PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
ESCUELA DE INGENIERIA

AN ANALYSIS OF SERVICE LEVEL IMPACT ON DELIVERY COST ROUTING WITH TIME WINDOWS

PHILIPPE BURQ

Thesis submitted to the Office of Research and Graduate Studies in partial fulfillment of the requirements for the Degree of Master of Science in Engineering

Advisor:
RICARDO GIESEN

Santiago de Chile, (January, 2012)

PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
ESCUELA DE INGENIERIA

AN ANALYSIS OF SERVICE LEVEL IMPACT ON DELIVERY COST ROUTING WITH TIME WINDOWS

PHILIPPE BURQ

Members of the Committee:

RICARDO GIESEN

JUAN CARLOS MUÑOZ

FELIPE DELGADO

SERGIO MATURANA

Thesis submitted to the Office of Research and Graduate Studies in partial fulfillment of the requirements for the Degree of Master of Science in Engineering

Santiago de Chile, (January, 2012)

To those who always believed

ACKNOWLEDGMENTS

This work is the final stone to my pre-grad studies. Several years of engineering courses that led me from France to Chile and gave me the opportunity to meet wonderful people. I would like to say a special thank you to those who made this possible, first and foremost my family that always supported me, Ecole Centrale and Pontificia Universidad Católica de Chile, the institutions that allowed me to pursue a wonderful joint degree, my cousin Rodrigo whose steps I followed specializing in transportation and logistics and Juan de Dios Ortúzar who welcomed me in the department and gave me the opportunity to extend my program and follow a Master's Degree.

I would like to gratefully thank Ricardo Giesen, my adviser, who suggested me this subject and assisted me all the way through its development. I also would like to mention Juan Carlos Muñoz who has always been there to answer doubts, questions and share his amazing knowledge and experience.

I also want to mention my gratitude to the whole transportation and logistics department of Universidad Católica. The students and teachers group is simply amazing and they make studying hours so much fun. Big thanks to the gang, Sebastían R, Felipe Z, Tomás P, Felipe D, Felipe H, Daniel R, Luís A, Pablo M, José Ignacio B, Francisca N, Marco M to only name a few….

A special mention to Doktor Freyss which has shared most of those years with me and has been a great friend all the way, thanks bud!

Finishing this work has not always been easy and I owe a very special mention to those who supported me in the darkest hours, Nacho Vargas and Jaime Besa who helped me make the right decision and give a final boost, Patrick Neise and my colleagues at McKinsey that have been unconditional supporters during those last two years, thanks! And finally, I would like to mention my uncle Memo which would ask for the thesis status every two weeks and invariably insist on our weekly lunches that I really should finish this….

TABLE OF CONTENTS

Pág.

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

The home delivery market is taking every day a more important place in the retail industry and in the life of the families as it is often a simple, practical solution to get the groceries task done.

However, the market is still relatively new and the players not very experienced. There is already some famous success and failure stories and the operations implications for a retailer are not yet fully understood.

In this thesis we propose a model to understand a specific part of the home delivery operation: the tradeoffs between costs and service level. The proposed model builds routes of clients and explicitly considers the truck’s speed and the time spent at the client during the delivery to estimate the percentage of deliveries dispatched on time in a given time window.

Main results include a better understanding of the cost structure of service level according to the number of trucks available, the number of clients served and the length of the time window. Some high level recommendations are also proposed to help on decisions such as fleet sizing.

Keywords Service level, Home Delivery, Vehicle Routing Problem with Time Windows

1. INTRODUCTION

This chapter will introduce the context and motivation of this thesis as well as the main objectives and scope of the study. It will then summarize the problem statement and methodology used and will conclude by describing the structure of the thesis.

1.1. Context and Motivation

The home delivery market has significantly increased in the last 10 years and has an important growth potential in the next few years. Despite initial failures, the growth rate is very important nowadays and is expected to continue at a double digit rate through 2014. Advancements in technology, the spread of broadband and the tendency of shoppers to research products prior to purchase are the key drivers of online market growth. A report published in 2010 by just-food forecast that in 2014 the UK online grocery market will have more than tripled the levels seen in 2009. During the same period the US market is expected to increase up to 75% reaching a value of US$13.55bn. Other large markets in Europe, like France and Germany, are forecast to increase in size over the next few years. As for smaller European countries such as those in Northern and Eastern Europe, sales are expected to boom towards the end of the decade. Finally, China and its large, affluent urban and increasingly sophisticated consumer base will be an ideal target market in the coming years.

Despite there being a significant amount of literature about the Vehicle Routing Problem (VRP) –for a recent review see Laporte (2009)- in the majority of the cases studied constant travel times throughout the day are assumed. Thus most of the previous research has not addressed the congestion problem. However, in most big cities, it is very common to find recurrent congestion at certain times of the day and variation of speed has to be considered at the moment of building routes. Ebensperger (2009) proposed a methodology to build routes with real time information considering hard time windows and focusing on the determination of the best operations strategy to minimize the costs.

Delivery time is also a variable that is usually assumed constant. However, it may vary between a highly populated zone which is more likely to have housings with lots of floors implying time spend in the elevator and a less dense zone where the access to a house will be much more straightforward and less time consuming. Moreover, data shows (Table 3-4: Contribution of Time spent at Client's) that delivery time can represent up to 40% of the time spend on a tour and thus variation in that dimension will have strong impact in the tour total length.

Another important motivation for this thesis is that most other literature considers the optimization of the routes. However this fails to take into account the cases of starting or small businesses that cannot afford to implement complicated software to optimize their routes. In these cases it is actually the driver that chooses the route and makes the difference. In this thesis, no expensive routing model is necessary to obtain the results and with the right data, any carrier can determine his most efficient operating point.

Finally, most businesses do not actually look for the best solution to their problem. Following the Pareto principle they rather look for more simple measures that are easy to apply and will be near enough to the optimum solution to assure substantial gains without being too costly be it in investments or implementation.

1.2. Research objectives and scope

The main objective of this thesis is to develop a better understanding of the tradeoff between cost of dispatch and service level and analyze the possible variations according to the time window length.

Through a case study, based on real data, we will understand what the main drivers of the route length are and then we will develop a model that take them explicitly into account to study their variation and impact in cost.

The objectives are to understand the effect of the variation of speed, time windows length and delivery time on the cost of dispatch. For different size of operations (number of trucks and time windows, we will be able to visualize the tradeoff between cost and service level, represented by the percentage of late deliveries.

This will generate simple recommendation to the market player as to how to choose the service level they want to provide to their client and have a better understanding of their cost structure.

The main limitations of this thesis are that we only consider costs, we do not consider explicitly the revenues generated from the dispatches. It is neither in the scope of this study to evaluate different pricing strategies according to the width of the time windows or the hour of the day nor any matter related with revenue management. The main objective is to have a better understanding of the cost structure only.

1.3. Problem statement and methodology

The problem studied in this thesis is the Vehicle Routing Problem with Time Windows. As a better grasp of the tradeoff between cost and service level is wanted, this study will aim at generating curves showing this tradeoff for different time windows and speeds.

Based on census data in Santiago and level of purchase in supermarket per region of the city we will randomly generate clients. Then using a modified Clarke and Wright savings algorithms, we build routes solving the Traveling Salesman Problem and assigning sub-sets of clients to trucks. Then with speeds and time spent at clients' values obtained from a Chilean retailer operation, we calculate the time of arrival at each client and deduce the percentage of late deliveries.

1.4. Thesis structure

The next chapter presents a review of related research in the literature. Previous work on routing problems is detailed and a summary of the different works in the area is proposed.

The third chapter presents the business case. The data we have from a Chilean e-retailer is analyzed and the main drivers of the route total length specified.

The fourth chapter defines the model that is used for the purpose of this thesis. The model is justified according to the objectives and the algorithm used will be explained.

The fifth chapter presents the simulation that will be realized. This first part will explain how the instances were generated and the second part presents the results. These are the main contributions of this work as they describe the tradeoff between cost and service level in retail-applied home delivery service.

Finally, the last chapter summarizes the main contributions of this study and proposes extensions for further investigation.

2. LITERATURE REVIEW

2.1. An overview of the Vehicle Routing Problem

The vehicle routing problem (VRP) is crucial for any distribution management and it is a problem faced every day by thousands of companies. Much progress has been made since the first article on the issue by Dantzig and Ramser (1959). Thanks to the development of computer systems, the last decade witnessed an increased utilization of transportation optimization. Problems that were impossible to tackle with more than a few clients have now been extensively studied and very comprehensive heuristics to solve them have been successfully developed.

The VRP is a very popular problem in combinatorial optimization. It generalizes the Traveling Salesman Problem (TSP) and is thus NP-hard. In its classical formulation the VRP looks to minimize the total travel cost of a set of routes serving a given set of clients with a deterministic, given in advance demand. Each customer is visited exactly once by one route and each route starts and ends at the depot. Adding a third restriction, that the total demand of the customer served by a route does not exceed the vehicle capacity, gives the Capacitated constrained VRP (CVRP).

However many different, more specific problems arise from this initial formulation. They differ from the initial problem for a variety of reasons, such as variations in the characteristics of the demand, the customers, the fleet, the conditions on the network and formulation of the objectives.

Later on some classic extensions of the VRP are described:

- The VRP with Time Windows (VPRTW). This is an extension of the CVRP where each client is associated with a time windows during he will receive his delivery (Daganzo, 1987a, b, Figliozzi, 2009).
- VRP with Backhauls (VRPB). In this problem the customer set is divided in two, one half contains customers who need a delivery, the other half need a pick-up (Deif and Bodin, 1984, Toth and Vigo, 1997, Brandão, 2006).

VRP with Pick-up and Delivery (VRPPD). In this version, each customer is associated with a quantity to be delivered and another one to be picked-up (Min, 1989, Montane y Galvão, 2006, Bianchessi and Righini, 2007).

- Any combination of the former extensions is a new type of problem.

A detailed survey of these problems and the methods used to approach them can be found in Toth and Vigo (2002).

Figure 2-1sums-up the different relations between the problems just presented.

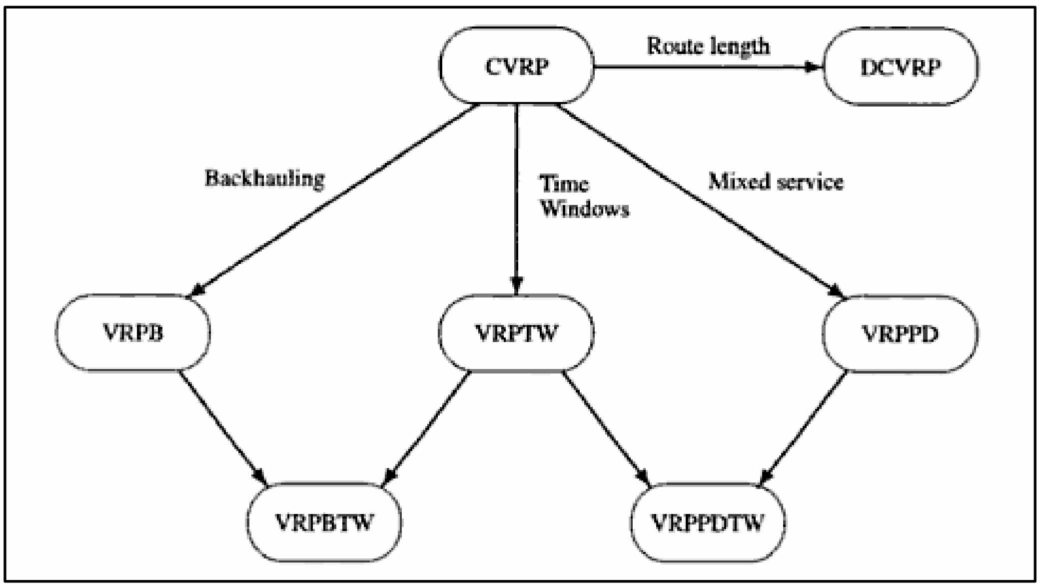

Figure 2-1: The basic problem of the VRP and their connection

2.2. VRP applied to On-line Home Delivery.

Most of the literature around extensions of the VRP is based on deterministic and static hypothesis. However, in the last decades some work has been done in an important extension of the VRP: stochastic VRP (SVPR). These problems try to have a better grasp of reality by inserting a degree of randomness into certain components. The three most common cases are: stochastic customers, stochastic demand and stochastic times. Jaillet (1985) laid the foundation of stochastic customer's problems. More recently Beraldi et al. (2005) investigated the Pick-up and Delivery Traveling Salesman Problem with Stochastic

Customers. The Vehicle Routing Problem with Stochastic Demands has been the most studied stochastic VRP. In this problem, customer demands are random and usually independent. Laporte et al. (2002) and Secomandi (2003) are some of the most recent works in the area. In the Vehicle Routing Problem with Stochastic Travel Time (VRPSTT), travel times are stochastic. Leipälä (1978) first computed the expected length of tours with random travel times and only one vehicle. Laporte et al. (1992) were probably the first to provide an exact algorithm for the VRPSTT.

A related problem is the Time-Dependent Vehicle Routing Problem (TDVRP). In this extension of the VRP, a vehicle fleet of fixed capacity serves customers of fixed demand from a central depot. Customers are assigned to a vehicle and the objective is to minimize the total routing time. The travel time between different points in the network depends on the distance between the points and the time of the day so as to include/incorporate the effects of urban congestion. This is a critical problem when customers are continually requesting a better service in the form of more reliable delivery times and narrower time windows and when recurrent congestion in big cities is a critical factor.

Allowing different travel times, the TDVRP extends the VRP (Christofides (1976)) to account for urban congestion.

Malandraki and Daskin (1992) propose heuristic algorithms to solve the TDVRP. Fleischmann et al. (2004), tackle the problem of derivation of travel time data from real traffic information. Zhong et al. (2004) offer an interesting approach, considering driver learning as a factor in travel time.

As the level of service required by customers increases, problems with time windows arise. Daganzo (1987a, b) gave one of the first models of VRP with time windows by dividing the day into time periods of equal length and then assigning clients to one of these periods according to their initial time window. Figliozzi (2009) contributed recently to the evaluation of the average length of vehicle routing with time windows.

From these formulations, naturally comes the investigation of the challenges and opportunities in home delivery. Agatz et al. (2007) offers a good insight of this problem. Agatz et al. (2007, 2008) discusses the importance of time.

2.3. Classification

See Table 2-1 for a sum-up of the most relevant literature for our problem.

Table 2-1: Previous Research in Home Delivery Problems

Reference	Time windows	Time spent at client's	Vehicle capacity	Objective	Comments
Figliozzi, 2009	Yes for some clients	Not considered	Yes	Minimize route length	
Agatz, 2009	Yes	Constant	Yes	Profit maximization	All time slots have the same demand
Campbell, 2006	Yes	Constant (20min)	Yes	Profit maximization	Can reject deliveries Time windows probabilistically chosen Monetary incentives
Lin, 2002	Yes	Constant (10min)	No	Minimize operation costs	
Campbell 2005	Yes	Not considered	Yes	Profit maximization	Can reject deliveries

3. OPERATIONAL DATA ANALYSIS

With the objective of having a better sense of the challenges that a real operation faces, we have studied the operation of a Chilean retailer entering the market of home delivery. In order to have a manageable amount of data, we've only considered a set of regions in Santiago where clients are dispatched from the same depot. We have more than 12 000 deliveries spread over 12 months, between March 2008 and April 2009. The main insights are detailed in the following sections.

It is important to stress out that this data has been collected manually during the operation (for example time and counter kilometers) by the operators (dispatcher and driver) and thus cannot be perfect. Some data cleaning has been done and in particular the most obvious outliers had been eliminated but some marginal errors may be found.

For confidentiality reasons, neither the names of the regions nor a map are disclosed.

This chapter presents the different data analyses that were performed. Section 1 presents the rates of late deliveries, section 2 shows the time spent in the actual delivery and section 3 presents its contribution in the route total time. Then section 4 presents the variation of speed throughout the day and finally section 5 summarizes the primary conclusions of these analyses.

3.1. Late deliveries

An important component of a home delivery operation is the service level, of which two key factors are the punctuality and time windows lengths offered. In our example there is only one time window possibility: 6-hours' time window that can be between 9am and 3pm ("am period") or between 3pm and 9pm ("pm period"). In a further section of this thesis we will study the costs of changing this structure.

In Table 3-1: are the percentages of late delivery from our retailer. We distinguish two types of them: short ones, less than an hour, and the longer ones, more than 1h. Late delivery means that for an “am period”, the truck arrives later than 3pm or later than 4pm in the case of a long delay.

Table 3-1: Percentage of late deliveries

	Short delays (<1h)	**Long delays (≥1h)**
Region 1	17,7%	6,5%
Region 2	11,2%	3,6%
Region 3	7,9%	3,0%
Region 4	14,7%	4,2%
Region 5	11,8%	4,7%
Region 6	15,9%	6,4%
Total	**12,8%**	**4,5%**

In order to try to better understand these late deliveries, we split them in am and pm deliveries in Table 3-2::

Table 3-2: Percentage of late deliveries, am vs. pm

	AM		**PM**	
	Short delays (<1h)	**Long delays (≥1h)**	**Short delays (<1h)**	**Long delays (≥1h)**
Region 1	21,6%	9,6%	11,4%	1,4%
Region 2	12,7%	4,9%	8,9%	1,7%
Region 3	11,3%	4,6%	1,6%	0,0%
Region 4	18,8%	5,9%	8,7%	1,6%
Region 5	15,6%	6,8%	6,0%	1,7%
Region 6	21,0%	9,4%	5,1%	0,0%
Total	**15,7%**	**6,4%**	**8,2%**	**1,3%**

This analysis shows that these late deliveries are much more significant in the morning than in the afternoon. Further more, the am represents 62% of the deliveries, the impact of the delay in the morning is thus multiplied. Observation on the field shows that the main explanation to that asymmetry is that all the orders are prepared the same day. Thus, with the depot opening like a regular supermarket around 7am, the operators do not have enough time to prepare all the morning orders such that the truck is able to leave the depot at 9am. Our data shows that for am deliveries, the average departure time from the depot is 12:04 and the average arrival to the first client is 12:27. However, on one side there are some goods –like frozen ones- that cannot be prepared in advanced and on the other side it may be logistically impossible or too expensive to open the depot earlier.

Real data analysis of late deliveries confirms the purpose of this thesis. Service level is not very good and a better understanding of its cost to the dispatcher will certainly help retailers make better decisions about it.

My conviction is that even if the number of late deliveries in the morning is probably over the number of late deliveries actually due to bad routing decisions, the real number is still around 10% which seems very high for 6 hours time windows. Moreover a better understanding of the cost of service level will allow different possible decisions for the morning and afternoon dispatch.

3.2. Time spent at client's

A key component in the dispatch time length is the time spent at the client's. Intuitively we may expect different delivery times according to the dispatch region: in denser areas where the order is delivered to apartments, you should expect problems to park the truck, longer times of handling (wait for elevator for example), whereas in less dense areas you shouldn't have any of those issues. Moreover, as the truck is able to park easier than is denser areas, the driver should be able to help during the handling process and transportation phase from the truck to the client's house. Another parameter that can influence the time at client's is the nature of the dispatch. Fresh goods do not need as much time to deliver as household appliances like a refrigerator. In our case we are considering the dispatch from a supermarket and we will suppose that the nature of goods dispatched is the same during the whole day. In Table 3-3: the average time at client's, standard deviation and count separated by dispatch region are presented.

Table 3-3: Time at client's

	Average ($\frac{min'sec''}{Client}$)	**Stand. Dev.**	**Count**
Region 1	10'21"	6'25"	339
Region 2	10'25"	5'37"	566
Region 3	10'50"	5'15"	185
Region 3	11'05"	7'45"	83
Region 4	11'01"	5'00"	89
Region 5	11'05"	5'13"	58
Total	**11'10"**	**5'53"**	**1320**

Data analysis shows that the variation of time at client's does not vary significantly across the dispatch region. A mere 7,4% between the fastest and slowest region.

Further analysis and data gathering is necessary to conclude this part. Either the data has been wrongly taken (for example rounding every stop at the client to either 10min or 5min) or the effect of "elevator" is counterbalanced by the fact people living in apartments tend to order less (they order more frequently and the family size is smaller) than people living in houses.

The authors of this thesis do not have access to that type of information and thus this study is proposed as an extension for further investigation. However we can study the proportion of the tour time length that is spent at the client's versus spent on the road.

3.3. Contribution of time at client's

We want to determine the contribution of the time at the client in the total time of dispatch. This is important as it will indicate us where it is worth putting our efforts to improve the forecasting of routing time. Table 3-4: presents the percentage of time spent at the client's out of the total time of dispatch:

Table 3-4: Contribution of Time spent at Client's

	Contribution of Time at client's in total dispatch time
Region 1	40,9%
Region 2	41,5%
Region 3	39,3%
Region 4	35,6%
Region 5	34,5%
Region 6	35,6%
Total	**39,9%**

Time at the client represent a significant part of the total time of the route: 40% on average and up to 42% in some regions. Given this results, we will explicitly integrate this component in our model.

3.4. Variation of speed

Another interesting variable that we wanted to examine was the speed of the trucks during the day and across the different dispatch region. To isolate the effect of the speed, we took out from the total time in tour the time spent at client. That way we only study the time actually spent on the road.

The first result is that there are significant differences of speed between the different dispatch regions (up to 16% faster). The results are shown in Table 3-5::

Table 3-5: Vehicle speed on the road according to dispatch region

	Average speed (km/h)
Region 1	24,1
Region 2	22,0
Region 3	20,3
Region 4	23,0
Region 5	20,4
Region 6	23,8
Total	**22,9**

The second interesting result is that the speed clearly changes during the day. The fastest hour is 15,2% faster than the slowest. This implies that a better modeling of the speed (instead of one fixed value for the whole day for example) should lead to clear improvements at the moment of planning the routes and forecasting the time of dispatch. The following figure shows the evolution of the speed along the day. We can see clearly that the peak hour in the afternoon is decreasing dramatically the speed of the trucks. This should be considered in the case of applying an hourly pricing strategy as it is much less efficient for the retailer to dispatch at this time of the day.

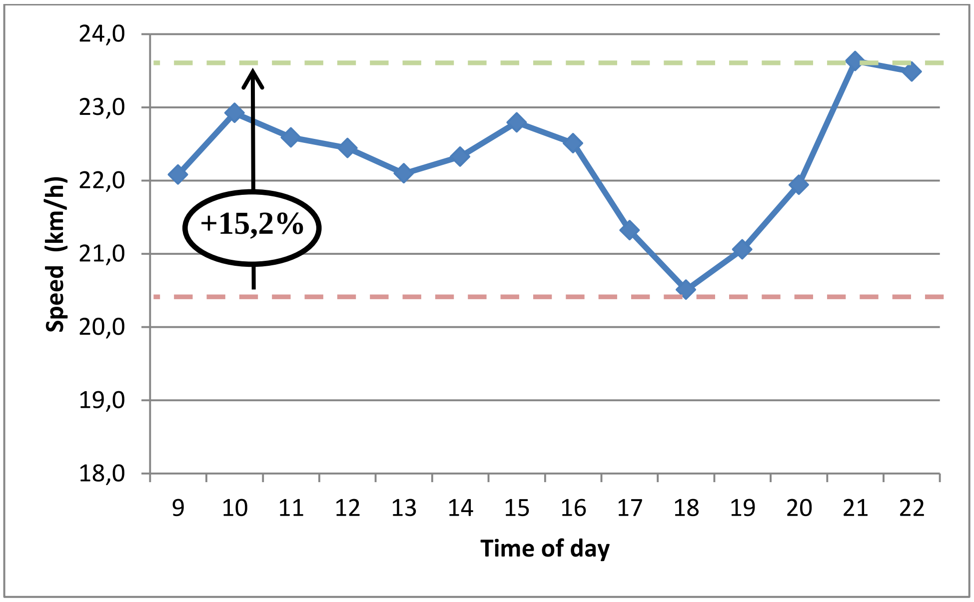

Figure 3-1 Evolution of speed along the day in all regions

Then we graphic the truck speed according to their drivers in Figure 3-2:

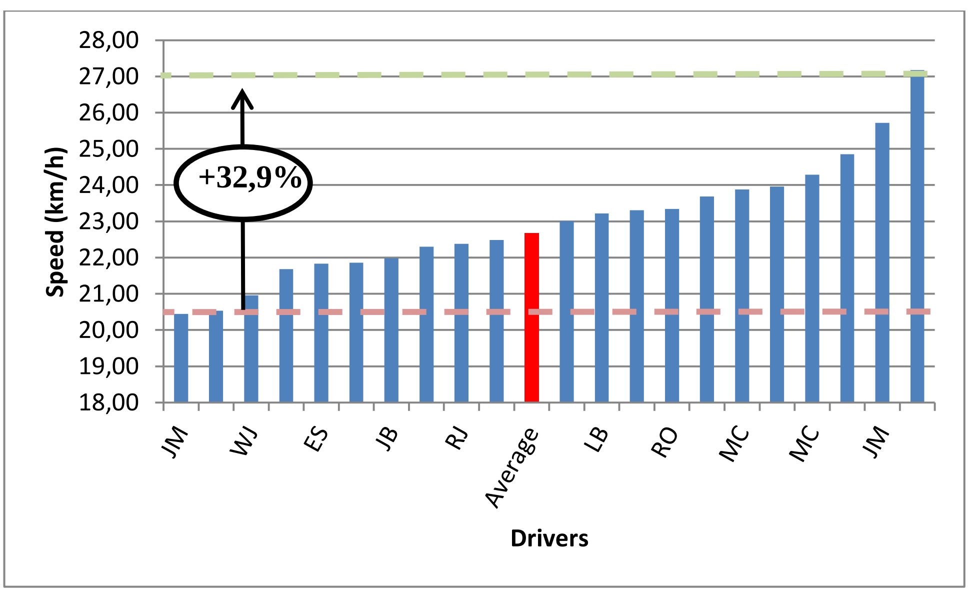

Figure 3-2 Difference of speed among drivers

Clearly the influence of the drivers is very important, without considering specific regions or time, the fastest driver can go up to 33% faster than the slowest. Here again, with a clearer understanding of the real costs of service level, a company will be able to leverage their drivers different abilities and assign them to different routes according to their personal abilities. Moreover, Figure 3-3 shows that there is no correlation between the time spent at the client's and the speed among the different drivers.

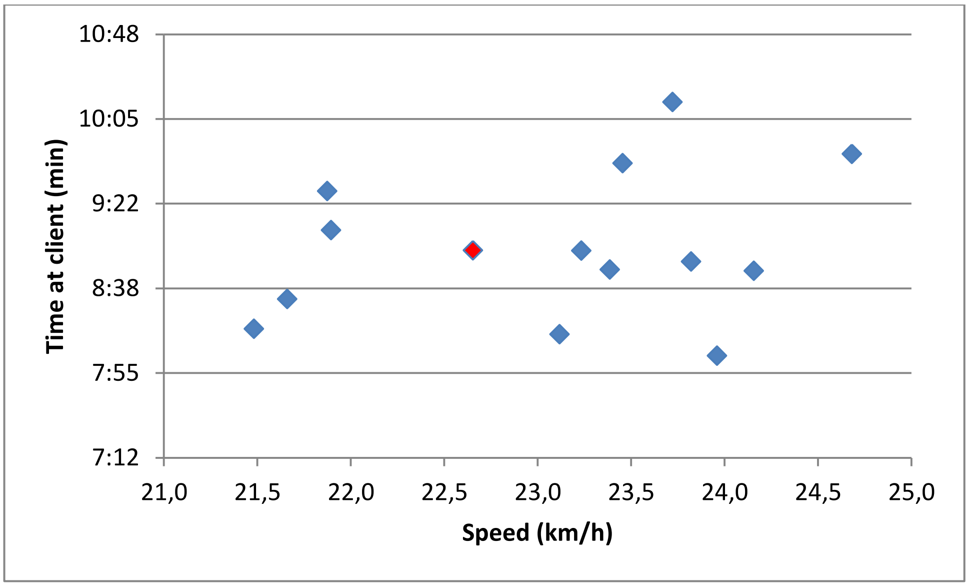

Figure 3-3 Time at client vs Speed, for each driver[1]

3.5. Preliminary conclusions

The main conclusions of this part are the following:

- There are a significant percentage of deliveries that are late or even very late (2-3% of the deliveries are more than 1h late which is a very bad service considering that the time windows is pretty large: 6h). This is a major issue as people are usually waiting at home for their delivery and punctuality is often a very important factor from a client's point of view. In some cases, if the delivery is late, the client is unable to receive it and the delivery has to be rescheduled. Usually prices in the

[1] Outliers have been eliminated: time at client is ≤30min, speed is between 10 and 60 km/h, drivers serve more than 15 clients

retail industry are very homogenous and quality of service (punctuality as well as time window size) may be key for the client at the moment of choosing among the different players in the market.

- Even if it may seem counter-intuitive, the time spent at the client is a very important variable as it may represent up to 40% of the total time of the route. Thus we think it is important to consider it explicitly in our model.
- Finally, speed is another factor that appears as significant in this data analysis. It varies significantly during the day and according to the different drivers. Thus a clear understanding of the tradeoffs between cost of resources and service level will allow to consciously build much more realistic routes and predict more accurately their duration, ensuring thus a better, predictable client service.

4. MODEL DEFINITION

In this chapter we will describe the model and the different variables we will use to fulfill the purposes of this thesis. We will also describe the methodology that we will use to obtain the final results.

4.1. Home delivery operations

For the purpose of this thesis we will be considering a classical home delivery operation with one depot serving multiple clients. In our experience this is an appropriate modeling of what is happening in developing countries like Chile where typically well-established retail stores chain are entering the home delivering market by using one of their stores as depot to cover a large area of the city.

Client can usually book up several weeks prior to the delivering date and up to the night before. At midnight the booking for the following day is closed and the next morning an operator consolidates the demand and builds the routes for the different delivering trucks.

4.2. Model formulation

As discussed in the introduction, we want to study the influence of service level in the dispatch costs. For that purpose, we need to build routes and compare them. Given that real data analyzed of home delivery operation suggests that a single depot usually serves between 20 and 100 clients a day, we choose to use a heuristic to build the routes since the 100 clients barrier has only been broken recently and exact algorithms may be very resource consuming (Laporte, 2010). A very well proven heuristic for the single-depot VRP is the “savings” algorithm of Clarke and Wright (1967). The savings algorithm is a heuristic and therefore it will not provide the optimal solution to the routing problem. However it obtains very good results: less than 2.2% deviation from the best-known solution for instances ranging from 25 to 42 nodes and less than 6,37% deviation for 100 nodes instances (Golden, 1980). Applied to the CMT set of benchmark proposed in 1979 by Christofides, Mingozzi and Toth, the deviation is about 7% (Laporte, 2010).

Notice that in order to include the effect of speed variation and time spent at the delivery site, we will use a modified version of this heuristic.

In this thesis we will only present an overview of the algorithm. However, a more complete version can be found in the original paper of Clark, Wright (1964).

This method is a “greedy” type algorithm, at each stage it will look to find the locally optimal choice with the hope of finding a very good solution. In the particular case of Clark and Wright, the process is the following:

a. We consider one depot *D* and *n* demand points (the clients). The initial solution is to go back and forth to each client from the depot, thus using *n* trucks. Then the total length of the solution is:

$$2 \times \sum_{i=1}^{n} d(D, i) \tag{1}$$

Where d(i,j) is the distance between i and j.

b. Now the algorithm starts looking for savings: if a truck serves two clients, for example i and j, in one single trip instead of the initial two, the total distance travelled is reduced by:

$$s(i,j) = 2 \times d(D,i) + 2 \times d(D,j) - [d(D,i) + d(i,j) + d(D,j)] \tag{2}$$

$$s(i,j) = d(D,i) + d(D,j) - d(i,j) \tag{3}$$

The following figure shows the trucks routes:

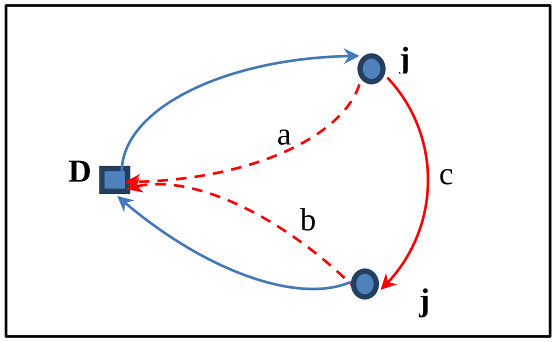

Figure 4-1: Illustration of a saving s(i,j) = a+b-c

The larger s(i,j), the more desirable to combine i and j, however, nodes i and j are combined only if they do not violate constraints from the VRP problem (that is without deleting a previously established direct connection between tow customers points and without exceeding the total capacity of the vehicle). To implement the algorithm, the savings s(i,j) are calculated for every pair (i,j) of clients and listed in descending order of magnitude –that way the algorithm always starts testing the most convenient couple (i,j) (that produces the highest savings). The algorithm will process the savings list starting with the biggest savings.

c. We consider three different situations where a new route is created:
- Neither i nor j have been assigned to a route, that is to say, they belong to their initial routes (D,i,D) and (D,j,D). Then both routes are eliminated and a new route (D,i,j,D) is created to take advantage of the savings.
- Only one of the two points (i or j) has already been included in a route and is not an interior point in its own route (that means it is adjacent to the depot, either as first client served or last). In that case link (i,j) is added at the beginning of the tour (or at the end according to the position of the adjacent point).
- Both points have already been included in two different routes and neither one of

them is an interior point in its route. Then both routes are merged.
- In any other case, the link(i,j) is not created.

d. The algorithm proceeds through the savings list till the end of it.

The end state of the algorithm is a set of routes that delivers all the clients under the restrictions previously stated.

4.3. Theoretical results

In this section we describe the methodology used to obtain the cost versus service level curves with a theoretical modeling.
We begin by explaining how to generate clients, then we describe how to calculate the delivery time and the service level. Finally we present the theoretical model's results.

4.3.1. Generation of clients and routes

In order to simulate the operation of a retailer, we need to generate clients. We suppose that the tour is divided in two subparts: a line-haul distance that is needed to reach the center of gravity of the clients in the dispatch zone and a local distance that is travelled into the dispatch zone to go from one client to another.
In this model, based on real data analysis, we suppose that the line-haul distance represents 30% of the total length of the tour. For simplification purposes we will also suppose that the clients are equally distributed in the local distance subpart. With these two hypotheses, we can simulate client tours. The tours length varies according to the density of the zone: if the zone is heavily populated, the distance between clients will be short, on the contrary, in a less dense zone, distances will be longer. In and Table 4-2 we show tours for respectively 3 and 10 clients. The first line of both tables indicates the different tour length whereas the first column indicates the clients in the tour. For example, the fourth column of (in blue) shows a tour length of 50km and the rows indicate the distance of the nth client from the

depot for different route lengths. For example, for the 50km route, client 1 will be visited 7,50 km from the depot, client 2 after 25,0 km and client 3 after 42,5km.

Table 4-1: Dispatch distance from depot for 3 clients' tours (Extract)

# of Clients	Tour Length (km) 10	30	50	70	90	100
1	1,50	4,50	7,50	10,50	13,50	15,00
2	5,00	15,00	25,00	35,00	45,00	50,00
3	8,50	25,50	42,50	59,50	76,50	85,00

Table 4-2 Dispatch distance from depot for 10 clients' tours (Extract)

# of Client	Tour Length (km) 10	30	50	70	90	100
1	1,50	4,50	7,50	10,50	13,50	15,00
2	2,38	7,13	11,88	16,63	21,38	23,75
3	3,25	9,75	16,25	22,75	29,25	32,50
4	4,13	12,38	20,63	28,88	37,13	41,25
5	5,00	15,00	25,00	35,00	45,00	50,00
6	5,88	17,63	29,38	41,13	52,88	58,75
7	6,75	20,25	33,75	47,25	60,75	67,50
8	7,63	22,88	38,13	53,38	68,63	76,25
9	8,50	25,50	42,50	59,50	76,50	85,00

We build these tables for tours ranging from 3 to 12 clients and for lengths between 10 and 100 km in 5km intervals.

4.3.2. Route length

After generating clients, we need to determine the length of the different part of the route to determine when the clients are dispatched. We will consider that the truck's time to travel any of the different kilometers in the route are independent variables and follow a normal distribution N(μ,σ) where μ[2] is the average pace (inverse of the speed) and σ the standard deviation. We also suppose that the times spent at the different clients along the routes are also independent variables that follows normal distribution $N(\mu_t, \sigma_t)$.

Thus given that we are only considering independent random variables that are normally distributed, their sum is also normally distributed with its mean being the sum of the two means and its variance being the sum of the variances.

Extending this to the sum of L kilometers, the time of arrival at a client located L kilometers from the depot will be a random variable normally distributed:

$$N\left(\frac{L}{v_s}, \sqrt{L}\sigma\right) \quad \textbf{(4)}$$

where

$$\frac{1}{v_s} = \frac{\sum_i t_i}{\sum_i d_i} \quad \textbf{(5)}$$

Where t_i is the time travelled in route i and d_i is the length of that same route i extracted from the sample data that we have. We calculate the average for all routes we have in the data (~1800 over 9 months).

We also calculate

$$Var\left(\frac{1}{v_s}\right) = \frac{\sum_i d_i \times \left(\frac{t_i}{d_i} - \frac{1}{v_s}\right)^2}{\sum_i d_i} \quad \textbf{(6)}$$

Similarly we calculate the average and variance of the time spent at the client.

Then we can deduce that the time of dispatch of client number j located on kilometer L of a given route is a variable that is normally distributed:

$$N\left(\frac{L}{v_s} + j \cdot \mu_t, \sqrt{L \cdot \sigma^2 + (j-1) \cdot {\sigma_t}^2}\right) \quad \textbf{(7)}$$

---

[2] To simplify the calculus, we will suppose that μ is constant throughout the day, we will thus not consider the speed variation during the day.

We can then determine the probability that each client will be dispatched out of the time window in all of the case scenarios.

4.3.3. Costing

If we suppose that we have a 100% utilization of the trucks then, by dividing the daily cost of a truck by the number of client it can serve during the day, we obtain the fixed cost of a truck per client. Additionally supposing a reasonable use of gasoline and average price of it, we can calculate the variable cost for each route.

The cost can thus be expressed as

$$Cost\ per\ client = \frac{Fixed\ daily\ cost}{\#\ clients\ served\ daily} + \frac{Gasoline\ Cost}{km\ yield \cdot \#Client} \qquad \textbf{(8)}$$

4.3.4. Intermediary results

Figure 4-2 presents the results of our analysis. The horizontal axis represents the service level in percentage (100% means that the truck delivers all the clients in the time window). The vertical axis represents the transportation cost per client calculated as shown in equation (9).

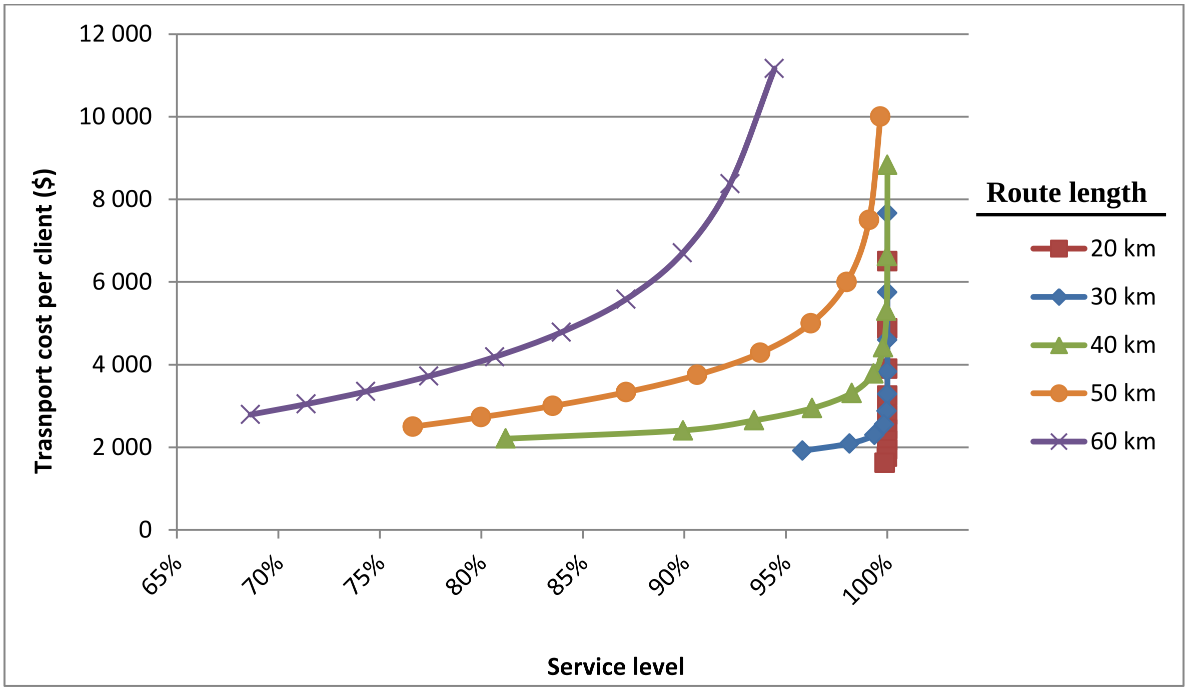

Figure 4-2: Transport cost vs service level

The main conclusions are:

- The curves are all convex and the better the service level we propose, the higher the marginal cost. Retailers should calculate the derivative function very thoroughly and according to how much they value the service level provided to the client they choose their position in the curve. This type of computation is only useful to segment clients and transfer the service level cost. Hence retailers can determine analytical guidelines to pricing a better service level.
- The longer the route the less convex the curve and thus the less sharp the evolution of marginal cost when improving the service level. That means that for less dense areas (equivalent of same number of clients but longer routes) the marginal increase of service level is not as expensive as more densely populated areas. This is due to the fact that in this latest type of regions, the proportion of the total time that is spent doing the delivery at the client's home is much higher. However, the longer the route, the lower the service level. Here retailers have to

solve a tradeoff problem between having more trucks (and thus serving clients in shorter routes) and service level. In the next section we study the cost of having an extra truck for different situation of delivery.

4.3.5. Further analyses

In this section we analyze the relationship between service level and number of clients served for different number of trucks.

This time, the lengths of routes are not fixed and we want to evaluate the service level for a certain amount of client served. In this section we will use a formulation mentioned in Daganzo (2004) for local distance between clients.

$$Local\ distance \approx k\sqrt{|R|N} \qquad \textbf{(9)}$$

Where:

N   is the number of destination/customers

k   is a dimensionless constant that depends on the metric (we will use a value of 0,57 as Santiago has a street grid very similar to Euclidian metric)

|R| is the surface area of R, the service region

This can be interpreted as the distance a truck has to travel once it is in the area where the clients are.

We can thus approximate the distance between two successive clients to

$$Distance\ between\ clients \approx k\sqrt{\frac{|R|}{N}} \qquad \textbf{(10)}$$

Similar to the previous section, we then build distances for different numbers of clients served. The more clients we serve, the nearer they are from each other because as the number of clients increases (for a constant zone size) it is easier to build the routes. Table 4-3 presents an extract from the clients we build. The first column represents the numbers of clients we will serve and then each following column indicates the distance of client i, indicated in the first row, from the depot. In this example, we added a 5km length long haul approach before adding the local distances between the clients.

Table 4-3 Dispatch distance from depot (Extract)

# Client	**2**	**5**	**10**	**15**
20	6,80	12,21	21,22	30,23
30	6,47	10,89	18,25	25,60
40	6,27	10,10	16,47	22,84
50	6,14	9,56	15,26	20,96
60	6,04	9,16	14,37	19,57
70	5,96	8,85	13,67	18,49
80	5,90	8,60	13,11	17,62
90	5,85	8,40	12,65	16,90
100	5,81	8,22	12,25	16,29

Just as in the first formulation of the model, we calculate the probability of arriving late at the different clients' locations according to their position on the route.

Finally, to build the relationship between service level and number of clients we suppose that the clients are evenly distributed between the trucks. For example if the company is serving 40 clients with 6 trucks, we suppose that each truck is serving 7 clients. For a time window of 120min, results are presented in Figure 4-3. In this figure, the vertical axis represents the service level (100% means that all clients are served in the time window). The horizontal axis indicates the number of clients that are served in the time window. This particular figure shows results for a 120min time window. The different curves represent different number of trucks available to realize the operation. For example the purple curves shows that with 5 trucks you can serve 40 clients with a service level of 98.7% or 70 clients with a service level of 65.63%

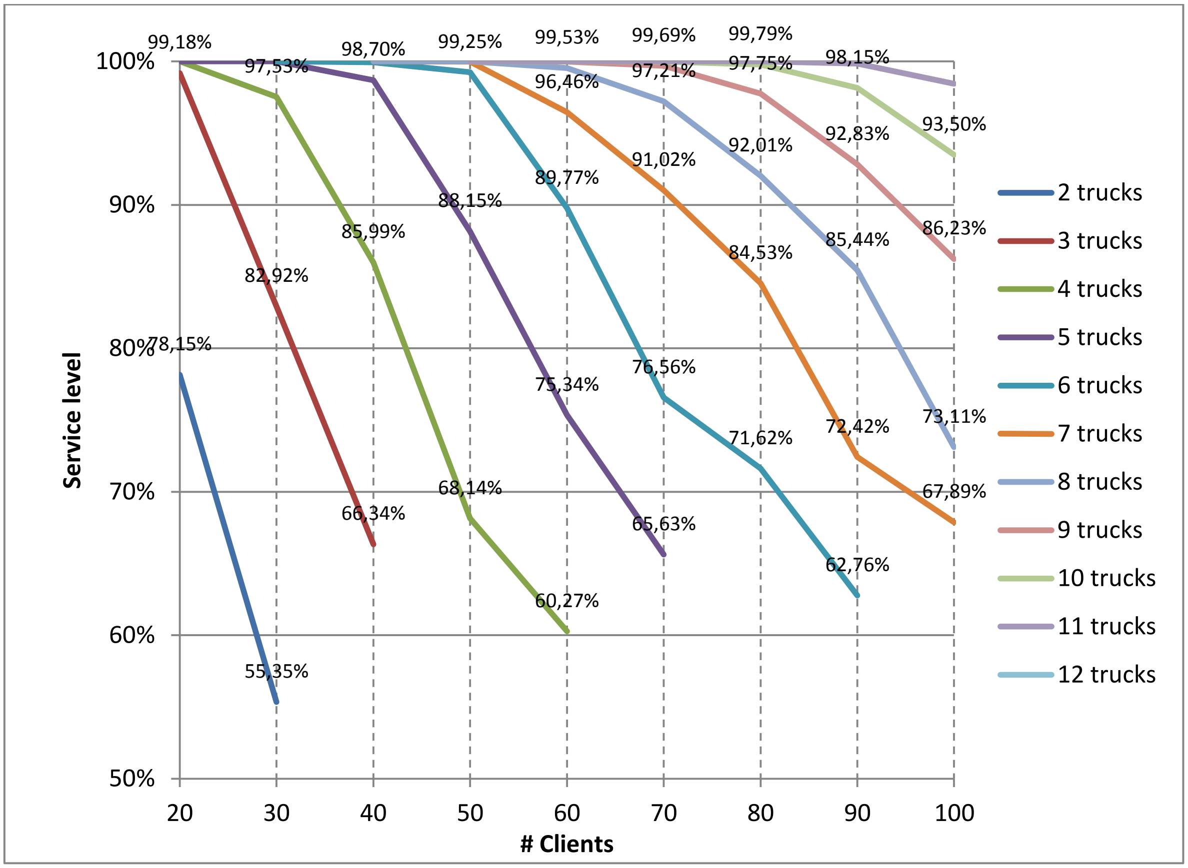

Figure 4-3: Service level vs Quantity of clients served, 120min Time window

The main conclusions are the following:

- If the density of clients, the speed of the trucks and the time window length are fixed, the relationship between service level and number of clients can be modeled by a linear function whose slope depends on the number of clients and the number of trucks.

- As we intuitively expected, the service level improves drastically with the numbers of trucks available to dispatch the clients' orders. However, the marginal benefit of a truck is a decreasing function that depends on the number of clients served and the number of trucks. That means that the more trucks you have and the more clients you serve, the less benefit an extra truck brings to the operation. Clearly this reflects an economy of scale. For example, if you can manage to always serve between 60 and 100 clients with a 120 minutes time window, then with 9 trucks you can guarantee to have at least a 85% service level.

To push further these analyses, we model the slope of the service level curve, supposing that the relationship between the service level and the number of clients is linear:

$$Service\ level = K - A \cdot N_{Clients} \qquad \textbf{(11)}$$

Where:

K is the intersect with the vertical axis

A is the slope of the curve, and depends on the number of trucks available (A≥0)

$N_{Clients}$ is the number of clients served

The constant K seems to depend on the time window length and the particular situation of the problem (speed, area surface etc…). Another way of calculating K is by studying the intersect of the curve with the horizontal axis at 100% service level. Given the situation of our particular situation, this occurs for about 7 clients. That means that if the truck serves 7 clients or less, they will be delivered on time, every extra marginal client over 7 will decrease the service level.

To determine the relationship between A and the number of trucks we suppose that for the ranges where the service level is not 100%, the curves are linear functions of the number of clients. We evaluate the slopes of the different curves in Figure 4-3 only considering the ranges where the service level is different than 100% (When the service level is 100% then the curve is flat and A=0, independent of the number of trucks) and we plot those values against the number of trucks in Figure 4-4. The horizontal axis represents the number of trucks and the vertical axis represents the value of the slope of the corresponding curve in Figure 4-3. For example, for 4 trucks, fitting a linear regression, the value of the slope of the green curve in Figure 4-3 is 0,0129. As intuited we find a clear relationship that can be expressed by the following expression with a R^2 of 0,977.

$$A = 0{,}0207 - 0{,}0018 \cdot \#trucks \quad \textbf{(12)}$$

Reinserting equation (12) in (11) we obtain an expression of the service level according to the number of trucks

$$Service\ level = K + N_{Client} \cdot (0{,}0018 \cdot \#trucks - 0{,}0207) \quad \textbf{(13)}$$

Rearranging the terms:

$$Service\ level = K - 0{,}0207 \cdot N_{Client} + 0{,}0018 \cdot \#trucks \cdot N_{Client} \quad \textbf{(14)}$$

The relationship deduced here seems coherent: the service level decreases with the number of clients (ceteris paribus, the more clients I have to serve, the worse will be my service level) and it increases with the number of trucks (ceteris paribus, the more trucks I have available to serve my clients, the better will be the service level).

Additionally this confirms that the value of the slope decreases with the number of trucks: the more trucks available the less costly in terms of marginal service level it is to serve extra clients. In other words, with a big operation going on, the company will absorb extra clients with fewer impact in its overall service level.

Notice that the relationship is linear and not inverted as we could have supposed doing a parallel with Queuing theory where the number of servers appears in the denominator.

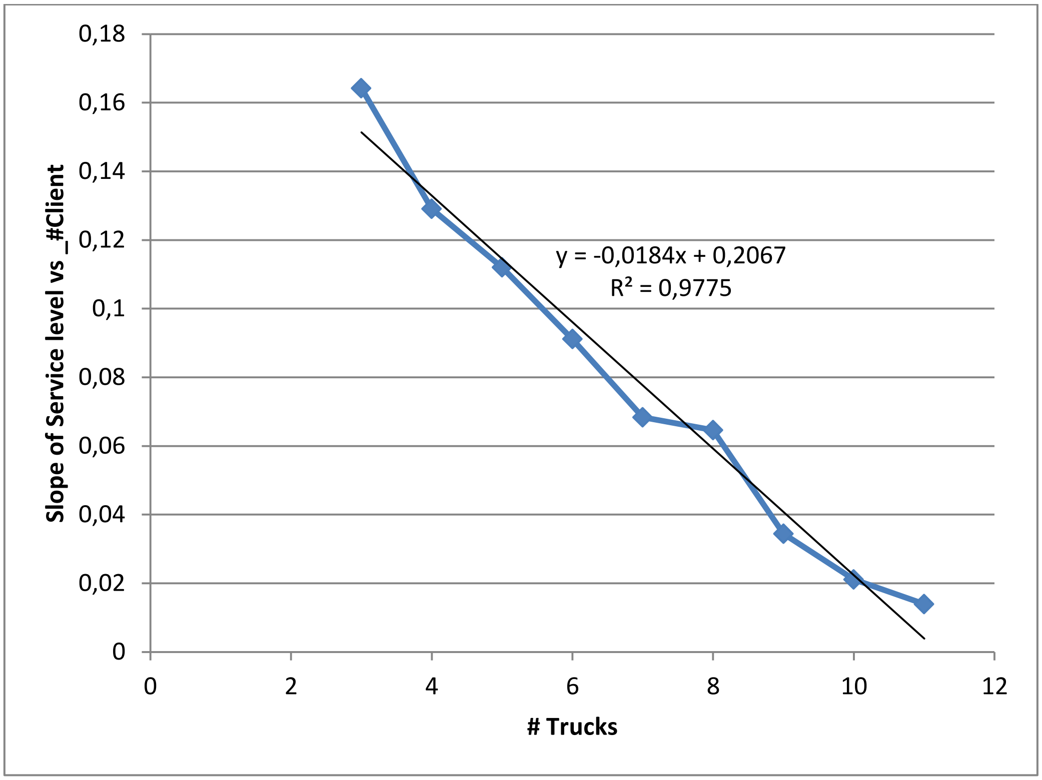

Figure 4-4: Constant A in function of #trucks, 120min Time window

In figures Figure 4-34, Figure 4-5 and Figure 4-6 are presented the service level (parameter A) vs the number of trucks for different time windows. Notice that the value of the slope depends on the length of the time window. We did the analysis for time windows of 1h30, 2h and 2h30. The bigger the time window, the steeper is the slope. Thus for smaller time windows, the parameter A change with the number of trucks is less significant and thus the service level is much more impacted by the variable number of clients.

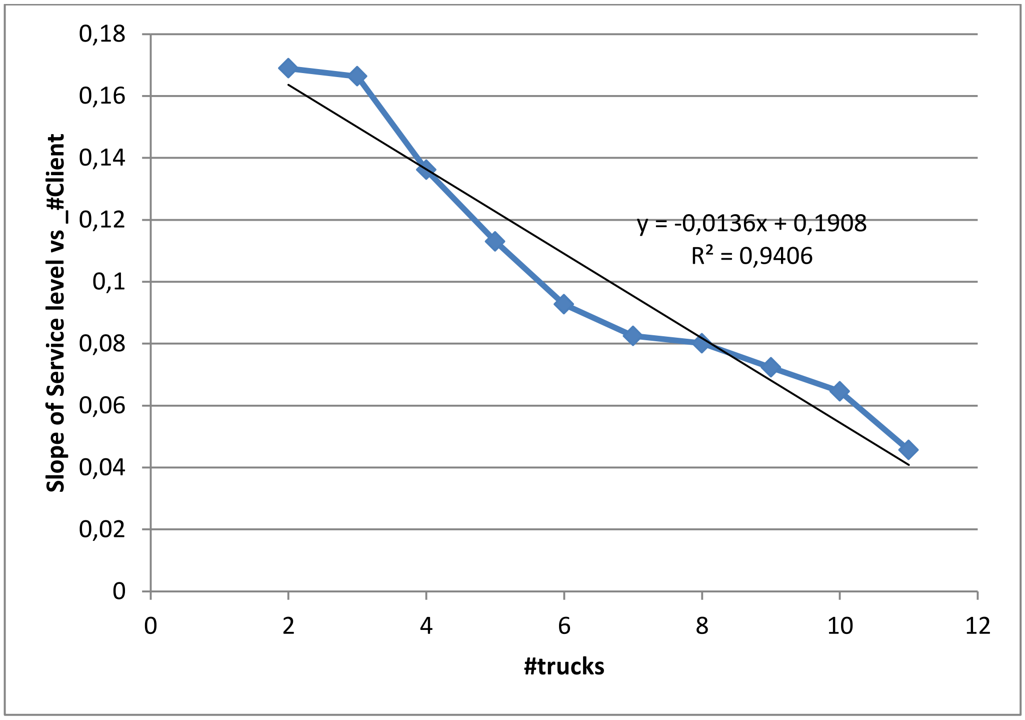

Figure 4-5: Constant A in function of #trucks, 90min Time window

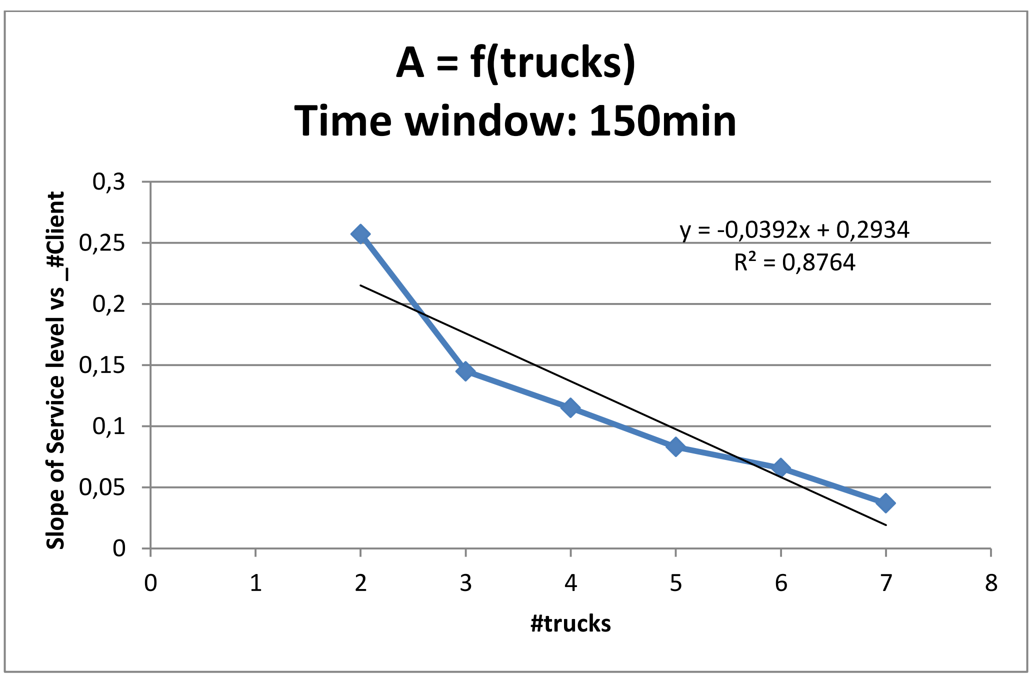

Figure 4-6: Constant A in function of #trucks, 150min Time window

This type of analysis is thus very useful for a manager who wants to dimension his trucks fleet. According to the number of client he expect to serve in a given time window, he can decide, according to the service level that he wants to provide, his fleet's size.
Another way to use this tool is to trigger the punctual rent of an extra truck. Let us say that for example the fleet is sized so as to guarantee a certain service level, whenever more clients order, the tool can indicate where the threshold to maintain the same service level is so as to rent an extra truck.

In the next chapter we simulate those results based on the data from the operation of a Chilean retailer. The objective is to validate these theoretical results.

5. SIMULATION EXPERIMENT

In order to validate our theoretical results, we simulate the same situation scenarios with real data and compare these with our model results.

In the first section of this chapter we explain how we randomly generated instances of demand (clients geographically placed), speed and time spent at client based on real data distribution. Then we explain how we simulate the dispatch operation and finally the third section presents the results we obtained.

5.1. Instances generation

5.1.1. Client generation

In order to simulate the operation of a retailer, we need to generate clients. We used the 2002 Santiago census from INE (Chilean National Institute of Statistics). We were able to obtain that data for every block of the studied zone. This represents approximately 5.700 different zones which are on average 30.000 m^2 each. However, the zone under study is pretty large (roughly 170 km^2) and we also want to include socio economics and behavioral differences. That is why we weight those numbers with the nominal annual sales in supermarkets per habitant that we obtained from a 2009 survey from INE. That way we obtain 5700 zones with a weighted value that should be representative of the shopping reality of the clients.

Then according to the density of the areas, we generate 30 instances of 20, 30, 40, 50, 60, 80 and 100 clients, each client being represented by the centroid of his zone. These quantities of clients reflect what have been observed in the real data for 2h time windows.

5.1.2. Speeds value and service time generation

For each set of instances, we randomly generated a speed according to the real distribution (not adjusted) calculated from the data we have. The distribution is presented in Figure 5-1.

The horizontal axis represents the speed in km/h in the routes and the vertical axis represents the frequency of a particular value in the sample.

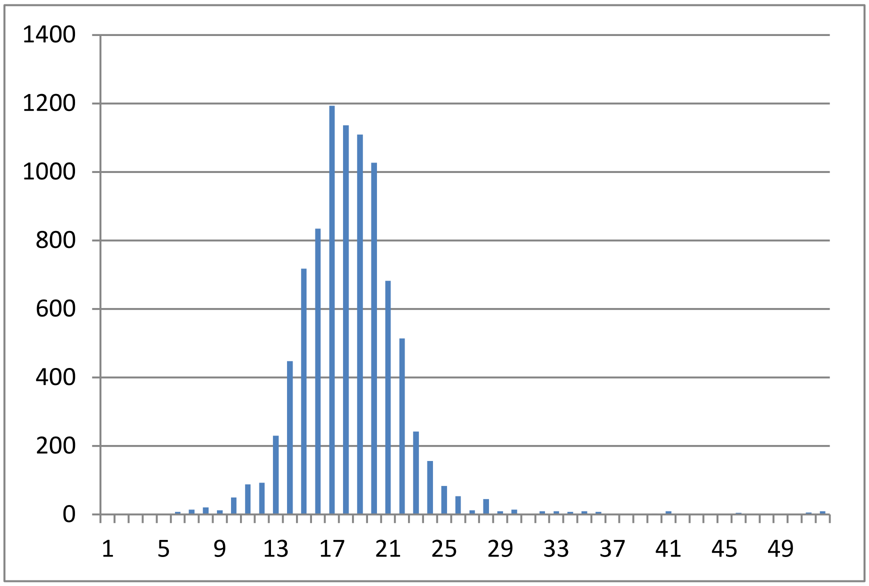

Figure 5-1: Speed value distribution in the sample, peak time

Similarly, we randomly generate service time at the client's home according to the histogram generated from the operation data provided. The distribution is presented in Figure 5-2.

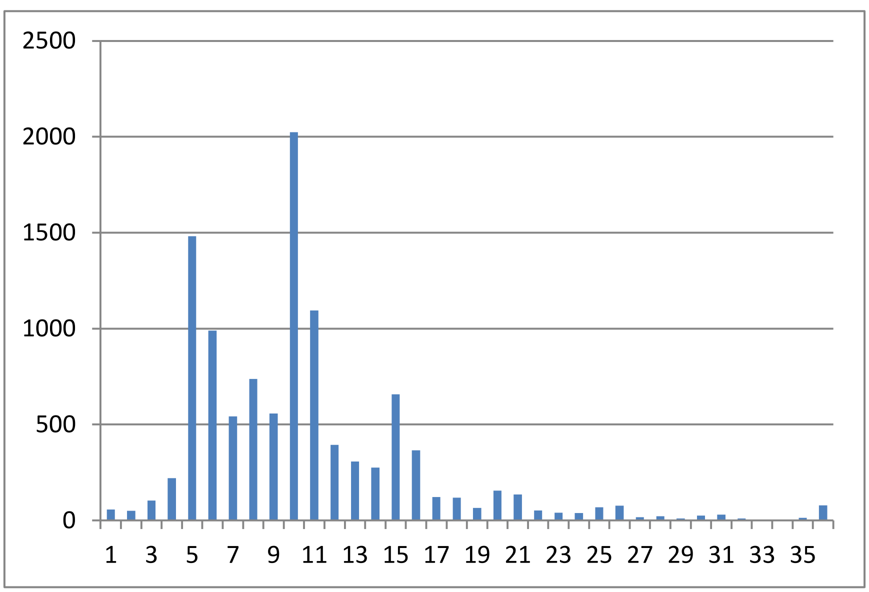

Figure 5-2: Time at client's distribution in sample

Just as in the previous figure, the horizontal axis represents the time spent at the client's home for the delivery and the vertical axis represents the frequency of a particular value in the sample. Please note that unfortunately this data is biased because of the manual input: clearly drivers tend to round the time at client to 5,10 or 15 minutes.

5.2. Simulation procedure

We then proceed to the simulation following five main steps

1. Using the Clark and Wright savings algorithm describe in the previous chapter, we solve the Travel Salesman Problem for 30 different instances: we determine the shortest path to visit each client of the instance from the depot exactly once.
2. We divide this route into sub-route according to the number of trucks available. We repeat this step for different scenarios: from 1 to 10 trucks available and we add a 15-client capacity restriction (this is the capacity of the trucks analyzed). We use a simple a simple sequential method: we determine the average customer per truck and then assign clients in the route order determined in step 1 to trucks. For example if we have 80 clients and 7 trucks, we assign clients 1 to 12 to truck 1, 13 to 24 to truck 2 etc…
3. For each sub-route we successively calculate the arrival time at the clients' home according to the truck's speed and the time spent at client's home previously generated. We suppose that the truck is leaving the depot at the beginning of the time window, thus it's important to note that it's not possible to arrive in advance (ie before the start of the time window).
4. We compare the time of arrival at each client's home with the time window constraint. For each instance we obtain the proportion of clients that are not dispatched on time

5. Finally, given a number of clients and trucks available, the service level is calculated as the average of the percentage of clients dispatched on time among the 30 instances simulated.

5.3. Main results

The results of the simulation are presented in Figure 5-3. As in Figure 4-3, the vertical axis represents the service level and the horizontal axis indicates the number of clients that are served in the time window. This particular figure shows results for a 120min time window. The different curves represent different number of trucks available to realize the operation.

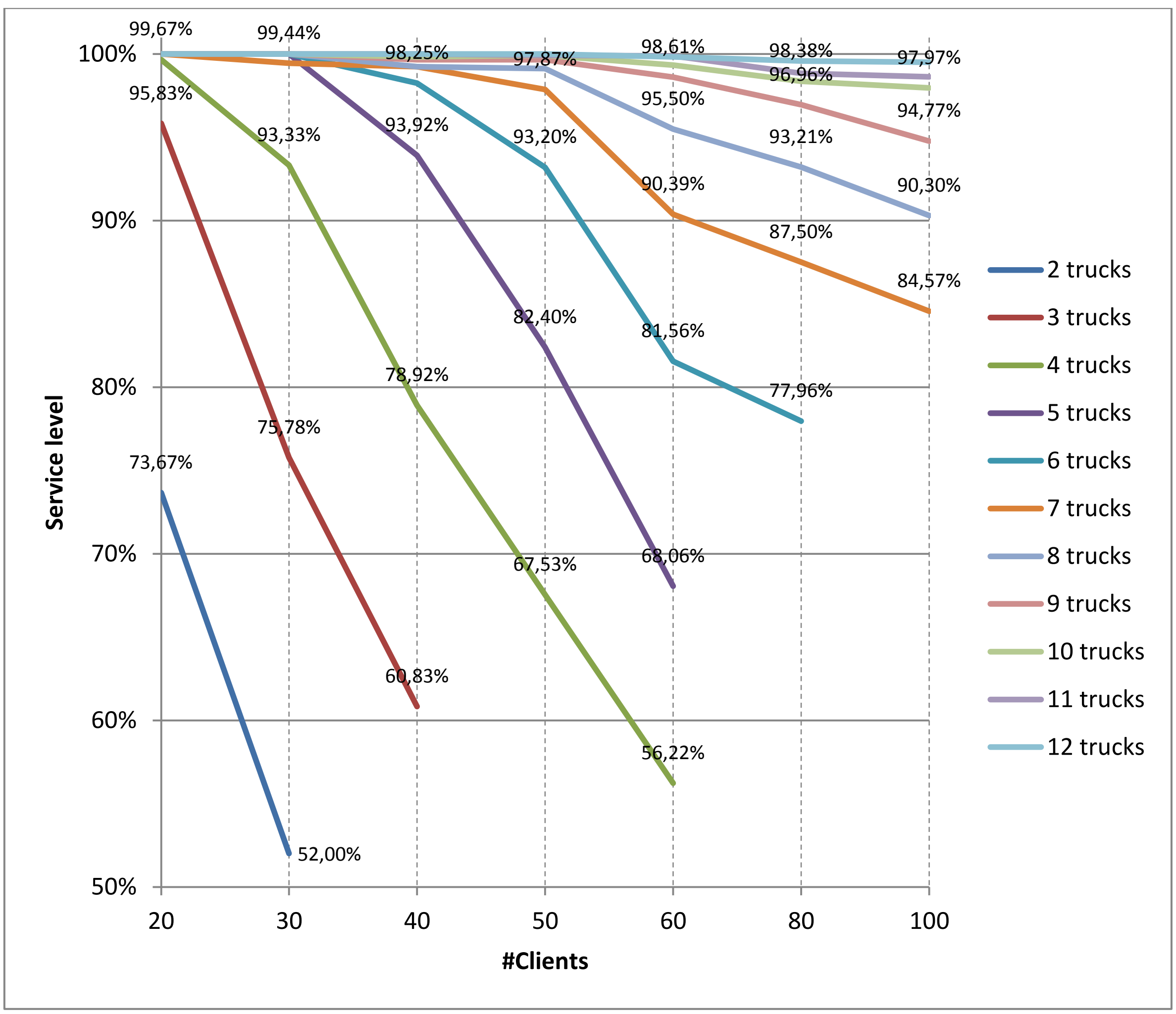

Figure 5-3. Simulation - Service level vs Quantity of clients, 120 min time window

The simulation yielded results very similar to the theoretical model:

- The service level improves with the numbers of trucks available to dispatch the clients' orders and the marginal benefit of a truck is a decreasing function that depends on the number of clients served and the number of trucks
- The relationship between service level and quantity of client is a linear function whose slope also depends on the number of clients and the number of trucks.

In order to compare with the theoretical model, we plotted the slope of the service level against the number of trucks available for the dispatch. Here again, in Figure 5-4, we find a rather good linear relationship ($R^2 = 0{,}88$) between these two parameters. The vertical axis represents the slope of the service level curve of Figure 5-3, whereas the horizontal one represents the number of trucks.

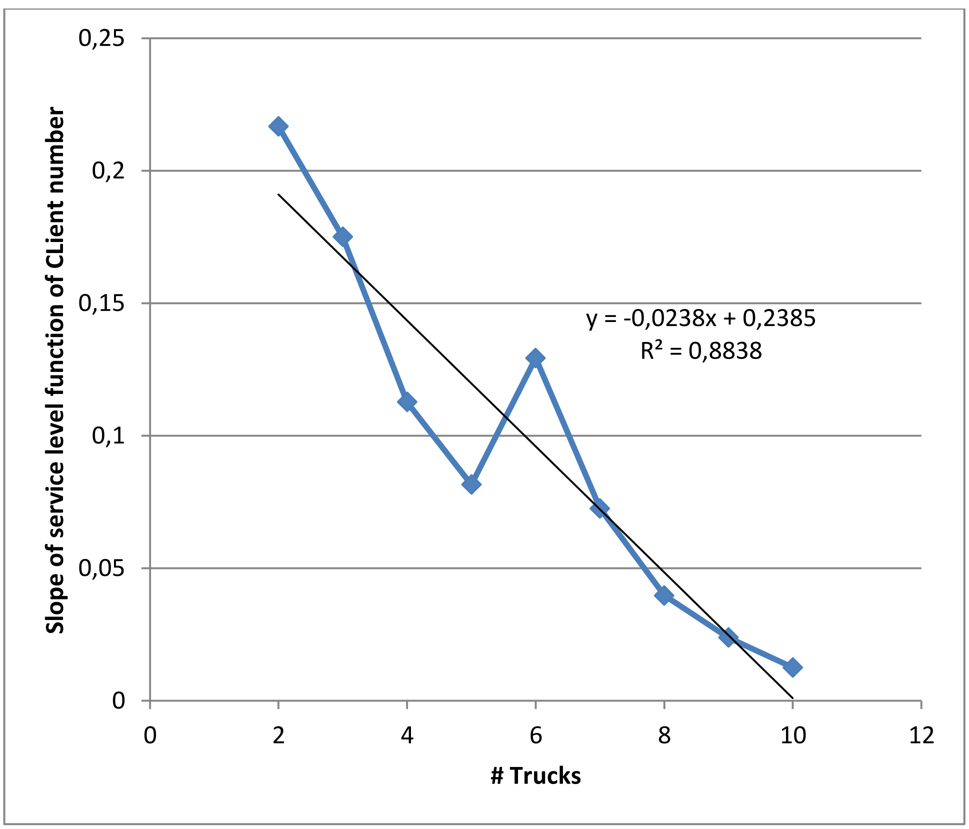

Figure 5-4: Simulation - Variation of service level vs Number of trucks, 120 min time windows

In Figure 5-5 we compare both curves. Even with a different sets of client (one is randomly simulated, the other is modeled) we have very similar results and we obtain a correlation of 1,007 between the curves with a R^2 of 0,91. Moreover, if we take out the value for 6 trucks –which seems to be an outlier, the R^2 goes up to 0,965 with a correlation of 1,022.

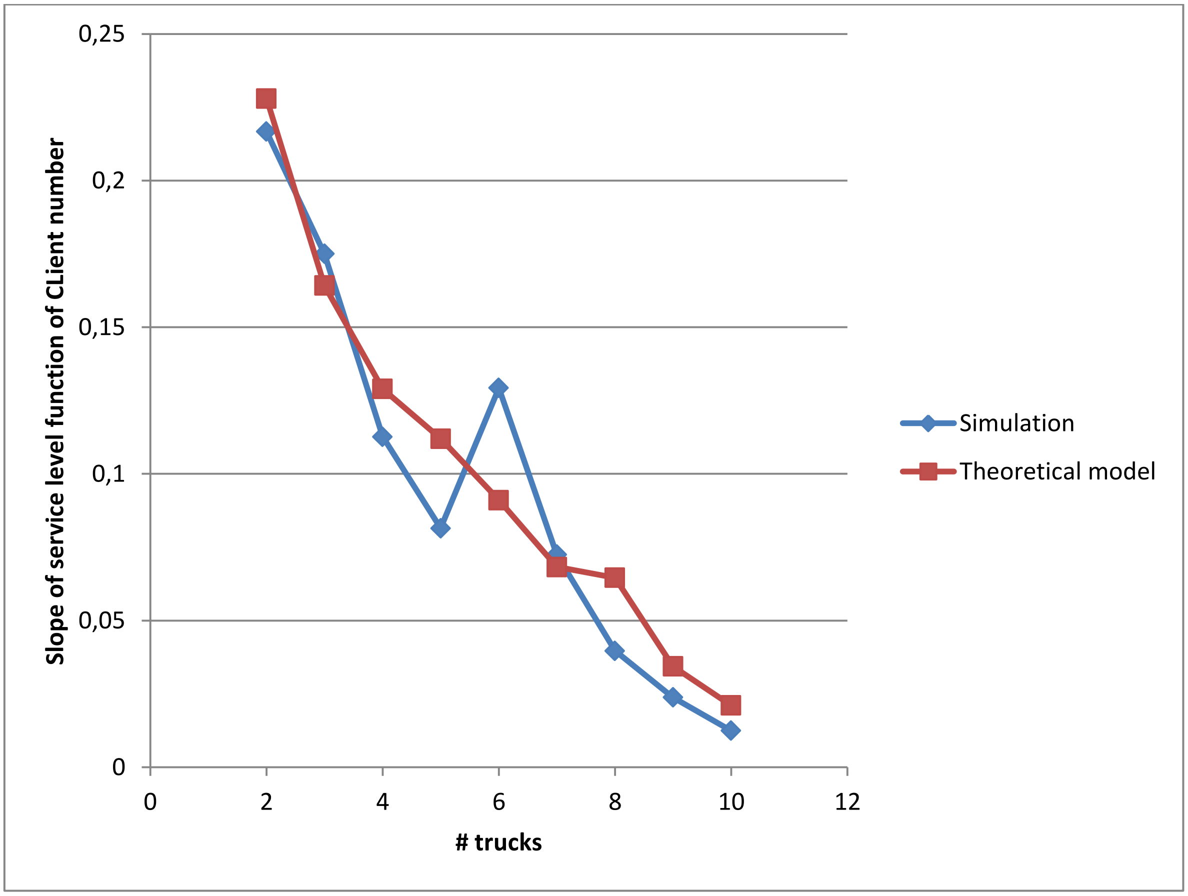

Figure 5-5: Simulation vs Model - Variation of service level vs Number of trucks, 120 min time window

We observe that the simulation line is below the theoretical one (hence a correlation value >1). This means that the slope of the service level curve of the simulated model y less steep than the one from the theoretical model. The marginal loss in service level due to an extra client is thus lower in the simulated model. An explanation for this effect is that the construction of the routes in the theoretical model is sub-optimal. When solving the TSP with real clients location, the most probable scenario is that the routes defined serve clients that are located nearby from one another (and in particular, closer than in the theoretical model) thus explaining the smaller impact at the moment of adding an extra client to the route.

All these results seem to validate the theoretical model proposed and the main results deducted in chapter 4 are valid in the simulated approach. The proposed model –with calibrated parameters according to the specific operation- could thus be used to study the dispatch operation of another retailer and help determine the size of the fleet according to the service level desired and the available budget.

In Figure 5-5: Simulation vs Model - Variation of service level vs Number of trucks, 120 min time windowFigure 5-5 we present an alternative way of using the results: plotting the service level against the number of trucks available instead of the number of clients. Usually the number of trucks is less dynamic than the number of clients served. In the first case the retailer need to buy or lease trucks, in the second case it only depend on the demand level of a particular day. This new representation gives an insightful view of the operation to a COO for example. Given a number of trucks available, he is able to visualize the service level he can offer according to the number of clients he will serve. That kind of result can be very useful at the moment of deciding to close or not a time window. That is to say, given a certain capacity and a minimum service level the company wants to guarantee, this graph indicates what should be the maximum number of clients you should agree to serve in a given time window.

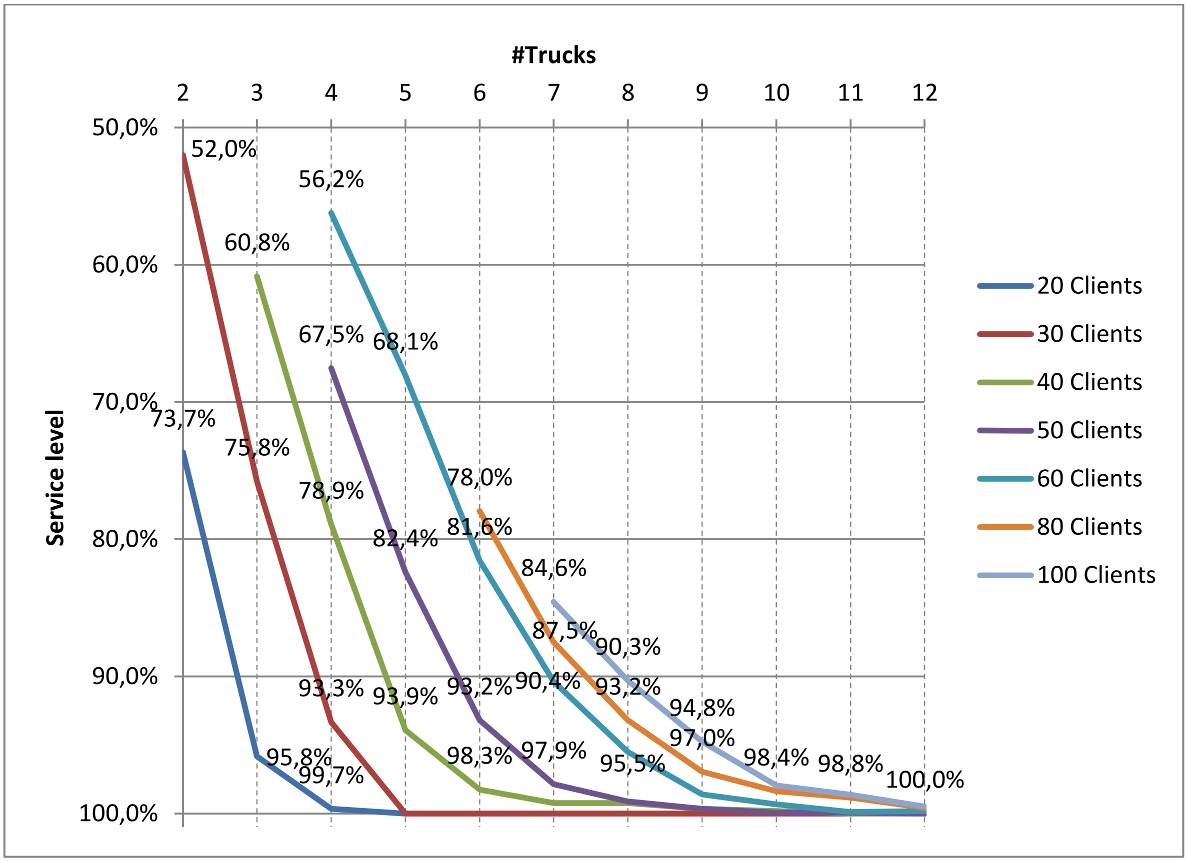

Figure 5-6 Service level vs #Trucks, 120min time window

6. CONCLUSIONS

This thesis has analyzed the tradeoffs involved in managing a home delivery operation service level. The main results are detailed in the next section and some possible extensions are proposed by the author.

6.1. Results and contributions

In this thesis we identified different parameters that influence the service level of a home delivery operation dispatch. Several conclusions were made throughout the investigation:

- The main conclusion, even if it may seem obvious, is that service level has a price and that price varies according to the different parameters of the situation.
- The easiest lever to improve the service level is the quantity of trucks available for client dispatch. The more available trucks, the better the service level. However, the marginal benefit of an extra truck varies according to the situation. For small operations, an extra truck will drive high impact but will represent a higher proportional cost in the operation. On the other side, for big operations, an extra truck will not have that much impact on the overall service level. That is explained by the fact that the marginal effect of an extra client on service level is a linear decreasing function of the number of trucks.
- High service level is expensive. As we have seen in this thesis, the curve Cost vs Service level is convex and thus the last marginal improvement in service level is very expensive.

In addition to these conclusions, we would like to lay down some high level recommendations for the COO in charge of such operations systems

- As the service level has a cost, it is essential for a company to have a clear strategy on the service level they want to propose to their client and how much money they are willing to invest in it.

- Once that strategy is clearly established, the model proposed in this thesis can be used to efficiently and consciously position the company in the exact operation point desired. For a given expected number of orders in a time window, and a number of trucks currently available, the responsible of the operation can use the model to understand the service level that the operation can deliver. Based on that, he has all the elements necessary to quantify:
    - Current level of service proposed to the client
    - Possible service level with a different number of trucks. Considering this factor and the structure cost of ownership or renting a truck will be of great help whenever taking a decision about fleet size and pricing of renting a truck "on the spot" when needed.
    - Effect on service level of an extra client
- Moreover, by recalibrating the model with different speeds and time at clients' homes, the retailer can tailor a specific dispatch strategy for different times and zones. Genta and Muñoz (2007) show the possibilities of leveraging the different characteristics of the operators (driving speed and dispatching time) to improve the efficiency of the operation. By using a similar methodology, it is possible to improve the service level with the exact same resources, understanding which routes are more critical and focusing better operation level on them.

6.2. Extensions suggestions

In this section we propose some extensions:

- Revenue management: with the tools proposed in this thesis and revenue management methods, we propose a tailored pricing to the client (according to day of the week, hour of the day, time window length) in order to capture what the client is actually willing to pay.
- Dynamic pricing: vary the price of the dispatch according to number of clients already confirmed. With the model presented, it is possible to understand the service

level and cost for the marginal client. Using this information it is possible to define the marginal price for a given service level

BIBLIOGRAFY

Agatz, N., Campbell, A., Fleischmann, M. and Savelsbergh, M., (2008) Time Slot Management in Attended Home Delivery. *ERIM Report Series Reference No. ERS-2008-022-LIS*

Agatz, Niels A.H., Fleischmann, Moritz and Van Nunen, J.A.E.E. (2006). E-Fulfillment and Multi-Channel Distribution - A Review . *ERIM Report Series Reference No. ERS-2006-042-LIS*

Beraldi, P., Ghiani, G., Laporte, G., Musmanno, G., (2005). Effiient neighbourhood search for the probabilistic pickup and delivery salesman problem. *Networks* 46, 195-198.

Boyer, K., G, T. H., & Frohlich, M. (2003). An exploratory analysis of extended grocery supply chain operations and home delivery. *Journal of Manufacturing Technology Management, 14*(8), 652-663.

Campbell, A., Savelsbergh, M., (2005). Decision Support for Consumer Direct Grocery Initiatives. *Transportation Science* 39, 313-327.

Campbell, A., Savelsbergh, M., (2006). Incentive Schemes for Attended Home Delivery Routing Problems. *Transportation Science* 40, 226-234.

Daganzo, C.F., (1987a). Modeling distribution problems with time Windows 1. *Transportation Science 21 (3), 171-17*
}9.

Daganzo, C.F., (1987b). Modeling distribution problems with time Windows 2. -2 Customer types. *Transportation Science 21 (3), 180-187.*

Dantzig, G.B., Ramser, J.M., (1959). The truck dispatching problem. *Management Science 6*, 81-91.

Ebensperger, M., Giesen, R., (2009). Una formulación para el problema de ruteo de vehículos con tiempos de viaje dependientes del tiempo para la actualización de rutas con información en tiempo real. Available http://200.29.152.69/xmlui/handle/123456789/22948

Figliozzi, M., A., (2009). Planning approximations to the average length of vehicle routing problems with time window constraints. *Transportation Research Part B 43 (2009) 438–44.*

Figliozzi, M., A., Mahmassani, H., S., Jaillet, P., (2007). Pricing in Dynamic Vehicle Routing Problems. *Transportation Science* Vol. 41, No. 3, pp. 302-318

Fleischmann, B., Gnutzmann, S., Sandvoß, E., (2004). Dynamic Vehicle Routing Based on Online Traffic Information. *Transportation Science* Vol. 38, No. 4, pp.420-433

Fleischmann, B., Gietz, M. y Gnutzmann, S. (2004). Time-Varying Travel Times in Vehicle Routing. *Transportation Science,* 38 (2), 160-173.

FOCUS: UK, US central to online grocery sales growth [Online, consulted November 2010]. Available http://www.just-food.com/analysis/uk-us-central-to-online-grocery-sales-growth_id109940.aspx

Genta, S. and Muñoz, J. C., (2007). "On assigning drivers for a home-delivery system on a performance basis," *Ann. Oper. Res*., v155, pp. 107-117, 2007.

Ghiani, G., Guerriero, F., Laporte, G., Musmanno, R., (2003). Real-time vehicle routing: Solution concepts, algorithms and parallel computing strategies. *European Journal of Operational Research 151, 1–11*

Laporte, G., (2009). Fifty Years of Vehicle Routing. *Transportation Science 2009 43:408-416*

Jaillet, P., (1985) Probabilistic traveling salesman problem. PhD Thesis, Operation Research Cneter, Massachussets Institute of Technology, Cmbridge, MA.

Solomon, M.M. (1987). Algorithms for the vehicle-routing and scheduling problems with time window constraints. *Operations Research 35 (2), 254–265*.

Lin, I., Mahmassani, H., (2002). Can Online Grocers Deliver ? Some Logistics considerations. *Transportation Research Record N°1817, Tranportation Planning and Analysis, 17-24*.

Van Woensel, T., Kerbache, L., Peremans, H., Vandaele, N., (2008). Vehicle routing with dynamic travel times: A queueing approach. *European Journal of Operational Research 186, 990–1007*

Zhong, H., Hall, R., W., Dessouk, M., (2007). Territory Planning and Vehicle Dispatching with Driver Learning. *Transportation Science, v.41 n.1, p.74-89.*