PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE

ESCUELA DE INGENIERIA

COMMODITY INDEX RISK PREMIUM

MAXIMILIANO IGNACIO ROJAS NAVARRETE

Thesis submitted to the Office of Research and Graduate Studies in partial fulfillment of the requirements for the Degree of Master of Science in Engineering

Advisor:

GONZALO CORTÁZAR SANZ

Santiago de Chile, October 2020

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Gratefully to my parents, sister, Jorn and Clemente

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ABSTRACT

Increasingly commodities have become an asset class in a process called financialization. Many institutional investors, looking for ways to expand their diversification opportunities, are holding positions in a commodity futures index and use them as a performance benchmark. Thus, institutional commodity holdings in commodities has expanded significantly.

In this thesis, we estimate the risk premium of a commodity index using analyst´s forecasts and futures prices of each of the commodities included in the index. We estimate futures and expected spot price curves using a no-arbitrage multifactor stochastic pricing model and a Kalman Filter. The model includes time-varying risk premia thus allowing for the exploration of macro variables that could explain their time variation for each family of commodities.

The proposed model has already shown to be effective for single commodities, such as oil and copper, but this thesis extends its use to analyze indices of the four main commodity sectors: energy, industrial metals, precious metals and agriculture, as well as a global portfolio that mimics the S&P GSCI Index. This allows for a better understanding of how some macro variables affect different commodity sectors providing useful information for institutional investors of a commodity index.

Keywords: Commodity Portfolios; Risk Premiums; GSCI Index; Futures; Expected Prices;

RESUMEN

Los *commodities* se han ido transformando en un tipo de activo financiero a través de un proceso llamado *financialization*.

Muchos inversionistas institucionales, buscando formas de expandir sus oportunidades de diversificación, están manteniendo posiciones en índices de futuros de *commodities* y usándolos como referencia de desempeño. De esta forma, las posiciones de inversionistas institucionales en *commodities* se ha expandido significativamente.

En esta tesis, se estiman los premios por riesgo para un índice de *commodities* usando las predicciones de analistas y precios futuros para cada uno de los commodities incluidos en el índice. Se estiman precios esperados y precios de contratos futuros utilizando un modelo estocástico multifactor de valorización de no arbitrariedad y Filtro de Kalman. El modelo incluye premios por riesgo variables en el tiempo permitiendo la exploración de variables macro que podrían explicar la variación en el tiempo de los premios para cada familia de *commodity*.

El modelo propuesto ya ha mostrado ser efectivo en *commodities* a nivel individual, tal como en el caso del petroley y cobre, pero en este paper se extiende su uso al análisis de índices de los cuatro principales sectores de commodities: energía, metales industriales, metales preciosos y agricultura, incluyendo además un portafolio que imita el índice S&P GSCI Index. Lo anterior permite un mejor entendimiento de como algunas variables macro afectan diferentes sectores de *commodities* proporcionando información útil para inversionistas institucionales con interés en índices de *commodities.*

Palabras Claves: Portafolio de commodities; Premios por riesgo; Índice GSCI; Contratos futuros; Precios esperados

1. GENERAL OVERVIEW

During the last decades, a great interest has been seen in commodity investments as investors were looking for diversification from stocks as both asset class had little co-movement between each other (Tang & Xiong, 2012). This interest triggered funds inflows in the commodity market by institutional investors without precedents. Institutional Investor's capital increased from $15 to 200 billion between 2003 and 2008 (Silvennoinen & Thorp, 2013). For this reason, commodities has being increasing their participation in investors' portfolio, rising the number of commodity's futures contract transactions, a process known as financialization of commodities (Basak & Pavlova, 2016).

As mentioned before, institutional investors in their finding of diversification, hold commodities through commodity index as the Goldman Sachs Commodity Index (GSCI) and the Dow Jones UBS Commodity Index (Basak & Pavlova, 2016).). Given this, according to Tang and Xiong (2012), after 2004 non-indexed commodities has become increasingly different in terms of behavior from indexed commodities.

Due to the relevance that commodity indices have achieved in the last time, our study will be focused on them. As many investors have diversification as a goal, we will work with commodity portfolios instead of single assets.

Given the status of a financial asset, studying commodities' returns has been gaining relevance among authors (Hong and Yogo, 2009; Shahzad et al., 2017; Chevallier & Sévi; 2013). One approach to calculate the commodity return is through the estimation of the risk premia. Keynes (1930) and Hicks (1939) mentioned that, given the imbalance between market players willing to buy and sell futures, speculators join the market requesting an additional payment in order to compensate this difference. This payment is defined as risk premia.

In this thesis, to estimate the risk premia we will use the futures contracts' prices and analyst' expected prices. Given this context, the risk premia is the difference between both recently mentioned prices. Risk premia in futures prices is not new in literature and has been a matter of great interest for researchers. However, risk premia are not commonly used for the estimation of returns given that some practitioners use futures prices as a simple way to estimate expected prices.

In the literature, different authors have tried to estimate the risk premia using different methodologies and most of them have done it via future prices. However, the lack or poor information about expectations in future prices maybe a reason of unsatisfactory performance in the estimation (Cortazar et al., 2019a). Given this, we include analysts' expectations in order to incorporate specialists' view about possible price behaviors. The methodology used was developed by Cortazar et al. (2019a) to estimate oil's risk premia. Nonetheless, we added extra steps to the above-mentioned methodology in order to estimate the risk premia of a commodity index.

The purpose of this thesis is to achieve reliable estimations of risk premia for a commodity portfolio. Furthermore, we will consider macroeconomic variables widely used in the literature in order to understand the main drivers that explain the risk premia behavior.

We think that the estimation of risk premia for a commodity index is a matter of great relevance and can guide future investigations.

2. INTRODUCTION

Many institutional investors, looking for ways to expand their diversification opportunities, have been increasing their positions in a commodity futures index and using it as a performance benchmark. There are estimates of more than a tenfold increase in institutional commodity holdings from 2003 to 2008 (Basak & Pavlova, 2016).

In this paper we estimate the risk premium of a commodity index using analyst´s forecasts and futures prices of each of the commodities included in the index. We analyze a commodity portfolio that mimics the GSCI-Goldman Sachs Commodity Index. We also explore the individual and joint explanatory power of market variables that may affect expected returns of portfolios that mimic the commodity index on four sectors: energy, industrial metals, precious metals and agriculture.

The relevance of commodity prices has been growing during recent years and will probably continue to do so with high demand for raw materials due in part to population growth (Lübbers & Posch, 2016). Also, the use of fossil fuels is expected to keep rising until 2040, even with the increase of renewable fuel production (EIA, 2016). As a result, commodity prices are due to continue to play a major role in the economy.

Moreover, commodities have increasingly been considered as an investment asset, a process known as financialization of the commodity markets. This has led to an increase in the number of transactions and the reduction in their heterogeneousness (Yang, 2013; Silvennoinen & Thorp, 2013; Daskalaki et al., 2013). Among the reasons that may have triggered this process are the low correlations between commodity returns and the return of stocks and bonds prior to the early 2000s (Bhardwaj et al., 2015; Brooks et al., 2013;

Erb & Harvey, 2006). This low correlation to other financial assets has led investors to trade commodities to diversify their portfolios, impacting commodity prices (Tang & Xiong, 2012; Sanders & Irwin, 2017; Silvennoinen & Thorp, 2013). Notwithstanding financialization being a widely accepted phenomenon, it is not clear yet how it may have affected each commodity price or return (Hamilton & Wu, 2014).

Given the financial nature of commodity investment, studying their return behavior is gaining interest among researchers (Hong and Yogo, 2009; Shahzad et al., 2017; Chevallier & Sévi; 2013). The risk premium for a given maturity may be defined as the expected spot price over the futures contract price for that maturity (Hsieh & Kulatilaka, 1982). Thus, understanding risk premia is a way to analyze expected returns from commodity investments.

The nature of risk premia is controversial. According to Keynes (1930) and Hicks (1939) in the normal backwardation theory, risk premia arise because of an imbalance between buyers and sellers of an asset, which is counterbalanced by speculators who require a payment to take their positions. Currently, there is no consensus whether risk premia are positive or negative (Singleton, 2014; Bakshi et al., 2015; Gorton et al., 2013). Moreover, some authors have recently found evidence to support a time varying risk premium (Fama & French, 2016; Szymanowska et al., 2014; Hamilton & Wu, 2014).

Since the financial crisis in 2008, financialization of the commodity markets has produced a reduction in the heterogeneity of different commodities returns, increasing their correlation, especially for indexed commodities (Bhardwaj et al., 2015). Indexed commodities are gaining relevance, when compared to non-indexed ones, because of their higher liquidity and the existence of derivatives which are demanded by speculative investors, who trade in and out of all commodities in a given index (Tang & Xiong, 2012; Boyd et al., 2018).

In this article we propose a new approach to estimate the risk premium of a commodity index by using filtered analyst´s forecasts and futures prices of each of the commodities included in the index.

The use of analyst' expectations is not new in the literature. For instance, Cortazar et al. (2019b) compare the average analysts' expectations of future spot prices to futures prices, to estimate a constant commodity risk premium term structure. Bianchi & Piana (2017) use expectations to determine risk premia of four different commodities. Orphanides & Kim (2005) use short-term interest rate expectations to estimate long-term interest rates. Altavilla et al. (2017) also uses analyst' expectations to improve estimations of interest rate curves.

In this study we implement the multifactor stochastic pricing model used in Cortazar et al. (2019a) and in Cifuentes et al. (2020) to estimate the time-varying risk premium term structure of oil and copper, respectively. We extend this approach to estimate the risk premium of a commodity index by analyzing the behavior of portfolios of four different industries: energy, industrial metals, precious metals and agriculture, and of a global portfolio that includes them all. Each portfolio is composed of a set of commodities of its industry weighted using the same weights as in the Standard and Poor´s Goldman Sachs Commodity Index (S&P GSCI Index). Futures prices, are from NYMEX, LME, COMEX, ICE or CBOT, depending on the commodity. Analysts' expectations data is obtained from Bloomberg. Data is from January 2014 to June 2018.

Once we have obtained the time varying risk premia of the different commodity portfolios, we explore the ability of a number of market variable to explain the time varying risk premia. The market variables that we examine are: S&P500 returns, VIX Index, NASDAQ Emerging Market Index (EMI) Returns, Term Premium, Default Premium and 5-Year Treasury Bill (Fama & French, 1989; Bhar & Lee, 2011; Hang & Yogo, 2012; Basu & Mifre, 2013; Bianchi & Piana, 2017; Szymanowska, De Roon, et al., 2014; Silvennoinen & Thorp, 2013; Daskalaki & Kostakis, 2014; among others). The joint analysis for several commodity sectors allows us to study not only which market variables are able to explain a given portfolio's risk premia, but also the correlation among different commodity sectors.

The paper is organized as follows. Section 2 describes the construction of the portfolios that represent the different commodity sectors. Section 3 describes the data. Section 4 develops the model used to estimate the risk premia and Section 5 presents the results. Finally, Section 6 concludes.

3. CONSTRUCTING THE COMMODITY PORTFOLIOS

3.1 Portfolio weights

In this section we define the individual commodity weights in each commodity portfolio. We define 5 portfolios as representative of indexed commodity investments: Energy, Industrial Metals, Precious Metals, Agriculture and a Global Portfolio which includes the four sectors.

To determine the commodity weights in each of the portfolios we use the S&P GSCI Index[1]. This index is widely used in the literature as a commodity market benchmark (Silvennoinen and Thorp, 2013; Daskalaki et al., 2014; Tang & Xiong, 2012; Basu and Miffre; 2013). The index determines commodity weights depending on the production of each raw material.

Table III-1 shows the commodity weights, in each portfolio from 2014 to 2018, according to the S&P GSCI Index. The Energy Portfolio accounts for at least 59% of the Global portfolio, reaching a maximum of 70% in 2014. In terms of the individual commodities, WTI has the highest weight in the Global Portfolio with an average weight of 32%.

3.2 Portfolio representation

There are two approaches to model a commodity portfolio. The first one is to model and calibrate each commodity individually. In this way the futures and expected prices curves are individually obtained for each commodity, and then weighted to represent the curves of the commodity portfolio. This procedure, however, has some problems both in terms

[1] We do not consider the livestock commodity sector because of the lack of analysts´ forecasts which are required to obtain risk premium estimations. Also, commodities that had a low weight and lack of data were also eliminated. Portfolio weights were adjusted to reflect the missing data.

of efficiency (15 individual calibrations) and consistency (some of commodities with little data could generate very volatile curves producing unreliable portfolio averages).

An alternative approach which we follow in this paper is to model and calibrate the portfolios directly. Data in each commodity portfolio is weighted according to the weights in Table 3.1, thus synthetic futures and synthetic expected prices (proxied by analysts' forecasts) for each portfolio are generated. Using the synthetic data each portfolio model is calibrated. In this way not only effort is reduced (only 5 calibrations, one for each portfolio, are needed), but also more consistent results may be achieved.

We now explain in greater detail how we compute the synthetic prices. Synthetic futures are determined using the following expression:

$$F_t^T = \sum_{i=1}^{n} N_{it} FP_{it}^T \quad \forall T, t \tag{3.1}$$

where F_t^T is the portfolio synthetic futures price at time t with maturity T, N_{it} is the number of contracts of commodity i at time t and FP_{it}^T is the futures price of commodity i at time t with maturity T.

The number of contracts N_{it} is determined by:

$$N_{it} = \frac{\frac{w_{it}}{\sum_i^n w_{it}} Investment_t}{FP_{it}^{T^*}} \tag{3.2}$$

where w_{it} is the weight of commodity i at time t in the corresponding portfolio. $Investment_t$ is the amount[2] in US$ dollars investment in every commodity portfolio to

[2] Arbitrarily, an amount of US$1,000 is assumed for the starting $Investment_0$ on each commodity.

create the synthetic futures portfolio and $FP_{it}^{T^*}$ is the closest-to-maturity futures contract price[3] of commodity i at time t, with maturity T^*.

Table 3.1: Commodity Weights[4] in the S&P GSCI Index 2014-2018

	2014	2015	2016	2017	2018
Energy					
Brent	32.8%	31.6%	26.5%	22.9%	24.4%
Natural Gas	3.7%	4.0%	4.2%	5.1%	4.0%
WTI	33.7%	31.3%	29.9%	31.5%	34.5%
Total Energy	70.2%	67.0%	60.7%	59.4%	62.9%
Industrial Metals					
Aluminium	2.7%	3.6%	4.2%	4.6%	4.6%
Copper	4.4%	4.9%	5.2%	5.7%	5.6%
Lead	0.6%	0.9%	0.8%	0.9%	1.0%
Nickel	0.7%	0.8%	0.9%	1.2%	1.0%
Zinc	0.9%	1.2%	1.3%	1.7%	1.7%
Total Industrial Metals	9.2%	11.3%	12.3%	14.1%	14.0%
Precious Metals					
Gold	4.0%	4.1%	4.2%	5.3%	5.4%
Silver	0.6%	0.5%	0.5%	0.7%	0.7%
Total Precious Metals	4.6%	4.6%	4.7%	6.0%	6.1%
Agriculture					
Corn	4.7%	5.4%	6.9%	6.3%	5.5%
Cotton	1.4%	1.5%	1.9%	2.1%	1.9%
Soybeans	3.8%	3.7%	4.8%	4.6%	4.0%
Sugar	2.0%	2.0%	3.2%	3.2%	1.9%
Wheat	4.1%	4.5%	5.3%	4.3%	3.7%
Total Agriculture	16.0%	17.2%	22.2%	20.5%	17.1%

To adjust the portfolio, rebalancing is made on a monthly basis, or when a futures contract has expired. In order to rebalance, a new closest-to-maturity futures contract must be used. Let t^* be time to rebalance, then:

[3] The S&P GSCI methodology uses the closest-to- maturity futures.
[4] Weights are normalized leaving out commodities with low representation and lack of information

$$Investment_{t*} = \sum_i^n N_{it^*-1}\, FP_{it^*}^{T^*} \tag{3.3}$$

$$N_{it*} = \frac{\frac{w_{it*}}{\sum_i^n w_{it*}} Investment_{t*}}{FP_{it*}^{T^*}} \tag{3.4}$$

Where $FP_{it*}^{T^*}$ is the closest-to-maturity futures prices of commodity i at time t^*. Finally, the synthetic futures price, with maturity T, becomes:

$$F_{t^*}^{T} = \sum_{i=1}^{n} N_{it^*} FP_{it^*}^{T} \qquad \forall T, t = t^* \tag{3.5}$$

The synthetic expected price of a portfolio for maturity T is computed from weighting each commodity's expected price for that maturity, using the following expression:

$$E_t^T = \sum_{i=1}^{n} N_{it} EP_{it}^{T} \quad \forall T, t \tag{3.6}$$

where E_t^T is the synthetic expected price of the portfolio at time t for maturity T, EP_{it}^T is the expected price of commodity i at time t with maturity T. By using the same number of contracts, N_{it}, synthetic futures and synthetic expected prices are rebalanced at the same date.

4. DATA

To estimate the risk premia of the commodity portfolios, data on futures and expected prices on each commodity are used.

Weekly futures prices, with maturities every 6 months from January 2014 to June 2018, are obtained from Bloomberg[5]. With this data synthetic futures for all five portfolios, Energy, Industrial Metals, Precious Metals, Agriculture and the Global Portfolio, are computed using the methodology described in the previous section.

Figure 4.1 presents the evolution of the weekly synthetic futures for the Energy Portfolio. Data is for synthetic futures with maturities every 6 months from 2014 to 2018. Notice that even though these synthetic futures prices are constructed by adding several individual commodities these aggregate prices have a relatively smooth behavior as a time series and across maturities.

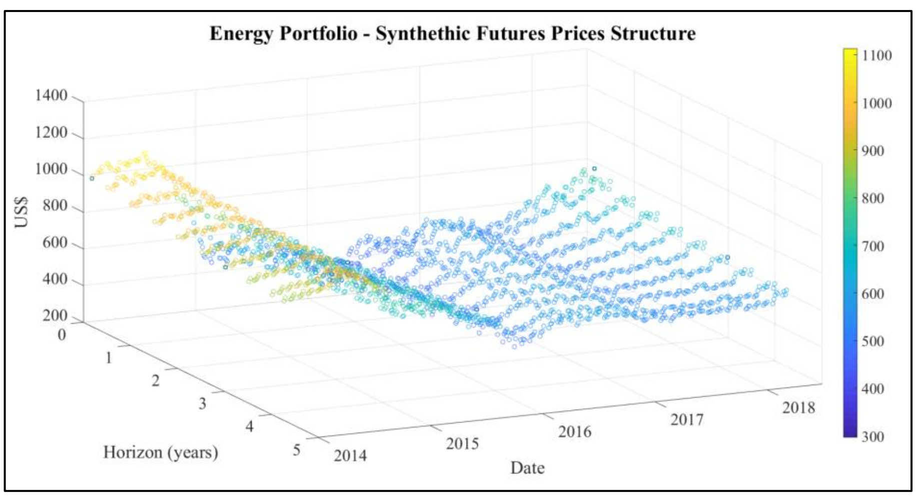

Figure 4.1: Synthetic futures prices data for the Energy Portfolio from 0.5 to 5 years (2014 to 2018).

[5] Data is originally from the ICE, NYMEX, CME, LME and COMEX exchanges.

Table 4.1 summarizes the synthetic futures data of the Energy Portfolio. It can be seen that prices are on average in contango and that the number of observations is relatively similar but decreasing across maturities.

Daily analysts' expected prices were obtained from Bloomberg. Expected prices come in two formats: quarterly and annually. Quarterly forecasts are for a maximum of 6 quarters and annually forecasts up to 5 years ahead. Given the relative scarcity of forecasts (compared to futures data), following Cortazar et al. (2019a) all weekly averages for the same maturity are added to generate the weekly data.

Table 4.1: Energy portfolio synthetic futures data 01/2014 to 06/2018

Maturity bucket (years)	Nº of observations	Mean price (US$)	Price SD (US$)	Max. price (US$)	Min. price (US$)	Mean maturity (years)
0-1	586	642.1	193.7	1,113.4	296.5	0.4751
1-2	477	649.9	161.9	1,040.3	370.1	1.5030
2-3	491	651.7	148.0	988.8	414.3	2.4957
3-4	468	656.3	170.7	965.3	442.3	3.5111
4-5	357	658.6	136.0	955.8	465.8	4.1860

Figure 4.2 and Table 4.2 show similar information to the presented above for futures, but now for the synthetic expected prices.

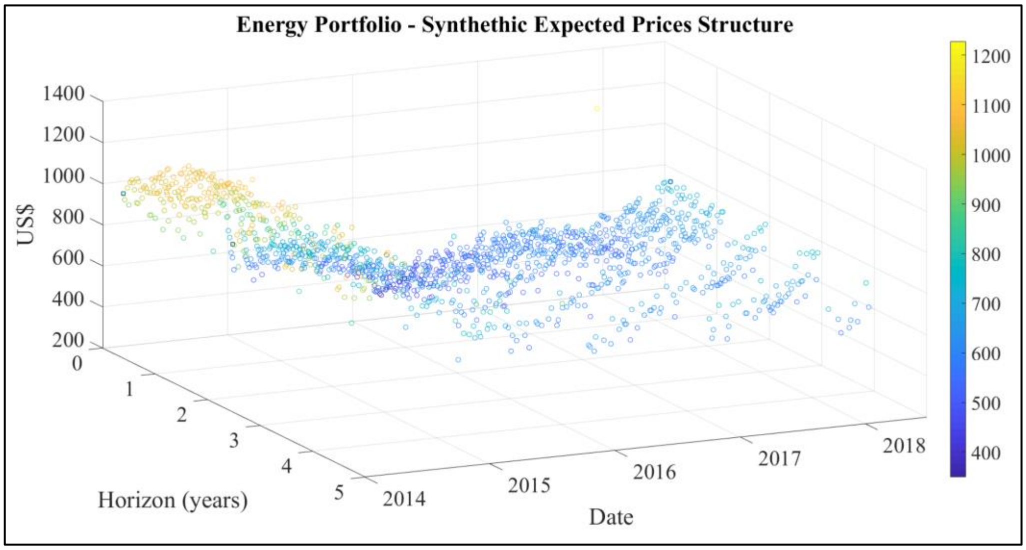

Figure 4.2: Synthetic expected prices data for Energy portfolio from 0.5 to 5 years (2014 to 2018).

Table 4.2: Energy portfolio synthetic expected prices 01/2014 to 06/2018

Maturity bucket (years)	Nº of observations	Mean price (US$)	Price SD (US$)	Max. price (US$)	Min. price (US$)	Mean maturity (years)
0-1	674	662.2	182.5	1,228.2	349.2	0.5576
1-2	393	705.5	170.4	1,118.8	453.8	1.3362
2-3	148	726.8	156.1	1,134.0	508.1	2.4496
3-4	114	723.7	146.1	1,088.2	517.4	3.4606
4-5	51	754.6	154.4	1,078.2	560.8	4.2646

Energy synthetic expected prices also are in contango, as futures did. One difference, however, is that now the number of observations declines heavily with maturity.

Information of synthetic futures and synthetic expected price data for the other 4 portfolios (Industrial Metals, Precious Metals, Agriculture and Global portfolio) is given in **Appendix A.**

With the information of futures and expected prices we can obtain the risk premium. Following Hsieh & Kulatilaka (1982) we can estimate the empirical annualized risk premium using the following expression:

$$\pi_t(T-t) = \frac{ln\left(\frac{E_t(S_T)}{F_t(T)}\right)}{T-t} \tag{4.1}$$

where $\pi_t(T-t)$ is the annualized risk premium at time t over the period from t to T, $E_t(S_T)$ is the expected spot price at time t for maturity T and $F_t(T)$ is the futures price at time t for maturity T.

Table 4.3 shows the average empirical risk premia for the 5 commodity portfolios. In 3 of the 5 portfolios risk premia is positive and decreases with maturity. Another interesting fact is that for the Precious Metals Portfolio the risk premium is negative. Finally, notice that given the limited availability of futures for agricultural commodities this portfolio and the Global do not have an empirical risk premium for a horizon over 3 years.

Table 4.3: Average empirical risk premiums for commodity portfolios

Maturity bucket (years)	Mean Energy Risk Premium	Mean Industrial Metals Risk Premium	Mean Precious Metals Risk Premium	Mean Agriculture Risk Premium	Mean Global Portfolio Risk Premium
0-1	7.01%	3.51%	-2.12%	0.59%	5.81%
1-2	6.45%	2.03%	-0.20%	-0.68%	4.11%
2-3	4.57%	2.25%	-0.30%	0.17%	3.47%
3-4	3.32%	1.69%	-0.62%	-	-
4-5	2.75%	1.65%	-1.25%	-	-

5. THE MODEL

5.1 Model definition

As was stated earlier, we extend the use of the Cortazar et al. (2019a) and Cifuentes et al. (2020) model for individual commodities to estimate the risk premium of commodity portfolios.

The model has 3 stochastic factors with time varying risk premium and is estimated using a Kalman Filter.

Let S_t be the commodity spot price at time t, then

$$ln(S_t) = Y_t = h' x_t \tag{5.1}$$

$$dx_t = \left(-Ax_t + \begin{bmatrix} b_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \right) dt + dw_t \tag{5.2}$$

where h is an $n\ x\ 1$ vector of constants, x_t is an $n\ x\ 1$ vector of state variables, b_1 is an scalar vector, A is an $n\ x\ n$ upper triangular matrix with its first diagonal element being zero the other diagonal elements all different and strictly positive. Let dw_t be an $n\ x\ 1$ vector that represent uncorrelated Brownian motions:

$$dw_t dw_t{}' \quad = Idt \tag{5.3}$$

where I is an $n\ x\ n$ identity matrix.

Let RP_t be the risk premium at time t and assuming the following expression:

$$RP_t = \lambda + \Lambda x_t \tag{5.4}$$

Then, we have the following risk adjusted equation:

$$dx_t = \left(-(A+\Lambda)x_t + \begin{bmatrix} b_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} - \lambda \right) dt + dw_t^Q \tag{5.5}$$

where λ is an $n \ x \ 1$ vector, Λ is an $n \ x \ n$ matrix that is not diagonal or triangular necessary and dw_t^Q is the Brownian motion under risk neutral measure. Further restrictions on λ and Λ are not necessary.

Futures prices are the expected spot price S_t under the risk-neutral measure Q. Given the lognormal distribution of the adjusted price, futures prices are defined as follows:

$$F_t(T) = e^{h' \ E_t^Q(x_T) + \frac{1}{2}h' \ Cov^Q(x_T)h} \tag{5.6}$$

with

$$E_t^Q(x_T) = e^{-(A+\Lambda)(T-t)}x_t + \left(\int_0^{T-t} e^{-(A+\Lambda)\tau} d\tau\right)(b-\lambda) \tag{5.7}$$

$$Cov^Q(x_T) = \int_0^{T-t} e^{-(A+\Lambda)\tau}(e^{-(A+\Lambda)\tau})' \ d\tau \tag{5.8}$$

Analogously the expected spot price satisfies the following equations:

$$E_t(S_T) = e^{h' \ E_t(x_T) + \frac{1}{2}h' \ Cov(x_T)h} \tag{5.9}$$

$$E_t(x_T) = e^{-A(T-t)}x_t + (\int_0^{T-t} e^{-A\tau}d\tau)b \tag{5.10}$$

$$Cov(x_T) = \int_0^{T-t} e^{-A\tau}(e^{-A\tau})' \ d\tau \tag{5.11}$$

The risk premium is:

$$\pi_t(T-t) = \frac{h' \ \left(E_t(x_T) - E_t^Q(x_T)\right) + \frac{1}{2} h' \ \left(Cov_t(x_T) - Cov_t^Q(x_T)\right) h}{T-t} \tag{5.12}$$

Finally, model implicit volatilities of futures prices σ_F and expected prices σ_E can be determined by the following expressions:

$$\sigma_F = \sqrt{h' \ \ e^{-(A+\Lambda)(T-t)} e^{-(A+\Lambda)(T-t)'} \ \ h} \tag{5.13}$$

$$\sigma_E = \sqrt{h' \ \ e^{-A(T-t)} e^{-A(T-t)'} h} \tag{5.14}$$

5.2 Model estimation

We estimate the parameters of the model and the time series of state variables using a Kalman Filter (Kalman, R., 1960).

The Kalman Filter estimates the optimum value of the state variables at each point in time and of all parameters using all past information and maximum likelihood.

The Kalman Filter can be defined by two equations. The first one:

$$z_t = H_t x_t + d_t + v_t \quad v_t \sim N(0, R_t) \tag{5.15}$$

where z_t is an $m_t \ x \ 1$ vector that contains futures and expected log-prices observations at time t. H_t is a $m_t \ x \ n$ matrix, d_t is an $m_t \ x \ 1$ vector and v_t is a measurement error vector of $m_t \ x \ 1$ dimension with zero mean and covariance R_t. x_t is an $n \ x \ 1$ vector of state variables. In the model, m_t depends on the number of observations at each time, so the dimension of $z_t, H_t, d_t, v_t \ y \ R_t$ change at each time period.

Matrix R_t is defined as:

$$R_t = \begin{bmatrix} {\sigma_f}^2 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & {\sigma_f}^2 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & {\sigma_e}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & {\sigma_e}^2 \end{bmatrix} \tag{5.16}$$

The second equation in the Kalman Filter is:

$$x_{t+1} = \bar{A}x_t + \bar{c} + w_t \quad w_t \sim N(0, Q_t) \tag{5.17}$$

where $\bar{A}$ is an $n\ x\ n$ matrix, and $\bar{c}$ is an $n\ x\ 1$ vector. $\bar{A}$ and $\bar{c}$ represent the discretization of the process. In the above expression, w_t is a vector of random variables with zero mean and an $n\ x\ n$ variance-covariance matrix Q_t.

6. RESULTS

6.1 Model fit

As was described before, the model uses two sets of data, synthetic futures and synthetic analysts´ expected prices, computed by weighting the data from each commodity in the portfolio. Using both sets of data, the model estimates two curves, a future and an expected price curve.

As an illustration, Figure 6.1 shows the data and curves for the Energy Portfolio corresponding to June 24th of 2015. The model fits much better the futures prices than the expected prices. This behavior occurs for all 5 portfolios as can be seen in Table 6.1 where we show the Mean Absolut Percentage Error (MAPE) between model prices and market prices for each portfolio

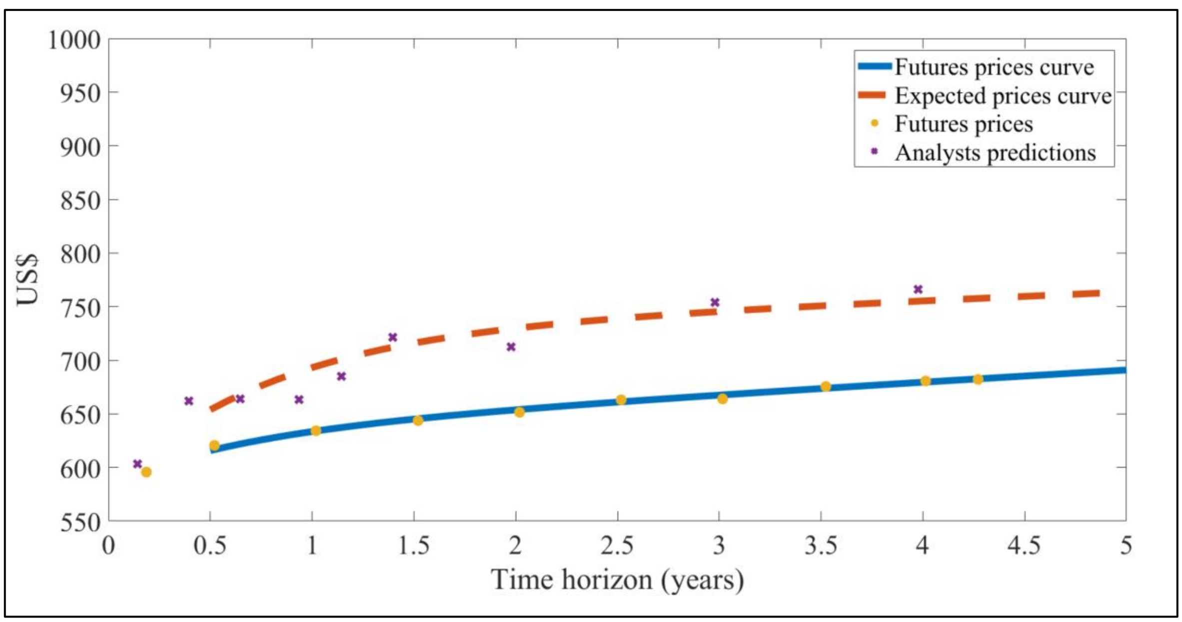

Figure 6.1: Futures and expected prices: Synthetic data and curves for the Energy Portfolio, June 24th, 2015

Table 6.1: Mean Absolut Percentage Error (MAPE) for each portfolio

	Energy	Industrial Metals	Precious Metals	Agriculture	Global
Futures	0.33%	0.06%	0.16%	0.47%	0.24%
Expected prices	5.90%	5.25%	6.65%	1.89%	3.57%

6.2 Risk premia results

In this section we report several characteristics of the risk premia for each of the five portfolios: Energy, Industrial Metals, Precious Metals, Agricultural, and the Global portfolios from 2014 to 2018. For the first 3 portfolios, risk premia are analyzed up to a 5 year horizon. Given that data for the Agricultural portfolio is available for maturities only up to 2.5 years, the analysis on the risk premia for this portfolio and for the Global portfolio is done only up to 2.5 years. In addition, a term structure of risk premia correlation between the Global portfolio and each of the commodity portfolios is presented.

The first set of results is shown in Figure 6.2 where the term structure of risk premia for the 5 portfolios is presented. Second, Figure 6.3 compares the model versus the empirical mean risk premia for each portfolio. Third, Figure 6.4 shows the volatility term structure of the risk premia, for the 5 portfolios. Finally, Figure 6.5 shows the correlation of the risk premia term structure between the Global Portfolio and each commodity sector portfolio. Several observations can be drawn from the figures.

First, the annual risk premia range between 8% and -2%, being highest for the Energy portfolio and lowest for the Precious Metals portfolio.

Second, the risk premium is positive for all maturities for the Energy, Industrial Metals and Global portfolios, negative for the Precious Metals portfolio and mixed for the Agriculture portfolio.

Third, the risk premium is decreasing with maturity for the Energy, Industrial Metals and Global portfolios and mixed for the Precious Metals and Agriculture portfolios.

Fourth, the annual volatility term structure of the risk premia is similar for the Energy, Industrial Metals, Agriculture and Global portfolios with short term values ranging 60 to 80% and 2.5 horizon values around 10 to 20%.

Fifth, the annual volatility term structure of the risk premium for the Precious Metals portfolio is flat at less than 10%.

Finally, the correlation term structure between the Global portfolio and each commodity sector portfolio is decreasing with maturity for the Energy and the Precious Metals portfolio and flat for Industrial Metals and Agriculture portfolios.

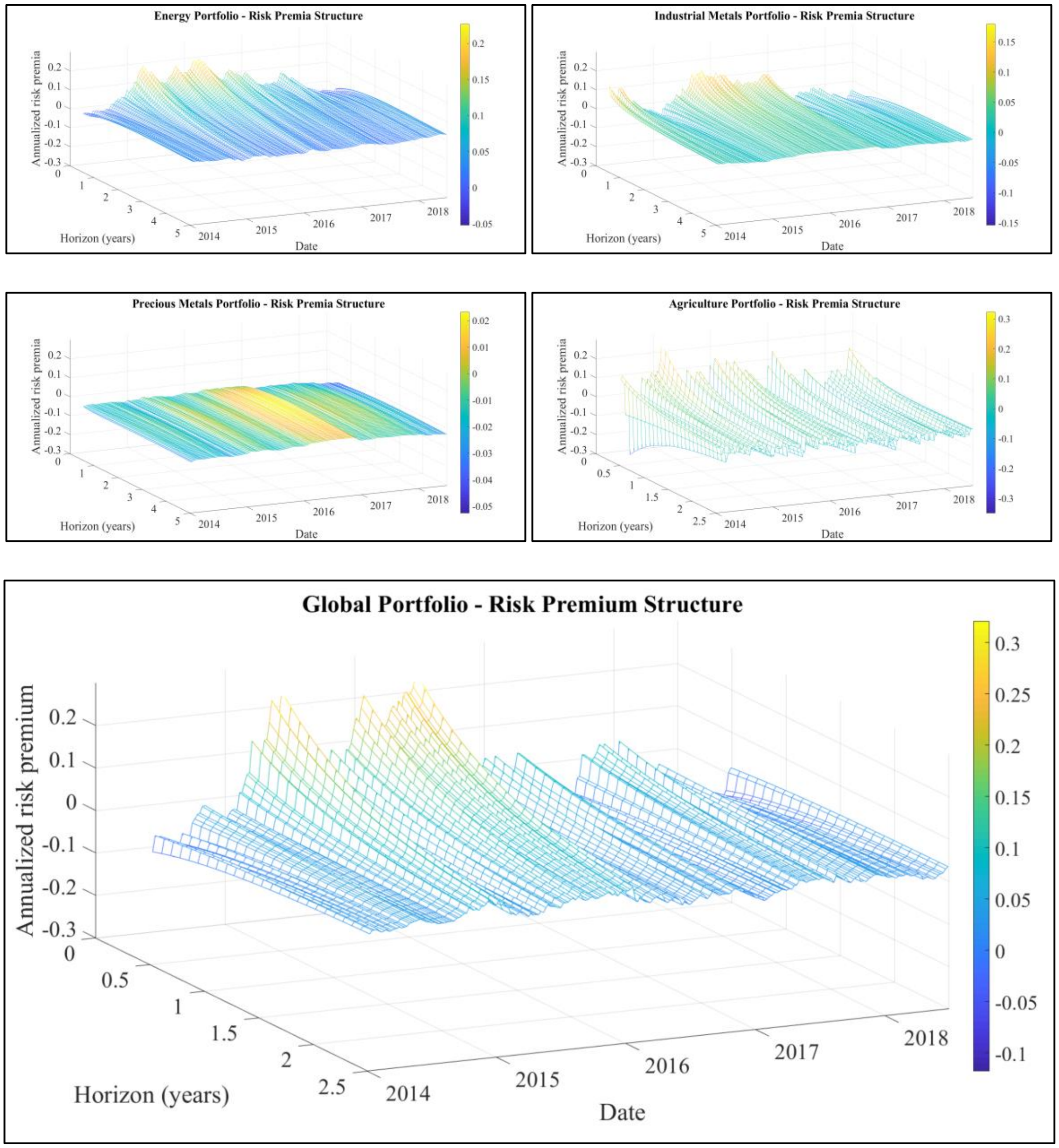

Figure 6.2: Risk Premium Term Structures for the 5 commodity portfolios: a) Energy, b) Industrial Metals, c) Precious Metals, d) Agricultural, and e) Global

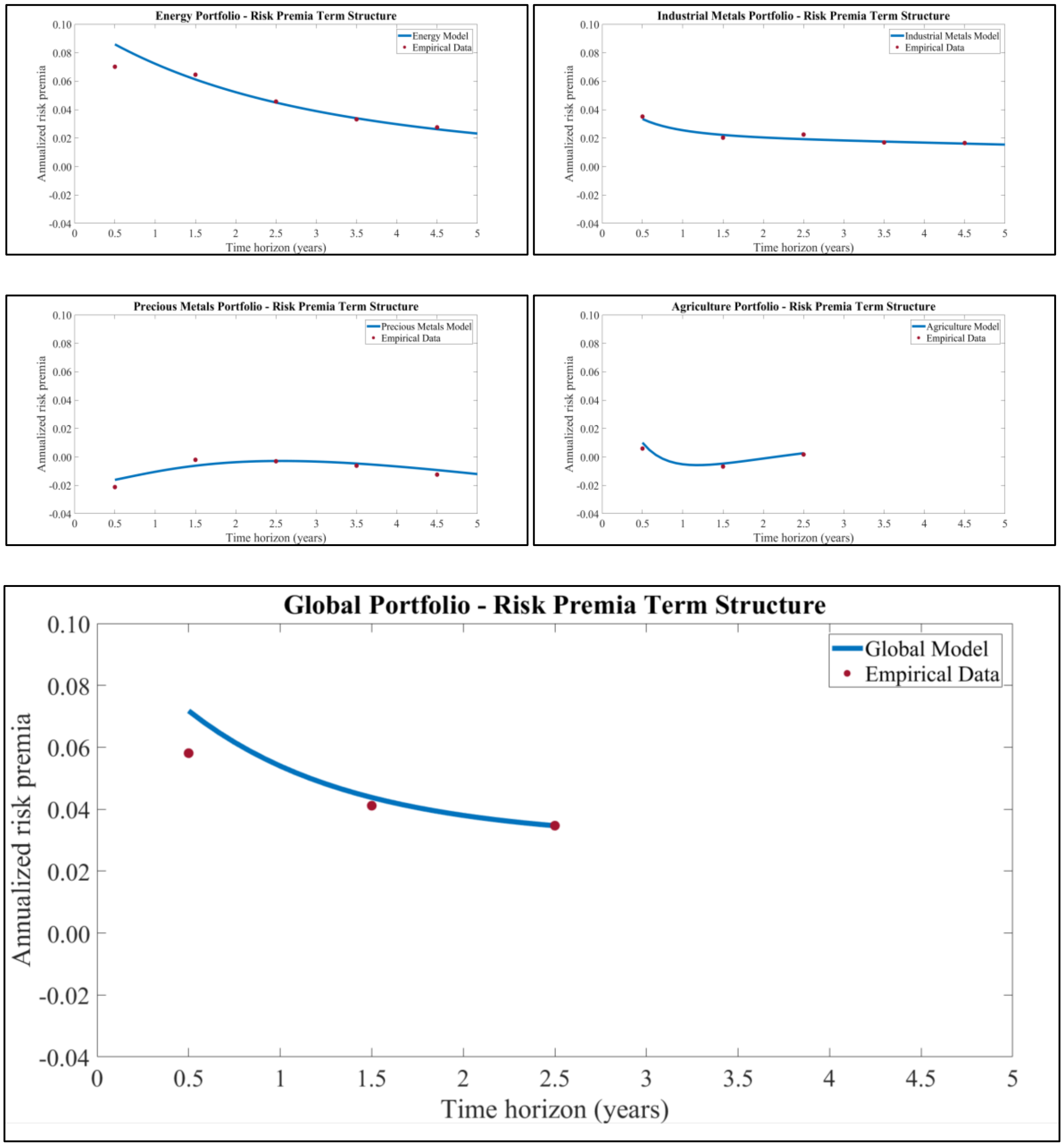

Figure 6.3: Empirical and Model Term Structure Mean Commodity Risk Premium for the 5 portfolios: a) Energy, b) Industrial Metals, c) Precious Metals, d) Agricultural, and e) Global

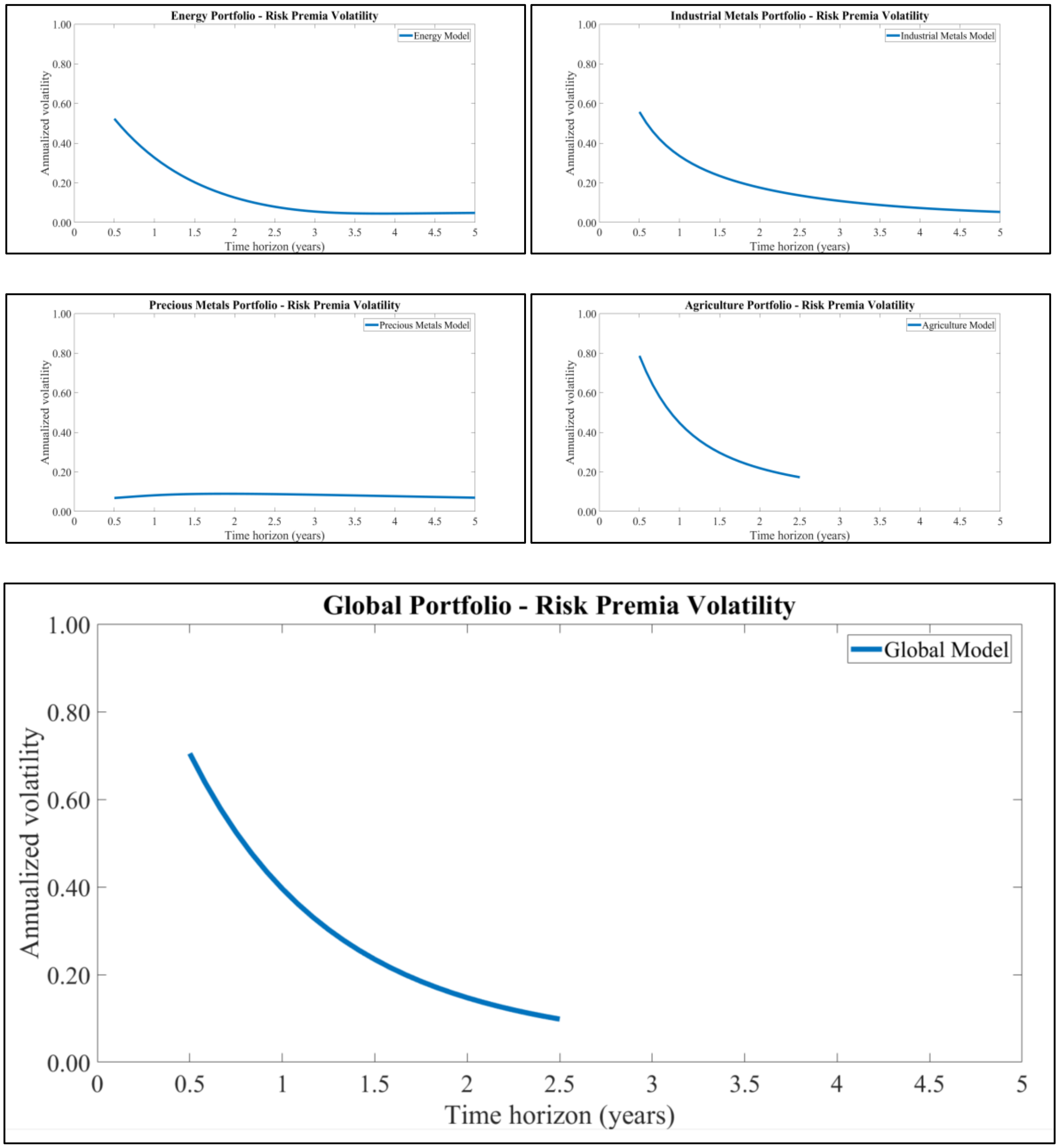

Figure 6.4: Annual Commodity Risk Premium Volatility for the 5 portfolios: a) Energy, b) Industrial Metals, c) Precious Metals, d) Agricultural, and e) Global

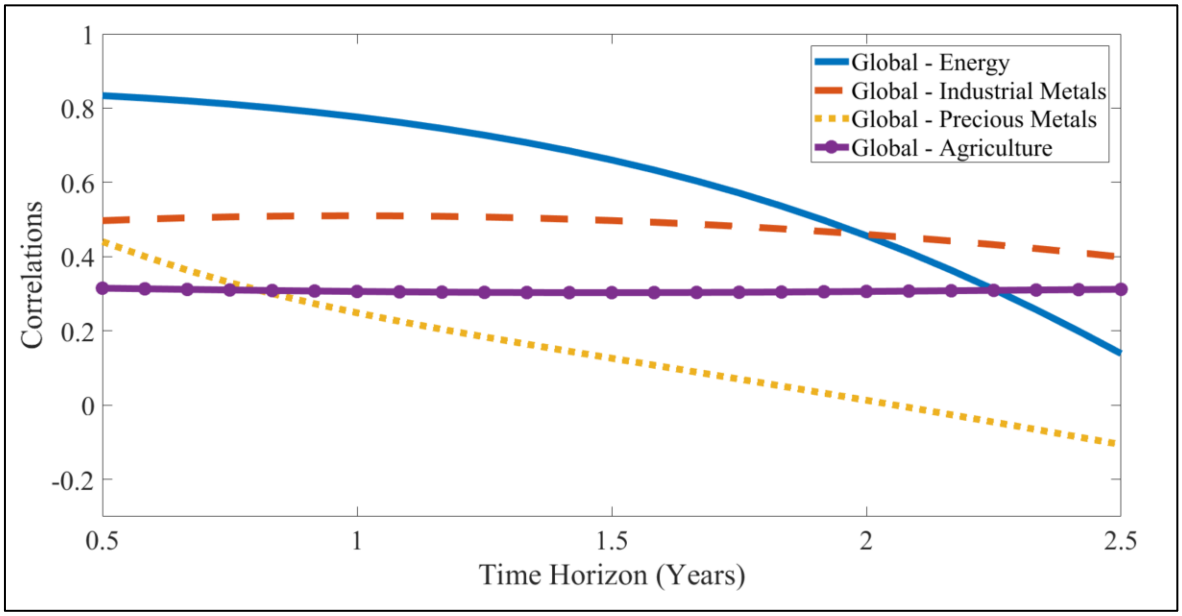

Figure 6.5: Risk premium term structure correlations between the Global portfolio and each of commodity sector portfolios: Energy, Agriculture, Industrial Metals and Precious Metals

7. MARKET DETERMINANTS OF COMMODITY PORFOLIO RISK PREMIA

7.1 Market variables data

In order to study the determinants of commodity risk premia for each commodity portfolio we consider the market variables that have been most commonly proposed in the literature for this purpose (Basu and Mifre, 2013; Bianchi and Piana, 2017; Szymanowska et al, 2014; among others). The market variables considered are: S&P500 Returns, VIX Index, NASDAQ Emerging Market Index (EMI) Returns, Term Premium, Default Premium and 5-Year Treasury Bill. We describe each one of them in what follows.

S&P500 Returns are used as a proxy of the state of the US economy. One of the reasons this market variable has been used in previous studies is because it may directly affect energy commodities (de Roon et al., (2000); Bianchi and Piana, 2017).

VIX, or Volatility Index, is a measure of the volatility expectations of the S&P500 for the next 30 days. It is commonly used as a measure of global uncertainty (Bhar and Lee. 2011), which may directly impact the analysts' forecasts.

EMI is used as a proxy of the state of emerging markets economies. Some emerging economies play a major role in the commodity market because of their role as large importers and exporters of raw materials.

Term Premium and **Default Premium** have been used as predictors of stocks and bonds excess returns (Fama and French. 1989; Hong and Yogo. 2012). Given the recent financialization of the commodities markets these variables may also affect the commodity risk premia. Term premium is estimated as the spread between the 10 Years

Treasury Bond yield and the interest rate of a 3-month US Treasury Bill. Default premium is estimated as the spread between BAA and AAA corporate bond yields.

The **5 Year Treasury Bill** is used as a proxy of the medium-term interest rates.

Table 7.1 and Figure 7.1 summarize the weekly time series of these variables from January 2014 to June 2018.

Table 7.1: Market variables summary January 2014 to June 2018

	Min	Max	Mean	Std
S&P500 Returns	-6.92%	3.97%	0.16%	1.56%
VIX	9.15	30.32	14.61	3.92
NASDAQ EMI Returns	-6.98%	7.24%	0.04%	2.10%
Term Premium	0.90%	2.86%	1.78%	0.53%
5 Year Treasury Bill	0.95%	2.94%	1.72%	0.42%
Default Premium	0.54%	1.54%	0.88%	0.24%

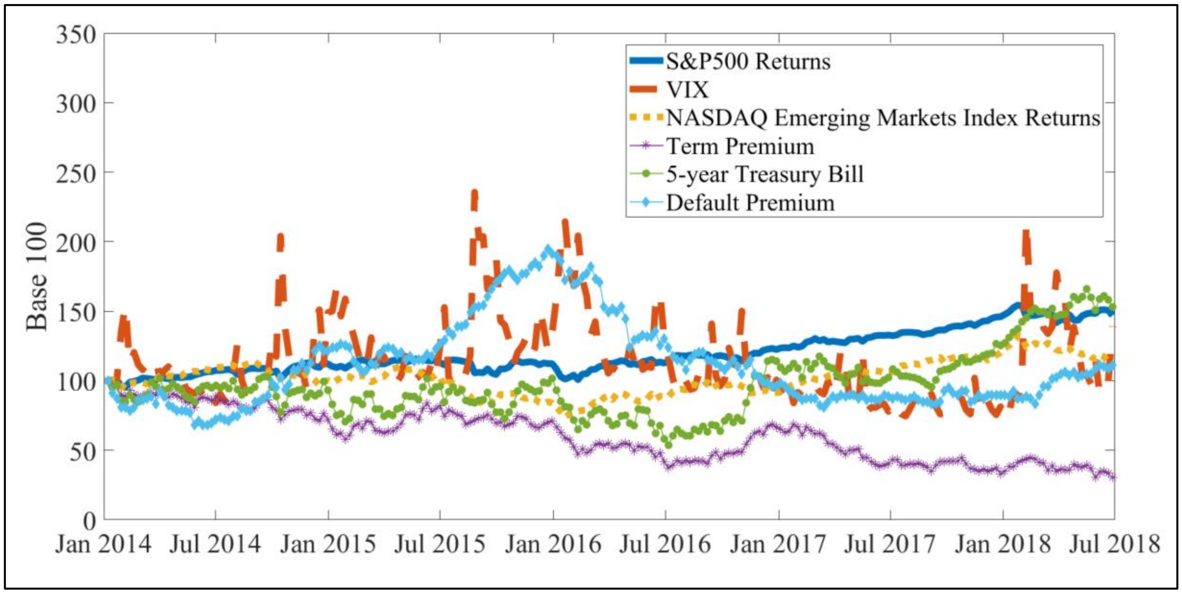

Figure 7.1: Market variables from January 2014 to June 2018. (Initial value is set to 100)

7.2 The regression model

In order to determine the market variables' explanatory power we run the following multivariate regression on all the market variables, X_t, plus maturity, T, for each of the 5 commodity portfolios (k):

$$RP_{t,T}^{k} = \hat{\beta}_0^k + \hat{\beta}_1^k X_t + \hat{\beta}_2^k T \ + \hat{\varepsilon}_{t,T}^{k} \tag{7.1}$$

where $\hat{\beta}_0^k$ is the regression intercept of portfolio k, $\hat{\beta}_1^k$ is the vector of coefficient for each market variable and $\hat{\beta}_2^k$ is the coefficient for maturity variable and $\varepsilon_{t,T}^{k}$ is the regression error of portfolio k. The reason that we included maturity in the regression is that in our preliminary analysis we performed 6 independent regressions, for 6, 9, 12, 18, 24 and 30 month maturity, using the same market variables, but without the maturity variable. After analyzing those results it became clear that the risk-premium maturity was a relevant variable and should be included in our model. (See **Appendix B**).

Table 7.2 shows the results of the multivariate regression for each of the commodity portfolios.

Table 7.2: Multivariate risk premium regression results for each of the five commodity portfolios. Significance levels are ***1%, **5% and *10%

	Energy	Industrial Metals	Precious Metals	Agriculture	Global
Intercept	0.0834***	0.0176***	0.0426***	-0.0334***	0.0028
S&P500 Returns	-0.0221	-0.1953***	0.0089	-0.5862***	-0.0477
VIX	0.0001	-0.0006***	-0.0004***	-0.0024***	0.0002
EMI Returns	-0.0298	-0.0598***	-0.0006	0.0416	-0.2683***
Term Premium	-0.0033***	0.0170***	-0.0107***	0.0292***	0.0154***
5-Year T-Bill	-0.0336***	-0.0417***	-0.0242***	-0.0106***	-0.0201***
Default Premium	0.0827***	0.0751***	0.0070***	0.0370***	0.0850***
Maturity	-0.0204***	-0.0059***	0.0067***	0.0004	-0.0175***
R2	0.6183	0.6153	0.8235	0.1281	0.4006

Several observations can be drawn from Table VII-2.

First, in terms of the model´s explanatory power, for three of the commodity portfolios (Energy, Industrial Metals and Precious Metals portfolios) the R2 is rather high (over 60%), for the Global Portfolio is over 40% while for the Agricultural Portfolio it amounts to less than 13%. This last portfolio had less data available so results may be less reliable.

Second, the 5-year T-Bill and Default Premium are the most consistent variables across all 5 Portfolios, both significant at the 1% level. The 5-year T-Bill has negative coefficients while the Default Premium is positive for all 5 portfolios.

Third, the Maturity variable is significantly negative (at the 1% level) for the Energy, Industrial Metals and Global portfolios, while its coefficient is significantly positive for the Precious Metals Portfolio and not statistically significant for the Agricultural portfolio. This shows that for most portfolios the longer the maturity the smaller the risk premium.

Fourth, the Term Premium is statistically significant (at the 1% level) for all 5 portfolios, but positive for The Industrial Metals, Agricultural and Global Portfolios, and negative for the Energy and Precious Metals portfolios.

Fifth, the EMI Returns is negative but significant only for the Industrial Metals and Global portfolios.

Sixth, VIX is statistically significant (at the 1% level) and negative for the Industrial Metals, Precious Metals and Agricultural portfolios.

Finally, the S&P500 Returns is only significant (at the 1% level) and negative for the Industrial Metals and the Agricultural portfolios.

More detailed regressions for each different maturity can be found at **Appendix B**.

7.3 Discussion

Our results show that Term Premium, 5 Year Treasury Bill and Default Premium are significant in almost every regression. In particular, Default Premium is significant and positive in every commodity portfolio which is in line with our expectations since this variable represents short-term uncertainty inducing investors to demand a higher risk premium.

It is worth noting that the S&P500 index seems to not have a major effect over commodity risk premia. Even though financialization increased the amount of funds invested in commodities in the early 2000s, it is not clear if a positive relationship exists between equities and commodities. Different studies have documented different findings. Some studies have found a high correlation between equities and commodities (e.g., Silvennoinen & Thorp, 2013; Tang & Xiong, 2012; Mensi et al., 2013; Delatte & Lopez, (2013); among others). On the other hand, authors like Zhu et al. (2014), Chong and Miffre (2009), Graham et al. (2013), Arouri and Nguyen (2010) among others, found that commodities are a good asset in terms of diversification when considering equities, which means that those assets do not have a high correlation between each other. Even more, Graham et al. (2013) find no evidence of co-movements between our benchmark commodity index S&P GSCI and S&P500.

Regarding NASDAQ Emerging Market Index, which is a measure of the equity markets in emerging economies, it is significantly negative in the Global portfolio. This is consistent with the de Boyre and Pavlova (2018) study between GSCI and MSCI Emerging Market Index.

With respect to the Maturity variable, it is worth noticing that, except for the Agriculture portfolio, it is significant. This regression results are consistent with our previous results respect to the risk premia term structure curves. In the Energy, Industrial Metals, and Global portfolio Maturity has a negative coefficient when trying to explain risk premiums. It seems that long-term expectations are closer to long-term maturity future prices, and risk premia decrease with maturity.

Regarding Precious Metals, this portfolio may be seen as a safe investment by investors in turmoil periods, thus investing in it is an insurance, resulting in a negative risk premium. As maturity increase, risk premiums become positive as turmoil expectations start to disappear and expected prices are a little higher than future prices, explaining our regression results of a positive coefficient in Maturity variable. Finally, in the Agriculture portfolio, the effect of Maturity is not a significant variable in this scenario. Lack of data and a consequent high volatility may be the reason why the Agriculture results are not significant in terms of Maturity.

The six market variables chosen —S&P500, VIX, EMI, Term Premium, 5-Year T-Bill and Default Premium— are able to explain over 60% of the risk premium variations for most of the portfolios. These results may be useful to investors who want to estimate future spot prices by adding risk premiums to futures prices as a new source of expected prices (Pierdzioch et al., 2013; United Nations, 2011).

8. CONCLUSION

A process called financialization has recently made commodities a new investment asset class. In this context institutional investors, looking for ways to expand their diversification opportunities, hold positions in a commodity futures index and use them as a performance benchmark. Thus, it has become important to estimate the magnitude and drivers of the risk premium of different commodity portfolios This paper estimates the risk premia using futures and analyst´s forecasts, extending the analysis to the main commodity sectors: energy, industrial metals, precious metals and agriculture. A global commodity portfolio, represented by the S&P GSCI index, is also analyzed.

To estimate the risk premia the Cortazar el al. (2019a) N-factor model is implemented using three stochastic factors, calibrated with a Kalman Filter and data on futures and analyst´s forecasts for each commodity sector.

Results show that risk premiums are positive and decreasing with maturity for the Energy, Industrial Metals and Global portfolios. Regarding the risk premium volatility, we find that it is high in the short run and decreasing with maturity. The correlation between the Global portfolio and each commodity sector is decreasing with maturity for the Energy and the Precious Metals Portfolio and flat for Industrial Metals and Agriculture Portfolios. Finally, we analyze the relationship between some market variables and the estimated risk premia in order to explain its behavior. We explore the variables S&P500, VIX, EMI, Term Premium, 5-Year T-Bill and Default Premium. We find that the 5-Year T-Bill, Default Premium and Term Premium are statistically significant across all 5 portfolios. We show that the six market variables are able to explain over 60% of the risk premium variations for most of the portfolios. These results may also be useful to investors who want to estimate future spot prices.

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APPENDIX

APPENDIX A: SYNTHETIC FUTURES AND SYNTHETIC EXPECTED PRICES

In this Appendix we present the synthetic futures and synthetic expected prices for each portfolio (excluding the Energy portfolio presented in the main text).

Figures A.1 to A.4 show the different synthetic futures term structures for each portfolio with observations each 6 month from 0.5 to 5 years in the Industrial Metals and Precious Metals portfolios. For the Agriculture portfolio synthetic futures prices are obtained only until 2.5 years due to lack of data of futures prices. Given that the Global portfolio includes Agriculture commodities, the same restriction applies.

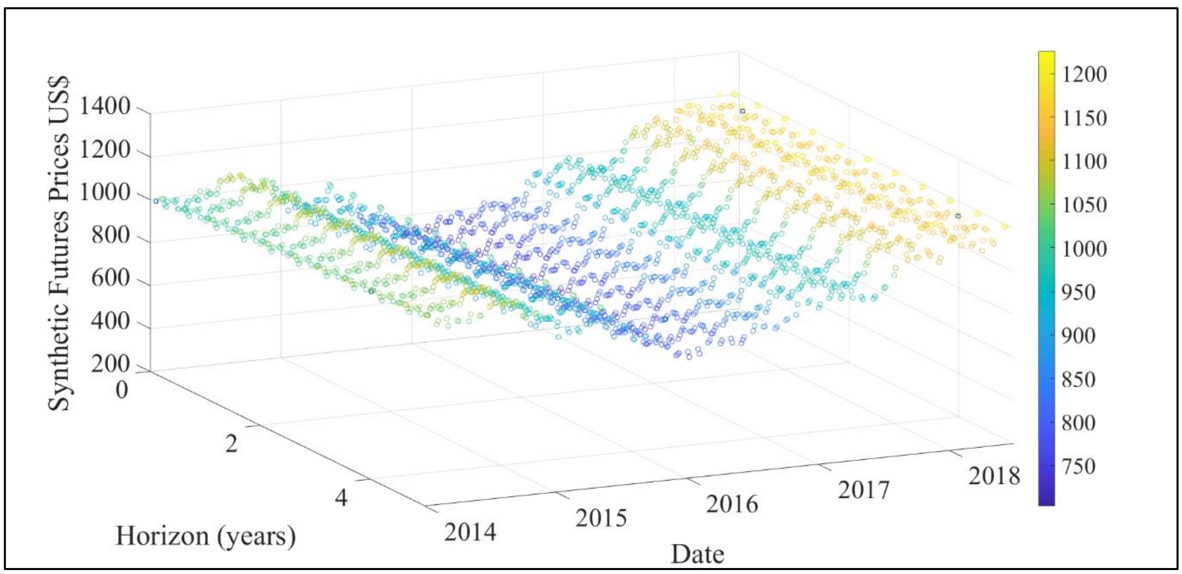

Figure A.1: Synthetic futures prices for the Industrial Metals Portfolio - 0.5 to 5 years (2014 to 2018)

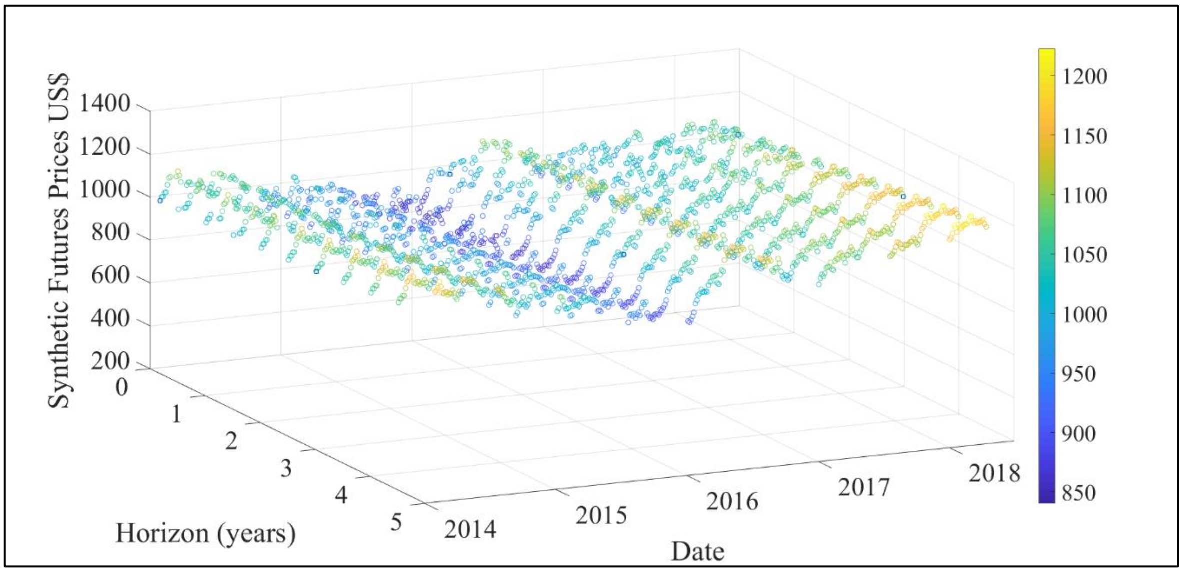

Figure A.2: Synthetic futures prices for the Precious Metals Portfolio - 0.5 to 5 years (2014 to 2018)

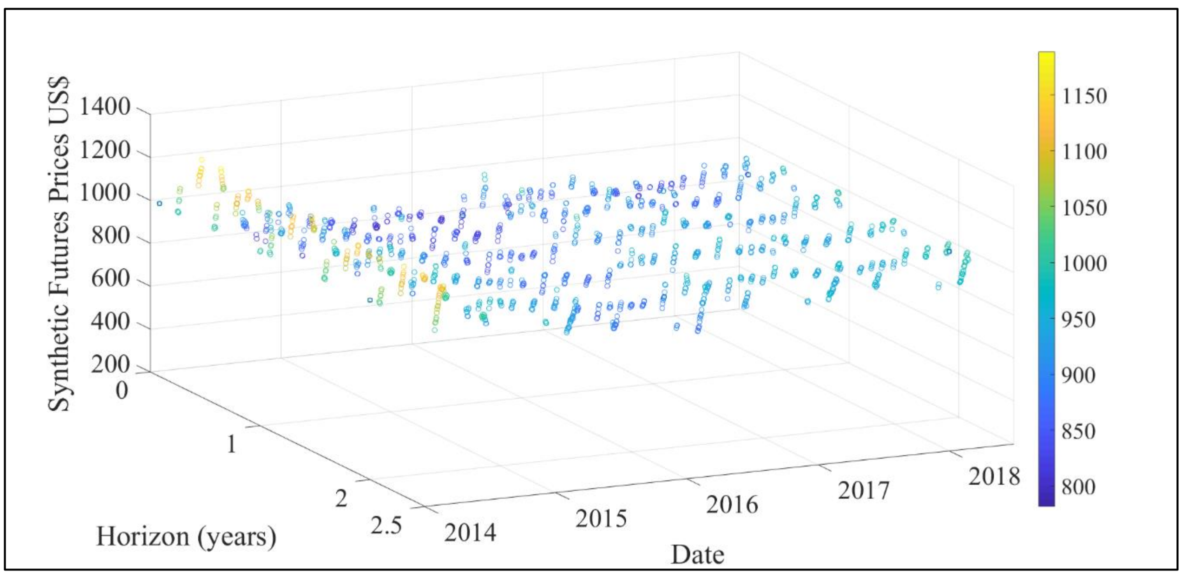

Figure A.3: Synthetic futures prices for the Agriculture Portfolio - 0.5 to 2.5 years (2014 to 2018)

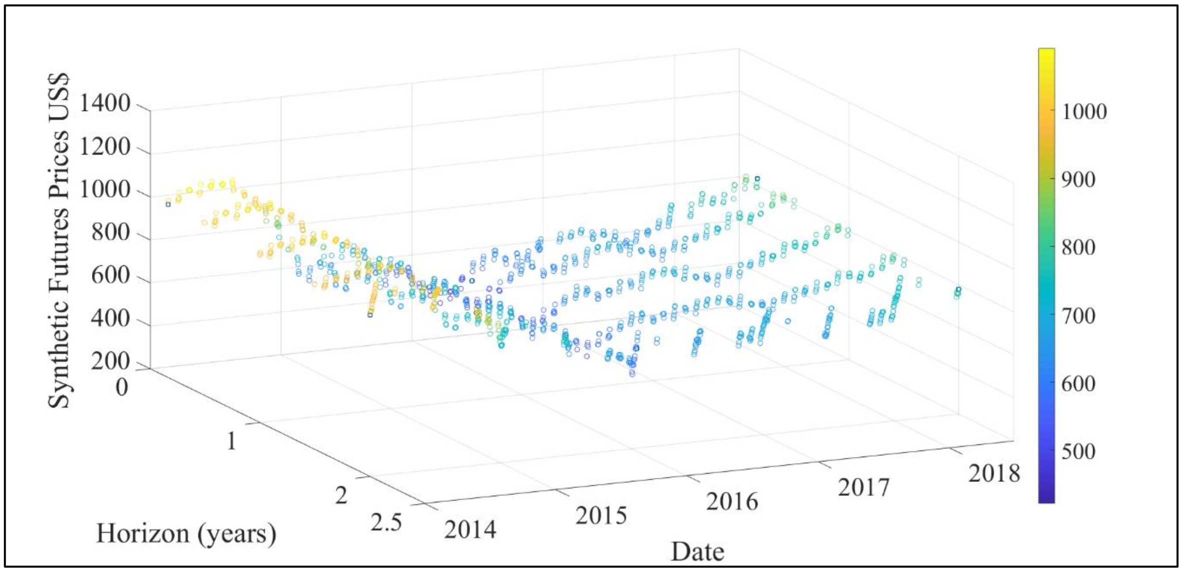

Figure A.4: Synthetic futures prices for the Global Portfolio - 0.5 to 2.5 years (2014 to 2018)

Tables A.1 to A.4 summarizes the different synthetic futures data of each portfolio.

Table A.1: Industrial Metal portfolio synthetic futures data 01/2014 to 06/2018

Maturity bucket (years)	Nº of observations	Mean price (US$)	Price SD (US$)	Max. price (US$)	Min. price (US$)	Mean maturity (years)
0-1	586	951.6	131.0	1,225.4	703.4	0.4350
1-2	477	961.9	130.1	1,220.1	709.5	1.5030
2-3	491	966.6	126.3	1,213.2	719.3	2.4957
3-4	468	971.8	121.8	1,210.2	731.8	3.511
4-5	469	977.7	118.8	1,213.9	745.7	4.4931

Table A.2: Precious Metal portfolio synthetic futures data 01/2014 to 06/2018

Maturity bucket (years)	Nº of observations	Mean price (US$)	Price SD (US$)	Max. price (US$)	Min. price (US$)	Mean maturity (years)
0-1	583	1,000.6	62.5	1,117.8	840.2	0.4238
1-2	501	1,012.4	68.1	1,130.8	844.5	1.4876
2-3	452	1,028.2	70.5	1,160.9	852.4	2.5114
3-4	466	1,044.8	74.7	1,191.2	862.3	3.5005
4-5	444	1,064.1	79.4	1,222.8	874.7	4.4751

Table A.3: Agriculture portfolio synthetic futures data 01/2014 to 06/2018

Maturity bucket (years)	Nº of observations	Mean price (USD)	Price SD (USD)	Max. price (USD)	Min. price (USD)	Mean maturity (years)
0-1	588	906.3	76.3	1,118.8	781.7	0.4516
1-2	475	947.8	59.5	1,166.5	827.2	1.5049
2-3	368	956.7	54.3	1,149.1	860.0	2.2932

Table A.4: Global portfolio synthetic futures data 01/2014 to 06/2018

Maturity bucket (years)	Nº of observations	Mean price (US$)	Price SD (US$)	Max. price (US$)	Min. price (US$)	Mean maturity (years)
0-1	586	729.0	156.3	1,091.7	421.7	0.4728
1-2	524	744.2	132.1	1,053.9	489.5	1.4903
2-3	115	740.6	105.2	1,021.7	602.7	2.0627

It can be seen from the tables above that every portfolio, excluding Global, are in contango. This results in the Global portfolio may be driven by the lack of data as the number of observations decrease significantly in the last maturity bucket.

Regarding synthetic expected prices, each portfolio term structure is presented from Figures A.5 to A.8. In the expected prices, weekly data is presented from 0.5 to 5 years. In this case, the Agriculture portfolio has data even for the longest maturity, thus not restricting the Global portfolio.

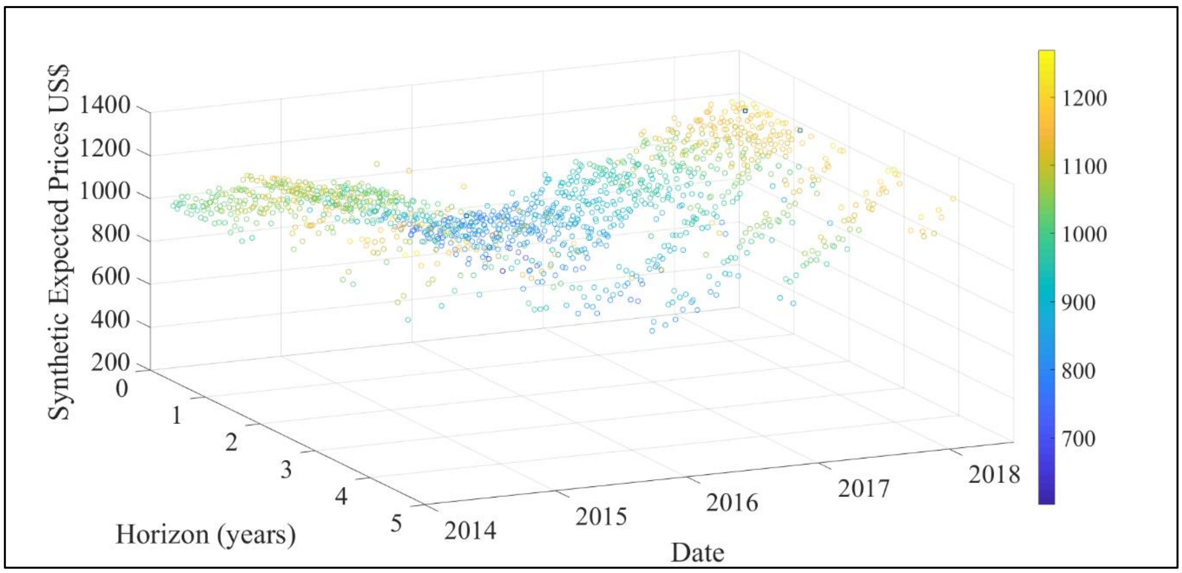

Figure A.5: Synthetic expected prices for the Industrial Metals Portfolio -0.5 to 5.0 years (2014 to 2018)

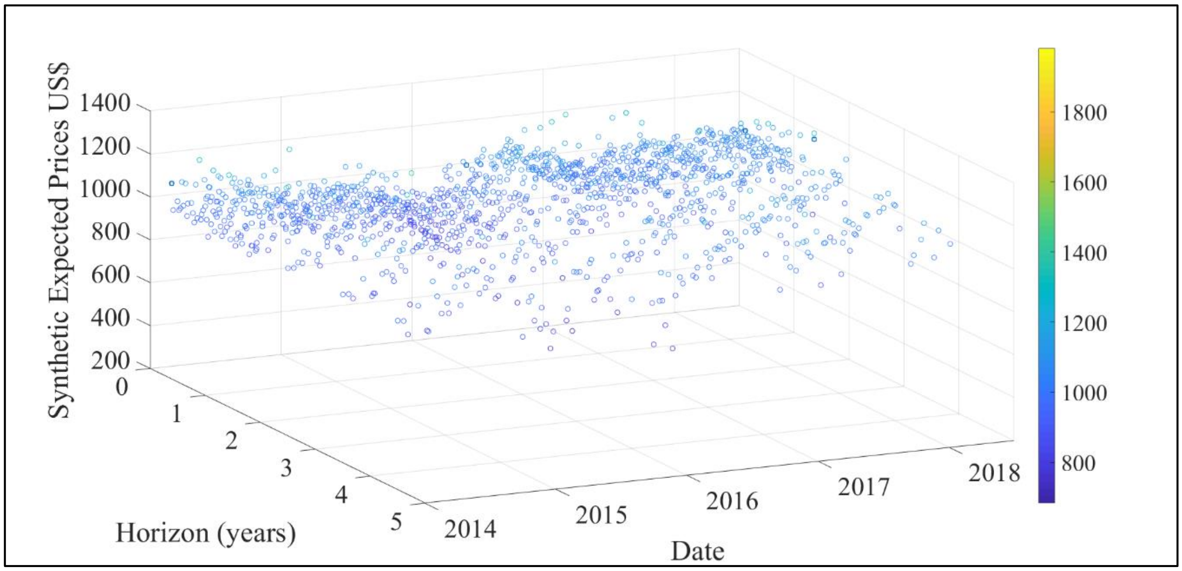

Figure A.6: Synthetic expected prices for the Precious Metals Portfolio - 0.5 to 5.0 years

(2014 to 2018)

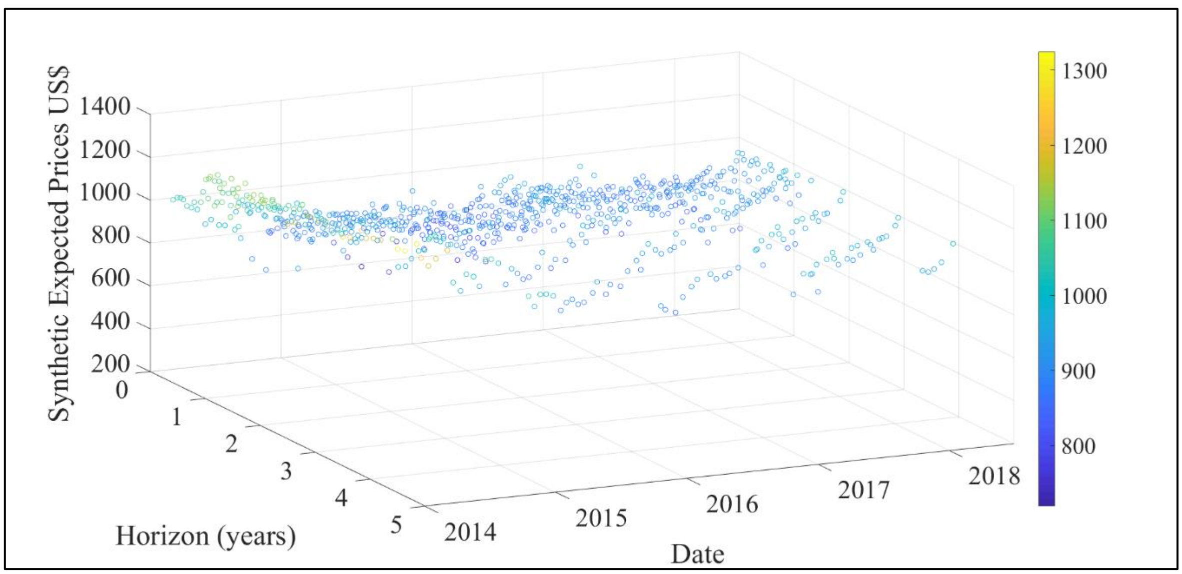

Figure A.7: Synthetic expected prices for the Agriculture Portfolio - 0.5 to 5.0 years

(2014 to 2018)

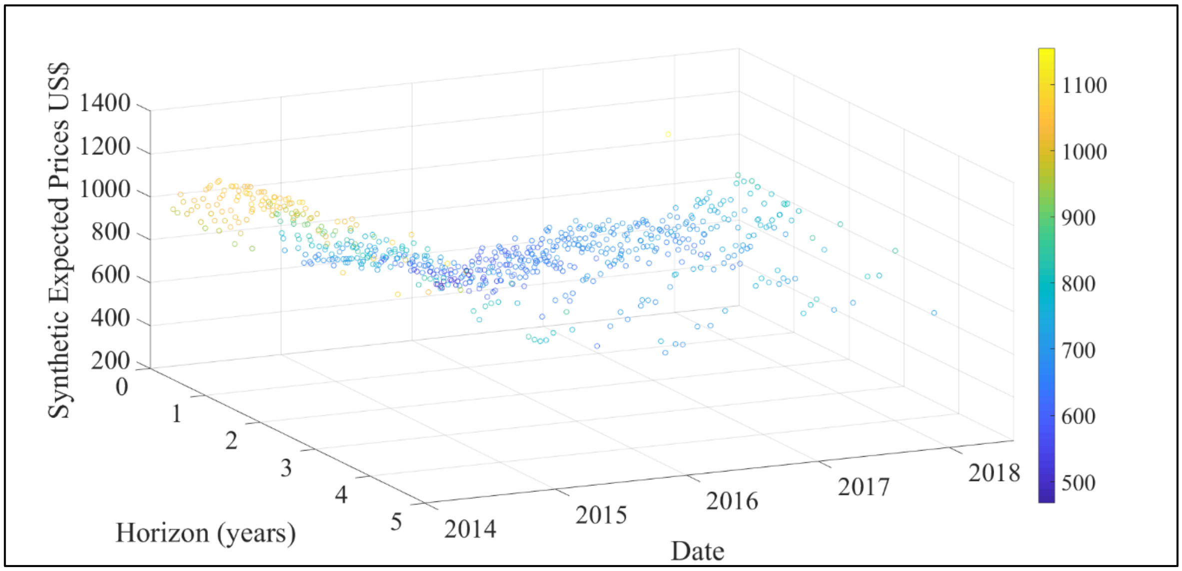

Figure A.8: Synthetic expected prices for the Global Portfolio - 0.5 to 5.0 years (2014 to 2018)

Tables A.5 to A.8 summarizes the different synthetic expected data of each portfolio.

Table A.5: Industrial Metals synthetic expected prices 01/2014 to 06/2018

Maturity bucket (years)	Nº of observations	Mean price (US$)	Price SD (US$)	Max. price (US$)	Min. price (US$)	Mean maturity (years)
0-1	652	965.4	120.2	1,214.8	601.5	0.5587
1-2	373	981.2	117.0	1,222.7	667.0	1.3402
2-3	135	1.009.4	119.7	1,235.3	735.2	2.4782
3-4	115	1.030.7	127.4	1,267.5	759.9	3.4598
4-5	51	1.030.8	120.0	1,269.3	819.0	4.2751

Table A.6: Precious Metals synthetic expected prices 01/2014 to 06/2018

Maturity bucket (years)	Nº of observations	Mean price (US$)	Price SD (US$)	Max. price (US$)	Min. price (US$)	Mean maturity (years)
0-1	724	990.7	76.7	1,295.8	753.5	0.5602
1-2	415	1,015.0	111.8	1,608.7	763.3	1.3453
2-3	150	1,028.7	134.1	1,886.3	756.7	2.4804
3-4	113	1,030.3	169.2	1,981.5	725.8	3.4543
4-5	51	1,001.7	161.5	1,549.6	683.4	4.2913

Table A.7: Agriculture synthetic expected prices 01/2014 to 06/2018

Maturity bucket (years)	Nº of observations	Mean price (US$)	Price SD (US$)	Max. price (US$)	Min. price (US$)	Mean maturity (years)
0-1	426	904.7	70.9	1,134.9	718.9	0.5574
1-2	222	933.4	73.2	1,142.6	756.0	1.3273
2-3	66	964.2	81.5	1,190.5	840.2	2.4679
3-4	54	980.0	101.3	1,255.6	855.0	3.4572
4-5	26	1,013.2	134.0	1,324.3	859.2	4.2187

Table A.8: Global synthetic expected prices 01/2014 to 06/2018

Maturity bucket (years)	Nº of observations	Mean price (US$)	Price SD (US$)	Max. price (US$)	Min. price (US$)	Mean maturity (years)
0-1	383	734.3	148.8	1,154.5	467.5	0.5545
1-2	178	769.6	138.3	1,095.3	569.9	1.3072
2-3	37	778.6	113.5	1,089.1	631.6	2.4746
3-4	26	813.1	132.2	1,116.9	620.3	3.4314
4-5	15	803.3	117.6	1,097.8	676.2	4.2385

In the synthetic expected prices term structure, contango is present for the Industrial Metals and Agriculture portfolios. For thee Precious Metals and Global portfolios the decrease in their last maturity bucket (4-5 years) may be driven by the lack of data.

APPENDIX B: REGRESSION RESULTS EXCLUDING MATURITY AS A VARIABLE

As discussed in Section 7.2, the Maturity variable plays an important role in explaining the risk premiums over the different portfolios. In this Appendix we do not use Maturity as a variable, but instead perform independent regressions for different maturities (6, 9, 12, 18, 24 and 30 months were considered). Tables B.1 to B.5 present the results for each of the five portfolios.

Table B.1: Multivariate risk premium regression results for the Energy Portfolio. Maturities from 6 to 30 months are considered. Significance levels are ***1%, **5% and *10%

	M6	M9	M12	M18	M24	M30
Intercept	0.0911***	0.0775***	0.0663***	0.0492***	0.037***	0.0279***
S&P500 Returns	-0.0011	-0.0008	-0.0005	-0.0001	0.0001	0.0003
VIX	-0.0006	-0.0003	-0.0001	0.0002	0.0004	0.0005***
EMI Returns	-0.0017	-0.0011	-0.0007	-0.0001	0.0002	0.0004
Term Premium	-0.0004	-0.0025	-0.0036	-0.0043**	-0.0037***	-0.0027***
5-Year T-Bill	-0.0857***	-0.0656***	-0.0497***	-0.0274***	-0.0134***	-0.0047***
Default Premium	0.1741***	0.141***	0.1138***	0.073***	0.0453***	0.0265***
R2	0.7801	0.781	0.7794	0.7625	0.7077	0.5715

Table B.2: Multivariate risk premium regression results for Industrial Metals Portfolio. Maturities from 6 to 30 months are considered. Significance levels are ***1%, **5% and *10%

	M6	M9	M12	M18	M24	M30
Intercept	0.0122	0.0087	0.0075	0.0077	0.0089	0.0102*
S&P500 Returns	-0.0049*	-0.0034*	-0.0025*	-0.0015	-0.0011	-0.0008
VIX	-0.0016	-0.0011	-0.0008	-0.0005	-0.0004	-0.0003
EMI Returns	-0.0012	-0.0009	-0.0007	-0.0005	-0.0004	-0.0003
Term Premium	0.0365***	0.0271***	0.0213***	0.0145***	0.0107***	0.0082***
5-Year T-Bill	-0.0841***	-0.0643***	-0.0518***	-0.0366***	-0.0275***	-0.0214***
Default Premium	0.1426***	0.1125***	0.0926***	0.0674***	0.0515***	0.0403***
R2	0.6590	0.6925	0.7170	0.7470	0.7622	0.7693

Table B.3: Multivariate risk premium regression results for Precious Metals Portfolio. Maturities from 6 to 30 months are considered. Significance levels are ***1%, **5% and *10%

	M6	M9	M12	M18	M24	M30
Intercept	0.0207***	0.0365***	0.0471***	0.0586***	0.0625***	0.0622***
S&P500 Returns	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
VIX	-0.0001*	-0.0002***	-0.0003***	-0.0004***	-0.0005***	-0.0005***
EMI Returns	0.0001	0.0001	0.0000	0.0000	0.0000	-0.0001
Term Premium	-0.0016***	-0.0062***	-0.0092***	-0.0122***	-0.0133***	-0.0135***
5-Year T-Bill	-0.0214***	-0.0234***	-0.0245***	-0.0251***	-0.0245***	-0.0236***
Default Premium	0.0047***	0.006***	0.0067***	0.0074***	0.0075***	0.0075***
R2	0.9351	0.8856	0.8522	0.8191	0.8048	0.7977

Table B.4: Multivariate risk premium regression results for Agriculture Portfolio. Maturities from 6 to 30 months are considered. Significance levels are ***1%, **5% and *10%

	M6	M9	M12	M18	M24	M30
Intercept	-0.031	-0.0402	-0.0415	-0.0355	-0.0272	-0.0196
S&P500 Returns	-0.0131**	-0.0097**	-0.0074**	-0.0049**	-0.0035**	-0.0028**
VIX	-0.0057**	-0.0041**	-0.003**	-0.0019**	-0.0013**	-0.001**
EMI Returns	-0.0001	0.0002	0.0004	0.0005	0.0005	0.0005
Term Premium	0.0502***	0.0416***	0.0353***	0.0268***	0.0213***	0.0176***
5-Year T-Bill	-0.0213	-0.0164	-0.0132	-0.0092	-0.007	-0.0057
Default Premium	0.0829**	0.0609**	0.0467**	0.0306**	0.0224**	0.0177**
R2	0.0945	0.1118	0.1293	0.1601	0.1822	0.1957

Table B.5: Multivariate risk premium regression results for Global Portfolio. Maturities from 6 to 30 months are considered. Significance levels are ***1%, **5% and *10%

	M6	M9	M12	M18	M24	M30
Intercept	-0.0839**	-0.056*	-0.0374	-0.0154	-0.0033	0.004
S&P500 Returns	-0.0012	-0.0009	-0.0007	-0.0004	-0.0002	-0.0001
VIX	0.0004	0.0003	0.0003	0.0002	0.0002	0.0001
EMI Returns	-0.0061**	-0.0046**	-0.0035**	-0.0022**	-0.0015**	-0.0011**
Term Premium	0.0325***	0.0244***	0.0192***	0.0131***	0.0099***	0.008***
5-Year T-Bill	-0.0567***	-0.0412***	-0.0297***	-0.0151***	-0.007**	-0.0026
Default Premium	0.2162***	0.1604***	0.1196***	0.0673***	0.0382***	0.0216***
R2	0.5414	0.5344	0.5217	0.4795	0.4174	0.3446

From above tables we can see that market variable coefficients are different on the regressions for different maturities, but their sign is in general consistent among them. Also, the 5-Year Treasury Bill, the Default Premium and the Term Premium coefficients seem to be the most significant market variables.