# MESHED LOW VOLTAGE DISTRIBUTION SYSTEM PLANNING WITH SPECIAL RELIABILITY STANDARDS 

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Thesis submitted to the Office of Research and Graduate Studies in partial fulfillment of the requirements for the degree of Master of Science in Engineering

Advisor:
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Santiago de Chile, May 2014
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To my family

## ACKNOWLEDGEMENTS

I would like to dedicate this work to my parents. It is with their help that I was able to go to college, and they provided me with support as I conducted this investigation. I also want to thank my brothers for their recommendations and corrections.

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#### Abstract

A method for planning underground low voltage meshed distribution networks is presented, determining required transformers, the medium voltage radial network and the low voltage meshed network. The optimization goal is to find the lowest possible total cost of conductors, transformers, conduits, and energy and power losses. Technical constraints, i.e. conductor capacity limits, voltage drops, and maximum transformer aging, must be fulfilled. For reliability purposes the network must cope with the fault of a complete medium voltage feeder, and the low voltage network must be able to confront the temporary failure of a small group of transformers. The planning model is formulated in a heuristic structure, which uses genetic algorithms in several of its steps. The results obtained from planning a real electric distribution system with 2241 loads are presented.


Keywords: power distribution planning, network planning, genetic algorithm, power flow.

## RESUMEN

Un método para la planificación de sistemas de distribución de baja tensión subterráneos es presentado, determinando los transformadores, la red de media tensión radial y la red de baja tensión enmallada requeridos. La meta de la optimización es encontrar el costo total más bajo posible, lo que incluye conductores, transformadores, canalizaciones y pérdidas de energía y potencia. Restricciones técnicas como los límites de capacidad de conductores, las caídas de tensión y el envejecimiento máximo de los transformadores deben ser cumplidas. Por propósitos de confiabilidad de suministro, la red en su conjunto debe ser capaz de sobrellevar la falla de un alimentador completo. A su vez, la red de baja tensión debe poder enfrentar la falla temporal de un pequeño grupo de transformadores. El modelo de planificación es formulado mediante pasos heurísticos, que usan algoritmos genéticos en varias etapas. Los resultados obtenidos de la planificación de una red de distribución eléctrica real con 2241 consumos son presentados.

Palabras Claves: Planificación de sistemas de distribución de potencia, planificación de redes eléctricas, algoritmos genéticos, flujos de potencia.

## 1. INTRODUCTION

Electricity distribution is the last link of the energy delivery chain which includes generation and transmission systems. In Chile, distribution companies are private enterprises that work as natural monopolies on specific geographic areas. As no market is available, the regulator sets the price that the power distribution monopoly can charge customers for its service. Several South American regulations establish a yardstick competition approach to set this price. Using an economic model, the regulator sets the price that an efficient firm would charge in a competitive environment (Rudnick \& Donoso, 2000). Distribution system planning (DSP) is a key component of this process. In downtown Santiago, Chile, a small meshed secondary network coexist within multiple radial systems and the challenge is how to design the optimal meshed network within DSP.

The objective of DSP is to find the optimal infrastructure needed to meet the power demand of a group of loads given a set of constraints. It specifies the optimal allocation and capacity of High Voltage - Medium Voltage (HV-MV) substations and Medium Voltage - Low Voltage (MV-LV) distribution transformers, the layout and capacity of feeders and secondary circuits, and how loads are allocated. DSP typical constraints include Kirchhoff's current and voltage laws; voltage drops; conductor, substation and transformers capacity limits; and supply reliability. Supply reliability can be assessed by indices or costs. Common indices are system average interruption duration index (SAIDI), system average interruption frequency index (SAIFI), customer average interruption duration index (CAIDI), customer average interruption frequency index (CAIFI), average service availability index (ASAI), customer interruption duration (CID), customer interruption frequency (CIF), expected energy not supply (EENS). To assess the customer interruption cost (CIC) (i.e. outage cost), the value of lost load (VOLL) has to be known for every customer. This value depends on the economic costs that outages produce to customers, and is commonly calculated with surveys. CIC can be a function of outage duration, outage frequency, and energy and power not supplied (Allan, Dialynas, \& Homer, 1979; Billinton
\& Billinton, 1989; Lotero \& Contreras, 2011; Tang, 1996; Skok, Krajcar, \& Skrlec, 2005). DSP can include radial or meshed topologies. In many studies planning under emergency conditions is also considered (Khator \& Leung, 1997). Depending on the size of the problem, it is solved using mathematical optimization or meta-heuristic techniques.

Mathematical optimization determines the best solutions to mathematically defined problems. It searches the vector that maximizes or minimizes an objective function. It selects that vector from a space of possible solutions that is given by the constraints of the problem. Mathematical optimization guarantees the discovery of the best solution, if it exist. However the required time to solve complex problems with high numbers of variables and restrictions may be too high. Examples of mathematical optimization algorithms are simplex method, branch \& bound, dynamic programming, cutting-plane method and barrier-Newton method.

Meta-heuristic techniques are solution methods that mix local improvement procedures with higher level strategies to escape from local optima, performing a robust search of the solution space. Meta-heuristic do not guarantee that the globally optimal solution can be found. However they give good solutions in a short processing time. It is common to use meta-heuristic techniques to solve complex DSP problems, with thousands of lines and loads.

Radial network planning has been a topic of much more interest than meshed network planning, because radial configurations are widely used due to their cost efficiency. Radial feeder design using a direct search technique and considering reliability indices is addressed in (Samui, Samantaray, \& Panda, 2012). In (Navarro \& Rudnick, 2009) distribution transformers are allocated and secondary circuits are designed in order to provide energy to a large number of loads using tabu search. In (Cossi, Romero, \& Mantovani, 2009) the planning problem of secondary distribution circuit and support structures is solved via tabu search. Load balancing among phases is also considered. In (Moreira,

Miguez, Vilacha, \& Otero, 2011) a branch-exchange algorithm is used for layout optimization and a dynamic programming for the conductor optimization of secondary radial circuits supplying a large number of loads. High-, medium- and low- voltage networks are planned in (Domingo, Roman, Sanchez-Miralles, Gonzalez, \& Martinez, 2011), using a branch-exchange technique, and considering distributed generation. A genetic algorithm (GA) for feeder routing and optimal substation location and sizing in a large scale distribution system is used in (Najafi, Hosseinian, Abedi, Vahidnia, \& Abachezadeh, 2009). In (Jimenez-Estevez, Vargas, \& Marianov, 2010) the optimal layout and capacity of radial feeders is calculated via GA based on generation of spanning trees. Simulated annealing technique is used in (Nahman \& Peric, 2008) for planning a radial distribution network.

Another strand of the literature studies meshed networks. Meshed networks have been found to be more reliable than other topologies, especially if the distribution network features distributed generation. In (Celli, Ghiani, Loddo, \& Pilo, 2005) a hill-climbing heuristic is used for meshed MV network planning. With two interrelated genetic algorithms open-loop MV networks are planned in (Skok et al., 2005). A hybrid structure that mixes radial and partially meshed feeders is solved via an heuristic algorithm based on the Traveling Salesman Problem in (Alvarez-Herault et al., 2011).

No planning methodology for a meshed low voltage network has been presented in the literature so far. The present paper closes this gap by developing such a methodology and applying it to the network located in downtown Santiago. This network supplies power to important public offices and private firms. It is a system with high reliability standards that was designed with meshed underground secondary mains and radial feeders.

The characteristics of a similar distribution system are described as in (Corporation, 1942). A grid of secondary mains, located under the streets or sidewalks, serve LV loads. Secondary mains are carried in duct systems, i.e. conduits. Under normal operation conditions, power comes from both ends of the secondary main. When a network transformer at one end of the conductor is out, the power can be supplied from the other end.

Two or more radial MV feeders are used. When one of them is out of service, the power can be supplied by the remaining feeder or feeders, not affecting the system. When all feeders are supplied by one substation which guarantees the availability of power at its LV bus, it is reasonable to accept that the worst emergency condition is the loss of one of the feeders. Under normal conditions, all feeders carry a similar share of the total load.

Network transformers are connected to the grid of secondary mains through network protectors, an electrically-operated air circuit breaker. Transformers are installed using an interlaced primary-feeder arrangement to diminish the maximum load they are exposed to when one feeder is out of service. This arrangement establishes that each transformer connected to one feeder is surrounded by transformers connected to other feeders. The higher the transformer impedance, the higher the load division between transformers is, but the higher the voltage drops too.

Most faults in secondary mains are either arcs or have less thermal capacity than the conductor, therefore are cleared fast without interrupting power supply. Few faults have high thermal capacity and must be cleared by fusing the conductor between the fault and adjacent junction points in the grid. Such faults result in service interruption for the loads connected to that section of the grid, however they are infrequent.

Feeder faults are cleared by opening the breaker between its supply end and the substation bus and opening all network protectors from transformers and loads connected to that feeder. A transformer fault is isolated the same way as a feeder fault.

The objective of this paper is to plan and design an optimized version of the meshed LV network, coupled to the radial MV network. The reliability standard of this optimal network is defined as the ability to cope with complete feeder failures, hereinafter contingencies. According to (Khator \& Leung, 1997) this research is categorized as planning for emergency. The design includes placing and sizing of primary feeders, distribution transformers and secondary circuits on a given sidewalk layout. Maximum current limits and voltage drops are recognized in cases of both normal operation and contingencies.

This problem is NP-hard, due to its combinatorial characteristics (Ramirez-Rosado \& Bernal-Agustin, 1998). The proposed algorithm uses a GA adapted for spanning trees reproduction to determine the layout of medium voltage conductors, and GA with integer codification for the selection of the network's conductors. A novel heuristic method is proposed for the positioning of transformers, while their sizing is calculated via GA.

This paper is structured as follows. A quick overview of the proposed solving method, and objective functions and constraints of the problem are presented in Section 2. The proposed method is described in detail in Sections 3 and 4. The results are presented in Section 5 and finally Section 6 concludes and discusses future work.

## 2. OPTIMIZATION FRAME

The steps of the proposed heuristic algorithm are shown in Fig. 2.1. First all loads are split into 5 different subzones. Then the following steps are taken for each subzone independently: The candidate transformers point, i.e. positions where transformers could possibly be installed, are defined. The LV network is designed next with a closest facility analysis and a power flow study is performed with this network and candidate transformers to know what transformers supply what amount of power. Then the algorithm iterates over a loop that designs the MV network and chooses the best locations and capacities of transformers. After that the LV trunk network, i.e. the conductors that handle the highest power flows, is defined. Finally conductors are selected for the MV and LV networks.

The problem is highly dimensional and therefore it is not computationally feasible to find the global solution. Instead, the problem is divided into parts, which are sequentially solved. The optimal solution of a given step enters all following optimization problems as a fixed variable. The separation of the problem into lower dimensional sub problems and the use of GAs ensure that a good solution rather than the optimal one is found.

All the steps of the algorithm feature similar optimization problems. In this section, a detailed overview of the objective functions, restrictions and solving methodologies is provided.

### 2.1. Objective Functions

The goal of greenfield planning is to create a minimum cost network given set of restrictions. The costs considered in this paper are: investments costs of conductors, conduits and transformers and costs related to energy and power losses. These cost functions are used in all genetic algorithms and in the conductor selecting model for the radial MV network. For information about investments costs and electrical properties see Appendix A.


Figure 2.1. Flowchart of the proposed algorithm. Dark rectangles show that a GA was used in that step.

### 2.1.1. Conductors Costs

Installing a conductor involves an investment cost $I N V$ that depreciates $D E P R$ each year, leaving a residual value $R V$ at the last year of the evaluation period $N_{e}$. During depreciation a tax rate $\tau$ is considered. The present value of a fixed annual expense over a
period on $N_{e}$ years at a discount rate of $r$ is:

$$
\begin{equation*}
P V_{1}=\frac{1-\left(\frac{1}{1+r}\right)^{N_{e}}}{r} \tag{2.1}
\end{equation*}
$$

Leaving a present value of investment costs of conductors $I C C$ as:

$$
\begin{equation*}
I C C=I N V-(D E P R \cdot \tau) P V_{1}-\frac{R V}{(1+r)^{N_{e}}} \tag{2.2}
\end{equation*}
$$

A current $I$ flowing through a line of resistance $R$ generates an operational cost $O P C$ due to energy $E L C$ and power $P L C$ losses:

$$
\begin{align*}
& E L C=C_{E}\left(I^{2} \frac{R}{1000} f_{c p}\right) 8760  \tag{2.3}\\
& P L C=C_{P}\left(I^{2} \frac{R}{1000} f_{c o n}^{2}\right) 12 \tag{2.4}
\end{align*}
$$

Where:

- $f_{c p}$ : Loss factor, ratio of average power loss over power loss at maximum demand.
- $C_{E}$ : Cost of energy losses CLP/kWh.
- $C_{P}$ : Cost of power losses CLP/kW.
- $f_{\text {con }}$ : Coincident factor, the ratio of the simultaneous maximum demand of a group of loads to the sum of their maximum individual demands.

The loss factor can be calculated as a function of the load factor $\left(f_{c}\right.$, ratio of average load over maximum load in a given time period) as (Moreira et al., 2011):

$$
\begin{equation*}
f_{c p}=0.3 f_{c}+0.7 f_{c}^{2} \tag{2.5}
\end{equation*}
$$

The operational cost $O P C$ increases as the demand for electricity grows over time. According to (Mandal \& Pahwa, 2002), a rate of peak load growth $g_{d}$ implies a rate of losses growth $g_{l}$ according to the expression $g_{l}=g_{d}^{2}+2 g_{d}$ because losses are proportional
to the square of the peak load. The present value of a fixed annual expense that grows with a rate $g_{l}$ is:

$$
\begin{equation*}
\sum_{i=1}^{N_{e}+1} \frac{\left(1+g_{l}\right)^{i-1}}{(1+r)^{i}} \tag{2.6}
\end{equation*}
$$

However conductors are sized to cope with the current $I$ of the end of the planning period $N_{p}$. As a result demand is supplied without overloading any conductor or transformer and with adequate costs. If the first or the last year of the evaluation period $N_{e}$ are used instead, the network would be too undersized or too oversized respectively. Therefore year $N_{p}$ gives a balanced result, where $1 \leq N_{p} \leq N_{e}$.

Therefore the present value of operational costs is:

$$
\begin{equation*}
O P C=(E L C+P L C)(1-\tau) P V_{2} \tag{2.7}
\end{equation*}
$$

Where:

$$
\begin{equation*}
P V_{2}=\sum_{i=1}^{N_{e}+1} \frac{\left(1+g_{l}\right)^{i-N_{p}}}{(1+r)^{i}} \tag{2.8}
\end{equation*}
$$

The current $I$ flowing through each conductor and transformer for year $N_{p}$ is obtained by solving a power flow study with the demand adjusted by the rate of peak load growth as $\left(1+g_{d}\right)^{N_{p}}$.

### 2.1.2. Transformers Costs

The installation and operation of transformers generate analogous investment costs $I C T$ (2.2) and operational costs $O P T$ (2.7) as in the case of a conductor. However energy losses in the iron core, which are constant during the period of study, are also considered. The present value $T L C$ of a power loss $P_{m} \mathrm{~kW}$ is:

$$
\begin{equation*}
T L C=C_{E}\left(P_{m} \cdot 8760\right) P V_{1} \cdot(1-\tau) \tag{2.9}
\end{equation*}
$$

### 2.1.3. Conduits Costs

All lines in the studied planning zone are buried underground. Conduits are the physical structures that contain one or more subterranean cables. It is assumed that conduits do not require maintenance and hence operational costs are zero. Installation costs per km ICCond depend on the number of cables that the conduit contains. Given an estimate of these values, investment costs can be computed as in (2.2). Medium voltage and low voltage cables are installed in independent conduits.

The more cables are installed in the same segment, the cheaper the conduit is. This cost signal leads the MV network design process described in Section 4.4 to install cables in the same edge when possible. This affects the layout of the MV network, in the sense that feeders will have similar shapes. The LV network design process does not change the layout of the network because of this signal. It only works as an additional investment cost for low impedance conductors, which are composed of two or more cables.

### 2.2. Constraints

Two types of constraints are considered. First, standard constraints: Voltage drop, maximum permissible carrying current of conductors and transformers aging limits. Second, special constraints given by the specific characteristics of the planning zone studied in this paper: A meshed low voltage network has to be designed; the network has to supply energy to all customers when dealing with a contingency, i.e. one complete feeder is out of service; and all feeders of one subzone must emerge from one substation.

It is important to note that power flow equations are solved at each stage of the algorithm. This means that power demands of all loads are automatically met.

### 2.2.1. Standard Constraints

Minimum and maximum values for voltage levels, $V^{\text {min }}$ and $V^{\text {max }}$, are specified. Let $N_{v}$ be the number of vertices. The voltage on every vertex $i, V_{i}$, has to satisfy:

$$
\begin{equation*}
V^{\min } \leq\left|V_{i}\right| \leq V^{\max }, \quad i=1, \ldots, N_{v} \tag{2.10}
\end{equation*}
$$

Conductors installed on the branch $l$ have a maximum permissible carrying current $I_{l}^{\max }$. Let $N_{b}$ be the number of branches. The current $I_{l}$ flowing through every branch $l$ has to satisfy:

$$
\begin{equation*}
I_{l} \leq I_{l}^{\max }, \quad l=1, \ldots, N_{b} \tag{2.11}
\end{equation*}
$$

Contingencies expose distribution transformers to loads that could excess their nameplate ratings. Overloads raise winding temperature causing faster insulation aging, and ultimately reducing the transformers lifespan. These effects can be avoided if transformers are installed in appropriate locations within the distribution network and the right nameplate ratings are chosen. Equations for $65^{\circ} \mathrm{C}$ rise mineral-oil-immersed distribution transformers of ("IEEE Guide for Loading Mineral-Oil-Immersed Transformers", 2012) are considered in this paper.

The aging acceleration factor $F_{A A}$ shows the relative aging of a transformer when exposed to a certain winding hottest-spot temperature $\Theta_{H}$ in ${ }^{\circ} \mathrm{C} . F_{A A}>1$ means that the transformer is aging faster than expected. It is calculated with the formula:

$$
\begin{equation*}
F_{A A}=\exp ^{\left[\frac{B}{383}-\frac{B}{\Theta_{H}+273}\right]} \tag{2.12}
\end{equation*}
$$

Where $B$ is an empirical constant equal to 15000 .
As $F_{A A}$ gives information about instants, the equivalent aging factor $F_{E Q A}$ is needed to integrate that data for a total time period. Considering that the whole planning period is divided into spans $\Delta t_{n}$, in which the transformer's aging factor is $F_{A A_{n}}$, the $F_{E Q A}$ is:

$$
\begin{equation*}
F_{E Q A}=\frac{\sum_{n=1}^{N} F_{A A_{n}} \Delta t_{n}}{\sum_{n=1}^{N} \Delta t_{n}} \tag{2.13}
\end{equation*}
$$

Where $\sum_{n=1}^{N} \Delta t_{n}$ is the total time period.
The hottest-spot temperature $\Theta_{H}$ consists of three components as following:

$$
\begin{equation*}
\Theta_{H}=\Theta_{A}+\Delta \Theta_{T O}+\Delta \Theta_{H} \tag{2.14}
\end{equation*}
$$

- $\Theta_{A}$ : Average ambient temperature, ${ }^{\circ} \mathrm{C}$
- $\Delta \Theta_{T O}$ : Top-oil rise over ambient temperature, ${ }^{\circ} \mathrm{C}$
- $\Delta \Theta_{H}$ : Winding hottest-sopt rise over top-oil temperature, ${ }^{\circ} \mathrm{C}$

The value of $\Delta \Theta_{T O}$ is calculated with the following formula:

$$
\begin{equation*}
\Delta \Theta_{T O}=\left(\Delta \Theta_{T O, U}-\Delta \Theta_{T O, i}\right)\left(1-\exp ^{-\frac{t}{\tau_{T O}}}\right)+\Delta \Theta_{T O, i} \tag{2.15}
\end{equation*}
$$

- $\Delta \Theta_{T O, U}$ : Ultimate top-oil rise over ambient temperature for load $\mathrm{L},{ }^{\circ} \mathrm{C}$
- $\Delta \Theta_{T O, i}$ : Initial top-oil rise over ambient temperature for $t=0,{ }^{\circ} \mathrm{C}$
- $\tau_{T O}$ : Oil time constant of transformer for any load L and for any specific temperature differential between $\Delta \Theta_{T O, U}$ and $\Delta \Theta_{T O, i}$, hours
- $t$ : Duration of load L, hours
$\Delta \Theta_{T O, U}$ is calculated as:

$$
\begin{equation*}
\Delta \Theta_{T O, U}=\Delta \Theta_{T O, R}\left[\frac{K^{2} R+1}{R+1}\right]^{n} \tag{2.16}
\end{equation*}
$$

- $\Delta \Theta_{T O, R}$ : Top-oil rise over ambient temperature at rated load, ${ }^{\circ} \mathrm{C}$
- $K$ : Ratio of load L to rated load, per unit
- R: Ratio of load loss at rated load to no-load loss
- $n$ : Empirically derived exponent used to calculate the variation of $\Delta \Theta_{T O}$ with changes in load
$\Delta \Theta_{T O, i}$ is the value of $\Delta \Theta_{T O}$ in the previous time period. For example, considering 1 hour intervals, if calculations are being made at the 13:00-14:00hrs period, $\Delta \Theta_{T O, i}$ is equal to $\Delta \Theta_{T O}$ at 12:00-13:00 hrs.

Then $\Delta \Theta_{H}$ is computed as:

$$
\begin{equation*}
\Delta \Theta H=\Delta \Theta_{H, R} K^{2 m} \tag{2.17}
\end{equation*}
$$

- $\Delta \Theta_{H, R}$ : Winding hottest-spot rise over top-oil temperature at rated load, ${ }^{\circ} \mathrm{C}$
- $m$ : Empirically derived exponent used to calculate the variation of $\Delta \Theta_{H}$ with changes in load.

Finally $\Delta \Theta_{T O, i}$ for the first hour of the load curve is calculated using (2.16). In that formula $K$ is equal to the RMS value of the 6 load ratios previous to this first hour.

The load curve showed in Fig. 2.2 is used to simulate the aging effect on transformers.
With the previously shown formulas and knowing the loads demanded for each transformer, it is possible to determine what type of transformer is the most economical one that ages slower than its rated lifespan. $F_{E Q A}$ calculations involve transformers facing normal operation conditions and contingencies.

All standard constraints are added as penalty functions to the objective function, so that genetic algorithms avoid unfeasible solutions.

### 2.2.2. Special Constraints

Downtown Santiago was the first neighborhood in Chile to have electricity back in 1883, leading the electrical development in the country. It is home to most government offices and the presidential palace. A meshed low voltage network was built in order to reach high reliability standard, minimizing energy outages.

In this paper a contingency is defined as the complete fault of one random feeder. The network's design must bear this contingency leaving all customers with power supply.


Figure 2.2. Load curve for all loads.

Assuming that each feeders fails $d$ days in a year, the network works normally $365-N_{f} \cdot d$ days. Thus the yearly cost of the system operation $O P C_{\text {Year }}$ is a weighted arithmetic mean of the base case cost $O P C_{B C}$, and each contingency case $O P C_{C o n}[i]$ as:

$$
\begin{equation*}
O P C_{Y e a r}=\frac{\left(365-N_{f} \cdot d\right) O P C_{B C}+\sum_{i=1}^{N_{f}} O P C_{C o n}[i] \cdot d}{365} \tag{2.18}
\end{equation*}
$$

At present this planning zone uses five feeders per subzone. Nevertheless this number is not necessarily optimal from a planning perspective. Hence an expression presented in (Jimenez-Estevez et al., 2010) that links voltage level $V_{f f}$, demand level and maximum carrying current of the biggest conductor $I_{f}^{\max }$ with the number of feeders that should
supply each subzone is used:

$$
\begin{equation*}
n_{f} \geq \frac{\sum_{i=1}^{N_{c}} d_{i}\left(1+g_{d}\right)^{N_{p}}}{\sqrt{3} \cdot V_{f f} \cdot I_{f}^{\text {max }}} \tag{2.19}
\end{equation*}
$$

Where $N_{c}$ is the number of loads (MV and LV), $d_{i}$ is the power demand of the $i$ th load and $N_{p}$ is the end year of the planning period. As the special reliability standard establishes that the network has to cope with the fault of one feeder, each subzone needs a minimum of $N_{f}=n_{f}+1$ feeders.

### 2.3. Solution Techniques

Genetic algorithms are used as a metaheuristic method to solve the various optimization problems discussed in Section 4. This type of algorithm works by creating an initial population of individuals (chromosomes) that represent potential solutions. The population evolves in a given environment (shaped by the objective function and the set of constraints) by applying the genetic operators: selection, reproduction and mutation. Evolution leads to the survival of the best individuals, which represent the best solutions of the optimization problem. The steps taken by a generic genetic algorithm are showed in Fig. 2.3.

Genetic algorithms fit perfectly with third-party libraries that solve power flows. An individual can be easily decoded into a certain network layout or structure, which is subsequently loaded into a solver. Python (programming language) and PyPower (a power flow solver), running a standard PC with an Intel Core i5 3.33Ghz processor, are used in this paper to implement the genetic algorithms and power flow studies respectively.

### 2.3.1. Genetic Algorithm for Selection of Transformers

The objective is to minimize installation and operational costs of transformers, which are discussed in Section 2.1.2. Additionally all standard restrictions from Section 2.2.1


Figure 2.3. Flowchart of a generic genetic algorithm.
must be satisfied. Also, contingencies are modeled as shown in Section 2.2.2. The objective function used for this algorithm is:

$$
\begin{align*}
& \sum_{i=1}^{N_{t}} I C T_{i}+O P T_{i}+T L C_{i} \\
& +\sum_{i=1}^{N_{t}} \max \left[0, F_{E Q A i}-1\right] \cdot M \\
& +\sum_{j=1}^{N_{N}}\left[\max \left[0, V^{\min }-V_{j}\right]+\max \left[0, V_{j}-V^{\max }\right]\right] \cdot M \\
& +\sum_{l=1}^{N_{B}} \max \left[0, I_{l}-I_{l}^{\text {max }}\right] \cdot M \tag{2.20}
\end{align*}
$$

Where $N_{t}$ is the number of transformers, $N_{N}$ is the number of nodes of the network, $N_{B}$ the number of conductors and $M$ is a very high number. All expressions multiplied by $M$ are constraints. When a constraint is not satisfied, $M$ forces the objective value of the individual to be very high.

An individual representing the set of transformers is a vector of length $N_{t}$. It is defined by:

$$
\begin{equation*}
\overrightarrow{x_{t}}=\left(x_{1}, x_{2}, \ldots, x_{r}\right) \tag{2.21}
\end{equation*}
$$

where $x_{i}$ are the genes of the individual and can take integer values from 0 to $T_{\text {types }}$, the number of available transformer types. Genes of the initial population are randomly drawn from a discrete uniform distribution.

A tournament selection method is used as in (Goldberg \& Deb, 1991). First, $n_{t}$ individuals are drawn from the population. Second, an individual is selected as follows. With probability $p_{t}$ the best individual from group defined on the first step is chosen, while with probability $1-p_{t}$ a random individual from the remaining $n_{t}-1$ is selected.

Local crossover (Dumitrescu, Lazzerini, Jain, \& Dumitrescu, 2000) is used because of its benefits in examining the search space when individuals are composed of integers numbers. Considering parents $P_{1}$ and $P_{2}$, for every pair of genes $g_{i, 1} g_{i, 2}$ from the $i$ th position, a new gene is created according to:

$$
\begin{equation*}
x_{i}=\alpha \cdot g_{i}^{1}+(1-\alpha) g_{i}^{2} . \tag{2.22}
\end{equation*}
$$

where $\alpha \sim U[0,1]$ is a random variable.


Figure 2.4. Local crossover diagram.

After reproduction, uniform mutation is used: each gene of the individual mutates with probability $p_{m}$. This gene is replaced for a random value that is chosen uniformly from the corresponding gene types available.

### 2.3.2. Genetic Algorithm for Selection of Conductors

The objective is to minimize the sum of all costs derived from the installation and operation of conductors and conduits discussed in Section 2.1, given the constraints listed in Section 2.2. The objective function used for this algorithm is:

$$
\begin{align*}
& \sum_{i=1}^{N_{c}} I C C_{i}+O P C_{Y e a r i}+\sum_{l=1}^{N_{B}} I C C o n d_{l} \\
& +\sum_{i=1}^{N_{t}} \max \left[0, F_{E Q A i}-1\right] \cdot M \\
& +\sum_{j=1}^{N_{N}}\left[\max \left[0, V^{\text {min }}-V_{j}\right]+\max \left[0, V_{j}-V^{\max }\right]\right] \cdot M \\
& +\sum_{l=1}^{N_{B}} \max \left[0, I_{l}-I_{l}^{\text {max }}\right] \cdot M \tag{2.23}
\end{align*}
$$

A set of conductors is a vector of length $N_{c}^{M V}+N_{c}^{L V}=N_{c}$. The first $N_{c}^{M V}$ elements represent MV branches, while the last $N_{c}^{L V}$ elements represent LV branches. The former take values from 0 to the number of MV conductor types, and the latter from 0 to the number of LV conductor types. Selection, reproduction and mutation operators are the same as the ones used in Section 2.3.1.

### 2.3.3. Genetic Algorithm for Spanning Trees

Here total costs associated to MV conductors and conduits are minimized given the constraints on voltage drop and maximum current limits. Contingency management is also considered, as explained in Section 4.4. The objective function is:

$$
\begin{align*}
& \sum_{i=1}^{N_{c}^{M V}} I C C_{i}+O P C_{Y e a r i}+\sum_{l=1}^{N_{B}^{M V}} \text { ICCond }_{l} \\
& +\sum_{j=1}^{N_{N}^{M V}}\left[\max \left[0, V^{\min }-V_{j}\right]+\max \left[0, V_{j}-V^{\text {max }}\right]\right] \cdot M \\
& +\sum_{l=1}^{N_{B}^{M V}} \max \left[0, I_{l}-I_{l}^{\max }\right] \cdot M \tag{2.24}
\end{align*}
$$

Each individual represents a certain spanning tree in a given graph. Thus considering a graph $G=(N, R)$ where $N$ is the set of network nodes and $R$ is the set of branches, $N-1$ genes of the individual are set to one, and the remaining $R-N+1$ are set to zero.

The initial population is generated with a slightly modified Kruskal algorithm. As in (Raidl \& Julstrom, 2003), this algorithm constructs random spanning trees on a graph. As trees with smaller total length tend to be better solutions, an heuristic bias is introduced to this algorithm. This bias favors short length branches when creating the trees. Initially the Kruskal algorithm sorts all routes by their lengths, and a spanning tree is created checking them in increasing length order. The remaining trees are created by randomly permuting the first $k$ branches before the list is checked again. The number of $k$ permutations increases with each generated tree according to:

$$
\begin{equation*}
k=\frac{\alpha_{K}(i-1) N_{N}}{P} \tag{2.25}
\end{equation*}
$$

where $P$ is the population size, $i$ is the index of the next tree $(i=1, \ldots, P), \alpha_{K}$ is a control parameter that defines the diversity of the initial population (the bigger the $\alpha_{K}$, the more diversity), and $N_{N}$ is the number of nodes of the graph.

Tournament selection is used as the selection method as in the previous steps.

Crossover and mutation methods that are specially designed for spanning trees are used. In (Jimenez-Estevez, Vargas, \& Palma-Behnke, 2007), different solution techniques are tested and path-interchange crossover is found to be the most efficient. It works by selecting two random vertices from the graph. The paths from one vertex to the other are then identified. These paths are interchanged between the two. This procedure is implemented as shown in (P. Carvalho, Ferreira, \& M., 1999; P. M. S. Carvalho, Ferreira, \& Barruncho, 2001). To explain how path interchange, a framework for spanning-tree genotype space needs to be defined. The genotype space is a partially ordered set $(A, \leq)$, that is a set where:
i) $a \leq a \forall a \in A$
ii) $a \leq b$ and $b \leq a$ implies $a=b \forall a, b \in A$
iii) $a \leq b$ and $b \leq c$ implies $a \leq c \forall a, b, c \in A$

The element $a$ is called direct precedent of $b$ in $A$ (denoted as $b \hookleftarrow a$ ) if and only if:
i) $a \neq b$
ii) $a \leq b$
iii) $\exists \not c \in A$ such that $a \leq c$ and $c \leq b$

Analogously the element $b$ is called direct follower of $a$. Spanning trees can be defined as partially ordered sets. Each node is preceded by one and only one node, except for the first element (root of the tree). Let $g$ be a function that codes $T$ as this genotype.

$$
\begin{equation*}
g: T \longrightarrow\{b \hookleftarrow a: \forall \operatorname{arc}(a b) \in T \text { with } a \leq b\} \tag{2.26}
\end{equation*}
$$

To successfully change arcs between spanning trees, both need to keep being partially ordered sets. To guarantee that, the three above mentioned properties i), ii) and iii) must hold for all arcs after changing. If the properties hold, the change is defined as consistent. Let $p(b)$ be the direct precedent of element $b$ in $T$, and $F(b)$ be the set of direct followers of $b$ in $T$. A non-consistent change can be identified when precedence $b \hookleftarrow a$ is taken over a tree ordered set $T$ where $b \leq a$. However multiple non-consistent precedence changes
could lead to a consistent ordered set $T$. Therefore consistency has to be specially tested after the last precedence change. When the change is consistent, precedence changes permits information crossover between spanning trees, guaranteeing radiality and connectivity. Path interchange allows interchange of important similarities about solutions, a key aspect for a good crossover method.

To interchange paths $P^{i}$ and $P^{i i}$ between solutions $T^{i}$ and $T^{i i}$ take following steps:
Step 1. Define a common root $r$ for both solutions. Represent the solutions $T^{i}$ and $T^{i i}$ as tree-ordered sets considering the selected root.

Step 2. Select two random nodes $a$ and $b$.
Step 3. Find the paths $P^{i}$ and $P^{i i}$ between $a$ and $b . P^{i}$ in $T^{i}$ and $P^{i i}$ in $T^{i i}$
Step 4. Submit $P^{i}$ to $T^{i i}$ and $P^{i i}$ to $T^{i}$ if possible. If not, go to Step 2.
To submit a path $P$ into a tree $T$, change $T$ by changing every precedence relation $x \hookleftarrow y$ of $T$ to $x \hookleftarrow z$ of $P$, if and only if $x \in T$ and changes $x \hookleftarrow z$ are consistent in $T$.

An example is showed in Fig. 2.5. Defining node $c$ as the root, the tree-ordered sets become:

$$
\begin{aligned}
& \text { - } T^{i}=\{d \hookleftarrow c, a \hookleftarrow c, b \hookleftarrow a, e \hookleftarrow b, f \hookleftarrow e\} \\
& \text { - } T^{i i}=\{e \hookleftarrow c, b \hookleftarrow e, a \hookleftarrow b, d \hookleftarrow a, f \hookleftarrow b\}
\end{aligned}
$$

Selecting two random nodes $b$ and $c$, paths are defined as $P^{i}=\{a \hookleftarrow c, b \hookleftarrow a\}$ and $P^{i i}=$ $\{e \hookleftarrow c, b \hookleftarrow e\}$. Submitting $P^{i}$ to $T^{i i}$ produces $\{e \hookleftarrow c, b \hookleftarrow a, a \hookleftarrow c, d \hookleftarrow a, f \hookleftarrow b\}$ and $P^{i i}$ to $T^{i}$ produces $\{d \hookleftarrow c, a \hookleftarrow c, b \hookleftarrow e, e \hookleftarrow c, f \hookleftarrow e\}$.

The chosen mutation operator is presented in (Raidl \& Julstrom, 2003). It adds a branch to the tree selected for mutation, creating a loop. Then a random line, different from the one that was added, is removed from the loop. An example is showed in Fig. 2.6.

Further modifications to the algorithm to fit the needs of the problem at hand are presented in Section 4.4.


Figure 2.5. Path Interchange.


Figure 2.6. Tree mutation operator.

## 3. ZONE SPLITTING

The planning zone studied in this paper is divided in 5 subzones. Each subzone has an independent substation power supply and similar power demands. The design proposed here tries to maintain these historical characteristics. A slightly modified version of the clustering technique known as weighted k -means is used to divide a set of $m$ loads into $k$ groups. Loads power demands are used as weights. A proximity factor that increases or decreases the distance between centroids and loads is assigned to each cluster . The algorithm is the following:
(i) $k$ elements from the set of $m$ loads are chosen. These $k$ loads define the initial centroids. Proximity factors are set to one for each centroid.
(ii) Each load is assigned to the nearest centroid according to the Euclidean distance multiplied by its respective proximity factor. Therefore $k$ groups are created.
(iii) New centroids are defined for each cluster. Their locations are computed as the demand weighted average of loads coordinates.
(iv) Total power demand is calculated within each cluster. If a clusters total power demand is higher than average demand, its proximity factor is increased. Similarly, proximity factors of clusters with lower than the average demand are decreased.
(v) Steps 2, 3 and 4 are repeated until the standard deviation of the power demands within each cluster is below an exogenous threshold.

The proximity factor $p f_{i}$ for cluster $i$ whose total power demand $T P D_{i}$ is:

$$
\begin{equation*}
p f_{i}=p f_{i}+\left(T P D_{i}-\sum_{j=1}^{k} T P D_{i} / k\right) \cdot 10^{-5} \tag{3.1}
\end{equation*}
$$

Proximity factors serve the key role of allowing loads to move from high demand clusters to low demand clusters, resulting in the desired power balance between all subzones. An example is presented in Fig. 3.1.


Figure 3.1. Example of the adapted k-means algorithm.
Once all the subzones are designed, the next step is to assign one substation to each cluster. For this purpose an integer linear optimization problem is solved. Its objective function is to minimize the sum of the distances from each substation to the centroid of its associated cluster. These distances are computed using the zones street layout. The restrictions avoid using one substation for more than two subzones, and force every subzone to be connected to one substation only.

## 4. SUBZONE DESIGN

The design and location of transformers, feeders and conductors is treated as an independent problem within each subzone. The layout of streets and sidewalks enters the optimization problem as a constraint and therefore any feasible solution is compatible with the spatial arrangement of the planning zone.

### 4.1. Candidate Transformer Points

If transformers can be installed anywhere on the grid, the location problem is very hard to solve. Therefore to make the problem more tractable, the search space is shrunk by exogenously choosing a finite set of feasible locations. Given that the studied zone has a high load density, it is natural to allow transformers to be installed near street intersections and high demand loads. The developed algorithm is based on a closest facility analysis between loads and street intersections (Appendix B). Basic inputs for this analysis are the road layout, the positions of incidents and facilities and the number of facilities that each incident must supply. The nearest facilities are found via Dijkstra algorithm. Street intersections are set to facilities and loads to incidents, and only one facility can be linked to each incident. Total power demand at each street intersection is then computed as the sum of all loads connected to that facility. The developed algorithm works as follows:
(i) All street intersections are stashed in a candidate list. A threshold is defined and set to 50 kVA .
(ii) Using closest facility analysis, every load is matched to its nearest intersection on the candidate list.
(iii) Total power demand ( $P_{a c c}$ ) is calculated at each intersection.
(iv) Street intersections whose $P_{\text {acc }}$ is below the threshold are removed from the candidate list.
(v) The threshold is incremented by 50 kVA .
(vi) Steps 2, 3, 4 and 5 are repeated until the largest power demand among all street intersections is lower than the threshold.

Finally, the number of candidate transformer located on each street intersection is calculated as

$$
\begin{equation*}
\left\lceil\frac{P_{a c c}}{\min P_{t}} \cdot \frac{N_{f}}{N_{f}-1}\right\rceil \tag{4.1}
\end{equation*}
$$

where $N_{f}$ is the number of feeders that supply electricity to the subzone and $\min P_{t}$ is the smallest nominal capacity among available transformers.

Candidate transformers are located on the sidewalk of each street intersection. If there are more than four transformers to be located at a given intersection, the remaining are installed in front of high demand loads.

### 4.2. Low Voltage Network Layout Design

As the studied zone requires a meshed grid, a similar closest facility analysis as the one shown in subsection 4.1 is used. Low voltage loads are set as incidents and candidate transformer points as facilities. The number of facilities to be found by every incident is set high (The higher this number, the more interconnected the grid is). All routes are then integrated into one meshed grid, as in Fig. 4.1.

### 4.3. First Transformer Power Flow

After the transformers have been optimally located and the LV network has been designed, the next step is to compute the amount of power flowing through each transformer. Since this cannot be done without first specifying the medium voltage network, temporary feeders are arranged between each transformer and its associated substation. All feeders have the same length and impedance. All low voltage cables within the network are assumed to have the smallest impedance available. Power calculations are carried out taking


Figure 4.1. Meshed low voltage grid.
only into account low voltage loads, since MV loads do not affect the power transmitted through transformers.

### 4.4. Medium Voltage Network Design

Radial layout was used for MV, as a lower cost solution. In this study, it is assumed that $N_{f}$ radial feeders supply energy to each subzone. These feeders are designed with a genetic algorithm adapted to create steiner trees in graphs.


Figure 4.2. Feasible MV network routes.

First, feasible routes are traced and the medium voltage network design problem is expressed as a graph theory problem. A closest facility analysis is performed, using a substation, transformers and high voltage loads as both incidents and facilities. The resulting layout is a graph $G=(N, R)$ where N is the set of network nodes, including the elements mentioned before as well as auxiliary nodes used to replicate the shape of sidewalks. And $R$ is the set of branches obtained by the closest facility analysis. An example is showed in Fig. 4.2.

Note that all network nodes do not have to be reached by every feeder. To improve reliability, as explained in (Corporation, 1942), each transformer is matched to only one feeder, and each MV load to two feeders. This means that each feeder is connected to
approximately $N_{t} / N_{f}$ transformers. To avoid grouping transformers that are too close to a specific feeder, the following algorithm is run:
(i) Three empty lists $L_{I}, L_{V}$ and $L_{\text {cost }}$ are defined and a counter $i$ is set to zero.
(ii) An element not in $L_{I}$ is chosen from the set of transformers. A sequence is initialized by associating feeder number 0 to this transformer. This element is appended to $L_{I}$ and $L_{V}$ and it is set as the current transformer. Counter $i$ is increased by one.
(iii) Euclidean distance is calculated from the current transformer to all others not in $L_{V}$. The element with the smallest distance is chosen and appended to $L_{V}$. The sequence is extended by matching feeder number $i \% N_{f}$ (integer division) to this transformer. Then it is set as the current transformer and counter $i$ is increased by one.
(iv) Step 3 is repeated until all transformers are in $L_{V}$
(v) As all transformers are matched to one feeder, a quick cost calculation is performed using the GA algorithm discussed in Section 2.3.3. The cost is stored in $L_{\text {cost }}$.
(vi) If all transformers are stored in $L_{I}$, go to the next step. Otherwise, delete all elements in $L_{V}$ and go to step 2
(vii) The sequence with to lowest cost in $L_{\text {cost }}$ is chosen.

This division of transformers into clusters in addition to the substation selected for the subzone, make the different sets of steiner nodes that have to be visited by each feeder. After defining the sets and computing the power flows through each transformer, it is possible to run the genetic algorithm presented in Section 2.3.3. It is important to note that one spanning tree represents $N_{f}$ feeders. Each feeder is created by pruning the edges of the tree that are not necessary to connect all steiner nodes of a certain cluster.

### 4.5. Transformers Selection

After the optimal medium and low voltage networks are generated, a new selection of transformers is performed. For this purpose a genetic algorithm as the one presented in Section 2.3.1 is used. The number of genes is given by the current set of transformers. The conductor types considered for the medium voltage network are the same calculated in Section 4.4. As low voltage cables are not determined yet, the conductor with the lowest impedance available is selected for the whole low voltage network.

### 4.6. High Voltage Network Design - Transformers Selection Loop

One or more genes of the best transformer selection individual (Section 4.5) can be zero. When this is the case, the points associated to those genes are deleted from the transformer's set. Therefore the MV network has to be redesigned, because some wires may now lead to empty points.

A loop is performed between algorithms of Sections 4.4 and 4.5. It ends whenever the best transformer selection individual has no genes that are equal to zero. This way the set of transformers and the MV network are compatible.

### 4.7. Low Voltage Network's Trunk Identification

Due to reliability reasons, a trunk low voltage network is defined in this paper. This network gathers the conductors with the highest currents that interconnect all transformers. All the cables have the same conductor type in order to confront the temporary failure of a small group of transformers. The trunk network is defined as a graph $G_{T}=\left(N_{T}, R_{T}\right)$. In case of a wire fault in the low voltage grid, this network allows all transformers to get their energy to other transformer's nodes. This means that all vertices $N_{T}$ have degree two or more, unless a transformer is positioned in a leaf vertex, in which case it has degree one. See Appendix C for basic graph theory.

The power flow study characteristics for the trunk network identification are: MV conductors are the ones calculated in the previous step; low voltage conductors are set as the ones with the lowest impedance available; no loads are considered.

Considering the previous premises, the algorithm develops as follows:
(i) One transformer is disconnected and a load equal to its nominal capacity is connected to the secondary coil.
(ii) A power flow study is performed and the amount of power flowing through each branch is stored.
(iii) Repeat previous steps until all transformers are disconnected.
(iv) For each branch, compute the sum of absolute values of the power flows obtained in step 2.
(v) Two variables, $M V A_{\text {min }}$ and $M V A_{\max }$ are initialized. The first one is set to zero, and the second is set to the highest total power flow among all branches.
(vi) A random number $M V A_{\text {rand }}$ with lower bound $M V A_{\text {min }}$ and upper bound $M V A_{\max }$ is drawn. All branches whose power flows are higher than $M V A_{\text {rand }}$ are saved in an auxiliary graph $G_{\text {aux }}$.
(vii) If all transformers are in the same connected component and there are no bridges in $G_{\text {aux }}, M V A_{\text {rand }}$ is stored in $M V A_{\min }$. Otherwise $M V A_{\max }$ is set to $M V A_{\text {rand }}$.
(viii) Repeat steps 6 and 7 until $M V A_{\max }-M V A_{\min }<\epsilon$.
(ix) All branches whose power flows are higher than $M V A_{\text {min }}$ are marked as part of the trunk and are saved to $G_{\text {aux }}$.

Marked branches are sorted by power in ascending order. One by one, each branch is removed from $G_{a u x}$. If $G_{a u x}$ does not fulfill the restrictions mentioned in the previous step 7, the current branch is added again to the list. Once all branches are examined, the trunk network is obtained.

### 4.8. Conductors Selection

Once both network's layouts, positions and capacities of transformers and the low voltage trunk are defined, conductors types for both networks are calculated. For this purpose a GA as the one presented in section 2.3.2 is run. The power flow studies to be performed in that algorithm need to consider all network elements: medium- and low voltage loads and networks; transformers; and the associated substation. Considering building restrictions, it is assumed that low voltage conductors on a same block have the same diameter. Therefore several branches are grouped into one gene, reducing the search space. Also all branches that are part of the trunk network are controlled by one gene. Therefore, all trunk branches have the same cable type. The optimal MV conductors calculated in Section 4.4 are coded into genes, and are used as a good initial solution.

After selecting optimal conductors, a pruning process is executed. The power flow studies for this process also consider all network elements. Each low voltage network branch that is not part of the trunk network is represented by one binary gene. Each gene points out if its associated branch should exist or not ( 1 or 0 respectively). If the gene is one, the conductor type remains the same as calculated before. If it is zero, the branch becomes type zero. In this way the total length of wires is minimized.

## 5. RESULTS

Greenfield planning is solved for the planning zone described above, which has 2230 LV and 11 HV customers amounting to a total demand of 158.85 MVA . Over 70 km of sidewalks represent about 1500 route segments for installation of conductors.

Zone splitting described in Section 3 is applied to the set of customers. The results are shown on Fig. 5.1 and a summary of their characteristics is shown in Table 5.1.

The number of candidate transformer points and the length of the LV network layout for every subzone is shown in Table 5.2. Fig 4.1 shows their layout for one of the subzones.


Figure 5.1. Subzones created with weighted k-means adapted for power balancing

Table 5.1. Clusters' Data

| Subzone No. | Demand <br> $[\mathrm{kW}]$ | LV loads No. | MV loads No. | Area <br> $\left[\mathrm{m}^{2}\right]$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 29454 | 362 | 2 | 261339 |
| 1 | 29562 | 828 | 0 | 502664 |
| 2 | 29602 | 528 | 0 | 355024 |
| 3 | 29616 | 303 | 3 | 136064 |
| 4 | 29501 | 209 | 6 | 204800 |

TABLE 5.2. Candidate transformer points (CTP) and LV network length

| Subzone No. | CTP | LV network length <br> $[\mathrm{km}]$ |
| :---: | :---: | :---: |
| 0 | 68 | 10.78 |
| 1 | 84 | 20.24 |
| 2 | 79 | 12.31 |
| 3 | 83 | 6.316 |
| 4 | 47 | 8.301 |

The number of feeders $N_{f}$ is calculated using 2.19. Considering total demand of each subzone in MVA presented in Table 5.1 , demand growth $g_{d}$ equal to $2.75 \%, T=4$, $V_{f f}=13000$ Volts, $I_{f}^{\max }=632$ Amperes and a power factor of 0.93 , the number of feeders is:

$$
\begin{equation*}
N_{f}=n_{f}+1=\frac{\frac{29.5}{0.93} \cdot 10^{6} \cdot(1.0275)^{4}}{\sqrt{3} \cdot 13000 \cdot 632}+1=2.7+1=3.7 \approx 4 \tag{5.1}
\end{equation*}
$$

For all genetic algorithms discussed in Section 3, the size of the population and the number of generation are both set to 100 .

One of the resulting trunk networks is shown in Fig. 5.2. The five trunk networks go over 23.02 km , representing a $39.7 \%$ of the total LV network length.

Due to the high number of power flows studies that are performed in each GA, the total program execution time to get results for all five zones is about 28 hours.


Figure 5.2. Low voltage trunk network of one subzone

As the proposed algorithm is an heuristic that is governed by random variables, ten samples are run in order to assess the variability of results. This means that ten different solutions are obtained for each subzone, as each subzone design is an independent process. The results presented in this section are the mix of the solutions with lowest cost for each subzone. Fig. 5.3 shows the costs obtained by ten different samples and the minimum cost of their mix. The difference between the highest and the lowest costs is 986 MM CLP. Mixing all solutions brings additional savings of 389 MM CLP. However that saving increases running time by ten times, that means approximately 250 hours.


Figure 5.3. Total cost for each of ten samples run. The red dashed line represents the total cost of mixing the lowest cost subzones from all samples.

Total costs are shown in detail in Table 5.3. Costs of conductors are significantly higher than costs from energy losses. This is explained by the high reliability standards set in the problem, which force conductors to be oversized.

Total network costs reported by the regulator in the tariff setting process of 2012 accounts for 27714 MM CLP. This value does not consider cost of energy and power losses. The cost obtained with the proposed algorithm is 23912 MM CLP. The method used by the regulator uses no optimization tools and has plenty of heuristic steps that oversimplifies the problem.

TAble 5.3. Total costs for all subzones

| Item | Subzone 0 <br> [MM CLP] | Subzone 1 <br> [MM CLP] | Subzone 2 <br> [MM CLP] | Subzone 3 <br> [MM CLP] | Subzone 4 <br> [MM CLP] | All subzones [MM CLP] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cables LV | 1244 | 2194 | 1623 | 937 | 1078 | 7075 |
| Cables MV | 581 | 511 | 527 | 278 | 181 | 2078 |
| Conduits LV | 399 | 748 | 520 | 288 | 326 | 2282 |
| Conduits MV | 144 | 198 | 146 | 99 | 98 | 683 |
| Losses | 889 | 1090 | 1091 | 939 | 587 | 4596 |
| Transformers | 2265 | 3066 | 2731 | 2282 | 1449 | 11793 |
| Total | 5521 | 7807 | 6639 | 4823 | 3718 | 28508 |

The total length of conductors listed by diameter and number of parallel cables are presented for both HV and LV networks in Table 5.4 and Table 5.5 respectively. The trunk network forces a substantial part of the LV network to have the biggest available conductor. Therefore the conductor $240 \times 8$ accounts for $43.1 \%$ of the total LV network.

Table 5.4. Total length of high voltage conductors

| Diameter <br> $\left[\mathrm{mm}^{2}\right]$ | No. of <br> cables | Total length <br> $[\mathrm{km}]$ | \% of total |
| :---: | :---: | :---: | :---: |
| 0 | 1 | 0.2 | 0.3 |
| 35 | 1 | 17.5 | 31.9 |
| 120 | 1 | 11.3 | 20.5 |
| 240 | 1 | 6.3 | 11.5 |
| 400 | 1 | 16.0 | 29.2 |
| 400 | 2 | 3.6 | 6.6 |
| 400 | 3 | 0.04 | 0.1 |
| Total |  | $\mathbf{5 5}$ | $\mathbf{1 0 0}$ |

In addition to the base case, three cases that involve different input data are considered:

- Base case: The algorithm as described in this paper.
- NLT: The LV trunk network is not considered.
- RTS: The set of available transformers is reduced by deleting the 1000 kVA option.

Table 5.5. Total length of low voltage conductors

| Diameter <br> $\left[\mathrm{mm}^{2}\right]$ | No.of <br> cables | Total length <br> $[\mathrm{km}]$ | \% of total |
| :---: | :---: | :---: | :---: |
| 0 | 1 | 4.8 | 8.3 |
| 16 | 1 | 2.7 | 4.6 |
| 70 | 1 | 5.1 | 8.9 |
| 240 | 1 | 8.1 | 14 |
| 240 | 2 | 5.1 | 8.8 |
| 240 | 3 | 3.7 | 6.3 |
| 240 | 4 | 1.4 | 2.4 |
| 240 | 5 | 0.7 | 1.2 |
| 240 | 6 | 0.7 | 1.2 |
| 240 | 7 | 0.6 | 1.1 |
| 240 | 8 | 25 | 43.1 |
| Total |  | $\mathbf{5 7 . 9}$ | $\mathbf{1 0 0}$ |

- NLT \& RTS: LV trunk network is not considered and the 1000 kVA option is deleted.

In order to size the effects of some elements in the network, results for cases described above are presented in Table 5.6.

Table 5.6. Total costs for different problem settings

| Item | Base Case <br> [MM CLP] | NLT <br> [MM CLP] | RTS <br> [MM CLP] | NLT \& RTS <br> [MM CLP] |
| :--- | ---: | ---: | ---: | ---: |
| Cables LV | 7075 | 5747 | 6413 | 4844 |
| Cables MV | 2078 | 2186 | 2595 | 2956 |
| Conduits LV | 2282 | 1938 | 2032 | 1727 |
| Conduits MV | 683 | 681 | 765 | 759 |
| Losses | 4596 | 4589 | 4529 | 4439 |
| Transformers | 11793 | 11511 | 12588 | 12590 |
| Total | $\mathbf{2 8 5 0 8}$ | $\mathbf{2 6 6 5 2}$ | $\mathbf{2 8 9 2 2}$ | $\mathbf{2 7 3 1 5}$ |

Not designing the LV trunk network brings savings of 1856 MM CLP. This value represents a $6.5 \%$ of the total network cost. Reducing the pool of available transformers increases the network cost in $1.5 \%$, that is 414 MM CLP. The increment in transformer and

HV network costs is somewhat compensated by the decrease of low voltage conduit and conductor costs. The higher the number of transformer is, the narrower the LV conductors in order to cope with the demand. 7 Both NLT \& RTS effects lead to a network that is more economical than the base case network by 1193 MM CLP. The LV network of this case has the lowest cost of all analyzed cases.

The equivalent aging factor $F_{E Q A}$ constraints described in Section 2.2.1 are generally fulfilled in all computed cases. Hence ignoring those constraints does not change the outcome of the algorithm. This is explained by the high impedances of the available transformers (see Table A.1). The higher their impedances, the higher the load division between transformers, thus reducing the possibility of overloading and aging.

Table 5.7. Results of the proposed algorithm and the regulator's data for the tariff setting process of 2012

| Item | Proposed algorithm | Regulator's data |
| :--- | :---: | :---: |
| Cables LV [km] | 254.7 | 80.75 |
| Cables MV [km] | 58.4 | 62 |
| Conduits LV [km] | 57.9 | 53.5 |
| Conduits MV [km] | 16.4 | 23.2 |
| Transformers 500 kVA | 5 | 0 |
| Transformers 750 kVA | 84 | 99 |
| Transformers 1000 kVA | 124 | 119 |

Table 5.7 shows a comparison between results of the proposed algorithm versus the regulator's data for the tariff setting process of 2012. Results are very similar for all items, except for the kilometers of low voltage cables. This difference is partially explained by the trunk low voltage network that forces a portion of low voltage cables to be oversized. However, when the trunk network is not considered (case NLT), the kilometers of low voltage cables are 209.3. This value is still $259.2 \%$ higher than the one reported by the regulator. As the proposed algorithm does not allow any violation of reliability constraints, cables must have high ampacities to cope with high LV currents. High ampacities imply having more conductors in parallel, increasing the number of kilometers of LV cables.

## 6. CONCLUSIONS

This paper proposes a novel heuristic algorithm based on genetic algorithms to design a LV meshed grid, transformers and MV feeders, closing an algorithmic gap in distribution design, particularly in LV meshed networks. It allows greenfield planning for a real distribution company problem with 2241 customers. All network elements are successfully sized to cope with the failure of a feeder and to provide energy to all loads given some voltage and current constraints. The proposed algorithm may be used in model company design approaches to distribution rate calculations. The lower cost calculated by this algorithm (23912 MM CLP) in comparison with the cost given by the chilean regulator in 2012 (27 714 MM CLP) indicates the feasibility and usability of the proposed method to calculate tariffs.

No optimization of the algorithm CPU time was made (the resultant 28 -hour execution time is no constraint in a rate assessment). Nevertheless, the algorithm could be made more efficient by using a faster power flow study solver and by decreasing the number of loops necessary to solve for transformers positioning and sizing, and the layout of the MV network. Additionally, as each subzone is solved independently, multiple CPU cores are helpful to decrease execution time.

Other ways of improving network reliability could be incorporated, such as LV crossconnections and MV loops. This may lead to cheaper solutions. Additionally reliability indices could be used to improve the algorithm when dealing with contingencies, adding nuances to the fulfillment of the reliability standards. These indices would also help to measure the advantages of the proposed methodology against other methods found in the literature.

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## APPENDICES

## APPENDIX A. DATA TABLES

Table A.1. Types of Transformers

| Nominal Capacity <br> [kVA] | Cost <br> [MMCLP] | R <br> [ohms] | X <br> [ohms] | Iron Core Losses <br> [Watts] |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | $\infty$ | $\infty$ | 0 |
| 500 | 47.8 | 3.456 | 28.592 | 1100 |
| 750 | 50.3 | 2.304 | 19.061 | 1500 |
| 1000 | 59.1 | 1.728 | 14.296 | 1900 |

Table A.2. Types of high voltage conductors

| Diameter <br> $\left[\mathrm{mm}^{2}\right]$ | No.of <br> cables | Cost <br> [MMCLP/km] | Ampacity <br> [Amperes] $]$ | R <br> $[$ ohms $/ \mathrm{km}]$ | X <br> $[\mathrm{ohms} / \mathrm{km}]$ | B <br> $[\mathrm{mS} / \mathrm{km}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | $\infty$ | $\infty$ | 0 |
| 35 | 1 | 6.1 | 196 | 0.524 | 0.140 | 0.074 |
| 120 | 1 | 25.15 | 330 | 0.153 | 0.128 | 0.110 |
| 240 | 1 | 47.17 | 488 | 0.075 | 0.124 | 0.148 |
| 400 | 1 | 76.54 | 632 | 0.047 | 0.123 | 0.179 |
| 400 | 2 | 153.08 | 1264 | 0.024 | 0.062 | 0.358 |
| 400 | 3 | 229.62 | 1896 | 0.015 | 0.041 | 0.537 |

Table A.3. Types of low voltage conductors

| Diameter <br> $\left[\mathrm{mm}^{2}\right]$ | No.of <br> cables | Cost <br> [MMCLP/km] | Ampacity <br> [Amperes] $]$ | R <br> $[\mathrm{ohms} / \mathrm{km}]$ | X <br> $[\mathrm{ohms} / \mathrm{km}]$ | B <br> $[\mathrm{mS} / \mathrm{km}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 16 | 1 | 4.54 | 127 | 1.21 | 0.15 | 0.065 |
| 70 | 1 | 11.02 | 285 | 0.268 | 0.132 | 0.091 |
| 240 | 1 | 33.92 | 576 | 0.075 | 0.124 | 0.148 |
| 240 | 2 | 67.84 | 1152 | 0.038 | 0.062 | 0.295 |
| 240 | 3 | 101.76 | 1728 | 0.025 | 0.041 | 0.443 |
| 240 | 4 | 135.67 | 2304 | 0.019 | 0.031 | 0.591 |
| 240 | 5 | 169.59 | 2880 | 0.015 | 0.025 | 0.738 |
| 240 | 6 | 203.51 | 3456 | 0.013 | 0.021 | 0.886 |
| 240 | 7 | 237.43 | 4032 | 0.011 | 0.018 | 1.034 |
| 240 | 8 | 271.35 | 4608 | 0.009 | 0.016 | 1.181 |

Table A.4. Types of high voltage conduits

| No. of Cables <br> Capacity | Cost <br> [MMCLP/km] |
| :---: | :---: |
| 0 | 0 |
| 1 | 27.13 |
| 2 | 33.51 |
| 3 | 37.07 |
| 4 | 42.16 |
| 5 | 45.73 |
| 6 | 54.6 |
| 7 | 56.3 |
| 8 | 66.13 |
| 9 | 67.3 |
| 10 | 76.48 |
| 12 | 88.44 |

Table A.5. Types of low voltage conduits

| No. of Cables <br> Capacity | Cost <br> [MMCLP/km] |
| :---: | :---: |
| 0 | 0 |
| 1 | 20.49 |
| 2 | 30.42 |
| 3 | 33.98 |
| 4 | 40.56 |
| 5 | 44.13 |
| 6 | 52.96 |
| 7 | 75.28 |
| 8 | 77.87 |

The following parameters are used to calculate all equivalents aging factors:

- $\Theta_{A}: 30^{\circ} \mathrm{C}$
- $\Delta \Theta_{T O, R}: 36^{\circ} \mathrm{C}$
- $\Delta \Theta_{H S, R}: 28.6^{\circ} \mathrm{C}$
- $R: 4,87$
- $\tau_{T O}: 3.5$ hours
- $n: 1 ; m: 1$


## APPENDIX B. CLOSEST FACILITY ANALYSIS

The closest facility analysis quantifies the cost of going from facilities to incidents and establishes which are nearest to one other over a certain route layout. The user defines the set of points which are modeled as facilities, another set of points as incidents and the number of facilities that have to be found for each incident. The cost is defined according to the characteristics of the problem to solve. Distance in kilometers is used in this paper. Closest facility analysis uses Dijkstra algorithm to find the closest routes.

Constraints can be specified. For instance distance cutoffs can be set, so that found incidents are closer than a given number of kilometers. No cutoffs were used in this paper. This algorithm is commonly used for transportation problem. However, for electric optimization problems, knowing which loads, transformers and substations are near to one other is an adequate starting point to minimize the cost of energy supply.

## APPENDIX C. GRAPH THEORY NOTES

Graphs are mathematical structures that show connectivity relationships between two or more elements. These elements are called vertices (i.e. nodes) and the link that connects two vertices are called edges (i.e. branches).

Some graph theory definitions that are used throughout this paper are explained.

- Connected components: A connected component is a subgraph of a graph composed of two or more vertices that are connected to each other, but not to other subgraphs.
- Degree: The degree of a vertex is the number of edges incident to that vertex.
- Bridges: A bridge is an edge whose removal increases the number of connected components. Bridges are always connected to vertices with degree one.

Fig. C. 1 shows an example of a graph where all mentioned definitions are presented.


Figure C.1. A graph composed of three connected components. Bridges are pictured as dashed lines. The number in each vertex shows its degree.

