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ENSAYOS EN DEUDA SOBERANA Y DEFAULT

por

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ESSAYS ON SOVEREIGN DEBT AND DEFAULT

by

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A thesis submitted in partial fulfillment of the requirements for the degree of
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A mi esposo y mi familia, por su apoyo incondicional.

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Abstract

After the Russian default in 1998 and the subsequent new generation of debt crises, the economic literature of sovereign debt was eager to regain momentum.[1] Three new topics captured academics' attention. The exclusion from capital markets, which proved not to be permanent. The possibility to legally enforce the contracts to avoid increasing high profile litigations. The outbreak of collective action problems at debt restructuring processes on disintermediated debt. Thenceforth, the main advances in literature collected into the following branches: theoretic quantitative models of debt, legal and microeconomic analysis of debt contracts and empirical analysis to different aspects of sovereign debt.

This dissertation contents two theoretical approaches nested within those topics, both focusing on processes that occur after default. The first one, aims to understand how the recent disintermediation of the sovereign debt market impacted on the outcome of debt restructuring. The second one, explores the non-negligible exclusion period after default, and the mechanisms that might explain the inactivity of the economy at capital markets once it successfully restructured defaulted debt.

The first chapter begins discussing some evidence of the recent disintermediation of sovereign debt market and explores the effects that this phenomenon produced on the restructuring process. In this new background, defaulting countries have had to confront mostly atomistic, unconnected bondholders when engaging on restructuring negotiations. According to the data, the nature of creditors did have an impact on the outcome of restructuring, and counterintuitively, the new conditions played against the government by reducing the effective investors' concession.

Hence, I propose a model to study the determination of the restructuring outcome (the haircut) when bondholders play a coordination game (a game in which the greatest benefits occur when players align their decisions). The resulting equilibrium multiplicity is solved using the global games approach. The main finding is that this new market setting entails a cost for the defaulting government: it works as an additional con-

[1] Literature survey at Panizza, Sturzenegger and Zettelemeyer (2009).

straint which forces it to reduce the concession asked to creditors in order to increase the probability of the program's acceptance. I then compare my results with the restructuring outcome obtained with Nash bargaining, which is the main solution technique at quantitative debt models with endogenous restructuring. I find that under certain conditions, the haircut with the coordination feature stays below the Nash bargaining one for each possible value of the bargaining power. Finally, I run simulations with calibrated parameters for illustrative purposes and find that coordination costs account for a significant portion of the total haircut reduction observed after the sovereign debt disintermediation process started.

The second chapter addresses the determinants of the exclusion period from capital markets which constitutes one of the main costs of defaults. The empirical literature states that almost half of this period is explained by the time it takes for the countries to re-access capital markets after they have restructured their debts. In particular, I focus on this second stage, which according the same authors, takes on average more than 7 years to solve. Henceforth, this chapter proposes a model to study the effects of issuance costs in the delay of reentry once the economy has already restructured its debt. I build on a standard quantitative model of sovereign debt with return to capital markets introducing two modifications. First, I simplify the restructuring process which I am not targeting in the analysis and second, I introduce both fixed and variable issuance costs in the period of reentry.

In a calibrated version of the model, I find that both fixed and variable issuance costs at reentry help to better match the length of defaults observed in the data. First, these costs reduce the incentives to default ex ante. Thus, issuance costs result an interesting feature to introduce default costs other than the standard output loss function used in the literature. Second, the duration of the autarky period experiences a significant increase approaching the empirical levels. Indeed, as the economy does not control the endowment process, the available decision is whether this is the proper moment to reentry capital markets given both the total amount of debt that can be raised and market prices. The delay here allows the economy to process a recovery in the endowment that results in the possibility of both issue more debt and get better prices. Finally, the model suggests that other default costs, that were not considered in restructuring negotiations and the economy expects to fund through new debt, might end up imposing a heavy burden that compromises the reentry to capital markets thus extending the term of the financial exclusion.

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Chapter 1

Coordinating in the Haircut: a model of sovereign debt restructuring in secondary markets

1.1 Introduction

After 1980, government debt in emerging economies experienced a dramatic disintermediation process.[1] Indeed, by the end of the century, a group of sparse, uncoordinated bondholders at capital markets had substituted international commercial banks as governments' main creditors. This transition embedded a coordination feature into many debt related processes. In some cases, such as the issuance and pricing of the bonds, the market alone did a good job avoiding coordination failures. In others, such as the restructuring after default, a general decision stage with no public information available, turns coordination a critical factor in the process.

In this paper, I propose a model to determine the outcome of the restructuring process when defaulted debt is entirely composed of bonds traded at capital markets. The solution consists of an equilibrium in the strategic interaction between a constrained sovereign and a continuum of uncommunicated bondholders playing a strategic complementaries coordination game.

[1] Andritzky (2006), De Brun and Della Mea (2003), Das et al (2012).

Three milestones paved the way for this transition from loans to bonds (Andritzky, 2006). First, the establishment of a high yield market to host the trading of the (more risky) emerging sovereign debt. Second, the Brady Plan in 1989. This strategy proposed a general securitization of defaulted bank loans to solve the Latin American debt crisis. The new assets thus created were poured into the high yield market adding extra volume and liquidity to it. Third, the liberalization of capital markets which fueled investment flows into assets abroad. As a consequence, he process attained considerable proportions. For instance, the contribution of bank loans to the composition of private stocks of emerging sovereign debt plummeted from 80 per cent in 1980 to 26 per cent in 2000, replaced by bonds (figure 1.1a). In the same direction, there is a sharp increase in the proportion of bonds among the assets subject to default and restructuring in that period (figure 1.1b).

Ex ante, governments might have impulsed this change with the idea that stripping away bargaining power from the creditors would reduce costs, conditions, and would save negotiating time and resources both at issuances and at eventual restructuring processes. However, at least in the case of restructurings, data suggests that the result was the opposite. Bai and Zhang (2012) on Benjamin and Wright database find that the change in the sovereign debt setting reduced the restructuring outcome by 14%. In line with their results, an inspection on Cruces and Trebesch database[2] suggests that, on average, the restructuring outcome reduced from 66% to 39% after its transition from bank loans to bonds.[3] Moreover, regression analysis on this data cannot reject a negative relationship between the type of creditor and the restructuring outcome. (See appendix 1.A). Bai and Zhang attribute the reduction in the restructuring outcome to the contraction in total bargaining time as capital markets reveal creditors' outside option previously kept as private information. I rather simplify the time dimension and focus instead on the effects of the alignment of strategies of uncoordinated investors, finding that it entails a cost for the government in the restructuring outcome.

In the rest of the paper I will refer to the restructuring outcome as the *haircut*. While renegotiating with creditors the restructuring terms on defaulted debt, the government can offer a plan that includes one or many instruments such as increasing maturities, buybacks, cash tenders, bond exchanges, face value reduction among others. The haircut summarizes the total equivalent percentage loss of the investment in terms of the net present value.

This paper analyses the effects of investors' strategic behavior on the haircut. The bondholders deciding whether they accept or reject government's restructuring proposal play a coordination game of strategic complements in which the highest payoffs are obtained when most players simultaneously align their strategies. In this particular case, acceptance when most reject implies a participation cost (for instance reputational)

[2] Cruces and Trebesch updated restructuring database (2014).

[3] The selected period expands from 1998 to 2014 and avoids restructuring processes inside Brady plan as they might have had special features that might add noise to the comparison.

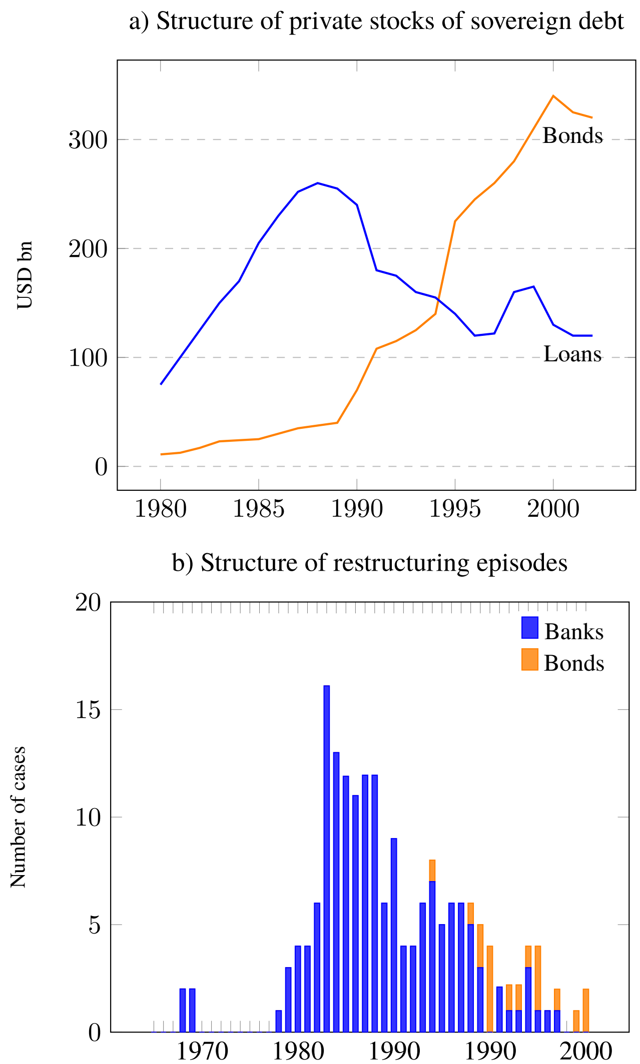

Figure 1.1: Bonds *versus* loans. (a) Structure of external public debt in emerging market countries (stocks of privately held debt). (Borensztein et al., 2004). (b) Finalized restructurings per year. (Anritzky, 2012).

and rejection when most accept entails a lower payment at the secondary market. But while most agents would then prefer to coincide in their decisions, the process itself makes it difficult to happen. On the one hand, once the proposal has been released there is a window of time for investors to decide in which there is not official information about the total acceptance rate achieved. On the other hand, there are thousands of bondholders[4] and they are spread enough so as to successfully connect the necessary amount to secure an agreed position.[5]

Theoretical models of coordination games bring about multiple Nash equilibria. In this particular case, we have a high payoff equilibrium where most investors accept the proposal, and a low payoff equilibrium in which most reject it. Bondholders aligning into the no participating choice would consist of a coordination failure as that is the less beneficial among them (Pareto inferior). Multiple equilibria are theoretical events but do not occur in reality, where only one of the alternative options is observed. So in order to select an equilibrium (to predict which one will be played) I introduce global games. Proposed by Carlsson and Van Damme (1993), this scheme restricts complete information assumption to solve for one equilibrium using iterative deletion of dominated strategies.

The model in this paper is a three stages game that starts with a defaulting government that proposes a restructuring plan to bondholders in stage one. Information regarding the state of the economy at this stage is normalized to zero and only after the proposal has been released to the market, the nature draws and reveals new information about an economic fundamental.[6] In the second stage, bondholders confront a coordination game, deciding its participation in government's proposal. We need to move from complete to incomplete information to introduce the global games scheme that allows us to project the unique equilibrium that solves the restructuring process. Finally, in the third stage, the government observes the equilibrium of the second stage, expressed in a percentage of total agreement, and the information of the fundamental of the economy and decides whether it pays as proposed or continues on default. As multiplicity in the coordination game at the second stage was solved, we can use backwards induction to find the solution to the complete model from the third to the first stage and thus obtain the optimal haircut.

I find that the coordination feature actually restricts government's negotiating power reducing the haircut even when the atomistic nature of creditors would suggest the opposite (for the loss of bargaining power). An illustration using simulation exercises to compare the results of the model with those obtained from a coordination-free case (using Nash bargaining with all negotiation power in government) suggests consis-

[4] As were the cases of Dominica (2004), Pakistan (1999), Uruguay (2003), Seychelles (2009) and even hundred of thousands such as the cases of Ukraine in 2000 (100 thousands) or Argentina in 2005 (600 thousands), (Andritzky, 2006).

[5] According to Das et al (2012) there exist few experiences of successful representative groups in sovereign bond restructuring events.

[6] This information sequence allows us to simplify the signaling feature of the restructuring process: responses of the market to information embedded in government's announcements.

tently that coordination restricts the haircut by 0-10 per cent depending on the value of the parameters used.

Finally, I would like to introduce a pair of comments about the model described below. First, it assumes homogeneous agents which results in a solution with full or none agreement. This simplifying assumption, works properly to explain massive retail behavior, and allows us to work with *plain-vanilla* instruments (with no contract amendments). On the other hand, it prevents the possibility of interior equilibria and, as a consequence, it cannot predict the participation rate. Second, I am modeling stakeholders decisions after default (which is the initial state), so the model does not include an assessment of sovereign's debt status at preliminar stages.

This paper relates with the increasing literature that studies different technical aspects of sovereign debt restructuring. Pitchford and Wright (2007) present an n investors bargaining model of alternative offers to study delay as a consequence of holdout and free riding strategies. They use their model to evaluate how contractual innovations can help solving distortions. Benjamin and Wright (2009) also study delays in restructuring processes and find that they are functional to both, investors and the government, as they allow them to increase their payoffs. As mentioned before, Bai and Zhang (2012) use a private information model to analyze how the disintermediation of sovereign debt has reduced the length of the negotiations. they argue that the secondary market replaced the endless bargaining process between sovereign and investors as a method to reveal each other's outside option. All these papers share in common that they assume a representative bondholder negotiating directly with the government (on behalf of the universe of bondholders) through one or multiple rounds. From another perspective, Bi, Chaumont and Zettlemeyer (2016) focus on the embedded coordination problem in the new market setting but they avoid equilibrium multiplicity proposing an adjustment of investors' payoffs. In another section, they use their model to endogenize the haircut now using sunspots to project the equilibrium at the investors' coordination game. This paper aims to contribute to this literature by addressing an unexplored dimension: the cost on the restructuring outcome of the coordination effort. Besides, unlike previous studies, the novel use of global games to project the final equilibrium allows me to solve multiplicity in terms of the observed fundamental avoiding the use of other tools such as sunspots.

Carlsson and Van Damme (1993) first propose global games to solve multiplicity in coordination games using information constraints. They demonstrate that by introducing some noise into private information, players are forced to estimate others' participation in terms of the distribution of a fundamental then allowing us to solve equilibrium multiplicity by iterative deletion of strictly dominated strategies. This paper constitutes a first application of this technology to solve multiplicity in theoretical models of sovereign debt restructuring.

Other applications include financial markets, taxes, business cycles, prices and other public signals.[7]

Finally, this paper also relates to the broad literature on sovereign debt with endogenous default originated in Eaton and Gersovitz (1981) and in particular to the sub set of models containing endogenous restructuring. The first approach is proposed in Yue (2010), obtaining the haircut as an output of the model and applied to replicate Argentina's crisis in 2001. Asonuma (2016) adds to the process the negotiation over the return of the bond exchanged for the defaulted securities, and Dvorkin et al (2018) a negotiation on the maturity of the new securities. Asonuma and Joo (2019) use one such model to analyze the dynamics of public investment during both default and the restructuring processes. All these quantitative models use Nash bargaining to solve debt renegotiation and, as a consequence, they simplify the n-dimensional feature of the investors and the resulting coordination effects on the outcome. The contribution to these works is a methodology to determine the haircut that takes into account the multiplicity of the investors set. Indeed, incorporating the coordination feature I find a unique haircut in terms of the fundamental and the specific parameters of the model (recovery at secondary market, participation costs, holdouts provisions). Then I take it further and compare the *coordination haircut* I obtain to the Nash bargaining haircut for each possible bargaining power of the agents. I find that both solutions coincide in a scenario where the government possesses complete bargaining power, and differ at the opposite one, where investors own all the bargaining power. In such case, besides, the Nash bargaining solution situates above the coordination one, due to the fact that the coordination of investors end up transferring a cost to the government in terms of the haircut.

The paper is organized as follows. Section 2 presents the model. The structure advances through two successive specifications: a complete information approach with multiple equilibria and a global games approach which introduces incomplete information to refine multiple equilibria. In section 3 I discuss the results using comparative statics. Then I add an illustration to estimate possible coordination costs, and a comparison against the main tool in the standard literature of endogenous haircut in defaulted debt (Nash bargaining). Section 4 concludes and discusses further analysis roads.

1.2 Model

The model consists of a three stages game in which agents interact strategically to determine the haircut of the defaulted debt. There are two kinds of agents, the government (or sovereign) who defaulted on a set of outstanding debt and a continuum of investors holding one unit of defaulted bond each. Bondholders' decisions are modeled within a coordination game which entails multiple equilibria with complete information.

[7] See Taylor and Uhlig (2016) for a detailed survey.

I then introduce incomplete information à la Morris and Shin (1998) using the global games approach as an equilibrium selection device. This setting allows us to identify a threshold in economic fundamental that determines strict dominance regions ruling investors' decisions, and then an optimal haircut that solves the game.

1.2.1 The game in the time line

Figure 1.2 presents the flow of the model. At stage zero, the government defaults on a subset of total outstanding bonds.[8] In the first stage, the government proposes a haircut h, corresponding to the total concession asked to the creditors of defaulted debt. Then nature draws a value for random variable θ which represents sovereign's current payment capacity and reveals it to all the agents. At the second stage, a continuum of creditors with a unit face value bond each, observe both the announced h and the fundamental θ, and determine an individual binary action a_i of acceptance (rejection) of government's proposal. In the last stage, the government knows θ and the acceptance rate (ℓ) and determines the result of the negotiation R. The process ends at this stage in any case. If acceptance rate reaches a required threshold (in line with government's constraint), government pays bondholders and holdouts and exits default; otherwise, it exits negotiation without switching the state. We will solve this game using backwards induction from the last stage to the first one.

1.2.2 Agents' payoffs

Sovereign

In its final decision stage, the government uses all the available information to evaluate the result of the restructuring proposal. The information set at third stage includes both the economic fundamental θ and the aggregate acceptance rate $\ell \in [0,1]$ for the program. We will denote this set with $\mathcal{I}^3_{gov} = \{\theta, \ell\}$, using super scripts to index the stage and under scripts to index the agent of reference. θ represents sovereign's current payment capacity. It is drawn by nature from a probability density with boundaries $[\check{\theta}, \hat{\theta}] \in \mathbb{R}$. The haircut h represents the total equivalent percentage loss on investment for bondholders (as in Sturzenegger and Zettlemeyer (2008)).

The expression in (1.1) describes government's normalized payoffs $G(\ell, h, \theta)$ as a function of the acceptance rate, the haircut and the observed payment capacity. In the first line, the government proposed a restructuring plan with haircut h which gathered total acceptance level ℓ. When the restructuring proposal is successful

[8] These are plain-vanilla contracts, with no special provisions in the event of default.

Nature
Releases a draw of θ

	Stage 0	Stage 1	Stage 2	Stage 3
agent	**Sovereign**	**Sovereign**	**Investors**	**Sovereign**
information available		$\mathcal{I}^1_{gov} = \{\emptyset\}$	I. Complete $\mathcal{I}^2_{inv} = \{h, \theta\}$ II. Incomplete $\mathcal{I}^2_{inv} = \{h, x_i\}$	$\mathcal{I}^3_{gov} = \{\ell, \theta\}$
action	Defaults on a set of bonds	Determines and releases h	Decide $a_i \in \{0,1\}$ then $\ell = \int_{i \in I} a_i di$ is determined	Decide $R \in \{0,1\}$ restructuring result

Figure 1.2: Time line.

($R = 1$) the sovereign pays h to participating bondholders as announced and provisions ν for those rejecting the plan.[9] In consequence, it exits default receiving a boost of $\xi \in [0,1]$ in payment capacity θ.[10] Such impulse could derive from recovering access to capital markets, ease of international sanctions, bailout funds received, implementation of structural reforms, political and financial distress amongst others. In the second line, the proposal gathers a low market acceptance (below governments required threshold), the renegotiation fails ($R = 0$) and government remains in default state with fundamental θ.

$$G(\ell, h, \theta) = \begin{cases} \theta(1+\xi) - [\ell(1-h) + (1-\ell)\nu] & \text{if } R = 1 \\ \theta & \text{if } R = 0 \end{cases} \tag{1.1}$$

For the government to engage in a restructuring plan, it is required that $\theta\xi \geq \ell(1-h-\nu)+\nu$ in (1.1), which implies that post restructuring assets gain has to exceed the total compromised payments.

Let us assume exogenous holdouts payment provision ν lays inside $(1-h, 1]$.[11] When $\nu = 1$ the government

[9] Using data from US corporate debt, holdout premium against early settlers situated at 11% in 115 restructurings between 1992 and 2000 (Fridson and Gao, 2002) and at 30% in 202 restructurings between 1980 1992 (Altman and Eberhart, 1994).
[10] Annual median growth in restructuring countries increases from 1.5% previous the final agreement to 4 to 5% after it (Das, et al (2012) on Trebesch (2011) data set).
[11] This total value would eventually be settled by the government or a judge in court.

pays the full bond value to the share of holdout investors $1-\ell$. Were $\nu = 1-h$, the government would pay the same amount to all debtors whether they accept or not, so that participation rate would turn irrelevant. In this case, government could announce $h = 1$ and yet exit default without any repayment to bondholders. Thus, let us assume $\nu > 1-h$, to abstract from this trivial scenario.

Note that when determining the haircut at stage one government's information set is normalized to $\mathcal{I}^1_{gov} = \{\emptyset\}$ forcing it then to use the distribution of θ.

Bondholders

A bondholder i from a continuum set of measure one of investors with one unit of bond each, chooses an individual action $a_i \in \{0,1\}$, which represents rejection or acceptance of the proposed repayment program. Each bondholder is characterized by a utility function $u(a,\ell,\theta) : \{0,1\} \times [0,1] \times [\check{\theta},\hat{\theta}] \to \mathbb{R}$ in (1.2).

$$
\begin{aligned}
u(0,\ell,\theta) &= \begin{cases} \delta\nu & \text{if} \quad R = 1 \\ 0 & \text{if} \quad R = 0 \end{cases} \\
u(1,\ell,\theta) &= \begin{cases} 1-h-m & \text{if} \quad R = 1 \\ -m & \text{if} \quad R = 0 \end{cases}
\end{aligned}
\tag{1.2}
$$

Both the proposal success (assessed by the government in the last stage) and bondholders payoffs will depend on achieving a minimal level of aggregate acceptance ex-post ($\ell > \ell^*$). However, each agent's information set at decision stage is the singleton $\mathcal{I}^2_{in} = \{\theta\}$. As the aggregate acceptance level is unknown while deciding, each agent uses a uniform prior over others' actions (assigns the same probability to each acceptance rate level $\ell \in (0,1)$).

Rejecting agents ($a_i = 0$ in (1.2)), expect a recovery value of $\delta\nu$ ($\delta \in [0,1]$) for unit of bond in a successful proposal and 0 otherwise. Agents accepting ($a_i = 1$ in (1.2)) receive $1-h$ when negotiation prospers or 0 otherwise, spending in any case participation costs $m > 0$ (for example to acquire the information).

Note that holdouts payments and receipts do not coincide. Here, $1-\delta$ is a non participating loss which should be interpreted as litigation expenses, the probability of not receiving that amount or the wait until that happens.[12] Then $\delta\nu$ is the expected recovery value and as such, should coincide with the bid price of the defaulted bond at the junk market.

[12] Wright (2011) calibrates restructuring costs in its Nash bargaining model as 3.5% of renegotiated debt, from which 90% falls upon lead investor.

This utility function may portray the bulk of investors for whom expected gains might not compensate litigation costs (low δ). The opposite case is the small group of professional holdouts[13] that buys the defaulted debt at secondary market and affords many years of litigation with sovereign at international courts with considerable return. I do not exhaustively include them in this model, as these group generally weights less than 10% of total outstanding (Das et al. 2012), and their behavior does not coincide with the mass of bondholders.

1.2.3 Full information and multiplicity

In this section I use backwards induction to solve the model starting from the last stage to the first one. Complete information entails multiple equilibria in stage 2 which will derive towards incomplete information to solve the game into a unique equilibrium.

Stage 3

Imposing government indifference condition in (1.1) we obtain a minimal threshold $\ell^*(\theta)$ for investors' acceptance ℓ in terms of the fundamental. Only for ℓ values above that level, the government pays as agreed and exits default.

$$\ell^*(\theta) = \frac{\nu - \xi\theta}{\nu - (1-h)} \tag{1.3}$$

Lemma 1 *Government softens the required acceptance level in the case of better economic conditions (higher θ), when it commits to a lower repayment (higher h) or when exiting boost (ξ) is higher. On the contrary, $\ell^*(\theta)$ raises when payments to holdouts ν are higher (if $\theta > \frac{1-h}{\xi}$, and lower otherwise).*

Proof: The cases of ξ, θ and h are trivial.

$$\frac{\partial \ell^*(\theta)}{\partial \nu} = \frac{-(1-h) + \xi\theta}{(\nu - (1-h))^2} \geq 0 \text{ for } \theta \geq \frac{1-h}{\xi} \blacksquare$$

Thus given h, the response to increases in holdout payments is not linear. A compromised government position (θ below $\frac{1-h}{\xi}$) forces it to reduce $\ell^*(\theta)$ each time ν augments with the purpose of gathering additional acceptance to finance the more expensive holdout payments. On the contrary, high fundamental draws allow the government to increase the threshold in response to increases in ν if it estimates the exit might be too expensive.

[13] Some of them are Water Street, Elliot Associates, Cerberus, Davidson Kempner, Aurelius Capital.

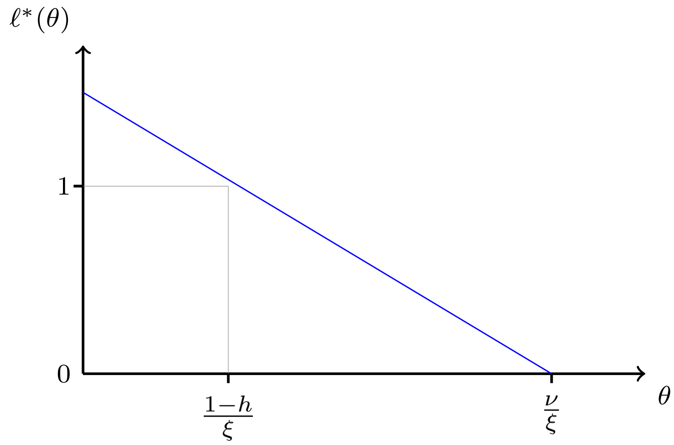

Figure 1.3: Threshold $\ell^*(\theta)$.

Evaluating $\ell^*(\theta)$ in the boundaries of $\ell \in [0,1]$ we can identify strict dominance regions in government's strategies as a function of fundamental θ (see figure 1.3). Thus $\underline{\theta} \equiv \frac{1-h}{\xi}$ and $\bar{\theta} \equiv \frac{\nu}{\xi}$ denote the pair of benchmark values of θ which demands limit levels of engagement, $\ell^*(\underline{\theta}) = 1$ and $\ell^*(\bar{\theta}) = 0$ respectively, to exit default. As a consequence, for $\theta \in [\check{\theta}, \underline{\theta})$ the default is the best response regardless of ℓ due to the extremely reduced payment capacity, while for $\theta \in [\bar{\theta}, \hat{\theta}]$ abundant assets make exiting default the best strategy even at null acceptance level.

Intuitively, using these limits in the utility function (1.1), low values of θ request each circulating bond to accept the haircut h ($\ell^* = 1$) in order to exit default, and in this case all the assets gain $\xi\theta$ would be applied to comply with the program $(1-h)$. On the contrary, high fundamental draws such as $\bar{\theta}$ can afford every obligation and even the holdouts at the highest rate at which it would apply only the asset gain $\xi\theta$.

Stage 2

This stage constitutes a symmetric binary action coordination game. I will use a general version with a continuum of agents. The reader can find a full-information-two-investors illustration in appendix 1.B.

With complete information, each value of θ determines a subgame where investors decide their strategy $a_i(\theta)$ as a best response to others' behavior in the corresponding scenario. We then obtain a pair of equilibrium strategies, depending on variable h and on parameters m, δ and ν. Strategy sets are denoted as follows: each element (parenthesis) represents a range for the observed fundamental value separated by the boundaries $\underline{\theta}$ and $\bar{\theta}$; inside parenthesis there are the responses to low and high aggregate acceptance level (against the threshold ℓ^*) respectively. Thus we have two possibilities:

$$\delta\nu > 1 - h - m \qquad a_i(\theta) = \{(0,0),(0,0),(0,0)\}$$

$$\delta\nu < 1 - h - m \qquad a_i(\theta) = \{(0,0),(0,1),(1,1)\}$$

For $\theta \in [\check{\theta}, \underline{\theta})$ sovereign continues on default regardless of the participation rate ℓ. Participation payoff is strictly below non-participation's : $u(1,\ell,\theta) = -m < u(0,\ell,\theta) = 0$. The strictly dominant strategy is to reject proposal if $m > 0$, for any $\delta\nu$ at both possible scenarios of acceptance (low and high), and as a result $\ell = 0$.

For $\theta \in [\underline{\theta}, \bar{\theta})$ sovereign's decision depends on overall acceptance ℓ. If $\delta\nu < 1 - h - m$ agents should all respond accepting when acceptance is low and rejecting otherwise so we can get both $\ell = 1$ or $\ell = 0$. In other words, there are two pure strategy Nash equilibria in this zone: full and zero investor participation, with the first of them Pareto preferable to the second ($1 - h - m > 0$). If $\delta\nu > 1 - h - m$ agents should reject proposal for all ℓ.

For $\theta \in [\bar{\theta}, \hat{\theta}]$ sovereign pays and exits default $\forall \ell \geq 0$, so agents' participation would not risk any loss (in others' decisions). Agent's best response depends on payoff parameters. When $\delta\nu$ is such that $1-h-m > \delta\nu$, accepting is the payoff dominant strategy, $\ell = 1$ yielding a full participation unique equilibrium. On the contrary, if $1-h-m < \delta\nu$, rejecting the proposal is payoff dominant and $\ell = 0$, in a unique no participation equilibrium. Finally if $1 - h - m = \delta\nu$ agents are indifferent between accepting or rejecting and we obtain a unique mixed strategies equilibrium with $\ell \in [0, 1]$.

Lemma 2 *WLG, we can assume that in equilibrium $\delta\nu < 1 - h - m$ so that holdouts expected payment imposes to the government a limit when determining the haircut level which turns stage 2 into a two equilibria coordination game.*

Proof: If $\delta > 1-h-m$ there is general rejection with $\ell = 0$, and unless $\theta > \bar{\theta}$, government would remain on default. A participating equilibrium would then be bounded below by off proposal payoff: $\delta\nu \leq 1 - h - m$. For agents to be indifferent, government policy should be to pay $1 - h = \delta\nu + m$, and in this case the game delivers an internal solution ($\ell \in (0, 1)$). That equilibrium, however has zero probability of occurrence, because even for a small ϵ fixing the proposal so that $1-h+\epsilon-m = \delta\nu$ gets full acceptance. So government's convenience will drive to a full participating equilibrium by setting $1 - h - m > \delta\nu$ ■

A consequence of Lemma 2, is that the incentive to hold out the acceptance in this model results from agents avoiding to lose $-m$ in case the proposal fails, and not from an extra expected payment ($\delta\nu < 1 - h - m$ is obviously not a preferred payoff).

Summing up, in stage 2, each value of fundamental θ determines a bayesian subgame that solves into unique zero and full participation Nash equilibria outside $(\underline{\theta}, \bar{\theta}]$. However, values of θ inside that range produce multiplicity with both full participating and no participating simultaneous equilibria.

Stage 1

In this stage the government determines the optimal haircut h as the value that maximizes its expected utility $G^*(h)$ in θ. Remember that the exact value of the fundamental θ has not been revealed yet and, as a consequence, the government assumes it takes some value in $[\check{\theta}, \hat{\theta}]$.

In the previous stage, we could identify dominance regions in bondholder's strategic behavior. These however, yield multiple equilibria (all bondholders participate and sovereign exists default and no one participates and sovereign remains in default) which does not allow us to determine a unique result for the government's problem.

1.2.4 Incomplete information

In this section I introduce some noise in investors' information using a global games approach to refine multiplicity. The uncertainty about θ still remains, but once the nature makes a draw from the distribution of θ, the government observes it directly, while investors only receive a noisy signal of it. Then uncertainty affects both types of agents but incomplete information will only affect investors. The new features do not modify stage 3 so this section will focus on the dynamics of the model over stages two to one.

Stage 2

Now investors' information set is $\mathcal{I}^2_{in} = \{x_i\}$ where $x_i = \theta + \sigma\varepsilon_i$ corresponds to a noisy private realization of fundamental θ with ε_i independent standard normal perturbations and $\sigma > 0$ a scaling parameter. In this setting, a strategy $s_i(x_i)$, for creditor i is a decision rule that maps from the space of signals to that of actions: $\mathbb{R} \rightarrow \{0, 1\}$. Accordingly, an equilibrium is a profile of strategies that maximizes each creditor's expected payoff, conditional on the information available.

It is important to note that investor's payoffs still depend on the realized value of θ not on his private signal which literature calls common values model. Morris and Shin (2002), propose a general framework to solve such games consisting of a series of sufficient conditions for the payoffs gain function: $u(1,\ell,\theta)-u(0,\ell,\theta)$. For those payoff functions compliant, it is possible to obtain a strategy profile s that conforms a unique Bayesian Nash equilibrium of this game.

Proposition 1 *Let θ^* be defined as in (1.4).*

$$\theta^*(h)=\frac{1}{\xi}\left(1-h+m\frac{\nu-(1-h)}{1-h-\delta\nu}\right) \tag{1.4}$$

For any $\tau>0$, there exists $\bar{\sigma}>0$ such that for all $\sigma\leq\bar{\sigma}$, if strategy s_i survives iterated deletion of strictly dominated strategies, then $s_i(x_i)=0$ for all $x_i\leq\theta^*-\tau$, and $s_i(x_i)=1$ for all $x_i\geq\theta^*+\tau$.

Proof: Appendix 1.C.

A take away from the demonstration of Proposition 1, is threshold $\theta^*(h)\in[\underline{\theta},\bar{\theta}]$ in (1.4) which discriminates regions of strict dominance in the game: when noise converges to zero, agents with signals $x_i>\theta^*(h)$ accept proposal and those that observe signals below that level reject it.

Characterization of the threshold $\theta^*(h)$

Some algebra on (1.4) allows us to express the threshold $\theta^*(h)$ as a convex combination of government payments to each set of investors (accepting and rejecting proposal) in (1.5), with the ratio of net costs to benefits of accepting as weightings.

$$\theta^*(h)=\frac{1}{\xi}\left((1-h)(1-\frac{m}{1-h-\delta\nu})+\nu\frac{m}{1-h-\delta\nu}\right) \tag{1.5}$$

In this expression, we can see that low values of participation cost m move the threshold away from holdouts payment as they increase the probability of a high acceptance result.

Although $\theta^*(h)$ is determined by the complete set of parameters in the model, not all of them affect it through the same channel. For instance, m and δ contribute directly via agents' payoff function. ξ has an indirect effect on $\theta^*(h)$ by affecting government's threshold $\ell^*(\theta)$, and then modifying the total acceptance required to shift the state, and the probability of different payoffs with it. Finally, the haircut h exerts both a direct (through agents decisions) and an indirect (through $\ell^*(\theta)$) effect on the threshold. These relations are the key content of the corollary below.

Corollary 1 $\frac{\partial\theta(h)^*}{\partial\xi} \leq 0$, $\frac{\partial\theta(h)^*}{\partial\nu} \geq 0$, $\frac{\partial\theta(h)^*}{\partial\delta} \geq 0$, $\frac{\partial\theta(h)^*}{\partial} \geq 0$.

Proof: Appendix 1.E.

Increases in holdout receipts, holdouts payments or participation costs discourage investors acceptance while higher after-restructuring boost (ξ) encourages it.

When participation costs m or holdout receipts δ rise, net participation benefit reduces (for higher costs or a better outside option) increasing the probability of rejection (as $\theta^*(h)$ expands). Note that high δ or m values compresses the haircut to the minimum, and even result in complete rejection in the limit, when the outside option is competitive or participation is too expensive.

Better after-restructuring conditions (higher ξ) make the program more attractive for the government whom then reduces the participation threshold $\ell^*(\theta)$ to augment its probability of success. This, in turn, reduces the chance of losing m for accepting a failed program and then encourages investor participation (by compressing θ^*). In the case of holdouts payments ν, they produce the opposite effect through the same channel, due to the fact that increases in ν force the government to tighten ℓ^*. As ν converges to $1-h$, $\theta^*(h)$ converges to $\underline{\theta} = \frac{1-h}{\xi}$, its lowest boundary, expanding the acceptance zone towards its maximum extension. ν at its lowest boundary, implies lower obligations for the government that reduces $\ell^*(\theta)$ in order to encourage participation to the maximum (as $\frac{\partial\ell^*(\theta)}{\partial\nu} \geq 0$ in $\theta \geq \frac{1-h}{\xi}$). On the contrary, $(1-h)$ converging towards ν pushes θ^* to $\bar{\theta}$ reducing the acceptance region to the minimum: in this case there would be little gain from the program and the government must require $\ell^*(\theta) = 1$, decreasing the probability of success.

θ as a function of the haircut

Corollary 2 $\theta^*(h)$ *is a U-shaped convex function that minimizes at:*

$$h_{in} = 1 - \delta\nu - \sqrt{m(1-\delta)\nu}$$

Proof: Appendix 1.E.

So there is first an indirect effect through government's threshold ℓ^*. As h increases, government commits to pay a smaller share of defaulted debt and the relief allows it to reduce the minimal acceptance threshold for the proposal (1.3). For investors, that implies a contraction in the probability of losing m (when entering a failed proposal). As a consequence, the acceptance region increases by decreasing the lower limit $\theta^*(h)$. The direct effect, on the contrary, affects $\theta^*(h)$ through investors' payoff: a higher h, determines a reduction in marginal utility of accepting the proposal thus discouraging investors that in consequence extend the non acceptance region (increasing $\theta^*(h)$).

Drawing upon convexity we can optimize $\theta^*(h)$ in h (appendix 1.E). In this unique value, which we denominate h_{in}, the threshold reaches its minimum, implying that the accepting region expands most (and the probability of a successful proposal with it) for a given set of parameters.

$$1 - h_{in} - \delta\nu = \sqrt{m(1-\delta)\nu} \tag{1.6}$$

This result contrasts with the idea that every sufficiently low h coordinates investors in the full acceptance equilibrium: here, however, there is a trade off for h that originates in strategic behavior. Although bondholders' investment recovery rate increases with a lower haircut (so we would expect higher participation rate), at the same time government situation deteriorates and the threshold $\ell^*(\theta)$ is raised, demanding a higher market support for the program which in turn increases the probability of failure and the lose of m for those whom participated (determining a lower participation rate).

That h_{in} level, however, would not be attainable by the government. For instance, replacing (1.6) in (1.3) with $\ell^* = 1$ we obtain the lowest payment capacity required to exit default when the haircut is set at h_{in} level, which we will denote by θ_0.

$$\theta_0 = \frac{1}{\xi}\left(\delta + \sqrt{m(1-\delta)\nu}\right) > \frac{1}{\xi}\left(\delta + 2\sqrt{m(1-\delta)\nu} - 1\right) = \theta^*_{in} \tag{1.7}$$

As it shows in (1.7),[14] had the government set the haircut in h_{in}, it would enter default for every observed $\theta \in [\check{\theta}, \theta_0]$ which is larger than $[\check{\theta}, \theta_{in}]$. So at least at this level of haircut, the government constraint adds $\theta_{in} - \theta_0$ to the probability of keeping at default.

Another consequence of convexity in $\theta^*(h)$ is that for any h not equal to h_{in} there is a pair of values that results in the same $\theta^*(h)$, and as a consequence in the same acceptance level. The effect of this duality in government decision is the main content of Proposition 2.

[14] $\theta_{in} > \theta_0$ requires $\sqrt{m(1-\delta)\nu} > 1$ which would be false due to $1 \geq \nu > \delta \geq 0$ and m small.

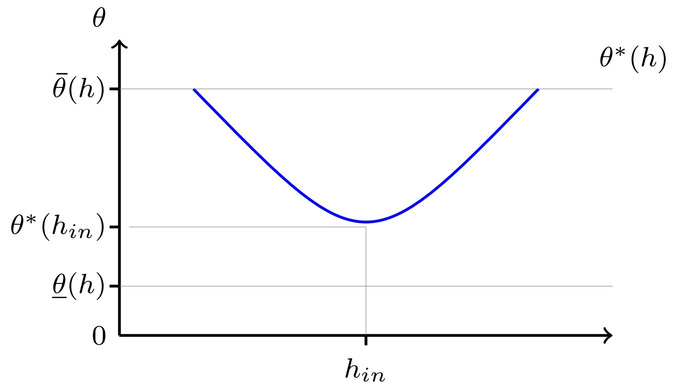

Figure 1.4: Threshold as a function of haircut.

Proposition 2 *The set of feasible haircut values* $\mathfrak{h}$ *for the government to solve its maximization problem is* $\mathfrak{h} = [h_{in}, 1 - \delta\nu - m)$.

Proof: Trivial as government's utility is an increasing function on h■

Observe finally, that the presence of m is critical to determine the optimal h. In this model, it is the existence of participation costs which forces both extreme values of h_{co} to differ from $1 - \delta\nu$. So increasing costs demand a lower haircut to keep investors entrance.

Stage 1

Having identified investors' optimal responses to θ, now the government computes the optimal haircut, as the value that maximizes its expected utility in (1.1) conditional to θ.

$$G^*(h) = \int_{\check{\theta}}^{\theta^*(h)} \theta f(\theta) d\theta + \int_{\theta^*(h)}^{\hat{\theta}} (\theta(1+\xi) - (1-h)) f(\theta) d\theta \tag{1.8}$$

We reduce the expression (1.8) by applying both simple and truncated expectation formulas.

$$G^*(h) = E[\theta] + \big(1 - F(\theta)\big)\big(\xi E[\theta|\theta > \theta^*] - (1-h)\big) \tag{1.9}$$

Differentiating (1.9) and solving for h, we obtain h_{co}, the haircut level that maximizes government's utility.

Proposition 3 *h_{co} solves:*

$$\frac{1 - F(\theta^*(h))}{f(\theta^*(h))} = \frac{\partial\theta^*(h)}{\partial h}\big(\xi\theta^*(h) - (1-h)\big) \tag{1.10}$$

Proof: Appendix 1.F.

From the second derivative of (1.9) we extract the conditions for a unique solution, which relay explicitly on the distribution function of θ.

Proposition 4 *Necessary conditions to solve government's maximization problem into a unique value h_{co}.*

1. $\frac{\partial f(\theta^*(h))}{\partial\theta^*(h)} \geq 0$
2. $\frac{\frac{\partial f(\theta^*(h))}{\partial\theta^*(h)}}{f(\theta^*(h))} \leq \frac{\frac{\partial^2\theta^*(h)}{\partial^2 h}}{\left(\frac{\partial\theta^*(h)}{\partial h}\right)^2}$

Proof: Appendix 1.G.

The haircut as a function of model parameters

Corollary 3 *Under conditions stated in Corollary 4 h_{co} is: decreasing in ν, δ and increasing in ξ.*

Proof: Appendix 1.H.

The haircut reduces with holdouts expectation δ and participation costs m as the government tries to compensate bondholders whose outside option and costs increased to secure its participation. In the limit, zero costs allow the government to situate the haircut at the highest possible value $h_{co} = 1 - \delta\nu$.

Holdouts payments ν, also exerts a negative effect on the haircut through both government's and investor's strategic decisions. Unlike the previous case, increases in ν do not only affect investors utilities but the threshold $\theta^*(h)$: higher payments to holdouts reduce the probability of success which expands the rejection region. In addition, a higher ν erodes government net assets position which then boosts required participation ratio ℓ^*. In both cases, the government compensates discouraged investors by reducing h. This double negative effect shows in figure 1.5, where it is evident that δ has a slightly lower effect on h_{co}.

The haircut increases with post exit boom, via again both investors' and government's channels. In the first case, the exit boom ξ affects the threshold θ^* by increasing the acceptance zone once the government reduced

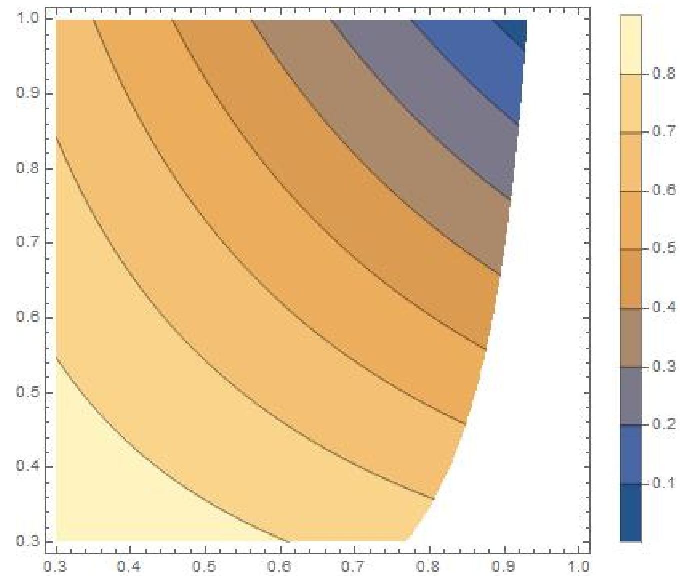

Figure 1.5: Simulated haircut on model parameters. Contour plot: h_{co} as a function of δ (abscissa) and ν (ordinate), with $m = 0.013$, $\xi = 0.04$, assuming an exponential distribution of θ with parameter $\lambda = 0.12$. White zone is restricted for $h > 1 - \delta\nu - m$.

$\ell^*(\theta)$ with the expected better results after the negotiation. Besides, the increase in post-negotiation boom encourages the government to reduce the haircut in order to secure the agreement and obtain a higher utility.

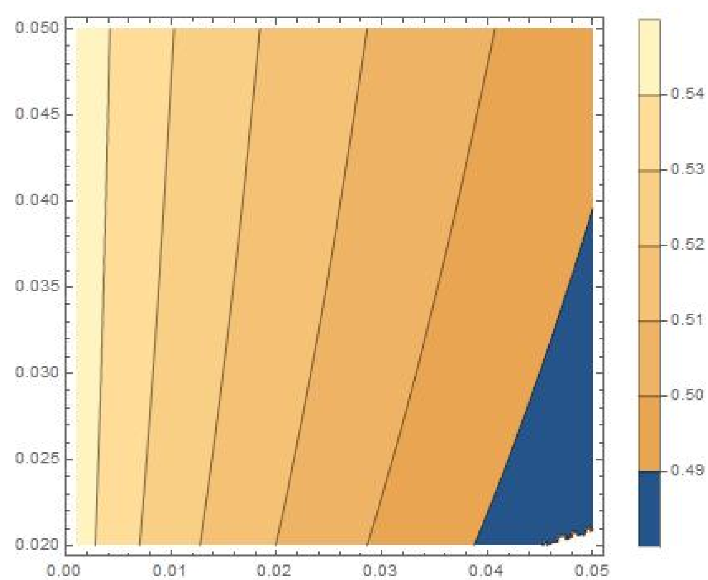

Figure 1.6: Simulated haircut on model parameters. Contour plot h_{co} as a function of m (abscissa) and ξ (ordinate), with $\delta = 0.66$, $\nu = 0.66$, assuming an exponential distribution of θ with parameter $\lambda = 0.16$. White zone is restricted for $h > 1 - \delta\nu - m$.

1.3 Final assessment

This section dedicates to the appraisal of the model. In the first part, there is a comparison between the coordination haircut and the haircut widely used in restructuring literature, obtained using Nash bargaining. I find that under certain conditions the Nash bargaining haircut overstates the coordination haircut. In the second part, I propose a procedure to assess the cost for the government of changing from bank financing towards market financing, which we will call coordination costs. We find that these costs can reach a 7% level in simulations, which accounts for almost a quarter of the average difference between banks and bonds financing.

1.3.1 Coordination vs Nash bargaining haircut

Most endogneous haircut models of sovereign debt default use Nash bargaining to determine the equilibrium recovery value. This section aims to analyze whether coordination strategies amongst investors introduce any difference in the regular outcome. For this objective, I compute a regular Nash bargaining solution and another that incorporates coordination. Then, I compare the resulting haircut in both of them.

Early endogenous sovereign default models such as Aguiar and Gopinath (2006) and Arellano (2008) assumed an exogenous recovery value after default (equal to zero or randomly set). Yue (2010) proposes an extension to endogenize the haircut through a Nash bargaining solution. After Yue (2010), many endogenous haircut quantitative models use one-shot Nash bargaining, to determine equilibrium haircut.[15] This model, although a helpful simple tool, relies critically on participants' bargaining power, which is not observable in the data.

We can think of two main resulting caveats. The first one, is that when the model is used ex-ante to estimate an equilibrium haircut, the result relies critically on the bargaining power assumed. We show in this section that another market configuration can affect the bargaining power thus yielding different restructuring results. The second one is when the model is used ex post, calibrating the bargaining power to obtain an observed haircut. This is not trivial as it might have significant effects on the variables; for example, in Yue (2010) variations in the bargaining power of about 18% produce variations of 3% in the average debt to output ratio, 13% in the average recovery rates and significant changes in correlations. Quantitative estimations of evolution after default assume a fixed bargaining power across time, which might not capture accurately sovereings' conditions. The model in this paper provides an endogenous determination of such bargaining

[15] See for instance Asonuma (2012), Bai, Zhang (2012).

power in terms of tractable parameters.

We start computing the regular Nash bargaining haircut, which we will denote by h_{nb}, in terms of the parameters of this model. (See Appendix 1.J). This is the h value that maximizes the Nash product of expected utilities (in θ) below.

$$\omega(h) = \alpha\ell\big((1+\xi)\bar{\theta} - (1-h)\ell - \nu(1-\ell) - \bar{\theta}\big)^{\alpha}\big(1-h-m-\delta\nu\big)^{1-\alpha} \tag{1.11}$$

In (1.11) I am using α and $1-\alpha$ to denote government's and investors' bargaining power respectively, and $\bar{\theta}$ for the mean of fundamental θ. Government's payoff function in (1.1) provides the gain of restructuring and the outside option available, θ. In the case of bondholders, the outside option to participate yields a payoff of $\delta\nu$ as no agent alone can change the overall restructuring result (to get 0 in case proposal fails). The Nash bargaining haircut in this setting is presented in (1.12) and the corresponding bargaining power α_{nb} in (1.13). Note that homogeneity in investors' utility function implies that if proposal succeeds $\ell = 1$.

$$h_{nb} = \alpha(1-m-\delta\nu) - (1-\alpha)(\xi\bar{\theta} - 1) \tag{1.12}$$

$$\alpha_{nb} = \frac{\xi\bar{\theta} - (1-h)}{\xi\bar{\theta} - (m+\delta\nu)} \tag{1.13}$$

For the Nash bargaining haircut with coordination strategies among investors, I use the result in Proposition 1. The main difference with the general model is that now the Nash product $\omega(h)$ includes the truncated mean of the fundamental $\mathring{\theta}(\theta^*(h))$, instead of $\bar{\theta}$, adding a new dependence on h to consider during the maximization.

$$\omega(h) = \alpha\big((1+\xi)\,\mathring{\theta}(\theta^*(h)) - (1-h)\ell - \nu(1-\ell)\big)^{\alpha}\big(1-h-m-\delta\nu\big)^{1-\alpha} \tag{1.14}$$

I will call $h_{nb,co}$ the coordination haircut that solves equation (1.15), and $\alpha_{nb,co}$ its associated bargaining power in (1.16).

$$\alpha\big(1-h-m-\delta\nu\big)\left(\xi\frac{\partial\,\mathring{\theta}(h)}{\partial\theta^*(h)}\frac{\partial\theta^*(h)}{\partial h} + 1\right) - (1-\alpha)\big(\xi\,\mathring{\theta}(\theta^*(h)) - (1-h)\big) = 0 \tag{1.15}$$

$$\alpha_{nb,co} = \frac{\xi\,\mathring{\theta}(\theta^*(h)) - (1-h)}{\xi\,\mathring{\theta}(\theta^*(h)) + \big(1-h-m-\delta\nu\big)\xi\frac{\partial\,\mathring{\theta}(h)}{\partial\theta^*(h)}\frac{\partial\theta^*(h)}{\partial h} - \delta\nu} \tag{1.16}$$

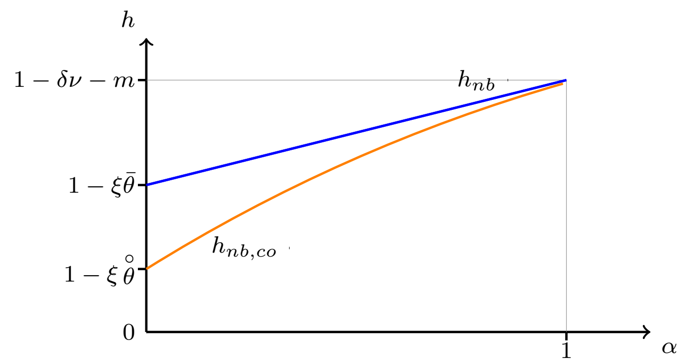

Figure 1.7: Nash and coordination haircut.

Lemma 3 *Under certain conditions, $h_{nb,co}$ and h_{nb} are both monotonous increasing functions of α.*

Proof: Appendix 1.K.

Lemma 3 discusses monotonocity and slope properties of the haircuts obtained through the Nash bargaining process. Now we can use the previous results to map each haircut h_{nb} and $h_{nb,co}$ in the α space and compare them. In the next proposition, we state that there is a decreasing wedge between haircuts, and that the haircut that considers coordination strategies remains below the general one.

Proposition 5 *Under conditions in Lemma 3,*

1. $h_{nb,co} \leq h_{nb}$ *for all* $\alpha \in [0, 1]$.

2. $h_{nb} - h_{nb,co}$ *is decreasing in* α.

Proof: Appendix 1.J.

Using proposition 5 and the limit values of both measures of haircut we plotted h_{nb} and $h_{nb,co}$ against α in figure 1.7. Now let us look at the limit values. When government's bargaining power is complete, $\alpha = 1$, both measures yield a haircut of $1 - \delta\nu - m$, the maximum feasible value for h_{co} according to Lemma 2. On the opposite extreme, when $\alpha = 0$ we can have one or many values for h depending on the distribution function of θ. Under Lemma 3 conditions, we can find monotonous h_{co} functions increasing over both $\alpha = 0$ and $\alpha = 1$ which intersect h_{nb} in $\alpha = 1$. In those cases, the Nash bargaining haircut situates above the coordination haircut.

This statement means that when using Nash bargaining to obtain an equilibrium outcome, we could be overestimating the haircut if the debt was issued in capital markets where investors play coordination strategies,

Parameter		Value	Source
Holdout payment	ν	0.835	Holdout premium
Discount on holdout payment	δ	0.375	Moody's avge. prices
Recovery after restructuring	ξ	0.004	Das (2012)
Participation costs	m	0.010	

Table 1.1: Model parameters in simulations

and where there is no room for communication between them (to coordinate strategies at least among the main players). The size of this error increases with the bargaining power of investors.

Wrapping up, a more detailed micro founded model signals that there is a loss of bargaining power that derives from the bondholders playing a coordination game.

1.3.2 Nash bargaining and coordination costs

This section amis to reach a measure for coordination costs. I start by measuring the haircut in a scenario in which the government keeps all the bargaining power, but it confronts a unique investor instead of a continuous of unorganized agents. These features might be captured by a Nash bargaining haircut with government's full bargaining power, which entails $\alpha = 1$. So we get:

$$h_{nb,\alpha=1} = 1 - \delta\nu - m \tag{1.17}$$

Which coincides with the net outside option value for investors and the upper bound for h according to lemma 2. It then can be calculated coordination costs as the difference in both haircuts:

$$coordination\, costs = h_{nb,\alpha=1} - h_{co} \tag{1.18}$$

Parameters in the simulations are presented in table 1.1. The value of ν was set following the average recovery value in Cruces and Trebesch database plus the weighted average of holdout premium in Fridson and Gao (2002) and Altman and Eberhart. δ value was selected in order to target market price $\nu\delta = 0.3$ as in the weighted average presented in Moody's investors service data report (2017) for 30-day post-default price or distressed exchange trading price. The value of ξ was set to 0.04 in what we understand would be a conservative value if we consider Das (2012) average economy's recovery rates after restructuring. m is a non observable participating cost it was calibrated to be 1% in total investment $m = 0.010$.

Using the previous calibration, I simulate those coordination costs (figure 1.8) and we can observe that they situate in 0-3.5% range. The coordination costs reduce with discount δ but increase with holdout payment ν. $h_{nb,\alpha=1}$ has a fixed reduction rate of δ with ν. The increase in coordination costs derive from a higher reduction rate of haircut h.

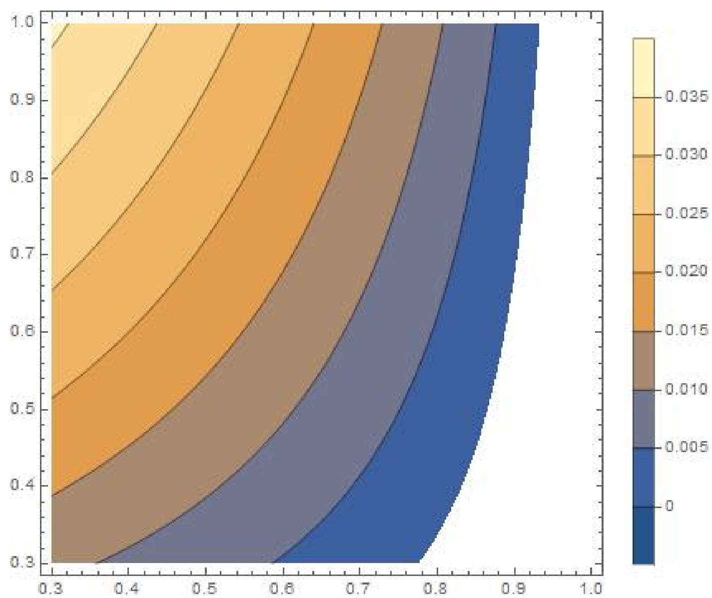

Figure 1.8: Coordination costs on model parameters. Contour plot: coordination costs as a function of δ (abscissa) and ν (ordinate), with $m = 0.01$, $\xi = 0.04$, assuming an exponential distribution of θ with parameter $\lambda = 0.16$. White zone is restricted for $h > 1 - \delta\nu - m$.

In figure 1.9, I present simulated coordination costs as a function of participation costs and recovery after default, ranging in 1-7% values. Although with some non linearities, for the higher ranges, costs reduce with ξ. This is an expected result because the haircut increases in ξ (3) while this variable does not affect $h_{nb,\alpha=1}$. In the case of participation costs, they have an increasing effect on coordination costs. As they affect constantly $h_{nb,\alpha=1}$ (upwards) we can say that increases in m have an increasing reduction power on the haircut.

It is important to note that these results are in line with the fact that the disintermediation in sovereign debt financing determined a reduction in the haircut. In other terms, in this new market setting, the government has a cost derived from the fact that it does not bargain directly with its counter parties and instead it has to target spread investors and their higher order beliefs with the proposed haircut (all agents has to think that the rest will participate as it is an appropriate proposal). Costs around 7% count for almost 25% of the difference in banks vs bonds debt, which results a significant figure in terms of money losses for a government in distress.

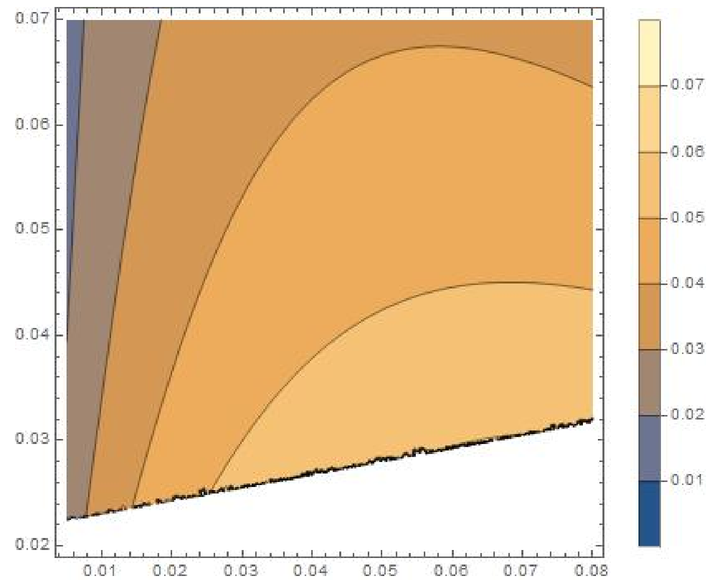

Figure 1.9: Coordination costs on model parameters. Contour plot: coordination costs as a function of m (abscissa) and ξ (ordinate), with $\delta = 0.375$, $\nu = 0.835$, assuming an exponential distribution of θ with parameter $\lambda = 0.16$. White zone is restricted for $h > 1 - \delta\nu - m$.

1.4 Conclusions

After the strong financial disintermediation process that started in 1980, negotiating an exit from defaulted debt turned a complex process. It requires to align the decisions of many thousands of investors with limited information regarding the progress of the process in a relative short period of time. The final result relies on the aggregated behavior of that mass of spread and mostly unconnected agents which are simultaneously elaborating their strategies. Surprisingly, instead of benefiting the sovereign, investor coordination lent the terms and outcomes of the restructuring towards the bondholders, with shorter negotiations and lower haircuts.

Game theory applied to sovereign debt restructuring entails using backward induction into a multi stage game in which one of the stages contains a multiple equilibria coordination scheme. These particularly occur over non limit economic conditions, where internal non trivial solutions seem most plausible. I then apply global games to solve multiplicity, which requires among other things, allocating small participation costs and noise in investors' information sets. With multiplicity thus solved, the model endogenizes the haircut which is uniquely set in terms of the parameters, including the distribution of an economic fundamental. Working with homogeneous bondholders yields full participation avoiding contract coordination devices (special majorities, exit consents, etc).

I embed my model into a Nash bargaining structure, and find that under certain conditions, a pure Nash bargaining result obtains higher haircut values than the one with coordination for each possible level of bargaining power. This might be an important drawback to consider when using Nash bargaining to determine equilibrium outcomes in this new market setting (spread investors).

We can conclude that by introducing coordination strategies, we set a *de facto* constrain on government's bargaining power, as it is forced to reduce its proposal in order to coordinate bondholders towards the participation option (trying to align second order beliefs: here, it is what each investor think that other's think about accepting/rejecting the proposal). Using simulated results, coordination costs range in 0-7.0% of the proposed haircut, explaining a significant portion of the difference in banks vs bonds haircuts observed in data.

1.5 References

Aguiar, Mark; Gita, Gopinath (2006): Defaultable Debt, Interest Rates and the Current Account. *Journal of International Economics* 69 (1), pp. 64-83.

Altman, Edward; Eberhart, Allan (1994): Do Seniority Provisions Protect Bondholders' Investments? *The Journal of Portfolio Management* Summer 1994, 20 (4), pp. 67-75

Andritzky, Jochen (2006): Sovereign Default Risk Valuation: Implications of Debt Crises and Bond Restructurings. Volume 582 of *Lecture Notes in Economics and Mathematical Systems*, Vol. 582. Springer Berlin Heidelberg.

Arellano, Cristina (2008): Default Risk and Income Fluctuations in Emerging Economies. *American Economic Review* 98 (3), pp. 690–712.

Asonuma, Tamon; Trebesch, Christoph (2016): Serial Sovereign Defaults and Debt Restructurings. *IMF Working Papers* 16/66.

Asonuma, Tamon; Joo, Hyungseok (2019): Sovereign Debt Overhang, Expenditure Composition and Debt Restructurings. *Forthcoming in the Journal of the European Economic Association.*

Bai, Yan; Zhang, Jing (2012): Duration of sovereign debt renegotiation. *Journal of International Economics*, 86 (2), pp. 252–268.

Benjamin, David; Wright, Mark (2009): Recovery before redemption: A theory of delays in sovereign debt renegotiations. Unpublished.

Bi, Ran; Chamon, Marcos; Zettelmeyer, Jeromin (2016): The Problem that Wasn't. Coordination Failures in Sovereign Debt Restructurings. In IMF Econ Rev 64 (3), pp. 471–501.

Borensztein, Eduardo; Chamon, Marcos; Jeanne, Olivier; Mauro, Paolo; Zettelmeyer, Jeromin (2004): Sovereign Debt Structure for Crisis Prevention. *IMF Occasional Paper 237.*

Borensztein, Eduardo; Panizza, Ugo (2009): The Costs of Sovereign Default. *IMF Staff Papers* 56 (4), pp. 683–741.

Carlsson, Hans; van Damme Eric (1993): Global Games and Equilibrium Selection. *Econometrica* 61 (5), pp. 989–1018.

Cole, Harold; Kehoe, Timothy (2000): Self-Fulfilling Debt Crises. *Review of Economic Studies* 67 (1), pp. 91–116.

Cruces, Juan J.; Trebesch, Christoph (2013): Sovereign Defaults. The Price of Haircuts. *American Economic Journal: Macroeconomics*, 5 (3), pp. 85–117.

Das, Udaibir; Papaioannou, Michael; Trebesch, Christoph (2012): Sovereign Debt Restructurings 1950–2010: Literature Survey, Data, and Stylized Facts. *IMF Working papers* 12/203.

De Brum, Julio; Della Mea, Umberto (2003): Una aproximación de mercado a la reestructuración de la deuda soberana: lecciones de la experiencia uruguaya. *Revista de Economía - Segunda Epoca Vol. X N 2 - Banco Central del Uruguay*

Dvorkin, Maximiliano; Sanchez, Juan; Sapriza, Horacio; Yurdagul, Emircan (2018): Sovereign Debt Restructurings. *FRB St. Louis Working Paper* No. 2018-13.

Eaton, Jonathan; Gersovitz, Mark (1981): Debt with Potential Repudiation: Theoretical and Empirical Analysis. *Review of Economic Studies* 48 (2), pp. 289–309.

Fridson, Martin; Gao, Yan (2002): Defaulted bond returns by seniority class. *Journal of Fixed Income* Fall 2002, 12 (2), pp.50-57

Moody's (2017): Sovereign Default and Recovery Rates, 1983-2016. *Data report 30 June 2017*

Morris, Stephen; Shin, Hyun Song (2003): Global games: theory and applications. *Advances in Economics and Econometrics* (Proceedings ofthe Eighth World Congress of the Econometric Society). Cambridge University Press.

Morris, Stephen; Shin, Hyun Song (2004): Coordination risk and the price of debt. *European Economic Review* 48 (1), pp. 133–153.

Panizza, Ugo; Sturzenegger, Federico; Zettelmeyer, Jeromin (2009): The Economics and Law of Sovereign Debt and Default. *Journal of Economic Literature* 47 (3), pp. 651–698.

Pitchford, R.; Wright, M. L. J. (2007): Restructuring the Sovereign Debt Restructuring Mechanism. In Federal Reserve of Minneapolis.

Pitchford, R.; Wright, M. L. J. (2012): Holdouts in Sovereign Debt Restructuring. A Theory of Negotiation in a Weak Contractual Environment. *Review of Economic Studies* 79 (2), pp. 812–837.

Sturzenegger, Federico (2004): Tools for the analysis of debt problems. *Journal of Reconstructing Finance* 1 (1), pp. 201–203.

Sturzenegger, Federico; Zettelmeyer, Jeromin (2008): Haircuts. Estimating investor losses in sovereign debt restructurings, 1998–2005. *Journal of International Money and Finance* 27 (5), pp. 780–805.

Taylor, John; Uhlig, Harald (ed., 2016): Handbook of Macroeconomics. *Elsevier*, edition 1, 2 (1), pp 1-1344.

Trebesch, Christoph (2011): Sovereign Default and Crisis Resolution. *Dissertation Manuscipt, Free University of Berlin*.

Wright, Mark (2001): Coordinating Creditors. *New Perspectives on Reputation and debt* 95 (2), pp. 288–392.

Yeyati, Eduardo Levy; Panizza, Ugo (2011): The elusive costs of sovereign defaults. *Journal of Development Economics* 94 (1), pp. 95–105.

Yue, Vivian (2010): Sovereign Default and Debt renegotiation. In *Journal of International economics* 80 (2), pp. 176-187.

Appendix

1.A Haircuts, some empirical facts

In this section I analyze some empirical facts from haircuts using the base of Cruces & Trebsech (2014 updated). Our variable of interest is the restructuring haircut (a measure of the total concession from debtors to creditors). The preferred measure is the one proposed in Sturzenegger & Zettelmeyer (2006 and 2008) in which the haircut summarizes the present value of investors' losses:

$$H^i_{SZ} = 1 - \frac{\text{Present value of New debt}(r^i_t)}{\text{Present value of old debt}(r^i_t)} \tag{1.19}$$

Where r^i_t is the yield prevailing at the time of the restructuring, which is considered a good proxy of debtor's default risk after the restructure.[16]

	Observations	Mean	SD	Mean	Max
SZ Haircut	187	.40	.28	-.098	.97
By type of creditor					
Bank debt restructuring	165	.37	.28	-.098	.97
Bond debt restructuring	22	.37	.22	.04	.76
By era					
1978–1989	99	.25	.19	-.098	.93
1990–1997	48	.51	.28	.03	.92
1998–2013	40	.52	.32	-.08	.97

Table 1.A.1: Haircut summary. Haircut measured as in Sturzenegger Zettelmeyer (2006) by type of creditor and era. Source: Cruces and Trebsesch (2013) updated with 2014 new data.

Table 1.A.1, presents a summary of results. As it can be noted, there were 187 sovereign restructurings during

[16] Cruces and Trebesch (2013).

1978-2013 with a mean haircut of 40%. A break up by type of creditor shows that there are only 22 cases of bond restructuring, although the average haircut does not seem to differ with its characteristic. Finally, the period when the process took place seems to be an important element. In fact, before 1990 haircuts where set around 25% while after that date they doubled up to 50% on average. These results suggest that the year in which the process took place will be a variable to control for in order to better understand the determinants of the haircut.

In Table 1.A.2, splitting the sample by era, I compare haircut means by type of creditor. With the whole sample, consistent with previous result, both groups bank and bond restructuring show no significant differences on mean averages. Splitting the sample using the variable era, I find that most bond restructurings situate at recent years (19 out of 22 cases in 1998-2020). Within this period there is almost a 30% difference in the haircut negotiated in restructuring, which results a significant difference according to the test.

	Bank	Bond	Difference	p-value
All sample	0.37	0.38	0.00	0.96
N	165	22		
1998-2013	0.66	0.39	0.27	0.01
N	19	21		

Table 1.A.2: Mean haircut by type of creditor and era.

Table 1.A.3 presents regression analysis results for the logarithm of haircut when using bonds against bank loans controlling by era, and others (logarithms of GDP pc and GDP). As the reader can note, once controlled by year, creditors nature results significant at least at 5% level and with a persistent negative sign. A little algebra on results indicates that the ratio of haircuts in bonds to banks situates at 43% and 52% in models (1) and (2) respectively.

Haircut	(1)	(2)
Bond dummy	-0.84***	-0.64*
	(0.25)	(0.25)
Constant	Yes	Yes
Decade	Yes	Yes
Other controls	No	Yes
N	180	162
adj. R^2	0.16	0.29

Table 1.A.3: Model estimates for the logarithm of Haircut. Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Other controls: GDP and GDP per capita in logarithmic units.

1.B Two investors illustration

Consider a two investors game where each of them has full information about economic fundamental θ. The game is sketched in the tables below, with investor one's strategies presented in rows and investor two's in columns. Payoffs are ordered in rows inside each cell respectively.

Government proposes a concession program whose result depends on observed fundamental θ. This variable in time, determines the payoff matrix and finally game equilibria as detailed below.

If $\theta < \underline{\theta}$ then $\ell^*(\theta) > 1$ in (1.3), meaning that the program fails even at 100% acceptance. In this case, there are no benefits for accepting investors: acceptance has a cost of $-m$ while rejection costs 0. Payoffs matrix is presented in Table 1.B.1 that shows weakly dominance in the unique Nash equilibrium in pure strategies, $s = \{s_1, s_2\} = \{Reject, Reject\}$.

		Investor 2: Accepts	Investor 2: Rejects
Investor 1	Accepts	$-m$ $-m$	$-m$ $\underline{0}$
	Rejects	$\underline{0}$ $-m$	$\underline{0}$ $\underline{0}$

Table 1.B.1: Payoffs matrix $\theta \in [\check{\theta}, \underline{\theta})$

Draws of $\theta \in [\bar{\theta}, \hat{\theta}]$ produce $\ell^*(\theta) \leq 0$, unconditional restructuring. The game pays $1-h-m$ to the accepting investors and δ to the rejecting ones (assume here that $\delta < 1-h-m$). This game in Table 1.B.2, solves into a unique pure strategies Nash equilibrium, with weakly dominance in $s = \{s_1, s_2\} = \{Accept, Accept\}$.

		Investor 2: Accepts	Investor 2: Rejects
Investor 1	Accepts	$\underline{1-h-m}$ $\underline{1-h-m}$	$\underline{1-h-m}$ $\delta\nu$
	Rejects	$\delta\nu$ $\underline{1-h-m}$	$\delta\nu$ $\delta\nu$

Table 1.B.2: Payoffs matrix $\theta \in [\bar{\theta}, \hat{\theta}]$

Finally, Table 1.B.3 portrays $\theta \in [\underline{\theta}, \bar{\theta})$ and then $\ell(\theta)^* \in [0, 1]$. Suppose we can identify a θ_0 value under which $\ell^*(\theta) > 0.5$: government requires both investors accepting to exit default. In this case (left box in the table) we get two equilibria in strict dominant strategies with mirror behavior. Above that level of θ government exits default even if only one investor participates (right box in the table). Now acceptance is a weakly dominant strategy and we get an all participating equilibrium.

Investor 1		Investor 2: Accepts	Investor 2: Rejects
	Accepts	$\underline{1-h-m}$ $\underline{1-h-m}$	$-m$ 0
	Rejects	0 $-m$	$\underline{0}$ $\underline{0}$

θ low $\rightarrow \ell^* > 0.5$

Investor 1		Investor 2: Accepts	Investor 2: Rejects
	Accepts	$\underline{1-h-m}$ $\underline{1-h-m}$	$\underline{1-h-m}$ $\delta\nu$
	Rejects	$\delta\nu$ $\underline{1-h-m}$	$\delta\nu$ $\delta\nu$

θ high $\rightarrow \ell^* = 0.5$

Table 1.B.3: Payoffs matrix $\theta \in [\underline{\theta}, \theta]$

Remember a strategy is an action plan for every contingency. Then we have:

$$a_i = \{(Reject, Reject), (Accept, Reject), (Accept, Accept), (Accept, Accept)\}$$

Where elements represent contingent fundamental subsets $\{[\check{\theta}, \underline{\theta}), [\underline{\theta}, \theta_0), (\theta_0, \bar{\theta}], [\bar{\theta}, \hat{\theta}]\}$, and inside each parenthesis we portray optimal responses to others' accept and reject decisions respectively.

The multiplicity originates at the second region, where we can get indistinctly all accepting and all rejecting equilibria.

1.C Equilibrium uniqueness

From (1.2) we construct in (1.20) the action gain function $\pi(\ell, \theta) : [0, 1] \times \mathbb{R}^+ \rightarrow \mathbb{R}$ as $\pi(\ell, \theta) = u(1, \ell, \theta) - u(0, \ell, \theta)$.

$$\pi(\ell, \theta) \equiv \begin{cases} 1 - h - \delta\nu - m & \text{if} \quad \ell \geq \ell^* \\ \\ -m & \text{if} \quad \ell < \ell^* \end{cases} \tag{1.20}$$

According to (1.20), agents make a $1 - h - \delta\nu - m$ net profit for accepting the agreement when it proceeds, and a $-m$ net loss for accepting it when it fails. Now we demonstrate equation (1.20) compliance with each condition:

- **C1: Action monotonicity: incentive to choose action $a = 1$ is increasing in ℓ.**
  $\pi(l, \theta)$ is a step function in ℓ, discontinuous at $\ell = \ell^*(\theta)$. If $1 - h - \delta\nu - m > -m$ then $\pi(\ell^{*+}, \theta) = 1 - h - \delta\nu - m > \pi(\ell^{*-}, \theta) = -m$ and the function is increasing in other players' actions, implying strategic complementarity in the game. If $1 - h - \delta\nu - m < -m$, $\pi(\ell, \theta)$ is decreasing in ℓ: rejecting government proposal is a strictly dominant strategy and the game turns into a strategic

substitutes structure. Coordination equilibrium requires $1-h-\delta\nu-m>0>-m$ to ensure action monotonicity■

- **C2: State monotonicity: the incentive to choose $a=1$ is non decreasing in fundamental θ.**
  $\ell^*_\theta(\theta)<0$ in (1.3), implies that increases in θ reduce the lower bound of $\pi(\ell^+,\theta)$, expanding the region where action $a=1$ is a dominant strategy (given C1 compliance: $\pi(\ell^{*+},\theta)>\pi(\ell^{*-},\theta)$). This can be interpreted as: worse economic conditions weaken government requirement of minimal acceptance which increases probability of a successful process■

- **C3: Strict Laplacian state monotonicity: ensures there is a unique crossing for a player with Laplacian beliefs.** Intuitively, it derives from the fact that players in the game assume a uniform distribution to the proportion ℓ of other players choosing action $a=1$.

$$\int_{\ell=0}^{\ell=1}\pi(\ell,\theta)d\ell=\int_{\ell=0}^{\ell=\ell^*(\theta)}-m\,d\ell+\int_{\ell=\ell^*(\theta)}^{\ell=1}1-h-\delta\nu-m\,d\ell=0 \tag{1.21}$$

  Equation (1.21) can be expressed as a linear function on θ with a unique solution in θ^* (appendix 1.D).

$$\theta^*=\frac{1}{\xi}\left(1-h+m\frac{\nu-(1-h)}{1-h-\delta\nu}\right) \tag{1.22}$$

- **C4: Uniform limit dominance:** There exists a pair of values $\{\theta_0,\theta_1\}\in\mathbb{R}$, and $\varepsilon\in\mathbb{R}_{++}$, such that $[1]$ $\pi(\ell,\theta)\leq-\varepsilon$ for all $\ell\in[0,1]$ and $\theta\leq\theta_0$; and $[2]$ there exists θ_1 such that $\pi(\ell,\theta)>\varepsilon$ for all $\ell\in[0,1]$ and $\theta\geq\theta_1$.
  From (1.3) we know that $\ell^*(\theta)$ is a linear function with value 1 at $\underline{\theta}=\frac{1-h}{\xi}$ (figure 1). Using $\ell^*_\theta(\theta)<0$ in lemma 1 we can define $\theta_0=\frac{1-h}{\xi}-\varepsilon$, such that $\ell^*(\underline{\theta})=1+\varsigma$ (for ς very small) and as a consequence $\pi(\ell,\theta)=-m$ for all $\ell\in[0,1]$ and every $\theta\leq\theta_0$■

  The analogue demonstration can be done for $[2]$, taking in this case $\theta_1=\frac{\nu}{\xi}+\varepsilon$.

- **C5: Continuity:**$\int_{\ell=0}^{\ell=1}g(\ell)\pi(\ell,x)d\ell$ **is continuous with respect to signal x and density g.**
  $\pi(\ell,x)$ presents only one point of discontinuity at $\ell=\ell^*$. Thus for a continuous density $g(\ell)$, discontinuity acquires zero mass, then the integral is weakly continuous■

- **C6: Finite expectations of signals:** $\int_{z=0}^{z=+\infty}zf(z)dz$ **is well defined for integration.**
  With $z=\frac{x-\theta}{\sigma}$, $f(z)$ is defined as a continuous density function with $\int_{z=0}^{z=1}f(z)<+\infty$■

As payoff gain function complies with conditions C1-C6, Proposition 1 states that the game can be solved to deliver a unique equilibrium in terms of fundamental θ. (Morris and Shin (2002))

1.D Laplacian state monotonicity proof

There is a single cross on θ^* for action gain function π.

$$\int_{\ell=0}^{\ell=1} \pi(\theta,\ell)d\ell = \int_{\ell=0}^{\ell=\ell^*(h)} -m\,d\ell + \int_{\ell=\ell^*(h)}^{\ell=1} 1-h-\delta\nu-m\,d\ell = 0$$

$$-m\ell^*(h) + (1-h-\delta\nu-m)(1-\ell^*(h)) = 0$$

Now we replace ℓ^* for the expression in (1.3).

$$(1-h-\delta\nu-m) - (1-h-\delta)\frac{\nu-\theta\xi}{\nu-(1-h)} = 0$$

$$(1-h-\delta\nu-m)(\nu-(1-h)) - (1-h-\delta)(\nu-\theta\xi) = 0$$

$$\theta = \frac{-(1-h-\delta\nu-m)(\nu-(1-h)) + (1-h-\delta)\nu}{(1-h-\delta)\xi}$$

$$\theta^* = \frac{1}{\xi}\left(1-h+m\frac{\nu-(1-h)}{1-h-\delta\nu}\right) \blacksquare$$

1.E Analysis of threshold θ^* partial derivatives

To determine (1.23), (1.24) and(1.26) I use $\nu > 1-h \geq \delta$, $m > 0$ and $\xi \leq 1$:

$$\frac{\partial\theta^*(h)}{\partial m} = \frac{1}{\xi}\left(\frac{\nu-(1-h)}{1-h-\delta\nu}\right) \geq 0 \tag{1.23}$$

$$\frac{\partial\theta^*(h)}{\partial\xi} = -\frac{1}{\xi^2}\left(1-h+\frac{m(\nu-(1-h))}{1-h-\delta\nu}\right) \leq 0 \tag{1.24}$$

$$\frac{\partial \theta^*(h)}{\partial \delta} = \frac{1}{\xi}\left(\frac{m(\nu-(1-h))}{(1-h-\delta\nu)^2}\right) \geq 0 \tag{1.25}$$

$$\frac{\partial \theta^*(h)}{\partial \nu} = \frac{1}{\xi}\left(m\frac{(1-\delta)(1-h)}{(1-h-\delta\nu)^2}\right) \geq 0 \tag{1.26}$$

$$\frac{\partial \theta^*(h)}{\partial h} = \frac{1}{\xi}\left(-1+\frac{m(1-\delta)\nu}{(1-h-\delta\nu)^2}\right) \tag{1.27}$$

Convexity of $\theta^*(h)$ in h allows us to find optimal value h_{in} in (1.28) by equating its derivative to zero:

$$1-h_{in}-\delta\nu = \sqrt{m(1-\delta)\nu} \tag{1.28}$$

1.F Government optimal haircut

We want to find $h \in [0,1]$ that maximizes government's expected utility $G^*(h) = E_\theta[G(h)]$ with $\theta \sim p[\check{\theta}, \hat{\theta}]$:

$$h_{co} = argmax_{h\in[0,1]}\{G^*(h)\} \tag{1.29}$$

Government expected utility is presented in (1.31). Note that for values of θ above θ^* all agents accept, so that $\ell = 1$.

$$G^*(h) = \int_{\check{\theta}}^{\theta^*} \theta f(\theta)d\theta + \int_{\theta^*}^{\hat{\theta}} (\theta(1+\xi)-(1-h))f(\theta)d\theta \tag{1.30}$$

Some rearranging of terms yields:

$$G^*(h) = \int_{\check{\theta}}^{\hat{\theta}} \theta f(\theta)d\theta + \xi\int_{\theta^*}^{\hat{\theta}} \theta f(\theta)d\theta - (1-h)\int_{\theta^*}^{\hat{\theta}} \theta f(\theta)d\theta \tag{1.31}$$

$$G^*(h) = E[\theta] + \big(1-F(\theta^*(h))\big)\big(\xi E[\theta|\theta>\theta^*] - (1-h)\big) \tag{1.32}$$

Taking first derivative on (1.32) in h:

$$\frac{\partial G^*(h)}{\partial h} = -f(\theta^*(h))\frac{\partial \theta^*(h)}{\partial h}\big(\xi E[\theta|\theta>\theta^*(h)] - (1-h)\big)...$$

$$+(1-F(\theta^*(h))\left(\underbrace{\frac{f(\theta^*(h))}{1-F(\theta^*(h))}\frac{\partial\theta^*(h)}{\partial h}\xi\big(E[\theta|\theta>\theta^*(h)]-\theta^*(h)\big)}_{(a)}+1\right)=0$$

Where I am using in (a) differentiation properties of truncated expectation.

$$\frac{\partial G^*(h)}{\partial h}=1-F(\theta^*(h))+f(\theta^*(h))\frac{\partial\theta^*(h)}{\partial h}\left(1-h-\xi\theta^*(h)\right) \tag{1.33}$$

So the optimal haircut h_{co} is the h that solves the equation in (1.34).

$$1-F(\theta^*(h))=f(\theta^*(h))\frac{\partial\theta^*(h)}{\partial h}\big(\xi\theta^*(h)-(1-h)\big) \tag{1.34}$$

1.G Concavity of government's problem

Taking the second derivative in (1.33) with respect to h:

$$\frac{\partial^2 G^*(h)}{\partial^2 h}=-f(\theta^*(h))\frac{\partial\theta^*(h)}{\partial h}+\frac{\partial f(\theta^*(h))}{\partial\theta^*(h)}\left(\frac{\partial\theta^*(h)}{\partial h}\right)^2\big(1-h-\xi\theta^*(h)\big)$$

$$+f(\theta^*(h))\frac{\partial^2\theta^*(h)}{\partial^2 h}\big(1-h-\xi\theta^*(h)\big)+f(\theta^*(h))\frac{\partial\theta^*(h)}{\partial h}\left(-1-\xi\frac{\partial\theta^*(h)}{\partial h}\right)$$

From proposition 2, we know that government will set $h\in[h_{in},1]$ where $\frac{\partial\theta^*(h)}{\partial h}\geq 0$. In this model, a successful proposal obtains $\ell=1$ which, using indifference condition of equation (1), implies that $\xi\theta>1-h$. This inequality will hold for θ^* and all the θ values above it (where the economy exists default).

$$\frac{\partial^2 G^*(h)}{\partial^2 h}=\underbrace{-f(\theta^*(h))\underbrace{\frac{\partial\theta^*(h)}{\partial h}}_{\geq 0}\left(\underbrace{2+\xi\frac{\partial\theta^*(h)}{\partial h}}_{>0}\right)}_{\leq 0}$$

$$+\left(\frac{\partial f(\theta^*(h))}{\partial\theta^*(h)}\underbrace{\left(\frac{\partial\theta^*(h)}{\partial h}\right)^2}_{\geq 0}+\underbrace{f(\theta^*(h))\frac{\partial^2\theta^*(h)}{\partial^2 h}}_{\geq 0}\right)\underbrace{\big(1-h-\xi\theta^*(h)\big)}_{\leq 0}$$

From the last expression, we can extract conditions for a global maximum that solves government's problem with uniqueness. Note that $\frac{\partial f(\theta^*(h))}{\partial h}$ plays a critical role.

1.H Analysis of h_{co} partial derivatives

To analyze partial effects of model parameters on optimal haircut over $[\check{\theta}, \hat{\theta}]$, I will use implicit function theorem.

For this section, let us denote a vector **x** of model parameters such that $\mathbf{x} = [\xi, \nu, \delta, m]$ using x_i to refer to one of its elements. Thus, equation (10) can be written as follows:

$$1 - F(\theta^*(h, \mathbf{x})) = f(\theta^*(h, \mathbf{x}))\frac{\partial \theta^*(h, \mathbf{x})}{\partial h}\big(\xi\theta^*(h, \mathbf{x}) - (1-h)\big) \tag{1.35}$$

Total differentiating (1.35) with respect to x_i yields:

$$-f(\theta^*(h, \mathbf{x}))\left(\frac{\partial \theta^*(h, \mathbf{x})}{\partial h}\frac{dh}{dx_i} + \frac{\partial \theta^*(h, \mathbf{x})}{\partial x_i}\right) =$$

$$\begin{aligned}
&\frac{\partial f(\theta^*(h, \mathbf{x}))}{\partial \theta^*(h, \mathbf{x})}\left(\frac{\partial \theta^*(h, \mathbf{x})}{\partial h}\frac{dh}{dx_i} + \frac{\partial \theta^*(h, \mathbf{x})}{\partial x_i}\right)\left(\frac{\partial \theta^*(h, \mathbf{x})}{\partial h}\right)\big(\xi\theta^*(h, \mathbf{x}) - (1-h)\big) \\
&+f(\theta^*(h, \mathbf{x}))\left(\frac{\partial^2 \theta^*(h, \mathbf{x})}{\partial^2 h}\frac{dh}{dx_i} + \frac{\partial \theta^*(h, \mathbf{x})}{\partial h \partial x_i}\right)\big(\xi\theta^*(h, \mathbf{x}) - (1-h)\big) \\
&+f(\theta^*(h, \mathbf{x}))\frac{\partial \theta^*(h, \mathbf{x})}{\partial h}\big(\xi\left(\frac{\partial \theta^*(h, \mathbf{x})}{\partial h}\frac{dh}{dx_i} + \frac{\partial \theta(h, \mathbf{x})}{\partial x_i}\right) + \frac{dh}{dx_i}\big)
\end{aligned}$$

Now let us solve for $\frac{dh}{dx_i}$ and observe the signs for the general case in (1.36):

$$\frac{dh}{dx_i} = \frac{\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial \theta^*(h,\mathbf{x})}\frac{\partial \theta^*(h,\mathbf{x})}{\partial x_i}\underbrace{\frac{\partial \theta^*(h, \mathbf{x})}{\partial h}}_{\geq 0}\gamma + \underbrace{f(\theta^*(h, \mathbf{x}))}_{\geq 0}\left(\frac{\partial^2 \theta^*(h,\mathbf{x})}{\partial h \partial x_i}\underbrace{\gamma}_{\geq 0} + \underbrace{\frac{\partial \theta^*(h, \mathbf{x})}{\partial h}}_{\geq 0}\big(1 + \xi\frac{\partial \theta^*(h,\mathbf{x})}{\partial x_i}\big)\right)}{-\underbrace{f(\theta^*(h, \mathbf{x}))}_{\geq 0}\left(\underbrace{\frac{\partial^2 \theta^*(h, \mathbf{x})}{\partial^2 h}\gamma}_{>0} + \underbrace{\frac{\partial \theta^*(h, \mathbf{x})}{\partial h}\left(\xi\frac{\partial \theta^*(h, \mathbf{x})}{\partial h} + 2\right)}_{\geq 0}\right) - \frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial \theta^*(h,\mathbf{x})}\underbrace{\left(\frac{\partial \theta^*(h)}{\partial h}\right)^2\gamma}_{\geq 0}} \tag{1.36}$$

Where I am using Proposition 2 in $\frac{\partial \theta^*(h,\mathbf{x})}{\partial h} \geq 0$. I am also using the fact that in equilibrium $\ell = 1$ and that the proposal must comply indifference condition on government's utility in (1.1) so that $\xi\theta^*(h, \mathbf{x} - (1-h))$, which I will refer as γ in the rest of this section, is higher or equal than zero . In addition, I use the property of positive probability density functions.

Following, I discuss the response of coordination haircut according to the slope of the density function of fundamental θ.

1.H.1 Density derivative $\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial \theta^*(h,\mathbf{x})}$ equals zero

In this case, the results will depend on the signs of $\frac{\partial \theta^*(h,\mathbf{x})}{\partial x_i}$ and the crossed derivative of $\theta^*(h, \mathbf{x})$ on parameters, all of them affecting the numerator of the expression in (1.37), while the denominator is negative. I additionally use the fact that $\theta^*(h, \mathbf{x})$ is a U-shaped parabola in the second derivative respect to h.

$$\frac{dh}{dx_i} = \frac{\left(\frac{\partial^2 \theta^*(h,\mathbf{x})}{\partial h \partial x_i} \underbrace{\gamma}_{\geq 0} + \frac{\partial \theta^*(h,\mathbf{x})}{\partial x_i} \left(\underbrace{\frac{\partial \theta^*(h,\mathbf{x})}{\partial h} \xi + 1}_{>0} \right) \right)}{\underbrace{\underbrace{\frac{\partial^2 \theta^*(h,\mathbf{x})}{\partial^2 h}}_{\geq 0} \gamma + \underbrace{\frac{\partial \theta^*(h,\mathbf{x})}{\partial h} \left(\xi \frac{\partial \theta^*(h,\mathbf{x})}{\partial h} + 2 \right)}_{\geq 0}}_{<0}} \tag{1.37}$$

Lemma 4 *Regarding the cross-second derivatives of $\theta^*(h, \boldsymbol{x})$:*

$$\frac{\partial^2 \theta^*(h,\boldsymbol{x})}{\partial h \partial m} \geq 0, \ \frac{\partial^2 \theta^*(h,\boldsymbol{x})}{\partial h \partial \nu} \geq 0, \ \frac{\partial^2 \theta^*(h,\boldsymbol{x})}{\partial h \partial \delta} \geq 0, \ \frac{\partial^2 \theta^*(h,\boldsymbol{x})}{\partial h \partial \xi} \leq 0$$

Proof: Appendix 1.I

1. Sub-case $\frac{\partial \theta^*(h,\mathbf{x})}{\partial x_i} > 0$ corresponds to m, ν, δ according to Corollary (1). Because of Lemma 4 we know that crossed derivatives are greater or equal than zero and as a consequence $\frac{dh}{dm} \leq 0$, $\frac{dh}{d\nu} \leq 0$, $\frac{dh}{d\delta} \leq 0$.
2. Sub-case $\frac{\partial \theta^*(h,\mathbf{x})}{\partial x_i} \leq 0$, which corresponds to $x_i = \xi$ according to Corollary 1. Using Proposition 2, we know that numerator in (1.37) is negative and so $\frac{dh}{d\xi} \geq 0$.

1.H.2 Density derivative $\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial\theta^*(h,\mathbf{x})}$ is positive

$$\frac{dh}{dx_i}=\underbrace{\frac{\underbrace{\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial\theta^*(h,\mathbf{x})}}_{>0}\frac{\partial\theta^*(h,\mathbf{x})}{\partial x_i}\underbrace{\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}}_{\geq 0}\gamma+f(\theta^*(h,\mathbf{x}))\left(\frac{\partial^2\theta^*(h,\mathbf{x})}{\partial h\partial x_i}\underbrace{\gamma}_{\geq 0}+\frac{\partial\theta^*(h,\mathbf{x})}{\partial x_i}\left(\underbrace{\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}}_{>0}\xi+1\right)\right)}{-\underbrace{f(\theta^*(h,\mathbf{x}))}_{\geq 0}\left(\underbrace{\frac{\partial^2\theta^*(h,\mathbf{x})}{\partial^2 h}}_{\geq 0}\gamma+\underbrace{\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}\left(\xi\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}+1\right)}_{>0}\right)-\underbrace{\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial\theta^*(h,\mathbf{x})}}_{>0}\underbrace{\left(\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}\right)^2}_{\geq 0}\gamma}}_{<0} \tag{1.38}$$

Using lemma 4, as in the previous case we get $\frac{dh}{d\nu}\leq 0, \frac{dh}{dm}\leq 0$, $\frac{dh}{d\delta}\leq 0$ and $\frac{dh}{d\xi}\geq 0$.

1.H.3 Density derivative $\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial\theta^*(h,\mathbf{x})}$ is negative

In this case, Proposition 5 for a unique solution to government's maximization problem (conditions for a concave government's expected utility) secures that denominator in (1.39) will always be negative, so let us now look the numerator to conclude.

$$\frac{dh}{dx_i}=\frac{\underbrace{\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial\theta^*(h,\mathbf{x}))}}_{<0}\frac{\partial\theta(h,\mathbf{x})}{\partial x_i}\underbrace{\frac{\partial\theta^*(h,\mathbf{x}))}{\partial h}}_{\geq 0}\gamma+f(\theta^*(h,\mathbf{x}))\left(\frac{\partial^2\theta^*(h)}{\partial h\partial x}\underbrace{\gamma}_{\geq 0}+\frac{\partial\theta^*(h,\mathbf{x})}{\partial x_i}\left(\underbrace{\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}}_{>0}\xi+1\right)\right)}{\underbrace{-\underbrace{f(\theta^*(h,\mathbf{x}))}_{\geq 0}\left(\underbrace{\frac{\partial^2\theta^*(h,\mathbf{x})}{\partial^2 h}}_{\geq 0}\gamma+\underbrace{\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}\left(\xi\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}+1\right)}_{\geq 0}\right)-\underbrace{\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial\theta^*(h,\mathbf{x})}}_{<0}\underbrace{\left(\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}\right)^2}_{\geq 0}\gamma}_{<0}} \tag{1.39}$$

In this case, we get again $\frac{dh}{d\nu}\leq 0, \frac{dh}{dm}\leq 0$, $\frac{dh}{d\delta}\leq 0$ and $\frac{dh}{d\xi}\geq 0$ only in the case that condition (1.40) holds.

$$\frac{\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial\theta^*(h,\mathbf{x})}}{f(\theta^*(h,\mathbf{x}))}\leq\frac{\frac{\partial^2\theta^*(h,\mathbf{x})}{\partial h\partial x_i}\gamma+\frac{\partial\theta^*(h,\mathbf{x})}{\partial x_i}(\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}\xi+1)}{\frac{\partial\theta^*(h,\mathbf{x})}{\partial x_i}\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}\gamma} \tag{1.40}$$

Meaning that in those cases in which the density function of θ presents a negative slope, for the partial derivatives of haircut respect to parameters to hold, it is needed a regular behavior in peakedness[17] of the

[17] This characteristic is not equivalent to the kurtosis as this last one regards also the tails of the distribution while peakedness don't.

distribution function. In other words, not a small set of values in the distribution with too high frequencies.

1.I Cross derivatives

In this appendix I demonstrate Lemma 4 of cross derivatives of $\theta^*(h, \mathbf{x})$. Using $\mathbf{x}$ as a vector of parameters ad x_i denoting one of its elements.

$$\frac{\partial^2 \theta^*(h, \mathbf{x})}{\partial h \partial m} = \frac{1}{\xi} \frac{(1-\delta)\nu}{(1-h-\delta\nu)^2} \geq 0$$

The case of $\frac{\partial^2 \theta^*(h, \mathbf{x})}{\partial h \partial m} = 0$ is for $\delta = 1$.

$$\frac{\partial^2 \theta(h, \mathbf{x})}{\partial h \partial \nu} = \frac{1}{\xi} \frac{m(1-\delta)(1-h-\delta\nu)^2 - 2(1-h-\delta\nu)(-\delta)m(1-\delta)\nu}{(1-h-\delta\nu)^4}$$

$$\frac{\partial^2 \theta(h, \mathbf{x})}{\partial h \partial \nu} = \frac{m(1-\delta)(1-h-\delta\nu)}{\xi} \left(\frac{1-h+\delta\nu}{(1-h-\delta\nu)^4} \right) \geq 0$$

So the cases of $\frac{\partial^2 \theta(h, \mathbf{x})}{\partial h \partial \nu} = 0$ are for $\delta = 1$ or $m = 0$.

$$\frac{\partial^2 \theta(h, \mathbf{x})}{\partial h \partial \delta} = \frac{1}{\xi} \frac{-m\nu(1-h-\delta\nu)^2 - 2(1-h-\delta\nu)(-\nu)m(1-\delta)\nu}{(1-h-\delta\nu)^4}$$

$$\frac{\partial^2 \theta(h, \mathbf{x})}{\partial h \partial \delta} = \frac{m\nu(1-h-\delta\nu)}{\xi} \left(\frac{\nu-(1-h)+(1-\delta)\nu}{(1-h-\delta\nu)^4} \right) \geq 0$$

Where the case of $\frac{\partial^2 \theta(h, \mathbf{x})}{\partial h \partial \delta} = 0$ is for $m = 0$.

$$\frac{\partial^2 \theta(h, \mathbf{x})}{\partial h \partial \xi} = -\frac{1}{\xi^2} \left(-1 + \frac{m(1-\delta)\nu}{(1-h-\delta\nu)^2} \right) \frac{\partial^2 \theta(h, \mathbf{x})}{\partial h \partial \xi} = -\frac{1}{\xi^2} \left(\frac{-(1-h-\delta\nu)^2 + m(1-\delta)\nu}{(1-h-\delta\nu)^2} \right) \leq 0$$

1.J Nash bargaining haircut

In this section we introduce our setting into a Nash generalized bargaining process, to now obtain the haircut as the allocation that maximizes Nash product in (1.41). $\omega(h)$ represents net output (restructuring success against failure) to distribute between agents: government and bondholders. Each of them has some bargaining power α and $1-\alpha$ respectively which will determine its share of the total product.

$$\omega(h) = (\bar{\theta}(1+\xi) - [(1-h)\ell + \nu(1-\ell)] - \theta)^{\alpha}(1-h-m-\delta\nu)^{1-\alpha} \tag{1.41}$$

It is important to note that if one investor i does not participate, it would not change the result of the agreement. So its individual outside option is to receive $\nu\delta$ and not zero (as the case in which the proposal fails).

1.J.1 General case

I obtain the haircut as the allocation that maximizes the generalized Nash product $\omega(h)$ of expected utilities in θ.

$$h = argmax\{\omega(h)\}$$

I am using $\bar{\theta}$ to denote the mean of the random variable θ. Now let us take the derivative of (1.41) in h and then solve for h.

$$\alpha\ell(\xi\bar{\theta} - (1-h)\ell - \nu(1-\ell))^{\alpha-1}(1-h-m-\delta\nu)^{1-\alpha} = \\ (1-\alpha)(\xi\bar{\theta} - \nu - \ell(1-h-\nu))^{\alpha}(1-h-m-\delta\nu)^{-\alpha} \tag{1.42}$$

Using that in a succesful proposal participation rate is $\ell = 1$, I get:

$$h_{nb} = \alpha(1-m-\delta\nu) - (1-\alpha)(\xi\bar{\theta} - 1) \tag{1.43}$$

We can get the α value for the equilibrium h as well.

$$\alpha = \frac{\xi\bar{\theta} - \nu - \ell(1-h-\nu)}{(\ell(-m-\delta\nu) + \xi\bar{\theta} - \nu - \ell(-\nu))} \tag{1.44}$$

With $\ell = 1$:

$$\alpha_{nb} = \frac{\xi\bar{\theta} - (1-h)}{\xi\bar{\theta} - (m+\delta\nu)} \tag{1.45}$$

1.J.2 Coordination case

We can think of a Nash Bargaining case where we introduce the coordination feature. In that case, we would use the truncated mean of the payment capacity θ instead of the plain mean, because for the agreement to succeed we need agents participating and that only occurs for those values of θ above or equal $\theta^*(h)$. This feature adds a new dependence link of $\omega(h)$ to h through $\theta^*(h)$. I will denote $\mathring{\theta}(\theta^*(h))$ the truncated mean and will take again the first derivative on (1.41) considering the new features.

$$\alpha\big(\xi\,\mathring{\theta}(\theta^*(h)) - (1-h)\ell - \nu(1-\ell)\big)^{\alpha-1}\big(1-h-m-\delta\nu\big)^{1-\alpha}\left(\xi\frac{\partial\,\mathring{\theta}(h)}{\partial\theta^*(h)}\frac{\partial\theta^*(h)}{\partial h} + \ell\right)$$
$$-(1-\alpha)\big(\xi\,\mathring{\theta}(\theta^*(h)) - (1-h)\ell - \nu(1-\ell)\big)^{\alpha}\big(1-h-m-\delta\nu\big)^{-\alpha} = 0$$

After some arrangements, using equilibrium $\ell = 1$, we get:

$$\alpha\big(1-h-m-\delta\nu\big)\left(\xi\frac{\partial\,\mathring{\theta}(h)}{\partial\theta^*(h)}\frac{\partial\theta^*(h)}{\partial h} + 1\right) - (1-\alpha)\big(\xi\,\mathring{\theta}(\theta^*(h)) - (1-h)\big) = 0 \tag{1.46}$$

From were it is possible to obtain a new $\alpha = \alpha_{nb,co}$ that incorporates the coordination aspect of this problem:

$$\alpha_{nb,co} = \frac{\xi\,\mathring{\theta}(\theta^*(h)) - (1-h)}{\xi\,\mathring{\theta}(\theta^*(h)) + \big(1-h-m-\delta\nu\big)\xi\frac{\partial\,\mathring{\theta}(h)}{\partial\theta^*(h)}\frac{\partial\theta^*(h)}{\partial h} - \delta\nu}$$

1.K Slopes of the haircuts under Nash bargaining model

In the general case, we can directly take the derivative of h_{nb} in α:

$$\frac{\partial h_{nb}}{\partial\alpha} = -m - \delta\nu + \xi\bar{\theta} > 0$$

In the coordination case, we have to use the implicit derivative of (1.46). As in appendix 1.H, I denote $\gamma = 1-h-\delta\nu-m$ to simplify notation. I obtain $\frac{dh}{d\alpha}$ below.

$$\frac{dh}{d\alpha} = -\frac{\gamma\left(\xi\frac{\partial\,\mathring{\theta}(h)}{\partial\theta^*(h)}\frac{\partial\theta^*(h)}{\partial h}\right) + \xi\,\mathring{\theta}(\theta^*(h)) - (1-h)}{\gamma\xi\alpha\left(\frac{\partial^2\,\mathring{\theta}(h)}{\partial\theta^*(h)}\left(\frac{\partial\theta^*(h)}{\partial h}\right)^2 + \frac{\partial\,\mathring{\theta}(h)}{\partial\theta^*(h)}\frac{\partial^2\theta^*(h)}{\partial h}\right) - \left(\xi\frac{\partial\,\mathring{\theta}(\theta^*(h))}{\partial\theta^*(h)}\frac{\partial\theta^*(h)}{\partial h} + 1\right)}$$

The numerator in $\frac{dh}{d\alpha}$ is always non-negative. We have $\gamma > 0$ for Lemma 2, then the parenthesis is non negative for the definition of truncated mean and Corollary 2, and finally we have that $\xi \overset{\circ}{\theta}(\theta^*(h)) > 1-h$. To see that, I expand the expressions using the derivative of the truncated mean of a random variable and denoting $H(\theta) = \frac{f(\theta)}{1-F(\theta)}$ to denote the hazard rate.

$$\gamma H(\theta^*(h))\big(\xi \overset{\circ}{\theta}(\theta^*(h)) - \xi\theta^*(h)\big)\frac{\partial\theta^*(h)}{\partial h} + \xi \overset{\circ}{\theta}(\theta^*(h)) - (1-h)$$

Note that $h = max\{0, 1-\xi \overset{\circ}{\theta}(\theta^*(h))\}$. If $\xi \overset{\circ}{\theta}(\theta^*(h)) = 1-h$, using the expression for $\theta^*(h)$ then we get $\overset{\circ}{\theta}(\theta^*(h) - \theta^*(h) < 0$ in (1.47), which contradicts the definition of truncated mean, thus implying that $\xi \overset{\circ}{\theta}(\theta^*(h)) > 1-h$.

$$\xi\big(\overset{\circ}{\theta}(\theta^*(h)) - \theta^*(h)\big) = \big(\xi \overset{\circ}{\theta}(\theta^*(h)) - \xi\theta^*(h)\big) = -m\frac{\nu-(1-h)}{1-h-\delta\nu} < 0 \tag{1.47}$$

So up to now, we know that the coordination haircut as a function of α is increasing when $\alpha = 0$ and that it can have both an increasing or decreasing slope when $\alpha = 1$, where $h_{co} = 1-\delta\nu - m$. In those cases in which (1.48) holds, $h_{\alpha,co}$ is a convex increasing function in α.

$$\gamma\xi\alpha\frac{\partial^2 \overset{\circ}{\theta}(h)}{\partial\theta^*(h)}\left(\frac{\partial\theta^*(h)}{\partial h}\right)^2 < 1 + \xi\frac{\partial \overset{\circ}{\theta}(\theta^*(h))}{\partial\theta^*(h)}\left(\frac{\partial\theta^*(h)}{\partial h} - \gamma\alpha\frac{\partial^2\theta^*(h)}{\partial h}\right) \tag{1.48}$$

1.L Comparison of cases inside Nash bargaining model

Using (1.43) and (1.46) we obtain $h_{nb}(\alpha = 0) = max\{0, 1-\xi\bar{\theta}\}$ and $h_{nb,co}(\alpha = 0) = max\{0, 1-\xi \overset{\circ}{\theta}\}$. For all those cases in which $1-\xi\bar{\theta} > 0$, as $\bar{\theta} \leq \overset{\circ}{\theta}$ then $h_{nb}(\alpha = 0) > h_{nb,co}(\alpha = 0)$.

As I did before, we can get $h_{nb}(\alpha = 1) = 1-m-\delta\nu$. For the coordination case, note first that using Corollary 2 and truncated mean properties we can get in (1.15) :

$$\xi\frac{\partial \overset{\circ}{\theta}(\theta^*(h))}{\partial\theta^*(h)}\frac{\partial\theta^*(h)}{\partial h} + 1 = \left(\frac{f(\theta^*(h))}{1-F(\theta^*(h))}\big(\overset{\circ}{\theta}(\theta^*(h)) - \theta^*(h)\big)\right)\frac{\partial\theta^*(h)}{\partial h} + 1 > 0\forall h$$

So now we can state that when $\alpha = 1$ there is a unique h value in which (1.15) holds: $h_{nb,co}(\alpha = 1) = 1-m-\delta\nu$.

Finally, under Lemma 3 condition, intersection between both haircut functions is unique in $\alpha \in [0,1]$ and in particular it occurs when $\alpha = 1$. As a consequence $h_{nb,co} \leq h_{nb}$ $\forall \alpha \in [0,1]$. Convexity in $h_{nb,co}$ and the values in $\alpha = 0$ and $\alpha = 1$ allow us to confirm that the difference is decreasing in α.

Chapter 2

Time To Tap:

Reentry costs and length of default

2.1 Introduction

It is widely accepted that default costs provide the required incentives for sovereign debt markets to exist (Dooley, 2008, Boresztein and Panizza, 2009). Understanding the nature of these costs is a key issue in the sovereign debt literature and one of tantamount importance for policy makers. However, it has been challenging both to identify the sources of such costs and to measure them. That is the case, for instance of the exclusion period from capital markets following a default,[1] which nevertheless constitutes one of the main components of these costs.[2] This paper investigates issuance costs as a novel source of delay in reentering sovereign debt markets.

Using the full re-access notion[3] the exclusion from capital markets after default lasts on average 16 years (Uribe and Schmitt-Grohé, 2017). Cruces and Trebesch (2013) distinguish two subperiods during the process: the first one starts from the beginning of default and covers the bargaining during the restructuring process until its conclusion, the second one extends from the end of the restructuring process until the coun-

[1] An economy enters in default status the year it skips at least one scheduled debt service (coupons or maturity payments) and exits the year it pays debt and arrears according the conditions negotiated in the restructuring agreement with investors.

[2] Other costs of default are reputational effects on the cost of debt, direct sanctions (as trade embargoes), spillovers on the banking sector, political crises. See Borensztein and Panizza (2009) for an empirical assessment of default costs.

[3] Re-access is considered full when inflows exceed 1% of GDP (taking then a longer time to occur) and partial when there are any positive flows.

try re-accesses sovereign debt markets. Using sovereign default data in 1980-2010 and the full re-access notion they find that on average the first subperiod lasts 8 years and the second 7.4 years.[4,5] Most of the literature has focused on explaining the length of the restructuring process (the first subperiod) but there has not been much progress understanding the final phase of the exclusion period. In this paper, I fill this gap in the literature analyzing the contribution of issuance costs to explain this portion of the exclusion period.

My paper extends the seminal model of debt with potential repudiation in Eaton and Gersovitz (1981) to study and quantify the effects of issuance costs on the length the exclusion period, and on other variables such as default frequency, debt to GDP ratio and spread. The model includes as additional ingredients non-zero recovery rates studied in Yue (2010) and endogenous reentry as in Bi (2009) and Benjamin and Wright (2012). Unlike those papers, I focus in the final phase of the exclusion period. In addition, here the sovereign must pay different types of issuance costs when endogenously decides to re-access the capital markets. As a consequence, the delay derives from the endowment process and the time it takes to compensate for total exit expenses, whose components are restructured debt and issuance costs.

The model builds over the standard sovereign debt model with endogenous capital markets reaccess. Each period, agents decide the optimal new debt issuance and whether to pay due debt next period or to default. In case of default, the economy compares the value of autarky with the value of solving debt distress and resuming good stance dynamics. To reentry, the economy should repay the defaulted debt according restructuring conditions and rise new funds at a costly rate. Issuance rates payed after default take the form of both fixed or variable fees on new debt.

It is important to note that fixed costs here would capture all other expenses outside issuance costs themselves that might confront the sovereign at reaccess, such as fiduciary sanctions, litigation costs or financial bailouts. Even other non observable costs resulting from information or market frictions.

In addition to the issuance costs, the model in this paper incorporates the haircut and the level of indebtedness as determinants of the length of default. The haircut (the debt concession) plays two opposing effects in a regular default. On the one hand, a higher proposed haircut difficults negotiations which might take longer thus extending the total autarky period.[6] On the other hand, it alleviates the repayment effort for the country in distress thus abbreviating the reentry process. In this paper, as I focus on the final phase of default, only the second mechanism will be working: a higher haircut reduces the time to reentry the market. Regarding

[4] When partial re-access is considered both subperiods last 8 and 5.1 years respectively.

[5] Richmond and Dias (2009) with data in 1980-2005 find that the subperiods have an average duration of 8 and 8.4 years respectively when considering full re-access (8 and 5.7 with partial re-access). In Gelos et al (2011) partial re-access to the market is attained in 4.7 for total default time with data in 1980-2000.

[6] See Cobas (2019) for the determination of the haircut in restructuring debt issued at secondary markets.

the level of debt, it has the same effects in the model as in empirical literature.[7] The higher the debt level, the more expensive it is repayment and then the longer it takes to the endowment process to reach the required level to exit from default.

In the quantitative section I assess the effects of issuance costs at reentry on the duration of default. I run three different settings, the first two analyze separately variable and fixed costs while the last one interacts them at selected levels. In all cases there is a significant reduction in the frequency of defaults and a non negative response in the duration.

At low costs levels, the economy compensates additional costs by increasing debt rising which is almost costless when previous debt level is low, and then there is almost no response in duration. At high cost levels, the economy needs to wait for endowment to compensate for the costs as an increase in debt is very costly. This is the main mechanism that explains the increase in delay, both at extensive (more long cases) and intensive margins (longer episodes). I find that fixed costs and variable costs at reentry add 0.5 and 2.5 years to default after restructuring process at costs levels within observed rates.

Literature Review

Empirical studies suggest that the length of defaults has a significant and heterogeneous duration across time and countries.[8] As discussed above, with different samples, Gelos et al (2011), Cruces and Trebesch (2013) and Richmond and Dias (2009) obtain non negligible (complete) exclusion periods of 4.7, 16.4 and 15.4 years, respectively. Regarding time heterogeneity, Richmond and Dias (2009) partition data by decade, and find that during the 1980s full access after restructuring was attained in 8.0 years in median, which reduced to 6.0 and 3.0 years afterwards. The same authors inform that the largest processes were observed in Middle East, Africa and Asia Pacific with median lengths of 9.0 to 8.0 years while the shortest processes were located in Latin America and Europe with 5.0 and 3.0 years respectively.

These studies also investigate the determinants of the length of market exclusion period. Among others, they emphasize: natural disasters, global market conditions, long term market expectations and the size of the haircut (Richmond and Dias, 2009); population size, fiscal balance, debt to GDP ratio, GDP per capita, and credit rating (Cruces and Trebesch, 2013); international reserves, short term debt to total debt, stock of debt with private creditors, and the quality of institutions (Gelos et al, 2011). In this paper, I investigate theoretically the role of issuance costs as determinants of the exclusion period.

[7] Cruces and Trebesch, (2013).
[8] See Uribe and Schmitt-Grohe (2017) for a survey.

From a theoretic point of view, the literature that considers the exclusion period can be categorized into two main groups. The first one assumes an exogenous duration. Eaton and Gersovitz (1981), Yue (2010) and Cole and Kehoe (2000) regard it as definitive, Cole and Kehoe (2000) as a fixed number of quarters and finally Arellano (2008), and Cuadra and Sapriza (2008) and Dvorkin et al. (2018) consider it as a draw from a reentry distribution function.

The second group assumes it is endogenous and attributes it fully to the length of the negotiations during the restructuring process (the first subperiod in empirical literature). Following Bi (2009) and Benjamin and Wright (2012) these authors include multiple round Nash bargaining into the general model to generate the exclusion period while repayment conditions are settled. For example, Asonuma and Trebesch (2016) find shorter length of exclusion in preemptive compared to post defaults. Asonuma and Joo (2019) show that the exclusion period increases when we substitute neutral by risk averse investors. D'Erasmo (2011) combines the bargaining in sovereign debt model with private information to evaluate how political uncertainty increases debt to GDP ratio and finds that in addition it extends the length of default. In all these papers, the delay is an opportunity to "increase the cake" (Bi, 2009) and get better stakes for incumbents. Another approach is that used in Bai and Zhang (2012) where the delay is a consequence of the multiple bargaining rounds the agents use to screen private information regarding each other's reservation value.

This paper relates to the literature that endogenizes the exclusion period in default, but instead of explaining the restructuring process, it intends to shed some light on the the final phase of the process after the restructuring is solved (the second subperiod). This is a particularly striking period, because once the repayment conditions are set there should not be elements to keep economies outside capital markets and nevertheless we see that it takes near eight years to reentry. I propose to investigate the costs of tapping the markets as one element that might contribute to explain this phenomenon.

The rest of the paper is structured as follows. Section 2 includes a brief discussion about issuance costs and their magnitude in the data. Section 3 describes the model and the algorithm used to numerically solve it. Section 4 specifies the functional forms and the calibrating strategy and presents the main quantitative results. Section 5 concludes.

2.2 Empirical evidence on issuance costs

Tapping into international bond markets is a complex process that posts non negligible costs to the issuing economy. These costs include legal, rating, listing and management fees necessary to accomplish the operation. In case of default, there might arise other expenses that add to the total bill, such as direct sanctions, litigating costs or financial bailouts. We now briefly review and provide a rough assessment for each of them following Van der Wansem et al (2019).

At the starting point, the sovereign contracts an international financial institution to organize the process. The lead managers mainly cover the determination of a proper structure, legal counseling and the management of the ordering book at issuing date. Delineating the asset implies to choose its currency, duration, interest type, coupon, redemption style, settlement date, and the objective market. Second, a legal team elaborates the bond contracts which include a set of specific payment, jurisdiction, and exit conditions (among others). These groups articulate with a sales team that operates the order book at issuance date, managing the demand both domestic and abroad. Lead managing fees depend on the size and complexity of the operation, but normally they range from 5 basis points in a 5 years bond to 22.5 basis points in a 30 years bond payed upfront the face value of the new issuance.

The sovereign should contract independent legal and financial advisory for experienced international counsel protecting its own interests. These costs accumulated would range from 3 to 8 basis points payed upfront the face value of the new issuance.

Then, it is necessary to contract one or multiple rating agencies which elaborate and release an assessment of the new issuance. They usually charge 10 basis points payed upfront the face value of the new issuance. Other expenses include the listing on the market, roadshows to present the bond, and other advisory services.

According to Van der Wansem at all (2019) all these expenses sum around 30 to 50 basis points payed upfront the face value of the new issuance, depending on the complexity of the operation.

On top of this, we should add other significant costs that might present once as fixed costs. That for example might be the case of bailout plans for the financial system, the costs of financial sanctions or both the expenses and litigation fees of restructuring process. These costs would compromise a significant portion of the new bond. For instance, Pitchford and Wright (2007) report that only the restructuring operation costs between 0.5% and 3% of the restructured debt.[9]

Finally, note that this costs we introduce in the model will be capturing other non-explicit costs that participate in the process. This is for example the case of informational costs, and other frictions that actually account for the delay in re-access.

In this paper I investigate whether issuance costs, fixed and/or variable, can contribute to explain the exclusion

[9] See Mitchener and Weidenmier (2005) for sanctions and supersanctions during the golden standard era.

period after restructuring. In the affirmative case, we are interested to analyze the magnitude required for their contribution to be significant.

2.3 Model

The model consists of a small open economy with a stochastic endowment process and access to international credit to smooth consumption. Debt takes the form of one period bonds sold to risk neutral investors abroad which eventually can be repudiated. In case of default, the country loses access to capital markets and faces default costs modeled as an endowment loss. The economy can exit default paying restructuring conditions and both fixed and variable costs at reentry.

2.3.1 General background

Consider a small open economy populated by identical households that maximize lifetime utility described in (2.1).

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{2.1}$$

c_t denotes consumption in period t, $\beta \in (0, 1)$ represents the subjective discount parameter and $u(.)$ the one period utility function, both strictly increasing and strictly concave. In this economy, there is a benevolent sovereign which takes optimal decisions (consumption, debt, default and reentry) on behalf of the households. I will then use indistinctly the terms economy, sovereign and households below.

Each period, households receive an endowment y_t of a non storable consumption good, which follows a stochastic process subject to independent, identically distributed shocks $N(0, 1)$. There are two credit market status, open and closed, according to payment decisions (pay or default respectively). Consumption smoothing is an option only when international credit market is available.

In open capital markets status, the economy learns its due debt and chooses whether to pay or to default it. If it pays, households can issue new debt in the form of one period bonds and sell them to investors abroad in order to reduce consumption volatility. Households receive the endowment y_t and issue new debt d_{t+1} taking price q_t as given. Total income in (2.2) is used to pay due debt d_t and optimal consumption c_t.

$$c_t + d_t = y_t + d_{t+1} q_t \tag{2.2}$$

When the economy decided to skip debt obligations, capital markets are closed. In this state, the economy can remain in autarky and consume default endowment as stated in (2.3) or repay according the restructuring program and exit default.

$$c_t = y_t - L(y_t) \tag{2.3}$$

At reentry, the economy must cancel the defaulted debt net of the haircut (concession h_t) as agreed at restructuring. In this case, total income is autarky endowment and new debt is charged with both variable $\phi^v \in [0, 1]$ and fixed costs $\phi^f \in \mathbb{R}_{\geq 0}$.

$$c_t + d_t(1 - h_t) = y_t - L(y_t) + (1 - \phi_v^b) d_{t+1} q_t - \phi_f \tag{2.4}$$

2.3.2 Recursive formulation with issuance costs

I will now proceed to the recursive formulation to focus on recursive Markov equilibria, representing borrower's infinite decision problem as a dynamic programming problem. The state variables in this problem are debt level, endowment, default status and haircut (d, y, z, h). These variables determine optimal decisions regarding first, the levels of consumption and debt issuance, and second, debt payment and capital markets reentry. Initial debt and endowment levels are given.[10]

State variables and timing

There are four state variables in this economy: current debt level d, endowment y, default status z and haircut h. Beginning each quarter, the sovereign observes the set (d, z, y) if it is in good stance and (d, z, y, h) otherwise, and then decides z' and d'.

New debt $d' \in D \subset \mathbb{R}_{\geq 0}$ can be issued during both good and bad standing but in the last case only in the period when the economy repays according the restructuring conditions.

The endowment $y \in Y \subset \mathbb{R}^+$ follows a stochastic process with an autorregressive component. Ending each quarter the new level is realized. During default, the corresponding endowment is punished through an

[10] In what follows, I shift to standard recursive notation in which next period variables are denoted with a tilde and current variables receive no time indicatives.

increasing and convex cost function $L(y)$ so that the available endowment is $y - L(y)$.

The state variable $z \in \{0, 1\}$ indicates default status. When $z = 1$ the economy defaulted and lost access to capital markets and when $z = 0$ the economy payed due debt thus remaining in good stance. The economy determines its status z' each period comparing the values of the alternative options then.

The haircut $h \in (0, 1)$ is drawn from a probability distribution and revealed just after the economy defaults. The value thus set will remain constant ($h = h'$) until the economy decides to re-access the capital markets.

Let us now quickly review the dynamics in the model. Beginning each period, the economy acknowledges its current endowment, due debt, financial markets status and restructuring conditions (in the state of default). Let us assume for instance, it starts in good financial standing. In this case, they decide optimally whether to keep access to capital markets (receive the complete endowment and issue new debt) or skip debt service and default. Once at the state of default agents consume autarky endowment while decide whether to pay due debt as agreed and return to capital markets or remain in default.

Value functions

Figure 2.1 illustrates the decision map with a detailed dynamic of the value functions and state variables. $V^c(d, y, 0)$ and $V^c(d, y, 1, h)$ represent the value functions of economies accessing capital markets during good and bad stance respectively while $V^b(d, y, h)$ represents the value function of an economy in default.

$V^c(d, y, 0)$ in (2.5) is the value function associated to an economy with access to capital markets ($z = 0$), debt level d and endowment y. If it decides to pay due debt, the economy receives good stance utility and issues new debt at the given prices $q(d', y)$. $V^f(d', y', 0)$ represents the value of next period's optimal choice which will be explained below. I denote $D(d, y, 0) = \{d'\}$ the policy function associated to this current debt status.

$$V^c(d, y, 0) = \max_{\{d'\}} \left\{ u(c) + \beta E_y[V^f(d', y', 0)] \right\} \tag{2.5}$$
$$s.t. \quad c + d = y + q(d', y)d'$$

The value of resuming access to capital markets $V^c(d, y, 1, h)$ for an economy in default ($z = 1$) with debt d, endowment y and haircut h is presented in (2.6). In this state, total income consists of default endowment and new issuances at market value net of both variable and fixed costs, while the expenses are consumption

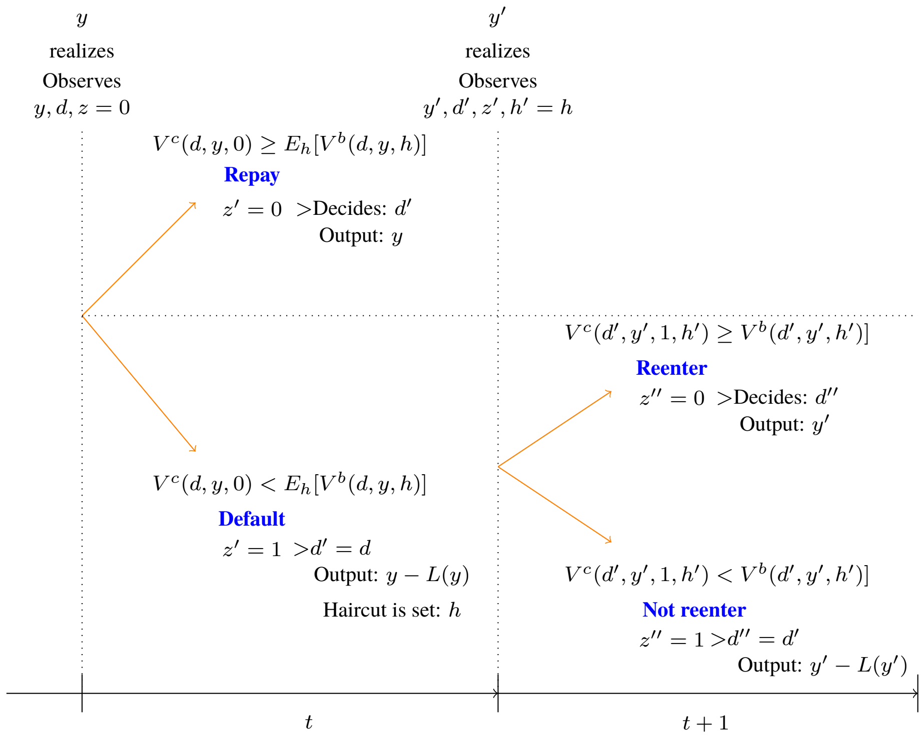

Figure 2.1: Decision map.

and the previously defaulted assets, net of the negotiated haircut h. The next period value of optimal decision is represented by $V^f(d', y', 0)$.

$$V^c(d, y, 1, h) = \max_{\{d'\}} \left\{ u(c) + \beta E_y \left[V^f(d', y', 0) \right] \right\} \tag{2.6}$$

$$s.t. \quad c + d(1 - h) = y - L(y) - \phi^f + (1 - \phi^v) q(d', y') d'$$

We will assume that there exist two possible values for the haircut, low and high, and that the high value occurs with probability δ. Endogenous negotiation literature suggests that the result of the restructuring process depend on which proposer (lenders or the borrower) has the highest bargaining power. In line with this, we can interpret here that when the draw corresponds to the high (low) haircut, the sovereign (investors) reached the negotiation with the highest bargaining power. In the model, the haircut is set once the economy defaults, and remains fixed $h = h'$ until it decides to reenter capital markets. Each haircut $h \in \{h_L, h_H\}$ determines a different constraint and as a consequence generates both specific value $V^c(d, y, 1, h)$ and policy $D(d, y, 1, h)$ functions.

The value function for an economy in default $V^b(d, y, h)$ with endowment y, due debt d and haircut h is presented in (2.7). It is compounded by the utility derived from autarky consumption now and the optimal decision for next period $V^f(y', d', 1, h)$ whether to pay according to restructuring conditions and issue new debt or stay in default.

$$V^b(d, y, h) = u(c) + \beta E_y\big[V^f(y', d', 1, h)\big] \tag{2.7}$$

$$s.t. \quad c = y - L(y)$$

Let us discuss now the values of financial standing $V^f(d, y, 0)$ and $V^f(d, y, 1, h)$, where f stands for default flag. In (2.8), the economy is in good financial standing and faces the alternatives of paying due debt and keep access to capital markets or skip the payment and transit to default state, with values $V^c(d, y, 0)$ and average $V^b(d, y, h)$ respectively. I will assume that the sovereign chooses to continue in good standing when payoffs coincide. Let us denote $Z(d, y, 0) \in \{0, 1\}$ the policy function related to this value function.

$$V^f(d, y, 0) = \max_{\{z'\}}\{(1 - z')\, V^c(d, y, 0) + z' \left(\delta V^b(d, y, h_H) + (1 - \delta)V^b(d, y, h_L)\right)\} \tag{2.8}$$

When the economy is already in default, it compares in (2.9) the value pay defaulted debt as agreed and recover access to capital markets $V^c(d, y, 1, h)$ to the value of remaining one more period in default $V^b(d, y, h)$.

$$V^f(d, y, 1, h) = \max_{\{z'\}}\{(1 - z')V^c(d, y, 1, h) + z'\, V^b(d, y, h)\} \tag{2.9}$$

Again in case of equality I will assume the sovereign prefers to resume market access. $Z(d, y, 1) \in \{0, 1\}$ represents the policy function related to this value.

Finally, we will review the price and recovery functions in (2.5) and (2.6). Let us start by denoting with α the event of repayment in the next period, which occurs when the value of repaying exceeds that of remaining in autarky.

$$\alpha = \{E_y\big[V^c(d', y', 0)\big] \geq \delta E_y\big[V^b(d', y', h_H)\big] + (1 - \delta)E_y\big[V^b(d', y', h_L)\big]\}$$

Risk neutral investors equate the expected yield of investing in new debt to that of a risk free asset with return r^*. Numerators in expression (2.10) represent the expected return of sovereign bonds which compare against risk free return in denominator. Thus, in the first ratio we have one times the probability of repayment and in the second ratio we have the expected recovery rate after default $\gamma(b', y')$ times the probability of default.

$$q(d',y) = \frac{Prob(\alpha)}{1+r^*} + \frac{1-Prob(\alpha)}{1+r^*}\Big[E_y\big[\gamma(d',y',h_L)\,|\alpha\big] + E_y\big[\gamma(d',y',h_H)\,|\alpha\big]\Big] \tag{2.10}$$

Let us define the repayment set $R(d,h) \subset Y$ for output levels that effectively exit default given debt d and haircut h.

$$R(d,h) = \{y \in Y \,|\quad V^c(d,y,1,h) \geq \; V^b(d,y,h)\}$$

The expected recovery rate for defaulted debt with haircut h in (2.11) is $1-h$ when it is optimal to exit default. The final term in this recursive formula updates it for the cases where the economy remains one extra period in default.

$$\gamma(d',y',h) = \frac{1}{1+r^*}\Big((1-h)\,\mathbb{1}_{y'\in R(d',h)} + E_{y'}\big[\gamma(d',y'',h)\,|\alpha\big]\,\mathbb{1}_{y'\notin R(d',h)}\Big) \tag{2.11}$$

Equilibrium

A recursive equilibrium in this model is a set of value and policy functions, for the government and a bond price function and a recovery rate for defaulted bonds such that given the initial value of debt, the international interest rate, the distribution of haircut values and the endowment process: the sovereign's value functions, and decisions over consumption, debt level, and capital market access status solve the problem in (2.5) to (2.11).

2.3.3 Computational algorithm

The computational algorithm follows the general framework in standard models of endogenous default with endogenous return proposed in Benjamin and Wright (2013). The main difference is the timing of reentry. While in their work it occurs immediately when the bargaining is solved, in this paper there is a post restructuring period during which the economy in autarky continues assessing its options until reentry.

The first step consists of obtaining for once both the distribution function and the transition matrix for the endowment process. For that purpose I follow the simple procedure proposed in Schmitt-Grohe and Uribe (2009). I start simulating 10 million steps of the AR(1) process and generating the vector of output values of the predetermined size (50 nodes in this case). Then I assign each observation in the series inside a bucket (delimited by two nodes) in order to obtain a numerical approximation to the probability distribution using the observed frequency of the buckets. Finally, starting with a zeros matrix with dimensions 50×50, I pick each pair of correlative elements (according the node vector) and thus obtain the observed frequency of transitions, registering it in the matrix by adding 1 to the element in the corresponding position (in program *matPi.m*).

I list below the steps followed in the algorithm to compute the model (in program *NestadosEGcostos.m*):

1. We start by choosing initial values for next period value functions $V^f(d', y', 0)$, $V^f(d', y', 1, h)$, prices $q(d', y')$ and the recovery function $\gamma(d', y', h)$.
2. Then compute the first iteration of the value functions $V^c(d, y, 0)$, $V^c(d, y, 1, h)$, $V^b(d, y, h)$ and the policy functions $D(d, y, z)$ and $D(d, y, z, h)$.
3. With the value functions of the previous step, update $V^f(d, y, 1, h)$ and $V^f(d, y, 0)$, the policy functions $Z(d, y, z)$ and the sets $R(d, h)$.
4. Use $R(d, h)$ and the value functions $V^c(d, y, 1, h)$ and $V^b(d, y, h)$ in an inner loop to obtain the recovery rate $\gamma(d', y', h)$.
5. Finally update the price using the Gauss-Seidel algorithm.
6. Iterate until convergence of the main components.

Once the value functions, policy functions, prices and recovery rates are determined, I obtain 100,000 simulations of 400 quarters (after discarding the first 40% of the total run) to get the predictions for the first and second moments targeted (in program *sim_horizontal.m*). To simulate the dynamics of the main variables in a default episode, I work with a simulation of 1.000.000 quarters (again after discarding the first 40% steps) and extract twenty five quarters windows centered in observed default (in program *sim_vertical.m*).

2.4 Quantitative Analysis

2.4.1 Functional forms and calibration

I assume one period corresponds to one quarter. Households' utility function takes a CRRA form with intertemporal elasticity of substitution $\sigma = 2$ as it is standard in literature.

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

Calibrated parameters		
β	0.9112	Subjective discount factor
a_0	0.00	Output loss function
a_1	-0.20	Output loss function
a_2	0.43	Output loss function
Parameters selected directly		
σ	2	Inv. intertemporal elasticity of cons.
r^*	0.01	World interest rate
ρ	0.9317	Serial correlation of $ln\, y_t$
σ_ϵ	0.037	St. dev. of innovation ϵ_t
h_L	0.22	Low value of haircut
h_H	0.53	High value of haircut
δ	0.50	Probability of high haircut
n_y	50	Output grid points
n_d	200	Debt grid points
$[\underline{y}, \bar{y}]$	[0.6523,1.533]	Output range
$[\underline{d}, \bar{d}]$	[0.0,1.5]	Debt range

Table 2.1: Calibrated default parameters.

The endowment follows the stochastic process in (2.12) with autocorrelation $\rho \in [0, 1)$, errors ϵ_t distributed iid $N(0, 1)$, and standard deviation of the shocks $\sigma_\epsilon > 0$. All the parameters in the process are calibrated following Uribe and Schmitt-Grohe (2017), whom replicate tradable output in Argentina from 1983:I to 2001:IV.[11] The series thus obtained is both highly persistent (serial autocorrelation of 0.93) and volatile (standard deviation around 10%) which incentivates agents to use credit to smooth consumption.

$$ln\, y_t = \rho\, ln\, y_{t-1} + \sigma_\epsilon \epsilon_t \tag{2.12}$$

Output values are discretized in a bounded set $[\underline{y}, \bar{y}] \subset \mathbb{R}^+$ with 50 nodes as in Chatterjee and Eyigungor (2012).[12,13] The extreme values in the grid are set 4.2 standard deviations from the unconditional mean (in logarithm).

$L(y)$ in (2.13) represents the endowment punishing function proposed in Chatterjee and Eyigungor (2012). The parameters a_0, a_1 and a_2 are set altogether with β targeting three data first moments as in Uribe and Schmitt-Grohe (2017): a debt to output ratio at good stance of 60.0%, default costs of 7% and a frequency of default of 2.6 times per century.

$$L(y) = \max\{0, a_0 - a_1 y + a_2 y^2\} \quad \text{with } a_1 \geq 0 \tag{2.13}$$

[11] This category groups agriculture, manufacturing, fishing, mining and forestry outputs.

[12] The size of the grid is placed between the models in Arellano (2008) and Uribe and Schmitt-Grohe that uses 21 and 200 nodes respectively.

[13] See Hatchondo, Martinez and Sapriza (2010) for an analysis on how the accuracy of the model in terms of default probability is affected by the nodes compounding the grid.

The referred authors target a 15% debt to GDP ratio recovered per quarter as observed in Argentina in 1982 to 2001 which determines a 60% annual ratio. Regarding default costs, the value represents the lower range of their estimates using Zarazaga's methodology in both the Argentinian defaults in 1982 and 2001. Finally the default frequency corresponds to the observed 5 episodes in 1824 to 2014 or 2.6 times each century.

I obtain thus a β value of 0.92 which is low for quarterly data but situates at a standard level in this literature. For quarterly data $\beta = 0.9530$ in Arellano (2008), Chaterjee and Eyigungor (2012) use $\beta = 0.9542$ and Uribe and Scmitt-Grohe use $\beta = 0.8500$.

The interest rate r^* is set to 1% quarterly, or 4% a year. The haircut values and its probabilities were assigned in order to match the empirical haircut properties in Cruces and Trebesch (2013). Using data in 1970 to 2010 for 180 default episodes in 68 countries they find an average haircut of 37% with a standard deviation of 22%.[14]

2.4.2 Numerical equilibrium properties

Table 2.2 compares selected empirical first and second moments of data with those obtained with the calibrated model. An evident feature is that the benchmark model is not capable to generate long default processes. Average time outside the market reaches 1.07 years. Note however, that 6.5 years in data consider total default and not only the second subperiod. Using as a rule of thumb that each phase takes half the total time in default (from empirical findings discussed below) then the observed time would reduce to 3.25 years and the benchmark model accounts for one third of it.

Spread reaches 18% the level observed in data with a three times higher correlation to output rate, yet keeping the negative sign. The model does a better job in terms of spread volatility.

Moments	Data	Benchmark
Def. Freq.	2.60	2.60
$E(d/y)$	58.00	57.30
Years in def.	6.5	1.07
$E(r - r^*)$	7.4	1.36
$\sigma(r - r*)$	2.9	3.48
$corr(r - r^*, y)$	-0.06	-0.18

Table 2.2: Selected first and second moments: data and model predictions.

Figure 2.1 presents the value functions of accessing capital markets in good and bad stance, as a function

[14] Benjamin and Wright (2013) report average loss to creditors weighted by total outstanding of 38%, considering 90 default episodes in 73 countries during 1989-2006.

of current obligations for the average endowment value in simulations. The negative slope derives from the fact that higher debt implies heavier repayment obligations which then reduce the available resources for consumption. The same intuition explains the fact that the incentives to default increase with current debt level.

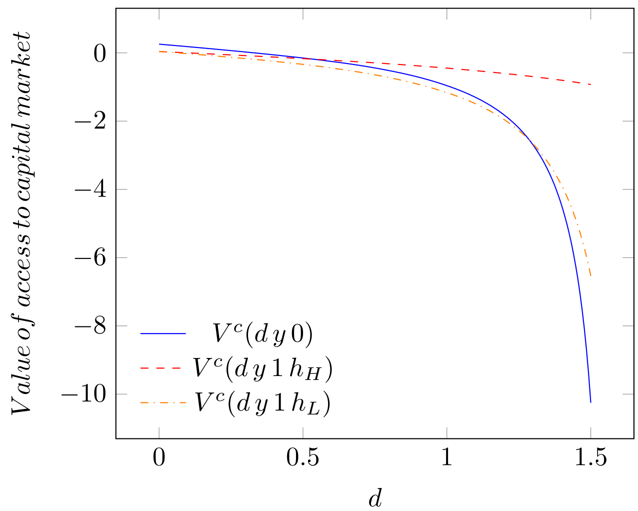

Figure 2.1: Value functions of accessing capital markets as a function of debt for average endowment level at simulations.

Comparing across the state z, the value functions of issuing from default settle below that of issuing in good stance at zero debt level. This is a result of the endowment cost $y - L(y)$ affecting defaulting economy at reentry. Once in default, a comparison across haircuts shows that the value of issuing new debt is higher for h_H because the economy attained to better repayment conditions (a higher haircut) at restructuring defaulted debt.

The policy function $Z(d, y, 0)$, as a function of debt and endowment is plotted in figure 2.2 panel a. Note that the resulting areas comply with two standard properties in default literature. First, defaults are more frequent at low values of output, reflecting that the "economy defaults in bad times".[15] As expected, the response here is monotonic. For instance, given a level of debt d_0, if the economy defaults for some y_0, it will default for all endowment values $y < y_0$ below that one. Second, default increases with higher debt levels. A higher repayment burden make it more attractive to skip payments and then spike short term utility on consumption.

Reentry policies $Z(d, y, 1, h)$ as functions of debt and endowment are presented in figure 2.2 for low and high haircut rates (panels b and c respectively). The decision to reentry is much more frequent for high values of haircut that imply a stronger concession on defaulted debt. In other terms, the higher haircut allows the

[15] Eaton and Gersovitz (1981).

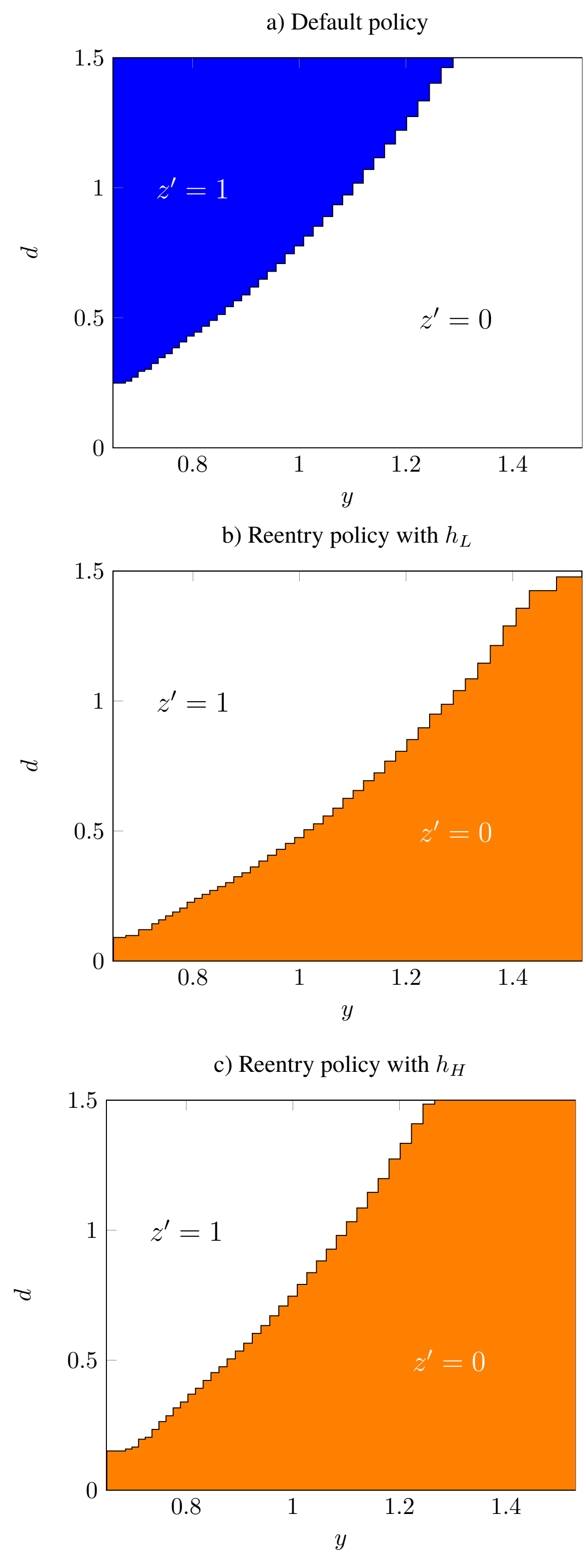

Figure 2.2: Flag policy functions. (a) Default policy $Z(d, y, 0)$. (b) Reentry policy with low haircut $Z(d, y, 1, h_L)$. (c) Reentry policy with high haircut $Z(d, y, 1, h_H)$.

economy to return the markets from weaker situations in terms of debt and or endowment.

New debt levels as a function of due obligations are plotted in figure 2.3. Policy functions increase on current (or previous if defaulted) obligations as they require rising additional funds for repayment. An interesting feature here is that for low debt levels (gross debt below 0.5 in this calibration), issuances after reentry rise more funds than those in good state. In this model, this is a consequence of the punishment on endowment during default, which determines that, although there is a concession on repayment the defaulting economy (still in the reentry period), it counts with lower resources for consumption and repayment so it rises additional funds at capital markets (enhanced in the case of the low haircut).

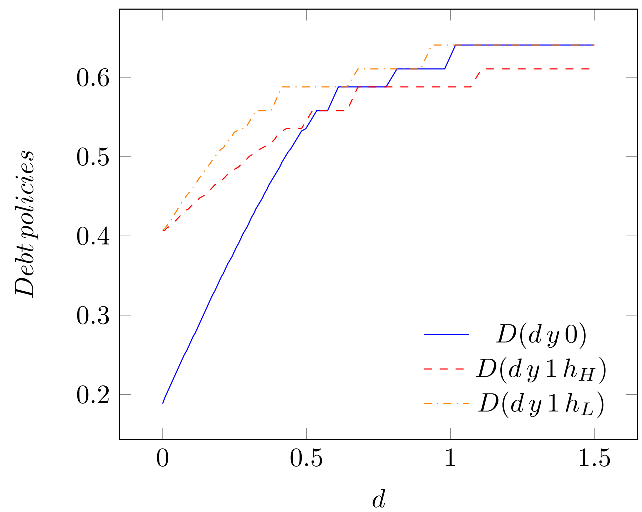

Figure 2.3: Debt policy functions at both good and default state.

This high funds-rising need after default will have two impacts on the economy. First, the average debt level outside default would increase with default frequency, in the cases of low indebted defaulting and low haircut at every previous debt level. Second, given debt level at default, higher fund rising (compared to issuance in good stance) will receive lower prices unless there is some recovery of endowment to compensate it. Hence, the economy will prefer to wait for recovery before issuing thus extending the time to tap (more in the case of the low haircut). This last relation between debt and the time to tap aligns with Cruces and Trebesch (2013) empirical findings.

Price schedules in figure 2.4 are increasing in endowment and decreasing in issued debt. The autorregressive feature of the model implies that a low endowment today increases the probability of a low endowment tomorrow and then the probability of default which presses downwards on prices. Likewise, a higher current debt level increases the probability of default as skipping the payment would yield short term gains on consumption and utility. So high debt levels now press downwards on the prices of new issued debt.

Returning to the gap in issuances after default mentioned before, note that the highest difference occurs at the lowest portion of debt. This is because in that portion prices are less responsive to increases in debt (due to a lower probability of default there) and so filling the output loss with additional debt is less expensive.

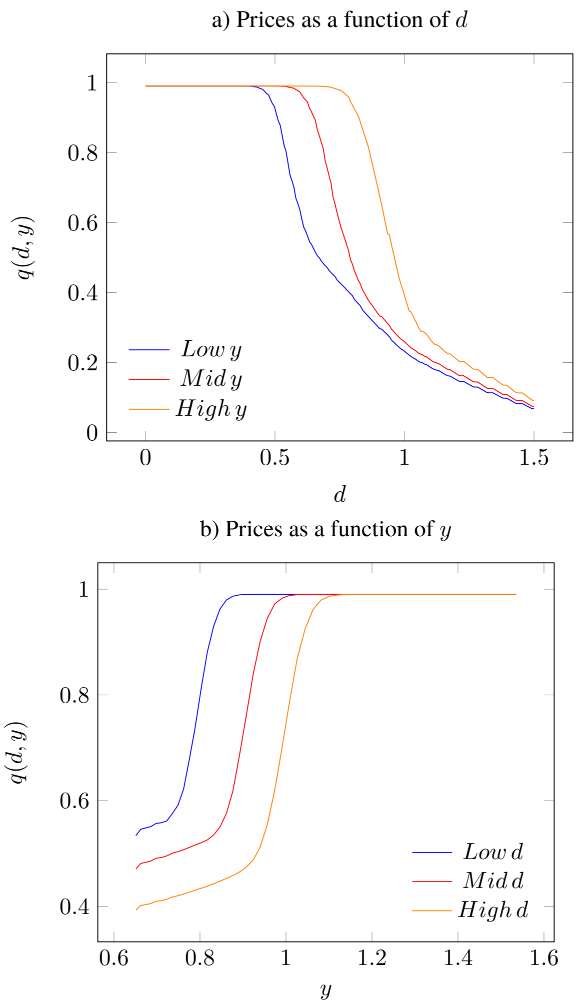

Figure 2.4: (a) Prices as a function of debt, for low, middle and high endowment values. (b) Prices as a function of endowment, for low, middle and high debt values. High and low values are ± 0.25 from mean value observed at simulations.

Table 2.3 presents first moments of selected variables distributed as a function of time. Almost 65% of the cases in the benchmark model concentrate in episodes of one to two quarters of duration. The short duration results from the fact that these cases present relatively low debt and received a high haircut (value above average) easing the reentry. This allows them to exit default with the lowest issuances and endowment levels. Episodes of three to nine quarters weight near 25%. In this group, there is more frequency of low haircuts which in turn required both higher issuances at reentry and endowment level. The longest processes weight only 11% of the cases. These episodes group the events with highest levels of debt at default which received the lowest haircut, which in turn required to wait in order to get higher issuances and output at exit. To sum up, we can say that in the model the length of the episodes is related to the total resources needed for repayment as agreed. The higher the resources needed in the form of output or issuances, due to a higher amount of debt at default or a lower haircut, the longer it takes to the economy to exit default.

Figure 2.5 presents the dynamics of typical default episodes, for both short and medium durations. It takes in general three negative shocks below the average endowment to enter default (0.7% of standard deviation) and once there the economy needs more than three years to recover the previous output level. As in Yeyati and Panizza (2009), it is the quarter of default the one that marks the trough of the economic contraction.

Sharp increases in default risk before default prevents distressed economies to rise additional funding at the market to compensate for the endowment contraction, and so we see that new debt remains about 60% in terms of GDP in that period. During default capital markets remain closed and so $d' = 0$.

Consumption follows the endowment trend. There is a short term spiking recovery after default due to the skip in payment (compensated via trade balance) and a stability at low values afterwards. In the shortest episodes, the haircut concession at reentry allows that consumption reaches even higher values than before, with a fast return to previous levels. This is not the case of longer episodes characterized by a slower convergence from below.

Prices start falling with the worsening of endowment as the probability of default increases peaking at default en $t = 0$. The economy upsurges with lower debt due to the haircut and so prices over compensate the fall experienced below.

2.4.3 Introducing costs of market reaccess following default

I now assume that the economy needs to pay fixed and variable costs when issuing new debt at capital markets reentry. I find that there is a reduction in the observed frequency of episodes, due to the fact that this new feature works as an increase in the costs of default. Variable costs produce significant effects on duration

Length	Frequency	$E[d]$	$E[d']$	$E[y_{exit}]$	$E[h]$
$1q - 2q$	64.65	0.43	0.45	0.67	0.46
$3q - 9q$	24.67	0.43	0.49	0.68	0.36
$\geq 10q$	10.67	0.47	0.53	0.69	0.22

Table 2.3: Average properties of episodes according to time, in a simulated timeline of 1 million years.

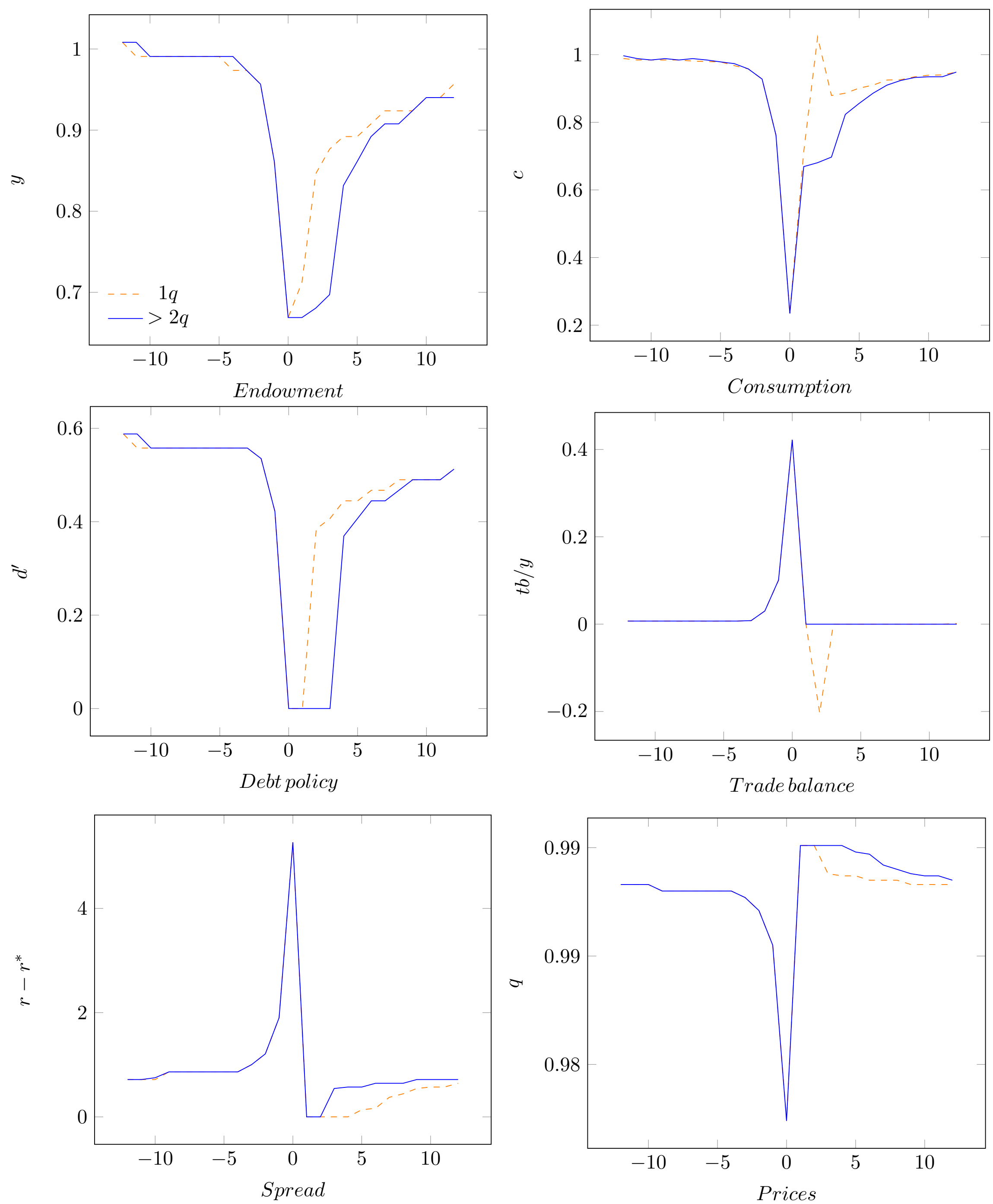

Figure 2.5: Dynamics in a typical default episode. Median of 25 quarters windows centered at default episode occurred in a simulated 1 million years timeline. Solid and dashed lines represent the median of episodes with durations above two quarters and of one quarter respectively.

only at high values. On the contrary, fixed costs rise the time to reentry within its observed empirical levels. Finally, average debt level outside default presents opposing responses to each type of costs, increasing when fixed costs are introduced.

Variable costs at reentry, $\phi^v > 0$**,** $\phi^f = 0$

Table 2.4 presents the main statistics after simulations of the model with variable issuance costs. The table shows that most of the variation concentrate in costs above 5pp of new debt.

Issuance costs make it more expensive the reentry process thus increasing the costs of default. As a consequence, the frequency of defaults in a hundred years period reduce from 2.60 to 1.43 episodes. In addition, the reduction in default frequency tightens average spreads by almost 7bps.

Issuance costs in percentage points of new debt issuance							
Moments	0.00	0.1	0.3	0.5	5.0	10	30
(a) Selected first and second moments							
Def. Freq.	2.60	2.60	2.56	2.56	2.33	2.19	1.43
Years in def.	1.07	1.07	1.07	1.07	1.46	1.94	7.43
$E[d/y]$	57.30	57.30	57.28	57.28	58.57	60.20	66.18
$E[h]$	23.70	23.70	23.70	23.70	23.90	23.96	24.98
$E[r - r^*]$	1.36	1.36	1.35	1.35	1.33	1.32	1.29
$\sigma[r - r*]$	3.48	3.48	3.46	3.46	3.49	3.53	3.91
$corr[r - r^*, y]$	-0.18	-0.18	-0.18	-0.18	-0.16	-0.15	-0.10
(b) Simulated distribution of default duration in percentage points							
1 q	48.87	48.87	48.87	48.87	46.99	41.51	2.70
2 q	15.78	15.78	15.78	15.78	11.28	11.02	30.18
3-9 q	24.67	24.67	24.67	24.67	25.67	24.39	18.35
$\geq$10 q	10.67	10.67	10.69	10.69	16.06	23.08	48.77

Table 2.4: Variable costs ϕ^v in issuances after default.

While variable costs remain below 50bps, there is a slight reduction in the frequency of cases. It mostly focuses in the number of short term episodes yet keeping the distribution by duration almost unaltered (as these are the most frequent cases). Also, in line with this, there is a mild contraction in debt levels during good times. First, because the economy takes less debt to avoid risky positions that would drive it to default. Second, as discussed in subsection 4.2, short term defaults characterize by having less debt, where there is an important difference in the size of issuances in good times and after default. In this case, less frequent defaults reduce high fund rising issuances at reentry.

As issuance costs continue increasing, they affect both the intensive and the extensive margins of the duration of default. At 5pp, we can see that the relative frequency of long term episodes increases to 42% and further. Besides, the average length of the longest cases increase from 20.14 quarters to 25.50 quarters.

Longer episodes occur in highly indebted economies which require large issuances to exit default. In these cases, any additional fund rise strategy is costly both for the effect of price elasticity (increasing with debt level, as showed in as depicted in figure 2.4) and the issuance costs. So the government increasingly appeals to “increase the cake” strategies: wait outside the market output to recover in order to obtain more debt at better prices. Note that the haircut increases with higher issuance costs. Without them, we saw that short episodes received the high haircut and long episodes the low one (that is one of the reasons they are longer). But as costs increase, it is more costly to reentry the market even when the economy received high haircuts at default.

To conclude, the model predicts that variable issuance costs extend the duration of default. indeed, for example 5pp increase by almost 50% the duration of the subperiod after restructuring. However, that occurs in values above those observed in data (30 to 50bps) which might us think that these costs might be capturing other non observed expenses that add to market costs.

Fixed costs at reentry, $\phi^v = 0$, $\phi^f > 0$

As in the previous case, the addition of fixed costs at market reentry determines a significant reduction in default frequency (see table 2.5, panel a), now even more severe than with variable costs. For instance, at $\phi^f = 0.1$ (where the weight on average debt is 0.15) the default frequency reduces from 2.60 to 1.66, above the 2.19 observed in the $\phi^v = 0.1$ case. Regarding the duration of default, it increases for almost all considered values, again exceeding the levels in the variable case up to $\phi^f = \phi^v = 0.3$ (see panel a) in figure 2.6).

The introduction of fixed costs imposes an additional hurdle to return to the capital markets, but now independent of the amount of debt raised at reentry. The default policy area in figure 2.6 contracts to the north west, in a rather parallel movement along debt or endowment. Here we have the price effect discussed above (price elasticity) but not the additional cost over debt, so the economy more frequently will cover part of reentry expenses with additional debt boosting average debt level.

As in the previous case, at very low debt levels, issuing additional debt has low impact on prices (because of the low probability of default) and then the economy “inexpensively” funds the fixed costs at reentry with additional debt (now without ϕ^v effect on debt quantity). Then we see little movement of the default policy

	Issuance costs in absolute terms						
	0.00	0.1	0.3	0.5	5.0	10	30
(a) Selected first and second moments							
Def. Freq.	2.60	2.57	2.56	2.56	2.15	1.66	1.39
Years in def.	1.07	1.07	1.14	1.17	1.73	2.51	6.66
$E[d/y]$	57.30	57.29	57.51	57.70	61.58	65.83	76.83
$E[\phi_f/d]$	0.00	0.17	0.52	0.53	8.26	15.55	42.02
$E[h]$	23.70	23.70	23.81	23.84	23.97	24.16	25.03
$E[r-r^*]$	1.36	1.35	1.36	1.36	1.29	1.18	1.25
$\sigma[r-r*]$	3.48	3.47	3.50	3.49	3.43	3.29	3.83
$corr[r-r^*,y]$	-0.18	-0.18	-0.17	-0.17	-0.16	-0.15	-0.12
(b) Simulated distribution of default duration in percentage points							
1 q	48.87	48.88	48.86	48.82	43.33	29.48	0.45
2 q	15.78	15.79	14.42	13.52	10.20	18.51	6.85
3-9 q	24.64	24.65	24.90	25.27	25.22	20.87	29.08
$\geq$10 q	10.67	10.64	11.83	12.39	21.25	31.11	63.62

Table 2.5: Fixed costs ϕ^f in issuances after default.

in this area.

When previous debt levels are higher, additional fund rising impacts negatively on prices. Default is now more expensive to overcome which even reduces the incentives to declare it at each combination of debt and endowment. Finally, while fixed costs remain below 5pp we cannot observe significant changes in out-of-default debt levels (although it increases).

As costs keep increasing, there is a very low frequency of defaults, because once the economy falls in default it is very costly to reentry the markets even for those cases in which it received the high haircut. As as consequence it needs to wait much more time to rise the necessary amount of debt at low prices to compensate for the additional costs which make the episodes much longer and with a lot more debt than before.

To sum up, fixed costs has to reach significant values in order to overcome the contraction in the frequency. Interestingly, increases between 10pp to 30pp (which might seem comparable to sanctions or financial bailout costs in empirical evidence) cause an increase in the time in default of 2.5 years. In addition there is a weaker discouraging effect at fund rising (only price effect) which determines higher debt levels than before.

Variable and fixed costs interacting and model recalibrated

In this section I introduce both variable and fixed costs jointly in the benchmark model and recalibrate it to match target moments with results presented in table 2.6.

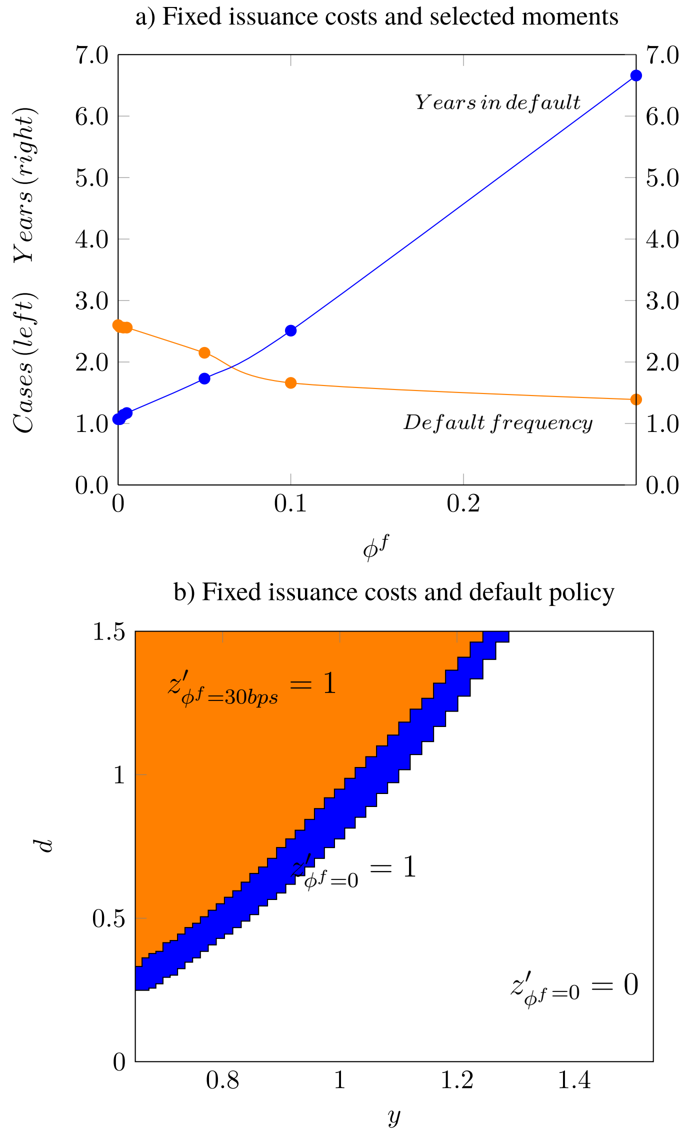

Figure 2.6: a) Effects of fixed costs in selected first moments. b)Default policy before and after fixed issue costs, at $\phi^f = 30pp$. Policy functions obtained at 100.000 simulations of 100 years windows.

With the benchmark calibration, grouping costs determine a more acute drop in default frequency than the one observed when costs are separately considered. Years in default triple those in the benchmark model and the effect is additive different to the case in frequency. One term episodes have a much reduced presence and the average haircut tends towards the empirical level.

I then recalibrate the model using the same target moments as before.[16] In this specification the years in default continue to triple those in benchmark model. The spread increases yet remains below the data, while

[16] The new parameter set is $(\beta, a_0, a_1, a_2) = (0.9112, 0.0, -0.2750, 0.53)$.

	Models with issuance costs			
	Data	Benchmark		Recalibrated
(ϕ^v, ϕ^f)	$(0.0, 0.0)$	$(0.05, 0.10)$	$(0.05, 0.10)$	$(0.05, 0.10)$
(a) Complete simulated sample				
Def. Freq.	2.60	2.60	1.54	2.60
Years in def.	6.08	1.07	3.29	3.13
$E[d/y]$	58.00	57.30	67.84	57.50
$E[\phi_f/d]$	n.a.	0.00	15.22	19.39
$E[h]$	37.00	23.70	32.64	31.54
$E[r-r^*]$	7.40	1.36	1.19	1.51
$\sigma[r-r*]$	2.90	3.48	3.40	4.02
$corr[r-r^*, y]$	-0.06	-0.18	-0.14	-0.14
(b) Simulated distribution of default duration				
1 q	n.a.	48.87	18.07	7.58
2 q	n.a.	15.78	25.05	26.07
3-9 q	n.a.	24.64	19.03	31.58
≥10 q	n.a.	10.67	37.84	34.76

Table 2.6: Fixed and variable costs in issuances after default.

both the volatility and correlation with output slightly fall behind the benchmark. In this case, the average haircut remain near 30%.

We can conclude then, that adding issuance costs helps the model to better match moments in the data. In particular, the duration of defaults. This feature contributes to explain the behavior of long processes, with average restructuring results but with important associated costs aside the proper repayment of defaulted debt.

2.5 Conclusions

The exclusion period from capital markets is considered one of the main costs after sovereign default. On average, it lasts more than 10 years. Empirical literature coincide in that only half this time is explained by the restructuring process for which there has been a lot of theoretical advances. On the contrary, at the moment there is much to learn about what explains the other half.

The aim of this paper is to investigate the determinants of the length of the period following debt restructuring until the country finally issues new debt. Thus, it contributes to the theoretical literature by introducing issuance costs into an otherwise standard model of sovereign debt. Empirical evidence shows that variable costs range from 30bps to 50bps at capital markets. Regarding fixed costs, they might have many sources but in most cases add a significant burden at the initial funds rising when returning the markets.

I extend a standard model of sovereign debt with endogenous return adding issuance costs both fixed and variable to issuances after default. In a simulated environment, I find that all cases add to the costs of default reducing the frequency of these episodes. At low cost values and indebtedness level, the economy compensate issuance costs with additional debt, and we see little impact in the years in default. With high indebtedness, a price effect aggregates to the issuance costs making very difficult to exit and triggering "increase the cake" strategies that generate longer default episodes.

Finally I find that variable costs increase by 50% the duration of default at plausible levels, and almost duplicate the autarky period for costs above 10pp. In the case of fixed costs, they boost the time in default by 2.5 times considering costs of 17% of average observed debt. This case also delivers higher debt levels outside default a constant caveat in this literature.

It is important to understand what drives market reaccess following default. For policy makers, this helps assessing the actual costs of default and planning ahead following one of these episodes. Thus my paper contributes to gain a better understanding of these phenomena by showing that the exit from default might be very expensive *per se*. Adding other costs to the bill, even if they are funded with fresh flows might be very expensive in terms of the time outside the market waiting until the economy and/or the market conditions ease thus allowing one such new issuance.

2.6 References

Arellano, Cristina (2008): Default Risk and Income Fluctuations in Emerging Economies. *American Economic Review* 98 (3), pp. 690-712.

Asonuma, Tamon; Trebesch, Christoph (2016): Sovereign debt restructurings: preemptive or post default. *Journal of the European Economic Association* 14(1), pp 175–214.

Asonuma, Tamon; Joo, Hyungseok (2019): Sovereign Debt Restructurings: Delays in Renegotiations and Risk Averse Creditors. *Journal of the European Economic Association*, forthcoming.

Benjamin, David; Wright, Mark (2013): Recovery Before Redemption? A Theory of Delays in Sovereign Debt Renegotiations. *Working paper, Federal Reserve Bank of Chicago and State University of New York Buffalo*.

Borensztein, Eduardo; Panizza, Ugo (2009): The Costs of Sovereign Default. *IMF Staff Papers* 56 (4), pp.

683-741.

Cuadra, Gabriel; Sapriza, Horacio (2008): Sovereign default, interest rates and political uncertainty in emerging markets. *Journal of International Economics*, 76 (1), pp. 78-88.

Chatterjee, Satyajit; Eyigungor, Burcu (2012): Maturity, indebtedness, and default risk. *The American Economic Review* 102 (6), pp. 2674-2699.

Cobas, Adriana (2019): Coordinating in the haircut. A model of dovereign debt restructuring in secondary markets. Mimeo.

Cole, Harold L.; Kehoe, Timothy (2000): Self-Fulfilling Debt Crises. *The Review of Economic Studies* 67 (1), pp. 91-116.

Cruces, Juan J.; Trebesch, Christoph (2013): Sovereign Defaults: The Price of Haircuts. *American Economic Journal: Macroeconomics* 5 (3), pp. 85–117.

Dvorkin, Maximiliano; Sánchez, Juan; Sapriza, Horacio; Yurdagul, Emircan (2018): Sovereign Debt Restructurings. *IMF Working Papers*.

Eaton, Jonathan; Gersovitz, Mark (1981): Debt with Potential Repudiation: Theoretical and Empirical Analysis. *Review of Economic Studies* 48 (2) pp. 289-309.

Gelos, R. Gaston; Sahay, Ratna; Sandleris, Guido (2011): Sovereign borrowing by developing countries: What determines market access? *Journal of International Economics* 83 (2), pp. 243–254.

Hatchondo, Juan; Martinez, Leonardo; Sapriza, Horacio (2010): Quantitative properties of sovereign default models: solution methods. *Review of Economic Dynamics*, 13 (4) pp. 919-933.

Mitchener, Kris James; Weidenmier, Marc D. (2005): Supersanctions and Sovereign Debt Repayment. *NBER Working Papers* No. 11472.

Richmond, Christine; Dias, Daniel (2009): Duration of Capital Market Exclusion: An Empirical Investigation. Mimeo.

Shmitt-Grohé, Stephanie; Uribe, Martín (2009): Finite-State approximation of VAR processes: a simulation approach. Mimeo.

Uribe, Martin; Schmitt-Grohé, Stephanie (2017): Open Economy Macroeconomics. *Princeton University Press.*

Van Der Wansem, Patrick B. G.; Jessen, Lars; Rivetti, Diego (2019): Issuing International Bonds, A Guidance Note. Discussion Papers, MTI Global Practice, *World Bank Group*.

Yeyati, Eduardo Levy; Panizza, Ugo (2011): The elusive costs of sovereign defaults. *Journal of Development Economics* 94 pp. 95–105.

Yue, Vivian Z (2010): Sovereign default and debt renegotiation. *Journal of International Economics* 80 (2) pp. 176-187.