PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE

SCHOOL OF ENGINEERING

ROBUST OPTIMIZATION IN PRODUCTION PLANNING PROBLEM

PAMELA PAZ ALVAREZ MARAMBIO

Thesis submitted to the Office of Graduate Studies in partial fulfillment of the requirements for the Degree of Doctor in Engineering Sciences

Advisor:
JORGE VERA ANDREO

Santiago de Chile, June, 2016

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DEDICATION

To my wonderful family

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I want to thanks all the persons involved in this research. First thanks to my advisor PhD. Jorge Vera by his patience and support in all the process. Also recognition to the complete committee for the time dedicated and the useful comments.

Finally, gratefulness to my family, all of them. To my beloved husband and my three beautiful kids. To my mother and all her help in uncountable situations and opportunities, to my sister and of course, to my father who is supporting me always and ever.

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RESUMEN

La Optimización Robusta (RO, por sus siglas en inglés) es una metodología para incluir la incertidumbre en modelos de optimización. La incorporación de la incertidumbre en los modelos de toma de decisiones es de interés. Esto se debe a que asumir datos conocidos puede llevar a decisiones incorrectas, que se traducen en altos costos, importantes pérdidas e incluso infactibilidades para aplicar las acciones a seguir.

En esta tesis, se exploró la hipótesis de que la RO es una herramienta eficiente para la planificación bajo incertidumbre. Específicamente, se estudió el uso de esta metodología en la industria forestal. El objetivo de la investigación fue analizar la aplicabilidad de la RO en la Gestión de la Cadena de Abastecimiento.

Para alcanzar el objetivo y validar la hipótesis, se abordaron dos problemas típicos de la industria: la planificación de la producción de aserraderos y de bosques. Ambos problemas se abordaron a nivel táctico y la incertidumbre considerada afecta los rendimientos y disponibilidad de bosques respectivamente. Para evaluar la metodología, los problemas se formularon, resolvieron y compararon en sus versiones deterministas y robustas en términos de optimalidad, factibilidad y estabilidad (estructura) de la solución.

Adicionalmente, ya que la planificación de la producción incluye decisiones en distintos niveles jerárquicos, que interactúan entre sí; se resolvió un caso específico para dos niveles de planificación (táctico y operativo). Esto con el objetivo de analizar la RO como herramienta para mejorar la coordinación y coherencia entre distintos niveles jerárquicos de planificación. En este sentido, se comparó el desempeño de modelos deterministas en ambos niveles, con una combinación de un modelo robusto a nivel táctico y uno determinista a nivel operativo.

Los resultados obtenidos indican que se validó la hipótesis y se cumplió el objetivo de la tesis. Por lo tanto, se concluyó que la RO: i) es una herramienta eficiente para el manejo de la incertidumbre, ii) es de simple aplicación, iii) no aumenta la dificultad de resolución de los modelos respecto al problema determinista ya que conserva la estructura del problema original, iv) facilita la interacción entre distintos niveles de planificación, v) a nivel de optimalidad no genera importantes pérdidas, vi) a nivel de factibilidad, mejora este indicador y vii) a nivel de estabilidad, genera soluciones en cierta medida constantes que facilitan la operación.

Palabras Claves: Optimización robusta, incertidumbre, cadena de abastecimiento forestal.

ABSTRACT

Robust Optimization (RO) is a methodology for the inclusion of uncertainty in mathematical optimization models. The incorporation of information regarding uncertainly in decision-making models is a topic of great interest. This is because the assumption that the data which feeds these models is known can lead to incorrect decisions, which translates into high costs, important losses and even infeasibilities to apply some actions.

In this thesis, the hypothesis is that RO is an efficient tool for planning under uncertainty. Specifically, the use of this methodology was explored in a particular sector, the forestry industry. The objective of this thesis was to analyze the applicability of RO in forestry supply chain management.

To validate the hypothesis and achieve the proposed objective, two typical problems in the forestry industry were studied, which correspond to production planning for sawmills and forest respectively. Both problems were dealt with on a tactical level and the uncertainty considered affected the performance and availability of the forests. To evaluate the RO methodology, deterministic and robust models were formulated, solved and compared in terms of the optimality, feasibility and stability (structure) of the solution.

Additionally, given that the production planning problem includes decision making in distinct hierarchical levels, which interact between each other, this thesis focused on a specific case for two levels of planning (tactical and operational). The aim of this was to analyze RO as a tool to improve coordination and coherence between different levels of planning hierarchy. To achieve this, the performance of deterministic models was compared, on both hierarchical levels, with the combination of a robust model on a tactical level and a deterministic model on an operational level.

The results obtained in every situation indicate that the hypothesis and the objective were fulfilled. Then it was concluded that RO: i) is an efficient tool for managing uncertainty, ii) is simple to apply, iii) does not increase the difficulty of resolving models relative to deterministic problems, because the original structure is preserved, iv) facilitates the interaction between distinct planning levels, v) in terms of optimality, it does not generate important losses, vi) improves the level of feasibility, and vii) regarding stability, generates solutions which are constant to some extent, and which facilitate operations.

Keywords: Robust optimization, uncertainty, forestry supply chain

1. INTRODUCTION

The forestry industry in Chile is one of the most important sectors on a national level in economic terms, accounting for 3.1% of GDP (Gross Domestic Product). The industry is the country's second largest exporter, and the largest based on renewable natural resources. Around 40 million cubic meters of wood are harvested annually, and the Central Bank indicates that forestry industry accounts for 7.8% of the country's total annual exports, which reached US$53 billion in 2009.

Given the importance of the forestry industry, the sector has been a pioneer in the use of tools associated with Operations Research to support decision making. Operations Research has produced a significant and positive change in the sector, generating annual savings in the order of US$13 million (Epstein et al., 1999), and there is still an extensive margin for improvement (Epstein et al., 2007).

Optimization models have been characterized as being the principal tool associated with Operations Research in the industry. These models have been successfully implemented to support decision making on various problems, such as transport planning and trucks assignment, harvest planning, the machinery localization, road construction, logging, etc. (Epstein et al., 2007).

To formulate and subsequently solve the models, a large quantity of data must be used. As in many other areas, the data is considered as known, but this assumption is usually incorrect. Generally, various elements in the models are uncertain, and this uncertainty can be due to distinct factors. In some cases there can be natural variability in the data, in others estimation errors can occur due to the difficulty and cost of obtaining high-quality information to estimate parameters, and in some cases assumptions must be made about the data which is used due to the lack of available information.

Particularly, the uncertainty associated to natural resources is very important because there are biological processes that affect the data.

Traditionally, decision makers tend to use estimated average values, most probable values, and other methods to avoid this difficulty. In some cases this can cause associated decisions to be incorrect, which leads to unnecessary costs and even impracticable decisions.

For this reason, the incorporation of uncertainty in mathematical models is essential to guarantee that the decisions based on the model are feasible or assure some level of feasibility, according to certain criteria, despite what the data may indicate in reality. Although techniques to incorporate uncertainty in mathematical models exist, only some of them have been applied in the forestry sector. These techniques can be divided into two types of approach: one of them using probability based models, and the other applying fuzzy models. The latter use numerous approximates as parameters of the model and the constraints are dealt with as fuzzy sets (Ramik and Vlach, 2002; Rommelfanger, 1996), and as such certain violations of the constraints are permitted. However, fuzzy optimization has few applications in this sector. On the other hand, some approaches use probabilistic information such as stochastic programming, linear programming with probabilistic constraints, scenario analysis, Markov decision models and optimal control theory (Weintraub and Bare, 1996; Martell et al. 1998). For example, Weintraub and Vera (1991) present an algorithm to solve a linear programming problem with probabilistic constraints, that is to say, problems with random coefficients associated with the constraints. This algorithm was subsequently applied in forestry planning models by Weintraub and Abramovich (1995) but not extensively.

One of the main difficulties of applying stochastic programming more extensively is the increase in complexity of the problems (both in terms of mathematical formulation and the application of algorithms to solve them) and the difficulties in adjusting reliable probability functions due, in part, to the lack of real and detailed data. As such, while various theoretical methods are available, only a few of them have been applied in practice (Weintraub and Romero, 2006). This leads to the possibility of using other optimization techniques which do not require detailed knowledge of the probabilities, such as those which allow the calculation of robust solutions. That is to say, solutions which are less sensitive to uncertainty and which are, to some extent, independent of the variability of the data, at least within a certain range. Robust Optimization (RO) is one of the techniques developed along these lines. This methodology aims to determine solutions which remain feasible for all or almost all of the possible scenarios of the defined data in a certain set, and to prevent the failure to satisfy constraints (Ben-Tal and Nemirovski 1999; El Ghauoi et al. 1998, Bertsimas and Sim 2003).

Kazemi et al. (2010b) apply the robust optimization approach, and they compare it with stochastic programming in a multi-period and multi-product production planning problem in sawmills, with uncertainty in process performance and product demand. The results indicate the high quality of the model and provide evidence for the advantages of the robust optimization approach over the stochastic programming approach.

Following these lines, the current research pose the hypothesis that RO is an efficient tool for planning under uncertainty; and performs an applicability analysis of RO methodology in supply chain management, through the proposal and resolution of robust optimization models for particular cases in the forestry supply chain.

As such, the objective of this thesis is supported by the fact that, in practice, the incorporation of uncertainty in the models which support decision making is limited. There are studies which use diverse methods to incorporate uncertainty, however due to the increase in complexity of formulating and resolving of the models, they are not practical to implement.

This thesis therefore aims to provide information regarding a methodology which is simple to apply and which does not significantly increase the difficulty of resolving the problems involved.

Specifically in this research, RO methodology is applied to two typical problems in the forestry sector: the sawmill planning problem and the harvest planning problem. Both problems require robust tactical planning, due to variability in the performance and availability of the forests, which can result in suboptimal or unfeasible solutions. Additionally, it is not practicable to use the stochastic approaches due to the lack of high-quality probabilistic information and the difficulty of the resultant models.

Additionally, an intertemporal production planning problem is solved for two planning horizons (tactical and operational), in which the performance of RO as a tool to facilitate coordination between distinct hierarchical planning levels is reviewed. This is due to the possible inconsistencies that could occur given the uncertainty between what is planned on a tactical level and what it is effectively possible to achieve on an operational level. This is particularly important in the forestry industry, as planning horizons present rather extensive time intervals, which further complicates the task of intertemporal coordination.

In this sense, the contribution of this thesis is to apply and demonstrate that the use of RO can be introduced efficiently as a supporting tool in companies' decision making processes. As an additional advantage, RO provides the possibility of easily analyzing the tradeoff between robustness and optimality, a calculation which can be used to measure the impact of uncertainty on the decision making process. That is to say, it is possible to manage the degree of conservatism of the solution without necessarily using protecting against the worst case, which is not very likely to occur. Using a set of uncertainty ensures this (Gabrel et al., 2014). This is one of the main benefits of RO.

As such, this research is a contribution to the world of Operations Research as it presents experiments which demonstrate the efficiency of the mentioned methodology. Additionally, it provides an important contribution to the forestry industry, delivering new tools to support the decision making process. As mentioned above, in this industry it is important the study of uncertainty as, in the management of natural resources, decisions are typically over long time horizons and there are biological processes that act, which cannot be predicted with accuracy.

This thesis is structured as follows. The following chapter presents the hypothesis and the objectives. The chapter 3 describes the operations planning process in the forestry sector, covering the sawmilling, harvest planning and intertemporal planning areas. Then, in chapter 4, information is provided regarding how uncertainty has been managed in optimization models, including details of the RO methodology. Chapters 5, 6 and 7 present the three applications respectively which were undertaken and the results that were obtained. The conclusions and recommendations for future research are in chapters 8 and 9 respectively.

2. HYPOTHESIS AND OBJECTIVES

2.1. Hypothesis

The hypothesis to be tested in this research is that Robust Optimization is an efficient tool for planning under uncertainty.

2.2. Objective

2.2.1. General Objective

Analyze the applicability of RO in forestry supply chain management.

2.2.2. Specific Objectives

- Evaluate the performance of RO as a tool to tackle problem of planning production with uncertainty.
- Evaluate the usefulness of RO as a methodology to improve coordination between distinct time horizons.

3. FORESTRY OPERATIONS PLANNING

Chile is a developing country where the majority of its trade is in raw materials. As has already been mentioned, the forestry industry is the second most important sector for Chilean exports, according to CORMA (Corporación Chilena de la Madera). More than 70% of the Chilean forestry sector's production is exported (Maturana et al., 2010).

The Chilean forestry sector is completely private, and is mainly based on large companies which possess vertically integrated plantations, with pulp mills and sawmills. The Arauco holding company and Forestal Mininco (CMPC), the two largest companies, are among the largest forestry companies in the world (Epstein et al. 1999). In 2011, Arauco and CMPC occupied the second and fourth place in global cellulose production respectively[1].

Including all of the aspects related to forest planning (plantation, road construction, recollection, transport and the production of sawn timber, pulp, paper and heating plants) presents great difficulty. The diversity of the problems and the scale of task planning have increased over time.

A common opinion in the forestry industry is there is potential to improve the integration between different parts of the flow (of timber) and the use of sophisticated techniques to increase the utilization of raw materials and production capacity. At the same time, client orientation is currently the center of attention. The idea behind this approach is that the appropriate type of products (or raw material) should be delivered to the client in the appropriate quantity and time (Rönnqvist, 2003).

[1]http://www.economiaynegocios.cl/noticias/noticias.asp?id=80828.

3.1. Components of flow in the forestry supply chain

The forestry supply chain (**Figure 3.1-1**) begins in the felling areas (stands) of the forests. Here, the trees are harvested (cut) and the branches are removed. In the majority of cases, the first cutting or "bucking" (the process of cutting felled trees into logs) is undertaken directly (**Figure 3.1-2**). When a tree is logged, there are often many possible types of logs which can be obtained in accordance with characteristics such as diameter, length, and the quality of the shaft (tree trunk). The logs are subsequently extracted from the forests. In some cases, the entire trees (with branches removed) are transported directly to sawmills, where the logging process can be conducted in a more efficient way (Rönnqvist, 2003).

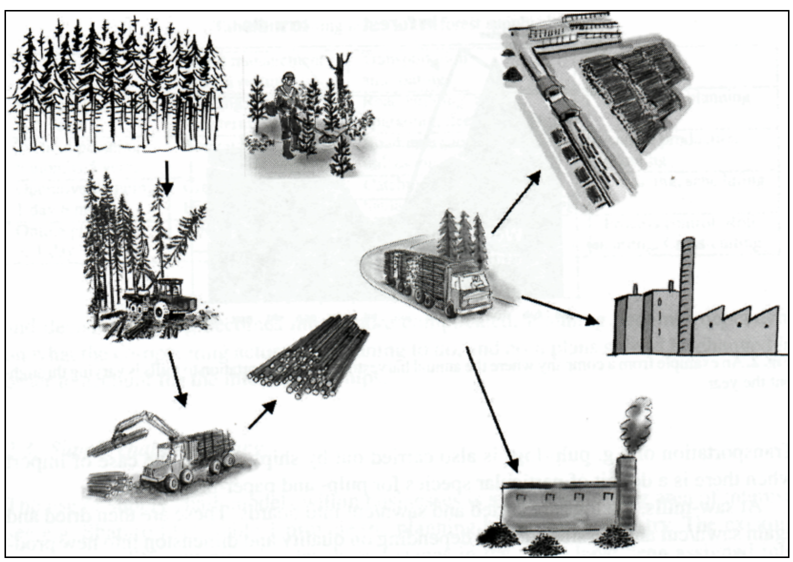

Figure 3.1-1: Flow in the forestry supply chain (Rönnqvist, 2003).

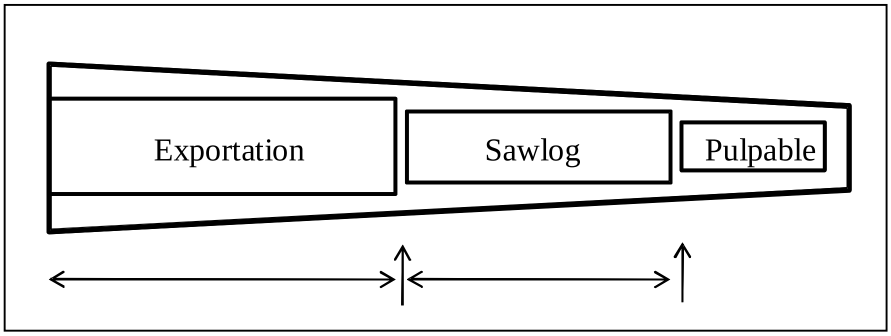

Figure 3.1-2: Example of “bucking” (Weintraub and Epstein, 2005).

The transportation can be conducted in one or two steps depending on the season. When the roads are in good condition, it is possible to transport directly to the industry. In this case, the logs are taken to sawmills or ports for direct exportation; the pulp to the cellulose plants and/or paper plants and the wood fuel to the heating plants. This latter product type is transported by special vehicles, which generally require the wood to be chipped. On the other hand, when the weather conditions are poor or in periods of thawing, the logs are frequently transported to terminals for intermediate storage (Rönnqvist, 2003).

To better illustrate the forestry supply chain, Figure 3.1-3 shows the physical flow of timber.

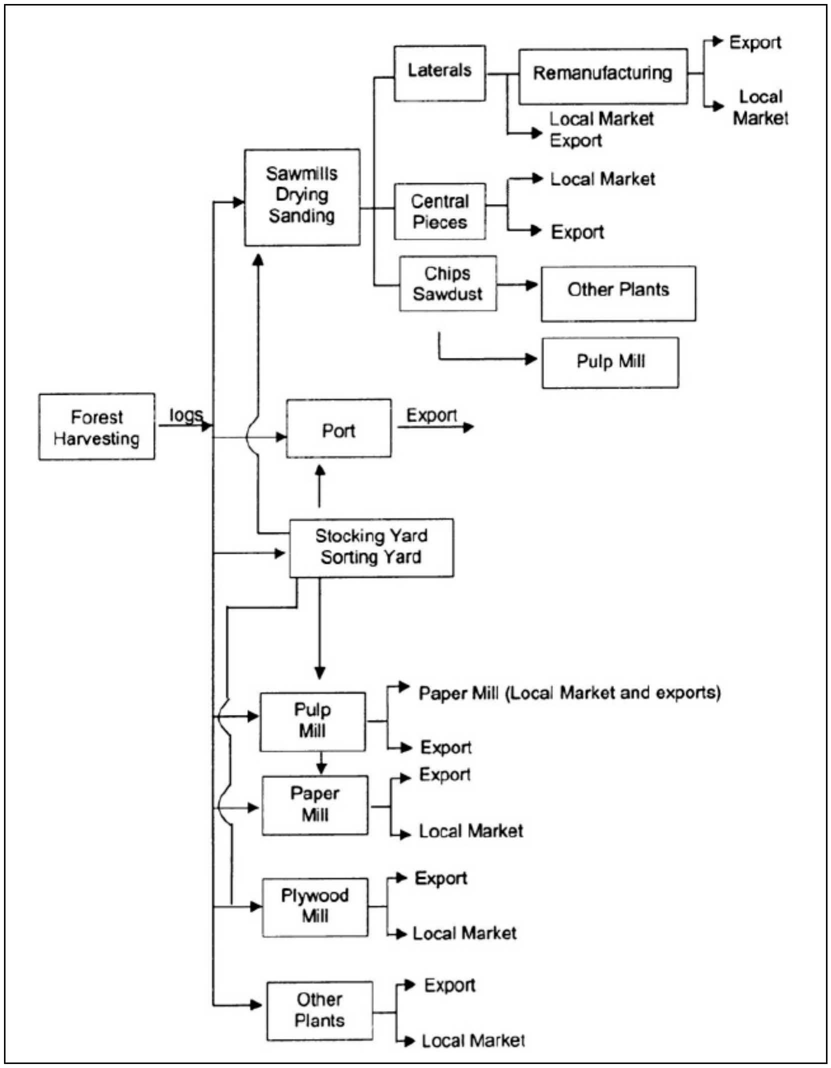

Figure 3.1-3: Physical forest supply chain (Weintraub and Epstein, 2005).

3.2. Actors in the chain

There are various actors involved in the flow of the forestry supply chain (Rönnqvist, 2003).

The main ones can be grouped as:

- Forestry industry companies with large investments in forests, which also possess their own cellulose, paper and sawmilling industries. They can be privately or state-owned.
- Forest owners associations, which represent private entities and have their own cellulose plants and sawmills.
- Independent sawmills, without any type of assets in large forests.
- Owners of independent forests which are not related to any industry.

Apart from these primary agents which represent producers and consumers, there are also timber merchants and transporters, which collect and transport the timber from the forests.

Although all of the actors involved recognize the importance of cooperation and integration throughout the flow chain, it is easy to see that different agents make decisions for their own benefit, which obstructs integration and cooperation (Rönnqvist, 2003).

3.3. Intertemporal Planning

As Bitran et al. (1982) signal, production management involves making complex decisions between a large numbers of alternatives. Given that, the production process is not immediate and must be anticipated with planning, a hierarchical structure is typically employed.

Hierarchical planning aims to simplify complex planning problems, which include different objectives and resources and which cover different temporal scales. In a hierarchical system, the decisions are taken sequentially, and each decision is associated with a level of information aggregation (Hax and Meal, 1973). This strategy aims to divide a large and complex problem into smaller and more manageable parts (Gelders and Van Wassenhove, 1981).

The number of decision levels depends on the specific application (Gelders and Van Wassenhove, 1981).Traditionally, three planning levels are considered: strategic, tactical and operational. The higher levels establish activities and define the resources available for the lower levels. The lower levels must implement the activities established by the higher levels and provide feedback regarding feasibility to the higher levels. Maintaining adequate feedback between the planning levels is a fundamental process to ensure the efficient and viable use of resources (Bitran and Tirupati, 1993).

The hierarchical approach methodology for production planning is used to organize coordination between distinct planning levels effectively (Bitran, 1982). The main difficulty is that due to lack of reliable data, forecasts or estimations must be made which can be erroneous.

As such, there are three main benefits of hierarchical planning, according to Boyland (2003):

- The reduction of problem complexity as a result of division into smaller sub problems and management of the required level of detail.
- Better management of uncertainty by postponing decisions for as long as possible.
- Increasing the specialization of each planning level.

In the particular case of the forestry industry, the hierarchical approach is used in supply chain management, with three levels, according to the impact on the company and the time periods for planning. These approaches are:

- Strategic level planning: Strategic plans refer to long term (30-50 years) and the large scale of resource assignment. The large scale nature of strategic plans allows for the use of aggregated data to simplify the plans. Large investments in factories are planned to increase production or improve the quality of the products. These investments are made with regard to the market situation and availability of timber.

- Tactical level planning: Tactical plans are for activities throughout the medium scale and medium term (1-5 years) zones. Tactical planning is mainly based on the reviewing of annual and quarterly budgets. The plan is reviewed whenever necessary.

- Operational level planning: Operational plans are the lowest level of planning (1-30 days) which detail exactly how each of the activities will be carried out. A central part of operational planning is the monthly adjustment to the production plan that is carried out with the aim of determine exactly when to change from one product to another. This depends on the results of recent production, as well as current inventory and orders. Operational plans are required to program the labor force and machinery for each of the activities planned on a tactical level.

In particular, three operational problems of great importance exist, which are: the daily programming of trucks, the location of harvesting machinery, and the short term harvest. Distribution of the operating costs is typically 30% for harvesting, 42% for transport 14% for road construction, 4% for loading and 10% for other processes (Epstein et al. 1999).

Despite the extensive use of the hierarchical approach in production planning, it has some drawbacks. This approach can lead to suboptimality, inconsistencies and even infeasibility. This is due to incorrect coordination between distinct hierarchical levels, errors in the aggregation and disaggregation of information and the existence of conflicting objectives between the distinct levels (Beaudoin et al., 2008).

For this reason, correct interactions between the decision models used by different levels are fundamental to assure the consistency of the global production plan. According to Bitran and Tirupati (1993), there is a lack of good models for systems characterized by uncertainty.

As such, due to the possible variability and uncertainty present in the forestry sector, it is necessary to consider distinct methodologies to achieve coordination in intertemporal planning and production.

4. MANAGING UNCERTAINTY

As has already been mentioned, the data used in optimization problems is not known exactly, and cannot be anticipated with certainty. This translates into the possibility that variations could be produced which significantly affect the solutions of the models; that is to say, the solutions could cease being optimal and could even become unfeasible due to small variations or disturbances in the nominal values considered (Shapiro, 2008). Because of this, it is desirable to consider uncertainty in the mathematical optimization models.

In Operations Research, the treatment of uncertainty was first studied in the 1950s in a work developed by Dantzig (1955). The author treats uncertainty using different scenarios, each one with a certain probability of occurring. This study is considered as one of the first steps in stochastic programming. Specifically, in this work Dantzig analyzed general mathematical models in which the activities could be divided in two stages or states. The first stage would require the activities to be undertaken to be determined. However, in the second stage the activities, as they depend on the first stage, did not necessarily have to be determined in advance. Following the previously mentioned study, optimization in uncertain conditions has evidenced extensive development, both from a theoretical and algorithmic point of view (Sahinidis, 2004), mainly for linear problems, but not so for nonlinear problems or integer programming. It is important to highlight that the inclusion of uncertainty in mathematical problems to support decisions has few real applications and has been limited largely to case studies. This is due to the difficulties in implementation, such as the determination of decision makers' preferences and the calculation of probabilities associated with certain events (Bjørndal et al., 2012).

The principal methods developed to consider the lack of certainty in optimization models will now be briefly described.

4.1. Methods for managing uncertainty

4.1.1. Using Average Values

Random parameters are replaced by the most probable value or average value, and as such the mathematical model is transformed in a deterministic model. However, the variability of the values in relation to the expected value is not considered, and as such the solution which is obtained could be substantially suboptimal and not very representative of reality (Kouvelis and Yu, 1997). Because of this, decision making becomes quite myopic.

However, in the case of problems in which the level of uncertainty is low, or the impact of uncertainty is not relevant, this method can be used (Vladimirou and Zenios, 1997). In fact, it is extensively employed given its simplicity when considering a deterministic model.

4.1.2. Using the Worst Case

As in the previous case, it is possible to replace the value of uncertain parameters with their worst value and use this as a deterministic model. However, the use of this methodology leads to very conservative and high cost results (Vladimirou and Zenios, 1997) due to the loss of optimality of the solutions.

4.1.3. Using Scenarios

This method involves using a finite number of scenarios and resolving each one of them independently. However, this technique is only useful for comparing the solutions obtained given distinct scenarios and selecting one of them in accordance with the criteria of the decision maker. Maturana and Contesse (1998) present an application for the mixed integer programming model for the case of sulfuric acid logistics in Chile. In the forest sector, it is also been applied this method (Klenner et al., 2000; von Gadow, 2000; Peter and Nelson, 2005). Nevertheless, a large number of scenarios are need, so the applicability is reduced.

The use of scenarios can also be combined with stochastic programming, associating probabilities of the distinct scenarios and considering each one of them as a particular manifestation of reality, as can be observed in Mulvey et al. (1995).

4.1.4. Sensitivity and Post-Optimal Analysis

Sensitivity and post-optimal analysis measures the impact which disturbances in the input data produce in the model's solutions. This is, therefore, a reactive approach which does not provide any mechanism with which sensitivity can be managed (Hillier and Lieberman, 1997).

This analysis is focused in identify key parameters in the models. Key parameters are those that generate big impact in the solutions when they change. So, in real problems with uncertainty, this method is not useful (Pickens and Dress, 1988).

4.1.5. Stochastic Programming

This methodology arises as an extension of linear and non-linear optimization models, where the uncertain coefficients are represented according to a probability distribution, either with discrete or continuous functions. It aims to maximize (minimize) some measure of expected behavior and every situation is weighted for its probability of occurring (Birge and Loveaux, 1997). However, it ignores higher order moments and the preferences of the decision maker regarding risk (Vladimirou and Zenios, 1997).

This methodology can be divided in:

- Programming with recourse:

In this case, decision variables are divided into two stages, the first of which corresponds to the variables which must be determined prior to the implementation of uncertain parameters. The second stage corresponds to the recourse variables. These variables can be considered as corrective measures or recourses against any unfeasibility due to a particular implementation. As such, the objective is to choose the first stage variables (here and now) in such a way that the sum of costs of the first stage plus the expected value of costs in the second stage is minimized (Sahinidis, 2004). The recourse concept can be applied in linear, nonlinear and integer programming.

- Probabilistic Programming:

Unlike the previous method, in probabilistic programming or “chance constraint” some infeasibilities are permitted with determined penalties (Shapiro et al., 2009). However, this methodology is difficult to manage, both numerically and from the point of view of modeling (Shapiro, 2008).

Another standard formulation in stochastic programming is the use of decision models in multiple stages with adaptive decisions. In a typical two-stage stochastic programming approach the set of decisions can be divided in two groups: the decisions which must be taken “here and now”, before the uncertain events are implemented, and the decisions which are taken following the events and, therefore, adapt and are sequential in character. This approach is powerful but, as Chen and Zhang (2009) note, it could give rise to large scale problems which are very difficult to resolve in terms of optimality. As well as this, stochastic programming requires detailed knowledge of the distribution of uncertain data, which is normally not known or difficult to obtain.

4.1.6. Fuzzy Programming

An alternative to stochastic programming is fuzzy programming, which makes a distinction between randomness and imprecision, and the main difference with the stochastic approach being that in practice instead of assuming probability functions, ownership functions are assumed (Jensen and Maturana, 2002). That is to say, the means of modeling uncertainty differs between the two approaches (Sahinidis, 2004).

4.1.7. Dynamic Programming

This term was proposed by Bellman (1956) to describe the mathematical theory he developed to deal with multistage decision problems (Sahinidis, 2004). In a system, it is possible to inspect a finite number of stages. Frequently, the stages are considered as points in time, which is the reason why the term “dynamic” is used. Furthermore, in any stage of the system it is possible to be in one state out of many possibilities. Additionally, in any moment a decision will have influence over the state of the system in a subsequent stage. As such, a return function associated with the decision is generated (Kall and Wallace, 1982). That is to say, with dynamic programming methodology an optimal solution is found for a problem with n variables broken down in n stages, and as such each stage corresponds to a sub-problem composed of a single variable. Now, given that the solution of a sub-problem is used as data for the following sub-problem, the resolution of dynamic programming problems is undertaken in a recursive way

(Taha, 2004). Specifically, in the case of probabilistic dynamic programming, the stages and returns or rewards in each stage are probabilistic.

4.1.8. Robust Programming

Specifically, Mulvey et al. (1995), consider robust programming as a stochastic programming approach which provides solutions which are less sensitive to variations in the data of the model, but which have a higher cost (Vladimirou and Zenios, 1997). Robust models explicitly incorporate random parameters, minimizing both the expected cost value as well as certain penalties for infeasibilities.

As such, the general model used in this methodology is presented in the following way (4.1.1):

$$
\begin{aligned}
Min \quad & \sigma\left(x, y_1, \ldots, y_S\right) + \omega\rho\left(z_1, \ldots, z_S\right) \\
s.a. \quad & Ax = b \\
& B_s x + C_s y_s + z_s = e_s \qquad \forall s \in \Omega \\
& x \geq 0, y_s \geq 0 \qquad \forall s \in \Omega
\end{aligned}
\tag{4.1.1}
$$

The variables *x* are named design variables and are not subject to uncertainty, while the variables *y*, or control variables, are subject to uncertainty. Furthermore, a set of scenarios $\Omega = \{1, 2, \ldots, S\}$ is defined in problem (4.1.1), and each scenario is $s \in \Omega$ associated with a set of possible outcomes $\{d_s, B_s, C_s, e_s\}$ The variables *z* correspond to variables which measure the infeasibility allowed for the control constraints under scenario *s*.

In this robust programming approach, a solution is robust if the optimal solution is maintained sufficiently optimal when the input data changes. Furthermore, the model is defined as robust if the optimal solution is maintained sufficiently feasible in the face of variations in the data.

Applications of this methodology and resolutions methods can be found in Mulvey et al. (1995), Albornoz (1998), Bai et al. (1997), Leung et al. (2002), Takriti and Ahmed (2004), Yang and Zenios (1997) and Yu (1996), as well as various others.

4.1.9. Simulation

In cases where developing mathematical or analytical models is too complex, there is the possibility of using systems simulation, where a system is defined as a set of entities (people, machines, etc.) which act and interact to achieve a goal (Law and Kelton, 2000). Systems simulation corresponds to a computational tool in which the system being studied is designed to scale and each time the computer program is run it represents an implementation of the real system.

In this way, when the simulation program is run various times, we can obtain the most probable value of one or more performance variables (Gazmuri, 1994). As such, the simulation is a statistical experiment and in consequence the results must be interpreted with the appropriate statistical tests (Taha, 2004).

Because of the disadvantages and difficulties in using the techniques mentioned above, RO methodology is now described. This method associates with each constraint with uncertain information, a protective function that improves the robustness of the solutions. This methodology, as is proposed in the literature should be an interesting tool for decision making under uncertainty.

4.2. Robust Optimization

The need to obtain robust solutions, that is to say, solutions which are immune to variability in the data, has been a relevant issue for a long time, particularly in the formulation of mathematical programming models. Soyster (1973) was the first to investigate this area. He proposed an approach to generate robust solutions in a linear model with the assumption that the parameters would vary within certain intervals, and consider all possible scenarios within those intervals. The resulting model provides a high protection level against uncertainty, but it is a very conservative model and it can generate an optimal value quite a lot worse than the deterministic problem's optimal value (which does not consider uncertainty).

In recent years some important advances have been made in RO (Ben-Tal and Nemirovski 1999; Bertsimas and Sim 2003), with different approaches and applications. As such, the approach developed by Ben-Tal and Nemirovski (1999) and independently by El Ghaoui et al. (1998) considers linear programming problems in the form expressed in (4.2.1).

$$\underset{x_0 \in R, x \in R^n}{Min} \quad \left\{ x_0 : f_0(x,\zeta) - x_0 \le 0,\ f_i(x,\zeta) \le 0, i = 1,\dots,m \right\} \tag{4.2.1}$$

In line with Ben-Tal and Nemirovski's (2002) notation, in this model x is the design vector, the functions f_0 (objective function) and $f_1,\dots,f_m$ are structural elements of the problem, and ζ represents the specific data of a particular instance.

The entry data is generally considered uncertain to some degree in the real world, and as such it is necessary to deal with this uncertainty in some way.

To differentiate it from the approach developed by Mulvey et al. (1995), Ben-Tal and Nemirovski (1999) consider the existence of "hard" constraints, which means constraints that must be satisfied independently of the specific implementation of the data. In this way, the candidate solution (x_0, x) must satisfy a semi-infinite system of constraints (4.2-2):

$$f_0(x,\zeta) \le x_0,\ f_i(x,\zeta) \le 0, i = 1,...,m \qquad \forall\left(\zeta \in U\right) \tag{4.2-2}$$

Where U is the uncertain data set (Ben-Tal and Nemirovski 2002). With this assumption, the original linear programming problem can be reformulated as:

$$\underset{x_0,x}{Min} \quad \left\{x_0 : f_0(x,\zeta) \le x_0,\ f_i(x,\zeta) \le 0, i = 1,...,m \qquad \forall\left(\zeta \in U\right)\right\} \tag{4.2.3}$$

Problem (4.2.3) is known as the "robust counterpart" of the original problem. This is a semi-infinite linear problem which appears to be computationally intractable. However, depending on the specific set U, the robust counterpart could be a tractable convex mathematical problem. Typically, the robust equivalent is a linear problem or a conic quadratic problem (Ben-Tal et al. 2005) which can be resolved using algorithms for linear problems, or interior point methods (Ben-Tal and Nemirovski 1998).

Some applications of this methodology can be found in Ben-Tal and Nemirovski (1999, 2000 and 2002), Ben-Tal et al. (2000) and Ben-Tal et al. (2004). Ben-Tal and Nemirovski (1999) develop an example of portfolio optimization in which the methodology is explained and furthermore, the results are compared with those obtained by the methodology proposed by Mulvey et al. (1995). What is more, Ben-Tal and Nemirovski (2000) used the methodology for the construction of robust solutions for the Netlib bookstore problems, as well as demonstrating that for many of these problems the robust solutions presented a low loss of optimality. Ben-Tal et al. (2000) presented an example of a portfolio problem and compared the results through the application of a linear programming model which substitutes the uncertain data with their expected values, as well as with stochastic programming, with RO resulting to be the superior approach.

A different approach, but one based on the same idea, is that developed by Bertsimas and Sim (2003, 2004). These authors propose a reformulation which conserves the linear structure of the problem, which is very attractive due to its applicability.

The authors consider linear problems in the form (4.2.4):

$$\begin{aligned} Max \quad & c^T x \\ s.t. \quad & Ax \leq b \\ & l \leq x \leq u \end{aligned} \tag{4.2.4}$$

Without losing generality, it is assumed that uncertainty affects the coefficients a_{ij} of the matrix A of problem (4.2.4). Each entry a_{ij} can be modeled as a symmetric and bounded random variable $\tilde{a}_{ij}$ which takes values of $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$. The advantage of use this uncertain data set is to maintain the structure of the original problem (Palma and Nelson, 2014). On the other hand, J_i is the set of coefficients a_{ij}, $j \in J_i$, which are subject to uncertainty. For each row i the parameter Γ_i is introduced. This parameter indicates the protection level in the model, as this can be associated with the amount of uncertain coefficients in each constraint. It is important to note that only a subset of the coefficients which can vary will do so. This approach assumes previous behavior, but what is more, if more coefficients change, the robust solution continues to be feasible with a high probability.

To obtain the robust counterpart to problem (4.2.4), each constraint with uncertainty is reformulated, adding a protection function, in the following way (4.2.5):

$$\sum_j a_{ij} x_j + \beta_i\left(x, \Gamma_i\right) \leq b_i \quad \forall i \tag{4.2.5}$$

The protection function corresponds to expression (4.2.6):

$$\beta_i\left(x, \Gamma_i\right) = \max_{\left\{S_i \cup \{t_i\} \mid S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\right\}} \left\{ \sum_{j \in S_i} \hat{a}_{ij} \left|x_j\right| + \left(\Gamma_i - \lfloor \Gamma_i \rfloor\right) \hat{a}_{it_i} \left|x_{t_i}\right| \right\} \tag{4.2.6}$$

Note that when $\Gamma_i = 0$, $\beta_i\left(x, \Gamma_i\right) = 0$ represents the original situation of the deterministic problem. On the other hand, when $\Gamma_i = |J_i|$ it is being faced with the greatest protection, and it coincides with Soyster's method. In this way, when Γ_i varies between $[0, |J_i|]$ the protection level can be adjusted and the constraint i is partially protected against uncertainty.

Additionally, it is possible associated approximate boundaries linking protection parameter (gamma) with a constraint satisfaction probability (Bertsimas et al., 2004; Bertsimas and Sim, 2004). Palma and Nelson (2009) note that as in chance constraint programming, RO methodology uses the idea of increasing the probability of satisfying the constraints. Particularly in Li and Li (2015) the relationship between the two methods (RO and chance constraint problem) is proposed.

The protection function mentioned previously is equivalent to linear programming problem (4.2.7):

$$\beta_i(x,\Gamma_i) = \quad \max \quad \sum_{j\in J_i} \hat{a}_{ij}\left|x_j\right| z_{ij} \tag{4.2.7}$$
$$s.a. \quad \sum_{j\in J_i} z_{ij} \leq \Gamma_i$$
$$0 \leq z_{ij} \leq 1 \qquad \forall j \in J_i$$

When the dual problem from the previous model is written, it results in problem (4.2.8):

$$\begin{aligned} Min \quad & \Gamma_i z_i + \sum_{j\in J_i} p_{ij} \\ s.t. \quad & z_i + p_{ij} \geq \hat{a}_{ij}\left|x_j\right| && \forall i, j \in J_i \\ & p_{ij} \geq 0 && \forall i, j \in J_i \\ & z_i \geq 0 && \forall i \end{aligned} \tag{4.2.8}$$

Replacing the previous dual problem in the protected constraint, Bertsimas and Sim (2004) propose the following problem (4.2.9) as a robust counterpart to the original model (4.2.4):

$$\begin{aligned} Max \quad & c^T x \\ s.t. \quad & \sum_j a_{ij}x_j + z_i\Gamma_i + \sum_{j\in J_i} p_{ij} \leq b_i \qquad \forall i \\ & z_i + p_{ij} \geq \hat{a}_{ij} y_j \qquad \forall i, j \in J_i \\ & -y_j \leq x_j \leq y_j \qquad \forall j \\ & l_j \leq x_j \leq u_j \qquad \forall j \\ & p_{ij} \geq 0 \qquad \forall i, j \in J_i \\ & y_j \geq 0 \qquad \forall j \\ & z_i \geq 0 \qquad \forall i \end{aligned} \tag{4.2.9}$$

As can be observed, the original linear structure is conserved, so the solution technique is the same used in the original problem, and it is possible to control the degree of conservatism in satisfying a constraint. What is more, this approach can be extended to discrete mathematical programming problems without large modifications. Various applications have been developed, for example in network design (Bertimas and Sim, 2003; Ordóñez and Zhao, 2007), in engineering (Ben-Tal and Nemirovski, 2002) and in inventory theory (Bertsimas and Thiele, 2006).

Also the methodology has been applied in natural resources planning. Bohle et al. (2010) use this approach in a grape harvest programming problem for fermentation. Varas et al. (2014) used the RO to solve a scheduling production problem in a sawmill. Palma and Nelson (2009 and 2014) solve forestry problems (harvest-scheduling and road-building models), Carlsson et al. (2014) work in the distribution and inventory planning in a pulp producer and they compare RO with a traditional deterministic approach using safety stock policy. The objective of safety stock is to consider the uncertainty in demand and transportation capacity. The advantage of this methodology is the size of the problem, but is needed determine the level of the safety stock. Carlsson et al. (2014) indicates that RO is considerably better than the best deterministic approaches that include safety stock levels.

So, all these applications demonstrate that robust optimization is a useful methodology to deal with data variability.

Specifically, in this research, we work with the approach of Bertsimas and Sim (2004) because we want to find a methodology of easy implementation and low complexity. Other approaches such as Ben-Tal and Nemirovski (2000) results in models that are computationally more complex and in some cases intractable due to the uncertain data sets used to model uncertainty (Bertsimas et al., 2011).

5. FIRST APPLICATION: PRODUCTION PLANNING PROBLEM IN SAWMILLS

5.1. Problem description

In a typical forestry company in Chile, the supply chain is divided into areas, such as the forestry division, the sawmill division, the cellulose plant division, etc. In this context, a complete set of interrelated decisions must be dealt with.

In particular, the sawmill division, possibly with various installations, aims to maximize the revenues which come from the sale of sawn timber. This timber can consist of “dry” or “green” boards in accordance with the percentage of humidity. There are specific markets for both types.

The raw material of sawmill consists of logs which come from the forests or stands. The logs are classified according to diameter, length and quality. Depending on the products demanded and the production process, each sawmill will have to determine the quantity of different types of logs required to satisfy the demand for boards. These requirements for logs must be satisfied by the company’s forestry division, although, in general, the decision processes in both divisions are not coordinated (Epstein et al. 2007). If the forestry division cannot fulfill the demand with its own forests, the sawmill can acquire logs from smallholders. It is important to point out that in Chile, the large forestry companies possess large expanses of forest, principally of radiata pine plantations.

The logs are transported by trucks from the forests, and once they arrive at the sawmill they are stored according to diameter and quality. The logs remain stored until they are used. When they are required in the process, the logs enter the sawmilling line where they are cut in accordance with a previously defined cutting pattern which is appropriate for the diameter of the log. This sawmilling process transforms the logs into rectangular cuts (Singer and Donoso, 2007) which depend on the characteristics of the log, producing boards of different sizes (**Figure 5.1-1**). In some cases, the process is divided in two, in which some boards “in process” go directly to the reprocessing phase and are eventually stored, resulting in “green” boards. Another quantity of boards “in process” go to the drying unit where the humidity content is regulated, resulting in “dry” boards. In some cases, the drying process can be subcontracted. Finally, some of the “dry” boards can go to the reprocessing plant.

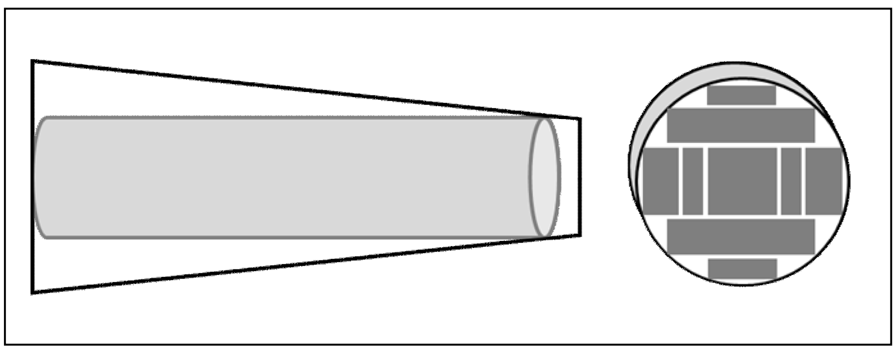

Figure 5.1-1: Example cutting pattern.

The process is concluded when the boards are ready to be dispatched, either to the local market or to ports to be shipped to different destinations around the world. Each process has costs associated with it and limited capacity. The same thing occurs in the inventory phases. **Figure 5.1-2** represents the general schema of sawmill operations. This schema is modeled in the following diagram.

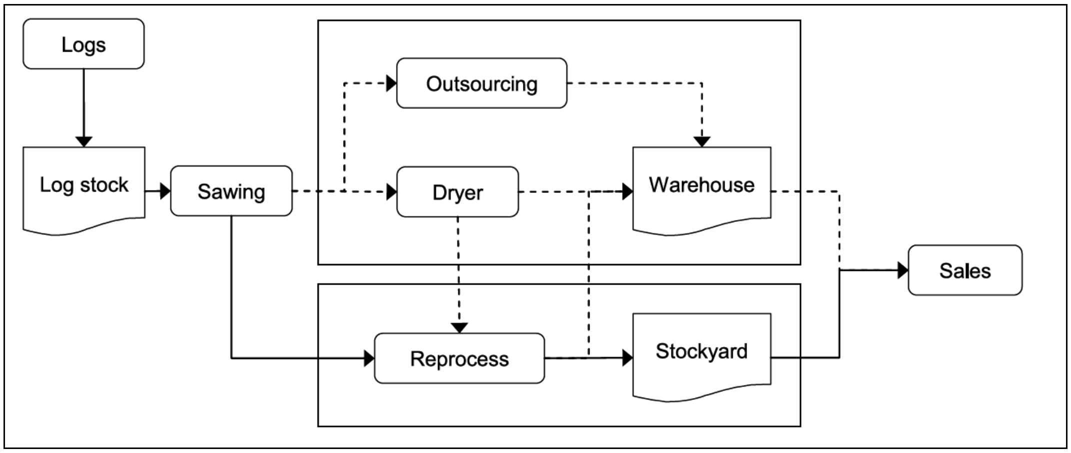

Figure 5.1-2: Schematic of a sawmill processes. (Source: Authors).

5.2. Deterministic production planning model

The production planning problem in sawmills can be modeled as a linear program (Weintraub and Epstein 2005; Singer and Donoso 2007). The deterministic model considered will now be presented. In this model uncertainty is not considered, although there are several sources of variability, the decisions are dangerous in the sense of the demand may not be fulfilled.

The consideration of uncertainty in this type of problem has been carried out through a combination of optimization with real time simulations (Kazemi et al. 2010a), although this type of analysis takes a lot of time.

The following notation is used in the formulation of the model:

- **Sets**

J Set of sawmills.

M Set of types of boards.

K Set of types of logs.

T Set of time periods.

E Set of cutting patterns.

- **Variables**

x_{jmt} Volume of boards type $m \in M$ (“green” o “dry”) for sale in the sawmill $j \in J$ in the time period $t \in T$ (m^3).

y_{jkt} Volume of logs type $k \in K$ demanded by the sawmill $j \in J$ in the time period $t \in T$ (m^3).

z_{jkt} Volume of logs type $k \in K$ in the sawmill’s inventory $j \in J$ in the time period $t \in T$ (m^3).

w_{jmt} Volume of boards type $m \in M$ in the sawmill’s inventory $j \in J$ in the time period $t \in T$ (m^3).

v_{jmt} Volume of boards type $m \in M$ sent to outsourcing by the sawmill $j \in J$ in the time period $t \in T$ (m^3).

r_{jmt} Volume of boards type $m \in M$ to be produced by the sawmill $j \in J$ in the time period $t \in T$ (m^3).

s_{jekt} Volume of logs type $k \in K$ to be processed with the cutting pattern $e \in E$ in the sawmill $j \in j$ in the time period $t \in T$ (m^3).

- **Parameters**

δ_t Discount factor for the time period $t \in T$.

A_m Price of boards type $m \in M$ (US$/ m^3).

a_{jkt} Purchasing cost of logs type $k \in K$ in the sawmill $j \in J$ in the time period $t \in T$ (US$/ m^3).

b_j Inventory cost of logs in the sawmill $j \in J$ (US$/ m^3).

c_j Inventory cost of boards in the sawmill $j \in J$ (US$/ m^3).

d Cost of outsourcing (US$/ m^3).

f_j Reprocessing cost in the sawmill $j \in J$ (US$/ m^3).

F Fraction of wood which is reprocessed (%).

g_j Drying cost in the sawmill $j \in J$ (US$/ m^3).

h_j Cost of sawmilling in the sawmill $j \in J$ (US$/ m^3).

R_{jekm} Performance of the cutting pattern $e \in E$ in the sawmill $j \in J$, applied to logs type $k \in K$, given that as boards type $m \in M$ produced (m^3 boards/ m^3 logs).

φ_j Sawmilling capacity in the sawmill $j \in J$ (m^3).

ψ_j Log inventory capacity in the sawmill $j \in J$ (m^3).

μ_j Board inventory capacity in the sawmill $j \in J$ (m^3).

θ_j Drying capacity in the sawmill $j \in J$ (US$/ m^3).

η_j Reprocessing capacity in the sawmill $j \in J$ (m^3).

B_{mt} Estimated demand for boards type $m \in M$ in the time period $t \in T$ (m^3).

- **Optimization model**

$$Max\ z = \sum_{t \in T} \delta_t \left[\begin{array}{l} \sum_{j \in J} \sum_{m \in M} A_m \cdot x_{jmt} - \sum_{j \in J} \sum_{k \in K} a_{jkt} \cdot y_{jkt} - \sum_{j \in J} \sum_{k \in K} b_j \cdot z_{jkt} - \\ \sum_{j \in J} \sum_{m \in M} c_j \cdot w_{jmt} - \sum_{j \in J} \sum_{m \in M} d \cdot v_{jmt} - \sum_{j \in J} \sum_{m \in M} F \cdot f_j \cdot r_{jmt} - \\ \sum_{j \in J} \sum_{m \in M} g_j \cdot r_{jmt} - \sum_{j \in J} \sum_{k \in K} \sum_{e \in E} h_j \cdot s_{jekt} \end{array} \right] \quad (5.2.1)$$

Subject to:

$$z_{jkt-1} + y_{jkt} = z_{jkt} + \sum_{e \in E} s_{jekt} \qquad \forall j \in J, k \in K, t \in T \quad (5.2.2)$$

$$w_{jmt-1} + r_{jmt} + v_{jmt} = w_{jmt} + x_{jmt} \qquad \forall j \in J, m \in M, t \in T \quad (5.2.3)$$

$$\sum_{e \in E} \sum_{k \in K} R_{jekm} \cdot s_{jekt} = r_{jmt} \qquad \forall j \in J, m \in M, t \in T \quad (5.2.4)$$

$$\sum_{e \in E} \sum_{k \in K} s_{jekt} \le \varphi_j \qquad \forall j \in J, t \in T \quad (5.2.5)$$

$$\sum_{k \in K} z_{jkt} \le \psi_j \qquad \forall j \in J, t \in T \quad (5.2.6)$$

$$\sum_{m \in M} w_{jmt} \le \mu_j \qquad \forall j \in J, t \in T \quad (5.2.7)$$

$$\sum_{m \in M} r_{jmt} \le \theta_j \qquad \forall j \in J, t \in T \quad (5.2.8)$$

$$\sum_{m \in M} r_{jmt} \le \eta_j \qquad \forall j \in J, t \in T \quad (5.2.9)$$

$$\sum_{j \in J} x_{jmt} \ge B_{mt} \qquad \forall m \in M, t \in T \quad (5.2.10)$$

$$x_{jmt}, y_{jkt}, z_{jkt}, w_{jmt}, v_{jmt}, r_{jmt}, s_{jekt} \ge 0 \qquad \forall j \in J, m \in M, e \in E, k \in K, t \in T \quad (5.2.11)$$

Expression (5.2.1) specifies the objective function of the problem, maximizes the economic benefit obtained from the sale of boards and considers the following costs: purchasing of logs, inventory, outsourcing, drying, reprocessing and sawmilling. Transport costs are not considered. Constraints (5.2.2) and (5.2.3) correspond to inventory constraints of logs and boards (“green” and “dry”), respectively. The set of constraints (5.2.4) transforms the logs into boards depending on the specific performance of sawmill. This performance is a function of the type of log, cutting pattern, sawmill and boards which need to be produced. Constraints (5.2.5), (5.2.6), (5.2.7), (5.2.8) and (5.2.9) correspond to the capacity limits of each phase (sawmilling, log and plank inventory, drying and reprocessing). The fulfillment of demand is represented by the set of constraints (5.2.10). Finally, constraints (5.2.11) are specifications of the nature of the variables.

It can be seen that the model includes various potentially uncertain parameters, such as demand, price, costs, performance or conversion efficiency in sawmills. Carino and Willis (2001a, 2001b) present a linear model to solve the production-inventory problem, and the results demonstrate that the model is highly sensitive to changes in conversion efficiency in sawmills. Additionally, as Rönnqvist et al. (2000) indicate, the logs which arrive at the sawmill are not straight or cylindrical, they can have curvatures or defects which cause losses in performance. In the current study, these results are followed and uncertainty in performance parameters (R_{jekm}) is considered. The RO methodology is proposed as a means of obtaining solutions which are immune, to some extent, to this variability.

It is assumed that the forestry division is capable of delivering all of the volume required by the sawmill. There is no loss of generality in this assumption, as the sawmill can buy logs from other producers if necessary.

5.3. Methodology to tackle the sawmill production planning problem

To evaluate the performance of robust optimization as a tool to tackle problem of planning production with uncertainty, the following steps will be taken:

- Formulate a robust counterpart to the deterministic sawmill planning problem
- Solve the deterministic program for a particular instance.

- Solve the robust counterpart for the same case.
- Compare the obtained results in both cases. The comparison considers three aspects: optimality, feasibility and solution structure.

In the case of optimality, the possible losses in value of the objective function when using robust optimization to consider uncertainty in the performance data is evaluated. On the other hand, in relation to feasibility, the behavior of the solutions regarding possible data instances is to be analyzed, for which is used Monte–Carlo simulation by generating 600 scenarios for different values of performance of logs for the production of boards. Also different levels of protection will be tested.

Finally, the analysis of the solution structure aims to determine the stability of the solution and its applicability when faced with uncertain data.

5.4. Robust production planning model

To formulate the robust counterpart to the problem presented, the first thing to do is substitute the decision variable r_{jmt} using equation (5.2.4). This substitution alters the objective function and several of its constraints, but it eliminates the equality constraint (which limits the use of RO by not incorporating protection functions), leaving the parameter uncertain in terms of restriction associated with demand (inequality). Although the objective function is affected by performance uncertainty, the construction of a robust counterpart is not considered. This is justified on the basis that the objective is to determine robust solutions. As such, the study is focused on the satisfaction of the constraints and the objective function is subordinated to this goal.

The principal constraint modified by the substitution is the demand constraint (5.2.10). Following the transformation and application of the robust specification previously described, the demand constraint is substituted for the following inequalities (5.4.1), resulting in a robust model:

$$\sum_{j\in J}\left(w_{jmt-1}+\sum_{e\in E}\sum_{k\in K}R_{jekm}\cdot s_{jekt}+v_{jmt}-w_{jmt}\right)-\alpha_{mt}\Gamma_{mt} \tag{5.4.1}$$

$$-\sum_{j\in J}\sum_{e\in E}\sum_{k\in K}\rho_{jekm}\geq B_{mt} \qquad \forall m,t$$

$$\alpha_{mt}+\rho_{jekm}\geq \hat{a}_{jekm}\beta_{jekt} \qquad \forall j,m,e,k,t$$

$$-\beta_{jekt}\leq s_{jekt}\leq \beta_{jekt} \qquad \forall j,e,k,t$$

$$\beta_{jekt},\alpha_{mt},\rho_{jekm}\geq 0 \qquad \forall j,m,e,k,t$$

The previous expression contains new variables (β_{jekt}, α_{mt}, ρ_{jekm}) introduced to generate the robust reformulation, in accordance with the approach of Bertsimas and Sim (2003). The parameter Γ_{mt} represents the degree of robustness for each constraint.

Is important to indicate that the parameter R_{jekm} not depend the period of time, so the robust counterpart correspond a simplify model based on Bertsimas and Sim (2003).

5.5. Data instances and size of the problems

A prototype problem instance corresponding to a typical situation in a Chilean forestry company was considered. The structure consists in a firm which manages three sawmills. Each of them is capable of processing 6 different types of logs with 3 cutting patterns. There are a total of 7 types of boards to be sold to clients, and the planning horizon considered is one year, divided into monthly periods.

As has already been mentioned, the parameter considered is subject to uncertainty, and corresponds to the performance of transforming logs in boards

(R_{jekm}). This parameter represents the performance of a log type *k* when the cutting pattern *e* is applied to it in sawmill *j* for the production of boards type *m* (m^3 plans / m^3 logs). This parameter affects sawmill production, which indicates an exchange rate between the logs and boards for each sawmill, and obviously, the variations in performance affect total production.

Regarding uncertainty, distinct scenarios are assumed in which the performance parameters could vary by 5%, 10%, 15% or 20% from the average value. Furthermore, variation in the "uncertainty budget" was also considered, which is represented by the parameter Γ. This parameter can have values of between 0 (deterministic case) and 104 (worst case) in the specific instance considered. Under these assumptions, the behavior of the robust counterpart is analyzed and the results are compared with those of the deterministic model. In the following subsections the results are presented in terms of optimality, feasibility and solution structure.

Given these instances, the size of the deterministic model is 1,296 decision variables and 1,848 constraints. When the expressions to formulate the robust counterpart are included, the model considers 3,972 decision variables and 13,776 constraints. Although the robust model is bigger than the deterministic problem, the linear structure is maintained and as such so are its advantages.

5.6. Results and comparison

The results of the deterministic and robust sawmill production planning models described in the previous sectors were represented through AMPL and were resolved using CPLEX software. Resolution times were not registered, as the models maintained their linear structure in the robust and deterministic case, which does not alter the complexity of the resolution.

The main decision variables which the problem considers correspond to the volume of logs type k to be processed with cutting pattern e in sawmill j in the time period t (s_{jekt}). With this decision variable, and given equation (5.2.4) of the model, it is possible to determine the production of each sawmill. This also permits the determination of the volume of logs type k required by each sawmill in each period (y_{jkt}).

For example, for the deterministic model, **Table 5.6-1** indicates the type of layout required for logs in each sawmill to achieve the maximization of benefits.

Table 5.6-1: Type of logs demanded by sawmills

Sawmill	**Log´s type**					
	L1	L2	L3	L4	L5	L6
1				⊗	⊗	⊗
2	⊗		⊗			
3	⊗					

As can be observed, not all sawmills require the same types of logs, there is “specialization” based on productivity. Furthermore, the type of log L2 is not necessary in any sawmill. This is due to the fact that L2 logs only participate in the production of 4 types of boards with low performance, while the other types of logs have a more extensive register to produce boards with better performance. **Table 5.6-2** shows the relationships which exists between logs and boards, that is to say, the types of boards that can be obtained from distinct types of logs.

Table 5.6-2: Relationship log-board

Log	**Board type**						
	B1	B2	B3	B4	B5	B6	B7
L1	⊗		⊗		⊗		⊗
L2	⊗		⊗		⊗		⊗
L3	⊗	⊗	⊗	⊗	⊗	⊗	
L4	⊗	⊗	⊗	⊗	⊗	⊗	
L5	⊗	⊗	⊗		⊗		
L6	⊗	⊗	⊗		⊗		⊗

Finally, **Table 5.6-3** shows the types of boards to be produced by each sawmill for the deterministic model, which is calculated in line with equation (5.2.4) for an arbitrary period of time.

Table 5.6-3: Boards to produce

Sawmill	**Board type**						
	B1	B2	B3	B4	B5	B6	B7
1	⊗	⊗	⊗	⊗	⊗	⊗	
2	⊗	⊗	⊗	⊗	⊗	⊗	⊗
3	⊗		⊗		⊗		⊗

a) Optimality

The results for the deterministic model indicate that the maximum benefit which can be obtained (optimal) is 3,252,270 U$$. In the case of the robust model, there will be variations according to the scenarios considered and the degree of robustness defined. As such, Figure 5.6-1 shows the optimal values for the different uncertainty scenarios (variation of the performance coefficient of 5%,10%, 15% and 20%) and according to the protection level considered (Γ).

The protection level was made to vary in a regular way from 0 (without protection) up to 104 (maximum protection).

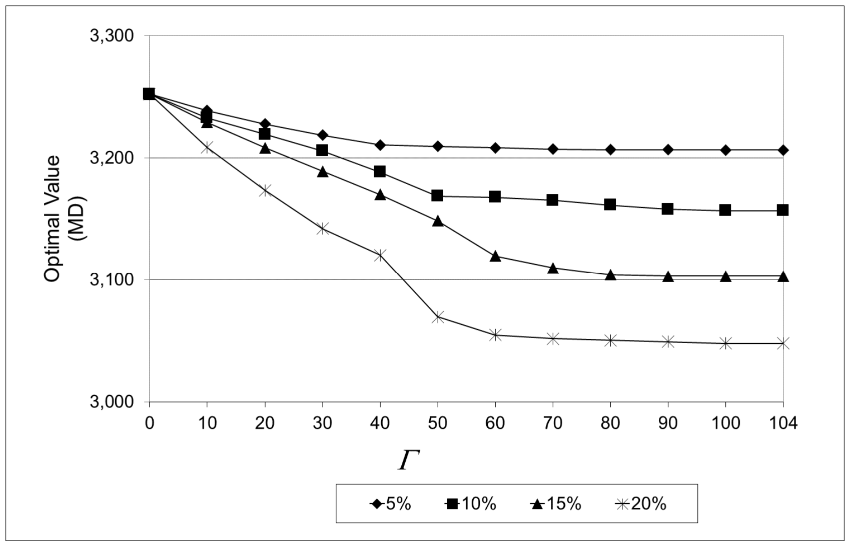

Figure 5.6-1: Optimal values – Robust model

As can be observed in the case of the least protection (when Γ is equal to 0), the optimal solution coincides with the optimal solution of the deterministic model, as in this case no protection is considered.

On the other hand, as can be expected, the graph shows that the reduction in value of the objective function is greater in cases with high variability. At the other extreme, if all the performance parameters were to reach their worst case simultaneously, this would be in line with the approach of Soyster (1973). **Table 5.6-4** shows the optimal value obtained with the greatest degree of protection possible against uncertainty (when Γ is equal 104), for each variability scenario in the performance parameters.

Table 5.6-4: Optimal value for the worst case

Variability (%)	Optimal Value (US$)	Reduction (%)
0	3,252,270	-
5	3,206,530	1.41
10	3,156,790	2.94
15	3,102,940	4.59
20	3,047,810	6.29

Another important result is that from a certain protection level onwards, there is no significant difference in the optimal value, even with respect to the worst case. In this data instance, when Γ (protection level) is around 60 or higher, the curve is fairly stable. This means that guarantee 100% of robustness is not significantly more expensive than guarantee at 60%.

Also, it is possible indicate, that, despite we are talking about costs, the protection against almost the worst case is achieved with even an intermediate amount of protection against variability.

The decision maker must select the tradeoff between optimality and protection, and if the demand is not fulfil evaluate the options in terms of costs and applicability to compensate the loss in optimality.

b) Feasibility

It is important to analyze the obtained solutions in terms of an a posteriori feasibility analysis. The robust solutions guarantee the 100% feasibility of the solution only when Γ reaches its maximum value (in this case 104). In any intermediate case, the possibility exists that in some scenarios the chosen solution could be infeasible. Through the Monte-Carlo simulation the behavior of solutions for distinct values of Γ and distinct variability was observed. In the simulation, 600 scenarios were considered for the parameter, which was subject to uncertainty R_{jekm}. Of these scenarios, 300 were generated following a uniform distribution in the variation interval, and for each of the variability scenarios (5%, 10%, 15% and 20%). The other 300 scenarios correspond to a normal distribution which is defined in such a way that 95% of the probability is within the interval.**Figure 5.6-2** and **Figure 5.6-3** show the results of the simulation for both distributions. The numbers indicate the percentage of scenarios in which the robust solution calculated is feasible. Feasibility is considered as the full satisfaction of the model's constraints.

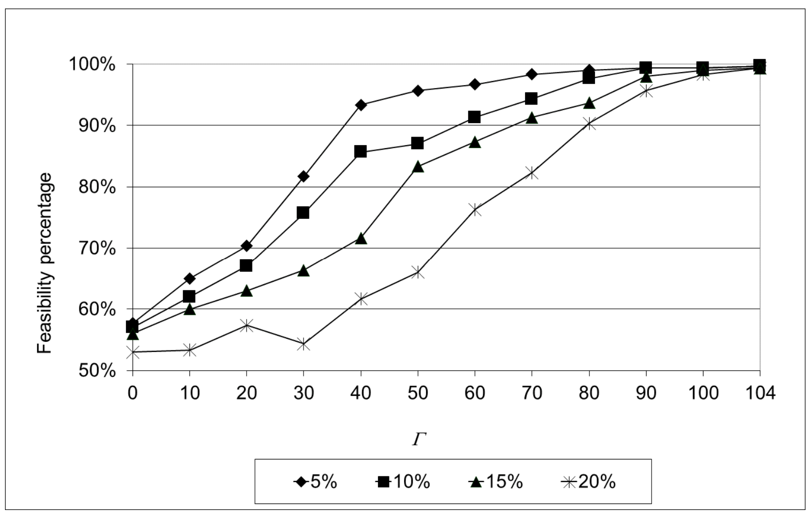

Figure 5.6-2: Simulations results under uniform distribution.

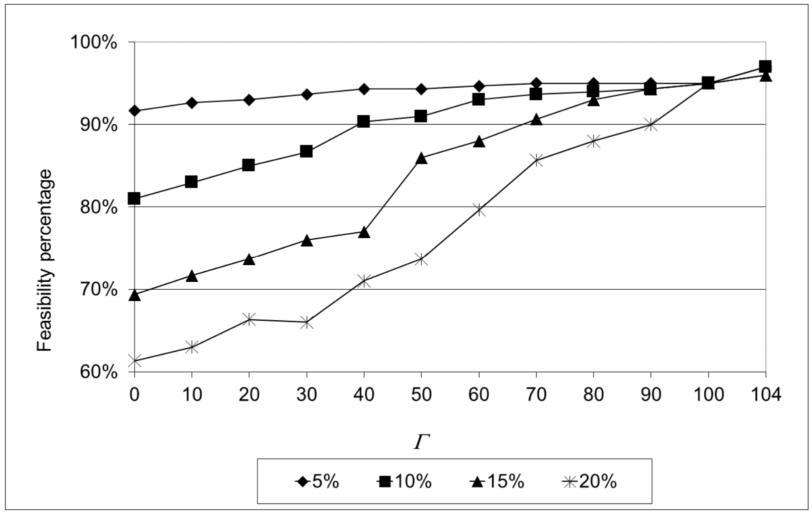

Figure 5.6-3: Simulations results under normal distribution.

The previous figures are very useful when combined with Figure 5 -3. These figures can determine an acceptable value of Γ for the calculation of robust solutions. The experience of the decision maker and their degree of risk aversion will affect the selection of a specific value of Γ, but for both distributions it can be seen that if $\Gamma = 50$, the estimated probability of infeasibility is less than 65% and furthermore, in any case, the loss of optimality never exceeds 7%. This percentage is considered not too high because if the demand is not meet, some actions could be taken.

With a higher value of Γ, the estimated feasibility of the solutions is closer to 100%, particularly in the case of normal distribution, as is expected given that normal distribution is more concentrated closer to the average. The conclusion is that a highly acceptable solution can be obtained with a value of Γ in the middle range, without being excessively conservative.

It is also important to precisely quantify the percentage of demand which is not satisfied when the solution is infeasible. **Table 5.6-5** shows, the percentage of lost viability in the demand constraints for three values of Γ. This is calculated by comparing unsatisfied demand with total demand, and determining the proportion. The table shows the results for the scenarios generated using normal distribution; the results for uniform distribution are similar.

Table 5.6-5: Percentage of unsatisfied demand.

	Variability			
Γ	**5%**	**10%**	**15%**	**20%**
10	35%	36%	40%	40%
60	17%	20%	20%	23%
100	10%	14%	15%	18%

It can be observed that for high variability and low values of Γ, the percentage of infeasibility is higher. This result is to be expected, as a low value of Γ provides low protection in the face of uncertainty. However, an intermediate range of uncertainty generates robust solutions which work reasonable well in terms of feasibility. It is important not to forget that the scenarios are based on a normal distribution, which covers only 95% of the range, and as such, could include infeasibility by design.

A deeper analysis can be undertaken by classifying the infeasibilities in accordance with the percentage of unsatisfied demand. Unfeasibility can be considered as significant if the percentage of unsatisfied demand is greater than 5% of total demand. This is considered reasonable in practice, as it is possible for the administration to manage these situations on an operative level, either by buying the demand which is lacking from other sawmills or accepting a penalty for unsatisfied demand. **Table 5.6-6** shows the percentage of scenarios which are infeasible within 5% of total demand.

The results are consistent with the previous conclusion that an increase in protection level generates fewer infeasibilities.

Table 5.6-6: Percentage of unsatisfied demand over 5%.

	Variability			
Γ	**5%**	**10%**	**15%**	**20%**
10	23%	22%	25%	27%
60	7%	9%	10%	14%
100	2%	3%	5%	8%

c) Structure of the solution

Another important aspect to analyze is the structure of the solutions in terms of their robustness. It is not desirable in a management context for production plans to change in an important way when uncertainty is taken into account. Robust solutions also have the goal of obtaining a stable production plan in spite of the variability in the data. The stability of the solution also helps, independently of variability in the data, to define the equipment setup in order to facilitate work on an operational level.

The main decision variables mentioned previously are y_{jkt} and s_{jekt}, which represent the volume of logs demanded and the volume of logs processed in the sawmills. **Table 5.6-7** shows, for variable y_{jkt}, the comparison between the production plan based on the deterministic solution and the plan generated using the robust model for a variability of 10% and a Γ value of 60.

Table 5.6-7: Deterministic and Robust Solutions.

	Log type											
Sawmill	Deterministic solution VOF: 3,252,270						Robust solution VOF: 3,138,840					
	L1	L2	L3	L4	L5	L6	L1	L2	L3	L4	L5	L6
1				⊗	⊗	⊗				⊗	⊗	⊗
2	⊗		⊗				⊗		⊗			
3	⊗						⊗			⊗		

This behavior is similar to that of other solutions. The decision variables which take values are repeated in at least 51% of the combinations of logs and sawmills, which in the worst case is a much higher percentage than in the intermediate cases.

Of course, the values of the decision variables change in the different scenarios analyzed. The sawmills demand more logs and process a greater volume to satisfy demand, but the specific logs which are required from the forestry division by the sawmills follows, in general, the same pattern as in Table 4-1. That is to saw, of the 6 types of logs, 5 are demanded in every uncertainty scenario and for any value of the parameter Γ. This, as has already been pointed out, is due to the fact that these logs have high levels of performance and can be used to produce more types of boards, which gives them greater flexibility.

On the other hand, the volume of logs processed by the sawmills depends to a great extent on the logs which come from the forest. The interesting thing is that the cutting patterns used for each log are independent of the uncertainty scenario used. This is shown in **Table 5.6-8**, where it can be observed that cutting patterns e_1 and e_2 are the most frequently used in the planning problem.

Table 5.6-8: Comparison of cutting patterns.

		Log type											
	Cuting pattern	Deterministic solution VOF: 3,252,270						Robust solution VOF: 3,138,840					
		L1	**L2**	**L3**	**L4**	**L5**	**L6**	**L1**	**L2**	**L3**	**L4**	**L5**	**L6**
Sawmill 1	**e1**					⊗	⊗					⊗	⊗
	e2				⊗						⊗		
	e3												
Sawmill 2	**e1**	⊗						⊗					
	e2			⊗						⊗			
	e3			⊗						⊗			
Sawmill 3	**e1**	⊗						⊗			⊗		
	e2												
	e3												

5.7. Sawmill production planning problem conclusions

In this application, we have shown how the RO approach can be used to improve the performance and reliability of the solutions obtained from a linear programming model to support production planning in a sawmill.

The approach permits natural variability induced by variation in the performance of different cutting patterns used in operations to be considered. The main advantage of this approach is that it allows the equilibrium between the requirements for robustness and the loss of optimality to be studied. The approach which we have followed, that of Bertsimas and Sim (2003, 2004), also conserves the original linear structure of the problem.

The main conclusion is that the “Price of robustness” for this particular problem is not very high, as the objective function did not deteriorate by more than 7% in all of the scenarios considered (5%, 10%, 15% and 20% of total variability, taking into account the most conservative protection against uncertainty). This is, of course, a conclusion based on a particular case which we have tested for this problem, but one which could reflect a property of the model. The concrete case presents high flexibility in obtaining boards from logs. What is more, the solution obtained guarantees certain immunity with respect to variations in performance, which are variations which correspond approximately to what is observed in practice. It is also interesting to note that deterioration of the worst case is obtained for protection values against uncertainty which are in the middle range, and as such more losses are not generated as a result of being conservative.

These conclusions are supported by the results of the simulation as the robust solutions obtained conserve a high percentage of feasibility, as can be observed in the Figures 5-4 and 5-5. In fact, the feasibility percentage is always above 50% and rapidly increases in the extent to which the protection level is increased, reaching practically 100%.

When feasibility is measured using loose dimensions, that is to say, dimensions which allow a violation of up to 5% of the demand constraints, the percentage of feasibility increases to a minimum of approximately 70%. Feasibility also increases rapidly in the extent to which the protection level is increased, as is shown in table 5-6. This result is very important as it indicates that a robust solution can be selected, for example, with an intermediate protection range, and guarantee good performance of the solution with a price of robustness which is never greater than 7% of the objective function.

This suggests that the RO methodology is a good tool to improve the current deterministic model, with the aim of permitting the inclusion of variability and uncertainty in the model. This does not involve a significant computational cost or the need to use highly specialized software. The use of this methodology allows for robust solutions to be obtained, and at the same time, for the sensitivity of the model to tactical decisions related to uncertainty in the performance parameters to be evaluated.

Another important conclusion is that the decision principals do not present significant changes in structure, that is to say, the types of logs required by the sawmills and the types of plank which are produced do not alter significantly with respect to the deterministic or nominal solution. Furthermore, the cutting patterns used tend to be the same in different variability scenarios. This is important as it indicates that the model is robust and can be used with confidence in a real management environment. This further reinforces the applicability of this methodology in the problems associated with the supply chain of Chilean forests.

6. SECOND APPLICATION: HARVEST PRODUCTION PLANNING PROBLEM

6.1. Problem description

The harvest planning problem consists of determining the surface of each forest or stand, in terms of the age of the harvest, and to cut in each period in such a way as to maximize the financial benefits. Revenues come from the sale of distinct products (logs) which can be obtained from each stand and the operational costs are associated with the harvest, transport and maintenance or inventory of the logs in intermediate locations. This is a tactical/operational problem in nature. **Figure 6.1-1** presents a diagram of the problem.

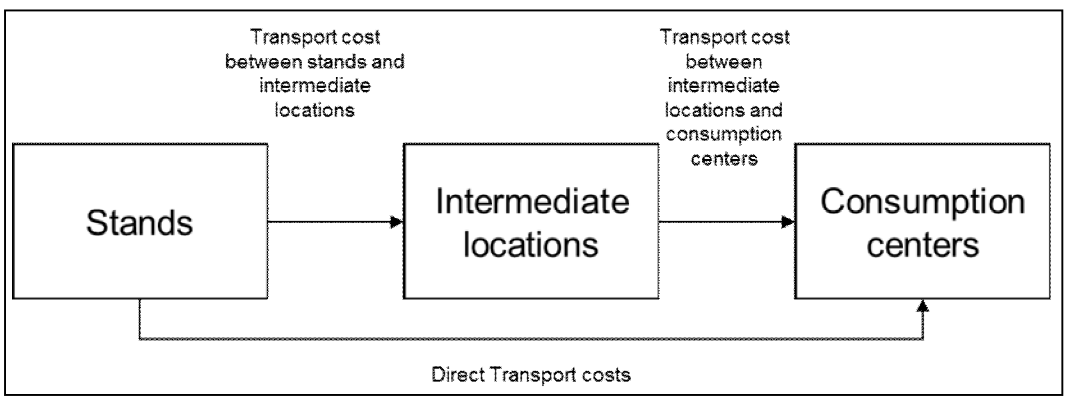

Figure 6.1-1: Flows in the harvest planning problem.

Given the existing transport costs, the volume to be transported from the stands or intermediate locations to the demand centers must also be determined; and from the intermediate locations to the consumption centers, assuring that demand will be fulfilled in each of these areas.

Additionally, to determine the surface area to be harvested proximity constraints, which aim to reduce the visual and environmental impact of the harvest, are also incorporated. This means that if a stand is harvested, none of the neighboring stands can be harvested in the same period, given a certain definition of what is considered as neighboring.

6.2. Deterministic model for harvest production planning

In the following section, a deterministic mathematical model which represents the previously described problem is presented. It is important to signal that the inclusion of proximity constraints implies the formulation of a model which includes binary variables in order to be resolved.

The notation used and the formulation of the model are as follows:

- **Sets**

I Sets of stands.

T Sets of time periods.

J Sets of intermediate locations.

P Sets of produce.

K Sets of consumption centers.

- **Variables**

x_{it} Surface area of the stand to be harvested $i \in I$ in the time period $t \in T$ (ha).

w_{ijpt} Volume of produce $p \in P$ to be transported from the stand $i \in I$ to the intermediate location $j \in J$ in the time period $t \in T$ (m^3).

w_{jkpt} Volume of produce $p \in P$ to be transported from the intermediate location $j \in J$ to the consumption center $k \in K$ in the time period $t \in T$ (m^3).

w_{ikpt} Volume of produce $p \in P$ to be transported from the stand $i \in I$ to the consumption center $k \in K$ in the time period $t \in T$ (m^3).

r_{jpt} Volume of produce $p \in P$ which is stored in the intermediate location $j \in J$ in the time period $t \in T$ (m^3).

v_{it} Binary variable, which takes the value of 1 if the stand $i \in I$ is harvested in the time period $t \in T$, and 0 if this is not the case.

- **Parameters**

B_{pkt} Selling price of the produce $p \in P$ to the consumption center $k \in K$ in the time period $t \in T$ ($/m^3).

H_{it} Cost of harvesting the stand $i \in I$ in the time period $t \in T$ ($/ha).

W_{jt} Storage costs in the intermediate location $j \in J$ in the time period $t \in T$ ($/m^3).

T^1_{ijt} Transport costs from the stand $i \in I$ to the intermediate location $j \in J$ in the time period $t \in T$ ($/m^3).

T^2_{jkt} Transport costs from the intermediate location $j \in J$ to the consumption center $\in K$ in the time period $t \in T$ ($/m^3).

T^3_{ikt} Transport costs from the stand $i \in I$ to the consumption center $k \in K$ in the time period $t \in T$ ($/m^3).

C^1_t Maximum production capacity in the time period $t \in T$ (ha).

C^2_{jt} Maximum storage capacity in the intermediate location $j \in J$ in the time period $t \in T$ (m^3).

R_{pi} Performance of one hectare of the stand $i \in I$ in terms of volume of produce $p \in P$ (m^3/ha).

D_{pkt} Demand from the consumption center $k \in K$ for product p in the time period $t \in T$ (m^3).

S_i Total surface area of the stand $i \in I$ (ha).

P Activation cost of the harvest variable, which is the same for every stand ($).

- **Optimization model**

$$Max\ z = \sum_{t \in T} \left[\begin{array}{l} \sum_{i \in I} \sum_{k \in K} \sum_{p \in P} \sum_{j \in J} B_{pkt} \cdot \left(w_{ikpt} + w_{jkpt} \right) - \sum_{p \in P} \sum_{i \in I} \sum_{j \in J} T^1_{ijt} \cdot w_{ijpt} - \\ \sum_{p \in P} \sum_{k \in K} \sum_{j \in J} T^2_{jkt} \cdot w_{jkpt} - \sum_{p \in P} \sum_{k \in K} \sum_{i \in I} T^3_{ikt} \cdot w_{ikpt} - \\ \sum_{j \in J} \sum_{p \in P} W_{jt} \cdot r_{jpt} - \sum_{i \in I} H_{it} \cdot x_{it} - \sum_{i \in I} P \cdot v_{it} \end{array} \right] \tag{6.2.1}$$

Subject to:

$$v_{it} + v_{vt} \le 1 \qquad \forall i \in I, v \in N(i), t \in T \tag{6.2.2}$$

$$\sum_{t \in T} x_{it} \le S_i \qquad \forall i \in I \tag{6.2.3}$$

$$\sum_{j \in J} w_{jkpt} + \sum_{i \in I} w_{ikpt} \ge D_{pkt} \qquad \forall p \in P, k \in K, t \in T \tag{6.2.4}$$

$$\sum_{i \in I} x_{it} \le C^1_t \qquad \forall t \in T \tag{6.2.5}$$

$$R_{pi} \cdot x_{it} \ge \sum_{j \in J} w_{ijpt} + \sum_{k \in K} w_{ikpt} \qquad \forall i \in I, t \in T, p \in P \tag{6.2.6}$$

$$x_{it} \le S_i \cdot v_{it} \qquad \forall i \in I, t \in T \tag{6.2.7}$$

$$\sum_{i \in I} w_{ijpt} - \sum_{k \in K} w_{jkpt} + r_{jpt-1} = r_{jpt} \qquad \forall j \in J, t \in T, p \in P \tag{6.2.8}$$

$$\sum_{p \in P} r_{jpt} \le C^2_{jt} \qquad \forall j \in J, t \in T \tag{6.2.9}$$

$$x_{it}, w_{ijpt}, w_{ikpt}, w_{jkpt}, r_{jpt} \geq 0 \qquad \forall i \in I, j \in J, p \in P, t \in T \qquad (6.2.10)$$
$$v_{it} \in \{0,1\} \qquad \forall i \in I, t \in T$$

Expression (6.2.1) represents the objective function of the problem, which aims to maximize the revenues of the Division. The group of constraints (6.2.2) corresponds to the proximity limitations, as it is not possible to harvest neighboring forests in the same time period, or to harvest greater surface area than that of each stand (6.2.3). The set of constraints (6.2.4) model the satisfaction of demand and necessary flows to meet requirements. Constraint (6.2.5) models the maximum production capacity in each period and constraints (6.2.9) model the maximum storage capacity in each period and sawmill. The constraints (6.2.6) represent the harvest flows according to production. The inventory in the intermediate locations is represented by equations (6.2.8), and the set of constraints (6.2.7) which indicate that it is only possible to harvest a certain surface area of a stand if it is included in the stands to be harvested. Finally, the nature of the variables is reflected in the relations (6.2.10).

6.3. Methodology to tackle the harvest production planning problem

To evaluate the performance of Robust Optimization as a tool to tackle the production planning problem with uncertainty, the following steps will be taken:

- Formulate a robust counterpart to the deterministic harvest production planning problem
- Resolve the deterministic problem.
- Resolve the robust counterpart for the same data.
- Compare the results obtained for both cases. The comparison considers three aspects: optimality, feasibility and structure of the solution.

As in the previous application, in the case of optimality, the aim is to evaluate the possible losses in value of the objective function when using robust optimization to consider uncertainty in the performance data. On the other hand, in relation to feasibility, the behavior of the solutions regarding possible data instances is to be analyzed, for which scenarios are generated via Monte-Carlo simulation and distinct protection levels are tested.

Finally, the analysis of the structure of the solution aims to determine the stability of the solution.

6.4. Robust harvest production planning model

To formulate the robust counterpart to the problem presented, the two groups of constraints that contain the uncertain parameter R_{pi} are reformulated. As such, constraints (6.2.6) are substituted for constraints (6.4.1) to generate the robust counterparts.

$$\begin{aligned}
&R_{pi} \cdot x_{it} - z_{1ipt} \cdot \Gamma_1 - \rho_{1ipt} \geq \sum_{j \in J} w_{ijpt} + \sum_{k \in K} w_{ikpt} && \forall i \in I, t \in T, p \in P \\
&z_{1ipt} + \rho_{1ipt} \geq \hat{a}_{pi} \cdot y_{1it} && \forall i \in I, t \in T, p \in P \\
&-y_{1it} \leq x_{it} \leq y_{1it} && \forall i \in I, t \in T \\
&z_{1ipt}, y_{1it}, \rho_{1ipt} \geq 0 && \forall i \in I, t \in T, p \in P
\end{aligned} \tag{6.4.1}$$

6.5. Data instances and size of the problems

As an application, data from a typical situation of a forestry company was used. The company is assumed to have 380 stands of different sizes, 5 intermediate locations, 17 consumption centers, 5 products (trimmed, thick saw wood, thin saw wood, short saw wood, pulpwood) and two periods (winter – summer) to be planned.

The uncertain parameter corresponds to the performance of the forests (R_{pi}). This parameter represents the performance of one hectare of forest or stand i to obtain a certain volume quantity of produce p (m^3/ha).

With respect to uncertainty, it is considered that performance can vary by 10% following the previously defined uncertainty model U. As in the previous application, variation in the "uncertainty budget" is considered, and is represented by the parameter Γ. As there are two constraints which contain the uncertain parameter, a parameter Γ is defined for each constraint. As such, for this case in particular Γ_1 can take values between 0 and 5 to adjust the uncertainty assumed. Under these assumptions, the performance of the robust counterpart is analyzed and the results are compared with the deterministic model. In the following subsections the results are presented in terms of optimality, feasibility and structure of the solution.

Given these instances, the size of the deterministic model is 86,020 decision variables and 91,355 constraints. When the expressions to formulate the robust counterpart are included, this model considers 94,380 decision variables and 103,113 constraints. Although the robust model is bigger than the deterministic problem, its linear structure is maintained and as such so are its advantages. It is important to highlight that although this is a linear problem (both the deterministic and the robust problem), they are also mixed problems which contain binary variables in their formulation, which makes their resolution more difficult.

6.6. Results and comparison

Table 6.6-1 presents the main results of the forest harvest planning model.

Table 6.6-1: Deterministic model results.

Index	**Results**		
	Period 1	**Period 2**	**Total**
Number of stands harvested (N°)	105	78	183
Area harvested (ha)	1,050	1,050	2,100
Inventory (m^3)	10,000	0	10,000
Sales (m^3)	197,905	278,281	476,185

Given the data instance, the results reflect the fact that both harvest capacity and inventory storage capacity are limited, and as such are scarce resources. The quantity of stands to be harvested is approximately 50% of the available stands.

a) Optimality

Table 6.6-2 shows the cases analyzed for distinct combinations of the protection level, depending on the constraints subject to uncertainty. **Figure 6.6-1** presents the optimal value for each of the cases.

Table 6.6-2: Gamma values.

Case	Γ_1	Γ_2
Deterministic (0)	-	-
1	0.1	0.5
2	0.2	1
3	0.3	1.5
4	0.4	2
5	0.5	2.5
6	0.6	3
7	0.7	3.5
8	0.8	4
9	0.9	4.5
10	0.95	4.75

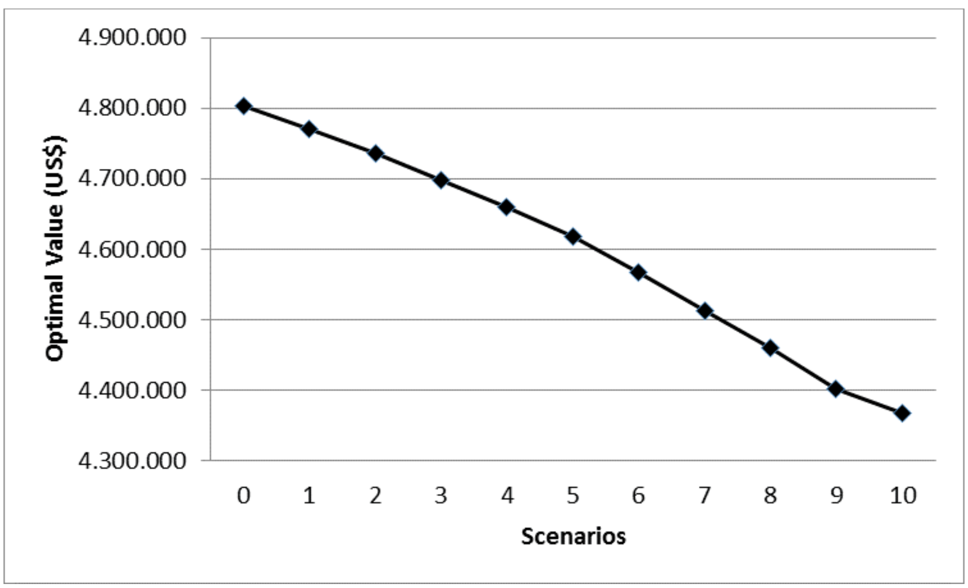

Figure 6.6-1: Optimal values in different cases.

As can be appreciated in the previous figure (**Figure 6.6-1**) and in **Table 6.6-3**, when the protection level is increased, the optimal value decreases. However, the difference when compared with the deterministic case does not exceed 10% even with the highest protection level considered.

Table 6.6-3: Percentage change in the optimal value.

Case	Variation respect the deterministic model (%)
Deterministic (0)	-
1	0.7%
2	1.4%
3	2.2%
4	3.0%
5	3.8%
6	4.9%
7	6.0%
8	7.1%
9	8.3%
10	9.1%

In the extent to which uncertainty increases, the value of the objective function decreases because the costs and quantity of stands harvested increases as a result of lower performance. In **Table 6.6-4** the results are presented for case 9.

Table 6.6-4: Robust model results.

Index	**Results**		
	Period 1	**Period 2**	**Total**
Number of stands harvested (N°)	101	105	206
Area harvested (ha)	1,050	1,050	2,100
Inventory (m^3)	10,000	0	10,000
Sales (m^3)	197,905	275,661.10	473,566.10

As in the previous case, the harvest capacity and inventory storage capacity are limited resources.

b) Feasibility

To test the robustness of the solution in terms of feasibility, that is to say, in terms of how the obtained solutions behave in the face of distinct data instances, diverse data instance scenarios were stimulated and the robust model solution was tested. 100 scenarios are created randomly within the uncertainty set.

To evaluate feasibility, the constraints which contain uncertainty parameters are revised, considering one group of constraints with 1,900 constraints, which correspond to constraints (6.2.6). The results are shown in **Table 6.6-5**.

Table 6.6-5: Summary feasibility.

Percentage of scenarios infeasible (%)	**Average infeasibility (%)**
10.0	0.71

In the previous table, it can also be observed that on average, the percentage of infeasibility (cubic meters of produce which are lacking in order to meet demand) is 0.71%.

c) Structure of the solution

When the solutions obtained are analyzed, it can be observed that the deterministic model solution is large part repeated in the different uncertainty scenarios, that is to say, the stands harvested and the quantity to be harvested are fairly homogenous. For example, in uncertainty case 1, 94.51% of the stands harvested (and their quantity in hectares) are the same as the stands to be harvested according to the deterministic model solution. **Table 6.6-6** shows the percentage of stands and quantities which are repeated with regard to the deterministic model in the different uncertainty cases.

Table 6.6-6: Robustness in the structure of the solution.

Case	**Maintenance of the structure (%)**
1	94.5
2	91.8
3	85.1
4	78.1
5	78.0
6	77.4
7	77.8
8	75.2
9	74.2
10	74.0

This is interesting as the solution demonstrates robust behavior and does not vary to a large extent when the protection level is increased in the face of uncertainty.

It can be concluded that in the extent to which uncertainty increases, the value of the variable associated with the harvest does not vary largely, and as such the solution is stable and robust.

6.7. Harvest production problem conclusions

With the results obtained, the analysis undertaken and conditions given, it should be highlighted that the results of the problem for all cases are limited by the harvest quantity and storage capacity in each period. Naturally, in the extent to which uncertainty increases, the number of stands harvested increases, and as such the costs associated with the harvest increase and the value of the objective function decreases.

The OR methodology is, in this sense, an excellent option. It is a methodology which is easy to implement, its solutions are robust and stable, with a high rate of feasibility and stability in the conservation of the deterministic solution structure.

7. THIRD APPLICATION: INTERTEMPORAL PLANNING PROBLEM

7.1. Problem description

The problem corresponds to the sawmill planning problem explained in previous sections. Specifically, the coordination problem between the tactical and operational planning levels is considered. The decisions which are taken on a tactical level affect performance on an operational level. Given the uncertainty in performance, or the availability of logs, the solution could be altered on an operational level or be infeasible.

Typically, on a tactical level decisions are taken on an aggregate level, and in this particular case, these decisions correspond to the volume of logs requested from the forestry division to satisfy the demand of the sawmill. In this way, the requested logs should arrive at the sawmill and with this information, the operative model should be used to determine the volumes to be cut with determined cutting patterns, with the aim of satisfying final demand.

In this way, trying to coordinate decisions between different decision levels is a complex task due to the variability and limited knowledge of the probabilistic behavior of the data.

7.2. Methodology to tackle intertemporal production planning problem

To evaluate the usefulness of RO as a methodology to improve coordination between distinct time horizons, a comparison of intertemporal planning is performed using only the deterministic model versus the utilization of a robust model which explicitly includes uncertainty in its formulation.

As a first step, the deterministic model is solved on a tactical level. With this model the quantity of logs which must be requested to the forestry division in monthly periods is obtained. On this level (tactical), the performance of the logs is on an aggregated level. The logs are transformed in boards and the boards must satisfy demand, which on a tactical level is also considered in an aggregated way.

Subsequently, on an operational level, where the boards must be produced to satisfy real demand, the first thing which occurs is the arrival of the requested logs to the sawmill from the forest. As the level of work is operational, the information is disaggregated in weekly periods in which the requirements must arrive. This information generates the real availability of the logs on an operational level, with which the operation planning problem is solved. In this problem, the information of specific cutting pattern performance is also disaggregated, and in this way boards are produced with the available logs.

On this short term level, although it could be expected that monthly demand for logs (determined on a tactical level) would be distributed in a homogenous way in each week, on an operational level this is not very likely. On the one hand, the logs which are actually sent each week from the forests are not necessarily distributed in a uniform way and, what is more, do not necessary meet the total quantity agreed, and as such lower than planned volume can arrive at the sawmills. That is to say, the forestry division does not necessarily satisfy the requested quantities, even on average. It is presupposed that the forestry division takes its own decisions and sends certain quantities to the sawmill.

To test the performance of RO methodology, simulation was used 100 data scenarios are run, varying the availability of logs on a weekly level and the percentage of unfulfilled demand during the first month of planning.

With these quantities (scenarios), which are the quantities of logs which are actually available, the sawmill executes its deterministic operational model.

Therefore, this is the point at which uncertainty emerges, in the real availability of specific logs on an operational level. Evidently, this affects real performance in terms of the final products which it is possible to obtain.

The results of the resolution, first of the deterministic tactical model and then the operational model, which is also deterministic, are analyzed.

Subsequently, to consider the uncertainty which could present itself on an operational level due to the real availability of logs, possible variability in aggregated performance is incorporated on a tactical level. In this way the robust tactical model and subsequently the deterministic operational model are resolved, and the behavior of the data scenarios generated is analyzed.

In this way, the methodology to evaluate RO in intertemporal planning is presented in the **Figure 7.2-1**.

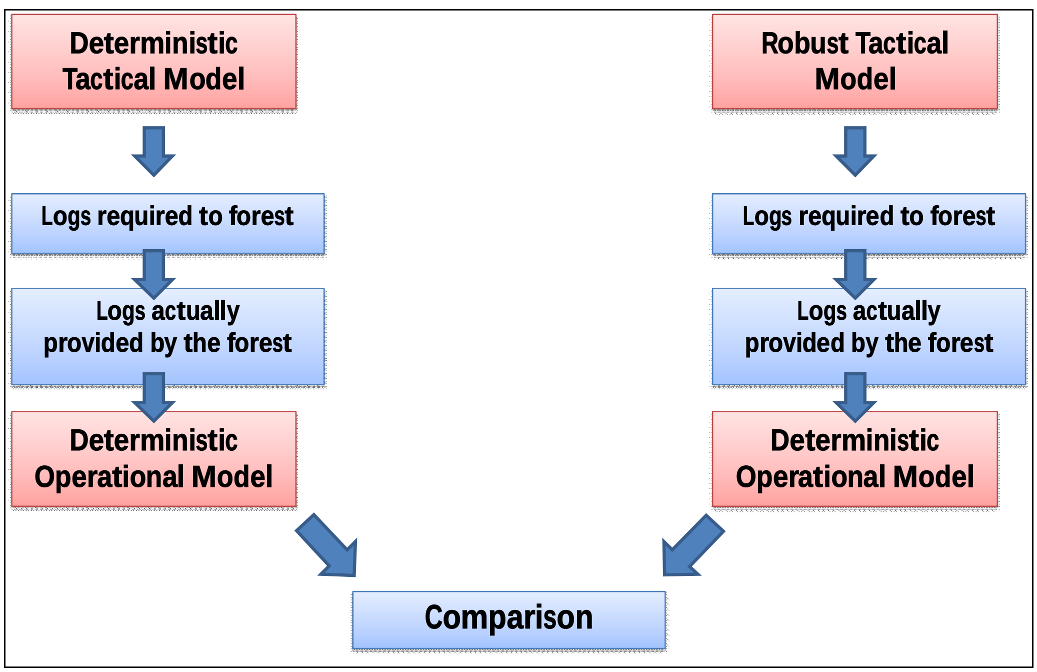

Figure 7.2-1: Flows in the planning problem.

7.3. Deterministic and robust tactical model

Firstly, the mathematical sawmill production planning model previously described is considered. However, some simplifications are considered and the information is aggregated to differentiate the tactical and operational models.

Specifically, the simplifications consider reduce the number of sawmill (used only 1 sawmill), eliminate constraints of drying capacity, reprocessing and not use cutting patterns.

a) Deterministic model

The notation used and formulation of the model are as follows:

- **Sets**

M Set of types of boards.

K Set of types of logs.

T Set of time periods.

- **Variables**

s_{kt} Volume of logs type $k \in K$ demanded and processed by the sawmill in the time period $t \in T$ (m^3).

w_{mt} Volume of board type $m \in M$ in inventory in the sawmill in the time period $t \in T$ (m^3).

- **Parameters**

C^T_{kt} Buying cost of logs type $k \in K$ in the sawmill in the time period $t \in T$ (US\$/ m^3).

$C^{A}{}_{kt}$ Sawmilling cost of logs type $k \in K$ in the sawmill in time period $t \in T$ (US$/ m^3).

$C^{B}{}_{mt}$ Inventory cost of boards type $m \in M$ in the sawmill in the time period $t \in T$ (US$/ m^3).

R_{km} Performance of logs type $k \in K$ to produce boards type $m \in M$ (m^3 boards/ m^3 logs).

$P^{A}{}_{t}$ Sawmilling capacity in the sawmill in time period $t \in T$ (m^3).

$P^{B}{}_{t}$ Inventory capacity in the sawmill in time period $t \in T$ (m^3).

D_{mt} Estimated demand for boards type $m \in M$ in the time period $t \in T$ (m^3).

- **Optimization model**

$$Min\ z = \sum_{t \in T}\left[\sum_{k \in K}\left(C_{kt}^{T} + C_{kt}^{A}\right)\cdot s_{kt} + \sum_{m \in M} C_{mt}^{B}\cdot w_{mt}\right] \qquad (7.3.1)$$

Subject to:

$$\sum_{k \in K} s_{kt} \le P_t^A \qquad \forall t \in T \qquad (7.3.2)$$

$$\sum_{m \in M} w_{mt} \le P_t^B \qquad \forall t \in T \qquad (7.3.3)$$

$$w_{mt-1} + \sum_{k \in K} R_{km}\cdot s_{kt} - w_{mt} \ge D_{mt} \qquad \forall m \in M, \forall t \in T \qquad (7.3.4)$$

$$w_{mt}, s_{kt} \ge 0 \qquad \forall m \in M, \forall k \in K, \forall t \in T \qquad (7.3.5)$$

The deterministic tactical model considers in expression (7.3.1) the minimization of cost of acquiring and sawmilling the logs, and the inventory costs of the boards. Constraints (7.3.2) and (7.3.3) correspond to the capacity constraints of sawmilling and inventory, respectively. The set of constraints (7.3.4) reflects the fulfillment of demand for boards in each period. Finally, constraints (7.3.5) are specifications of the nature of the variables.

In this study, uncertainty in performance parameters (R_{km}) is considered and the utilization of RO methodology is proposed to obtain solutions which are immune, to some extent, to this variability in order to improve coordination between tactical and operational planning levels.

b) Robust model

Considering that constraint (7.3.4) contains the uncertain parameter, it is that constraint which is protected, obtaining the following robust model:

$$Min\ z = \sum_{t \in T} \left[\sum_{k \in K} \left(C_{kt}^{T} + C_{kt}^{A} \right) \cdot s_{kt} + \sum_{m \in M} C_{mt}^{B} \cdot w_{mt} \right] \tag{7.3.6}$$

Subject to:

$$\sum_{k \in K} s_{kt} \leq P_t^A \qquad \forall t \in T \tag{7.3.7}$$

$$\sum_{m \in M} w_{mt} \leq P_t^B \qquad \forall t \in T \tag{7.3.8}$$

$$\left(w_{mt-1} + \sum_{k \in K} R_{km} \cdot s_{kt} - w_{mt} \right) - \phi_{mt}\Gamma - \sum_{k \in K} \omega_{kmt} \geq D_{mt} \qquad \forall m \in M, \forall t \in T \tag{7.3.9}$$

$$\phi_{mt} + \omega_{kmt} \geq \widehat{R}_{km} \mu_{kt} \qquad \forall m \in M, \forall k \in K, \forall t \in T \tag{7.3.10}$$

$$-\mu_{kt} \leq s_{kt} \leq \mu_{kt} \qquad \forall k \in K, \forall t \in T \tag{7.3.11}$$

$$w_{mt}, s_{kt}, \phi_{mt}, \omega_{km}, \mu_{kt} \geq 0 \qquad \forall m \in M, \forall k \in K, \forall t \in T \tag{7.3.12}$$

7.4. Operational model

At operational level also is a simplified model, but the information is more disaggregate in shorter periods of time and the use of cutting patterns.

The notation used and formulation of the model is as follows:

- **Sets**

M Set of types of boards.

K Set of types of logs.

T Set of time periods.

E Set of cutting patters.

- **Variables**

$s°_{ekt}$ Volume of logs type $k \in K$ processed with cutting pattern $e \in E$ by the sawmill in the time period $t \in T$ (m^3).

$w°_{mt}$ Volume of boards type $m \in M$ in inventory in the sawmill in time period $t \in T$ (m^3).

$z°_{kt}$ Volume of logs type $k \in K$ in inventory in the sawmill in the time period $t \in T$ (m^3).

- **Parameters**

C^P_{kt} Inventory cost of logs type $k \in K$ in the sawmill in the time period $t \in T$ (US$/ m^3)

C^A_{kt} Sawmilling cost of logs type $k \in K$ in the sawmill in time period $t \in T$ (US$/ m^3).

C^B_{mt} Inventory cost of boards type $m \in M$ in the sawmill in time period $t \in T$ (US$/$m^3$).

D^T_{kt} Availability of logs type $k \in K$ in the sawmill in time period $t \in T$ (US$/$m^3$).

$R°_{ekm}$ Performance of logs type $k \in K$ to produce boards type $m \in M$ using cutting pattern $e \in E$ (m^3 boards/ m^3 logs).

$P^A{}_t$ Sawmilling capacity in the sawmill in time period $t \in T$ (m^3).

$P^B{}_t$ Inventory capacity of boards in the sawmill in time period $t \in T$ (m^3).

$P^P{}_t$ Inventory capacity of logs in the sawmill in time period $t \in T$ (m^3).

$D°_{mt}$ Estimated demand for boards type $m \in M$ in the time period $t \in T$ (m^3).

- **Optimization model**

$$Min\ z = \sum_{t \in T} \left[\sum_{m \in M} C^B_{mt} \cdot w^o{}_{mt} + \sum_{e \in E} \sum_{k \in K} C^A_{kt} \cdot s^o{}_{ekt} + \sum_{k \in K} C^P_{kt} \cdot z^o{}_{kt} \right] \quad (7.4.1)$$

Subject to:

$$z^o{}_{kt-1} + D^T{}_{kt} - z^o{}_{kt} \geq \sum_{e \in E} s^o{}_{ekt} \qquad \forall k \in K, \forall t \in T \quad (7.4.2)$$

$$w^o{}_{mt-1} + \sum_{k \in K} \sum_{e \in E} R^o{}_{ekm} \cdot s^o{}_{ekt} - w^o{}_{mt} \geq D^o{}_{mt} \qquad \forall m \in M, \forall t \in T \quad (7.4.3)$$

$$\sum_{e \in E} \sum_{k \in K} s^o{}_{ekt} \leq P^A_t \qquad \forall t \in T \quad (7.4.4)$$

$$\sum_{k \in K} z^o{}_{kt} \leq P^P_t \qquad \forall t \in T \quad (7.4.5)$$

$$\sum_{m \in M} w^o{}_{mt} \leq P^B_t \qquad \forall t \in T \quad (7.4.6)$$

$$w^o{}_{mt} = w^p{}_{mt} - w^m{}_{mt} \qquad \forall m \in M, \forall t \in T \quad (7.4.7)$$

$$w^o{}_{mt}, s^o{}_{ekt}, z^o{}_{kt}, w^p{}_{mt}, w^m{}_{mt} \geq 0 \qquad \forall e \in E, \forall m \in M, \forall k \in K, \forall t \in T \quad (7.4.8)$$

As in the tactical model, the operational model considers the minimization of costs as its objective function (7.4.1). The costs considered correspond to the sawmilling and inventory costs of the logs, and the inventory cost of the boards. The constraints (7.4.2) model the volume of logs which are processed in each period in the sawmill. These logs can come from the available inventory in the sawmill or directly from the forest. The set of constraints (7.4.3) represents the fulfillment of demand for boards in each period. Constraints (7.4.4), (7.4.5) and (7.4.6) correspond to the capacity constraints of sawmilling and inventory of logs and boards, respectively. The set of constraints (7.4.7) is used to segment the inventory of boards and to be able to determine if breaking points are produced which will not allow demand to be satisfied. Finally, constraints (7.4.8) are specifications of the nature of the variables.

7.5. Data instances and size of the problems

Data from a typical situation of a forestry company was used. The company is assumed to have 6 types of logs and 7 types of measured timber both on a tactical and operational level. For the tactical planning level a time horizon of 12 months is considered, divided in monthly periods.

The uncertainty parameter, on a tactical level, is that which corresponds to the performance of the logs (R_{km}). This parameter represents the performance of a log to obtain a certain volume in quantity of boards.

With respect to uncertainty, it is considered that performance can vary by 5% in line with the previously defined uncertainty model *U*. As in the previous applications, variation in the “uncertainty budget” is considered, and is represented by the parameter Γ. This parameter can take values between 0 and 6, to adjust to the uncertainty assumed.

On an operational level, the planning horizon considered is monthly, and is divided into weeks. Additionally, 5 cutting patterns are proposed.

7.6. Results

a) Results of the deterministic tactical model

Table 7.6-1 presents the results of the tactical deterministic model, regarding the quantity of logs (m^3) to be requested from the forest each month.

Table 7.6-1: Logs demanded to forest (m^3) as tactical deterministic model.

Period	Logs demanded to forest (m^3)					
	LOG1	LOG2	LOG3	LOG4	LOG5	LOG6
1	5,585	0	0	0	17,340	2,573
2	7,140	0	0	0	2,885	15,474
3	7,911	0	0	0	5,617	11,971
4	5,420	0	0	0	20,079	0
5	3,770	0	0	0	21,729	0
6	8,425	0	0	0	7,419	9,655
7	5,400	0	0	0	20,099	0
8	7,874	0	0	0	11,176	6,449
9	7,491	0	0	0	0	18,008
10	8,517	0	0	0	0	16,982
11	0	0	5,982	0	18,649	0
12	0	0	4,869	0	20,439	0

The results obtained on an operational level (**Table 7.6-2**) show that not considering possible variability on a tactical level can lead to demand not being fulfilled on an operational level, putting business continuity at risk. As such, of the 100 scenarios simulated, only 66.6% of them were feasible given the availability of logs. On the other hand, in these feasible scenarios, a cost increase is produced, which on average was 3.03%.

Table 7.6-2: Summary of scenarios at the operational level.

Unmet demand (%)	Average increase in the value of the objective function (%)
33.3	3.03

b) Results of the tactical robust model

Similarly, **Table 7.6-3** presents the results of the tactical model. In this case the robust counterpart for Γ is equal to 6, which as such considers the highest protection level possible. The numbers represent the quantity of logs (m^3) to be requested from the forest each month.

Table 7.6-3: Logs demanded to forest (m^3) as tactical robust model.

Period	Logs demanded to forest (m^3)					
	LOG1	**LOG2**	**LOG3**	**LOG4**	**LOG5**	**LOG6**
1	7,822	1,962	0	0	0	15,714
2	2,397	6,952	0	0	826	15,324
3	0	11,674	0	0	715	13,110
4	6,052	0	0	0	19,447	0
5	7,399	0	0	0	5,053	13,047
6	10,385	0	0	0	0	15,114
7	9,753	0	0	0	0	15,746
8	9,156	0	0	0	7,347	8,996
9	7,909	0	0	0	0	17,590
10	8,989	0	0	0	0	16,510
11	1,175	0	5,314	0	19,010	0
12	3,125	0	2,511	0	19,862	0

Following the same exercise as previously, 100 scenarios are simulated, varying the weekly availability of logs and the percentage of unfulfilled demand for the first month of planning.

When uncertainty is incorporated on a tactical level, the operational level becomes more robust with 100% of scenarios being feasible, and therefore satisfying all of the demand. No cost increases were generated; on the contrary, there is an average reduction of 10% on an operative level. And on a tactical level, for the most conservative case, the increase in the objective value is 12.6%.

For distinct protection levels (Γ) the results are similar.

c) Comparison and conclusions

As can be observed in Table 7.6-4, the quantities required on a tactical level, either using the deterministic model or the robust model, independently of the protection level, are the same. However, the specific volumes and types of logs would vary, in order to favor those logs assigned to a greater quantity of final products (boards).

Table 7.6-4: Comparison between robust and deterministic tactical model.

Case	**Period**	**Logs demanded to forest (m^3)**						**Total Volume (m^3)**
		LOG1	**LOG2**	**LOG3**	**LOG4**	**LOG5**	**LOG6**	
Deterministic	**1**	5,585	0	0	0	1,7340	2,573	**2,550**
Robust (Γ=6)	**1**	7,822	1,962	0	0	0	15,714	**2,550**
Robust (Γ=5)	**1**	7,822	1,962	0	0	0	15,714	**2,550**
Robust (Γ=4)	**1**	7,822	1,962	0	0	0	15,714	**2,550**
Robust (Γ=3)	**1**	8,075	1,436	0	0	833	15,154	**2,550**
Robust (Γ=2)	**1**	8,955	327	0	0	802	15,415	**2,550**
Robust (Γ=1)	**1**	8,177	0	0	0	5,960	11,361	**2,550**

On the other hand, regarding costs on an operational level, when variability is introduced in an explicit way through the robust counterpart it can be observed that the increase in costs, when comparing the robust approach with the deterministic approach, is not more than 12.6% (**Table 7.6-5** and **Figure 7.6-1**).

Table 7.6-5: Optimal value – Deterministic and Robust model.

Case	Optimal value (US$)	Variation (%)
Deterministic	47,652,700	-
Robust (Γ=1)	50,747,000	6.5
Robust (Γ=2)	52,247,500	9.6
Robust (Γ=3)	53,387,900	12.0
Robust (Γ=4)	53,676,200	12.6
Robust (Γ=5)	53,676,200	12.6
Robust (Γ=6)	53,676,200	12.6

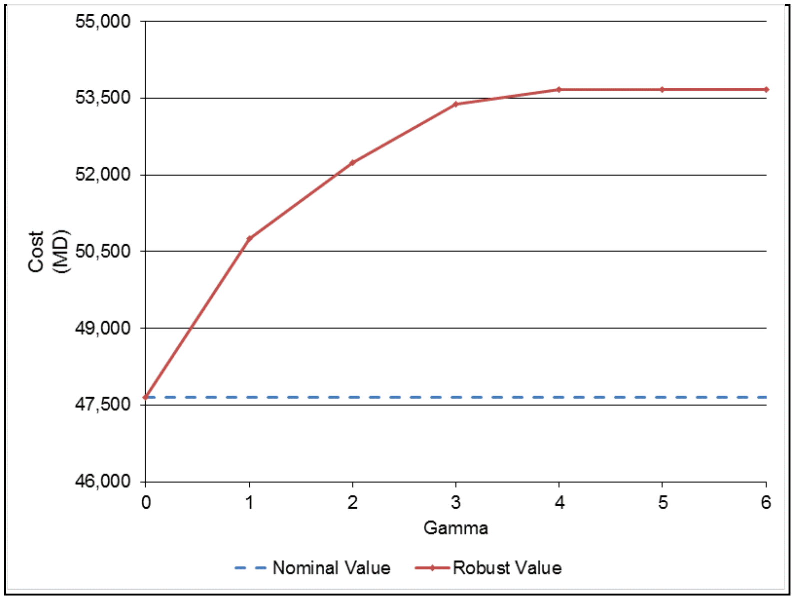

Figure 7.6-1: Optimal value – Tactical model.

It can be concluded that when the robust model is used on a tactical level in the medium term, short term decisions are more consistent, thus guaranteeing customer service. Additionally, losses in optimality on a tactical level are not significant, do not exceed 12.6% and, at least in the instances studied, the optimal value on an operational level improve using RO.

8. GENERAL CONCLUSIONS AND DISCUSSION

In this research the applicability of the RO is explored in some applications of production planning problem in the supply chain in a major national industries, the forest industry. In particular, three exploratory examples were developed to test the hypothesis and achieve the objectives.

A first model examines how the production plan for a sawmill is affected due the uncertainty in the performance of different cutting patterns. The uncertainty is because the variability in the logs from forests.

The second model, studies the problem of harvest stands to provide logs, considering the variability in productivity per hectare of forest for different types of product (export, sawlog and pulpwood).

Finally, a production problem in two hierarchical levels is analyzed, and the methodology used a robust model at the tactical level to analyze the impact of uncertainty at the operational level.

The main conclusion which is obtained from this study is effectively that the Robust Optimization methodology, following the approach of Bertsimas and Sim, is a very good tool to support decision making in uncertain situations. In problems where the probability distribution of the data is not known or cannot be determined, RO and its simplicity in the construction of uncertain sets facilitates the incorporation of uncertainty, notably improving the performance of the solutions.

This methodology, as was expected, is very simple to apply, and given that the robust models maintain the same structure as the original deterministic models, does not increase resolution difficulty. Particularly, in the applications presented in this research, deterministic models and their robust counterpart are linear. This represents a significant advantage associated with this method. With the applications or exploratory examples developed in this study it was shown that the RO can be used in actual practice in large problems. By keeping the same structure to the original model, it can be solved using the same techniques of resolution without increasing the complexity of analysis to users.

As can be observed in the cases studied, the loss of optimality due to the inclusion of protection against uncertainty is low and can be disregarded. It never exceeded 10% and what is more, with this methodology a tradeoff between optimality and robustness can be established. Therefore, it is possible to manage the loss of optimality within the limits which the decision maker considers appropriate.

Regarding robustness, the methodology turns out to be robust, with the percentage of feasibility of the solutions increasing between the deterministic model and the robust model in all of the cases analyzed. Similarly, in terms of optimality, it is possible for the decision maker to define how robust they expect the solution to be.

This feature of the RO is relevant because the decision maker can choose how conservative or risky to be. Through the parameter gamma it is possible to move from insuring against the worst case (conservative) and, in the other extreme, to not consider the uncertainty at all and using the deterministic model (risky). Between these points, it is possible choose the desired level of robustness, which is possible to associate with percentages of feasibility according to the literature.

Finally, in relation to the stability of the solutions, it is observed that a fairly high percentage of the decisions are maintained stable. This is desirable and can even be considered a further demonstration of the robustness of this methodology. The stability of the solution also helps, independently of the data variability, to define the equipment setups which will facilitate work on an operational level.

Additionally, in the intertemporal production planning case studied, it is observed that RO is also a tool to support the coordination of distinct hierarchical planning levels in a more efficient way. This provides greater versatility to this methodology.

Particularly, the use of the RO to study the coherence between different decisions hierarchical level opens an alternative way to study this important issue. This problem is still not solved completely. So, this research is becoming an initial methodological contribution to explore in future research. In fact, this thesis and in particular the third application associated with decision making between different hierarchical levels led to an undergraduate thesis (Espinoza, 2013).

Finally, through production models present general structure makes it possible to extend these results to supply chains in other sectors of economic activity (retail, energy production, etc.)

9. FUTURE RESEARCH

Some of the challenges for future research which have been identified in this thesis correspond to the use of a moving horizon for the robust model, in order to be able to adjust the decisions which are taken through various periods, incorporating the possibility of modifying them when uncertainty disappears given that the present time has been executed. In this way, an action plan can be adjusted in each period, with the objective of incorporating performances which are currently taking place. As such, only the decision regarding the optimal solution for the first period would be adopted, and from then on modifications can be incorporated in accordance with what occurs in reality. This, of course, is an approximation to introduce some adaptability in the decisions, but it does not optimize in time in the same way as a formulation of dynamic stochastic programming. However, the utilization of decision models in moving horizons is a habitual practice.

On the other hand, the uncertain data in our case studies (performance coefficients) is also present in the objective function, and in various constraints which are repeated. This indicates that an implicit relationship exists between the constraints and the objective function. This relationship was not considered in this study, and the solution obtained is probably more conservative than is really necessary. The question which should be investigated is whether or not it would really be relevant to consider these relationships between the coefficients. Future research on this issue should include extensions to the RO methodology to manage this situation.

Finally, it is important to evaluate the applicability of this RO methodology for the forestry supply chain as a whole, integrating solutions which connect different problems in diverse divisions (sawmills, forests, etc.).

As a final comment regarding the approach of this thesis; the models presented are static, which means that all of the decisions must be taken before the implementation of uncertain data (Chen and Zhang, 2009). This assumption could produce solutions which are too conservative and restrictive. For example, for the sawmill production planning model, some of the decisions, as is the case with the product inventory decisions, could be adapted according to the information that is known. A two stage stochastic problem would allow for this. However, in this study the robust model presented has been considered as it has the advantage of easy applicability. However, the authors Ben-Tal et al. (2004) have developed extensions to the robust optimization approach which consider adaptable decisions. They introduce the concept of an Adjustable Robust Counterpart (ARC) to include decisions which must be taken after the implementation of uncertain events. ARC generates less conservative solutions, but a price is paid as the resulting problems are computationally difficult to resolve (NP-hard). However, they also propose some simplifications which could be implemented and applied to the models presented in this thesis. Therefore, future research could compare the adaptive robust approach with the more traditional approach in two phases of stochastic programming.

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11. ANNEXES

ANNEX A: MATHEMATICAL MODEL SAWMILL PLANNING PROBLEM

```
##################################################
#       Conjuntos aserraderos
##################################################

set ASERRADERO;                          # Aserraderos de la división
set TROZA;                         # Tipos de trozas
set CORTES;                        # Tipos de corte en aserradero
set TABLA;                         # Especificación de tablas
set TABLAVERDE;                          # Tablas sin secado
set TABLASECA;                           # Tablas con secado

##################################################
#       Parametros generales
##################################################

param T>=0;                        # Períodos
param descuento {i in 1..T};             # Descuento


###################################################
#        Parametros aserradero
###################################################

param PR{TABLA}>=0;                      # Precio tabla
param CA{ASERRADERO} default 99999;            # Costo aserrío
param CS{ASERRADERO} default 99999;            # Costo secado
param CR{ASERRADERO} default 99999;            # Costo reproceso
param CO default 99999;                        # Costo outsourcing
param CT{ASERRADERO} default 99999;            # Costo inventario
trozas
```

```
param CP{ASERRADERO} default 99999;          # Costo inventario
patio
param CB{ASERRADERO} default 99999;          # Costo inventario
bodega
param PF{ASERRADERO,TROZA} default 0;        # Precio troza
param CAPA{ASERRADERO}>=0;             # Capacidad aserradero
param CAPC{ASERRADERO}>=0;             # Capacidad cancha de trozas
param CAPP{ASERRADERO}>=0;             # Capacidad patio
param CAPS{ASERRADERO}>=0;             # Capacidad secado
param CAPB{ASERRADERO}>=0;             # Capacidad bodega
param CAPR{ASERRADERO}>=0;             # Capacidad reproceso
param DD{TABLA, t in 1..T}>=0;               # Estimación de
demanda
param RM{ASERRADERO,CORTES,TROZA,TABLA} default 0; # Rendimiento a
tabla
param VD >=0;                          # Parámetro para variar
demanda
param CTS>=0;                          # Parámetro para variar
capacidad bodega
param CTV>=0;                          # Parámetro para variar
capacidad patio
param CAPOT{ASERRADERO}>=0;            # Capacidad outsourcing
param VR>=0;                           # Parámetro que hace variar
el rendimiento

###################################################
#        Parametros iniciales
###################################################

param IT0 {ASERRADERO,TROZA}>=0;       # Inventario inicial troza
param IP0 {ASERRADERO, TABLAVERDE}>=0;       # Inventario inicial
patio
param IB0 {ASERRADERO, TABLASECA}>=0;        # Inventario inicial
bodega
```

```
########################################################
#   Variables aserradero
########################################################

var IT{ASERRADERO,TROZA, t in 0..T}>=0;      # Inventario  troza
var IP{ASERRADERO,TABLAVERDE, t in 0..T}>=0; # Inventario patio
var IB{ASERRADERO,TABLASECA, t in 0..T}>=0;  # Inventario bodega
var AT{ASERRADERO,CORTES,TROZA, t in 1..T}>=0;     # Trozas aserrar
var PT{ASERRADERO,TABLA, t in 1..T}>=0;      # Producción tabla
var OT{ASERRADERO,TABLASECA, t in 1..T}>=0;  # Outsourcing
var VV{ASERRADERO,TABLA, t in 1..T}>=0;      # Ventas estimadas
var DT{ASERRADERO,TROZA, t in 1..T};         # Demanda por trozas
var TT>=0;                        # Cota (FO)

######################################################
#   FUNCION OBJETIVO_1
######################################################

maximize GANANCIA_1:
      TT;

###################################################################
#             RESTRICCIONES ASERRADERO
###################################################################

subject to INVENTARIO_CANCHA {j in ASERRADERO, k in TROZA, t in
1..T}:
   DT[j,k,t] =IT[j,k,t] + sum {e in CORTES} AT[j,e,k,t] - IT[j,k,t-
1];

subject to CAP_ASERR {j in ASERRADERO, t in 1..T}:
    sum {e in CORTES, k in TROZA} AT[j,e,k,t]<= CAPA[j];
```

```
subject to CAP_CANCHA {j in ASERRADERO, t in 1..T}:
    sum {k in TROZA} IT[j,k,t]<=CAPC[j];

subject to CAP_PATIO {j in ASERRADERO, t in 1..T}:
    sum {m in TABLAVERDE} IP[j,m,t]<= CTV*CAPP[j];

subject to CAP_BODEGA {j in ASERRADERO, t in 1..T}:
    sum {m in TABLASECA} IB[j,m,t]<= CTS*CAPB[j];

#### Condiciones iniciales #######

subject to INVENTARIO_TROZA_INICIAL {j in ASERRADERO, k in TROZA}:
    IT[j,k,0]= IT0[j,k];

subject to INVENTARIO_PATIO_INICIAL {j in ASERRADERO, m in
TABLAVERDE}:
    IP[j,m,0]= IP0[j,m];

subject to INVENTARIO_BODEGA_INICIAL {j in ASERRADERO, m in
TABLASECA}:
    IB[j,m,0]=IB0[j,m];

#### Con sustitución ########

subject to COTA_FO_C:
 TT<=
 sum {t in 1..T} descuento[t]*(
 +sum {j in ASERRADERO,m in TABLAVERDE}
 PR[m]*(IP[j,m,t-1]+ sum {e in CORTES, k in TROZA}
AT[j,e,k,t]*VR*RM[j,e,k,m] - IP[j,m,t])           # Ventas
estimadas
 +sum {j in ASERRADERO,m in TABLASECA}
```

```
 PR[m]*(IB[j,m,t-1]+ sum {e in CORTES, k in TROZA}
AT[j,e,k,t]*VR*RM[j,e,k,m] + OT[j,m,t] - IB[j,m,t])     # Ventas
estimadas
 -sum {j in ASERRADERO, e in CORTES, k in TROZA} CA[j]*AT[j,e,k,t]
           # Costos Aserrío
 -sum {j in ASERRADERO, m in TABLASECA} CS[j]*(sum {e in CORTES, k
in TROZA} AT[j,e,k,t]*VR*RM[j,e,k,m])          # Costo secado
 -sum {j in ASERRADERO, m in TABLASECA} 0.2*CR[j]*(sum {e in
CORTES, k in TROZA} AT[j,e,k,t]*VR*RM[j,e,k,m])      # Costo
reproceso seca
 -sum {j in ASERRADERO, m in TABLAVERDE} CR[j]*(sum {e in CORTES, k
in TROZA} AT[j,e,k,t]*VR*RM[j,e,k,m])         # Costo reproceso
verde
 -sum {m in TABLASECA} CO*(sum {j in ASERRADERO} OT[j,m,t])
      # Costo Outsourcing
 -sum {j in ASERRADERO, k in TROZA} CT[j]*IT[j,k,t]
# Costo inventario trozas
 -sum {j in ASERRADERO, m in TABLAVERDE} CP[j]*IP[j,m,t]
      # Costo inventario patio
 -sum {j in ASERRADERO, m in TABLASECA} CB[j]*IB[j,m,t]
      # Costo inventario bodega
 -sum {j in ASERRADERO, k in TROZA} PF[j,k]*DT[j,k,t]
      # Precio de las trozas
 );

subject to CAP_SECADO_C {j in ASERRADERO, t in 1..T}:
    sum {m in TABLASECA, e in CORTES, k in TROZA}
AT[j,e,k,t]*VR*RM[j,e,k,m]<= CAPS[j];

subject to CAP_REPROC_C {j in ASERRADERO, t in 1..T}:
    0.2*(sum {m in TABLASECA,e in CORTES, k in TROZA}
AT[j,e,k,t]*VR*RM[j,e,k,m]) +
    (sum {m in TABLAVERDE,e in CORTES, k in TROZA}
AT[j,e,k,t]*VR*RM[j,e,k,m]) <= CAPR[j];
```

```
subject to REPROCESO {j in ASERRADERO, t in 1..T}:
    sum {m in TABLASECA} OT[j,m,t] <= CAPOT[j];

subject to DDA_EST_VERDE_C {m in TABLAVERDE , t in 1..T}:
    sum {j in ASERRADERO} (IP[j,m,t-1]+ sum {e in CORTES, k in
TROZA} AT[j,e,k,t]*VR*RM[j,e,k,m]
    - IP[j,m,t])>=VD*DD[m,t];

subject to DDA_EST_SECA_C {m in TABLASECA , t in 1..T}:
     sum {j in ASERRADERO} (IB[j,m,t-1]+ sum {e in CORTES, k in
TROZA} AT[j,e,k,t]*VR*RM[j,e,k,m] + OT[j,m,t]
     - IB[j,m,t])>=VD*DD[m,t];
```

ANNEX B: MATHEMATICAL MODEL HARVEST PLANNING PROBLEM

```
MODEL: !MODELO ASERRADERO ROBUSTO;

SETS:
ASERRADERO/MULCHEN,NACTO,BUCA/:CA,CS,CR,CT,CP,CB,CAPA,CAPC,CAPP,CAPS,CA
PB,CAPR,CAPOT;
TROZA/C32,C38,M26,M34,I24,I34/:;
CORTES/E1,E2/;
TABLAVERDE/MAS,TAP/:PRV;
TABLASECA/MSH,MUE,REM,EST,MAK/:PRS;
PERIODOS/1..12/:DESCUENTO;
ARCO_JK(ASERRADERO,TROZA):PF;
ARCO_MT(TABLAVERDE,PERIODOS):DDV,ZMT4,GAM4;
ARCO_NT(TABLASECA,PERIODOS):DDS,ZNT5,GAM5;
ARCO_JEKM(ASERRADERO,CORTES,TROZA,TABLAVERDE):RMV,AV,RO3,RO4;
ARCO_JEKN(ASERRADERO,CORTES,TROZA,TABLASECA):RMS,AS,RO1,RO2,RO5;
ARCO_JNT(ASERRADERO,TABLASECA,PERIODOS):OT,IB;
ARCO_JKT(ASERRADERO,TROZA,PERIODOS):IT,DT;
ARCO_JEKT(ASERRADERO,CORTES,TROZA,PERIODOS):AT,YJ1,YJ2,YJ3,YJ4,YJ5;
ARCO_JMT(ASERRADERO,TABLAVERDE,PERIODOS):IP;
ARCO_JT(ASERRADERO,PERIODOS):ZJT1,ZJT2,ZJT3,GAM1,GAM2,GAM3;

ENDSETS

DATA:
DESCUENTO,VD,CTS,CTV,VR,CO,CA,CS,CR,CT,CP,CB,PRS,PRV,PF,CAPA,CAPC,CAPP,
CAPS,CAPB,CAPR,CAPOT,DDS,DDV,RMS,RMV,GAM1,GAM2,GAM3,GAM4,GAM5,AS,AV=@OL
E('C:\Users\Cata\Desktop\Datos_aserradero.xls');
@OLE('C:\Users\Cata\Desktop\Datos_aserradero.xls')=AT,IT,OT,DT,IP,IB,ZJ
T1,ZJT2,ZJT3,ZMT4,ZNT5,RO1,RO2,RO3,RO4,RO5,YJ1,YJ2,YJ3,YJ4,YJ5;
ENDDATA

MAX=@SUM(ASERRADERO(J):@SUM(TABLAVERDE(M):DESCUENTO(1)*PRV(M)*(0+@SUM(C
ORTES(E):@SUM(TROZA(K):AT(J,E,K,1)*VR*RMV(J,E,K,M)))-
IP(J,M,1))))+@SUM(ASERRADERO(J):@SUM(TABLAVERDE(M):DESCUENTO(2)*PRV(M)*
```

```
(IP(J,M,1)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,2)*VR*RMV(J,E,K,M)))-
IP(J,M,2))))+@SUM(ASERRADERO(J):@SUM(TABLAVERDE(M):DESCUENTO(3)*PRV(M)*
(IP(J,M,2)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,3)*VR*RMV(J,E,K,M)))-
IP(J,M,3))))+@SUM(ASERRADERO(J):@SUM(TABLAVERDE(M):DESCUENTO(4)*PRV(M)*
(IP(J,M,3)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,4)*VR*RMV(J,E,K,M)))-
IP(J,M,4))))+@SUM(ASERRADERO(J):@SUM(TABLAVERDE(M):DESCUENTO(5)*PRV(M)*
(IP(J,M,4)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,5)*VR*RMV(J,E,K,M)))-
IP(J,M,5))))+@SUM(ASERRADERO(J):@SUM(TABLAVERDE(M):DESCUENTO(6)*PRV(M)*
(IP(J,M,5)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,6)*VR*RMV(J,E,K,M)))-
IP(J,M,6))))+@SUM(ASERRADERO(J):@SUM(TABLAVERDE(M):DESCUENTO(7)*PRV(M)*
(IP(J,M,6)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,7)*VR*RMV(J,E,K,M)))-
IP(J,M,7))))+@SUM(ASERRADERO(J):@SUM(TABLAVERDE(M):DESCUENTO(8)*PRV(M)*
(IP(J,M,7)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,8)*VR*RMV(J,E,K,M)))-
IP(J,M,8))))+@SUM(ASERRADERO(J):@SUM(TABLAVERDE(M):DESCUENTO(9)*PRV(M)*
(IP(J,M,8)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,9)*VR*RMV(J,E,K,M)))-
IP(J,M,9))))+@SUM(ASERRADERO(J):@SUM(TABLAVERDE(M):DESCUENTO(10)*PRV(M)
*(IP(J,M,9)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,10)*VR*RMV(J,E,K,M)))
-
IP(J,M,10))))+@SUM(ASERRADERO(J):@SUM(TABLAVERDE(M):DESCUENTO(11)*PRV(M
)*(IP(J,M,10)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,11)*VR*RMV(J,E,K,M)
))-
IP(J,M,11))))+@SUM(ASERRADERO(J):@SUM(TABLAVERDE(M):DESCUENTO(12)*PRV(M
)*(IP(J,M,11)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,12)*VR*RMV(J,E,K,M)
))-
IP(J,M,12))))+@SUM(ASERRADERO(J):@SUM(TABLASECA(N):DESCUENTO(1)*PRS(N)*
(0+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,1)*VR*RMS(J,E,K,N)))+OT(J,N,1)
-
IB(J,N,1))))+@SUM(ASERRADERO(J):@SUM(TABLASECA(N):DESCUENTO(2)*PRS(N)*(
IB(J,N,1)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,2)*VR*RMS(J,E,K,N)))+OT
(J,N,2)-
IB(J,N,2))))+@SUM(ASERRADERO(J):@SUM(TABLASECA(N):DESCUENTO(3)*PRS(N)*(
IB(J,N,2)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,3)*VR*RMS(J,E,K,N)))+OT
(J,N,3)-
IB(J,N,3))))+@SUM(ASERRADERO(J):@SUM(TABLASECA(N):DESCUENTO(4)*PRS(N)*(
IB(J,N,3)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,4)*VR*RMS(J,E,K,N)))+OT
(J,N,4)-
```

```
IB(J,N,4))))+@SUM(ASERRADERO(J):@SUM(TABLASECA(N):DESCUENTO(5)*PRS(N)*(
IB(J,N,4)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,5)*VR*RMS(J,E,K,N)))+OT
(J,N,5)-
IB(J,N,5))))+@SUM(ASERRADERO(J):@SUM(TABLASECA(N):DESCUENTO(6)*PRS(N)*(
IB(J,N,5)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,6)*VR*RMS(J,E,K,N)))+OT
(J,N,6)-
IB(J,N,6))))+@SUM(ASERRADERO(J):@SUM(TABLASECA(N):DESCUENTO(7)*PRS(N)*(
IB(J,N,6)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,7)*VR*RMS(J,E,K,N)))+OT
(J,N,7)-
IB(J,N,7))))+@SUM(ASERRADERO(J):@SUM(TABLASECA(N):DESCUENTO(8)*PRS(N)*(
IB(J,N,7)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,8)*VR*RMS(J,E,K,N)))+OT
(J,N,8)-
IB(J,N,8))))+@SUM(ASERRADERO(J):@SUM(TABLASECA(N):DESCUENTO(9)*PRS(N)*(
IB(J,N,8)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,9)*VR*RMS(J,E,K,N)))+OT
(J,N,9)-
IB(J,N,9))))+@SUM(ASERRADERO(J):@SUM(TABLASECA(N):DESCUENTO(10)*PRS(N)*
(IB(J,N,9)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,10)*VR*RMS(J,E,K,N)))+
OT(J,N,10)-
IB(J,N,10))))+@SUM(ASERRADERO(J):@SUM(TABLASECA(N):DESCUENTO(11)*PRS(N)
*(IB(J,N,10)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,11)*VR*RMS(J,E,K,N))
)+OT(J,N,11)-
IB(J,N,11))))+@SUM(ASERRADERO(J):@SUM(TABLASECA(N):DESCUENTO(12)*PRS(N)
*(IB(J,N,11)+@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,12)*VR*RMS(J,E,K,N))
)+OT(J,N,12)-IB(J,N,12))))-
@SUM(ARCO_JEKT(J,E,K,T):DESCUENTO(T)*CA(J)*AT(J,E,K,T))-
@SUM(PERIODOS(T):DESCUENTO(T)*@SUM(ARCO_JEKN(J,E,K,N):CS(J)*AT(J,E,K,T)
*VR*RMS(J,E,K,N)))-
@SUM(PERIODOS(T):DESCUENTO(T)*@SUM(ARCO_JEKN(J,E,K,N):(0.2)*CR(J)*AT(J,
E,K,T)*VR*RMS(J,E,K,N)))-
@SUM(PERIODOS(T):DESCUENTO(T)*@SUM(ARCO_JEKM(J,E,K,M):CR(J)*AT(J,E,K,T)
*VR*RMV(J,E,K,M)))-@SUM(ARCO_JNT(J,N,T):DESCUENTO(T)*CO*OT(J,N,T))-
@SUM(ARCO_JKT(J,K,T):DESCUENTO(T)*CT(J)*IT(J,K,T))-
@SUM(ARCO_JMT(J,M,T):DESCUENTO(T)*CP(J)*IP(J,M,T))-
@SUM(ARCO_JNT(J,N,T):DESCUENTO(T)*CB(J)*IB(J,N,T))-
@SUM(ARCO_JKT(J,K,T):DESCUENTO(T)*PF(J,K)*DT(J,K,T));
```

```
!1;
@FOR(ARCO_JK(J,K):DT(J,K,1)=IT(J,K,1)+@SUM(CORTES(E):AT(J,E,K,1))-0);
@FOR(ARCO_JK(J,K):DT(J,K,2)=IT(J,K,2)+@SUM(CORTES(E):AT(J,E,K,2))-
IT(J,K,1));
@FOR(ARCO_JK(J,K):DT(J,K,3)=IT(J,K,3)+@SUM(CORTES(E):AT(J,E,K,3))-
IT(J,K,2));
@FOR(ARCO_JK(J,K):DT(J,K,4)=IT(J,K,4)+@SUM(CORTES(E):AT(J,E,K,4))-
IT(J,K,3));
@FOR(ARCO_JK(J,K):DT(J,K,5)=IT(J,K,5)+@SUM(CORTES(E):AT(J,E,K,5))-
IT(J,K,4));
@FOR(ARCO_JK(J,K):DT(J,K,6)=IT(J,K,6)+@SUM(CORTES(E):AT(J,E,K,6))-
IT(J,K,5));
@FOR(ARCO_JK(J,K):DT(J,K,7)=IT(J,K,7)+@SUM(CORTES(E):AT(J,E,K,7))-
IT(J,K,6));
@FOR(ARCO_JK(J,K):DT(J,K,8)=IT(J,K,8)+@SUM(CORTES(E):AT(J,E,K,8))-
IT(J,K,7));
@FOR(ARCO_JK(J,K):DT(J,K,9)=IT(J,K,9)+@SUM(CORTES(E):AT(J,E,K,9))-
IT(J,K,8));
@FOR(ARCO_JK(J,K):DT(J,K,10)=IT(J,K,10)+@SUM(CORTES(E):AT(J,E,K,10))-
IT(J,K,9));
@FOR(ARCO_JK(J,K):DT(J,K,11)=IT(J,K,11)+@SUM(CORTES(E):AT(J,E,K,11))-
IT(J,K,10));
@FOR(ARCO_JK(J,K):DT(J,K,12)=IT(J,K,12)+@SUM(CORTES(E):AT(J,E,K,12))-
IT(J,K,11));
!2;
@FOR(ASERRADERO(J):@FOR(PERIODOS(T):@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E
,K,T)))<=CAPA(J)));
!3;
@FOR(ASERRADERO(J):@FOR(PERIODOS(T):@SUM(TROZA(K):IT(J,K,T))<=CAPC(J)))
;
!4;
@FOR(ASERRADERO(J):@FOR(PERIODOS(T):@SUM(TABLAVERDE(M):IP(J,M,T))<=(CTV
*CAPP(J))));
!5;
@FOR(ASERRADERO(J):@FOR(PERIODOS(T):@SUM(TABLASECA(N):IB(J,N,T))<=(CTS*
CAPB(J))));
```

```
!6 (INCERTIDUMBRE);
@FOR(ASERRADERO(J):@FOR(PERIODOS(T):@SUM(TABLASECA(N):@SUM(CORTES(E):@S
UM(TROZA(K):AT(J,E,K,T)*VR*RMS(J,E,K,N))))+(ZJT1(J,T)*GAM1(J,T))+@SUM(T
ABLASECA(N):@SUM(CORTES(E):@SUM(TROZA(K):RO1(J,E,K,N))))<=CAPS(J)));
@FOR(PERIODOS(T):@FOR(ARCO_JEKN(J,E,K,N):(ZJT1(J,T)+RO1(J,E,K,N))>=(AS(
J,E,K,N)*YJ1(J,E,K,T))));
@FOR(ARCO_JEKT(J,E,K,T):(-YJ1(J,E,K,T))<=AT(J,E,K,T));
@FOR(ARCO_JEKT(J,E,K,T):YJ1(J,E,K,T)>=AT(J,E,K,T));
@FOR(ARCO_JEKT(J,E,K,T):YJ1(J,E,K,T)>=0);
@FOR(ARCO_JEKN(J,E,K,N):RO1(J,E,K,N)>=0);
@FOR(ARCO_JT(J,T):ZJT1(J,T)>=0);

!7(INCERTIDUMBRE);
@FOR(ASERRADERO(J):@FOR(PERIODOS(T):(0.2)*@SUM(TABLASECA(N):@SUM(CORTES
(E):@SUM(TROZA(K):AT(J,E,K,T)*VR*RMS(J,E,K,N))))+(ZJT2(J,T)*GAM2(J,T))+
@SUM(TABLASECA(N):@SUM(CORTES(E):@SUM(TROZA(K):RO2(J,E,K,N))))+@SUM(TAB
LAVERDE(M):@SUM(CORTES(E):@SUM(TROZA(K):AT(J,E,K,T)*VR*RMV(J,E,K,M))))+
(ZJT3(J,T)*GAM3(J,T))+@SUM(TABLAVERDE(M):@SUM(CORTES(E):@SUM(TROZA(K):R
O3(J,E,K,M))))<=CAPR(J)));
@FOR(PERIODOS(T):@FOR(ARCO_JEKN(J,E,K,N):(ZJT2(J,T)+RO2(J,E,K,N))>=(AS(
J,E,K,N)*YJ2(J,E,K,T))));
@FOR(PERIODOS(T):@FOR(ARCO_JEKM(J,E,K,M):(ZJT3(J,T)+RO3(J,E,K,M))>=(AV(
J,E,K,M)*YJ3(J,E,K,T))));
@FOR(ARCO_JEKT(J,E,K,T):(-YJ2(J,E,K,T))<=AT(J,E,K,T));
@FOR(ARCO_JEKT(J,E,K,T):YJ2(J,E,K,T)>=AT(J,E,K,T));
@FOR(ARCO_JEKT(J,E,K,T):(-YJ3(J,E,K,T))<=AT(J,E,K,T));
@FOR(ARCO_JEKT(J,E,K,T):YJ3(J,E,K,T)>=AT(J,E,K,T));
@FOR(ARCO_JEKT(J,E,K,T):YJ2(J,E,K,T)>=0);
@FOR(ARCO_JEKT(J,E,K,T):YJ3(J,E,K,T)>=0);
@FOR(ARCO_JEKN(J,E,K,N):RO2(J,E,K,N)>=0);
@FOR(ARCO_JEKM(J,E,K,M):RO3(J,E,K,M)>=0);
@FOR(ARCO_JT(J,T):ZJT2(J,T)>=0);
@FOR(ARCO_JT(J,T):ZJT3(J,T)>=0);

!8;
```

```
@FOR(ASERRADERO(J):@FOR(PERIODOS(T):@SUM(TABLASECA(N):OT(J,N,T))<=CAPOT
(J)));
!9(INCERTIDUMBRE);
@FOR(TABLAVERDE(M):@SUM(ASERRADERO(J):(0+@SUM(CORTES(E):@SUM(TROZA(K):A
T(J,E,K,1)*VR*RMV(J,E,K,M)))-IP(J,M,1)))-(ZMT4(M,1)*GAM4(M,1))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO4(J,E,K,M))))
>=VD*DDV(M,1));
@FOR(TABLAVERDE(M):@SUM(ASERRADERO(J):(IP(J,M,1)+@SUM(CORTES(E):@SUM(TR
OZA(K):AT(J,E,K,2)*VR*RMV(J,E,K,M)))-IP(J,M,2)))-(ZMT4(M,2)*GAM4(M,2))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO4(J,E,K,M))))>=VD*DDV
(M,2));
@FOR(TABLAVERDE(M):@SUM(ASERRADERO(J):(IP(J,M,2)+@SUM(CORTES(E):@SUM(TR
OZA(K):AT(J,E,K,3)*VR*RMV(J,E,K,M)))-IP(J,M,3)))-(ZMT4(M,3)*GAM4(M,3))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO4(J,E,K,M))))>=VD*DDV
(M,3));
@FOR(TABLAVERDE(M):@SUM(ASERRADERO(J):(IP(J,M,3)+@SUM(CORTES(E):@SUM(TR
OZA(K):AT(J,E,K,4)*VR*RMV(J,E,K,M)))-IP(J,M,4)))-(ZMT4(M,4)*GAM4(M,4))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO4(J,E,K,M))))>=VD*DDV
(M,4));
@FOR(TABLAVERDE(M):@SUM(ASERRADERO(J):(IP(J,M,4)+@SUM(CORTES(E):@SUM(TR
OZA(K):AT(J,E,K,5)*VR*RMV(J,E,K,M)))-IP(J,M,5)))-(ZMT4(M,5)*GAM4(M,5))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO4(J,E,K,M))))>=VD*DDV
(M,5));
@FOR(TABLAVERDE(M):@SUM(ASERRADERO(J):(IP(J,M,5)+@SUM(CORTES(E):@SUM(TR
OZA(K):AT(J,E,K,6)*VR*RMV(J,E,K,M)))-IP(J,M,6)))-(ZMT4(M,6)*GAM4(M,6))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO4(J,E,K,M))))>=VD*DDV
(M,6));
@FOR(TABLAVERDE(M):@SUM(ASERRADERO(J):(IP(J,M,6)+@SUM(CORTES(E):@SUM(TR
OZA(K):AT(J,E,K,7)*VR*RMV(J,E,K,M)))-IP(J,M,7)))-(ZMT4(M,7)*GAM4(M,7))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO4(J,E,K,M))))>=VD*DDV
(M,7));
@FOR(TABLAVERDE(M):@SUM(ASERRADERO(J):(IP(J,M,7)+@SUM(CORTES(E):@SUM(TR
OZA(K):AT(J,E,K,8)*VR*RMV(J,E,K,M)))-IP(J,M,8)))-(ZMT4(M,8)*GAM4(M,8))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO4(J,E,K,M))))>=VD*DDV
(M,8));
```

```
@FOR(TABLAVERDE(M):@SUM(ASERRADERO(J):(IP(J,M,8)+@SUM(CORTES(E):@SUM(TR
OZA(K):AT(J,E,K,9)*VR*RMV(J,E,K,M)))-IP(J,M,9)))-(ZMT4(M,9)*GAM4(M,9))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO4(J,E,K,M))))>=VD*DDV
(M,9));
@FOR(TABLAVERDE(M):@SUM(ASERRADERO(J):(IP(J,M,9)+@SUM(CORTES(E):@SUM(TR
OZA(K):AT(J,E,K,10)*VR*RMV(J,E,K,M)))-IP(J,M,10)))-
(ZMT4(M,10)*GAM4(M,10))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO4(J,E,K,M))))>=VD*DDV
(M,10));
@FOR(TABLAVERDE(M):@SUM(ASERRADERO(J):(IP(J,M,10)+@SUM(CORTES(E):@SUM(T
ROZA(K):AT(J,E,K,11)*VR*RMV(J,E,K,M)))-IP(J,M,11)))-
(ZMT4(M,11)*GAM4(M,11))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO4(J,E,K,M))))>=VD*DDV
(M,11));
@FOR(TABLAVERDE(M):@SUM(ASERRADERO(J):(IP(J,M,11)+@SUM(CORTES(E):@SUM(T
ROZA(K):AT(J,E,K,12)*VR*RMV(J,E,K,M)))-IP(J,M,12)))-
(ZMT4(M,12)*GAM4(M,12))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO4(J,E,K,M))))>=VD*DDV
(M,12));
@FOR(PERIODOS(T):@FOR(ARCO_JEKM(J,E,K,M):(ZMT4(M,T)+RO4(J,E,K,M))>=(AV(
J,E,K,M)*YJ4(J,E,K,T))));
@FOR(ARCO_JEKT(J,E,K,T):(-YJ4(J,E,K,T))<=AT(J,E,K,T));
@FOR(ARCO_JEKT(J,E,K,T):YJ4(J,E,K,T)>=AT(J,E,K,T));
@FOR(ARCO_JEKT(J,E,K,T):YJ4(J,E,K,T)>=0);
@FOR(ARCO_JEKM(J,E,K,M):RO4(J,E,K,M)>=0);
@FOR(ARCO_MT(M,T):ZMT4(M,T)>=0);

!10(INCERTIDUMBRE);
@FOR(TABLASECA(N):@SUM(ASERRADERO(J):(0+@SUM(CORTES(E):@SUM(TROZA(K):AT
(J,E,K,1)*VR*RMS(J,E,K,N)))+OT(J,N,1)-IB(J,N,1)))-
(ZNT5(N,1)*GAM5(N,1))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO5(J,E,K,N))))>=VD*DDS
(N,1));
@FOR(TABLASECA(N):@SUM(ASERRADERO(J):(IB(J,N,1)+@SUM(CORTES(E):@SUM(TRO
ZA(K):AT(J,E,K,2)*VR*RMS(J,E,K,N)))+OT(J,N,2)-IB(J,N,2)))-
(ZNT5(N,2)*GAM5(N,2))-
```

```
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO5(J,E,K,N))))>=VD*DDS
(N,2));
@FOR(TABLASECA(N):@SUM(ASERRADERO(J):(IB(J,N,2)+@SUM(CORTES(E):@SUM(TRO
ZA(K):AT(J,E,K,3)*VR*RMS(J,E,K,N)))+OT(J,N,3)-IB(J,N,3)))-
(ZNT5(N,3)*GAM5(N,3))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO5(J,E,K,N))))>=VD*DDS
(N,3));
@FOR(TABLASECA(N):@SUM(ASERRADERO(J):(IB(J,N,3)+@SUM(CORTES(E):@SUM(TRO
ZA(K):AT(J,E,K,4)*VR*RMS(J,E,K,N)))+OT(J,N,4)-IB(J,N,4)))-
(ZNT5(N,4)*GAM5(N,4))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO5(J,E,K,N))))>=VD*DDS
(N,4));
@FOR(TABLASECA(N):@SUM(ASERRADERO(J):(IB(J,N,4)+@SUM(CORTES(E):@SUM(TRO
ZA(K):AT(J,E,K,5)*VR*RMS(J,E,K,N)))+OT(J,N,5)-IB(J,N,5)))-
(ZNT5(N,5)*GAM5(N,5))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO5(J,E,K,N))))>=VD*DDS
(N,5));
@FOR(TABLASECA(N):@SUM(ASERRADERO(J):(IB(J,N,5)+@SUM(CORTES(E):@SUM(TRO
ZA(K):AT(J,E,K,6)*VR*RMS(J,E,K,N)))+OT(J,N,6)-IB(J,N,6)))-
(ZNT5(N,6)*GAM5(N,6))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO5(J,E,K,N))))>=VD*DDS
(N,6));
@FOR(TABLASECA(N):@SUM(ASERRADERO(J):(IB(J,N,6)+@SUM(CORTES(E):@SUM(TRO
ZA(K):AT(J,E,K,7)*VR*RMS(J,E,K,N)))+OT(J,N,7)-IB(J,N,7)))-
(ZNT5(N,7)*GAM5(N,7))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO5(J,E,K,N))))>=VD*DDS
(N,7));
@FOR(TABLASECA(N):@SUM(ASERRADERO(J):(IB(J,N,7)+@SUM(CORTES(E):@SUM(TRO
ZA(K):AT(J,E,K,8)*VR*RMS(J,E,K,N)))+OT(J,N,8)-IB(J,N,8)))-
(ZNT5(N,8)*GAM5(N,8))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO5(J,E,K,N))))>=VD*DDS
(N,8));
@FOR(TABLASECA(N):@SUM(ASERRADERO(J):(IB(J,N,8)+@SUM(CORTES(E):@SUM(TRO
ZA(K):AT(J,E,K,9)*VR*RMS(J,E,K,N)))+OT(J,N,9)-IB(J,N,9)))-
(ZNT5(N,9)*GAM5(N,9))-
```

```
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO5(J,E,K,N))))>=VD*DDS
(N,9));
@FOR(TABLASECA(N):@SUM(ASERRADERO(J):(IB(J,N,9)+@SUM(CORTES(E):@SUM(TRO
ZA(K):AT(J,E,K,10)*VR*RMS(J,E,K,N)))+OT(J,N,10)-IB(J,N,10)))-
(ZNT5(N,10)*GAM5(N,10))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO5(J,E,K,N))))>=VD*DDS
(N,10));
@FOR(TABLASECA(N):@SUM(ASERRADERO(J):(IB(J,N,10)+@SUM(CORTES(E):@SUM(TR
OZA(K):AT(J,E,K,11)*VR*RMS(J,E,K,N)))+OT(J,N,11)-IB(J,N,11)))-
(ZNT5(N,11)*GAM5(N,11))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO5(J,E,K,N))))>=VD*DDS
(N,11));
@FOR(TABLASECA(N):@SUM(ASERRADERO(J):(IB(J,N,11)+@SUM(CORTES(E):@SUM(TR
OZA(K):AT(J,E,K,12)*VR*RMS(J,E,K,N)))+OT(J,N,12)-IB(J,N,12)))-
(ZNT5(N,12)*GAM5(N,12))-
@SUM(ASERRADERO(J):@SUM(CORTES(E):@SUM(TROZA(K):RO5(J,E,K,N))))>=VD*DDS
(N,12));
@FOR(PERIODOS(T):@FOR(ARCO_JEKN(J,E,K,N):(ZNT5(N,T)+RO5(J,E,K,N))>=(AS(
J,E,K,N)*YJ5(J,E,K,T))));
@FOR(ARCO_JEKT(J,E,K,T):(-YJ5(J,E,K,T))<=AT(J,E,K,T));
@FOR(ARCO_JEKT(J,E,K,T):YJ5(J,E,K,T)>=AT(J,E,K,T));
@FOR(ARCO_JEKT(J,E,K,T):YJ5(J,E,K,T)>=0);
@FOR(ARCO_JEKN(J,E,K,N):RO5(J,E,K,N)>=0);
@FOR(ARCO_NT(N,T):ZNT5(N,T)>=0);

!11;
@FOR(ARCO_JNT(J,N,T): OT(J,N,T)>=0);
@FOR(ARCO_JNT(J,N,T): IB(J,N,T)>=0);
@FOR(ARCO_JMT(J,M,T): IP(J,M,T)>=0);
@FOR(ARCO_JKT(J,K,T): IT(J,K,T)>=0);
@FOR(ARCO_JKT(J,K,T): DT(J,K,T)>=0);
@FOR(ARCO_JEKT(J,E,K,T): AT(J,E,K,T)>=0);

END
```

ANNEX C: MATHEMATICAL MODEL INTERTEMPORAL PLANNING PROBLEM

```
##################################################
#       MODELO TACTICO DETERMINISTA, OPERATIVO Y ROBUSTO
##################################################

##################################################
#       Conjuntos aserraderos
##################################################

set TROZA;                                          # Tipos de trozas
set TABLA;                                          # Especificación de
tablas
set CORTE;                                          # Patron de corte


####################################################
#        Parametros aserradero
####################################################

param Tp>=0;                                              # Períodos a
planificar
param CTt{TROZA,t in 1..Tp} default 99999;         # Costo compra
trozas
param CBt{TABLA,t in 1..Tp} default 99999;         # Costo
inventario tablas
param CAt{t in 1..Tp} default 99999;               # Costo aserrío
param Rt{TROZA,TABLA} default 0;             # Rendimiento troza-
tabla
param PAt{t in 1..Tp}>=0;                          # Capacidad
proceso de aserrío
```

```
param PBt{t in 1..Tp}>=0;                         # Capacidad
bodega de tablas
param D{TABLA, t in 1..Tp}>=0;                    # Estimación de
demanda
param To>=0;                                      # Períodos a
planificar operativo
param CB{TABLA,t in 1..To} default 99999;         # Costo
inventario tablas
param CP{TROZA,t in 1..To} default 99999;         # Costo
inventario trozas
param CA{t in 1..To} default 99999;               # Costo aserrío
param R{CORTE,TROZA,TABLA} default 0;        # Rendimiento troza-
tabla por patron
param PA{t in 1..To}>=0;                          # Capacidad
proceso de aserrío
param PB{t in 1..To}>=0;                          # Capacidad
bodega de tablas
param PP{t in 1..To}>=0;                          # Capacidad
bodega de trozas
param Do{TABLA, t in 1..To}>=0;                   # Estimación de
demanda
param DT{TROZA, t in 1..To}>=0;              # Disponibilidad de
trozas

###################################################
#        Parametros iniciales
###################################################

param w0t {TABLA}>=0;                        # Inventario inicial
tabla tactico
param w0 {TABLA}>=0;                         # Inventario inicial
tabla operativo
param z0 {TROZA}>=0;                         # Inventario inicial
troza operativo
```

```
###################################################
#        Parametros ROBUSTO
###################################################

#param GAMMA{TABLA,t in 1..Tp} >=0;
param GAMMA >=0;
param RR >=0;

#######################################################
#   Variables aserradero
#######################################################

var wt{TABLA, t in 0..Tp}>=0;                     # Inventario
tabla tactico
var rt{TABLA, t in 1..Tp}>=0;                     # Producción
tablas tactico
var st{TROZA, t in 1..Tp}>=0;                     # Trozas aserrar
tactico

var w{TABLA, t in 0..To};                    # Inventario tabla
operativo
var wp{TABLA, t in 0..To}>=0;                     # Inventario
tabla operativo
var wm{TABLA, t in 0..To}>=0;                     # Faltante tabla
operativo

## var r{TABLA, t in 1..To}>=0;                   # Producción
tablas operativo
var s{CORTE,TROZA, t in 1..To}>=0;                # Trozas aserrar
operativo
var so{TROZA, t in 1..To}>=0;                # Trozas aserrar
operativo sin diferenciar corte
```

```
var z{TROZA, t in 0..To}>=0;                 # Inventario trozas
operativo

#######################################################
#   Variables robusto
#######################################################

var phi{TABLA, t in 1..Tp}>=0;
var omega{TROZA, TABLA,t in 1..Tp}>=0;
var mu{TROZA, t in 0..Tp}>=0;


#####################################################
#   FUNCION OBJETIVO
#####################################################

minimize COSTOS_TAC:
      sum {t in 1..Tp} (
       +sum {k in TROZA} CTt[k,t]*st[k,t]          # Costo compra
trozas
       +sum {m in TABLA} CBt[m,t]*wt[m,t]          # Costo
inventario tablas
       +sum {k in TROZA} CAt[t]*st[k,t]      # Costo aserrío
       );

minimize COSTOS_OP:
      sum {t in 1..To} (
       +sum {m in TABLA} CB[m,t]*w[m,t]                  # Costo
inventario tablas
       +sum {k in TROZA,e in CORTE} CA[t]*s[e,k,t]      # Costo
aserrío
       +sum {k in TROZA} CP[k,t]*z[k,t]                  # Costo
aserrío
        +sum {m in TABLA} (100000)*wm[m,t]
```

```
       );

minimize COSTOS_OP1:
      sum {t in 1..To} (
       +sum {m in TABLA} CB[m,t]*w[m,t]                  # Costo
inventario tablas
       +sum {k in TROZA} CA[t]*so[k,t]                      # Costo
aserrío
       +sum {k in TROZA} CP[k,t]*z[k,t]                  # Costo
aserrío
        +sum {m in TABLA} (100000)*wm[m,t]
       );

##################################################################
#             RESTRICCIONES
##################################################################

#### Condiciones iniciales #######

subject to INVENTARIO_PATIO_INICIAL_T {m in TABLA}:
    wt[m,0]= w0t[m];

#### Restricciones #######

subject to CAP_ASERR {t in 1..Tp}:
    sum {k in TROZA} st[k,t]<= PAt[t];

subject to CAP_BODEGA {t in 1..Tp}:
    sum {m in TABLA} wt[m,t]<= PBt[t];


subject to DDA_TABLAS {m in TABLA, t in 1..Tp}:
     wt[m,t-1]+ sum {k in TROZA} Rt[k,m]*st[k,t]-wt[m,t]>=D[m,t];
```

```
subject to TABLAS {m in TABLA, t in 1..Tp}:
     sum {k in TROZA} Rt[k,m]*st[k,t]=rt[m,t];

     #### Condiciones iniciales operativo #######

subject to INVENTARIO_PATIO_INICIAL {m in TABLA}:
    wp[m,0]= w0[m];

subject to INVENTARIO_TROZA_INICIAL {k in TROZA}:
    z[k,0]= z0[k];

#### Restricciones #######

subject to INVENTARIO_TABLAS {m in TABLA, t in 1..To}:
   w[m,t-1]+sum {e in CORTE, k in TROZA} R[e,k,m]*s[e,k,t]-w[m,t]
>= Do[m,t];

subject to INVENTARIO_TABLAS1 {m in TABLA, t in 1..To}:
   w[m,t-1]+sum {k in TROZA} Rt[k,m]*so[k,t]-w[m,t] >= Do[m,t];

subject to INVENTARIO_TROZAS {k in TROZA, t in 1..To}:
   z[k,t-1] + DT[k,t] - z[k,t] = sum {e in CORTE} s[e,k,t];

subject to INVENTARIO_TROZAS1 {k in TROZA, t in 1..To}:
   z[k,t-1] + DT[k,t] - z[k,t] =  so[k,t];

subject to CAP_ASERR_O {t in 1..To}:
    sum {k in TROZA, e in CORTE} s[e,k,t]<= PA[t];

subject to CAP_ASERR_O1 {t in 1..To}:
    sum {k in TROZA} so[k,t]<= PA[t];

subject to CAP_BODEGA_O {t in 1..To}:
    sum {m in TABLA} wp[m,t]<= PB[t];
```

```
subject to CAP_PATIO {t in 1..To}:
      sum {k in TROZA} z[k,t]<= PP[t];

subject to DEF_W {m in TABLA, t in 0..To}: w[m,t] = wp[m,t] -
wm[m,t];

#### Restricciones ROBUSTAS #######

subject to DDA_TABLAS_R {m in TABLA, t in 1..Tp}:
     (wt[m,t-1]+ sum {k in TROZA} Rt[k,m]*st[k,t]-wt[m,t])-
phi[m,t]*GAMMA - sum {k in TROZA} omega[k,m,t] >=D[m,t];

subject to R1 {m in TABLA, k in TROZA, t in 1..Tp}:
     phi[m,t] + omega[k,m,t] >= RR*Rt[k,m]*mu[k,t];

#subject to R2 {k in TROZA, t in 1..Tp}:
#    -mu[k,t]<=st[k,t];

subject to R3 {k in TROZA, t in 1..Tp}:
    st[k,t]<=mu[k,t];
```