PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
ESCUELA DE INGENIERIA

# INTERTEMPORAL STOCHASTIC SAWMILL PLANNING: MODELING AND MANAGERIAL INSIGHTS

**ALFONSO ANDRÉS LOBOS RUIZ**

Tesis presentada a la Dirección de Investigación y Postgrado
como parte de los requisitos para optar al grado de
Master of Science in Engineering

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Santiago de Chile, July 2015

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*To my friends and family*

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# RESUMEN

Los modelos de optimización y simulación han sido usados por más de cincuenta años en la industria forestal. En esta área, al igual que en otras, estos modelos han apoyado la toma de decisiones en problemas de planificación de distinta índole y que involucran distintos períodos de tiempo. Una complejidad inherente a tomar decisiones que involucran períodos distintos de tiempo es el cómo garantizar la consistencia de las decisiones, es decir, que cuando sean llevadas a la práctica no impliquen grandes costos extras a la empresa y que está pueda garantizar un nivel de servicio adecuado. En este trabajo se estudiará un problema de aserradero donde se deben encargar troncos para ser usados en los próximos cuatro meses de producción, los que luego serán cortados para cumplir con las demandas semanales por tablas que tenga agendadas la empresa. El problema de consistencia está en que los troncos encargados diferirán de los que serán recibidos, y sin importar este hecho la empresa deberá satisfacer las demandas incurridas, ya sea pidiendo insumos extras, externalizando trabajo o negociando un atraso en la entrega de parte de las demandas. Este trabajo presenta distintos modelos de optimización estocástica de dos etapas que resuelven el problema de aserradero y que se diferencian en cómo modelan la interacción de inventarios, y en los diversos patrones de cortes que pueden usar en un tronco para producir distintas tablas. Para comparar como las diferencias entre los modelos afectarían en una operación real se utilizó un modelo de horizonte rodante con el que se pudo simular años de operación y para el cuál fue necesario crear escenarios de demandas de tablas y de incertidumbre en el suministro de troncos. Los resultados muestran como disminuye el costo de la operación al estar disponibles más patrones de corte para cada tronco y las diferencias en el comportamiento de compra de los troncos al usarse distintas formas de modelar la interacción de inventarios.

**Palabras Claves:** Optimización de planificación en aserraderos, programación estocástica, formulaciones de dos estapas, horizonte rodante

# ABSTRACT

Optimization models have long been used in the Forest Industry. Here, as well as in other areas, models are used in different time horizons to support planning and scheduling. Guaranteeing the consistency of the production policies in those different time periods is highly relevant for efficiency and demand fulfilment. This paper presents a set of Sawmill Planning Models that cover tactical planning, as well as operational planning. Aggregated planning decisions are modeled in order to determine the log supply for the sawmill. At the operational level, detailed weekly production plans are defined using the actual log supply, which might not be consistent with what was originally planned, due to several variabilities. We address the issue of coordinating short-term decisions with mid-term planning using a two-stage stochastic optimization formulation. Various models with certain variations are proposed in order to simulate all of the complexities that are present in the Sawmill Planning Problem. To test the models, we simulated a special Rolling Horizon method using different demand scenarios. Finally, we present results and managerial insights regarding the effects of uncertainty.



**Keywords:** Sawmill Planning, Stochastic Programming, Two-Stage Formulations, Rolling Horizon

# 1. ARTICLE BACKGROUND

## 1.1 Introduction

Many applications in administration, production, and other areas require decision-making at different points in time and relying on differing levels of information. For example, a manager may have to decide today the amount of supply to purchase to last his or her company throughout the year. If the quantity was not sufficient, more can be ordered but at a higher cost. We will call decision planning of this nature intertemporal planning.

The difficulty of making decisions in advance is that they are based on assumptions on or oversimplifications of future events. In a vast majority of cases simplified models and estimates are not sufficient to adequately prepare for the future. This deviation between what is expected and what occurs translates to additional costs incurred in transactions.

This work focuses on facing this problem of intertemporal planning for a sawmill. This problem involves a company deciding the amount of logs to purchase and labor to hire for the following four production months in order to satisfy the demand for the company's different types of lumber. This demand should be satisfied regardless of the initial order of logs and labor the company has made. In the event of the initial order not being sufficient to fulfill demand, the company may purchase more supplies at a higher cost, due to urgency and lack of advance notice, outsource part of the work, or negotiate a delay in delivery for a percentage of the demand. Works related to this topic are Alvarez and Vera (2014), Maturana et al. (2010), and Kazemi Zanjani et al. (2010).

The particularity of the problem outlined in this article is that the type and amount of logs that are received by the company will deviate from the original order. In general terms, there is an uncertainty in the supply of stock. To our knowledge, this topic has not yet been treated in this area yet.

The main objective of this work is to present models that are capable of simulating this situation. To this end, two models were developed that were capable of simulating the complexities of the sawmill problem presented in the article. Each model consists of a version in which each log can be cut using several cutting patterns (the disaggregate version) or one in which only one cut can be used to produce lumber (the aggregate version). The models were named First Model with Aggregation (FMA) or with Disaggregation (FMD), Second Model with Aggregation (SMA) or with Disaggregation (SMD).

All the models proposed are stochastic two-stage models. Stochastic two-stage optimization make it possible to simulate situations where decisions must take place before and after the values of specific uncertainty parameters are known. We assume uncertainty in the fact that the amount and type of logs received will differ from the ones ordered, though if one wishes to order logs at a lower price this must be done before the quantity and type of the actual supply is known.

The secondary objective of this work is to show how the operation of a plant over long periods of time can be simulated in order to anticipate situations that might occur during actual operation. This was achieved by using a rolling horizon scheme that required the creation of scenarios of demand and disturbance in the supply of logs.

The data used in this work are intended to simulate common situations in this industry. For this reason, the obtained results replicate interesting situations that may occur in this industry, though numerically they are not derived from nor represent a real operation. In particular, the obtained results aid in showing the weaknesses, strengths, and behavior of the models (FMA, FMD, SMA, SMD), were they to be used in a real operation for long periods of time.

### 1.2 Main Objectives

1. To develop optimization models capable of simulating the sawmill problem explained in the above introductory section. In particular, to show why the stochastic two-stage optimization framework is an adequate model for the sawmill problem raised in the article.

2. To show how the behavior of a model over long periods of time can be simulated before used in a real operation and why it is important to do so.
3. To explain theoretically and empirically the differences between the models proposed to solve the sawmill problem raised in the article. These models are explained in the section 2.3.

## 1.3 Literature Review

### 1.3.1 Intertemporal planning and uncertainty management

According to Haas et al. (1981), in intertemporal production problems there are both tactical and operational decisions to be made. A tactical decision pertains to long time periods, in our case, how many logs and how much labor to order each month, while an operational decision is how many logs to cut and which cutting patterns to use to satisfy a weekly demand. The objective of intertemporal planning is to connect the tactical model (based solely on tactical decisions) with the associated operational model based only on operational decisions that uses tactical decisions as fixed data.

Zipkin (1980a), and Mendelssohn (1980) also studies the feasibility of disaggregating a tactical solution in order to use it in the associated operational problem, though the study is based on the structures of the tactical and operational problem matrix. Mathematical bounds and theorems are obtained in these works.

A topic related to the problem of the connection between the tactical and associated operational problem is the treatment of uncertainty in connecting the two. The two-stage stochastic optimization Birge and Louveaux (1997) is used for problems in which certain parameters involve uncertainty. In this methodology, two kinds of decisions must be made: in the first stage, decisions made before the value of uncertain parameters are known and in the second stage, decisions made when the value of all parameters are known. This idea is generalized in the multi-stage stochastic optimization (Birge and Louveaux (1997)).

In the two-stage stochastic optimization process, decisions in the first and second stage are resolved in the same optimization problem. In this problem, the first stage decisions can be considered tactical decisions and those of the second stage as operational decisions. Examples of this methodology in use for planning problems include Kazemi Zanjani et al. (2010), and Gupta and Maranas (2000).

In stochastic optimization in two or more stages is assumed that there is a finite number of values that uncertain parameters can take, or that a probability distribution is known for them *apriori*. In the methodology known as robust optimization, these uncertain parameters are assumed to belong to a certain range, which is known (Bertsimas and Sim (2004), Ben-Tal and Nemirovski (1998) and Soyster (1973)). One way of solving tactical and operational problems in conjunction, using a robust optimization scheme is proposed in Thiele et al. (2009). Another idea, developed in Alvarez and Vera (2014), is to solve a tactical problem using differing levels of robustness, increasing the cost of the solution, but empirically showing the increased feasibility of the associated operational problem.

An intertemporal problem only simulates in detail the time that its operational model encompasses, that is to say, if the tactical model simulates four months and the operational model only one, then only one month is simulated in detail. To simulate in detail periods lasting several months or years, a technique called rolling horizon can be used. This technique simulates the periods chronologically, using results obtained in one period as input for the following. In a simple example of production planning, the remaining stock at the end of a month becomes the initial stock of the following. This methodology can be used with actual results of an operation to feed in to the following period (see Brown et al. (2001)), or by creating fictional result scenarios to feed in to the model like is done in Ho * (2005), Boulaksil et al. (2009), and the method used in the article presented in this work. A compilatory work on the rolling horizon methodology is Sahin et al. (2013).

### 1.3.2 Forestry optimization and sawmill planning

Optimization and simulation techniques have been used for over 50 years in the forest industry (D'Amours et al. (2009), Rönnqvist (2003)). Several topics have been discussed, some of which include:

1. Appropriate ways of tree planting and soil usage so that: a) An appropriate composition of nutrients is maintained over time, b) Forest fires can be easily contained or their impact limited, c) Appropriate quantities and types of trees are chosen for planting in order to minimize the effect of price fluctuation of wood pulp and wood on company profits.
2. How to construct paths between forests and sawmills.
3. What type and quantity of logs to order/buy in order to satisfy demand for lumber over different periods of time.
4. Various other implementations as seen in D'Amours et al. (2008), Rönnqvist (2003), and Epstein et al. (2010).

Decisions in forest industry can be divided into different categories according to the time period they encompass (according to Rönnqvist (2003)). Strategic decisions are those that involve periods of ten to one hundred years, tactical decisions involve periods of six months to five years, and operational decisions involve periods of one day to six months and online planning involving periods of less than one day.

| | Forest management and harvesting | Transportation and routing | Production |
|---|---|---|---|
| Strategic planning $> 5$ years | Planting, Evaluation, long term harvesting | Road building, road upgrading, fleet management | Investment planning |
| Tactical planning 6 months 5 years | Annual harvest plans | Road upgrade, Equipment utilization | Annual production planning |
| Operative planning 1 day 6 months | Crew scheduling, Harvest sequencing | Catchment areas, back-haulage planning, scheduling | Lot sizing, scheduling |
| Online planning $< 1$ day | Bucking | Truck dispatching | Process control, Roll cutting, Cross-cutting |

FIGURA 1.1. Table 1 as appears in Rönnqvist (2003)

a) Sawmill Planning

Sawmill planning seeks to determine the appropriate quantity of logs to order and labor to hire in order to satisfy a company's demand for lumber. In order to produce the lumber the logs can be cut using different cutting patterns.

Usual difficulties in sawmill planning are: 1.- that logs differ in type and diameter, 2.- that each log can be cut using various patterns which results in different combinations of lumber obtained, 3.- that demand for lumber can be uncertain, 4.- that production can be planned for periods ranging from days to months.

In the article presented here and the rest of the literature review, each log is assumed to be able to be categorized into types whose possible cutting patterns are known. There are companies that lack the infrastructure or preparedness to categorize each log into a known type and thus prefer to cut each log in a way that maximizes the volume of useable lumber produced (Saadatyar (2013), Faaland and Briggs (1984)). The latter way of cutting the logs will not be used in this work.

In Maturana et al. (2010), a linear programming model is used in order to decide the quantity of logs to purchase and the way of cutting them in order to satisfy demand for a six-week period. In Kazemi Zanjani et al. (2010), the amount of logs to purchase is not set, though log yield uncertainty is simulated, that is, the quantity and type of lumber produced when cutting a log is uncertain. The problem is resolved as a stochastic two-stage problem where a certain kind of robustness or stability is introduced by using a modification of ideas proposed in Mulvey et al. (1995).

Zanjani et al. (2010) extends the work done in Kazemi Zanjani et al. (2010) using a stochastic two-stage optimization scheme, assuming uncertainty in log yield and lumber demand. Alvarez and Vera (2014) also simulates uncertainty in log yield though using robust, rather than stochastic, optimization in order to solve the problem.

### 1.4 Conclusions

The main result of this work was to show how a sawmill problem with uncertainty in its supply could be modeled. Two models were developed that were capable of simulating the problem, with each model having two versions as mentioned in the section 1.1.

The usage of a methodology that could replicate several months or years of operation before a model is actually used in a sawmill plant was crucial in this work. In particular, the rolling horizon methodology presented in the article in section 2.4 would not only aid in predicting the model that would incur the least cost during real operation, but also impart understanding of the strengths and weaknesses of each model. This is due to the fact that this methodology can simulate months or years of operation, in comparison to four months using aggregation and one month in detail when running a model once. The most significant results obtained when using the rolling horizon scheme were as follows:

1. Of the models outlined in section 2.3, the best results were obtained by SMD. Following was FMD, obtaining results that were 1.62% more costly than SMD, and FMA and SMD which were 15.16% and 30.64% respectively more costly than SMD.
2. How to aggregate variables is a topic that merits careful study. In the example shown in the article, the cost of aggregating cutting patterns was 13.36% for the first model and 30.64% for the second. In a real operation, the amount of variables used (types of logs, lumber, possible cuts) can be prohibitive for optimization software or too extensive to be dealt with in detail by the company. As such, how to reach an adequate balance between aggregating variables and the controlled deterioration of the target function should be studied.
3. The SMD model behaves more stochastically than the models FMD and FMA, which attempt to have an increased control over uncertainty. This is because the SMD model takes into account that a good scenario will receive more stock than expected, while the FMD and FMA only take into account that there will be no losses. The former situation is reflected in the fact that 48.8% of the cost of buying unexpected logs is incurred during actual operation in the SMD model, while the same percentage is only 5.96% and 1.52% in the FMD and FMA models respectively. The real operation is simulated by the operational model in the rolling horizon scheme shown in Figure 2.1.
4. The SMD and FMD models are not capable of keeping their stock stable over time. This is because demand for certain types of lumber in the long term is less than what is produced when cutting the logs.

The results obtained show that in every model, aggregating cuts translates to large incurred costs for the company (over 15% in both models), but the difference between each inventory system used does not seem to increase cost significantly (FMD is 1.62% more costly than SMD). The decision between using FMD or SMD lies on the basis of how much the company values increased control over uncertainty, as the SMD model is less costly, but purchases more last-minute logs than the FMD model in order to satisfy demand. Having a better estimate of stock needed at the start of a month allows the sales department to be more confident in offering contracts with clients and to better understand the functioning of the plant during this period. Whether this knowledge is sufficient to prefer FMD to SMD is up to the company.

Naturally, the results can change if other data is used when simulating the sawmill problem. Nevertheless, we consider our simulations to be valuable for a company's planning by shedding light on certain behaviors that follow if a certain model were to be used to plan operations.

### 1.5 Future Work

The most important further work to be done on this topic is using data from an actual sawmill operation. Using real data would help to understand the computational complexity of the models presented in the article and to develop scenarios adequate for the use of the rolling horizon methodology.

A second work proposed is to carry out a sensitivity analysis of the models, which would answer the following two questions: 1.- How does the solution of a model change using different levels of uncertainty?, 2.- How does a solution behave when x% of uncertainty was expected and the actual operation has y% of uncertainty?

Given the small amount of variables and restrictions used in the models presented in the article (FMA, FMD, SMA, SMD), discussing computational complexity and methods of solution was not relevant. In turn, if hundreds or even thousands of variables or hundreds or thousands of uncertainty scenarios would have been used, the method of solution would have been relevant. In the presented article the Sample Average Approximation (SAA) method was used to solve the stochastic two-stage models Birge and Louveaux (1997). The basis of SAA is to solve in a single optimization model the entire stochastic two-stage problem. Methods such as L-Shaped (Birge and Louveaux, 1997, p.156-158) treat every uncertainty scenario as an independent problem and obtained solutions are fed back to a guide problem, that is, they divide the problem into several sub-problems and a guide problem, which are used in an iterative scheme. Large-scale problems that cannot be dealt with using the SAA method can be solved using the L-Shaped method.

One idea we are developing is using first order optimization (Nemirovski et al. (2009), Lan (2012), Nesterov (2013)) in order to solve the SMD model using a computational cluster. The first order methods, like the L-Shaped method, independently solve each uncertainty scenario in independent optimization problems and use the results of these in a projection problem, which feeds back to them in turn. In the SMD model the projection problem is resolved using a simple heuristic which makes it ideal for solving in a cluster of hundreds of processors, as each processor can solve a problem associated with a different uncertainty scenario, and as such the greater part of the computational load of the SMD model would be perfectly parallelized.

# 2. INTERTEMPORAL STOCHASTIC SAWMILL PLANNING: MODELING AND MANAGERIAL INSIGHTS

## 2.1 Introduction

Intertemporal decision problems are common in many areas of management. A typical intertemporal problem is planning production using monthly data, while the actual production takes place on a daily or weekly basis. In this sense, problems such as these present different levels of aggregation in the different time spans that are adopted. The aggregation of data and other elements is used for two main reasons. The first is because no precise or detailed information is available, especially when referring to the future. The second reason is in order to simplify the problem. One of the main challenges when working with intertemporal problems is achieving a certain level of consistency between decisions that are made using different levels of aggregation and in different time horizons. While an optimization model might predict excellent performance using aggregated monthly data, if demand and production have to be decided on a daily or weekly basis then the detailed production plan might be a complete failure. Furthermore, various sources of uncertainty add to the lack of information, making it difficult to make truly consistent decisions.

This study focuses on questions regarding intertemporal consistency in the context of a real problem taken from the forest industry. The problem consists of planning operations at a sawmill. In this case, the supply of raw materials and manual labor is planned on a monthly basis, whereas production is planned from week to week, depending on the actual availability of the raw materials. This immediately raises questions regarding consistency; especially as monthly planning uses aggregated information and therefore the actual supply of raw materials might differ from the original plan.

Forestry companies have been active users of Operations Research methodologies. In fact, Optimization and Simulation have been widely used in the forestry industry over the last 50 years (See, for example, D'Amours et al. (2009), Rönnqvist (2003)). In this sense, several issues have already been addressed. These issues include how to make best use of the land and improve planting decisions, as well as how to build roads in order to optimize the transportation of logs from the forests to the sawmill/pulp plants and other facilities. Optimization and Simulation has also been used to look at production planning in sawmills, pulp plants and processed wood facilities, in order to satisfy demands for products in different time periods. Many of such applications are described in D'Amours et al. (2009), Rönnqvist (2003), and Epstein et al. (2010).

According to Rönnqvist (2003), decisions in forestry optimization can be divided into three categories. The first of these are strategic decisions, which have a long-term effect (several decades), such as planting and facility location. The second category is tactical decisions, involving periods of approximately 6 months to 5 years, such as harvesting planning. The final category consists of operational decisions, which cover a time span ranging from a few days to several weeks and determine the details of the operations. Given this, several issues arise regarding intertemporal planning, and coordinating these decisions represents a significant problem.

The problem of sawmill production planning is particularly challenging because of various different complexities. With this type of problem, a sawmill company has to decide on the amount and type of logs that it has to buy in order to satisfy a given demand for lumber (the final product). Lumber is produced when logs are cut following certain patterns in order to obtain boards. The yield is the amount and type of lumber that is produced when a cubic meter of a given type of log is cut using a certain cutting pattern. One common difficulty is the level of uncertainty involved in predicting demand. However, there is also variability among production yields because of the inherent irregularities that come from the biological nature of the raw material. Furthermore, sawmills are inserted in a network of complex logistic operations, which can lead to additional variability.

Mathematical models for decision making in a sawmill plant are not new. For example, a model is presented in Maturana et al. (2010) to decide which patterns (cuts) will be applied over a six-week period. In this study, the log supply is fixed and there is no uncertainty in the data (yields, demands). In Kazemi Zanjani et al. (2010) the situation is different; the yields are considered uncertain and the problem is modeled as a two-stage problem.

This problem has also been studied over longer periods of time, such as Alvarez and Vera (2014) and Zanjani et al. (2010), where operations were modeled over a period of several months and years. In Alvarez and Vera (2014) the amount of logs to be purchased is not fixed and there is no uncertainty in the log supply (i.e. the amount and type of logs that were ordered matched those that were received). In Zanjani et al. (2010) uncertainty was simulated for both the yield and the demand, while log supply was considered as fixed for each period. The first problem was solved using robust optimization, while the latter was a multistage stochastic model that was solved using a scenario approach.

In this study, uncertainty will not be considered for yield or demand. Instead, it will address short-term uncertainty in the supply of raw materials. Supply uncertainty and variability occur because the purchase of logs involves cutting down certain areas of a forest and the harvest schedule might not be in sync with the demand. Additionally, logistic considerations in the forest operations might lead to changes in the harvest and transportation schedule and the demand for raw material might not be met exactly. Therefore, only an estimate can be given for the amount and type of logs that will be obtained (to the best of our knowledge, this type of uncertainty has yet to be addressed by the literature). This particular situation was observed by one of the authors in a large forest company in Chile and prompted the questions that are explored in this paper.

To help coordinate decisions, we have modeled the hierarchical planning process using a two-stage stochastic approach. This approach uses tactical decisions to calculate the log requirements. Using recourse, these requirements are calculated by taking into account the potential impact they will have on operational decisions, which in turn consider uncertainty in the actual supply. The two-stage model is then simulated in a Rolling Horizon (RH) setting.

The paper is organized as follows: The second section presents an introduction to intertemporal planning problems and looks at how they can be modeled using Stochastic

Optimization. The third section explains the sawmill planning problem that is central to this study and details the models that are used to solve it. Section 2.4 explains the RH method, as well as detailing its implementation and how the demand and log supply scenarios were defined. Section 2.5 presents the computational results for the sawmill planning models and provides managerial insights. Finally, we present our conclusions and recommendations for future work in section 2.6.

### 2.2 Intertemporal Planning and multistage decisions

Intertemporal methods focus on using suitable techniques to allow long-term aggregate planning to "communicate" with short-term disaggregated planning. The popular "Rolling Horizon" (RH) method is one way to achieve this and is widely used in practice. This method supposes that aggregated information is available for long-term planning and that when the short term arrives, real or more up-to-date information can be used for detailed (disaggregated) planning. The results of this detailed planning, as well as any new information, allow the assumptions for the aggregated planning to be updated, with the plan now covering a horizon that has advanced one period in time. An example of a study involving the RH method can be found in Sethi and Sorger (1991).

One of the most obvious ways of modeling these relations is to acknowledge the hierarchy of decisions in different time horizons. Hierarchical planning (see Haas et al. (1981) and Bitran et al. (1982)) is an approach that was created for solving problems that are too large or complex to be solved by a computer, or that are logically solved by a company in stages. In this approach, an aggregated problem is first solved before the solution to this problem is then disaggregated and used to solve a more detailed problem, which in turn represents the real problem. Zipkin (1980b) addressed the issue of obtaining solutions from an aggregated problem and developed bounds to show when a solution to an aggregated problem can provide feasible solutions to a disaggregated problem. Recent studies using this approach include Aghezzaf et al. (2011) and De Araujo et al. (2007).

Another way to study intertemporal problems involving uncertainty is to use Robust Optimization. This approach is typically used for problems where it is only known that the uncertain parameters can take values within a certain interval range (Soyster (1973), Ben-Tal and Nemirovski (1998), and Bertsimas and Sim (2004)). An example of how Robust Optimization is used in an intertemporal problem can be found in Alvarez et al. (2015). In the aforementioned study Alvarez et al. (2015), an aggregated planning problem was solved by calculating the log requirements. The solution was then used in a short-term operational problem involving uncertainty. This study illustrated that increased uncertainty could lead to operational infeasibility or excessive costs. Robust Optimization is used to compute tactical plans, which reveal improved empirical feasibility when the robust solutions are used in the operational problem. However, there are alternatives to Robust Optimization that also help coordinate such intertemporal decisions. In the present study, we use stochastic two-stage formulations. In fact, multistage formulations have already been used in the forest industry (a good example is Kazemi Zanjani et al. (2010)). These methods, which are studied in detail in Birge and Louveaux (1997), assume that decisions are taken in different time periods.

The difference between multistage formulations and the Rolling Horizon method is that the former incorporates beliefs as to what could happen in the future. Those beliefs are represented as different scenarios, which are assigned estimated probabilities of occurrence. Rolling Horizon is a method for simulating the decision making processes using the models over time. However, techniques to use uncertainty information in RH have also been developed in Sethi and Sorger (1991) and in Alden and Smith (1992). A relatively recent review paper about this technique is Sahin et al. (2013).

### 2.2.1 Two stage formulations

In two-stage formulations, the first stage variables are decisions that must be taken "here and now", while second stage variables are decisions that can be taken when "the data is revealed". This approach is stochastic because the formulation includes "beliefs" as to what the values of the uncertain parameters could be. These beliefs can be discrete or continuous. In the former case they are called discrete scenarios, while in the latter they follow a continuous probability distribution. A two-stage problem formulation can be stated as the following:

$$\begin{aligned} \min \ \ & c^T x + Q(x) \\ & Ax \leq b \end{aligned} \tag{2.1}$$

Where $Q(x) = \mathbb{E}\left(\varphi(x,\xi)\right)$ is called the recourse of the problem. $\xi$ represent the uncertainty, and $\varphi(x,\xi)$ is defined here as:

$$\begin{aligned} \varphi(x,\xi) = \min \ \ & d_\xi^T y \\ D_\xi x + E_\xi y & \geq e_\xi \end{aligned} \tag{2.2}$$

Here $A$, $D_\xi$, $E_\xi$ are real matrices and $d_\xi, e_\xi$ are vectors, all of them with appropriate dimensions (the subindex $\xi$ represents the dependence of the data on the uncertainty characteristics). Parameters shown in (2.2) may suffer from uncertainty depending on the specific problem. A problem is said to have fixed recourse when the matrix E is not subject to uncertainty. One of the most common and widely-used methods for solving two-stage formulations is Sample Average Approximation (SAA). This method creates one set of second stage variables for every possible scenario. It then solves a single large linear problem containing the information for the whole problem. The disadvantage of the SAA method is that it can be too expensive to solve if there are many scenarios or if the size of the second stage is large. Furthermore, in the case of continuous uncertainty, a sample of $\xi$ should be chosen in the hope that it accurately represents the uncertainty.

The SAA formulation of (2.1) with $n$ scenarios ($\xi_1$, .., $\xi_n$) with associated probabilities $(p_1, .., p_n)$ is:

$$\begin{aligned} \min \;\; c^T x + \sum_{i=1}^{n} p_i d_{\xi_i}^T y_{\xi_i} & \\ Ax \le b & \\ D_{\xi_1} x + E_{\xi_1} y_{\xi_1} \ge e_{\xi_1} & \\ \vdots & \\ D_{\xi_n} x + E_{\xi_n} y_{\xi_n} \ge e_{\xi_n} & \end{aligned} \tag{2.3}$$

This formulation can be tackled using Large Scale Optimization methods. Furthermore, the original formulation (2.1) can be also solved using other methods such as stochastic first order methods, as described in Birge and Louveaux (1997).

### 2.3 Modeling the Intertemporal Sawmill Planning Problem

The sawmill planning problem studied here can be seen as two different problems/models that should be solved consecutively and in the appropriate order. Both models are production planning problems. The objective in both models is to satisfy the demand for lumber at the lowest possible cost. The first model uses monthly information to determine the amount of logs to purchase and labor to hire in order to satisfy demand. In the second model, the logs and labor assigned by the first model will be used as input for the weekly planning during the first month. Extra logs and labor could also be added in the second model at a higher cost, or some capacity could also be outsourced. Inventory, capacity and others constraints are also modeled in both models.

The problem with using two different models in different time horizons is how to have them communicate with each other and how to coordinate decisions. Good coordination and consistency is needed so that the planning solution can be used in the weekly model without producing significant extra costs for the company. In our particular situation, weekly log supply is uncertain and we therefore need a way to incorporate the potential cost of inconsistencies into the planning model. As highlighted in the introduction, there are different ways to link these two models in order to ensure consistency. In this particular case, the actual log supply is only known when the logs have arrived at the sawmill. This led the authors to use two-stage decision models based on the feedback they can provide through the recourse function.

Below we present various models with certain variations. The main models will be called the First Model and the Second Model. Each of these models includes a version in which several cutting patterns can be used to process a log. There is also another version of each model where only an average cutting pattern is used. These models will therefore be called the First Model with Disaggregation (FMD), the First Model with Aggregation (FMA), the Second Model with Disaggregation (SMD) and the Second Model with Aggregation (SMA). In all of the models (FMA, FMD, SMD and SMA) the number of logs to purchase and amount of labor to hire are "here and now" decisions that are made on a monthly basis. Furthermore, another important difference between the models is the way in which they link the operational and tactical inventories.

The first models (FMA and FMD) ensure feasibility in average terms, *i.e.*, that an aggregated monthly model of the problem will be feasible. It also assumes future uncertainty represented in the cost of extra log purchases, additional outsourcing and demand postponement for the first month. However, there is no link between the inventories left at the end of the fourth week of the first month and the initial inventory for the second month. This model can be seen as having a long first stage, together with a short second stage. Monthly inventories are considered for the four months in the first stage, while a weekly inventory is considered in detail in the second stage for the first month.

The second models (SMA and SMD) link the inventories left at the end of the fourth week of the first month with the opening inventories of the second month. However, it does not ensure that demand will be fulfilled by basic production should the average supply scenario occur. Indeed, it may also require overtime and/or demand postponement. The second models can be viewed as a big recourse problem with a box-constrained domain for the first stage.

In addition to the aforementioned properties, all of the models also take into consideration inventory holding costs, as well as demand postponement costs. For the first detailed month, we also take into account the extra cost for log purchases and overtime, as well as the cost of outsourcing.

The FMD and SMD models are shown below. Following this, we explain the differences between these models and the FMA and SMA models.

a) FMD and SMD models

The two models use the same sets and have many common parameters. Given this, we first describe the shared sets and parameters, before describing the variables for each model and its constraints. In this work, the units for lumber and logs are cubic meters and costs are given in dollars.

**Sets used in both models:**

- $i \in \{1, 2, 3, 4\}$ indicates the weeks of the first month.
- $t \in \{1, \ldots, 4\}$ indicates the months.
- $c \in C$ indicates the different types of logs.
- $e \in E_c$ are the cutting patterns for log type $c$.
- $m \in M$ corresponds to the various types of lumber produced by the company.

**Shared first stage parameters:**

- $W_t$: Cost of each hour of manual labor hired in month $t$.
- $Craw_{ct}$: Cost of a type $c$ log bought in advance during period $t$.
- $PC$: Maximum monthly processing capacity of the plant, in cubic meters of logs.
- $UX$, $LX$: Upper and lower bound on number of hours of manual labor available.
- $ME_{ct}$: Upper bound on the amount of type $c$ logs the company can buy in period $t$.

**Shared second stage parameters:**

- $\alpha_m$: Percentage of demand for lumber $m$ for which the company can postpone delivery in a given week or month, depending on the case.
- $\beta'_{mi}$: Cost of delaying product $m$ in week $i$.
- $Ceraw'_{ci}$: Cost of buying an extra type $c$ log in week $i$.
- $EW'_i$: Cost of hiring one extra hour of manual labor in week $i$.
- $\rho_{cc'}$: Fraction of type $c$ logs that are ordered and substituted for type $c'$ logs, in any week during the first month.
- $d'_{mi}$: Demand for type $m$ lumber in week $i$.
- $O_c$: Outsourcing cost for one cubic meter of type $c$ lumber.

**Shared monthly parameters** (in the FMD model these parameters are used in the first stage; in the SMD model they are used in the second stage):

- $h_{mt}$: Inventory holding cost for type $m$ lumber in month $t$.
- $hw_{ct}$: Inventory holding cost for a type $c$ log in month $t$.
- $d_{mt}$: Demand for type $m$ lumber in month $t$.

**General parameters:**

- $\phi$: Productivity of manual labor, in cubic meters of logs processed per hour of manual labor.
- $Y_{cem}$: Yield of the cutting pattern: amount of type $m$ lumber obtained when a type $c$ log is processed using cutting pattern $e$.
- $z0_m = z'_{m0}$: Initial inventory of type $m$ lumber.
- $w_{c0} = w'_{c0}$: Initial inventory of type $c$ logs.
- $b'_{m0}$: Initial amount of type $m$ lumber already delayed at the beginning of the planning horizon.

<u>FMD Model</u>

**First stage variables:**

- $X_t$: Number of hours of manual labor assigned for month $t$.
- $raw_{ct}$: Number of type $c$ logs procured for month $t$. These are the logs that are ordered in advance.
- $r_{cet}$: The amount of type $c$ logs processed with cutting pattern $e$ in month $t$.
- $or_{cet}$: The amount of type $c$ logs outsourced to be processed with cutting pattern $e$ in month $t$.
- $z_{mt}$: Inventory of type $m$ lumber in month $t$.
- $w_{ct}$: Inventory of type $c$ logs in month $t$.

**Second stage variables.** To differentiate second stage variables from first stage ones, the second stage variables are indexed with a $'$.

- $r'_{cei}$: Type $c$ logs processed with cutting pattern $e$ in week $i$.
- $eraw'_{ci}$: Extra amount of type $c$ logs bought in week $i$.
- $or'_{cei}$: Amount of type $c$ logs outsourced to be processed with cutting pattern $e$ in week $i$.
- $ex'_i$: Extra hours (overtime) assigned at week $i$.
- $b'_{mi}$: Amount of type $m$ lumber in week $i$ whose delivery is postponed to the following week.
- $RR'_{ci}$: Actual amount of type $c$ logs received in week $i$. This is determined by the first stage variables.
- $z'_{mi}$: Inventory of type $m$ lumber in week $i$.
- $w'_{ci}$: Inventory of type $c$ logs in week $i$.
- $extraOr'$: The amount of additional outsourced work beyond what was planned in the first stage.

The only extra parameter that the FMD model needs is $O$ cost. This is the cost of the outsourced cuts that were not anticipated in the first stage.

**First stage constraints:**

1. The labor assigned in month $t$ must be between bounds. This constraint is a simplification of the ability of the company to hire and fire workers.

$$LX \leq X_t \leq UX \ \ \forall t \in [1, \ldots, 4] \tag{2.4}$$

2. Production cannot exceed the sawmill's maximum capacity (capacity in terms of machines and physical constraints in general).

$$\phi \cdot X_t \leq PC \ \ \forall t \in [1, \ldots, 4] \tag{2.5}$$

3. The amount of type $c$ logs purchased in month $t$ cannot exceed a given upper bound.

$$raw_{ct} \leq ME_{ct} \ \ \forall c \in C, t \in [1, \ldots, 4] \tag{2.6}$$

4. Monthly log inventory. Here, $w_{c0}$ is the initial inventory.

$$w_{ct} = w_{c,t-1} + raw_{ct} - \sum_{e \in E_c} (r_{cet} + or_{cet}) \ \ \forall c \in C, t \in [1, \ldots, 4] \tag{2.7}$$

5. Monthly lumber inventory. Here, $z_{m0}$ is the initial inventory.

$$z_{mt} = z_{m,t-1} + \sum_{c \in C} \sum_{e \in E_c} Y_{cem} \cdot (r_{cet} + or_{cet}) - d_{mt} \ \ \forall m \in M, t \in [1, \ldots, 4] \tag{2.8}$$

6. The amount of logs processed in month $t$ is bounded by the productivity of the available labor.

$$\sum_{c \in C} \sum_{e \in E_c} r_{cet} \leq \phi X_t \ \ \forall t \in [1, \ldots, 4] \tag{2.9}$$

7. The amount of logs processed in month $t$ cannot exceed the plant's maximum capacity. Here, capacity is expressed in terms of machinery and infrastructure.

$$\sum_{c \in C} \sum_{e \in E_c} r_{cet} \leq PC \ \ \forall t \in [1, \ldots, 4] \tag{2.10}$$

The first stage objective function considers costs associated with lumber inventory, the purchase of logs, outsourcing, log inventory, labor, and the cost associated with the recourse of the short-term problem. This final item corresponds to the expected cost of the second stage objective function, explained below. Here, $X$ and $raw$ are the first stage variables.

$$\sum_{t=1}^{4}\left(\sum_{m\in M}(h_{mt}z_{mt})+\sum_{c\in C}\left(Craw_{ct}raw_{ct}+\sum_{e\in E_c}O_c or_{cet}+hw_{ct}w_{ct}\right)+W_tX_t\right)+\mathbb{E}(Q(X,raw,\xi)) \tag{2.11}$$

**Second stage constraints:**

1. Amount of type $c$ logs that were received in week $i$.

$$RR'_{ci}=\sum_{c'\in C}\rho_{cc'}raw_{c'1}\ \ \forall c\in C, i\in[1,\dots,4] \tag{2.12}$$

2. Weekly log inventory for the weeks of the first month. The logs can be processed locally or outsourced. Recall that the parameters $w_{c0}$ and $w'_{c0}$ represents the same quantities.

$$w'_{ci}=w'_{c,i-1}+RR'_{ci}+eraw'_{ci}-\sum_{e\in E_c}(r'_{cei}+or'_{cei})\ \ \forall c\in C, i\in[1,\dots,4] \tag{2.13}$$

3. Weekly lumber inventory for the four weeks of the first month, considering outsourcing and postponement.

$$z'_{mi}=z'_{m,i-1}+\sum_{c\in C}\sum_{e\in E_c}Y_{cem}\cdot(r'_{cei}+or'_{cei})+b'_{mi}-b'_{m,i-1}-d'_{mi}\ \ \forall m\in M, i\in[1,\dots,4] \tag{2.14}$$

4. The amount processed in week $i$ is bounded by the capacity calculated from available labor, including overtime.

$$\sum_{c\in C}\sum_{e\in E_c}r'_{cei}\le\phi\left(\frac{X_1}{4}+ex'_i\right)\ \ \forall i\in[1,\dots,4] \tag{2.15}$$

5. The amount processed in week $i$ cannot exceed the plant's maximum capacity.

$$\sum_{c\in C}\sum_{e\in E_c}r'_{cei}\le\frac{PC}{4}\ \ \forall i\in[1,\dots,4] \tag{2.16}$$

6. The proportion of demand postponed to the next period cannot cannot exceed a given fraction of the demand for said period.

$$b'_{mi} \leq \alpha_m d'_{mi} \;\; \forall m \in M, i \in [1, \ldots, 4] \tag{2.17}$$

7. The balance of processed work, including outsourcing.

$$\sum_{i=1}^{4} \sum_{c \in C} \sum_{e \in E_c} (or'_{cei}) - extraOr' \leq \sum_{c \in C} \sum_{e \in E_c} or_{ce1} \tag{2.18}$$

The second stage objective function penalizes for the additional costs that were not forecasted at the tactical level. Following this logic, the idea is not to include here inventory costs, purchase costs (logs and labor) or processing costs. The costs that are included are overtime, delayed products, extra logs that are purchased and the cost of outsourcing.

$$\sum_{i=1}^{4} \left( EW'_i ex'_i + \sum_{m \in M} (\beta'_{mi} b'_{mi}) + \sum_{c \in C} (Ceraw'_{ci} eraw'_{ci}) \right) + O \cdot extraOr' \tag{2.19}$$

SMD Model

**First stage variables:**

- $X_t$: Number of hours assigned for month $t$.
- $raw_{ct}$: Amount of type $c$ logs purchased for month $t$. These are the logs that are ordered in advance.

**Second stage variables, for the first four weeks.**

- $r'_{cei}$: Amount of type $c$ logs processed with cutting pattern $e$ in week $i$.
- $or'_{cei}$: Amount of type $c$ logs outsourced to be processed with cutting pattern $e$ in week $i$.
- $ex'_i$: Extra hours (overtime) assigned at week $i$.
- $eraw'_{ci}$: Amount of extra type $c$ logs purchased for week $i$.

- $b'_{mi}$: Amount of type $m$ lumber in week $i$ postponed to the next period.
- $RR'_{ci}$: Actual amount of type $c$ logs received in week $i$.
- $z'_{mi}$: Inventory of type $m$ lumber in week $i$.
- $w'_{ci}$: Inventory of type $c$ logs in week $i$.

**Second stage variables for the second to the fourth months.**

- $r_{cet}$: Amount of type $c$ logs processed with cutting pattern $e$ in month $t$ at the sawmill.
- $or_{cet}$: Amount of type $c$ logs outsourced to be processed with cutting pattern $e$ in month $t$.
- $rraw_{ct}$: Extra type $c$ logs ordered for month $t$.
- $ex_t$: Extra labor hired for month $t$.
- $b_{mt}$: Amount of type $m$ lumber in month $t$ postponed to the following period.
- $z_{mt}$: Inventory of type $m$ lumber in month $t$.
- $w_{ct}$: Inventory of type $c$ logs in month $t$.

Only two new parameters must be added for the SMD model. In this model, extra logs and additional labor can be purchased in the months included in the second stage. The cost of buying a type $c$ log in month $t$ in the second stage is $Crraw_{ct}$ and hiring one hour of labor in month $t$ is $EW_t$. Here, we assume that the weekly storage costs for the first month are one quarter of the monthly storage costs for that month.

**First Stage constraints for the SMD model:**

1. The number of hours of manual labor hired in month $t$ must be between a lower and an upper bound

$$LX \leq X_t \leq UX \ \ \forall t \in [1, \ldots, 4] \tag{2.20}$$

2. Production cannot exceed the sawmill's maximum capacity.

$$\phi X_t \leq PC \;\; \forall t \in [1, \ldots, 4] \tag{2.21}$$

3. The amount of type $c$ logs that were bought in month $t$ cannot exceed an upper bound.

$$raw_{ct} \leq ME_{ct} \;\; \forall c \in C, t \in [1, \ldots, 4] \tag{2.22}$$

The first stage objective function is the cost of labor and logs purchased during this stage, as well as the value of the recourse. $X$ and $raw$ values for the first month are used in the recourse.

$$\sum_{t=1}^{4} \left( \sum_{c \in C} (Craw_{ct} raw_{ct}) + W_t X_t \right) + \mathbb{E}(Q(X, raw, \xi)) \tag{2.23}$$

**Second stage constraints:**

1. Amount of type $c$ logs received in week $i$.

$$RR'_{ci} = \sum_{c' \in C} \rho_{cc'} raw_{c'1} \;\; \forall c \in C, i \in [1, \ldots, 4] \tag{2.24}$$

2. Weekly log inventory for the weeks of the first month, considering internal processing and outsourcing.

$$w'_{ci} = w'_{c,i-1} + RR'_{ci} + eraw'_{ci} - \sum_{e \in E_c} (r'_{cei} + or'_{cei}) \;\; \forall c \in C, i \in [1, \ldots, 4] \tag{2.25}$$

3. Weekly lumber inventory for the weeks of the first month, considering the amount of lumber that is outsourced and/or postponed. $z'_{m0}$ and $b'_{m0}$ are initial inventories.

$$z'_{mi} = z'_{m,i-1} + \sum_{c \in C} \sum_{e \in E_c} Y_{cem} \cdot (r'_{cei} + or'_{cei}) + b'_{mi} - b'_{m,i-1} - d_{mi} \;\; \forall m \in M, i \in [1, \ldots, 4] \tag{2.26}$$

4. Processing capacity in week $i$ is determined by productivity of available labor, including overtime.

$$\sum_{c \in C} \sum_{e \in E_c} r'_{cei} \leq \phi \left( \frac{X_1}{4} + ex'_i \right) \;\; \forall i \in [1, \ldots, 4] \tag{2.27}$$

5. The amount processed in week $i$ is limited by the plant's capacity.

$$\sum_{c\in C}\sum_{e\in E_c} r'_{cei} \leq \frac{PC}{4} \ \ \forall i \in [1,\ldots,4] \tag{2.28}$$

6. The amount of type $m$ lumber postponed in week $i$ to the following period cannot exceed a fraction of that week's demand.

$$b'_{mi} \leq \alpha_m d_{mi} \ \ \forall m \in M, i \in [1,\ldots,4] \tag{2.29}$$

7. Inventory of logs for the second to the fourth months. The initial inventory for the second month is the inventory left after the fourth week of the first month.

$$w_{c2} = w'_{c4} + raw_{c2} + rraw_{c2} - \sum_{e\in E_c}(r_{ce2} + or_{ce2}) \ \ \forall c \in C \tag{2.30}$$

$$w_{ct} = w_{c,t-1} + raw_{ct} + rraw_{ct} - \sum_{e\in E_c}(r_{cet} + or_{cet}) \ \ \forall c \in C, t \in [3,4] \tag{2.31}$$

8. Inventory of lumber for the second to fourth months. The initial inventory for the second month is the inventory left after the fourth week of the first month.

$$z_{m2} = z'_{m4} + \sum_{c\in C}\sum_{e\in E_c} Y_{cem}\cdot(r_{ce2} + or_{ce2}) + b_{m2} - b'_{m4} - d_{m2} \ \ \forall m \in M \tag{2.32}$$

$$z_{mt} = z_{m,t-1} + \sum_{c\in C}\sum_{e\in E_c} Y_{cem}\cdot(r_{cet} + or_{cet}) + b_{mt} - b_{m,t-1} - d_{mt} \ \ \forall m \in M, t \in [3,4] \tag{2.33}$$

9. The amount processed in month $t$ in the sawmill cannot exceed the maximum capacity, calculated from the available labor, including overtime.

$$\sum_{c\in C}\sum_{e\in E_c} r_{cet} \leq \phi\,(X_t + ex_t) \ \ \forall t \in [2,\ldots,4] \tag{2.34}$$

10. The amount processed in month $t$ cannot exceed plant capacity.

$$\sum_{c\in C}\sum_{e\in E_c} r_{cet} \leq PC \ \ \forall t \in [2,\ldots,4] \tag{2.35}$$

11. The amount of type $m$ lumber postponed in month $t$ cannot exceed a fraction of the monthly demand.

$$b_{mt} \leq \alpha_m d_{mt} \;\; \forall m \in M, t \in [2, \ldots, 4] \tag{2.36}$$

The second stage objective function considers the following costs for the first four weeks: postponement, lumber inventory, extra logs purchased, log inventory, outsourcing and overtime. The costs for the second to fourth months are the following: postponement, lumber inventory, log inventory, extra logs purchased, and additional labor.

$$\sum_{i=1}^{4} \left( \sum_{m \in M} \left( \beta'_{mi} b'_{mi} + \frac{h_{m1}}{4} z'_{mi} \right) + \sum_{c \in C} \left( Ceraw_{ci} eraw'_{ci} + \frac{hw_{c1}}{4} w'_{ci} + \sum_{e \in E_c} \left( O_c or'_{cei} \right) \right) + EW_i ex'_i \right) + \\ + \sum_{t=2}^{4} \left( \sum_{m \in M} \left( \beta_{mt} b_{mt} + h_{mt} z_{mt} \right) + \sum_{c \in C} \left( hw_{ct} w_{ct} + Crraw_{ct} rraw_{ct} + \sum_{e \in E_c} O_c or_{cet} \right) + EW_t ex_t \right) \tag{2.37}$$

<u>FMA and SMA models</u>

As explained previously, the FMA and SMA models are almost identical to the FMD and SMD models described above. The only difference is that in some parts the FMA and SMA models can only use one average cut for each log. In those parts, the yield that is obtained when cutting one cubic meter of a type $c$ log is the average yield of the cuts in the set $E_c$.

The FMA model only uses the average yield in the first stage of the FMD model. In other words, variables $r_{cet}$ and $or_{cet}$ are changed for $r_{ct}$ and $or_{ct}$ and $Y_{cm}$ is used instead of $Y_{cem}$. The SMA model only uses the average cutting pattern in the second stage for the second to the fourth month.

### 2.3.1 Modeling log supply uncertainty

As mentioned previously, the uncertain effect we want to research in this study is related to irregularities in the actual supply of logs to the sawmill. In this setting, the quantities planned in advanced for the first month, $raw_{c1}$, might not be distributed appropriately during the four weeks of the month. Moreover, different factors might also affect the detailed distribution of the different types of logs. We explain how we model this below.

In an "ideal" situation, we would have that $RR'_{ci} = raw_{c1}/4$. The actual amount of type $c$ logs that will be received in week $i$ of the first month ($RR'_{ci}$) will, instead be given by the following expression:

$$RR'_{ci} = \sum_{c' \in C} \rho_{cc'} raw_{c1} \;\; \forall c \in C, i \in [1, \ldots, 4] \tag{2.38}$$

where $\rho_{cc'} \geq 0 \, \forall c, c' \in C \times C$ is the parameter that will define how the supply deviates from the ideal. We will represent two effects:

1. Due to various factors, forest companies can sometimes dispatch more or fewer logs than were actually requested.
2. A percentage of the type $c$ logs that are ordered can end up being substituted by type $c'$ logs.

The first effect is simulated by associating the parameters $\rho_{cc'} \;\; \forall c, c' \in C \times C$ with a realization of a uniform random variable. This uniform random variable is $u_1 \sim Unif(1 - \delta_1, 1 + \delta_1)$. Intuitively, the sawmill manager can receive a minimum of $(1 - \delta_1)\%$ of the logs that are ordered, or a maximum of $\delta_1\%$ additional logs. The second effect is simulated by assuming that $\rho_{c,c'} = u_{2,c,c'} \sim Unif(0, \delta_2) \, \forall c' \neq c$ with $\delta_2 > 0$ and maintaining the fact that in each row the sum of its members must be 0.25.

Both of these effects are then simulated together using $\rho_{c,c'} = u_1 \cdot u_{2,c,c'} \; \forall c' \neq c$ and $\rho_{c,c} = u_1 \times \left(0.25 - \sum_{\{c' \in C, c' \neq c\}} u_{2,c,c'}\right) \; \forall c \in C$. We latter explain how this uncertain behavior is introduced into our Rolling Horizon simulation.

## 2.4 Rolling Horizon Approach

In this section, we explain the Rolling Horizon (RH) method used in this study (Figure 2.1). The process solves the two-stage optimization model in a sequence, using Sample Average Approximation. This is done in order to obtain a tactical policy which (through the recourse) considers the response of the operational. Following this, the tactical policy is then evaluated using an operational model for the first month. Subsequently, the horizons “roll” over by a month. In this case, the test models that will be applied are the ones described in the previous section (FMA, SMA, FMD and SMD), while the operational model is explained in 2.4.3.

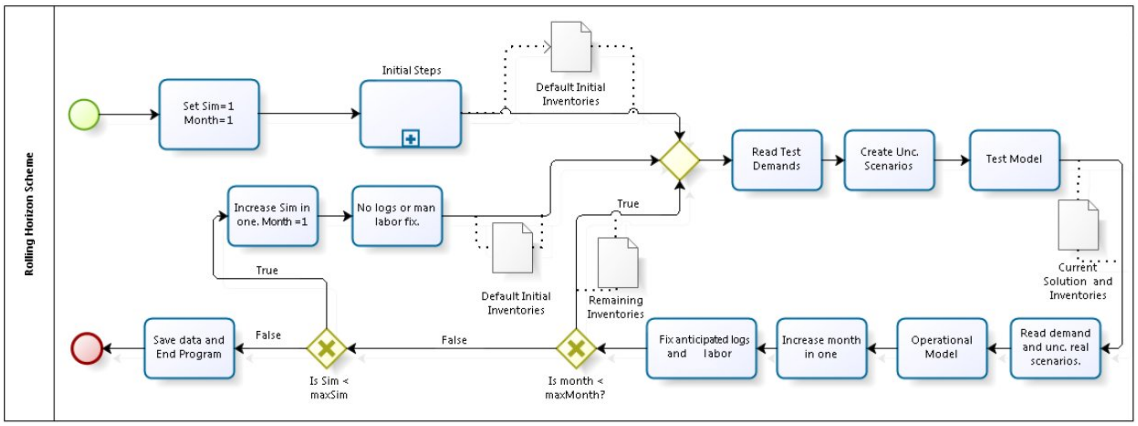


FIGURA 2.1. Rolling Horizon method applied in this work

### 2.4.1 Decision persistence

In a sawmill plant operation, the amount of logs ordered should have a certain level of continuity between one month and the followings. The same is true for labor. This continuity is represented in Figure 2.1 in the box “Fix anticipated logs and labor”. This continuity allows management to purchase extra logs or hire additional labor. This is done at a higher price, once an “anticipated” amount has been purchased/hired. This continuity is typically required in practice in order to reduce “nervousness”. In production planning, nervousness is observed when small changes in expected demands, or other external factors, lead to tremendous changes in the production plan. Correct handling of nervousness can be achieved by adapting the production plan to unexpected changes in such a way that it can be handled by the company. Studies relating to nervousness include Blackburn et al. (1986), de Kok and Inderfurth (1997), Kazan et al. (2000).

To model persistence/continuity, two new first stage variables $rraw1_{c,t}$ $\forall c \in C, t \in 1,2,3$ and $Xextra_t$ $t \in 1,2,3$ were created and added to the models. These new variables play the same role as the first stage variables $raw$ and $X$. The difference is that in any of the test models used here (FMA, FMD, SMA, SMD) the variables $raw$ and $X$ remain fixed for the first three months and free for the fourth month. The two new variables will not be fixed, but they will be more costly than $raw$ and $X$. Variables $rraw1$ and $Xextra$ represent the extra amount of logs and labor that are purchased/hired in the first stage. These variables are more expensive than $raw$ and $X$, but are cheaper than obtaining logs/labor in the recourse. Logs and labor purchased/hired in the recourse can be seen as last minute decisions, while $rraw1$ and $Xextra$ are decisions made with a certain amount of notice. As an example, the amount of type $c$ logs that the operational model receives from the first stage in month $t \geq 3$ comes from two variables. The first of these variables is $raw$, which are the logs ordered three months in advance. The second variable, $rraw1$ are the logs ordered during the same month, or one or two months in advance. We have defined the costs so that $rraw1_{c,t}$ will have a 25% premium, $rraw1_{c,t-1}$ 17.5% premium, and $rraw1_{c,t-2}$ a 10% premium. All premiums are calculated in relation to $raw$ costs. The same percentages and relationships are true for $Xextra$.

The inventories that are used as input in Figure 2.1 are lumber, logs and the amount that is delayed in the fourth week of the operational model. This last quantity should be satisfied the next time the operational model is executed.

### 2.4.2 Demand and Log Supply generation

To be able to compare the models, the same supply and demand scenarios were used in every test. Three types of demand instances, each one consisting of 10 independent simulations of 50 months of demand were used. To calculate $\mathbb{E}_{\xi}[Q(x)]$, each month in the test model had 96 logs supply scenarios. Each test model simulated four months of operation in order to plan accurately (the "current" month and the following three months). The demand that the model "sees" for the first month (*i.e.* the current month) is certain, but for the other months there is a degree of uncertainty. Each simulation used the same log supply scenarios. A different group of 96 log supply scenarios was used for each month, following the description given in subsection 2.3.1.

The demand faced by the company could be any of the three demand scenarios shown in Figure 2.2. Every demand scenario follows the same pattern: for the first 4 months there are no changes, there are then 8 months where a peak or some other behavior occurs, followed by 38 months of stability.

The demand scenarios in the RH method (Figure 2.1) are used in the following way: firstly, a demand scenario and a test model is chosen before executing the method. Secondly, it is supposed that the method is currently in month t. The test model will then "see" the actual demand for month $t$. For month $t+1$ the test model will receive the actual demands multiplied by a uniform random variable $Unif(0.95, 1.05)$ (the demand for each product is multiplied by a different realization of this random variable). The demand it "sees" for the month $t+2$ is constructed in the same way as for month $t+1$, but now the uniform random variable is $Unif(0.925, 1.075)$. The demand it "sees" for month $t+3$ is constructed in the same way as before, but using the random variable $Unif(0.9, 1.1)$. Following this idea, the test model will "see" the actual demand for the current month and only an estimate for the following three months.

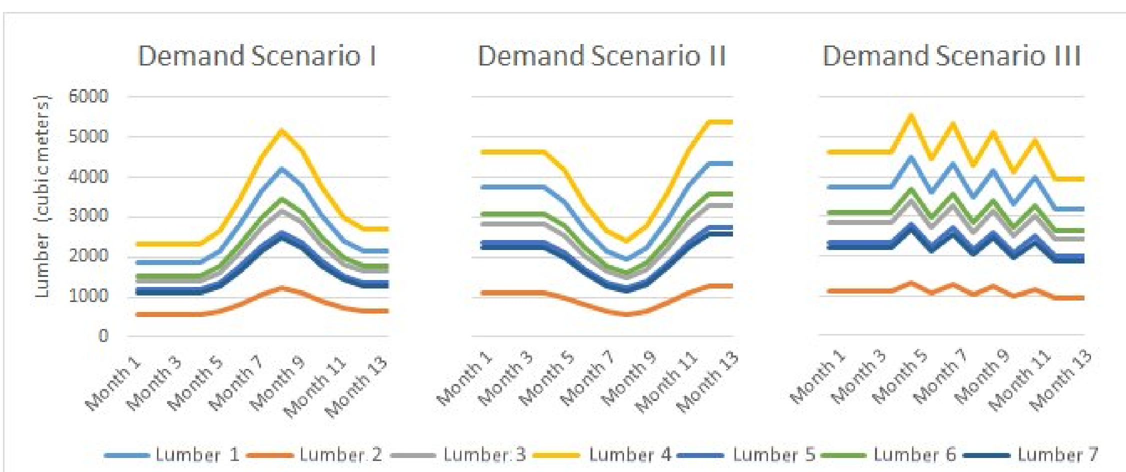


FIGURA 2.2. Demand Scenarios used in the RH scheme

### 2.4.3 Operational Model

The operational model used to test the result of the tactical policy in the first month will replicate the sawmill operation over one month, disaggregated into the four weeks of the month. For this operational model we use the second stage of the SMD model for the four weeks of the first month, but receive the logs and labor that were ordered in the test model as input. The amount of logs and labor received are the same as those calculated for the first month in the test model. All of the other parts, *i.e.*, the objective function, constraints, sets and variables, are the same as those used in the second stage of the SMD model for the four weeks of the first month. The operational model only uses one log supply scenario and sees the real demand for the four weeks of the first month. The log supply scenario used was calculated in the same way as the SAA scenarios.

## 2.5 Computational results and managerial analysis

We will now present our computational experience with the models. We tested the RH approach on a problem based on a real industrial situation, albeit simplified. Our test consists of six different types of logs and seven types of lumber. We also used four different cutting patterns for each type of log.

The whole RH method was developed and implemented in C++ and the optimization models were executed using Gurobi 5.6. The methods were executed on a computer with an Intel i5-2450M processor with 2.5GHz core speed and 4 GB of RAM.

Since the models were not particularly big, execution times are not relevant. However, it is worth noting that the FMD and FMA models scale much better than the SMD and SMA models if a solution method such as SAA is used. The difference between the models is the size of the recourse problem. With hundreds of cuts, logs, lumber types, SAA scenarios, as well as other possible constraints, the SMD and SMA models would have had much longer execution times than the FMD and FMA models.

Table 2.1 shows the costs generated by the models in the three demands scenarios shown in Figure 2.2. The cost consists of the purchase of logs and labor that are effectively

TABLA 2.1. Average monthly costs per demand scenario, in US$

| | Average Cost | SMD | SMA | FMD | FMA |
|---|---|---|---|---|---|
| Dem. S.1 | First 4 months | 3083178 | 4766701 | 3348211 | 3719232 |
| | Peak (5-12 months) | 4907265 | 6288658 | 4959859 | 5802120 |
| | Period of Stability (13-50 months) | 3655192 | 4722878 | 3694323 | 4158074 |
| Dem. S.2 | First 4 months | 3086009 | 4142369 | 3334553 | 3755871 |
| | Peak (5-12 months) | 2386849 | 3407162 | 2394311 | 2782534 |
| | Period of Stability (13-50 months) | 3635857 | 4699604 | 3682487 | 4166584 |
| Dem. S.3 | First 4 months | 3087634 | 4446804 | 3337649 | 3740286 |
| | Peak (5-12 months) | 3094691 | 3982264 | 3111759 | 3652852 |
| | Period of Stability (13-50 months) | 2685905 | 3468795 | 2715770 | 3048606 |

used in the current month, plus the costs incurred in the operational model. Recall that each demand scenario has three different sections: the first four months, the peak, and the period of stability (Table 2.1). We observe that the SMD model returns the best results in terms of costs in every demand scenario and for each section of the total horizon. In terms of performance, the SMD model, is then followed by the FMD model, the FMA model and, finally, the SMA model. Table 2.2 shows how the cost is distributed across the different categories for the first demand scenario. It can be seen that the cost of purchasing logs represents more than an 80% of the total. The remaining costs after logs and labor represent less than 10% of the total cost (demand scenario 2 and 3 reveal similar distributions). We can therefore see that, in general, the SMD and FMD models provide planning policies that lead to lower costs in the operations planning. This indicates that these models have a greater capacity for coordinating intertemporal decisions..

### 2.5.1 Managerial analysis of the first demand scenario

We will now look at the behavior of inventories and log purchases. The results will be focused on the two best models, SMD and FMD, as these outperformed the other two models. Tables 2.3 and 2.4 show the distribution of log purchases.

Several interesting insights can be obtained from these results:

TABLA 2.2. Average monthly costs, by origin, for each model in Demand Scenario I, in US$ M

| Costs | SMD | SMA | FMD | FMA |
|---|---|---|---|---|
| Total Cost of Purchasing Logs | 3.111M | 4.102M | 3.161M | 3.851M |
| Total Manual Labor | 0.503M | 0.795M | 0.521M | 0.521M |
| Log Inventory | 0.004M | 0.053M | 0.012M | 0.012M |
| Lumber Inventory | 0.176M | 0.027M | 0.163M | 0.001M |
| Backlog Cost | 0.001M | 0.000M | 0.001M | 0.000M |
| Outsourcing Cost | 0.013M | 0.000M | 0.011M | 0.000M |
| Total | 3.810M | 4.977M | 3.869M | 4.386M |

TABLA 2.3. Log purchase distribution between planned and unplanned purchases for Demand Scenario I

| Log Costs | SMD | SMA | FMD | FMA |
|---|---|---|---|---|
| Planned Log Purchase Cost | 85.2% | 37.2% | 85.7% | 87.3% |
| Unplanned Log Purchase Cost | 14.8% | 62.8% | 14.3% | 12.7% |
| → Extra Operational | 48.80% | 0.0% | 5.96% | 1.52% |
| → Extra First Month (First Stage) | 41.86% | 99.11% | 76.88% | 45.90% |
| → Months 2,3,4 (First Stage) | 9.333% | 0.89% | 17.17% | 52.58% |

1. The SMA model presents a significant problem, which explains its behavior. The model only has the first month with disaggregated cutting patterns. The SMA model therefore buys an excessive amount of logs to be used in the same month. This is reflected in the fact that 63% of the cost of purchasing logs comes from $rraw1$ logs to be used in the current month. The SMA model does not take into account that logs purchased for future months will also be processed in the operational model, which uses disaggregated cutting patterns. In other words, all logs have equally good yields regardless of the moment in which they were ordered.
2. The percentage of the cost for unplanned log purchases in the SMD model is similar to the FMA and FMD models. The difference lies in how this cost is distributed. The SMD model knows that there are favorable scenarios in which it will have a surplus of logs, a fact that the FMA and FMD models are not able to

TABLA 2.4. How the different types of logs were bought in the FMD and SMD models for Demand Scenario I

| | | Log Type 1 | Log Type 2 | Log Type 3 | Log Type 4 | Log Type 5 | Log Type 6 |
|---|---|---|---|---|---|---|---|
| SMD | raw | 0.0% | 5.9% | 0.0% | 14.0% | 71.4% | 8.7% |
| | rraw1 | 0.2% | 6.7% | 0.7% | 14.5% | 19.6% | 58.2% |
| | eraw | 7.9% | 24.5% | 0.0% | 34.4% | 2.2% | 31.0% |
| FMD | raw | 0.1% | 7.2% | 0.0% | 14.9% | 67.6% | 10.2% |
| | rraw1 | 37.3% | 0.2% | 0.0% | 6.3% | 23.2% | 33.0% |
| | eraw | 14.0% | 16.1% | 0.0% | 44.8% | 3.6% | 21.6% |

foresee. This can be seen in the fact that 48.8% of the unplanned cost comes in the "operational model" for the SMD model (in comparison to 5.86% and 1.52% for the FMD and FMA models, respectively). The SMD model works in a more "stochastic" way than the FMD and FMA models, which exercise more "control" over the uncertainty.

3. The fact that the FMA model does not see disaggregated cuts in the first month leads to a difference in the amount of $rraw1$ purchased by the FMD model (76.88%) and FMA model (45.9%) for the first month. From the recourse, the FMD and FMA models perceive when the operational model will probably need to procure extra logs. The difference is that the FMD model knows that one extra log purchased in the first month can be processed using one of several patterns, while the FMA model can only see the average cut. For this reason it is cheaper for the FMD model to buy $rraw1$ for the current month than the FMA model.

Table 2.4 shows that most of the planned logs that are purchased are type five, with type four a distant second.

a) How the models manage their inventories

The combined inventory costs (lumber + logs) over time are shown in Figure 2.3. A more detailed analysis (which is not included here due to space limitations) indicates that the SMD and FMD models have problems with lumber types one and two. The cutting patterns used to process the logs during the RH method produce more of those products than is required in the long run. This type of problem can only be perceived using an approach such as an RH simulation conducted over a long period of time.

The problem of an increasing inventory happens because four months is a relatively short time window. The FMD and SMD models do not react to this because in each time window the increase is only small (and therefore cheap). For this reason, the problem is only visible using simulations over a long period of time, such as the RH method. The use of the four-month time window stems from the actual industrial situation which serves as a basis for this study. In this case, a four month rolling horizon was adopted by the company. A larger time window will probably allow for a more stable process.

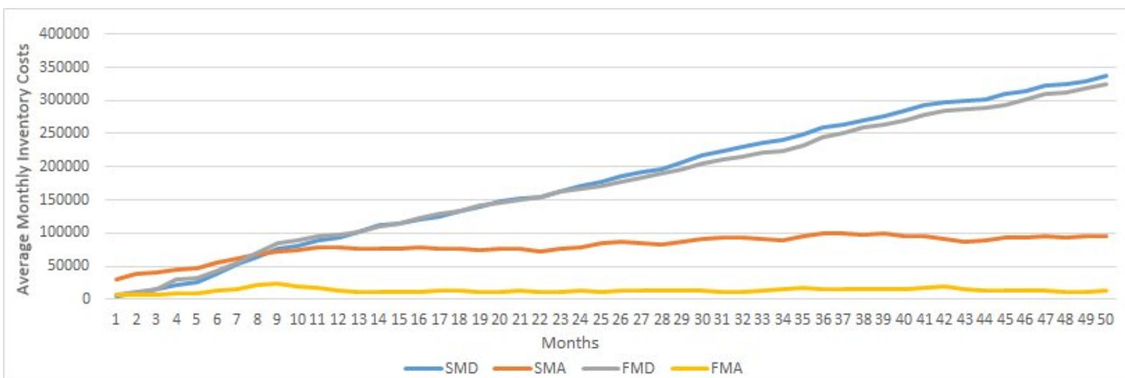


FIGURA 2.3. Monthly inventory costs of lumber and logs

b) The cost of aggregation

The cost of aggregation is obtained by calculating the difference in optimal value between the FMA and FMD models, as well as for the SMD and SMA models. This is done for the first or second model, respectively. What is interesting about this question is how costly it can be for the company to aggregate. Aggregating in the first model made the sawmill plant produce solutions that were 13.36% more expensive, while in the second model it was 30.64% more expensive on average.

In actual sawmill operations there will be around 20 different types of logs, several hundred cutting patterns and dozens of types of lumber that could be produced. This makes the problem of how costly is to aggregate parameters particularly relevant. A good exercise would be to simulate results using different degrees of aggregation. A manager should stop aggregating cuts, logs and lumber when the estimated function cost remains almost invariant to increases in disaggregation. This will allow them to identify the cutting patterns that really affect the revenue of the operations. The same idea of the cost that is incurred when aggregating variables can also be easily used in other industries and operations.

An interesting question is how to aggregate variables or constraints correctly. This is a difficult question that involves both managerial and mathematical insights. The managerial insight is to aggregate variables that are similar in terms of the products or processes they represent. The mathematical insight is to try to estimate a *priori* and a *posteriori* bounds on the deterioration of the objective function due to the aggregation of variables. Some insights regarding the second question can be found in Zipkin (1980b) and Mendelssohn Mendelssohn (1980).

### 2.6 Conclusions and Future Work

The study described in this paper had two objectives. The first objective was to find a good sawmill planning model that could simulate all of the complexities that are present in this kind of industry. This includes some of the uncertainties, variabilities, and inconsistencies that can occur between decisions that are made in different time horizons. Two models were presented, each with two different versions. These four models simulated all of the conditions that are highlighted in section three (SMD, SMA, FMD, FMA). The second objective was to test how these models would behave in a real operation over long periods of time. For this purpose, a Rolling Horizon (RH) method was used and the operational policies that were computed by the models were evaluated using an operational decisions model.

The results indicate that models using disaggregated information provide tactical policies that can be implemented more effectively (in this case, log procurement quantities). In this case, effectiveness refers to short-term costs, where uncertainty arises with regards to the actual supply of logs. This indicates that those models are better suited for coordinating decisions in a hierarchical setting such as the one studied in this paper.

On the other hand, from the use of the RH technique it is clear how important it is to test the models dynamically rather than statically. The Rolling Horizon made it possible to see the level of impact that aggregation could have on the quality of the solutions. In the case of the first model, the estimated increase in cost for a company from aggregation was 13.36%, with an increase of 30.64% for the second model. Furthermore, the use of the Rolling Horizon method made it possible to appreciate how inventories would behave over time. The FMD and SMD models were not able to maintain a steady inventory in the long run. The Rolling Horizon method revealed how the log purchase was distributed according to type and whether it was planned or unplanned. In terms of the moment at which the models purchased the logs, the four models behaved differently. The Second Model with Aggregation performed awfully, purchasing only 37.2% of its logs in advance. The other three models purchased more than 85% of the logs in advance. The SMD model seems to wait until the last moment to purchase additional logs. The reason for this is that the SMD model can "see" that the supply scenario could be good or bad in the future and therefore protects itself to a certain extent. The first model (FMD and FMA) only protects itself against bad scenarios and is not able to "see" the surplus that comes with good scenarios. For these reasons, the second model works closer to stochastic optimization, while the first model works closer to robust optimization.

In a different industry, with different data, the behavior of the models could of course be different than the case described in this paper. However, we consider that the value of our simulations lies in showing how management could identify the different behaviors of the model, select the best modeling approach to better coordinate their intertemporal decisions, and plan for actions in advance.

We still have to apply the models and Rolling Horizon method using the full set of data from a real sawmill operation problem. Furthermore, a sensitivity analysis could also be used to understand how to improve the behavior of a model. This analysis could help management better understand the behavior of the models under different levels of uncertainty.

Only SAA was used to solve the two-stage problems in this study. With the right structure of the recourse, first order methods (such as stochastic subgradient methods) could have been used, as well as the L-Shaped method. When several scenarios are used in two-stage problems, first order methods might be more easily parallelized than the SAA method. In addition to this, it would not be possible to ensure that the FMD and FMA models would scale better than the SMD and SMA models. This will be the topic of a future article.

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