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Configuration Ray Representations in Non-Abelian Quantum Kinematics and Dynamics

J. Krause[1]

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Ray extensions of the regular representation of noncompact non-Abelian Lie groups are examined as generalizations of the Cartesian coordinate representation of ordinary quantum mechanics to the case of generalized non-Cartesian coordinates and generalized noncommuting momenta. (The momenta are in fact the generators of the representation, and so they satisfy the Lie algebra of the group.) The concept of configuration ray representation is introduced within this new kinematic formalism as subrepresentations of the regular representation which are embossed with the "relativity theory" of a given system. The main features of the mathematical formalism leading to these representations in configuration spacetime are discussed, and their importance for non-Abelian quantum kinematics and dynamics is emphasized. Two miscellaneous examples on the calculus of phase functions for configuration ray representations are given.

1. INTRODUCTION

In a previous paper (Krause, 1987; hereafter referred as paper I) we have presented a rather simple formalism of 2-cocycle calculus for unitary ray representations of Lie groups. This formalism may be especially attractive to physicists, in general, since it can be used in quantum theories without requiring a working knowledge of cohomology (Michel, 1964). In this paper we wish to examine this matter further, from the special standpoint of its applications in non-Abelian quantum kinematics and dynamics.

Although the method used in paper I is not specific to a particular unitary representation, the analysis and calculations made in that paper were anchored on the group manifold. The group manifold is usually not the carrier space of the relevant realizations of Lie groups in physics.

[1]Facultad de Física, Pontificia Universidad Católica de Chile, Casilla 306, Santiago, Chile.

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Nevertheless, as a matter of fact, our approach to 2-cocycles is general enough and means no restriction on the *method* of exponent factor calculus (see also Houard, 1977).

The same approach has been used recently for studying a possible connection between non-Abelian quantum kinematics and dynamics for noncompact Lie groups (Krause, 1993*c*). However, a systematic, general, and detailed discussion of the *mathematical tools* one need to use in order to apply the ideas of quantum kinematics to quantum dynamics is still missing in the literature. We devote this paper to filling this gap.

So, the present work has a purely instrumental character. It bears some interest for physics, however, for it presents in a simple fashion new powerful group-theoretic techniques which may have many useful applications in quantum theories (see, for instance, Krause, 1986, 1988). For the physical motivation of non-Abelian quantum kinematics, in general, we refer the reader to our previous work (Krause, 1993*b*, and references quoted therein).

There are two well-known facts we would like to recall concerning this issue. First, the functions $g(q'; q)$ (which combine the parameters q' and q according to the composition law of the group) may themselves be considered as defining two separate point symmetry groups (the *first-parameter group* G_1 and the *second-parameter group* G_2), which are in fact isomorphic with the original group G because they have the same group manifold $M(G) = \{q\}$ and because the laws of composition are essentially the same for G_1, G_2, and G. Strictly speaking, G_2 is antiisomorphic with G, since it is isomorphic when the group elements are taken in the reverse order. [For these and other details see Racah (1965).] Moreover, the group manifold itself is the homogeneous carrier space of these self-realizations of G; so one does not need to consider an extraneous space for visualizing the action of the group.

The other fact concerns the regular representation of Lie groups, which is a subject that has been amply studied by mathematicians (Naimark and Stern, 1982). Indeed, the regular representation has the peculiarity of being another self-contained faithful construct whose building blocks belong exclusively to the group structure itself, since the carrier Hilbert space $\mathscr{H}(G)$ is defined on the group manifold $M(G)$ by means of the invariant (right and left) measures obtained from the composition law $g(q'; q)$. This feature makes the regular representation an outstanding group-theoretic notion. Because of this same feature, however, the regular representation has not yet played an explicit role in quantum theory, since it appears as a formalism that lies far away from the concrete physical pictures on which Lie groups operate.

Notwithstanding this fact, there is one good reason to choose the regular representation as the cornerstone of quantum kinematics if one