PHYSICAL REVIEW B, VOLUME 64, 201405

Dynamic instability in resonant tunneling

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(Received 18 June 2001; revised manuscript received 22 August 2001; published 2 November 2001)

We show that a novel instability may be present in resonant tunneling through a quantum well in one, two, and three dimensions, when the resonance lies near the emitter Fermi level. A simple semiclassical model which simulates the resonance and the projected density of states by a nonlinear conductor, the Coulomb barrier by a capacitance, and the time evolution by an iterated map, is used. The model reproduces the observed hysteresis in such devices, and exhibits a series of bifurcations leading to fast chaotic current fluctuations.

DOI: 10.1103/PhysRevB.64.201405 PACS number(s): 73.40.Gk,72.15.Gd

Resonant tunneling through a double barrier, originally a rather elementary academic exercise, has already shown to be an extremely rich source of new and surprising physics for over three decades.[1] One instance is the bistable region in the I–V characteristic that some samples exhibit, leading to hysteresis as the curve is traced first increasing and then decreasing the bias.[2–8] This effect is currently understood as caused by the interaction of the current flowing through the device with the charge trapped in the well formed between the barriers.[3] Calling Q this charge, its effect on the incoming electrons may be viewed as an increase in the local potential by Q/C, where C is the capacitance, lifting the resonance level in the same amount. The latter can thus be somewhat above or below the emitter conduction band edge at the same bias, depending on whether the well is charged or uncharged. Since in the first case there is current flow through the well and in the second there is none, both cases are physically consistent, and in fact, observed experimentally. As an application, it has been suggested that in the bistable region the device may act as a THz detector and fast switch.[9,10]

Another instance are the THz oscillations that may arise in the presence of a magnetic field in the direction of the current flow, which become chaotic if the field is sufficiently strong.[11–13] In contrast with the hysteresis effect described above, these oscillations are associated with passage of the resonance across the emitter Fermi level. Assuming the resonance is initially above the emitter Fermi sea and off resonance, it will eventually enter the latter as the bias is increased, allowing electrons to tunnel into the well. Thus, with a certain time constant, charge begins to build up between the barriers. The Coulomb repulsion between this charge layer and the incoming electrons effectively causes an upward shift of the bottom of the well, and hence, also of the resonance state. This may drive the system off-resonance again until the charge, due to the finiteness of the barrier and the associated finite lifetime of the resonance state, has tunneled out again into unoccupied states on both the emitter and collector sides. The resonance then is no longer sustained above the Fermi energy by the decreased charge in the well, the resonance falls again and a new cycle begins. Whether this oscillation is damped away or not, is determined by the strength of the interaction, which in turn depends on the projected density of states in the emitter at or near the Fermi energy. By changing the profile in the density of states the magnetic field enhances this coupling, and, if large enough, induces the system to perform sustained oscillations. The purpose of this work is to show that, under favorable conditions, the oscillations may also be present in the absence of a magnetic field, as well as in situations of lower dimensionality such as tunneling through a quantum wire or dot.

We model the system by a simple circuit that captures the essence of the system under discussion, shown in Fig. 1. As follows from the discussion below, it includes the following features that appear to be essential in the description of the device: (1) the existence of a resonance state with an associated lifetime, which results in a time delay between the bias voltage and the buildup or decay of the charge in the well (“tunneling time”); (2) the existence of a Fermi sea on the emitter side, with a density of states depending on the dimensionality of the device (bulk, planar, or linear), and on the magnetic field, if present; (3) the existence of the Coulomb repulsion between the charge trapped in the well and the incoming electrons. The nonlinear element in the circuit represents transmission from the emitter side into the well, allowing a charge

$$\Delta Q(t,t+\tau)=\tau I_{\max}f(V) \tag{1}$$

to leave the source U_0 in the time interval τ. The dimensionless nonlinear function $f(V)$, depending on the voltage $V=U_0-U$ between emitter and well, is the convolution of the transmissivity of the resonance with the number of occupied states in the emitter, available for tunneling. This latter quantity may be written as

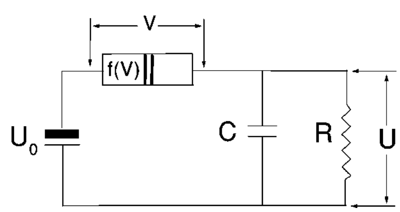

FIG. 1. Model circuit for a double barrier device. The function $f(V)$ represents the response for tunneling into the well formed by the barriers.

$$N(V)=\theta(eV-E_r+E_F)[1-\theta(eV-E_r)]\rho_d(V), \qquad (2)$$

where $\theta(x)$ is the Heaviside step function, E_r is the energy of the resonance level at zero bias, and E_F, the Fermi energy. All energies and voltages are assumed to be in electron-volts and measured with respect to the bottom of the conduction band on the emitter side. The quantity $\rho_d(V)$ represents the number of states in the emitter side, with the component of kinetic energy along the current flow, equal the resonance energy shifted by the bias V. It is given by

$$\rho_d(V)=\alpha_d(eV+E_F-E_r)^{d-1/2}. \qquad (3)$$

Here α_d is a constant depending on the dimensionality $d=1,2$, or 3 of the emitter (the well has dimension $d-1$). The values are, $\alpha_1=2, \alpha_2=2L\sqrt{2m^*}/(\pi\hbar)$, $\alpha_3=m^*L^2/(\pi\hbar^2)$, where L is the emitter width and m^* the effective mass of the carriers.

The capacitance C represents the Coulomb barrier. As charge flows through the nonlinear element, entering the well, the voltage drop $V=U_0-U$ is reduced. At the same time charge is allowed to leave the well through the collector represented by the load resistor R. These elements define a time constant $\tau_0=RC$ characterizing the rate at which the well may be emptied. With these definitions the conservation of charge provides an equation of motion for the voltage $U(t)$ on the output side of the device,

$$C[U(t+\tau)-U(t)]=\Delta Q-\tau\frac{U(t)}{R}. \qquad (4)$$

Defining $V_n=V(t+n\tau)$ one obtains from Eqs. (1) and (4) the following iterated map,

$$V_{n+1}=(1-\gamma)V_n-\gamma RI_{\max}f(V_n)+\gamma U_0, \qquad (5)$$

where $\gamma=\tau/\tau_0$ and we assume that $\tau_0>\tau$ holds. The map has fixed points given by

$$V^*=U_0-RI_{max}f(V^*). \qquad (6)$$

A simple linear analysis in the neighborhood of one of these fixed points shows that for it to be stable the condition

$$-1<RI_{\max}\frac{df}{dV}\Big|_{V^*}<2/\gamma-1 \qquad (7)$$

must be satisfied. It follows that for either an increasing or decreasing function $f(V)$ at the fixed point, an instability occurs provided the local derivative is sufficiently large in absolute value for the above condition to be violated. For a sharp resonance the function $f(V)$ will essentially follow the profile given by Eq. (3) within the window $E_r-E_F\leqslant V\leqslant E_r$, becoming negligibly small elsewhere. The finite resonance width smoothens the edges defined by the top and bottom of the Fermi sea. Assuming a lorentzian form for the transmissivity $T(V)=\Gamma^2/[(E-E_r+eV)^2+\Gamma^2]$ one has for $\Gamma\ll E_F$,[14]

$$f(V)\approx\rho_d(V)\left(\mathrm{arctg}\left(\frac{eV-E_r+E_F}{\Gamma}\right)-\mathrm{arctg}\left(\frac{eV-E_r}{\Gamma}\right)\right). \qquad (8)$$

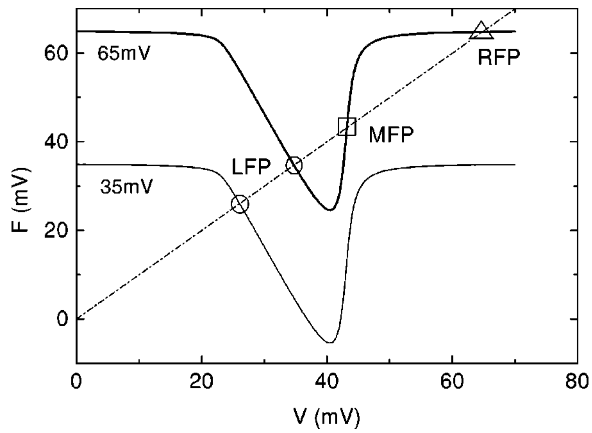

FIG. 2. The two sides of Eq. (6) for two values of the external bias: $U_0=35$ mV and 65 mV. Symbols mark intersections determining positions of the fixed points.

The expression in parenthesis is a positive definite funtion of V resembling a hat, giving $f(V)$ such shape for $d=1$ since then $\rho_d(V)$ is a constant; for $d=2$ and 3, $f(V)$ rather resembles an asymmetric hat. For convenience we call it the ''hat function'' in what follows.

The fixed points, as determined by Eq. (6), may be found graphically as the intersection of the straight line through the origin, $F=V$, and the curve representing the right hand side of that equation. Because of the inverted hat form of the latter one can easily verify that there are either one, or three fixed points, as illustrated in Fig. 2, depending on the size of the external bias U_0. The figure is for the $d=3$ case, with parameters as given below. When there are three solutions, the one in the middle (MFP, square in the figure) occurs at the rather steep fall of the arctg function, yielding a likely unstable fixed point according to the criterion given in Eq. (7). The intersection furthest to the right (RFP, triangle in the figure) occurs in the flat portion of the hat function and is therefore stable.

We are interested in the leftmost fixed point (LFP, the circles in the figure), representing the intersection located in the left side of the hat function. As U_0 varies and the overall curve raises, the intersection closely follows the functional form of the number of states available for tunneling, Eq. (3). We first consider the $d=1$ case, for which $\rho_1(V)$ first rises abruptly as a step function, thereafter remaining constant in the relevant region. The LFP then traces the contour of the arctg function, whose steep variation will generally violate the stability condition (7). The situation is as in the $d=3$ case with magnetic field, since the latter modifies the spectra in the emitter transforming it into a series of quasi-one-dimensional dispersion laws, and for which the instability is known to exist.[11] For $d=2$, there is a square root dependence at the LFP intersect, and if the resonance width Γ is sufficiently small the diverging local derivative will cause the stability condition again to be violated near the edge (the Fermi energy).

The $d=3$ case needs closer attention because of the linear form of $\rho_3(V)$. Calculating the total emitter current flowing through the resonance and relating the time constant τ with the resonance width through $\tau \sim 2\pi\hbar/\Gamma$ one gets, away from the hat edges,

$$RI_{\max}\frac{df}{dV}\Big|_{V*} \sim \frac{e^2 m^* D_z}{\epsilon\epsilon_0 \hbar^2 \gamma}, \tag{9}$$

where D_z is the separation between the barriers, and ϵ, ϵ_0 are the dielectric constants of the enclosed material and the vacuum, respectively. Using the values appropriate for GaAs ($m^* = 0.068$ m, $\epsilon = 12.5$), one gets from (7) the simple stability condition

$$1.3D_z(\text{nm}) < 2 - \gamma, \tag{10}$$

where the distance D_z is to be given in nanometers. This condition is easily violated in actual samples.

We have done numerical studies in order to verify the presence of the instability, and have found it to occur in all three cases discussed above. Figure 3 shows the I–V curves for the (a) $d=1$, (b) $d=2$ and (c) $d=3$ cases, in the absence of a magnetic field. The parameters used in the figure are $\gamma = 0.5$, $\Gamma = 1$ meV, $E_F = 20$ meV, $E_r = 43$ meV, and $D_z = 13$ nm, appropriate for typical devices based on $Al_xGa_{1-x}As$. It is also necessary to specify some short cross sectional dimensions in cases (a) and (b), chosen as $D_xD_y = 100$ nm^2 and $D_x = 10$ nm in the figure, respectively. For these values, at low external bias U_0 the device is always stable. Yet, as the voltage increases and current starts to flow due to the entrance of the resonance in the Fermi sea, the iteration (5) does not settle to a fixed point but rather fluctuates in steps τ, first regularly (bifurcations region in Fig. 3) and then, at larger bias, in a chaotic fashion. This shows that the system becomes unstable. At still larger voltages the bifurcation process unfolds until the system is stable again. The data shown are for an increasing bias. If the external voltage is then decreased, the system remains in the RFP stable state (triangle in Fig. 2) for as long as there are three solutions to Eq. (6). The values of the critical bias at which these solutions decrease in number from three to one for panels (a), (b), and (c) of Fig. 2 are 49 mV,60 mV, and 53 mV, respectively. The case with magnetic field and $d=3$, not shown, produces a similar figure for large enough field, as expected from results reported previously.[11,12]

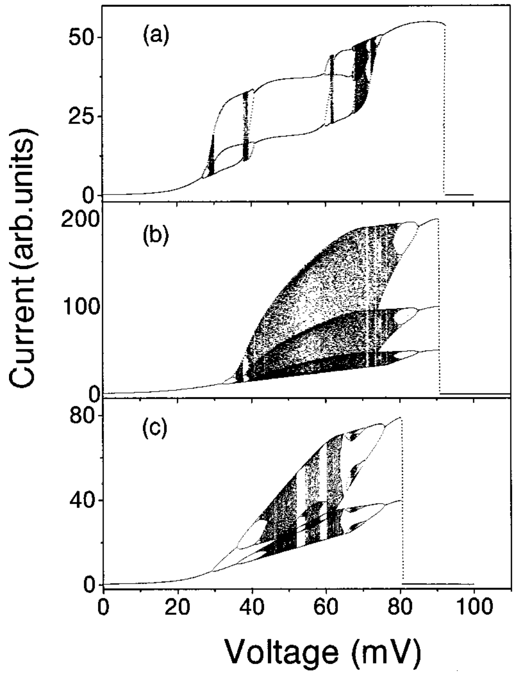

FIG. 3. Current-voltage iterates for a quantum well embedded in a (a) one-dimensional, (b) two-dimensional and (c) three-dimensional conductor.

In summary, we have shown that an instability induced by the interaction of the current with the trapped charge may appear in resonant tunneling through constrictions in one, two, and three dimensions, when the resonance is close to the Fermi level. Instead of integrating the quantum mechanical equations of motion as done previously for the $d=3$ case with magnetic field,[11] we have used a circuit model with the advantage of mathematical simplicity and the possibility of an analytical discussion of the stability in all three dimensions. Experimental accessibility of the instability may require testing emission or absorption of THz radiation. Usual electronics will normally just register a time average of the oscillations because of the small value of their typical period, which, for a resonance of 1 meV width in a GaAs quantum well would be of the order of 4 ps.

This research was supported in part by a Cátedra Presidencial en Ciencia and FONDECYT grant 1990425.

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