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Salazar, Grondona, Reinaldo Tomás Enrique

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Occupational safety, wages and labor turnover in the mining industry

Reinaldo Tomás Enrique Salazar Grondona

Comisión

Alexandre Janiak
Pablo Celhay

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Reinaldo T.E. Salazar Grondona †

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Abstract

The objective of this paper is to identify the channels that allow us to numerically replicate the following scenarios: the first, in which firms compensate through wages an increase in the level of risk associated with employment, better known as compensating wage differentials; the second, in which firms do not compensate for risk, and there are jobs with low risk and high wages, and jobs with high risk and low wages. For this we use a labor turnover model with search frictions based on Burdett and Mortensen (1998), which also includes a job risk variable that depends on the technology adopted by the firm. This novel extension influences both the firm's decisions, since it faces a trade-off between wage and safety technology to be adopted, and the workers' decisions, since they must not only make a wage decision but also a safety decision. Given this framework, we obtain the equilibrium distributions of wages, technology and number of employees per firm for different numerical cases, based on which we generate simulations that allow us to replicate the above scenarios.

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†Comments to rtsalazar@uc.cl

1 Introduction

The theory of compensating wage differentials, initially mentioned by Adam Smith in his work The Wealth of Nations, establishes that there are wage differentials that compensate for differences in the level of risk associated with a job. Specifically, Smith (1776) states that "the wages of labor vary with the ease or hardship, the cleanliness or dirtiness, the honourableness or dishonourableness of the employement" (p. 112). There is a vast literature associated with this topic and the calculation of VSL (value of a statistical life) which, according to Ruser and Butler (2009), is defined as the amount a society is willing to pay to avoid a loss of a statistical life. Ruser (1991) had already mentioned that the increase in workers' compensation benefits was associated with an increase in the accident rate, i.e., the employee was given higher compensation for accepting a higher level of risk. Aldy and Viscusi (2003) find evidence of this relationship between level of risk taken by workers and wages, predicting that there is a positive correlation between the two variables. In addition, the work of Leeth and Ruser (2003) identifies that women have three times greater compensating wage differentials than men, while Hersch (1998) concludes that there is no evidence of compensation for men, but there is for women.

This positive correlation between wages and the associated risk level, confirmed by Aldy and Viscusi (2003), will not necessarily hold true for all industries. Figure 1 shows the relationship between firm size and the ratio of accidents per worker in the Chilean mining industry. It can be seen that as the size of the firm increases, so does the accident ratio. However, after a certain size, this ratio begins to decline, suggesting that larger firms (in terms of employees) have a lower accident ratio.

According to the yearbook published by Sernageomin (2010), the average salaries of large Chilean mining companies far exceed those of medium-sized mining companies: US $69,639 versus US$24,585 per worker (annual values). Under the assumption that larger firms offer higher wages than smaller firms (since higher wages attract a larger number of employees), in the case described above, the prediction of the theory of compensating wage differentials does not hold. The firms that pay the highest wages (the large ones) are also the ones that have the least associated risk. Then, how can we explain that both types of firms can coexist given the fact that there are firms that pay more and have a lower level of associated risk? We are faced with a scenario in which up to a certain point the prediction of the theory of compensating wage differentials is fulfilled, but then it ceases to be valid.

According to Ruser and Butler (2009), common criticisms of this approach are based on the assumption of employee rationality and complete information. If workers underestimate the risks associated with the job under consideration, or if they are unaware of it, they will not demand a wage premium congruent with the risk that this job entails. This leads to an underestimation of the VSL and thus to a lower wage. One of the works that tries to explain this is the one by Gegax et al. (1991), in which the authors estimate wage compensation using workers' perception of accident rates through a survey. However, there seem to be no theoretical tools that can replicate this scenario in which there is both the world confirmed by Aldy and Viscusi (2003) of compensating wage differentials and the case in which higher wages are associated with less risky jobs.

The objective of this study is to test theoretically the existence of both scenarios. Brown (1980) finds no evidence of compensating wage differentials even after using panel data controlling for individual fixed effects. According to Manning (2003), this is because Brown does not consider the existence of frictions in the labor market, since even workers with identical skills would receive different levels of utility.

In this way we intend to contribute, firstly, to the literature in the area of occupational health and safety economics by identifying a channel through which these scenarios are generated. For this, the inclusion of search frictions in a labor turnover model, as mentioned by Manning (2003), will be considered. Secondly, it is expected to make a contribution in the area of labor economics, specifically in the literature related to search frictions and labor turnover models, through the inclusion of a risk variable. To this end, we consider extending the Burdett and Mortensen (1998) model proposed in the paper entitled Wage Differentials, Employer Size, and Unemployment. In this paper the authors,

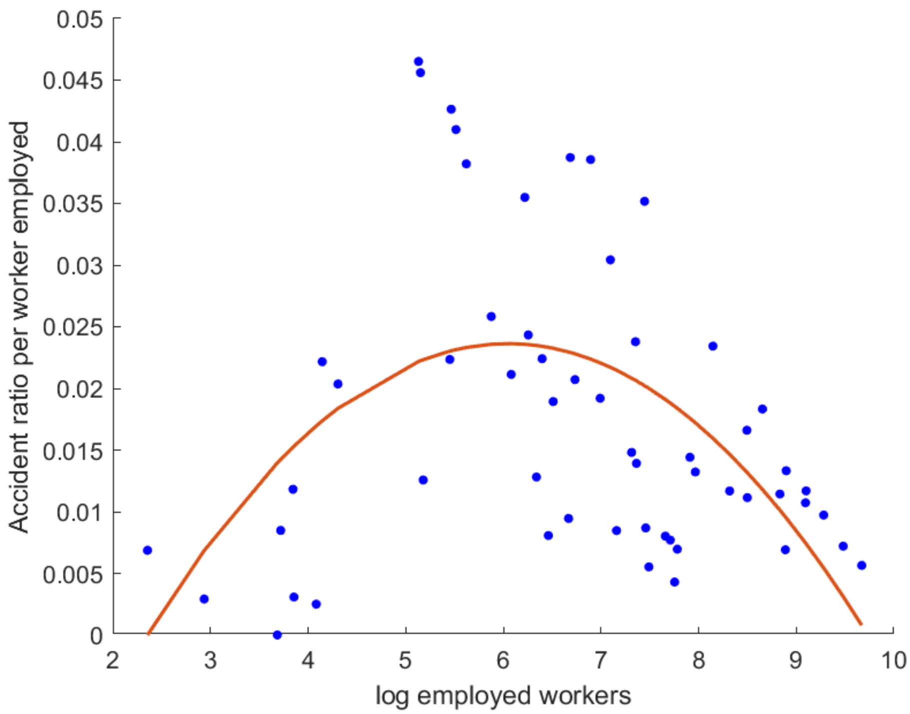

Figure 1: Ratio of accidents per worker vs. logarithm of total workers per site, average over all years of the sample (2010 - 2021). Each point represents a mining site. Source: SONAMI (2022)

based on labor turnover, find that in equilibrium there is a non-degenerate distribution of wages (wage dispersion) despite the fact that in the given economy all agents (workers and firms) are equal ex-ante. Furthermore, they show that a minimum wage can exist and that it increases the level of employment (as long as it does not exceed marginal product). Thus, this model will be extended using a labor risk variable, which will depend on the technology adopted by the firm, and the scenarios observed in the mining industry will be generated numerically.

With respect to the firms' decision on which technology to use, a paper in this area is that of Acemoglu and Shimer (2000), in which the firms in the model with search frictions they propose must decide which technology to commit to and the wage level to offer to workers. The authors find that, first, when labor search is costly, in equilibrium there will be wage dispersion among identical workers, even when firms use the same technology. Second, with the *framework* they propose to analyze firm decisions, they show that the forces that generate wage dispersion also generate technology dispersion. Third, they show that in equilibrium the firms that offer higher wages are those that also adopt the best technologies.

A paper in the line of the literature related to hedonic wages (dealing with the relationship between wage and job characteristics) is the one by Hwang et al. (1998) in which they investigate the consequences of labor market search on that theory. For this, they extend Mortensen's (1990) model in which they reach a non-degenerate distribution of wages in equilibrium, adding a non-wage component that influences agents' decision making. The main difference with this work is that in the model presented here, agents will make decisions based on values discounted by the probability of death, while in Hwang et al. they discount at a fixed rate, since the wage component they include does not affect the future values observed by workers.

The models of Burdett and Mortensen (1998) and Acemoglu and Shimer (2000) do not include an occupational risk variable that influences the decisions of agents, both firms and workers. Both papers use wage as a method of employee retention, but not the level of risk. How do the distributions of labor turnover, wage, and technology change in equilibrium when considering risk in decisions? It is then hoped to contribute to the field of labor economics through the inclusion and analysis of this variable in a model of labor search with frictions.

Therefore, in order to develop this research, the job search model proposed by Burdett and Mortensen (1998) will be extended to generate numerical scenarios like those observed in the mining industry. The extension of this model is mainly based on adding the probability of accidents through the inclusion of technologies, which will determine the safety level of the firm. The better the technology adopted by the firm, the safer its operation will be, and therefore the probability of an accident will be lower. However, the safer the technology, the higher the cost for the company. Thus, the company will have to make two decisions: the salary to offer (since this is the standard decision made by firms in these models) and the technology to use. The agents will observe these decisions and discount the value of working in the firms considering the wage paid and the probability of death, and with this information they will make their decisions.

Finally, numerical results were obtained for different cases, among which both the positive and negative relationship between the salary offered by the firms and the level of associated risk observed in the mining industry, an example used as a framework in this research, was confirmed. In this way, it was possible to identify a mechanism by which both types of firms can coexist: both those that offer high wages and safety levels, and those that offer low wages and safety levels. Thus, it is possible to observe both the world where the theory of compensating wage differentials is fulfilled, and the world where it does not exist.

It is expected that future research will focus on calibrating the parameters used in the model, since in the present work they were used in such a way that they generate strong labor frictions. It is interesting to see what happens in a more competitive labor market.

The rest of the paper is structured as follows: Section 2 presents the model developed, the assumptions behind it and a numerical algorithm to solve it. Section 3 describes the results obtained for different parameterizations of the model. Finally, section 4 summarizes the main conclusions of this research and possible next steps.

2 Model

The model to be used is presented below. This is an extension of the model proposed by Burdett and Mortensen (1998), in which they generate wage dispersion in the equilibrium of an economy where all agents, both firms and workers are equal to each other ex-ante, respectively.

A continuum of both firms and workers will be assumed. Workers are equally productive ex-ante and value leisure at b. The number of workers is defined by the variable m, while the measure of firms is set to 1, for simplicity. These are infinitely small (individually) relative to the entire market. Workers may be employed or unemployed, and will randomly search firms for jobs. They will choose those that offer a higher wage than they receive if they are employed, or one that exceeds the value of leisure if they are unemployed, but they will also consider the level of risk associated with such employment, which will be characterized by the technology adopted by the firm. All workers receive offers at a rate of λ_i, where $i \in \{employed, unemployed\}$. That is, the arrival of job offers depends on the employment status of the worker. On the other hand, employed workers may lose their jobs at a δ rate.

Firms set wages w and the level of technology k to maximize their profits. All workers in the same firm receive equal wages w. The technology k adopted by the firm will define the level of labor risk faced by employees, and the marginal product received by the firm, defined by $f(k)$. The joint distribution of wages and technology offered by firms is given by $F_{w,k}(w, k)$.

The novelty lies in the inclusion of a risk variable that will affect workers' decision making. Firms, by including in their decision set the level of technology to lease, influence the level of risk associated with the job they offer. The higher the technology expenditure (higher level of k), the lower the risk associated with the job offered. For such reasons, the rate $d(k)$ is defined as the probability of death by occupational accident of an employed worker. In addition, we define the variable $\bar{d}$ as the probability of death by natural causes of each worker, and n as the birth rate of new workers, who will enter the labor market in an unemployed state.

Given that the worker faces both the wage level w and the technology level k (the latter through the probability of death by occupational accident $d(k)$) offered by the firm to make his decision, the first part of the analysis will focus on characterizing the equilibrium levels of this economy considering that the worker, instead of deciding by observing (w, k), will do so by directly observing the value that these combinations can give him if he is employed, i.e. $V_1(w, k) = V_1$.

In other words, in order to make his decision to be employed or to continue looking for a job, the worker will face the combination (w, k) through the variable V_1, which comes from a distribution defined as:

$$F_{V_1}(V_1) = F_{V_1}(V_1(w, k))$$

Once the distribution $F_{V_1}(V_1)$ has been characterized, we will proceed to the second stage, which consists of characterizing the individual distributions for wages w and technology k in equilibrium. With these, we will try to generate with numerical exercises the scenario observed in the mining industry, in which there is initially a positive correlation between wage and labor risk, and then a negative one. Any factor that generates dispersion in V_1 is expected to contradict what is predicted by the theory of compensating wage differentials and, therefore, generate a negative or non-existent correlation between wages and risk level.

The following is the problem that the worker must face.

2.1 Worker problem

We define V_0 as the value the worker obtains from being unemployed. Conversely, let $V_1(w,k) = V_1$ be the value he obtains from being employed in a firm that pays a wage w and possesses a technology level k. The rate at which workers discount future benefits, r_w, includes the rate of death from natural causes, $\bar{d}$. Then,

$$r_w = \tilde{r} + \bar{d}$$

where $\tilde{r}$ is the discount rate. In addition, λ_0 is the rate at which unemployed workers receive a job offer, and λ_1 is the rate at which employed workers receive offers.

2.1.1 Unemployment value

Equation (1) represents the expected value of a worker's discounted income when unemployed.

$$r_w \cdot V_0 = b + \lambda_0 \cdot [\int [\max(V_0, \tilde{V_1}) - V_0]\, dF_{V_1}(\tilde{V_1})] \tag{1}$$

In other words, the opportunity cost of looking for a job while unemployed, the interest given by $r_w \cdot V_0$, is equal to income while unemployed, b, plus the capital gain attributable to finding and accepting a job (only if accepting the offer yields more value than remaining unemployed and continuing the search).

2.1.2 Employment value

Equation (2) represents the expected value of a worker's discounted income when employed.

$$(r_w + d(k)) \cdot V_1 = w + \lambda_1 \cdot [\int [\max(V_1, \tilde{V_1}) - V_1]\, dF_{V_1}(\tilde{V_1})] + \delta \cdot [V_0 - V_1] \tag{2}$$

The interpretation of this equation is similar to (1): the opportunity cost of seeking employment while employed, the interest given by $(r_w + d(k)) \cdot V_1$, is equal to income while employed, w, plus the capital gain attributable to finding and accepting a job (only if accepting the offer yields more value than remaining in the same job and continuing to search), minus the value associated with a possible loss of value from job destruction, given by $\delta \cdot [V_0 - V_1]$.

Equation (2) shows two channels through which the desired scenarios can be generated. The first one shows the positive correlation between salary and risk identified by Aldy and Viscusi (2003). For a fixed V_1, it is observed that there is a positive relationship between the wage w and the level of risk represented by $d(k)$. An increase (decrease) in the risk level must be compensated by an increase (decrease) in the wage level for the employed worker to remain indifferent. This is just what the theory of compensating wage differentials states. Thus, the following proposition arises:

Proposition 1 (Compensating wage differentials) *Given a value of V_1, the higher the risk $d(k)$ associated with the job, the higher the wage w with which the firm must compensate the worker.*

However, since the base characteristics of the Burdett and Mortensen (1998) model hold, dispersion in the distribution of V_1 is expected. If this happens, the positive relationship between wage and risk may break down, as an increase (decrease) in the level of risk through $d(k)$ may be offset by a decrease (increase) in V_1 without altering the wage w, and still keep the employed worker indifferent. This is the second channel, in which firms do not compensate through the wage for a higher level of associated risk.

Proposition 2 (Risk without salary compensation) *Due to the existing dispersion in V_1, a higher risk $d(k)$ associated with employment will not necessarily be offset by a higher wage w.*

Moreover, $V_1(.)$ is increasing in w and V_0 is independent of w. Let R and C be the reservation wage and reservation capital, respectively. An unemployed worker will not accept a job offer that pays a wage less than R and has a capital level less than C.

Then,

$$V_1(R, C) = \underline{V}_1 \tag{3}$$

where $\underline{V}_1$ represents the reserve value below which no worker will accept to be employed. In equilibrium it will be satisfied that $\underline{V}_1 = V_0$.

The analysis of the worker's problem is based on the use of equations (1), (2) and (3). Considering the worker's indifference condition between choosing V_1 y $\tilde{V_1}$, and under the condition stated in equation (3), the following expression can be obtained, the development of which can be found in Appendices (2) and (3):

$$R - \frac{r_w + d(C)}{r_w} \cdot b = [\frac{r_w + d(C)}{r_w} \cdot \lambda_0 - \lambda_1] \cdot [\int_{\underline{V}_1}^{\infty} [1 - F_{V_1}(\tilde{V_1})] \, d(\tilde{V_1})] \tag{4}$$

Equation (4) represents an optimal strategy that an unemployed worker must follow to accept a job: that it offers a wage greater than or equal to R, which is defined on the basis of the distribution $F_{V_1}(V_1)$.

On the other hand, by developing the equation (2), w can be obtained as a function of V_1 and k (see Appendix 5.3),

$$w(V_1, k) = V_1 \cdot (r_w + d(k) + \lambda_1 \cdot (1 - F_{V_1}(V_1)) + \delta) - \lambda_1 \cdot [\int \tilde{V_1} \, dF_{V_1}(\tilde{V_1})] - \delta \cdot V_0 \tag{5}$$

From equation (5) we can obtain the marginal rate of substitution of $\frac{\partial w(V_1,k)}{\partial d(k)} = V_1$. The higher V_1 offered by the firm, the higher the wage compensation $w(V_1, k)$ must be for the worker to remain indifferent.

2.1.3 Equilibrium flows

We will now calculate the flow of workers in and out of unemployment in equilibrium. Let u be the number of unemployed workers. The flow of workers out of unemployment is: $\lambda_0 \cdot (1 - F_{V_1}(\underline{V}_1)) \cdot u + \bar{d} \cdot u$ where $\bar{d}$ is the probability of dying from natural causes. The flow of workers entering unemployment is: $\delta \cdot (m - u) + n$, where n is the number of worker births.

Equating the flows gives the number of unemployed workers in equilibrium,

$$u = \frac{m \cdot \delta + n}{(1 - F_{V_1}(\underline{V}_1)) \cdot \lambda_0 + \bar{d} + \delta} \tag{6}$$

where the exogenous rate of births of unemployed workers n can be defined as n.

$$n = m \cdot \bar{d} + (m - u) \cdot \int_C^{\infty} d(\tilde{k}) \, dG_k(\tilde{k}) \tag{7}$$

where $\tilde{d} = \int_C^{\infty} d(\tilde{k}) \, dG_k(\tilde{k})$, and $G_k(k)$ is the proportion of workers employed in a firm that offers a level of technology no greater than k. From this, an expression for m can be obtained,

$$m = \frac{n + u \cdot \tilde{d}}{\bar{d} + \tilde{d}} \tag{8}$$

Replacing this expression in u, we obtain:

$$u = \frac{n \cdot (\delta + \bar{d} + \tilde{d})}{[(1 - F_{V_1}(\underline{V}_1)) \cdot \lambda_0 + \bar{d} + \delta] \cdot \bar{d} + ((1 - F_{V_1}(\underline{V}_1)) \cdot \lambda_0 + \bar{d}) \cdot \tilde{d}} \tag{9}$$

It can be seen from this expression that the level of unemployment rises with an increase in δ, but falls with increases in $\bar{d}$ and $\tilde{d}$ (see Appendix 5.4).

On the other hand, we define $G_{V_1}(V_1, t)$ as the proportion of workers employed at t receiving a value no greater than V_1. Let $G_{V_1}(V_1, t) \cdot (m - u(t))$ be the number of workers employed receiving a value less than or equal to V_1 at t.

Its derivative can be defined as follows:

$$\begin{aligned} \frac{\partial G_{V_1}(V_1, t) \cdot (m - u(t))}{\partial t} = \lambda_0 \cdot \max(F_{V_1}(V_1) - F_{V_1}(\underline{V}_1), 0) \cdot u(t) - [\lambda_1 \cdot [1 - F_{V_1}(V_1)] + \\ + \delta + \bar{d} + \tilde{d}] \cdot G_{V_1}(V_1, t) \cdot (m - u(t)) \end{aligned} \tag{10}$$

The first term represents the flow in t of workers from being unemployed to employed in firms paying less than or equal to V_1. The second term represents the flow in t of workers who move from being employed in a firm paying less than or equal to V_1 to unemployed; who move to a firm offering higher value; who die from natural causes; and who die from occupational accidents.

Considering that in the steady state both flows equalize, that $V_1 > \underline{V}_1$ and equation (9), we obtain the following expression for $G_{V_1}(V_1)$ (see development in Appendix 5.5)

$$G_{V_1}(V_1) = \frac{(F_{V_1}(V_1) - F_{V_1}(\underline{V}_1))}{\lambda_1 \cdot [1 - F_{V_1}(V_1)] + \delta + \bar{d} + \tilde{d}} \cdot \frac{\delta + \bar{d} + \tilde{d}}{(1 - F_{V_1}(\underline{V}_1))} \tag{11}$$

The equation (11) then represents the steady-state distribution of V_1 values earned by workers who are employed, such that $V_1 > \underline{V}_1$.

2.1.4 Number of workers per firm

As in Manning (2003), let $N(V_1, k, F)$ be the number of workers in a firm offering V_1 given the distribution $F_{V_1}(V_1)$. Let $s(V_1, k, F)$ be the separation rate of the same firm offering V_1 and $R(V_1, F)$ be the recruitment flow. In equilibrium it is satisfied that

$$s(V_1, k, F) \cdot N(V_1, k, F) = R(V_1, F) \tag{12}$$

where

$$s(V_1, k, F) = \delta + \lambda_1 \cdot [1 - F_{V_1}(V_1)] + \bar{d} + d(k) \tag{13}$$

and

$$R(V_1, F) = \lambda_0 \cdot u \cdot (1 - F_{V_1}(\underline{V}_1)) + (m - u) \cdot G(V_1, F) \cdot \lambda_1 \tag{14}$$

Then, to determine the number of workers, $N(V_1,k,F)$ is cleared, and the expressions for $s(V_1,k,F)$ and $R(V_1,F)$ are replaced. Under the assumption that $F(\underline{V}_1)=0$ we obtain (see Appendix 5.6).

$$N(V_1,k,F)=\left[\frac{n\cdot\lambda_0\cdot(\delta+\bar{d}+\tilde{d})\cdot(\lambda_1+\delta+\bar{d}+\tilde{d})}{([\lambda_0+\bar{d}+\delta]\cdot\bar{d}+(\lambda_0+\bar{d})\cdot\tilde{d})\cdot(\lambda_1\cdot[1-F_{V_1}(V_1)]+\delta+\bar{d}+\tilde{d})}\right]\cdot\frac{1}{\delta+\lambda_1\cdot[1-F_{V_1}(V_1)]+\bar{d}+d(k)} \tag{15}$$

For firms offering the lowest value $V_1=\underline{V}_1$, the number of workers they will hire is given by the following expression

$$N(\underline{V}_1,k,F)=\left[\frac{n\cdot\lambda_0\cdot(\delta+\bar{d}+\tilde{d})}{[\lambda_0+\bar{d}+\delta]\cdot\bar{d}+(\lambda_0+\bar{d})\cdot\tilde{d}}\right]\cdot\frac{1}{\delta+\lambda_1+\bar{d}+d(k)} \tag{16}$$

In the next section, the problem faced by a profit maximizing firm is presented.

2.2 Firm Problem

The firm in this economy faces a profit maximization problem π. For this, it must decide the value V_1 it offers to workers. This decision will be subject to a choice of safety technology k. The value it decides to offer will directly affect its profit through the wage level itself; through the number of workers it attracts, as higher value increases the number of workers; through the technology adopted; and through the marginal product $f(k(V_1))$.

The problem can be defined as follows

$$\pi=\max_{V_1}[f(k(V_1))-w(V_1,k(V_1))]\cdot N(V_1,k(V_1),F)-r_f\cdot k(V_1) \tag{17}$$

subject to

$$k(V_1)=arg\max_{\hat{k}}(f(\hat{k})-w(V_1,\hat{k}))\cdot N(V_1,\hat{k},F)-r_f\cdot\hat{k} \tag{18}$$

whose first order condition with respect to k is

$$\Big(\frac{\partial f(\hat{k})}{\partial\hat{k}}-\frac{\partial w(V_1,\hat{k})}{\partial\hat{k}}\Big)\cdot N(V_1,\hat{k},F)+(f(\hat{k})-w(V_1,\hat{k}))\cdot\frac{\partial N(V_1,\hat{k},F)}{\partial\hat{k}}=r_f \tag{19}$$

where $\frac{\partial f(\hat{k})}{\partial\hat{k}}\geq 0$.

Equation (19) presents new terms that differentiate it from the case of perfect competition, in which it is obtained that $\frac{\partial f(k)}{\partial k}\cdot N(V_1,k)=r_f$. On the left-hand side of the equation (19) (which in perfect competition is the marginal product), the new term that is added is $(f(k)-w(V_1,k))\cdot\frac{\partial N(V_1,k)}{\partial k}-\frac{\partial w(V_1,k)}{\partial k}\cdot N(V_1,k)$. It is then observed that there are 3 effects that the capital decision has for a firm:

Proposition 3 (Effects of security technology on the firm) *There are 3 effects of the security technology decision:*

1. ***A direct effect through the firm's marginal product:*** $\frac{\partial f(k)}{\partial k}\cdot N(V_1,k)$. *This is the effect that can also be found in the case of perfect competition, and is the mechanism through which capital is productive for the firm.*

2. ***A labor security effect, which helps the firm to have a larger scale and thus generate higher profits:*** $(f(k) - w(V_1, k)) \cdot \frac{\partial N(V_1,k)}{\partial k}$*. Through this mechanism, the firm can influence the number of workers it hires. Workers value security positively, so increases (decreases) in the level of* k *will generate increases (decreases) in the level of employees.*

3. ***A compensating wage differential effect:*** $\frac{\partial w(V_1,k)}{\partial k} \cdot N(V_1, k)$*. This effect is of interest for the study, because if it is negative, it can be affirmed that there are compensating wage differentials.*

These are the 3 effects that affect the firm when it makes its decision on the level of technology k to adopt.

2.3 Equilibrium

Under the assumption that $F(k, N) = f(k) \cdot N(V_1, k) = k^\alpha \cdot N(V_1)^{1-\alpha}$, with $0 \leq \alpha \leq 1$, the problem to be solved can be rewritten as follows.

$$\pi = \max_{V_1} k(V_1)^\alpha \cdot N(V_1, k(V_1), F)^{1-\alpha} - w(V_1, k(V_1)) \cdot N(V_1, k(V_1), F) - r_f \cdot k(V_1) \tag{20}$$

subject to

$$k(V_1) = arg \max_{\hat{k}} \hat{k}^\alpha \cdot N(V_1, \hat{k}, F)^{1-\alpha} - w(V_1, \hat{k}) \cdot N(V_1, \hat{k}, F) - r_f \cdot \hat{k} \tag{21}$$

The first-order condition of (21) with respect to $\hat{k}$ is

$$\begin{aligned}\alpha \cdot \hat{k}^{\alpha-1} \cdot N(V_1, \hat{k}, F)^{1-\alpha} + \hat{k}^\alpha \cdot (1-\alpha) \cdot N(V_1, \hat{k}, F)^{-\alpha} \cdot \frac{\partial N(V_1, \hat{k}, F)}{\partial \hat{k}} - \\ - \frac{\partial w(V_1, \hat{k})}{\partial \hat{k}} \cdot N(V_1, \hat{k}, F) - w(V_1, \hat{k}) \cdot \frac{\partial N(V_1, \hat{k}, F)}{\partial \hat{k}} = r_f\end{aligned} \tag{22}$$

where $N(V_1, k, F)$ is given by the equation (15) and its derivative with respect to k is

$$\frac{\partial N(V_1, k, F)}{\partial k} = N(V_1, k, F) \cdot \frac{-d'(k)}{\delta + \lambda_1 \cdot [1 - F(V_1)] + \bar{d} + d(k)} \tag{23}$$

Both expressions obtained for $N(V_1, k, F)$ y $\frac{\partial N(V_1,k,F)}{\partial k}$ are replaced in the first order condition to obtain the solution of $k(V_1)$. Due to the complexity of the solution, a numerical analysis is necessary to find the desired solutions. The algorithm used for the numerical solution is presented below.

2.4 Resolution algorithm

Given the complexity of the problem to be solved, the following algorithm is used to numerically determine a solution.

1. We perform a guess of the distribution of values offered in equilibrium by the firms, $F(V_1)$, and of the minimum that workers accept to be employed, V_0.
2. The solution to the capital choice problem $k(V_1) = arg \max_{\hat{k}} (f(\hat{k}) - w(V_1, \hat{k})) \cdot N(V_1, \hat{k}, F) - r_f \cdot \hat{k}$ is found. For this, the first order condition of the problem is used.
3. Using the equation (5) we determine $w(V_1, k(V_1))$.

4. The firm's profits are calculated with equation (20).

5. In Burdett and Mortensen (1998) it is obtained that in equilibrium all firms receive the same profit π. Thus, if for every element of the distribution $F(V_1)$ such that $V_1 > V_0$ the firms get the same profit π, then a solution has been found. If not, one must return to the first step.

6. The obtained distribution $F(V_1)$ must be fitted to the equation (1) with V_0. For this, the distribution is fitted with $Pr(V_1|V_1 \geq V_0)$, and based on this result the value of V_0 is updated with the equation (1).

7. If the updated value for V_0 is not equal to the initial realized guess, then use a combination of the initial guess and the updated value of V_0 as the new guess and return to step 1.

8. If the updated value for V_0 is equal to the initial guess realized, then an equilibrium has been found.

9. Once the equilibrium distributions of wages, capital and employment are obtained, they can be used to generate simulations. With these, we try to replicate the scenario proposed in Figure 1. For this purpose, a random sample of 1000 firms is generated, which choose wage and capital levels according to the equilibrium distributions obtained. These simulations are performed for different values of α.

2.5 Model parameterization

This section lists the parameters used to perform the numerical exercises and the values assigned to them. In general, these were adjusted in an ad hoc manner in order to replicate the case of the mining industry, in which the risk-wage ratio is positive in one section and negative in another.

Since the numerical analysis requires the use of grids, the first parameters presented in the table are the points in each grid and the upper bound for the variables k and V_1. Parameters b, λ_0, δ and $\bar{d}$ were kept constant in all cases.

Table 1: Parameters per case - Numerical analysis

Parameters	**Values**				
	α=0	α=1/4	$\alpha = 0.5$	$\alpha = 3/4$	$\alpha = 0.9$
k grid points	10000	1000	1000	1000	1000
Maximum k	300	300	300	300	300
V grid points	30	30	30	30	30
Maximum V	150	150	100	150	100
b	0.1	0.1	0.1	0.1	0.1
r	0.01	0.05	0.15	0.2	0.01
n	0.5	0.3	0.2	0.1	0.5
λ_0	0.1	0.1	0.1	0.1	0.1
λ_1	0.3	0.5	0.5	0.5	0.3
δ	0.05	0.05	0.05	0.05	0.05
$\bar{d}$	0.01	0.01	0.01	0.01	0.01
V_0 guess	67	46	24	22	1
Simulated firms	1000	1000	1000	1000	100

The results of these numerical exercises are presented below.

3 Results

3.1 Firm problem when $\alpha = 0$

When $\alpha = 0$, the first point of the proposition 3 ceases to have an effect, since it is multiplied by the parameter α (see development in Appendix (5.7)). The following figures show the simulations performed with the equilibrium distributions. In particular, the relationship between the level of risk taken by firms and the wages they pay is presented. The objective was to replicate the case of the mining industry presented in the first section, in which an increasing and also a decreasing risk-wage relationship was observed.

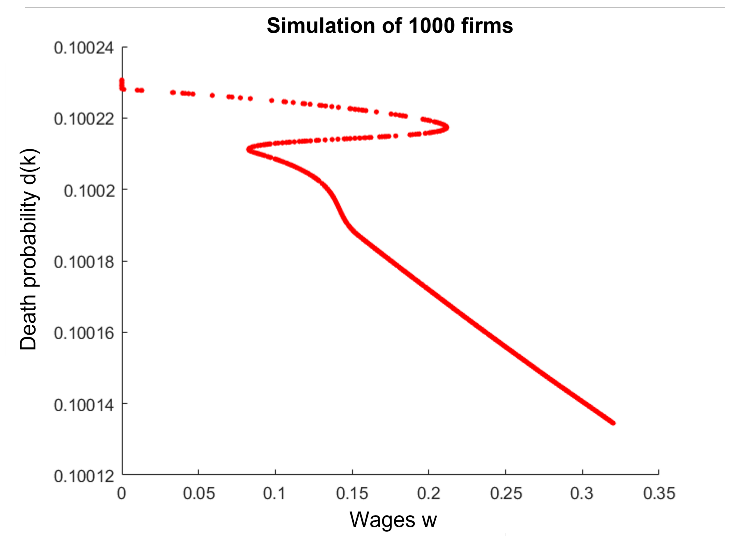

Figure 2: Probability of death $d(k)$ vs wage w offered by firms when $\alpha = 0$. Each point represents one firm.

Figure 2 shows the relationship between the level of risk, given by the variable $d(k)$, and wages $w(V_1, k)$. It can be seen that, in general, the relationship between the two variables is negative. This result implies that, for the parameters used, the firms that pay the best wages are at the same time the safest.

However, it can also be observed that part of this relationship is indeed positive. This does not rule out the possibility that these firms are complying with the theory of compensating wage differentials, so that the framework presented for the mining industry in which the two types of firms coexist could be replicated.

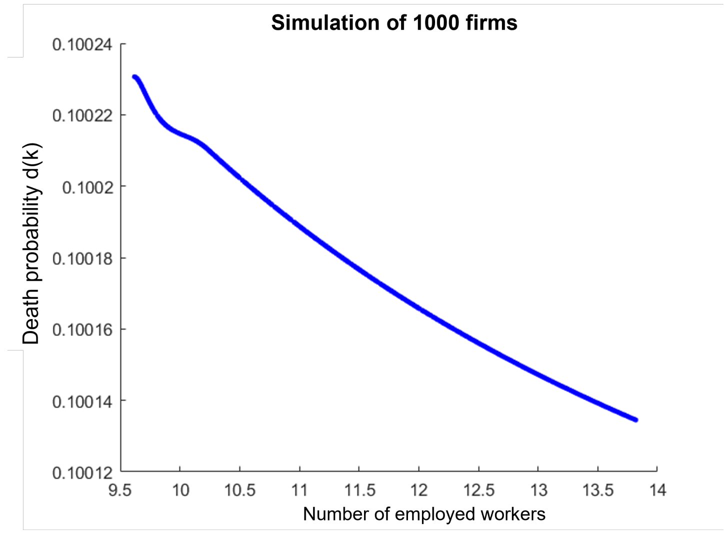

Figure 3: Probability of death $d(k)$ vs number of employees when $\alpha = 0$. Each point represents one firm.

Since figure 1 shows the relationship between risk and number of employees, it is interesting to observe this same relationship but in the model. This is shown in figure 3. This again shows this decreasing relationship observed in figure 2, i.e., the largest firms have the lowest probability of death.

3.2 Firm problem when $\alpha = \frac{1}{4}$

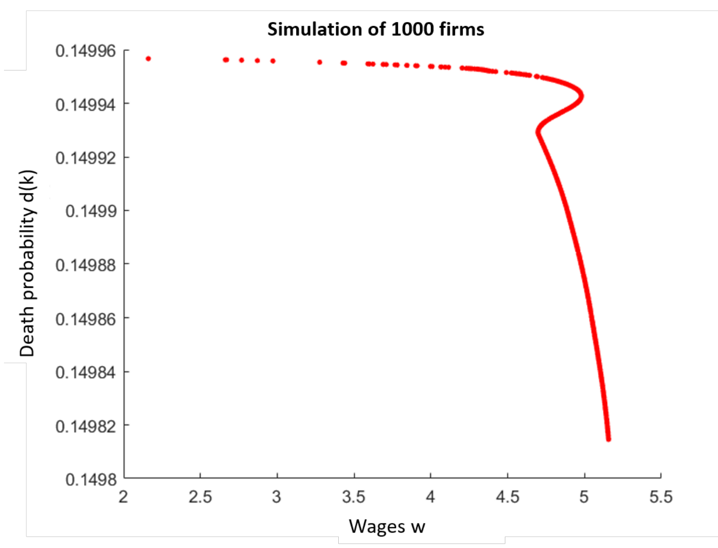

Figure 4: Probability of death $d(k)$ vs wage w offered by firms when $\alpha = \frac{1}{4}$. Each point represents one firm.

Figure 4 shows the relationship between the level of risk, given by the variable $d(k)$, and wages $w(V_1, k)$. It can be seen that, in general, the relationship between the two variables is negative. In fact, the slope of this case is first relatively constant, and then falls faster than in the case of $\alpha = 0$. This result implies that, for the parameters used, the firms that pay the best wages are at the same time the safest.

However, there is again a section in which there are firms that have a positive relationship between risk level and wages, which does not rule out the possibility that the theory of compensating wage differentials is fulfilled in this section.

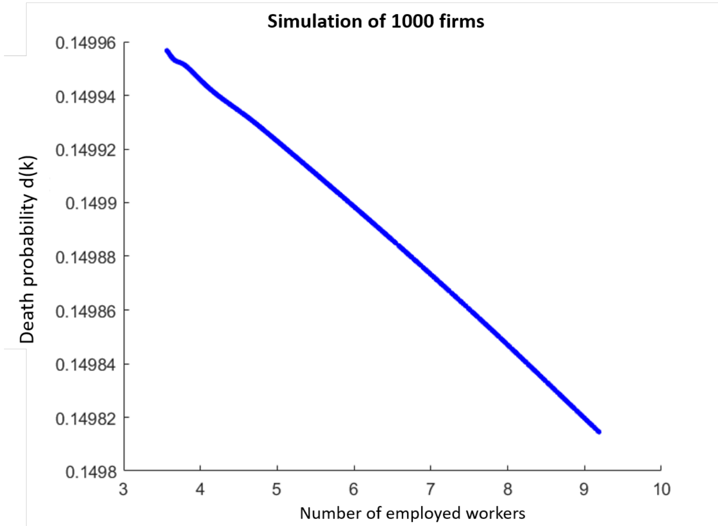

Figure 5: Probability of death $d(k)$ vs number of employees when $\alpha = \frac{1}{4}$. Each point represents one firm.

Figure 5 shows the relationship between risk and number of employees. This again shows the decreasing relationship observed in figure 4, i.e., the largest firms have the lowest probability of death. However, the slope falls with a constant speed, unlike the previous case.

3.3 Firm problem when $\alpha = \frac{1}{2}$

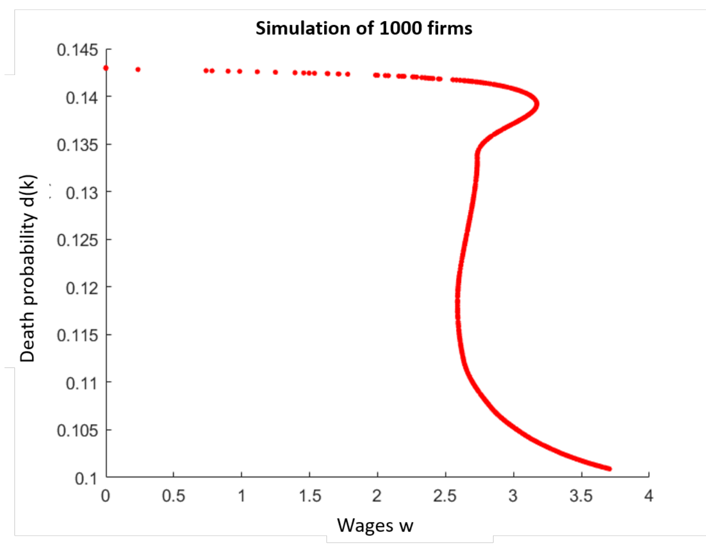

Figure 6: Probability of death $d(k)$ vs wage w offered by firms when $\alpha = \frac{1}{2}$. Each point represents one firm.

Figure 6, like the previous ones, shows the relationship between the level of risk, given by the variable $d(k)$, and wages $w(V_1, k)$. It is observed that, in general, firms pay high wages, as they accumulate on the right-hand side of the figure. It is also observed that the slope changes strongly, as it has steep inflection points that invert the relationship between wages and risk. The upward slope of this figure is higher than in the previous cases, which may imply the presence of risk compensation: the higher the probability of an accident, the higher the wage paid.

It can be observed that firms reach a minimum wage level (which is among the highest) before they start to decrease the risk associated with the job they offer. Once this level is reached, the risk drops sharply.

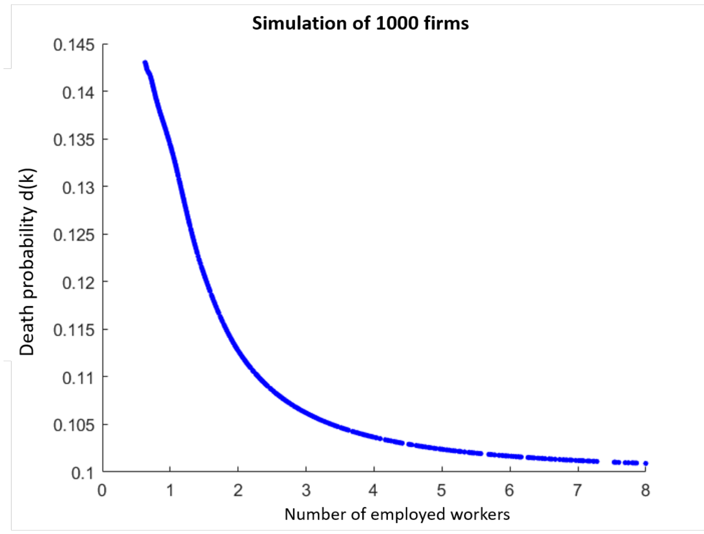

Figure 7: Probability of death $d(k)$ vs number of employees when $\alpha = \frac{1}{2}$. Each point represents one firm.

Figure 7 shows the relationship between risk and number of employees. This curve differs from the previous cases in that it now takes a convex shape, whereas the others were linear. At the beginning, smaller firms can reduce their risk which allows them to slowly increase the number of workers they employ. However, as the firm decreases its risk further, the increase in the number of employees starts to become larger, because the rate of substitution between the two increases.

3.4 Firm problem when $\alpha = \frac{3}{4}$

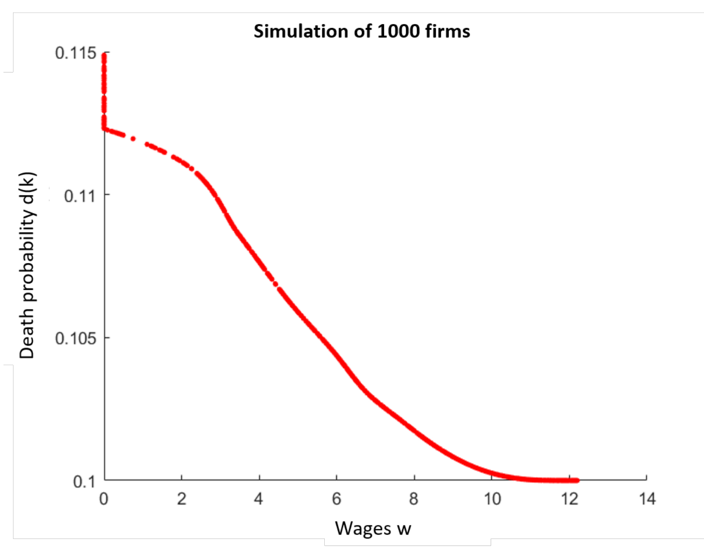

Figure 8: Probability of death $d(k)$ vs wage w offered by firms when $\alpha = \frac{3}{4}$. Each point represents one firm.

Figure 8 presents the relationship between the risk level given by $d(k)$ and the wages obtained from the simulation. Unlike the previous cases, for these parameters this relationship is decreasing. This implies that no trade-offs are paid for risk level, but that the firms that pay the best are at the same time the firms that have the least associated risk.

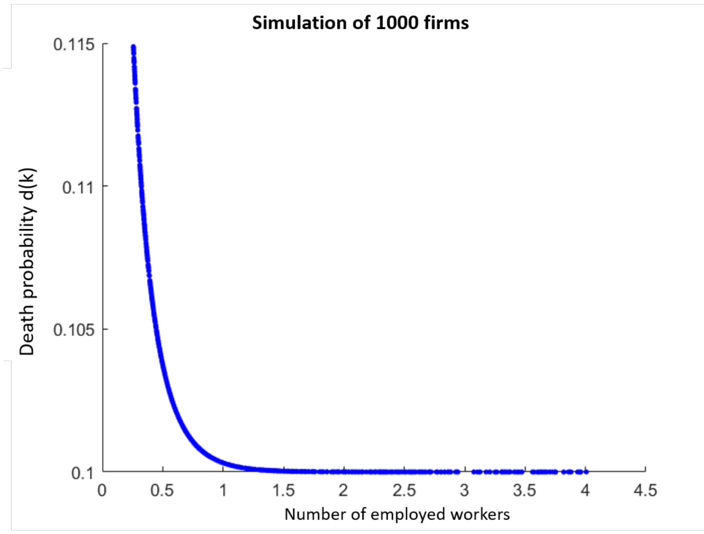

Figure 9: Probability of death $d(k)$ vs number of employees when $\alpha = \frac{3}{4}$. Each point represents one firm.

The figure 9 follows the same pattern as in the previous case ($\alpha = \frac{1}{2}$), as it becomes even more convex. The notion is similar to that of the previous case.

3.5 Firm problem when $\alpha = 0.9$

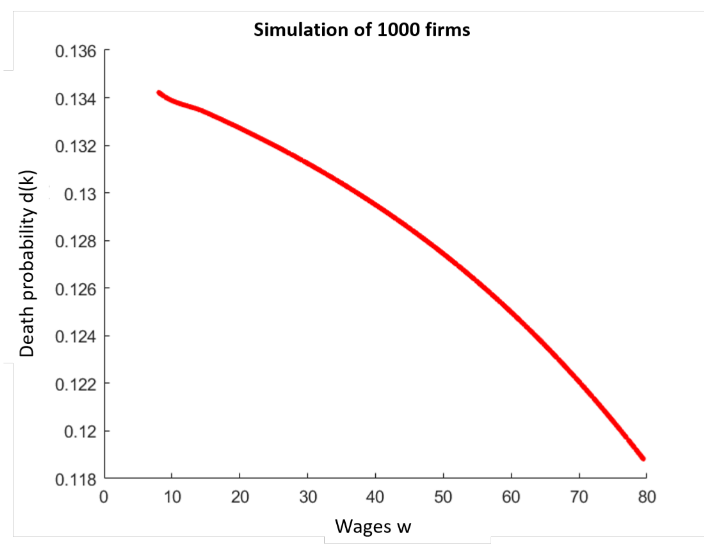

Figure 10: Probability of death $d(k)$ vs wage w offered by firms when $\alpha = 0.9$. Each point represents one firm.

Finally, figure (10) shows a concave relationship between the level of risk and wages. In general, the findings of this figure do not differ from the previous cases in that there is no section where the relationship is increasing. Thus, for this parameterization, no evidence of the presence of compensating wage differentials is found.

Figure (11) is also presented, but it is similar to that of the previous cases.

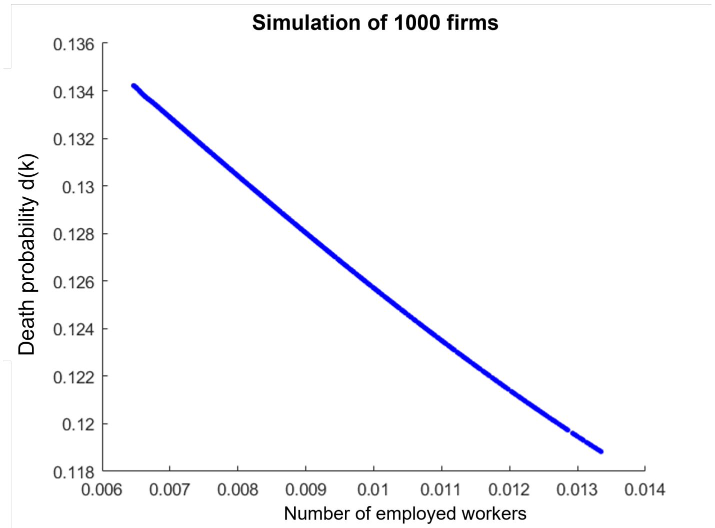

Figure 11: Probability of death $d(k)$ vs number of employees when $\alpha = 0.9$. Each point represents one firm.

3.6 Firm problem when $d(k) = d$

In the latter case we analyze what would happen if a firm cannot affect the probability of death through the technology level k. The simplicity of this model allows for a theoretical development. If it is assumed that $d(k) = d \Rightarrow \tilde{d} = d \;\forall\; k$, the firm's problem is defined as follows.

$$\pi = \max_{w}(f(k(w)) - w) \cdot N(w, F) - r \cdot k(w) \tag{24}$$

subject to

$$k(w) = arg \max_{\hat{k}}(f(\hat{k}) - w) \cdot N(w, F) - r \cdot \hat{k} \tag{25}$$

The firm no longer attracts workers through capital. For this reason, the only decision that influences the number of employees a firm has is the wage w it offers. Replacing the assumption made in the previous cases in that $f(k(w)) \cdot N(w, F) = k^{\alpha} \cdot N^{1-\alpha}$, the above problem can be rewritten as follows.

$$\pi = \max_{w} k(w)^{\alpha} \cdot N(w, F)^{1-\alpha} - w \cdot N(w, F) - r \cdot k(w) \tag{26}$$

subject to

$$k(w) = arg \max_{\hat{k}} \hat{k}^{\alpha} \cdot N(w, F)^{1-\alpha} - w \cdot N(w, F) - r \cdot \hat{k} \tag{27}$$

Obtaining the CPO of the second problem with respect to $\hat{k}$ we obtain

$$\alpha \cdot \hat{k}^{\alpha-1} \cdot N(w, F)^{1-\alpha} = r \tag{28}$$

where

$$\hat{k}^{*} = N(w, F) \cdot \left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}} = k(w) \tag{29}$$

To solve the first problem, we use the same procedure as Burdett and Mortensen (1998), in which they obtain that in equilibrium all firms receive the same π. Thus, a firm that offers the lowest wage $\underline{w}$ will obtain a profit level $\underline{\pi}$, which will be equal to the profit of a firm that offers a wage $w \geq \underline{w}$. Then $\pi = \underline{\pi}$, which implies that (see development in Appendix 5.9)

$$N(w,F) = N(\underline{w},F) \cdot \left[\frac{\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}}{\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - w} \right] \tag{30}$$

Then, using the definition of $N(w,F)$ one can clear $F(w)$, which is similar to that obtained by Burdett and Mortensen (1998).

$$F_w(w) = \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot \left[1 - \left(\frac{(\frac{\alpha}{r})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - w}{(\frac{\alpha}{r})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}} \right)^{\frac{1}{2}} \right] \tag{31}$$

The density function for wages is obtained by deriving (41) with respect to w.

$$f_w(w) = \frac{dF_w(w)}{dw} = \frac{1}{2} \cdot \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot \frac{1}{[(\frac{\alpha}{r})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - w]^{\frac{1}{2}} [(\frac{\alpha}{r})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}]^{\frac{1}{2}}} \tag{32}$$

Then, to obtain the job distribution $F(N)$ we use $f(w)$ and the equation (40), and we obtain the following expression

$$f_N(N) = \frac{1}{2} \cdot \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot \left(\frac{N(\underline{w},F)}{N^3} \right)^{\frac{1}{2}} \tag{33}$$

To obtain the cumulative function

$$F_N(N) = \int_{-\infty}^{N} f(\tilde{N}) d\tilde{N}$$

$$F_N(N) = \frac{1}{2} \cdot \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot N(\underline{w},F)^{\frac{1}{2}} \cdot (-2) \cdot N^{-\frac{1}{2}} \tag{34}$$

To obtain the distribution of k we use the expression obtained for $f(N)$ and the CPO, and we obtain the following expression

$$f_k(k) = \frac{1}{2} \cdot \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot \left[\left(\frac{N(\underline{w},F)}{k^3} \right) \cdot \left(\frac{\alpha}{r} \right)^{\frac{1}{1-\alpha}} \right]^{\frac{1}{2}} \tag{35}$$

$$F_k(k) = \int_{-\infty}^{k} f(\tilde{k}) d\tilde{k}$$

$$F_k(k) = \frac{1}{2} \cdot \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot N(\underline{w},F)^{\frac{1}{2}} \cdot \left(\frac{\alpha}{r} \right)^{\frac{\frac{1}{2}}{1-\alpha}} \cdot (-2) \cdot k^{-\frac{1}{2}} \tag{36}$$

Thus, we have obtained expressions for all distributions of interest in equilibrium.

4 Conclusion

The literature associated with the theory of compensating wage differentials establishes that workers exposed to higher risk should be compensated with higher wages compared to workers in the same jobs but with lower levels of risk. Authors such as Aldy and Viscusi (2003) found evidence of this positive relationship between wages and occupational risk. However, Brown (1980) found no evidence after using even panel data controlling for individual fixed effects. Manning (2003) asserts that this is so because Brown does not consider the existence of job search frictions.

The objective of this paper is precisely to test the validity of this theory using a model of labor turnover with job search frictions. For this, we extended the model of Burdett and Mortensen (1998) in which agents are ex ante equal, but the equilibrium wage distribution is non-degenerate. A risk variable was also added, which is observed by workers as well as the wage through an aggregating variable defined by V_1. Workers make their employment decision based on risk-wage combinations expressed through V_1. Given a value of V_1, the higher the risk associated with employment, the higher the wage with which the firm must compensate the worker to remain employed in that firm. However, given that there is dispersion in V_1, this positive correlation can be broken. This would generate the scenario where firms do not compensate workers for facing a higher level of risk.

The risk variable depends on the technology adopted by the firm, which has 3 effects on the firm: a direct effect through marginal product, as in the case of perfect competition; a labor security effect, which helps the firm to have a larger scale and thus generate higher profits; and a compensating wage differential effect.

Numerical results were obtained for different cases, among which both the positive and negative relationship between the wage offered by firms and the level of associated risk observed in the mining industry, the example used as a framework in this research, was confirmed. Thus, the most important conclusion is that it was possible to identify a mechanism by which both types of firms can coexist: both those that offer high wages and security levels, and those that offer low wages and security levels. Therefore, it is possible to observe both the world where the theory of compensating wage differentials is fulfilled, and the world where it does not exist.

Finally, the parameters used make the simulated labor market one where there are many labor frictions. A next step is to calibrate the parameters in a way that diminishes the effect of these frictions, to see how the results vary in a more competitive labor market.

5 Appendix

5.1 Development of equation (4)

Under the assumption that the support of the distribution $F_{V_1}(V_1)$ is found in the interval $[\underline{V_1}, \overline{V_1}]$

$$\frac{1}{r+d(C)} \cdot [R+\lambda_1 \cdot [\int_{\underline{V}_1}^{\overline{V_1}} [\max(V_1, \tilde{V_1}) - V_1]\, dF_{V_1}(\tilde{V_1})] + \delta \cdot [V_0 - V_1]] = \frac{1}{r} \cdot [b + \lambda_0 \cdot [\int_{\underline{V}_1}^{\overline{V_1}} [\max(V_0, \tilde{V_1}) - V_0]\, dF_{V_1}(\tilde{V_1})]]$$

$$R - \frac{r+d(C)}{r} \cdot b = \frac{r+d(C)}{r} \cdot \lambda_0 \cdot [\int_{\underline{V}_1}^{\overline{V_1}} [\tilde{V_1} - V_0]\, dF_{V_1}(\tilde{V_1})] - \lambda_1 \cdot [\int_{\underline{V}_1}^{\overline{V_1}} [\tilde{V_1} - V_1]\, dF_{V_1}(\tilde{V_1})]$$

$$R - \frac{r+d(C)}{r} \cdot b = [\frac{r+d(C)}{r} \cdot \lambda_0 - \lambda_1] \cdot [\int_{\underline{V}_1}^{\overline{V_1}} [\tilde{V_1} - V_0]\, dF_{V_1}(\tilde{V_1})] \tag{37}$$

because $\tilde{V_1} = V_0 = V_1$. Note that the integral of the equation (37) can be treated with integration by parts as follows:

$$\int_{\underline{V}_1}^{\overline{V_1}} [\tilde{V_1} - V_0]\, dF_{V_1}(\tilde{V_1}) = \int_{\underline{V}_1}^{\overline{V_1}} [1 - F_{V_1}(\tilde{V_1})]\, d(\tilde{V_1})$$

Replacing this expression in (37), the following expression is obtained:

$$R - \frac{r+d(C)}{r} \cdot b = [\frac{r+d(C)}{r} \cdot \lambda_0 - \lambda_1] \cdot [\int_{\underline{V}_1}^{\overline{V_1}} [1 - F_{V_1}(\tilde{V_1})]\, d(\tilde{V_1})]$$

5.2 Integration by parts

To demonstrate that

$$\int_{\underline{V}_1}^{\overline{V_1}} [\tilde{V_1} - V_0]\, dF_{V_1}(\tilde{V_1}) = \int_{\underline{V}_1}^{\overline{V_1}} [1 - F_{V_1}(\tilde{V_1})]\, d(\tilde{V_1})$$

Let

$$u = \tilde{V_1} - V_0$$
$$dv = dF_{V_1}(\tilde{V_1})$$

Then

$$du = d\tilde{V_1}$$
$$v = F_{V_1}(\tilde{V_1})$$

And

$$\int_{\underline{V}_1}^{\overline{V_1}} [\tilde{V_1} - V_0]\, dF_{V_1}(\tilde{V_1}) = (\tilde{V_1} - V_0) \cdot F_{V_1}(\tilde{V_1}) \Big|_{\underline{V}_1}^{\overline{V_1}} - \int_{\underline{V}_1}^{\overline{V_1}} F_{V_1}(\tilde{V_1})\, d\tilde{V_1}$$
$$\int_{\underline{V}_1}^{\overline{V_1}} [\tilde{V_1} - V_0]\, dF_{V_1}(\tilde{V_1}) = (\tilde{V_1} - V_0) \cdot F_{V_1}(\tilde{V_1}) \Big|_{\underline{V}_1}^{\overline{V_1}} - \int_{\underline{V}_1}^{\overline{V_1}} F_{V_1}(\tilde{V_1})\, d\tilde{V_1}$$

Considering that when evaluated over the entire support $F_{V_1}(\overline{V_1}) = 1$ and that when evaluated at $F_{V_1}(\underline{V}_1) = 0$, then

$$\int_{\underline{V}_1}^{\overline{V_1}} [\tilde{V_1} - V_0]\, dF_{V_1}(\tilde{V_1}) = \int_{\underline{V}_1}^{\overline{V_1}} d\tilde{V_1} - \int_{\underline{V}_1}^{\overline{V_1}} F_{V_1}(\tilde{V_1})\, d\tilde{V_1}$$

$$\int_{\underline{V}_1}^{\overline{V_1}} [\tilde{V_1} - V_0]\, dF_{V_1}(\tilde{V_1}) = \int_{\underline{V}_1}^{\overline{V_1}} [1 - F_{V_1}(\tilde{V_1})]\, d(\tilde{V_1})$$

5.3 Development of equation (5)

$$(r_w + d(k)) \cdot V_1 = w + \lambda_1 \cdot [\int_{-\infty}^{+\infty} [\max(V_1, \tilde{V_1}) - V_1]\, dF_{V_1}(\tilde{V_1})] + \delta \cdot [V_0 - V_1]$$

$$(r_w+d(k))\cdot V_1 = w+\lambda_1\cdot \int_{-\infty}^{V_1} [\max(V_1, \tilde{V_1})-V_1]\, dF_{V_1}(\tilde{V_1})+\lambda_1\cdot \int_{V_1}^{+\infty} [\max(V_1, \tilde{V_1})-V_1]\, dF_{V_1}(\tilde{V_1})+\delta\cdot[V_0-V_1]$$

$$(r_w + d(k)) \cdot V_1 = w + \lambda_1 \cdot \int_{-\infty}^{V_1} [V_1 - V_1]\, dF_{V_1}(\tilde{V_1}) + \lambda_1 \cdot \int_{V_1}^{+\infty} [\tilde{V_1} - V_1]\, dF_{V_1}(\tilde{V_1}) + \delta \cdot [V_0 - V_1]$$

$$(r_w + d(k)) \cdot V_1 = w + \lambda_1 \cdot \int_{V_1}^{+\infty} [\tilde{V_1} - V_1]\, dF_{V_1}(\tilde{V_1}) + \delta \cdot [V_0 - V_1]$$

$$(r_w + d(k)) \cdot V_1 = w + \lambda_1 \cdot \int_{V_1}^{+\infty} \tilde{V_1}\, dF_{V_1}(\tilde{V_1}) - \lambda_1 \cdot \int_{V_1}^{+\infty} V_1\, dF_{V_1}(\tilde{V_1}) + \delta \cdot [V_0 - V_1]$$

$$(r_w + d(k)) \cdot V_1 = w + \lambda_1 \cdot \int_{V_1}^{+\infty} \tilde{V_1}\, dF_{V_1}(\tilde{V_1}) - \lambda_1 \cdot V_1 + \lambda_1 \cdot V_1 \cdot (1 - F_{V_1}(V_1)) + \delta \cdot [V_0 - V_1]$$

$$(r_w + d(k)) \cdot V_1 = w + \lambda_1 \cdot \int_{V_1}^{+\infty} \tilde{V_1}\, dF_{V_1}(\tilde{V_1}) - \lambda_1 \cdot V_1 + \lambda_1 \cdot V_1 \cdot F_{V_1}(V_1) + \delta \cdot [V_0 - V_1]$$

$$(r_w + d(k)) \cdot V_1 = w + \lambda_1 \cdot \int_{V_1}^{+\infty} \tilde{V_1}\, dF_{V_1}(\tilde{V_1}) - \lambda_1 \cdot V_1 \cdot (1 - F_{V_1}(V_1)) + \delta \cdot [V_0 - V_1]$$

$$w(V_1, k) = V_1 \cdot (r_w + d(k) + \lambda_1 \cdot (1 - F_{V_1}(V_1)) + \delta) - \lambda_1 \cdot [\int \tilde{V_1}\, dF_{V_1}(\tilde{V_1})] - \delta \cdot V_0$$

5.4 Unemployment level u

Equation (9) determines the level of unemployment in equilibrium,

$$u = \frac{n \cdot (\delta + \bar{d} + \tilde{d})}{[\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}}$$

To show how unemployment varies when its main parameters vary, the derivative is applied.

$$\begin{aligned}
\frac{\partial u}{\partial \delta} &= n \cdot \frac{(\lambda_0 + \bar{d} + \delta) \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d} - (\delta + \bar{d} + \tilde{d}) \cdot \bar{d}}{\Big([\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}\Big)^2} \\
&= n \cdot \frac{(\delta + \bar{d} + \tilde{d}) \cdot \bar{d} + \lambda_0 \cdot (\bar{d} + \tilde{d}) - (\delta + \bar{d} + \tilde{d}) \cdot \bar{d}}{\Big([\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}\Big)^2} \\
&= n \cdot \frac{\lambda_0 \cdot (\bar{d} + \tilde{d})}{\Big([\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}\Big)^2} \\
&\geq 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial u}{\partial \bar{d}} &= n \cdot \frac{(\lambda_0 + \bar{d} + \delta) \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d} - (\delta + \bar{d} + \tilde{d}) \cdot (\bar{d} + (\lambda_0 + \bar{d} + \delta) + \tilde{d})}{\Big([\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}\Big)^2} \\
&= n \cdot \frac{(\lambda_0 + \bar{d} + \delta) \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d} - (\delta + \bar{d} + \tilde{d}) \cdot \bar{d} - (\delta + \bar{d} + \tilde{d}) \cdot (\lambda_0 + \bar{d} + \delta) - (\delta + \bar{d} + \tilde{d}) \cdot \tilde{d}}{\Big([\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}\Big)^2} \\
&= n \cdot \frac{(\lambda_0 + \bar{d} + \delta) \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d} - (\delta + \bar{d} + \tilde{d}) \cdot \bar{d} - (\delta + \tilde{d}) \cdot (\lambda_0 + \bar{d} + \delta) - \bar{d} \cdot (\lambda_0 + \bar{d} + \delta) - (\delta + \bar{d} + \tilde{d}) \cdot \tilde{d}}{\Big([\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}\Big)^2} \\
&= n \cdot \frac{(\lambda_0 + \bar{d}) \cdot \tilde{d} - (\delta + \bar{d} + \tilde{d}) \cdot \bar{d} - \delta \cdot (\lambda_0 + \bar{d} + \delta) - \tilde{d} \cdot (\lambda_0 + \bar{d}) - \delta \cdot \tilde{d} - (\delta + \bar{d} + \tilde{d}) \cdot \tilde{d}}{\Big([\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}\Big)^2} \\
&= n \cdot \frac{-(\delta + \bar{d} + \tilde{d}) \cdot \bar{d} - \delta \cdot (\lambda_0 + \bar{d} + \delta) - \delta \cdot \tilde{d} - (\delta + \bar{d} + \tilde{d}) \cdot \tilde{d}}{\Big([\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}\Big)^2} \\
&= (-1) \cdot n \cdot \frac{(\delta + \bar{d} + \tilde{d}) \cdot \bar{d} + \delta \cdot (\lambda_0 + \bar{d} + \delta) + \delta \cdot \tilde{d} + (\delta + \bar{d} + \tilde{d}) \cdot \tilde{d}}{\Big([\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}\Big)^2} \\
&\leq 0
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial u}{\partial \tilde{d}} &= n \cdot \frac{(\lambda_0 + \bar{d} + \delta) \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d} - (\delta + \bar{d} + \tilde{d}) \cdot (\lambda_0 + \bar{d})}{\left([\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}\right)^2} \\
&= n \cdot \frac{(\lambda_0 + \bar{d}) \cdot \bar{d} + \delta \cdot \bar{d} + \delta \cdot \lambda_0 - \delta \cdot \lambda_0 + (\lambda_0 + \bar{d}) \cdot \tilde{d} - (\delta + \bar{d} + \tilde{d}) \cdot (\lambda_0 + \bar{d})}{\left([\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}\right)^2} \\
&= n \cdot \frac{(\lambda_0 + \bar{d}) \cdot \bar{d} + (\bar{d} + \lambda_0) \cdot \delta - \delta \cdot \lambda_0 + (\lambda_0 + \bar{d}) \cdot \tilde{d} - (\delta + \bar{d} + \tilde{d}) \cdot (\lambda_0 + \bar{d})}{\left([\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}\right)^2} \\
&= n \cdot \frac{(\delta + \bar{d} + \tilde{d}) \cdot (\lambda_0 + \bar{d}) - \delta \cdot \lambda_0 - (\delta + \bar{d} + \tilde{d}) \cdot (\lambda_0 + \bar{d})}{\left([\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}\right)^2} \\
&= n \cdot \frac{-\delta \cdot \lambda_0}{\left([\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}\right)^2} \\
&= (-1) \cdot n \cdot \frac{\delta \cdot \lambda_0}{\left([\lambda_0 + \bar{d} + \delta] \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot \tilde{d}\right)^2} \\
&\leq 0
\end{aligned}
$$

5.5 Obtaining $G(V_1)$

Considering the steady state, both flows are equalized

$$\lambda_0 \cdot \max(F_{V_1}(V_1) - F_{V_1}(\underline{V}_1), 0) \cdot u = [\lambda_1 \cdot [1 - F_{V_1}(V_1)] + \delta + \bar{d} + \tilde{d}] \cdot G_{V_1}(V_1) \cdot (m - u)$$

Under the assumption that $V_1 > \underline{V}_1$ and considering the equation (9) we obtain the following expression for $G_{V_1}(V_1)$:

$$G_{V_1}(V_1) = \frac{\lambda_0 \cdot (F_{V_1}(V_1) - F_{V_1}(\underline{V}_1))}{\lambda_1 \cdot [1 - F_{V_1}(V_1)] + \delta + \bar{d} + \tilde{d}} \cdot \frac{u}{m - u}$$

$$G(V_1) = \frac{\lambda_0 \cdot (F_{V_1}(V_1) - F_{V_1}(\underline{V}_1))}{\lambda_1 \cdot [1 - F_{V_1}(V_1)] + \delta + \bar{d} + \tilde{d}} \cdot \frac{\delta + \bar{d} + \tilde{d}}{(1 - F_{V_1}(\underline{V}_1)) \cdot \lambda_0}$$

$$G_{V_1}(V_1) = \frac{(F_{V_1}(V_1) - F_{V_1}(\underline{V}_1))}{\lambda_1 \cdot [1 - F_{V_1}(V_1)] + \delta + \bar{d} + \tilde{d}} \cdot \frac{\delta + \bar{d} + \tilde{d}}{(1 - F_{V_1}(\underline{V}_1))}$$

To obtain $\frac{u}{m-u}$, we start by replacing the equation (9) in the expression for m (equation (8))

$$m = \frac{n + u \cdot \tilde{d}}{\bar{d} + \tilde{d}} = \frac{n + \frac{n \cdot (\delta + \bar{d} + \tilde{d})}{[(1 - F_{V_1}(\underline{V}_1)) \cdot \lambda_0 + \bar{d} + \delta] \cdot \bar{d} + ((1 - F_{V_1}(\underline{V}_1)) \cdot \lambda_0 + \bar{d}) \cdot \tilde{d}} \cdot \tilde{d}}{\bar{d} + \tilde{d}}$$

Ordering terms,

$$m = \frac{n}{\bar{d}+\tilde{d}} \cdot \frac{[(1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d}+\delta]\cdot\bar{d}+((1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d})\cdot\tilde{d}+\tilde{d}\cdot(\delta+\bar{d}+\tilde{d})}{[(1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d}+\delta]\cdot\bar{d}+((1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d})\cdot\tilde{d}}$$

From this expression for m we subtract u, whose expression was obtained in the equation (9)

$$m-u = \frac{n}{\bar{d}+\tilde{d}} \cdot \frac{[(1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d}+\delta]\cdot\bar{d}+((1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d})\cdot\tilde{d}+\tilde{d}\cdot(\delta+\bar{d}+\tilde{d})}{[(1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d}+\delta]\cdot\bar{d}+((1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d})\cdot\tilde{d}} -$$

$$- \frac{n\cdot(\delta+\bar{d}+\tilde{d})}{[(1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d}+\delta]\cdot\bar{d}+((1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d})\cdot\tilde{d}}$$

Rearranging terms it can be obtained

$$m-u = \frac{n\cdot(1-F_{V_1}(\underline{V}_1))\cdot\lambda_0}{[(1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d}+\delta]\cdot\bar{d}+((1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d})\cdot\tilde{d}}$$

Then, $\frac{u}{m-u}$ can be obtained

$$\frac{u}{m-u} = \frac{\delta+\bar{d}+\tilde{d}}{(1-F_{V_1}(\underline{V}_1))\cdot\lambda_0}$$

5.6 Number of employed workers

$$N(V_1,F) = \frac{R(V_1,F)}{s(V_1,k,F)}$$

$$N(V_1,k,F) = \frac{\lambda_0\cdot u\cdot(1-F_{V_1}(\underline{V}_1))+(m-u)\cdot G_{V_1}(V_1,F)\cdot\lambda_1}{\delta+\lambda_1\cdot[1-F_{V_1}(V_1)]+\bar{d}+d(k)}$$

$$N(V_1,k,F) = \Big(\lambda_0\cdot\frac{n\cdot(\delta+\bar{d}+\tilde{d})}{[(1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d}+\delta]\cdot\bar{d}+((1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d})\cdot\tilde{d}}\cdot(1-F_{V_1}(\underline{V}_1))+$$

$$+\frac{n\cdot(1-F_{V_1}(\underline{V}_1))\cdot\lambda_0}{[(1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d}+\delta]\cdot\bar{d}+((1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d})\cdot\tilde{d}}\cdot G_{V_1}(V_1,F)\cdot\lambda_1\Big)\cdot\frac{1}{\delta+\lambda_1\cdot[1-F_{V_1}(V_1)]+\bar{d}+d(k)}$$

$$N(V_1,k,F)=\Big(\lambda_0\cdot\frac{n\cdot(\delta+\bar{d}+\tilde{d})}{[(1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d}+\delta]\cdot\bar{d}+((1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d})\cdot\tilde{d}}\cdot(1-F_{V_1}(\underline{V}_1))+$$

$$+\frac{n\cdot(1-F_{V_1}(\underline{V}_1))\cdot\lambda_0}{[(1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d}+\delta]\cdot\bar{d}+((1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d})\cdot\tilde{d}}\cdot\frac{(F_{V_1}(V_1)-F_{V_1}(\underline{V}_1))}{\lambda_1\cdot[1-F_{V_1}(V_1)]+\delta+\bar{d}+\tilde{d}}\cdot\frac{[\delta+\bar{d}+\tilde{d}]}{(1-F_{V_1}(\underline{V}_1))}\cdot\lambda_1\Big)\cdot$$

$$\cdot\frac{1}{\delta+\lambda_1\cdot[1-F_{V_1}(V_1)]+\bar{d}+d(k)}$$

Rearranging the terms, we obtain

$$N(V_1,k,F)=\left[\frac{n\cdot\lambda_0\cdot(\delta+\bar{d}+\tilde{d})\cdot[(1-F_{V_1}(\underline{V}_1))\cdot(\lambda_1\cdot(1-F_{V_1}(V_1))+\delta+\bar{d}+\tilde{d})+\lambda_1\cdot(F_{V_1}(V_1)-F_{V_1}(\underline{V}_1)]}{([(1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d}+\delta]\cdot\bar{d}+((1-F_{V_1}(\underline{V}_1))\cdot\lambda_0+\bar{d})\cdot\tilde{d})\cdot(\lambda_1\cdot[1-F_{V_1}(V_1)]+\delta+\bar{d}+\tilde{d})}\right]\cdot$$

$$\cdot\frac{1}{\delta+\lambda_1\cdot[1-F_{V_1}(V_1)]+\bar{d}+d(k)}$$

Assuming $F(\underline{V}_1)=0$,

$$N(V_1,k,F)=\left[\frac{n\cdot\lambda_0\cdot(\delta+\bar{d}+\tilde{d})\cdot(\lambda_1+\delta+\bar{d}+\tilde{d})}{([\lambda_0+\bar{d}+\delta]\cdot\bar{d}+(\lambda_0+\bar{d})\cdot\tilde{d})\cdot(\lambda_1\cdot[1-F_{V_1}(V_1)]+\delta+\bar{d}+\tilde{d})}\right]\cdot\frac{1}{\delta+\lambda_1\cdot[1-F_{V_1}(V_1)]+\bar{d}+d(k)}$$

5.7 Firm problem with $\alpha = 0$

$$\pi=\max_{V_1}k(V_1)^{\alpha}\cdot N(V_1,k(V_1),F)^{1-\alpha}-w(V_1,k(V_1))\cdot N(V_1,k(V_1),F)-r_f\cdot k(V_1) \tag{38}$$

subject to

$$k(V_1)=arg\max_{\hat{k}}\hat{k}^{\alpha}\cdot N(V_1,\hat{k},F)^{1-\alpha}-w(V_1,\hat{k})\cdot N(V_1,\hat{k},F)-r_f\cdot\hat{k}$$

Obtaining the CPO of the second problem with respect to $\hat{k}$ we obtain

$$\alpha\cdot\hat{k}^{\alpha-1}\cdot N(V_1,\hat{k},F)^{1-\alpha}+\hat{k}^{\alpha}\cdot(1-\alpha)\cdot N(V_1,\hat{k},F)^{-\alpha}\cdot\frac{\partial N(V_1,\hat{k},F)}{\partial\hat{k}}-\frac{\partial w(V_1,\hat{k})}{\partial\hat{k}}\cdot N(V_1,\hat{k},F)-w(V_1,\hat{k})\cdot\frac{\partial N(V_1,\hat{k},F)}{\partial\hat{k}}=r_f$$

Using the expressions obtained for $N(V_1,k,F)$ and $\frac{\partial N(V_1,k,F)}{\partial k}$, and considering that from the equation (2) it was obtained that $w(V_1,k)=V_1\cdot(r_w+d(k)+\lambda_1\cdot(1-F_{V_1}(V_1))+\delta)-\lambda_1\cdot[\int\tilde{V}_1\,dF_{V_1}(\tilde{V}_1)]-\delta\cdot V_0$, and using the assumption that $\alpha=0$, the CPO can be rewritten as

$$\frac{\partial N(V_1,\hat{k},F)}{\partial\hat{k}}-\frac{\partial w(V_1,\hat{k})}{\partial\hat{k}}\cdot N(V_1,\hat{k},F)-w(V_1,\hat{k})\cdot\frac{\partial N(V_1,\hat{k},F)}{\partial\hat{k}}=r_f$$

The first thing to notice is that the level of technology chosen by the firm, $k(V_1)$, is no longer productive, since the first effect mentioned in proposition 3 is lost. The other two effects remain valid.

$$N(V_1,k,F)\cdot\frac{-d'(k)}{\delta+\lambda_1\cdot[1-F(V_1)]+\bar{d}+d(k)}-d'(k)\cdot V_1\cdot N(V_1,\hat{k},F)-$$
$$-w(V_1,\hat{k})\cdot N(V_1,k,F)\cdot\frac{-d'(k)}{\delta+\lambda_1\cdot[1-F(V_1)]+\bar{d}+d(k)}=r_f$$

Reorganizing terms,

$$r_f=-d'(k)\cdot N(V_1,k,F)\cdot\left[V_1+\frac{1-w(V_1,\hat{k})}{\delta+\lambda_1\cdot[1-F(V_1)]+\bar{d}+d(k)}\right]$$

In addition, from equation (2),

$$(r_w+d(k))\cdot V_1=w+\lambda_1\cdot[\int_{-\infty}^{+\infty}[\max(V_1,\tilde{V}_1)-V_1]\,dF_{V_1}(\tilde{V}_1)]+\delta\cdot[V_0-V_1]$$

$$(r_w+d(k))\cdot V_1=w+\lambda_1\cdot\int_{-\infty}^{V_1}[\max(V_1,\tilde{V}_1)-V_1]\,dF_{V_1}(\tilde{V}_1)+\lambda_1\cdot\int_{V_1}^{+\infty}[\max(V_1,\tilde{V}_1)-V_1]\,dF_{V_1}(\tilde{V}_1)+\delta\cdot[V_0-V_1]$$

$$(r_w+d(k))\cdot V_1=w+\lambda_1\cdot\int_{-\infty}^{V_1}[V_1-V_1]\,dF_{V_1}(\tilde{V}_1)+\lambda_1\cdot\int_{V_1}^{+\infty}[\tilde{V}_1-V_1]\,dF_{V_1}(\tilde{V}_1)+\delta\cdot[V_0-V_1]$$

$$(r_w+d(k))\cdot V_1=w+\lambda_1\cdot\int_{V_1}^{+\infty}[\tilde{V}_1-V_1]\,dF_{V_1}(\tilde{V}_1)+\delta\cdot[V_0-V_1]$$

$$(r_w+d(k))\cdot V_1=w+\lambda_1\cdot\int_{V_1}^{+\infty}\tilde{V}_1\,dF_{V_1}(\tilde{V}_1)-\lambda_1\cdot\int_{V_1}^{+\infty}V_1\,dF_{V_1}(\tilde{V}_1)+\delta\cdot[V_0-V_1]$$

$$(r_w+d(k))\cdot V_1=w+\lambda_1\cdot\int_{V_1}^{+\infty}\tilde{V}_1\,dF_{V_1}(\tilde{V}_1)-\lambda_1\cdot V_1+\lambda_1\cdot V_1\cdot(1-F_{V_1}(V_1))+\delta\cdot[V_0-V_1]$$

$$(r_w+d(k))\cdot V_1=w+\lambda_1\cdot\int_{V_1}^{+\infty}\tilde{V}_1\,dF_{V_1}(\tilde{V}_1)-\lambda_1\cdot V_1+\lambda_1\cdot V_1\cdot F_{V_1}(V_1)+\delta\cdot[V_0-V_1]$$

$$(r_w+d(k))\cdot V_1=w+\lambda_1\cdot\int_{V_1}^{+\infty}\tilde{V}_1\,dF_{V_1}(\tilde{V}_1)-\lambda_1\cdot V_1\cdot(1-F_{V_1}(V_1))+\delta\cdot[V_0-V_1]$$

$$V_1=\frac{w(V_1,k)+\lambda_1\cdot[\int\tilde{V}_1\,dF_{V_1}(\tilde{V}_1)]+\delta\cdot V_0}{r_w+d(k)+\lambda_1\cdot(1-F_{V_1}(V_1))+\delta}$$

Replacing this equation in the CPO, recalling that $r_w=\tilde{r}+\bar{d}$ and under the assumption that $\tilde{r}=0$, one has that,

$$r_f=-d'(k)\cdot N(V_1,k,F)\cdot\left[\frac{1+\lambda_1\cdot[\int\tilde{V}_1\,dF_{V_1}(\tilde{V}_1)]+\delta\cdot V_0}{\delta+\lambda_1\cdot[1-F_{V_1}(V_1)]+\bar{d}+d(k)}\right]$$

5.8 Firm problem with $\alpha = 0.9$

As an example, all the results are presented for one of the cases presented above, that of $\alpha = 0.9$.

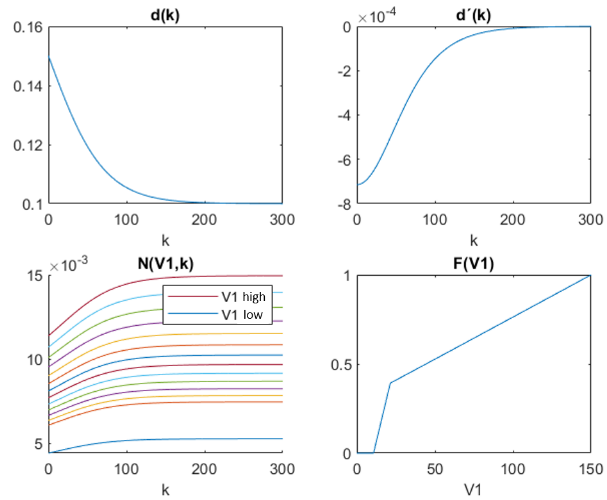

Figure 12: Probability of death $d(k)$ and its derivative $d'(k)$, number of employees $N(V_1, k)$ for different levels of V_1 and k, and distribution of V_1 offered at equilibrium given by $F(V_1)$ for $\alpha = 0.9$.

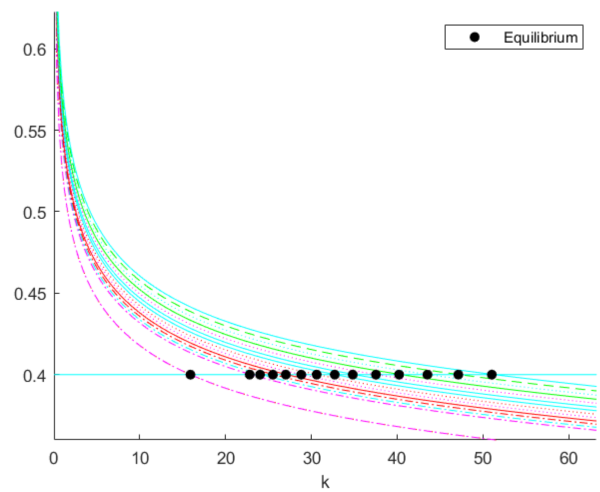

Figure 13: First-order condition of the firm problem.

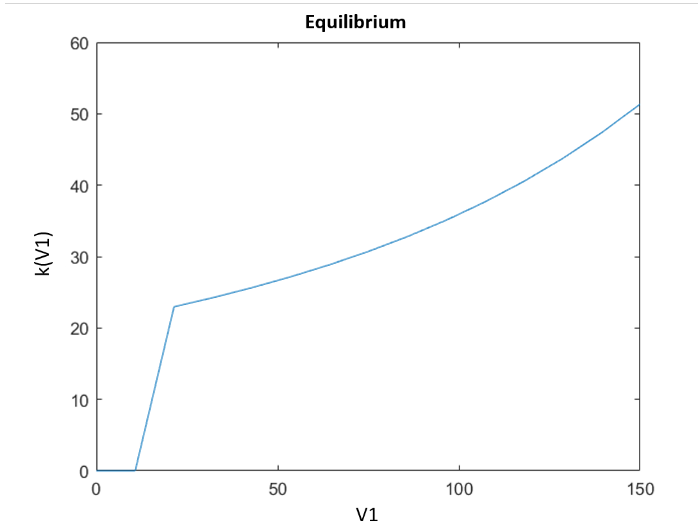

Figure 14: Levels of k that solve the first-order condition of the firm problem.

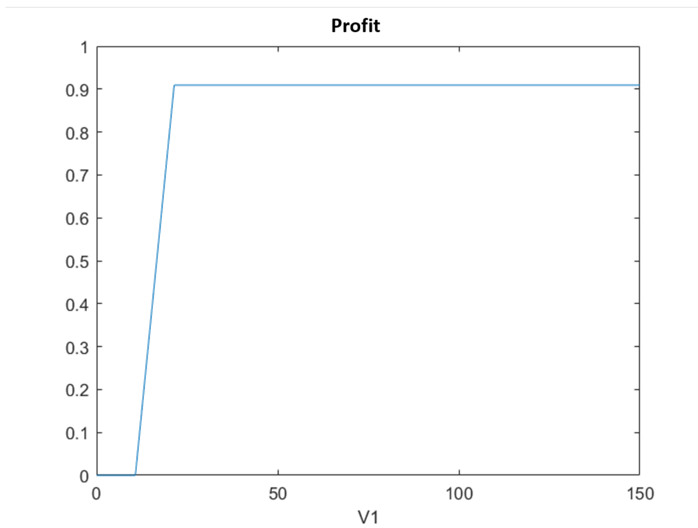

Figure 15: Level of benefits obtained by firms for each V_1 offered.

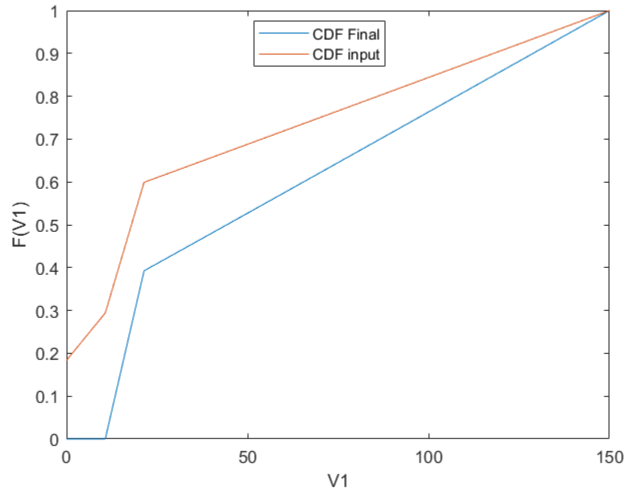

Figure 16: Initial guess for the distribution $F(V_1)$, and $F(V_1)$ in equilibrium.

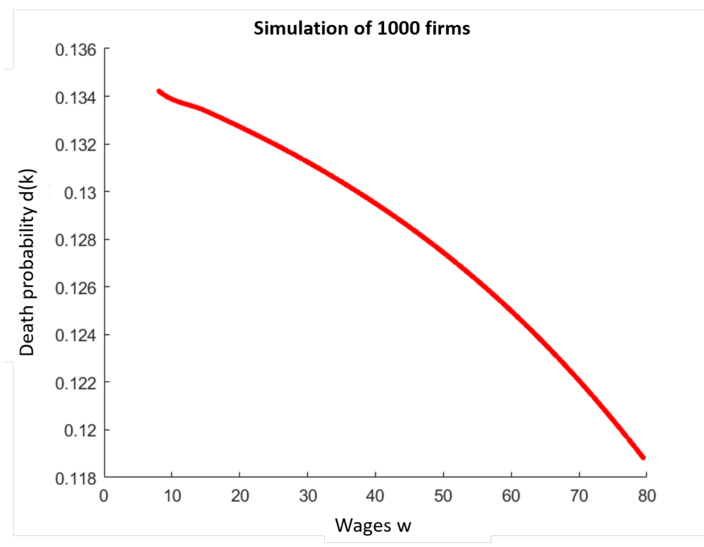

Figure 17: Probability of death $d(k)$ vs wage w offered by firms when $\alpha = 0.9$. Each point represents one firm.

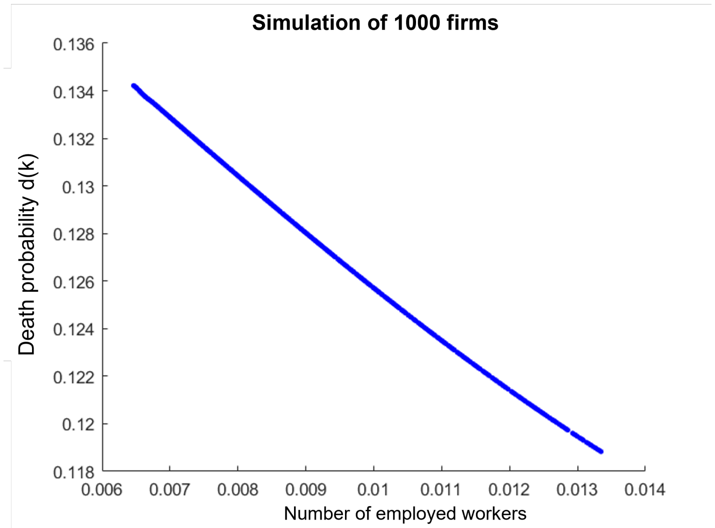

Figure 18: Probability of death $d(k)$ vs wage w offered by firms when $\alpha = 0.9$. Each point represents one firm.

5.9 Obtaining distributions when $d(k) = d$

$$\pi = \underline{\pi} \tag{39}$$

$$k(w)^{\alpha} \cdot N(w,F)^{1-\alpha} - w \cdot N(w,F) - r_f \cdot k(w) = k(\underline{w})^{\alpha} \cdot N(\underline{w},F)^{1-\alpha} - \underline{w} \cdot N(\underline{w},F) - r_f \cdot k(\underline{w})$$

$$N(w,F)^{\alpha} \cdot \left(\frac{\alpha}{r_f}\right)^{\frac{\alpha}{1-\alpha}} \cdot N(w,F)^{1-\alpha} - w \cdot N(w,F) - r_f \cdot N(w,F) \cdot \left(\frac{\alpha}{r_f}\right)^{\frac{1}{1-\alpha}} = k(\underline{w})^{\alpha} \cdot N(\underline{w},F)^{1-\alpha} - \underline{w} \cdot N(\underline{w},F) - r_f \cdot k(\underline{w})$$

$$N(w,F) \cdot \left[\left(\frac{\alpha}{r_f}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - w\right] = N(\underline{w},F) \cdot \left[\left(\frac{\alpha}{r_f}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}\right]$$

$$N(w,F) = N(\underline{w},F) \cdot \left[\frac{\left(\frac{\alpha}{r_f}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}}{\left(\frac{\alpha}{r_f}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - w}\right] \tag{40}$$

Then, using the definition of $N(w,F)$, $F(w)$ can be obtained.

$$F_w(w) = 1 - \frac{\left(\frac{n \cdot \lambda_0 \cdot (\delta + \bar{d} + d)(\lambda_1 + \delta + \bar{d} + d)}{(\lambda_0 + \bar{d} + \delta) \cdot \bar{d} + (\lambda_0 + \bar{d}) \cdot d} \cdot \frac{1}{N(\underline{w},F)} \cdot \left[\frac{\left(\frac{\alpha}{r_f}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - w}{\left(\frac{\alpha}{r_f}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}}\right]\right)^{\frac{1}{2}} - \delta - \bar{d} - d}{\lambda_1}$$

replacing the definition of $N(\underline{w},F)$ gives

$$F_w(w) = 1 - \frac{\Psi - \delta - \bar{d} - d}{\lambda_1}$$

where

$$\Psi = (\lambda_1 + \delta + \bar{d} + d) \cdot \left(\frac{\left(\frac{\alpha}{r_f}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - w}{\left(\frac{\alpha}{r_f}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}}\right)^{\frac{1}{2}}$$

Rearranging terms, we obtain an expression similar to that obtained by Burdett and Mortensen (1998) as an expression of the wage distribution in equilibrium.

$$F_w(w) = \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot \left[1 - \left(\frac{(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - w}{(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}} \right)^{\frac{1}{2}} \right] \tag{41}$$

The density function for wages is obtained by deriving (41) with respect to w.

$$f_w(w) = \frac{dF_w(w)}{dw} = \frac{1}{2} \cdot \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot \frac{1}{[(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - w]^{\frac{1}{2}} [(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}]^{\frac{1}{2}}} \tag{42}$$

Then, to obtain the distribution of jobs $F_N(N)$ we use $f_w(w)$ and the equation (40), and we obtain the following expression

$$f_N(N) = \frac{1}{2} \cdot \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot \frac{g'(N)}{[(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - g(N)]^{\frac{1}{2}} [(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}]^{\frac{1}{2}}} \tag{43}$$

where

$$g(N) = (\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \frac{N(\underline{w}, F)}{N} \cdot [(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}]$$

and

$$g'(N) = \frac{N(\underline{w}, F)}{N^2} \cdot [(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}]$$

Rearranging the terms of $f_N(N)$,

$$f_N(N) = \frac{1}{2} \cdot \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot$$
$$\cdot \frac{\frac{N(\underline{w},F)}{N^2} \cdot [(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}]}{[(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - ((\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \frac{N(\underline{w},F)}{N} \cdot [(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}])]^{\frac{1}{2}} [(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}]^{\frac{1}{2}}}$$

$$f_N(N) = \frac{1}{2} \cdot \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot$$
$$\cdot \frac{\frac{N(\underline{w},F)}{N^2} \cdot [(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}]^{\frac{1}{2}}}{[(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - (\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) + \frac{N(\underline{w},F)}{N} \cdot [(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}]]^{\frac{1}{2}}}$$

$$f_N(N) = \frac{1}{2} \cdot \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot \frac{\frac{N(\underline{w},F)}{N^2} \cdot [(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}]^{\frac{1}{2}}}{(\frac{N(\underline{w},F)}{N} \cdot [(\frac{\alpha}{r_f})^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha) - \underline{w}])^{\frac{1}{2}}}$$

$$f_N(N) = \frac{1}{2} \cdot \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot \left(\frac{N(\underline{w}, F)}{N^3} \right)^{\frac{1}{2}} \tag{44}$$

To obtain the cumulative function

$$F_N(N) = \int_{-\infty}^{N} f_N(\tilde{N}) d\tilde{N}$$

$$F_N(N) = \frac{1}{2} \cdot \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot N(\underline{w}, F)^{\frac{1}{2}} \cdot (-2) \cdot N^{-\frac{1}{2}}$$

To obtain the distribution of k we use the expression obtained for $f_N(N)$ and the CPO, and we obtain the following expression

$$f_k(k) = \frac{1}{2} \cdot \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot \left[\left(\frac{N(\underline{w}, F)}{k^3} \right) \cdot \left(\frac{\alpha}{r} \right)^{\frac{1}{1-\alpha}} \right]^{\frac{1}{2}} \tag{45}$$

$$F_k(k) = \int_{-\infty}^{k} f(\tilde{k}) d\tilde{k}$$

$$F_k(k) = \frac{1}{2} \cdot \frac{\lambda_1 + \delta + \bar{d} + \tilde{d}}{\lambda_1} \cdot N(\underline{w}, F)^{\frac{1}{2}} \cdot \left(\frac{\alpha}{r} \right)^{\frac{\frac{1}{2}}{1-\alpha}} \cdot (-2) \cdot k^{-\frac{1}{2}}$$

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