for each valid value of l. $\square$

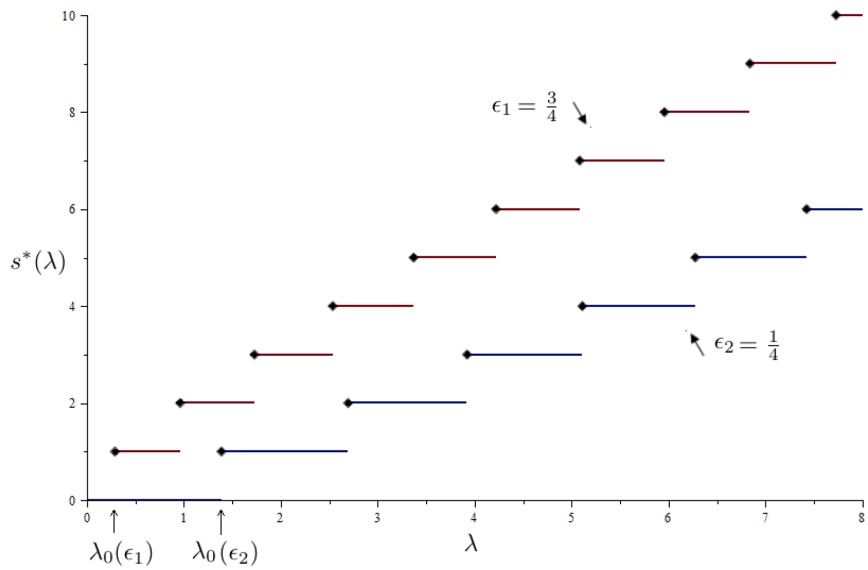

FIGURE A.1. Behavior of $s^*(\lambda)$ for $\lambda \in [0, 8]$ and $\epsilon = \{\frac{3}{4}, \frac{1}{4}\}$.

TABLE A.1. Roots of the transcendental equations (A.19) for $l \in \{0..10\}$.

ϵ	λ_{-1}	λ_0	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}
$\frac{3}{4}$	0	0.288	0.961	1.727	2.535	3.369	4.219	5.083	5.956	6.838	7.726	8.620
$\frac{1}{4}$	0	1.386	2.693	3.920	5.109	6.274	7.423	8.558	9.684	10.802	11.914	13.020

Due to Lemma A.1, $s^*(\lambda)$ is an increasing step function in λ. Besides, close form expressions are available only for $\lambda_0 = -\ln(\epsilon)$ and $\lambda_1 = -1 - W_{-1}(-\frac{\epsilon}{e})$, where $W_{-1}(x)$ is the lower branch of the Lambert W function. See Disney & Warburton (2012) for a discussion about the Lambert W function in the context of EOQ.

For the sake of the exposition, Fig. A.1 shows the relationship between $s^*(\lambda)$ and λ, for $\epsilon = \{\frac{3}{4}, \frac{1}{4}\}$, when $\bar{\lambda} = 8$. Table A.1, shows the unique roots of the transcendental equations (A.19) for $l \in \{0..10\}$. For $\epsilon = \frac{3}{4}$, the partition of $\mathbb{S}(\bar{\lambda} = 8)$ is given by $\cup_{l=0}^{9}[\lambda_{l-1}, \lambda_l[\cup [\lambda_9, 8]$, whereas for $\epsilon = \frac{1}{4}$, the partition is given by $\cup_{l=0}^{5}[\lambda_{l-1}, \lambda_l[\cup [\lambda_5, 8]$.

Note that although $s^*(\lambda)$ is non-decreasing in λ, and constant in any interval of the partition $\mathbb{S}\left(\overline{\lambda}\right)$, it is interesting to also characterize $h^*(\lambda)$, the optimal value parametrized in λ.

A.4. Behavior of the optimal value $h^*(\lambda)$

In this section, we first characterize $h^*(\lambda)$ on the partition of $\mathbb{S}(\bar{\lambda})$, and then we analyze its behavior for all λ. Consider the following lemma.

Lemma A.2. *For any interval of the partition of $\mathbb{S}(\bar{\lambda})$, $h^*(\lambda)$ is continuous, convex and increasing in λ.*

Proof: Convexity was proved by Rossi et al. (2014, Appendix B). The increasing behavior was proved by Jain et al. (2012, Lemma 2). For the sake of the explanation, we briefly describe both proofs.

First, note that $p(x, \lambda)$ is a continuous function of λ. Thus, $f(s, \lambda)$ is also continuous in λ.

Second, note that $f(s, \lambda)$ can be wrote as,

$$f(s, \lambda) = (C_o + C_u) \sum_{j=0}^{s-1} F(j, \lambda) + C_u(\lambda - s) \tag{A.22}$$

now,

$$\frac{\partial}{\partial \lambda} f(s, \lambda) = -(C_o + C_u) F(s-1, \lambda) + C_u \tag{A.23}$$

whereas,

$$\frac{\partial^2}{\partial \lambda^2} f(s, \lambda) = (C_o + C_u) p(s-1, \lambda) \tag{A.24}$$

Note that $\frac{\partial^2 f(s,\lambda)}{\partial \lambda^2} \geq 0,\ \forall \lambda > 0$, and given $s \in \mathbb{Z}^+$, so $f(s, \lambda)$ is convex in λ.

Moreover, let $\lambda^* = \arg\min_{\lambda > 0} f(s, \lambda)$ be the unique root of $\sum_{x=0}^{s-1} e^{-\lambda^*} \frac{(\lambda^*)^x}{x!} = \epsilon$. Hence, $f(s, \lambda) \geq f(s, \lambda^*),\ \forall \lambda \geq \lambda^*$. Now consider the interval $[\lambda_{l-1}, \lambda_l[$. If $\lambda \in [\lambda_{l-1}, \lambda_l[$, then by Lemma A.1, $s^*(\lambda)$=argmin$_{s \in \mathbb{Z}^+} f(s, \lambda) = l$. But recall that if $s^*(\lambda) = l$, $f(l, \lambda)$ reach its minimum at λ_{l-1}. Thus, $f(l, \lambda) \geq f(l, \lambda_{l-1})$. Therefore, $h^*(\lambda) = f(l, \lambda)$ is continuous, convex and increasing in $[\lambda_{l-1}, \lambda_l[,\ \forall l = 0..\bar{s} - 1$, and also for $[\lambda_{\bar{s}-1}, \lambda_{\bar{s}}]$. □

But another interesting question remains: is $h^*(\lambda)$ continuous in $\mathbb{S}(\bar{\lambda})$? If so, then, by Lemma A.2, $h^*(\lambda)$ is increasing in the whole interval $[0, \bar{\lambda}]$. Consider the following,

Lemma A.3. *For any interval $[\lambda_{l-1}, \lambda_l[$, $0 \leq l < \bar{s}$, it holds that $h^*(\lambda_l) = f(l, \lambda_l) = f(l + 1, \lambda_l)$, which implies $s^*(\lambda_l)$=argmin$_{s \in \mathbb{Z}^+} f(s, \lambda_l)$={l, l+1}.*

TABLE A.2. Slope of the linear interpolants $\left(m_l = \frac{h^*(\lambda_l)-h^*(\lambda_{l-1})}{\lambda_l-\lambda_{l-1}}\right)$ joining consecutive points of set $P = \{(\lambda_l, h^*(\lambda_l)) : l \in \mathbb{Z}^+ \cup \{-1\}\}$, for $\epsilon = \{\frac{3}{4}, \frac{1}{4}\}$.

	l	-1	0	1	2	3	4	5	6	7	8	9	10
$\epsilon = \frac{3}{4}$	λ_l	0.000	0.288	0.961	1.727	2.535	3.369	4.219	5.083	5.956	6.838	7.726	8.620
	$h^*(\lambda_l)$	0.000	0.863	1.413	1.832	2.183	2.490	2.766	3.020	3.256	3.477	3.685	3.884
	m_l	-	3.000	0.817	0.547	0.434	0.368	0.325	0.294	0.270	0.251	0.235	0.222
$\epsilon = \frac{1}{4}$	λ_l	0.000	1.386	2.693	3.920	5.109	6.274	7.423	8.558	9.684	10.802	11.914	13.020
	$h^*(\lambda_l)$	0.000	1.386	1.963	2.390	2.744	3.053	3.331	3.586	3.822	4.044	4.253	4.452
	m_l	-	1.000	0.442	0.347	0.298	0.265	0.242	0.224	0.210	0.198	0.188	0.180

Proof: We need to prove that,

$$f(l, \lambda_l) = f(l+1, \lambda_l) \tag{A.25}$$

now consider (A.22),

$$(C_o + C_u)\sum_{j=0}^{l-1} F(j, \lambda_l) + C_u(\lambda_l - l) = (C_o + C_u)\sum_{j=0}^{l} F(j, \lambda_l) + C_u(\lambda_l - l - 1) \tag{A.26}$$

rearranging terms,

$$C_u = (C_o + C_u)F(l, \lambda_l) \tag{A.27}$$

that is equivalent to,

$$\epsilon = F(l, \lambda_l) \tag{A.28}$$

and this last equality holds due to Lemma A.1. □

Due to Lemmas A.2 and A.3, $h^*(\lambda)$ is both continuous and increasing in $\mathbb{S}(\bar{\lambda})$, but not convex. Note that $h^*(\lambda)$ is the lower envelope of convex functions $\{f(s, \lambda)\}_{s \in \mathbb{Z}^+}$[2]. Therefore, $h^*(\lambda)$ is piecewise-convex in $\mathbb{S}(\bar{\lambda})$ (Tsevendorj, 2001). Moreover, $h^*(\lambda)$ it is not differentiable in $\mathbb{S}(\bar{\lambda})$, particularly, at the unique roots of the transcendental equations $\sum_{i=0}^{l} e^{-\lambda_l} \frac{\lambda_l^i}{i!} = \epsilon$, $\forall\ 0 \leq l < \bar{s}$, where the optimum of problem $\min_{s \in \mathbb{Z}^+} f(s, \lambda)$, given λ, is not unique.

Fig. A.2 shows how the expected cost increases in parameter λ. On this, it is interesting to point out that the linear interpolation of the points $P = \{(\lambda_l, h^*(\lambda_l)) : l \in \mathbb{Z}^+ \cup \{-1\}\}$ is piecewise linear, decreasing, and concave. Table A.2 show this for $\epsilon = \{\frac{3}{4}, \frac{1}{4}\}$. One possible explanation of this behavior is by the decreasing in the coefficient of variation of the Poisson distribution as λ rises.

[2]Because $h^*(\lambda)$ can be expressed as $\min\{f(s, \lambda) : s \in \mathbb{Z}^+\}$.

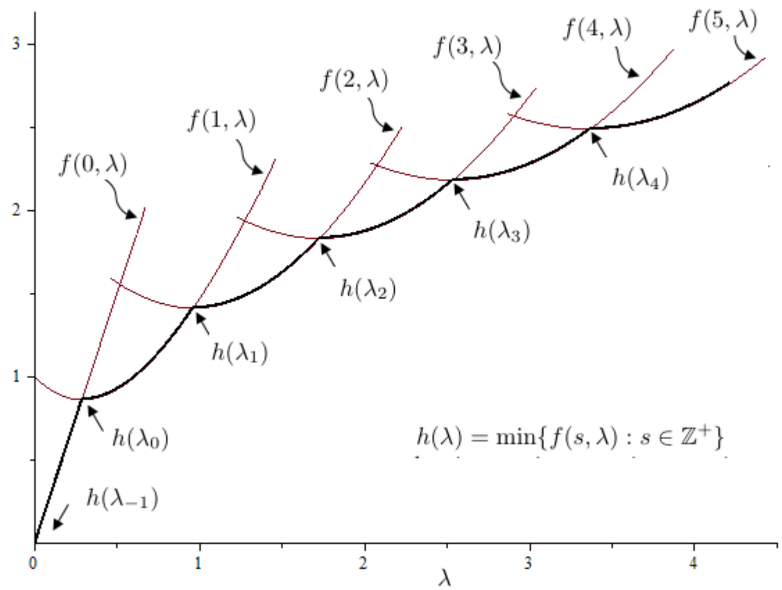

FIGURE A.2. Behavior of $f(s,\lambda) : s \in \{0,1,2,3,4,5\}$ and $h^*(\lambda)$ for $\epsilon = \frac{3}{4}$.

A.5. Concluding Remarks

We have characterized the newsvendor problem with Poisson demand. We show that the objective function is discretely convex in the order quantity, for parameter λ fixed. Then, we derive both optimality conditions and bounds on the order quantity s. After that, we show that the optimal order quantity is non-decreasing in the parameter λ and constant within the intervals of partition $\mathbb{S}(\lambda)$. Finally, we characterize the optimal value h^* as a piecewise convex, increasing and continuous function of parameter λ. We also show its global continuity.