Death and revival of the quantum discord and the measurement-induced disturbance

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Three different quantifiers, entanglement of formation (ENT), quantum discord (QD), and measurement-induced disturbance (MID), are used to measure the quantum correlations of two qubits in a common squeezed bath. A subspace was found for initial conditions in a squeezed bath, where the system experiences no decoherence. We relate the three measurements with the "distance" from the initial condition to the decoherence free subspace, in order to study the effect of the decoherence in the quantum correlations. We show examples of a system with quantum correlations even when entanglement is null. Furthermore, we study the necessary conditions for the system to become truly classical. We found that, under certain initial conditions and at specific times, the system becomes classical and both the QD and the MID vanish, thus observing the phenomena of sudden death and revival of the quantum correlations. Finally, we observe discontinuities in the QD.

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1. INTRODUCTION

The phenomenon of decoherence is caused when, in nature, an initially pure state interacts, intentionally or unexpectedly, with the environment (other quantum degrees of freedom). The system loses information to the environment, resulting a nonunitary evolution [1–3] of the reduced density matrix of the system, which becomes a mixed state. This process, named decoherence, has the undesired effect of producing the death or degrading of the entanglement. The entanglement is useful for diverse tasks, such as teleportation [4], cryptography [5,6], and quantum computation. However, recently, several researchers have found that entanglement does not exhaust the realm of quantum correlations. As it was found that some separable states have quantum correlations [7–12], two new quantifiers emerged, based on entropy measurement of information, quantum discord (QD) [13,14] and measurement-induced disturbance (MID) [15].

QD determines whether a state is a semiquantum state, which is immune to a partial measurement in one party of a composite quantum system. A bad feature of the QD is that it is asymmetrical with respect to which subsystem is measured. Moreover, in some cases, the nullity of QD is also asymmetrical.

The MID defines a classical state as a state that remains unperturbed under a complete measurement in both parties, in the case of a quantum bipartite state. Even if a proper minimization procedure has not been found, we present the conditions under which the MID vanishes. This may happen even in the presence of coherences. In this paper, we include a discussion about the relation between decoherence and QD and also MID. We use entanglement of formation (ENT) and these two new quantifiers to measure the correlations between two two-level atoms interacting with a common squeezed reservoir. We put special interest in studying the conditions for zero quantum correlations, as when QD, MID, and ENT vanish, the system becomes completely classical. Our present system has a decoherence free subspace (DFS) that consists in a two-dimensional plane, within the four-dimensional Hilbert space [16]. The term "decoherence free subspaces" was used by Lidar and Whaley [17] to refer to robust states against perturbations, in the context of Markovian master equations. One uses the symmetry of the system–environment coupling to find a quiet corner in the Hilbert space not experiencing this interaction.

In this work, we make use of an initial state that has a variable component in the DFS, giving us a way of monitoring the various decoherence effects as a function of the "distance" from the DFS. In this way, we hope to find "distances" for which all correlations are zero. While the study of the effect of decoherence on the entanglement for this specific model was made by Hernandez and Orszag in [18], we add here the analysis of QD and MID, confirming that, in most cases, the QD and MID are not null, even when the entanglement vanishes. We observed that, when the system is initially in the DFS, there is no degrading effect in the quantum correlations due to the reservoir. However, as we get far from DFS, we found that, besides entanglement, also QD and MID present the phenomenon of death and revival, as depicted in Fig. 1. For some specific times in the dynamical evolution, we found that both the QD and the MID vanish and the system becomes classical, whereas the quantum coherences spontaneously disappear and the density matrix becomes diagonal in the computational basis. However, it seems that the squeezed reservoir manages to create quantum coherences again.

We also study the discontinuities that are present in the QD, when choosing the measurement that disturbs the less the overall quantum system.

2. CORRELATIONS

A. Entanglement of Formation

For a given ensemble of pure states $\{p_i, |\psi_i\rangle\}$, the ENT is the average entropy of entanglement over a set of states that minimizes this average over all possible decompositions

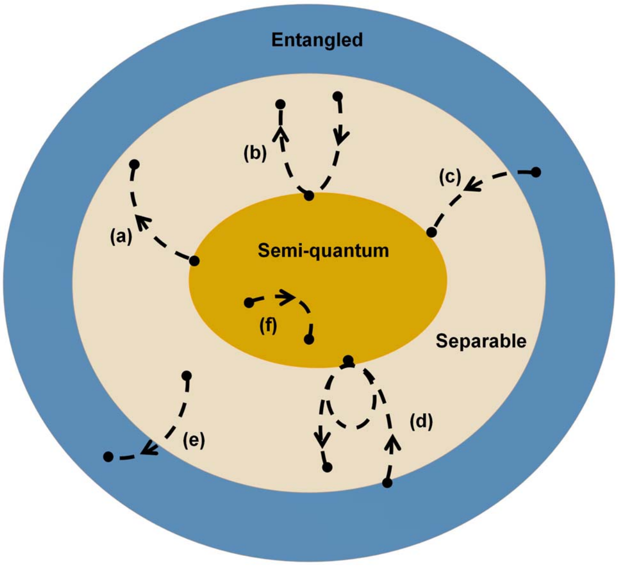

Fig. 1. (Color online) All the possible time evolutions for QD. (a) Birth of QD, (b) simultaneous death and revival of QD, (c) asymptotic death of QD, (d) several points of death and revival of QD, (e) QD never vanishes, and (f) zero QD. The same evolutions are possible for MID, but we must replace the inside region by classical states.

of ρ, [19], defined as $E(\rho) = \min \sum_i p_i E(\psi_i)$, where the entanglement $E(\psi)$ is defined as the von Neumann entropy of either of the two subsystems $E(\psi) = S(\rho_A) = S(\rho_B)$, with $S(\rho) = -\mathrm{tr}(\rho \log_2 \rho)$.

However, it is very difficult to know which ensemble $\{p_i, \Psi_i\}$ is the one that minimizes the average. A concept closely related to the entanglement of formation is the concurrence [20,21].

For a general mixed state ρ_{AB} of two qubits, we define $\tilde{\rho}$ to be the spin-flipped state $\tilde{\rho}_{AB} = (\sigma_y \otimes \sigma_y)\rho^*_{AB}(\sigma_y \otimes \sigma_y)$ where ρ^* is the complex conjugate of ρ, and σ_y is the Pauli matrix. The concurrence is defined as

$$C'(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{1}$$

where $\{\lambda_i\}$ are the square roots, in decreasing order, of the eigenvalues of the non-Hermitian matrix $\rho\tilde{\rho}$.

Finally, the entanglement of formation is related to concurrence, via $E(\rho) = H\left[\frac{1}{2} + \frac{1}{2}\sqrt{1 - C'^2}\right]$ with $H(x) = -x\log_2 x - (1-x)\log_2(1-x)$.

The entanglement vanishes for a *separable* state, defined as:

$$\rho = \sum_i p_i \rho_i^A \otimes \rho_i^B, \tag{2}$$

and it is equal to 1 for maximally entangled states.

B. Quantum Discord

The total correlation of a quantum system is quantified by the quantum mutual information $I(\rho) = S(\rho^A) + S(\rho^B) - S(\rho)$, which can be separated into classical and quantum correlations $I(\rho) = C(\rho) + Q(\rho)$.

In search of a formula for classical correlation, Henderson and Vedral proposed a list of conditions that a classical correlation must satisfy [14]. The obtained expression that fulfills all the conditions is $C(\rho^{AB}) = \max_{\{B_k\}}[S(\rho^A) - S(\rho|\{B_k\})]$, with the quantum conditional entropy defined as $S(\rho|\{B_k\}) = \sum_k p_k S(\rho_k)$, where $\{\rho_k, p_k\}$ is the ensemble of all possible results for the outcome, after a set of von Neumann measurements $\{B_k\}$, made in subsystem B. Also $\rho_k = \frac{1}{p_k}(I \otimes B_k)\rho(I \otimes B_k)$ is the system state after a measurement, where $p_k = \mathrm{tr}(I \otimes B_k)\rho(I \otimes B_k)$ is the probability for obtaining the outcome ρ_k after the measurement. The maximization in the classical correlation is done over all possible measurements of B, although we can choose to measure in A obtaining a different result due to the asymmetry of QD. However, this problem disappears for systems where $S(\rho^A) = S(\rho^B)$.

With this definition for the classical correlation, we get the QD as

$$Q(\rho) = I(\rho) - C(\rho). \tag{3}$$

In order to identify the zeros of QD, a new simple criterion is proposed in [22]. Given a general matrix ρ_{AB} of dimensions $N \times M$, this matrix will have zero discord with respect to system B, which can be written in the following way:

$$\rho = \sum_{ik} p_{ik}\rho_i^A \otimes \Pi_k^B, \tag{4}$$

where $\Pi_k = |k_B\rangle\langle k_B|$ is a complete basis of orthogonal projectors. This matrix is called *semiquantum* with respect to B, because we can always find a measurement $B_k = |k_B\rangle\langle k_B|$ that does not affect the initial quantum state.

To verify if our matrix can be written in this way, we just have to take the submatrices

$$\rho^{(i_A j_A)} = \langle i_A|\rho_{AB}|j_A\rangle = \sum_{k=1}^{M}\sum_{l=1}^{M}\langle i_A k_B|\rho_{AB}|j_A l_B\rangle |k_B\rangle\langle l_B|, \tag{5}$$

where $\{|i\rangle, |j\rangle, |k\rangle, |l\rangle\}$ are $|0\rangle$ or $|1\rangle$, and then verify that (a) they are normal, $[\rho^{(i_A j_A)}, (\rho^{(i_A j_A)})^\dagger] = 0$, and (b) that they all commute with each other.

For real matrices, and performing the measurement in B, these conditions can be expressed as the following equalities:

$$\begin{aligned} \rho_{14} &= \rho_{23}, \\ \rho_{12}(\rho_{13} - \rho_{24}) &= \rho_{23}(\rho_{11} - \rho_{22}), \\ \rho_{34}(\rho_{13} - \rho_{24}) &= \rho_{23}(\rho_{33} - \rho_{44}). \end{aligned} \tag{6}$$

For an X-form matrix, this reduces to $\rho_{14} = \rho_{23}$, $\rho_{11} = \rho_{22}$, and $\rho_{33} = \rho_{44}$.

The same analysis can be done if we measure in A. But in this case, in the conditions of Eq. (6), we must exchange $2 \leftrightarrow 3$. For an X-form matrix, this reduces to $\rho_{14} = \rho_{23}$, $\rho_{11} = \rho_{33}$, and $\rho_{22} = \rho_{44}$.

The last condition under which QD vanishes is the trivial one, when we have no coherences, and the matrix is just in a diagonal form, without any condition on the diagonal elements except that the trace is one.

C. Measurement-Induced Disturbance

Luo [15] defined a classical system as one that is not disturbed by a measurement, so the quantumness of a system depends on how much is changed after the measurement. Let $\{\Pi_i^A\}$ and $\{\Pi_i^B\}$ be complete projective measurements for parties

A and B. Then, after the measurement, the state ρ changes to $\rho \to \Pi(\rho) = \sum_{ij}\Pi_i^A \otimes \Pi_j^B \rho \Pi_i^A \otimes \Pi_j^B$. If the final state remains unchanged $\Pi(\rho) = \rho$, then ρ is called a classical state. If it changes, then a natural measure of quantum correlations is

$$M(\rho) = \min_{\Pi}\{I(\rho) - I(\Pi(\rho))\},$$

where I is the quantum mutual information. It is found that the measurement that keeps invariant the reduced density matrices is the one induced by the spectral decompositions of the marginal states, $\rho^A = \sum_i p_i^A \Pi_i^A$ and $\rho^B = \sum_i p_i^B \Pi_i^B$. While it is not certain that this measurement minimizes the impact of measurement, we use it in order to compare the effect of the MID with the QD. This choice of measurement makes MID not well defined when the reduced matrices are multiples of the identity.

MID vanishes for a *classical* state, defined as

$$\rho = \sum_{ij} p_{ij}\Pi_i^A \otimes \Pi_j^B, \tag{7}$$

which is unperturbed under measurement in both parties. This happens when the conditions for B [Eq. (6)] and the analogous conditions for A are satisfied simultaneously. For an X-form matrix, this means $\rho_{11} = \rho_{22} = \rho_{33} = \rho_{44}$ and $\rho_{23} = \rho_{14}$. This condition is independent of the measurement basis.

For pure states, the calculation of quantum correlations is simpler, the density matrix of any bipartite pure state, $\rho = |\phi\rangle\langle\phi|$, can be written in the Schmidt decomposition, where the state is $|\phi\rangle = \sum_j \alpha_j |j\rangle \otimes |j\rangle$. Thus the distribution of quantum information corresponds to

$$I(\rho) = 2S, \qquad C(\rho) = S, \qquad Q(\rho) = S, \qquad M(\rho) = S, \tag{8}$$

where $S = -\sum_j |\alpha_j|^2 \log_2 |\alpha_j|^2$.

In the case of a product state, $S = 0$, and, for a maximally entangled state, $S = 1$.

3. MODEL

We consider two two-level atoms that interact with a *common* squeezed reservoir, and we will focus on the evolution of the entanglement and QD, using as a basis, the DFS states, as defined in [16,17,23].

We write now a general master equation for the density matrix in the interaction picture, assuming that the correlation time between the atoms and the reservoirs is much shorter than the characteristic time of the dynamical evolution of the atoms, so that the Markov approximation is valid:

$$\begin{aligned}\frac{\partial\hat\rho}{\partial t} = \frac{\gamma}{2}\sum_{i,j=1}^{2}\big[&(N+1)(2\sigma_i\hat\rho\sigma_j^\dagger - \sigma_i^\dagger\sigma_j\hat\rho - \hat\rho\sigma_i^\dagger\sigma_j)\\ &+ N(2\sigma_i^\dagger\hat\rho\sigma_j - \sigma_i\sigma_j^\dagger\hat\rho - \hat\rho\sigma_i\sigma_j^\dagger)\\ &- M(2\sigma_i^\dagger\hat\rho\sigma_j^\dagger - \sigma_i^\dagger\sigma_j^\dagger\hat\rho - \hat\rho\sigma_i^\dagger\sigma_j^\dagger)\\ &- M^*(2\sigma_i\hat\rho\sigma_j - \sigma_i\sigma_j\hat\rho - \hat\rho\sigma_i\sigma_j)\big],\end{aligned} \tag{9}$$

where γ is the decay constant of the qubits, and $\sigma_i = |1\rangle_i\langle 0|$ and $\sigma_i^\dagger = |0\rangle_i\langle 1|$ are the raising $(+)$ and lowering $(-)$ operators of the ith atom. It should be pointed out that, in Eq. (9), the $i = j$ terms describe the atoms interacting with independent local reservoirs, while the $i \neq j$ terms denote the couplings between the modes induced by the common bath.

It is simple to show that this master equation can also be written in the Lindblad form with a single Lindblad operator S:

$$\frac{\partial\rho}{\partial t} = \frac{1}{2}\gamma(2S\rho S^\dagger - S^\dagger S\rho - \rho S^\dagger S), \tag{10}$$

with

$$\begin{aligned}S &= \sqrt{N+1}(\sigma_1 + \sigma_2) - \sqrt{N}e^{i\Psi}(\sigma_1^\dagger + \sigma_2^\dagger)\\ &= \cosh(r)(\sigma_1 + \sigma_2) - \sinh(r)e^{i\Psi}(\sigma_1^\dagger + \sigma_2^\dagger),\end{aligned} \tag{11}$$

where the squeeze parameters are Ψ, and $N = \sinh^2|r|$, $r = |r|e^{i\Psi}$. Here we consider $M = \sqrt{N(N+1)}$. The DFS consists of the eigenstates of S with zero eigenvalue. The states defined in this way form a two-dimensional plane in Hilbert space and are not affected by decoherence when the system interacts with the environment. Two orthogonal vectors in this plane are

$$|\phi_1\rangle = \frac{1}{\sqrt{N^2+M^2}}(N|++\rangle + Me^{-i\Psi}|--\rangle), \tag{12}$$

$$|\phi_2\rangle = \frac{1}{\sqrt{2}}(|-+\rangle - |+-\rangle). \tag{13}$$

We can also define the states $|\phi_3\rangle$ and $|\phi_4\rangle$ orthogonal to the $\{|\phi_1\rangle, |\phi_2\rangle\}$ plane:

$$|\phi_3\rangle = \frac{1}{\sqrt{2}}(|-+\rangle + |+-\rangle), \tag{14}$$

$$|\phi_4\rangle = \frac{1}{\sqrt{N^2+M^2}}(M|++\rangle - Ne^{-i\Psi}|--\rangle). \tag{15}$$

We solve analytically the master equation by using the $\{|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle, |\phi_4\rangle\}$ basis; however, we use the standard basis to calculate the concurrence and discord. For simplicity, we will consider $\Psi = 0$.

For a *thermal reservoir* $N = n$, where n is the mean number of thermal photons. For a *squeezed reservoir*, $N = \sinh^2 r$ is the average number of squeezed photons. $N = 0$ for a *vacuum reservoir*.

4. RESULTS

Now we present some analytical results obtained by taking as initial conditions states of the form

$$|\Psi_1\rangle = \epsilon|\phi_1\rangle + \sqrt{1-\epsilon^2}|\phi_4\rangle, \tag{16}$$

$$|\Psi_2\rangle = \epsilon|\phi_2\rangle + \sqrt{1-\epsilon^2}|\phi_3\rangle, \tag{17}$$

which evolve with the master equation [Eq. (10)], with the density matrix maintaining its original X-form matrix, for all times. This allows us to study if the effect of global noise on the entanglement, QD, and MID decay may depend on whether the initial two-party state belongs to a DFS or not.

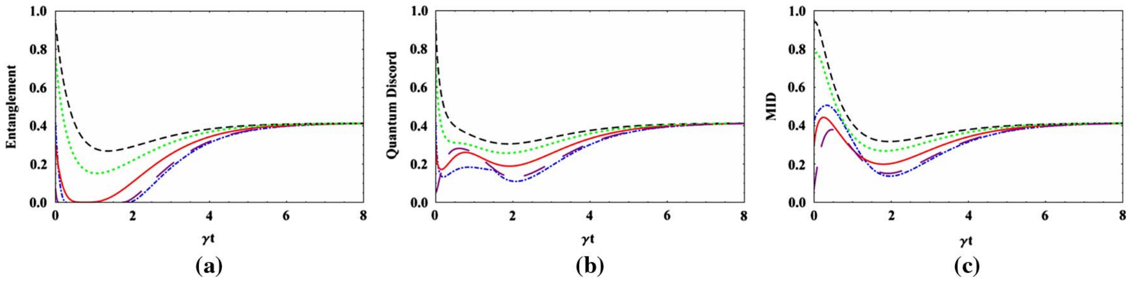

Fig. 2. (Color online) Quantum correlations for initial condition $|\Psi_1\rangle$ in squeezed reservoir. (a) Entanglement for the initial condition $|\Psi_1\rangle$, with $N = 0.1$, and different values of ϵ: $\epsilon = 0.0001$ (dotted–dashed), $\epsilon = 0.2$ (long dashed), $\epsilon = 0.5$ (solid), $\epsilon = 0.7$ (dotted), and $\epsilon = 0.8$ (dashed). (b) QD for the initial condition $|\Psi_1\rangle$, with $N = 0.1$, and different values of ϵ: $\epsilon = 0.0001$ (dotted–dashed), $\epsilon = 0.2$ (long dashed), $\epsilon = 0.5$ (solid), $\epsilon = 0.7$ (dotted), and $\epsilon = 0.8$ (dashed). (c) MID for the initial condition $|\Psi_1\rangle$, with $N = 0.1$, and different values of ϵ: $\epsilon = 0.0001$ (dotted–dashed), $\epsilon = 0.2$ (long dashed), $\epsilon = 0.5$ (solid), $\epsilon = 0.7$ (dotted), and $\epsilon = 0.8$ (dashed).

We analyze this feature by moving the parameter ϵ, which moves the initial condition to the DFS. First, the analytical formulas for the different quantifiers for an X-form matrix are presented here. The density matrix, written in the base $|1\rangle = |11\rangle$, $|2\rangle = |10\rangle$, $|3\rangle = |01\rangle$, and $|4\rangle = |00\rangle$, is

$$\begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}. \tag{18}$$

For this kind of density matrix, the concurrence can be easily found [24] as

$$C'(\rho) = \max\{0, C'_1(\rho), C'_2(\rho)\}, \tag{19}$$

where

$$C'_1(\rho) = 2\left(\sqrt{\rho_{23}\rho_{32}} - \sqrt{\rho_{11}\rho_{44}}\right), \tag{20}$$

$$C'_2(\rho) = 2\left(\sqrt{\rho_{14}\rho_{41}} - \sqrt{\rho_{22}\rho_{33}}\right), \tag{21}$$

which, as we mentioned, is related to entanglement.

In order to evaluate the QD, we follow the procedure of [25–27]. We found three possibilities for the minimum for the expression $S(\rho|\{B_k\}) = p_0 S(\rho_0) + p_1 S(\rho_1)$ in the classical part of Eq. (3). Thus, for QD, we have

$$\min_{\{B_k\}} Q = \min_{\{B_k\}} \{Q_1, Q_2, Q_3\}, \tag{22}$$

where

$$\begin{aligned} Q_1 &= S(\rho_B) - S(\rho_{AB}) + -\rho_{11} \log \frac{\rho_{11}}{\rho_{11} + \rho_{33}} - \rho_{33} \log \frac{\rho_{33}}{\rho_{11} + \rho_{33}} \\ &\quad - \rho_{22} \log \frac{\rho_{22}}{\rho_{22} + \rho_{44}} - \rho_{44} \log \frac{\rho_{44}}{\rho_{22} + \rho_{44}}, \\ Q_{2,3} &= S(\rho_B) - S(\rho_{AB}) + 1 - \frac{1}{2}\left(1 - \sqrt{\Gamma^2 + 4\Theta_{2,3}}\right) \\ &\quad \times \log\left(1 - \sqrt{\Gamma^2 + 4\Theta_{2,3}}\right) - \frac{1}{2}\left(1 + \sqrt{\Gamma^2 + 4\Theta_{2,3}}\right) \\ &\quad \times \log\left(1 + \sqrt{\Gamma^2 + 4\Theta_{2,3}}\right), \end{aligned} \tag{23}$$

where $\Gamma = \rho_{11} - \rho_{33} + \rho_{22} - \rho_{44}$ and for real density matrices $\Theta_2 = (\rho_{14} + \rho_{23})^2$ and $\Theta_3 = (\rho_{14} - \rho_{23})^2$.

The exact formula for MID is the result of comparing the matrix ρ with the diagonal matrix diag$[\rho_{11}, \rho_{22}, \rho_{33}, \rho_{44}]$, obtaining:

$$M(\rho) = -S(\rho^{AB}) + \sum_i \rho_{ii} \log \rho_{ii}. \tag{24}$$

A. Decoherence Free Subspace for a Squeezed Reservoir

After the computation of the dynamical evolution of the quantum correlations, we present the main results obtained. The first observation arises when starting from an initial state in the DFS plane ($\epsilon = 1$). The local and nonlocal coherences

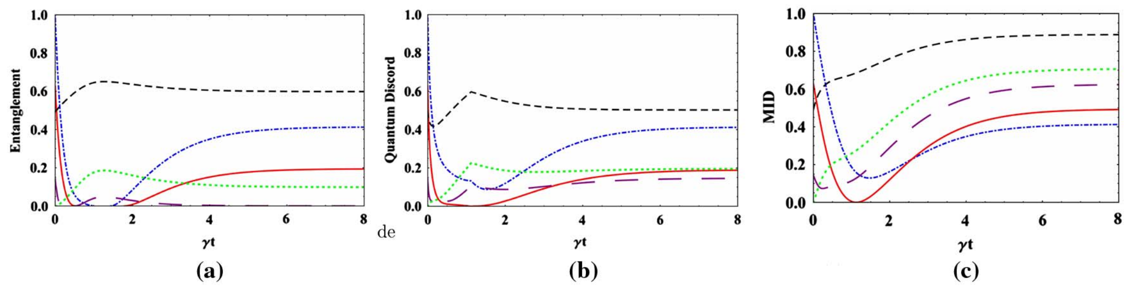

Fig. 3. (Color online) Quantum correlations for initial condition $|\Psi_2\rangle$ in a squeezed reservoir. (a) Entanglement for the initial condition $|\Psi_2\rangle$, with $N = 0.1$, and different values of ϵ: $\epsilon = 0.0001$ (dotted–dashed), $\epsilon = 0.369192$ (solid), $\epsilon = 0.6$ (long dashed), $\epsilon = 1/\sqrt{2}$ (dotted), and $\epsilon = 0.9$ (dashed). (b) QD for the initial condition $|\Psi_2\rangle$, with $N = 0.1$, and different values of ϵ: $\epsilon = 0.0001$ (dotted–dashed), $\epsilon = 0.369192$ (solid), $\epsilon = 0.6$ (long dashed), $\epsilon = 1/\sqrt{2}$ (dotted), and $\epsilon = 0.9$ (dashed). (c) MID for the initial condition $|\Psi_2\rangle$, with $N = 0.1$, and different values of ϵ: $\epsilon = 0.0001$ (dotted–dashed), $\epsilon = 0.369192$ (solid), $\epsilon = 0.6$ (long dashed), $\epsilon = 1/\sqrt{2}$ (dotted), and $\epsilon = 0.9$ (dashed).

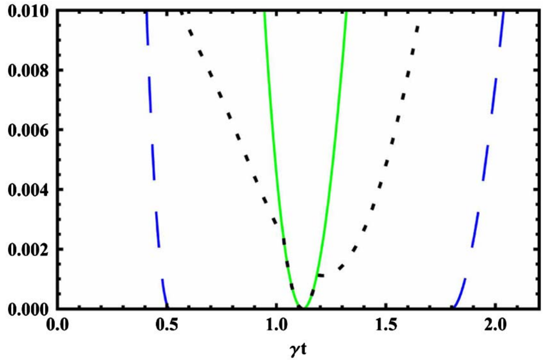

Fig. 4. (Color online) Quantum correlations for the initial condition $|\Psi_2\rangle$ with $\epsilon = 0.369192$ and $N = 0.1$. Entanglement (dashed), QD (solid), and MID (dotted).

are not affected by the environment, thus it experiences no decoherence and the quantum correlations stay constant in time [18].

For the initial state $|\phi_1\rangle$, the quantum correlations do increase with the squeeze parameter N, getting a maximally entangled Bell state $|\phi_1\rangle \to \frac{1}{\sqrt{2}}(|++\rangle + |--\rangle)$ for $N \to \infty$. In fact, in the information distribution of Eq. (8), S increases with N, getting the maximum value $S = 1$ when $N \to \infty$.

This means that this reservoir is not acting as a thermal one, in the sense of introducing randomness. On the contrary, a common squeezed bath tends to enhance the quantum correlations, as we increase the parameter N.

On the other hand, for $|\Psi_2\rangle$, if we start with the initial state $|\phi_2\rangle$, this state is independent of N and it is also maximally entangled, so $C' = 1$ and again $S = 1$ for all times and all N.

Now, we consider as initial states the superpositions given in Eqs. (16) and (17), where we vary ε between 0 and 1 for a fixed value of the parameter $N = 0.1$.

From Fig. 2, we can see that the asymptotic limit with initial $|\Psi_1\rangle$ does not depend on ϵ, and is of the form $|\Psi_1\rangle_{\lim_{t\to\infty}} = \alpha|11\rangle + \beta|00\rangle$, and is given by

$$\alpha = \sqrt{\frac{N}{1+2N}},$$
$$\beta = \sqrt{\frac{-1 + 11N + 24N^2 + 12N^3}{(1+2N)(-1+12N+12N^2)}}.$$

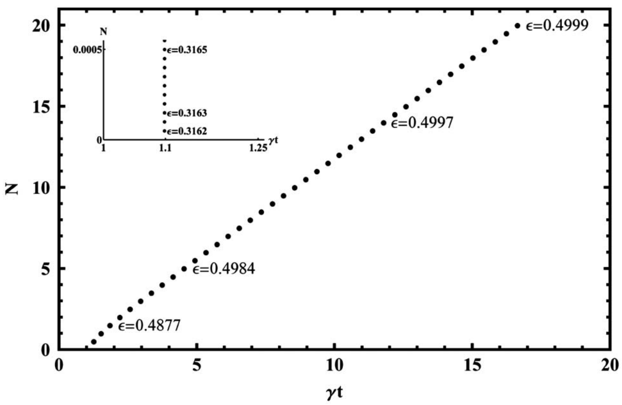

Fig. 5. The points are the zeros of QD and MID; each of them corresponds to a different value of ϵ.

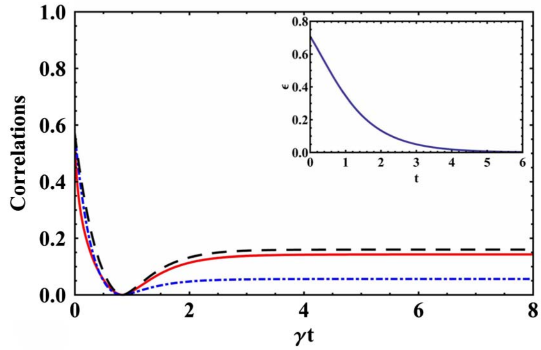

Fig. 6. (Color online) Quantum correlations for the initial condition $|\Psi_2\rangle$ in a vacuum reservoir ($N = 0$), for $\epsilon = 0.4$, entanglement (dotted–dashed), QD (solid), and MID (dashed). The inner plot represents the point where the quantum correlations are zero as we vary ϵ.

Since the above state is pure, the entropy is given by

$$S = -\beta^2 \log_2 \beta^2 - \alpha^2 \log_2 \alpha^2, \tag{25}$$

and all the quantum correlation MID, QD, and ENT are the same.

When we have $|\Psi_2\rangle$ as the initial state, our steady state is mixed, and depends on ϵ.

It is noteworthy that, for the initial state $|\Psi_1\rangle$, we obtain the match $\rho_{22} = \rho_{33}$, therefore, $S(\rho^A) = S(\rho^B)$. This equality implies that QD is independent of the subsystem we choose to measure. On the other hand, for an initial state $|\Psi_2\rangle$, we obtain $\rho_{22} \neq \rho_{33}$, therefore, we do not get the same result when measuring in A or B. Here, we chose to measure at subsystem B.

B. Sudden Death and Revival of QD and MID

We have a special interest to study under which conditions the quantum correlations vanish. In some cases, the QD can vanish even with finite (nonzero) coherences. The conditions for zero QD are the ones given by Eq. (6) and the equivalent conditions when we measure A, also when both conditions (for A and B) are satisfied MID will be zero. An example of null QD that satisfies the conditions for an X-form matrix is given by [28]. This state corresponds to a *semiquantum* state [Eq. (4)].

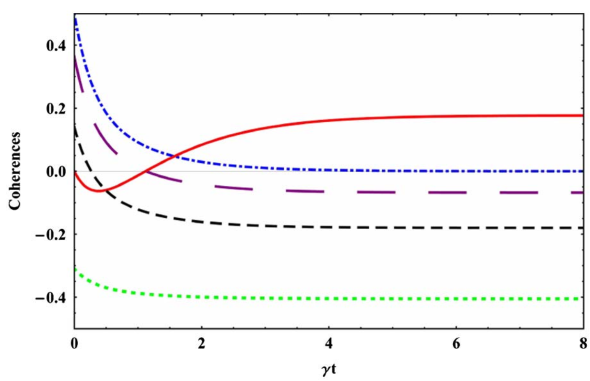

Fig. 7. (Color online) Coherences ρ_{14} and ρ_{23} for the initial condition $|\Psi_2\rangle$, with $N = 0.1$, and different values of ϵ: ρ_{23}, $\epsilon = 0.0001$ (dotted–dashed), ρ_{23}, $\epsilon = 0.369192$ (long dashed), ρ_{23}, $\epsilon = 0.6$ (dashed), ρ_{23}, $\epsilon = 0.9$ (dotted), ρ_{14}, and $\epsilon = 0.9$ (solid).

As we see from Figs. 2 and 3, there are several curves for which entanglement presents sudden death and revival, where there is a time interval where the state remains separable. This effect was studied in [18] for the same initial conditions $|\Psi_1\rangle$ and $|\Psi_2\rangle$, and they showed that, as we get *near* to the DFS ($\varepsilon_c < \varepsilon \leq 1$), the system shows no disentanglement and these phenomena of entanglement sudden death and revival disappear.

Also, another case where the system becomes semiquantum was studied in [28] and corresponds with curve (b) in Fig. 1.

The interesting case is the initial condition $|\Psi_2\rangle$, where, besides the death and revival of ENT, we also observe another phenomena, the death and revival of QD and MID (Fig. 3). For the case of $N = 0.1$, this value corresponds to $\epsilon = 0.369192$ and it happens at $t = 1.120824$.

As expected, the point of zero discord and MID belongs to the separable interval where ENT is zero, as shown in Fig. 4. This phenomena is represented graphically in Fig. 1 by curve (b), which touches the classical region in a single point.

As we vary the squeeze parameter N, we have a similar behavior for the zero of QD and MID, except that the point of zero quantum correlations shifts to the right, as one can see from Fig. 5. Also, each point in this curve is determined by a different value of ϵ, which varies as $0.3 < \epsilon \leq 0.5$. For large N, all these points correspond to $\epsilon = 0.5$. For a vacuum reservoir ($N = 0$), the system shows a classical point for every $\epsilon \leq 0.707$, and coincides with the points described in [18] for entanglement (Fig. 6). The time of simultaneous death and revival is given by $t = 1/2 \log[(1 - \epsilon^2)/\epsilon^2]$. In this case, the state goes directly from an entangled to a classical state without any gap between them.

In particular, for the case of a squeezed reservoir, the phenomenon of death and revival for QD and MID is produced by the squeezed bath. The reservoir destroys the interaction via one photon (ρ_{23}), but at the same time enhances the interaction via two photons (ρ_{14}), as we can see in Fig. 7. When both coherences coincide with the horizontal axis $\rho_{14} = \rho_{23} = 0$, we get a *classical* system. It seems that the system suffers from decoherence, but somehow the environment generates new coherences through pairs of photons.

This phenomenon, in some cases, is also the responsible for the points with a discontinuity in the slope of the QD [Fig. 3(b)], as the minimum changes between the different measurements Q_2 and Q_3 in Eq. (23), depending on the relative sign of the coherences.

This behavior is produced by the change between Θ_2 and Θ_3. When ρ_{14} and ρ_{23} have the same sign, then the minimum is obtained by $S(\rho|\{B_2\})$. On the contrary, when the coherences have different sign, the term Θ increases the value of $S(\rho|\{B_k\})$ and the minimum is obtained with $S(\rho|\{B_3\})$ (see Fig. 7).

This analysis is valid when the minimum is achieved by $S(\rho|\{B_{2,3}\})$, but, also for both initial conditions, there are almost unnoticed discontinuities when the minimum changes between $S(\rho|\{B_1\})$ and $S(\rho|\{B_{2,3}\})$. The phenomenon of discontinuity was observed experimentally in [29].

5. CONCLUSIONS

In this work we study the behavior of ENT, QD, and MID for two two-level atoms interacting via a squeezed reservoir. The time evolution of this system is given by the Lindblad master equation. This system presents a special set of initial conditions that are not affected by decoherence. We show that the behavior of the three quantifiers (ENT, QD, MID) is similar when we take an initial condition belonging to the DFS, but far from the DFS the behavior is very different. We further observe that ENT vanishes during some time periods while the QD and MID are different from zero. These states are separable states. Also, we have shown that, for some states with a nonzero "distance" from the DFS, there exists a specific value of N (the mean number of thermal photons) for which QD, MID, and entanglement are zero at a particular time. This result shows that, at some point, the system becomes truly classical (with zero QD and MID), but immediately after, the reservoir starts again generating quantum correlations. The system also presents discontinuities in the QD generated by the relative sign between the coherences of the density matrix.

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