Counterterms and dual holographic anomalies in CS gravity

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Abstract

The holographic Weyl anomaly associated to Chern-Simons gravity in $2n+1$ dimensions is proportional to the Euler term in $2n$ dimensions, with no contributions from the Weyl tensor. We compute the holographic energy-momentum tensor associated to Chern-Simons gravity directly from the action, in an arbitrary odd-dimensional spacetime. We show, in particular, that the counterterms rendering the action finite contain only terms of the Lovelock type.

The AdS/CFT correspondence[1, 2, 3] provides a prescription to compute vacuum expectation values of CFT operators in terms of dual classical fields on AdS. This prescription has been checked successfully in many examples. In particular, the CFT energy momentum tensor T^{ij} is dual to the spacetime metric governed by Einstein's equations with a negative cosmological constant. One can use the correspondence to compute the CFT Weyl anomaly. This calculation was indicated in [3], and carried on in detail in [4] with the expected result

$$\langle T^{\mu}_{\mu} \rangle = \mathcal{A} = \frac{N^2}{32\pi^2}\left(R^{ij}R_{ij} - \frac{1}{3}R^2\right), \tag{1}$$

for $\mathcal{N}=4$ $SU(N)$ SYM theory in four dimensions which is dual to type IIB string theory on $AdS_5 \times S^5$.

For a CFT (in $d=4$) with n_s real scalars, n_f Dirac spinors and n_v vectors the anomaly is [5]

$$\mathcal{A} = c\,C^2 - a\,E_4 \tag{2}$$

with

$$a = \frac{1}{360(4\pi)^2}\left(n_s + 11n_f + 62n_v\right) \tag{3}$$

$$c \;=\; \frac{1}{120(4\pi)^2}\left(n_s + 6n_f + 12n_v\right), \tag{4}$$

and the curvature invariants E_4 and C^2 are

$$E_4 = R^{ijkl}R_{ijkl} - 4R^{ij}R_{ij} + R^2, \qquad C^2 = C^{ijkl}C_{ijkl}. \tag{5}$$

Of course, (7) reduces to (1) for the $\mathcal{N}=4$ multiplet with $n_s = 6$, $n_f = 2$ and $n_v = 1$ [4].

In higher (even) dimensions, the anomaly is always characterized by the coefficient multiplying the Euler density and an increasing number of coefficients c_i [1] multiplying curvature invariants which transform homogeneously under Weyl rescaling of the metric. This classification of anomalies has been identified in [8] where they were called type A and type B.

One general feature of any four-dimensional CFT whose dual gravity theory is the Einstein-Hilbert action with cosmological constant is that the two anomaly coefficients a and c are equal [4]. For a generic CFT this is, however, not the case. Specific examples are the theories constructed in [9]. This is reflected in the dual gravity theory by the presence of higher derivative terms. For the theories of [9] they arise from similar terms on the world-volume of D7 branes which are needed for their construction.

Consider, for example, the following action in five dimensions

$$I = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-G}\left(\hat{R} - 2\Lambda + \alpha\hat{R}^2 + \beta\hat{R}^{\mu\nu}\hat{R}_{\mu\nu} + \gamma R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho}\right) \tag{6}$$

Hatted objects refer to spacetime tensors. The Weyl anomaly, associated to the CFT dual to (6) was calculated in [10, 11, 12] with the result,

$$\mathcal{A} = \frac{R^2_{AdS_5}}{16\kappa_5^2}\left\{(1-40\alpha-8\beta+4\gamma)C^2 - (1-40\alpha-8\beta-4\gamma)E_4\right\}. \tag{7}$$

Using the AdS/CFT relations between the string theory and the gauge theory parameters one finds for the theories constructed in [9] that the coefficients in front of the higher derivative terms are $\mathcal{O}(1/N)$ and consequently $a - c \sim N$ (with $a, c \sim N^2$). However, for realistic CFT's such as (S)QCD inside its conformal window one has $a - c \sim N^2$. So far no critical string theory dual for such theories has been found (for attempts within the context of non-critical strings, see [13, 14]).

The higher derivative gravity theories we will consider in this note cannot be considered as duals to realistic conformal field theories as they are necessarily non-unitary [16]. But they exhibit peculiar features as far as the anomalies are concerned since all type B anomalies vanish.

[1] $i = 1$ for $d = 4$, $i = 1, 2, 3$ for $d = 6$ [6], $i = 1, \ldots, 12$ for $d = 8$ [7], while the number of Weyl-invariants in general dimension is still unknown.

For the five-dimensional theory (arbitrary odd dimensions will be considered below), Chern-Simons gravity arises at the point of enhanced gauge symmetry of the action (6). This corresponds to the choice [2]

$$\Lambda = -3, \beta = -4\alpha = -4\gamma = -1, \tag{8}$$

for which I reduces to the five dimensional Chern-Simons form [15]

$$I = \int \mathrm{Tr}\left(AdAdA + \frac{3}{2} dAA^3 + \frac{3}{5} A^5 \right) \tag{9}$$

for the group $SO(4,2)$.

The particular choice (8) has several consequences which were analyzed in detail in [16]. First, the coefficient c of the anomaly vanishes identically, as it can be readily checked by inserting (8) in (7). The anomaly is then given by the the Euler term alone. Since Chern-Simons gravities exist in all odd dimensions [15, 17, 18], one may wonder whether the anomaly associated to all Chern-Simons theories is the pure Euler term in the corresponding dimension. This turns out to be true, as indicated in [16] by an argument based on the equations of motion.

The goal of this note is to prove this statement by computing, directly from the action, the Chern-Simons-AdS holographic energy momentum tensor and its corresponding anomaly in any odd dimension $2n+1$.

We start by explaining the Hamiltonian method to compute the holographic energy momentum tensor (see [19, 20, 21] for other Hamiltonian approaches). In the ADM parametrization using r as "time" in $D = d+1$ dimensions,

$$ds^2 = N^2\, dr^2 + h_{ij}(r,x)(dx^i + N^i dr)(dx^j + N^j dr). \tag{10}$$

the gravitational action can be cast in the Hamiltonian form [3],

$$I_0 = \int dr \int d^d x \left(\pi^{ij} h'_{ij} + N\mathcal{H} + N^i \mathcal{H}_i \right). \tag{11}$$

This form of the action is universal and follows by geometrical considerations. The explicit formulae for the constraints vary depending on the particular theory. But for any invariant theory of gravity, the action can always be cast into the form (11). The action I_0 needs boundary terms and counterterms to be well defined. For the problem at hand, these can be computed as follows.

[2] The AdS radius l has been set to unity.

[3] We choose to start with the Hamiltonian action for convenience, but one may well start with the Lagrangian action. In this context, it would be interesting to explore how the conformal anomaly appears in the regularization scheme for CS-AdS gravity action proposed in [22].

The on-shell variation of I_0 (assuming that the bulk fields satisfy the equations of motion) is

$$\delta I_0 = \int_{r=\varepsilon} d^d x \, \pi^{ij} \, \delta h_{ij}. \tag{12}$$

Now, we make a definite choice for the lapse N and shift N^i functions at infinity, and assume the FG form for the asymptotic metric

$$ds^2 = \frac{dr^2}{4r^2} + \frac{1}{r} \, g_{ij}(x^i, r) dx^i dx^j \tag{13}$$

which corresponds to $N = \frac{1}{2r}$, $N^i = 0$ and $h_{ij} = \frac{1}{r} g_{ij}$. We shall not need to solve the asymptotic equations, nor assume a particular expansion[4] for $g_{ij}(r, x^i)$. We only assume that the limit $r \to 0$,

$$g_{ij}(x^i, r) \;\longrightarrow\; g_{(0)ij}(x^i) \tag{14}$$

exists. Under these conditions we will show that the variation (12) can be rewritten in the form,

$$\delta I_0 = \int_{r=\varepsilon} d^d x \, \left(\frac{1}{2} \sqrt{g_{(0)}} \; T^{ij} \, \delta g_{(0)ij} + \delta B \right) \tag{15}$$

where T^{ij} is *finite*, and B is a (divergent) local functional of g_{ij}.

From (15) we conclude that the correct gravitational action is obtained by passing the term B to the left hand side. We define the renormalized action

$$I \equiv I_0 - \int d^d x \; B. \tag{16}$$

From (15) we see that its variation with respect to $g_{(0)ij}$ is well defined and finite. Our goal is now to compute the counterterm B, and the coefficient T^{ij} which becomes the holographic energy momentum tensor.

This procedure was carried out in [16] for Einstein gravity, and five-dimensional Chern-Simons gravity. We shall now extend these results for Chern-Simons gravity in any odd dimension $D = 2n + 1$. In particular, we shall compute the explicit formula for B, which turns out to be a Lovelock action in $2n$ dimensions[5].

It is worth mentioning here that the standard procedure to find the 1-point function (see [27] for a review), by solving the asymptotic equations, inverting the series, and varying with respect to the regularized metric becomes unfeasible in Chern-Simons gravity due to the higher

[4]For standard gravity [23], $g_{ij} \simeq g_{(0)ij} + r \, g_{(1)ij} + r^2 \, g_{(2)ij} \cdots$. The coefficient $g_{(1)}$ is universal and locally related to $g_{(0)}$. The energy momentum tensor depends [24, 25] on $g_{(n)}$ which is non-locally related to $g_{(0)}$.

[5]A note of caution is in order here. As shown in [25, 26], when matter fields are present, the counterterm action contributes to the finite piece in a non-trivial way. Here, we restrict the discussion to the matter free action, and leave for the future a more general analysis.

powers in the curvature tensor and the resulting complicated equations of motion. The method, described above, for finding the variation of the action becomes extremely powerful if one deals with complicated actions of gravity.

Let us apply this procedure to Chern-Simons gravity in arbitrary dimensions. Chern-Simons gravities are particular cases of Lovelock gravities. The Lovelock action is [28],

$$I = \sum_{2p<D} \alpha_p I_{(p)} \tag{17}$$

where the terms $I_{(p)}$

$$I_{(p)} = \frac{1}{2p!} \int dr\, d^d x \sqrt{-G}\, \delta^{[\mu_1 \ldots \mu_{2p}]}_{[\mu_1 \ldots \mu_{2p}]} \hat{R}^{\mu_1 \mu_2}_{\mu_1 \mu_2} \ldots \hat{R}^{\mu_{2p-1}\mu_{2p}}_{\mu_{2p-1}\mu_{2p}} \tag{18}$$

represent the dimensional continuation of the Euler densities of the lower dimensions.

The Hamiltonian structure of this action was studied in [29]. For this theory, the "velocities" h'_{ij} cannot be inverted as functions of the momenta. But the relation $\pi^{ij}(h'_{kl})$ does exists [29],

$$\pi^i_j = \frac{1}{4} \sum_{p \geq 0} \alpha_p \sum_{s=0}^{p-1} C_{s(p)} \left(\pi_{s(p)}\right)^i_j \tag{19}$$

where

$$\left(\pi_{s(p)}\right)^i_j = \sqrt{-h}\, \delta^{[i i_1 \ldots i_{2s} \ldots i_{2p-1}]}_{[j j_1 \ldots j_{2s} \ldots j_{2p-1}]} \hat{R}^{j_1 j_2}_{i_1 i_2} \ldots \hat{R}^{j_{2s-1} j_{2s}}_{i_{2s-1} i_{2s}} K^{j_{2s+1}}_{i_{2s+1}} \ldots K^{j_{2p-1}}_{i_{2p-1}}, \tag{20}$$

and K_{ij} is the extrinsic curvature of the $r = const.$ submanifolds. The coefficients $C_{s(p)}$ are

$$C_{s(p)} = \frac{4^{p-s}}{s!\,[2(p-s)-1]!!}. \tag{21}$$

For the particular case of Chern-Simons gravity, the coefficients α_p entering in (17) are fixed to

$$\alpha_p = \frac{n!\,[2(n-p)]!}{2^{p-1}(n-p)!}. \tag{22}$$

For this choice, the Lagrangian in (17) becomes a Chern-Simons form satisfying $d\mathcal{L} = F^{n+1}$, with $F \in SO(2n,2)$. We shall not, however, make use of the "gauge theory" formulation.

For the choice (22), the momenta can be rewritten in a more compact form, written in terms of a continuous parameter $t \in [0,1]$,

$$\begin{aligned} \pi^i_j &= n\sqrt{-h} \int_0^1 dt\, \delta^{[i i_1 \ldots i_{2n-1}]}_{[j j_1 \ldots j_{2n-1}]} K^{j_1}_{i_1} \left(\frac{1}{2} R^{j_2 j_3}_{i_2 i_3}(h) - t^2 K^{j_2}_{i_2} K^{j_3}_{i_3} + \delta^{j_2}_{i_2} \delta^{j_3}_{i_3} \right) \times \ldots \\ &\quad \times \left(\frac{1}{2} R^{j_{2n-2} j_{2n-1}}_{i_{2n-2} i_{2n-1}}(h) - t^2 K^{j_{2n-2}}_{i_{2n-2}} K^{j_{2n-1}}_{i_{2n-1}} + \delta^{j_{2n-2}}_{i_{2n-2}} \delta^{j_{2n-1}}_{i_{2n-1}} \right). \end{aligned} \tag{23}$$

where we have used the Gauss-Codazzi relation in the radial foliation

$$\hat{R}^{ij}_{kl} = R^{ij}_{kl}(h) - K^i_k K^j_l + K^i_l K^j_k. \tag{24}$$

For notational simplicity in what follows we shall omit all indices. For example, the expression (23) for ${\pi^i}_j$ will be written simply as,

$$\pi = n\sqrt{-h}\int_0^1 dt\, K\left(\frac{1}{2}R(h) - t^2 KK + 1\right)^{n-1}. \tag{25}$$

It is straightforward to reinsert the indices. Note also that since all tensors have the same number of covariant and contravariant indices, no signs will be lost when manipulating expressions as (25).

We are now ready to compute the on-shell variation appearing in (12), $\delta I_0 = \int {\pi^i}_j\, h^{jk}\delta h_{ki} = \int {\pi^i}_j\, g^{jk}\delta g_{ki}$, for Chern-Simons gravity. Note that the extrinsic curvature ${K^i}_j$ in the adapted frame (13) takes the simple form

$$ {K^j}_i = \delta^j_i - r{k^j}_i, \qquad \text{with} \qquad k^j_i = g^{jl}g'_{li} \tag{26}$$

and the Riemann tensor

$$R^{ij}_{kl}(h) = r\, R^{ij}_{kl}(g). \tag{27}$$

Inserting this form for K and $R(h)$ into (12), and using (25), we obtain

$$\delta I_0^{(n)} = \int_{r=\varepsilon} d^{2n}x\, \frac{n}{r^n}\int_0^1 dt\, \sqrt{-g}\,(1-rk)\left(\frac{r}{2}R(g) - t^2(1-rk)^2 + 1\right)^{n-1} g^{-1}\delta g, \tag{28}$$

This formula has a remarkable structure. As an example we display here the first few values of $n = 2, 3, 4$ corresponding to dimensions $D = 5, 7, 9$ (keeping only the finite and divergent terms in the limit $\varepsilon \to 0$),

$$\begin{aligned}
\delta I_0^{(2)} &= \int_{r=\varepsilon} d^4x\,\sqrt{-g}\,\left(k^2 + \frac{1}{2}Rk - \frac{1}{2r}R - \frac{2}{3r^2}\right) g^{-1}\delta g \\
\delta I_0^{(3)} &= \int_{r=\varepsilon} d^6x\,\sqrt{-g}\,\left(\frac{1}{4}kR^2 + k^2R + \frac{4}{3}k^3 - \frac{1}{4r}R^2 - \frac{2}{3r^2}R - \frac{8}{15r^3}\right) g^{-1}\delta g \\
\delta I_0^{(4)} &= \int_{r=\varepsilon} d^8x\,\sqrt{-g}\,\left(\frac{3}{4}k^2R^2 + \frac{1}{8}kR^3 + 2k^3R + 2k^4 - \frac{1}{8r}R^3 - \frac{1}{2r^2}R^2 - \frac{4}{5r^3}R - \frac{16}{35r^4}\right) g^{-1}\delta g \\
\delta I_0^{(5)} &= \ldots
\end{aligned} \tag{29}$$

We note that the divergent terms (as $\varepsilon \to 0$) only depend on g_{ij} and its associated curvature $R(g)$, but not on k. This means that these terms can be written as total variations. In fact, reinserting the indices, these terms contain the product of p curvatures which can easily be written as a total variation,

$$\begin{aligned}
\sqrt{-g}\,R^p\,g^{-1}\delta g &= \sqrt{-g}\delta^{[ii_1\ldots i_{2p-1}]}_{[jj_1\ldots j_{2p-1}]}R^{j_1j_2}_{i_1i_2}\ldots R^{j_{2p-2}j_{2p-1}}_{i_{2p-2}i_{2p-1}}g^{jl}\delta g_{li} \\
&= 2\delta\left(\sqrt{-g}\delta^{[i_1\ldots i_{2p}]}_{[j_1\ldots j_{2p}]}R^{j_1j_2}_{i_1i_2}\ldots R^{j_{2p-1}j_{2p}}_{i_{2p-1}i_{2p}}\right) \qquad (30)
\end{aligned}$$

with $p = 0\ldots n-1$. Note that the second line is valid up to a boundary term. The counterterm B is thus a local functional of g_{ij} of the Lovelock type in $2n$ dimensions. We give the explicit form below.

Let us now prove that the structure displayed in (29) is a general property of Chern-Simons gravities present for all dimensions. We go back to Eq. (28). Our aim is to prove that the divergent terms in this expression do not contain k. To this end, we shall take the derivative of (28) with respect to k, and prove that it gives a finite quantity. We compute,

$$\begin{aligned}
\frac{\partial(\delta I_0)}{\partial k} &= -n\int_{r=\varepsilon}d^{2n}x\,\frac{\sqrt{-g}}{r^{n-1}}\int_0^1 dt\left[\left(\frac{r}{2}R - t^2(1-rk)^2+1\right)^{n-1} - \right. \\
&\quad \left. -\,2t^2(1-rk)^2\left(\frac{r}{2}R - t^2(1-rk)^2+1\right)^{n-2}\right]g^{-1}dg \qquad (31)
\end{aligned}$$

We see that the integrand is a total derivative respect to t

$$\frac{d}{dt}\left[t\left(\frac{r}{2}R - t^2(1-rk)^2+1\right)^{n-1}\right]. \qquad (32)$$

This means that the integral over t can be perform explicitly and we get

$$\frac{\partial(\delta I_0)}{\partial k} = -n\int_{r=\varepsilon}d^{2n}x\,\sqrt{-g}\left(\frac{1}{2}R + 2k + rk^2\right)^{n-1}g^{-1}dg \qquad (33)$$

which is explicitly finite in the limit $\varepsilon \to 0$.

The piece in δI_0 that depends on k is thus finite, and can be evaluated at $\varepsilon = 0$ directly. Integrating (33) we find a simple formula for the finite piece

$$\delta I_{fin} = -n\int d^{2n}x\sqrt{-g_{(0)}}\int_0^1 dt\,k\left(\frac{1}{2}R_{(0)} + 2tk\right)^{n-1}g^{-1}_{(0)}\delta g_{(0)} \qquad (34)$$

where $g_{(0)ij} = g_{ij}$ evaluated at $r=0$. Putting back all indices and varying with respect to $g_{(0)ij}$, we finally reach at the general formula for the holographic energy momentum tensor,

$$\begin{aligned}
T^i_j = \frac{g_{(0)jl}}{2\sqrt{-g_{(0)}}}\frac{\delta I_{fin}}{\delta g_{(0)li}} &= n\int_0^1 dt\delta^{[ii_1\ldots i_{2n-1}]}_{[jj_1\ldots j_{2n-1}]}k^{j_1}_{i_1}\left(\frac{1}{2}R^{j_2j_3}_{i_2i_3}(g) + 2tk^{j_2}_{i_2}\delta^{j_3}_{i_3}\right)\times\ldots \\
&\quad \times\left(\frac{1}{2}R^{j_{2n-2}j_{2n-1}}_{i_{2n-2}i_{2n-1}}(g) + 2tk^{j_{2n-2}}_{i_{2n-2}}\delta^{j_{2n-1}}_{i_{2n-1}}\right). \qquad (35)
\end{aligned}$$

It is direct to see that this formula reproduces the finite pieces in the above variations. This formula is in full agreement with the result of [16]. As shown in that reference, using the equations of motion, the trace of T^{ij} can be written purely in terms of $g_{(0)ij}$, and it is equal to the $2n$-dimensional Euler density

$$T^i_{\ i} = \frac{1}{2^n}\delta^{[i_1 \dots i_{2n}]}_{[j_1 \dots j_{2n}]} R^{j_1 j_2}_{i_1 i_2}(g_{(0)}) \dots R^{j_{2n-1} j_{2n}}_{i_{2n-1} i_{2n}}(g_{(0)}). \tag{36}$$

We end by displaying the explicit formula for B. Reinserting all indices the formula reproducing the divergent pieces in δI_0 is,

$$\begin{aligned} \delta B &= \frac{n}{2^{n-1}} \frac{\sqrt{-g}}{\varepsilon^n} \int_0^1 dt \delta^{[i i_1 \dots i_{2n-1}]}_{[j j_1 \dots j_{2n-1}]} \delta^{j_1}_{i_1} \left(\varepsilon R^{j_2 j_3}_{i_2 i_3} + 2\left(1 - t^2\right) \delta^{j_2}_{i_2} \delta^{j_3}_{i_3} \right) \times \dots \\ & \quad \times \left(\varepsilon R^{j_{2n-2} j_{2n-1}}_{i_{2n-2} i_{2n-1}} + 2\left(1 - t^2\right) \delta^{j_{2n-2}}_{i_{2n-2}} \delta^{j_{2n-1}}_{i_{2n-1}} \right) g^{jl} \delta g_{li}. \end{aligned} \tag{37}$$

Now, (37) is exactly the variation of an action of the Lovelock type. In fact, this can be integrated to yield,

$$B = 2n(n-1)! \sqrt{-g} \sum_{p=0}^{n-1} \frac{2^{n-2p-1}(2(n-p)-1)!!}{\varepsilon^{n-p}} \delta^{[i_1 \dots i_{2p}]}_{[j_1 \dots j_{2p}]} R^{j_1 j_2}_{i_1 i_2} \dots R^{j_{2p-1} j_{2p}}_{i_{2p-1} i_{2p}} \tag{38}$$

which has exactly the form (17). We finally note that this counterterm action can be expressed in terms of the metric $h_{ij} = g_{ij}/\varepsilon$, and all dependence on the cutoff parameter ε disappears. This is presumably related to the character of the anomaly which is of type A, with no contributions from the Weyl tensor.

To summarize, we have shown that Chern-Simons gravity in $D = 2n+1$-dimensional spacetimes has special features which allow the computation of the holographic energy momentum tensor explicitly for all n. We have also isolated the general form of the counterterm that renders the action finite, and show that it has the form of a Lovelock action in $2n$ dimensions.

Acknowledgments

MB was partially supported by Fondecyt grants # 1020832 and # 7020832, RO by Fondecyt grant #3030029 and ST by GIF - the German-Israeli Foundation for Scientific Research.

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