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A decision framework for a farmer who is risk averse in the Arrow-Pratt sense and downside risk averse

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ABSTRACT: This study provides a decision framework to analyze optimal production diversification decisions under uncertainty for a farmer who is risk averse in the Arrow-Pratt sense and downside risk averse. The decision model accounts for the third central moment of the joint distribution of portfolio returns. This is relevant for asymmetrical return distributions. This general model contains the classical decision model as a special case. The benefit of the proposed generalization is that each competing behavioral hypothesis is discerned econometrically through the significance of the agent's coefficient of absolute and/or relative downside risk aversion.

KEYWORDS: Downside risk, extension of E-V model, prudence, risk aversion, skewness preference.

JEL classification: D21, D81, G11.

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Un modelo de decisión para un agricultor que es averso al riesgo en el sentido Arrow-Pratt y averso al riesgo downside

RESUMEN: Este estudio proporciona un marco de decisión para analizar las decisiones óptimas de diversificación productiva en condiciones de incertidumbre para un agricultor que es averso al riesgo en el sentido Arrow-Pratt y averso al riesgo downside. El modelo de decisión incorpora el tercer momento central de la distribución conjunta de los retornos del porfolio. Esto es especialmente relevante para distribuciones de retornos asimétricos. Este modelo incluye el modelo de decisión clásica como un caso especial. El beneficio de la generalización es que permite discernir cada hipótesis de comportamiento econométricamente a través de la estimación del coeficiente de aversión al riesgo y al downside.

PALABRAS CLAVE: Riesgo downside, extensión del modelo E-V, prudencia, aversión al riesgo, preferencia por skewness.

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1. Introduction

Risk is present in all agricultural management decisions, as a result of price, yield and resource uncertainty. Furthermore, the attitudes of producers toward risk influence acreage. Empirical results show that agricultural producers are risk averse (Just, 1974, 1975, 2003; Antle, 1987, 1989, 2010; Pope and Just, 1991; Gómez-Limón *et al.,* 2003; Picazo-Tadeo and Wall, 2011). Additionally, risk aversion has been found to influence farmer's technology adoption (Marra *et al.,* 2003).

Extending the estimation of risk aversion coefficients to include downside risk aversion, Antle (1987, 1989) finds empirical support that farmers are risk averse in the Arrow-Pratt sense and downside risk averse. More recently, Di Falco and Chavas (2006, 2009), Hennessy (2009) and Antle (2010) incorporate downside risk aversion on input use decisions, showing the importance of input's asymmetric effects on output distributions on the optimal input use decision. Many other authors present evidence supporting the hypothesis that agents prefer a distribution that is more right skewed (Tronstad and McNeill, 1989; Eeckhoudt and Schlesinger, 2006; Yang *et al.*, 2010).

In light of the empirical evidence which shows that agricultural production does not follow a normal distribution or, for that matter, a symmetric probability distribution (e.g., Day, 1965; Gallagher, 1987; and Just and Weninger, 1999) it is restrictive to model farmer's decision under risk using the expected value-variance (E-V) decision rule (Grootveld and Hallerbach, 1999; Bouyssou *et al.,* 2000; and Ehrgott *et al.,* 2004). Restrictive assumptions must be made about the probability distribution (*e.g.*, normally distributed random variables) and about the agent's utility function (*e.g.,* quadratic utility functions) to ensure consistency of the optimal choices between expected utility maximization and E-V analysis.

This suggests that in order to explicitly account for the farmer's preference for right skewed distributions (downside risk aversion) when using a moment-based approximation of the agent's von-Neumann Morgenstern utility function, it is necessary to employ a third-order moment-based Taylor's series expansion of the farmer's expected utility function. Hence, it is necessary to select the optimal production portfolio on the basis of the first three moments of the probability distribution, rather than on the first two. Menezes and Wang (2004, 2005) and Eeckhoudt and Schlesinger (2006) provide a general choice theoretic characterization of the decision of an investor with preferences for skewness. More recently, Garlappi and Skoulakis (2011) provide conditions under which the approximate expected utility of a given portfolio based on a third order Taylor Series Expansion converges to its exact counterpart.

The purpose of this paper is to provide a decision framework to analyze production diversification decisions under uncertainty for a farmer who is risk averse in the Arrow-Pratt sense and downside risk averse, which explicitly allows for the analysis of the trade-off between expected returns, variance and skewness. A higher moment-based approximation than a third order Taylor's series approximation will not be employed since as Brockett and Kahane (1992) show, in empirical studies the fourth moment showed significance only in a few cases, and higher moments were always insignificant. This general model (E-V-S) contains the classical E-V portfolio model as a special case. The contribution of this generalization is that estimation of the structural econometric model allows each competing behavioral hypothesis to be discerned econometrically through the significance of the agent's coefficient of absolute and relative downside risk aversion.

The paper is structured as follows. The next section develops a moment-based approximation of the farmer´s expected utility function. The producer´s decision model which explicitly allows for the analysis of the trade-off between expected returns, variance and skewness, is presented in Section 3. Section 4 derives the properties of the farmer´s optimal acreage allocation functions. The proposed structural econometric model is derived in Section 5. Conclusions are drawn in Section 6.

2. A moment-based approximation of the farmer's expected utility function

In what follows, we develop a parametric optimization model for investment decisions under uncertainty for a farmer who is risk averse in the Arrow-Pratt sense and downside risk averse explicitly accounting for the third central moment of the joint distribution of the portfolio's returns.

The producer's expected utility of wealth is modeled as

$$EU\left(W_0 + \alpha'\tilde{\pi}\right) = \int_{\Theta} U\left(W_0 + \alpha'\tilde{\pi}\right) f\left(\tilde{\pi}\middle|\mathbf{x}\right) d\tilde{\pi} \qquad [1]$$

where W_0 denotes initial wealth, $\alpha = [\alpha_1, \alpha_2, ..., \alpha_\mu]'$ is a vector of non-negative acreage allocations to different outputs which satisfy the adding up constraint $\mathbf{l}'\alpha \leq A$ where A is the total available acreage, and $\tilde{\pi} = \left[\tilde{\pi}_1, ..., \tilde{\pi}_m\right]'$ is a vector of stochastic output returns. Taking a third order Taylor's series expansion of $U\left(W_0 + \alpha'\tilde{\pi}\right)$ about expected final wealth, $\overline{W} = W_o + \mu_1$, yields

$$\begin{gathered} V\left(W_o, \mu_1, \mu_2, \mu_3\right) = \mu_1 - \frac{1}{2} R_a\left(\overline{W}\right)\mu_2 + \frac{1}{6} D_a\left(\overline{W}\right)\mu_3, \\ R_a\left(W_o\right) = -\frac{U^2\left(\overline{W}\right)}{U^1\left(\overline{W}\right)}, \\ D_a\left(W_o\right) = \frac{U^3\left(\overline{W}\right)}{U^1\left(\overline{W}\right)} \end{gathered} \qquad [2]$$

where μ_1 is the output portfolio's expected profits, μ_k represents the kth central moment of the portfolio's returns, $k \in [2,3]$, $R_a\left(\overline{W}\right)$ and $D_a\left(\overline{W}\right)$ denote the agent's Arrow-Pratt coefficient of absolute risk aversion and coefficient of absolute downside risk aversion, respectively, and $V\left(W_o, \alpha, \mu_1, \mu_2, \mu_3\right)$ is the agent's expected utility function. The expected returns of the output portfolio and the kth central moment of the portfolio's returns, $k \in [2,3]$ are given by

$$\mu_1(\mathbf{x}) = \sum_{i=1}^{m} \alpha_i \mu_{1i} \qquad [2a]$$

$$\mu_2(\mathbf{x}) = \sum_{i=1}^{m} \alpha_i^2 \mu_{2i} + \sum_{i=1}^{m} \sum_{\substack{j=1 \\ j \neq i}}^{m} \alpha_i \alpha_j \sigma_{ij}. \qquad [2b]$$

$$\mu_3 = \sum_{i=1}^{m} \alpha_{3i}^3 \mu_{3i} + \sum_{i=1}^{m} \sum_{\substack{j=1 \\ j \neq i}}^{m} \sum_{\substack{k=1 \\ k \neq i}}^{m} \alpha_i \alpha_j \alpha_k \sigma_{ijk}. \qquad [2c]$$

Thus, μ_1, the conditional first raw moment of the portfolio, is the weighted sum of expected profits of each output, μ_{1i}, where the weights are given by the acreage allocated to each output (α_i).

Equation [2b] shows that the conditional second central moment of the output portfolio's profits is composed of the weighted sum of individual output return variances and covariances between output profits. In general, the farmer is capable of reducing the output portfolio's conditional variance by diversifying the assets included in the portfolio. In the special case where all the covariances are negative, it is possible to eliminate the total conditional variance (*i.e.,* $\mu_2 = 0$). However, not all of the output portfolio's variance can be eliminated through diversification since, in general, some of the covariances between outputs are positive. Even though not all of the portfolio's variance is completely diversifiable, the overall conditional portfolio variance may be lower than the variance of any of the single outputs (Farrar, 1962; Elton and Gruber, 1987).

The conditional third central moment, μ_3, or measure of skewness of the output portfolio consists of the weighted sum of the third central moment of each output's returns and of the joint movement of the deviations of returns of outputs *i, j,* and *k*. The coskewness between returns of asset *i, j*, and *k*, $\sigma_{i,j,k}$ (.), represents the conditional expectation of the joint movement of the deviations of profits of outputs *i, j*, and *k*. The coskewness is positive whenever the returns of the three outputs move together; *i.e.*, when the (un)favorable profits for all three outputs occur together. In contrast, $\sigma_{i,j,k}$ (.) is negative whenever the profits of one output are inversely related with the outcomes of the other two outputs.

As with the variance of the output portfolio, [2c] implies that the amount of skewness that is diversifiable depends on the coskewness of all the outputs included in the portfolio. Beedles and Simkowitz (1978) explain the observed investor´s choice of less than well-diversified portfolios by the fact that increased diversification leads to a progressive loss in right skewness of the portfolio, a positive attribute for downside risk averse investors. Right skewness of the portfolio tends to occur when the conditional third central moment of returns are positive and with positive coskewness between asset returns.

The farmer´s expected utility is non-decreasing in a variance and skewness preserving increase in the portfolio's expected value; that is $V_{\mu 1} \geq 0$. Additionally, the expected utility of a risk averse and downside risk farmer is non-increasing and non-decreasing in a mean and skewness preserving increase in portfolio's variance and in a mean and variance preserving increase in portfolio's skewness, respectively. Therefore, when the farmer is risk averse and downside risk averse then $V_{\mu 2} < 0$ and $V_{\mu 3} > 0$.

Additionally, when the farmer is risk averse, his expected utility function is concave in the portfolio's expected value; *i.e.* $V_{\mu_1\mu_1} \leq 0$. Expected utility increases at a decreasing rate as the expected value of the portfolio increases. The cross derivative of expected utility with respect to the portfolio's expected value and variance ($V_{\mu_1\mu_2} = V_{\mu_2\mu_1}$) is ambiguous and depends on the effect of the farmer´s wealth on the coefficients of risk aversion and downside risk aversion. Under DARA, CARA, and IARA[1], an increase in portfolio's variance increases, does not affect, and decreases the marginal effect of the expected value of the portfolio's returns on expected utility. This is evidenced by differentiating [2] with respect to μ_1 and μ_2 which yields $V_{\mu_1\mu_2} = -0.5R'_a(\overline{W})$ which will be positive, zero, and negative whenever the farmer´s coefficient of absolute risk aversion is decreasing, constant, or increasing in the level of wealth. Similarly, $V_{\mu_1\mu_3} = V_{\mu_3\mu_1}$ will be positive, zero, and negative whenever the agent's coefficient of absolute downside risk aversion is increasing, constant, or decreasing in the level of wealth (IDRA, CADRA, and DDRA). This result is derived by differentiating [2] with respect to μ_1 and then with respect to μ_3, which yields $V_{\mu_1\mu_3} = 0.\overline{33}D'_a(\overline{W})$.

Let $S_{12} = -V_{\mu_2}/V_{\mu_1}$ and $S_{13} = -V_{\mu_3}/V_{\mu_1}$ denote the slopes of the farmer´s indifference curves between and μ_1 and μ_2 between μ_1 and μ_3, respectively. These indifference curves are upward sloping and downward sloping, respectively, under risk aversion and downside risk aversion; *i.e.,* $S_{12} \geq 0$ and $S_{13} \leq 0$ since $V_{\mu_1} \geq 0$, $V_{\mu_2} < 0$ and $V_{\mu_3} > 0$. The marginal rate of substitution between μ_1 and μ_2 ($MRS_{\mu_1\mu_2}$), for a given level of μ_3, represents the increase in μ_1 which is necessary to maintain the farmer´s expected utility level invariant for a given increase in μ_2 (*i.e.,* $MRS_{\mu_1\mu_2} = d\mu_1/d\mu_{2|d\mu_3=0} = S_{12}$). On the other hand, the marginal rate of substitution between μ_1 and μ_3 ($MRS_{\mu_1\mu_3}$), for a given level of μ_2, represents the decrease in μ_1 which is necessary to maintain the farmer´s expected utility level invariant for a given increase in μ_3 (*i.e.,* $MRS_{\mu_1\mu_3} = d\mu_1/d\mu_{3|d\mu_2=0} = -S_{13}$).

Results (I.1) - (I.6), proven in Propositions A.1 and A.2 of the annex, summarize the properties of these indifference curves.

(I.1) $\partial S_{12}/\partial\mu_1 \geq 0$,

(I.2) $\partial S_{13}/\partial\mu_1 \leq 0$,

(I.3) $\partial S_{12}/\partial\mu_2 >, =, < 0$ under IARA,CARA, DARA,

(I.4) $\partial S_{13}/\partial\mu_2 <, =, > 0$ under IARA,CARA, DARA.

(I.5) $\partial S_{12}/\partial\mu_3 <, =, > 0$ under IADRA,CADRA, DADRA

(I.6) $\partial S_{13}/\partial\mu_3 >, =, < 0$ under IADRA,CADRA, DADRA

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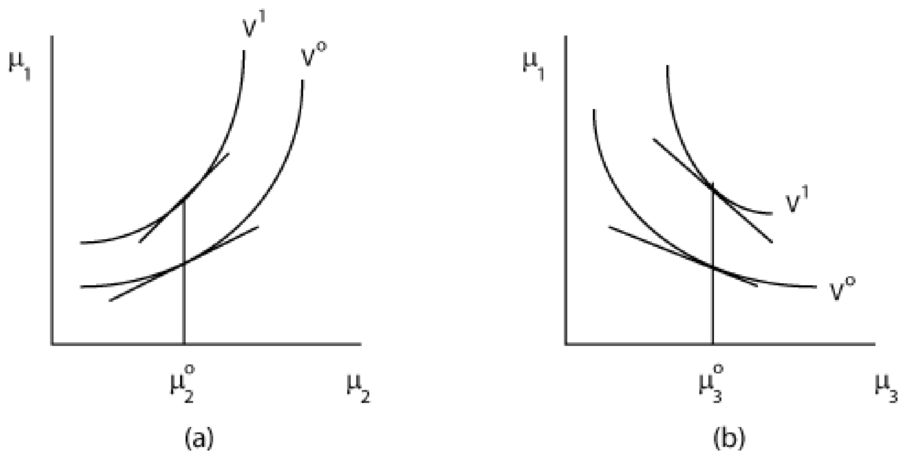
[1] DARA is decreasing absolute risk aversion, CARA is constant absolute risk aversion, and IARA is increasing absolute risk aversion.

Results (I.1) and (I.2) state that the slopes of the indifference curves between μ_1 and μ_2 and between μ_1 and μ_3 become steeper under a variance and skewness preserving increase in the first raw moment of the portfolio's returns; that is, as Figures 1(a) and 1(b) show, as μ_1 increases, *ceteris paribus,* the marginal rates of substitution between μ_1 and μ_2 and between μ_1 and μ_3 increase.

FIGURE 1

Effect of an increase in μ_1 on the slopes of the agent's indifference curves between μ_1 and μ_2 and between μ_1 and μ_3

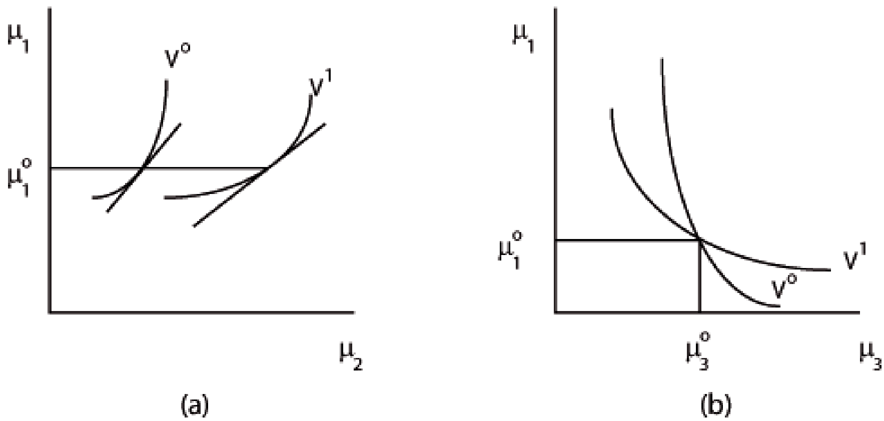


Source: Own elaboration.

The effect of an increase in the conditional second central moment of output returns however, is not so clear. Results (I.3) and (I.4) state that when the farmer´s Arrow-Pratt coefficient of absolute risk aversion is decreasing, constant, or increasing in wealth, the slope of the indifference curve between μ_1 and μ_2 decreases, remains constant, or increases and the slope of the indifference curve between μ_1 and μ_3 increases, remains unchanged, or decreases as μ_2 increases. Figure 2 presents the case of decreasing absolute risk aversion. As the conditional second central moment increases, *ceteris paribus*, the agent moves to a new indifference curve between μ_1 and μ_2, V^1, where the marginal rate of substitution between these two moments is lower (Figure 2(a)). On the other hand, the increase in μ_2 produces a rotation of the indifference curve between μ_1 and μ_3, on the initial point (μ_1^o, μ_3^o), such that the marginal rate of substitution between μ_1 and μ_3 decreases (Figure 2(b)); the initial indifference curve rotates from V^0 to V^1 which has a lower slope at (μ_1^o, μ_3^o).

FIGURE 2

Effect of an increase in μ_2 on the slopes of the agent's indifference curves between μ_1 and μ_2 and between μ_1 and μ_3 under DARA

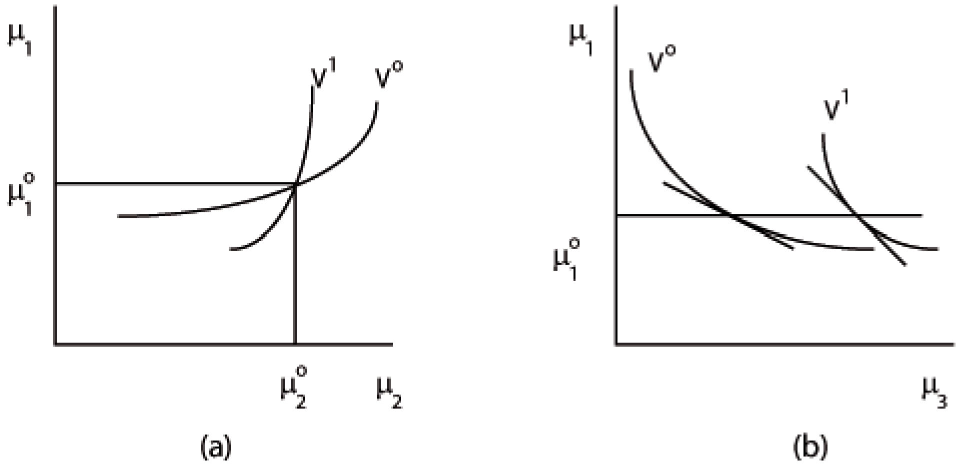


Source: Own elaboration.

The effects of an increase in the conditional third central moment on the agent's marginal rates of substitution between the moments of the distribution of the portfolio's returns depend on the rate of change of the coefficient of absolute downside risk aversion. From (I.5) and (I.6), under increasing, constant, or decreasing absolute downside risk aversion, the marginal rate of substitution between μ_1 and μ_2 and between μ_1 and μ_3 decreases, does not change, or increases as μ_3 increases. The case of decreasing downside risk aversion is depicted in Figure 3. As Figure 3(a) shows, an increase in the conditional third central moment produces a rotation of the indifference curve between μ_1 and μ_2 such that the marginal rate of substitution between these moments increases. Further, the increase in μ_3 leads to an increase in the marginal rate of substitution between μ_1 and μ_3 as the agent shifts to a higher indifference curve, see Figure 3(b).

FIGURE 3

Effect of an increase in μ_3 on the slopes of the agent's indifference curves between μ_1 and μ_2 and between μ_1 and μ_3 under DADRA



Source: Own elaboration.

3. Selection of the optimal portfolio of assets

The farmer selects the optimal output portfolio by determining the acreage allocation of each output, α^*, so as to

$$\begin{aligned} &\underset{\alpha}{Max}\left\{ \mu_1 - \frac{1}{2} R_a(\overline{W})\mu_2 + \frac{1}{6} D_a(\overline{W})\mu_3 \right\} \\ &\text{s.t.} \\ &\qquad \mathbf{l}'\alpha \leq A \\ &\qquad \alpha_i \geq 0, \quad \forall \ i = 1,...m \end{aligned} \qquad [3]$$

where l is an (m*1) unit vector, defined by [3a], represents the expected returns, and μ_2 and μ_3 denote the second and third central moments, given by equations [3b] and [3c], respectively. The necessary Kuhn-Tucker first order conditions (FOC) which characterize the optimal acreage allocations, for *i = 1,...,m* are:

$$\frac{\partial L(\alpha^*, \mathbf{x}^*, \lambda^*)}{\partial \alpha_i} = (\mu_{1i}) - \left(S_{12}\frac{\partial \mu_2}{\partial \alpha_i}\right) - \left(S_{13}\frac{\partial \mu_3}{\partial \alpha_i}\right) - \tilde{\lambda}^* \leq 0 \quad [3a]$$

$$\frac{\partial L(\alpha^*, \mathbf{x}^*, \lambda^*)}{\partial \alpha_i}\alpha_i^* = \alpha_i^*\left((\mu_{1i}) - \left(S_{12}\frac{\partial \mu_2}{\partial \alpha_i}\right) - \left(S_{13}\frac{\partial \mu_3}{\partial \alpha_i}\right) - \tilde{\lambda}^*\right) = 0 \quad [3b]$$

$$\alpha_i^* \geq 0 \quad [3c]$$

where $\tilde{\lambda}^* = \lambda^*/V_{\mu_1}$.

The first term of the right hand side of equations [3a] and [3b] represents the effects of the choice variables on the conditional first raw moment of total portfolio profits. The second and third terms, on the other hand, represent the impact of the choice variables on the conditional second and third central moments of random profits, respectively. In the special case of CARA and when the agent is not downside risk averse, equations [3a] - [3c] reduce to the usual Kuhn-Tucker first order conditions associated with the expected-variance approach[2].

Equations [3a] - [3c] characterize the optimal acreage allocations for i = 1,...,m. Equation [3a] implies that output i will be included in the portfolio (*i.e.*, $\alpha_i^* > 0$) if and only if its marginal expected utility evaluated at $\alpha_i^* = 0$ is greater than $\tilde{\lambda}^*$.[3] The tendency of an output to be included in the optimal portfolio is affected by several factors. First, output i tends to be in the optimal output mix whenever the marginal contribution of output i to the overall expected net return of the portfolio is more positive (*i.e.,* $\mu_{1i}(\cdot) > 0$). Second, it tends to be included if the marginal contribution of output i to the conditional second central moment of the portfolio of outputs, evaluated at $\alpha_i = 0$, is non-positive. Third, output i is included in the optimal portfolio mix whenever its marginal contribution to the third central moment of the portfolio of outputs, evaluated at $\alpha_i = 0$ is non-negative.

Additionally, the coefficients of absolute risk aversion and of absolute downside risk aversion are positively related to the benefits of diversification; the higher these coefficients are, the greater is the importance of the marginal contribution of output i to the portfolio's conditional second and third central moments. For the extreme case where the farmer is risk neutral (*i.e.* $S_{12} = 0$ and $S_{13} = 0$), he chooses only one asset. Finally, the farmer's initial wealth level affects the decision of whether to include an output in the optimal portfolio through its effect on the farmer´s coefficients of absolute risk aversion and absolute downside risk aversion.

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[2] Note that $S_{12} = 0.5R_a$ under CARA and $S_{13} = 0$ under downside risk neutrality.

[3] The marginal expected utility of output i, $EU^1(.)\tilde{\pi}_i$, is decreasing in α_i since second order conditions imply $EU^2(.)\tilde{\pi}_i{}^2 \leq 0$.

Suppose that $\alpha_i^* > 0$ is satisfied for $i = 1,...,\tau$, then the necessary Kuhn-Tucker first order conditions can be rewritten as

$$\alpha_i^* = 0 \quad \forall \text{ i} > \tau \qquad [4a]$$

$$\mu_{1i}\left(\mathbf{x}_i^*, \varepsilon_i\right) - S_{12}\frac{\partial \mu_2}{\partial \alpha_i} - S_{13}\frac{\partial \mu_3}{\partial \alpha_i} = \tilde{\lambda}^* \quad \forall \text{ i} \leq \tau \qquad [4b]$$

$$\sum_{i=1}^{\tau} \alpha_i = A \qquad [4c]$$

Under the assumptions that the land constraint is binding, the sufficient second order conditions for a maximum of (3) over α_i $i \leq \tau$, is that the $((\tau+1) \times (\tau+1))$ bordered Hessian matrix of second order derivatives, $\overline{H}$,

$$\overline{H} = \begin{bmatrix} 0 & \nabla_{\lambda\alpha} L\left(\alpha^*, \lambda^*\right) \\ \nabla_{\alpha\lambda} L\left(\alpha^*, \lambda^*\right) & \nabla_{\alpha\alpha} L\left(\alpha^*, \lambda^*\right) \\ & \end{bmatrix} \qquad [5]$$

is negative definite, where $L(\alpha^*,\lambda^*)$ denotes the Lagrangian function evaluated at the optimal values of the choice variables[4].

4. Properties of optimal output acreage allocations

In principle, by the implicit function theorem, one can solve [4a] - [4c] for the optimal acreage allocations and the optimal shadow value of the land constraint, $\left(\alpha_i^*, \lambda^*\right)$ $\forall i \leq \tau$, as functions of the parameters of the decision model[5]. Substituting these into [4a] - [4c] and differentiating with respect to a parameter of the model yields the fundamental equation of comparative statics

[4] The second order conditions for all those assets not included in the optimal portfolio are automatically satisfied.

[5] The implicit function theorem states that first order conditions can be solved for the optimal asset allocations as functions of the parameters of the problem, if the Bordered Hessian matrix of second order conditions is negative definite.

$$\begin{bmatrix} \partial\lambda^*/\partial\gamma \\ \partial\alpha_1^*/\partial\gamma \\ \vdots \\ \partial\alpha_m^*/\partial\gamma \end{bmatrix} = -\overline{H}^{-1}\begin{bmatrix} \nabla_{\lambda\gamma} L(\alpha^*, \lambda^*) \\ \nabla_{\alpha\gamma} L(\alpha^*, \lambda^*) \end{bmatrix} \qquad [6]$$

where $\gamma \in \Gamma = \{W_o, A, \mu_{1i}\ \forall i, \mu_{2i}\ \forall i, \mu_{3i}\ \forall i, \sigma_{cv}, \sigma_{cs}\}$, $\sigma_{cv} = [\sigma_{12},\ldots,\sigma_{1m},\ldots,\sigma_{m,m-1}]$ $\forall\, i \le \tau$ and $\sigma_{cs} = [\sigma_{112},\ldots\sigma_{11m},\ldots,\sigma_{mm,m-1}]\ \forall\, i \le \tau$ denote vectors of distinct covariances and coskewness coefficients, respectively, $\overline{H}$ is the negative definite bordered Hessian matrix of second order conditions[6]. Isolating $\partial\alpha_i^*/\partial\gamma$ from [6] yields

$$\frac{\partial\alpha_i^*}{\partial\gamma} = -\left|\overline{H}\right|^{-1}\sum_{k=1}^{m+1}\overline{H}_{ki}\frac{\partial^2 L}{\partial\alpha_k\partial\gamma} \qquad [7]$$

where $\overline{H}$ represents the negative-definite bordered Hessian matrix of second order conditions and $\overline{H}_{ki}$ denotes the ki^{th} cofactor of $\overline{H}$.

For the case where the conditional first raw moment of the returns of output i changes (*i.e.* $\gamma = \mu_{1i}$), [7] becomes

$$\frac{\partial\alpha_i^*}{\partial\mu_{1i}} = \left|\overline{H}\right|^{-1}\left[-\overline{H}_{ii} + \sum_{k=1}^{m}\alpha_i^*\overline{H}_{ki}\left(\frac{\partial S_{12}}{\partial\overline{W}}\frac{\partial\mu_2}{\partial\alpha_k} + \frac{\partial S_{13}}{\partial\overline{W}}\frac{\partial\mu_3}{\partial\alpha_k}\right)\right]\ \forall\ i \le \tau \qquad [8]$$

Note, additionally, that the effect of a change in the farmer's final expected wealth on the optimal acreage allocated to output *i* is

$$\frac{\partial\alpha_i^*}{\partial\overline{W}} = \left|\overline{H}\right|^{-1}\left[\sum_{k=1}^{m}\overline{H}_{ki}\left(\frac{\partial S_{12}}{\partial\overline{W}}\frac{\partial\mu_2}{\partial\alpha_k} + \frac{\partial S_{13}}{\partial\overline{W}}\frac{\partial\mu_3}{\partial\alpha_k}\right)\right] \qquad [9]$$

Equation [9] implies that the net effect on the optimal acreage allocations of a change in the first raw moment of returns of output *i* is given by

$$\frac{\partial\alpha_i^*}{\partial\mu_{1i}} = \left[\frac{\partial\alpha_i^c}{\partial\mu_{1i}} + \alpha_i^*\frac{\partial\alpha_i^*}{\partial\overline{W}}\right] \qquad [10]$$

[6] In the derivation of the fundamental equation of comparative statics, the marginal change in the exogenous variable is implicitly assumed not to induce a discrete change in the optimal asset allocations; that is, the marginal change does not induce the investor to cease investment of an asset or to include an asset that was not originally included in the optimal asset mix.

where $\frac{\partial \alpha_i^c}{\partial \mu_{1i}} = -\frac{\bar{H}_{ii}}{|\bar{H}|} > 0$ denotes the increase in the wealth compensated output allocation due to an increase in its own first raw moment of returns[7]. Hence, the net change in the optimal acreage allocation of output *i* due to an increase in its own first raw moment of returns is not unambiguous and depends on the effects of an increase in the final expected level of wealth.

The effect of μ_{1i} is unambiguously positive on α_i^* when the farmer is risk averse and downside risk averse, since an increase in the agent's initial expected wealth leads to an unambiguous increase in the use of an input which decreases the conditional variance and increases the conditional third central moment of per acre profits. This result is derived from [9] and the fact that under risk aversion and downside risk aversion, the agent's preferences are convex and, thus, $\frac{\partial S_{12}}{\partial \mu_1} \geq 0$ and $\frac{\partial S_{13}}{\partial \mu_1} \geq 0$. Hence, this effect is due to the increase in the marginal rate of substitution between μ_1 and μ_2 and between μ_1 and μ_3. This result is unambiguous in the sense that it is independent of the rate of change of the agent's coefficients of absolute risk aversion and absolute downside risk aversion.

Additionally, the effect of μ_{1i} is unambiguously positive on α_i^* when: (i) the farmer is risk neutral, since $\partial \alpha_i^* / \partial \bar{W} = 0$ under risk neutrality, and (ii) under CARA and CADRA, since $\partial S_{12} / \partial \bar{W} = 0$ under CARA and $\partial S_{13} / \partial \bar{W} = 0$ under CADRA. In the special case of CARA and when the farmer is not downside risk averse, equation [10] reduces to

$$\frac{\partial \alpha_i^*}{\partial \mu_{1i}} = \frac{\partial \alpha_i^c}{\partial \mu_{1i}} > 0 \quad \forall \ \mathrm{i} \ \leq \tau \qquad [11]$$

which is the standard result associated with the expected-variance approach.

Proposition 1 summarizes the effects of a conditional mean and conditional third central moment preserving increase in the conditional variance of portfolio´s returns (μ_2) and of a conditional mean and conditional variance preserving increase in the conditional third central moment (μ_3) on optimal acreage allocation in output *i* for a risk averse in the Arrow-Pratt sense and downside risk averse investor.

Proposition 1 For a risk averse and downside risk averse farmer, if an output decreases the variance and increases the third central moment of the portfolio's returns then

(a) $\partial \alpha_i^* / \partial \mu_2 \geq 0$ under IARA and CARA,
(b) $\partial \alpha_i^* / \partial \mu_3 \leq 0$ under IADRA and CADRA.

Result (a) of Proposition 1 establishes that the effect of a conditional mean and conditional third central moment preserving increase in the conditional variance of

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7 When $\bar{H}$ is negative-definite, then $sign\left(\left|\bar{H}\right|\right) = (-1)^m$ and $sign\left(\bar{H}_{ii}\right) = (-1)^{m-1}$ (Takayama, 1985, pg: 162-163).

portfolio's returns is ambiguous; it can be signed however, under certain assumptions on the rate of change of the agent's risk attitudes. The direct effect of an increase in μ_2 is to increase the importance of the marginal reduction in the conditional variance caused by including output *i* (α_1) in the optimal portfolio, thus encouraging an increased acreage allocation in output *i*. Under increasing or constant absolute risk aversion, the indirect effect is also non-negative; an increase in μ_2 increases the marginal rate of substitution between μ_1 and μ_2 (Property I.3) thus increasing the importance of the risk reducing properties of α_i. Under DARA, on the other hand, an increase in μ_2 reduces the marginal rate of substitution between μ_1 and μ_2 implying that it is optimal to reduce α_i, while the direct effect leads to an increase in the optimal input use; hence, the final effect is ambiguous.

The effect of a conditional mean and conditional variance preserving increase in the conditional third central moment, as result (b) shows, is also ambiguous[8]. The importance of the positive marginal effect of α_i on the conditional third central moment decreases as the conditional third central moment of per acre profits increases; this direct effect implies that a decrease in α_i is optimal. The indirect effect of the reduction in downside risk, on the other hand, depends on the rate of change of the agent's coefficient of absolute downside risk aversion. When the coefficient of absolute downside risk aversion is increasing or constant in wealth, the indirect effect of an increase in μ_3 is a decrease in the marginal rate of substitution between μ_1 and μ_2 and between μ_1 and μ_3, leading to a decreased use of the input (Properties I.5 and I.6). Hence, under IADRA or CADRA, the indirect effect reinforces the direct effect and the optimal use of α_i will decrease. Under DADRA, on the other hand, the indirect effect opposes the direct effect of a decrease in downside risk so the final effect is ambiguous.

5. The empirical model

The econometric specification of the model is derived by employing a first-order Taylor's series approximation of the optimal acreage and of the conditional first raw moment and conditional second and third central moments of per acre revenues, for all farmers and time periods, which yields

$$\alpha_{ift} = \begin{cases} 0 \;\; \forall i > \tau \\ a_{io} + a_{iW} W_{oft} + a_{iA} A_{ft} + \sum_{k=1}^{n} a_{ik} w_{kt} + \sum_{k=1}^{m} \left(a_{i1k} \mu_{1kft} + a_{i2k} \mu_{2kft} + a_{i3k} \mu_{3kft} \right) + \\ \sum_{j=1}^{m} \sum_{k \neq j}^{m} \left(a_{ijk} \sigma_{jkft} + a_{ijjk} \sigma_{jjkft} \right) + \sum_{j=1}^{m} \sum_{k \neq j}^{m} \sum_{l \neq k}^{m} a_{ijkl} \sigma_{ijkft} + \zeta_{ift} \;\; \forall i \leq \tau \end{cases} \qquad [12a]$$

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[8] A conditional mean and variance preserving increase in the conditional skewness leads to a distribution of per acre profits which has less downside risk.

$$\mu_{1ift} = \beta_{io} + \sum_{j=1}^{n} \beta_{ij} x_{ijft} + \sum_{e=1}^{m} \beta_{ie} \varepsilon_{eft}, \forall i \qquad [12b]$$

$$\mu_{2ift} = \gamma_{io} + \sum_{j=1}^{n} \gamma_{ij} x_{ijft} + \sum_{e=1}^{m} \gamma_{ie} \varepsilon_{eft}, \forall i \qquad [12c]$$

$$\mu_{3ift} = \delta_{io} + \sum_{j=1}^{n} \delta_{ij} x_{ijft} + \sum_{e=1}^{m} \delta_{ie} \varepsilon_{eft}, \forall i \qquad [12d]$$

$$\sigma_{ijft} = \phi_{o}^{ij} + \sum_{k=1}^{n} \left(\phi_{ik}^{ij} x_{ikft} + \phi_{jk}^{ij} x_{jkft} \right) + \phi_{ei}^{ij} \varepsilon_{ift} + \phi_{ej}^{ij} \varepsilon_{jft}, \forall i \neq j \qquad [12e]$$

$$\sigma_{ijkft} = \psi_{o}^{ijk} + \sum_{l=1}^{n} \left(\psi_{il}^{ijk} x_{ilft} + \psi_{jl}^{ijk} x_{jlft} + \psi_{kl}^{ijk} x_{klft} \right) + \psi_{ei}^{ijk} \varepsilon_{ift} + \psi_{ej}^{ijk} \varepsilon_{jft} + \psi_{ek}^{ijk} \varepsilon_{kft}, \quad \forall i \neq j \neq k. \qquad [12f]$$

Equations [12b] - [12f] determine the moments of the joint distribution of per acre profits.

The estimation of *m* equations established in [12a] is feasible when data is available on acreage allocations. However, there are drawbacks to estimating [12a] - [12f] even when the necessary data is available. No structure of the decision process has been imposed on the acreage allocation equations. This is a serious drawback since the lack of structure in the estimation leads to a loss of efficiency of the econometric estimates (Just *et al.*, 1983). Moreover, direct estimation of [12a] does not ensure that the estimated acreage allocations are mutually consistent with the farmer's underlying decision problem. The necessary structure can be imposed by estimating equations [12a] - [12f] jointly with the producer's first order conditions. This approach imposes on the optimal acreage allocations the theoretical restrictions and properties implied by the decision model. Additionally, this approach is a source of additional non-redundant information that helps identify the structural parameters. Specifically, [12a] - [12f] can be jointly estimated with the following 2m + 1 equations

$$\alpha_{ift} \left(\mu_{1ift} - \mathbf{w}_{t}'\mathbf{x}_{ift} - S_{12ft} \frac{\partial \mu_{2ft}}{\partial \alpha_{ift}} - S_{13ft} \frac{\partial \mu_{3ft}}{\partial \alpha_{ift}} - \tilde{\lambda}_{ft} \right) = 0 \quad \forall i,f,t \qquad [13a]$$

$$\alpha_{ift} \left(\alpha_{ift} \left(\frac{\partial \mu_{1ift}}{\partial x_{ijft}} - w_{jft} \right) - S_{12ft} \frac{\partial \mu_{2ft}}{\partial x_{ijft}} - S_{13ft} \frac{\partial \mu_{3ft}}{\partial x_{ijft}} \right) = 0 \quad \forall i,j,f,t \qquad [13b]$$

$$\sum_{i=1}^{m} \alpha_{ift} = A_{ft} \qquad [13c]$$

In order to estimate the 2m+1 equations included in equations [12] and [13], data on α, **w**, x, and λ are necessary. In general, however, the shadow value of land is not observable. In order to overcome this empirical obstacle, the land constraint can be substituted into the producer's decision problem by solving for α_{mft}. This procedure yields the following set of FOC to be estimated jointly with equations [12].

$$\alpha_{ift}\begin{bmatrix} \mu_{1ift} - \mu_{1mft} - \mathbf{w}_t'\left[\mathbf{x}_{ift} - \mathbf{x}_{mft}\right] - \\ S_{12ft}\left(\frac{\partial \mu_{2ft}}{\partial \alpha_{ift}} - \frac{\partial \mu_{2ft}}{\partial \alpha_{mft}}\right) - S_{13ft}\left(\frac{\partial \mu_{3ft}}{\partial \alpha_{ift}} - \frac{\partial \mu_{3ft}}{\partial \alpha_{mft}}\right) \end{bmatrix} = 0, \forall i, f, t \qquad [14a]$$

$$\alpha_{mft} = A - \sum_{i=1}^{m-1} \alpha_{ift} \qquad [14b]$$

$$\frac{\partial \mu_{2ft}}{\partial \alpha_{ift}} = 2\alpha_{ift}\mu_{2ift} + 2\sum_{\substack{j=1 \\ j\neq i}}^{m} \alpha_{jft}\sigma_{ijft} \qquad [14c]$$

$$\frac{\partial \mu_{3ft}}{\partial \alpha_{ift}} = 3\alpha_{ift}^2\mu_{3ift} + 6\sum_{\substack{j=1 \\ j\neq i}}^{m} \alpha_{ift}\alpha_{jft}\sigma_{iijft} + 6\sum_{\substack{j=1 \\ j\neq i}}^{m}\sum_{\substack{k=1 \\ k\neq i}}^{m} \alpha_{jft}\alpha_{kft}\sigma_{ijkft} \qquad [14d]$$

These equations depend on the farmer's coefficients of absolute risk aversion and absolute downside risk aversion which are functions of the expected end of period wealth. The farmer's Arrow-Pratt coefficient of absolute risk aversion is defined as

$$R_a\left(\overline{W}_{ft}\right) = \eta_1\left(\overline{W}_{ft}\right)^{\eta_2} \qquad [15]$$

where $\overline{W}_{ft} = W_{oft} + \alpha_{ft}'\,\overline{\pi}_{ft}$. This specification is flexible enough to permit risk-loving, risk-neutral, and risk-averse behavior (Bar-Shira *et al.,* 1997);

$$if\ \eta_1 \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0\ \ then\ \text{the producer is risk} \begin{Bmatrix} \text{averse} \\ \text{neutral} \\ \text{loving} \end{Bmatrix} \qquad [15a]$$

Furthermore, the rate of change of the producer's coefficient of absolute risk aversion can be tested with the above functional specification of $R_a(\overline{W}_{ft})$; the sign of the elasticity of absolute risk aversion with respect to wealth, η_2, determines whether the agent's degree of risk aversion decreases, remains unchanged, or increases as the level of wealth increases. For a risk averse farmer ($\eta_2 > 0$), for example,

$$\text{if } \eta_2 \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0, \text{ then } \mathrm{R_a}(\overline{W}_{ft}) \text{ is } \begin{Bmatrix} \text{increasing} \\ \text{constant} \\ \text{decreasing} \end{Bmatrix} \text{ in } \overline{\mathrm{W}}_{\mathrm{ft}} \quad [15b]$$

The agent's coefficient of absolute downside risk aversion can be specified as

$$D_a(\overline{W}_{ft}) = \eta_1^2 (\overline{W}_{ft})^{2\eta_2} - \eta_1 \eta_2 (\overline{W}_{ft})^{\eta_2 - 1} \quad [16]$$

The rates of change of these coefficients of absolute downside risk aversion are

$$D_a'(\overline{W}_{ft}) = 2\eta_1^2 \eta_2 \overline{W}_{ft}^{2\eta_2 - 1} - \eta_1 \eta_2 [\eta_2 - 1] \overline{W}_{ft}^{\eta_2 - 2} \quad [16a]$$

Hence, the signs of η_1 and η_2 also determine whether the producer is downside risk averse and the rate of change of the coefficients of absolute and relative downside risk aversion. Table 1 presents all plausible signs of η_1 and η_2 and their implications for the agent's risk attitudes. For example, if $\eta_1 > 0$ and $-1 < \eta_2 < 0$ then the farmer is risk averse in the Arrow-Pratt sense and downside risk averse and their coefficients of absolute (relative) risk aversion and downside risk aversion are decreasing (increasing) in $\overline{W}$. In some cases a qualitative analysis of the signs of η_1 and η_2 is not sufficient to determine the characteristics of the producer's downside risk attitudes; in these cases it is necessary to analyze the relative magnitude of each term of [16a].

Finally, estimation of the structural econometric model allows us to test whether producers are risk averse and/or downside risk averse through the significance of the agent's coefficient of absolute and relative downside risk aversion.

TABLE 1

Implications of the signs of parameters of the agent's coefficients of risk aversion and downside risk aversion

η_1	η_1	$R_a(\overline{W})$	$R'_a(\overline{W})$	$R'_r(\overline{W})$	$D_a(\overline{W})$	$D'_a(\overline{W})$	$D'_r(\overline{W})$
$\eta_1 < 0$	$\eta_2 > 1$	-	-	-	+	+	+
$\eta_1 < 0$	$\eta_2 = 1$	-	-	-	+	+	+
$\eta_1 < 0$	$0 < \eta_2 < 1$	-	-	-	+	?	+
$\eta_1 < 0$	$\eta_2 = 0$	-	0	-	+	0	+
$\eta_1 < 0$	$-1 < \eta_2 < 0$	-	+	-	?	-	?
$\eta_1 < 0$	$\eta_2 = -1$	-	+	0	?	?	0
$\eta_1 < 0$	$\eta_2 < -1$	-	+	+	?	?	?
$\eta_1 < 0$	$-\infty < \eta_2 < \infty$	0	n.a.	n.a.	0	n.a.	n.a.
$\eta_1 < 0$	$\eta_2 > 1$	+	+	+	?	?	?
$\eta_1 < 0$	$\eta_2 = 1$	+	+	+	?	+	?
$\eta_1 < 0$	$0 < \eta_2 < 1$	+	+	+	?	+	+
$\eta_1 < 0$	$\eta_2 = 0$	+	0	+	+	0	+
$\eta_1 = 0$	$-1 < \eta_2 < 0$	+	-	+	+	-	+
$\eta_1 > 0$	$\eta_2 = 1$	+	-	0	+	-	0
$\eta_1 > 0$	$\eta_2 < -1$	+	-	-	+	-	-

n.a. = Not applicable.
Source: Own elaboration.

6. Conclusions

This study provides a general framework to analyze the trade-off between expected returns, variance and skewness for a farmer who is risk averse in the Arrow-Pratt sense and downside risk averse. The decision model for a risk averse and downside risk averse farmer developed in this paper explicitly accounts for the conditional third central moment of the joint distribution of outputs. This general model contains the classical E-V portfolio model as a special case. The benefit of this generalization is that estimation of the structural econometric model allows each competing behavioral hypothesis to be discerned econometrically through the significance of the agent's coefficient of absolute and relative downside risk aversion.

Endogeneity of the optimal output mix is also explicitly considered. The model establishes that an output tends to be included in the optimal portfolio whenever its marginal contribution to the overall expected value and third central moment is positive and when its marginal contribution to the overall second central moment is negative. Additionally, the output choice decision depends on the investor's coefficients of risk and downside risk aversion.

Given the decision framework, the farmer´s optimal portfolio choice depends on the farmer´s initial wealth and the moments of the joint distribution of portfolio returns. The results show that an output *i* tends to be included in the optimal portfolio whenever its marginal contribution to the overall conditional expected return and conditional third central moment (conditional second central moment) of the portfolio of assets is more positive (negative). Furthermore, the results indicate that a higher acreage will be allocated to an output whose expected returns are increasing as long as final expected wealth has a non-negative effect on optimal acreage allocations. In general, however, the effect of an exogenous variable on the optimal acreage allocations is ambiguous. More importantly, many of these ambiguities, which are important in policy analysis, can be resolved only empirically.

An interesting area of further research is the application of the decision model to the analysis of the firm's risk management decisions. One of the elements of this decision is whether to hedge in the futures and/or options markets which have different downside risk implications; futures markets increase the downside risk protection while options markets additionally provide a higher potential for upside gain than futures markets. Thus, it is necessary to include downside risk in the optimal risk management decisions of the firm.

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Annex

Proposition A.1:

$$\frac{\partial S_{12}}{\partial \mu_1} \geq 0, \frac{\partial S_{12}}{\partial \mu_2} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \text{ under} \begin{Bmatrix} \text{DARA} \\ \text{CARA} \\ \text{IARA} \end{Bmatrix} \text{ and } \frac{\partial S_{12}}{\partial \mu_3} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \text{ under } \begin{Bmatrix} \text{DDRA} \\ \text{CDRA} \\ \text{IDRA} \end{Bmatrix}$$

Proof: Under risk aversion and downside risk aversion, the agent’s preferences are convex; that is, $\wp = \{(\mu_1, \mu_2, \mu_3) : V(\mu_1, \mu_2, \mu_3) \geq V^o\}$ is a convex set which implies that $dS_{12}/d\mu_2 \geq 0$. Totally differentiating S_{12} with respect to μ_2 yields

$$\frac{dS_{12}}{d\mu_2} = \frac{\partial\left(-\frac{V_{\mu_2}\left(\mu_1(\mu_2), \mu_2, \mu_3\right)}{V_{\mu_1}\left(\mu_1(\mu_2), \mu_2, \mu_3\right)}\right)}{\partial\mu_2} = \frac{\partial S_{12}}{\partial\mu_1}\frac{\partial\mu_1}{\partial\mu_2} + \frac{\partial S_{12}}{\partial\mu_2} \qquad [A.1]$$

where

$$\frac{\partial S_{12}}{\partial\mu_1} = \frac{-V_{\mu_1}V_{\mu_2\mu_1} + V_{\mu_2}V_{\mu_1\mu_1}}{\left(V_{\mu_1}\right)^2} \qquad [A.2]$$

$$\frac{\partial\mu_1}{\partial\mu_2} = -\frac{V_{\mu_2}}{V_{\mu_1}} = S_{12} \geq 0 \qquad [A.3]$$

$$\frac{\partial S_{12}}{\partial\mu_2} = \frac{V_{\mu_2}V_{\mu_1\mu_2}}{\left(V_{\mu_1}\right)^2} = -S_{12}\frac{V_{\mu_1\mu_2}}{V_{\mu_1}} \qquad [A.4]$$

Equation [A.2] implies that $\partial S_{12}/\partial\mu_1 > (=) 0$ for the cases of IARA and CARA since $V_{\mu 1} \geq 0$, $V_{\mu 2} \leq 0$, and $V_{\mu 1 \mu 1} \leq 0$ respectively, and $V_{\mu 1 \mu 2} < (=)\ 0$ under IARA and CARA. Under DARA, on the other hand, $V_{\mu 1 \mu 2} > 0$ so, apparently, the sign is ambiguous. It can, however, be unambiguously signed by employing the fact that preferences are convex since A.1 implies that

$$\frac{\partial S_{12}}{\partial\mu_1} \geq \frac{V_{\mu_1\mu_2}}{V_{\mu_1}} \qquad [A.5]$$

therefore, under DARA $\frac{\partial S_{12}}{\partial\mu_1} \geq 0$.

Equation [A.4] implies that $\frac{\partial S_{12}}{\partial\mu_1} <,=,> 0$ as $V_{\mu_1\mu_2} >,=,< 0$ which will occur under DARA, CARA, IARA. The last result is obtained by differentiating S_{12} with respect to μ_3 which yields

$$\frac{\partial S_{12}}{\partial\mu_3} = -S_{12}\frac{V_{\mu_1\mu_3}}{V_{\mu_1}} \qquad [A.6]$$

Expression [A.6] will be less than, equal to, and greater than zero as $V_{\mu 1 \mu 3} >, =, < 0$ *i.e.*, $\frac{\partial S_{12}}{\partial\mu_3} <,=,> 0$ under IDRA, CDRA, DDRA.

Proposition A.2: $\frac{\partial S_{13}}{\partial\mu_1} \leq 0$, $\frac{\partial S_{13}}{\partial\mu_3}\begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0$ under $\begin{Bmatrix} \text{DDRA} \\ \text{CDRA} \\ \text{IDRA} \end{Bmatrix}$, and $\frac{\partial S_{13}}{\partial\mu_2}\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$ under $\begin{Bmatrix} DARA \\ CARA \\ IARA \end{Bmatrix}$.

Proof: Convexity of the agent's preferences implies

$$\frac{dS_{13}}{d\mu_3} = \frac{\partial\left(-\frac{V_{\mu_3}(\mu_1(\mu_3), \mu_2, \mu_3)}{V_{\mu_1}(\mu_1(\mu_3), \mu_2, \mu_3)}\right)}{\partial\mu_3} = \frac{\partial S_{13}}{\partial\mu_1}\frac{\partial\mu_1}{\partial\mu_3} + \frac{\partial S_{12}}{\partial\mu_3} \quad \text{[A.7]}$$

where

$$\frac{\partial S_{13}}{\partial\mu_1} = \frac{-V_{\mu_1}V_{\mu_3\mu_1} + V_{\mu_3}V_{\mu_1\mu_1}}{\left(V_{\mu_1}\right)^2} \quad \text{[A.8]}$$

$$\alpha_{mft} = A - \sum_{i=1}^{m-1}\alpha_{ift} \qquad \frac{\partial\mu_1}{\partial\mu_3} = -\frac{V_{\mu_3}}{V_{\mu_1}} = S_{13} \leq 0 \quad \text{[A.9]}$$

and

$$\frac{\partial S_{13}}{\partial\mu_3} = \frac{V_{\mu_3}V_{\mu_1\mu_3}}{\left(V_{\mu_1}\right)^2} = -S_{13}\frac{V_{\mu_1\mu_3}}{V_{\mu_1}} \quad \text{[A.10]}$$

[A.8] implies that $\frac{\partial S_{13}}{\partial\mu_1} \leq 0$ under IDRA and CDRA since $V_{\mu1\mu2} < (=)\ 0$. Furthermore, note that convexity of preferences also implies that

$$\frac{\partial S_{13}}{\partial\mu_1} < \frac{V_{\mu_1\mu_3}}{V_{\mu_1}} \quad \text{[A.11]}$$

and, thus, $\frac{\partial S_{13}}{\partial\mu_1} < 0$ under DDRA since $V_{\mu1\mu3} < 0$.

From [A.10] it is clear that $\partial S_{13}/\partial\mu_3 >, =, < 0$ as $V_{\mu1\mu3} >, =, < 0$ which is the case when the agent's coefficient of absolute downside risk aversion is increasing, constant, decreasing in the level of wealth. Finally, differentiating S_{13} with respect to μ_2 yields

$$\frac{\partial S_{13}}{\partial\mu_2} = -S_{13}\frac{V_{\mu_1\mu_2}}{V_{\mu_1}} \quad \text{[A.12]}$$

which will be <, =, > 0 under IARA, CARA, DARA.