Nº 207
Marzo 2002

Documento de Trabajo

ISSN (edición impresa) **0716-7334**
ISSN (edición electrónica) **0717-7593**

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www.economia.puc.cl

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January, 3^{rd} 2002

Abstract

This paper deals with the view that income elasticity of health care is larger than one, as argued by empirical results on the literature. We build a theoretical model that shows that endogenous demographic transition may play a fundamental role on this result. It is argued that families must choose the number and the life expectancy of their members. Due to limited resources, there is a trade-off between those two variables though. It is shown that increases on income may produce a demographic transtion that allows a larger allocation of resources to health care.

JEL classification: I10, J11

Keywords: Health share of income, life expectancy, demographic transition.

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1. Motivation

The health share of income has been rising steadily since the sixties on the OECD countries. This observation has led to the notion of health care being a luxury good. Large empirical work has been done on this topic to determine if in fact health care is a luxury good. The results are mixed and they usually depends on the data set (cross sectional evidence among countries or time series data) or the methodology used. See Blomqvist and Carter (1997), Gerdtham (1992), Gerdtham and Jönsson (1991) and Parkin, McGuire and Yule(1987).

This paper will provide a theoretical model explaining this observation. The main idea on the model built on a family that must choose the number of its members and their life expectancy. Due to limited resources there will be a trade-off between those two chosen variables. This is the usual trade-off between quantity and quality of the family members in the human capital literature. See Becker (1981) and Becker, Murphy and Tamura (1993). It will be shown that relative prices of health (fertility) may depend positively on fertility (health). The intuition is basically that the family values all their members in similar way and therefore when deciding if accumulating an additional unit of health on children, the family must spent the same amount on each child. Thus, the larger is the number of children, the larger is the marginal cost of an additional unit of health per capita. In that case, as a country develops and a demographic transitions occurs, the accumulation of health occurs due not only to the usual income effect but also to the decrease on the relative price of health. This last effect reinforces the initial effect of income which is stretched out. Even if the health income elasticity is smaller or equal than one, the effect of income over health may seem larger than one due to the substitution effect above described. It is also shown that smaller depreciation rates of health may also increase the health share of income. Thus countries with larger fraction of elderly people should have a larger health share of income.

The paper is developed in the following way. Section 2 develops the model while section 3 solves the transitional dynamics of this economy and provides some simulations under alternative scenarios. Section 4 concludes.

2. The model

The framework used in this model is the one of overlapping generations. There is a family that is composed by young, middle age and older individuals. The young individuals will be assumed to be newborns that do not work. Newborns receive from their parents some health expenditure that determine their health capital stock. This health capital stock will determine if they reach their middle age or die before that. We are going to assume that the survival likelihood from the young age to the middle age is an increasing function of the health stock they carried over from their childhood. Let H_{mt} be the health capital stock obtained from parents during the childhood and let $\lambda(\bullet)$ be a constant survival density function over ages. Thus $\lambda(H_{mt})$ is the survival density function for the individuals from childhood to middle age. We will assume that $\lambda(h) = Ah^{\theta}$, where $A,\theta>0$. This function is increasing but concave on health stock and Inada conditions apply. No other activity will be realized during childhood, meaning that individuals do not consume or work during their childhood.

At middle age or adulthood, individuals work and obtain y_t units of physical good as a return. Income, y_t, growth at rate $g>0$. Those units of physical good may be used as consumption good during adulthood (c_{mt}), as savings to consume during their old age(s_t), as investment on health stock for their old age (I_{mt}) or as investment on their children health stock ($n_t H_{mt+1}$), where n_t denotes the number of children the middle age individual chooses and H_{mt+1} the stock of health capital of each of her children when they reach middle age. Thus, fertility rates is also a chosen variable for the middle age individual. Savings are invested on the capital market. There is no constraint on savings, meaning that savings can be negative with no bounds and thus capital markets are perfect. Finally, after working and after choosing the allocation of resources, the uncertainty is resolved, meaning that individuals either die or remain alive. If they remain alive they consume the amount they chose (c_{mt}) and they obtain some current level of utility $u(c_{mt})=\frac{c_{mt}^{1-\sigma}}{1-\sigma}$, where $\sigma > 0$. If they die they obtain some constant level of utility u_0. This last level of utility is normalized to zero, e.g. $u_0 = 0$. Hence the expected level of utility obtained during adulthood is $u(c_{mt})\lambda(H_{mt})$.

If still alive at the beginning of the last period of life -retirement age-, individuals get back their savings from the capital market with a return equal to r_{t+1}. Individuals consume at the end of the period as above. However, they may die before that. In fact they reach the end of the period with probability $\lambda(H_{ot+1})$,conditional on being alive at the beginning of the retirement period where H_{ot+1} is the health stock of the retired individual. This health stock (H_{ot+1}) is linear on initial stock, (H_{mt}) and health investment, I_{mt}. The initial health stock depreciates at rate δ.

The individual are altruists and hence they care about the well-being of their descendants, as in Becker, Murphy and Tamura (1993). Let $\alpha(n_t) = \frac{\alpha}{1-\varepsilon}n_t^{-\varepsilon}$ be the constant elasticity altruism-discount factor per children where $\alpha, \varepsilon > 0$ and let β the time period discount factor. Finally, the problem faced by children when they become adults is the same as the one faced by parents currently. Thus we will use a recursive setup, where we multiply the discounted utility level of each child by the number of children, n_t.The individual's problem can be stated now. The problem is:

$$
\begin{aligned}
V_t(H_{mt}) &= \max_{s_t, I_{mt}, n_t, H_{mt+1}} u(c_{mt})\lambda(H_{mt}) + \beta u(c_{ot+1})\lambda(H_{mt})\lambda(H_{ot+1}) \\
&\quad + \beta\alpha(n_t) n_t V_{t+1}(H_{mt+1}) && (1) \\
c_{mt} &= y_t - s_t - n_t H_{mt+1} - I_{mt} && (2) \\
c_{ot+1} &= (1 + r_{t+1}) s_t && (3) \\
H_{ot+1} &= H_{mt}(1 - \delta) + I_{mt} && (4)
\end{aligned}
$$

Where $V_t(H_{mt})$ is the value function of a middle age individual at time t. Notice that this value function has H_{mt} as state variable. This variable is exogenous to the individuals' problem (it is decided by her parents) but it may determine her decisions, as larger initial health stock may require less health investment to obtain a given level of life expectancy (survival probability function). To characterize the problem the following two assumption will be stated:

$$
\begin{aligned}
\textit{Assumption 1} &: \quad \sigma = 1 \\
\textit{Assumption 2} &: \quad \frac{\beta\alpha}{1 - \varepsilon} < 1
\end{aligned}
$$

Assumptions 1 and 2 have different roles. Assumption 1 will allow us to write the current utility as a separable function. Also, notice that expected utility at middle age under assumption 1 will simplify to:

$$\begin{aligned}
\lim_{\sigma\to 1} u(c_{mt})\lambda(H_{mt}) &= \lim_{\sigma\to 1}\frac{c_{mt}^{1-\sigma}}{1-\sigma}AH_{mt}^{\theta} = \lim_{\sigma\to 1}\frac{A}{1-\sigma}c_{mt}^{1-\sigma}e^{\theta\ln(H_{mt})} \\
&= \lim_{\sigma\to 1}\frac{A}{1-\sigma}c_{mt}^{1-\sigma}e^{\widetilde{\theta}(1-\sigma)\ln(H_{mt})} = \ln(c_{mt}) + \widetilde{\theta}\ln(H_{mt}) \quad (5)
\end{aligned}$$

Condition (5) uses L'Hopital' s rule. The constant $\widetilde{\theta}$ is a value such that $\theta = \widetilde{\theta}(1-\sigma)$. A similar expression can be found for current utility at retirement age. Separability on the current utility function allows a useful simplification of the problem as first order conditions will depend only on the chosen variable, as it will be shown below.

Assumption 2 basically assures that the problem under analysis is bounded and hence a solution to the problem exists. The next lemma states this result.

Lemma 2.1. *Under assumption 2, there exists a unique continuous value function V satisfying (1) to (4). Further, the set of optimal policies* $\{s_t, I_{mt}, n_t, H_{mt+1}\}_{t=0}^{\infty}$ *is non empty.*

Proof: See mathematical appendix

The above lemma allows us to characterize the individuals's optimal decisions, as we know that this set is non empty and therefore the set of optimal solutions must exist. Further, since there is a unique value function V satisfying the problem, the a unique maximum is attained. Hence, assumption 2 assures that a solution to the problem exists and that a unique maximum is attained.

To characterize the problem notice that using condition (5) and assumption (1), current utility functions are separable between consumption and health stock. Thus, we obtain the following set of first order conditions that characterize the individuals' problem:

$$\beta(1+r_{t+1})\frac{1}{c_{ot+1}} = \frac{1}{c_{mt}} \tag{6}$$

$$\beta\alpha\frac{n_t^{1-\varepsilon}}{1-\varepsilon}\frac{\partial V_{t+1}}{\partial H_{mt+1}} = \frac{n_t}{c_{mt}} \tag{7}$$

$$\beta\alpha n_t^{-\varepsilon} V_{t+1} = \frac{H_{mt+1}}{c_{mt}} \tag{8}$$

$$\frac{\beta\widetilde{\theta}}{H_{ot+1}} = \frac{1}{c_{mt}} \tag{9}$$

Equations (6) to (9) have the traditional interpretation of marginal benefit being equal to marginal cost. On the equations, the left hand side is the marginal benefit while the right hand side is the marginal cost. Equation (6) is the optimality condition for savings. It states that the marginal cost of an additional unit of savings is the marginal utility of consumption at middle age while its marginal benefit is the discounted gain of marginal utility at retirement. Equation (7) presents the optimality condition for health investment on children. The marginal benefit of increasing marginally health investment on children is given by the change on the level of utility per children, $\frac{\partial V_{t+1}}{\partial H_{mt+1}}$, discounted by the relevant discount factor that includes the altruism component, $\beta \frac{\alpha}{1-\varepsilon} n_t^{-\varepsilon}$, and multiply by the number of children, n_t. The marginal cost is basically given by the number of children, as we spent the same amount on each child. This marginal cost is measured in terms of utility level at the middle age. Equation (8) presents the optimality condition for children. Basically the marginal benefit of an additional child is given by the utility level attained by children times the altruism parameter that indicates the value of children for parents. Its marginal cost is given by the investment made on each children, H_{mt}, measured in utility terms. Finally, equation (9) is the optimality condition for own health investment. The marginal benefit is given by the discounted change on the survival likelihood at the old age while the marginal cost is determined by the opportunity cost of consumption at middle age.

The conditions on equations (7) and (8) present a characteristic that will provide some interesting results later. In fact, the marginal cost of an additional unit of health capital provided to each child and the marginal cost of children are not constant. Further, the marginal cost of health capital (children) depends on the number of children (level of health capital). Those effects are quite important because they stretch out any initial changes on fertility or health investment. Notice that the main observation we want to address is the fact that as income increases, health expenditure increase proportionally more. This effect may be explained, at least at a part, by the characteristic of those relative prices. For instance, suppose that income rises on 1% and it produces an increase of 1% on H_{mt} and I_{mt}. In that case, total health expenditure increase on 1% and obviously, its associated elasticity with respect to income is 1. However notice that as H_{mt} rises, the marginal cost of per child also rises and therefore the number of children decreases. But as fertility rate decreases, the marginal cost of health stock on equation (7) decreases also, increasing further H_{mt} due to a substitution effect towards health. Thus, we stretch out the effect on H_{mt}. Thus we may observe health expenditure increasing proportionally more than income even when its elasticity with respect to income is one.

In summary, the model presents individuals choosing the number of children and the investment on health on each of them. Hence, they choose the size of their family and the expected extension of their life expectancy. They also affect their own life expectancy. Income may directly affect optimal decisions. A second effect is that relative prices of fertility and health capital of children are not constant and they may be distorted as income changes. This last effect produce further changes on optimal decisions due to substitution and thus, even when the income health elasticity may not be larger than one, we may observe a proportionally larger change on health expenditure than the observed change on income.

Next section will characterize the dynamic behavior of the economy.

3. Dynamics of the economy

3.1. The relationship between health expenditure and the demographic transition

3.1.1. The main variables

Last section indicated that changes in income may produce large effects on health expenditure due to the fact that relative prices of health and fertility may be distorted. In this section the dynamic behavior of the economy is characterized. It is shown that as income increases, total health expenditure raises proportionally more than income and fertility rates decrease. On the long run, there is an equilibrium where fertility becomes constant.

To characterize the dynamics notice that we may obtain a reduced form solution for the value function $\mathrm{V}_t(H_{mt})$. Using equations (7) and (8), we have:

$$\frac{\partial V_{t+1}(H_{mt+1})}{\partial H_{mt+1}} \frac{H_{mt+1}}{V_{t+1}(H_{mt+1})} = 1 - \varepsilon \Rightarrow V_{t+1}(H_{mt+1}) = K H_{mt+1}^{1-\varepsilon}, \nabla t \qquad (10)$$

Where K is a constant of integration with the same sign as 1-ε and thus the value function is an increasing but concave function of health capital stock. This reduced form will be useful in characterizing equations (7) and (8), namely the first order conditions of health capital stock on children and fertility rate respectively. In fact, using (8) and (10), we get:

$$\frac{(n_t H_{mt+1})^{\varepsilon}}{c_{mt}} = \beta\alpha K \tag{11}$$

This condition provides a first approach to understand the evolution of the health share of income. The right hand side of the equation is constant thus the left hand side must be also constant. Notice that $n_t H_{mt+1}$ is total health expenditure on young individuals and suppose that consumption good have an income elasticity equals to one. In that case, as income increases health expenditure on young individuals will increase proportionally more than income if $\varepsilon < 1$. The intuition for this condition is the following. An exogenous increase on y_t produces an income effect over total health expenditure on young individuals. Hence H_{mt+1} and n_t become larger due to the income effect. But from above, we know that larger values of H_{mt+1} are associated with higher marginal cost of children. This is a substitution effect towards lower fertility rate that provides larger incentives to accumulate H_{mt+1}. This substitution effect will raise the initial effect on total health expenditure on young individuals -$n_t H_{mt+1}$. Thus the larger is the substitution effect over fertility, the larger must be the effect over H_{mt+1}, for a given increase on consumption, such that equation (11) holds.

Notice that the substitution effect is directly related to the parameter ε. In fact, the altruism-discount factor has a constant elasticity of substitution form with elasticity of substitution of children over time equal to $\frac{1}{\varepsilon}$. As $\varepsilon < (>)1$, the elasticity of substitution is larger (smaller) than one, meaning that parents are willing (not willing) to substitute children over time and the substitution effect will be large (small). Hence, if $\varepsilon < 1$, the substitution effect offsets the income effect over fertility rate and we require a larger increase on H_{mt+1}, implying a larger increase on total health expenditure compared to consumption. The way to obtain this larger increase on H_{mt+1} is throughout the change in relative prices of health stock and as indicated in equation (7), since smaller fertility rate is associated with smaller marginal cost of health capital stock. The contrary holds when $\varepsilon > 1$.

Further when ε approaches 1, the income and the substitution effect over fertility rate balance each other and the relative price of health capital is no distorted. In that case, an increase on consumption is accompanied by a proportionally equal increase on health, meaning that both goods have a elasticity of income equal to one. This last result is not surprising in the following sense.

Suppose fertility is constant -which is similar to the fact that income and substitution effect of fertility rate offset each other- and thus the individuals choose only health investment (H_{mt+1} and I_{mt}) and consumption goods (c_{mt} and c_{ot+1}). As the utility function is logarithmic on each those arguments, the income elasticity must be one for each of them.

At this point, it is important to characterize the evolution of fertility rate. It will be shown that fertility rate may present some transitional dynamics on the short run until reaching some long run level. During this transition, relative prices of health will be distorted producing large accumulation of health and the health share of income will be increasing over time.

Notice that the envelope condition of the individual's problem is:

$$\frac{\partial V_t(H_{mt})}{\partial H_{mt}} = \frac{(1+\beta)\widetilde{\theta}}{H_{mt}} + \frac{\beta\widetilde{\theta}(1-\delta)}{H_{ot+1}} \tag{12}$$

This condition basically indicates that a larger initial health stock has a positive effect over life expectancy at the middle and old age. This last effect depends on the law of motion of health stock. Using (12) and (11), we get:

$$n_t^{\varepsilon} = \frac{\beta\alpha\widetilde{\theta}}{(1-\varepsilon)}[\frac{(1+\beta)c_{mt}}{H_{mt+1}} + \frac{(1-\delta)}{\widetilde{\theta}(1+g)}]$$

$$= \quad \frac{\beta\alpha\widetilde{\theta}}{(1-\varepsilon)}[\frac{1+\beta}{\beta\alpha K}n_t c_{mt}^{\frac{\varepsilon-1}{\varepsilon}} + \frac{(1-\delta)}{\widetilde{\theta}(1+g)}] \tag{13}$$

Equation (13) characterizes the evolution of fertility rate over time. Some interesting characteristics of this equation are the followings. First consider $\varepsilon = 1$. In that case, we can easily solve for fertility rate, and it does not have any dynamics, as it does not depend on c_{mt}. Consider now $\varepsilon < 1$ and consumption being a normal good. This case will present some dynamics over time characterized by the fact that fertility rate may initially increase but later, fertility rates must decrease until stabilizing at $n_{ss} = \frac{(1-\delta)\beta\alpha}{(1+g)(1-\varepsilon)}$. To obtain this result take some given level of $n_t > 0$ and perturb c_{mt}. The effect over fertility rate of the increase on c_{mt} is given by the following elasticity:

$$\frac{\partial n_t}{\partial c_{mt}}\frac{c_{mt}}{n_t} = \frac{\frac{\varepsilon-1}{\varepsilon}}{\frac{\widetilde{\theta}(1+\beta)}{\varepsilon(1-\varepsilon)K}\frac{c_{mt}^{\frac{1-\varepsilon}{\varepsilon}}}{n_t^{1-\varepsilon}} - 1} \tag{14}$$

When $\varepsilon < 1$ and $c_{mt} \to 0$, the elasticity converges to $\frac{1-\varepsilon}{\varepsilon} > 0$. However when $c_{mt} > n_t^{\frac{1}{\varepsilon}}\Omega$ where $\Omega = [\frac{\varepsilon(1-\varepsilon)K}{\widetilde{\theta}(1+\beta)}]^{\frac{\varepsilon}{1-\varepsilon}}$, the elasticity becomes negative and further as $c_{mt} \to \infty$, the elasticity converges to zero. If c_{mt} is a normal good, this results imply that for a given level of fertility, the elasticity on (14) may be positive only for small values of consumption. Later fertility rates must decrease and converge to some long run level, as output continues to raise. The intuition for this result is the following. The maximized utility level is an increasing function of fertility rate when $\varepsilon < 1$ and thus fertility rate is also a normal good. However the health investment on children also raises over time, producing an increase on the relative price of children. Over time this last substitution effect offsets the income effect over children, producing the demographic transition towards a lower fertility rate. Notice that H_{mt+1} increases unambiguously though. In fact as fertility rate decreases, the relative price of children health investment also decreases. Hence, the substitution and the income effect reinforce each other in the case of H_{mt+1}. It follows that as a country develops, we observe large increases on children health investment and on population life expectancy.

Notice that those last results assume that c_{mt} is a normal good. It is straightforward to show that this is the case. Since the capital markets is perfect, we may write the intertemporal budget constraint faced by parents as:

$$
\begin{aligned}
y_t &= c_{mt} + \frac{c_{ot+1}}{1+r_{t+1}} + n_t H_{mt+1} + I_{mt} \\
&= c_{mt}(1+\beta+\beta\widetilde{\theta}) + (\beta\alpha K c_{mt})^{\frac{1}{\varepsilon}} - H_{mt}(1-\delta) \\
&= c_{mt}(1+\beta+\beta\widetilde{\theta}) + (\beta\alpha K c_{mt})^{\frac{1}{\varepsilon}}(1 - \frac{1-\delta}{n_t(1+g)}) \qquad (15)
\end{aligned}
$$

Where we assume a stable growth path and we use I_{mt}=H_{ot+1}-H_{mt}(1-δ),

$n_t H_{mt+1} = (\beta\alpha K c_{mt})^{\frac{1}{\varepsilon}}$ from (11) and $H_{ot+1} = \beta\widetilde{\theta}c_{mt}$ from (9). Equations (13) and (15) determine two implicit functions for consumption at middle age and fertility rate: $c_{mt} = c_{mt}(\beta, \widetilde{\theta}, \alpha, \varepsilon, \delta, g, y_t); n_t = n_t(\beta, \widetilde{\theta}, \alpha, \varepsilon, \delta, g, y_t)$. The properties of those implicit functions can be established by comparative statics.

Lemma 3.1. *The implicit function for consumption at middle age and fertility rates defined by (13) and (15) have the following properties:*

$$c_{mt} = c_{mt}(\underset{-}{\beta}, \underset{-}{\alpha}, \underset{-}{\widetilde{\theta}}, \underset{?}{\varepsilon}, \underset{+}{\delta}, \underset{+}{g}, \underset{+}{y_t})$$

$$n_t = n_t(\underset{+}{\beta}, \underset{+}{\alpha}, \underset{+}{\widetilde{\theta}}, \underset{?}{\varepsilon}, \underset{-}{\delta}, \underset{-}{g}, \underset{?}{y_t})$$

Proof: See mathematical appendix

Basically the intuition for the results on the implicit demand function for consumption is the following. First, the larger is the discount factor (β, α), the more the individuals are willing to postpone current consumption and they may increase future oriented goods such as children. Also, a larger $\widetilde{\theta}$ is associated with a larger marginal increase on life expectancy for a given increase on health stock, thus we are willing to substitute away from consumption to accumulate more health stock. By complementarity on the utility function between health investment on children and fertility rate, we also obtain a positive effect over fertility rates. A decrease on the depreciation rate have a positive impact over consumption at middle age while a negative impact on fertility rates. Lower δ

provides more incentives to allocate resources on health stock, increasing health share of income. As above due to the complementarity on the utility function between children and children's health, we increase fertility also. The larger is the economy growth rate, the more resources will be available for future generations and therefore future generations may invest larger resources on their own health. In that case, less resources may be invested by parents on their children's health. Thus fertility rate decreases due to the usual complementarity on the utility function. Exogenous changes on ε have different effects though. In fact, consider a decrease on ε. In this case, the elasticity of substitution of children over time raises and parents are willing to have less children today. Basically due to the complementarity on the utility function, we would observe also a smaller health share of income today. In the future we should have more children and thus a larger health share of income. Finally, consumption goods are normal goods as we expected. This last conclusion, e.g. consumption being a normal good, shows that the above results for the evolution of fertility and health investment hold.

Further, we may also obtain the properties of health share. Notice that this economy has only two goods, namely consumption and health, the health share of income (SH_t) may be defined on a stable growth path as:

$$SH_t = 1 - \frac{c_{mt} + c_{ot}}{y_t} = 1 - \frac{c_{mt}(1 + \frac{\beta(1+r_{t+1})}{(1+g)})}{y_t} \tag{16}$$

Where the second equality uses the condition $\frac{c_{ot+1}}{c_{ot}} = 1+g$ on the stable growth path. From this condition, we may obtain the effects of exogenous changes on the parameter of the model over the health share of income. The following lemma indicates the main effects of exogenous changes on the parameter of the model over SH$_t$.

Lemma 3.2. *The health share of income have the following properties:*

$$SH_t = SH_t(\underset{+}{\alpha}, \underset{+}{\widetilde{\theta}}, \underset{?}{\varepsilon}, \underset{-}{\delta}, \underset{+}{y_t}, \underset{-}{r_{t+1}})$$

Proof: See mathematical appendix

The health share of income has in general the opposite results of consumption, due to the properties of equation (16). The effects of g and β are omitted from the lemma because they are ambiguous. In the case of a larger β, the individuals face less incentives to consume at middle age but larger incentives to consume at retirement age. Notice that as above ε has an ambiguous effect over the health share of income. The intuition is the same as above. Disturbing ε produces a change on the willingness to substitute children over time which affect consumption and thus the health share of income. In fact, a smaller ε provides larger willigness to substitute children over time and it would decrease the current number of children and increase current consumption as shown above. Hence, the health share of income would be negatively affected. In the future, the effect must be the opposite and therefore we should observe a larger health share of income later. Among others results reported above, further consideration will be now given to the result showing that the health share of income is increasing on output. As we have only two goods, we might analyze consumption's income elasticity instead. If the income elasticity of consumption is smaller (larger) than one, the income elasticity of health must be larger (smaller) than one and the health share of income must be increasing (decreasing) on output. Hence let' s consider the income elasticity of consumption. This elasticity might be written as[1]:

$$\frac{\partial c_{mt}}{\partial y_t}\frac{y_t}{c_{mt}} = 1 + \left[\frac{1}{\frac{\varepsilon(1+\beta+\beta\widetilde{\theta})}{(\beta\alpha K)^{\frac{1}{\varepsilon}}}c_{mt}^{\frac{\varepsilon-1}{\varepsilon}}+1}\right]\left[\varepsilon(1-\frac{1-\delta}{n_t(1+g)})-1\right] \tag{17}$$

This elasticity provides two interesting insights. First, notice that the elasticity is smaller than one whenever $\varepsilon \leq 1$. Second, this elasticity converges on the long run[2] to $\varepsilon(1-\frac{(1-\varepsilon)}{\beta\alpha})$. Further, this long run elasticity converges to one

[1] In fact, we have:

$\frac{\partial c_{mt}}{\partial y_t}\frac{y_t}{c_{mt}} = \frac{c_{mt}(1+\beta+\beta\widetilde{\theta})+(\beta\alpha K c_{mt})^{\frac{1}{\varepsilon}}-H_{mt}(1-\delta)}{c_{mt}(1+\beta+\beta\widetilde{\theta})+\frac{1}{\varepsilon}(\beta\alpha K c_{mt})^{\frac{1}{\varepsilon}}}$

Using $\frac{H_{mt+1}}{H_{mt}} = (1+g)$ on a stable growth path and equation (11), we get the result.

[2] In that case, in a growing economy we have that $\underset{t\to\infty}{lim}$ $c_{mt} \to \infty$ and $\underset{t\to\infty}{lim}$ $n_t \to \frac{(1-\delta)\beta\alpha}{(1+g)(1-\varepsilon)}$. Thus:

$\underset{t\to\infty}{lim}\left[\frac{1}{\frac{\varepsilon(1+\beta+\beta\widetilde{\theta})}{(\beta\alpha K)^{\frac{1}{\varepsilon}}}c_{mt}^{\frac{\varepsilon-1}{\varepsilon}}+1}\right] \to 1,\ \underset{t\to\infty}{lim}\left[\varepsilon(1-\frac{1-\delta}{n_t(1+g)})-1\right] \to \varepsilon(1-\frac{1-\varepsilon}{\beta\alpha}).$

only as $\varepsilon \to 1$. Intuitively when $0<\varepsilon < 1$, the value function is concave on H_{mt+1} and therefore we always allocate resources on children's health capital, deviating resources from consumption. When $\varepsilon = 1$, the value of future health capital does not affect utility,as it can be shown in (10). Hence, there is no point in accumulating health capital and thus we do not deviate resources. This long run result, meaning the elasticity being equal to one when $\varepsilon = 1$, also holds in the short run. In that case, we simply do not have dynamics and we reach immediately the long run equilibrium. When $\varepsilon < 1$, fertility rate plays a fundamental role on the elasticity of consumption. The larger is fertility rate, ceteris paribus, the more likely the income elasticity of consumption approaches to one. This result follows from our earlier analysis. In fact, larger fertility rate is associated with a larger relative price of health investment on children and thus individuals face less incentive to invest on their children's health and they allocate more resources to their own consumption.

3.1.2. Discussion

The model developed above assume a log function on both consumption and health stock. This utility form is useful as we require the income elasticity being equal to one on both goods. However, we observe that the health share of income rise as output increases. This effect depends on two basic properties of the model.

First, there is a complementarity on the utility function between children and health investment on them and second, there is a trade-off between quantity and quality of children on the budget constraint.

The results show that when $0<\varepsilon < 1$ and output rises over time, the health share of income may be increasing due to changes on relative prices of health and fertility. The relative price of health depends on fertility rate and thus, as long as there is a demographic transition with decreasing fertility rates, there may be larger incentive to accumulate health. The demographic transition is endogenous also. In fact, as the economy develops the relative price of children increases and fertility rate decreases. Hence, we observe an economy extending the life expectancy of their inhabitants and switching its age pyramid through time. The intuition for this results is that a family may face a trade-off between the number of their members and the life expectancy of each of them, due to limited resources.

The case $\varepsilon = 1$ presents no transitional dynamics though. Basically, the utility function presents no complementarity between children and health investment. Thus initial changes on fertility are not stretch out, as health increases. Relative prices do not change and we observe an equilibrium that is reached instantaneously.

Some other interesting properties of the health share of income is that it is an increasing function of $\widetilde{\theta}$ and a decreasing function of δ. Consider the case of a country that improves its health system such that for the same level of health stock, life expectancy is larger. This may be the case of countries eradicating infectious diseases. That case is similar to an increase $\widetilde{\theta}$ on the model and therefore we would predict that the amount of resources spent on health, as a fraction of income, increases too. Thus some exogenously improvement on health technology may produce larger improvement on life expectancy. The depreciation rate provides also other source of variation on health share. Countries with larger depreciation rate on health stock may be countries with a larger fraction of old individuals. Those countries should also have a larger health share of income, as shown above, as they need to spent more resources on health to obtain some given level of life expectancy.

3.2. Simulation

In this subsection, we specify the value of the parameters of the model and we specify a computational algorithm that allows us to obtain the evolution of the economy over time. We will specify a baseline case and later we report results in three alternative scenarios: (1) lower depreciation rate, (2) larger $\widetilde{\theta}$ and (3) lower ε.

The model has three periods of time: childhood (young individuals), adulthood (middle age individuals) and retirement age. We assume that adulthood and retirement age may last at most 35 years. Suppose that childhood lasts 20 years. In that case, the maximum life span is 90 years old. However, some individuals may die before the 90^{th} year, as indicated by the uncertainty that embodied the model. Those time spans allow us to determine some parameters of interest in the model. The baseline scenario will be the following. First, the time discount parameter β, will be set equal to 0.98 per year. Similar values can be found on a large set of economic studies such as Aiyagari (1994) and Imrohoroglu (1989), among others. The depreciation rate and per capita income growth rate will be set equal to one per cent per year while the interest rate will be set equal to 5% per year. Those values are converted onto a 35-years horizon period. Finally, $\widetilde{\theta}$,ε and α are set equal to 0.5, 0.75 and 0.99 respectively. Three alternative scenarios are also considered. The first alternative scenario will set depreciation rate equals to zero, the second alternative scenario will set $\widetilde{\theta}$ equals to 0.6 while the third alternative scenario will set ε equals to 0.73.

The simulation will be implemented by the following algorithm. We first, fix some level of income, y_t. We guess some initial value for fertility rate, n_t, and we compute the initial level of consumption at middle age c_{mt} using equation (15). Given c_{mt} we may compute fertility rate using equation (13). If the distance between value of fertility rate compute in this last step and the initial guess for fertility rate is smaller than ξ,-ξ was set arbitrarily equal to0.001- we use c_{mt} and n_t as our final estimates of consumption at middle age and fertility rate at time t. If the distance is larger, we use equation (13) to obtain a new value for fertility rate at time t, $\overline{n_t}$. Given $\overline{n_t}$, we iterate in the same way as above until we reach some equilibrium values $\widehat{n_t}$ and $\widehat{c_{mt}}$. We compute the health share of income at time t using equation (16). Finally, to implement the algorithm we set an initial value for y_t at t=0 and we use the per capita income growth rate to calculate the subsequent values of y_t.

The results of the simulation show that the health share of income presents a sustained increase over time -see figure 1. During the initial periods of time, the increase of health share is larger. Further as we expected, the health share of income is larger compared to the baseline case, when $\widetilde{\theta}$=0.6 and $\delta = 0$. In all the cases, the largest movement on health share of income are associated with larger changes on fertility rates, as it can be seen on figure 2. Hence the change on the relative price of health produces large effects on health share of income. The alternative scenario ε=0.73 shows that the health share of income becomes smaller than the one obtained on the baseline case on the initial periods of time considered but it becomes larger later. The main cause of this behavior can be seen on the evolution of fertility rate in this case. Fertility rate is much flatter and at a smaller level than the one of the baseline case. Hence by complementarity of the utility function, we observe a small level of health share of income initially. Smaller fertility rates are also associated with smaller relative price of health which produce a larger health share of income later, when this substitution effect becomes important enough.

[Insert figures 1 and 2]

4. Summary

The theoretical model built in this paper shows that endogenous demographic transition may play a fundamental role on explaining why the health share of income raises over time. Basically, we argue that families must choose the number of their members and their life expectancy and limited resources produce a trade-off between those two chosen variables. The trade-off appears on the relative prices of those variables. In fact, the relative price of health will depends on fertility while the relative price of fertility will depend on health. Thus, relative prices will not be constant and will be affected by income. In that scenario, changes on income will have a direct effect over health throughout the traditional income effect but it may produce an additional effect throughout the distortion on relative prices. Those two effects reinforce each other and they produce large effects over health expenditure. This result depends crucially on the substitution effect that reinforce the income effect. This substitution effect appears as a country develops and the demographic transition occurs.

5. References

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6. Mathematical Appendix

6.1. Proof of lemma 2.1

Notice that $\mathrm{H}_{mt} \in \mathrm{R}_{+}, \nabla t$. Hence the set of possible values for the state variable and its correspondence $\Gamma(H_{mt})$ is a non empty, continuous, convex and compact set. Further assumption 2 assures that the effective discount factor faced by parents is smaller than one. Hence assumptions 4.3 and 4.4 on Stockey, Lucas and Prescott (1989) are satisfied. Also notice that $\mathrm{F}(\mathrm{H}_{mt},\circ) \equiv u(c_{mt})\lambda(H_{mt}) + \beta u(c_{ot+1})\lambda(H_{mt})\lambda(H_{ot+1})$ under assumption (1) is a simple sum of continuous functions and therefore it is a continuous function. Guess that V_{t+1} is continuous. Hence, it follows that the function to be maximized on (1) is continuous over the compact set $\Gamma(H_{mt})$.

By the theorem of the Maximum, a supremum V_t must be attained and further V_t must be continuous. Applying Blackwell's conditions for a contraction, that are satisfied since assumption 2 assures that the discount factor is smaller than one, it follows that there is a unique value function V satisfying equation (1). Since $\mathrm{V}=\mathrm{V}_t, \nabla t$ and V_t is continuous, the guess for continuity on V_{t+1} is corroborated and V must be continuous.

Finally, by the theorem of Maximum, the optimal policy function is non empty.

Q.E.D.

6.2. Proof of lemma 3.1 and 3.2

Using the implicit functions obtain from equations (13) and (15), we may obtain the following comparative statics:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \partial c_m/\partial\alpha \\ \partial n/\partial\alpha \end{bmatrix} = \begin{bmatrix} b_{1\alpha} \\ b_{2\alpha} \end{bmatrix} \Leftrightarrow \Omega \begin{bmatrix} \partial c_m/\partial\alpha \\ \partial n/\partial\alpha \end{bmatrix} = B$$

where $a_{11}=(1+\beta+\beta\widetilde{\theta})+\frac{(\beta\alpha K)^{\frac{1}{\varepsilon}}}{\varepsilon}c_{mt}^{\frac{1-\varepsilon}{\varepsilon}}(1-\frac{1-\delta}{n_t(1+g)}), a_{12}=\frac{(1-\delta)(\beta\alpha K c_{mt})^{\frac{1}{\varepsilon}}}{(1+g)n_t^2}$,
$a_{21}=\frac{\varepsilon-1}{\varepsilon}c_{mt}^{\frac{\varepsilon-1}{\varepsilon}-1}n_t(\frac{\theta(1+\beta)}{(1-\varepsilon)K})$, $a_{22}=(\frac{\theta(1+\beta)}{(1-\varepsilon)K})c_{mt}^{\frac{\varepsilon-1}{\varepsilon}}-\varepsilon n_t^{\varepsilon-1}$ and
$b_{1\alpha}=-\frac{(\beta K c_{mt})^{\frac{1}{\varepsilon}}\alpha^{\frac{1}{\varepsilon}-1}}{\varepsilon}(1-\frac{1-\delta}{n_t(1+g)}),\ b_{2\alpha}=-\frac{\beta(1-\delta)}{(1-\varepsilon)}$.

Thus we have $a_{11},a_{12}>0$; $a_{21},a_{22},b_{1\alpha},b_{2\alpha}<0$ and $\det|\Omega|<0$. Hence,

$$\begin{aligned} \partial c_m/\partial\alpha &= \frac{1}{det|\Omega|}[b_{1\alpha}a_{22}-b_{2\alpha}a_{12}]<0 \\ \partial n/\partial\alpha &= \frac{1}{det|\Omega|}[b_{2\alpha}a_{11}-b_{1\alpha}a_{21}]>0 \end{aligned}$$

Similar comparative statics systems yield:

$$\begin{aligned}
\partial c_m/\partial\beta &= \frac{1}{det|\Omega|}[b_{1\beta}a_{22}-b_{2\beta}a_{12}]<0,\ \partial n/\partial\beta=\frac{1}{det|\Omega|}[b_{2\beta}a_{11}-b_{1\beta}a_{21}]>0\\
\partial c_m/\partial\widetilde{\theta} &= \frac{1}{det|\Omega|}[b_{1\widetilde{\theta}}a_{22}-b_{2\widetilde{\theta}}a_{12}]<0,\ \partial n/\partial\widetilde{\theta}=\frac{1}{det|\Omega|}[b_{2\widetilde{\theta}}a_{11}-b_{1\widetilde{\theta}}a_{21}]>0\\
\partial c_m/\partial\delta &= \frac{1}{det|\Omega|}[b_{1\delta}a_{22}-b_{2\delta}a_{12}]>0,\ \partial n/\partial\delta=\frac{1}{det|\Omega|}[b_{2\delta}a_{11}-b_{1\delta}a_{21}]<0\\
\partial c_m/\partial(1+g) &= \frac{1}{det|\Omega|}[b_{1g}a_{22}-b_{2g}a_{12}]>0,\ \partial n/\partial(1+g)=\frac{1}{det|\Omega|}[b_{2g}a_{11}-b_{1g}a_{21}]<0\\
\partial c_m/\partial\varepsilon &= \frac{1}{det|\Omega|}[b_{1\varepsilon}a_{22}-b_{2\varepsilon}a_{12}]>(<)0 \qquad when\ c_{mt}\ small\ (l\arg e)\\
\partial n/\partial\varepsilon &= \frac{1}{det|\Omega|}[b_{2\varepsilon}a_{11}-b_{1\varepsilon}a_{21}]>(<)0 \qquad when\ c_{mt}\ small\ (l\arg e)
\end{aligned}$$

Since simple calculations show that $b_{1\beta}, b_{2\beta} < 0$; $b_{1\widetilde{\theta}}, b_{2\widetilde{\theta}} < 0$; $b_{1g}, b_{2g} > 0$; $b_{1\delta}a_{22} - b_{2\delta}a_{12} < 0$; $b_{2\delta}a_{11} - b_{1\delta}a_{21} > 0$. Finally,

$$\frac{\partial c_{mt}}{\partial y_t} = \frac{a_{22}}{det|\Omega|} > 0$$

Using this last result and condition (14), we obtain $\frac{\partial n_t}{\partial y_t} > 0$ when c$_{mt}$ is small and $\frac{\partial n_t}{\partial y_t} < 0$ when c$_{mt}$ is large.

The results of lemma 3.2 follows from simply differentiation of condition (16) and the results just proved on consumption.

Q.E.D.