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A finite quantum gravity field theory model

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Abstract
We discuss the quantization of delta gravity, a two symmetric tensors' model of gravity. This model, in cosmology, shows accelerated expansion without a cosmological constant. We present the $\tilde{\delta}$ transformation which defines the geometry of the model. Then, we show that all delta-type models live at one loop only. We apply this to general relativity and we calculate the one-loop divergent part of the effective action showing its null contribution in vacuum, implying a finite model. Then, we proceed to study the existence of ghosts in the model. Finally, we study the form of the finite quantum corrections to the classical action of the model.

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1. Introduction

In the 20th century, two major revolutions in physics changed forever the way in which we understand nature and the world. One of these refers to quantum mechanics which bares its form to physicists like Niels Bohr, Werner Heisenberg, Erwin Schroedinger and Paul Dirac to name a few. This framework has been used successfully to describe the physics of the small, i.e. from atoms to quarks, their forces and interactions. The second one is Einstein's general theory of relativity [1] which describes the physics of the very large, from the motion of planets in the solar system to the motion of galaxies in the Universe, that is, the gravitational interaction. General relativity, or GR in short, is an excellent classical theory as it agrees with the classical tests of GR [2] and it provides a beautiful geometric interpretation of gravity. However, in spite of their individual successes, there is a problem. In fact, GR is not renormalizable. This means among other things that we cannot compute quantum corrections to the classical results, black hole thermodynamics cannot be understood in statistical terms (we cannot count states) and near the Big Bang, GR breaks down, i.e. quantum effects dominate the evolution of the Universe and perhaps this could explain inflation. To solve these problems, physicist have tried to find a unified theory that can encompass all phenomena in all scales and, in the particular case of the gravitational interaction, a quantum theory of gravity.

For the first half of the past century, practically no one worked on such a thing, but in the second half, increasing amounts of people began to address the problem of finding a theory of quantum gravity. The oldest approaches to quantized gravity were methods developed by people like Gerard 'tHooft [3] and Bryce DeWitt [4] such as the path integral quantization and canonical quantization, respectively. Recently, the results of these efforts have resulted in different theories which stand like candidates to solve the problem of quantum gravity. These are, among others, superstring theory [5, 6], loop quantum gravity (LQG) [7], twistors [8] and non-commutative geometry [9], but until now none of the above has produced satisfactory results that can single out one from the others.

The prime candidate for a consistent description of quantum gravity is string theory [5, 6]. Strings appeared around 1970 to explain the confinement of quarks inside hadrons. But later it was realized that in a closed string theory, a spin 2 particle that can be identified with the graviton naturally appears. Even in an open string theory, higher order computations will involve closed strings as intermediate states, so the inclusion of gravity in string theory is not only natural but may also be unavoidable. Moreover, the finiteness of string models make the assertion plausible that the model so formulated provides a consistent theory of quantum gravity. But string theory has a plethora of consistent vacua, so unique phenomenological predictions are difficult to obtain [10].

A second candidate to quantize gravity has been developed in recent years. It is called LQG, where a canonical treatment of GR is implemented [7]. New variables (loops) are introduced, in order to exploit the analogy with non-Abelian Yang–Mills fields. They automatically solve some of the constraints of the theory. In a suitable Hilbert space (spin network states), it is possible to diagonalize geometrical objects such as volume and area, implementing its quantization. This recent progress has permitted the computation of the black hole entropy using purely combinatoric methods [11], up to an arbitrary real number, the Barbero–Immirzi parameter. In this formalism, the semiclassical limit is difficult to obtain [12].

Less widely explored, but interesting possibilities, are the search for non-trivial ultraviolet fixed points in gravity (asymptotic safety [13]) and the notion of induced gravity [14]. The first possibility uses exact renormalization-group techniques [15], and lattice and numerical techniques such as Lorentzian triangulation analysis [16]. Induced gravity proposed that gravitation is a residual force produced by other interactions.

In a recent paper [17], a two-dimensional field theory model explore the emergence of geometry by the spontaneous symmetry breaking of a larger symmetry where the metric is absent. Previous work in this direction can be found in [18], [19] and [20].

Nevertheless, coming back to the original approaches, which are still pursued today, via direct path integral quantization it is known that, to one loop, the Einstein–Hilbert theory is, in vacuum and without a cosmological constant, finite on-shell [3, 21], but at two loops or more is non-renormalizable [22–24]. This means that if we truncate the theory to one loop and evaluate in the equations of motion, what we will have is a finite model of gravitation. But how can we achieve this?

In [25] a modified non-Abelian Yang–Mills model is shown, which from now on will be refer to as $\tilde{\delta}$ Yang–Mills. It is a natural and almost unique extension of non-Abelian Yang–Mills theory whose main characteristics are that it preserves the classical equations of the original model at the quantum level, introduces new symmetries, which are a natural extension of the former symmetry, is renormalizable, preserves the property of asymptotic freedom and lives at one loop in the sense that higher loop corrections are absent. An important point is that the quantum correction result is the double of the usual case.

Taking from the $\tilde{\delta}$ Yang–Mills case and knowing the finiteness on-shell for GR at one loop, the questions we raise are: Can we apply the approach of $\tilde{\delta}$ Yang–Mills to GR? Will it live at one loop? Will it give a finite model of quantum gravity? The answer to the first question is yes (see [26]) and in the case of $\tilde{\delta}$ GR, we have a natural and almost unique extension of GR that has two tensor fields. These are the graviton field $g_{\mu\nu}$ which transforms as a rank 2 covariant tensor under general coordinate transformations (GCTs), plus $\tilde{g}_{\mu\nu}$ which transforms as a two covariant tensor under GCTs and under an extra symmetry. The classical aspects of the model were explored in [26], where it is shown that $\tilde{\delta}$ GR preserve the classical equations of the former metric $g_{\mu\nu}$. The equations of motion for both fields are second order, the Newtonian limit is compatible with experiments, the equivalence principle is satisfied and, in cosmology, the accelerated expansion of the Universe is obtained without introducing a cosmological constant.

In this work, we show that all delta theories live at one loop. We fix the gauge using the Becchi–Rouet–Stora–Tyutin (BRST) method. We compute the divergent part of the effective action and it results the double of what was found in [3]. As the model lives at one loop, this is the exact effective action. Since the equation for the original field is preserved, this means that the quantum corrections of the model are on-shell in the $g_{\mu\nu}$ fields so that the divergent part of the effective action vanishes. This implies that in our model, the effective action at one loop is exact and finite in vacuum so that it does not need to be renormalized.

The problem that this model has is the apparently inevitable appearance of ghosts. Due to them, it may not be unitary or stable. This in turn implies difficulties with the quantization of the model, but in [27–31], phantom fields are used to explain the accelerated expansion of the Universe as an alternative to the cosmological constant and quintessence, a feature that our model presents [26] and that our model seems to introduce in a natural way. It would be possible that our ghosts could be related to phantom fields in $\tilde{\delta}$ GR. This connection may be far reaching because the phantom idea has gained great popularity as an alternative to the cosmological constant. The present model could provide an arena to study the quantum properties of a phantom field, since the model has a finite quantum effective action. Moreover, the advantage of being a gauge-type model opens the possibility of fixing a gauge in which the model is unitary or impose a condition to restrict the physical Hilbert space in such a way that the model defined on this subspace is unitary. On the other hand, as [30] mentions, a choice could be made of having either ghosts or instabilities. There the author explains that in order to save unitarity we are forced to choose instabilities which would imply having a Hamiltonian not bounded from below.

Naturally, a theory of gravitation without matter is incomplete, but it serves as a motivation for future works where the research on these types of models can lead us to more realistic results. A possible solution is to use $\tilde{\delta}$ supergravity models that contain matter fields [32] and could cure the phantom instability.

In section 2, we give the definition of $\tilde{\delta}$ transformation, present the GCTs and its corresponding extensions and define the new gauge transformations and the generalizations of the covariant derivative. Then, in section 3, we show the general form of the invariant action for general $\tilde{\delta}$ theories, present and demonstrate the invariance of $\tilde{\delta}$ gravity action and give the general form of the classical equations of motion for general fields. In section 4, we compute the effective action for a generic $\tilde{\delta}$ model and show that all of them live at one loop. Section 5 is the most important section of this work; here we applied what was seen in the previous sections to the particular case of Einstein–Hilbert theory. We show the classical equations of motion for the two fields and give solutions for the particular cases of Schwarzschild and Freedman–Robertson–Walker (FRW) metrics. We apply the background field method (BFM) and give the relevant quadratic total Lagrangian. We also calculate the divergent part of the effective action at one loop using an algorithm developed in [21]. In section 6, using the gauge fixing of the previous section, we explore the Hamiltonian formalism, redefine the fields, the creation and annihilation operators and see the existence of ghosts. Finally in section 7, we analyze the form of the finite quantum corrections to the effective action and we show the modification of the equations of motion due to the simplest type of corrections [33–36].

In appendix A, we describe the gauge fixing procedure for the modified model of GR and obtain the Faddeev–Popov for this case using the BRST method in detail [37]. In appendix B, we give a review of the background field method following [38]. Finally, in appendix C, we give a brief review of the algorithm developed in [21] for the computation of the divergent part of the effective action at one loop and we indicate the values of the parameters used in our case.

It is important to note that we work with the $\tilde{\delta}$ modification to general relativity, based on the Einstein–Hilbert theory. From now on, we will refer to this model as $\tilde{\delta}$ gravity.

Motivated by simplicity, we will use the cosmological constant $\Lambda = 0$. We will use the Riemann tensor given in [21]:

$$R^{\alpha}{}_{\beta\mu\nu} = \partial_\mu \Gamma_{\nu\beta}{}^{\alpha} - \partial_\nu \Gamma_{\mu\beta}{}^{\alpha} + \Gamma_{\mu\gamma}{}^{\alpha}\Gamma_{\nu\beta}{}^{\gamma} - \Gamma_{\nu\gamma}{}^{\alpha}\Gamma_{\mu\beta}{}^{\gamma} \tag{1}$$

with the Ricci tensor $R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}$, the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$ and

$$\Gamma_{\mu\nu}{}^{\alpha} = \tfrac{1}{2} g^{\alpha\beta}(\partial_\nu g_{\beta\mu} + \partial_\mu g_{\nu\beta} - \partial_\beta g_{\mu\nu}). \tag{2}$$

2. $\tilde{\delta}$ Transformation

In this work, we will study a modification of models that consist in the application of a variation that we will define as $\tilde{\delta}$. This variation will produce new elements that we define as $\tilde{\delta}$ fields. We take throughout our work the convention that a tilde tensor is equal to the $\tilde{\delta}$ transformation of the original tensor associated with it when all its indices are covariant. We raise and lower indices using the metric g.

In this form we will have

$$\tilde{S}_{\mu\nu\alpha\ldots} \equiv \tilde{\delta}(S_{\mu\nu\alpha\ldots}) \tag{3}$$

and, for example

$$\begin{aligned} \tilde{\delta}\,(S^{\mu}{}_{\nu\alpha\ldots}) &= \tilde{\delta}(g^{\mu\rho}S_{\rho\nu\alpha\ldots}) \\ &= \tilde{\delta}(g^{\mu\rho})S_{\rho\nu\alpha\ldots} + g^{\mu\rho}\tilde{\delta}(S_{\rho\nu\alpha\ldots}). \end{aligned} \tag{4}$$

It is known that $\delta(g^{\mu\nu}) = -\delta(g_{\alpha\beta})g^{\mu\alpha}g^{\nu\beta}$, so

$$\tilde{\delta}\,(S^{\mu}{}_{\nu\alpha\ldots}) = -\tilde{g}^{\mu\rho}S_{\rho\nu\alpha\ldots} + \tilde{S}^{\mu}{}_{\nu\alpha\ldots}. \tag{5}$$

2.1. General coordinate transformation

With the previous notation in mind, we can work out the general transformations $\tilde{\delta}$ for any tensor with all its indices covariant (for mixed indices, please see (5)). We begin by considering GCTs or diffeomorphism in its infinitesimal form

$$\begin{aligned} x'^{\mu} &= x^{\mu} - \xi_0^{\mu}(x) \\ \delta x'^{\mu} &= -\,\xi_0^{\mu}(x), \end{aligned} \tag{6}$$

where δ is the GCT. Now, we define

$$\xi_1^{\mu}(x) \equiv \tilde{\delta}\xi_0^{\mu}(x). \tag{7}$$

Moreover, we postulate that $\tilde{\delta}$ commutes with δ. Now we see some examples.

(I) A scalar $\Phi(x)$:

$$\Phi'(x') = \Phi(x)$$
$$\delta\Phi(x) = \xi_0^{\mu}\Phi_{,\mu}; \tag{8}$$

noting that $\tilde{\delta}$ commutes with δ, we can read the transformation rule for $\tilde{\Phi} = \tilde{\delta}\Phi$:

$$\delta\tilde{\Phi}(x) = \xi_1^{\mu}\Phi_{,\mu} + \xi_0^{\mu}\tilde{\Phi}_{,\mu} . \tag{9}$$

(II) A vector $V_{\mu}(x)$:

$$\delta V_{\mu}(x) = \xi_0^{\beta}V_{\mu,\beta} + \xi_{0,\mu}^{\alpha}V_{\alpha}; \tag{10}$$

therefore, using (5), our new transformation will be

$$\delta\tilde{V}_{\mu}(x) = \xi_1^{\beta}V_{\mu,\beta} + \xi_{1,\mu}^{\alpha}V_{\alpha} + \xi_0^{\beta}\tilde{V}_{\mu,\beta} + \xi_{0,\mu}^{\alpha}\tilde{V}_{\alpha}. \tag{11}$$

(III) Rank 2 covariant tensor $M_{\mu\nu}$:

$$\delta M_{\mu\nu}(x) = \xi_0^{\rho}M_{\mu\nu,\rho} + \xi_{0,\nu}^{\beta}M_{\mu\beta} + \xi_{0,\mu}^{\beta}M_{\nu\beta} \tag{12}$$

and for $\tilde{M}_{\mu\nu}$,

$$\delta\tilde{M}_{\mu\nu}(x) = \xi_1^{\rho}M_{\mu\nu,\rho} + \xi_{1,\nu}^{\beta}M_{\mu\beta} + \xi_{1,\mu}^{\beta}M_{\nu\beta} + \xi_0^{\rho}\tilde{M}_{\mu\nu,\rho} + \xi_{0,\nu}^{\beta}\tilde{M}_{\mu\beta} + \xi_{0,\mu}^{\beta}\tilde{M}_{\nu\beta}. \tag{13}$$

We can define the new GCTs so that δ_0 is the transformation in ξ_0 and δ_1 in ξ_1. This new transformation is the basis of this type of model.

2.2. *Symmetry, algebra and gauge*

2.2.1. Gauge transformations. In gravitation, we have a model with two fields. The first is just the usual gravitational field $g_{\mu\nu}(x)$ and a second one $\tilde{g}_{\mu\nu}(x)$ which corresponds to the $\tilde{\delta}$ variation of the first. We will have two gauge transformations associated with the GCT given by (12) and (13):

$$\delta g_{\mu\nu}(x) = \xi_{0\mu;\nu} + \xi_{0\nu;\mu} \tag{14}$$

$$\delta\tilde{g}_{\mu\nu}(x) = \xi_{1\mu;\nu} + \xi_{1\nu;\mu} + \tilde{g}_{\mu\rho}\xi_{0,\nu}^{\rho} + \tilde{g}_{\nu\rho}\xi_{0,\mu}^{\rho} + \tilde{g}_{\mu\nu,\rho}\xi_0^{\rho}, \tag{15}$$

where $\xi_0^{\mu}(x)$ and $\xi_1^{\mu}(x)$ are infinitesimal contravariant vectors of the gauge transformations. Studying the algebra of these transformations, we see

$$[\delta_{\bar{\xi}_0}, \delta_{\xi_0}]g_{\mu\nu}(x) = \zeta_{0\mu;\nu} + \zeta_{0\nu;\mu} = \delta_{\zeta_0}g_{\mu\nu} \tag{16}$$

with

$$\zeta_0^{\lambda} = \bar{\xi}_{0,\rho}^{\lambda}\xi_0^{\rho} - \xi_{0,\rho}^{\lambda}\bar{\xi}_0^{\rho} \tag{17}$$

and

$$[\delta_{\bar{\xi}}, \delta_{\xi}]\tilde{g}_{\mu\nu}(x) = \zeta_{1\mu;\nu} + \zeta_{1\nu;\mu} + \tilde{g}_{\mu\rho}\zeta_{0,\nu}^{\rho} + \tilde{g}_{\nu\rho}\zeta_{0,\mu}^{\rho} + \tilde{g}_{\mu\nu,\rho}\zeta_0^{\rho} = \delta_{\zeta}\tilde{g}_{\mu\nu}(x), \tag{18}$$

where ζ_0 is as before and

$$\zeta_1^{\lambda} = \bar{\xi}_{0,\rho}^{\lambda}\xi_1^{\rho} + \bar{\xi}_{1,\rho}^{\lambda}\xi_0^{\rho} - \xi_{0,\rho}^{\lambda}\bar{\xi}_1^{\rho} - \xi_{1,\rho}^{\lambda}\bar{\xi}_0^{\rho}; \tag{19}$$

it can be seen from the above equations that both transformations form a closed algebra.

2.2.2. Covariant differentiation. As is usual, we define the covariant derivative as

$$\nabla_\nu A_\alpha = D_\nu A_\alpha = A_{\alpha;\nu} = A_{\alpha,\nu} - \Gamma_{\alpha\nu}{}^{\lambda} A_\lambda, \tag{20}$$

where A_α is a covariant vector. Now we generalize the definition of the covariant derivative when it acts on 'tilde' tensors, e.g.

$$\nabla_\nu \tilde{A}_\alpha = \tilde{\delta}(D_\nu A_\alpha) = \tilde{A}_{\alpha,\nu} - \Gamma_{\alpha\nu}{}^{\lambda}\tilde{A}_\lambda - \tilde{\delta}(\Gamma_{\alpha\nu}{}^{\lambda})A_\lambda, \tag{21}$$

where $\tilde{A}_\alpha = \tilde{\delta}A_\alpha$, and we reserve the D notation for the usual covariant derivative and ∇ for the generalized one so that

$$\nabla_\nu \tilde{A}_\alpha = D_\nu \tilde{A}_\alpha - \tilde{\delta}(\Gamma_{\alpha\nu}{}^{\lambda})A_\lambda, \tag{22}$$

where

$$\tilde{\delta}\big(\Gamma_{\alpha\nu}{}^{\lambda}\big) = \tfrac{1}{2}g^{\lambda\rho}(D_\nu \tilde{g}_{\rho\alpha} + D_\alpha \tilde{g}_{\nu\rho} - D_\rho \tilde{g}_{\alpha\nu}); \tag{23}$$

further, the infinitesimal transformation of the modified connection is

$$\delta\big(\tilde{\delta}\Gamma_{\mu\nu}{}^{\varepsilon}\big) = \nabla_\mu\nabla_\nu\xi_1^{\varepsilon} + R^{\varepsilon}{}_{\nu\gamma\mu}\xi_1^{\gamma} + \tilde{\delta}\big(R^{\varepsilon}{}_{\nu\gamma\mu}\big)\xi_0^{\gamma} \tag{24}$$

with

$$\tilde{\delta}(R^{\varepsilon}{}_{\nu\gamma\mu}) = D_\gamma\big[\tilde{\delta}\big(\Gamma_{\mu\nu}{}^{\varepsilon}\big)\big] - D_\mu\big[\tilde{\delta}\big(\Gamma_{\gamma\nu}{}^{\varepsilon}\big)\big]. \tag{25}$$

As $D_\nu A_\alpha$ is a two covariant tensor, $\nabla_\nu\tilde{A}_\alpha$ is a tilde tensor of rank 2 and transforms according to equation (13). This definition of the covariant derivative will be used in section 5. We note that an analogous type of generalization of the covariant derivative was used in [25].

Now that we have established the notation and the definitions, we can start to look for the structure of the modified models. In the following section, we will define the new invariant action and find the classical equations of motion.

3. Modified model

As the GCTs were extended, we can look for an invariant action. We start by considering a model which is based on a given action $S_0[\phi_I]$ where ϕ_I are generic fields, and then we add to it a piece which is equal to a $\tilde{\delta}$ variation with respect to the fields and we let $\tilde{\delta}\phi_J = \tilde{\phi}_J$ so that we have

$$S[\phi,\tilde{\phi}] = S_0[\phi] + \kappa_2\int \mathrm{d}^4x \frac{\delta S_0}{\delta\phi_I(x)}[\phi]\tilde{\phi}_I(x) \tag{26}$$

with κ_2 an arbitrary constant and the index I can represent any kind of indices. For more details of the definition of $\tilde{\delta}$, please see appendix A of [26]. This new defined action shows the standard structure which is used to define any modified element or function for $\tilde{\delta}$ type models, for example, the gauge fixing and Faddeev–Popov. Next, we verify that this form of action is indeed the correct one for $\tilde{\delta}$ gravity and so is invariant to the new GCT.

3.1. The modified model's invariance

In this paper, we will investigate the $\tilde{\delta}$ gravity action, obtained by the procedure sketched above:

$$S[g,\tilde{g}] = \int \mathrm{d}^dx\sqrt{-g}\left(-\frac{1}{2\kappa}R + \mathcal{L}_M\right) + \kappa_2\int\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R - \kappa T^{\mu\nu}\right)\sqrt{-g}\tilde{g}_{\mu\nu}\,\mathrm{d}^dx, \tag{27}$$

where $\mathcal{L}_M$ is some matter Lagrangian and

$$T^{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_M)}{\delta g_{\mu\nu}} = -2\frac{\delta\mathcal{L}_M}{\delta g_{\mu\nu}} - g^{\mu\nu}\mathcal{L}_M \tag{28}$$

is the energy–momentum tensor. Now we must verify that (27) is invariant under the following transformations:

$$\delta g_{\mu\nu}(x) = g_{\mu\rho}\xi^{\rho}_{0,\nu} + g_{\nu\rho}\xi^{\rho}_{0,\mu} + g_{\mu\nu,\rho}\xi^{\rho}_0 = \xi_{0\mu;\nu} + \xi_{0\nu;\mu}$$
$$\delta\tilde{g}_{\mu\nu}(x) = \xi_{1\mu;\nu} + \xi_{1\nu;\mu} + \tilde{g}_{\mu\rho}\xi^{\rho}_{0,\nu} + \tilde{g}_{\nu\rho}\xi^{\rho}_{0,\mu} + \tilde{g}_{\mu\nu,\rho}\xi^{\rho}_0.$$

We can see that (27) is obviously invariant under transformations generated by ξ^{ρ}_0, since these are GCTs, and we declared $\tilde{g}_{\mu\nu}$ to be a rank 2 covariant tensor. Under transformations generated by $\xi^{\rho}_1(\delta_1)$, $g_{\mu\nu}$ does not change, so we have

$$\begin{aligned}\delta_1 S(g,\tilde{g}) &= \kappa_2\int\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R - \kappa T^{\mu\nu}\right)\sqrt{-g}(\delta_1\tilde{g}_{\mu\nu})\,\mathrm{d}^d x\\ &= \kappa_2\int\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R - \kappa T^{\mu\nu}\right)\sqrt{-g}(\xi_{1\mu;\nu} + \xi_{1\nu;\mu})\,\mathrm{d}^d x\\ &= -2\kappa_2\int\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R - \kappa T^{\mu\nu}\right)_{;\nu}\sqrt{-g}\xi_{1\mu}\,\mathrm{d}^d x = 0.\end{aligned} \tag{29}$$

We note that we must impose conservation of the energy–momentum tensor $T^{\mu\nu}{}_{;\nu} = 0$ so that (29) is fulfilled.

3.2. Classical equation

Now that we know that our action is invariant we can start to study the model. To begin with, we will see how are the classical equations of motion. When varied (26) with respect to $\tilde{\phi}_I$, we obtain the classical equation of ϕ_I:

$$\frac{\delta S_0}{\delta\phi_I(x)}[\phi] = 0, \tag{30}$$

and when varied with respect to ϕ_I, we obtain the $\tilde{\phi}_I$'s equation:

$$\frac{\delta S_0}{\delta\phi_I(y)}[\phi] + \kappa_2\int \mathrm{d}^4x\frac{\delta^2 S_0}{\delta\phi_I(y)\delta\phi_J(x)}[\phi]\tilde{\phi}_J(x) = 0. \tag{31}$$

Simplifying this equation using (30), we obtain

$$\int \mathrm{d}^4x\frac{\delta^2 S_0}{\delta\phi_I(y)\delta\phi_J(x)}[\phi]\tilde{\phi}_J(x) = 0, \tag{32}$$

where we note that $\frac{\delta^2 S_0}{\delta\phi_J\delta\phi_I}[\phi]$ is a differential operator acting on $\tilde{\phi}_J$. $\tilde{\phi}_J$ belongs to the kernel of this differential operator. It turns out that the kernel is not zero, a fact that can be clearly seen in this paper, for the case of gravitation, in equation (44) and below.

Having studied the classical model, we can begin to look for its quantum aspects. In the following section, we will compute the quantum corrections using a path integral approach.

4. Quantum modified model

In this section, we derive the exact effective action for a generic $\tilde{\delta}$ model and apply the result to the Einstein–Hilbert action in section 5. We saw that the classical action for a $\tilde{\delta}$ model is (26). This in turn implies that we now have two fields to be integrated in the generating functional of Green's functions:

$$Z(j,\tilde{j}) = \mathrm{e}^{\mathrm{i}W(j,\tilde{j})} = \int \mathcal{D}\phi\mathcal{D}\tilde{\phi}\, \mathrm{e}^{\mathrm{i}(S_0+\int \mathrm{d}^N x \frac{\delta S_0}{\delta \phi_I}\tilde{\phi}_I + \int \mathrm{d}^N x (j_I(x)\phi_I(x)+\tilde{j}_I(x)\tilde{\phi}_I(x)))}. \tag{33}$$

We can readily appreciate that because of the linearity of the exponent on $\tilde{\phi}_J$ what we have is the integral representation of a Dirac's delta function so that our modified model, once integrated, over $\tilde{\phi}_J$ gives a model with a constraint making the original model live on-shell:

$$Z(j,\tilde{j}) = \int \mathcal{D}\phi\, \mathrm{e}^{\mathrm{i}\left(S_0+\int \mathrm{d}^N x j_I(x)\phi_I(x)\right)} \delta\left(\frac{\delta S_0}{\delta \phi_I(x)} + \tilde{j}_I(x)\right). \tag{34}$$

A first glance at equation (34) could lead us to believe that this model is purely classical. But we can see by doing a short and simple analysis that this is not so. For this, we follow [39]. See also [40].

Let φ_I solve the classical equation of motion

$$\left.\frac{\delta S_0}{\delta \phi_I(x)}\right|_{\varphi_I} + \tilde{j}_I(x) = 0. \tag{35}$$

We have

$$\delta\left(\frac{\delta S_0}{\delta \phi_I(x)} + \tilde{j}_I(x)\right) = \overset{-1}{\det}\left(\left.\frac{\delta^2 S_0}{\delta\phi_I(x)\delta\phi_J(y)}\right|_{\varphi_I}\right)\delta(\phi_I - \varphi_I). \tag{36}$$

Therefore,

$$\begin{aligned} Z(j,\tilde{j}) &= \int \mathcal{D}\phi\, \mathrm{e}^{\mathrm{i}(S_0+\int \mathrm{d}^N x j_I(x)\phi_I(x))} \delta\left(\frac{\delta S_0}{\delta \phi_I(x)} + \tilde{j}_I(x)\right) \\ &= \mathrm{e}^{\mathrm{i}(S_0(\varphi)+\int \mathrm{d}^N x j_I(x)\varphi_I(x))}\, \overset{-1}{\det}\left(\left.\frac{\delta^2 S_0}{\delta\phi_I(x)\delta\phi_J(y)}\right|_{\varphi_I}\right). \end{aligned} \tag{37}$$

Note that φ is a functional of $\tilde{j}$. The generating functional of the connected Green function is

$$W(j,\tilde{j}) = S_0(\varphi) + \int \mathrm{d}^N x j_I(x)\varphi_I(x) + \mathrm{i}\mathrm{Tr}\left(\log\left(\left.\frac{\delta^2 S_0}{\delta\phi_I(x)\delta\phi_J(y)}\right|_{\varphi_I}\right)\right). \tag{38}$$

Define

$$\begin{aligned} \Phi_I(x) &= \frac{\delta W}{\delta j_I(x)} \\ &= \varphi_I(x) \\ \tilde{\Phi}_I(x) &= \frac{\delta W}{\delta \tilde{j}_I(x)}. \end{aligned}$$

The effective action is defined by

$$\Gamma(\Phi,\tilde{\Phi}) = W(j,\tilde{j}) - \int \mathrm{d}^N x\{j_I(x)\Phi_I(x) + \tilde{j}_I(x)\tilde{\Phi}_I(x)\}.$$

We get, using equations (35) and (38),

$$\Gamma(\Phi,\tilde{\Phi}) = S_0(\Phi) + \int \mathrm{d}^N x \frac{\delta S_0}{\delta \Phi_I(x)} \tilde{\Phi}_I(x) + i\mathrm{Tr}\left(\log\left(\frac{\delta^2 S_0}{\delta \Phi_I(x)\delta \Phi_J(y)}\right)\right). \tag{39}$$

This is the exact effective action for $\tilde{\delta}$ theories. In this demonstration, we have assumed that all the relevant steps for fixing the gauge have been made in (33), so S_0 includes the gauge fixing and Faddeev–Popov Lagrangian, in the manner that the new gauge fixing and Faddeev–Popov will be the original ones plus their $\tilde{\delta}$ variation. For more details on this and in the case of gravitation, see equations (A.6) and (A.16).

Comparing equation (16.42) of [39] with equation (39), we see that the one-loop contribution to the effective action of δ theories is exact and the $\tilde{\delta}$-modified model lives only to one loop because higher corrections simply do not exist. Finally, it is twice the one-loop contribution of the original theory from which the $\tilde{\delta}$ model was derived. This results from having doubled the number of degrees of freedom. We also see that this term does not depend on the $\tilde{\phi}_I$ fields.

These conclusions can also be achieved through a topological analysis with Feynman diagrams, but due to its length and complexity we have chosen to leave it out. See [25] for the particular case of non-Abelian $\tilde{\delta}$ Yang–Mills.

We see from equation (39) that the equations of motion for the original field $\Phi_I(x)$ do not receive quantum corrections:

$$\begin{aligned} &\frac{\delta}{\delta \tilde{\Phi}_I(z)} \Gamma(\Phi,\tilde{\Phi}) = 0 \\ &\frac{\delta S_0}{\delta \Phi_I(z)} = 0. \end{aligned} \tag{40}$$

On the other side, when varying with respect to ϕ_I, one obtains that the equations of motion for the new field $\tilde{\phi}_I$ do receive quantum corrections:

$$\begin{aligned} &\frac{\delta}{\delta \Phi_I(x)} \Gamma(\Phi,\tilde{\Phi}) = 0 \\ &\int \mathrm{d}^N x \frac{\delta^2 S_0}{\delta \Phi_I(z)\delta \Phi_J(x)} \tilde{\Phi}_J(x) + \mathrm{i}\frac{\delta}{\delta \Phi_I(z)} \mathrm{Tr}\left(\log\left(\frac{\delta^2 S_0}{\delta \Phi_I(x)\delta \Phi_J(y)}\right)\right) = 0. \end{aligned} \tag{41}$$

In conclusion, the quantum corrections behave as a source that only affects the equations of the new field remaining the ones of the original field unchanged. This is clearly seen when we compare (40) and (41) with (30) and (32).

In general, $\mathrm{Tr}\big(\log\big(\frac{\delta^2 S_0}{\delta\Phi_I(x)\delta\Phi_J(y)}\big)\big)$ could be divergent and need to be renormalized (see [25]). From equation (39), we see that the $\tilde{\delta}$ model will be renormalizable if the original theory is renormalizable. But, due to equation (40), originally non-renormalizable theories could be finite or renormalizable in its $\tilde{\delta}$ version. This term can be calculated in many ways, for example, by the Zeta function regularization (see, for instance, [41]), perturbation theory (Feynman diagrams), etc. For gravitation, the calculation of this term is quite difficult in any of the above methods so we will use an alternative method developed in [21].

In this work, the $\tilde{\delta}$ gravity model contains two dynamical fields, $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$, which are important to describe the gravitational field in this approach. Please see [26]. So, we must consider the effective action of the model for the two fields. We saw that $g_{\mu\nu}$ satisfies the classical equations always. This is the meaning of equation (34). But, the equation of motion for $\tilde{g}_{\mu\nu}$ do receive quantum corrections. Moreover, one-particle irreducible (1PI) graphs containing $g_{\mu\nu}$ external legs are nontrivial and subjected to quantum effects.

In the following section, we will study $\tilde{\delta}$ gravity. We will see that the divergent part of the quantum corrections to the effective action give a null contribution to the equations of motion for pure gravity and without a cosmological constant, which means that under these conditions we have a finite model of gravity.

5. $\tilde{\delta}$ Gravity

Until now, we have studied $\tilde{\delta}$ models in general. We found the invariant action given by (26), with the classical equations of motion (30) and (32). Then, we demonstrated that $\tilde{\delta}$ models live only to one loop and the effective action is given by (39). In this section, we apply these results to gravity. In the first part, we will present the classical equations of motion for both fields and show the solutions in two cases. Then, we will apply the background field method (BFM) to obtain the quadratic Lagrangians and finally we calculate the divergent part of the effective action for $\tilde{\delta}$ gravity.

5.1. Classical equations of motion and solutions

Now we are ready to study the modifications to gravity. In this case, we have that $\phi_I \to g_{\mu\nu}$ and $\tilde{\phi}_I \to \tilde{g}_{\mu\nu}$. So, using (26), we obtain

$$\begin{aligned} L_0[g_{\mu\nu}] &= \sqrt{-g}\left(-\frac{1}{2\kappa}R + \mathcal{L}_M\right) \\ L[g_{\mu\nu}] &= \sqrt{-g}\left[-\frac{1}{2\kappa}R + \mathcal{L}_M + \kappa_2'\left(G^{\mu\nu} - \kappa T^{\mu\nu}\right)\tilde{g}_{\mu\nu}\right] \end{aligned} \tag{42}$$

with $\kappa = \frac{8\pi G}{c^4}$, $\kappa_2' = \frac{\kappa_2}{2\kappa}$, $\mathcal{L}_M$ some matter Lagrangian and

$$\begin{aligned} T^{\mu\nu} &= -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_M\right)}{\delta g_{\mu\nu}} \\ &= -2\frac{\delta\mathcal{L}_M}{\delta g_{\mu\nu}} - g^{\mu\nu}\mathcal{L}_M \end{aligned} \tag{43}$$

is the energy momentum tensor. Recall from (29) that we need $T^{\mu\nu}$ to be conserved. If we variate this action, we obtain the equations of motion:

$$\begin{aligned} &G^{\mu\nu} = \kappa T^{\mu\nu} \\ &\frac{1}{2}\left(R^{\mu\nu}\tilde{g}^{\sigma}_{\sigma} - R\tilde{g}^{\mu\nu}\right) + F^{(\mu\nu)(\alpha\beta)\rho\lambda}D_\rho D_\lambda \tilde{g}_{\alpha\beta} = \kappa\frac{\delta T_{\alpha\beta}}{\delta g_{\mu\nu}}\tilde{g}^{\alpha\beta} \end{aligned} \tag{44}$$

with

$$\begin{aligned} F^{(\mu\nu)(\alpha\beta)\rho\lambda} &= P^{((\rho\mu)(\alpha\beta))}g^{\nu\lambda} + P^{((\rho\nu)(\alpha\beta))}g^{\mu\lambda} - P^{((\mu\nu)(\alpha\beta))}g^{\rho\lambda} - P^{((\rho\lambda)(\alpha\beta))}g^{\mu\nu} \\ P^{((\alpha\beta)(\mu\nu))} &= \frac{1}{4}\left(g^{\alpha\mu}g^{\beta\nu} + g^{\alpha\nu}g^{\beta\mu} - g^{\alpha\beta}g^{\mu\nu}\right), \end{aligned} \tag{45}$$

where $(\mu\nu)$ shows that the μ and ν are in a totally symmetric combination. An important thing to note is that both equations are of second order in derivatives which is needed to preserve causality. In this paper, we will work in the vacuum, this is $\mathcal{L}_M = 0$, so that (44) simplifies to

$$\begin{aligned} &R^{\mu\nu} = 0 \\ &F^{(\mu\nu)(\alpha\beta)\rho\lambda}D_\rho D_\lambda \tilde{g}_{\alpha\beta} = 0. \end{aligned} \tag{46}$$

Some particular solutions to equations (44) and (46) are as follows.

For the vacuum, we have for example the case of Schwarzschild

$$g_{\mu\nu} = \begin{pmatrix} -\left(1-\frac{\alpha}{r}\right) & 0 & 0 & 0 \\ 0 & \frac{1}{1-\frac{\alpha}{r}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin(\theta) \end{pmatrix} \tag{47}$$

which has a solution for $\tilde{g}_{\alpha\beta}$ of the form

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} -\left(1-\frac{2\alpha+\beta}{r}\right) & 0 & 0 & 0 \\ 0 & \frac{1+\frac{\beta}{r}}{\left(1-\frac{\alpha}{r}\right)^2} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin(\theta) \end{pmatrix}, \tag{48}$$

where it has been imposed that $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ approach Minkowski space when $r \to \infty$, and α and β are determined by boundary conditions.

Another interesting case is for the case of Friedman–Robertson–Walker under a density $\rho(t)$ with an equation of state $p(t) = \omega\rho(t)$:

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & R(t) & 0 & 0 \\ 0 & 0 & R(t)r^2 & 0 \\ 0 & 0 & 0 & R(t)r^2\sin(\theta) \end{pmatrix}, \tag{49}$$

where $R(t) = R_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+\omega)}}$. The solution for $\tilde{g}_{\alpha\beta}$ is

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} -\tilde{A}(r) & 0 & 0 & 0 \\ 0 & \tilde{B}(r) & 0 & 0 \\ 0 & 0 & \tilde{B}(r)r^2 & 0 \\ 0 & 0 & 0 & \tilde{B}(r)r^2\sin(\theta) \end{pmatrix} \tag{50}$$

with $\tilde{A}(r) = 3\omega l_2\left(\frac{t}{t_0}\right)^{\frac{\omega-1}{\omega+1}}$, $\tilde{B}(r) = R_0^2 l_2\left(\frac{t}{t_0}\right)^{\frac{3\omega-1}{3(1+\omega)}}$ and l_2 a free parameter. This case was analyzed in [26], where accelerated expansion of the Universe was obtained without a cosmological constant.

5.2. BFM and quadratic Lagrangians

We proceed to calculate the quadratic Lagrangians for $\tilde{\delta}$ gravity and Faddeev–Popov. These expressions are needed to obtain the one-loop corrections of the model. For this, we use the background field method (see appendix B), with $\mathcal{L}_M = 0$. That is $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$ and $\tilde{g}_{\mu\nu} \to \tilde{g}_{\mu\nu} + \tilde{h}_{\mu\nu}$. So, (42) reduces to

$$\begin{aligned} L_0[g_{\mu\nu} + h_{\mu\nu}] &= \frac{\sqrt{-g}}{2\kappa}\left(-\bar{R} - \frac{1}{2}C^2\right) \\ L[g_{\mu\nu} + h_{\mu\nu}] &= \frac{\sqrt{-g}}{2\kappa}\left(-\bar{R} - C^\mu H_\mu + \kappa_2 \bar{G}^{\mu\nu}\tilde{g}_{\mu\nu}\right) \end{aligned} \tag{51}$$

with $\bar{R} = R[g+h]$ and $\bar{G}^{\mu\nu} = G^{\mu\nu}[g+h]$. We have included the original gauge fixing $C_\mu = h^\nu_{\mu;\nu} - \frac{1}{2}h^\nu_{\nu;\mu}$ and the new part $H_\mu = \frac{1}{2}\left(1 + \frac{\kappa_2}{2}\tilde{g}^\alpha_\alpha\right)C_\mu + \kappa_2\left(\tilde{C}_\mu - \frac{1}{2}\tilde{g}_{\mu\rho}C^\rho\right)$ (see appendix A). When we calculate the quadratic part in the quantum gravitational fields, $h_{\mu\nu}$ and $\tilde{h}_{\mu\nu}$, we obtain

$$L_{\text{quad}} = \frac{1}{2}\sqrt{-g}\vec{h}^T_{(\alpha\beta)}P^{((\alpha\beta)(\mu\nu))}\left(\left[K^{(\gamma\varepsilon)}_{(\mu\nu)}\right]^{(\lambda\eta)}\nabla_\lambda\nabla_\eta + \left[W^{(\gamma\varepsilon)}_{(\mu\nu)}\right]\right)\vec{h}_{(\gamma\varepsilon)} \tag{52}$$

and

$$\vec{h}_{(\alpha\beta)} = \begin{pmatrix} h_{\alpha\beta} \\ \tilde{h}_{\alpha\beta} \end{pmatrix} \tag{53}$$

$$\begin{aligned}\left[K^{(\gamma\varepsilon)}_{(\mu\nu)}\right]^{(\lambda\eta)} &= \frac{1}{2\kappa} g^{\lambda\eta} \begin{pmatrix} \left(1+\frac{\kappa_2}{2}\tilde{g}^{\sigma}_{\sigma}\right)\delta^{\gamma\varepsilon}_{\mu\nu} + \kappa_2 P^{-1}_{((\mu\nu)(\sigma\rho))}\tilde{\delta}(P^{((\sigma\rho)(\gamma\varepsilon))}) & \kappa_2\delta^{\gamma\varepsilon}_{\mu\nu} \\ \kappa_2\delta^{\gamma\varepsilon}_{\mu\nu} & 0 \end{pmatrix} \\ &\quad - \frac{\kappa_2}{2\kappa}\tilde{g}^{\lambda\eta}\delta^{\gamma\varepsilon}_{\mu\nu}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\end{aligned} \tag{54}$$

$$\begin{aligned}&\left[W^{(\gamma\varepsilon)}_{(\mu\nu)}\right] \\ &= \frac{1}{\kappa}\begin{pmatrix} \left(1+\frac{\kappa_2}{2}\tilde{g}^{\sigma}_{\sigma}\right)X^{(\gamma\varepsilon)}_{(\mu\nu)} + \kappa_2\tilde{\delta}(X^{(\gamma\varepsilon)}_{(\mu\nu)}) + \kappa_2 P^{-1}_{((\mu\nu)(\sigma\rho))}\tilde{\delta}(P^{((\sigma\rho)(\alpha\beta))})X^{(\gamma\varepsilon)}_{(\alpha\beta)} & \kappa_2 X^{(\gamma\varepsilon)}_{(\mu\nu)} \\ \kappa_2 X^{(\gamma\varepsilon)}_{(\mu\nu)} & 0 \end{pmatrix},\end{aligned} \tag{55}$$

where

$$\begin{aligned}X^{(\gamma\varepsilon)}_{(\mu\nu)} = \tfrac{1}{2}\big(&R^{\gamma\varepsilon}_{\mu\nu} + R^{\varepsilon\gamma}_{\mu\nu} + \tfrac{1}{2}\big(\delta^{\gamma}_{\mu}R^{\varepsilon}_{\nu} + \delta^{\varepsilon}_{\mu}R^{\gamma}_{\nu} + \delta^{\gamma}_{\nu}R^{\varepsilon}_{\mu} + \delta^{\varepsilon}_{\nu}R^{\gamma}_{\mu}\big) \\ &- \delta^{\gamma\varepsilon}R_{\mu\nu} - \delta_{\mu\nu}R^{\gamma\varepsilon} - \tfrac{1}{2}R\big(\delta^{\gamma}_{\mu}\delta^{\varepsilon}_{\nu} + \delta^{\varepsilon}_{\mu}\delta^{\gamma}_{\nu} - \delta_{\mu\nu}\delta^{\gamma\varepsilon}\big)\big),\end{aligned} \tag{56}$$

where $P^{((\alpha\beta)(\mu\nu))}$ is defined in (45) and $\delta^{\gamma\varepsilon}_{\mu\nu}$ is the symmetrized Kronecker delta. Moreover, the covariant derivative works on the $\vec{h}_{(\gamma\varepsilon)}$ vector like

$$\nabla_{\lambda}\vec{h}_{(\gamma\varepsilon)} = \partial_{\lambda}\vec{h}_{(\gamma\varepsilon)} - \left[\Gamma_{\lambda\gamma}{}^{\beta}\right]\vec{h}_{(\beta\varepsilon)} - \left[\Gamma_{\lambda\varepsilon}{}^{\beta}\right]\vec{h}_{(\gamma\beta)} \tag{57}$$

with

$$\left[\Gamma_{\lambda\gamma}{}^{\beta}\right] = \begin{pmatrix} \Gamma_{\lambda\gamma}{}^{\beta} & 0 \\ \tilde{\delta}\left(\Gamma_{\lambda\gamma}{}^{\beta}\right) & \Gamma_{\lambda\gamma}{}^{\beta} \end{pmatrix}. \tag{58}$$

And using the BRST method, we obtain the Faddeev–Popov:

$$L_{FP} = \vec{\bar{c}}^{T}_{\mu}\sqrt{-g}\left(\left[K^{\mu\lambda}_{FP}\right]^{(\rho\nu)}\nabla_{\rho}\nabla_{\nu} + \left[W^{\mu\lambda}_{FP}\right]\right)\vec{c}_{\lambda} \tag{59}$$

where

$$\vec{c}_{\lambda} = \begin{pmatrix} c_{\lambda} \\ \tilde{c}_{\lambda} \end{pmatrix} \tag{60}$$

$$\left[K^{\mu\lambda}_{FP}\right]^{(\rho\nu)} = \mathrm{i}g^{\nu\rho}\begin{pmatrix} \frac{1}{2}\left(1+\frac{\kappa_2}{2}\tilde{g}^{\sigma}_{\sigma}\right)g^{\mu\lambda} - \frac{\kappa_2}{2}\tilde{g}^{\mu\lambda} & \kappa_2 g^{\mu\lambda} \\ g^{\mu\lambda} & 0 \end{pmatrix} - \mathrm{i}\kappa_2\tilde{g}^{\nu\rho}g^{\mu\lambda}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{61}$$

$$\left[W^{\mu\lambda}_{FP}\right] = \mathrm{i}\begin{pmatrix} \frac{1}{2}\left(1+\frac{\kappa_2}{2}\tilde{g}^{\sigma}_{\sigma}\right)R^{\mu\lambda} - \kappa_2\tilde{g}_{\alpha\beta}R^{\mu\alpha\lambda\beta} - \frac{\kappa_2}{2}\tilde{g}^{\mu\alpha}R^{\lambda}_{\alpha} - g^{\alpha\beta}g^{\mu\gamma}\tilde{\delta}\left(R^{\lambda}{}_{\alpha\beta\gamma}\right) & \kappa_2 R^{\mu\lambda} \\ R^{\mu\lambda} & 0 \end{pmatrix} \tag{62}$$

with

$$\nabla_{\lambda}\vec{c}_{\mu} = \partial_{\lambda}\vec{c}_{\mu} - \left[\Gamma_{\lambda\mu}{}^{\beta}\right]\vec{c}_{\beta}. \tag{63}$$

The details of the Faddeev–Popov Lagrangian is presented in appendix A.

5.3. Divergent part of the effective action

In section 4, we demonstrated that the quantum corrections to the effective action do not depend on the tilde fields, in this case $\tilde{g}_{\mu\nu}$. On the other side, the renormalization theory tells us that its divergent corrections can only be local terms. So, by power counting and invariance of the background field effective action under GCTs, we know that the divergent part to L loops is [3, 24]

$$\Delta S^L_{\text{div}} \propto \int \mathrm{d}^4x\sqrt{-g}R^{L+1}, \tag{64}$$

where R^{L+1} is any scalar contraction of $(L+1)$ Riemann's tensors. As our model lives only to one loop,

$$L^{\text{div}}_Q = \sqrt{-g}(a_1R^2 + a_2R_{\alpha\beta}R^{\alpha\beta}). \tag{65}$$

We do not use $R_{\alpha\beta\gamma\lambda}R^{\alpha\beta\gamma\lambda}$ because we have the topological identity in four dimensions:

$$\sqrt{-g}(R_{\alpha\beta\gamma\lambda}R^{\alpha\beta\gamma\lambda} - 4R_{\alpha\beta}R^{\alpha\beta} + R) = \text{total derivative}. \tag{66}$$

To calculate the divergent part of the effective action in our model (i.e. a_1 and a_2 in (65)), we made a FORM program [42] to implement the algorithm developed in [21], obtaining in our case (see appendix C)

$$\begin{aligned} L^{\text{div}}_{Q,grav} &= \sqrt{-g}\frac{\hbar c}{\varepsilon}\left(\frac{7}{12}R^2 + \frac{7}{6}R_{\alpha\beta}R^{\alpha\beta}\right) \\ L^{\text{div}}_{Q,ghost} &= -2\times\sqrt{-g}\frac{\hbar c}{\varepsilon}\left(\frac{17}{60}R^2 + \frac{7}{30}R_{\alpha\beta}R^{\alpha\beta}\right) \\ L^{\text{div}}_{Q} &= \sqrt{-g}\frac{\hbar c}{\varepsilon}\left(\frac{1}{60}R^2 + \frac{7}{10}R_{\alpha\beta}R^{\alpha\beta}\right) \end{aligned} \tag{67}$$

with $\varepsilon = 8\pi^2(N-4)$. When we compare it with the usual result in gravitation [3, 21] we can see that we obtain twice the divergent term of general relativity. Divergences also double in Yang–Mills [25].

Moreover, since Einstein's equations of motion are exactly valid at the quantum level,

$$\left(\frac{\delta\Gamma(g,\tilde{g})}{\delta\tilde{g}_{\mu\nu}}\right) = R^{\mu\nu} = 0, \tag{68}$$

where $\Gamma(g,\tilde{g})$ is the effective action in the BFM. It follows that the contribution of (67) to the equation of motion vanishes:

$$\hbar c\left[\frac{\sqrt{-g}}{\varepsilon}\left(\frac{1}{2}g^{\mu\nu}\left(\frac{1}{60}R^2 + \frac{7}{10}R_{\alpha\beta}R^{\alpha\beta}\right)\right.\right. \\ \left.\left. + \frac{1}{30}R\frac{\delta R}{\delta g_{\mu\nu}} + \frac{7}{10}R_{\alpha\beta}\frac{\delta R^{\alpha\beta}}{\delta g_{\mu\nu}} + \frac{7}{10}R^{\alpha\beta}\frac{\delta R_{\alpha\beta}}{\delta g_{\mu\nu}}\right)\right]_{R_{\alpha\beta}=0} = 0. \tag{69}$$

Therefore, $\tilde{\delta}$ gravity is a finite model of gravitation if we do not have matter and a cosmological constant. The finiteness of our model implies that the Newton constant does not run at all, nor with time or energy scale which would be supported by the very stringent experimental bounds set on its change [43, 44]. We must note that this model is finite only in four dimensions because we need (66). Moreover, in more dimensions, there could appear more terms in (65) that contain $R^{\mu_1\mu_2\cdots\mu_N}$ with N the dimension of space that gives a nonzero contribution to the equations of motion.

In spite of these apparent successes, there seems to be a problem with this model and this is the possible existence of ghosts. This issue will be dealt with in the following section.

6. Ghosts

In this section, we discuss the possibility that our model has ghosts and the lost of unitarity due to them. In order to proceed with this endeavor, we first write the quadratic Lagrangian (52) for a noninteracting model (that is, with the backgrounds both equal to the Minkowsky metric tensor) and calculate from it the canonical conjugated momenta to the quantum fields. It is important to note that for the Lagrangian (52) a gauge has been chosen. Then, it is possible to show that under these conditions and in this gauge, the quantum fields obey the wave equation and an expansion in plane waves is possible where the Fourier coefficients are promoted to creation and annihilation operators much in the same way as can be done for the electromagnetic potential. We use the canonical commutation relations for fields and momenta to work out the corresponding canonical commutation relations for the creation and annihilation operators. We also show first the Hamiltonian in terms of fields and momenta and then in terms of annihilation and creation operators.

To study the existence of ghosts in the model we will study small perturbations to flat space. This is done by taking expression (52) and putting the backgrounds equal to the Minkowski metric $g_{\mu\nu} = \eta_{\mu\nu}$ and $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$, thus obtaining

$$S[h, \tilde{h}] = -\frac{1}{2\kappa}\int \mathrm{d}^4x P^{((\alpha\beta)(\mu\nu))}\left(\frac{(1-\kappa_2)}{2}\partial_\rho h_{\alpha\beta}\partial^\rho h_{\mu\nu} + \kappa_2\partial_\rho\tilde{h}_{\alpha\beta}\partial^\rho h_{\mu\nu}\right) \tag{70}$$

where now

$$P^{((\alpha\beta)(\mu\nu))} = \tfrac{1}{4}(\eta^{\alpha\mu}\eta^{\beta\nu} + \eta^{\alpha\nu}\eta^{\beta\mu} - \eta^{\alpha\beta}\eta^{\mu\nu}) \tag{71}$$

and the equations of motion for the fields are

$$\begin{aligned}\partial^2 h_{\mu\nu} &= 0\\ \partial^2\tilde{h}_{\mu\nu} &= 0\end{aligned} \tag{72}$$

with $\partial^2 = \eta^{\rho\lambda}\partial_\rho\partial_\lambda$. This corresponds to the wave equation with energy $E_{\mathbf{p}} = |\mathbf{p}|$. Here, we note that in order to obtain these equations, we have made use of a particular gauge fixing term (A.10) in the Lagrangian (52).

It is well known that for a diffeomorphism invariant Lagrangian, the canonical Hamiltonian is zero. This is so in delta gravity as well as in general relativity: the total Hamiltonian is a linear combination of the first class constraints (see [4]). After gauge fixing, the Hamiltonian is

$$\begin{aligned}H = \int \mathrm{d}^3x &\left(\frac{2\kappa}{\kappa_2}P^{-1}_{((\alpha\beta)(\mu\nu))}\left(\tilde{\Pi}^{\alpha\beta}\Pi^{\mu\nu} - \frac{(1-\kappa_2)}{2\kappa_2}\tilde{\Pi}^{\alpha\beta}\tilde{\Pi}^{\mu\nu}\right)\right.\\ &\left.+\frac{\kappa_2}{2\kappa}P^{((\alpha\beta)(\mu\nu))}\left(\partial_i\tilde{h}_{\alpha\beta}\partial_i h_{\mu\nu} + \frac{(1-\kappa_2)}{2\kappa_2}\partial_i h_{\alpha\beta}\partial_i h_{\mu\nu}\right)\right)\end{aligned} \tag{73}$$

with

$$P^{-1}_{((\alpha\beta)(\mu\nu))} = \eta_{\alpha\mu}\eta_{\beta\nu} + \eta_{\alpha\nu}\eta_{\beta\mu} - \eta_{\alpha\beta}\eta_{\mu\nu} = 4P_{((\alpha\beta)(\mu\nu))} \tag{74}$$

and where the conjugate momenta are

$$\begin{aligned}\Pi^{\mu\nu} &= \frac{\delta\mathcal{L}}{\delta\dot{h}_{\mu\nu}}\\ &= \frac{1}{2\kappa}P^{((\alpha\beta)(\mu\nu))}((1-\kappa_2)\dot{h}_{\alpha\beta} + \kappa_2\dot{\tilde{h}}_{\alpha\beta})\end{aligned} \tag{75}$$

$$\begin{aligned}\tilde{\Pi}^{\mu\nu} &= \frac{\delta\mathcal{L}}{\delta\dot{\tilde{h}}_{\mu\nu}}\\ &= \frac{\kappa_2}{2\kappa}P^{((\alpha\beta)(\mu\nu))}\dot{h}_{\alpha\beta}.\end{aligned} \tag{76}$$

We can write our fields h and $\tilde{h}$ in the following way:

$$h_{\mu\nu}(\mathbf{x},t) = \int \frac{\mathrm{d}^3p}{\sqrt{(2\pi)^3 2E_{\mathbf{p}}}}\left[\chi^{(AB)}_{(\mu\nu)}(\mathbf{p})a_{(AB)}(\mathbf{p})\,\mathrm{e}^{\mathrm{i}p\cdot x} + \chi^{(AB)}_{(\mu\nu)}(\mathbf{p})a^{+}_{(AB)}(\mathbf{p})\,\mathrm{e}^{-\mathrm{i}p\cdot x}\right]\Big|_{p_0=E_{\mathbf{p}}}$$

$$\tilde{h}_{\mu\nu}(\mathbf{x},t) = \int \frac{\mathrm{d}^3p}{\sqrt{(2\pi)^3 2E_{\mathbf{p}}}}\left[\chi^{(AB)}_{(\mu\nu)}(\mathbf{p})\tilde{a}_{(AB)}(\mathbf{p})\,\mathrm{e}^{\mathrm{i}p\cdot x} + \chi^{(AB)}_{(\mu\nu)}(\mathbf{p})\tilde{a}^{+}_{(AB)}(\mathbf{p})\,\mathrm{e}^{-\mathrm{i}p\cdot x}\right]\Big|_{p_0=E_{\mathbf{p}}}, \tag{77}$$

where $\chi^{(AB)}_{(\mu\nu)}(\mathbf{p})$ is a polarization tensor and $a_{(AB)}(\mathbf{p})$ and $\tilde{a}_{(AB)}(\mathbf{p})$ are promoted to annihilation operators when we quantize it. $a^{+}_{(AB)}(\mathbf{p})$ and $\tilde{a}^{+}_{(AB)}(\mathbf{p})$ correspond to the creation operators. A and B are indices of polarization that work like Lorentz indices, that is, they go from 0 to 3 and are moved up and down with η^{AB}. As these indices are presented symmetrically we will have ten polarization tensors, enough to make a complete basis. For the quantization of the model, we must impose the canonical commutation relations; the only nonvanishing commutators are

$$[h_{\mu\nu}(t,\mathbf{x}), \Pi^{\alpha\beta}(t,\mathbf{y})] = [\tilde{h}_{\mu\nu}(t,\mathbf{x}), \tilde{\Pi}^{\alpha\beta}(t,\mathbf{y})] = \mathrm{i}\delta^{\alpha\beta}_{\mu\nu}\delta^3(\mathbf{x}-\mathbf{y}), \tag{78}$$

and when expressed using (77) the non-vanishing commutators are

$$[a^{AB}(\mathbf{p}), \tilde{a}^{+}_{CD}(\mathbf{p}')] = [\tilde{a}^{AB}(\mathbf{p}), a^{+}_{CD}(\mathbf{p}')] = \frac{4\kappa}{\kappa_2}\delta^{AB}_{CD}\delta^3(\mathbf{p}-\mathbf{p}') \tag{79}$$

$$[\tilde{a}^{AB}(\mathbf{p}), \tilde{a}^{+}_{CD}(\mathbf{p}')] = -\frac{4\kappa(1-\kappa_2)}{\kappa_2^2}\delta^{AB}_{CD}\delta^3(\mathbf{p}-\mathbf{p}'); \tag{80}$$

there is a slight subtlety in calculating the above commutators. Basically the expression that appears at one stage of the calculus is

$$\sum_{ABCD}\chi^{(AB)}_{(\mu\nu)}P^{(\alpha\beta)}_{(\gamma\epsilon)}\chi^{(\gamma\epsilon)}_{CD} = \sum_{ABCD}\chi^{(AB)}_{(\mu\nu)}\frac{1}{2}\delta^{(\alpha\beta)}_{(\gamma\epsilon)}\chi^{(\gamma\epsilon)}_{CD} - \frac{1}{4}\eta^{\alpha\beta}\chi^{(AB)}_{(\mu\nu)}Tr(\chi) \tag{81}$$

and since we have the completeness relation

$$\sum_{ABCD}\chi^{(AB)}_{(\mu\nu)}\chi^{(\alpha\beta)}_{(CD)}\delta^{(CD)}_{(AB)} = \delta^{(\alpha\beta)}_{(\mu\nu)}, \tag{82}$$

we must impose $Tr(\chi) = 0$ which in turn means that $Tr(h) = Tr(\tilde{h}) = 0$. This can always be done because the gauge fixing being used does not fix entirely the gauge freedom and this further condition can be imposed (see [45]).

The Hamiltonian expressed in terms of creation and annihilation operators is

$$H = \int \frac{\mathrm{d}^3p}{4\kappa}E_{\mathbf{p}}\left((1-\kappa_2)a^{+}_{AB}a^{AB} + \kappa_2 a^{+}_{AB}\tilde{a}^{AB} + \kappa_2\tilde{a}^{+}_{AB}a^{AB}\right), \tag{83}$$

where we have subtracted an infinite constant. Looking at this Hamiltonian, we note that it has cross-products of operators, which obscures its physical interpretation. Something analogous happens when we observe the commutators (79) and (80) and so it is difficult to define their action over states. Because of this we redefine our annihilation (and therefore also the creation) operators; for this, we return to our action (70) and define

$$h_{\mu\nu} = A\bar{h}^1_{\mu\nu} + B\bar{h}^2_{\mu\nu}$$

$$\tilde{h}_{\mu\nu} = C\bar{h}^1_{\mu\nu} + D\bar{h}^2_{\mu\nu}, \tag{84}$$

where A, B, C and D are real constants so that the new fields, $\bar{h}^1$ and $\bar{h}^2$, are real fields. On replacing this in (70), we obtain

$$S[\bar{h}^1,\bar{h}^2] = \frac{1}{2\kappa}\int \mathrm{d}^4x P^{((\alpha\beta)(\mu\nu))}\left(\frac{A}{2}(A-\kappa_2 A+2\kappa_2 C)\bar{h}^1_{\alpha\beta}\partial^2\bar{h}^1_{\mu\nu} + \frac{B}{2}(B-\kappa_2 B + 2\kappa_2 D)\bar{h}^2_{\alpha\beta}\partial^2\bar{h}^2_{\mu\nu}\right) + P^{((\alpha\beta)(\mu\nu))}(AB-\kappa_2 AB+\kappa_2 AD+\kappa_2 BC)\bar{h}^1_{\alpha\beta}\partial^2\bar{h}^2_{\mu\nu} \tag{85}$$

and with the objective of decoupling the new fields, we make the last term in (85) null. It can be demonstrated that imposing the above criteria, it is inevitable that one (and only one) of the two fields will be a ghost. We make the choice of $\bar{h}^2$ as the corresponding ghost. Taking the above considerations plus the condition that (85) has the usual form of an action with real fields, we impose that the coefficients of the first and second terms in it be $\frac{1}{2}$ and $-\frac{1}{2}$ respectively. This means that

$$\begin{aligned} A &= B \\ C &= \frac{1-(1-\kappa_2)B^2}{2\kappa_2 B} \\ D &= -\frac{1+(1-\kappa_2)B^2}{2\kappa_2 B}, \end{aligned} \tag{86}$$

where B is left as an arbitrary real constant. Here we make the point that if we had chosen $\bar{h}^1$ as the ghost, then the real constants would change such that $C \leftrightarrow D$.

Then, the action we are left with finally is

$$S[\bar{h}^1, \bar{h}^2] = \frac{1}{2\kappa}\int \mathrm{d}^4x P^{((\alpha\beta)(\mu\nu))}\left(\frac{1}{2}\bar{h}^1_{\alpha\beta}\partial^2\bar{h}^1_{\mu\nu} - \frac{1}{2}\bar{h}^2_{\alpha\beta}\partial^2\bar{h}^2_{\mu\nu}\right). \tag{87}$$

Following this same line of reasoning, we can find the destruction operators for $\bar{h}^1$ and $\bar{h}^2$:

$$b^1_{AB}(\vec{p}) = \frac{1+B^2(1-\kappa_2)}{2B}a_{AB}(\vec{p}) + \kappa_2 B\tilde{a}_{AB}(\vec{p}) \tag{88}$$

$$b^2_{AB}(\vec{p}) = \frac{1-B^2(1-\kappa_2)}{2B}a_{AB}(\vec{p}) - \kappa_2 B\tilde{a}_{AB}(\vec{p}), \tag{89}$$

where we have used (84). It can be verified that the only nonvanishing commutators are now

$$[b^{1(AB)}(\vec{p}), b^{1+}_{CD}(\vec{p}')] = 4\kappa\delta^{AB}_{CD}\delta^3(\vec{p}-\vec{p}') \tag{90}$$

$$[b^{2(AB)}(\vec{p}), b^{2+}_{CD}(\vec{p}')] = -4\kappa\delta^{AB}_{CD}\delta^3(\vec{p}-\vec{p}'). \tag{91}$$

These commutators indicate that b^1 and b^2 have a vanishing inner product and that b^2 is the annihilation operator for the ghost. On the other hand, the Hamiltonian expressed in terms of these operators is

$$H = \int \frac{\mathrm{d}^3p}{4\kappa}E_{\mathbf{p}}\left(b^{1+}_{AB}b^{1AB} - b^{2+}_{AB}b^{2AB}\right). \tag{92}$$

Due to the existence of the ghost it is possible that this model will not be unitary. To analyze this in greater depth, it is necessary to do a more profound study of the S-matrix, but to do this for gravitation is a colossal task that would take us beyond the original scope of this paper. On the other side, the existence of ghost or phantom fields has been proposed by some authors to explain the accelerated expansion of the Universe [27–31], a feature that our model presents [26]. The problem with these models is that when they are quantized, either there is a lost of unitarity or there is negative energy which means loss of stability. Looking at (87), we find that the propagators of $\bar{h}^1$ and $\bar{h}^2$ are respectively

$$-2\kappa P^{-1}_{((\alpha\beta)(\mu\nu))}\frac{\mathrm{i}}{p^2 - \mathrm{i}\varepsilon} \tag{93}$$

$$2\kappa P^{-1}_{((\alpha\beta)(\mu\nu))}\frac{\mathrm{i}}{p^2 \pm \mathrm{i}\varepsilon}, \tag{94}$$

where $\pm$ in the phantom propagator, $\bar{h}^2$, will decide whether unitary and negative energy solutions or nonunitary and positive energy solutions will be present in the model [30].

The advantage that our model has against other models that use scalar fields for the phantoms is that being a gauge model, there remains open the possibility of fixing a gauge in which the model is unitary keeping the model's good attributes, as in the BRST canonical quantization [46]. On the other hand as a possible solution to the case of instability, we may consider $\tilde{\delta}$ supergravity which may solve the unboundedness of the Hamiltonian from below. The last argument comes from the fact that in supersymmetry one defines the Hamiltonian as the square of a Hermitian charge, making it positive definite [47, 48].

Having explained the problem that our model has, now we would like to discuss the new physics that our model may predict. For this, we will analyze the type of some finite quantum corrections and how the most simplest ones affect the equations of motion of the model.

7. Finite quantum corrections

The finite quantum corrections to our modified model of gravity can be separated into two groups. The first are the non-local terms, which are characterized by the presence of a logarithm in the form [36]

$$\begin{aligned}
&\sqrt{-g}R_{\mu\nu}\ln\left(\frac{\nabla^2}{\mu^2}\right)R^{\mu\nu}\\
&\sqrt{-g}R\ln\left(\frac{\nabla^2}{\mu^2}\right)R,
\end{aligned} \tag{95}$$

where $\nabla^2 = g^{\alpha\beta}\nabla_\alpha\nabla_\beta$, ∇_β being the covariant derivative. There are no terms like the above ones but quadratic in the Riemann tensor because these terms always occur like

$$\frac{1}{\epsilon} + \ln\left(\frac{\nabla^2}{\mu^2}\right) \tag{96}$$

and it is known that the terms that appear with the pole are purely Ricci tensors and Ricci scalars [3, 21] (see also equation (67)), which in turn is due to (66). Now, when looking at the quantum corrections and equation (68), we need to care about the variations of (95) with respect to $g_{\mu\nu}$. Taking this into consideration, for the nonlocal terms we have

$$\begin{aligned}
\delta\left(\sqrt{-g}\right)R_{\mu\nu}\ln\left(\frac{\nabla^2}{\mu^2}\right)R^{\mu\nu} &= 0\\
\sqrt{-g}R_{\mu\nu}\delta\left(\ln\left(\frac{\nabla^2}{\mu^2}\right)R^{\mu\nu}\right) &= 0\\
\sqrt{-g}\delta(R_{\mu\nu})\ln\left(\frac{\nabla^2}{\mu^2}\right)R^{\mu\nu} &= 0\\
\delta\left(\sqrt{-g}\right)R\ln\left(\frac{\nabla^2}{\mu^2}\right)R &= 0\\
\sqrt{-g}R\delta\left(\ln\left(\frac{\nabla^2}{\mu^2}\right)R\right) &= 0\\
\sqrt{-g}\delta(R)\ln\left(\frac{\nabla^2}{\mu^2}\right)R &= 0
\end{aligned} \tag{97}$$

because our model lives on-shell, i.e. $R_{\mu\nu} \equiv 0$ and $R \equiv 0$. So we see that the only relevant quantum corrections will come from the second group, that is, from the local terms which correspond to a series expansion in powers of the curvature tensor. The linear term is basically R, which corresponds to the original action, and the quadratic terms when taking into account their contribution are null due to (66). The next terms to be considered are cubic in the Riemann tensor. In principle any power of the curvature tensor will appear, but now we want to discuss only the cubic ones because they are simpler to be dealt with [33]. The most general form of these corrections is

$$L_Q^{\mathrm{fin}} = \sqrt{-g}\big(c_1 R_{\mu\nu\lambda\sigma} R^{\alpha\beta\lambda\sigma} R^{\mu\nu}{}_{\alpha\beta} + c_2 R^{\mu\nu}{}_{\lambda\sigma} R_{\mu\alpha}{}^{\lambda\beta} R^{\alpha\sigma}{}_{\nu\beta} + c_3 R_{\mu\nu} R^{\mu\alpha\beta\gamma} R^{\nu}{}_{\alpha\beta\gamma} + c_4 R R_{\mu\nu\lambda\kappa} R^{\mu\nu\lambda\kappa}\big). \tag{98}$$

These types of corrections will affect the equations of motion for $\tilde{g}_{\mu\nu}$. So, using (41), we obtain

$$F^{(\mu\nu)(\alpha\beta)\rho\lambda} D_\rho D_\lambda \tilde{g}_{\alpha\beta} = -\frac{1}{\kappa_2}\big(M^{(\mu\nu)} + c_1 N^{(\mu\nu)} + c_2 B^{(\mu\nu)} + 3\{D_\rho\, , D_\sigma\} E^{[\sigma\mu][\nu\rho]}\big) \tag{99}$$

with

$$M^{(\mu\nu)} = \tfrac{1}{2}\left(D_\alpha D^\nu A^{(\alpha\mu)} + D_\alpha D^\mu A^{(\alpha\nu)} - D_\alpha D^\alpha A^{(\mu\nu)} - g^{\mu\nu} D_\alpha D_\beta A^{(\alpha\beta)}\right) \tag{100}$$

$$A^{(\mu\nu)} = c_3 R^{\mu\alpha\beta\gamma} R^{\nu}{}_{\alpha\beta\gamma} + c_4 g^{\mu\nu} R^{\alpha\beta\gamma\epsilon} R_{\alpha\beta\gamma\epsilon} \tag{101}$$

$$N^{(\mu\nu)} = \tfrac{1}{2} g^{\mu\nu} R_{\rho\epsilon\lambda\sigma} R^{\lambda\sigma\alpha\beta} R_{\alpha\beta}{}^{\rho\epsilon} + 3 R_{\rho\epsilon\lambda\sigma} R_{\alpha}{}^{\nu\epsilon\rho} R^{\alpha\mu\lambda\sigma} \tag{102}$$

$$B^{(\mu\nu)} = \tfrac{1}{2} g^{\mu\nu} R_{\rho\epsilon\lambda\sigma} R^{\rho\alpha\lambda\beta} R_{\alpha}{}^{\sigma\epsilon}{}_{\beta} + 3 R_{\rho\epsilon\lambda\sigma} R^{\nu\sigma\rho}{}_{\beta} R^{\mu\epsilon\beta\lambda} \tag{103}$$

$$E^{[\sigma\mu][\nu\rho]} = c_1 R^{\sigma\mu}{}_{\alpha\beta} R^{\alpha\beta\nu\rho} + \tfrac{1}{2} c_2 \big(R^{\nu}{}_{\alpha}{}^{\sigma}{}_{\beta} R^{\rho\beta\alpha\mu} - R^{\rho}{}_{\alpha}{}^{\sigma}{}_{\beta} R^{\nu\beta\alpha\mu}\big), \tag{104}$$

where $[\mu\nu]$ means that μ and ν are in a antisymmetric combination, and $F^{(\mu\nu)(\alpha\beta)\rho\lambda}$ was defined in (45). Obviously, if we do not have quantum correction, i.e. $c_1 = c_2 = c_3 = c_4 = 0$, (99) is transformed into (46). It is possible to demonstrate that one solution to (46) is $\tilde{g}_{\mu\nu} = g_{\mu\nu}$, a fact that is necessary so that the predictions of the original theory of Einstein–Hilbert are still fulfilled in vacuum. This means that the solution of (99) must come to be small perturbations to $g_{\mu\nu}$.

$\tilde{\delta}$ Gravity will provide finite answers for the constants c_i. Due to the general structure of the finite quantum corrections, they will be relevant only at very short distances and strong curvatures. So the natural scenario to test the predictions of the model is the inflationary epoch of the Universe. The computation of the c_i and the phenomenological implications of quantum $\tilde{\delta}$ gravity will be discussed elsewhere.

7. Conclusions

We have shown following [25] that the $\tilde{\delta}$ transformation, applied to any theory, produces physical models that live only at one loop. This is achieved introducing new fields that generate a new constraint through a functional Dirac's delta inside the path integral (34). We have seen that the original symmetries are generalized when we apply the $\tilde{\delta}$ transformation. Moreover, the modified model is invariant under the generalized symmetries.

Now, going to $\tilde{\delta}$ gravity, we calculated the divergent part of the action to one loop and we obtained twice the well-known result of [3]. We see that this factor of 2 also appears in [25]. The divergent part at one loop is zero in the absence of matter and on-shell, so $\tilde{\delta}$ gravity is a finite quantum model, in four-dimensional spacetime. This in turn implies that the Newton gravitational constant does not run with scale, which agrees with the very stringent experimental bounds that restrict its variation [43, 44].

We have shown that perturbing around the Minkowsky vacuum and using a particular Lorentz invariant gauge, we can redefine the gravitational fields in such a way that the free part of the action is decoupled. In this redefinition, it is seen that one of the new fields is a ghost. Despite that this may bring unitary or unstable problems (negative energies), these ghosts (phantoms) can explain the accelerated expansion of the Universe [26] at a classical level. Scalar phantoms have been introduced in order to explain dark energy in [27] and discussed in many papers. See, for instance, [28–31]. This connection may be far reaching because the phantom idea has gained great popularity as an alternative to the cosmological constant. The present model could provide an arena to study the quantum properties of a phantom field, since the model has a finite quantum effective action. In this respect, the advantage of the present model is that being a gauge model, it could give us the possibility of solving the problem of lack of unitarity using standard techniques of gauge theories as the BRST method. This is something that needs to be studied further but goes beyond the original scope of this paper.

We want to point out that supergravity with matter is finite at the one-loop level [32]. According to the general argument developed in this paper, $\tilde{\delta}$ supergravity will be a one-loop model which has a strong possibility to be a finite quantum model of gravity plus matter and also it may solve the instability of negative energies since in supersymmetry one has a Hermitian charge whose square is equal to the Hamiltonian operator meaning that the Hamiltonian is positive definite [47, 48].

Finally, we have shown that the contribution of quadratic local and non-local logarithmic terms is zero due to the on-shell condition of the modified model. We have also shown how the cubic corrections in the Riemann tensor affect the equation of motion (99). Given the general form of the quantum corrections in quantum $\tilde{\delta}$ gravity, they may be important during the inflationary epoch of the Universe.

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Appendix A. BRST formalism

First we give the BRST transformations $\bar{\delta}$ of our model:

$$\xi_0^\mu(x) = \lambda c_0^\mu(x)$$
$$\xi_1^\mu(x) = \lambda c_1^\mu(x), \tag{A.1}$$

where λ is a Grassmann constant and c_0^μ, c_1^μ are the two ghosts of our model. Starting from the gauge transformations for our quantum fields $h_{\mu\nu}$ and $\tilde{h}_{\mu\nu}$ [49], we obtain to zero order in h and $\tilde{h}$

$$\bar{\delta}h_{\mu\nu} = c_{0\mu;\nu} + c_{0\nu;\mu} \tag{A.2}$$

$$\bar{\delta}\tilde{h}_{\mu\nu} = c_{1\mu;\nu} + c_{1\nu;\mu} + \tilde{g}_{\mu\nu;\lambda}c_0^\lambda + \tilde{g}_{\mu\lambda}c^\lambda_{0;\nu} + \tilde{g}_{\nu\lambda}c^\lambda_{0;\mu} \tag{A.3}$$

and we also have

$$\begin{aligned}\bar{\delta}c_0^\mu &= c_0^\rho c^\mu_{0,\rho}\\ \bar{\delta}c_1^\mu &= c_0^\rho c^\mu_{1,\rho} + c_1^\rho c^\mu_{0,\rho}\\ \bar{\delta}\bar{c}_0^\mu &= \mathrm{i}b_0^\mu(x)\\ \bar{\delta}\bar{c}_1^\mu &= \mathrm{i}b_1^\mu(x)\end{aligned} \tag{A.4}$$

for the corresponding anti-ghosts $\bar{c}$ and where the b's are the auxiliary Nakanishi–Lautrup fields which satisfy

$$\bar{\delta}b^\mu_{0,1} = 0. \tag{A.5}$$

It has been verified that these transformations are nilpotent. Now, we choose for our gauge fixing term

$$\mathrm{GF} = -\sqrt{-g}\frac{C^2}{2} - \tilde{\delta}\left(\kappa_2\sqrt{-g}\frac{C^2}{2}\right) \tag{A.6}$$

and we see that this is a good choice for our gauge fixing since it is invariant under both transformations δ_0 and δ_1 (see section 3.1), where [3, 21]

$$\begin{aligned}C^2 &= g^{\alpha\beta}C_\alpha C_\beta\\ C_\mu &= D_\nu h^\nu_\mu - \tfrac{1}{2}D_\mu h^\nu_\nu.\end{aligned} \tag{A.7}$$

In this way, we have

$$\begin{aligned}\mathrm{GF} = &-\sqrt{-g}\left[\left(1+\frac{\kappa_2}{2}g^{\alpha\beta}\tilde{g}_{\alpha\beta}\right)\frac{C^2}{2} + \kappa_2\tilde{\delta}\left(\frac{g^{\mu\rho}C_\mu C_\rho}{2}\right)\right]\\ = &-\sqrt{-g}\left[\left(1+\frac{\kappa_2}{2}g^{\alpha\beta}\tilde{g}_{\alpha\beta}\right)\frac{C_\mu C^\mu}{2} + \kappa_2\left(\tilde{C}_\mu C^\mu - \frac{\tilde{g}_{\mu\beta}C^\mu C^\beta}{2}\right)\right],\end{aligned} \tag{A.8}$$

where

$$\tilde{C}_\mu = \tilde{\delta}C_\mu = \tilde{\delta}\left[D_\nu h^\nu_\mu - \tfrac{1}{2}D_\mu h^\nu_\nu\right] = g^{\nu\rho}\left[\nabla_\nu\tilde{h}_{\rho\mu} - \tfrac{1}{2}\nabla_\mu\tilde{h}_{\rho\nu}\right] - \tilde{g}^{\nu\rho}\left[D_\nu h_{\rho\mu} - \tfrac{1}{2}D_\mu h_{\rho\nu}\right], \tag{A.9}$$

and this can be written in the form

$$\mathrm{GF} = -\sqrt{-g}H_\mu C^\mu \tag{A.10}$$

with

$$H_\mu = \left[\left(1+\frac{\kappa_2}{2}\tilde{g}^\alpha_\alpha\right)\frac{C_\mu}{2} + \kappa_2\left(\tilde{C}_\mu - \tilde{g}_{\mu\beta}\frac{C^\beta}{2}\right)\right]. \tag{A.11}$$

Having established the form of the gauge fixing term, we can now by a standard procedure (the BRST method) find the associated Faddeev–Popov Lagrangian. Following [37], now we do

$$\mathcal{L}_{\mathrm{GF+FP}} = -\mathrm{i}\bar{\delta}(P), \tag{A.12}$$

where P in our case is

$$P = \bar{c}_0^\mu H_\mu + \bar{c}_1^\mu C_\mu + \beta_1\bar{c}_1^\mu b_{0\mu} + \beta_2\bar{c}_0^\mu b_{1\mu}, \tag{A.13}$$

where the β's are arbitrary constants to be fixed shortly, so we have

$$\mathcal{L}_{\mathrm{GF+FP}} = -\mathrm{i}\big(\mathrm{i}b_0^{\mu}H_{\mu} + \mathrm{i}b_1^{\mu}C_{\mu} + \mathrm{i}(\beta_1+\beta_2)b_1^{\mu}b_{0\mu} - \bar{c}_0^{\mu}(\bar{\delta}H_{\mu}) - \bar{c}_1^{\mu}(\bar{\delta}C_{\mu})\big) \tag{A.14}$$

and so

$$\mathcal{L}_{\mathrm{GF}} = b_0^{\mu}H_{\mu} + b_1^{\mu}C_{\mu} + (\beta_1+\beta_2)b_1^{\mu}b_{0\mu} \tag{A.15}$$

$$\mathcal{L}_{\mathrm{FP}} = \mathrm{i}\big(\bar{c}_0^{\mu}(\bar{\delta}H_{\mu}) + \bar{c}_1^{\mu}(\bar{\delta}C_{\mu})\big). \tag{A.16}$$

Now, for the gauge fixing part, we can use the equations of motion for the auxiliary fields to make them disappear:

$$\begin{aligned} \frac{\partial\mathcal{L}_{\mathrm{GF}}}{\partial b_1^{\mu}} &= C_{\mu} + (\beta_1+\beta_2)b_{0\mu} = 0 \longrightarrow b_{0\mu} = -\frac{C_{\mu}}{(\beta_1+\beta_2)} \\ \frac{\partial\mathcal{L}_{\mathrm{GF}}}{\partial b_0^{\mu}} &= H_{\mu} + (\beta_1+\beta_2)b_{1\mu} = 0 \longrightarrow b_{1\mu} = -\frac{H_{\mu}}{(\beta_1+\beta_2)}; \end{aligned} \tag{A.17}$$

substituting it in $\mathcal{L}_{\mathrm{GF}}$ we get

$$\mathcal{L}_{\mathrm{GF}} = -\frac{C^{\mu}H_{\mu}}{(\beta_1+\beta_2)} - \frac{C^{\mu}H_{\mu}}{(\beta_1+\beta_2)} + \frac{(\beta_1+\beta_2)C^{\mu}H_{\mu}}{(\beta_1+\beta_2)^2} = -\frac{C^{\mu}H_{\mu}}{(\beta_1+\beta_2)} \tag{A.18}$$

so we see that we recover our initial gauge fixing if we set $(\beta_1+\beta_2) = 1$. Now for the Faddeev–Popov Lagrangian we have

$$\mathcal{L}_{\mathrm{FP}} = \mathrm{i}\left(\bar{c}_0^{\mu}(\bar{\delta}H_{\mu}) + \bar{c}_1^{\mu}(\bar{\delta}C_{\mu})\right) \tag{A.19}$$

and it is well known that [3, 21]

$$\bar{\delta}C_{\mu} = D_{\nu}D^{\nu}c_{0\mu} + R_{\mu\nu}c_0^{\nu} \tag{A.20}$$

and using

$$\begin{aligned} \bar{\delta}h_{\nu\rho} &= D_{\nu}c_{0\rho} + D_{\rho}c_{0\nu} \\ \bar{\delta}\tilde{h}_{\nu\rho} &= \nabla_{\nu}c_{1\rho} + \nabla_{\rho}c_{1\nu} \end{aligned} \tag{A.21}$$

we get

$$\begin{aligned} \bar{\delta}H_{\mu} &= \left[\left(1+\frac{\kappa_2}{2}\tilde{g}^{\alpha}_{\alpha}\right)\frac{\bar{\delta}C_{\mu}}{2} + \kappa_2\left(\bar{\delta}\tilde{C}_{\mu} - \tilde{g}_{\mu\beta}\frac{\bar{\delta}C^{\beta}}{2}\right)\right] \\ \bar{\delta}\tilde{C}_{\mu} &= \nabla_{\nu}\nabla^{\nu}c_{1\mu} + R_{\mu\nu}c_1^{\nu} - g^{\rho\nu}\tilde{\delta}(R^{\alpha}{}_{\rho\nu\mu})c_{0\alpha} - \tilde{g}^{\nu\rho}\big[D_{\nu}D_{\rho}c_{0\mu} + c_{0\sigma}R^{\sigma}{}_{\rho\mu\nu}\big]. \end{aligned} \tag{A.22}$$

So, evaluating in (A.19), we will obtain (59).

Appendix B. Background field method

The BFM is a mechanism utilized to calculate the effective action at any order of perturbation theory without losing explicit gauge invariance. This simplifies the calculations and the comprehension of the model. The importance of the effective action is due to the fact that it contains all the quantum information of the theory and that from it all 1PI Feynman diagrams can be computed. Stringing them together, we can compute all connected Feynman diagrams in a more efficient manner [38] and from them the S-matrix can be calculated.

Next we calculate the effective action Γ for a general model using the BFM. One begins by defining the generating functional of disconnected diagrams $Z[J]$:

$$Z[J] = \int \mathcal{D}\varphi\, \mathrm{e}^{\mathrm{i}(S[\varphi]+J\cdot\varphi)}, \tag{B.1}$$

where S is the action of the system and where we will be using the notation $J \cdot \varphi \equiv \int J\varphi \, \mathrm{d}^4x$. In the BFM, we do the identification $\varphi \to \varphi + \phi$ inside the action, where ϕ is an arbitrary background. So now we have

$$\hat{Z}[J, \phi] = \int \mathcal{D}\varphi \, \mathrm{e}^{\mathrm{i}(S[\varphi+\phi]+J\cdot\varphi)}. \tag{B.2}$$

Now the generating functional of connected diagrams $W[J]$ is

$$W[J] = -\mathrm{i} \ln Z[J] \tag{B.3}$$

so we define

$$\hat{W}[J, \phi] = -\mathrm{i} \ln \hat{Z}[J, \phi] \tag{B.4}$$

and

$$\bar{\varphi} = \frac{\delta W}{\delta J} \tag{B.5}$$

so here

$$\hat{\varphi} = \frac{\delta \hat{W}}{\delta J}; \tag{B.6}$$

with all these definitions, it is possible to give the formula for the usual effective action:

$$\Gamma[\bar{\varphi}] = W[J] - J \cdot \bar{\varphi} \tag{B.7}$$

and the background field effective action:

$$\hat{\Gamma}[\hat{\varphi}, \phi] = \hat{W}[J, \phi] - J \cdot \hat{\varphi}; \tag{B.8}$$

now we do the shift $\varphi \to \varphi - \phi$, so that

$$\hat{Z}[J, \phi] = Z[J] \, \mathrm{e}^{-\mathrm{i}J\cdot\phi} \tag{B.9}$$

from which it follows (after taking logarithms) that

$$\hat{W}[J, \phi] = W[J] - J \cdot \phi; \tag{B.10}$$

taking now the functional derivative with respect to J,

$$\hat{\varphi} = \bar{\varphi} - \phi \tag{B.11}$$

but now we can appreciate that

$$\begin{aligned} \hat{\Gamma}[\hat{\varphi}, \phi] &= W[J] - J \cdot \phi - J \cdot \hat{\varphi} \\ &= W[J] - J \cdot \phi - J \cdot (\bar{\varphi} - \phi) \\ &= W[J] - J \cdot \bar{\varphi} \\ \hat{\Gamma}[\hat{\varphi}, \phi] &= \Gamma[\hat{\varphi} + \phi]. \end{aligned} \tag{B.12}$$

In particular, if we take $\hat{\varphi} = 0$, we have

$$\hat{\Gamma}[0, \phi] = \Gamma[\phi]. \tag{B.13}$$

This means that the effective action of the theory Γ can be computed from the background field effective action $\hat{\Gamma}$ by taking the quantum field to zero and with the presence of the background ϕ. Since the derivatives of the effective action with respect to the fields generate the 1PI diagrams, the last equation means that if we treat ϕ perturbatively what we will have will be diagrams with external legs corresponding to the background field ϕ and with internal lines corresponding to the quantum field φ.

And so, to study the quantum effects it only suffices to carry out an expansion in the quantum fields in the action S or in the Lagrangian L using the identification of the BFM. This means that

$$\begin{aligned}\phi_I &\to \phi_I + \varphi_I \\ \tilde{\phi}_I &\to \tilde{\phi}_I + \tilde{\varphi}_I.\end{aligned} \tag{B.14}$$

We use (B.14) in $\tilde{\delta}$ gravity, where $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$ and $\tilde{g}_{\mu\nu} \to \tilde{g}_{\mu\nu} + \tilde{h}_{\mu\nu}$.

Appendix C. Divergent part of the effective action at one loop

As was mentioned in section 4, there are various ways to calculate the divergent part of the effective action at one loop, but they are quite complicated. So, we have resolved to follow an algorithm developed in [21].

The effective action Γ to one loop can be written as

$$\Gamma[\phi] = S[\phi] + \frac{i}{2}\hbar \mathrm{Tr} \ln D + O(\hbar^2), \tag{C.1}$$

where

$$D_i{}^j = \frac{\delta^2 S}{\delta\phi_i \delta\phi_j}[\phi] \tag{C.2}$$

is a differential operator depending on the background field ϕ_i. Its most general form is

$$\begin{aligned}D_i{}^j = {} & K^{\mu_1\mu_2\ldots\mu_L j}{}_i \nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_L} + S^{\mu_1\mu_2\ldots\mu_{L-1} j}{}_i \nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_{L-1}} \\ & + W^{\mu_1\mu_2\ldots\mu_{L-2} j}{}_i \nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_{L-2}} + N^{\mu_1\mu_2\ldots\mu_{L-3} j}{}_i \nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_{L-3}} \\ & + M^{\mu_1\mu_2\ldots\mu_{L-4} j}{}_i \nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_{L-4}} + \cdots,\end{aligned} \tag{C.3}$$

where K, S, W, N, M are parameters which must be specified for each model and ∇_μ is a covariant derivative:

$$\nabla_\alpha T^\beta{}_i{}^j = \partial_\alpha T^\beta{}_i{}^j + \Gamma_{\alpha\gamma}{}^\beta T^\gamma{}_i{}^j + \omega_\alpha{}^k{}_i T^\beta{}_k{}^j - \omega_\alpha{}^j{}_k T^\beta{}_i{}^k \tag{C.4}$$

$$\nabla_\mu \Phi_i = \partial_\mu \Phi_i + \omega_\mu{}^j{}_i \Phi_j; \tag{C.5}$$

here,

$$\Gamma_{\mu\nu}{}^\alpha = \tfrac{1}{2} g^{\alpha\beta}(\partial_\mu g_{\nu\beta} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu}) \tag{C.6}$$

and $\omega_\mu{}^j{}_i$ is a connection on the principle bundle. The computation of the divergent part of the effective action at one loop is done through a lengthy and cumbersome calculation that consists of the sum of a finite number of one-loop divergent Feynman diagrams; the details are given in [21] and the result by equation (30) in the same reference. This last result is too large to be shown here, but it depends on the parameters involved in $D_i{}^j$ (C.3). So basically, what we need is the quadratic part of the Lagrangian of the model to obtain the divergent part of the effective action.

In $\tilde{\delta}$ gravity, we have

$$\phi_i \to \vec{h}_{(\alpha\beta)}, \tag{C.7}$$

where $\vec{h}$ is defined in (53). As the covariant derivative acting on $\vec{h}$ is given by (57), this means $\mathrm{I} \to (\alpha\beta)$:

$$\omega_\mu{}^j{}_i \to -([\Gamma_{\mu\alpha}{}^\rho]\delta^\nu_\beta + [\Gamma_{\mu\beta}{}^\rho]\delta^\nu_\alpha), \tag{C.8}$$

where $[\Gamma_{\mu\alpha}{}^{\rho}]$ is given by equation (58). The other relevant parameters in our model are given by

$L = 2$
$K^{\mu_1\mu_2\ldots\mu_L j}{}_i$ given by (54)
$S^{\mu_1\mu_2\ldots\mu_{L-1} j}{}_i = 0$
$W^{\mu_1\mu_2\ldots\mu_{L-2} j}{}_i$ given by (55).

On the other side of the Faddeev–Popov ghosts, we have

$$\phi_i \to \vec{c}_\alpha, \tag{C.9}$$

where $\vec{c}_\alpha$ is defined by (60) and the covariant derivative (63) shows that

$$\omega_\mu{}^j{}_i \to -[\Gamma_{\mu\alpha}{}^{\rho}]. \tag{C.10}$$

Finally, the other parameters are given by

$L = 2$
$K^{\mu_1\mu_2\ldots\mu_L j}{}_i$ given by (61)
$S^{\mu_1\mu_2\ldots\mu_{L-1} j}{}_i = 0$
$W^{\mu_1\mu_2\ldots\mu_{L-2} j}{}_i$ given by (62).

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