PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
ESCUELA DE INGENIERÍA

ON THE USE OF FUZZY REAL OPTIONS HYBRID SIMULATION: ANALYSIS AND INTERPRETATION

MATÍAS JOSÉ LÓPEZ-ESTÉVEZ GUERRA

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Master of Science in Engineering

Advisors:
DANIEL OLIVARES QUERO
TOMÁS REYES TORRES

Santiago de Chile, November 2016

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Gratefully to my parents

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I would like to dedicate this work to my parents, Juan José and María José. They have supported me in every decision of my life, only by their efforts I have been granted the chance to complete my studies and develop this research work.

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ABSTRACT

A step-by-step application of fuzzy real options hybrid simulation to evaluate risky project is outlined. Real option values are estimated by Least Squares Monte Carlo simulation, capturing the value of managerial flexibility in a project's course of action. Possibility theory and fuzzy sets are used to represent the opinion of experts regarding uncertain parameters which do not have sufficient historical information to be correctly represent by probability density functions. A methodology to process and display fuzzy-random results from the simulation by confidence levels referring to the experts' opinion accuracy is proposed. In order to show the suitability of the framework two study cases are develop; an illustrative project evaluation example and a real case application concerning two hydropower plants evaluation comparison.

Keywords: Fuzzy Real Options, Fuzzy Sets, Real Option Analysis, Decision Analysis, Least Squares Monte Carlo Simulation, Project Evaluation, Uncertainty Representation.

RESUMEN

Una guía para la aplicación de opciones reales difusas por medio de simulación híbrida para evaluar proyectos riesgosos es presentada. El valor de las opciones reales es estimado a través de simulación de Monte Carlo por mínimos cuadrados, capturando el valor de flexibilidad de gestión dentro de un proyecto. Teoría de posibilidad y sets difusos son usados para representar la opinión de expertos sobre parámetros inciertos que no poseen suficiente información histórica para ser correctamente representados mediante una función de densidad de probabilidad. Una metodología para procesar y presentar resultados de carácter aleatorio-difuso por medio de niveles de confianza referidos a la precisión de la opinión de los expertos es propuesta. Para poder mostrar los beneficios de esta metodología dos casos de estudio son desarrollados; un ejemplo ilustrativo de evaluación de proyecto y una aplicación real sobre la comparación de evaluaciones entre dos generadoras hidroeléctricas.

Keywords: Opciones Reales Difusas, Sets Difusos, Análisis de Opciones Reales, Análisis de Decisión, Simulación de Monte Carlo por Mínimos Cuadrados, Evaluación de Proyectos , Representación de Incertidumbre.

1. INTRODUCTION

The pursuit for establishing new means to represent and handle project's intricate variables in its evaluation is continuously ongoing. Even though deterministic analysis as Net Present Value (NPV) and Internal Rate of Return (IRR) offer a clear view of the profitability of a project, they fall short at considering other scenarios were uncertainty is present (Fraser et al., 2008).

Real Option Valuation (ROV) is currently the standard technique to address the evaluation of projects under uncertainty of one or more of its parameters. It acknowledges the existence of managerial flexibility on the project timeline, allowing changes on its course of action based on uncertainty realizations (Dixit & Pindyck, 1994). Enlarging the investment if the market situation improves or ceasing operations if they are unprofitable are common examples of this type of actions (Kodokula & Papudesu, 2006). The added value obtained by recognizing this alternatives can be decisive in high risk projects were the expected value suggested by a classical NPV would just show one negative outcome (Yeo & Qiu, 2003; Bowman & Moskowitz, 2001).

In ROV, a stochastic analysis is performed based on the nature and characterization of random parameters. Even though quite effective to model variables with known Probability Density Functions (PDFs), ROV does not provide a systematical way to include variables for which PDFs are unknown. In such cases, it is necessary to use alternative methods to model uncertain parameters that can capture some known features without recurring to arbitrary assumptions on PDFs (Sandri et al., 1995). In this regard, accounting for the experts' opinion on the characterization of uncertain parameters in risky projects remains an open problem.

Fuzzy sets and possibility theory, first introduced by Zadeh (1978), provide a suitable framework to model expert elicitation on uncertain outcomes. By means of assigning a degree of membership to the value of a parameter, experts' opinion can be used to create a fuzzy set that describes their believes regarding the most and least possible future scenarios.

The use of fuzzy sets to model uncertain parameters in ROA derives in Fuzzy Real Options (FRO), which allows to consider both randomness and fuzziness in the evaluation of a project (Carlsson & Fuller, 2003).

FRO have been used in the past to assess the impact of uncertainty that can only be characterized by experts' opinions in the evaluation of risky projects (Wang et al., 2009; Cheng & Lee, 2007). Previous researches have also explored the use of hybrid variables, where certain moments of the PDF of a random variable are treated as fuzzy variables. In Sun et al. (2015), the wind speed for wind turbines generation is represented by a Weibull distribution (Ahmadi & Ghasemi, 2012), which distributions parameters are indicated by fuzzy numbers, expressing probabilistic and possibility uncertainty. Also, Wang et al. (2010) use fuzzy sets to indicate the volatility in the random process of the remediation and redevelopment costs of brownfields evaluation.

Fuzzy logic and systems has been previously used to asses risks and make decision in the electricity markets area. In S. Medina (2007), fuzzy logic is used to consider external factors in the Colombian electricity market such as regulatory changes and socio-political issues. Optimality in energy contracts decision making is a recurring theme when assessing risk in electricity market. Schmutz et al. (2002) measure the economic performance of contract portfolios considering multiple aspects of risk, using fuzzy set theory to tackle vagueness in information. A self-scheduling approach for large electricity consumers is developed based on a probabilistic fuzzy system by Zarif et al. (2012), specifying strategies for different levels of uncertainty set by fuzzy α-cuts. Lastly, a FRO approach for the operation of generation assets is investigated by Yu & Sheble (2003), taking into account impacts of de-regulation on the valuation of such assets.

Methodologies for solving FRO are characterize by the models used to evaluate the real option's value. In Liao & Ho (2010) a binomial lattice approach is developed to conduct an investment project valuation, which considers managerial flexibilities and the complexity of estimating future cash flows in uncertain decision making environments. The Black-Scholes formula to value real options has also been used under a fuzzy environment to consider assumptions over key parameters of its calculation (Wu, 2004).

In order to obtain numerical solutions via FRO simulation it is indispensable to separate randomness and fuzziness. Wang et al. (2011) extend the Least Squares Monte Carlo Method (LSM) to simulate a hybrid process obtaining adequate results. The procedure is divided into two steps: (i) generating fuzzy samples from the fuzzy set provided by the experts, and (ii) using LSM to compute the value of the real option for each fuzzy sample. This provides –per fuzzy sample– a fuzzy real option compose of a project crisp (scalar) value calculated by the LSM and a degree of membership set by the fuzzy variable used.

A relevant inconvenience of the previous approach resides on the fact that all the resulting information is fuzzy, which difficult a direct interpretation by the decision maker (DM). Besides choosing a suitable defuzzification method, the challenge extends to capturing the random nature of FRO and also displaying results for different levels of confidence on the accuracy of the experts' opinion. Additionally, given the number of parameters that could be regarded as fuzzy variables in a given project, the methodology needs to support a multi-fuzzy set environment. Applying the methodology in Wang et al. (2011) with multiple fuzzy variables leads to a significant increase in the number of fuzzy samples, creating new challenges in presenting, interpreting, and building judgments based on FRO results.

In this thesis a framework to solve FRO via hybrid simulation is proposed. The main contributions of this work are:

- We propose a methodology to process, display, and interpret fuzzy-random results per confidence level to facilitate judgment by DMs.
- We outline a step-by-step application of FRO hybrid simulation considering multiple parameters as fuzzy variables.
- We demonstrate the use of FRO hybrid simulation in two illustrative examples, highlighting key aspect of the applicability and suitability of the framework.

The rest of the thesis is organized as follows, Section II reviews the essential concepts and notions of ROA and fuzzy reasoning. Section III explains the suggested framework resolution steps, followed by an illustrative project evaluation example using FRO hybrid simulation in Section IV, and a realistic case study of two hydropower plant projects in Section V. Final remarks and future work conclude the paper in Section VI.

2. RANDOMNESS AND FUZZINESS WITHIN A PROJECT

2.1. Real Options and LSM Simulation

Real options recognize the value provided by management flexibility through actions such as waiting to execute, expand, or abandon the project during its lifetime. To implement ROA, the inputs to model the stochastic behavior of a random variable are typically obtained from historical data. The predominant numerical methods used to calculate ROV are the Black-Scholes formula (Black & Scholes, 1973), binomial lattices (Cox et al., 1979) and Monte Carlo simulation (Boyle, 1977). Each alternative delivers an adequate approximation of the option's value, contrasting for the most part on their implementation process. In particular, following Wang et al. (2011), the LSM algorithm designed by Longstaff & Schwartz (2001) is adopted in this work.

LSM essentially decides optimally, for each simulated path and period, between executing in the present time and waiting for the next period. Thus LSM supports the valuation of discrete American options by simulation, allowing its exercise any period. The optimal decision at each period is based on an estimation of the evolution of all possible paths.

The initial step consists in creating discrete sample paths based on the random variable. Geometric Brownian Motion (GBM) is a suitable tool for their generation, applying $S(t + \Delta t) = S(t)(\mu\Delta t + \sigma\kappa\sqrt{\Delta t})$. Value changes of the variable S on each Δt follow the previous formula, with μ being the drift rate, σ the volatility rate and κ a normal variable distributed as N(0,1).

Subsequently, for every path, a prediction for the stopping rule instructing if and when the option should be exercised is made utilizing a recursive method. Commencing at the last period t, the maximum project value among executing it or not is selected, a direct task since it is the final instance to make a decision (European option).

Afterwards, for $t - 1$, maximum project values are selected again for that period and stored in vector $\vec{X}$. The previously known values of t are discounted one period and stored in vector $\vec{Y}$. To decide if the option should be exercised or it is preferable to wait depending on the values of $t-1$, the expected value of waiting until t is obtained by regressing $\vec{Y}$ on a function of $\vec{X}$.

The function form used in this work is $\vec{Y} = c_0 + c_1\vec{X} + c_2\vec{X}^2 + c_3\vec{X}^3$, which proves to give satisfactory estimations for the projects analyzed. Conducting a least square estimation to find out the constants (c_0, c_1, c_2, c_3) grants the parameters for the conditional expectation function $E[Y|X]$. Individually for every path, its value on $\vec{X}$ is compared to the one obtained using $E[Y|X]$. Depending which one is greater, it will instruct to execute the option or wait another period.

This comparison is then done to periods $t-2$ with $t-1$, $t-3$ with $t-2$, until reaching the initial one. Once the decision of using the option in a period takes place –since it can be executed only once–, it can't be used on future periods. In such way, if the comparison of the first periods for a path indicates that it should use the option at the beginning, it does not matter how good the values become in the future for that particular path after the next period, it will always exercise it on the first period. This acknowledges the fact that management decisions do not have confident long-term future information and judgments are based over the present and a near future approximation for the next period. The process ends specifying a stopping rule for each path indicating in which period the option is exercised.

Once the stopping rule is retrieved, the value of each path is obtained by exercising –or not– the option when it is specified. Finally the expected value for the real option is attained by taking the mean of its value in all paths. Also, a PDF can be obtained when considering the outcomes of every path. Therefore, different metrics concerning the ROV's random nature are also available for a broader analysis.

2.2. Possibility Theory and Fuzzy Sets Analysis

2.2.1. Possibility and Probability

When an uncertain variable does not have sufficient historical data to get a representative PDF, a purely probabilistic ROA would require arbitrary assumptions on PDFs derived from expert elicitation. Possibility theory can be applied in such realm of imprecise probabilities, acknowledging the insufficient historical data available for the variable (Dubois & Prade, 1988; Yager, 1992).

There is a central distinctness between the information that possibility and probability distributions offer; while the former measures the likelihood of a variable taking a certain value using degrees of membership, the latter measures the probability based on abundant historical data. Both approaches aim at describing the behavior of an uncertain variable; however, only possibility distributions might be appropriate in instances with limited information (Dubois & Prade, 1993; Dubois et al., 2000).

2.2.2. Fuzzy Variable

Fuzzy variables are central to the use of possibility theory. Such variables are characterize by a degree of membership $\mu_X(x)$ that measures how possible it is for a variable X to take a value of x; thus a fuzzy variable is defined by the tuple $[x, \mu_X(x)]$. $\mu_X(x_a) = 1$ implies that there are no values considered more possible than $X = x_a$ and $\mu_X(x_b) = 0$ that $X = x_b$ is considered impossible. In possibility theory, the degree of membership $\mu_X(x)$ is defined equal to the possibility of the variable X taking the value x (Zadeh, 1978).

Possibility measures are non-additive, which has implications on the algebra of multiple fuzzy variables. In specific, the possibility union of non-interactive (unrelated) events is calculated as the maximum value of their individual possibilities, and as the minimum in the case of intersection (Zadeh, 1978).

2.2.3. Fuzzy Sets

Equivalent to a possibility distribution, fuzzy sets specify the degree of membership assigned to a range of values. Triangular and trapezoidal membership functions are the most common shapes to build fuzzy set (see Fig 2.1).

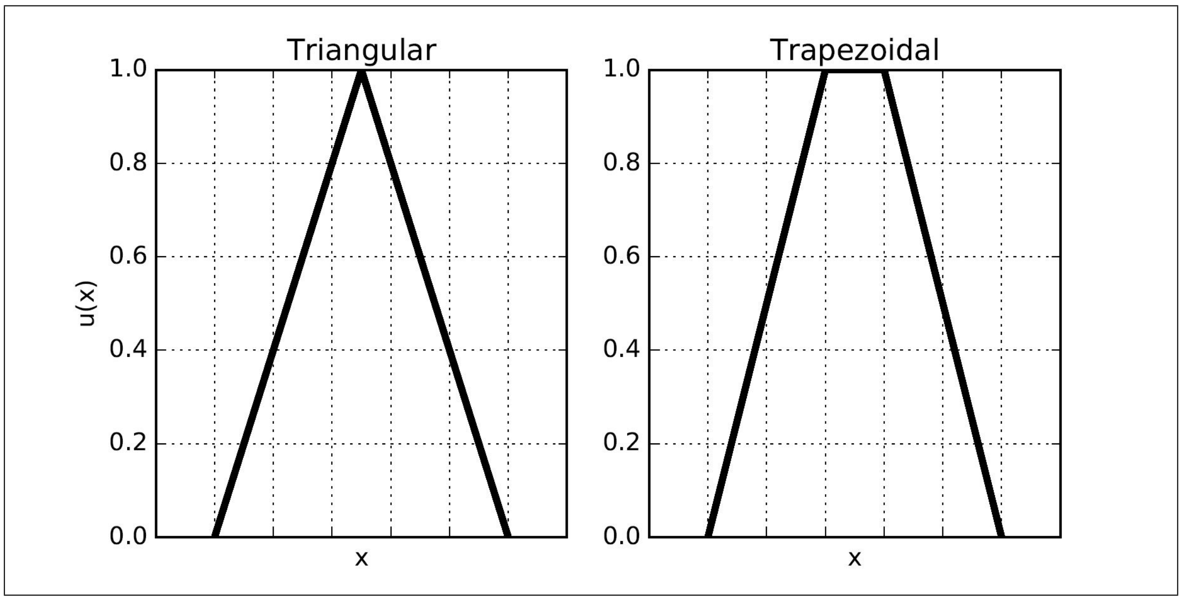

Figure 2.1. Triangular and trapezoidal fuzzy sets.

2.2.4. Defuzzification

Solving a problem with fuzzy variables as inputs culminates with fuzzy results as outputs. Converting these outputs to a crisp form is the process known as defuzzification. Defuzzification is of utmost importance to enable the interpretation of fuzzy results and further handling for decision making.

Common recent alternatives to find a single crisp value to represent and rank a fuzzy set include the center of gravity, center of maxima, and median methods (Brunelli & Mezei, 2013). In this work, we make use of the fuzzy expected value (FEV) methodology proposed by Liu & Liu (2002), since this is specifically designed for discrete fuzzy simulations. This technique uses the so-called credibility measures (Liu, 2008) to calculate the FEV the same way a probability measure is used to obtain the expected value of a random variable.

2.2.5. γ-level Fuzzy Subsets (γ-set)

Fuzzy sets can be reduced to fuzzy subsets by imposing a minimum degree of membership to be considered. A γ-set of a fuzzy set X contains only the elements of the original set which possess a $\mu_X(x) \geq \gamma$ for $\gamma \in [0, 1]$ (Carlsson & Fuller, 2001). The range of values from the elements get narrower as γ gets closer to 1, admitting only higher and higher possibilities. Different γ-sets for the same triangular membership functions are illustrated in Fig 2.2.

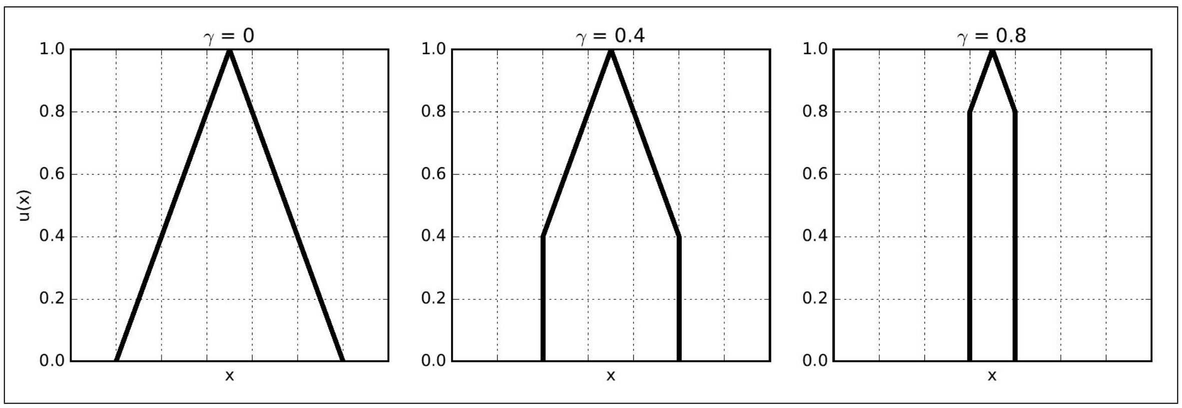

Figure 2.2. γ-level fuzzy subsets.

2.2.6. Confidence Levels

Running simulations for a given γ-set implies that realizations of fuzzy variables with degrees of membership below γ are considered to be impossible. Hence, the DM can set a confidence level associated to how accurate the experts' opinion is weighed. Ignoring scenarios under a γ-level is a judgment made by the DM regarding his own conviction over the experts' believes. For example, a DM might choose to ignore realizations under a certain possibility depending on how conservative the experts' opinion is considered.

The same type of information available for the original fuzzy set ($\gamma = 0$) can be obtained for different γ-sets, to account for several confidence levels. Particularly in this work, the information computed for different γ-sets are the FEV, and minimum and maximum crisp values.

Selecting a confidence level of $\gamma = 1$ for a triangular membership function means that only the value considered as most possible by the experts is accounted for. This is equivalent to conducting a non-fuzzy analysis and obtaining a single value for ROV, since all values but one are considered impossible. For a trapezoidal function, the use of $\gamma = 1$ would yield a segment of equi-possible values, translating into a typical sensibility analysis for ROV.

In this regard, a FRO analysis displaying results by confidence levels becomes a complementary tool for ROV. It allows the DM to judge information from $\gamma = 1$ levels (normal ROV) until $\gamma = 0$, including scenarios which are considered less possible but yet not impossible by the experts.

3. FRAMEWORK WALK-THROUGH

The general framework proposed in this work to perform a FRO hybrid analysis is depicted in Fig 3.1, and described in detail in this section.

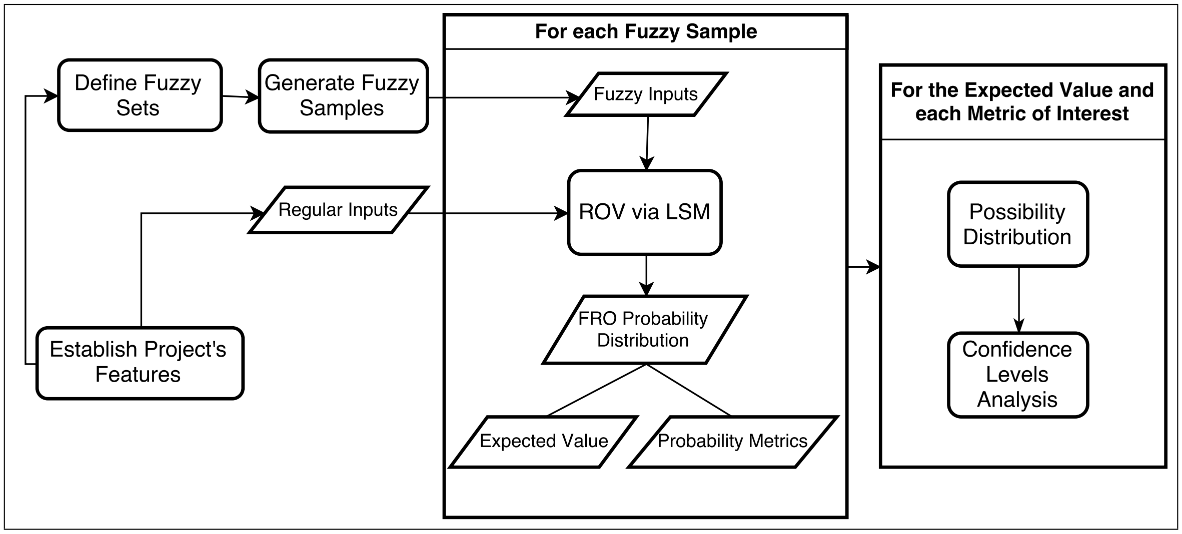

Figure 3.1. Framework's conceptual map.

3.1. Establishing Project's Features

This task consists of setting the parameters required to forecast project cash flows. Also the random variables that influence the forthcoming payoffs must be identified, together with the real options available –such us waiting for or abandoning– the execution of the project.

Then, the parameters that will be handled as fuzzy variables must be chosen. Even though, numerous aspects of the project could be argued to be influenced by experts' opinion to some extent, it is advisable to treat only crucial variables with fuzziness in order to limit simulation running times. This framework only contemplates the use of non-interactive fuzzy variables.

3.2. Defining Fuzzy Sets

Fuzzy sets are the tools used to capture experts' opinion about possible future outcomes of an uncertain variable. Its construction begins by establishing the most possible value (or values) of a variable in the experts opinion, assigning them a membership degree of 1. Minimum and maximum limits are then set by assigning them, and every value outside those bounds, a membership degree of 0.

Triangular and trapezoidal fuzzy sets are used throughout the examples. Also, as an extra mean to improve this representation, Bézier quadratic curves (Jowers, 2007) can be employed to represent an optimistic or pessimistic viewpoint as presented in Fig 3.2.

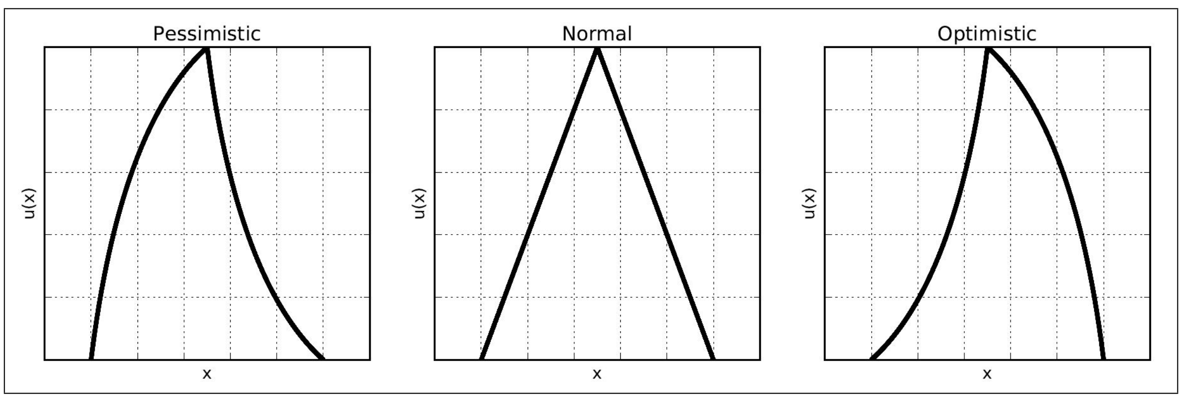

Figure 3.2. Pessimistic and optimistic triangular fuzzy sets using bézier quadratic curves.

3.3. Generating Fuzzy Samples

In the case of a single fuzzy set, a fuzzy sample corresponds to a fuzzy variable obtained from that set. A sample can be created by selecting a crisp value from a uniform distribution between the boundaries of the fuzzy set, with its respective degree of membership. When sampling multiple fuzzy sets, crisp values are likewise retrieved independently for each set, where the membership degree of the sample corresponds to the minimum of the membership degrees of each value retrieved, which represents the intersection of non-interactive fuzzy events.

One must be careful of the trade-off between using abundant fuzzy samples and the associated computational burden. An adequate fuzzy sample size to capture the information of one fuzzy set lies between 100 and 200 (Wang et al., 2011); however, this sample size grows exponentially with the number of variables consider as fuzzy in the simulation. Anyhow, parallel computing can be applied to reduce simulation times.

3.4. Estimating the Real Option via LSM for each Fuzzy Sample

The procedure continues by calculating the value of the real option via LSM for each fuzzy sample; that is, fuzzy variables are fixed to their crisp values in each simulation. Hence, a fuzzy real option is attained with a value given by the ROV and a degree of membership equal to the one of the corresponding fuzzy sample. The appropriate number of sample paths to run LSM will depend on the desired level of accuracy in the ROV and the time limit imposed for its simulation.

The expected value of ROV (E[ROV]) is typically the most important piece of information used by DMs, and is determined from the LSM simulation as the mean of all the paths' ROVs. However, other relevant metrics of interest are readily available from the implied PDFs shaped by the sample paths. In particular, Conditional Value at Risk (CVaR) (Rockafellar & Uryasev, 2002) and cumulative probabilities are used in this work for further analysis and comparison; nonetheless, other metrics could also be chosen. In this regard, it is important to note that the fuzzy real option calculated by LSM returns a fuzzy PDF, not only a single fuzzy value.

3.5. Possibility Distribution and Confidence Level Analysis

The results of the aforementioned procedure enclose information from several fuzzy-random simulations, which must be organized and processed in order to produce meaningful insights for DMs. As an alternative framework to address this issue, an analysis based on possibility distributions and a γ confidence levels is propose in this work.

The use of multiple fuzzy sets yields a big amount of fuzzy variables combinations. As a result, similar E[ROV] values could be obtained from high- or low-possibility fuzzy samples as depicted by the dots in Fig 3.3.

The results obtained from the fuzzy real options simulated can be used to estimate an underlying continuous possibility distribution as illustrated by the solid lines in Fig 3.3. The left part of the distribution corresponds to the possibility of being lower than or equal to a given value of E[ROV], ranging from the minimum possible E[ROV] to the lowest E[ROV] with a possibility of 1. Similarly, the right side corresponds to the possibility of being greater than or equal to a given value of E[ROV], ranging from the largest E[ROV] with a possibility of 1 to the maximum possible E[ROV].

The abovementioned possibility distribution can be readily used to derive minimum and maximum crisp values limits for any given γ-set. Then, the FEV associated with these γ-sets can be calculated by applying equations (1) and (2) from Liu & Liu (2002).

For a given fuzzy set A with n fuzzy variables $[x_i, \mu(x_i)]$ sorted so that $x_1 < x_2 < \ldots < x_n$; the FEV is given by:

$$FEV(A) = \sum_{i=1}^{n} x_i p_i \tag{3.1}$$

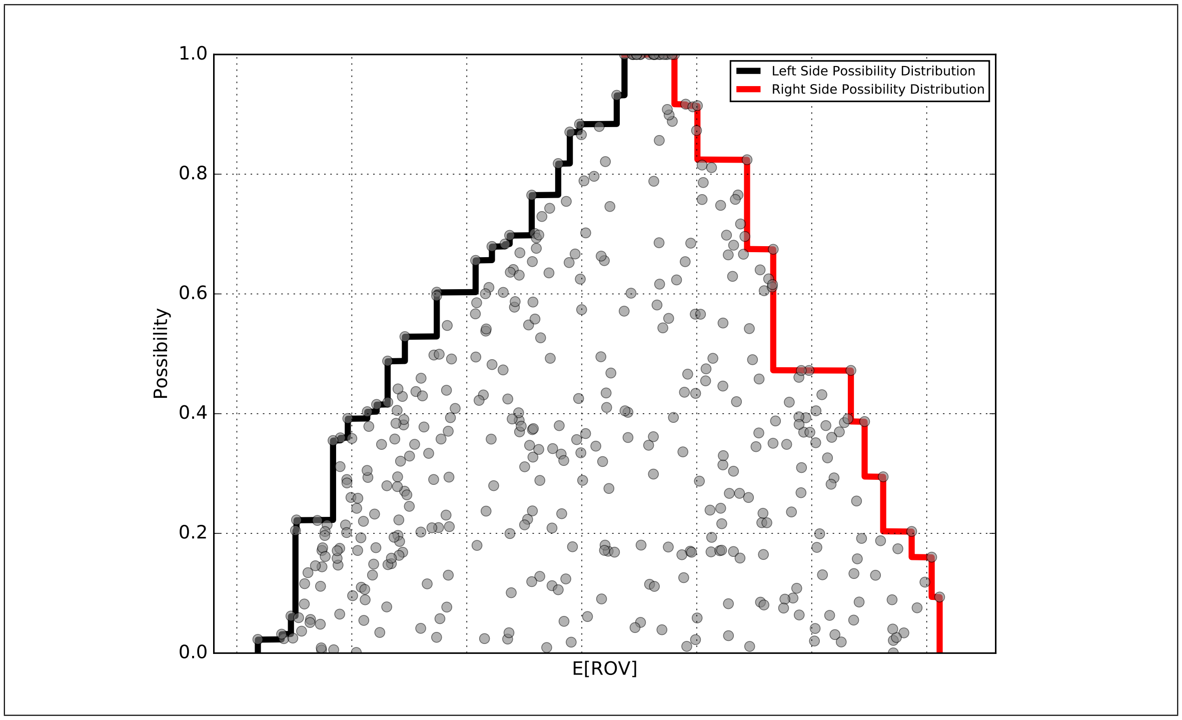

Figure 3.3. Continuous possibility distribution estimated from fuzzy real options simulated.

$$p_i = \frac{1}{2}(\vee_{j=i}^{n}\mu_j - \vee_{j=i+1}^{n+1}\mu_j) + \frac{1}{2}(\vee_{j=1}^{i}\mu_j - \vee_{j=0}^{i-1}\mu_j)$$
$$\mu_0 = \mu_{n+1} = 0 \quad \text{for} \quad i = 1, 2, \ldots, n \tag{3.2}$$

Noteworthy, calculating the FEV with the set of all fuzzy real options in Fig 3.3 is equivalent to considering only the set of fuzzy real options that define the underlying possibility distribution. Thus, using the latter yields benefits in terms of computation times without loss of accuracy, which is particularly important in instances with a large number fuzzy samples.

The same previously mentioned procedure can be used to obtain FEVs for any other metric (e.g., CVaR and cumulative probabilities). That is, for each metric of interest, the possibility distribution must be recomputed from the entire set of fuzzy real options for that particular metric to then compute its FEVs.

In sum, value limits and FEVs by confidence level, for each selected metric of interest, will be available as a result. This allows a more comprehensive analysis and interpretation by the DM, taking into account the fuzzy-random nature of uncertainty.

4. AN ILLUSTRATIVE NUMERICAL EXAMPLE

The following example aims to facilitate the comprehension of the previously explained procedure, and exemplify the results, analysis, and insights accessible to the DM.

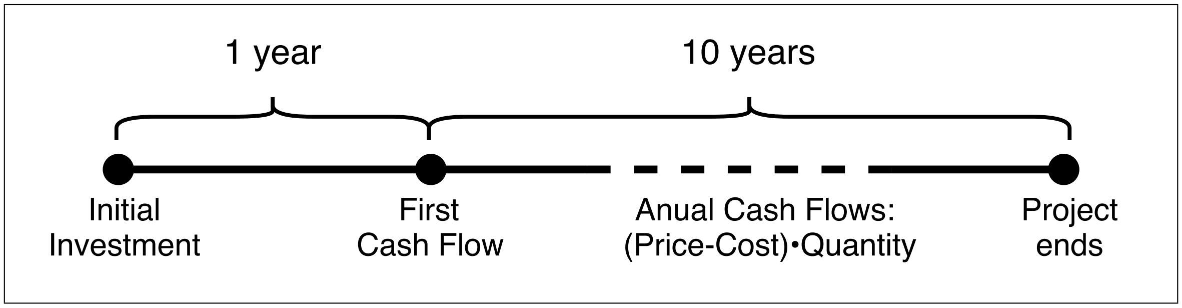

Figure 4.1. Project timeline.

The illustrative example considers a simple project with the timeline shown in Fig 4.1. An initial investment is made in period 0, which yields annual incomes that last 10 years and start in period 1. These annual incomes are assumed to come from selling q units of a product, at a unit price p and a unit cost c.

In this example, the price p is assumed to be a random variable that follows a random walk process specified by a GBM with time steps of one year, drift μ_p, and volatility σ_p. The real option available in the project consists of the ability to fix the price p to its present realization for all future periods. In other words, the developer has the option to sign a contract to keep p fixed if the actual price p is deemed favorable over its continuation. The specific project parameters used are presented in Table 4.1.

Initial investment I and price drift μ_p parameters are considered as fuzzy variables. Some reasons to assign a fuzzy nature to the investment I may include the risk of a cost overrun, delays, and the occurrence of other unexpected events which are subject to experts' opinion. Treating the price drift as a fuzzy variable gives a hybrid structure to the price process, which is justified by an unknown response of the market to the product being offered. Therefore the price drift μ_p is estimated by market studies and experts' assessment.

Table 4.1. Example Project Parameters

Parameter		Value
Discount rate	r	0.1
Quantity	q	100k
Cost	c	3.2 USD
Initial price	p_0	5 USD
Annual price volatility	σ_p	0.05
Annual price drift	μ_p	fuzzy
Initial investment	I	fuzzy

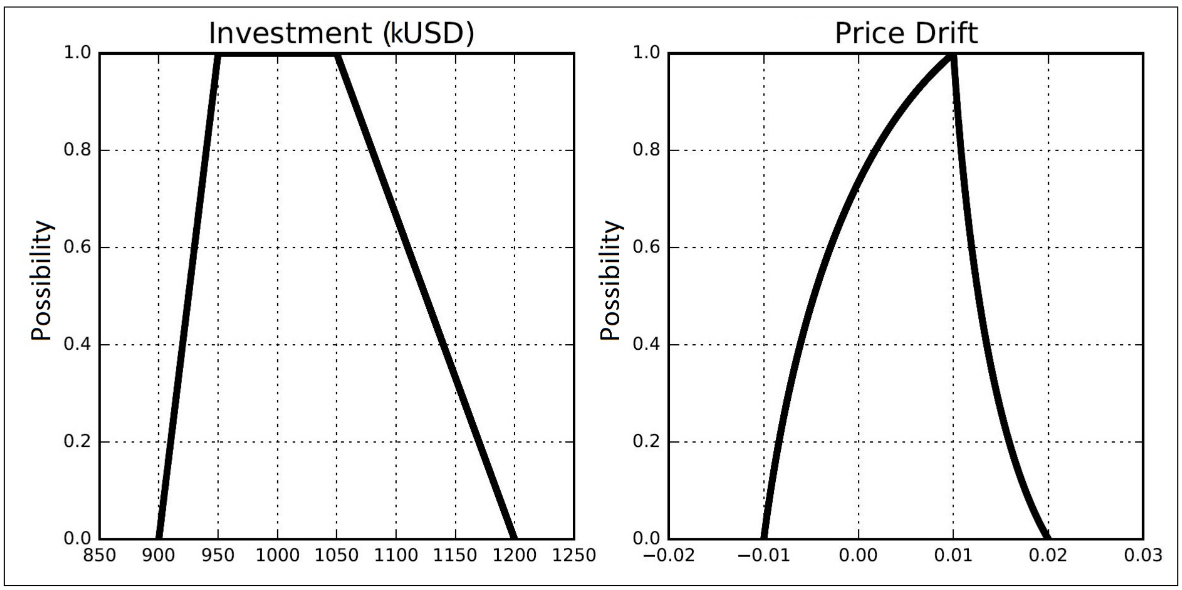

Figure 4.2. Fuzzy sets for investment and price drift.

The trapezoidal fuzzy set for the initial investment I, shown in Fig 4.2, indicates that the experts believe the most possible outcome value for the variable is in the range 950-1050 kUSD, without preferences for any values within that range. Investments below 900 kUSD and beyond 1200 kUSD are deemed impossible by the experts. The pessimistic triangular fuzzy set for the price drift μ_p, shown in Fig 4.2, considers only one value (0.01) with maximum possibility, and a pessimistic stance towards better (higher) drift values.

The FRO simulation is performed using 22,500 fuzzy samples (150 per each of the two fuzzy sets) and the ROV is estimated for each sample by LSM using 100 GBM price paths. As a consequence, the ROV will be computed 2.25 million times, once for each combination of fuzzy samples and random paths. The adequate amount of fuzzy samples and price paths depends on the evaluation characteristics, for this particular example tolerable convergence rates of FEVs are observed using the amounts indicated.

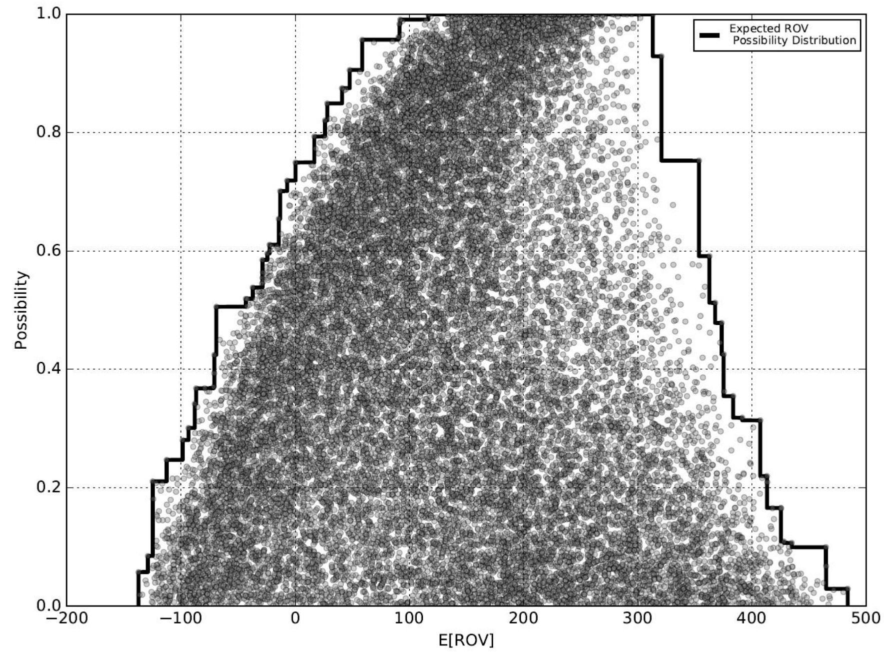

Figure 4.3. Possibility distribution estimated from fuzzy real options simulated ($\sigma = 0.05$).

Figure 4.3 displays all 22,500 fuzzy real options outputs arranged by their E[ROV] and possibility (membership value) for this case, in which price volatility is high at $\sigma_p = 0.05$. The solid line represents the underlying continuous possibility distribution of E[ROV]. Similarly a possibility distribution is also obtained for the metrics of CVaR-10% and CVaR-5%, these are displayed together with E[ROV] (CVaR-100%) in Fig 4.4. Additionally, for each of the aforementioned metrics, Table 4.2 shows FEVs and maximum and minimum crisp values for different confidence levels (γ-sets).

Thus, if the DM deems reasonable only those scenarios with a possibility of 0.8 or higher according to experts' opinion ($\gamma = 0.8$), the FEV of E[ROV] would be 176.9 kUSD,

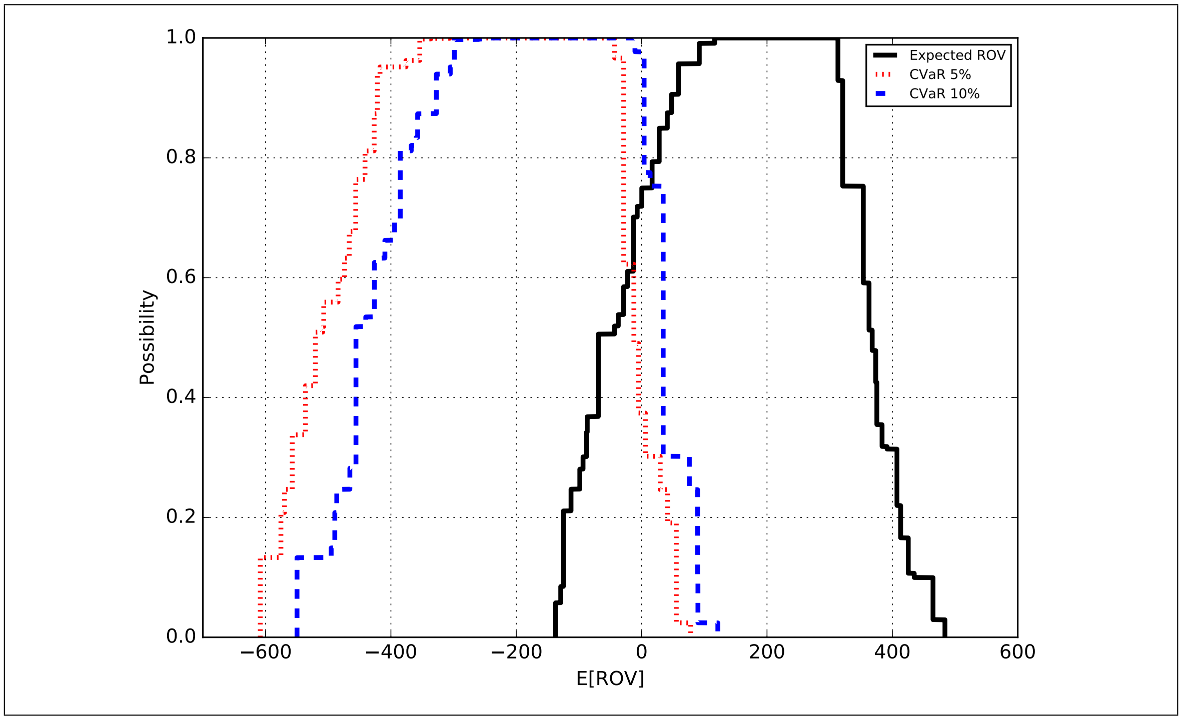

Figure 4.4. Possibility distributions for E[ROV] and CVaRs ($\sigma = 0.05$).

Table 4.2. E[ROV] and CVaRs per Confidence Level ($\sigma = 0.05$)

	E[ROV] (kUSD)			CVaR 10% (kUSD)			CVaR 5% (kUSD)		
γ	FEV	Min	Max	FEV	Min	Max	FEV	Min	Max
1	214.9	116.6	313.2	-126.0	-241.3	-10.7	-174.7	-306.4	-43.1
0.8	176.9	28.0	320.8	-185.4	-385.1	3.9	-231.9	-441.3	-28.6
0.6	170.9	-22.5	353.6	-187.3	-426.3	34.4	-238.4	-473.9	-12.4
0.4	164.5	-69.1	375.2	-195.1	-455.7	34.4	-252.7	-536.3	-4.9
0.2	162.3	-124.8	413.2	-191.5	-489.5	89.3	-251.4	-575.5	41.5
0	165.2	-137.2	483.9	-195.1	-549.8	121.6	-251.9	-608.4	78.3

with a possible range of variation between 28 kUSD and 320.8 kUSD (see Table 4.2). However, despite the positive E[ROV], the results for the CVaR metrics indicate that significant losses may still occur. It is also worth mentioning that the specific fuzzy samples that produced the minimum and maximum values for each metric and confidence level are, in general, not the same.

It can be observed from Table 4.2 that the project evaluation tends to produce worse FEVs of E[ROV] when considering lower confidence levels, these lower values can be attributed to the pessimistic/conservative characteristic of the fuzzy set in Fig 4.2.

The DM might also be interested in knowing the chances of the project having a ROV below a certain threshold. If so, possibility distributions of cumulative probabilities of ROV can be calculated. In particular for this example, we have selected the cumulative probability of ROV being below 0 kUSD (CP0) and 100 kUSD (CP100) as metrics of interest. With this information the DM might reconsider its judgment based on commercial strategies, risk aversion policy, etc.

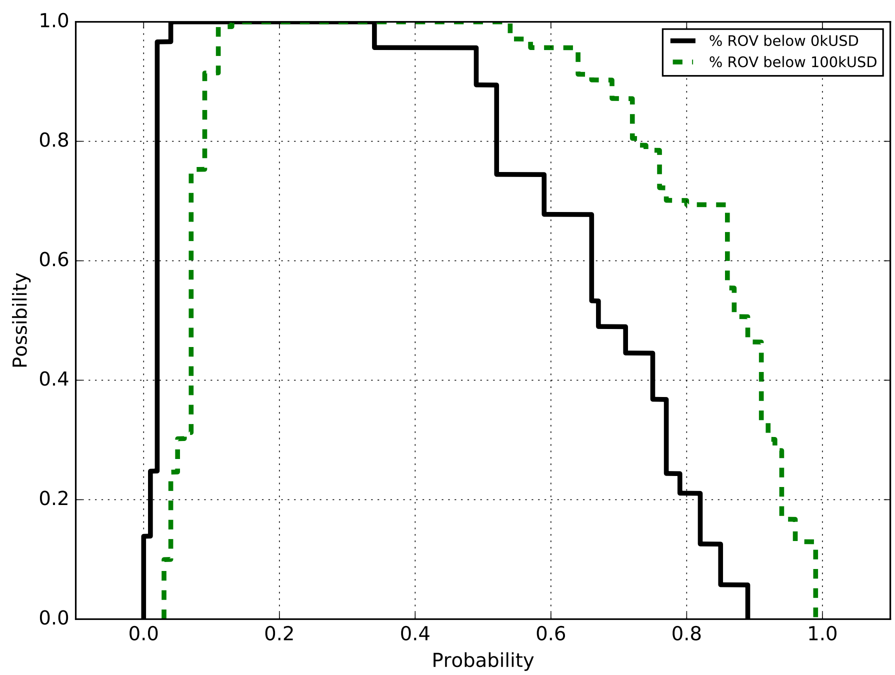

Figure 4.5. Possibility distributions for CP0 and CP100 ($\sigma = 0.05$).

The possibility distribution and detailed numerical results for CP0 and CP100 metrics are displayed in Fig 4.5 and Table 4.3, respectively. It can be observed from the results that FEVs of both CP0 and CP100 increase as the confidence level considered gets closer to $\gamma = 0$, which is consistent with the results of E[ROV] and CVaR metrics in Table 4.2.

Table 4.3. Cumulative Probabilities per Confidence Level ($\sigma = 0.05$)

	CP0			CP100		
γ	FEV	Min	Max	FEV	Min	Max
1	19.0%	4%	34%	33.5%	13%	54%
0.8	26.6%	2%	52%	40.3%	9%	73%
0.6	31.5%	2%	65%	44.3%	7%	86%
0.4	33.6%	2%	75%	45.5%	7%	91%
0.2	34.5%	2%	82%	45.6%	4%	94%
0	34.7%	0%	89%	45.9%	3%	99%

Also, when considering the most possible scenario ($\gamma = 1$) the project presents a high FEV of CP0 (19.0%), warning about high chances of the project yielding net losses.

Even though the use of hybrid FRO analysis already offers interesting insights to the DM, the methodology is particularly resourceful when used to compare two or more alternative projects. To illustrate this, the evaluation is repeated for a case of low price volatility ($\sigma_p = 0.03$), which results are shown in Figs 4.7 and 4.8 and Tables 4.4 and 4.5.

Table 4.4. E[ROV] and CVaRs per Confidence Level ($\sigma = 0.03$)

	E[ROV] (kUSD)			CVaR 10% (kUSD)			CVaR 5% (kUSD)		
γ	FEV	Min	Max	FEV	Min	Max	FEV	Min	Max
1	209.4	132.5	286.2	-26.3	-128.7	76.1	-53.4	-154.6	47.8
0.8	167.1	20.0	303.7	-45.8	-198.0	102.0	-76.2	-239.2	83.0
0.6	163.0	-21.5	332.3	-51.2	-264.8	146.1	-86.4	-305.5	118.1
0.4	159.4	-66.0	362.7	-57.6	-300.6	153.0	-93.0	-351.1	135.0
0.2	161.6	-96.9	406.3	-59.9	-347.3	182.8	-93.3	-392.8	170.1
0	162.8	-145.9	461	-57	-361.2	249.1	-92.4	-408.3	232.0

Possibility distributions of E[ROV] are quite similar for the cases of low (Fig 4.7) and high price volatility (Fig 4.4). In particular, comparing Table 4.4 and 4.2 show that the FEVs of E[ROV] for the case with low volatility, even though lower, are within a margin of only 10 kUSD from the high volatility case for any given confidence level. However, in terms of CVaR-5% and CVaR-10%, differences of 3 to 4 times lower expected losses appear due to the narrower price paths. These significant differences, which would not be

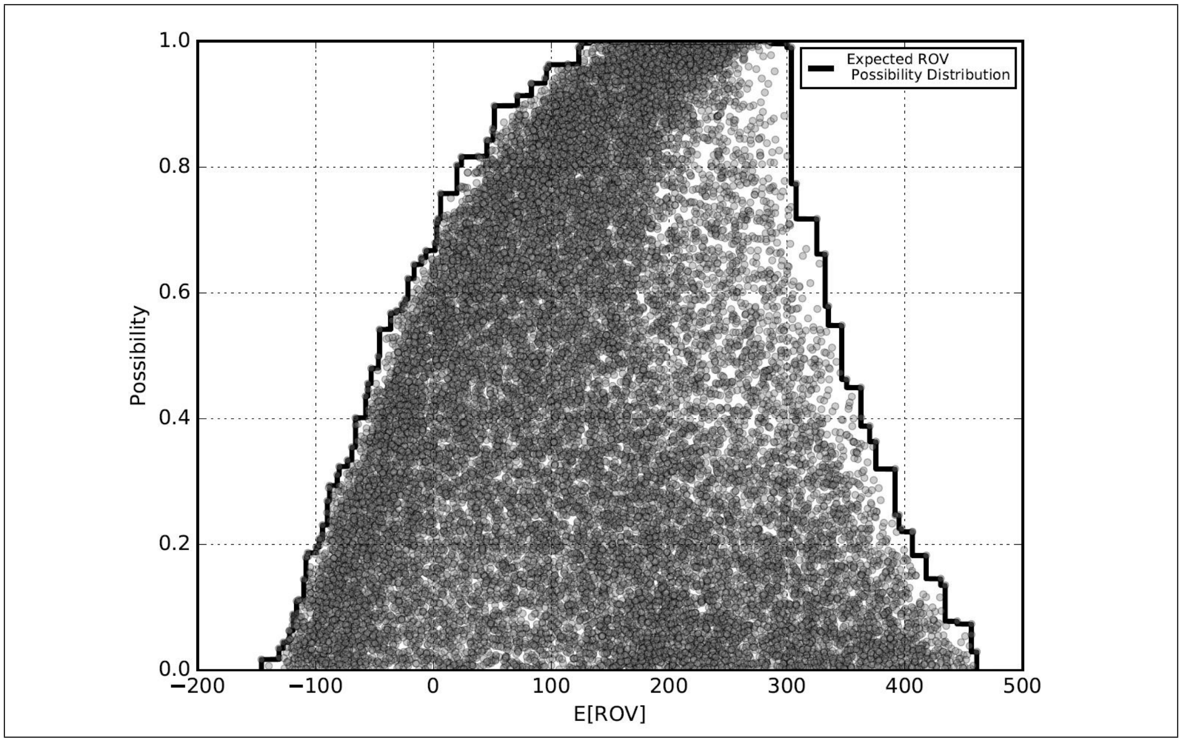

Figure 4.6. Possibility distribution estimated from fuzzy real options simulated ($\sigma = 0.03$).

observed if only E[ROV] provided by the LSM were analyzed, support the usefulness of a hybrid analysis with multiple metrics of interest.

Table 4.5. Cumulative Probabilities per Confidence Level ($\sigma = 0.03$)

	CP0			CP100		
γ	FEV	Min	Max	FEV	Min	Max
1	7.5%	0%	15%	23%	5%	41%
0.8	21.3%	0%	45%	39.9%	2%	79%
0.6	28.0%	0%	65%	44.1%	2%	93%
0.4	31.6%	0%	80%	45.2%	0%	99%
0.2	33.1%	0%	90%	45.2%	0%	99%
0	33.3%	0%	96%	45.2%	0%	99%

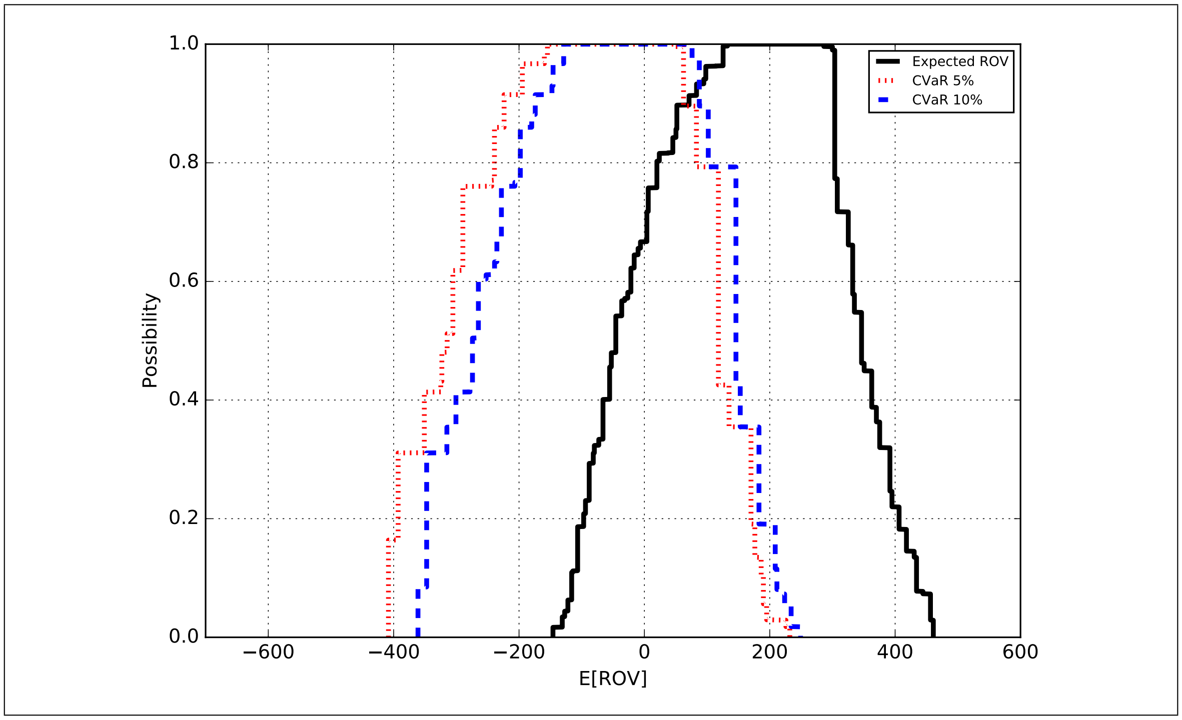

Figure 4.7. Possibility distributions for E[ROV] and CVaRs ($\sigma = 0.03$).

The analysis of cumulative probabilities also indicates lower risks associated with the low volatility case, which is more apparent for γ closer to 1, as shown in Table 4.5. Displaying the results by confidence level is particularly helpful in this example due to the similar FEVs both cases have for low γ levels.

In sum, each project responds differently to its own random and fuzzy risks. This different response is unveiled by arranging crisp value limits and FEVs for any metric of interest by confidence levels. The outlined framework captures this behavior and allows for a direct comparison between projects by the DM.

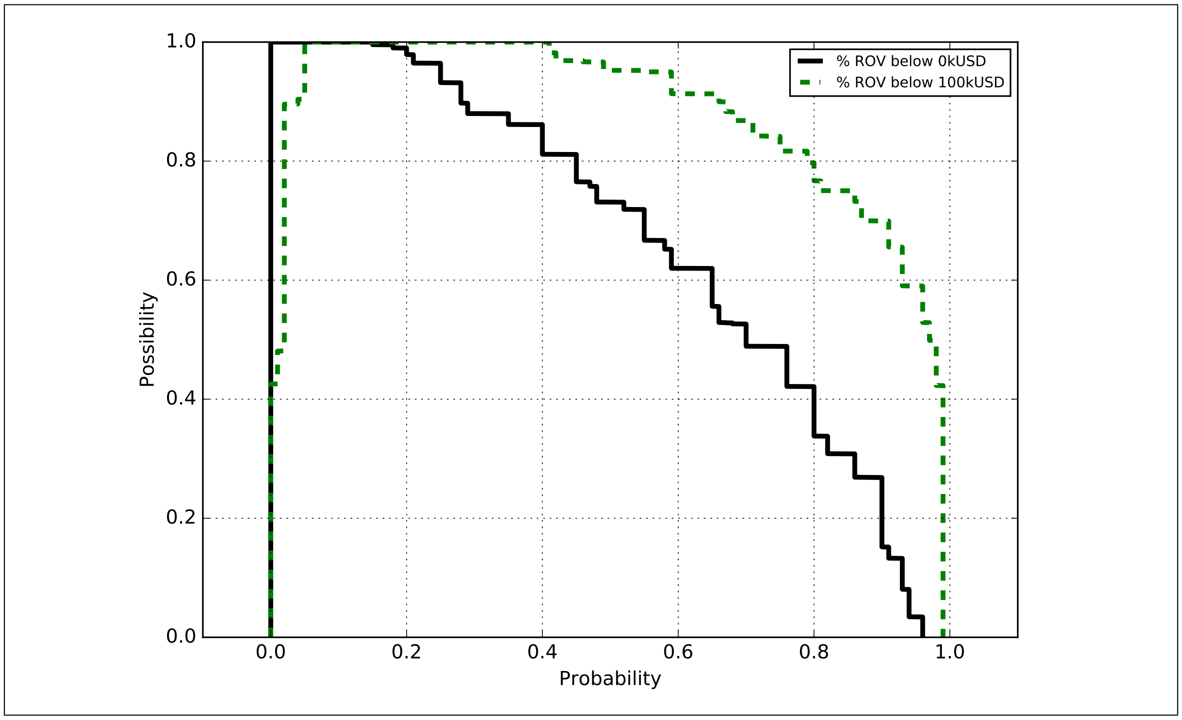

Figure 4.8. Possibility distributions for CP0 and CP100 ($\sigma = 0.03$).

5. HYDROPOWER PLANTS EVALUATION APPLICATION

Hydropower refers to the energy generated by the use of fast flowing or falling water, usually by the construction of dams or run-of-the-river applications. Hydropower projects are characterized for their high initial investment and low operational costs, by the virtue of having water as fuel. Consistently, their early development usually withstands several field studies to determine the future costs of infrastructure. Also, studies are required to apply for an environmental permit, which could prevent the projects continuation if denied.

Even after preliminary studies are completed (having the certainty of investments costs and obtaining an environmental permit) high risks still exist associated with the electricity market.The high investments and low operation costs make hydropower plants projects very sensible to energy prices, since the energy associated to their capacity factor and installed capacity will be produced independently. Because of this, Power Purchase Agreements (PPAs) are often signed between a generator and an electricity distributor or final industrial client to set fixed the energy price of future transactions for a portion (or the whole) of the plants energy generation. Therefore, a Final Investment Decision (FID), after the preliminary studies,is often made taking into account the expected investment costs, permitting results, and PPAs available.

In this regard, the timelines of hydropower projects may experience changes depending on preliminary studies outcomes and electricity market circumstances. Options of abandoning, investing or waiting are present since the studies are finalize and until they expire. Thus, ROV is a convenient tool to value these managerial flexibilities along the project timeline.

In Fig 5.1, the project timeline describes four key stages of a hydropower plant evaluation. Stage A refers to the preliminary studies that must be done before the FID. Once this stage has ended, there is certainty about the investment costs of the project. Also, the continuation of the project is affected by a probability of failure of not obtaining the environmental permit approved or having the project rejected by socio-political reasons. Having

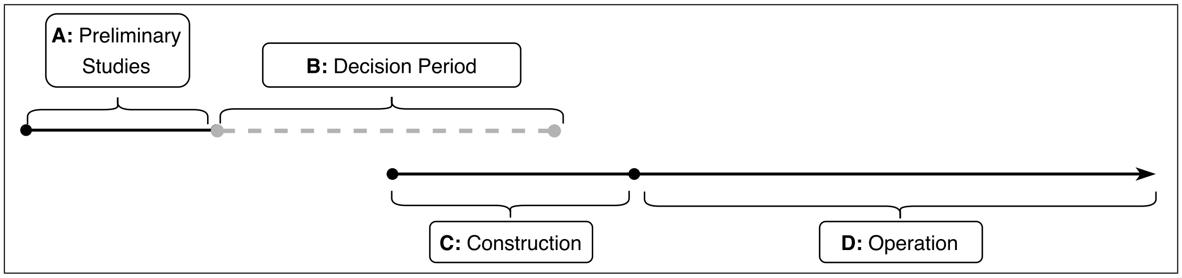

Figure 5.1. Hydropower plant project timeline.

the permit rejected forces the abandonment of the project and the studies cost are assume as losses. Having the permit accepted allows the project to be developed from that moment and until the expiration of the studies (5 years).

Stage B is the decision period, if the environmental permit was accepted, then the FID can choose when to execute the project. In this thesis, it is assumed that the whole energy sales for the entire plant lifespan are cover by a PPA, thus settling all the future cash flows. The PPA is treated as a random variable, creating random walk paths with GBM for time steps of one semester (10 periods total) for a given volatility and drift. Hence, the company will be able to decide in each semester to invest on the project or wait for a better PPA depending on the present project profitability.

Stage C starts when the option of investing is exercised, and relates to the construction period of the hydropower plant. Investment costs are according to the preliminary studies and a defined yearly payment outlaid for its duration.

Finally, Stage D refers to the cash flows obtained by the plants operation in its lifespan. The evaluation considers that the project is receiving energy payments and capacity payments through time. The former are computed using the PPA and the hydropower plants maximum capacity with its corresponding yearly capacity factor, which indicates the ratio with which the plant operates at its full capacity. The latter payment refers to the capacity that can be guaranteed to be provided by the plant to the network at any time, calculated by multiplying the assigned firm capacity and with the yearly firm capacity price. Low operational costs compared with the income generated by these two payments are incurred each year. A tax rate over the earnings is considered together with the tax shield created by the depreciation of the investment incurred in Stage C for a determined duration.

The investment, probability of failure and initial PPA are parameters considered as fuzzy due to lack of historical data, therefore they are estimated by experts opinion. The investment is uncertain before the field studies are completed; consequently it is necessary to estimate its possible value for the present evaluation. The possible complications associated with the construction of the plant, caused by the geographic characteristics of the location and potential over- or under-costs, can be assessed by experts in order to estimate the investments value using a fuzzy set.

Probability of failure associated with environmental or socio-political issues avoiding the development of the hydropower plant project is a difficult parameter to estimate, highly based in experts opinion. Past history concerning the development of similar projects in the area can be taken into account if available. However, the manner in which the projects developers plan to work in combination with local authorities and communities highly affects the chances of the project being accepted.

The initial PPA available after finishing the preliminary studies is also considered as fuzzy. Even though the parameter possesses historical data, it fails to be enough to confidently estimate its future value. The electricity market is very volatile and has multiple factors which can cause big changes in PPAs values. Technology changes and competitors behavior are two main uncertain factors which are finally addressed by experts elicitation. Renewable energy insertion to a network can have big price implications because of its low operational costs. PPAs conducted by public biddings largely depend on the price strategy adopted by the other generators in the network. As a result, the future PPA obtainable for the project has a fuzzy nature which is ultimately estimated by experts.

Two projects, HydroA and HydroB, are evaluated by FRO hybrid simulation in this section, with their parameters being specified in Table 5.1, Fig 5.2 and Fig 5.3 (detailed information in Appendix A) . Standard medium size run-of-the-river hydropower plants characteristics are considered for both cases. Their main difference resides on the better capacity factor that HydroB has over HydroA, with the trade-off of also having a higher possible investment and failure probabilities. This makes HydroA a more conservative project when compared to HydroB, having a minor initial investment but not as high energy and capacity payments due to its lower average power generated through the year.

Table 5.1. HydroA and HydroB Parameters

Parameter	HydroA	HydroB
Discount rate	0.06	
Preliminary Studies Duration	3 years	
Preliminary Studies Cost	3 MMUSD/year	
Probability of Failure	fuzzy	
Construction Duration	4 years	
Total Investment	fuzzy	
Investment Annual Disbursement	30%-40%-20%-10%	
Installed Capacity	100 MW	
Capacity Factor	0.55	0.7
Firm Capacity	30 MW	50 MW
Firm Capacity Price	95 USD/KW-year	
Operational Cost	1.5 MMUSD/year	
Plant Lifespan	50 years	
Tax Rate	0.27	
Investment Depreciation	30 years	
Initial PPA (GBM)	fuzzy	
PPA Volatility (GBM)	0.1	
PPA Drift (GBM)	0	
Maximum PPA (GBM)	80 USD/MWh	
Minimum PPA (GBM)	50 USD/MWh	

Possibility distribution for expected ROV values (E[ROV]) and CVaRs for the 10% worst ROVs cases calculated by the FRO hybrid simulation for both projects are depicted in Fig 5.4 and Fig 5.5. Information per confidence levels obtainable from these distributions are specified in Table 5.2a and Table 5.2b.

E[ROV] values suggests HydroB is a more profitable project to invest on. Its FEV is better across almost every confidence level; therefore it has a better fuzzy expected value

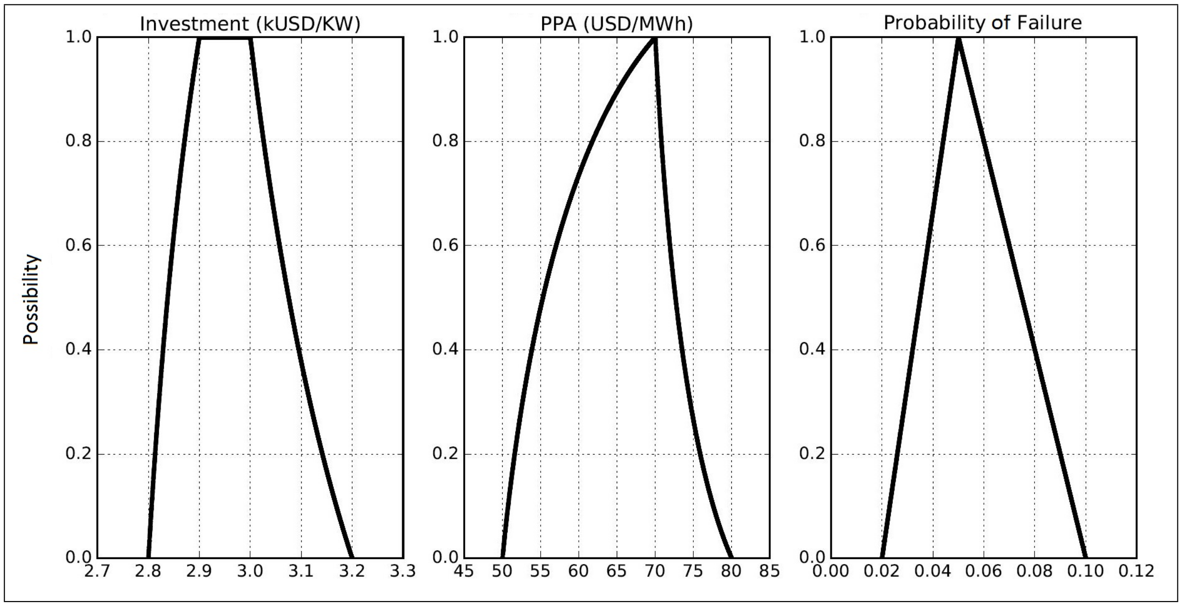

Figure 5.2. HydroA fuzzy sets.

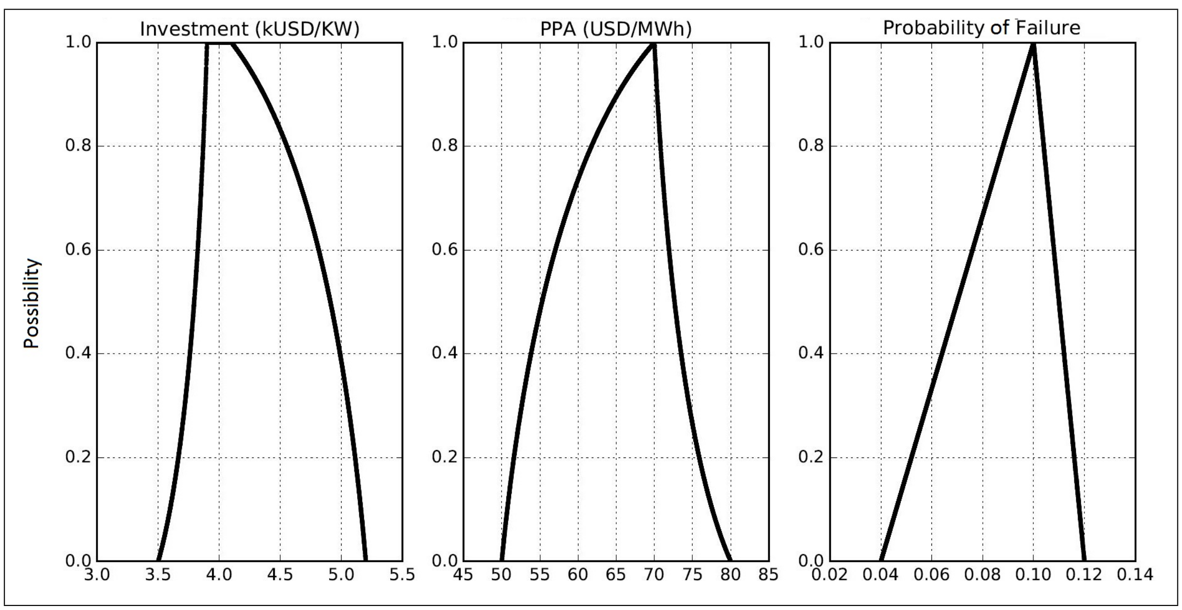

Figure 5.3. HydroB fuzzy sets.

independent of how much does the DM trusts the experts opinion accuracy. However, by looking at the wider interval limits of E[ROV] it is clear that HydroB has a more volatile nature than HydroA. This is explained by its higher energy generation (better capacity factor), and therefore this plant is more dependent on the fuzzy-random PPA to generate future

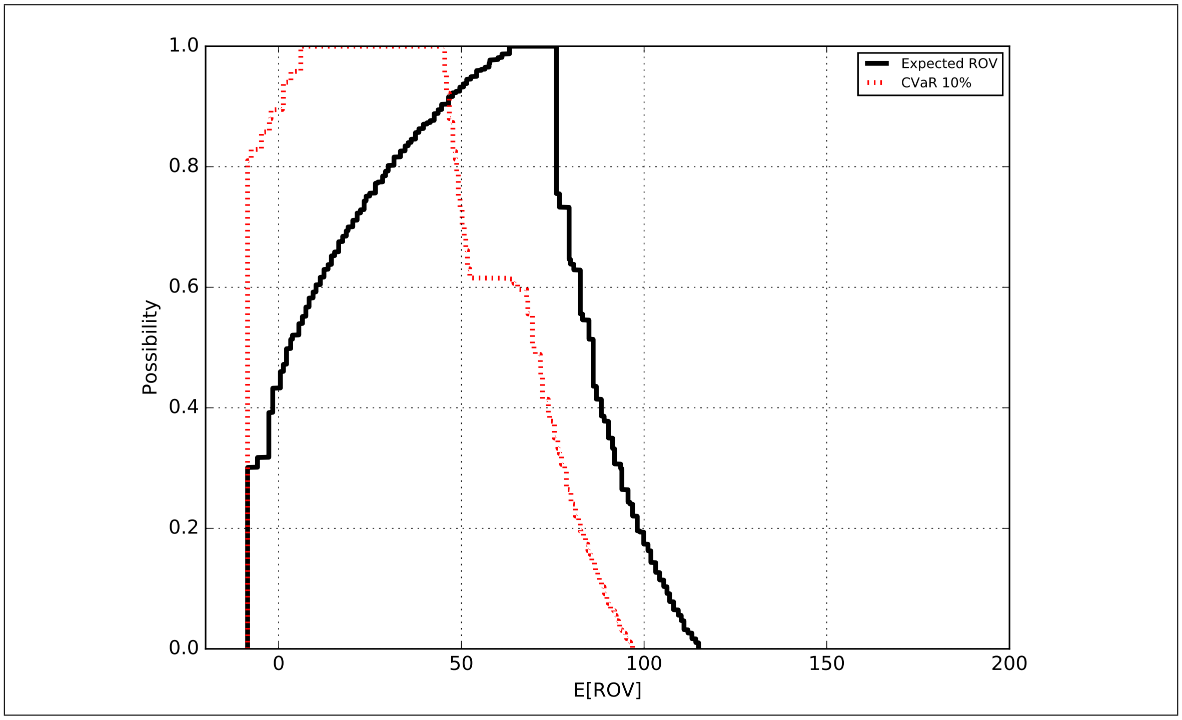

Figure 5.4. [ROV] and CVaR-10% possibility distributions for HydroA.

earnings. Thus, a low PPA may not be enough to cover the superior investment needed; meanwhile a favorable PPA can cause much higher profits. CVaRs calculated support this analysis by showing lower ROV worst scenarios for HydroA in comparison with HydroB, especially for higher confidence levels.

The left side of the possibility distributions of E[ROV] in Fig 5.4 and Fig 5.5 illustrates a limit possibility from where the distribution is fixed in a minimum E[ROV]. Given the nature of the evaluation, the minimum ROV possible is equivalent to the losses caused by performing the preliminary studies and then deciding not to invest on the project. Hence, that possibility limit denotes the scenario where the fuzzy parameters take unfavorable values that will always indicate not to invest in the project, independently of the PPAs evolution in Stage B. HydroB exhibits such scenario for a higher possibility than HydroA, suggesting that it is less likely to be developed due to risks associated with its fuzzy parameters.

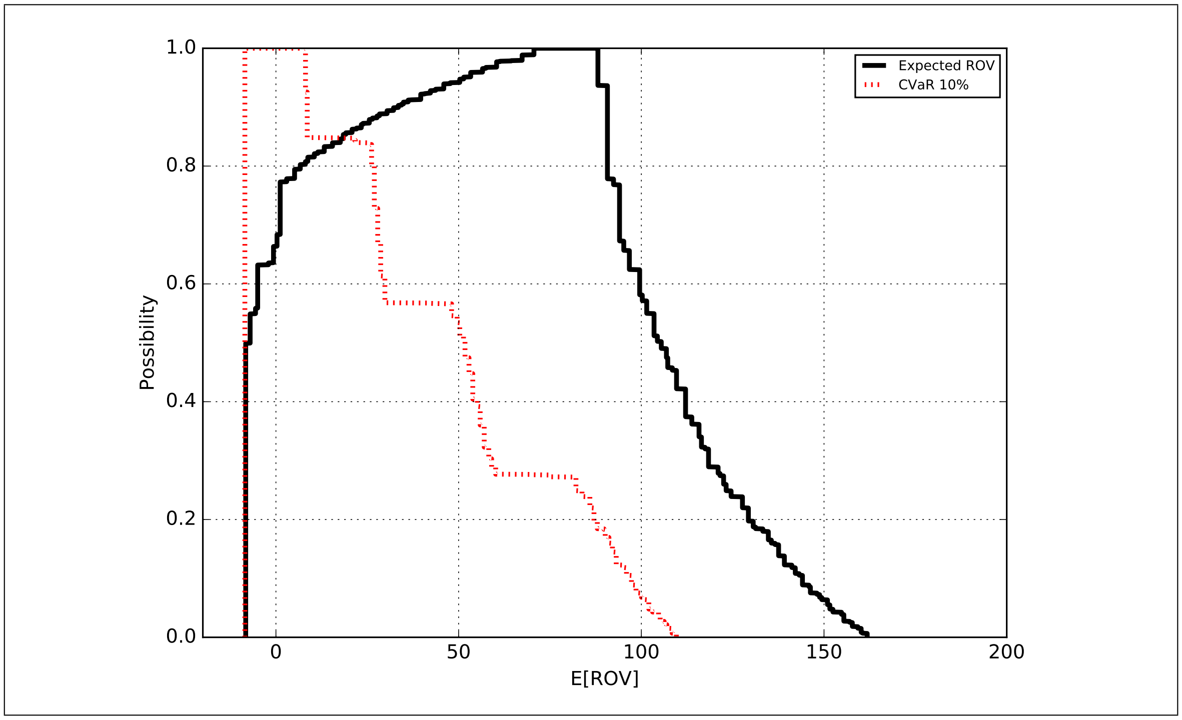

Figure 5.5. [ROV] and CVaR-10% possibility distributions for HydroB.

Interestingly the FEVs of E[ROV] change from being higher for HydroB for $\gamma = 1$ to higher in HydroA for $\gamma = 0.8$ due to higher exposure of HydroB to changes in fuzzy parameters. Then the FEVs again become higher in HydroB for $\gamma = 0.6$ and lower, due to the saturation of losses to a maximum level (-8.5 MMUSD). Therefore, considering larger possible ranges for the fuzzy parameters do not necessarily affect the FEVs for both projects in the same magnitude.

Possibility distribution for CP0 are calculated by the FRO hybrid simulation for both projects and depicted in Fig 5.6 and Fig 5.7. Information per confidence levels obtainable from these distributions is specified in Table 5.2c. Results reinforce the risky nature of HydroB over HydroA, with double or triple the FEVs chances of having a negative ROV across all the confidence levels. A DM could easily change his judgment related to the preference of the projects when knowing that the random nature of the PPA price evolution implies a high FEV of CP0 of 37.6% for HydroB and only 14.3% for HydroA for a confidence level of $\gamma = 0.6$.

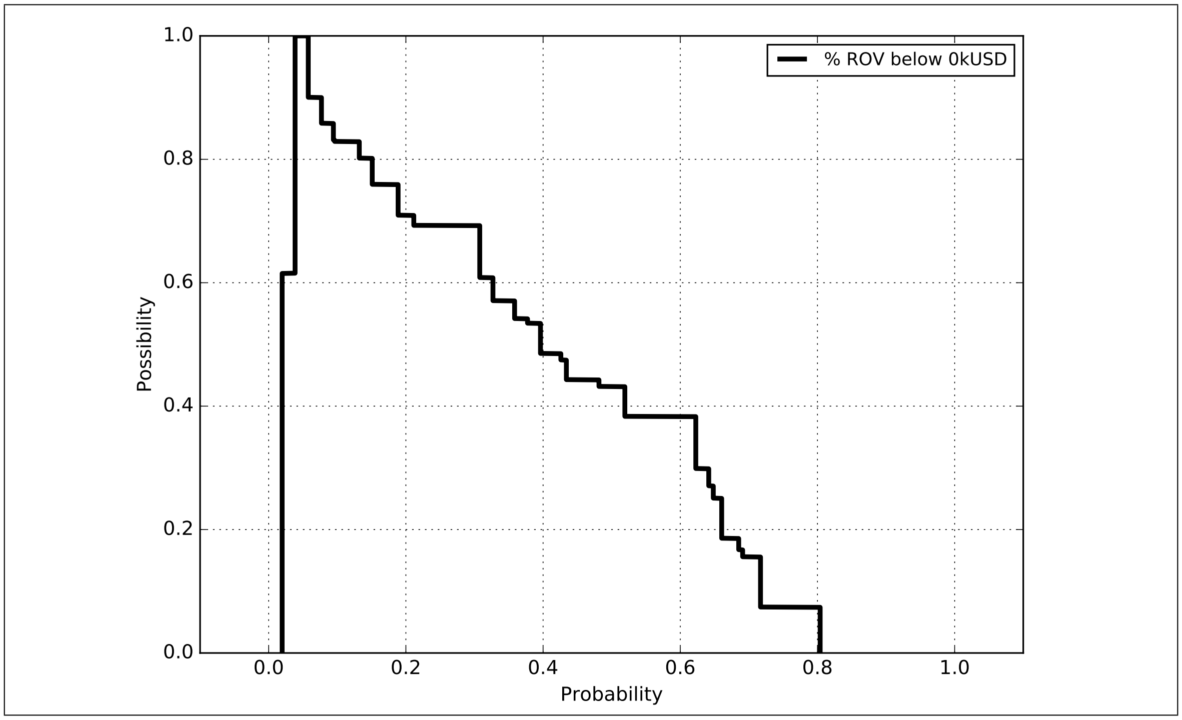

Figure 5.6. CP0 possibility distributions for HydroA.

The results produced by the hybrid FRO analysis provides the DM with fuzzy-random information per confidence level which would not be available if a classical FRO analysis for E[ROV] and $\gamma = 0$ had been performed. For example, for the case of a risk averse profit maximizing DM, might make different decisions based on $\gamma = 0$ as compared with $\gamma = 0.8$. In the former case the DM observes higher expected profits for HydroB at moderately higher risks based on CVaR-10% metric; however in the latter HydroA features both higher expected profits and lower risks.

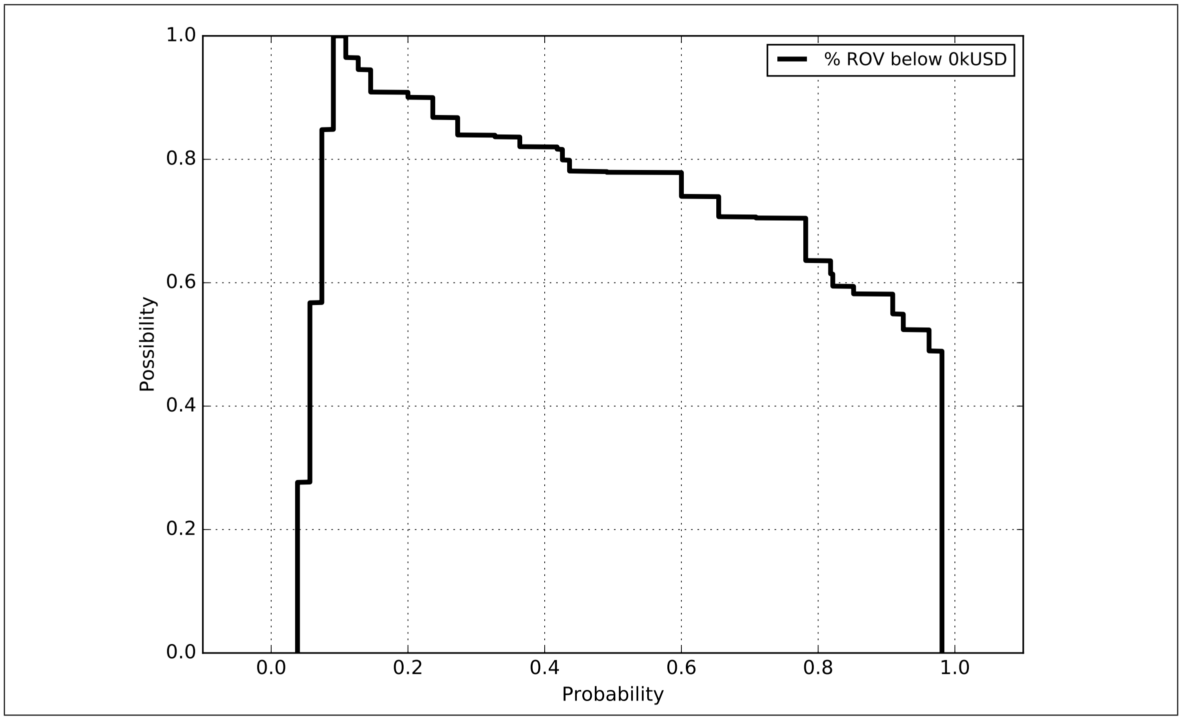

Figure 5.7. CP0 possibility distributions for HydroB.

Table 5.2. Result per Confidence Level for HydroA and HydroB

	HydroA			HydroB		
γ	FEV	Min	Max	FEV	Min	Max
1	69.6	63.1	76	79.4	70.6	88.1
0.8	54.5	29.9	76	51.5	6.6	90.7
0.6	49.9	10.2	82.5	50.4	-4.9	99.6
0.4	48.2	-1.6	88.3	52.7	-8.5	112.1
0.2	48.6	-8.5	98.1	55.1	-8.5	129.3
0	49.4	-8.5	114.9	56.7	-8.5	161.8

(a) E[ROV] (MMUSD)

	HydroA			HydroB		
γ	FEV	Min	Max	FEV	Min	Max
1	25.8	6.1	45.5	-0.2	-8.5	8.1
0.8	20.1	-8.5	48.3	7.8	-8.5	26.9
0.6	26.3	-8.5	65.4	8.7	-8.5	29.8
0.4	28.5	-8.5	74.1	15.6	-8.5	55.1
0.2	29.7	-8.5	82.1	20.1	-8.5	87
0	30.4	-8.5	96.2	21.1	-8.5	110

(b) CVaR-10% (MMUSD)

	HydroA			HydroB		
γ	FEV	Min	Max	FEV	Min	Max
1	4.8%	3.8%	5.7%	10%	9.1%	10.1%
0.8	8.7%	3.8%	15.1%	23%	7.4%	42.6%
0.6	14.3%	1.9%	32.3%	37.6%	7.4%	82.1%
0.4	19%	1.9%	51.9%	41.5%	5.6%	98.1%
0.2	21.5%	1.9%	66%	41.3%	3.8%	98.1%
0	22.3%	1.9%	80%	41.3%	3.8%	98.1%

(c) CP0

6. FINAL REMARKS AND FUTURE WORK

This work has presented a detailed framework for application, interpretation, and analysis of hybrid FRO simulations, taking into account multiple fuzzy variables. The use and calculation of alternative metrics of interest other than E[ROV], such as CVaR-X% and CPX, has been proposed and illustrated. Furthermore, an analysis of the results based on different γ-sets allowing a customized interpretation of them by the DM is thoroughly discussed.

The results and interpretation of the study cases demonstrate the value of the proposed framework when conducting an FRO hybrid simulation to evaluate a risky project. The DM can use the metrics of interest resulting FEVs and value limits to base his judgment in accordance with a given commercial strategy goal, risk aversion policy, etc. Furthermore, a DM can choose to ignore possible scenarios below a certain possibility by using γ confidence levels. Thus, obtaining results with less exposure to risks associated to fuzzy variables, which assists the DM in situations where he heavily trusts the accuracy of the experts or deems their opinion to be over conservative.

Future work will concentrate on extending the proposed methodology to incorporate risk constraints over different metrics of interest (e.g., CVaR-X% and CPX) when exercising the option in LSM for each fuzzy sample. Furthermore, such constraints could even be set differently for each confidence level, depending on the DM preferences.

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APPENDICES

APPENDIX A. FUZZY SETS INPUT DATA

Table A.1. Input Fuzzy Sets Value Limits per Confidence Level, Numerical Example ($\sigma = 0.05$)

	Price Drift		Investment (kUSD)	
γ	Min	Max	Min	Max
1	0.01	0.01	950	1050
0.8	0.002	0.0108	940	1080
0.6	-0.003	0.012	930	1110
0.4	-0.006	0.014	920	1140
0.2	-0.008	0.016	910	1170
0	-0.01	0.02	900	1200

Table A.2. Input Fuzzy Sets Value Limits per Confidence Level, Numerical Example ($\sigma = 0.03$)

	Price Drift		Investment (kUSD)	
γ	Min	Max	Min	Max
1	0.01	0.01	950	1050
0.8	0.002	0.0108	940	1080
0.6	-0.003	0.012	930	1110
0.4	-0.006	0.014	920	1140
0.2	-0.008	0.016	910	1170
0	-0.01	0.02	900	1200

Table A.3. Input Fuzzy Sets Value Limits per Confidence Level, HydroA

	Investment (kUSD		PPA (USD/MWh)		Failure Probability	
γ	Min	Max	Min	Max	Min	Max
1	2.9	3	70	70	0.05	0.05
0.8	2.87	3.02	61.8	70.8	0.044	0.059
0.6	2.84	3.06	57	71.9	0.038	0.069
0.4	2.82	3.1	53.8	73.5	0.032	0.079
0.2	2.81	3.14	51.6	75.9	0.026	0.089
0	2.8	3.1	50	80	0.02	0.1

Table A.4. Input Fuzzy Sets Value Limits per Confidence Level, HydroB

	Investment (kUSD		PPA (USD/MWh)		Failure Probability	
γ	Min	Max	Min	Max	Min	Max
1	3.9	4.1	70	70	0.1	0.1
0.8	3.86	4.55	61.8	70.8	0.088	0.103
0.6	3.82	4.81	57	71.9	0.076	0.108
0.4	3.76	4.99	53.8	73.5	0.064	0.112
0.2	3.67	5.11	51.6	75.9	0.052	0.116
0	3.5	5.2	50	80	0.04	0.12