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Superparticle mass spectra from $SO(10)$ grand unified models with Yukawa coupling unification

Howard Baer,[1] Marco A. Díaz,[1,2] Javier Ferrandis,[1,3] and Xerxes Tata[4]
[1]*Department of Physics, Florida State University, Tallahassee, Florida 32306*
[2]*Facultad de Física, Universidad Católica de Chile, Av. Vicuña Mackenna 4860, Santiago, Chile*
[3]*Departament de Física Teòrica, Universitat de València, Spain*
[4]*Department of Physics and Astronomy, University of Hawaii, Honolulu, Hawaii 96822*

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We examine the spectrum of superparticles obtained from the minimal $SO(10)$ grand unified model, where it is assumed the gauge symmetry breaking yields the minimal supersymmetric standard model (MSSM) as the effective theory at $M_{GUT}\sim 2\times 10^{16}$ GeV. In this model, unification of Yukawa couplings implies a value of $\tan\beta\sim 45$–55. At such high values of $\tan\beta$, assuming universality of scalar masses, the usual mechanism of radiative electroweak symmetry breaking breaks down. We show that a set of weak scale sparticle masses consistent with radiative electroweak symmetry breaking can be generated by imposing non-universal GUT scale scalar masses consistent with universality within $SO(10)$ plus extra D-term contributions associated with the reduction in rank of the gauge symmetry group when $SO(10)$ spontaneously breaks to $SU(3)\times SU(2)\times U(1)$. We comment upon the consequences of the sparticle mass spectrum for collider searches for supersymmetry. One implication of $SO(10)$ unification is that the light bottom squark can be by far the lightest of the squarks. This motivates a dedicated search for bottom squark pair production at $p\bar{p}$ and e^+e^- colliders.

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Unification of the standard model (SM) of strong, weak and electromagnetic interactions within a single Lie group such as $SU(5)$ or $SO(10)$ has a long history and many attractive features [1]. $SU(5)$ is the smallest grand unifying group, and predicts the quantization of electric charge, the unification of gauge couplings and the unification of bottom and tau Yukawa couplings at scales of $Q=M_{GUT}\simeq 10^{15}$ GeV [2]. The $SO(10)$ theory incorporates all the matter fields of the SM into the 16-dimensional spinor representation, ψ_{16}, of $SO(10)$ [3]. In minimal $SO(10)$, not only the gauge couplings but *all* the Yukawa couplings (within a generation) are unified at $Q=M_{GUT}$. If the right-handed neutrino field present in ψ_{16} acquires a large Majorana mass, it decouples from the theory, and a small neutrino mass is induced via the see-saw mechanism [4].

The supersymmetric version of this model, with supersymmetry (SUSY) softly broken at a scale $\lesssim 1$ TeV, naturally stabilizes the hierarchy between the weak scale and the grand-unification scale. Supersymmetry also raises the unification scale to $M_{GUT}\simeq 2\times 10^{16}$ GeV, which helps reduce the rate for proton decay to below the level of experimental bounds. In addition, the introduction of supersymmetry with soft SUSY breaking (SSB) masses of order the weak scale allows for the near unification of gauge coupling constants [5]. In supergravity-based models, it is usually assumed that all scalar masses receive a common mass $m_Q=m_U=m_D=m_L=m_E=m_{H_u}=m_{H_d}\equiv m_0$ at M_{GUT}, while all gauginos receive a common mass $m_{1/2}$ and all trilinear SSB terms unify to A_0. The SSB masses and couplings are then evolved via renormalization group equations (RGEs) from M_{GUT} to $Q\sim M_{weak}$. The $m_{H_u}^2$ term is driven to negative values, which results in radiative breaking of electroweak symmetry, provided the top quark mass is large (e.g. 175 GeV).

In addition to the matter superfield $\hat{\psi}_{16}$, the minimal $SO(10)$ model includes a **10** dimensional Higgs superfield $\hat{\phi}_{10}$ that decomposes into a $\mathbf{5}+\overline{\mathbf{5}}$ representation of $SU(5)$, and includes the two Higgs superfields ($\hat{H}_u$ and $\hat{H}_d$) of the minimal supersymmetric standard model (MSSM). The superpotential includes the term $W\ni\lambda\hat{\psi}^T\hat{\psi}\hat{\phi}+\cdots$ responsible for quark and lepton masses, with λ the single Yukawa coupling in the low energy theory. The dots represent terms including for instance higher dimensional Higgs representations and interactions responsible for the breaking of $SO(10)$.

The mass spectrum of SUSY particles in minimal supersymmetric $SO(10)$ constrained by radiative electroweak symmetry breaking has been studied previously in a number of papers [6–16]. Unification of bottom, tau and top Yukawa couplings was found to occur at very large values of the parameter $\tan\beta\sim 50$–60, and specific spectra were generated for values of $m_t\sim 190$ GeV [7]. Assuming universality of soft SUSY breaking masses at M_{GUT}, it was found [8,10] that Yukawa unification consistent with radiative electroweak symmetry breaking could also occur for $m_t<170$ GeV as long as $m_{1/2}\gtrsim 300$ GeV. This generally leads to sparticle masses far beyond the reach of the CERN e^+e^- collider LEP2 or Fermilab Tevatron $p\bar{p}$ colliders. For values of $m_t\simeq 175$ GeV, solutions including radiative electroweak breaking were very difficult to achieve. In Ref. [15], the SUSY particle mass spectrum was investigated with *non-universal* SSB masses. Various solutions were found, but the non-universality in general broke the $SO(10)$ symmetry. In Ref. [16], it was argued that $SO(10)$ D-term contributions to scalar masses had the correct form to allow for successful radiative electroweak symmetry breaking and the computation of weak scale SUSY particle masses.

In this Rapid Communication, we explicitly calculate the sparticle mass spectrum for $SO(10)$ SUSY grand unified theory (GUT) models, taking the pole mass $m_t=175$ GeV.

We make the following assumptions. We assume the structure of minimal SUSY $SO(10)$ above the scale $Q=M_{GUT}$. We assume that SUSY $SO(10)$ directly breaks to the MSSM at M_{GUT}. Accordingly, there exist independent masses m_{16} and m_{10} for the matter and Higgs scalar fields. In the breakdown of $SO(10)$ to $SU(3)_C\times SU(2)_L\times U(1)_Y$, additional D-term contributions (parametrized by M_D^2 which can be either positive or negative) to the SSB scalar masses arise [17]:

$$m_Q^2=m_E^2=m_U^2=m_{16}^2+M_D^2$$

$$m_D^2=m_L^2=m_{16}^2-3M_D^2,\quad m_{H_{u,d}}^2=m_{10}^2\mp 2M_D^2.$$

Thus, the model is characterized by the following free parameters: m_{16}, m_{10}, M_D^2, $m_{1/2}$, A_0, sgn(μ). The value of $\tan\beta$ will be restricted by the requirement of Yukawa coupling unification, and so is *not* a free parameter.

Our procedure is as follows. We generate random samples of model parameters:

$$0<m_{16}<1500\text{ GeV},\quad 0<m_{10}<1500\text{ GeV},$$

$$0<m_{1/2}<500\text{ GeV},\quad -500^2<M_D^2<+500^2\text{ GeV}^2,$$

$$45<\tan\beta<55,$$

$$-3000<A_0<3000\text{ GeV and }\ \mu>0\text{ or }\mu<0.$$

We then calculate the non-universal scalar masses according to formulas given above, and enter the parameters into the computer program ISASUGRA. ISASUGRA is a part of the ISAJET package [18] which calculates an iterative solution to the 26 coupled RGEs of the MSSM.

To calculate the values of the Yukawa couplings at scale $Q=M_Z$, we begin with the pole masses $m_b=4.9$ GeV and $m_\tau=1.784$ GeV. We calculate the corresponding running masses in the $\overline{\rm MS}$ scheme, and evolve m_b and m_τ up to M_Z using 2-loop SM RGEs. At $Q=M_Z$, we include the SUSY loop corrections to m_b and m_τ using the approximate formulas of Pierce *et al.* [19]. A similar procedure is used to calculate the top quark Yukawa coupling at scale $Q=m_t$.

Starting with the three gauge couplings and t, b, and τ Yukawa couplings of the MSSM at scale $Q=M_Z$ (or m_t), ISASUGRA evolves the various couplings up in energy until the scale where $g_1=g_2$, which is identified as M_{GUT}, is reached. The GUT scale boundary conditions are imposed, and the full set of 26 RGE's for gauge couplings, Yukawa couplings and relevant SSB masses are evolved down to $Q\sim M_{weak}$, where the renormalization group improved one-loop effective potential is minimized at an optimized scale choice $Q=\sqrt{m_{\tilde t_L}m_{\tilde t_R}}$ and radiative electroweak symmetry breaking is imposed. Using the new spectrum, the full set of SSB masses and couplings are evolved back up to M_{GUT} including weak scale sparticle threshold corrections to gauge couplings. The process is repeated iteratively until a stable solution within tolerances is achieved. We accept only solutions for which the Yukawa couplings λ_t, λ_b and λ_τ unify to within 5%. This constraint effectively fixes the value of $\tan\beta$ typically to ~48. Yukawa unified solutions are found only for values of $\mu<0$. We also require the lightest SUSY particle to be the lightest neutralino, and that electroweak symmetry is successfully broken radiatively.

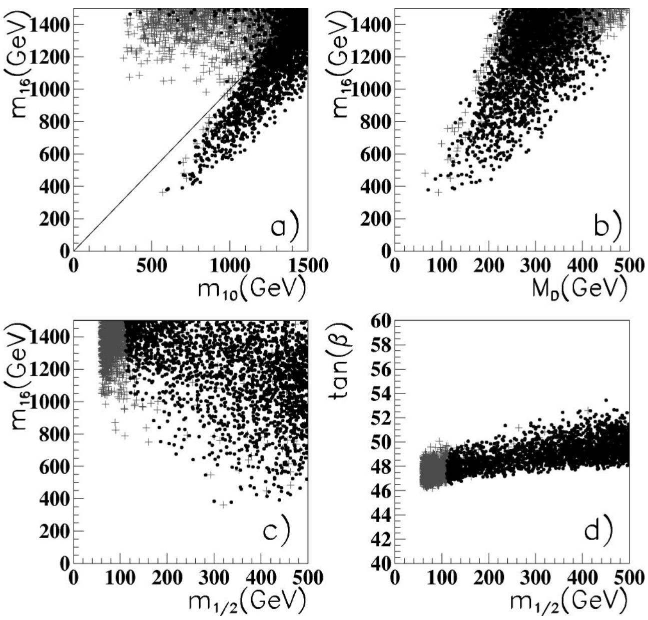

FIG. 1. Plots of regions of parameter space where valid solutions to minimal SUSY $SO(10)$ are obtained, consistent with Yukawa coupling unification to 5%, and radiative electroweak symmetry breaking.

We show in Fig. 1 the regions of model parameter space for which a SUSY mass spectrum can be calculated consistent with the above constraints. In Fig. 1a, we show the plane of m_{10} vs m_{16}. Each dot represents a point for which a solution was obtained. Points denoted by a cross are valid solutions, but with sparticle or Higgs masses below existing limits from LEP2. We require $m_{\tilde\tau_1}>73$ GeV, $m_{\tilde\chi_1^\pm}>95$ GeV and $m_h>85.2$ GeV [20]. From the distribution of points, we see that regions of model parameter space with $m_{16}<m_{10}$ are preferred, although for very large values of m_{16}, a few solutions are obtained for $m_{10}>m_{16}$. In Fig. 1b, we plot the M_D vs m_{16} parameter plane. In this frame, M_D actually stands for $\text{sgn}(M_D^2)\times\sqrt{|M_D^2|}$. No solutions were obtained for $M_D^2<0$, and in fact no solutions were obtained for $M_D^2=0$: this illustrates that non-zero D-term contributions to scalar masses are crucial for a valid sparticle mass spectrum in minimal $SO(10)$. The requirement of positive definite D-term contributions to scalar masses will leave, as we shall see, a distinctive imprint on the SUSY particle mass spectrum [17]. From the $m_{1/2}$ vs m_{16} plane in Fig. 1c, it can be seen that m_{16} is typically larger than $m_{1/2}$; otherwise $\tilde\tau_1$ becomes the lightest SUSY particle, in violation of cosmological limits on charged relic particles. Finally, in Fig. 1d, we show the range of $\tan\beta$ values for which solutions were generated versus the parameter $m_{1/2}$. We see that $46<\tan\beta<52$, with the slightly higher values of $\tan\beta$ being preferred when $m_{1/2}$ is large. The bounds on $\tan\beta$ are weakened if $\tau-b-t$ Yukawa unification is relaxed to more than 5%.

In Fig. 2, we show the range of selected sparticle and Higgs boson masses that are generated within minimal

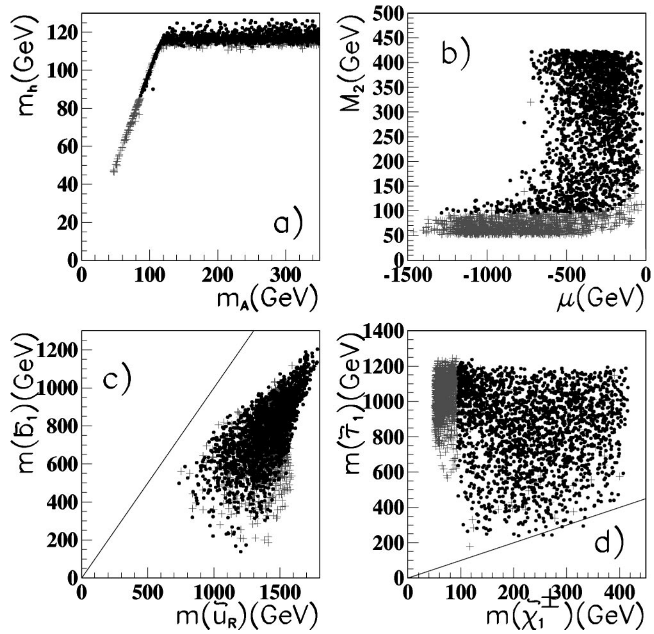

FIG. 2. The range of selected sparticle masses that are generated in minimal SUSY $SO(10)$ models with Yukawa coupling unification and radiative electroweak symmetry breaking.

$SO(10)$ with Yukawa coupling unification. In frame a, we see that the light Higgs boson h has mass generally bounded by $m_h < 125$ GeV. This range of light Higgs boson masses may well be accessible to Fermilab Tevatron Higgs boson searches [21]. Values of $m_h \lesssim 110$ GeV are associated with cases where m_A becomes comparable to or smaller than M_Z. In frame b, we plot solutions in the μ vs M_2 plane, where M_2 is the $SU(2)$ gaugino mass. Many solutions with $|\mu| < M_2$ exist, which generally implies that the lighter charginos and neutralinos have substantial Higgsino components. The solutions with large $|\mu|$ and $M_2 \sim 100$ GeV all correspond to values of $m_{16} > 1300$ GeV. In frame c, the bottom squark mass $m_{\tilde{b}_1}$ is plotted versus $m_{\tilde{u}_R}$. We see that although $m_{\tilde{u}_R}$ can be only as light as ~ 700 GeV, the $\tilde{b}_1$ mass can be as low as ~ 150 GeV. The bottom squark (mainly b_R) is generically much lighter than other squarks, because of the D-term contribution to m_D at $Q = M_{GUT}$ as well as b-Yukawa coupling effects which are significant for large values of $\tan\beta$ [22]. Finally, in frame d, we show the lightest tau slepton mass versus the light chargino mass. In $SO(10)$, the stau is the lightest of the sleptons, but as can be seen, solutions with $m_{\tilde{\tau}_1} < 200$ GeV are very difficult to generate, and almost always, $m_{\tilde{\tau}_1} > m_{\tilde{\chi}_1^\pm}$, so that two body decays such as $\tilde{\chi}_2^0 \to \tilde{\tau}_1 \tau$ or $\tilde{\chi}_1^\pm \to \tilde{\tau}_1 \nu$ almost never occur. A feature of minimal $SO(10)$ is that the light stau may contain a large left stau component, whereas in models with universality, the light stau is dominantly a right slepton. This could have an impact on the efficiency of detecting daughter tau leptons via their hadronic decay.

In Table I, we show sample weak scale sparticle and Higgs boson masses for five $SO(10)$ solutions with unified Yukawa couplings. It is possible to find solutions with sparticle masses potentially accessible to both LEP2 and Fermilab Tevatron searches, in contrast to previous studies assuming universality of scalar masses at M_{GUT}. For case 1, we take $(m_{16}, m_{10}, M_D, m_{1/2}, A_0) = (405.8, 680.3, 96.8, 427.2, 596)$ GeV. This solution requires $\tan\beta = 51.3$ to unify the Yukawa couplings. The evolution of gauge and Yukawa couplings for this case is shown in Fig. 3a. In our program, we do not require the $SU(3)$ gauge coupling to exactly unify with the $SU(2)$ and $U(1)$ gauge couplings, but rather attribute the near miss to unknown high scale physics. The Yukawa couplings diverge from their unification point and evolve to M_{weak}, with a kink in the curves coming from weak scale threshold effects. In Fig. 3b, we show the evolution of SSB Higgs boson masses and third generation SSB masses. We actually plot sgn(m_H^2)$\times\sqrt{|m_H^2|}$. In this case, both Higgs squared masses evolve to negative values, signaling the onset of radiative electroweak symmetry breaking. The GUT scale non-universality due to D-term contributions is evident. It usually results in left SSB slepton masses being close to or lighter than right slepton masses, and right sbottom masses lighter than the other squark masses.

The final weak scale sparticle masses are listed in Table I. For case 1, none of the sparticle or Higgs bosons are accessible to LEP2, while one or more of the Higgs bosons may be accessible to the Fermilab Tevatron running at maximal luminosity. An e^+e^- collider operating at $\sqrt{s} = 500$ GeV would find not only the various MSSM Higgs bosons, but also charginos and neutralinos (with substantial Higgsino

TABLE I. Weak scale sparticle masses and parameters (GeV) for five $SO(10)$ case studies.

Parameter	Case 1	Case 2	Case 3	Case 4	Case 5
m_{16}	405.8	1240.0	1022.0	414.8	629.8
m_{10}	680.3	1414.0	1315.0	735.7	836.2
M_D	96.8	410.6	329.8	171.9	135.6
$m_{1/2}$	427.2	136.5	232.0	449.1	348.8
A_0	596.0	-1100.0	-1350.0	576.7	-186.5
$\tan\beta$	51.3	47.0	48.6	51.3	52.1
$m_{\tilde{g}}$	1021.3	409.9	631.5	1069.3	864.8
$m_{\tilde{u}_L}$	983.7	1337.8	1178.5	1033.7	974.4
$m_{\tilde{d}_R}$	925.4	1057.6	970.1	934.9	910.8
$m_{\tilde{t}_1}$	718.4	737.9	512.3	754.5	618.7
$m_{\tilde{b}_1}$	735.6	140.6	187.1	721.7	636.8
$m_{\tilde{l}_L}$	478.7	1012.8	857.8	428.3	634.6
$m_{\tilde{l}_R}$	452.9	1321.1	1088.9	489.6	662.5
$m_{\tilde{\nu}_e}$	472.0	1009.7	854.1	420.7	629.5
$m_{\tilde{\tau}_1}$	233.2	790.1	623.6	272.6	427.8
$m_{\tilde{\nu}_\tau}$	386.0	787.4	619.5	314.6	519.1
$m_{\tilde{\chi}_1^\pm}$	159.5	110.5	122.9	177.5	106.3
$m_{\tilde{\chi}_2^0}$	166.7	110.3	131.6	195.1	126.1
$m_{\tilde{\chi}_1^0}$	129.0	56.8	84.0	152.3	87.5
m_h	113.7	115.5	118.8	116.4	93.7
m_A	115.8	645.0	479.9	277.9	93.9
m_{H^+}	152.2	652.3	490.2	295.1	137.1
μ	-157.2	-329.8	-150.5	-185.5	-113.9
$\langle\tilde{\tau}_1\vert\tilde{\tau}_L\rangle$	0.14	0.99	0.99	0.47	0.11

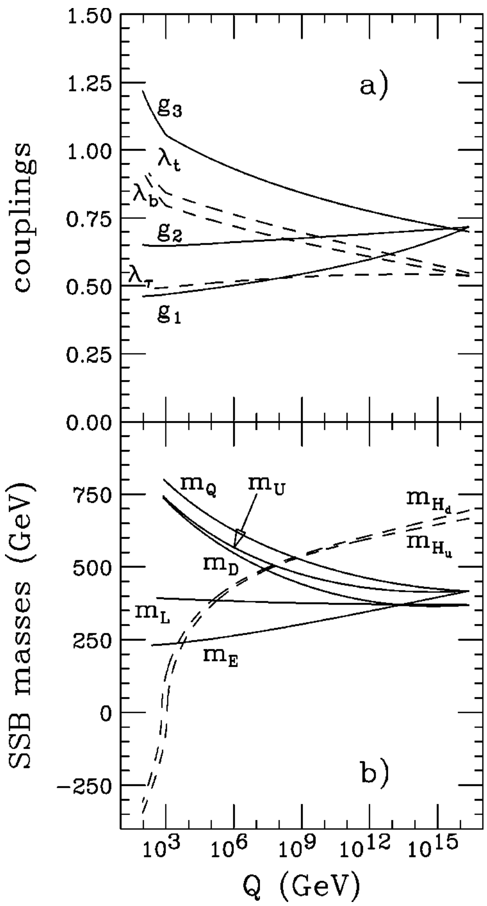

FIG. 3. For case 1 in Table I, we show (a) the running of both gauge and Yukawa couplings between $Q=M_{GUT}$ and $Q=M_{weak}$. In (b), we show the running of SSB Higgs masses (dashed curves) and third generation SSB masses (solid curves).

components) and light tau sleptons (in this case $\tilde{\tau}_1 \sim \tilde{\tau}_R$).

The second case study point shown has very large values of m_{16} and m_{10}, leading in general to a spectrum with very heavy scalars. The exception in this case is that the $\tilde{b}_1$ mass is only 140 GeV, and is directly accessible to Fermilab Tevatron collider searches, according to Ref. [23].

In this case, $\tilde{b}_1 \to b\tilde{\chi}_1^0$ with a branching fraction of $\sim 80\%$, so that $\tilde{b}_1\overline{\tilde{b}_1}$ production would be visible in $b\bar{b} + \not{E}_T$ events. The gluino is also relatively light and decays via $\tilde{g} \to b\tilde{b}_1$: it would be interesting to examine whether the improved b tagging at Tevatron upgrades would allow its detection in the multi-b plus $\not{E}_T$ channel. The light Higgs boson might be accessible to high luminosity Tevatron experiments, but the charginos and neutralinos would be difficult to see via the trilepton channel since $\tilde{\chi}_2^0$ dominantly decays to $b\bar{b}\tilde{\chi}_1^0$ and the $\tilde{\chi}_2^0 \to e\bar{e}\tilde{\chi}_1^0$ branching fraction is only 0.8%. A Higgs signal of $b\bar{b}l + \not{E}_T$ events from Wh production would contain substantial contamination from $\tilde{\chi}_1^{\pm}\tilde{\chi}_2^0$ events, which give rise to the same event topology. Contrary to models with universality, the left selectron (smuon) is significantly lighter than the right selectron (smuon). This distinctive feature of the $SO(10)$ model would be difficult to discern as the sleptons are very heavy. In the squark sector, the $\tilde{d}_R$ and $\tilde{s}_R$ are the lightest of the first two generations of squarks, owing to the D-terms.

In case 3, again the bulk of the scalars are quite heavy, and well beyond the reach of LEP2 or the Tevatron. Again the exception is the light bottom squark. In this case, however, $\tilde{b}_1$ decays with a 24% (8%) branching fraction to $b\tilde{\chi}_2^0$ ($b\tilde{\chi}_3^0$), so the event signatures will be more complicated. Since $\tilde{\chi}_2^0$ and $\tilde{\chi}_3^0$ decay with a large rate to $b\bar{b}\tilde{\chi}_1^0$, some of the $\tilde{b}_1\overline{\tilde{b}_1}$ events will contain final states with up to six b-jets plus $\not{E}_T$. If clean trilepton signatures are detected [24], they will contain a mixture of events from both $\tilde{\chi}_1^{\pm}\tilde{\chi}_2^0$ and $\tilde{\chi}_1^{\pm}\tilde{\chi}_3^0$ production.

In case 4, all strongly interacting sparticles including the bottom squark are quite heavy and accessible only at the CERN Large Hadron Collider (LHC). However, the various sleptons and sneutrinos are within reach of an e^+e^- linear collider operating at $\sqrt{s} \simeq 1000$ GeV. In this case, a very mixed $\tilde{\tau}_1$ whose composition may be measurable at a Linear Collider [25] may serve to distinguish this framework from models with universal soft masses. Moreover, $\tilde{\nu}_{eL}$ and the $\tilde{e}_L$ are measurably lighter [26] than $\tilde{e}_R$, again in contrast with expectations in models with universality. Note also that $|\mu| < M_2$, so that the light charginos and neutralinos have substantial Higgsino components, and further that there is only a small mass gap between $m_{\tilde{\chi}_2^0}$ or $m_{\tilde{\chi}_1^{\pm}}$ and $m_{\tilde{\chi}_1^0}$, so that -ino decay products will be soft.

Finally, in case 5, again the light charginos and neutralinos are Higgsino-like, and will be challenging to detect at the Fermilab Tevatron collider. This spectrum is characterized by a very light Higgs boson spectrum, and in fact 32% of top quark decays are to charged Higgs bosons. Indeed, both the light and pseudoscalar Higgs boson are at the edge of detectability at the LEP2 collider.

We have demonstrated that the inclusion of D-terms can lead to radiative electroweak symmetry breaking even in models with Yukawa coupling unification. As shown in Fig. 1b, we are unable to find corresponding solutions for models with scalar mass universality for the ranges of parameters studied here. We have not attempted to do an analysis of the phenomenological implications of the model. In a follow-up report, we will present results of calculations for the neutralino relic density, $b \to s\gamma$ decay rate, direct dark matter detection rate, and prospects for collider searches [27]. Parts of the parameter space as well as some of the case studies may well be excluded by experimental constraints. For instance, our preliminary results indicate that the predicted value for the decay $b \to s\gamma$ exceeds the experimental upper limit by a factor ~ 2–4 if $m_{1/2} \sim 200$–500 GeV. This is well-known to be a problem common to models with large tan β and $\mu < 0$ [28], the region of parameter space where Yukawa couplings unify.

Despite this phenomenological problem, we find it encouraging that it is possible to construct a calculable framework with gauge and Yukawa coupling unification. We can imagine other physics that may make it possible to circumvent experimental limits such as those from $b \to s\gamma$. For example, it has been pointed out [29] that if the right-handed neutrino mass is significantly below the GUT scale and if $m_{10}^2 = 2m_{16}^2$, third generation scalars would be radiatively driven to much lower masses than other matter scalars, and further, that when D-terms are included, radiative electroweak symmetry breaking is still possible [30]. In this case, since the degeneracy between squarks is badly broken, it is possible that gluino-mediated contributions to $b \to s\gamma$ (which are generally thought to be small) may be significant. Whether these are large enough (and of the correct sign) to cancel the chargino-mediated amplitudes remains to be investigated. Alternatively, one might imagine that the usual computation of $b \to s\gamma$ amplitudes may be altered by CP violating phases between various chargino amplitudes [31], or by large Yukawa coupling radiative corrections [32].

To summarize, we have shown that explicit evaluation of sparticle mass spectra is possible in the minimal SUSY $SO(10)$ model with Yukawa coupling unification and radiative electroweak symmetry breaking, by including non-universal SSB masses at $Q = M_{GUT}$ which are in accord with $SO(10)$ breaking to the gauge group of the MSSM. The resulting spectra reflect the influence of the D-term contributions to scalar masses. Characteristic features of the model can include a light sbottom, a $\tilde{\tau}_1$ which is mainly $\tilde{\tau}_L$, $m_{\tilde{l}_L} < m_{\tilde{l}_R}$ and lighter charginos and neutralinos with substantial (sometimes even dominant) Higgsino components.

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