Cooperative effects in a one-photon micromaser with atomic polarization

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(Received 15 February 1995; revised manuscript received 19 July 1996)

We study the cooperative effects in a one-photon micromaser with atomic polarization. We find that the injected polarization makes the micromaser more insensitive to cooperative effects as compared to the unpolarized case. Although probability diffusion in phase space takes place and spikes develop in the average photon number, the squeezing properties of the cotangent states are more affected by the cavity losses than by the presence of two atoms in the micromaser cavity. [S1050-2947(97)10002-6]

PACS number(s): 84.40.Ik

Cooperative effects have been shown to be important in micromasers. It was found that the effects of having two simultaneously and fully inverted atoms in the one-photon micromaser produces a strong disruption in the trapping states, for a low photon number [1,2]. On the other hand, it is known that when two-level atoms are injected into a micromaser in a coherent superposition of their levels, the field inside may evolve into a pure state, if the atom-field interaction time obeys a trapping condition [3,4]. These states, called *cotangent states,* were shown to exhibit squeezing on one of their quadratures for a wide range of parameters [5]. In the present communication we will deal with the problem of how the above-mentioned cotangent states are affected by cooperativity as well as cavity losses.

We consider the single-mode cavity into which a monoenergetic beam of two-level atoms is injected in a coherent superposition of its levels. The interaction of the atom-field system will be described by the Jaynes-Cummings Hamiltonian, which in the interaction picture and in the dipole and rotating-wave approximations is given by

$$V=\hbar g(a\sigma_{+}+a^{\dagger}\sigma_{-}),\tag{1}$$

where g is the coupling constant between the atom and field, σ_{-} (σ_{+}) is the lowering (raising) operator for the atom, and a ($a^{\dagger}$) is the annihilation (creation) operator for the field mode. We assume an exact resonance between the atomic transition and field frequencies.

Slosser and Meystre [4] showed that the field pumped with a monokinetic atomic beam will evolve into a cotangent state if the following conditions are full-filled: (a) the atomic flux is low enough that there is at most one atom at a time inside the cavity; (b) the atoms are injected into the cavity in a coherent superposition of their two levels, i.e., the initial state of every atom is described by the state vector

$$|\psi_a\rangle=c_g|g\rangle+c_e|e\rangle,\tag{2}$$

where $|g\rangle$ and $|e\rangle$ represent the ground and excited states of the atom, respectively, with $|c_g|^2+|c_e|^2=1$; (c) the decay time of the cavity is large enough compared to the atom-field interaction time, so that cavity losses may be neglected; (d) the interaction time τ obeys the trapping condition

$$g\tau\sqrt{N_u+1}=p\pi \quad \text{with } p,N_u \text{ integers;}\tag{3}$$

and (e) the initial field is such that the probabilities for $n>N_u$ are zero. If the above conditions are satisfied, then, in steady state, the nonvanishing field density-matrix elements will lie within a block in the Fock space bound by a maximum photon number N_u.

In a more realistic model of micromaser, the Poissonian injection of atoms must be considered. Then, if $t=t_i$ and t_{i+1} are the instants at which the ith and $(i+1)$th atoms enter the cavity, respectively, the probability distribution for the intervals $\Delta t_i\equiv t_{i+1}-t_i$ will obey

$$P(\Delta t_i)=\frac{e^{-\Delta t_i/\Delta t}}{\Delta t},\tag{4}$$

where Δt is the average time between consecutive atoms.

To simulate the operation of a micromaser, we assume that the intervals between atoms obey Eq. (4) and we consider events up to two atoms inside the cavity. Atomic fluxes lower than 500 atoms/s have been reached in experiments; interaction times are of the order of microseconds, giving an average close to 10^{-3} atoms simultaneously inside the cavity, so that the above assumption is quite reasonable [6].

In practice, the simulation is composed of the three basic events shown in Fig. 1. If $\Delta t_{i-1},\Delta t_i\geqslant\tau$ [Fig. 1(a)], the state of the field at the instant just after the ith atom has left the cavity will be described by

$$\rho_f(t_i+\tau)=\mathrm{Tr}_{\mathrm{at}}\, U_I(\tau)\rho_{af}(t_i)U_I^{\dagger}(\tau),\tag{5}$$

where U_I is the well-known evolution operator of the Jaynes-Cummings model and $\rho_{af}(t)=\rho_a\otimes\rho_f(t_i)$, where $\rho_a=|\psi_a\rangle\langle\psi_a|$, with $|\psi_a\rangle$ given by Eq. (2), represents the state of the atom-field system at the instant that the atom enters the cavity. When either Δt_i or Δt_{i-1} is smaller than τ, a two-atom event will occur. Let us suppose, without loss of generality, that $\Delta t_{i-1}\geqslant\tau$ and $\Delta t_i<\tau$, with $\Delta t_i+\Delta t_{i+1}>\tau$. During the interval $\Delta\tau_i=\tau-\Delta t_i$ the ith and $(i+1)$th atoms will interact simultaneously with the field. The state of the whole system at the instant that the ith atom leaves will be given by

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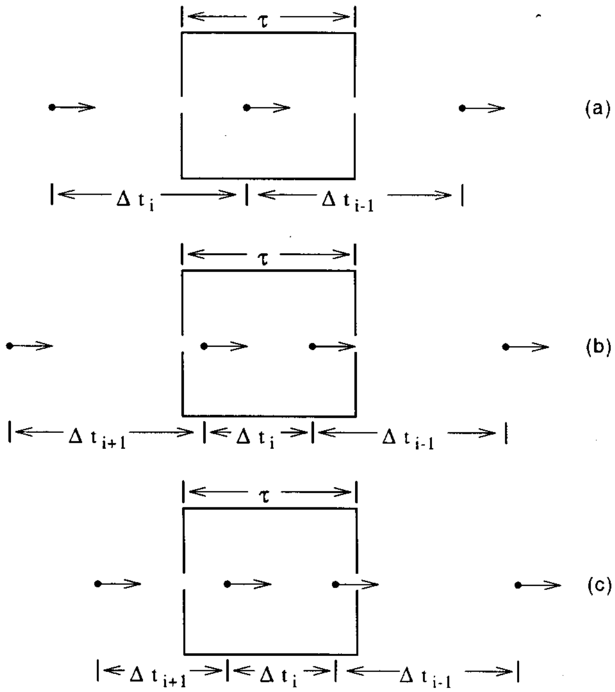


FIG. 1. Poissonian injection of atoms. (a) $\Delta t_i, \Delta t_{i-1} > \tau$ and we have either zero or one atom inside the cavity. (b) $\Delta t_i < \tau$ and $\Delta t_{i+1}, \Delta t_{i-1} > \tau$. Two atom events take place. (c) $\Delta t_i, \Delta t_{i+1} < \tau$ and $\Delta t_{i-1} > \tau$. Two-atom events occur twice.

$$\rho(t_i+\tau) = U_{II}(\Delta\tau_i)\rho_a \otimes [U_I(\Delta t_i)\rho_{af}(t_i)U_I^\dagger(\Delta t_i)]U_{II}^\dagger(\Delta\tau_i), \tag{6}$$

where U_{II} is the time evolution operator for the two-atom system interacting with the cavity field [1]. Now two possibilities arise: the $(i+1)$th atom may leave either before or after a new atom enters the cavity. In the first case [Fig. 1(b)], just after that atom has left, the state of the field will be

$$\rho_f(t_{i+1}+\tau) = \mathrm{Tr}_{\mathrm{at}(i+1)}\{U_I(\Delta t_i)\mathrm{Tr}_{\mathrm{at}(i)}[\rho(t_i+\tau)]U_I^\dagger(\Delta t_i)\}, \tag{7}$$

with $\rho(t_i+\tau)$ given in Eq. (6). But if a new atom enters at $t=t_{i+2}$, with $\Delta t_{i+1} < \tau$ [Fig. 1(c)], a two-atom event will again occur. The state of the whole system at that instant will be

$$\rho(t_{i+2}) = \rho_a \otimes U_I(\Delta t_i + \Delta t_{i+1} - \tau)\mathrm{Tr}_{\mathrm{at}(i)}[\rho(t_i+\tau)] \times U_I^\dagger(\Delta t_i + \Delta t_{i+1} - \tau), \tag{8}$$

and just after this last atom has left the resonator, the field inside will be described by

$$\rho_f(t_{i+2}+\tau) = \mathrm{Tr}_{\mathrm{at}(i+2)}\{U_I(\Delta t_{i+1})\mathrm{Tr}_{\mathrm{at}(i+1)} \times [U_{II}(\Delta\tau_{i+1})\rho(t_{i+2}) \times U_{II}^\dagger(\Delta\tau_{i+1})]U_I^\dagger(\Delta t_{i+1})\}. \tag{9}$$

Here we also include cavity losses by assuming that the relaxation time of the field γ^{-1} is much larger than the flight time of an atom, so that they are significant only while there are no atoms inside the resonator. In those intervals the field

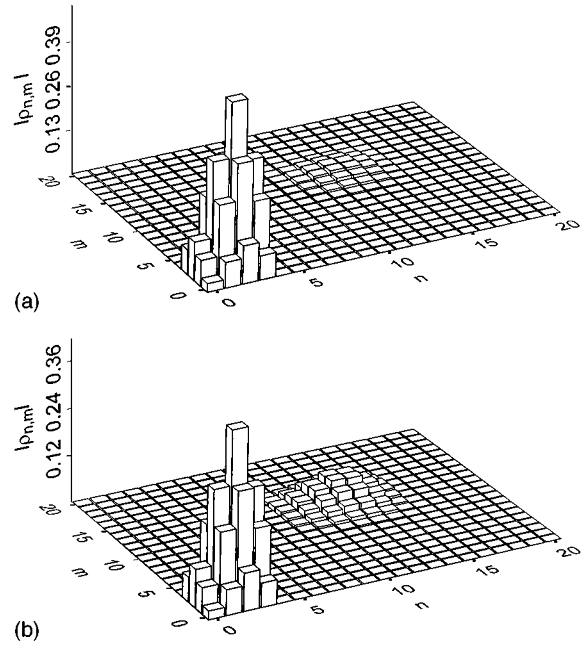


FIG. 2. Absolute value of the reduced field density matrix for an atomic flux of 1500 atoms/s, $N_u = 3$, and $|c_e|^2 = 0.9$ after (a) 800 and (b) 2500 atoms.

will decay according to the master equation of a damped harmonic oscillator, which, in the absence of thermal photons, is

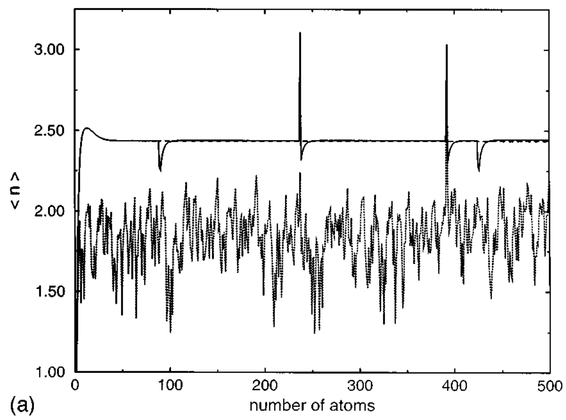
$$\dot{\rho}_{n,m} = -(\gamma/2)[(n+m)\rho_{n,m} - 2\sqrt{(n+1)(m+1)}\rho_{n+1,m+1}]. \tag{10}$$

It is not difficult to show that the solution of Eq. (10) is

$$\rho_{n,m}(t) = e^{-\gamma(n+m)t/2}\rho_{n,m}(0) + \sum_{j=1}^{\infty}\prod_{k=1}^{j}\sqrt{(n+k)(m+k)}e^{-\gamma(n+m)t/2} \times \frac{(1-e^{-\gamma t})^j}{j!}\rho_{n+j,m+j}(0). \tag{11}$$

We describe some numerical results. We took for all cases $g = 10^5$ [7]. In order to show the probability diffusion due to cooperative effects we have neglected losses and chosen a small N_u and a relatively large population in the atomic upper state. The atomic flux is such that there is an average of 0.024 atom simultaneously in the cavity. In Fig. 2(a) we show the field density-matrix elements (absolute value) after 800 atoms have crossed the cavity, and one can already see a small hill between $n = 10$ and 15, clearly indicating that the trap already has a small leak. This effect is of course more evident after 2500 atoms [Fig. 2(b)]. The fact that for larger N_u's the probability diffusion is smaller [1] can be attributed to the fact that the time spent by the two atoms within the cavity is now shorter and therefore the damaging effect in the trapping is diminished.

In Figs. 3(a) and 3(b), we have plotted $\langle n \rangle$ and $\langle(\Delta n)^2\rangle$, respectively, versus number of atoms for the following cases: (i) a cotangent state, (ii) the same cotangent state with coop-



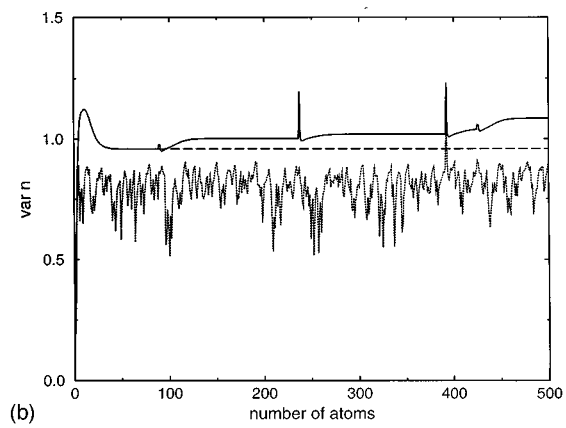


FIG. 3. (a) Average photon number and (b) photon number variance versus the number of atoms for the cotangent state (dashed line), with cooperative effects and no losses (full line) and cooperative effects and losses (dotted line). The parameters taken are $p=1$ [Eq. (3)], $N_u=5$, atomic flux equal to 1000 atoms/s, $|c_e|^2=0.8$, $Q=10^8$.

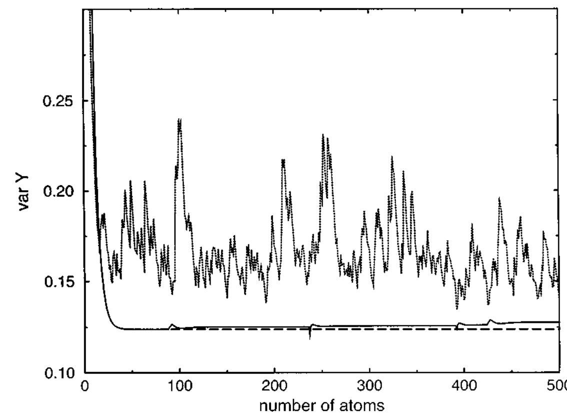


FIG. 4. Phase quadrature variance versus number of atoms for the cotangent state (dashed line), with cooperative effects and no losses (full line) and cooperative effects and losses (dotted line), for the same parameters as in Fig. 3.

erative effects, and (iii) the same as (ii) with cavity losses. We notice that cooperative effects produce spikes and fluctuations in the average photon number. However, the general trend of the $\langle n\rangle$ curve is not going up dramatically with time as compared to the unpolarized case [1]. This can be understood intuitively. In the case of inverted atoms the steady-state photon distribution of the field is a δ function placed in the upper corner of the trapping block, while in the case of polarized atoms the distribution is spread throughout this block and therefore it will take a longer time to diffuse from it. On the other hand, field losses lower the average photon number, as expected, and introduce noise in both $\langle n\rangle$ and $\langle(\Delta n)^2\rangle$, masking the cooperative effects.

Figure 4 shows the variance of the phase quadrature $Y=(a-a^\dagger)/2i$ as a function of the number of atoms for the same three cases of Fig. 3. We observe here that the squeezing properties of the cotangent states are extremely robust to cooperative effects, which otherwise seem to be very destructive. Squeezing is a delicate effect that is strongly affected by cavity losses, as shown in the curve with losses. We should stress that the cavity losses considered here are not particularly low since much better Q factors have been experimentally obtained [8]. A more detailed analysis of losses in a micromaser was made by Qamar and Zubairy [9].

Finally, in Fig. 5 we show the entropy versus the number of atoms corresponding to the parameters of Figs. 3 and 4. We observe that cooperative effects produce only minor changes in the purity of the trapped states. However, we can still recognize the cooperative spikes that tend to decay to almost, but not quite, the zero entropy value. On the other hand, cavity losses have a very destructive effect on the purity of the field.

It is striking to observe the difference between the large spikes seen in $\langle n\rangle$ [Fig. 3(a)] versus the insensitivity of the variance of Y at the instants at which two-atom events take place (Fig. 4). Keeping in mind the fact that, for typical values of atomic fluxes this kind of event is very rare and when one of them occurs it takes only a very small fraction of the total interaction time between one atom and the field, we may expand the time evolution operator for two atoms interacting with the field in terms of the small parameter $\epsilon \equiv(\pi/\sqrt{N_u+1})\Delta\tau/\tau$, where $\Delta\tau$ is the time the two atoms spend together in the cavity and τ the atomic flight time. We find that the additional contributions to $\langle n\rangle$ and $\langle(\Delta Y)^2\rangle$ introduced by a two-atom event are, respectively,

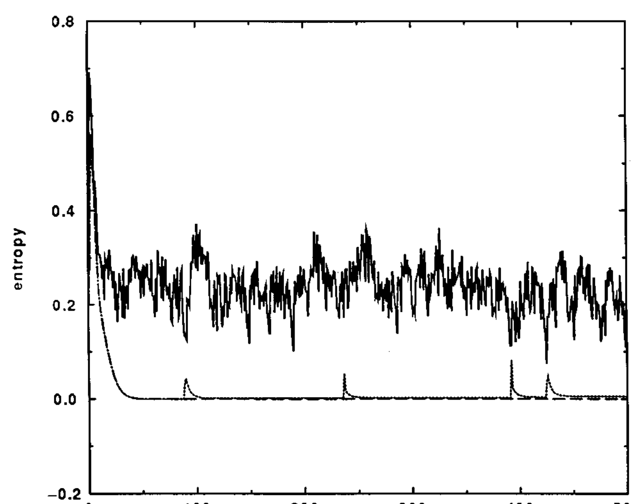


FIG. 5. Entropy versus the number of atoms for the cotangent state (dashed line), with cooperative effects (dotted line) and cooperative effects and losses (full line) for the parameters of Fig. 3.

$$\delta\langle n\rangle = -2\epsilon\ \mathrm{Im}\left[\sqrt{c_g^* c_e}\sum_m \sqrt{m+1}\,\overline{\rho_f^{m+1,m}}\right] + O(\epsilon^2) \tag{12}$$

and

$$\delta\langle(\Delta Y)^2\rangle = O(\epsilon^2), \tag{13}$$

where $\overline{\rho_f}$ is the field density matrix just before that event. As we can see, $\delta\langle n\rangle$ (linear in ϵ) is more affected by cooperative effects than $\delta\langle(\Delta Y)^2\rangle$ (quadratic in ϵ).

Summarizing the findings of the present work, we showed that, in agreement with the unpolarized case, cooperative effects cause the trapping blocks in phase space to leak and the original pure cotangent state becomes impure and develops a finite (nonzero) entropy. In spite of this, it is most remarkable, as shown above, that the squeezing properties of these states are only slightly altered by events of two atoms being simultaneously inside the cavity. As a matter of fact, we show in the present work that cavity losses act as a much more efficient squeezing-spoiling agent, as compared to cooperative effects.

This work was supported in part by the FONDECYT (Chile), Grants Nos. 2950067, 195081, and 1950790 and DIU de C94.11.10-1.1.

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