PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
ESCUELA DE INGENIERÍA

ELECTIVE SURGERY PLANNING WITH MULTIPLE PATIENT PRIORITIES USING COLUMN GENERATION

MATIAS EZEQUIEL IORDACHE

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Master of Science in Engineering

Advisor:
SERGIO MATURANA

Santiago de Chile, November, 2015

PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
ESCUELA DE INGENIERÍA

ELECTIVE SURGERY PLANNING WITH MULTIPLE PATIENT PRIORITIES USING COLUMN GENERATION

MATIAS EZEQUIEL IORDACHE

Members of the Committee:
SERGIO MATURANA
JORGE VERA
VICTOR ALBORNOZ
MARCOS SEPULVEDA

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Master of Science in Engineering

Santiago de Chile, November, 2015

Gratefully to my parents

ACKNOWLEDGEMENTS

I would like to thank Sergio Maturana for his advice, support and guidance. This research would not have been possible without him. Also, I would like to thank my family and friends for their support and dedication.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

The purpose of this paper is to study the use of mathematical models for scheduling elective surgeries and to derive guidelines from the results of these models. With the aim of improving the quality of service delivered, we proposed the development of mathematical programming models using different methodologies including the formulation of a mixed integer linear model which we solved in order to obtain the optimal allocation of resources involved, taking into account the specific constraints of the system in which it is implemented and also allowing to select patients using an objective and measurable criteria, a topic that often takes a backseat in many investigations.

We evaluated the model using various case scenarios with different amounts of people on the waiting lists. The number of operating rooms was also tested in our experiment, to analyze if they had to increase the number of operating rooms in order to deal with increased demand. We solved this model in a reasonable time, so that the decision makers could use it perfectly on a weekly basis, since the surgery scheduling takes place every Monday.

Keywords: column generation, scheduling, health care.

RESUMEN

El propósito de este trabajo es estudiar el uso de modelos de programación para agendar cirugías electivas y derivar lineamientos de la resolución de estos modelos. Con el objetivo de mejorar la calidad del servicio prestado, hemos propuesto el desarrollo de modelos de programación matemática utilizando distintas metodologías, incluyendo la formulación de un modelo lineal entero mixto que hemos resuelto con el fin de obtener la asignación óptima de los recursos involucrados, teniendo en cuenta las limitaciones específicas del sistema en el que se implementa y que permite seleccionar los pacientes que usan un criterio objetivo y medible, un tema que a menudo es dejado de lado en muchos estudios.

Se evaluó el modelo usando varios escenarios con distintas cantidades de personas en las listas de espera. El número de salas de operaciones también fue probado en nuestro experimento, para analizar si tuvieran que aumentar el número de salas de operaciones a fin de tratar con la demanda aumentada. Solucionamos este modelo en un tiempo razonable, de modo que las personas que toman decisiones lo pudieran usar perfectamente cada semana, ya que la programacin de la ciruga ocurre cada lunes.

Palabras Claves: generación de columnas, programación, sistema de salud.

1. INTRODUCTION

1.1. Motivation

Surgical care services in the Chilean public health system often have excessively long waiting lists, which is a serious problem for the population they serve; long waiting lists can be explained by the high demand of patients, the medical staff shortage, and the lack of standardization of processes and tools to support management.

Usually the task of scheduling surgeries has been carried out manually, without support systems, by medical staff. The surgery planning process is the set of activities that must be performed in order to schedule surgeries, in other words, the surgery scheduling process determines all aspects of surgery including: operating room (OR), date, patient, medical equipment and surgical procedure. This process should take into consideration available surgeon hours and the duration of each surgery since it is restricted by hospital capacity, availability of anesthetists, beds and other resources that are often scarce.

Unless current conditions undergo a substantial change, the alternative to reduce the waiting list, or at least keep the demand under control, would be to design and implement a systematic planning tool that considers all the relevant aspects of the problem, thus automating the process, in order to avoid human error that could cause delays and setbacks, which could cause high costs and unfavorable consequences for users. This new tool would ensure that there is an improvement in the decision making process, enabling a more efficient use of existing resources. If necessary, it could justify an increase in resources in order to increase system capacity.

1.2. Objectives

The objective of this thesis is to propose and test mathematical programming models to assign surgical operations to operating rooms in order to improve the operating room usage while taking into account the various resource-related constraints and the specifications of the operations processes. This model is developed specifically for elective surgeries (not emergencies). The specific objectives are the following:

(i) Study and analysis of the surgical planning process in the hospital and data collection.
(ii) Analysis of the current planning process in order to find weaknesses and opportunities for improvement.
(iii) Definition of performance indicators to measure the performance of the current method to be able to compare with the model.
(iv) Formulation of the model.
(v) Resolution of the model using different scenarios.
(vi) Evaluation of the results.

1.3. Hypothesis

Through the use of computational tools and optimization it is possible to improve the planning of elective surgeries, allowing the improvement of efficiency and the optimization of the usage of the scarce hospital resources, while making a fair patient selection.

1.4. Methodology

In order to meet the objectives previously defined a survey process was conducted at the Hospital del Salvador, specifically in the area responsible for scheduling surgeries. This was performed to study possible existing opportunities for improvement, which restrictions should be considered and to fully understand the process.

An analysis of the collected information and data provided by the hospital was made. With the aim of improving the quality of service delivered, we proposed to develop a scheduling methodology that allows for an optimal allocation of resources, taking into account the restrictions of the system in which it is implemented. This scheduling methodology would also allow the selection of patients using an objective and measurable criteria, a topic that usually takes a backseat in many investigations.

After solving the problem the results were analyzed by comparing them with the original schedule performance. For this we defined indicators to measure certain key variables considered in the study. Finally, we validate the obtained results, in collaboration with the Hospital del Salvador.

1.5. Case Study

1.5.1. Hospital description

We worked with the Hospital del Salvador, a tertiary referral public Hospital in Santiago. We conducted interviews with the staff of the hospital, especially with surgical area managers who are responsible for monitoring the surgical table, or the weekly schedule of surgeries to be performed.

In addition to that, they have the following responsibilities: first, they need to send surgical tables before 14:30 to the Pavilion Unit and schedule all surgeries after 15:30 in the medical management service with SAM - ANITA, a computer support system that helps keep track of the current status of the patient. Additionally, they need to review tables early in the day and register changes or deviations that could occur in order to program pending surgeries. Lastly, they need to file original tables by date.

1.5.2. General aspects

Inpatient means that the procedure requires the patient to be admitted to the hospital prior to the surgery, primarily so that he or she can be closely monitored during the procedure and afterwards, during recovery. Outpatient means that the procedure does not require hospital admission and may also be performed outside the premises of a hospital. Outpatients represent a significant percentage of the total number of patients. The process of scheduling should be similar for both, although there are some differences.

Deviations from the original plan occur when surgeries take longer than planned, certain equipment or personnel are not available, there are a large number of emergency cases, etc. In that sense there is greater certainty with the hospitalized patients, since the outpatients are liable to not appear or to arrive later, and dealing with deviations from the original plan is an important part of the job handled by the decision makers.

The surgical specialty is that of major influx inside the hospital. Some general topics that must be taken into consideration are:

- It is necessary to serve all the patients who arrive at the emergency department.
- Patients cited for a medical appointment, should arrive approximately thirty minutes before being treated. In the event that the file of the patient can not be found, the doctor can attend the patient without his file, in which case he or she has to create a bulletin that is added later to the file of the patient.
- Sometimes, many emergency patients may arrive at once or a patients medical condition may require the patient to be seen immediately. This may cause a delay for the patient to receive appropriate medical attention.

1.5.3. Identified problems

The main problems and difficulties in the process of scheduling surgeries are the following:

(i) Long waiting lists. Large number of elective patients waiting for surgical attention

(ii) High levels of cancellation of surgeries in some months it reaches up to 20%

(iii) Repetition of information in spreadsheets and their information system. Repetition of information is dangerous because information can be changed in one place and not the other, thus causing inconsistencies. Also, they waste time by processing the same information twice.

(iv) Wasting time due to manual scheduling, which according to personnel from the hospital, can also cause biases when scheduling patients. A tool does not currently exist in order to help with the scheduling.

(v) There are no appropriate indicators to measure the performance of surgery scheduling.

1.5.4. Flow process description

Patients begin the flow by providing the necessary documentation. In the event that their medical examinations are not current they will need to first update these. Depending on whether they are scheduled for hospitalization or surgery, the process will be different. In the case of hospitalization, this could be an emergency, an inpatient who requires to be admitted to the hospital prior to the surgery, or patients who require hospitalization for a clinical study.

Elective surgery is a surgery that is scheduled in advance because it does not involve a medical emergency. On the other hand, emergency surgery is one that must be performed without delay; the patient requires immediate surgery if they do not want to risk permanent disability or death.

In the case of surgical resolution the surgery can be classified as priority surgery, ambulatory surgery, a major surgery scheduled (CMA in Spanish) or an order of invasive procedure. If the patient is a priority, he or she enters the priority waiting list.

In the case that there is availability for surgery, the patient is removed from the waiting list and the surgery is scheduled in the surgical table. Otherwise the patient returns to the waiting list. Depending on whether or not he or she requires hospitalization, the patient

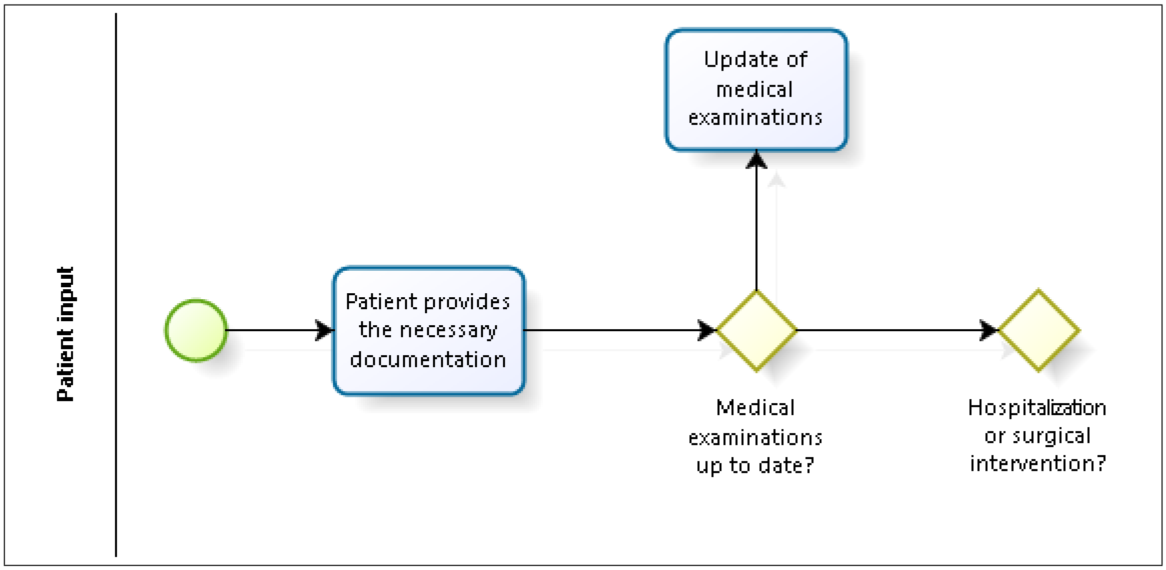

Figure 1.1. Patient enters the system.

is hospitalized and receives surgical intervention. Another possibility is that the patient enters as an emergency patient, in which case he or she is treated immediately.

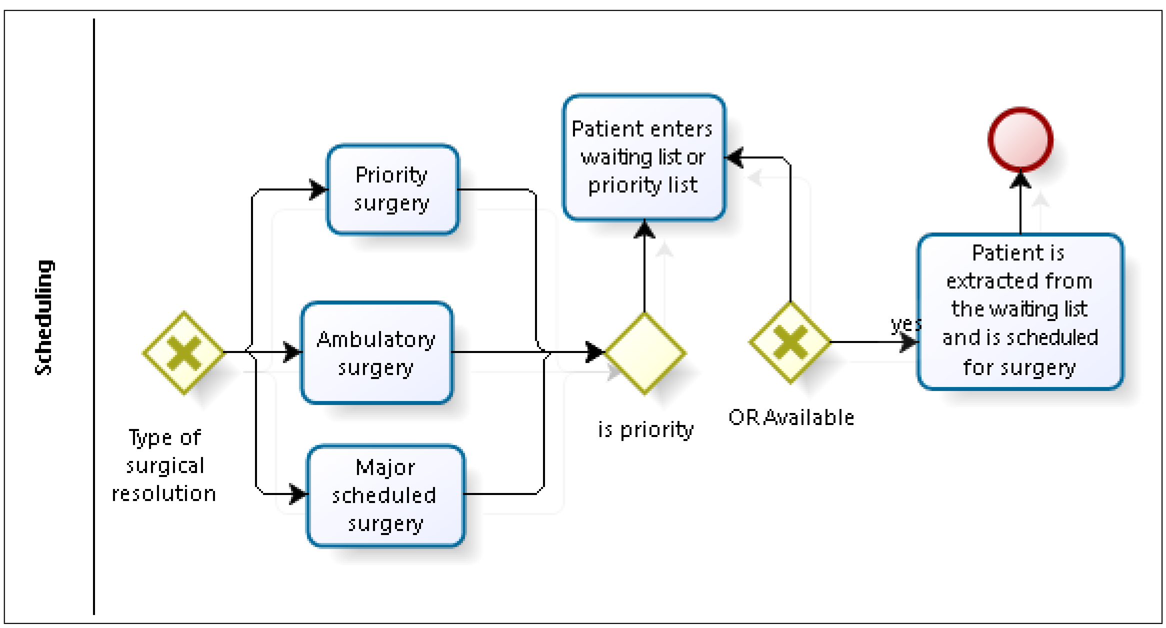

Figure 1.2. Surgical resolution and patient scheduling.

There are different outputs depending on the type of surgery the patient has received. In the case of outpatient surgery, they could be directly discharged. If they received anesthesia, they will need to recover and be stabilized, otherwise they will be transferred to the ICU (Intensive Care Unit) to be stabilized and later transferred to different areas depending on the type of surgery. If the surgery is not successful the patient may require a further surgery.

1.6. Results

We ran the models for different scenarios that we generated by changing the number of patients on the waiting list and increasing the number of ORs, obtaining high levels of operating room utilization. The patients were chosen using an objective and systematic procedure, reducing the bias that could be introduced by the decision makers.

The main contributions presented in this thesis are: the study of several methodologies to optimize elective surgery schedules and the optimal selection of patients using objective criteria. In contrast to other studies, we used the priority of the patient to make this decision and at the same time, we optimized the occupation of the operating rooms, thus integrating both problems. It is important to emphasize that this research seeks to put theory into practice specifically in the Chilean context in order to combat the important problems faced by the public health system. In order to do so, we, cooperated greatly with hospital staff, validating our research with them.

1.7. Conclusions

It is possible to conclude from the work done that it fulfilled all the targets we proposed initially in addition to obtaining a satisfactory impression from the hospital personnel of the hospital who worked with us throughout the entire investigation process, and who provided essential information for our research development and confirmation of the results obtained by the model.

The performance of the model was tested by solving randomly generated scenarios. We compared the solution quality and computation time of different scenarios with a real hospital in Chile. The number of patients and the number of operating rooms were also tested in our experiment. We solved this model in a reasonable time, so that decision makers could use it perfectly on a weekly basis, since surgery scheduling occurs every Monday.

We realized that the surgical scheduling is a complex task. The surgical table often deviates significantly from what really happens during a surgical day, these deviations occur when surgeries take longer than planned, certain equipment or personnel are not available, there are a large number of emergency cases, etc. In such an environment, a calendar is a guide for operational management rather than a precise schedule for procedures that are actually performed.

If the hospital defines better policies or if we give them better computational tools, it will be possible to use resources more effectively and efficiently, thus improving the planning of surgeries, minimizing costs and offering people a better quality of service.

The use of computational systems and mathematical programming models proved able to transform the process of programming elective surgeries in the hospital, making it much more efficient. In addition, it allowed medical staff to make more objective decisions eliminating arbitrary choices. Lastly, more time was allotted for those who were manually responsible for programming, thus allowing them to focus on other urgent tasks that were previously pushed aside.

1.8. Future research

It would be interesting for future investigations to create models that include several other models. That is to say, to find a way to bring together the different tools that have been developed in order to perform more integral planning by taking into consideration all the different aspects that are included in each individual model. These aspects include:

costs, hospital capacity, order of the surgeries in the surgical blocks, patients' selection, etc. The integration of stochastic variables in the model, for example the duration of the surgeries.

In addition, consider other resources in the model, such as doctors and nurses. We have mentioned the existence of emergency patients, it would be interesting to develop a model by taking these into account.

This paper is focused on the tactical level for operating room allocation. The operational level can be further studied. Other possible studies include the sequencing of patients in each OR and how to address the deviations that occur when surgeries take longer than planned, certain equipment or personnel are not available, there are a large number of emergency cases, etc.

Finally, it would be interesting for future studies to implement software platforms that will enable the hospital staff to easily create surgery schedules on a daily basis.

2. ELECTIVE SURGERY PLANNING WITH MULTIPLE PATIENT PRIORITIES USING COLUMN GENERATION

2.1. Introduction

A hospital is an institution whose mission is to provide both curative and preventive medical care to a population. This institution is a fundamental part of the public health system, which aims to deliver the best care possible by making efficient use of the scarce resources they usually have. This, however, implies complications which occur due to various reasons such as, the growing demand for medical care, an aging population both in developed and developing countries, new pathologies, and better informed patients demanding better service (Roland, Di Martinelly, Riane, & Pochet, 2010). This situation produces a high volume of patients, which can trigger failure in service delivery in certain peaks of demand. Additionally, this lack of capacity in certain surgical specialties may result in suboptimal treatment and an extended length of stay for surgical patients (Chow, Puterman, Salehirad, Huang, & Atkins, 2011). This lack of capacity to absorb the demand causes the system to become inefficient, which clearly affects the quality of service perceived by patients, particularly if there is high variability in the demand (McManus et al., 2003).

One way to address these problems is to make a systematic plan that allows a more efficient use of resources. According to research, operating rooms (ORs) are the most expensive resource in the hospital, typically consuming more than 10% of the budget provided for the institution (Gordon, Paul, Lyles, & Fountain, 1988). In addition, it should be noted that of the three major components of the health system (surgery, medical and mental health), the surgery department is the most sensitive to costs controlled through systematic management (Wickizer, 1991). Thus, there is evidence that making efficient use of the operating rooms and other resources involved would be advantageous, as the first is the most expensive area of the hospital and the latter is the most sensitive to cost control.

The surgical scheduling problem is defined as the selection of procedures to be performed, the allocation of time for those procedures, and the subsequent sequencing of the procedures to be performed within the time available. This work seeks primarily to address the first two the selection of the procedures and the allocation of the patients to operating rooms in the determined time horizon. The arrivals of emergency patients, patients who do not show up for surgery, and the variability in the duration of the procedures make the surgical planning problem highly complex, mainly because any plan can deviate significantly (May, Spangler, Strum, & Vargas, 2011). In summary, this is a complicated process involving many variables and problems that hinder decision-making.

There are different strategies that a hospital can adopt in order to create the surgery schedule, although only the following three types are normally considered:

- Open Scheduling Strategy: in this strategy the operating room (OR) planner allocates operating rooms to surgeons in the order of their requests. It is also known as the ”first come first served” strategy.
- Block Scheduling Strategy: in this strategy the number and types of surgical blocks be must defined. Afterwards, you must assign the number of patients who will be operated on each particular block. A block is the amount of time during which a specific sub-specialty is assigned to an OR.
- Modified Block Scheduling Strategy: this strategy is similar to the Block Scheduling Strategy, but leaves some blocks free for greater flexibility in the scheduling.

We are going to focus on the second and third strategy, considering that most hospitals in Chile mainly use those two for their daily operation.

The way that patients and dates are selected for surgery is induced by empirical rules that do not take into account all possible variables and complex interactions involved in this process. Furthermore, the programming process is time consuming due to the lack of a support decision system (Cardoen, Demeulemeester, & Beliën, 2009a). Even worse, this process, like the one discussed in this paper, is completely manual.

With the aim of improving the quality of service delivered, we proposed to develop a scheduling methodology that allows for an optimal allocation of resources, taking into account the restrictions of the system in which it is implemented. This scheduling methodology would also allow the selection of patients using an objective and measurable criteria, a topic that usually takes a backseat in many investigations.

2.2. Literature Review

The OR scheduling problem could be classified as: strategic for the long term, tactical for the medium term, and operational for the short term (May et al., 2011). Most of the literature concentrates on the tactical and operational levels; characterized by considering that the number of operating rooms is fixed.

The problems at the strategic level generally refer to those associated with capacity planning. In the case of ORs, the scheduling problem is usually called the Case Mix Problem (CMP). The CMP refers to the time allocated to each surgical specialty in the OR in order to minimize the total costs or maximize the total revenues. It can also refer to the problem of how the available OR time is divided between the surgeons. For example, in Choi and Wilhelm (2014c) a nonlinear stochastic model is proposed that solves the problem of demand planning using different methodologies.

In the tactical level the problem is usually called the Master Surgery Scheduling Problem (MSSP). In this problem, the OR's time is allocated to the surgical specialties over the scheduling window (typically, a week) in order to maximize resource utilization.

There is abundant literature on the development of methods needed to improve the scheduling of operating rooms. The problem of planning which surgeries are performed on a given day and in a short time horizon, usually a week, is solved in several papers.

They usually develop models of mixed integer programming (MILP), using techniques such as branch and price for its resolution which is a method for solving huge integer linear programming (ILP) and mixed integer linear programming (MILP) problems with many variables. The method is a hybrid of branch and bound, and column generation methods. We can find these methodologies in (Fei, Chu, & Meskens, 2009; Fei, Chu, Meskens, & Artiba, 2008b; Wang, Tang, & Fung, 2014; Molina-Pariente, Fernandez-Viagas, & Framinan, 2015; Meskens, Duvivier, & Hanset, 2012; Lamiri, Xie, & Zhang, 2008; Cardoen, Demeulemeester, & Beliën, 2009b; Cappanera, Visintin, & Banditori, 2014).

Column generation itself is a very popular and efficient algorithm for solving large linear programs, and it has been used in many papers in the area of health care (Brunner & Edenharter, 2011; Range, Lusby, & Larsen, 2014; Lamiri, Xie, & Zhang, 2008; Holte & Mannino, 2013; Fei, Chu, Meskens, & Artiba, 2008a; Augusto, Xie, & Perdomo, 2010).

Some of these models by (Berg, Denton, Ayca Erdogan, Rohleder, & Huschka, 2014; Choi & Wilhelm, 2014a; Lamiri, Grimaud, & Xie, 2009; Lamiri, Xie, Dolgui, & Grimaud, 2008) consider stochasticity in various aspects such as the length of the surgeries or the arrival of patients. It is usually necessary to use other methodologies to solve these models. Stochastic mixed integer programming models have been proposed for the surgery scheduling problem, although they have to make approximations in the length of surgical cases.

They have also applied stochastic dynamic programming methods in order to generate an optimal policy. In this approach various types of models of dynamic programming (DP) aimed at finding long-term policies for scheduling surgeries have been developed (Gerchak, Gupta, & Henig, 1996; Sauré, Patrick, Tyldesley, & Puterman, 2012; Min & Yih, 2010; Patrick, Puterman, & Queyranne, 2008). The aim of most of these models is to minimize the cost of overtime and the cost of conducting (or postponing) surgeries. Consequently, there is a trade-off between the two.

There is also a group that uses genetic algorithms and other heuristics to solve the problem of planning to perform surgeries and work (Roland et al., 2010; Beliën & Demeulemeester, 2007)

Finally, the Surgery Scheduling Problem (SSP) is defined for the operational level. The SSP refers to assigning each surgical case a start time while attempting to find the optimal sequence of surgeries to be performed during a given day, minimizing the waiting time and maximizing resource utilization while taking into account operational restrictions. This problem is addressed in the following work Zhao and Li (2014); Jebali, Alouane, and Ladet (2006); Choi and Wilhelm (2014b). This problem is also addressed by Fei et al. (2009) as a second phase based on the results obtained in the planning phase (first phase).

The objective of this research is to solve the problem of planning when each patient is going to be scheduled on and during which blocks, which can be considered part of the MSSP problem. The idea then is to build models that allow the creation of weekly schedules subject to the corresponding restrictions, with specific operating rooms dedicated to certain specialties, taking into consideration that we must first program the highest priority patients. The daily scheduling problem (SSP) is out of the scope of this work. As this problem is very large, we will use the column generation algorithm in order to solve it.

All the works we reviewed typically use the cost of occupation of the operating rooms. Instead we consider the costs associated with the patients, which are sometimes hard to measure but are very important to provide a better service, which is the main problem in our context.

Also we studied a real case study working with a Chilean hospital. We conducted several interviews with the staff of the hospital to help us better understand the problem and how different it is from other cases already studied.

2.3. Problem formulation

This section seeks to model the problem of planning surgeries within the period. This model is going to determine which day each surgery will be performed on and in which operating room.

2.3.1. Assumptions

In this work we will make the following assumptions. The planning horizon will be one week. Although the duration of surgeries with all of their components are stochastic, we are going to use the expected value, which is data known by the decision making team. According to Choi and Wilhelm (2014a) it is possible to assume that the length of the surgery is normally distributed given the central limit theorem. There has to be at least two doctors in a particular specialty for surgery available. We incorporated the availability of staff from different functions, i.e. not only of surgeon but also of anaesthetists, surgeon's assistant, auxiliary, etc. In one parameter considered in the master problem. We do not include emergency patients, since they are usually operated on immediately within exclusive operating rooms dedicated to emergencies. We also assume that there is a block scheduling strategy where the halls are reserved specific for types of surgeries. Furthermore, it is assumed that the pavilions are not multifunctional; there are specialized operating rooms for certain types of surgeries.

The model presented in this paper is formulated using the DantzigWolf decomposition as a set partitioning problem, which is an algorithm for solving linear programming problems with special structure. The model is also based on the set partitioning formulation of the models formulated by Fei et al. (2008a); Fei, Meskens, and Chu (2010). Unlike these models, we do not assume poly-functional operating rooms. In other words, each operating room is assigned to a certain surgery type during the block that is being scheduled. We also defined different priority groups for the patients that are associated with a maximum recommended waiting period. We take into account the medical criterion and their experience, which was suggested by the hospital staff because the doctors always want to have a certain degree of power in the decision of scheduling a certain patient. All of these factors were included in our objective function. Our focus in this research is to offer the best possible service for the patients, and to make a fair selection of the patients rather than to minimize the operational costs.

Column generation (CG) is used to solve the model, as it is an efficient algorithm for linear problems that are too large to solve otherwise. The idea is that in problems where there is a vast number of variables, it is oftentimes unnecessary to consider them all to find an optimal solution. Only the variables that have the potential to improve the objective function are generated.

In order to check the optimality of a solution, a sub problem called the pricing problem is solved. The original problem is split into two problems: the master problem and the pricing problem, which is a problem for the dual LP. Then that problem is solved to identify columns that may enter the basis. In the case that a new column is found, the master problem has to be re optimized. The linear relaxation problem is solved by a CG approach in which each column represents a plan for one operating room and is generated by solving the auxiliary problem, corresponding to a single operating room planning problem for a determined surgery type. For more information about the method of column generation, and branch and price, see Barnhart, Johnson, Nemhauser, Savelsbergh, and Vance (1998).

2.3.2. Sets

The model will use the following sets:

- d : Days of the week. $\{$1,.., D$\}$
- opr : Operating Rooms. $\{$1,.., OPR$\}$
- st : Types of surgeries. $\{$1,.., ST$\}$
- g : Priority groups of patients. $\{$1,.., G$\}$
- p : Patients. $\{$1,..,P$\}$

- j : Surgery plan, allocation of OR time blocks for each surgeon or specialty. {1,..,J}

2.3.3. Parameters

The following parameters will be used, in both the master problem and the auxiliary problem:

- $OR^d_{opr,st}$: Regular surgical time for the operating room opr during the day d for patients of type st
- OO^d_{opr}: Maximum surgical overtime for the operating room opr during the day d
- $AS_{st,d}$: Available surgeon hours for patients who require surgery type st during the day d
- WT_p: Time a patient p has been on the waiting list
- $MRWT_g$: The maximum recommended waiting time for patients who belong to the group g
- MC_p: Parameter that indicates the medical criterion on a determined surgery according to a multitude of factors that doctors determine through their experience and knowledge
- SD_p: Expected surgery duration for patient p, including preparation time, anesthesia and cleaning
- $CPS_{st,opr}$: 1 if operating room opr is available for patients that require surgery type st; 0 otherwise
- $PT_{p,st}$: 1 if patient p requires surgery type st; 0 otherwise
- $PG_{p,g}$: 1 if patient p belongs to the group g; 0 otherwise
- c_j : Total cost of assigning plan j
- α_g : Penalty for operating patients in the priority group g after the recommended maximum waiting time
- β_g : Penalty for postponing the surgery for a patient p in group g

- γ : Weight given to the medical criteria in the calculation of the score
- $u_{p,j}$: 1 if a patient p is assigned to the plan j; 0 otherwise
- v_j^d : 1 if the plan j is assigned on day d; 0 otherwise
- $w_{opr,j}$: 1 if plan j is assigned to the operating room opr; 0 otherwise

2.3.4. Master problem

In the master problem, each column corresponds to a feasible plan for an operating room on a given day, and each operating room can be used by, at most, one feasible plan in a given day. The objective function minimizes the total cost of the plans that will be performed during a given week. The variable x_j is the binary variable associated with the use of the plan j. The goal is to minimize the total cost associated with the selected plans.

$$\min \sum_j (c_j \cdot x_j) \tag{2.1}$$

$$\text{s.t.} \sum_j u_{p,j} \cdot x_j \leq 1, \forall p \tag{2.2}$$

$$\sum_j v_j^d \cdot w_{opr,j} \cdot x_j \leq 1, \forall opr, d \tag{2.3}$$

$$\sum_j v_j^d \cdot (\sum_p u_{p,j} \cdot SD_p) \cdot x_j \leq AS_{st}^d, \forall st, d \tag{2.4}$$

$$x_j \in \{0, 1\}, \forall j \tag{2.5}$$

The restriction 2.2 ensures that every patient is scheduled at most one time. The restriction 2.3 ensures that each operating room can be occupied by at most one accepted feasible plan in one given day. Restriction 2.4 prevents the assignment of more surgical time than is available for surgeons specialized in patients of type st.

2.3.5. Costs

The total cost associated with a determined plan includes costs from three sources. First, the penalty cost if a patient is scheduled beyond the maximum waiting time recommended for a patient of the group g. Second, there is the cost associated with the postponement of a patient who belongs to the group g. Lastly, there is the cost (benefit) of programming a patient associated with certain medical criterion.

First, we define $\kappa_{p,g}$ as $\kappa_{p,g} = \alpha_g \cdot \max(0, WT_p - MRWT_g)$ which is the penalty for each week a patient is scheduled beyond the maximum recommended waiting time. The total expected penalty cost if a patient is scheduled beyond the maximum recommended waiting time is defined as:

$$CWT_j = \sum_p \sum_g \kappa_{p,g} \cdot PG_{p,g} \cdot u_{p,j}$$

The cost for not scheduling a patient of the group g in the plan j is defined as:

$$CNS_j = \sum_g \beta_g \cdot \sum_p (1 - u_{p,j}) \cdot PG_{p,g}$$

The cost (or benefit) associated with the medical criteria for a specific patient, which is the priority given by the doctor with regard to the condition of the patient including a wide range of factors such as: age, weight, disease, etc, is defined as:

$$CMD_j = \sum_p \sum_p MC_p \cdot \gamma \cdot u_{p,j}$$

Now, we can define the total cost of scheduling plan j as follows:

$$c_j = CWT_j + CNS_j + CMD_j \tag{2.6}$$

Since these costs are difficult to quantify, certain assumptions must be made based on expert opinion and empirical experiments in order to correctly calibrate the model. The recommended waiting times are based on defined guidelines regarding reasonable waiting times.

2.3.6. Auxiliary problem

In order to define the objective function of the auxiliary problem, we must first define the reduced cost. Since the auxiliary problem must generate the columns of the master problem which correspond to feasible plans from solving this auxiliary problem, if such columns are found the master problem is re optimized. In the auxiliary problem the binary parameters of the master problem: $u_{p,j}$, v_j^d, $w_{opr,j}$ are variables that have to be determined from solving the problem. The reduced cost for the column A_j is defined as:

$$\sigma_j = c_j - \sum_p \pi_p \cdot u_{p,j} - \sum_d \sum_{opr} v_j^d \cdot w_{opr,j} \cdot \pi_{opr,d} - \sum_d \sum_{st} v_j^d \cdot (\sum_p u_{p,j} \cdot SD_p) \cdot \pi_{st,d} \tag{2.7}$$

Where π_p, $\pi_{opr,d}$ and $\pi_{st,d}$ represent the dual variables corresponding to the constraints of the master problem. We want to minimize the reduced cost to decide which column we enter into the master problem. It is also required that all feasible plans must satisfy the constraints of the problem, thus the auxiliary problem is

$$\min(\sigma_j) \tag{2.8}$$

$$\text{s.t.} \sum_{st} \sum_{p} u_{p,j} \cdot SD_p \cdot PT_{p,st} \leq \sum_{st} \sum_{d} \sum_{opr} v_j^d \cdot (OR_{opr,st}^d + OO_{opr}^d) \cdot w_{opr,j}, \forall j \quad (2.9)$$

$$\sum_{st} \sum_{d} \sum_{opr} v_j^d \cdot w_{opr,j} \cdot CPS_{opr,st} = 1, \forall j \quad (2.10)$$

The restriction 2.9 ensures that each surgical plan does not exceed the maximum time available for a certain type of patient. The restriction 2.10 ensures that each plan corresponds to a single operating room assigned to a specific type of surgery in one day.

2.3.7. The algorithm

The Dantzig-Wolfe decomposition can be described as follows:

(i) We start with a feasible solution to the reduced master problem, then formulate the objective function for each sub problem, and each sub problem will give solutions to improve the current value of the objective function

(ii) The sub problems are then re-solved for their new objective functions. Then each of their solutions are given to the master problem

(iii) The master problem incorporates the new columns generated by the sub problems, thus improving the value of the objective function

(iv) The program performs a certain number of iterations until the maximum number of iterations defined is reached or it cannot improve the current solution

(v) If the value of the objective function is improved, we should go to step (i), otherwise, if the master problem cannot be further improved, it finishes

2.3.8. Building initial feasible solution

It is necessary to obtain a basic feasible solution to determine the parameters of the objective function of the auxiliary problem. The better the initial solution, the less time it is going to take to find a solution. In order to achieve that, we describe the steps required.

(i) Patients must be ordered according to the priority group in which they belong, placing the highest priority patients at the top. If they are of equal priority, they are ordered by their waiting period, which is taken from the patient waiting list in descending order.

(ii) Each surgical case is assigned to any operating room with sufficient time available and appropriate equipment required for the specific surgery type.

(iii) If we cannot assign the patient due to a lack of open time available, a dummy plan is created for each case. Having all the surgeries assigned means that we have generated an initial feasible solution.

2.4. Results

In order to evaluate the methodology proposed in this research we used data from an important public hospital in Chile, which is described next.

2.4.1. Case study

This model was applied to the Hospital del Salvador. This is a public hospital located in Santiago, Chile, and is part of the service network of the East Metropolitan Health Service (SSMO). The SSMO serves approximately 1,182,923 people. This population grew by 10.1 % between 2008 and 2013. The communities served by the Hospital contain a large geriatric population: 17.4 % are more than 60 years old according to the public records of the Hospital del Salvador (2015). Their mission as a public health center is to provide health care in a timely, decisive and efficient manner. They also strive for, continuous improvement, especially concerning high complexity pathologies, and mainly serve an adult population.

The hours in which surgeries are performed are 8:00 to 14:30 for short blocks and 8:00 to 19:00 for long blocks, as defined by the hospital itself. They use a block scheduling strategy, which means that each block corresponds to just one type of surgery in a day.

The data obtained from the hospital shows that an average of 17% of the trauma surgeries are canceled, with a coefficient of variation of 29%. An average of 15% of general surgeries are canceled with a coefficient of variation of 40%, and finally, an average of 14% of the urology surgeries are canceled, with a coefficient of variation of 35%. We noted that there is a high percentage of surgeries that are canceled, but also the deviation is usually high.

2.4.2. Implementation

The proposed method was run using a program developed in Python to generate the input and create the initial feasible solutions. The NEOS Server was used in conjunction with IBM ILOG CPLEX Optimizer, implementing the master and the auxiliary problems in GAMS which is a language of mathematical modeling and high-level optimization. This language is specialized in large problems. For more details on the implementation of column generation in GAMS, see (Kalvelagen, 2003).

2.4.3. Indicators

To evaluate the model results we used the following indicators:

- Utilization Rate (UR): ratio of the number of open hours that are occupied by patients in the operating rooms with at least one surgical case assigned to their regular open hours in one week.

- Percentage of Scheduled Patients (PSP): ratio of the number of patients scheduled or assigned to the involved week to the number of patients awaiting the assignments in one week.
- Idle time (IT): total number of idle hours during the regular open period, for a determined time horizon.
- Percentage of Priority Patients Scheduled (PPPS): number of priority patients scheduled for one week divided by those on the waiting list.

2.4.4. Experimental results

We defined three groups of patients according to their different priorities (urgent, semi urgent and non urgent). We defined different surgery types as well. We used the same surgery types used by the hospital (urology, trauma, general surgery). For the surgery duration we assumed that the elective surgery duration for each patient follows log normal distribution according to Hancock, Walter, More, and Glick (1988). The data generated is between intervals [2, 5] expressed in integer minutes. The opening time for each OR depends on which type of block is assigned. We defined short blocks of 6 hours and long blocks of 10 hours, assigning each OR randomly to one of those blocks.

In this section we will present results concerning the performance of two different methods. The first one is based on the current method used by the hospital. We created an automated program that uses the same ideas applied by the decision makers when they built the weekly operating table, using real data provided by the hospital, which is fairly similar to how we build the initial feasible solution. We compared its performance with the mathematical model developed in this paper.

We provide the average number of scheduled patients for each method, the average utilization rate, Percentage of Priority Patients Scheduled and Percentage of Scheduled Patients and finally the average idle time, which correspond to the indicators defined previously. Some of these indicators are already being used by the hospital, but most of them were defined here due to the lack of appropriate indicators to measure the performance

Table 2.1. Comparison of current method and the model developed

Methodology	Scheduled Patients	UROR	PPPS	IT
	Scenario 1			
Current method	79	81.8%	34.1%	72
Mathematical model	93	92.9%	51.8%	28
	Scenario 2			
Current method	128	82.5%	31.5%	105.6
Mathematical model	154	93.2%	59.1%	45.6

of the method. Our results were obtained from solving both methods and then we will compare their performance.

In the table 2.1 we show the results for two scenarios. The first scenario reflects the current situation in the hospital where they have 10 ORs and a long waiting list which makes it impossible to meet given the current capacity. The idea is to study the system in regime, which means that they cannot schedule all the patients that are waiting. In this case we used a waiting list of 200 patients.

In a second hypothetical scenario, we considered 15 ORs and a waiting list of 400 patients. In this scenario we wanted to see how the model would behave with a bigger problem. It is possible to observe that as the number of ORs (problem size) increases, the number of scheduled patients grows. But, more interestingly the percentage of priority patients scheduled also increases, probably due to a larger number of options to choose from while the other indicators remained almost constant.

As we can see, the mathematical model proposed in this paper performs better than the current methodology. It is able to schedule more patients and, substantially enhance the utilization rate of the operating rooms. With this model, a greater proportion of the priority patients were scheduled and less available operating room time was wasted, which is crucial considering the extremely long waiting lists presented by the hospital.

The hospital was very interested in knowing whether they had sufficient capacity to operate on all the patients. In order to do that, we simulated different availability of operating room scenarios and we obtained the amount of patients that were possible to schedule for surgery for a given week. We can see the results in the figure 2.1.

Obviously, the increase in the number of operating rooms would allow the decision makers to schedule more surgeries. The question is, how many more? Figure 2.1 shows that the results are almost linear, assuming that we already have sufficient of the other resources involved.

The model took just a few minutes to solve (all the jobs were run with the GAMS default of 1000 seconds). We used a program coded in Python 3 in order to process the input. On the other hand, the mathematical model was coded in GAMS and using CPLEX to solve it, the proposed method ran on a Lenovo IdeaPad Z400 (Intel Core i5 Dual CPU: 2.6 GHz, RAM: 6 GB DDR3, Operating System: Windows 10). This is essential in the daily operation of the hospital. Changes in the schedule can occur and the model should obtain results in a reasonable time.

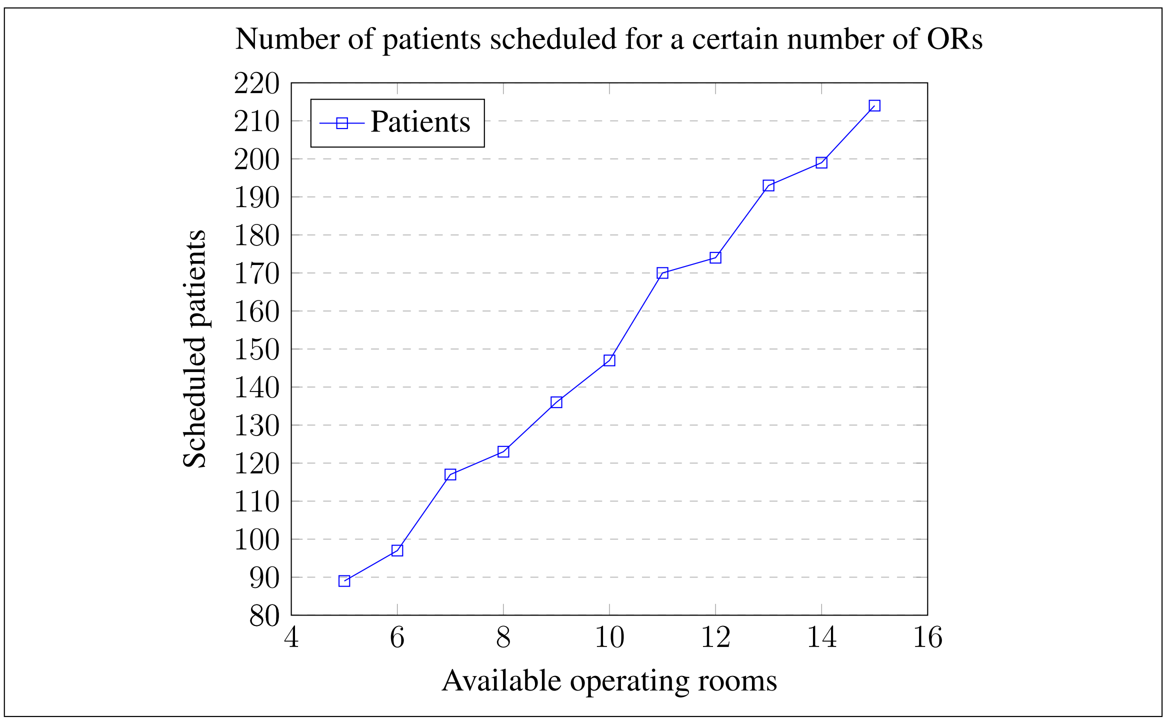

Figure 2.1. Simulation with different number of ORs.

2.5. Conclusions

In this paper, we developed and solved a weekly surgery planning problem. We used a set-partitioning model and solved it using a column generation based algorithm, so that each surgical case can be assigned to a surgery date.

We did not assume polyfunctional operating rooms, unlike what most of the literature has done, because the majority of the hospitals reserve and specialize ORs for a certain type of surgery. Furthermore, our idea is to make sure that the model would represent the reality in the most accurate way possible. In addition, we used a different approach for the cost defined, focusing on minimizing the cost of scheduling after the maximum recommended time, the cost for postponing, and the medical criteria which was highly important for the personnel responsible for the process of decision making.

The performance of the model was tested by solving randomly generated scenarios. We compared the solution quality and computation time of different scenarios with a real hospital in Chile. The number of patients and the number of operating rooms were also tested in our experiment. We solved this model in a reasonable time, so it could be perfectly used by the decision makers on a weekly basis, considering that they have to decide which surgeries to schedule every Monday.

Surgical scheduling is a complex task since, the surgical table often significantly deviates from what really happens during a surgical day. In such an environment, a surgical planning is a guide for the operational management rather than a precise timetable for the surgeries that are actually performed. If we provide the decision makers with better methodologies to solve this problem or if the hospital defines better policies, it will be possible to use resources effectively and efficiently, thus improving the planning of surgeries, minimizing costs and offering people a better quality of service.

Additionally, this paper is focused on the tactical level for surgical planning. The operational level can be further studied taking into account the inherent variability in the surgical context. Other possible further studies include the sequencing of patients in each OR and how to address the deviations that occur when a surgery takes longer than planned or an emergency arrives.

Acknowledgements

This research was funded by The Fund for the Promotion of Scientific and Technological Development (FONDEF) Grant: CA13I10319. We are also grateful to the staff from the Hospital del Salvador for their feedback and the data they provided.

REFERENCES

Augusto, V., Xie, X., & Perdomo, V. (2010, March). Operating theatre scheduling with patient recovery in both operating rooms and recovery beds. *Computers & Industrial Engineering*, *58*(2), 231–238. Retrieved from http://linkinghub.elsevier.com/retrieve/pii/S0360835209001193 doi: 10.1016/j.cie.2009.04.019

Barnhart, C., Johnson, E. L., Nemhauser, G. L., Savelsbergh, M. W. P., & Vance, P. H. (1998). Branch-and-Price: Column Generation for Solving Huge Integer Programs. *Operations Research*, *46*(3), 316–329. doi: 10.1287/opre.46.3.316

Beliën, J., & Demeulemeester, E. (2007, January). Building cyclic master surgery schedules with leveled resulting bed occupancy. *European Journal of Operational Research*, *176*(2), 1185–1204. Retrieved from http://linkinghub.elsevier.com/retrieve/pii/S0377221705006946 doi: 10.1016/j.ejor.2005.06.063

Berg, B. P., Denton, B. T., Ayca Erdogan, S., Rohleder, T., & Huschka, T. (2014, October). Optimal booking and scheduling in outpatient procedure centers. *Computers & Operations Research*, *50*, 24–37. Retrieved from http://linkinghub.elsevier.com/retrieve/pii/S0305054814000987 doi: 10.1016/j.cor.2014.04.007

Brunner, J. O., & Edenharter, G. M. (2011, July). Long term staff scheduling of physicians with different experience levels in hospitals using column generation. *Health care management science*, *14*(2), 189–202. Retrieved from http://www.ncbi.nlm.nih.gov/pubmed/21465250 doi: 10.1007/s10729-011-9155-x

Cappanera, P., Visintin, F., & Banditori, C. (2014, August). Comparing resource balancing criteria in master surgical scheduling: A combined optimisation-simulation approach. *International Journal of Production Economics*. Retrieved from http://linkinghub.elsevier.com/retrieve/

pii/S0925527314002606 doi: 10.1016/j.ijpe.2014.08.002

Cardoen, B., Demeulemeester, E., & Beliën, J. (2009a, June). Optimizing a multiple objective surgical case sequencing problem. *International Journal of Production Economics*, *119*(2), 354–366. Retrieved from http://linkinghub.elsevier.com/retrieve/pii/S0925527309000887 doi: 10.1016/j.ijpe.2009.03.009

Cardoen, B., Demeulemeester, E., & Beliën, J. (2009b, September). Sequencing surgical cases in a day-care environment: An exact branch-and-price approach. *Computers & Operations Research*, *36*(9), 2660–2669. Retrieved from http://linkinghub.elsevier.com/retrieve/pii/S0305054808002384 doi: 10.1016/j.cor.2008.11.012

Choi, S., & Wilhelm, W. E. (2014a). An approach to optimize block surgical schedules. *European Journal of Operational Research*, *235*(1), 138–148. Retrieved from http://dx.doi.org/10.1016/j.ejor.2013.10.040 doi: 10.1016/j.ejor.2013.10.040

Choi, S., & Wilhelm, W. E. (2014b). An approach to optimize block surgical schedules. *European Journal of Operational Research*, *235*(1), 138 - 148. Retrieved from http://www.sciencedirect.com/science/article/pii/S0377221713008631 doi: http://dx.doi.org/10.1016/j.ejor.2013.10.040

Choi, S., & Wilhelm, W. E. (2014c). On capacity allocation for operating rooms. *Computers & Operations Research*, *44*, 174–184. Retrieved from http://linkinghub.elsevier.com/retrieve/pii/S0305054813003201 doi: 10.1016/j.cor.2013.11.007

Chow, V. S., Puterman, M. L., Salehirad, N., Huang, W., & Atkins, D. (2011). Reducing Surgical Ward Congestion Through Improved Surgical Scheduling and Uncapacitated Simulation. *Production and Operations Management*, *20*(3), 418–430.

Fei, H., Chu, C., & Meskens, N. (2009). Solving a tactical operating room planning problem by a column-generation-based heuristic procedure with four criteria. *Annals of Operations Research*, *166*(1), 91–108. doi: 10.1007/s10479-008-0413-3

Fei, H., Chu, C., Meskens, N., & Artiba, a. (2008a). Solving surgical cases assignment problem by a branch-and-price approach. *International Journal of Production Economics*, *112*(1), 96–108. doi: 10.1016/j.ijpe.2006.08.030

Fei, H., Chu, C., Meskens, N., & Artiba, A. (2008b, March). Solving surgical cases assignment problem by a branch-and-price approach. *International Journal of Production Economics*, *112*(1), 96–108. Retrieved from http://www.sciencedirect.com/science/article/pii/S0925527307001296 doi: 10.1016/j.ijpe.2006.08.030

Fei, H., Meskens, N., & Chu, C. (2010, March). A planning and scheduling problem for an operating theatre using an open scheduling strategy. *Computers & Industrial Engineering*, *58*(2), 221–230. Retrieved from http://linkinghub.elsevier.com/retrieve/pii/S0360835209000801 doi: 10.1016/j.cie.2009.02.012

Gerchak, Y., Gupta, D., & Henig, M. (1996). *Reservation Planning for Elective Surgery Under Uncertain Demand for Emergency Surgery* (Vol. 42) (No. 3). doi: 10.1287/mnsc.42.3.321

Gordon, T., Paul, S., Lyles, A., & Fountain, J. (1988). Surgical unit time utilization review: Resource utilization and management implications. *Journal of Medical Systems*, *12*(3), 169–179. doi: 10.1007/BF00996639

Hancock, W. M., Walter, P. F., More, R. A., & Glick, N. D. (1988). Operating room scheduling data base analysis for scheduling. *Journal of Medical Systems*, *12*(6), 397–409. doi: 10.1007/BF00992688

Holte, M., & Mannino, C. (2013). The implementor/adversary algorithm for the cyclic and robust scheduling problem in health-care. *European Journal of Operational Research*, *226*(3), 551–559. Retrieved from http://dx.doi.org/10.1016/j.ejor.2012.10.029 doi: 10.1016/j.ejor.2012.10.029

Jebali, A., Alouane, A. B. H., & Ladet, P. (2006). Operating rooms scheduling. *International Journal of Production Economics*, *99*(12), 52 - 62. Retrieved from http://www.sciencedirect.com/science/

article/pii/S0925527304004517 (Control and Management of Productive SystemsControl and Management of Productive Systems) doi: http://dx.doi.org/10.1016/j.ijpe.2004.12.006

Kalvelagen, E. (2003). Column generation with GAMS. *GAMS Development Corp., Washington ...*, 1–9. Retrieved from http://www.amsterdamoptimization.com/pdf/colgen.pdf

Lamiri, M., Grimaud, F., & Xie, X. (2009). Optimization methods for a stochastic surgery planning problem. *International Journal of Production Economics*, *120*(2), 400–410. doi: 10.1016/j.ijpe.2008.11.021

Lamiri, M., Xie, X., Dolgui, A., & Grimaud, F. (2008). A stochastic model for operating room planning with elective and emergency demand for surgery. *European Journal of Operational Research*, *185*(3), 1026–1037. doi: 10.1016/j.ejor.2006.02.057

Lamiri, M., Xie, X., & Zhang, S. (2008). Column generation approach to operating theater planning with elective and emergency patients. *IIE Transactions*, *40*(9), 838–852. doi: 10.1080/07408170802165831

May, J. H., Spangler, W. E., Strum, D. P., & Vargas, L. G. (2011, May). The Surgical Scheduling Problem: Current Research and Future Opportunities. *Production and Operations Management*, *20*(3), 392–405. Retrieved from http://doi.wiley.com/10.1111/j.1937-5956.2011.01221.x doi: 10.1111/j.1937-5956.2011.01221.x

McManus, M. L., Long, M. C., Cooper, A., Mandell, J., Berwick, D. M., Pagano, M., & Litvak, E. (2003). Variability in surgical caseload and access to intensive care services. *Anesthesiology*, *98*(6), 1491–1496. doi: 10.1097/00000542-200306000-00029

Meskens, N., Duvivier, D., & Hanset, A. (2012, October). Multi-objective operating room scheduling considering desiderata of the surgical team. *Decision Support Systems*, *To appear*. Retrieved from http://linkinghub.elsevier.com/retrieve/pii/S0167923612002746 doi: 10.1016/j.dss.2012.10.019

Min, D., & Yih, Y. (2010). An elective surgery scheduling problem considering patient priority. *Computers and Operations Research*, *37*(6), 1091–1099. Retrieved from http://dx.doi.org/10.1016/j.cor.2009.09.016 doi: 10.1016/j.cor.2009.09.016

Molina-Pariente, J., Fernandez-Viagas, V., & Framinan, J. (2015, January). Integrated Operating Room Planning and Scheduling Problem with Assistant Surgeon Dependent Surgery Durations. *Computers & Industrial Engineering*. Retrieved from http://linkinghub.elsevier.com/retrieve/pii/S0360835215000157 doi: 10.1016/j.cie.2015.01.006

Patrick, J., Puterman, M. L., & Queyranne, M. (2008). Dynamic Multipriority Patient Scheduling for a Diagnostic Resource. *Operations Research*, *56*(6), 1507–1525. doi: 10.1287/opre.1080.0590

Range, T. M., Lusby, R. M., & Larsen, J. (2014). A column generation approach for solving the patient admission scheduling problem. *European Journal of Operational Research*, *235*(1), 252–264. Retrieved from http://dx.doi.org/10.1016/j.ejor.2013.10.050 doi: 10.1016/j.ejor.2013.10.050

Roland, B., Di Martinelly, C., Riane, F., & Pochet, Y. (2010, March). Scheduling an operating theatre under human resource constraints. *Computers & Industrial Engineering*, *58*(2), 212–220. Retrieved from http://linkinghub.elsevier.com/retrieve/pii/S0360835209000084 doi: 10.1016/j.cie.2009.01.005

Sauré, A., Patrick, J., Tyldesley, S., & Puterman, M. L. (2012). Dynamic multi-appointment patient scheduling for radiation therapy. *European Journal of Operational Research*, *223*(2), 573–584. doi: 10.1016/j.ejor.2012.06.046

Wang, Y., Tang, J., & Fung, R. Y. K. (2014). A column-generation-based heuristic algorithm for solving operating theatre planning problem under stochastic demand and surgery cancellation risk. *International Journal of Production Economics*, *158*, 28–36. Retrieved from http://dx.doi.org/10.1016/j.ijpe.2014.07.015 doi: 10.1016/j.ijpe.2014.07.015

Wickizer, T. M. (1991). Effect of hospital utilization review on medical expenditures in selected diagnostic areas: An exploratory study. *American Journal of Public*

Health, *81*(4), 482–484. doi: 10.2105/AJPH.81.4.482

Zhao, Z., & Li, X. (2014). Scheduling elective surgeries with sequence-dependent setup times to multiple operating rooms using constraint programming. *Operations Research for Health Care*, *3*(3), 160 - 167. Retrieved from http://www.sciencedirect.com/science/article/pii/S2211692314000320 doi: http://dx.doi.org/10.1016/j.orhc.2014.05.003

APPENDIX

A. FIGURES

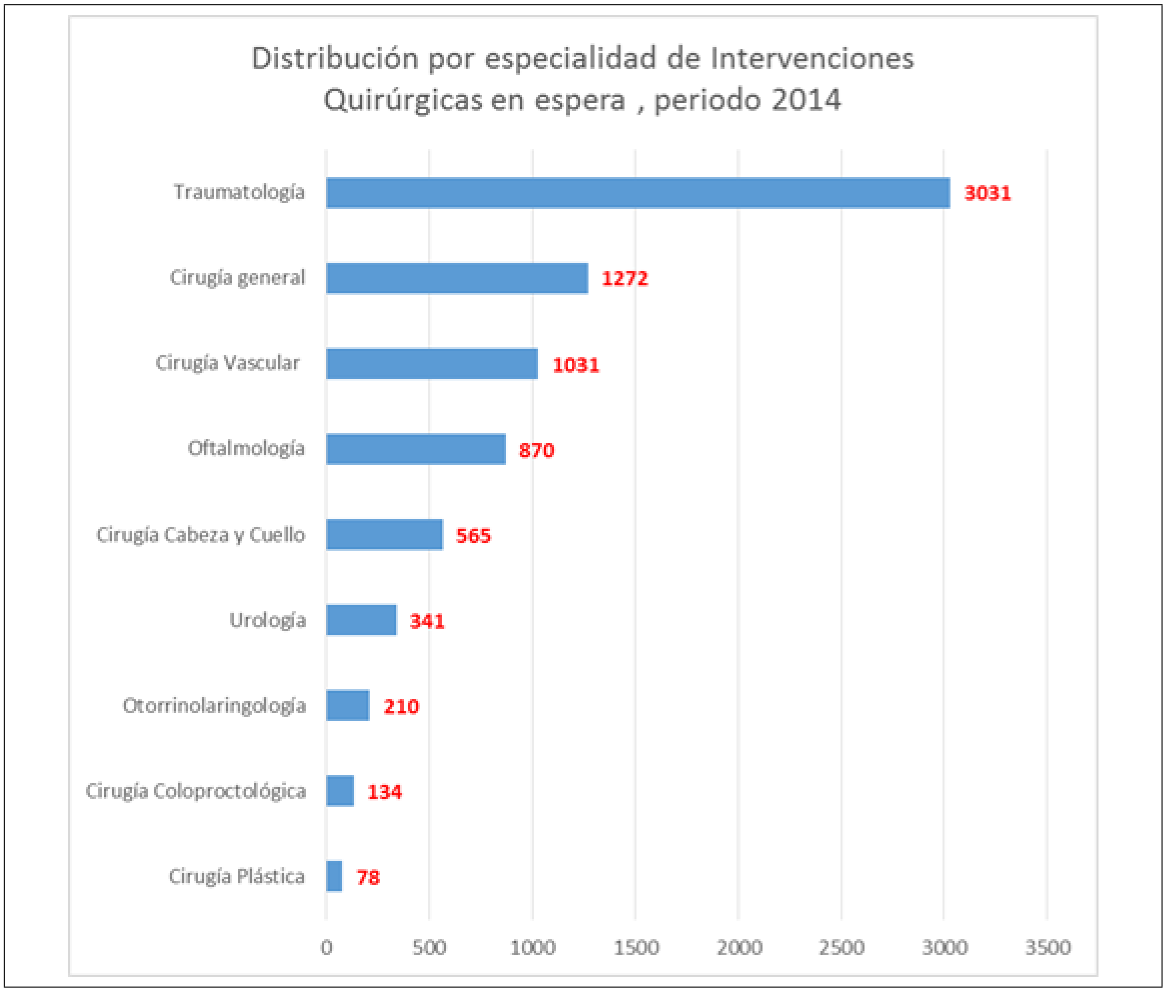

Figure A.1. Surgery waiting specialty distribution.