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# Is there an upper limit to black hole masses?

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## ABSTRACT

We make a case for the existence for ultra-massive black holes (UMBHs) in the Universe, but argue that there exists a likely upper limit to black hole (BH) masses of the order of $M \sim 10^{10}\,\mathrm{M}_{\odot}$. We show that there are three strong lines of argument that predicate the existence of UMBHs: (i) expected as a natural extension of the observed BH mass bulge luminosity relation, when extrapolated to the bulge luminosities of bright central galaxies in clusters; (ii) new predictions for the mass function of seed BHs at high redshifts predict that growth via accretion or merger-induced accretion inevitably leads to the existence of rare UMBHs at late times; (iii) the local mass function of BHs computed from the observed X-ray luminosity functions of active galactic nuclei predict the existence of a high-mass tail in the BH mass function at $z = 0$. Consistency between the optical and X-ray census of the local BH mass function requires an upper limit to BH masses. This consistent picture also predicts that the slope of the $M_{\rm bh}$–$\sigma$ relation will evolve with redshift at the high-mass end. Models of self-regulation that explain the co-evolution of the stellar component and nuclear BHs naturally provide such an upper limit. The combination of multiwavelength constraints predicts the existence of UMBHs and simultaneously provides an upper limit to their masses. The typical hosts for these local UMBHs are likely the bright, central cluster galaxies in the nearby Universe.

**Key words:** galaxies: active – galaxies: evolution – galaxies: nuclei – X-rays: galaxies.

## 1 INTRODUCTION

Observations of black hole (BH) demographics locally are increasingly providing a strong constraint on models that explain the assembly and growth of BHs in the Universe. The existence of a tight relation between the velocity dispersion of bulges and the mass of the central BH has been reported by several authors (Lehto & Valtonen 1996; Gebhardt et al. 2000; Merrit & Ferrarese 2001; Tremaine et al. 2002). This correlation is tighter than that between the luminosity of the bulge and the mass of the central BH (Magorrian et al. 1998). The physical processes that set up this correlation are not fully understood at the present time, although there are several proposed explanations that involve the regulation of star formation with BH growth and assembly in galactic nuclei (Haehnelt, Natarajan & Rees 1998; Natarajan & Sigurdsson 1998; Silk & Rees 1998; King 2005; Murray, Quataert & Thompson 2005).

Recent work by several authors has suggested that ultra-massive black holes (UMBHs)[1] ought to exist: Bernardi et al. (2006) show that the high velocity dispersion tail of the velocity distribution function of early-type galaxies constructed from the Sloan Digital Sky Survey (SDSS) had been under-estimated in earlier work suggestive of a corresponding high-mass tail for the central BH masses hosted in these nuclei. As first argued by Lauer et al. (2007a) and subequently by Bernardi et al. (2007) and Tundo et al. (2007), even when the scatter in the observed $M_{\rm bh}$–$\sigma$ correlation is taken into account it predicts fewer massive BHs compared to the $M_{\rm bh}$–$L_{\rm bulge}$ relation. While Bernardi et al. (2007) argue that this is due to the fact that the $\sigma$–$L_{\rm bulge}$ relation in currently available samples is inconsistent with the SDSS sample from which the distributions of $L_{\rm bulge}$ or $\sigma$ are based. From an early-type galaxy sample observed by *Hubble Space Telescope* (*HST*), Lauer et al. (2007b) argue that the relation between $M_{\rm bh}$–$L_{\rm bulge}$ is likely the preferred one for BCGs [Brightest Cluster Galaxies (BCGs)] consistent with the harbouring of UMBHs as evidenced by their large core sizes. The fact that the high-mass end of the observed local BH mass function is likely biased is a proposal that derives from optical data. Deriving the mass functions of accreting BHs from optical quasars in the Sloan Digital Sky Survey Data Release 3 (SDSS DR3), Vestergaard et al. (2008) also find evidence for UMBHs in the redshift range $0.3 \le z \le 5$.

In this paper, we show that UMBHs exist using X-ray and bolometric active galactic nuclei (AGN) luminosity functions (LFs) and

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[1] BHs with masses in excess of $5 \times 10^9\,\mathrm{M}_{\odot}$ are hereafter referred to as UMBHs.

for consistency with local observations of the BH mass density, an upper limit to their masses is required. To probe the high-mass end of the BH mass function, in earlier works the AGN LFs were simply extrapolated. This turns out to be inconsistent with local estimates of the BH mass function. Here, we focus on the high-mass end of the predicted local BH mass function, that is extrapolation of the $M_{\rm bh}$–$\sigma$ relation to higher velocity dispersions and demonstrate that a self-limiting cut-off in the masses to which BHs grow at every epoch reconciles the X-ray and optical views.

The outline of this paper is as follows: in Section 2, we briefly summarize the current observational census of BHs at high and low redshift including constraints from X-ray AGN. The pathways to grow UMBHs are described in Section 3. Derivation of the local BH mass function from the X-ray LFs of AGN is presented in Section 4. The argument for the existence of an upper limit to BH masses from various lines of evidence is presented in Section 5; the prospects for detection of this population is presented in Section 6 followed by conclusions and discussion. We adopt a cosmological model that is spatially flat with $\Omega_{\rm matter} = 0.3$; $H_0 = 70\,{\rm km\,s^{-1}\,Mpc^{-1}}$.

## 2 STATUS OF CURRENT CENSUS OF BHs AT HIGH AND LOW REDSHIFT

The demography of local galaxies suggests that every galaxy hosts a quiescent supermassive BH (SMBH) at the present time and the properties of the BH are correlated with those of the host. In particular, observational evidence points to the existence of a strong correlation between the mass of the central BH and the velocity dispersion of the host spheroid (Gebhardt et al. 2000; Merrit & Ferrarese 2001; Tremaine et al. 2002) in nearby galaxies. This correlation strongly suggests coeval growth of the BH and the stellar component via likely regulation of the gas supply in galactic nuclei (Haehnelt & Rees 1993; Silk & Rees 1999; Kauffmann & Haehnelt 2000; King 2003; Bromley, Somerville & Fabian 2004; Alexander et al. 2005; Begelman & Nath 2005; Murray, Quataert & Thompson 2005; Sazonov et al. 2005; Cattaneo et al. 2006).

BH growth is primarily powered by gas accretion (Lynden-Bell 1969) and accreting BHs that are optically bright are detected as quasars. The build-up of SMBHs is likely to have commenced at extremely high redshifts. Indeed, optically bright quasars have now been detected at $z > 6$ (e.g. Fan et al. 2001a; Fan et al. 2003) in the SDSS. There are also indications that high redshift quasar hosts are strong sources of dust emission (Omont et al. 2001; Carilli et al. 2002; Cox et al. 2002; Walter et al. 2003; Reuland et al. 2004) suggesting that quasars were common in massive galaxies at a time when galaxies were undergoing copious star formation. The growth spurts of SMBHs are also detected in the X-ray waveband. The summed emission from these AGN generates the cosmic X-ray Background (XRB), and its spectrum suggests that most black-hole growth is optically obscured (Di Matteo et al. 1999; Fabian 1999; Mushotzky et al. 2000; Hasinger et al. 2001; Barger et al. 2003, 2005; Worsley et al. 2005). There are clear examples of obscured black-hole growth in the form of 'Type-2' quasars, and the detected numbers are in agreement with some recent XRB models (Treister & Urry 2005; Gilli, Comastri & Hasinger 2007) and have the expected luminosity dependence of the obscured fraction. Additionally, there is tantalizing recent evidence from infra-red (IR) studies that dust-obscured accretion is ubiquitous (Martínez-Sansigre et al. 2005, 2007). At present it is unknown what fraction of the total mass growth occurs in such an optically dim phase as a function of redshift.

The build-up of BH mass in the Universe has been traced using optical quasar activity. The current phenomenological approach to understanding the assembly of SMBHs involves optical data from both high and low redshifts. These data are used to construct a consistent picture that fits within the larger framework of the growth and evolution of structure in the Universe (Haehnelt et al. 1998; Haiman & Loeb 1998; Kauffmann & Haehnelt 2000; Wyithe & Loeb 2002; Di Matteo et al. 2003; Volonteri, Haardt & Madau 2003; Steed & Weinberg 2004).

BH accretion histories derived from the quasar LF (e.g. Sołtan 1982; Haehnelt, Natarajan & Rees 1998; Salucci et al. 1999; Yu & Tremaine 2002; Marconi et al. 2004; Merloni, Rudnick & Di Matteo 2004; Shankar et al. 2004), synthesis models of the XRB (e.g. Comastri et al. 1995; Gilli, Risaliti & Salvati 1999; Ueda et al. 2003; Barger et al. 2005; Treister & Urry 2005; Gilli et al. 2007) and observations of accretion rates in quasars at different redshifts (McLure & Dunlop 2004; Vestergaard 2004) and composite models (Hopkins et al. 2005, 2006a,b) suggest that SMBHs spend most of their lives in a low efficiency, low accretion rate state. In fact, only a small fraction of the SMBHs lifetime is spent in the optically bright quasar phase, although the bulk of the mass growth occurs during these epochs. In this paper, we examine the consequences of such an accretion history for the high-mass end of the local BH mass function.

Surveys at X-ray energies allow us to obtain a more complete view of the AGN population, as they cover a broader range in luminosity and are simultaneously less affected by baises due to obscuration. While optical surveys of quasars, like the SDSS or 2dF, are used to obtain a large sample of unobscured and high-luminosity sources, it is with X-ray surveys that the obscured low-luminosity population can be well traced. In particular, surveys at hard X-ray energies, 2–10 keV, are almost free of selection effects up to columns of $N_{\rm H} \sim 10^{23}\,{\rm cm^{-2}}$. In the work of Ueda et al. (2003), the AGN X-ray LF is computed based on a sample of $\sim$250 sources observed with various X-ray satellites. One of the important conclusions of this paper is the confirmation of a luminosity-dependent density evolution, in the sense that lower luminosity sources peak at lower redshifts, $z < 1$, while only the high luminosity sources are significantly more abundant at $z \sim 2$, as observed in optical quasar surveys (e.g. Boyle et al. 2000). Additionally, using this X-ray LF and evolution it was possible for Ueda et al. (2003) to convincingly account for the observed properties of the extragalactic XRB.

Extending the argument presented by Sołtan (1982) to the X-ray waveband, AGN activity can be used to trace the history of mass accretion on to SMBHs (Fabian & Iwasawa 1999). Marconi et al. (2004) and Shankar et al. (2004) used the LF of Ueda et al. (2003) to calculate the spatial density of SMBHs inferred from AGN activity and compared that with observations. These authors reported in general a good agreement between observations and the density inferred from AGN relics, suggesting that there is little or no room for further obscured accretion, once Compton-thick AGN are properly accounted for. A similar conclusion was also obtained by Barger et al. (2005) from an independently determination of the LF, thus confirming this result.

## 3 PATHWAYS FOR GROWING UMBHs

Below, we discuss plausible scenarios for forming these UMBHs at low redshift. There are two feasible channels for doing so: (i) expect extremely rare UMBHs to form from the merging of BHs due to the merging of galaxies via the picture suggested by Volonteri et al. (2003); (ii) form from accretion on to high redshift 'seeds' with perhaps a brief period of Super-Eddington accretion, the descendants of the SMBHs that power the most luminous quasars at $z = 6$ as proposed recently by Volonteri & Rees (2005), Begelman, Volonteri & Rees (2006), Lodato & Natarajan (2007) and Volonteri, Lodato & Natarajan (2008). We discuss these two possible channels for growing UMBHs in more detail below.

### 3.1 Merging history of BHs

Following the merging dark matter (DM) hierarchy of haloes starting with seed BHs at $z = 20$, populating the $3.5-4\sigma$ peaks, Volonteri et al. (2003) are able to reproduce the mass function of local BHs as well as the abundance of the rare $10^9\,M_\odot$ BHs that power the $z = 6$ SDSS quasars. Proceeding to rarer peaks say, $6\sigma$ at $z = 20$ in this scheme yields the rarer $10^{10}\,M_\odot$ local UMBHs. In fact, the formation of a very small number density of UMBHs at $z = 0$ is inevitable in the standard hierarchical merging $\Lambda$ cold dark matter ($\Lambda$CDM) paradigm. A massive DM halo with mass, $M = 10^{13}\,M_\odot$ at $z = 0$ which is the likely host to an UMBH, is likely to have experienced about 100 mergers between $z = 6$ and 0, starting with $10^9\,M_\odot$ at $z = 6$.

Recently, a numerical calculation of the merger scenario mentioned above has been performed in simulations by Yoo et al. (2007). Focusing on the merger history of high-mass cluster-scale haloes ($M \sim 10^{15}\,M_\odot$). They find that in 10 realizations of haloes on this mass scale, starting with the highest initial BH masses at $z = 2$ of $\sim$ few times $10^9\,M_\odot$, four clusters contain UMBHs at $z = 0$. Therefore, rare UMBHs are expected in the local Universe. Yoo et al. (2007) argue that BH mergers can significantly augment the high end tail of the local BH mass function.

Similarly, using a model for quasar activity based on mergers of gas-rich galaxies, Hopkins et al. (2006a) showed that they could explain the observed local BH mass at low to intermediate BH masses ($10^6-10^9\,M_\odot$). However, at higher BH masses, their calculations overpredict the observed values even considering a possible change in the Eddington fraction at higher masses.

### 3.2 Growth from massive high-redshift seeds

Conventional models of BH formation and growth start with initial conditions at high redshift with seed BHs that are remnants of the first generation of stars in the Universe. Propagating these seeds via merger accompanied accretion events leading to mass growth for the BHs (Volonteri et al. 2003), it has been argued that in order to explain the masses of BHs powering the bright $z \sim 6$ quasars by the SDSS survey (Fan et al. 2004, 2006) that either a brief period of Super-Eddington accretion (Volonteri & Rees 2005) or more massive seeds are needed (Begelman et al. 2006; Lodato & Natarajan 2006, 2007). Massive seeds can alleviate the problem of assembling $\sim 10^9\,M_\odot$ BHs by $z = 6$ which is roughly 1 Gyr after the big bang in the concordance $\Lambda$CDM model. The local relics of such super-grown BHs are expected to result in UMBHs. We note here that following the evolution of the massive BHs that power the $z = 6$ quasars, in a cosmological simulation, Di Matteo et al. (2008) find that these do not necessarily remain the most massive BHs at subsequent times. Therefore, while UMBHs might not be direct descendants of the SMBHs that power the $z = 6$ quasars, there is ample room for UMBHs to form and grow.

Below, we briefly present scenarios that provide the massive BH seeds in the first place that will eventually result in a small population of UMBHs by $z = 0$. These physically plausible mechanisms are critical to our prediction of UMBHs at low redshift. Two models have been proposed, one that involves starting from the remnants of Population III stars with brief episodes of accretion on to them exceeding the Eddington rate to bump up their masses (Volonteri & Rees 2006) and the other that explains direct formation of massive BH seeds prior to the formation of the first stars (Lodato & Natarajan 2006, 2007).

Volonteri & Rees (2005) have proposed a scenario to explain the high BH masses $\sim 10^9\,M_\odot$ needed to power the luminous quasars detected $z = 6$ in the SDSS. This is accomplished they argue by populating the $4\sigma$ peaks in the DM density field at $z \sim 24$ with seed BHs which arise from the remnants of Population III stars in the mass ranges $20\,M_\odot < M_{\rm bh} < 70\,M_\odot$ and $130\,M_\odot < M_{\rm bh} < 600\,M_\odot$. These remnant BHs then undergo an episode of super-Eddington accretion from $6 < z < 10$. They argue that in these high redshift, metal-free DM haloes $T > 10^4$ $K$ gas can cool in the absence of $H_2$ via atomic hydrogen lines to about 8000 K. As shown by Oh & Haiman (2002), the gas at this temperature settles into a rotationally supported 'fat' disc at the centre of the halo under the assumption that the DM and the baryons have the same specific angular momentum. Further, these discs are stable to fragmentation and therefore do not form stars and exclusively fuel the BH instead. The accretion is via stable supercritical accretion at rates well in excess of the Eddington rate due to the formation of a thin, inner feeding disc. The accretion radius is comparable to the radiation trapping radius which implies that all the gas is likely to end up in the BH. Any further cooling down to temperatures of $10\,{\rm K} < T < 200\,{\rm K}$ for instance, halts the accretion, causes fragmentation of the disc which occurs when these regions of the Universe have been enriched by metals. This process enables the comfortable formation of $10^9\,M_\odot$ BHs by $z = 6$ or so to explain the observed SDSS quasars. In a $\Lambda$CDM Universe, the time available from $z = 6$ to 0 is $\sim$12.7 Gyr. To grow by an order of magnitude during this epoch requires an accretion rate of $<1\,M_\odot\,{\rm yr}^{-1}$ which is well below the Eddington rate; however, it requires a gas-rich environment.

In recent work, Lodato & Natarajan (2007) have shown that an ab initio prediction for the mass function of seed BHs at high redshift can be obtained in the context of the standard $\Lambda$CDM paradigm for structure formation combined with careful modelling of the formation, evolution and stability of pre-galactic discs. They show that in DM haloes at high redshifts $z \sim 15$, where zero metallicity pre-galactic discs assemble (prior to the formation of the first stars), gravitational instabilities in these discs transfer angular momentum out and mass inwards efficiently. Note that the only coolants available to the gas at this epoch are either atomic or molecular hydrogen. Taking into account the stability of these discs, in particular the possibility of fragmentation, the distribution of accumulated central masses in these haloes can be computed. The central mass concentrations are expected to form seed BHs. The application of stability criteria to these discs leads to distinct regimes demarcated by the value of the $T_{\rm vir}/T_{\rm gas}$ where $T_{\rm gas}$ is the temperature of the gas and $T_{\rm vir}$ is the virial temperature of the halo. The three regimes and consequences are as follows: (i) when $T_{\rm vir}/T_{\rm gas} > 3$ the disc fragments and forms stars instead of a central mass concentration; (ii) $2 < T_{\rm vir}/T_{\rm gas} < 3$, when both central mass concentrations and stars form; (iii) $T_{\rm vir}/T_{\rm gas} < 2$, when only central mass concentrations form and the discs are stable against fragmentation. Using the predicted mass function of seed BHs at $z \sim 15$, and propagating their growth in a merger driven accretion scenario we find that the masses of BHs powering the $z = 6$ optical quasars can be comfortably accommodated and consequently a small fraction of UMBHs is predicted at $z = 0$. Evolving and growing these seeds to $z = 0$, the abundance of UMBHs can be estimated (Volonteri, Lodato & Natarajan 2008).

## 4 THE LOCAL BH MASS FUNCTION DERIVED FROM X-RAY LFs OF AGN

The new evidence that we present in this work for the existence of a rare population of UMBHs stems from using X-ray LFs of AGN and the implied accretion history of BHs. Hard X-rays have the advantage of tracing both obscured and unobscured AGN, as the effects of obscuration are less important at these energies. In particular, we use the hard X-ray LF and luminosity-dependent density evolution presented by Ueda et al. (2003) defined from $z = 0$ to 3. We further assume that these AGN are powered by BHs accreting at the Eddington limit. In order to calculate bolometric luminosities starting from the hard X-ray luminosity, the bolometric corrections derived from the AGN spectral energy distribution library presented by Treister et al. (2006) are used. These are based mainly on observations of local AGN and quasars and depend only on the intrinsic X-ray luminosity of the source, as they are based on the X-ray to optical ratios reported by Steffen et al. (2006). To account for the contribution of Compton-thick AGN to the BH mass density missed in X-ray LFs, we use the column density distribution of Treister & Urry (2005) with the relative number of Compton-thick AGN adapted to match the spatial density of these sources observed by *INTEGRAL*, obtained from the AGN catalog of Beckmann et al. (2006). In order to account for sources with column densities $N_{\rm H} = 10^{25}$–$10^{26}$ cm$^{-2}$ which do not contribute much to the XRB, but can make a significant contribution to the BH mass density (e.g. Marconi et al. 2004), we multiply the BH mass density due to Compton-thick AGN by a factor of 2, i.e. we assume that they exist in the same numbers as in the $N_{\rm H} = 10^{24}$–$10^{25}$ cm$^{-2}$ range, in agreement with the assumption of Marconi et al. (2004) and consistent with the $N_{\rm H}$ distribution derived from a sample of nearby AGN by Risaliti, Maiolino & Salvati (1999). Under this assumption, the contribution of sources with $N_{\rm H} > 10^{25}$ cm$^{-2}$ to the total population of SMBHs is ~7 per cent.

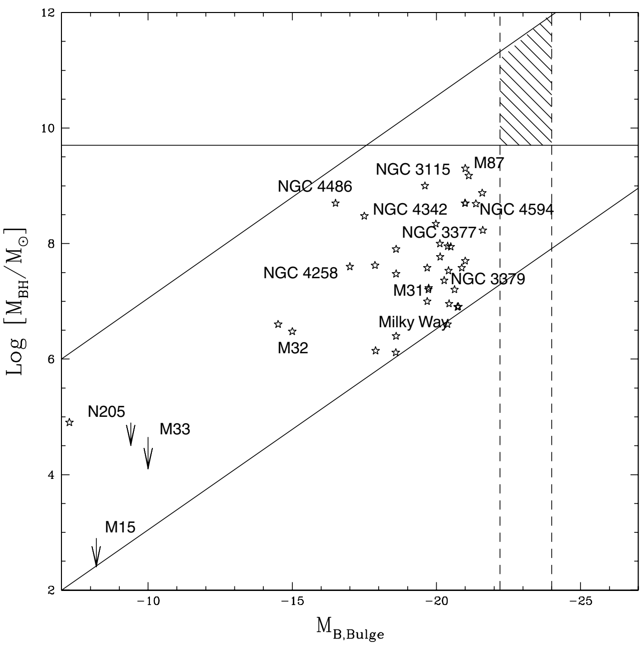


**Figure 1.** Relation between the inferred BH mass versus the host bulge luminosity; data taken from Magorrian et al. (1998); Ho (1998) and Gebhardt et al. (2002). The vertical dashed lines indicate the typical luminosity of cD galaxies and the hatched region is the parameter space for finding UMBHs.

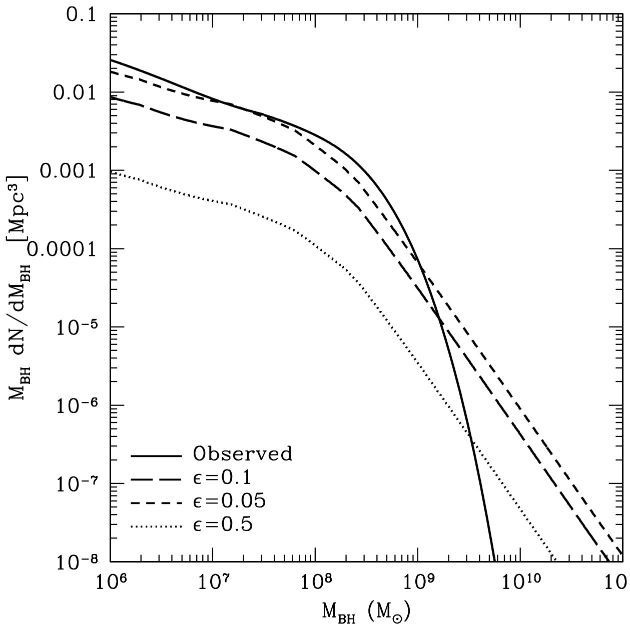


**Figure 2.** BH spatial density per unit mass as function of BH mass. Dashed lines show the values inferred by integrating the hard X-ray LF of Ueda et al. (2003) using the bolometric corrections described on the text for three different efficiencies: 0.05 (*dashed*), 0.1 (*long dashed*) and 0.5 (*dotted*). The *solid line* shows the derived number density of BHs from the SDSS local measured velocity function obtained using the Merritt & Ferrarese correlation between the BH mass and velocity dispersion of bulges.

We then convert these X-ray LF's to an equivalent BH mass function, and evolve these mass functions by assuming that accretion continues at the Eddington rate down to $z = 0$. The results of this procedure are shown in Fig. 2 for three different values of the accretion efficiency $\epsilon$. Note that we do not consider models in which the efficiency parameter varies with redshift or BH mass since such models merely add more unconstrained parameters. As can be seen clearly in Fig. 2, these simple models do not reproduce the observed local BH mass function at the high-mass end. The functional form adopted for the X-ray LF is a double power-law as proposed by Ueda et al. (2003)

$$\frac{{\rm d}\Phi(L_{\rm X}, z=0)}{{\rm dLog}L_{\rm X}} = A\left[(L_{\rm X}/L_*)^{\gamma 1} + (L_{\rm X}/L_*)^{\gamma 2}\right]^{-1}. \quad (1)$$

And the evolution is best described by the luminosity dependent density evolution (LDDE) model, where the cut-off redshift $z_{\rm c}$ is expressed by a power law of $L_{\rm X}$, consistent with observational constraints (see Ueda et al. 2003 for more details):

$$\frac{{\rm d}\Phi(L_{\rm X}, z)}{{\rm dLog}L_{\rm X}} = \frac{{\rm d}\Phi(L_{\rm X}, 0)}{{\rm dLog}L_{\rm X}} e(z, L_{\rm X}), \quad (2)$$

where

$$e(z, L_{\rm X}) = (1+z)^{p1} \quad [z < z_{\rm c}(L_{\rm X})], \quad (3)$$

$$e(z_{\rm c})\{(1+z)/[1+z_{\rm c}(L_{\rm X})]\}^{p2} \quad [z \geq z_{\rm c}(L_{\rm X})]. \quad (4)$$

This simple and conservative analysis predicts a population of UMBHs with a local abundance of $\sim 3 \times 10^{-6}$ Mpc$^{-3}$! This is fairly robust as this population is predicted for a large range of efficiencies. These LF's shown in Fig. 2 also simultaneously account for the cosmic XRB, as shown by several authors (for instance see Treister & Urry 2005 and Gilli et al. 2007 and references therein), suggesting that the X-ray view presents a fairly complete picture of the accretion and growth of BHs. Note that our estimates of the BH mass function are in general agreement with those of Marconi et al. (2004) [for a direct comparison see their Fig. 2, right-hand panel], the very slight difference arises due to an alternate choice of bolometric correction factors and our prescription for including Compton-thick AGN. Estimates by other authors are also in agreement with our treatment here out to masses of a few times $10^8\,M_\odot$. For BH masses $< 10^9\,M_\odot$, there appears to be consistency between the optical and X-ray views of BH growth. However, for $M_{\rm bh} > 10^9\,M_\odot$, all models that assume Eddington accretion with varying efficiencies systematically overestimate the local abundance of high-mass BHs.

As can be seen in Fig. 2, for a reasonable value of the efficiency, $\epsilon \gtrsim 0.05$, there is a good agreement between the BH mass density at $z = 0$, as obtained from the velocity dispersion of bulges, and the density inferred from AGN relics, for BH masses smaller than $\sim$2–$3 \times 10^9\,M_\odot$. However, for higher masses, in particular the UMBH mass range, independent of the value of $\epsilon$ assumed, the BH mass density from AGN relics is significantly higher than the observed value, indicating that UMBHs should be more abundant than current observations suggest. If there is a mass dependent efficiency factor for accretion such that higher mass BHs tend to accrete at higher efficiency and hence at lower rates, then our estimate of the high-mass tail would be an overestimate. There is, however, no evidence for such a mass dependence at lower masses (Hopkins, Narayan & Hernquist 2006).

The SDSS First Data Release covers approximately 2000 $\deg^2$ (Abazajian et al. 2003), yielding a comoving volume of a cone on the sky out to $z = 0.3$ of $3.34 \times 10^8\,{\rm Mpc}^3$. Given our predicted abundance above, we expect $\sim$1000 UMBHs in the SDSS volume, however only a few are detected. No combination of assumed accretion efficiency and Eddington ratio coupled with the X-ray AGN LF can reproduce the observed local abundance at the high-mass end.

### 4.1 Evidence for an upper limit to BH masses

However, we find that modifying one of the key assumptions made above brings the predicted abundance of local UMBHs into better agreement with current observations. In the modelling, we have extrapolated the observed X-ray AGN LF slope to brighter luminosities. We find that if this slope is steepened at the bright end, we can reproduce the observed UMBH mass function at $z = 0$ for $M \geq 10^9\,M_\odot$ as well. In order to reconcile the observationally derived local BH mass function at the high-mass end, the slope $\gamma_2$ in equation (1) needs to be modified. We find that the slope $\gamma_2$ for BH masses $M_{\rm bh} < 10^9\,M_\odot$ is $\sim 2.2$, which however, does not provide a good fit for higher masses. A slope steeper than $\gamma_2 = 5$ is required to fit BH masses in excess of $10^9$, we find that formally the best fit is found in reduced-$\chi^2$ terms for the value of $\gamma_2 = 6.9$. Such a steepening simulates the cut-off of a self-regulation mechanism that limits BH masses and sets in at every epoch.

In Fig. 3, the results of such a self-limiting growth model are plotted. The predicted abundance of UMBHs is now in much better agreement with observations at $z = 0$ and is consistent with the number of UMBHs detected by SDSS. In a self-regulated mass-growth model, we predict the abundance of UMBHs at $z = 0$ to be

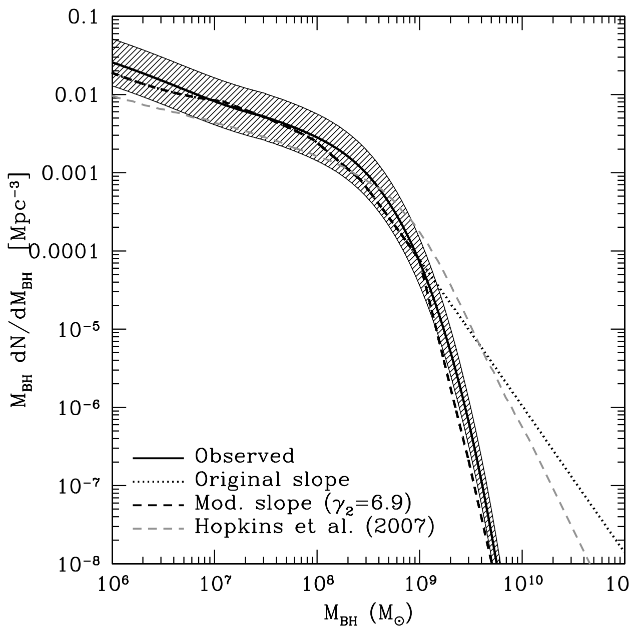


**Figure 3.** BH spatial density per unit mass as function of BH mass. The *solid line* shows the SDSS-derived values, as shown in Fig. 2, assuming a constant 30 per cent uncertainty (*shaded region*). The *dotted line* shows the values derived integrating the hard X-ray LF for an efficiency of 0.05, while the *gray dashed line* shows the relation reported by Hopkins et al. (2007, fig 10) using a bolometric LF. In order to match the observed relation, the slope of the hard X-ray LF was modified for masses higher than $10^9\,M_\odot$, as shown by the *black dashed line*.

$7 \times 10^{-7}\,{\rm Mpc}^{-3}$. Therefore, requiring consistency between the X-ray and optical views of BH growth and assembly with the observed number of UMBHs at $z = 0$, points to the existence of a self-regulation mechanism that limits BH masses. The self-regulation is implemented as a steepening of the X-ray AGN LF at the luminous end (the value of $\gamma_2$ needed is plotted in Fig. 4) and does not have an important effect on the XRB, since this change in slope only affects sources with X-ray luminosities $L$(2–10 keV) greater than $10^{45}\,{\rm erg\,s^{-1}}$, while most of the XRB emission is produced by sources with luminosities of $10^{43-44}\,{\rm erg\,s^{-1}}$, as found by Treister & Urry (2005). Note that the use of a bolometric LF, as reported by Hopkins, Richards & Hernquist (2007) does not match the slope at the high-mass end (shown as the dashed curve in Fig. 3). One key consequence of our model is that the slope of the $M_{\rm bh}$–$\sigma$ relation at the high-mass end likely evolves with redshift. Recent cosmological simulations find evidence for such a trend (Di Matteo et al. 2008). The functional form of this expected variation will depend on the specific model of self-regulation employed. Below, we explore physical processes that are likely to regulate the growth of BHs in galactic nuclei.

Converting the high end of the local BH mass function into the equivalent velocity dispersions of the host spheroids, we find values in excess of $350\,{\rm km\,s^{-1}}$. In the context of the currently popular hierarchical model for the assembly of structure, the most massive galaxies in the Universe are expected to be the central galaxies in clusters. High-$\sigma$ peaks in the density fluctuation field at early times seed clusters that assemble at later times, and hence these are the preferred locations for the formation of the most massive galaxies in a CDM dominated Universe.

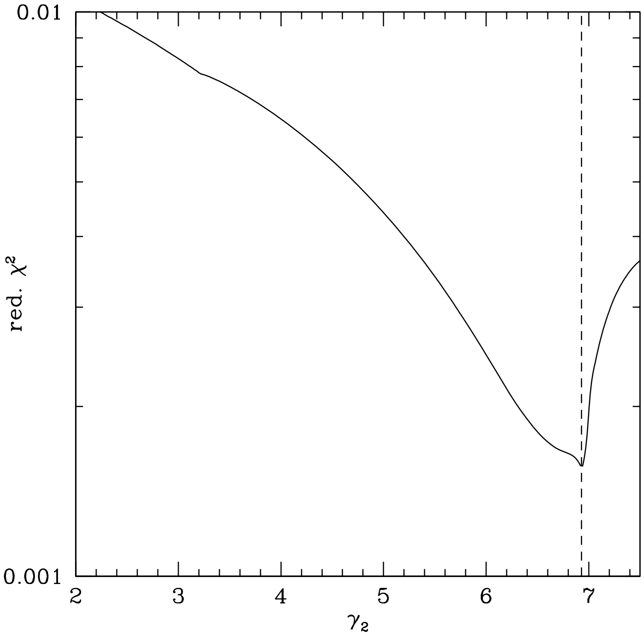


**Figure 4.** The reduced *chi*$^2$ for the index $\gamma_2$ in the X-ray AGN LF required to match the high-mass end of the local BH mass density. In order to match the observed relation, the slope of the hard X-ray LF was modified for masses higher than $10^9\,\mathrm{M_\odot}$.

## 5 THE UPPER LIMIT TO BH MASSES FROM SELF-REGULATION ARGUMENTS

While we predict above that a few, rare UMBHs are likely to exist at the centres of the brightest central galaxies in clusters, we further argue that there likely exists an upper limit to BH masses. Evidence for this is presented using several plausible physical scenarios that attempt to explain the coeval formation of the BH and the stellar component in galactic nuclei. Clearly the existence of UMBHs is intricately related to the highest mass galaxies that can form in the Universe.

Given that star formation and BH fueling appear to be coupled (e.g. Di Matteo, Springel & Hernquist 2005 and references therein; Silk & Rees 1998), it is likely that there is a self-limiting growth cycle for BHs and therefore a physical upper limit to their masses. Here, we present several distinct arguments that can be used to estimate the final masses of BHs (Haehnelt et al. 1998; Silk & Rees 1998; Fabian 1999, 2002; King 2005 and Murray et al. 2005). These involve self-limiting growth due to a momentum-driven wind, self-limiting growth due to the radiation pressure of a momentum-driven wind, and from an energy-driven superwind model.

Murray et al. (2005) argue that the feedback from momentum driven winds, limits the stellar luminosity, which in turn regulates the BH mass. They argue for Eddington limited star formation with a maximum stellar luminosity,

$$L_{\mathrm{M}} = \frac{4 f_{\mathrm{g}} c}{G}\,\sigma^4, \tag{5}$$

where, $f_{\mathrm{g}}$ is the gas fraction in the halo and $\sigma$ the velocity dispersion of the host galaxy. Star formation in this scheme is unlikely to evacuate the gas at small radius in the galactic nucleus, therefore, all the gas in the inner-most regions fuel the BH. The growing BH itself clears out this nuclear region with its accretion luminosity approaches $L_{\mathrm{M}}$. At this point, the fuel supply to the BH is shut off and this may shut off the star formation as well. The final BH mass is then given by

$$M_{\mathrm{BH}} = \frac{f_{\mathrm{g}}\kappa_{\mathrm{es}}}{\pi G^2}\,\sigma^4, \tag{6}$$

where $\kappa_{\mathrm{es}}$ is the electron scattering opacity. For the most massive, nearby early-type galaxies at the very tail of the measured SDSS velocity dispersion function with velocity dispersions of ~350–400 km s$^{-1}$ (Bernardi et al. 2005) this gives a final BH mass of $\sim 10^{10}\,\mathrm{M_\odot}$. Therefore, normal galaxies with large velocity dispersions are the presumptive hosts for UMBHs.[2] Furthermore, there appears to be a strong indication of the existence of an upper mass limit for accreting BHs derived from SDSS DR3 by Vestergaard et al. (2008) in every redshift bin from $z = 0.3$–5.

An alternative upper limit can be obtained when the emitted energy from the accreting BH back reacts with the accretion flow itself (Haehnelt et al. 1998). The final shut-down of accretion will depend on whether the emitted energy can back react on the accretion flow prior to fuel exhaustion. This arguement provides a limit,

$$M_{\mathrm{bh}} \sim 5.6 \times 10^9\,\mathrm{M_\odot}\,(f_{\mathrm{kin}}/0.0001)^{-1}\,j_{\mathrm{d}}^{-5}\left(\frac{\lambda}{0.05}\right)^{-5}\left(\frac{m_{\mathrm{d}}}{0.1}\right)^{5} \times \left(\frac{\sigma}{350\,\mathrm{km\,s^{-1}}}\right)^{5}\mathrm{M_\odot}, \tag{7}$$

where $f_{\mathrm{kin}}$ is the fraction of the accretion luminosity which is deposited as kinetic energy into the accretion flow (cf. Silk & Rees 1998), $\lambda$ is the spin parameter of the DM halo, $j_{\mathrm{d}}$ is the specific angular momentum of the disc $m_{\mathrm{d}}$ is the disc mass fraction. The back-reaction time-scale will be related to the dynamical time-scale of the outer parts of the disc and/or the core of the DM halo and should set the duration of the optically bright phase. It is interesting to note here that the accretion rate will change from super-Eddington to sub-Eddington without much gain in mass if the back-reaction time-scale is shorter than the Salpeter time. The overall emission efficiency is then determined by the value of $\dot{m}$ when the back-reaction sets in and is reduced by a factor $1/\dot{m}$ compared to accretion at below the Eddington rate (Begelman & Meier 1982). By substituting the value of the velocity dispersion of nearby cD's ~350 km s$^{-1}$, we obtain a limiting value of the mass, if we assume that the bulk of the mass growth occurs in the optically bright quasar phase. Due to the dependence on the spin parameter $\lambda$ of the DM halo, the desired UMBH mass range can arise preferentially in high velocity dispersion haloes with low spin.[3]

King (2005) presents a model that exploits the observed AGN-starburst connection to couple BH growth and star formation. As the BH grows, an outflow drives a shell into the surrounding gas which stalls after a dynamical time-scale at a radius determined by the BH mass. The gas trapped inside this bubble cools, forms stars and is recycled as accretion and outflow. Once the BH reaches a critical mass, this region attains a size such that the gas can no longer cool efficiently. The resulting energy-driven flow expels the remaining gas as a superwind, thereby fixing the observed $M_{\mathrm{bh}}$–$\sigma$ relation as well as the total stellar mass of the bulge at values in good agreement with current observations. The limiting BH mass

[2] Objects with high velocity dispersion as a consequence of superposition are not the hosts of UMBHs.

[3] The distribution of spins of DM haloes measured from *N*-body simulations is found to be a log-normal with a median value of 0.05, and since there is no significant halo mass dependence, a small fraction of the haloes do reside in this low-spin tail.

is given by

$$M_{\rm bh} = \frac{f_{\rm g}\,\kappa}{\pi\,G^2}\,\sigma^4, \qquad (8)$$

where $f_{\rm g}$ is the gas fraction ($\Omega_{\rm baryon}/\Omega_{\rm matter} = 0.16$), $\kappa$ the electron scattering opacity and $\sigma$ the velocity dispersion. This model argues that BH growth inevitably produces starburts and ultimately a superwind.

Note that both the Murray et al. (2005) model and the King (2005) model predict $M_{\rm bh} \propto \sigma^4$ while the Haehnelt et al. (1998) and Silk & Rees (1998) predict a $\sigma^5$ dependence. The current error bars on the observational mass estimates for BHs preclude discrimination between these two possibilities. Shutdown of star formation above a critical halo mass effected by the growing AGN has also been proposed as a self-limiting mechanism to cap BH growth and simultaneously explain the dichotomy in galaxy properties (Cattaneo et al. 2006; Croton et al. 2006).

## 6 PROSPECTS FOR DETECTION OF QUIESCENT UMBHs

UMBHs are expected to be rare in the local Universe, from our analysis of the X-ray LF of AGN, we predict an abundance ranging from $\sim$ few times $10^{-6}$–$10^{-7}$ Mpc$^{-3}$. These estimates are in good agreement with those obtained from optical quasars in the SDSS DR3 by Vestergaard et al. (2008). The results of the first attempts to detect and measure masses for UMBHs is promising. Dalla Bonta et al. (2007) selected three BCGs in Abell 1836, Abell 2052 and Abell 3565. Using Advanced Camera for Surveys (ACS) aboard the *HST* and the Imaging Spectrograph (STIS), they obtained high resolution spectroscopy of the H$\alpha$ and N II emission lines to measure the kinematics of the central ionized gas. They present BH mass estimates for two of these BCGs, $M_{\rm bh} = 4.8^{+0.8}_{-0.7} \times 10^9\,{\rm M}_{\odot}$ and $M_{\rm bh} = 1.3^{+0.3}_{-0.4} \times 10^9\,{\rm M}_{\odot}$ and an upper limit for the BH mass on the third candidate of $M_{\rm bh} \leq 7.3 \times 10^{10}\,{\rm M}_{\odot}$.

It is interesting to note that Bernardi et al. (2005) in a census of the most massive galaxies in the SDSS survey do find candidates with large velocity dispersions ($\geq 350\,{\rm km\,s^{-1}}$). The largest systems they find are claimed to be extremes of the early-type galaxy population, as they have the largest velocity dispersions. These $\sim$31 systems (see table 1 of Bernardi et al. 2006 for details on these candidates) are not distant outliers from the Fundamental Plane and the mass-to-light scaling relations defined by the bulk of the early-type galaxy population. Clear outliers from these scaling relations tend to be objects in superposition for which they have evidence from spectra and images. We argue that these extreme early-type galaxies might harbour UMBHs and likely their abundance offers key constraints on the physics of galaxy formation. Although the observations are challenging, a more comprehensive and systematic survey of nearby BCGs is likely to yield our first local UMBH before long. As discussed above, candidates from the SDSS are promising targets for observational follow-up as they are extremely luminous. Utilizing the *HST*, the light profile might show evidence for the existence of an UMBH in the centre (e.g. Lauer et al. 2002). In fact, for SDSS *J*032834.7+001050.1 and SDSS *J*161541.3+471004.3, it may be possible to measure spatially resolved velocity dispersion profiles even from ground-based facilities.

## 7 DISCUSSION

The interplay between the evolution of BHs and the hierarchical build-up of galaxies appears as scaling relations between the masses of BHs and global properties of their hosts such as the BH mass versus bulge velocity dispersion – the $M_{\rm bh}$–$\sigma_{\rm bulge}$ relation and the BH mass versus bulge luminosity $M_{\rm bh}$–$L_{\rm Bulge}$ relation. The low BH mass end of this relation has recently been probed by Ferrarese et al. (2006) in an ACS survey of the Virgo cluster galaxies. They find that galaxies brighter than $M_B \sim -20$ host a supermassive central BH whereas fainter galaxies host a central nucleus, referred to as a central massive object (CMO). Ferrarese et al. report that a common $M_{\rm CMO}$–$M_{\rm gal}$ relation leads smoothly down from the scaling relations observed for more more massive galaxies. Extrapolating observed scaling relations to higher BH masses to the UMBH range, we predict that these are likely hosted by the massive, high luminosity, central galaxies in clusters with large velocity dispersions. The velocity dispersion function of early-type galaxies measured from the SDSS points to the existence of a high velocity dispersion tail with $\sigma > 350\,{\rm km\,s^{-1}}$ (Bernardi et al. 2006). If the observed scaling relations extend to the higher mass end as well, these early-types are the most likely hosts for UMBHs.

Recent simulation work that follows the merger history of cluster-scale DM haloes and the growth of BHs hosted in them by Yoo et al. (2007) also predict the existence of a rare population of local UMBHs. However, theoretical arguments suggest that there may be an upper limit to the mass of a BH that can grow in a given galactic nucleus hosted in a DM halo of a given spin. Clearly the issue of the existence of UMBHs is intimately linked to the efficiency of galaxy formation and the formation of the largest, most luminous and massive galaxies in the Universe.

Possible explanations for the tight correlation observed between the velocity dispersion of the spheroid and BH mass involve a range of self-regulated feedback prescriptions. An estimate of the upper limits on the BH mass that can assemble in the most massive spheroids can be derived for all these models and they all point to the existence of UMBHs.

In this paper, we have argued that while rare UMBHs likely exist, there is nevertheless an upper limit of $\sim 10^{10}\,{\rm M}_{\odot}$ for the mass of BHs that inhabit galactic nuclei in the Universe. We first show that our current understanding of the accretion history and mass build up of BHs allows and implies the existence of UMBHs locally. This is primarily driven by new work that predicts the formation of massive BH seeds at high redshift (Lodato & Natarajan 2007) and their subsequent evolution (Volonteri et al. 2008). Starting with massive seeds and following their build-up through hierarchical merging in the context of structure formation in a CDM dominated Universe, we show that a viable pathway to the formation of UMBHs exists. There is also compelling evidence from the observed evolution of X-ray AGN for the existence of a local UMBH population. Convolving the observed X-ray LF's of AGN, with a simple accretion model, the mass function of BHs at $z = 0$ is estimated. Mimic-ing the effect of self-regulation processes that impose an upper limit to BH masses and incorporating this into the X-ray AGN LF, we find that the observed UMBH mass function at $z = 0$ is reproduced. This self-regulation limited growth is implemented by steepening the high-luminosity end of the AGN LF at the bright end. We estimate the abundance of UMBHs to be $\sim 7 \times 10^{-7}$ Mpc$^{-3}$ at $z = 0$. The key prediction of our model is that the slope of the $M_{\rm bh}$–$\sigma$ relation likely evolves with redshift at the high-mass end. Probing this is observationally challenging at the present time but there are several bright, massive early-type galaxies that are promising host candidates from the SDSS survey as well as a survey of bright central galaxies of nearby clusters. Observational detection of UMBHs will provide key insights into the physics of galaxy formation and BH assembly in the Universe.

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