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Hall effect in narrow channels

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Calculations of band structure and Hall resistance are performed for samples of small width in the independent-electron approximation. In the limit of strong fields, results show several Hall plateaus distorted by finite-width effects. In the low-field limit and a parabolic confinement potential, the classical Hall resistance is always obtained. The results in the weak-field limit follow from a partial occupation of the relevant subbands and should also be true for arbitrary shape of laterally confining potentials.

I. INTRODUCTION

The effect of finite width and interfaces on the integer Hall effect[1] is being increasingly researched recently.[2–5] Two important effects have been discovered. First the high-field plateaus become distorted and asymmetric in narrow samples.[6–8] Second, another recently found effect features a deviation from the $R_H \propto B$ classical result; the Hall resistance is smaller than this, and is more like $R_H \propto B^s$ with $s=2$ or higher. This is known as the quenching of the Hall effect.

Using the independent-electron approximation it will be argued in this work that the proportionality $R_H \propto B$ is always obtained in the weak-field limit, for strictly one-dimensional samples. Therefore the Hall quenching must be due to other effects, possibly as a result of mode mixing, which suppresses edge states in the vicinity of external lead contacts.[9] The classical result for strictly one-dimensional channels can be traced to having a partial occupation of the energetically overlapping subbands, a general characteristic for any one-dimensional model in the weak-field limit.

Any theory of these effects requires the crucial step of selecting the model of calculating transport properties and choosing the form for the unknown confining potential. The approach presented here is to assume external reservoirs which keep a constant Fermi level. Only extended states are used and only states below the Fermi level are relevant to the Hall conductance.[10–12] The main idea behind this is that for very wide samples the Hall conductance decreases in jumps as more Landau levels are pushed above the Fermi level when B increases. As far as the longitudinal conductance is concerned it is assumed that the material acts as a perfect conductor when E_F is in a gap. This is because then there would not be states just above E_F into which electrons could thermally or inelastically scatter. Hence the longitudinal resistance is zero in the region in which no states cross the Fermi level; that is, in a region of a plateau in the Hall conductance. In this model plateaus are broadened out by the effect of lateral confinement, since finite-width bands instead of narrow Landau levels move above the Fermi level as B increases. Hence there is no need to have disorder or localized states to explain this effect here. That is because confinement reduces the region over which the plateaus are effectively flat, as seen in experiments.[11]

In Sec. II the harmonic oscillator potential is adopted for the confinement in the x direction: $\frac{1}{2}k_x x^2$, where $k_x = m\omega_R^2$ is the spring constant. Making certain assumptions about the effective width and counting of states it is shown that reasonable results are obtained both in the strong-B limit and the weak-B limit. In Sec. III the limitations of our model are discussed.

II. HARMONIC OSCILLATOR POTENTIAL MODEL

The Hamiltonian of an electron moving in the xy plane in the presence of a magnetic field $B = \nabla \times \mathbf{A}$ and a harmonic confinement potential is

$$H = \frac{1}{2m}(p - eA)^2 + \tfrac{1}{2}k_x x^2 . \quad (2.1)$$

Choosing $\mathbf{A} = (0, Bx, 0)$ one gets

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m}\omega_R^2/\Omega^2 + \tfrac{1}{2}m\Omega^2(x - x_0)^2 , \quad (2.2)$$

where $\Omega^2 = \omega_R^2 + \omega_c^2$ and $\omega_c = eB/m$ is the cyclotron frequency; $\omega_R = (k_x/m)^{1/2}$ and $x_0 = eBP_y/m^2\Omega^2$ represents the guiding center for a state with momentum $P_y = \hbar k_y$.

The term in p_y^2 is entirely due to the lateral confinement effect, because of which Landau levels become bands.

For each value of the momentum p_y there is then a nondegenerate Landau spectrum of index n. In a given level n this momentum is assumed to have a maximum value $p_{\max}$ so that the filling factor for each level is then $\nu_n = p^{(n)}/p_{\max}$. If this ratio is equal to 1 the band is completely full, but it is partially full if it is less than 1.

It is physically clear that very large values of p_y lead to energies above the Fermi energy. For an energy given by the Fermi level E_F the value of $p^{(n)}$ is given from (2.2) as

$$E_F-(n+\tfrac{1}{2})\hbar\Omega=\frac{p^{(n)^2}}{2m}\omega_R^2/\Omega^2 \; . \tag{2.3}$$

The next problem at hand is to try to identify $p_{\max}$, which so far is only defined in relation to $\nu_n = p^{(n)}/p_{\max}$. However as a first step we first try to find the effective width.

The number of states per unit area for each Landau band of index n is

$$D=\frac{2p_{\max}}{\delta p_y}\frac{1}{WL}$$

where $\delta p_y = h/L$ is the smallest possible increment in p_y. Here W is the effective width and L is the sample length in the y direction.

Taking N full Landau bands and the rest partially full, the number of electrons per unit area is

$$n_s=DN+D\sum_{n=N}^{N_{\max}}\nu_n \; . \tag{2.4}$$

To identify the effective width W we first go to the weak-field limit $B\to 0$ or $\omega_c/\omega_R\to 0$. In this limit it is not hard to argue that a large number of Landau bands are only partially full (for $E_F \gg \hbar\omega_R$) with none completely full ($N=0$). Then

$$n_s=D\sum_{n=0}^{N_{\max}}\frac{p^{(n)}}{p_{\max}} \; . \tag{2.5}$$

Hence

$$n_s=\frac{2}{hW}\sum_{n=0}^{N_{\max}}p^{(n)} \tag{2.6}$$

which is independent of $p_{\max}$. This can be rewritten as

$$n_s=\frac{2}{W}\frac{E_F}{\hbar\omega_R}\frac{(2mE_F)^{1/2}}{h}\left[x\sum_{n=0}^{N_{\max}}\left[1-nx-\frac{x}{2}\right]^{1/2}\right] , \tag{2.7}$$

where $x=\hbar\Omega/E_F$ and the number of terms in the sum is $N_{\max}\sim 1/x$. The $B\to 0$ limit followed by the $\omega_R\to 0$ limit gives $x\to 0$. The limit in square brackets is of the form $[0\times\infty]$ which tends to

$$\int_0^1(1-x)^{1/2}dx=\tfrac{2}{3} \; .$$

Then

$$n_s\to\frac{2}{W\hbar}\frac{E_F}{h\omega_R}(2mE_F)^{1/2}\tfrac{2}{3} \; . \tag{2.8a}$$

However, this limit is also that of free electrons in a plane for which

$$n_s=\frac{k_F^2}{4\pi}=\frac{1}{4\pi}\left[\frac{2mE_F}{\hbar^2}\right] . \tag{2.8b}$$

Equating the two expressions gives

$$\frac{W}{2}=\frac{2}{3}\left[\frac{2E_F}{m\omega_R^2}\right]^{1/2} \tag{2.9}$$

which is $\frac{2}{3}$ of the classical turning point value. The turning point or something close to it has been used in other studies as well.[13,14]

Consider now just the classical motion of an electron in a magnetic field and parabolic confinement potential. Solution of the equation of motion shows that for the same velocity of the electron starting off perpendicular to the channel from the center, the turning point is smaller when there is a nonzero magnetic field. The turning point is $(2E_F/m\Omega^2)^{1/2}$ instead of $(2E_F/m\omega_R^2)^{1/2}$. hence

$$\frac{W}{2}=\frac{2}{3}\left[\frac{2E_F}{m\Omega^2}\right]^{1/2} \tag{2.10}$$

is a reasonable turning point criteria when $B\neq 0$.

In trying to see what happens for very strong magnetic fields one notes that the term $(p_y^2/2m)(\omega_R^2/\Omega^2)$ approaches zero as $B\to\infty$ provided some limit is placed on p_y. In the strong-field limit it is also valid to argue that $-W/2<x_0<W/2$ so that a limit on p_y must be $|p_y|<(W/2)(m^2\Omega^2/eB)$. A justification for this is that the correct degeneracy factor is recovered is the strong-field limit, as follows. Indeed, using this $p_{\max}$ gives (independently of W)

$$D=m^2\Omega^2/eBh \tag{2.11}$$

which in the limit $B\to\infty$ tends to

$$D=eB/h \tag{2.12}$$

states per unit area, the usual expression. Such type of limitation has been used in Ref. 15. The use of this maximum value of p_y in determining which bands are completely full is illustrated in Fig. 1. If

$$E_F>(n+\tfrac{1}{2})\hbar\Omega+(p_{\max})^2/2m\frac{\omega_R^2}{\Omega^2} \tag{2.13}$$

then level n is completely full. One notes that such completely full levels only arise in the strong-field limit. In this limit the expression for $p_{\max}$ is justifiable and the first term of (2.4) becomes important. In the weak-field limit this selection no longer holds, but it is no longer needed since the second term of (2.4) is independent of $p_{\max}$ and the first term is zero.

The remaining problem of finding the Hall conductance is now simple. Consider Fig. 2 which shows a block of material of height H, width W, and length L. Taking into account the compensation between the Hall field ϵ and the Lorentz term $\mathbf{v}\times\mathbf{B}$, the average drift velocity is just $v=\epsilon/B$. Current density is then $J=nev=(n_s/H)e(\epsilon/B)=I/A$, where $A=HW$ is the

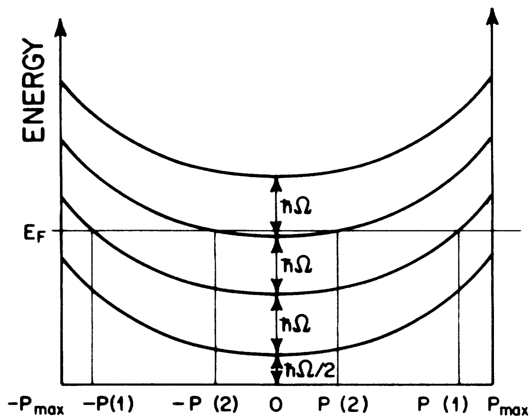

FIG. 1. This figure shows the bands corresponding to each Landau level in the interval $(-p_{\max},p_{\max})$. Here $n=0$ is full but $n=1$ and $n=2$ only partially occupied. Filling of partially full bands are contemplated through a factor of $p(n)/p_{\max}$.

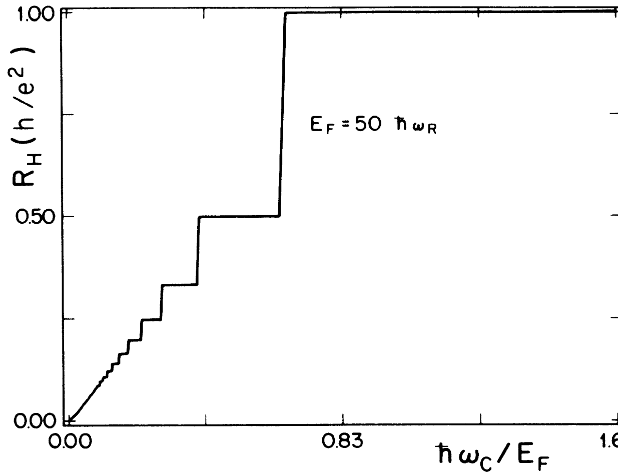

FIG. 3. This figure shows the Hall resistance R_H in units of h/e^2 plotted against $\hbar\omega_c/E_F$ for a wide channel ($E_F=50\hbar\omega_R$). Well-formed Hall steps can be seen.

cross section. Hence $G_H=en_s/B$ is the Hall conductance. For the very strong-field limit $n_s=DN=(eB/h)N$, where only the first term in (2.4) is considered. Then the magnetic field cancels out to give

$$G_H=(e^2/h)N ,$$

where N is the number of full levels beneath the Fermi energy. It is this cancellation which accounts for plateaus in the limit of strong fields. Results for the Hall conductance are shown in Figs. 3 and 4 with a variable effective width which depends on the ratio $\hbar\omega_R/E_F$. The full expression (2.4) is used in the numerical calculations.

III. DISCUSSION

A noteworthy characteristic of these results is that R_H varies as expected from the quantum Hall step regime as $B\rightarrow\infty$ to the classical linear dependence in the limit $B\rightarrow 0$. For lower values of E_F (narrow widths) the step structure of the plateaus is more rounded and stretches out over longer periods than the ideal values. The calculation is performed with just one spin per state over the entire range. Hence with two spins per state in the weak-B limit one needs to take $\frac{1}{2}$ the slope of the $R_H \propto B$ result.

Although the procedure followed here is very different from a Landauer multichannel approach, the results are quite similar to those of Peeters[16] who also sees the steps wash out for weak magnetic fields. The calculation uses independent electrons and no attempt is made to include electronic correlations or electronic polarization effects. These would modify the effective potential.[17]

Semiclassical considerations play an important role in selecting the width and limits for momentum. These seem unavoidable when selecting limits within which there is current. Models may differ on the precise way this is done but we think the present method is reasonable in this respect. For example, if no width limit at all is taken for p_y, then the second term in (2.4) is the only

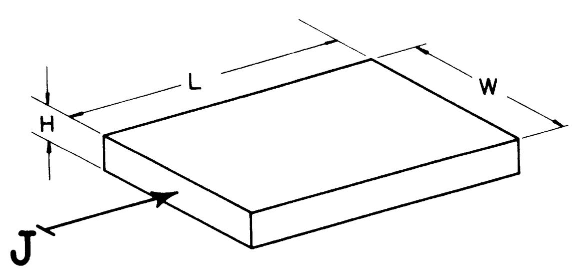

FIG. 2. This figure shows a block of material with a small height H, width W, and length L, through which a current density **J** is introduced.

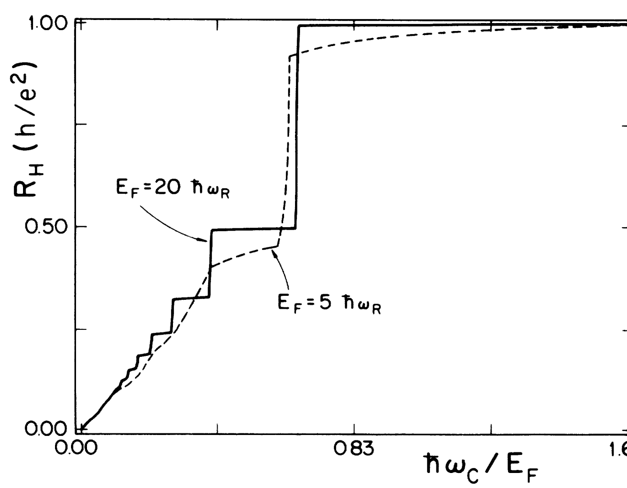

FIG. 4. R_H in units of h/e^2 plotted against $\hbar\omega_c/E_F$ for $E_F=20\hbar\omega_R$ and $E_F=5\hbar\omega_R$. This contrast shows a shift towards smaller R_H for the narrower channel. (See discussion in text.)

term. However it is equally clear that in that case the strong-field limit would never be properly recovered.

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[1] Klaus von Klitzing, Rev. Mod. Phys. **58**, 519 (1986).
[2] S. B. Kaplan and A. C. Warren, Phys. Rev. B **34**, 1346 (1986).
[3] M. L. Roukes, A. Scherer, S. J. Allen, Jr., H. G. Craighead, R. M. Ruthen, E. D. Beebe, and J. P. Harbison, Phys. Rev. Lett. **59**, 3011 (1987).
[4] W. Hansen, M. Horst, J. P. Kotthaus, U. Merkt, Ch. Sikorski, and K. Ploog, Phys. Rev. Lett. **58**, 2586 (1987).
[5] B. R. Snell, K. S. Chan, F. W. Sheard, L. Eaves, G. A. Toombs, D. K. Mande, J. C. Portal, S. J. Ban, Pl Claxton, G. Hill, and M. A. Pate, Phys. Rev. Lett. **59**, 3806 (1987).
[6] S. B. Kaplan and A. Hartstein, Phys. Rev. B **33**, 2909 (1986).
[7] M. A. Kastner, S. B. Field, J. C. Licini, and S. L. Park, Phys. Rev. Lett. **24**, 2535 (1988).
[8] G. Timp, A. M. Chang, P. Mankiewich, R. Behringer, J. E. Cunningham, T. Y. Chang, and R. E. Howard, Phys. Rev. Lett. **59**, 732 (1987).
[9] C. W. N. Beenaker and H. van Houten, Phys. Rev. Lett. **60**, 2406 (1988).
[10] T. Toyoda, V. Gudmundsson, and Y. Takahashi, Phys. Lett. **102A**, 130 (1984).
[11] H. Z. Zheng *et al.*, Phys. Rev. Lett. **55**, 1144 (1985).
[12] G. A. Baraff and D. C. Tsui, Phys. Rev. B **24**, 2274 (1981).
[13] F. Brinkop, W. Hansen, J. P. Kotthaus, and K. Ploog, Phys. Rev. B **37**, 6547 (1988).
[14] K. F. Berggren, T. J. Thornton, D. J. Newson, and M. Pepper, Phys. Rev. Lett. **57**, 1769 (1986).
[15] R. E. Prange, in *The Quantum Hall Effect*, edited by R. E. Prange and S. E. Girvin (Springer-Verlag, New York, 1987).
[16] F. M. Peeters, Phys. Rev. Lett. **61**, 589 (1988).
[17] S. E. Laux and F. Stern, Appl. Phys. Lett. **49**, 91 (1986).