Quantum Kinematic Theory of the Poincaré Group in Two-Dimensional Spacetime

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Abstract

Non-Abelian quantum kinematics is applied to thePoincare group $P_+^\uparrow(1, 1)$,as an example of the quantization-through-the-symmetryapproach to quantum mechanics. Upon quantizing thegroup, generalized Heisenberg commutation relations are obtained, and aclosed Heisenberg–Weyl algebra follows. Then,according to the general theory, the three basicquantum-kinematic invariant operators are calculated;these afford the superselection rules for diagonalizing theincoherent rigged Hilbert space $H(P_+^\uparrow)$ of the regularrepresentation. This paper examines only one of thesediagonalization schemes, while introducing a irreducible spacetime representation carried by isotopicplane-wave eigenvectors of two compatible superselectionoperators (which define a Poincare-invariant linear2-momentum). Thereafter, the principle of microcausality produces massive 2-spinor isotopic states in 1+ 1 Minkowski space. The Dirac equation is thus deducedwithin the quantum kinematic formalism, and the familiarJordan–Pauli propagation kernel in 2-dimensional spacetime is also obtained as a Hurwitzinvariant integral over the group manifold. The maininterest of this approach lies in the adoptedgroup-quantization technique, which is a strictlydeductive method and uses exclusively the assumed Poincaresymmetry.