PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE

ESCUELA DE INGENIERIA

ROBUST AND EFFICIENT DECISION MAKING FOR TRANSMISSION EXPANSION PLANNING UNDER UNCERTAINTY

CONSTANTIN SEBASTIÁN KLAUS VELÁSQUEZ GUERINO

Thesis submitted to the Office of Research and Graduate Studies in partial fulfillment of the requirements for the Degree of Master of Science in Engineering

Advisor:

HUGH RUDNICK VAN DE WYNGARD

Santiago de Chile, (January, 2017)

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A mi Papá,

Jaime Velásquez

ACKNOWLEDGEMENTS

I thank both professor Rudnick and Watts for teaching me, guiding me, and making me part of the Research Group which aided the National Energy Commission (CNE) in developing the Transmission Law enacted during 2016 by the Chilean Congress. I am much indebted to professor Rodrigo Moreno for his invaluable review, comments and contributions to this thesis. I am especially thankful to Professor Moreno for sharing with me his novel research ideas and helping me extend this thesis. Thanks to Dr. Luiz Barroso for suggestions to the conceptual framework of this thesis. Thanks to Fondecyt project 1141082.

Finally, I thank both my couple Paulina Pinochet, my parents Jaime Velásquez and Carolina Guerino, and the rest of my family. I would have never achieved my goals without their unconditional support. I deeply thank Jorge Urrutia, Rodrigo Pérez, Cristian Bustos, Marysol Ayala, Danilo Jara, and all the great people with whom I worked during the Transmission Law project, who helped me and taught me invaluable lessons, both professionally and in every other aspect of life.

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RESUMEN

La Planificación de la Expansión de la Transmisión (PET) es un problema complejo afecto a diversas incertidumbres, desde precios de combustibles hasta terremotos y oposición pública. A la fecha no se ha desarrollado ningún análisis sistemático de las diferentes incertidumbres presentes en la PET. Por otra parte, el debate sobre metodologías de planificación bajo incertidumbres de largo plazo, capturadas por escenarios estratégicos, carece de exhaustividad. En efecto, mientras que los defensores de la programación estocástica ignoran la indisponibilidad de probabilidades precisas en la práctica, los oponentes se basan en criterios alternativos como el minimax regret, sin verificar la eficiencia de la solución.

En ese contexto, esta tesis intenta una contribución cuádruple a la literatura. Primero, desarrollar un marco conceptual para investigadores, políticos y profesionales, sobre las diversas incertidumbres y riesgos relevantes para PET. Segundo, cuantificar la importancia práctica y el impacto de la ambigüedad de las probabilidades de los escenarios en la PET estocástica. Tercero, presentar un nuevo criterio de decisión de robustez distribucional para PET bajo ambigüedad. Cuarto, comparar la solución óptima bajo criterios alternativos de decisión para PET como el costo esperado, el minimax regret y la robustez distribucional, tanto teóricamente como mediante simulación, utilizando el concepto de eficiencia de Pareto.

Se aplica la metodología desarrollada al sistema IEEE RTS de 24-barras, bajo dos escenarios. En este caso, PET estocástica es relativamente insensible a las variaciones en las probabilidades de los escenarios. Sin embargo, también hace caso omiso de muchas soluciones eficientes en la parte cóncava de la frontera de Pareto. Aunque el minimax regret puede descubrir tales soluciones, se prueba que metodologías míopes de minimax regret (con respecto a la eficiencia Paretiana) tales como aquellas aplicadas en Reino Unido, PJM y Chile, pueden incurrir en pérdidas gratuitas de oportunidad de hasta el 4% de los costes totales. Se requiere el análisis de sistemas grandes y bajo más escenarios para proporcionar conclusiones sólidas que identifiquen fortalezas y debilidades de ambos métodos.

Palabras Claves: Análisis de decisiones, planificación de transmisión, programación estocástica, incertidumbre, optimización multiobjetivo

ABSTRACT

Transmission Expansion Planning (TEP) is a complex problem faced with a myriad of uncertainties ranging from fuel prices to earthquakes and public opposition. To date, no systematic analysis has been developed for the different underlying uncertainties in TEP. Moreover, the debate on planning methodologies for coping with long term uncertainties captured by strategic scenarios lacks completeness. Indeed, whilst advocates of stochastic programming ignore the unavailability of precise probabilities in practice, opponents rely on alternative criterions such as minimax regret without verifying solution efficiency.

Therefore, this thesis attempts a fourfold contribution to the literature. First, develop a conceptual framework for researchers, policy and practitioners to better understand and communicate the various uncertainties and risks relevant to TEP. Second, quantify the practical relevance and impact of ambiguous scenario probabilities in stochastic TEP. Third, present a novel distributional robust decision-making criterion for TEP under ambiguity. Fourth, compare the optimal solution under alternative TEP decision-making criterions such as expected cost, minimax regret and distributional robustness, by both theory and simulation based on the well-established concept of Pareto efficiency.

The developed methodology is applied to the IEEE 24 bus RTS system under two scenarios. In this example, stochastic TEP is relatively insensitive to variations in scenario probabilities. However, it also ignores many efficient solutions in the concave Pareto frontier. Although minimax regret can discover such solutions, it is proved that myopic minimax regret methodologies (with respect to Pareto efficiency) such as those applied in UK, PJM and Chile can incur in gratuitous opportunity losses of as much as 4% of total system costs. Further analyses of large-scale systems under more scenarios are needed in order to provide robust insights to identify strengths and weaknesses of both methods.

Keywords: Decision analysis, transmission planning, stochastic programming, uncertainty, multiobjective optimization

1. INTRODUCTION

The *electric transmission system* (or *power grid*) is the infrastructure which allows the transportation of large volumes of electricity from production centers (i.e. generators) to final consumers (e.g. large industrial customers, distribution companies which supplies households and others customers within their concession area).[1] Adaptations to the transmission system are required to cope with changing conditions in the power system. Particularly new investments are required to deliver electricity in the face of growing demand, as well as investment and retirement of generators. Therefore, efficient expansion of the transmission system is, and will remain for the foreseeable future, vital for solving the energy trilemma of ensuring a reliable, sustainable and affordable supply (Newbery, 2016; Rivier, Pérez-Arriaga, & Olmos, 2013).

In Chile, the *Electric Transmission Law enacted in 2016* allows the regulator to plan the expansion of the transmission system considering capacity slackness for long-term developments, and also to proactively develop generation hubs by means of transmission expansion.[2] Furthermore, costs of new transmission projects will be completely allocated to demand by means of a postage stamp methodology (i.e. demand pays proportionally to its consumption), instead of the current locational cost allocation methodology which shares costs between generators and demand depending on their location within the power system.[3] The transmission law was developed in a two-year timespan by the National Energy Commission (CNE) and the Pontifical Catholic University of Chile. The law was developed on the grounds of widespread public participation and building consensus among key stakeholders, whilst incorporating the lessons from past experiences with

[1] The transmission system comprises overhead and underground transmission lines, power transformers, substations, circuit breakers, measurement and control equipment, among others. However, transformers and overhead transmission lines are the focus of this thesis and of most long-term transmission planning studies.

[2] Proactive transmission expansion attempts to guide the development of generation by means of transmission investments. In Chile, this proactive approach attempts to enable exploitation of remotely located renewable generation resources which may find it difficult or impossible to enter the market without coordinating transmission investments (Ferreira, Rudnick, & Barroso, 2016).

[3] Optimal transmission cost and benefit allocation is a complex and very important issue in power markets (Hogan, 2011; Strbac, Konstantinidis, Moreno, Konstantelos, & Papadaskalopoulos, 2015). The discussion on alternative methodologies for transmission cost allocation is out of the scope of this thesis.

transmission laws (most notably 2011's CADE and the failed attempt to enact a law considering "Electric Highways").

The newly enacted law has raised *concerns regarding the risks of overbuilding the transmission system* on the grounds of capacity slackness and proactive generation hub developments. Moreover, less opposition to risky transmission expansion projects is expected since transmission costs are now fully allocated to demand, and demand is perceived to be less interested or able than generators to analyze expansion proposals and oppose to those proposals that affect them. Furthermore, the methodologies for transmission expansion under uncertainty used in Chile were heavily criticized during discussions of the project.

Hence, *analyzing transmission expansion planning under uncertainty is important for Chile today*, since the planning process undertaken by the Ministry and the regulator can have huge impacts in future and current generations alike. The public and policy-makers alike seek increasing integration of renewable energy sources in the electricity sector, both in Chile and in the rest of the world. The 2016 Transmission Law attempts to develop a more sustainable and forward-looking electricity sector in Chile. However, overbuilding the transmission system increases electricity prices today, hoping that economic benefits and increased sustainability in the long-term (e.g. 20 years from now) offset today's increases in prices due to overinvestment. The risks of over and underinvestment in the face of long-term uncertainty should therefore be analyzed from an engineering perspective in order to help the regulator in successfully implementing the transmission law.

This thesis attempts to contribute to the discussion on transmission expansion planning under uncertainty. A variety of engineering studies were possible in order to analyze the 2016 Transmission Law and its practical implications. For example, alternative cost allocation methodologies and their relation to market efficiency and incentives; proactive transmission expansion models; or the design of expansion processes which attempt to solve or ameliorate public opposition to new transmission projects; are some of the research subjects that emerged from the discussion of the Transmission Law. We chose to study uncertainty since it is an overarching and fundamental issue in the transmission planning process in Chile and the rest of the world.

The rest of this thesis is structured as follows. First, this introduction proceeds by explaining the basics of transmission expansion planning (section 1.1), and then stating the hypothesis and objectives of this thesis (section 1.2). Section 2 lays out a conceptual framework for TEP under uncertainty. Section 3 presents a thorough and in-depth literature review on optimization models for TEP under uncertainty, and identifies gaps in the literature. Section 4 presents the engineering methodology devised to study TEP under scenario uncertainty. Section 5 presents results of the methodology as applied to two small-scale test cases, and discusses those results. Section 0 concludes this thesis.

1.1. Basics of Transmission Expansion Planning

The expansion of the transmission system has been and will remain a regulated activity driven primarily by central planning, since the transmission business is a natural monopoly (Newbery, 2000).[4] Central transmission expansion planning is the process which determines the timing and basic properties of new investments in the transmission system, required to provide a reliable, sustainable and affordable electricity supply. Transmission expansion planning is (and must be) undertaken in every power system, whether extensive reforms to the Electricity Supply Industry (ESI) have taken place (e.g. England and Wales, Norway, California), intermediate reforms have emerged (e.g. Korea, Ukraine, Chile, South Africa) or the supply remains monopolized by a vertically integrated (state, public or privately owned) utility (e.g. Nigeria).[5]

The need for central planning of the transmission system expansion is based on the technical and economic properties of the transmission business and system, most notably:[6]

---

[4] The development of the transmission system by means of merchant transmission projects remains limited in practice (Joskow, 2005).

[5] Chile was the first country to reform its electricity supply industry. Today, the Chilean electricity market is based on centralized transmission expansion planning, and private, decentralized generation investment decisions.

[6] A thorough description of the technical and economic properties of the transmission system is out of the scope of this thesis. The reader is referred to excellent textbooks on transmission and electricity such as (Kirschen & Strbac, 2005; Pérez-Arriaga, 2013; Stoft, 2002), and (Baldick & Kahn, 1993; Hirst & Kirby, 2001; Hogan, 1999) for a discussion of key properties of the transmission system directly related to its expansion.

- Complex physical interactions take place in the transmission system, ruled by Kirchoff's and other laws. Important consequences of these interactions are the network externalities which preclude a simple definition of property rights for transmission lines, due to (Hogan, 1999):
    - the inability to trace the origin and destiny of power flows (e.g. to determine whether a given consumer is being supplied by a particular producer), and
    - the inability to accurately control and distribute power flow among alternative transmission paths. [7] This inability is commonly referred to as *loop flows*, which refers to the fact that power flows through every possible path between two given nodes.
- Extensions to the transmission system are long-lived (between 10 and 40 years), capital-intensive infrastructure investments, with long lead-times (from 2 and up to 10 or 20 years, mostly due to public opposition and approval processes) and minimal operation and maintenance costs (i.e. transmission costs are largely sunk). Furthermore, large economies of scale and the irreversible and lumpy nature of investments[8] preclude a fully competitive solution to develop the transmission network.
- Sunk costs in new transmission investments largely surpass short-run opportunity costs of transmission congestion (i.e. spot prices differentials in a given transmission line) (Perez-Arriaga, Rubio, Puerta, Arceluz, & Marin, 1995). Moreover, relieving congestion benefits generators and consumers instead of transmission owners who invest in new transmission capacity.

The aforementioned properties of the transmission business and system are key to understanding both the theoretical foundations of transmission planning, as well as real processes undertaken in the industry and the mathematical models which assess those

[7] It is possible to control the flow of power through the grid to some extent, albeit not remotely to the same extent as the flow of water through a system of pipes and valves.

[8] Transmission investments are irreversible since there is no market or alternative use for decommissioned elements of the transmission system. On the other hand, investments are lumpy since it is not possible to add an arbitrary level of additional capacity to the system. Conductors, transformers and other network elements available in the market follow a discrete set of standardized voltages and ratings (e.g. 100 MW @ 138 kV, or 200 MW @ 230 kV, but not every possible combination thereof).

processes. To this end, it is also worth noting that electricity is a rather particular service since supply and demand in the system needs to be continuously and precisely kept in balance in real-time by some entity (the independent / transmission system operator, or the vertically integrated utility). Other properties of the electricity supply such as the low elasticity of demand are also important to understand power systems and, therefore, transmission expansion planning. These and other properties will be further explained later as needed.[9]

Practical transmission expansion planning are complex and diverse processes, although everywhere transmission expansions are primarily driven by engineering analyses in order to ensure a reliable electricity service (Joskow, 2005). TEP processes entail stages of technical and economic analyses, regulatory review, and stakeholder or open consultations. Most transmission investments are often justified on the grounds of preserving high levels of reliability of the electricity supply.

However, TEP can (and often does) simultaneously pursue a variety of objectives other than preserving reliability, such as (ENTSO-E, 2015):

- Minimizing total costs of electricity supply, considering transmission investment and operation costs of the generation fleet. Such Integrated Resource Planning process is classic in the vertically integrated utility, and also common in cost-based power pools such as Chile) (Stoll, 1989).
- Maximizing socio-economic welfare (equivalent to minimizing total supply costs under perfect competition and perfectly inelastic demand).
- Reducing congestions or even achieving gold-plating standards in order to increase the scope and liquidity of regional markets as well as ensuring the integration of variable renewable energy sources (an approach adopted in Alberta, for example, and also as part of the planning practices in Europe and particularly in Germany) (Stoft, 2006; Weber et al., 2013).
- Increase the sustainability of the power system, reduce CO2 emissions, increase the integration of renewable energy sources (e.g. EU) and comply

[9] A more in-depth introduction to power systems and markets can be found elsewhere in textbooks such as (Hogan, 1998; Hunt, 2002; Pérez-Arriaga, 2013; Stoft, 2002).

with regulatory requirements such as Renewable Portfolio Standards (e.g. USA).

- Increate the security of supply and the resiliency of the system (i.e. ability of the system to withstand and / or recover from extreme conditions such as earthquakes).

Furthermore, a wider variety of benefits should be consider in assessing transmission expansion projects, for example, relieving reliability-must-run constraints (Chang, Pfeifenberger, & Hagerty, 2013).

A static version of transmission expansion planning is often used for long-term studies. The static TEP determines which transmission investments to commit to today, in order to reduce operation costs in the future, hence solving the fundamental trade-off between present investments and future operation costs. However, from an economic perspective, transmission expansion is a dynamic problem since commitment to investments can be taken today or in the future and, moreover, these investments can be scheduled to begin operations in one of many future years. In the face of growing demand and due to lumpy investments, it may be better to build larger (and more expensive) lines today which can be further expanded in the future (for example, by allowing a second circuit to be added to the base transmission line), instead of building small lines which require more investments in the future. Moreover, in the face of uncertainty transmission projects acquire an option value since it may be worth to *"wait-and-see"* before committing *"here-and-now"* to a particular transmission expansion project (Baldick & Kahn, 1993; Stoft, 2006). Furthermore, flexible alternatives to transmission expansion such as demand response, energy efficiency, or even transmission elements with low investment costs and lead-times (e.g. phase-shifters) can and are being used (mostly in US and EU) to defer commitments to large transmission investment projects (Konstantelos & Strbac, 2015; Watts & Rudnick, 2014; Wilson & Biewald, 2013). Although very important, the dynamic nature of the transmission expansion planning problem will be ignored in this thesis, following the current state of the art.

Optimization and simulation models to assist transmission expansion planning do not currently fulfill the needs of real planning processes. Only since the early 2000, the growth in computation capacity and improvements in solution methodologies have allowed the

solution of middle and large-scale transmission expansion planning problems, albeit with rather restrictive assumptions (most notably linearized power flow equations) (Bixby, 2015). Despite the development of a huge amount of high-quality papers, optimization and simulation tools for assisting decision-makers in transmission expansion planning, these tools do not yet fulfill the needs of planners. Instead, a reduced set of alternative expansion plans is assessed and constructed iteratively by the planner by repeatedly running electrical simulation software (e.g. DigSilent, Neplan) and production-cost simulations (e.g. Plexos, OSE2000). Therefore, there is still much work to do in order to develop workable and useful mathematical tools to support transmission expansion planning (Lumbreras & Ramos, 2016; Velasquez, Watts, Rudnick, & Bustos, 2016).

Despite the complexity of the transmission expansion planning problem and the many challenges faced by planners today (e.g. public opposition to new infrastructure), planning must be undertaken under significant uncertainties regarding future conditions of the power system and market. Decisions must be made without knowing the conditions under which these decisions will perform. Moreover, alternative assessment methodologies have been developed and applied to real TEP processes in order to assist this complex decision-making process. These alternative methodologies, the rationale for this thesis and its hypothesis and objectives are briefly stated next.

1.2. Hypothesis and Objectives

Scenario planning is one of the most widespread techniques to deal simultaneously with various uncertainties in transmission expansion planning, both in real processes and scientific papers (Lumbreras & Ramos, 2016; Watts & Rudnick, 2014). Stochastic programming and robust approaches such as minimax regret are alternative and fundamentally distinct methodologies to assess decision-making under scenarios. Stochastic programming is based on the well-founded expected utility theory (Savage, 1954; von Neumann & Morgenstern, 1944),[10] but it requires probability distributions

[10] Expected-utility theory is a foundational theory on rational decision-making. It is well known that common people does not make decisions guided by the expected-utility theory (Tversky & Kahneman, 1974). However, this thesis and most (if not all) transmission expansion planning studies focus on a normative

which are hard to precisely determine in practice (Miranda & Proenca, 1998a). On the other hand, approaches such as minimax regret do not require probabilities to be assigned to each scenario, but several issues do affect minimax regret (Bunn, 1984) (also see appendix B).

A comprehensive literature review (detailed in the section 3) reveals that, to this date, no practical nor scientific study on transmission expansion planning has simultaneously addressed the advantages, disadvantages and relationships among stochastic programing and minimax regret (Buygi et al, 2003; Latorre et al, 2003; Lee et al, 2006; Lumbreras & Ramos, 2016). Particularly, critiques to one or the other approach have not addressed the relevance nor the impact of scenario probabilities in stochastic transmission expansion planning. This thesis attempts to fill this gap in the literature, as stated in the following hypothesis and objectives.

The **hypothesis** of this thesis is that probabilities assigned to scenarios are relevant in stochastic TEP models, since variations in the probabilities are not negligible. Instead these variations can and do significantly impact the structure of the optimal expansion plan.

According to the proposed hypothesis, the **general objective** of this thesis is to study the optimal transmission expansion planning under scenario uncertainty, when scenario probabilities are hard to determine precisely (i.e. under *ambiguity*). The **specific objectives** are the following:

1) To propose a new decision criterion under uncertainty, accounting for the lack of precise scenario probabilities in long-term TEP studies.
2) To develop a computationally efficient and tractable methodology for TEP under scenario uncertainty by using the newly proposed decision criterion.
3) To analyze the relevance and the impact of scenario probabilities in TEP by studying test cases commonly used in the literature.
4) To analyze the new decision criterion and its relation to other criterions established in the literature.

2. CONCEPTUAL FRAMEWORK FOR TRANSMISSION EXPANSION PLANNING UNDER UNCERTAINTY[11]

TEP research and real processes for modern electricity markets are overwhelmed by a myriad of different uncertainties, thus arguing in favor of a conceptual framework designed specifically for TEP (Figure 2-1). Relevant uncertainties in TEP range from hourly wind speed to socioenvironmental aspects – for example, public opposition - comprising uncertainties as varied as demand growth, weather phenomenon like El Niño (which affects hydrology and thus water inflows to reservoirs), fuel prices, terrorism and the so-called Black Swans which refer to surprising, unforeseen events, such as the financial crisis of 2008. In the face of a wide variety of uncertainties in TEP, different management strategies and modeling techniques are available. Also, various properties may be identified.

Hence alternative groups or more general classifications of uncertainties are possible, and some have already been mentioned in TEP literature, with no further elaboration. However, the problem of dealing with uncertainty is not exclusive to power systems. Its modern history dates back at least to 1921's seminal economic work by Frank Knight. Over the course of the century, many new concepts and typologies have emerged to try to understand uncertainty and deal with its impacts, either providing a more general framework (Apostolakis, 1990; Keynes, 1952; Knight, 1921; Rowe, 1994; van Asselt & Rotmans, 2002; W.E. Walker et al., 2003; Warren E. Walker, Lempert, & Kwakkel, 2013) or addressing specific problems (Ascough, Maier, Ravalico, & Strudley, 2008; Beck, 1987). Today, attempts to categorize and conceptualize uncertainty stem from a variety of disciplines comprising economy, finance, social sciences, sustainability, policy making and military planning. Different typologies have also been developed to better serve the objectives of particular TEP papers (Chamorro, Abadie, de Neufville, & Ilic, 2012; Mejia-Giraldo & McCalley, 2014; van der Weijde & Hobbs, 2012). This sparsity argues in favor of developing a comprehensive conceptual framework for TEP under uncertainty. Moreover, as Walker et al puts it, *"a better understanding of the different dimensions of*

[11] A similar version of this chapter has been published in the July 2016 IEEE Power and Energy Magazine issue (Velasquez et al., 2016).

uncertainty and their potential impact … would help in identifying and prioritizing effective and efficient research and development activities for decision support." (2003).

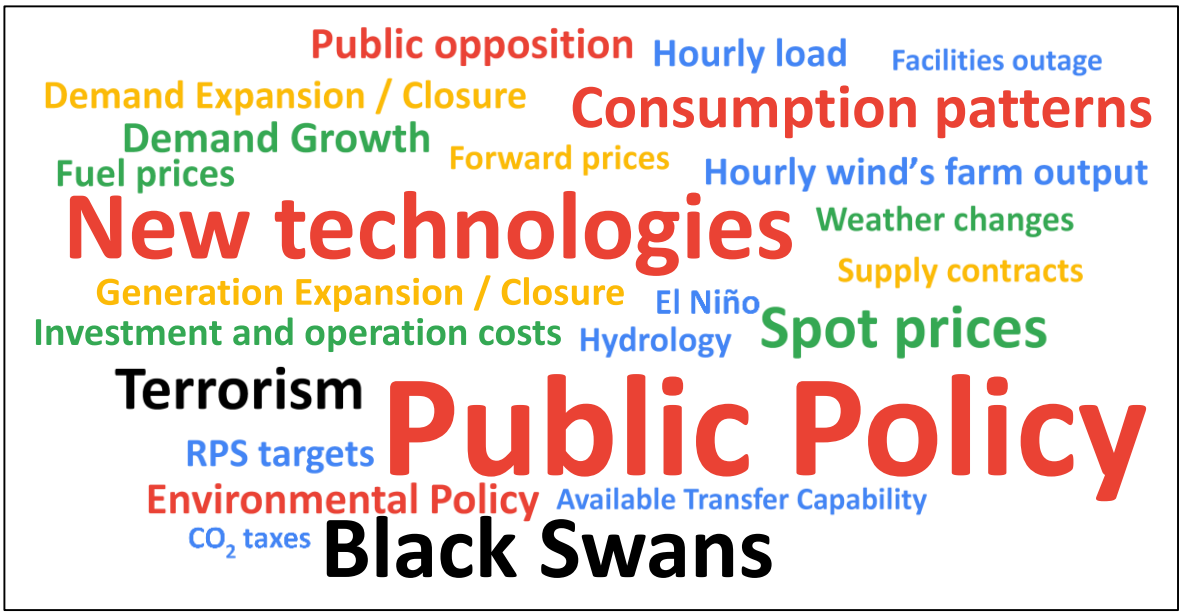

Figure 2-1 Wide variety of uncertainties in TEP suggests many different management strategies, modeling techniques, key properties and classifications.

More precisely, a conceptual framework for planning under uncertainty is required for three reasons.

- First, providing a common framework for effective dialogue is needed to improve communication between professionals of the electric industry and academy.
- Second, focusing future research efforts could be aided by a deeper understanding of the varied nature of uncertainty, its key properties and impacts in TEP.
- Third, an assessment of existing and future approaches to TEP under uncertainty requires further analysis of the properties of uncertainty, in order to identify dis/advantages and applicability of each approach.

The conceptual framework needed to fulfill this threefold purpose should properly define the concepts of uncertainty and risk. It should also characterize and classify the myriad of uncertainties relevant to TEP in a typology simultaneously practical and based upon solid theoretical foundations. A framework to address previously discussed issues

and exploit opportunities offered by an in-depth analysis of uncertainties in TEP is presented.

Clear, broad and practical definitions for the basic concepts of uncertainty and risk are the first and foremost component of the conceptual framework for TEP under uncertainty. Different definitions of uncertainty and risk are available on the literature (Nelson & Katzenstein, 2014). We adopt the definition that *"Uncertainty is a state … where* ***it is impossible to exactly describe existing state or future outcome****"*, (Hubbard, 2007), cited by (Gu & McCalley, 2010).[12] Thus, the proposed definition is broad enough to capture the main impact of uncertainty in planning, namely the impossibility to accurately and precisely predict the future. Hereinafter we refer to any uncertain quantity or proposition as an uncertainty.

Because of the impossibility to accurately and precisely predict the future, **planning under uncertainty inevitably carries risk**. Risk is defined as the impact of uncertainties in TEP, commonly deemed as a hazard. Simply put, uncertainty is the cause while risk is the consequence or impact on TEP (usually negative, see Figure 2-2), and we refer to any uncertain quantity or proposition as an uncertainty. For planning purposes, uncertain outcomes are to be modelled while risk is to be quantified and managed or hedged against. The quantification of risk relates to some measure of performance, most commonly costs and economic benefits, but also reliability, sustainability or other measures in a multicriteria framework, as discussed later (section 2.4).

[12] Hubbard's original definition of uncertainty was purposefully modified by removing the reference to "limited knowledge" since, as will be explained later, uncertainty does not necessarily stem from imperfect knowledge.

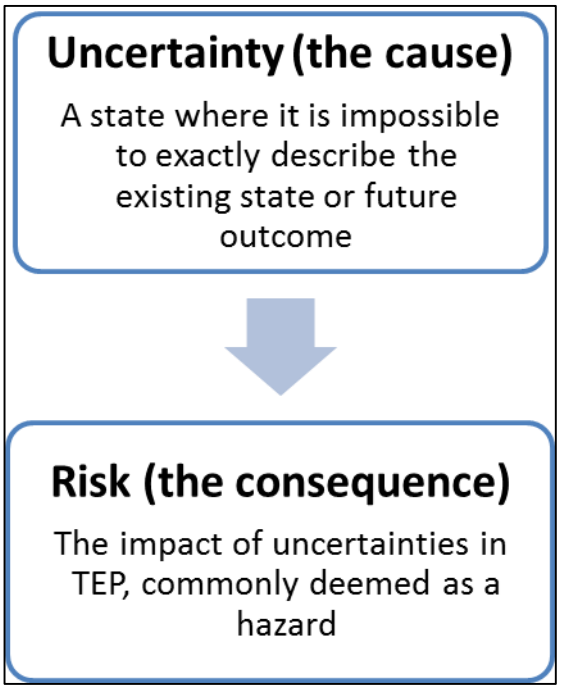

Figure 2-2 Definition of uncertainty as the cause and risk as the consequence.

The typology of TEP uncertainties consists of one conceptual foundation and three practical dimensions (Figure 2-3). The conceptual foundation is the distinction between two extreme natures of uncertainty, inherent variability or aleatory on the one hand, and lack of knowledge or epistemic on the other. Each of the three practical dimensions relates to a key property of TEP uncertainties, and is useful in categorizing uncertainties but also in identifying similar practical considerations regarding modeling and planning processes. We discuss each of the four parts of the typology in subsequent sections, beginning by the conceptual core, followed by the structure, timescale and source of uncertainty, in order. The practical significance of each dimension is discussed under each section. Note that the present typology may not directly apply to power system problems other than expansion planning. For example, unit commitment may require a far more detailed typology focused on uncertainties which are important on operational timescale.

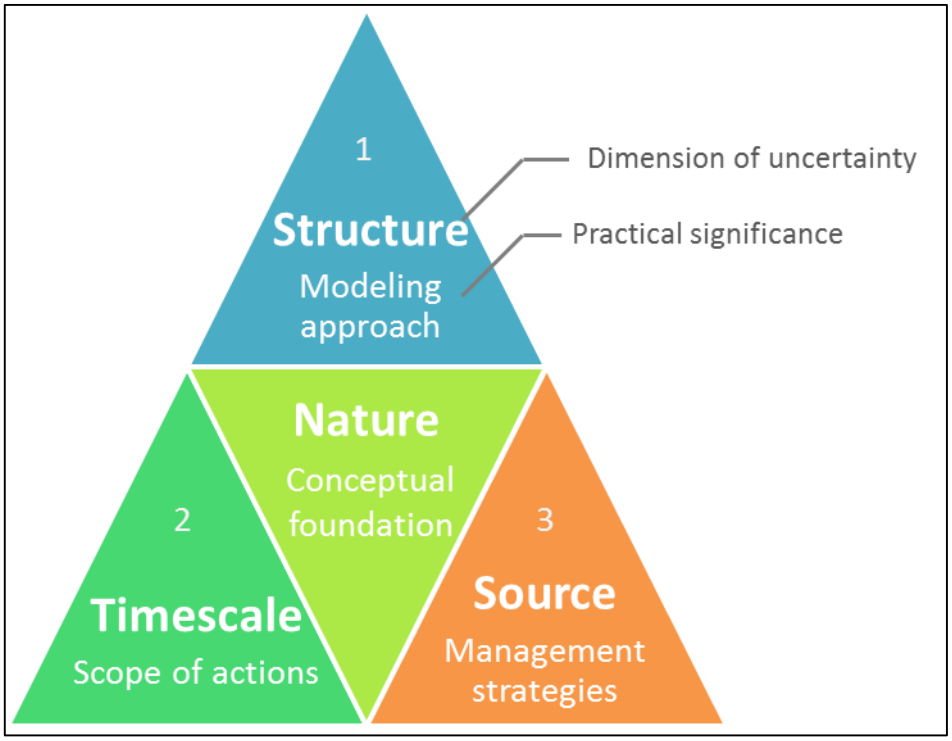

Figure 2-3 The proposed typology of TEP uncertainties consists of one conceptual foundation and three practical dimensions.

2.1. Nature of uncertainty: aleatory or epistemic?

The core of the proposed typology is the conceptual distinction between two extreme natures of uncertainty, aleatory or epistemic. Whereas *aleatory uncertainty* refers to a natural variability inherent to a particular process (e.g. hourly wind speed, human behavioral variability and earthquake incidents), which not always can be represented by a probability distribution; *epistemic uncertainty* is produced by imperfect knowledge (e.g. the model to describe competition in electricity markets, generation expansion and failure rates for new facilities) and, thus, could possibly be reduced by increased inquiry. The distinction between epistemic and aleatory nature has been regarded in sustainability literature as the more general and fundamental categorization of uncertainty.

In order to assert the importance and extent of this distinction in TEP, consider generation expansion in a deregulated industry (Figure 2-4). On one extreme, TEP studies often assume that generation expansion is not described by probability distributions. Hence, generation expansion is modeled by means of alternative scenarios, using a classic

"what-if" analysis, minimax regret, among others. On the other extreme, proactive TEP models treat generation expansion by means of an equilibrium model between generation firms. Thus generation expansion is not uncertain due to lack of knowledge of the underlying investment process but rather due to operating conditions, which are in turn characterized by publicly known and agreed upon probabilities. Thus, proactive models remove epistemic uncertainties by directly modeling investment behaviour, while aleatory uncertainty regarding system operation is irreducible –but easily described by probabilities–, i.e. inherent.

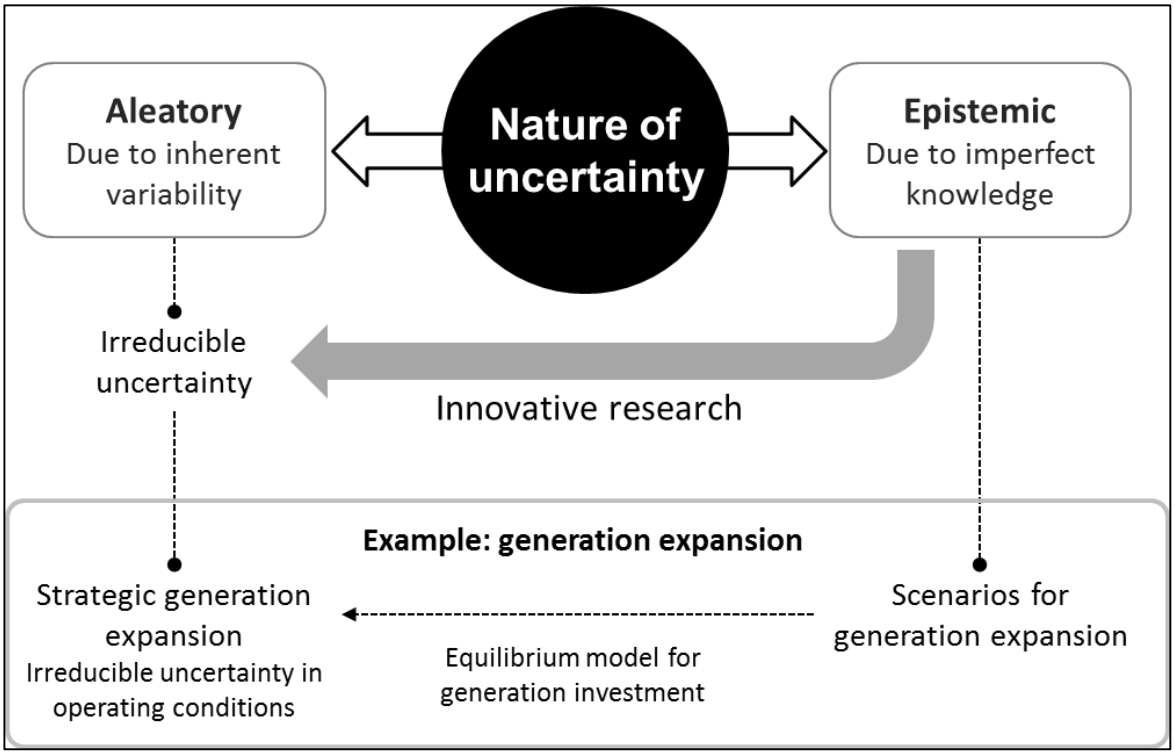

Figure 2-4 Fundamental nature of uncertainty ranges from aleatory to epistemic. Distinction becomes relevant only when innovative research enlightens understanding of some uncertainty.

Although we stress the importance of distinguishing epistemic and aleatory uncertainties, we include this distinction in a higher level with respect to more practical dimensions because of its conceptual importance and its lack of practical relevance outside innovative research. While we do not attempt to argue in favor of scenarios or equilibrium models to represent generation expansion in TEP, it is clear that complex uncertainties such as generation expansion are simultaneously composed of imperfect knowledge (i.e. the economic equilibrium describing generation investment by competing firms) and

inherent variability (e.g. uncertainty in operating conditions and fuel prices, present even if the equilibrium model of investment is completely specified). However, this distinction is irrelevant from a practical perspective until innovative research enlightens our understanding of the underlying process that produces uncertainty. Nevertheless, increased research efforts may never be able to explain many important sources of uncertainty such as public policy or earthquakes. Thus, modeling uncertainties in practice has not and should not focus on the nature of uncertainty.

2.2. Structure of uncertainty: Known, unknown and Unknowable

The structure of uncertainty is a practical modeling distinction between **K**nown, **u**nknown, and **U**nknowable uncertainties (**KuU**, respectively, see Figure 2-5), a categorization of uncertainties proposed by Diebold, Doherty & Herring in 2010. **K**nown uncertainties are those where a probabilistic representation is completely specified (e.g. hourly nodal load and yearly demand growth), and therefore structured mathematical tools such as stochastic programming are readily available. **u**nknown uncertainties are those for which probabilities are hard or impossible to assign to some events (e.g. investment costs, delays in starting operation and new facilities' outage), and are often modeled by sets bounding outcome (e.g. intervals centered on a nominal forecast) or other less-structured representation (e.g. fuzzy sets). **U**nknowable uncertainties are situations where even the events cannot be clearly identified in advance (e.g. long term scenarios and earthquakes), thus presenting the lowest level of mathematical structure. For dealing with poorly structured U uncertainties, strategic scenarios depicting divergent futures or possible singular events are used. Given the lesser mathematical structure of scenarios, robust and flexible expansion plans are designed, as discussed in section "Optimization approaches to TEP under uncertainty". We abstract from the close relationship between the extremes of perfect and poor structure and the nature of uncertainty previously discussed, in order to serve practice rather than theory.

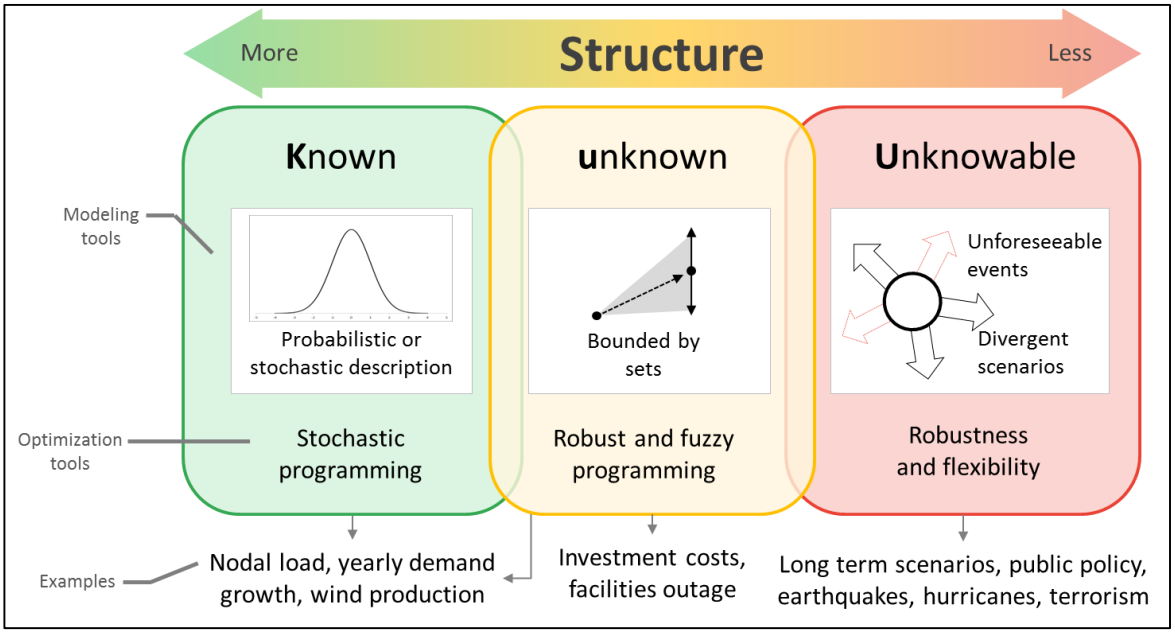

Figure 2-5 Structure of uncertainties, from known probabilistic or stochastic uncertainties to Unknowable or scenario uncertainty.

KuU extends the classical division between random and non-random uncertainties found in TEP literature by providing a more practical and consistent framework for modeling. The classical division states that *random* uncertainties are observable events that repeat frequently, so statistics may be derived from historical data for random uncertainties (e.g. yearly demand growth). Any other uncertainty is *non-random* and, some assert, to be treated by scenarios (Majid Oloomi Buygi, Shanechi, Balzer, Shahidehpour, & Pariz, 2006; M O Buygi, Balzer, Shanechi, & Shahidehpour, 2004). Although such division successfully identifies *random* uncertainties that may be modeled by probability distributions or stochastic processes, it fails to identify robust and fuzzy approaches applied to TEP under uncertainty, and also fails to acknowledge subjective probabilities. Therefore, we further distinguish non-random uncertainties between **u** and **U**, given the different modeling approaches applied in TEP research for each class (e.g. robust / fuzzy for nodal load and scenarios for long term state of the market, respectively). Also, **K** extends the group of random uncertainties by considering all those described by probabilities, whether these are derived from historical data, a stochastic process, or subjective beliefs. Although subjective probabilities are not commonly used in TEP, their existence must be acknowledged for completeness since they are rigorous mathematical

objects required for rational decision making under Bayesian statistical theory. Examples of subjective probabilities are those assigned to future scenarios by expert elicitation.

Some uncertainties may be categorized differently or move between categories depending on data availability, modeling decisions or knowledge, as we explain next with four examples. First, note that transmission facilities' failure because of weather conditions may be regarded as **u** and modeled by fuzzy techniques because accurate historical data is missing, even though repeatability may justify the application of probabilistic techniques if such data were available (i.e. a **K** uncertainty). Second, even though demand growth and nodal load is often labeled as random and treated by probabilistic and mathematical forecasting methods, other approaches such as fuzzy and robust programming have been applied for reasons such as tractability issues of stochastic optimization. Therefore, load growth may be either **K** or **u** depending on modeling choices. Third, although generation expansion is often modeled by scenarios, research on proactive models moves generation expansion from **U** or **u** to **K**, as discussed in the previous section. Fourth, increased understanding of regulatory uncertainties such as RPS targets will probably never be enough to move such uncertainty from **U** or **u** to **K**. Although **KuU** is connected to the nature of uncertainty, **KuU** is more practical and thus also related to practical considerations.

Unknowable uncertainties such as long term state, earthquakes and medium term Black Swans are highly unpredictable. Unfortunately, these uncertainties are also accompanied by huge risks from which hedging is difficult or even impossible, as discussed next for each example.

- First, the traditional approach for dealing with long term uncertainties is based on a reduced number of strategic scenarios with inherently high risks. However, these scenarios do not describe every important future. The purposes of strategic scenarios are to bound uncertainty and understand its impact rather than precisely forecasting future outcome, since accurate prediction is an impossible task in the long term. Because scenarios are only assumed to describe the future, it is not rare to miss important future scenarios not perceived plausible at the time the scenarios are constructed. Moreover, the future actually realized may well be a combination of constructed scenarios

with other unforeseen events and processes. Thus, the fact is that plausible futures are not identifiable. Furthermore, huge risks stem from the high degree of uncertainty in the long term, even when only considering the set of constructed scenarios.

- Second, predicting the timing and location of earthquakes and hurricanes is impossible, and their occurrence has enormous impacts on electricity systems. Thus, scenarios seem to be the appropriate modeling tool both because of unpredictability and the need to hedge against massive risks.
- Third, it is impossible to hedge against risks produced by completely unforeseeable Black Swans such as terrorist attacks, the financial crisis of 2008 or the Chilean crisis of 2004 produced by the breach of gas supply contracts by Argentina.

Although conceptual distinctions and practical considerations are involved in definitions of **KuU**, categories do not appropriately match modeling approaches when plausible alternative cases completely describe future outcomes. An **u**nknown uncertainty by definition comprises situations when outcome can be completely bounded to multiple plausible cases, without probabilities for each case. Under such extensive alternative cases, either robust programming or scenarios together with robustness and flexibility analysis may be used, even though the uncertainty is not **U**nknowable. However, the proposed definitions capture the conceptual distinction among different mathematical structures. Also, more often than not, scenarios are used in TEP to model **U**nknowable uncertainties. Nevertheless, the fact remains that care must be taken to apply **KuU** to alternative cases that extensively describe future outcomes.

2.3. Timescale of uncertainty: short, medium and long term

Uncertainty may be revealed in different timescales, ranging from hours to decades (Figure 2-6). We distinguish between short, medium and long term uncertainties mainly because decisions have different scopes in each timescale. In the short term (from hours to months), uncertainties such as hourly demand, equipment availability, wind/solar production, and spot fuel prices are revealed. Thus, operation decisions related to market clearing, economic dispatch and unit-commitment are taken on a given system and market.

In the medium term (from months to years) relevant uncertainties are demand growth, generation and demand expansion or closure (including generation siting), supply contracts and evolution of existing technologies (e.g. increase of photovoltaic plants' efficiency). A recent and relevant uncertainty has been delays in the commercial operation date of the “planned” assets themselves, including transmission facilities, resulting from the complex social-environmental challenges to obtain the needed permits to allow the project implementation. Planning decisions in medium term involve primarily expansion of transmission systems, changing the systems composition and topology but not its structure. In some markets such as the Brazilian, generation expansion is also partially guided by central planning decisions. The long term spans decades in the future and so new disrupting technologies (e.g. new storage and PV technology) and smart grids (e.g. vehicles to grid, large scale residential distributed generation and demand response), among others, become significant uncertainties.

In the highly uncertain long term environment, strategic decisions with less detail but dramatically more impact may be taken, for example, large transmission corridors, regional interconnections, market structure and policy making in general. The scope of strategic decisions in the long term is such that future scenarios may be partly normative in order to guide the evolution of the electric system and market towards desired goals. This guidance is particularly important where central planning by state agencies influences the development of generation systems by posing policy preferences in the energy auctions, as in the case of Brazil. Nevertheless, normative elements merely complement explorative scenarios of plausible futures, without replacing them. Although proactively constructing future by policy making and market structure are out of the scope of TEP, these actions are related in the long term, as discussed in the following section.

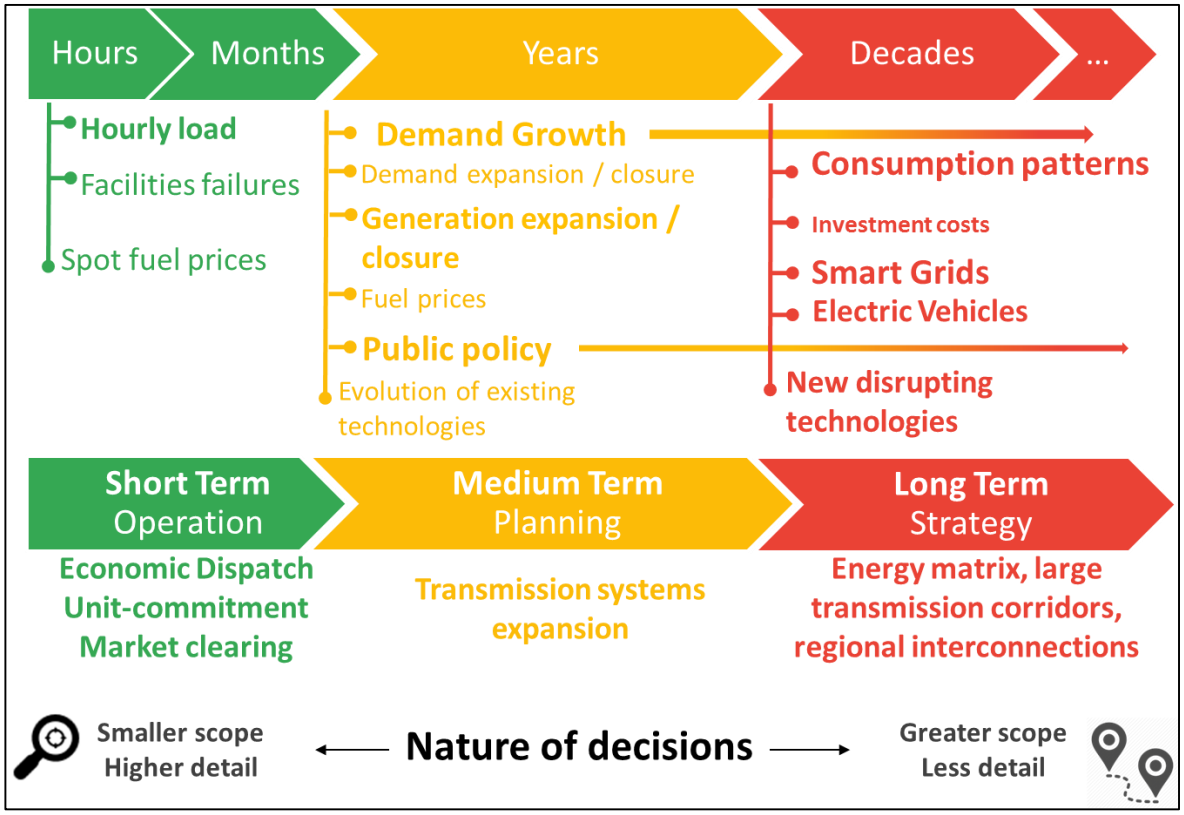

Figure 2-6 Timescale of uncertainties: short, medium and long term are related to operational, planning and strategic actions, respectively. While short term operation decisions require higher detail, strategic decisions in the long term are less detailed but have far greater scope and impact.

2.4. Source of uncertainty: from wind speed to public opposition

Uncertainties are produced by different processes and have different impacts on TEP. Similar sources of uncertainty are grouped based on impacts on TEP and to a lesser extent on process' similarities, thus emphasizing risks (consequences) rather than causes. Four primary sources of uncertainty and one aggregate category are defined next. Distinguishing between such sources has practical applications on identifying common models, relationships between different management strategies and assessment of expansion projects, as discussed subsequently after defining each source.

The four primary sources of uncertainty in TEP are physical, economic, regulatory-geopolitical and social-environmental. Physical uncertainties directly impact the physical infrastructure of a power system (i.e. transmission system or generation plants), cannot be controlled and occur or vary in small timescales ranging from seconds to a year. Examples

comprise outage of generation or transmission facilities, inelastic loads, wind speed, hydrology and natural disasters. Economic uncertainties are the result of some economic process such as market clearing or macroeconomic equilibrium, for example, fuel prices, investment costs and economic growth. Regulatory-geopolitical and social-environmental uncertainties result from complex political or social processes for which no quantitative model is available, such as public opposition to transmission projects or terrorism. Note that weather is not a primary source since the practical focus should be on risks and not causes. More precisely, different risks result whether uncertainty produces outage of facilities or public opposition to new transmission lines. Risks due to facilities outage are somewhat similar to those produced by natural disasters. Hence, weather related risks are grouped under physical or social-environmental sources.

Uncertainties from the four primary sources can be modeled and then fed into some power system and market model (Figure 2-7). Although modeling varies for each uncertainty within the same source, some common modeling approaches can be identified. For example, physical uncertainties such as wind speed, hydrology and hourly residential load are often modeled by probability distributions. Instead, many economic uncertainties are assumed to follow some stochastic process such as Geometric Brownian Motion or discrete Markov chains. Finally, strategic scenarios and alternative cases are used to model social-environmental and regulatory-geopolitical uncertainties.

Aggregate uncertainties in TEP are products of the interaction between different uncertainties. Such interactions are captured by a set of power system and market models. Electricity prices, power flows, reliability standards and total system costs in a regulated industry are clear examples of aggregate uncertainties. Complex uncertainties produced by electricity markets are harder to classify because of the lack of common information and models in a restructured environment. For example, bidding strategies, forward contracting and generation expansion could in principle be determined by directly modeling the equilibrium among profit-maximizing firms. However, generation expansion and bidding strategies are often modeled in TEP as exogenous economic uncertainties, in part because each participant of the market has different expectations and models.

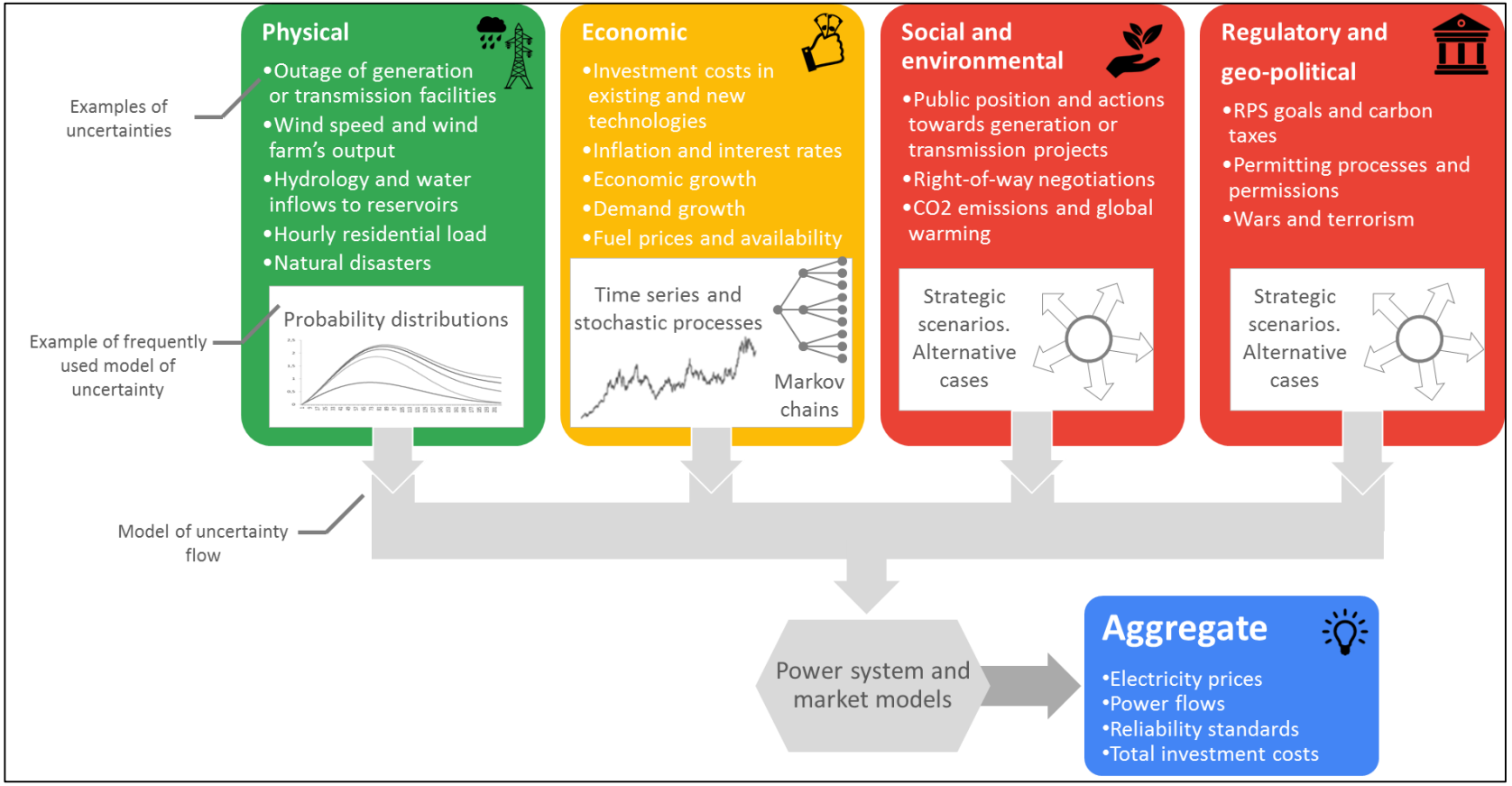

Figure 2-7 Sources of uncertainty in TEP related by power system and market models. Some common models for each source of uncertainty can be identified.

Transmission expansion is directly related to management strategies focused on hedging against one or more sources of uncertainty, sometimes in a bidirectional fashion. For example, TEP is a complement or constituent part of markets integration and resiliency, while energy efficiency and demand response are, to some extent, supplements to transmission investment. Also, feedback loops exist between policies guiding TEP and TEP findings that advice policy-making. For example, regulatory measures aimed at ensuring cost recovery may be triggered by significant transmission investments needed to comply with public policy objectives. The close relationship among strategies suggests that TEP is part of a broader guidance process for the sustainable and efficient development of electricity markets.

The assessment of strategies is inherently multicriteria since each strategy is affected by risks produced by multiple sources of uncertainty. For example, integration of regional markets can increase liquidity, competition, reliability and resiliency, thus addressing economic, physical and aggregate risks (e.g. complementary energy resource utilization, higher adequacy and more competitive electricity prices, respectively). Therefore, assessment of expansion projects and other actions must consider various performance metrics in a multicriteria decision framework. For example, Expected Energy Not Supplied and CO_2 emissions should not be completely monetized and added to economic measures of costs and benefits, since security of service and emissions reductions are objectives on their own. Other public policy objectives also need to be assessed independently, for example increased integration of renewable energy sources. Thus, compromise plans instead of optimal ones are to be constructed in the face of multiple, potentially conflicting objectives. Since the performance of a plan varies across scenarios, different risk measures must be considered for each different objective and its associated performance metric.

Holistic assessments of expansion projects are needed, considering relevant interactions to other complementary or supplementary strategies, and including different costs and benefits associated to different risks. A few management strategies and risk or performance metrics are depicted in Figure 2-8, with the purpose of illustration rather than comprehensiveness.

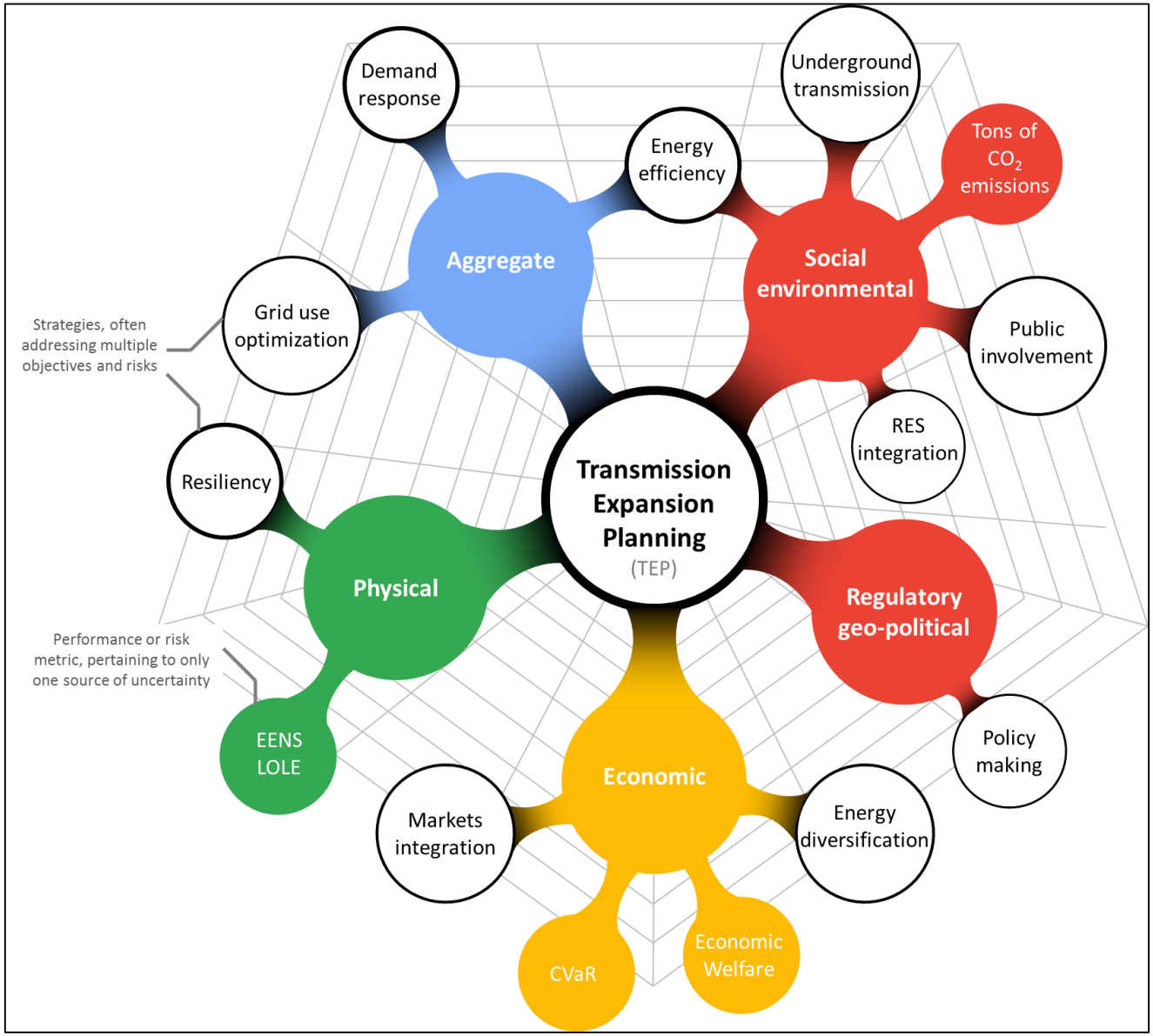

Figure 2-8 Depiction of the multidimensionality and complexity of TEP, when related to strategies and performance measures from varied sources of uncertainty.

2.5. Optimization approaches to TEP under uncertainty

Optimization approaches to TEP under uncertainty are currently unable to deal with multiple and varied uncertainties. Despite increasing research efforts on both methodological and conceptual contributions, no model is currently successful in simultaneously addressing short, medium and long term uncertainties from varied sources. Hence, practical TEP studies often rely on heuristics, simplified analysis and manual search for solutions based on production cost simulations, in spite of optimization models. More research and industrial applications of optimization are required to appropriately aid TEP decision making. However, future works should consider the main advantages and limitations of existing modeling approaches, assessed under the conceptual framework proposed here. Three main research lines are identified next: stochastic programming, robust programming and scenario planning.

Stochastic programming (SP) and robust optimization (RO) are, to a great extent, alternative approaches to deal primarily with medium term and relatively well-structured uncertainties. SP is the most powerful but scope-limited optimization tool available. It deals with **K**nown uncertainties for which probability distributions are completely specified for their temporal evolution. The successive revelation of uncertainty allows SP to exploit the trade-off between decisions to be made here-and-now and wait-and-see decisions that can be postponed to later stages. More importantly, SP can easily adopt expected utility as its objective function, resting on von Neumann and Morgenstern's economic theory of rational choice which constitutes the "correct" quantification and valuation of risks. However, tractability issues counterweight these theoretical advantages, because of the large number of discrete probability-weighted scenarios required for SP. On contrast, RO reduces tractability issues by modeling uncertainty by sets and not individual realizations. It has proven to be a powerful tool for addressing some decision-making problems involving uncertainty parameters whose true probability distribution is difficult to predict. RO's objective is constructing solutions that are feasible against every likely realization of uncertainty within the prescribed set, possibly resulting in overly conservationist solutions. Although conservationism of RO can be controlled, the fact remains that avoiding probabilities and a large number of discrete scenarios comes

at the price of a rather extreme valuation of risk, since worst-case objective functions or uncertainty realizations are considered when optimizing.

Although SP and RO may be applied to long term TEP, the traditional – and mostly implemented - approach assumes divergent strategic scenarios for bounding long term outcome. Instead of assuming probabilities or intervals to describe uncertainty, between 3 and 6 strategic scenarios are usually constructed to represent plausible divergent futures, relying more on expert knowledge and discussion than on quantitative forecasting tools. Because of the long term and qualitative nature of these scenarios, it is hard to assign probabilities to each scenario, also due to the practical difficulty on conveying consensus among stakeholders whom objectives and expectations are different and potentially in conflict. On the other hand, over-conservationism of RO may be more critical in the face of divergent scenarios.

Many alternative approaches, none of which is reliable on itself, are available to deal with **U**nknowable uncertainties in the form of strategic scenarios. Because strategic scenarios lack mathematical structure there is no theoretical criterion for rational decision making. Instead, generic properties of robustness and flexibility often guide the construction of expansion plans. Robustness can be defined as the ability of a plan to withstand multiple futures preserving near-optimality of the solution among these futures. Flexibility is the ability of a plan to be adapted quickly and at low costs to multiple futures. Many criterions have been developed in order to construct robust and flexible plans, most notably minimization of maximum regrets or adjustment cost. However, none of the available criterions, neither combinations of them, have extensively been proved to be reliable for applications in a wide variety of situations. Minimax regret is affected by known issues such as high sensitivity to extreme scenarios and forgoing significant benefits on several scenarios in exchange of a minor improvement in the regret on one particular scenario. Other less known or discussed issues include insensitivity to relative opportunity losses and the lack of coherence, in decision analytic terminology, because valuation of alternatives depends on considering all possible alternatives (more precisely, considering the truly optimal alternative for each scenario). On the other hand, issues and advantages of flexibility have not been extensively assessed, since few applications are available to this date.

In conclusion, no criterion or approach is currently widely accepted for long term scenario TEP, but rather conflicting evidence and arguments are presented in the literature. The hope is for a solid approach to be developed by researchers. However, we must admit the possibility that a reliable approach to find one "optimal" solution is simply never reached, acknowledging the lack of mathematical structures and solid theories to aid decision making under scenarios.

3. LITERATURE REVIEW OF TRANSMISSION EXPANSION PLANNING UNDER UNCERTAINTY

Transmission Expansion Planning (hereinafter TEP) is clouded by uncertainties. Timely and cost-effective transmission expansion is vital for ensuring secure and reliable electricity service to customers, enhancing competition and ensuring market efficiency in electricity markets. Given the irreversibility and long lifetimes of transmission investments, TEP necessarily requires addressing uncertainties on future system conditions years ahead. Because of these fundamental properties of transmission, the importance of developing tools and models to assist power system planning under uncertainty has long been recognized. More specifically, as any decision under uncertainty is made before the uncertainty is revealed, addressing uncertainties allows hedging against risks caused by the outcomes of decisions taken under uncertainty.

However, restructuring of the electricity sector increased and diversified the sources of uncertainty primarily regarding generation expansion, since it is no longer coordinated with transmission expansion. New sources of uncertainty include generation expansion plans but also bidding behavior of firms. More recently, higher integration of variable power sources such as wind farms and solar parks increased uncertainty in an operating timescale. Also, growing environmental concerns introduce uncertainty regarding public policy such as Renewable Portfolio Standards and CO_2 Emissions Allowance. Finally, growing trends on the electricity sector such as electric vehicles, distributed generation, smart grids, demand response and resiliency against natural disasters have also become important for long term planning.

A wide variety of modeling approaches have been applied to TEP under uncertainty. We discuss these approaches organized on three categories based on their underlying assumption on uncertainty quantification, namely: probabilistic assumption (including stochastic programming, real options and probabilistic TEP), limited quantification (mainly robust programming), and scenarios for large uncertainties (including minimax regret criterion and flexibility measures).

A comprehensive literature review on the optimization approaches to transmission expansion planning under uncertainty is presented next. Sections 3.1, 3.2, and 3.3

synthesize and analyze stochastic / probabilistic, robust, and robustness / flexibility approaches to TEP under uncertainty. For each approach we briefly describe the general characteristics; identify journal articles which apply each approach to TEP; describe uncertainty modeling; discuss dis/advantages both in modeling and solution; and suggest future work in TEP applications (Table 3-1). Section 3.4 discusses and summarizes some of the gaps in the literature and the contributions of this thesis in bridging those gaps.

Table 3-1 Summary of Modeling approaches to TEP under uncertainty.

Underlying assumption	Approach	Uncertainty modeling	Key Advantages	Main Disadvantages
Uncertainty is described by probability distributions.	**Stochastic programming**	Stochastic process' including GBM and mean-reverting process' for nodal load and fuel prices.	Detailed modeling of temporal evolution of uncertainties.	Curse of dimensionality. Requires discrete uncertainty description.
	Real options analysis		Detailed modeling of option value and timing of investments.	Often applied to valuation of specific individual transmission projects.
Limited quantification of uncertainty is available.	**Robust programming**	Uncertain parameters belong to a bounded set. Robust TEP focuses on description of nodal load and wind production as intervals.	Allows direct handling of continuous uncertainty. Requires statistical analysis instead of complete probability distributions.	Set description substantially affects tractability and conservationism. Unable to directly handle multistage problems. Treating over-conservationism via budget parameters is an ad-hoc approach.
Uncertainty is unknown but knowable	**Robustness analysis**	Multiple plausible future scenarios.	Dealing with divergent futures and structural changes in electricity markets.	No single theoretical decision criterion has proven to be sufficient.

3.1. Stochastic and probabilistic programming

Describing uncertainty by means of probability distributions allows the application of many different statistical, modeling and optimization techniques grounded on a solid body of literature. However, the solid foundation of the aforementioned techniques comes at the price of assuming well-structured uncertainties for which probability distributions (or stochastic processes) are known. Therefore, the applicability of such approaches is limited to a narrow subset of uncertainties, and arguably precluded for long-term strategic analysis.

Stochastic programming seeks to optimize sequential decisions under uncertain parameters that are revealed on the future, following a known probability distribution or stochastic process (Sen, 2013; Shapiro, Dentcheva, & Ruszczyński, 2009). Applications of stochastic programming to power systems planning and TEP often focus on nodal load uncertainty by setting a two-stage framework, where integer investment variables are solved before the realization of uncertainty (here-and-now decisions) while operation variables are fixed after the realization of uncertainty (wait-and-see decisions). Therefore, algorithms based on Generalized Benders Decomposition may be applied to solve the resulting optimization problem (Alvarez Lopez, Ponnambalam, & Quintana, 2007; Baughman, Siddiqi, & Zarnikau, 1995; Bustamante-Cedeño & Arora, 2008; Park & Baldick, 2013). Multistage stochastic TEP models have also been recently developed (Konstantelos & Strbac, 2015; Loureiro, Claro, & Pereira, 2015; Seddighi & Ahmadi-Javid, 2015). Multistage models are capable of capturing the option value of waiting for more information before committing to an expansion project, whether it is a transmission investment or other flexible alternative that allows further postponement of commitment (e.g. storage) (Konstantelos & Strbac, 2015). However, the detail of multistage dynamic models comes at the price of increased problem scale, thus requiring the application of decomposition algorithms.

Stochastic programming's solid economic foundation should be stressed. According to expected utility theory, rational decision makers should seek to maximize expected utility functions that explicitly incorporate risk preferences (von Neumann & Morgenstern, 1944). Therefore, neither "probabilistic choice" nor stochastic programming are inherently risky as opposed to the assertion of Miranda & Proenca (Miranda & Proenca, 1998a, 1998b), since rational risk-averse decision makers should not minimize expected costs disregarding risk. Moreover, the availability of probabilities, whether objective or subjective, mandates a rational decision maker to use such probabilities to maximize expected utility (Lindley, 1971; Raiffa, 1968). On the grounds of expected utility, mean-variance models inspired by Markowitz theory of financial portfolios (Markowitz, 1959) have been applied to TEP (Alvarez Lopez et al., 2007; Bae, Son, & Kim, 2010; Malcolm & Zenios, 1994), as well as constant risk-aversion functions (van der Weijde & Hobbs, 2012). However, risk-averse decision making on multistage

stochastic problems is not that simple, and several techniques are available, mostly based on CVaR (Homem-de-Mello & Pagnoncelli, 2016).

Stochastic TEP models uncertain parameters as simple probability distributions or more complex stochastic models. Given that most available stochastic programming approaches can only handle discrete uncertainties, discretization of continuous uncertainty models is often required (Birge & Louveaux, 2011, Chapter 2; Sen, 2013). Modeling of load, fuel prices and other uncertainties in TEP comprises diffusion processes taken from financial literature (Hull, 2012), Markov chains, and simple probability distributions. Nodal load has been modeled as a Inhomogeneous Geometric Brownian Motion (Chamorro et al., 2012), and as normal distribution. Regarding fuel prices, (Seddighi & Ahmadi-Javid, 2015) modeled natural gas prices as Geometric Brownian Motion, and (Chamorro et al., 2012) modeled coal and natural gas prices as an Inhomogeneous Geometric Brownian Motion. It should be noted that forecasting and statistical analysis of historical data is a tool used to model uncertainty. Using a point-estimate based on forecasts is a deterministic model of uncertainty, while fitting a probability distribution or stochastic process to historical data is a modeling approach. Little effort has been made to validate forecast and stochastic models for load uncertainty, inherent to power systems and thus, not addressed in other fields (as opposed to fuel prices).

Other approaches to TEP under probabilistic uncertainty are real options analysis (ROA) and chance-constrained programming. ROA attempts to replace net present value as a valuation framework by emphasizing the option value of postponing, modifying or abandon investment projects (Dixit & Pindyck, 1994). ROA has been applied to transmission expansion only as a valuation framework for specific projects (Hedman & Sheble, 2005; Osthues, Rehtanz, Blanco, & Negnevitsky, 2014; Pringles, Olsina, & Garces, 2014), using detailed diffusion models (Chamorro et al., 2012). Chance-constrained programming admits a given probability of violating certain constraints instead of enforcing constraint satisficing in every possible uncertainty realization (Shapiro et al., 2009, Chapter 4). mainly supplying nodal load and curtailment of wind power (Park & Baldick, 2013).

Whereas probabilities for a particular generation expansion may not be reliably assigned based on statistical analyses[13], repetition of events may provide some insight of the structure of the uncertainty, for even though each generation expansion project is unique, they share common characteristics. Moreover, aggregation may provide a basis for probability estimation, which is important for distributed generation. However, each regulatory process is "more unique" that each generation expansion, and therefore analysis of past events may not provide any information on future outcomes, as opposed to generation expansion.

3.2. Robust programming

Robust programming comprises a wide range of mathematical techniques for optimization under uncertainty (Gabrel, Murat, & Thiele, 2014). Recent robust TEP applications have focused on optimization under hard constraints, i.e. those that must be satisfied in any possible realization of the uncertain parameters (Ben-Tal & Nemirovski, 1999; Soyster, 1973). Uncertainty description is therefore deterministic and set-based, so no probability distribution is required (Bertsimas, Brown, & Caramanis, 2011). Allowing the decision maker to select a solution that is feasible for any realization of the uncertain parameter within the prescribed set (i.e. immunize the models against data uncertainty by solving the worst-case problem) has two important implications. First, hard-constraint satisficing for any realization of the uncertainty may produce overly conservative solutions. Consequently, robust programming models with adjustable conservationism levels have been developed (Bertsimas & Sim, 2004), and TEP applications have relied on uncertainty budget parameters to control this conservationism. Second, the construction of the set which describes and bounds uncertainty is crucial to the methodology and the final solution, and encompasses the tradeoff between over conservationism and guaranteeing that the uncertain parameter will effectively lie in the chosen set (Bertsimas et al., 2011).

[13] Subjective probabilities may well be assigned to particular expansion projects, based on the beliefs of the decision maker (DM) regarding some event. Bayesian decision theory provides a theoretical foundation for decision analysis based subjective probabilities, which do not inherently lack mathematical formality nor theoretical foundations. Moreover, rationality of the DM requires subjective probabilities to be rigorously estimated and used as any other probability, given the DM has some notion of likelihood of the events (Pratt et al., 1995; Raiffa, 1968). However, we assume no such probabilities are available, since existing literature often precludes such possibility.

Robust TEP attempts to derive a robust plans in the sense of feasibility for any foreseeable load level in the medium or long term (Alizadeh, Dehghan, Amjady, Jadid, & Kazemi, 2013; Chen, Wang, Wang, He, & Wang, 2014; Jabr, 2013; Mínguez & García-Bertrand, 2016; Ruiz & Conejo, 2015). Along load levels, uncertainties such as generation expansion / closure (Chen et al., 2014; Ruiz & Conejo, 2015), wind farm's output (Jabr, 2013), transmission investment costs (Alizadeh et al., 2013) and operational costs (Ng & Sy, 2014) have also been considered. In a different approach, (Sauma, Traub, & Vera, 2015) consider only delays in connection of new power plants to the grid.

All available models are static and, with the exception of (Ng & Sy, 2014; Sauma et al., 2015)[14], describe each uncertain parameter by a bounded interval. Only Ruiz & Conejo (2015) briefly comment the importance of building uncertainty sets that ensure, within a given confidence level, that the random variable will be contained in the resulting set. Although the selection and construction of uncertainty sets is an active research area (Bertsimas et al., 2011; Gabrel et al., 2014), Ruiz & Conejo do not apply such techniques because of the complexity of the resulting formulations. Instead, based on hourly load levels and power production up to the year 2014, the authors suggest the application of statistical techniques and forecast models to derive upper and lower bounds for uncertain parameters in the target year 2020.

Robust programming does not directly address uncertainty valuation, since no trade-off is considered as opposed to the probabilistic approaches. Ensuring feasibility against any possible realization of uncertain parameters does not directly measure the impact of each outcome. Robust programming approaches which consider uncertainty in objective function explicitly minimize the worst-case objective (Alizadeh et al., 2013; Ng & Sy, 2014; Ruiz & Conejo, 2015). Conservationism level of the solution is controlled by an uncertainty budget parameter which restricts the maximum allowed uncertainty deviation with respect to a certain reference value. Jabr (2013) consider an integer parameter that restricts the number of uncertain parameters which may simultaneously attain its extreme

[14] (Ng & Sy, 2014) assume load and operational costs to be equal to a nominal value plus a linear combination of some unspecified underlying factors called "primitive uncertainties", which indirectly correlate load and operational costs based on historical data. (Sauma et al., 2015) consider discrete delays in connection-to-the-grid of new power plants.

values. Alizadeh, Dehghan, Amjady, Jadid & Kazemi (2013) also allow real budgets to permit one parameter to vary in a subset of the original interval. Chen, Wang, Wang, He & Wang (2014) consider upper and lower bounds on total new generation power as well as intervals for each parameter. Ruiz & Conejo (2015) restrict the absolute percent deviation of aggregate load or power produced in a given region of the network. After the decision maker provides an uncertainty set and budget, the model produces a robust solution which may be highly dependent on the uncertainty level set by the budget parameter (Ruiz & Conejo, 2015). However, only (Alizadeh et al., 2013) discuss the selection of the uncertainty budget, based on simple upper bounds for the chance of constraint violation in terms of the budget parameter. Solution techniques include Benders Decomposition (Jabr, 2013), primal cutting planes (Ruiz & Conejo, 2015), scenario generation (Chen et al., 2014; Sauma et al., 2015), and direct solution of the full robust counterpart (Alizadeh et al., 2013). (Mínguez & García-Bertrand, 2016) analyzes and improves computational performance of previous approaches.

In the future, requirements of research on robust TEP are primarily related to uncertainty set construction and uncertainty budget selection; computational tractability of existing approaches; development of models and solution algorithms for multi-stage robust programming; and applications of other techniques such as distributional robustness or combinations of stochastic and robust programming to TEP (Gabrel et al., 2014).

Other approaches have been applied to TEP under **u**nknown uncertainties, such as fuzzy programming (Andreoni, Garcia-Agreda, Strada, & Saraiva, 2006; Choi, El-Keib, & Tran, 2005; Da Rocha & Saraiva, 2013; Kamyab, Fotuhi-Firuzabad, & Rashidinejad, 2012; Peng Wu, Haozhong Cheng, & Jie Xing, 2008; Tome Saraiva, Miranda, & Pinto, 1996), information-gap decision theory (Dehghan, Kazemi, & Amjady, 2014; Sarhadi & Amraee, 2015; Taherkhani & Hosseini, 2015) and connection numbers with grey theory (Jin & Cheng, 2008). However, the practical validity of these approaches has not been confirmed, due to the scarce applications available in the literature.

3.3. Robustness and flexibility analysis

Scenario approaches together with decision criterions such as minimax regret and adjustment cost have been applied to TEP under Unknowable uncertainties, since both the

stochastic and robust approaches are unable to model multiple divergent futures produced by structural changes in electricity markets. Uncertainties are captured by a given set of future scenarios, all deemed plausible but without any information regarding likelihood of each scenario, let alone probabilities[15]. Although many authors abstract from the content of the assumed scenarios, a common approach to construction of scenarios in both academy and industry is based on intuitive logics school (van der Weijde & Hobbs, 2012)[16], which conceives scenarios as a mean to bound, understand and communicate uncertainty, rather than accurately predicting or forecasting future outcome (Bradfield, Wright, Burt, Cairns, & Van Der Heijden, 2005; Myers & Kitsuse, 2000). Each scenario should present a trajectory to some future state in a narrative and compelling fashion, outlining the interaction between the most important uncertainties in an internally consistent manner. The set of considered scenarios should also be diverse enough to appropriately represent the range of possible future outcomes (Schoemaker, 1993). Assigning subjective probabilities to these scenarios is a complex or even impossible task, since such probabilities will hardly convey consensus among the different stakeholders whom objectives are potentially in conflict (E. O. Crousillat, Dorfner, Alvarado, & Merrill, 1993; Linares, 2002). Moreover, the foundation of such subjective probabilities is not clear since scenarios are not necessarily mutually exclusive and collectively exhaustive.

Since there is no theoretical method to aggregate the performance metric of a plan (e.g. total cost) under several possible future scenarios with no assigned probabilities, the objective becomes finding expansion plans that attain some generic desirable properties under uncertainty, such as robustness and flexibility (Andreoni et al., 2006; Maghouli, Hosseini, Oloomi Buygi, & Shahidehpour, 2011; W. Walker, Haasnoot, & Kwakkel, 2013). Robustness can be defined as the ability of a plan to withstand multiple futures, either preserving feasibility or near-optimality of the solution among these futures (Mulvey, Vanderbei, & Zenios, 1995). On the other hand, flexibility is the ability of a plan

[15] Assuming that all scenarios are plausible is not equivalent to assuming they're equally likely and, therefore, having equal probabilities.

[16] A review study prepared by the Pontifical University Catholic of Chile for the Chilean National Energy Commision (CNE) found that planning processes on United States, Europe and Latin America often employs strategic scenarios to represent long term uncertainties (Watts & Rudnick, 2014).

to be adapted quickly and at low costs to multiple possible futures (Zhao, Dong, Lindsay, & Wong, 2009), and thus may be understood as dynamic robustness (W. Walker et al., 2013). Other approaches to planning under such deep uncertainties have been proposed in the fields of military planning, policy making and sustainability, including assumption based planning that analyzes the impact of broken assumptions (Dewar, 2002; W. Walker et al., 2013), resilience and precautionary principles (Aven, 2015).

Minimization of maximum regret is by far the most common criterion used to develop robust transmission expansion plans under scenarios (B. Cedeño & Arora, 2011; Bustamante-Cedeño & Arora, 2008; M O Buygi et al., 2004; Charlin, Rudnick, & Araneda, 2015; Chen et al., 2014; De la Torre, Feltes, Román, & Merrill, 1999; de Silva, Rider, Romero, & Murari, 2006; Fang & Hill, 2003; Maghouli et al., 2011), although several other criterions are available (Bunn, 1984, Chapter 2; M O Buygi et al., 2003). Minimax regret is an extreme measure of robustness, and thus is highly sensitive to the choice of extreme scenarios (Higle & Wallace, 2002). Moreover, it is insensitive to opportunity losses other than the maximum, thus potentially forgoing significant benefits in several scenarios in favor of a reduced advantage in one extreme scenario. Also, minimax regret is insensitive to relative losses of opportunities, so normalized or relative regret has been proposed instead (Bagajewicz, n.d.; Motamedi, Zareipour, Buygi, & Rosehart, 2010). Finally, minimax regret is not independent on irrelevant alternatives. Indeed, the performance metric of every plan may change in the case a new plan becomes available, that is optimal under a certain scenario but not necessarily optimal in a minimax regret sense, thus contradicting the coherence principle of decision analysis (Bunn, 1984; Lindley, 1971)[17]. These issues with minimax regret are explained in detail in annex B.

Flexible TEP was introduced by (Zhao et al., 2009) by minimizing the maximum adjustment cost required to adapt a given plan to a change in the realized scenario. The authors implement goal programming to satisfice multiple conflicting objectives, therefore not *optimizing* but rather *satisficing* (Simon & A., 1956; Zeleny, 1982). Adjustment cost for a particular scenario is defined as the additional investment required

[17] The discipline of decision analysis is based on subjective probability theory, heavily criticizing minimax regret. The main subjects of decision analysis are decision making under uncertainty and multiple objectives (Schum, 2013).

to adapt a plan in order to satisfice a given target in the given scenario. The measure of flexibility is then defined as the maximum adjustment cost across scenarios. (Mejia-Giraldo & McCalley, 2014) extend the approach to a multistage framework combining strategic scenarios with uncertainty sets and adjustable robust programming to account for uncertainties within the boundaries of a given scenario, thus providing more flexible solutions that are also less dependent on the choice of scenarios. However, the model requires selecting an arbitrary parameter to include the adjustment cost directly in the objective function, thus requiring the decision maker to clearly quantify preferences on the property of flexibility, not on particular projects and its impact. Such quantification is an extremely difficult ex-ante requirement for a decision maker, who may be tempted to test different values of the parameter until achieving an acceptable solution. Therefore, the modified approach of (Mejia-Giraldo & McCalley, 2014) also remains in the field of satisficing, while a rational decision maker seeks to optimize its decisions. Moreover, the authors do not clearly state the difference between and adjustment cost and the traditional regret, and therefore both measures could be directly summed to provide a broader performance indicator. (Qiu et al., 2015; Zhao, Foster, Dong, & Wong, 2011) extended flexibility to expansion under stochastic and probabilistic uncertainties, although the flexibility approach is more valuable under scenarios without probabilities.

A few hybrid models attempt to mix different approaches to TEP under uncertainty, mainly between scenarios and probabilistic or robust approaches. (M O Buygi et al., 2004; Gu & McCalley, 2010) use many probabilistic scenarios produced by Monte Carlo simulation along each strategic scenario. Multiple decision criterions are simultaneously used in a fuzzy multiobjective framework in (Maghouli et al., 2011) and (Majid Oloomi Buygi et al., 2006). (Maghouli et al., 2011) considers maximum regret and adjustment cost, while (Majid Oloomi Buygi et al., 2006) considers maximum regret, average regret and a custom metric called “degree of robustness”, based on relaxing maximum regrets.

Attention should also be paid to the development of scenarios and their interactions to TEP models, even though TEP research correctly focuses on developing new optimization techniques. At least two main kinds of scenarios are considered in both academy and industry, case-driven and strategic. Case-driven scenarios describe many possible combinations of outcomes of some set of uncertainties such as winter / summer peak,

generation expansion or load growth rates (Bustamante-Cedeño & Arora, 2008; Majid Oloomi Buygi et al., 2006; Gorenstin, Campodonico, Costa, & Pereira, 1993; Mejia-Giraldo & McCalley, 2014; PJM, 2015a), while strategic scenarios follow the intuitive logic approach previously explained, thus emphasizing internal consistency by analyzing interactions among several uncertainties, and selecting between three and six scenarios to broadly represent plausible uncertainty realizations (Gu & McCalley, 2010; Linares, 2002; F D Munoz, Hobbs, Ho, & Kasina, 2014; National Grid, 2015a, 2015b; PJM, 2015b; Sanchis et al., 2015; van der Weijde & Hobbs, 2012). In order to improve scenario construction methodologies and integration with optimization approaches, further research is required regarding strategic scenarios, for stochastic reduction techniques may be readily applied to case-driven scenarios (Growe-Kuska, Heitsch, & Romisch, 2003). For example, assessment on the divergence of scenarios would allow quantifying how broad the set of selected scenarios is, thus signaling scenarios that may represent uncertainty too narrowly, or identify potentially high impact scenarios not considered. Also, it is not clear that planning for a reduced set of strategic scenarios provides successful hedges against risk in case an intermediate scenario occurs. Moreover, methodologies for optimization under both medium term case-driven scenarios and long term strategic scenarios have not been developed, since most papers focus on one of the two timescales.

3.4. Discussion, Gaps in the Literature and Contribution of this Thesis

Current TEP models do not successfully fulfill the needs of real TEP processes. Tremendous efforts have been carried during the past two decades in order to develop tractable and useful transmission expansion planning models which can assist decision makers facing such a complex problem. Despite amazing advances in a wide variety of areas related to TEP modeling and solution methodologies, tractable, detailed and sophisticated TEP models are still missing in both the state-of-the-art and professional solutions.

Furthermore, no one-size-fits-all approach to TEP under uncertainty is currently available in the literature. Instead, a variety of alternative modeling and solution approaches have been developed for dealing with particular kinds of uncertainties. The

three main branches of stochastic / probabilistic, robust, and robustness / flexibility assume different models of the underlying uncertainties and are therefore useful for different situations. Particularly, the tractability advantages of a robust formulation seem like a promising alternative for modeling a wide variety of medium and short term uncertainties (e.g. hourly wind output and load levels, and delays on connections of planned generation units). However, robust optimization does not currently seem appropriate for dealing with strategic long-term scenarios, since enforcing strict feasibility under the variety of these scenarios may be too costly. Furthermore, decision-makers often focus on the tradeoffs between multiple conflicting objectives under long-term strategic scenarios.

From a techno-economic perspective, we have classified approaches to TEP under strategic scenarios as ranging between the two extremes of stochastic programming, and robustness / flexibility approaches. **To date, the debate regarding stochastic programming and alternative approaches has been half-blind**. Stochastic TEP is clearly the superior methodology when the probability structure is assumed to be available for scenarios (F D Munoz et al., 2014; Wallace, Nationale, & Industriel, 1998).[18] Advocates of alternative methods argue in favor of other approaches that assume no scenario-weighting probabilities are available (Miranda & Proenca, 1998b). Particularly, minimax regret has long been employed in TEP literature and keeps on being used in practical TEP processes, for example in PJM, UK and Chile.

The issues affecting minimax regret criterion have not been thoroughly analyzed and discussed (Higle & Wallace, 2002) (those deficiencies are explained in appendix B). Therefore, it is currently unclear whether these theoretical deficiencies are relevant in practical TEP applications. On the other hand, despite theoretical robustness, the practical limitation of stochastic TEP regarding assumed scenario probabilities has only been pointed out but never before quantified nor analyzed. Instead, alternative methods have been proposed to manage risks without assuming probabilities, thereby abandoning

[18] Stochastic or probabilistic does not necessarily imply optimizing expected value but rather a general utility function considering risk preferences (often risk aversion). Expected cost would be a superior criterion only for a risk neutral decision maker.

stochastic TEP without further analysis.[19] Hence, it seems that no efforts have been made to bridge the gap between these two extreme approaches to TEP under long-term strategic scenarios.

This thesis contributes to bridging the gap between stochastic programming and robustness approaches by quantifying and analyzing the relevance and the impact of weighting probabilities assumed for long-term strategic scenarios in the optimal transmission expansion plan. It is the first step in shedding lights regarding alternative decision-making criterions for TEP under uncertainty.

Instead of simply avoiding probabilities, we assume these probabilities to be uncertain themselves. Therefore, we explicitly tackle the *ambiguity* that undermines decision-maker's confidence on scenario probabilities and on the methodologies that rely on such probabilities (Ellsberg, 1961). Explicitly modeling such ambiguity as a second-order probability distribution to describe the first-order probability distribution on scenarios has been proposed before but never applied to TEP (Borgonovo & Plischke, 2015).

In modeling ambiguity in TEP under scenario uncertainty, the contributions of this thesis are threefold:

- First, we quantify the practical relevance and impact of ambiguous scenario probabilities in stochastic TEP.
- Second, we present a novel distributionally robust decision-making criterion for TEP under ambiguity.
- Third, we compare the optimal solution under alternative TEP decision-making criterions such as expected cost, minimax regret and distributional robustness, by both theory and simulation based on the theory on multiobjective decision-making and the well-established concept of Pareto efficiency.

[19] It is worth mentioning that critiques to expected costs have been used to argue in favor of alternative methodologies, since expected costs do not account for risks (Miranda & Proenca, 1998a). The logic of the argument is flawed since other alternatives lie between expected cost and robustness criterions such as minimax regret. For example, considering risk-averse decision-making attitudes is possible in a stochastic TEP framework, and models in this direction have been developed (Alvarez Lopez et al., 2007). Therefore, the lack of explicit risk management in expected cost stochastic TEP does not necessarily argue in favor of minimax regret.

Hence, from a practical perspective this thesis allows for explicit quantification and analysis of the impact and relevance of scenario-weighting probabilities. Such novel analysis is without precedents in TEP literature. Also, from a theoretical perspective this thesis enables decision-makers to include more information in the model, thereby contributing to compliance with Savage's axioms and the Subjective Expected Utility Theory (Pratt, Raiffa, & Schlaifer, 1995; Savage, 1951, 1954). Thus, the theoretical foundation of TEP models as tools to assist rational decision-making is enhanced.[20]

The methodology developed to model ambiguity in TEP under scenario uncertainty is presented next.

[20] The purpose of this thesis is to shed lights on the prescriptive role of TEP models (i.e. recommending what should be rationally done) instead of the descriptive role of decision criterions (i.e. how people actually make decisions).

4. METHODOLOGY

The methodology proposed for this thesis is partly based on a two-stage stochastic TEP model. Our simplified model assumes commitment to transmission investment decisions must be made today (i.e. *here-and-now*), whilst the benefits of such decisions are realized when operating the system in the future (i.e. *wait-and-see*). Uncertainty on future conditions is captured by a reduced number of probability-weighted scenarios that describe the long-term trends in the power market (e.g. fuel and technology costs, disruptive technologies, and regulation). An example of such a model is studied in (F D Munoz et al., 2014).

However, unlike any previous work we explicitly model the inaccuracy of probabilities assigned to long-term scenarios by assuming that such probabilities are themselves uncertain and described by a probability distribution, which we call *second-order probability distribution*.[21] Therefore, no single transmission expansion plan can be readily identified as optimal, but rather a set of efficient expansion alternatives can be built and further analyzed. Such set of Pareto efficient alternatives is obtained by interpreting the costs under each scenario as a different objective of a multi-objective optimization problem (Miranda & Proenca, 1998b).

Also unlike any previous work, we quantify the robustness of each expansion plan with respect to the probabilities assumed for long-term scenarios (i.e. the scenario probabilities). Assuming a second-order probability distribution that describes the probabilities for scenarios, we define a new "second-order probabilistic robustness" measure as the likelihood that a given expansion plan is optimal.[22] To efficiently evaluate the second-order robustness we integrate the second-order probability density function on the set of probabilities for which a given expansion alternative remains optimal. Such a calculation is repeated for each expansion alternative in the convex hull of the efficient set of expansion alternatives.

[21] Hereinafter, we refer as “scenario probabilities” as the set of probabilities assigned to each scenario, i.e. the set of numbers $\{p_i : \sum_i^S p_i = 1 \ \wedge \ p_i \in [0,1]\}$.
[22] Note that such an approach is conceptually and practically different to a local sensitivity analysis.

In turn, the Pareto frontier of expansion plans is obtained by applying the specialized genetic algorithm NSGA-II, commonly used in multi-objective TEP literature (Arabali, Ghofrani, Etezadi-Amoli, Fadali, & Moeini-Aghtaie, 2014; Majid Oloomi Buygi et al., 2006; Maghouli et al., 2011) (Figure 4-1 depicts the proposed methodology). Explicit optimal solutions to the stochastic TEP are also used to seed the genetic algorithm in order to ensure at least attainment of the exact convex part of the Pareto frontier, and also improve the convergence rate of the genetic algorithm (section 4.6).

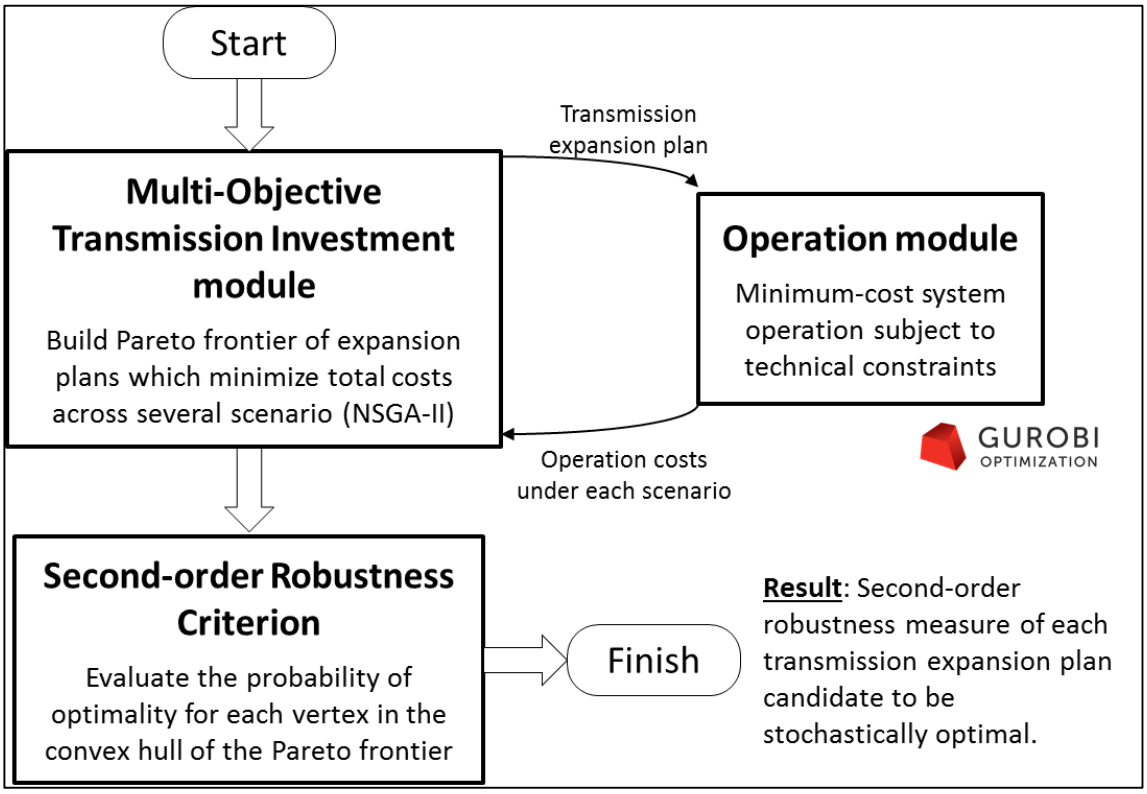

Figure 4-1 Summary of the proposed solution methodology.

Finally, the optimal solution under alternative decision-making criterions is compared. We consider stochastic programming with expected cost (thus assuming risk-neutral decision-makers), minimax regret, minimax cost, and two novel criterions: the aforementioned second-order stochastic robustness, and also a novel distributional robustness criterion. Since we prove that the optimal solution under these alternative criterions lies in the Pareto frontier, we first obtain the frontier and then obtain optimal solutions by evaluation of the reduced set of efficient alternatives.

After defining the nomenclature to be used, the methodology is explained in further detail in the rest of this section.

4.1. Nomenclature

4.1.1. Sets and Indices (Italic Font)

S	Scenarios, indexed s.
H	Hours of system operation, indexed h.
B	Buses, indexed b.
G	Generators, indexed k.
G_b	Generators at bus b.
L	Transmission lines, indexed l.
L_E	Existing transmission lines.
L_C	Candidate transmission lines.
Ω_l	Pair of nodes connected to line l.

4.1.2. Parameters (Normal Font)

$\mathrm{MC_{k,s}}$	Generation marginal cost.
VOLL	Value of Lost Load.
$\mathrm{IC_l}$	Investment cost in candidate transmission line.
$\mathrm{D_b}$	Demand at bus b.
$\mathrm{S_l}$	Line susceptance.
$\mathrm{M_l}$	Large positive number.
$\mathrm{G_k}$	Installed generation capacity.
$\mathrm{N_1}$	Target planning year (when the scenarios realize and system operation is simulated).
$\mathrm{N_2}$	Final year of planning horizon.
$\mathrm{d_h}$	Duration of load block (or representative hour) h.
$\Phi_{\mathrm{b,l}}$	Element of node-line incidence matrix.

4.1.3. Variables (Italic Font)

y_l	Transmission investment decision.
$g_{k,s,h}$	Generation.
$r_{l,s,h}$	Load curtailment.

$f_{l,s,h}$	Power flow in transmission lines.
$\theta_{b,s,h}$	Phase angle.
OC_s	Operating cost.
IC	Total investment cost.
TC_s	Total cost.
$\tilde{p}_s$	Scenario probability of occurrence.

4.2. Two-stage Stochastic Transmission Expansion Planning under Scenario Uncertainty based on Mixed Integer Linear Programming

A two-stage stochastic TEP model is presented next. The model is based on a linear approximation of the power flow equations (e.g. the DC power flow). Transmission investments as well as a full year of system operation is optimized. A set of system states is used to represent system operation. These states can represent blocks of a load duration curve (Wood, Wollenberg, & Sheblé, 2013), or hours in a full 8760-hour model. The assumptions of this model (e.g. DC power flow, no unit-commitment constraints, among others) are often adopted by academic references on long-term TEP.

The mathematical optimization model presented next is an always-feasible linear-programming problem based on the classical disjunctive formulation of two-stage stochastic TEP (Alguacil, Motto, & Conejo, 2003; Alvarez Lopez et al., 2007; Bahiense, Oliveira, Pereira, & Granville, 2001; Binato, Pereira, & Granville, 2001).

$$\min_{\{y,g,r,f,\theta\}} \sum_{s\in S} \tilde{\mathrm{p}}_{\mathrm{s}} \cdot TC_s \tag{1}$$

subject to:

$$TC_s = IC + OC_s \;\; \forall s \tag{2}$$

$$IC = \sum_{l} \mathrm{IC_l} y_l \tag{3}$$

$$OC_s = \sum_{h} \mathrm{d_h} \cdot \left[\sum_{k\in G} \mathrm{MC_{k,s}} g_{k,s,h} + \sum_{b\in B} \mathrm{VOLL} \cdot r_{b,s,h} \right] \tag{4}$$

$$\sum_{k\in G_b} g_{k,s,h} + r_{b,s,h} + \sum_{l} \Phi_{b,l}\, f_{l,s,h} = \mathrm{D_{b,s,h}} \;\; \forall b,h,s \tag{5}$$

$$f_{l,s,h} = \mathrm{S_l}\left(\theta_{b,s,h} - \theta_{p,s,h}\right) \;\; \forall\, (b,p) \in \Omega_l, l \in L_E, h, s \tag{6}$$

$$\left| f_{l,s,h} - \mathrm{S_l}\left(\theta_{b,s,h} - \theta_{p,s,h}\right) \right| \leq \mathrm{M_l}(1 - y_l) \;\; \forall\, (b,p) \in \Omega_l, l \in L_C, h, s \tag{7}$$

$$\left|f_{l,s,h}\right| \leq \mathrm{F_l} \quad \forall\, l \in L_E, h, s \tag{8}$$

$$\left|f_{l,s,h}\right| \leq \mathrm{F_l} y_l \quad \forall\, l \in L_C, h, s \tag{9}$$

$$0 \leq g_{k,s,h} \leq \mathrm{G_k} \quad \forall k, s, h \tag{10}$$

$$0 \leq r_{b,s,h} \leq \mathrm{D_b} \quad \forall b, s, h \tag{11}$$

$$y_l \in \{0,1\} \quad \forall l \in L_C \tag{12}$$

The objective is to minimize total investment and expected operation costs (eq. 1, 2, 3 and 3). Both investment and operation costs are weighted by the scenario probability $\tilde{\mathrm{p}}_\mathrm{s}$ for notational clarity with the following parts of the methodology. However, the expectation is only applied to operation costs since investment costs do not depend on the realized scenario (because investment decisions are taken before the realization of uncertainty). Indeed, note that the objective function is equivalent to the following:

$$\sum_{s \in S} \tilde{\mathrm{p}}_\mathrm{s} \cdot TC_s = IC \cdot \left(\sum_{s \in S} \tilde{\mathrm{p}}_\mathrm{s}\right) + \sum_{s \in S} \tilde{\mathrm{p}}_\mathrm{s} \cdot OC_s = IC + \sum_{s \in S} \tilde{\mathrm{p}}_\mathrm{s} \cdot OC_s \tag{13}$$

Technical constraints, described next, must be met for a transmission expansion plan to be feasible in terms of the physical operation of the power system. Equation 4 is the nodal power balance, which ensures that power is conserved in each bus of the system. Indeed, the total power incoming to a bus (composed of local generation and incoming power flow from transmission lines) plus load-shedding equals total power consumption in the same bus. Load-shedding enables some or all of the load in a given bus to be unmet, although at a high penalty cost given by the term with VOLL (set in this thesis to $10,000 per MWh of load unmet) in operating costs (eq. 4).

Power flow through existing and candidate transmission lines is given by the linear approximation of Kirchoff's laws (eqs. 4 and 5). Note that a sufficiently large disjunctive parameter M_l ensures that equation 5 for candidate transmission laws collapses to equation 4 if the corresponding candidate transmission line is built (i.e. $y_l = 1$), whereas the constraint is free otherwise (refer to (Bahiense et al., 2001) for further discussion). Equations 6 and 7 enforce thermal limits on existing and candidate transmission lines. Equations 8 and 9 bound generation and load curtailment, while equation 10 states the integrality of transmission investment decisions.

4.3. TEP under Scenarios with no Weighting Probabilities

The problem described in section 4.2 assumes that probabilities for long-term scenarios are known and accurate. If no weighting probabilities are available, (Miranda & Proenca, 1998b) suggests reinterpreting the transmission expansion planning problem as a multiobjective optimization problem where the decision maker seeks a solution which simultaneously minimizes total costs under each possible future scenario. The mathematical statement of the multiobjective problem is:

$$\min \begin{Bmatrix} TC_{s_1} = IC + OC_{s_1} \\ \vdots \\ TC_{s_S} = IC + OC_{s_S} \end{Bmatrix} \tag{14}$$

subject to constraints (2) through (12).

In such multiobjective optimization problem the decision maker would like to implement a solution that simultaneously attains the minimum total costs for each scenario (i.e. an ideal or utopia solution). However, such an ideal solution does not exist in general, especially in problems with conflicting objectives such as long-term transmission expansion planning under various scenarios.

Instead, a set of efficient expansion alternatives (also known as Pareto frontier) can be identified for multiobjective optimization problems (Zeleny, 1982).[23] The decision maker can then further analyze efficient expansion alternatives in order to choose one alternative for implementation. Although there are various approaches for solving multiobjective optimization problems (Marler & Arora, 2004; Zeleny, 1982), TEP literature often focuses on identifying the efficient set of expansion alternatives (M O Buygi et al., 2004; Maghouli et al., 2011; Motamedi et al., 2010). Such an approach enables greater decision-making flexibility by presenting a set of possible decision alternatives, which also contains the optimal solution under a variety of decision criterions (i.e. stochastic, minimax regret, minimax cost, minimum distance to the utopia point under several distance metrics).

Assuming scenario probabilities, a single optimal transmission expansion plan can be identified by solving problem (1) – (12). It can be shown that using scenario weighting

[23] An efficient alternative is defined as one for which no other alternative exists that is not inferior in any of the objectives, and that is strictly superior in at least one of the objectives.

probabilities the optimal solution will always belong to the convex hull of the Pareto frontier (see Figure 4-2 for an illustration). Furthermore, the convex hull of the Pareto frontier provides the set of all expansion alternatives that are the optimal solution of the stochastic problem (1) – (10), for any possible set of scenario probabilities.

Nevertheless, the decision maker may also be interested in further analyzing efficient solutions which are not optimal under any set of weighting probabilities. These solutions (such as solution C in Figure 4-2) cannot be discovered by minimizing a probability weighted sum of the objectives (hence they are called unsupported solutions), or solutions in the concave part of the Pareto frontier (Ehrgott & Gandibleux, 2003). Although irrelevant for a stochastic decision criterion, efficient but unsupported alternative may be selected by ex-ante decision criterions such as minimax regret (see appendix B.5).

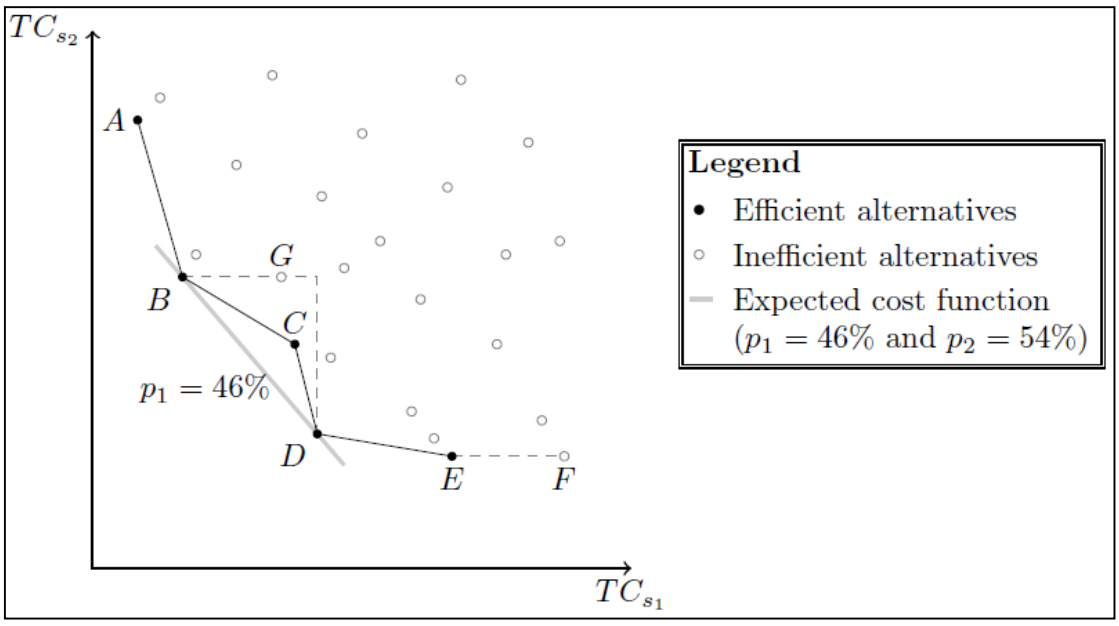

Figure 4-2 Depiction of a concave Pareto frontier for TEP under two scenarios, and the optimal solutions when minimizing a probability weighted-sum of the objectives.

Hence, discovering the full set of Pareto efficient solutions can enable more decision making flexibility and also allow easy comparison of alternative decision making criterions. Furthermore, such a Pareto frontier of solutions enables the analysis of alternative solutions under different sets of probabilities, an important analysis since these probabilities for long-term scenarios are unlikely to be readily available, or to be accurate and convey the consensus of all the stakeholders involved in the transmission expansion

planning process. A novel decision making criterion is introduced next in order to account for the ambiguity in scenario probabilities.

4.4. Second-Order Probabilistic Robustness Criterion

A second-order probabilistic robustness measure is defined next, as a measure of the robustness of a given expansion plan to the probabilities of the scenarios. Scenario probabilities are assumed to be uncertain themselves. We further assume that a joint probability distribution function is available to describe scenario probabilities. We call such joint PDF the *second-order PDF*. We then calculate the likelihood that a given expansion plan is the stochastic optimal, since the probabilities of the scenarios are not known precisely. The likelihood that an expansion plan is optimal is what we call the *second-order robustness measure* of an expansion plan.

We proceed now to the formal definition of the problem. Denote $STEP(p)$ as the stochastic optimization problem defined by equations (1) through (12), given a set of scenario probabilities p. For a given p, at least one (and potentially more) stochastically optimal expansion plan χ can be identified by solving:

$$\chi = \text{argmin}\left(STEP(p)\right) \tag{15}$$

Denote the set of all stochastically optimal expansion plans as Λ. We call these X the stochastically optimal plans. Formally:

$$\Lambda = \{\chi : \chi = \text{argmin}\, STEP(p),\ \ \forall\, p \in \mathbb{R}^S \ /\ 1^T p = 1 \wedge 0 \leq p \leq 1\} \tag{16}$$

The optimal expansion plan should be robust against the uncertainty in scenario probabilities p, since these probabilities are not accurately known. We quantify the relative robustness of each stochastically optimal plan against probability uncertainty, in order to understand the importance and the impact of the scenario probabilities in the optimal plan. Furthermore, a new decision criterion is proposed whereby the plan which is more robust against uncertainty on the scenario probabilities is chosen.

We assume scenario probabilities to be described by some known joint probability distribution function, which we call the second-order PDF. Formally, the second-order PDF is defined as:

$$f : p \in \Gamma \rightarrow \mathbb{R}_0^+ \tag{17}$$

$$\wedge \quad \int_{\Gamma} f(p)\, dp = 1$$

where p is a vector with the probabilities of $S-1$ scenarios, and Γ is the set of all possible probabilities.[24] Hence, $f(p)$ describes the relative likelihood that the scenario probabilities are determined by p. Furthermore, the absolute likelihood that the scenario probabilities belong to some set Ξ can be obtained as $\Pr\{p \in \Xi\} = \int_{\Xi} f(p)\, dp$.

We define the second-order robustness measure $r(\chi)$ as the probability that $\chi \in \Lambda$ is the optimal solution to the stochastic TEP. The formal definition is:

$$r(\chi) = \int_{\Xi(\chi)} f(p)\, dp \tag{18}$$

where $\Xi(\chi)$ is the set of probabilities for which $\chi \in \Lambda$ remains the stochastically optimal plan. Formally:

$$\Xi(\chi) = \{p : \chi = \text{argmin}\,(STEP(p))\} \tag{19}$$

The second-order robustness measure $r(\chi)$ is a quantitative indicator of the robustness of the stochastically optimal plan χ against uncertainty in the scenario probabilities. Therefore, the new decision criterion for TEP under scenario uncertainty is to choose the expansion plan which maximizes the second-order robustness:

$$\chi^* = \underset{\chi \in \Lambda}{\text{argmax}}\ r(\chi) \tag{20}$$

For example, in the case of STEP under two scenarios, the only second-order uncertain variable is the probability of occurrence of the first scenario, denoted p_{s_1}. We assume a symmetrical triangular PDF for describing the uncertainty in p_{s_1} (Figure 4-3). The peak of the triangular PDF is located in $p_{s_1} = 0.5$ (where $p_{s_2} = 1 - p_{s_1} = 0.5$). An stochastically optimal expansion plan X can be identified by solving $\min\,(STEP(0.5))$. Such plan remains the optimal solution to $STEP(p)$ for all $p \in \Xi(\chi) = [0.4, 0.75]$. The robustness measure $r(X)$ is then the area under the triangular PDF in the interval [0.4, 0.75]. We would like a solution to be the stochastic optimal plan for the most likely set of probabilities p_{Nom}, while simultaneously being robust against uncertainty in scenario probabilities.

[24] Note that probabilities sum to one, hence $S-1$ independent probabilities can be chosen provided they sum less than one, i.e. $\Gamma = \{p : 1^T p \leq 1 \wedge 0 \leq p \leq 1\}$.

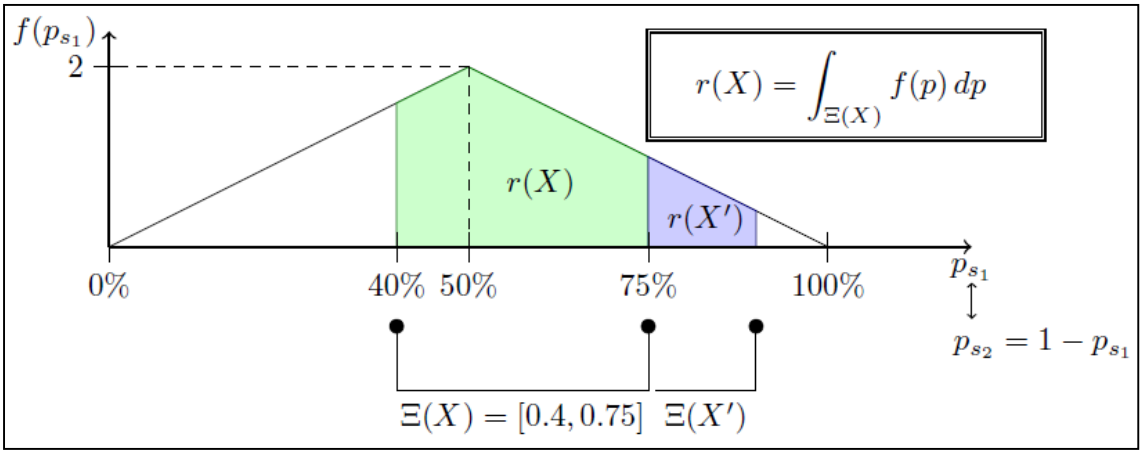

Figure 4-3 Second-Order Probabilistic Robustness Measure.

The second-order stochastic robustness measure defined previously is mostly useful for quantifying and analyzing the relevance and impact of scenario probabilities in the stochastic solution. Particularly, the proposed decision criterion which maximizes the second-order robustness measure is useful to identify the spread of the stochastic solutions in the Pareto frontier, with respect to a given second order probability distribution. For example, slight or significant differences between the equiprobable stochastic solution and the second-order robust solution can be readily identified by applying the second-order robustness criterion.

4.5. Calculation Methodology for the Robustness Criterion

The robustness measure is calculated by leveraging the geometry of the problem which allows us to use traditional KKT duality conditions for optimality (Figure 4-4).

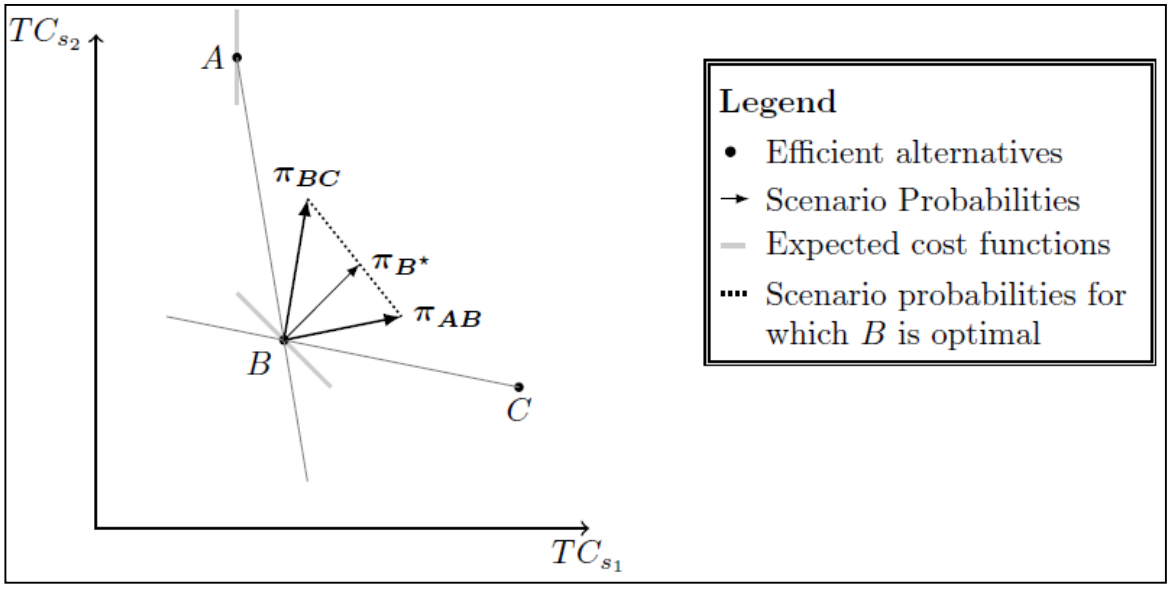

Figure 4-4 Duality concepts applied to scenario probabilities of a stochastically optimal transmission expansion plan.

By geometric arguments based on convex-cones it can be shown that the robustness measure $r(\chi)$ can be calculated by taking the integral (18) in the convex combination of the probabilities which define the adjacent facets of the convex hull of the Pareto frontier. In other words, the set of optimality $\Xi(\chi)$ comprises all probabilities which are a convex combination of the probabilities for which the solution χ ceases to be optimal. Although the underlying problem $STEP(p)$ is non-convex due to the integrality of investment variables, the arguments in this case are analogous to the duality conditions of linear programming, since a polytope describes the alternative solutions to the stochastic TEP (see Figure 4-4).

4.6. Distributional Robustness and other Decision Criterions

We propose a novel distributionally robust decision making criterion, which is also analyzed in this thesis for TEP under scenario uncertainty and ambiguity. Such a criterion assumes that scenario probabilities are known to belong to some *ambiguity set*. Then, the performance of each alternative transmission expansion plan is measured by calculating the worst possible expected cost that can result from scenario probabilities within this ambiguity set. Finally, the solution which minimizes the worst possible expected cost (within the ambiguity set) is selected by the decision maker for implementation.

This is a distributional robustness formulation since the scenario probabilities distribution is not known exactly. Instead, it is known that the scenarios probabilities distribution lies within the ambiguity set. The criterion proceeds to select the optimal solution based on a worst-case scenario analysis within the ambiguity set. It is worth noting that the distributionally robust criterion assumes a less structured ambiguity than the second-order robustness criterion proposed previously, since scenario probabilities are only assumed to lie within some set without specifying the likelihood of particular realizations of those scenario probabilities.

Distributional robustness is an approach developed in the context of the increasing literature on robust optimization (Gabrel et al., 2014), and has recently been applied to model wind farm uncertainty in the unit commitment problem (Xiong, Jirutitijaroen, & Singh, 2016). To our knowledge, distributional robustness has never before been applied to TEP under scenario uncertainty.

Two other alternative decision making criterions are also considered for comparison, namely minimax cost and minimax regret. Minimax cost is a worst-case approach, equivalent to the distributional robustness criterion when the ambiguity set of scenario probabilities is assumed to be unconstrained (i.e. the decision maker wishes to hedge against the worst case under any possible coherent set of probabilities). Minimax regret is a widely used decision making criterion for TEP under uncertainty. Minimax regret aims at minimizing the maximum regret or opportunity loss, with respect to the optimal solution that would have been implemented by the decision maker, had he known beforehand what scenario was going to occur. Both criterions –minimax cost and minimax regret– are defined precisely and analyzed in depth in appendix B.

All of the minimax-like criterions –distributional robustness, minimax cost and minimax regret– are minimized by efficient solutions (proofs in appendices B and C). This fact allows us to build the Pareto frontier and then select the optimal solution under each criterion from among the set of Pareto efficient solutions, simply by selecting the highest performing solution (i.e. an enumeration within the Pareto efficiency set). This approach is theoretically sound and also simple to implement and fast to calculate. The approach is effective in practical experiments since very few efficient solutions are often obtained,

and the alternative criterions are easily calculated from the information produced by the Pareto frontier production methodology.

4.7. Proposed Model and Solution Methodology for Finding Pareto-Efficient Transmission Expansion Plans

The methodology developed to compare alternative decision making criterions and to analyze the relevance and impact of scenario probabilities is a mixture of a multiobjective optimization genetic algorithm (NSGA-II) and the stochastic TEP formulation. The genetic algorithm allows full discovery of the Pareto frontier, including unsupported plans (i.e. plans in the concave part of the frontier). The stochastic model is used to seed the genetic algorithm, thus enhancing convergence rates. Moreover, optimizing the MILP stochastic model ensures efficiency for stochastic solutions, whereas our unsupported solutions are not ensured to be efficient due to the heuristic nature of genetic algorithms. Both procedures are briefly discussed next.

The full (approximate) set of Pareto-efficient expansion plans is built with Non-Dominated Sorting Genetic Algorithm – II (Deb, Pratap, Agarwal, & Meyarivan, 2002), which is commonly used in multiobjective TEP applications. NSGA-II is a genetic algorithm specialized for multiobjective optimization problems. Being a genetic algorithm, NSGA-II is easy to implement. Particularly in this problem, NSGA-II only requires simulation of the system operation, which is a continuous always-feasible linear program instead of the more complex and time consuming Mixed Integer Linear Programming TEP problem. The lack of binary investment decision variables results in a simpler problem which is easier to solve, therefore allowing for the simulation of more complex operation problems (e.g. (Maghouli, Hosseini, Buygi, & Shahidehpour, 2009; Maghouli et al., 2011; Motamedi et al., 2010)). However, being metaheuristics, genetic algorithms do not ensure convergence to globally optimal solutions (which always exist for the stochastic TEP model described by equations (1) – (12)) nor to the true Pareto frontier. Moreover, slow convergence to the Pareto frontier can occur in middle and large-scale problems. Genetic algorithms also rely on heuristics which must be chosen by the user, with no clear criterions to choose among alternative heuristics or parameters for those heuristics (e.g. whether to include random individuals in all generations, and the

probabilities of random mutation). More importantly, there is no clear termination criterion for multiobjective genetic algorithms, and the original NSGA-II simply terminates after a fixed number of generations despite the possibility of being stuck in the same (or very similar) Pareto frontier for many generations.[25]

The stochastic TEP optimization model is also solved in order to ameliorate some of the aforementioned weaknesses of NSGA-II. Solutions of the stochastic TEP model are fed to NSGA-II as the initial population. Thereafter, NSGA-II allows for easy discovery of Pareto efficient alternatives in the concave part of the Pareto frontier (whereas the stochastic TEP model only renders efficient alternatives in the convex part of the frontier). Our experiments show that seeding efficient solutions to NSGA-II significantly improves convergence rates. To our knowledge, this is the first attempt to seed NSGA-II with proven optimal solutions to the underlying TEP objective-weighted problem.

Apart from NSGA-II, many other methods exist for building Pareto efficient frontiers of multiobjective optimization problems (Ehrgott & Gandibleux, 2003; Marler & Arora, 2004). However, alternative methods are not further considered since NSGA-II seems sufficient to fulfill the objectives of this thesis. Nevertheless, in the future it might be worthwhile to implement alternative methods. One such alternative is the ε-constraint method, which optimizes a single objective whilst constraining all of the other objectives. Thus, efficient solutions in both the concave and convex part of the Pareto frontier can be discovered by exploring a grid within the feasible objective space. The ε-constraint method is an interesting alternative to NSGA-II since it allows for non-heuristic construction of the Pareto frontier whilst preserving simplicity and ease of implementation. Of course, more complicated methodologies might be interesting in the future as well.

A comparison among alternative decision-making criterions is also carried. The minimax regret solution is obtained among the Pareto efficient solutions, as discussed in

[25] Our experiments with the test cases described in section 5 suffered from these issues of a pure NSGA-II methodology, particularly slow convergence and the need to adjust parameters and parts of the algorithm itself in order to improve performance of the method.

appendix B. The solution which minimizes the maximum cost is also obtained among the efficient solutions.

The solution methodology is implemented in Python interfacing with the Gurobi MILP solver[26], and run on a PC with Intel Core i5 of 3.33 GHz CPU and 4 GB RAM. Operation simulation models (OPF) are used by NSGA-II, while stochastic transmission expansion optimization models (TEP) are also formulated and solved to seed initial individuals to NSGA-II.

[26] All the source codes developed and used in this thesis are publicly available online in http://github.com/csvelasq/TepUnderScenarios, under the free and open source MIT license.

5. RESULTS AND DISCUSSION

The methodology developed for this thesis and described in section 4 is applied to two test cases commonly used in TEP literature: Garver 6-bus system (section 5.1) and IEEE 24 bus Reliability Test System (section 5.2). Two scenarios are used in each case, and the second-order probability distribution is a triangular distribution centered on the equiprobable point (identical to the distribution depicted in Figure 4-3).

5.1. Small Scale Case Study: Garver 6 bus

Garver's 6 bus case study system was introduced by Len Garver in 1970 and has been used ever since as a small-scale case study for TEP models. The system, depicted in Figure 5-1, is composed of 5 interconnected buses, and one disconnected bus.

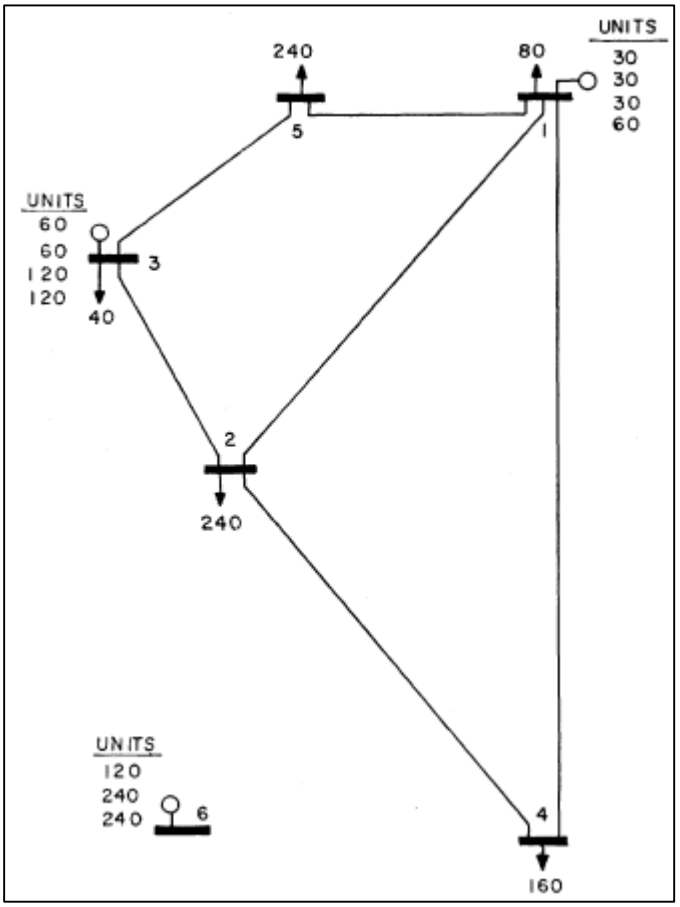

Figure 5-1 Garver 6 bus test-case system.
Source: (Garver, 1970).

Details of the transmission system, transmission investment alternatives and costs, and generation costs and capacities, were obtained from (Alguacil et al., 2003) and (Alvarez Lopez et al., 2007) (full details of this case study are found in Appendix A.1). Two scenarios were considered, one with the base load levels by Garver, and one with a 30% increase in load levels on each node. On the second scenario, additional generators were added in nodes 4 and 5, for a total of 300 additional MW. For each scenario, two load-blocks were considered for system operation in a whole year: peak, with a duration of 760 hours; and valley, with a duration of 8000 hours and a load equal to 75% of peak load. Investment costs are annualized to 15 years with a discount rate of 7%.

Each corridor is assumed to hold a maximum of 3 parallel transmission lines. However, for the sake of solution times, a subset of 12 transmission candidate lines was considered. The reduced number of candidate lines allowed full enumeration of all possible transmission expansion plans, thus allowing us to validate both the NSGA-II methodology as well as the TEP optimization model. The methodology was also applied to the full set of alternative transmission expansion plans, only to discover the same Pareto frontier found by restricting the set of candidate lines.

The results of this test case are depicted in Figure 5-2 and summarized in Table 5-1. Four efficient alternatives were found in this case. In this particular test case, the same alternative expansion plan (the second blue alternative in the figure, from left to right) is the optimal solution under our new second-order robustness criterion, under minimax regret, and is also the solution to the expected cost TEP with equally likely scenarios (however, the solution with the minimax cost criterion was the optimal solution to scenario 1). Furthermore, that optimal alternative was the only alternative under which no load-shedding occurred under either scenario. Hence, it is safe to say that, in this particular test case, there is one clear alternative expansion plan to be chosen under scenario uncertainty.

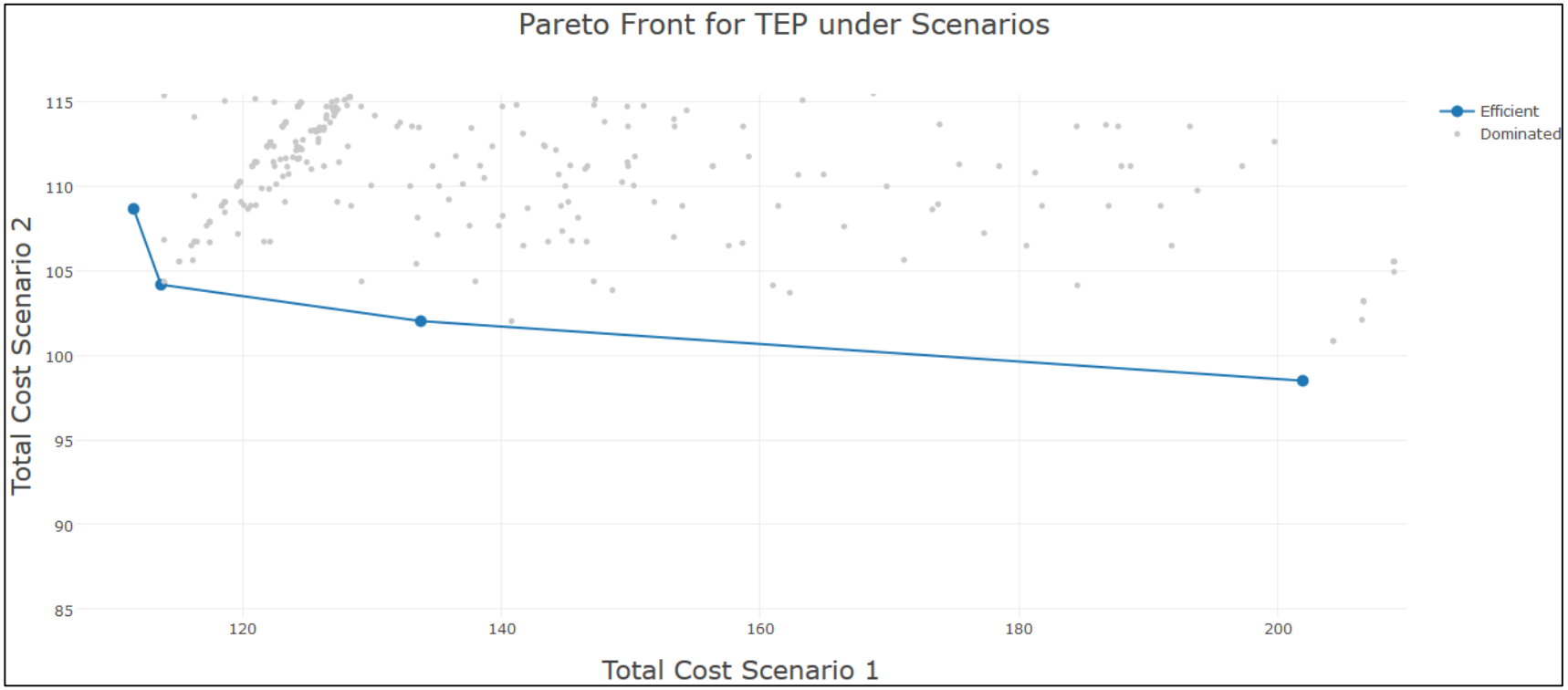

Figure 5-2 Pareto frontier for Garver 6-bus test case.

Table 5-1 Summary of results for Garver 6-bus test case.

Built Lines*	**Investment Cost [MMUS$]**	**Operation Costs S1 [MMUS$]**	**Total Costs S1 [MMUS$]†**	**Operation Costs S2 [MMUS$]**	**Total Costs S2 [MMUS$]**	**Probability range S1****	**Robustness Measure [%]*****
['TC3(B2-B3)', 'TC5(B2-B6)', 'TC6(B2-B6)', 'TC8(B3-B5)']	$100.00	$190.17	$201.92	$86.75	**$98.50**	[0.0% , 4.9%]	-
['TC3(B2-B3)', 'TC5(B2-B6)', 'TC6(B2-B6)', 'TC7(B2-B6)', 'TC8(B3-B5)']	$130.00	$118.51	$133.78	$86.75	$102.02	[4.9% , 9.7%]	1.4%
['TC3(B2-B3)', 'TC8(B3-B5)', 'TC10(B3-B6)', 'TC11(B4-B6)', 'TC12(B4-B6)']	$148.00	$96.31	$113.70	$86.79	$104.17	[9.7% , 68.0%]	77.6%
['TC3(B2-B3)', 'TC5(B2-B6)', 'TC8(B3-B5)', 'TC11(B4-B6)', 'TC12(B4-B6)']	$130.00	$96.31	**$111.58**	$93.39	$108.66	[68.0% , 100.0%]	-

†Optimal costs for deterministic TEP under each scenario are emphasized in bold font.
*See details of the test case in Annex A.1 for visualizing built transmission lines.
**Details the range of probabilities for scenario 1, for which each expansion plan is the optimal stochastic solution. Probabilities for scenario 2 are implied (since they both sum to unity).
***Robustness measure is not calculated for border solutions (i.e. optimal solutions for the deterministic TEP of each scenario).

The previously described test case is rather particular since load-shedding occurs in Pareto efficient solutions, and also because of the lack of discrepancies between multiples decision making criterions. However, the following observations can be made:

- The Pareto efficient frontier can be extremely small and, moreover, fully convex, in particular cases were alternative scenarios are very similar. In this case, only four solutions were efficient, and all of them lied in the convex part of the Pareto frontier. This lack of diversity on the Pareto frontier is obviously due to the similarity of the two scenarios considered, which results in a reduced subset of transmission candidate lines to be good projects. Indeed, the set of efficient alternatives is very similar in terms of candidate lines built. We know the similarity of optimal expansion plans is not valid in general. Moreover, transmission lines built in optimal determinist expansion plans may not even be part of the optimal stochastic solution (F D Munoz et al., 2014). However, it is interesting to see a toy case in which this similarity and simplicity of the problem does occur.
- A single, extremely robust solution can emerge in particular test cases, although this cannot be guaranteed in general. In this case, the criterions of minimax regret, expected cost, max second-order robustness and max reliability (i.e. minimizing loss-of-load across all scenarios) happened to agree. However, the minimax cost criterion suggested the decision-maker to choose another alternative.
- The proposed solution methodology is valid since the Pareto frontier coincides with full-enumeration. However, the proposed solution methodology can discover the Pareto efficient frontier in much less computation time than full enumeration. Moreover, full-enumeration is not even possible if more than 20 candidate transmission lines are considered (in that case, 2^{20} alternative expansion plans would need to be simulated). Nevertheless, improvements to the solution methodology are possible and may be interesting for future works, as discussed in section 4.6.

5.2. Medium Scale Case Study: IEEE 24 bus Reliability Test System

The IEEE 24 bus Reliability Test System, depicted in Figure 5-3, is considered as a medium scale case system.

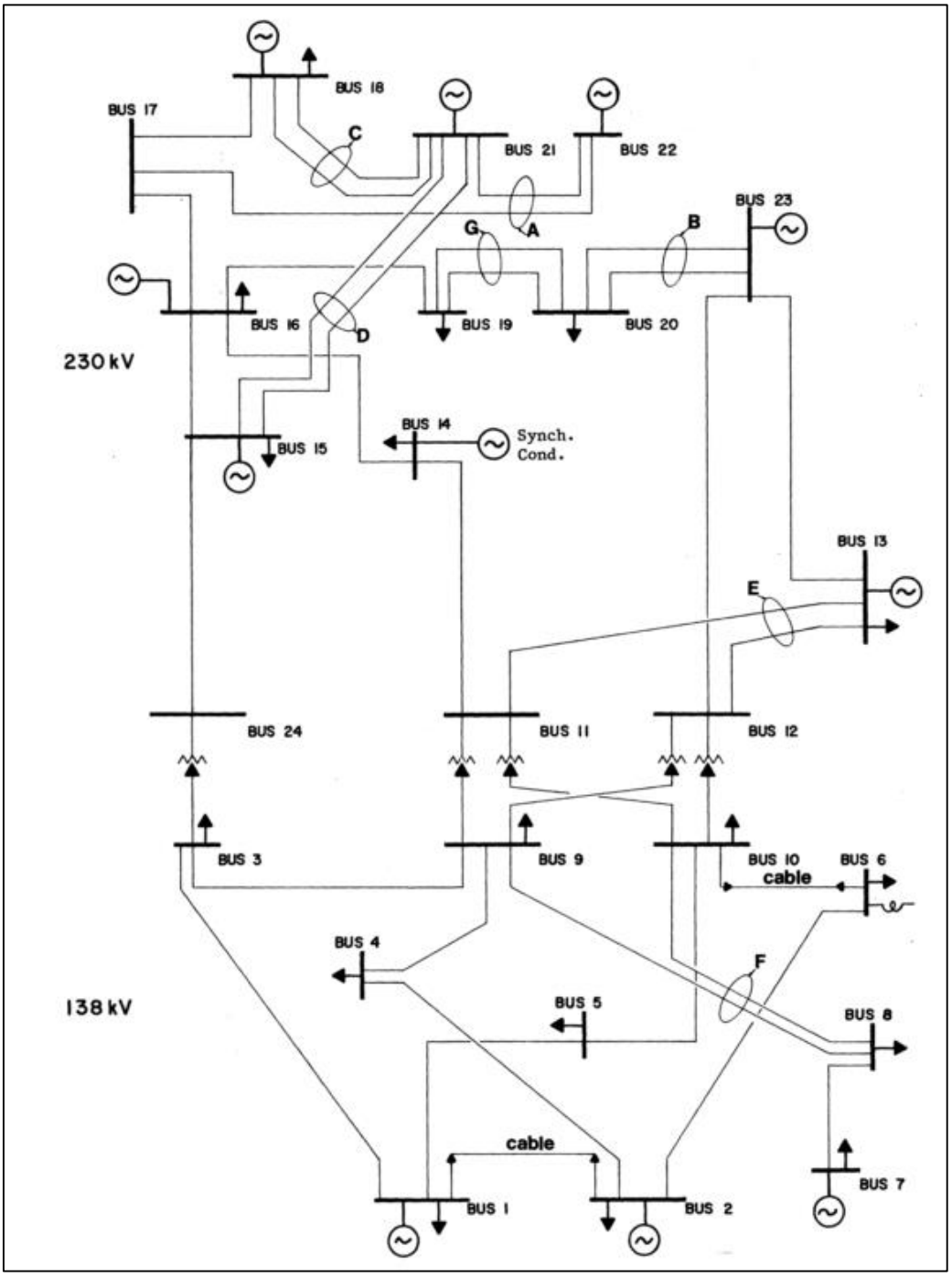

Figure 5-3 System topology for IEEE 24 bus Reliability Test System.

Source: (Reliability Test System Task Force of the Application of Probability Methods Subcommittee, 1979)

Transmission expansion is optimized for operating the IEEE 24 bus RTS system 15 years in the future (details in appendix A.2). Two scenarios were considered. In the high economic growth scenario, a 4% load growth per annum is assumed, yielding 5.1 GW in year 15. In the other scenario, characterized by low economic growth, 1% load growth per annum is assumed, yielding 3.3 GW in year 15. In both scenarios, the same load growth is assumed for each bus. Analogous to the small test case, two load-blocks were considered for system operation in the whole year in each scenario: peak, with a duration of 760 hours; and valley, with a duration of 8,000 hours and a load equal to 75% of peak load.

Under both scenarios, the existing fleet of generating power plants is assumed to grow by a factor proportional to total load growth in the 15-year period. Moreover, wind and solar PV plants are included in the high-growth scenario, for a total average injection of 650 MW of variable renewable energy sources in both load blocks (peak and valley). Hence, generating capacities of 4.7 GW and 8.0 GW are considered under the low and high growth scenarios, respectively. Solving the generation expansion planning problem under each scenario is a more sophisticate approach for determining the generating fleet to consider under each scenario. Moreover, endogenously determining the market's response to transmission investment in terms of generation investment is a more comprehensive framework for TEP under uncertainty (van der Weijde & Hobbs, 2012). However, manually devising the generating fleet is sufficient for the purposes of this thesis, which focuses on transmission investment. In particular, the approach applied in this thesis taken from (Fang & Hill, 2003).

Following (Fang & Hill, 2003), each corridor is assumed to hold a maximum of 3 parallel transmission lines. Unit investment costs of 10 and 20 MUS$/km are considered for transmission lines in 138 and 230 kV, respectively. All transformers are assumed to cost 50 MUS$. All investment costs are annualized to 15 years with a discount rate of 7%. Synchronous condensers and reactors are not considered in this test case.

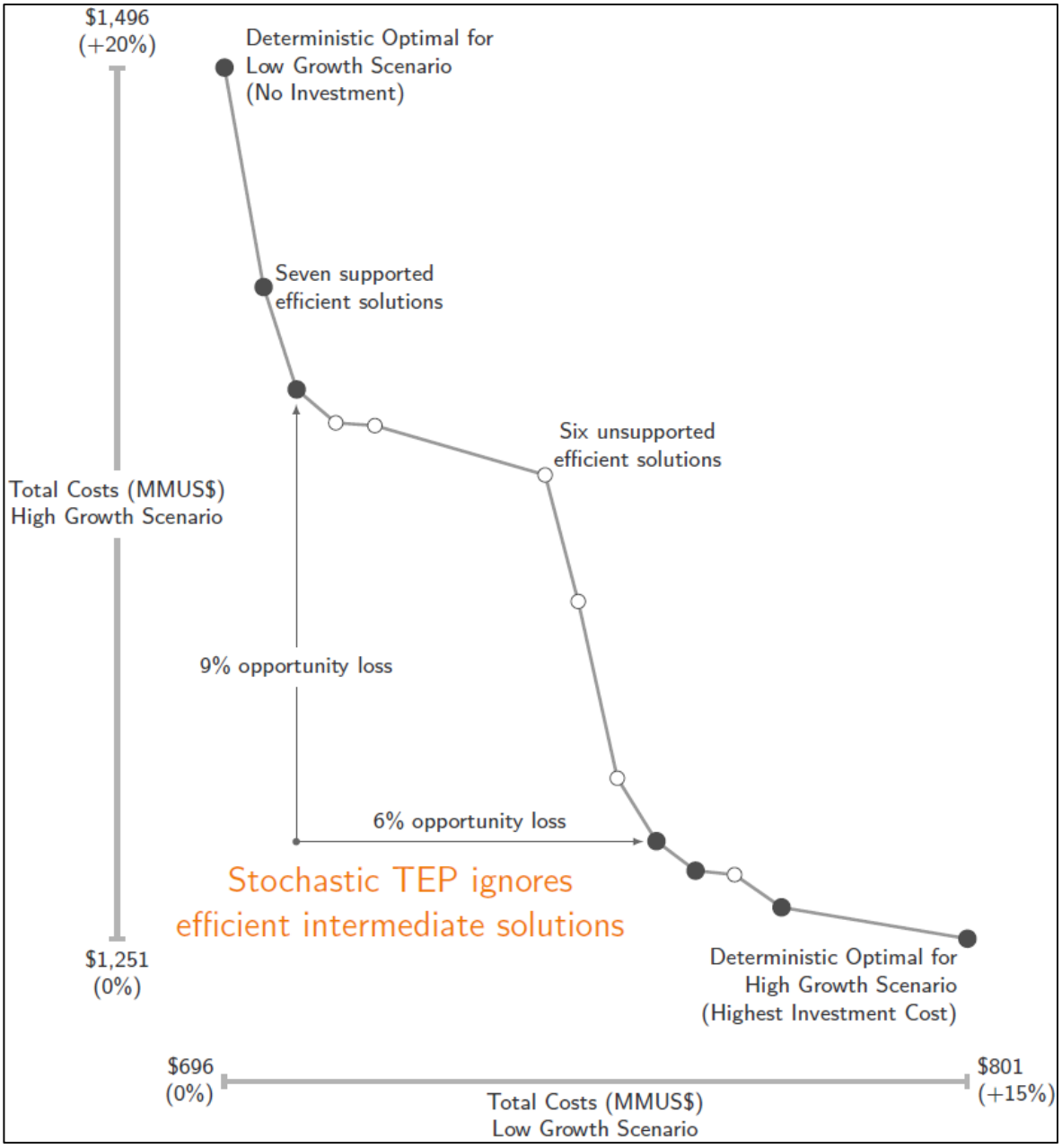

Figure 5-4 TEP Pareto frontier for the 24 bus test case under two scenarios.

Results are depicted in Figure 5-4 and summarized in Table 5-2 and Table 5-3. The Pareto frontier comprises 16 efficient expansion plans, 9 of which are supported and therefore solutions to the stochastic TEP problem under some set of scenario probabilities.[27] Total costs (hereinafter all costs are in millions of US dollars) of efficient

[27] Three weakly Pareto optimal solutions are also included for illustration purposes.

solutions vary 15% for the low growth scenario (between $696 and $801) and 20% for the high growth scenario (between $1,251 and $1,496). Transmission investment ranges from no new lines (in the deterministic optimal solution to the low growth scenario) to $950 in three new lines (in the deterministic optimal solution to the high growth scenario), with intermediate solutions building up to six new lines across only six of the 45 available corridors (see Table 5-3 for details). It is worth noting that efficient solutions build lines in three corridors that are not optimal in a deterministic sense under either scenario (corridors (10-12), (7-8) and (9-12)). Also, only generation operational costs and transmission investment costs are in compromise in this test case, since none of the efficient solutions incurs in load shedding (as opposed to the small scale test case).

In this case, all the evaluated criterions yielded different solutions. The stochastic solution with equiprobable scenarios (plan 12 in Table 5-2) remains optimal for probabilities between 46% and 60% for the low growth scenario[28], yielding a robustness measure of 26%[29]. However, the second-order robustness criterion suggests implementing a neighboring solution (plan 15) with robustness measure of 30%. It is worth noting that a change of only 4% of the scenario probabilities results in a different stochastic solution. Nevertheless, the equiprobable stochastic solution and the second-order robust solution differ by less than 2% in total costs and only by two built transmission lines.

Despite the similarity of the solutions suggested by the equiprobable stochastic and the second-order robustness criterion, the fact remains that both criterions diverge leaving the decision maker with no single solution alternative to implement with confidence in the face of irreversible and capital intensive transmission investments which raise electricity tariffs today in the hope that it will lower tariffs 15 years in the future. Moreover, one unsupported solution (plan 13) lies in between the equiprobable stochastic solution and the second-order robustness solution. Further examination of efficient solutions in the concave part of the Pareto frontier may well be worth. For example, a dynamic stochastic

[28] A probability range of [46%, 60%] in the low growth scenario is equivalent to a range of [40%, 64%] in the high growth scenario, since both probabilities must sum to one.

[29] Recall that the robustness measure is not equal to the range of probabilities (14%) but rather equal to the integral of the probability distribution (triangular in this case) over the probability range (26%, since a triangular distribution implies that the decision maker deems equiprobable scenarios to be more likely that one scenario being more likely than the other.

model or a flexibility analysis might drive the decision maker to implement plan 13. Other analyses such as sustainability and reliability may also argue for implementing plan 13 or even other plans in the concave Pareto frontier.

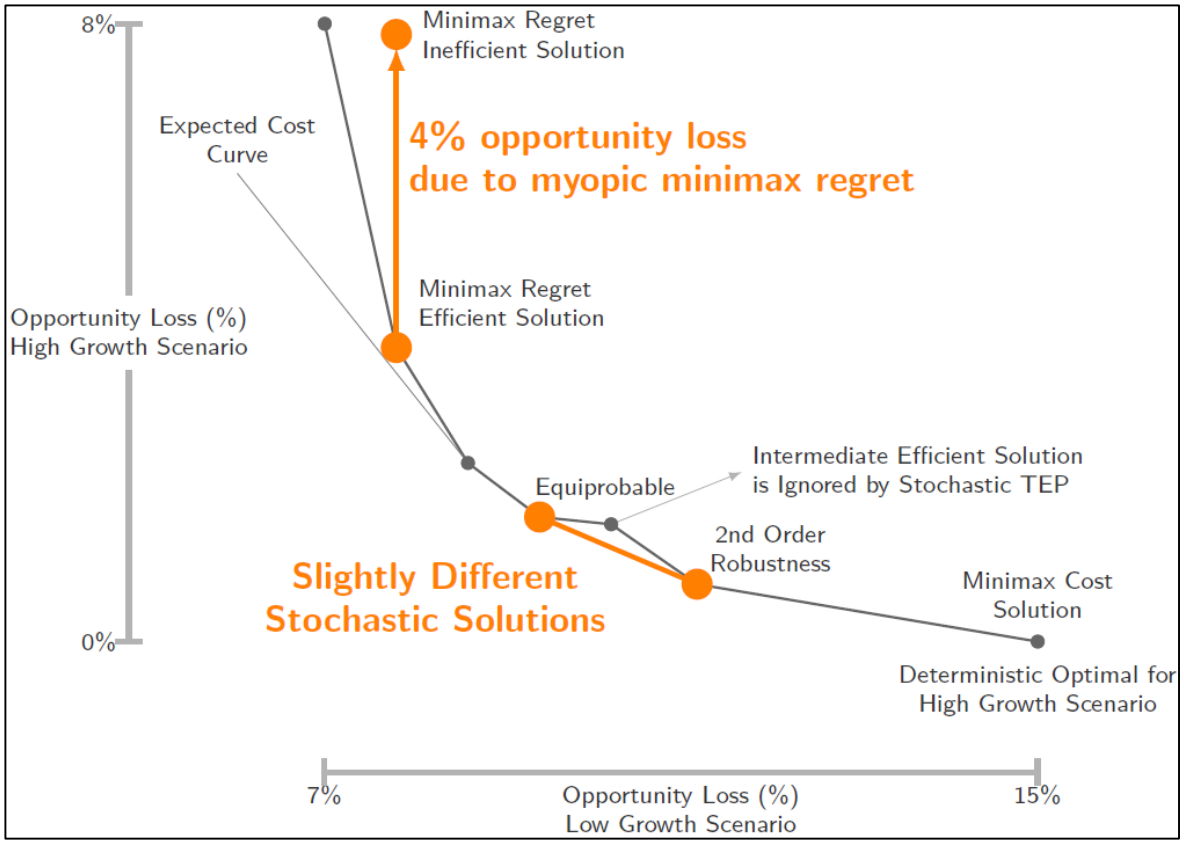

Figure 5-5 TEP solutions by alternative decision making criterions, for the 24 bus test case under two scenarios.

Minimax regret and minimax cost criterions also yield different solutions. In this two scenario case the minimax cost criterion obviously recommends implementing the deterministic optimal solution to the high growth scenario, which is also the efficient solution with highest investments (MMUS$950). In turn, the minimax regret criterion suggests a nonsupported efficient solution (plan 10) which lowers investment costs at the expense of higher operation costs under the high growth scenario (see Table 5-3). Recall that such nonsupported solution would never result from a stochastic TEP model despite the set of scenario probabilities assumed. Yet, this solution might also be interesting for the decision maker.

However, the theoretical deficiency of minimax regret with respect to solution efficiency is illustrated here by a practical TEP case, for the first time in the over 3 decades that the minimax regret criterion has been in use (Bunn, 1984; E. Crousillat, 1989; Gorenstin et al., 1993; Miranda & Proenca, 1998b). Indeed, note that the minimax regret criterion suggests two alternative solutions with the same regret (plans 9 and 10). However, only one of those two solutions is efficient (plan 10), since the other plan incurs in 4% higher costs in the high growth scenario, while attaining the same total costs in the low growth scenario (see Figure 5-5). Since the minimax regret criterion considers only the costs in one particular scenario (the one with highest opportunity loss for each plan), it cannot distinguish between two alternative solutions with the same cost in one scenario but different costs in other scenarios.

If guided by the minimax regret criterion, an inefficient solution could be implemented by the decision maker. Furthermore, the decision maker could be blind to the inefficiency of the implemented solution. For example, even if a MILP optimization model is used, such a model could find the inefficient plan first and stop having proved optimality. Yet, the decision maker would never know that the implemented solution incurs in gratuitous opportunity loss, since another solution with the same maximum regret but in the efficient set could be implemented instead.

In practice, minimax regret criterion is often applied to a restricted set of solution alternatives, such as the set of optimal deterministic solutions under each scenario. However, even an optimization procedure which endogenously builds the plan which minimizes the maximum regret could incur in myopic opportunity loss. The relevance of this finding can be fully appreciated by noting that practical transmission expansion planning processes worldwide employ minimax regret as the ex-ante criterion to choose a single solution under long-term uncertainty (e.g. National Grid in Great Britain, ENTSO-E in the European Union, PJM in USA, and the National Energy Commission in Chile).

Table 5-2 Summary of Pareto frontier for TEP under two scenarios in the IEEE 24 bus RTS test case.

Plan *	Total Costs Low Growth Scenario [MMUS$]	Relative Regret Low Growth Scenario [%]	Total Costs High Growth Scenario [MMUS$]	Relative Regret High Growth Scenario [%]	Probability Range Low Growth Scenario	Robustness Measure [%]	2nd order Robustness Solution	Myopic Stochastic Solution	Minimax Regret Solution
1	$ 696.3	0.0%	$ 1,495.8	19.6%	[92%, 100%]	1%			
2	$ 701.8	0.8%	$ 1,483.9	18.6%					
3	$ 701.8	0.8%	$ 1,434.2	14.6%	[86% , 92%]	3%			
4	$ 706.4	1.5%	$ 1,405.5	12.3%	[72% , 86%]	12%			
5	$ 711.9	2.2%	$ 1,396.1	11.6%					
6	$ 717.4	3.0%	$ 1,395.3	11.5%					
7	$ 741.3	6.5%	$ 1,381.4	10.4%					
8	$ 746.0	7.1%	$ 1,345.9	7.6%					
9	$ 751.5	7.9%	$ 1,344.3	7.4%	-	-	-	-	X
10	$ 751.5	7.9%	$ 1,296.3	3.6%	-	-	-	-	X
11	$ 757.0	8.7%	$ 1,278.6	2.2%	[60% , 72%]	16%	-	-	-
12	$ 762.4	9.5%	$ 1,270.3	1.5%	[46% , 60%]	26%	-	X	-
13	$ 767.9	10.3%	$ 1,269.2	1.4%					
14	$ 774.5	11.2%	$ 1,267.5	1.3%					
15	$ 774.5	11.2%	$ 1,260.0	0.7%	[25% , 46%]	30%	X	-	-
16	$ 800.6	15.0%	$ 1,251.2	0.0%	[0%, 25%]	13%	-	-	-

*Column "Plan" is a numeric index of alternative expansion plans to allow comparison of Figure 5-4, Table 5-2 and Table 5-3. Note that alternative expansion plans are presented in decreasing order of totals costs in the low growth scenario (which translates in left to right ordering in Figure 5-4).

Table 5-3 Details of Pareto efficient transmission expansion plans.

Plan	Total Built lines	Number of Lines Built in Each Corridor						Investment Cost [MMUS$]	Operation Costs S1 [MMUS$]	Operation Costs S2 [MMUS$]	Supported?
		(10-12)	(3-24)	(7-8)	(16-17)	(9-12)	(14-16)				
1	0	.	.	.	.	.	.	$0	$696.3	$1,495.8	Supported
2	1	1	.	.	.	.	.	$50	$696.3	$1,478.4	Inefficient
3	1	.	1	.	.	.	.	$50	$696.3	$1,428.7	Supported
4	1	.	.	1	.	.	.	$160	$688.9	$1,387.9	Supported
5	2	.	1	1	.	.	.	$210	$688.9	$1,373.0	Non-Supported
6	3	.	2	1	.	.	.	$260	$688.9	$1,366.8	Non-Supported
7	2	.	1	.	1	.	.	$410	$696.3	$1,336.4	Non-Supported
8	2	.	.	1	1	.	.	$520	$688.9	$1,288.8	Non-Supported
9	3	.	.	1	1	1	.	$570	$688.9	$1,281.7	Inefficient
10	3	.	1	1	1	.	.	$570	$688.9	$1,233.7	Non-Supported
11	4	.	2	1	1	.	.	$620	$688.9	$1,210.5	Supported
12	5	1	2	1	1	.	.	$670	$688.9	$1,196.7	Supported
13	6	2	2	1	1	.	.	$720	$688.9	$1,190.1	Non-Supported
14	5	1	1	2	1	.	.	$780	$688.9	$1,181.9	Inefficient
15	5	.	2	2	1	.	.	$780	$688.9	$1,174.4	Supported
16	3	.	1	.	1	.	1	$950	$696.3	$1,146.9	Supported

Finally, we calculate the solution to the distributionally robust criterion. Given two scenarios, the ambiguity set of plausible scenario probabilities is always a one dimensional range in the scenario probabilities in one single scenario, due to the constrain that probabilities sum to unity. For example, if the probability of low-growth scenario is assumed to be between 30% and 60%, then the probability of the high-growth scenario will be between 70% and 40%.

In this case, the distributionally robust solution can be obtained by selecting the efficient alternative which minimizes the worst of two expected cost, each one with an extreme of the assumed probability range.[30] For example, assuming a low-growth scenario probability between 30% and 60%, expected cost must be calculated with both these probabilities for each efficient expansion alternative. Then, for each efficient alternative the maximum of both expected cost is selected. Finally, the optimal solution is the efficient alternative which minimizes this maximum cost, from among all efficient alternatives.

The following table details the optimal solution under alternative probability ranges. We see that the same solution to our robustness criterion (solution 15) is optimal for ranges accumulating around equiprobability, a result which seems consistent with our assumed triangular distribution which weighs more cases near the 50% / 50% case. However, the minimax cost plan (plan 16) is obtained by weighing more the high growth scenario (range 20% , 50%) or by requiring distributional robustness under almost all possible cases (range 10% , 90%). These results seem reasonable and consistent with previous results.

Table 5-4 Details of Pareto efficient transmission expansion plans.

Low growth scenario probability range	**Optimal plan ID**	**Minmax Expected Cost [MMUS$]**
45% , 55%	15	$1,041.5
30% , 70%	15	$1,114.3
30% , 90%	15	$1,114.3
20% , 50%	16	$1,161.1
10%, 90%	16	$1,206.1

[30] The robust solution can be obtained by enumerating extreme vertex of the ambiguity set due to the geometry of such set, which under two scenarios is always a special case of a polytope, as discussed in the appendix.

6. CONCLUSIONS

Timely and efficient provision of transmission investment is vital for ensuring an affordable and sustainable electricity service whilst providing high levels of service reliability. In seeking to intensify competition in its electricity generation sector and also to facilitate the scale-up of renewable energy integration, Chile enacted in 2016 a new transmission law which allows the regulator to plan a transmission system with capacity slackness for long-term (i.e. more than 20 years in the future) demand requirements and hypothetical potential generation hubs. Furthermore, new transmission investments are now to be fully charged to demand, instead of allocating a portion of investment costs to generators. Hence, concerns have raised regarding the effectiveness of planning processes and the methodologies that assist decision making, so as to ensure that expected future savings do offset higher electricity tariffs today. Indeed, under the recently enacted law there are serious risks of overbuilding the transmission system and even building infrastructure which will never be used, in the case of generation hubs.

This thesis contributes to the ongoing debate regarding transmission expansion planning under uncertainty both in Chile and abroad. First, a comprehensive analysis of the state-of-the-art on transmission expansion planning (TEP) under uncertainty was developed in this thesis. A novel framework for TEP under uncertainty was developed, based on well agreed upon theoretical foundations but focused mostly on serving practical purposes by policymakers, practitioners and researchers alike. In the light of the proposed conceptual framework, methodologies for assessing TEP under uncertainty developed in the literature and in practice were analyzed in depth by characterizing the philosophy, advantages and disadvantages of alternative modeling approaches as well as identifying gaps in the literature.

Second, a novel decision making criterion was proposed and analyzed in this thesis, hoping to lay the first step in bridging the gap between stochastic androbustness / flexibility approaches to TEP under long term strategic scenario uncertainty. While stochastic TEP rests on solid theoretical foundations (namely von Neumann and Morgenstern’s expected utility theory), advocates of alternative methods point to the lack of precise and reliable probabilities for long term strategic scenarios as a downside of

stochastic TEP which, in turns, argues in favor of employing alternative and less understood approaches such as minimax regret.

The novel theoretical analyses and optimization results developed in this thesis indicate that scenario probabilities do alter the optimal stochastic TEP solution but only slightly, while the minimax regret criterion suffers from theoretical deficiencies never before thoroughly studied, and those deficiencies do arise in practice yielding inefficient results (from a Pareto perspective). Thus, this thesis proves that neither the very theoretical foundations of TEP under strategic scenario uncertainty, nor the practical implications of alternative decision making criterions, are thoroughly understood today.

Further examination of tools for assisting TEP under strategic scenario uncertainty is thus needed. A key question to address (related to our hypothesis) is whether the stochastic criterion remains relatively insensitive to scenario probabilities under high scale test cases with improved modeling of hourly generation resources. Furthermore, the analysis developed in this thesis might be particularly valuable for analyzing planning practices for efficiently integrating generation hubs to real power systems. As pointed out in this thesis, this is an important issue for policymakers and planning practitioners alike. For example, planning practices have been discussed for efficiently scaling-up off-shore wind generation in UK's north seas, and for integrating renewable generation hubs in Chile (Strbac, Moreno, Konstantelos, Pudjianto, & Aunedi, 2014). Analyzing the optimal expansion patterns yielded by alternative decision-making criterions could help in shedding light workable planning practices for practitioners and policymakers to efficiently integrate renewable generation hubs to power markets.

Conclusions on alternative decision making criterions for TEP under strategic scenario uncertainty are further discussed in section 6.1, while future works arising from this thesis are discussed in section 6.2.

6.1. Inquiry on the efficiency of alternative decision making criterions

The following conclusions can be drawn from both theoretical analyses and optimization results for the middle scale test case analyzed in this thesis.

First, the rich diversity of efficient solutions arising in TEP is not fully seized by existing methods, including deterministic planning, minimax regret, and stochastic

methods. Efficient solutions are not a combination of deterministic solutions under each scenario, since half of the corridors that are built in efficient solutions do not arise in deterministic solutions. Stochastic TEP with a single set of probabilities is also blind to the diversity of efficient solution alternatives due to the non-convexity of the problem stemming from the integrality of investment decision variables. Exploring alternative efficient solutions can well be worth the additional computational effort since practical planning processes apply other criteria and conduct further assessments in order to finally determine the expansion plan to be implemented.

Second, stochastic TEP is not immune to scenario probabilities and, in fact, variations of as little as 4% in the scenario probabilities can change the optimal solution. Although very slight variations distinguish the optimal expansion plans (i.e. two built transmission lines and less than 2% of total costs), the solution did change even in this simple case. Moreover, in our test case the robustness of the equiprobable solution to variations in the scenario probabilities hides the existence of efficient but unsupported solutions in the concave part of the Pareto frontier. Among these unsupported solutions lied the efficient solution to the minimax regret decision criterion. Hence, stochastic TEP is effectively blind to the set of efficient but unsupported solutions, as previously asserted by Miranda & Proenca for a different problem (1998a).

Whether or not the optimal solution varies slightly with scenario probabilities is not currently clear, since larger test cases and more complex scenarios and optimization models may well produce a more densely populated Pareto frontier with many alternative plans being optimal for only small sets of scenario probabilities. For example, (F D Munoz et al., 2014) finds many more alternative expansion plans in a 240-bus / 448-transmission elements test case.

Third, alternative decision criterions for TEP under scenario uncertainty diverge in general. Consequently, the decision maker faces a set of alternative expansion plans instead of a single optimal solution to the problem. Lacking a single theoretical decision criterion, the divergence of existing criterions in practice results in the inability to confidently choose a single transmission expansion plan to implement. In this study, minimax regret, minimax cost, equiprobable stochastic, and the novel second-order robustness criterion, all yielded different solutions. The irreversibility of transmission

investments requires firm commitments to be taken today. Unfortunately, it seems that TEP models are still unable to recommend a single transmission expansion planning. Hence, the dynamics of transmission investments are worth exploring (Konstantelos & Strbac, 2015), among other possible criterions for selecting the expansion plan to implement (ENTSO-E, 2015). Nevertheless, producing a set of efficient alternatives instead of one single solution can be itself advantageous since candidate expansion plans are assessed by multiple stages and models in practical transmission expansion planning processes, due to the inability of current optimization models to integrate the various dimensions which are relevant to TEP.

Fourth, myopic minimax regret methodologies (with respect to Pareto efficiency) such as those applied in the industry can incur in gratuitous opportunity losses. Considering costs in only one scenario to assess the performance of a plan (in the minimax regret case, the scenario which yields the highest opportunity loss) can inefficiently forego opportunity gains in other scenarios, since the decision maker could implement an alternative solution with the same maximum regret but lower costs in other scenarios. Hence the term "myopic", since minimax regret is blind to regrets other than the maximum and, in particular, can be blind to Pareto efficiency. Inefficient solutions could thus be recommended by optimization models and planning tools that fail to acknowledge the lack of solution efficiency in the minimax regret criterion. Moreover, the decision maker might never be aware about the inefficiency of the solution, even if an optimization model is solved to optimality in order to endogenously build the expansion plan which minimizes the maximum regret. Indeed, a MILP model would stop in an inefficient solution without necessarily identifying degenerate solutions and, in particular, the efficient solution that must exist for the minimax regret criterion.

More complex test cases and extensions to the simple model developed in this thesis should be analyzed in order to further study the impact of scenario probabilities, as will be discussed in the following section.

6.2. Future Works

Enhancements to the proposed methodology and further analyses are possible. Regarding the methodology, it might be interesting to explore alternative methods for

building Pareto efficient frontiers, such as the ε-constraint method. This and other methods may provide exact efficient solutions in the concave part of the Pareto frontier, instead of solutions approximated by genetic algorithms (NSGA-II). While the methodology developed in this thesis ensures finding exact solutions in the convex Pareto frontier by means of the stochastic TEP model, efficient solutions found by NSGA-II in the concave Pareto frontier are not ensured to be efficient (although it is highly likely these solutions are indeed efficient, given a fine-tuned genetic algorithm). Moreover, other solution methodologies such as ε-constraint have the advantage of not requiring heuristics and parameters set arbitrarily (or by experimentation) by the user, as opposed to NSGA-II.

Regarding the analyses performed in this thesis, in the future it might be interesting to study the behavior of the newly proposed second-order robustness criterion in real power systems, for example the Chilean or UK power grid, and under many scenarios (e.g. up to six scenarios). To study real and / or large-scale power systems require further work on data collection and scenario generation, as well as being more computationally challenging. No immediate modifications are required on the model nor on the solution methodology for these analyses to be developed. However, a methodology for calculating multidimensional integrals must be implemented to study more than two possible scenarios (e.g. sparse grids or Monte Carlo methods). Such extension is relatively simple since readily available software libraries exist that provide multidimensional integration.

In the IEEE 24 bus RTS test case, alternative solutions are very similar in terms of total costs and built lines. However, larger test cases and more complex models may produce more diversity in the set of alternative solutions. We hypothesize that more diverse solutions could be obtained primarily by increasing the system's size, since various tests with the IEEE 24 bus case produced little additional diversification of the efficient solutions with respect to the case presented in this thesis, despite the use of very different generation scenarios (in terms of operation costs, generation technologies and locations, and location of new generation hubs).

Moreover, investment costs were annualized in this thesis, thus inappropriately balancing investment and operation costs, since commitments to transmission investments raise electricity tariffs today in the hope that these raises will be offset by operation savings in the future. Hence, a correct economic valuation of transmission investment (i.e. based

on net present value) is needed to improve the methodology developed in this thesis, for example adopting the framework developed by (F D Munoz et al., 2014; van der Weijde & Hobbs, 2012). However, the conclusions stated above are still valid from a theoretical perspective.

Also, several extensions to this work are possible. First, the fundamental tradeoff between generation and transmission capacity expansions can be explored by extending the model in several ways. The simplest extension is to formulate a two-stage integrated generation and transmission expansion planning model, where system operation are the only second-stage decisions (Alvarez Lopez et al., 2007). A similar but more complex extension is to replicate the three-period proactive model proposed by (F D Munoz et al., 2014), thus modeling the relation and the long lag between transmission and generation investment in competitive power markets, as well as the ability to adapt the system by means of recourse expansion decisions once the long-term uncertainty is realized.

Second, further analyses on alternative decision making criterions are possible. For example, decisions under a variety of criterions may be compared by choosing among the Pareto efficient solutions, including Laplace or Hurwicz criterions (M O Buygi et al., 2003), minimizing the distance (euclidian or not) to the multiobjective utopia point[31], flexible expansion plans, mean-variance or CVaR-constrained stochastic TEP, among others.

Third, other uncertainties may be included in the model by extending the scenario tree both vertically and horizontally, and solving the resulting model by means of Benders decomposition or Progressive Hedging. Vertically, thousands of scenarios may be considered under each long-term strategic scenario in order to account for short-term uncertainties in system operation (e.g. hourly wind availability and load levels, yearly hydrological inflows, fuel and carbon emissions costs, and forced outages). Attempts in the literature to integrate short and long-term uncertainties are currently scarce and limited

[31] The utopia point is the point where the minimum possible value of each objective function is simultaneously attained. In the case of two-stage stochastic TEP, the utopia point would be the (often fictitious) solution where the deterministic optimal function value for each scenario is simultaneously attained. A plan which attains such a utopia point in the objective space does not exist for the problem studied in this thesis, unless the Pareto frontier is composed of only one expansion plan (which would be the optimal solution under all scenarios).

(Chamorro et al., 2012; Francisco D. Munoz & Watson, 2015). Hence, such an extension is likely to be computationally challenging but also a significant contribution to the state of the art.

On the other hand, a horizontal extension of the scenario tree entails modeling the dynamics of the transmission expansion planning problem by considering multiple periods when commitments can be taken and later adjusted according to revealed information. Such multiperiod TEP models under uncertainty are also scarce and limited in the existing literature (Gorenstin et al., 1993; Konstantelos & Strbac, 2015). However, these models are interesting in their ability to include flexible alternatives that can defer risky transmission investments. It might be worth considering robust formulations for both the vertical and horizontal extension of the scenario tree, in order to deal with short and medium term uncertainties whilst rendering a tractable optimization model. Particularly, it is interesting to study the relation between medium and long-term investment dynamics under different discount rates (Weitzman, 2001), so as to determine whether higher tariffs today due to firm commitments to investment are worthwhile given the (uncertain) expectations of future operating savings, or whether it is better to defer investments.

Fourth, multiple objectives can be considered in the transmission expansion planning problem, particularly maximizing reliability or security of supply, and maximizing the sustainability of the electricity industry. Considering these objectives along the minimization of total costs renders a multiobjective optimization model, since for each objective there are multiple possible scenarios and, hence, multiple objective values under each scenario. Therefore, the interaction between scenario probabilities and multiple non-monetizable objectives could be simultaneously studied. It is worth noting that long-term expansion planning processes in EU (ENTSO-E's TYNDP's) and also in Chile (after the enactment of the 2016 Transmission Law) attempt to assess investment decisions with a multicriteria framework. Solving the energy trilemma of affordability, reliability and sustainability is a vital problem in modern economies. The literature on these multiobjective models is also scarce.

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APPENDIX

A. TEST CASES

A.1. Garver 6 bus test case

Table A-1 Nodal data for base scenario of Garver 6-bus case study.

Node	Load level (MW)	Installed Generating Capacity (MW)	Generation Marginal Cost (US$/MWh)
B1	80	150	10
B2	240	0	0
B3	40	360	20
B4	160	0	0
B5	240	0	0
B6	0	600	30

Table A-2 Nodal data for second scenario of Garver 6-bus case study.

Node	Load level (MW)	Installed Generating Capacity (MW)	Generation Marginal Cost (US$/MWh)
B1	104	150	10
B2	312	0	0
B3	52	360	20
B4	208	200	5
B5	312	100	10
B6	0	600	30

Table A-3 Existing transmission lines data for Garver 6-bus case study.

Node From	Node To	Susceptance (pu)	Thermal Capacity (MW)
B1	B2	2.5	100
B1	B4	1.67	80
B1	B5	5	100
B2	B3	5	100
B2	B4	2.5	100
B3	B5	5	100

Table A-4 Candidate transmission lines data for Garver 6-bus case study.

Node From	Node To	Susceptance (pu)	Thermal Capacity (MW)	Investment Cost (MUS$)
B1	B2	2.5	100	40
B1	B5	5	100	20
B2	B3	5	100	20
B2	B4	2.5	100	40
B2	B6	3.33	100	30
B2	B6	3.33	100	30
B2	B6	3.33	100	30
B3	B5	5	100	20
B3	B5	5	100	20
B3	B6	2.08	100	48
B4	B6	3.33	100	30
B4	B6	3.33	100	30

A.2. IEEE 24 bus RTS test case

Table A-5 Bus and load data for IEEE 24 bus RTS test case.

Bus	Load Scenario 1 Peak (MW)	Load Scenario 1 Valley (MW)	Load Scenario 2 Peak (MW)	Load Scenario 2 Valley (MW)
1	125	94	226	169
2	113	84	203	152
3	209	157	376	282
4	86	64	155	116
5	82	62	148	111
6	158	118	284	213
7	145	109	261	196
8	199	149	358	268
9	203	152	366	274
10	226	170	408	306
11	0	0	0	0
12	0	0	0	0
13	308	231	554	416
14	225	169	406	304
15	368	276	663	497
16	116	87	209	157
17	0	0	0	0
18	387	290	696	522
19	210	158	378	284

20	149	111	268	201
21	0	0	0	0
22	0	0	0	0
23	0	0	0	0
24	0	0	0	0
	3,309	**2,482**	**5,959**	**4,469**

Table A-6 Generator data for IEEE 24 bus RTS test case.

Bus	Generator Type	Generation Marginal Cost ($/MWh)*	Installed Generating Capacity Scenario 1 (MW)	Installed Generating Capacity Scenario 2 (MW)
1	Oil	150	40	86
1	Coal	50	212	328
2	Oil	150	56	86
2	Coal	50	212	328
7	Oil	150	418	648
13	Oil	150	823	1,277
15	Oil	150	84	130
15	Coal	50	432	670
18	Nuclear	6	1,115	1,729
23	Coal	50	919	1,426
7	CCGT	50	418	648
24	Wind	0	-	216
17	Solar PV	0	-	432
			4,728	8,007

*Marginal costs are taken from (Konstantelos & Strbac, 2015), in pounds per MWh.

Table A-7 Transmission branches data for IEEE 24 bus RTS test case.

Bus From	Bus To	Susceptance (pu)	Thermal Capacity (MW)	Total Investment Cost (M$)	Existing Branches	Maximum New Branches
1	2	71.94	175	30	1	2
1	3	4.73	175	550	1	2
1	5	11.83	175	220	1	2
2	4	7.89	175	330	1	2
2	6	5.21	175	500	1	2
3	9	8.40	175	310	1	2
3	24	11.92	400	50	1	2
4	9	9.64	175	270	1	2
5	10	11.33	175	230	1	2
6	10	16.53	175	160	1	2
7	8	16.29	175	160	1	2

8	9	6.06	175	430	1	2
8	10	6.06	175	430	1	2
9	11	11.92	400	50	1	2
9	12	11.92	400	50	1	2
10	11	11.92	400	50	1	2
10	12	11.92	400	50	1	2
11	13	21.01	500	660	1	2
11	14	23.92	500	580	1	2
12	13	21.01	500	660	1	2
12	23	10.35	500	1340	1	2
13	23	11.56	500	1200	1	2
14	16	25.71	500	540	1	2
15	16	57.80	500	240	1	2
15	21	20.41	500	680	2	1
15	21	20.41	500	680	3	0
15	24	19.27	500	720	1	2
16	17	38.61	500	360	1	2
16	19	43.29	500	320	1	2
17	18	69.44	500	200	1	2
17	22	9.50	500	1460	1	2
18	21	38.61	500	360	2	1
18	21	38.61	500	360	3	0
19	20	25.25	500	550	2	1
19	20	25.25	500	550	3	0
20	23	46.30	500	300	2	1
20	23	46.30	500	300	3	0
21	22	14.75	500	940	1	2
1	8	7.44	175	350	0	3
2	8	7.89	175	330	0	3
6	7	5.21	175	500	0	3
13	14	22.37	500	620	0	3
14	23	16.13	500	860	0	3
16	23	12.17	500	1140	0	3
19	23	16.50	500	840	0	3

B. DETAILED ANALYSIS OF THE MINIMAX REGRET AND MINIMAX COST DECISION CRITERIONS

B.1. Definition of minimax regret decision criterion

Consider the following notation: there are n possible futures denoted $\{\theta_1, \theta_2, \dots, \theta_n\}$, and m possible decisions, denoted as $\{a_1, a_2, \dots, a_m\}$. Let y_{ij} be the value of action j (denoted a_j and hereinafter referred to as action, decision, choice, option or alternative, indistinctly) for the decision-maker, should scenario θ_i occur. The pay-off matrix $A_{n\times m}$ has columns related to actions and rows related to future scenarios, describing y_{ij} as depicted next:

	a_1	a_2	…	a_m
θ_1	y_{11}	y_{12}		y_{1m}
θ_2	y_{21}	y_{22}		y_{2m}
…			$\ddots$	
θ_n	y_{n1}	y_{n2}		y_{nm}

The best decision under the future scenario θ_i is given by the action related to the highest entry y_{ij} in row i of the pay-off matrix. The value of such an optimal decision is:

$$r_i^* = \max_j y_{ij}$$

Regret is a metric of the performance of an action under a given future scenario (known with certainty), and is defined as the opportunity loss that results from choosing the action a_j, when scenario θ_i occurs. This is:

$$regret\{a_j, \theta_i\} = r_{ij} = r_i^* - y_{ij} \geq 0$$

Note that this regret is zero if a_j is the optimal alternative under scenario θ_i. Choosing any other option yields a non-negative regret, since such an option is not optimal for the given scenario.

Maximum regret is a metric of the performance of an alternative when the future is not known with certainty, and is defined as the maximum regret of an alternative a_j across

all scenarios. Then, the minimax regret criteria result in choosing the action a_j that minimizes the maximum regret, that is, choosing a_j^* such that:

$$a_j^* = \left\{a_j \mid r_j = \min_{\mathrm{j}} \left\{\max_i r_{ij}\right\}\right\}$$

Hence, option a_q is optimal in a minimax regret sense if and only if:

$$r_q^* \leq r_j^* \quad \forall\, j$$

where $r_{ij} = \max_j y_{ij} - y_{ij}$ and $r_i^* = \max_j y_{ij}$.

The minimax regret decision criterion is particularly interesting when the decision-maker is appraised with hindsight (Bunn, 1984, p. 20). However, minimax regret also possesses some theoretical weaknesses which are described next.

B.2. Lack of Independence of Irrelevant Alternatives in the Minimax Regret Criterion

The main theoretical critique to minimax regret criterion, developed by Lindley (1971) and explained by Bunn (1984), refers to the *violation of the axiom of independence of irrelevant alternatives* (one of the axioms required from a decision criterion in Arrow's impossibility theorem). The independence of irrelevant alternatives refers to the prescription that a decision criterion should not alter the ranking of available decision alternatives should an additional decision become available, if such newly available decision is never preferred to previously available alternatives. In simple terms and to serve as example, if I do not prefer to go fishing rather than going to the movies or to a concert, then the decision of going to a movie or a concert should not depend on whether I consider (or have the possibility of) going fishing. Minimax regret violates such an axiom since deterministically optimal alternatives for particular scenarios may not be (and are not in general) optimal in a minimax regret sense.

For a mathematical example given by Bunn, consider the following pay-off matrix:

y_{ij}	a_1	a_2
θ_1	8	2
θ_2	0	4

The optimal alternative under scenario θ_1 is a_1 (since $8 > 2$), while the optimal alternative under scenario θ_2 is a_2 (since $4 > 0$). Then, if the decision-maker chooses a_2 but θ_1 occurs instead of θ_2, one obtains a payment of 2, when the best available decision a_1 would have provided a payment of 8. Hence, regret is $8 - 2 = 6$. Therefore, the regret or opportunity loss matrix is:

r_{ij}	a_1	a_2
θ_1	0	6
θ_2	4	0

In such situation, the optimal decision in a minimax regret sense is a_1.

Now consider a new alternative (i.e. previously unavailable to the decision-maker) a_3, which yields the following pay-off matrix:

y_{ij}	a_1	a_2	a_3
θ_1	8	2	1
θ_2	0	4	7

The regret matrix considering this new alternative is changed since under scenario 2 the optimal alternative is no longer a_2 but rather a_3:

r_{ij}	a_1	a_2	a_3
θ_1	0	6	7
θ_2	7	3	0

In this new situation the minimax regret criterion results in choosing a_2 instead of a_1. The optimal decision changed due to the introduction of a new alternative which is optimal under one of the scenarios, thus altering the regret of all alternatives in such scenario and, in this case, altering the minimax regret valuation. However, option a_3 is not optimal in the minimax regret sense[32].

Bunn points that a relaxation of the axiom of independence of irrelevant alternatives is equivalent to requiring that the valuation of alternatives is independent of dominated

[32] , a_3In this case, a_3 is never preferred under the minimax regret criterion (without a priori knowledge of the future).

alternatives. In practical terms, we can conclude that **a sound application of the minimax regret criterion requires considering the optimal alternative under each scenario** (or a sufficiently close-to-optimal alternative so that maximum regrets are not altered should the truly optimal decision become available).

At least in principle, the previous conclusion precludes using the minimax regret criterion when using any optimization method which does not ensure attainment of the optimal decision under each scenario, for example the commonly used methodology in TEP of genetic algorithms (Charlin et al., 2015). However, it is yet to be determined whether there exist conditions for ensuring that the search for a deterministically optimal solution can be stopped without altering the optimal minimax regret alternative significantly.

B.3. Insensitivity to relative opportunity losses in the minimax regret criterion

The minimax regret criterion is insensitive to relative opportunity losses since regret is a difference of pay-offs, therefore being possible that two pay-offs of different magnitude are deemed as equivalent. For example, $\$1.050 - \$1.000 = \$50$ but also $\$150 - \$100 = \$50$. Regret does not consider whether opportunity losses are more or less important relative to the magnitude of the optimal pay-off to which decision alternatives are compared (Bagajewicz, n.d.).

An alternative for considering relative opportunity losses is using relative regret instead of absolute regret. Relative regret is defined as:

$$relative\ regret\{a_j\} = \tilde{r}_j = \frac{r_i^* - y_{ij}}{r_i^*}$$

Then, the optimal alternative is that which minimizes the maximum relative regret. Such an approach has been applied to TEP by (Motamedi et al., 2010). However, it is not currently clear whether the optimal solution changes (and how much) when using relative instead of absolute regret in a minimax decision criterion.

B.4. Insensitivity to foregone opportunities other than the maximum loss in the minimax regret decision criterion

Since minimax regret is defined in terms of the máximum regret across all posible future scenarios, such a decision criterion is insensitive to opportunity losses other than the maximum for each alternative. Consider for example the following pay-off matrix (Bagajewicz, n.d.):

r_{ij}	a_1	a_2	a_3	a_4
θ_1	100	99	0	150
θ_2	0	95	100	85
θ_3	5	40	200	0
$\max_i r_{ij}$	100	99	200	150

In this case, the optimal alternative in a minimax regret sense is a_2 since its maximum regret, 99 (see last row of the matrix), is less than the maximum regret of a_1, equal to 100. However, one unit of regret in scenario θ_1 is saved at the cost of 95 units of opportunity loss in scenario 2, and 40 units of regret in scenario 3, compared to an opportunity loss of 0 and 5 for scenarios 2 and 3, respectively, when choosing a_1.

A proposal for overcoming this issue is to use the average regret across scenarios. However, it is easy to show that such an approach is equivalent to minimizing the expected value across future scenarios, considering all scenarios are equally likely (Miranda & Proenca, 1998b). Therefore, this methodology is only an unnecesary complication of the probabilistic criterion with equally-likely scenarios.

Indeed, the weighted regret across scenarios (with weighs w_i under each scenario) is:

$$\bar{r_j} = \sum_{i=1}^{n} w_i \cdot r_{ij} = \sum_{i=1}^{n} w_i \cdot \left(\max_j\{y_{ij}\} - y_{ij}\right)$$
$$= \sum_{i=1}^{n} w_i \cdot \max_j\{y_{ij}\} - \sum_{i=1}^{n} w_i \cdot y_{ij}$$

Since the first sum is constant, then:

$$\min_j \bar{r}_j \Leftrightarrow \max_j \sum_{i=1}^{n} w_i \cdot y_{ij}$$

Therefore, this approach is only a reorder of terms of a probabilistic approach with weighing probabilities w_i assigned to each scenario θ_i[33]. Despite the lack of a theoretical foundation or practical utility of weighted regrets, such a criterion has been applied in TEP literature (Motamedi et al., 2010), without any reference to or discussion of the previously described equivalence to expected value.

B.5. Search methods for minimax regret solutions and the relation of minimax regret to multiobjective efficiency

Different methods have been applied to find the transmission expansion plan which minimizes the maximum regret among scenarios. The most basic (and also the weakest) solution method is to choose the plan which minimizes the maximum regret, from among the optimal plans for each scenario. Other manual robustness criterions are possible and have been applied (e.g. in MISO), such as building transmission lines which simultaneously appear in multiple scenario deterministic optimal solutions. A further level of sophistication in the methodology are heuristic solutions to explore the decision space in search for diverse solutions (e.g. genetic algorithms (Charlin et al., 2015)). Both of the aforementioned solution methodologies can suffer from the lack of independence of irrelevant alternatives, discussed previously.

The weakness of minimax regret relating to the independence of irrelevant alternatives can be partially removed by building an optimization model which explicitly devises a transmission expansion plans which minimizes the maximum regret (Strbac et al., 2014). Considering the MILP disjunctive formulation given by equations (1) – (10), an equivalent linear program for minimizing maximum regret can be formulated by using the well-known transformation of the maximum function to linear programs, as follows:

$$\min_X \max_{s \in S} r_s(X) \equiv \min_X z \quad s.t.\ r_s(X) \leq z \ \ \forall\, s \in S \tag{21}$$

[33] Across this section the decision-maker has been assumed to maximize value,. If costs are to be minimized instead, minimizing the weighted-regret is equivalent to minimizing the weighted-costs across escenarioscenarios.

However, this reformulation does not change the fact that the availability of new transmission projects may change the regrets of all solutions and change the solution that minimizes the maximum regret, even if the new alternative is not the one that minimizes the maximum regret.

An important observation is that **a solution that minimizes the maximum regret always lies in the Pareto efficient frontier of the multiobjective problem for alternative scenarios**. Indeed, if a solution is not efficient then there exists some solution which dominates it and, hence, can only improve the regret across any scenario. Therefore, there is always an efficient solution which minimizes the maximum regret, even if there exists an inefficient solution which also minimizes the maximum regret.

Before formally stating the proposition and providing a proof, we introduce the following propositions which will be used in the proof.

Definition 1. A decision, alternative, choice, or solution, is a transmission expansion plan $x \in \Omega = \{0,1\}^{Lc}$.

Definition 2. The total costs of implementing decision alternative (transmission expansion plan) x, if scenario $s \in S$ occurs, is $TC_s(x) \in \mathbb{R}$. The problem's data ensures that

$$TC_s(x) \geq 0 \; \forall x \in \Omega$$

The optimal total cost under each scenario is:

$$TC_s^* = \inf_{x \in \Omega} TC_s(x) = TC_s(x_s^*)$$

Then, x_s^* is said to be the deterministically optimal solution under scenario s. It can be easily shown that the optimal solution x_s^* for each scenario lies in the Pareto frontier (proof left to the reader).

Definition 3. **(Minimax Regret)** The regret of alternative $x \in \Omega$ under scenario $s \in S$ is defined as:

$$r_s(x) = TC_s(x) - TC_s^*$$

The maximum regret across scenarios is simply $MR(x) = \max_{s \in S} r_s(x)$. The optimal solution in a minimax regret sense is then $x_{MR}^* = \operatorname{argmin}_{x \in \Omega} MR(x)$, and MR^* is the optimal (minimum) maximum regret. We say that x_{MR}^* minimizes the function MR.

Proposition 1. Let $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$ be two vectors such that $x_i \leq y_i \;\; \forall i = 1 \ldots n$. Then:

$$\max_{i=1\ldots n} x_i \leq \max_{i=1\ldots n} y_i$$

Proof:

$$x_j \leq \max_{i=1\ldots n} x_i \;\; \forall j = 1 \ldots n$$

$$y_j \leq \max_{i=1\ldots n} y_i \;\; \forall j = 1 \ldots n$$

Since $x_i \leq y_i \;\; \forall i = 1 \ldots n$, then:

$$\exists j \in [1, n]: \max_{i=1\ldots n} x_i \leq y_j$$

$$\Rightarrow \max_{i=1\ldots n} x_i \leq \max_{i=1\ldots n} y_i$$

□

Definition 4. **(Pareto dominance)** Let $x^*, x \in \Omega$ be two alternative decisions available to the decision maker. x^* is said to dominate (in a Pareto sense) x if and only if:

$$TC_s(x^*) \leq TC_s(x) \;\; \forall s \in S$$

$$\wedge \exists s \in S : TC_s(x^*) < TC_s(x)$$

That is, x^* performs equal to or better than decision x in every scenario, and performs strictly better than x in at least one scenario, where better performance corresponds to lower total costs. In this case we say that $x^* \prec x$, that is, x^* is preferred over x.

Proposition 2. **(Transitivity of Pareto dominance)** The partial order relation $\prec$ is transitive, that is, let $x, y, z \in \Omega$ such that $x \prec y$ and $y \prec z$, then $x \prec z$.

Proof: Since $x \prec y$ and $y \prec z$, then:

$$TC_s(x) \leq TC_s(y) \;\; \forall s \in S \;\; \wedge \exists s_1 \in S : TC_{s_1}(x) < TC_{s_1}(y)$$

$$TC_s(y) \leq TC_s(z) \;\; \forall s \in S \;\; \wedge \exists s_2 \in S : TC_{s_2}(y) < TC_{s_2}(z)$$

Therefore:

$$TC_s(x) \leq TC_s(z) \; \forall s \in S$$

Furthermore:

$$TC_s(x) \leq TC_s(y) \;\; \forall s \in S \;\; \Rightarrow \;\; TC_{s_2}(x) \leq TC_{s_2}(y) < TC_{s_2}(z)$$

Therefore:

$$TC_s(x) \leq TC_s(z) \; \forall s \in S \wedge \;\exists s_2 \in S : \; TC_{s_2}(x) < TC_{s_2}(z)$$

$$\Leftrightarrow x \prec z$$

□

Definition 5. **(Pareto efficiency)** Let $x^* \in \Omega$ be an available alternative for the decision maker. x^* is said to be an efficient alternative, or equivalently, Pareto dominant alternative, if and only if there is no other decision alternative that dominates x^*, that is:

$$\nexists x \in \Omega : x \neq x^*$$
$$\wedge\ TC_s(x) \leq TC_s(x^*)\ \ \forall s \in S$$
$$\wedge\ \exists s \in S : TC_s(x) < TC_s(x^*)$$

That is, there is no other decision x that performs equal or better than decision x^* in every scenario, and that performs strictly better than x^* in at least one scenario. In any other case, x^* is said to be an inefficient or Pareto dominated alternative.

Definition 6. **(Pareto efficiency)** The set $E \subseteq \Omega$ that contains all efficient alternatives is called the Pareto dominant set, or Pareto efficient frontier. That is, an alternative x is efficient if and only if $x \in E$, while and alternative x is inefficient if and only if $x \notin E$.

Proposition 3. **(Alternative definition of inefficiency)** An alternative $x \in \Omega$ is inefficient if and only if there exists an efficient alternative $y^* \in E$ that dominates x.

Proof: The sufficiency is proven by noting that if there exists an efficient $y^* \in \Omega$ that dominates x, then x is inefficient.

Necessity is proven as follows. x is inefficient if and only if there exists some $y \in \Omega$ that dominates x. If y is not efficient, then there exists $y_1 \in \Omega$ that dominates y and that, by transitivity, also dominates x. Extending this argument we can always find an efficient alternative y^* that dominates x, since in our problem there is a finite decision space (related to alternative transmission lines to be built).

□

Proposition 4. An optimal solution in the minimax regret sense, that is efficient, always exists.

Proof: If the optimal minimax regret solution is efficient, then the proposition is proven. However, we must prove that if the optimal minimax regret solution is inefficient, then there must exist another optimal minimax regret solution (i.e. which attains the same minimum maximum regret MR^*) that is efficient.

That is, we must prove that:

$$\{x^*_{MR} \text{ minimizes } MR \wedge\ x^*_{MR} \text{ is inefficient}\} \tag{22}$$

$$\Rightarrow \exists x_0 \in E : x_0 \text{ also minimizes } MR$$

By proposition 3:

$$x_{MR}^* \text{ is inefficient } \Leftrightarrow x_{MR}^* \notin \mathrm{E}$$

$$\Leftrightarrow \exists x_0 \in E: TC_s(x_0) \leq TC_s(x_{MR}^*)\ \forall s \quad \wedge \exists j: \quad TC_j(x_0) < TC_j(x_{MR}^*) \tag{23}$$

$$\Leftrightarrow \exists x_0 \in E: TC_s(x_0) - TC_s^* \leq TC_s(x_{MR}^*) - TC_s^*\ \forall s \quad \wedge \exists j: \quad TC_j(x_0) - TC_s^* < TC_j(x_{MR}^*) - TC_s^* \tag{24}$$

$$\Leftrightarrow \exists x_0 \in E: r_s(x_0) \leq r_s(x_{MR}^*)\ \forall s \quad \wedge \exists j: \quad r_j(x_0) < r_j(x_{MR}^*) \tag{25}$$

By proposition 1:

$$\Rightarrow \exists x_0 \in E: MR(x_0) \leq MR(x_{MR}^*) \tag{26}$$

Note that this inequality can be strict (i.e. it is not always necessarily equal) due to the second strict inequality on regrets.

$$\Leftrightarrow \exists x_0 \in E: [\{MR(x_0) = MR(x_{MR}^*)\} \vee MR(x_0) \leq MR(x_{MR}^*)] \tag{27}$$

$$\Leftrightarrow \exists x_0 \in E: \big[\{\ \{x_0 \in E \text{ and } x_{MR}^* \notin E \text{ both minimize } MR\} \vee \{\text{neither } x_0 \in E \text{ nor } x_{MR}^* \notin E \text{ minimize } MR\}\} \vee \{x_{MR}^* \text{ does not minimize } MR\}\big] \tag{28}$$

$$\Leftrightarrow [\exists x_0 \in E: \{x_0 \text{ and } x_{MR}^* \notin E \text{ both minimize } MR\}\,] \vee [\exists x_0 \in E: \{\text{neither } x_0 \text{ nor } x_{MR}^* \notin E \text{ minimize } MR\}\,] \vee [\exists x_0 \in E: \{x_{MR}^* \text{ does not minimize } MR\}\,] \tag{29}$$

$$\Rightarrow [\exists x_0 \in E: \{x_0 \text{ and } x_{MR}^* \notin E \text{ both minimize } MR\}\,] \vee [x_{MR}^* \notin E \text{ does not minimize } MR\,] \vee [x_{MR}^* \notin E \text{ does not minimize } MR\,] \tag{30}$$

$$\Leftrightarrow [\exists x_0 \in E: \{x_0 \text{ and } x_{MR}^* \notin E \text{ both minimize } MR\}\,] \vee [x_{MR}^* \notin E \text{ does not minimize } MR\,] \tag{31}$$

Then:

$$\{x_{MR}^* \text{ minimizes } MR \wedge x_{MR}^* \text{ is inefficient}\} \Leftrightarrow \{x_{MR}^* \text{ minimizes } MR\} \wedge \{x_{MR}^* \notin \mathrm{E}\} \tag{32}$$

$$\Rightarrow \{x_{MR}^* \text{ minimizes } MR\} \tag{33}$$
$$\wedge \{[\exists x_0 \in E: \{x_0 \text{ and } x_{MR}^* \notin E \text{ both minimize } MR\}\ ]$$
$$\vee\ [x_{MR}^* \notin E \text{ does not minimize } MR\ ]\}$$

$$\Leftrightarrow \big[\{x_{MR}^* \text{ minimizes } MR\} \tag{34}$$
$$\wedge \big\{\exists x_0 \in E: \{x_0 \text{ and } x_{MR}^* \notin E \text{ both minimize } MR\}\big\}\big]$$
$$\vee\ [\{x_{MR}^* \text{ minimizes } MR\} \wedge \{x_{MR}^* \notin E \text{ does not minimize } MR\ \}]$$

Since $p \wedge \sim p \Leftrightarrow False$, and $False \vee p \Leftrightarrow p$, then:

$$\Leftrightarrow \{\text{Both } x_{MR}^* \notin E \text{ and } x_0 \in E \text{ minimize } MR\} \vee \{False\} \tag{35}$$
$$\{x_{MR}^* \text{ minimizes } MR \wedge\ x_{MR}^* \text{ is inefficient}\} \tag{36}$$
$$\Leftrightarrow \{\exists x_0 \in E:\ \text{Both } x_{MR}^* \notin E \text{ and } x_0 \in E \text{ minimize } MR\}$$

□

Hence, a solution which minimizes the maximum regret can always be found in the Pareto frontier, but such a solution may also exist outside of the Pareto efficient frontier. Thus, it may be wise to first build he Pareto efficient frontier and then identify the solution that minimizes the maximum regret, instead of directly building the solution the minimizes the maximum regret (e.g. by the linear program presented in equation (19)). Such methodology ensures that the solution minimizes the maximum regret and is also efficient.

B.6. Second-order robustness of the minimax cost decision criterion

The minimax cost decision criterion selects the expansion alternative which minimizes the worst-case cost among scenarios, that is:

$$x_c^* = \min_x \max_{s \in S} TC_s(x) \tag{37}$$

The minimax cost solution x_c^* always lies in the Pareto efficient frontier.

An interpretation of minimax cost is that **the minimax cost solution is the most robust solution against variations in the scenario probabilities**. Indeed, consider a solution x and a set of probabilities p. In the face of second-order uncertainty, i.e. the

possibility of variations in the vector p of scenario probabilities, the worst possible total cost for the solution x due to variations in p is given by:[35]

$$\begin{aligned} (P) \qquad & \max_{\mathrm{p}} c^T p \\ \text{s.t.} \qquad & 1^T p = 1 \qquad : \pi \\ & 0 \leq p \leq 1 \qquad : y \end{aligned} \tag{38}$$

where $c^T = TC(x)$ is the set of total costs under each scenario for the expansion plan x. The dual of the linear problem (P) is:

$$\begin{aligned} (D) \qquad & \min_{\pi,\mathrm{y}} \pi + 1^T y \\ \text{s.t.} \qquad & \pi\, 1 + y \geq c \\ & y \geq 0, \pi \text{ free} \end{aligned} \tag{39}$$

The dual solution $\left\{y = 0; \pi = \max_{s \in S} c_s\right\}$ attains a cost of $\max_{s \in S} c_s$, which is also attained in the primal problem with a solution where the probabilities are all zero except for the scenario with worst performance, whose probability is equal to one. By strong-duality, the solution to the primal is hence the worse possible total cost across scenarios, which is by definition the minimax cost. Hence, minimizing the maximum total cost is equivalent to finding the solution which is more robust against variations in the scenario probabilities.

[35] In these equations, 1 stands for a vector of ones, of the size required for the corresponding matrix multiplication to be defined.

C. DISTRIBUTIONAL ROBUSTNESS CRITERION

The distributionally robust solution to TEP under scenario uncertainty is the solution of the following optimization problem.

$$\min_{\{\boldsymbol{y},\boldsymbol{g},\boldsymbol{r},\boldsymbol{f},\boldsymbol{\theta}\}} \max_{\boldsymbol{p}\in\mathbb{F}} \sum_{s\in S} \widetilde{\boldsymbol{p}}_{\boldsymbol{s}} \cdot TC_s \tag{40}$$

subject to constraints (2) through (11), where $\mathbb{F}$ is the ambiguity set.

This can be formulated as:

$$\min_{\{\boldsymbol{y},\boldsymbol{g},\boldsymbol{r},\boldsymbol{f},\boldsymbol{\theta}\}} EC \tag{41}$$

s.t.

$$EC \geq \sum_{s\in S} \widetilde{\boldsymbol{p}}_{\boldsymbol{s}} \cdot TC_s \quad \forall s \in S, \qquad \forall \boldsymbol{p} \in \mathbb{F} \tag{42}$$

The formulation and tractability of the robust counterpart will depend on the structure of the ambiguity set $\mathbb{F}$ chosen to describe the uncertainty on scenario probabilities (i.e. the distribution).

If the uncertainty set is a polytope described by a finite set of vertex points $\{\boldsymbol{p^i}\}$ (i.e. the intersection of finitely-many half-spaces), then the distributionally robust formulation yields the following MILP:

$$(PDR) \quad \min_{\{\boldsymbol{y},\boldsymbol{g},\boldsymbol{r},\boldsymbol{f},\boldsymbol{\theta}\}} EC \tag{43}$$

s.t.

$$EC \geq \sum_{s\in S} \widetilde{\boldsymbol{p}}_{\boldsymbol{s}} \cdot TC_s \quad \forall s \in S, \qquad \forall \boldsymbol{p} \in \{\boldsymbol{p^i}\} \tag{44}$$

and to equations (2) through (12).

Since $\mathbb{F} = \left\{\boldsymbol{p} : \boldsymbol{p} = conv\{\boldsymbol{p^i}\}\right\}$ (i.e. points within the convex hull of the vertex points $\boldsymbol{p^i}$), constraint (16) also holds for any point $\boldsymbol{p}$ within the convex hull of those points (it suffices to take the convex combination of all inequalities across $\boldsymbol{p^i}$ to prove this), thus ensuring that EC is the worst possible expected cost across all probability distributions $\boldsymbol{p}$ within the ambiguity set $\mathbb{F}$.

It is easy to show that the distributionally robust solution always lies in the Pareto efficient frontier (independent of the geometry of the ambiguity set). Indeed, let χ^* be the solution to the distributionally robust TEP (PDR), i.e. $\chi^* =$

$\text{argmin}_{\chi} \max_{p \in \mathbb{F}} E_p[TC_s(\chi)] \;\; s.t. \; (2) - (12)$, and $\mathbb{F}$ be such that $\forall \boldsymbol{p^0} : \exists p_s^0 = 0 \Rightarrow \boldsymbol{p^0} \notin \mathbb{F}$ (i.e. probabilities in $\mathbb{F}$ do not contain any corner of the probability space, meaning that no particular scenario is possibly conceived to be absolutely certain). If χ^* is non efficient, then:

$$\begin{aligned} &\exists \chi_0, s_0 : TC_s(\chi_0) \leq TC_s(\chi^*) \; \forall s \; \wedge TC_{s_0}(\chi_0) < TC_{s_0}(\chi^*) \\ \Rightarrow &\sum_{s \neq s_0} p_s TC_s(\chi_0) \leq \sum_{s \neq s_0} p_s TC_s(\chi^*) \; \wedge p_{s_0} TC_{s_0}(\chi_0) < p_{s_0} TC_{s_0}(\chi^*) \\ &\Rightarrow E[TC_s(\chi_0)] < E[TC_s(\chi^*)] \\ &\Rightarrow \max_{p \in \mathbb{F}} E_p[TC_s(\chi)] < \max_{p \in \mathbb{F}} E_p[TC_s(\chi^*)] \quad \rightarrow\leftarrow \end{aligned} \tag{45}$$

The result is hence proven by contraposition (we want to prove that if x is the optimal solution then it is efficient or $p \Rightarrow q$, so we assume it is inefficient and prove that it cannot be optimal or $\sim q \Rightarrow \sim p$). Since the solution χ^0 has strictly lower expected cost than solution χ^* under any possible $p \in \mathbb{F}$,[36] then it also has strictly lower worse expected cost than solution χ^*, arriving at a contradiction since then χ^* cannot be an optimal solution to (PDR). Hence, χ^* must be an efficient alternative.

[36] Note that $\{a \leq b \wedge c < d\} \Rightarrow \{a + c < b + d\}$ since $\{\{a < b \vee a = b\} \wedge c < d\} \Leftrightarrow \{\{a < b \wedge c < d\} \vee \{a = b \wedge c < d\}\} \Rightarrow \{\{a + c < b + d\} \vee \{a + c < b + d\}\}$.