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LIST PRICES AND COLLUSION IN INDUSTRIAL MARKETS

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List Prices and Collusion in Industrial Markets*

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Abstract

In several industrial markets secret discounts are the rule. Standard models suggest that if suppliers in these markets want to collude, they must develop an allocation structure to avoid discounts, but many antitrust cases are centered on their list price announcements. We build a testable theory that (i) rationalizes the publication of list prices, (ii) shows collusion through coordinated publication is possible, and (iii) informs the policy debate by pointing to indicators antitrust authorities should be concerned with.

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1 Introduction

Several industrial markets exhibit the following pricing model: (1) upstream suppliers with market power set a list price at which anyone can buy; (2) large downstream firms negotiate a secret, idiosyncratic discount off the list price; and (3) small buyers pay the list price. Call this the list-price-and-discount pricing model. Industries using this model have included threads in the EU and fiberglass and urethane in the US. As Stigler (1964) pointed out, secret discounts undermine the stability of cartels, so collusion in such industries is usually considered problematic.

Cartels may solve the secret discount problem by designing allocation and monitoring strategies that ensure firms' sales behave as agreed (Marshall and Marx, 2012). Examples are customer, geographic and market share allocation. For example, the Sugar Institute described by Genesove and Mullin (2001) provided an allocation structure that made collusion possible in a market with the list-price-and-discount model. In recent decades, however, industries using the model have been accused of colluding through list price coordination, without any allocation structure (Boshoff and Paha, 2017; Harrington Jr and Ye, 2019).[1]

A problem courts face in deciding these cases is that defendants can plausibly claim that, due to pervasive secret discounts, any ostensible list price coordination can have no anticompetitive effect. Indeed, defendants repeatedly argue this point.[2] Moreover, since discounts are negotiated, it is unclear whether list prices have any bearing on the market equilibrium. What we need is a theory of why list prices exist and how they affect markets. Absent such a theory, courts may issue contradictory opinions in similar cases.

US courts have already issued such opinions. While the Seventh Circuit Court pointed out that list prices would be an "awkward facilitator of price collusion" due to discounts, the Tenth Circuit Court argued that increases changed the baseline for negotiations, thus affecting all prices.[3] It is not obvious why this would be the case: buyers are professionals who would disregard higher list prices as cheap talk. Since the interests of sellers and buyers are opposed on this point, such cheap talk is irrelevant.[4] For example, in neither the Nash nor Rubinstein

[1]Threads (EU): Commission of the European Communities, 14.09.2005, Case COMP/38337/E1/PO/ Thread. Fiberglass (US): *Reserve Supply v. Owens-Corning Fiberglas* 971 F. 2d 37 (7th Cir. 1992) *[sic]*. Urethane (US): *In Re: Urethane Antitrust Litigation*, No. 13-3215, 10th Cir. Harrington Jr and Ye (2019) discuss these and other cases.

[2]Commission of the European Communities, 14.09.2005, Case COMP/38337/E1/PO/ Thread, 112, 159-60; *Reserve Supply v. Owens-Corning Fiberglas* 971 F. 2d 37 (7th Cir. 1992), para 61; Class Plaintiffs' Response Brief (February 14, 2014), *In Re: Urethane Antitrust Litigation,* No. 13-3215, 10th Cir.; pp. 8-9.

[3]*Reserve Supply v. Owens-Corning Fiberglas* 971 F. 2d 37 (7th Cir. 1992), para 62 and *In Re: Urethane Antitrust Litigation,* No. 13-3215 (10th Cir. Sep. 29, 2014); p. 7.

[4]In terms of a Crawford and Sobel (1982) setting, the opposing interests of buyer and seller make it unlikely

bargaining models does cheap talk affect results (Nash, 1950; Rubinstein, 1982), and there is evidence that buyers resist price increases.[5]

This paper contributes to the literature on collusion through list prices in different ways. First, we build a model of industrial markets with heterogeneous buyers and show that collusion in list prices is possible. Second, we show that in the presence of antitrust action, a fall in list prices is not indicative of collusion. Third, we specify which market indicators serve to discriminate between competition and collusion. Finally, since we characterize equilibria when list prices are published and when they are not, we are able to derive non-obvious, testable predictions about the dispersion of wholesale prices (list prices and discounted transaction prices) in the different equilibria.

The paper is organized as follows. Section 2 reviews and comments the literature on list prices. Section 3 presents a model of industrial markets in which suppliers publish their list prices, and analyzes the feasibility of collusion. The model yields consistent and intuitive results. Section 4 inquires what happens in the model when suppliers cannot commit to a public list price and compares the results with those of Section 3. Section 5 draws conclusions for competition policy, Section 6 describes robustness checks on our assumptions and Section 7 concludes.

2 Comments on Previous Literature

A small theoretical literature on the possibility of collusion through list prices has emerged recently. Raskovich (2007) presents a model where bargained discounts can result in higher list and transaction prices. Buyers are identical except for the exogenous opportunity to negotiate a discount, and collusion depends on suppliers coordinating the proportion of buyers who can negotiate.

Harrington Jr and Ye (2019) provide an incomplete information model of industrial markets where list prices may signal costs. Buyers are identical except that some are exogenously constrained to be one-stop shoppers. There is a separating competitive equilibrium and a pooling collusive one, and collusion works because high list prices make retailers bargain less aggressively. However, collusion depends on buyers having no memory.

that cheap talk would be believed. It is possible the Tenth Circuit Court has some version of the anchoring bias in mind (Tversky and Kahneman, 1974). Since buyers are experienced professionals, it is not obvious that they are subject to this bias.

[5]See evidence in Marshall and Marx (2012), chapters 2 and 6. See also the cited paper Marshall et al. (2008) and EC decision on the *Vitamins* case: Case COMP/E-1/37.512 - Vitamins, Comm'n Decision (Nov 21, 2001), para 325.

Raskovich (2007) and Harrington Jr and Ye (2019) have a similar problem, because we expect large retailers to have a say on whether they can negotiate. Gill and Thanassoulis (2016) also present a model where discounts raise equilibrium prices. However, they model consumer markets with take-it-or-leave-it discounts, and also assume identical buyers with an exogenous opportunity to use a discount.

Harrington Jr (2020) presents a model in which price sharing itself can be anticompetitive. The results extend to industrial markets where firm executives share list instead of transaction prices, which are an exogenous function of list prices. However, its focus is different from ours. His is a model which extends to list prices; it does not seek to explain why list prices exist nor how they can be used to collude when transaction prices include secret, idiosyncratic discounts.

Horn and Wolinsky (1988) present a model of industrial markets with suppliers and retailers in bilateral monopoly relationships. The model has two stages. In the first stage a supplier and a retailer bargain à la Nash for an input price.[6] In the second stage all input prices are revealed and retailers compete in quantities. Since failure to agree on an input price results in foreclosure, Horn and Wolinsky show that a monopoly supplier has a better outside option when retailers' products are substitutes. In that case, if negotiations fail the supplier will sell more through the other buyer. The opposite is true when retailers' products are complements.

Horn and Wolinsky face the problem that the monopoly supplier is engaged in two interdependent bargaining processes. They solve it assuming that a monopoly supplier bargaining with retailer R_i believes she has already reached the equilibrium agreement with R_{-i}, for all i. This is a Nash equilibrium in Nash equilibria, hence the name "Nash-in-Nash". Collard-Wexler et al. (2019) provide microfoundations for this equilibrium concept.

Horn and Wolinsky's model cannot account for the list-price-and-discount model. Although nothing stops suppliers from publishing arbitrarily high list prices (if it is costless), they have no effect on their profits, because neither their outside option nor their bargaining power changes. For that reason, collusion that changes profits cannot be achieved through coordination of list prices. This is what defendants in the cited cases argue. In the next section we show that adding a fringe of small buyers changes this result. Therefore, discounting small buyers as irrelevant is a mistake.

[6] Binmore et al. (1986) show that Nash bargaining can be microfounded by Rubinstein (1982)'s alternating offers model.

3 Model with Public List Prices

The model generalizes Horn and Wolinsky (1988) to include industrial markets that have small buyers. We generalize the model by adding an initial stage in which suppliers commit to list prices at which small buyers buy. Since our concern is the competitive effects of list prices, we consider only the case in which retailers' products are perfect substitutes in the downstream market.

Key to our model is that transaction costs, in the spirit of Coase (1937) and Williamson (1985), provide a rationale for the existence of list prices.[7] If there are small buyers with no bargaining power, and communicating prices to each one is costly, it is efficient for suppliers with market power to publish a uniform list price. This is true even for arbitrarily small transaction costs. Small, competitive retailers buy at the list price, and large retailers negotiate discounts.

The small buyers composing the fringe which can have several interpretations. They can be interpreted as small firms that buy from large suppliers. They can also be interpreted as a subset of consumers that buy directly from suppliers, so the model can be extended to the growing set of wholesale markets with direct-to-consumer sales.[8]

3.1 Setting

Upstream suppliers. There are two upstream suppliers U_j, $j \in \{1, 2\}$. Each has a local monopoly over a competitive fringe that buys q_j^F, a function of list price w_j^L.

Retailers. There are two large retailers R_i, $i \in \{A, B\}$. Retailer R_A (R_B) is in a bilateral monopoly relation with U_1 (U_2) ($i = A \iff j = 1$, so we will abuse that notation). In the retail market, R_A and R_B produce perfect substitutes and their only cost is the input price.

Market demand. The inverse demand function is $p(Q) = a - Q$, with $Q = q_1^F + q_2^F + q_A + q_B$ and $a > 0$.

Fringe input demand/downstream supply. Fringes' input demand function is

$$q_j^F(w_j^L) = \gamma(a - w_j^L) \quad j = 1, 2. \tag{3.1}$$

[7]Williamson noted that practices usually considered anticompetitive can be efficient once transaction costs are considered (Williamson, 1985, ch. 1).

[8]See Rhodes et al. (2020). Industries cited in footnotes 3 and 4 of the working paper are wine, (https://bit.ly/30qNQVq), traditional packaged goods, consumer electronics and travel services (https://bit.ly/2VKiB8e). Also from the paper: Nike's sales https://on.wsj.com/2ITY5wNandhttps://bit.ly/2DgV706; and the European Commission (2017)'s report and the report by consulting firm Oliver-Wyman (2018) at https://www.oliverwyman.com/our-expertise/hubs/retails-revolution.html.

Parameter γ indexes the size of the fringe. To microfound it, consider a fringe composed of small, competitive firms, like mom-and-pop shops that sell few products, while retailers are large stores with some market power, like supermarkets. Mom-and-pop shops are in no position to negotiate discounts with large suppliers. Differences in factory location, and transport or transaction costs can generate a local monopoly of suppliers over small shops.

Small and large stores and supermarkets serve different customers. Small shops sell to consumers that buy small quantities of a few products at a time, perhaps with high frequency, while supermarkets sell to one-stop shoppers, or to consumers that prefer to buy in bulk. It is not necessary that small stores and large retailers charge the same price, as they serve different segments of the market.

Alternatively, the fringe could be composed of consumers willing to incur the transaction costs necessary for buying directly from the suppliers, while retailers serve consumers with different preferences. Again, the sale price need not be the same, or even lower, for consumers in the fringe. Both interpretations of the fringe imply consumer heterogeneity.

In either case, γ indexes the size of the fringe. If $\gamma = 0$, our model is the same as Horn and Wolinsky (1988)'s. When $\gamma \geq 1$ non-positive prices appear, so we will only consider $\gamma \in (0, 1)$.(Non-positive prices are not possible in this setting when retailers and suppliers bargain à la Nash and suppliers have positive bargaining power.) Define

$$\tilde{a} = a(1 - 2\gamma) + \gamma(w_1^L + w_2^L). \tag{3.2}$$

Therefore, the inverse demand function can be expressed as

$$p = \tilde{a} - q_A - q_B. \tag{3.3}$$

Beliefs and equilibrium concept. Agents have passive beliefs (they have a fixed conjecture about offers being received by the other players, which is correct in equilibrium) and the equilibrium concept is subgame perfect equilibrium.

Timing. The stage game has three stages

- (τ_0) Supplier U_j publicly commits to a list price w_j^L at which fringe j buys.
- (τ_1) Retailer i bargains à la Nash for a transaction price $w_i \leq w_j^L$ (so the discount is $w_j^L - w_i$). The retailer's outside option is zero profits, and the supplier's is selling to the fringe at w_j^L. The suppliers' bargaining power is $\beta \in (0, 1)$.

- (τ_2) Transaction prices are revealed and retailers compete in quantities over the residual demand left by the fringes.

3.2 Competitive Equilibrium

Using backward induction, in τ_2 R_i solves

$$\max_{q_i \geq 0} \left[\tilde{a} - q_i - q_{-i} - w_i\right] q_i$$

so

$$q_i = \frac{\tilde{a} + w_{-i} - 2w_i}{3} \tag{3.4}$$

$$p = \frac{\tilde{a} + w_{-i} + w_i}{3} \tag{3.5}$$

$$\pi_i(w_i, w_{-i}) = \frac{\left[\tilde{a} + w_{-i} - 2w_i\right]^2}{9}. \tag{3.6}$$

Therefore, considering w_{-i} as fixed, for the Nash bargaining in τ_1 we have $\hat{\pi}_i(w_i) = \pi_i(w_i)$. Retailer R_i's outside option is $\bar{\pi}_i = 0$ because of the bilateral monopoly assumption.[9]

R_i buys an amount l_j of input from U_j. Since it maximizes profits, $l_j = q_i$. On the equilibrium path supplier U_j sells to R_i and the fringe, so $\hat{\pi}_j(w_i) = w_i \cdot q_i(w_i) + w_j^L \cdot q_j^F(w_j^L)$. Its outside option is selling to the fringe at the (already fixed) public list price, so $\bar{\pi}_j = w_j^L \cdot q_j^F(w_j^L)$. The bargaining problem is described by the set of payoffs $B_i^j = \{[\hat{\pi}_i, \hat{\pi}_j - \bar{\pi}_j] \,|w_i \geq 0\}$ and disagreement point $(0, 0)$. Because of passive beliefs, players act assuming w_{-i} is fixed.

Lemma 1. *The utility possibility set is convex.*

Proof. This and other proofs are relegated to Appendix A.

By Lemma 1 there is a unique solution to the Nash bargaining problem

$$\max_{w_i \in \mathbb{R}_+} \left[\frac{\left[\tilde{a} + w_{-i} - 2w_i\right]^2}{9}\right]^{1-\beta} \cdot \left[w_i^j \cdot \frac{\tilde{a} + w_{-i} - 2w_i}{3}\right]^{\beta}. \tag{3.7}$$

Since $\beta > 0$ we can use the logarithmic transformation. The first-order condition is

$$\frac{\beta}{w_i} = \frac{4 - 2\beta}{\tilde{a} + w_{-i} - 2w_i}$$

[9] We use Ide and Montero (2020)'s notation, where $\hat{\pi}$ denotes on-path profits and $\bar{\pi}$ denotes off-path profits.

$$\Rightarrow w_i = \frac{\beta(\tilde{a} + w_{-i})}{4}.$$

Remembering that $\tilde{a} = \tilde{a}(w_1^L, w_2^L)$, price and quantity in τ_1 are

$$w_i(w_1^L, w_2^L) = \frac{\beta\tilde{a}}{(4-\beta)} \tag{3.8}$$

and

$$l_j(w_1^L, w_2^L) = q_i(w_1^L, w_2^L) = \frac{(4-2\beta)\tilde{a}}{3(4-\beta)}. \tag{3.9}$$

So far we have generalized Horn and Wolinsky (1988), but note that all the endogenous variables are functions of the list prices. Given the list price of the competition, in τ_0 upstream supplier U_j sets list price w_j^L to solve

$$\max_{w_j^L \geq 0} \pi_j(w_j^L) = \left[\frac{\beta(4-2\beta)}{3(4-\beta)^2(1+2\gamma)}\right] \cdot \tilde{a}^2 + w_j^L \gamma(a - w_j^L). \tag{3.10}$$

In doing so U_j considers two forces: the bargaining and fringe effects. The bargaining effect measures how the list price affects U_j's bargaining position, and the fringe effect how much the list price affects U_j's profits from selling to its fringe. To simplify the following expressions define

$$\mu \equiv \mu(\beta) = \frac{4\beta - 2\beta^2}{3(4-\beta)^2}.$$

It is straightforward that

$$\frac{\partial\mu}{\partial\beta} > 0$$

for all $\beta \in (0, 1)$. We also have

$$\lim_{\beta \to 0} \mu = 0 \text{ and } \lim_{\beta \to 1} \mu = \frac{2}{27}.$$

Using (3.2), the first and second-order conditions of (3.10) are

$$\frac{\partial\pi_j}{\partial w_j^L} = \gamma \left\{ \underbrace{2\mu\left[a(1-2\gamma) + \gamma(w_1^L + w_2^L)\right]}_{\text{bargaining effect}} + \underbrace{(a - w_j^L) - w_1^L}_{\text{fringe effect}} \right\} = 0 \tag{3.11}$$

$$\frac{\partial^2 \pi_j}{\partial (w_j^L)^2} = \gamma \left\{ 2\mu\gamma - 2 \right\} < 0$$

where the negative sign comes from $\mu < 2/27$ for all $\beta \in (0, 1)$. The problem is concave and the best response function is

$$w_j^L = \frac{a\left[1 + 2\mu(1 - 2\gamma)\right]}{2 - 2\mu\gamma} + \frac{2\mu\gamma w_{-j}^L}{2 - 2\mu\gamma}. \tag{3.12}$$

Note the strategic complementarity between list prices: $\partial w_j^L / \partial w_{-j}^L \in (0, 1)$ for all β, $\gamma \in (0, 1)$. The solution to the resulting system is the symmetric equilibrium with list prices

$$w_j^L = \frac{a\left[1 + 2\mu(1 - 2\gamma)\right]}{2 - 4\mu\gamma}. \tag{3.13}$$

Plugging (3.13) into (3.11) we see that the bargaining effect is positive (it has to be, otherwise, the inverse demand intercept would be non-positive). So, the list price is always higher than the price that maximizes profits from the fringe, which U_j is trading for higher profits from bargaining with R_i. Using (3.8) and (3.9) competitive transaction prices and quantities are

$$w_i = \frac{2\beta a(1 - \gamma)}{(4 - \beta)(2 - 4\mu\gamma)} \tag{3.14}$$

and

$$q_i = \frac{(4 - 2\beta)2a(1 - \gamma)}{3(4 - \beta)(2 - 4\mu\gamma)}.$$

All prices and quantities are positive for $\gamma < 1$. Note that in all prices and quantities a is just a scale parameter, so profits are scaled by a^2, and we can normalize a to 1 for simplicity. We call this the (symmetric) *public competitive equilibrium.*

3.3 Collusive Equilibrium

We consider collusion in list prices, which is what defendants in the cited cases were accused of. All the variables in τ_1 and τ_2 are functions of the list price, so to find the collusive equilibrium we need only consider changes in τ_0. For easier notation, we will denote all equilibrium variables x in the collusive equilibrium with a check ($\check{x}$). Now suppliers maximize joint profits, so the problem is

$$\max_{(w_1^L, w_2^L)\in\mathbb{R}_+^2} \pi_1 + \pi_2 = 2\mu\tilde{a}^2 + w_1^L\gamma(a - w_1^L) + w_2^L\gamma(a - w_2^L).$$

The first-order condition with respect to any list price is

$$\frac{\partial(\pi_1 + \pi_2)}{\partial w_j^L} = \gamma\left\{\underbrace{2\mu\left[a(1-2\gamma) + \gamma(w_1^L + w_2^L)\right]}_{\text{bargaining effect+externality}} + \underbrace{(a - 2w\cdot{}_j^L}_{\text{fringe effect}}\right\} = 0.$$

Considering the externality, the bargaining effect is twice as large, so we expect a higher list price. The second partial derivatives are

$$\frac{\partial^2(\pi_1 + \pi_2)}{\partial(w_j^L)^2} = \gamma\left\{4\mu\gamma - 2\right\} < 0 \tag{3.15}$$

and

$$\frac{\partial^2(\pi_1 + \pi_2)}{\partial w_j^L \partial w_{-j}^L} = 4\mu\gamma^2 > 0. \tag{3.16}$$

The eigenvalues of the Hessian matrix are

$$k = \frac{\partial^2(\pi_1 + \pi_2)}{\partial(w_j^L)^2} \pm \frac{\partial^2(\pi_1 + \pi_2)}{\partial w_j^L \partial w_{-j}^L}.$$

Given the signs of the second derivatives, both eigenvalues will be negative if the sum is negative. The sum is

$$k = \gamma\left\{4\mu\gamma - 2\right\} - 4\mu\gamma^2 = -2\gamma < 0$$

so both eigenvalues are negative and the Hessian matrix is definite negative. Therefore the objective function is quasi-concave and the first-order conditions characterize the best collusive list prices. Equilibrium list and transaction prices

$$\check{w}_j^L = \frac{a\left[1 + 4\mu(1-2\gamma)\right]}{2 - 8\mu\gamma} \tag{3.17}$$

$$\check{w}_i = \frac{2\beta a(1-\gamma)}{(4-\beta)(2-8\mu\gamma)} \tag{3.18}$$

and retail supply

$$\check{q}_i = \frac{(4-2\beta)2a(1-\gamma)}{3(4-\beta)(2-8\mu\gamma)}.$$

As in the competitive equilibrium, all prices and quantities are scaled by a, so profits are scaled by a^2. Call this the (symmetric) *public collusive equilibrium*.

3.4 Sustainability of Collusion

Assume both suppliers have a common discount factor $\delta \in (0,1)$. We look for a strategy that can sustain collusion for any δ above some critical discount factor $\underline{\delta} \in (0,1)$. Consider the grim trigger strategy: continue colluding if everyone has always set the collusive list price in the past, otherwise set the competitive list price forever. Let π^C and π^M be a supplier's profits in competition and collusion, and $\pi^D(x)$ the profit of deviating to a list price x. The incentive compatibility constraint is

$$\frac{\pi^M}{1-\delta} \geq \pi(x) + \frac{\delta\pi^C}{1-\delta} \quad \forall x \geq 0 \tag{3.19}$$

so we only have to consider the best deviation list price. That price is given by the reaction function, expression (3.12). Let w^D be this list price. Plugging (3.17) into (3.12) we get

$$w^D = \frac{a\left[1+2\mu(1-2\gamma)\right]}{2-2\mu\gamma} + \frac{2\mu\gamma}{2-2\mu\gamma}\frac{a\left[1+4\mu(1-2\gamma)\right]}{2-8\mu\gamma}.$$

Since the cheater is playing its best response $\pi^D(w^D) \geq \pi^M > \pi^C$. Therefore, there exists $\underline{\delta} \in [0,1)$ such that, for all $\delta \geq \underline{\delta}$ collusion is sustainable with a grim trigger strategy. The critical discount factor is given by

$$\underline{\delta} = \frac{\pi^D(w^D) - \pi^M}{\pi^D - \pi^C} \in [0,1) \quad \forall \beta,\, \gamma \in (0,1),\, \forall a > 0. \tag{3.20}$$

Since w^D is also scaled by a, $\pi^D(w^D)$ is also scaled by a^2. Since both numerator and denominator are scaled by a^2, $\underline{\delta}$ is independent of a.

3.5 Welfare Effects of Collusion

Now we study the effects of collusion on prices and quantities.

Proposition 1. *For all $\beta, \gamma \in (0,1)$ collusion (i) raises list prices, (ii) raises transaction prices, (iii) reduces fringe supply, (iv) increases retailer supply, (v) reduces total supply, (vi) increases retail prices, (vii) increases suppliers' profits and (viii) increases retailers' profits.*

List prices increase due to strategic complementarity. This reduces fringe supply and, other things equal, will increase retailer supply. Therefore, there will be a bigger pie to split between retailers and suppliers, and transaction prices increase.

Retailer supply is affected in two ways. First, transaction prices increase, so supply falls. Second, fringe supply falls, and retailers increase their supply to substitute for it. Proposition 1 shows the second effect always dominates. Nevertheless, when we add all quantities, total supply always falls, so consumers are hurt by collusion. Retailers' profits increase because the effect of higher quantities and retail prices more than offsets the effect of higher input prices.

4 Model with Private List Prices

4.1 Setting

Assume suppliers do not make their list prices publicly available. For example, it may be forbidden by the antitrust authority. Instead, they privately communicate prices to potential buyers. We assume the cost of communication is zero in the intensive margin. For example, a supplier may hire personnel to communicate prices, or outsource the task, at a cost χ.

A priori, the private list prices may introduce two differences relative to public list prices. First, retailers cannot assume that the price suppliers offer to the fringes are the same as the public list prices, because a supplier may try to expropriate its retailer by offering a lower price to the fringe.

Second, supplier U_j cannot trust that supplier U_{-j} will set a price equal to the public list price. If that were true, U_{-j} could expropriate U_j by secretly lowering its price. These problems are similar to the commitment problem faced by the monopolistic supplier in Hart and Tirole (1990). Therefore a second possible rationale for the existence of public list prices can be that it solves the problem of committing to a bargaining position. To distinguish the private equilibrium variables x from those in the public competitive and public collusive equilibria, we mark them with a tilde ($\tilde{x}$). Although they are no longer public, to avoid confusion we will continue to call the prices paid by the fringe "list prices".

Beliefs and equilibrium concept. We continue using passive beliefs. The equilibrium concept is subgame perfect equilibrium.

Timing. The new stage game changes the order of the three stages

- (τ_0') Upstream supplier U_j bargains a la Nash with retailer R_i for a transaction price w_i, where the tilde denotes prices in the private model. Suppliers' bargaining power is $\beta \in (0, 1)$.
- (τ_1') U_j makes a take-it-or-leave-it offer of price w_j^L to the fringe.
- (τ_2') All input prices are made public and retailers compete in quantities in the downstream market.

We do not study collusion with private prices because the comparative statics we care about is a shock that (1) forces firms to stop publishing prices, and (2) ends the cartel, if there is one. Alternatively, the shock ends the cartel (if there is one) but allows firms to keep publishing list prices. This shock can be an action taken by an antitrust authority. In neither case is collusion with private list prices the focus.

4.2 Competitive Equilibrium

Given w_1^L, w_2^L, w_A and w_B, the equilibrium in τ_2' is the same as in the model with public prices, summarized in (3.4), (3.5) and (3.6). Using those functions, in τ_1', U_j knows w_i and solves

$$\max_{w_j^L \in \mathbb{R}_+} \pi_j(w_j^L) = \gamma w_j^L(a - w_j^L) + w_i \frac{\left(a(1-2\gamma) + \gamma w_j^L + \gamma w_{-j}^L + w_{-i} - 2w_i\right)}{3}.$$

The first and second-order conditions are

$$\frac{\partial \pi_j}{\partial w_j^L} = \gamma(a - 2w_j^L) + \frac{\gamma w_i}{3} = 0$$

and

$$\frac{\partial^2 \pi_j}{\partial (w_j^L)^2} = -2\gamma < 0.$$

So, the objective function is concave and the optimal price w_j^L as a function of w_i is

$$w_j^L = \frac{3a + w_i}{6}. \tag{4.1}$$

In τ_0' R_i correctly anticipates w_j^L. R_i's on-path and off-path profits are

$$\hat{\pi}_i = \left[\frac{\tilde{a}\left(w_1^L(w_A), w_2^L(w_B)\right) + w_{-i} - 2w_i}{3}\right]^2$$

$$\bar{\pi}_i = 0.$$

We assume that, if bargaining fails, U_j will set the list price according to (4.1) using the last discussed transaction price as the argument of the function. Thus, any incremental profit U_j receives from the negotiation comes from selling to the R_i.[10] So, U_j's on and off-path profits are

$$\hat{\pi}_j = \underbrace{\left[w_i \frac{\left(a(1-\gamma) + w_{-i}\left(\frac{6+\gamma}{6}\right) - w_i\left(\frac{12-\gamma}{6}\right)\right)}{3}\right]}_{w_i \cdot q_i(w_i)} + w_j^L \cdot q_j^F(w_j^L)$$

$$\bar{\pi}_j = w_j^L \cdot q_j^F(w_i).$$

The bargaining problem is described by the set of payoffs $\tilde{B}_i^j = \{[\hat{\pi}_i, \hat{\pi}_j - \bar{\pi}_j] \,|w_i \geq 0\}$ and disagreement point $(0, 0)$.

Lemma 2 *The utility possibility set is convex.*

By Lemma 2 there is a unique solution to the Nash bargaining problem. Plugging (4.1) into the profit functions we get

$$\max_{w_i \in \mathbb{R}_+} \left[\frac{\left(a(1-\gamma) + w_{-i}\left(\frac{6+\gamma}{6}\right) - w_i\left(\frac{12-\gamma}{6}\right)\right)^2}{9}\right]^{1-\beta} \cdot \left[w_i \frac{\left(a(1-\gamma) + w_{-i}\left(\frac{6+\gamma}{6}\right) - w_i\left(\frac{12-\gamma}{6}\right)\right)}{3}\right]^{\beta}.$$

With $\beta > 0$ the transaction price will always be larger than zero, so we can use the logarithmic transformation. The first-order condition is

$$\frac{\beta}{w_i} = \frac{(2-\beta)(12-\gamma)}{6a(1-\gamma) + w_{-i}(6+\gamma) - w_i(12-\gamma)}$$

[10] This is equivalent to saying that the off-path list price is set according to an on-path equilibrium *function*, (4.1), instead of an equilibrium *value*.

$$\iff w_i = \frac{\beta 6a(1-\gamma)}{24-4\gamma} + \frac{w_{-i}\beta(6+\gamma)}{24-2\gamma}.$$

Therefore, in the symmetric equilibrium, private transaction prices are

$$\tilde{w}_i = \frac{\beta 6a(1-\gamma)}{6(4-\beta)-\gamma(2+\beta)} \tag{4.2}$$

which is positive for β, $\gamma \in (0,1)$. Using (4.1) we get the equilibrium list price

$$\tilde{w}_j^L = a\left[\frac{6(4-\beta)-\gamma(2+\beta)+2\beta(1-\gamma)}{12(4-\beta)-2\gamma(2+\beta)}\right]. \tag{4.3}$$

Retailer supply is

$$\tilde{q}_i = \frac{3a(1-\gamma)\left(24-12\beta-2\gamma+\gamma\beta\right)}{9\left(24-6\beta-2\gamma-\beta\gamma\right)}. \tag{4.4}$$

Call this the (symmetric) *private equilibrium*.

4.3 Welfare Effects of Publishing List Prices

Lemma 3 sets up the proof for Proposition 2, which compares prices, quantities and profits in the public competitive equilibrium and the private equilibrium. (We consider supplier profits in the private equilibrium before discounting transaction costs χ.)

Lemma 3. *Letting π_j be U_j's profit in the competitive equilibrium with public prices and $\tilde{\pi}_j$ its profit in the equilibrium with private prices, for all $\gamma \in (0,1)$ we have*

$$\lim_{\beta\to 1}(\pi_j - \tilde{\pi}_j) > 0 \quad \text{and} \quad \lim_{\beta\to 0}(\pi_j - \tilde{\pi}_j) = 0.$$

Proposition 2. *Relative to the public competitive equilibrium, for all $\beta, \gamma \in (0,1)$ in the private prices model (i) list prices fall, (ii) transaction prices increase, (iii) retailer supply falls, (iv) fringe supply increases, (v) total supply increases, (vi) the retail price falls and (vii) retailers' profits fall. Moreover, for every $\gamma \in (0,1)$, there exists $\bar{\beta}(\gamma) \in (0,1)$ such that $\pi_j(\beta,\gamma) > \tilde{\pi}_j(\beta,\gamma)$ if $\beta > \bar{\beta}(\gamma)$ and $\pi_j(\beta,\gamma) < \tilde{\pi}_j(\beta,\gamma)$ if $\beta < \bar{\beta}(\gamma)$.*

The fall in list prices is what we expect when a supplier cannot commit to a price. The relative increase in transaction prices is not straightforward: given the discount, the fall in list prices results in lower transaction prices. But, holding β constant, discounts are a function of the bargaining positions. If a supplier can set its list price as an optimal reaction to the outcome of the negotiation with the retailer, its bargaining position improves, so discounts fall. Call this the optimal response effect. Proposition 2 shows the optimal response effect dominates.

Regarding $\bar{\beta}(\gamma)$, the interpretation is similar. Any value (for the supplier) of not publishing list prices comes from the optimal response effect. But the effect of bargaining over a bigger pie grows with suppliers' bargaining power, as it allows them to appropriate more of the pie. Therefore, any positive effect of not publishing list prices (before discounting χ) will be at its strongest for low values of β.

Proposition 3. *Relative to the public collusive equilibrium, in the private equilibrium*

I. For all $\beta, \gamma \in (0,1)$, (i) list prices are lower, (ii) fringe supply is higher, (iii) retailer supply is lower, (iv) total supply is higher, (v) the retail price is lower and (vi) retailers' profits are lower.

II. There exists $\beta^T \in (0,1)$ such that $\breve{w}_i > \tilde{w}_i$ if $\beta > \beta^T$, and $\breve{w}_i < \tilde{w}_i$ otherwise.

III. For every $\gamma \in (0,1)$, there exists $\underline{\beta}(\gamma) \in (0,1)$ such that $\breve{\pi}_j(\beta,\gamma) > \tilde{\pi}_j(\beta,\gamma)$ if $\beta > \underline{\beta}(\gamma)$ and $\breve{\pi}_j(\beta,\gamma) < \tilde{\pi}_j(\beta,\gamma)$ if $\beta < \underline{\beta}(\gamma)$.

IV. For all $\gamma \in (0,1)$, $\underline{\beta}(\gamma) < \bar{\beta}(\gamma)$.

The first set of results is straightforward considering Propositions 1 and 2. Regarding transaction prices, two forces operate. From Proposition 1, collusion in list prices helps suppliers bargain for higher transaction prices (collusion effect). From Proposition 2, however, we know that the ability to set list prices as an optimal response to the transaction price improves suppliers' outside option, resulting in higher transaction prices (optimal response effect). Proposition 3 shows that the optimal response effect dominates for low supplier bargaining power, where they cannot take full advantage of the bargaining benefits from collusion. When supplier bargaining power is high, the collusion effect dominates.

The intuition for the result on supplier profits is the same as that for transaction prices. Note, however, that public collusive transaction prices become higher than private ones before public collusive supplier profits become higher than private supplier profits ($\beta^T < \underline{\beta}(\gamma)$ for all γ). The fourth result follows directly from Proposition 1.

Define wholesale price dispersion as the difference between list prices and transaction prices.[11] Lemma 4 shows that dispersion always falls when list prices cease to be published.

[11] In the public equilibria, dispersion would be the discount. In the private equilibrium, however, there is no "list price" being discounted. Therefore, we use the more general term "dispersion".

Lemma 4. *For all β, $\gamma \in (0,1)$, $\tilde{w}_j^L - \tilde{w}_i < w_j^L - w_i$ and $\tilde{w}_j^L - \tilde{w}_i < \check{w}_j^L - \check{w}_i$.*

4.4 Discussion

Here we recapitulate the intuition behind the results, focusing on the public competitive and private equilibria. In the public competitive equilibrium, suppliers will raise list prices above the level that maximizes profits from monopolizing the fringe - see equation (3.11). As the fringe shrinks, retailers gain market share, so there is a bigger pie to be split during negotiations. We called this the *bargaining effect*. As we can see in (3.7) and (3.10), the bargaining effect operates exclusively through the size of the pie to be split, not the outside option.

When list prices become private, two things happen. First, the supplier cannot commit to a list price high above its monopoly level. Therefore, once the transaction price has been agreed upon, the supplier has an incentive to lower its list price. This commitment problem is similar to that in Hart and Tirole (1990), and will tend to lower list prices and suppliers' profits. It will also lower transaction prices, as lower list prices imply fewer sales for retailers - it is the bargaining effect in reverse.

However, when the retailer knows its supplier will set the list price as an optimal response to the outcome of the negotiation, the supplier's outside option is improving. This will push the transaction price upwards - the *optimal response effect*. The fall in the dispersion of list prices is the consequence of the lack of commitment and the optimal response effect. In Section 6 we review robustness exercises, and our results, including the fall in dispersion, are robust to different modeling assumptions.

5 Applications

5.1 Antitrust Policy and Welfare

Consider the following situations:

Situation 1. An antitrust authority correctly suspects that suppliers in and industrial markets are colluding by coordinating in published list prices, and brings them to trial. As a precautionary measure, before arriving at a decision the authority forbids the publication of list prices.

Situation 2. The same as Situation 1, except suppliers are competing.

Proposition 2 shows that when list prices cease to be published, they will fall regardless of whether suppliers were colluding or not.[12] This means the most intuitive indicator for discriminating between Situations 1 and 2 does not work. Ideally, we would be able to infer whether suppliers were colluding by comparing changes in other prices, or quantities or profits, induced by the prohibition of publication. For example, an equilibrium variable the value of which falls in Situation 1, but increases in Situation 2, (qualitatively) discriminates between both situations.

The best indicator we have is transaction prices. If suppliers are competing, transaction prices will increase when list price publication ceases. If they are colluding, they will only increase if, $\beta < \beta^T$. So, if suppliers' bargaining power is large enough, transaction prices can qualitatively discriminate between Situations 1 and 2.

Alternatively, given γ, if $\beta \in (\underline{\beta}(\gamma), \bar{\beta}(\gamma))$, then $\pi_j < \tilde{\pi}_j < \check{\pi}_j$, so supplier profits (before discounting χ) can be used to discriminate between Situations 1 and 2. However, $\beta^T < \underline{\beta}(\gamma)$ for all γ, so transaction costs are always a better indicator. For other parameter values, discrimination between both situations has to be done quantitatively.

As we have shown that collusion through list prices is possible, it is worth considering whether list price publication is always negative. To this end, note first that Proposition 2 shows that for a large range of β, publishing list prices is profitable even when suppliers compete. This is consistent with the problem of lack of commitment mentioned in Section 4.

Nevertheless, given γ, if $\beta \in (\underline{\beta}(\gamma), \bar{\beta}(\gamma))$, then list price publication only appears profitable if suppliers are colluding. However, this does not consider the transaction costs χ imposed on U_j when list prices are not published. Therefore, it is not obvious that list price publication is always a facilitating practice.

Regardless of whether it is a facilitating practice, policymakers may frown upon list price publication because it raises both list and retail prices even if suppliers are competing. Transaction costs χ must be considered in this welfare analysis, and traded-off against higher prices. If transaction costs are large enough, surplus may increase with list price publication, at least if suppliers are competing. A shock like the prohibition of list price publication could be used to estimate these transaction costs.

[12] We assume the cartel was not setting prices as a function of expected detection and punishment. Harrington Jr (2004) studies how cartels set prices when they trade off collusive profits against expected punishment, which is beyond the scope of this paper.

5.2 Empirical Testing

Our model has falsifiable empirical predictions. A novel and non-obvious prediction is the fall in wholesale price dispersion when list price publication ceases, regardless of whether suppliers were competing or not. A shock like that of Situations 1 or 2 could be used to test it.

6 Robustness

We are concerned by two assumptions we have made: the bilateral monopoly relations between retailers and suppliers and the functional form of the fringes' input demand/downstream supply function.

6.1 No Bilateral Monopoly

Regarding the bilateral monopoly assumption, it has been a concern since Horn and Wolinsky (1988). One reason is there is no straightforward way to relax it. If retailers with constant marginal costs can bargain with both suppliers, which also have constant marginal costs, it is not obvious how to define the outside option. Pricing at marginal cost could show up, but the Bertrand paradox implies there is no bargaining, even though we observe it in real markets.

To deal with these problems, in Appendix C we solve a model where retailers with increasing marginal costs buy from both suppliers in equilibrium. Increasing marginal costs, whether due to capacity constraints or decreasing returns to scale, avoid the Bertrand paradox. Proposition C.1 shows that all the results of Proposition 1 still stand.

Proposition C.2 compares the public competitive equilibrium with the private equilibrium. Again private list prices fall but, for high values of suppliers' bargaining power, public competitive transaction prices are higher than private ones. That is, the optimal response effect only dominates the higher list price effect when suppliers have little bargaining power. Retailer supply and the retail price are higher in the public competitive equilibrium, and fringe supply is lower. Before discounting transaction costs, supplier profits are higher in the private equilibrium when suppliers have little bargaining power. This suggests that the optimal response effect is instrumental in avoiding too large a fall in profits when there is no commitment.

Proposition C.3 compares the public collusive equilibrium with the private equilibrium. The only results that do not follow by transitivity from Propositions C.1 and C.2 are those for transaction prices and (pre-transaction costs) supplier profits. Public collusive transaction prices are higher than private ones beyond some bargaining power, so the optimal response effect still dominates the collusion effect when supplier bargaining power is low. Public collusive supplier profits are also higher when supplier bargaining power is above some threshold. (Both thresholds in Proposition C.3 are lower than those in Proposition C.2.)

The result that wholesale price dispersion falls when list prices cease to be published still holds (Lemma C.1). Overall, results are remarkably similar to those in the baseline model, although the parameter space in which we can (qualitatively) discriminate between Situations 1 and 2 is different. We conclude that the bilateral monopoly assumption is not critical for our results.

6.2 Responsive Fringe

The robustness check in Appendix D involves a new fringe input demand/downstream supply function. Now the fringe is composed of small firms pricing at marginal costs. We explicitly model how the fringe responds to changes in the retail price. Proposition D.1 shows that all the results of Proposition 1 are robust to this change.

Proposition D.2 compares the public competitive equilibrium with the public equilibrium. Private list prices are lower, but so are private transaction prices - the optimal response effect does not compensate for the fall in the list price in this model. As in the baseline model, retailer supply and the retail price are higher, and fringe supply is lower, in the public competitive equilibrium. Supplier profits are always higher in the public competitive equilibrium, even before discounting transaction costs.

The comparison between the public collusive equilibrium and the private equilibrium follows by transitivity from Propositions D.1 and D.2. That means there is no indicator in this model that can discriminate between Situations 1 and 2. Nevertheless, despite the private transaction price always being lower than the public competitive one, we still observe a fall in dispersion when list prices cease to be published, regardless of whether suppliers were competing (Lemma D.2). This result suggests the optimal response effect is still present. Instead of showing up in the level of prices, it shows up in the dispersion.

Overall, except for the lack of qualitative discrimination between Situations 1 and 2, the results from this model are notably similar to those in the baseline model.

7 Conclusion

Justifying antitrust action in the presence of secret discounts has been a problem over the years. To contribute to the policy debate, we develop a model of upstream market competition that rationalizes the existence of list prices and studies the possibility of collusion. Buyer heterogeneity, which has been argued to be irrelevant in some antitrust cases, explains the existence of list prices and makes collusion between suppliers possible.

We also characterize an equilibrium in which list prices are not published, and compare it to competition and collusion when list prices are published. This is important because antitrust authorities may want to forbid list price publication if they suspect it is a facilitating practice. In that case, understanding the welfare effects of publication, and the expected changes in prices and quantities, has direct policy relevance.

Our exercise shows several results. First, changes in list prices after their publication is prohibited are not a good indicator of collusion. Second, publication need not be a facilitating practice. Third, other prices and profits can discriminate between competition and collusion. Fourth, the theory has a non-obvious, testable prediction: wholesale price dispersion always falls when list prices cease to be published. These results are remarkably robust to changes in the baseline model's assumptions. To the best of our knowledge, no other theory provides these tools for antitrust authorities.

There are several avenues for future research. First, the results could be used to develop and estimate econometric models that can test our theory or find evidence of collusion. Our robustness checks point to different ways of modeling the fringe. Beyond controlling for changes in market data, like costs and demand, econometric modeling and estimation are important because there are not always clear qualitative indicators of collusion.

Second, we have taken fringe size as given. But recent growth in direct-to-consumer sales suggests it is endogenous and influenced by suppliers' decisions. Market expansion through higher fringe demand could be a rationale for list price publication, as suppliers try to reach consumers directly by lowering transaction costs.

Appendix A: Proofs

Proof of Lemma 1. A sufficient condition for the utility possibility set to be convex is that the payoff functions of both agents are quasi-concave. Given w_{-i}, R_i's profit as a function of w_i is

$$\hat{\pi}_i(w_i) = \frac{(\tilde{a} + w_{-i} - 2w_i)^2}{9}.$$

Remember $\tilde{a}$ is fixed in the second stage. The first derivative of the objective function is

$$\frac{\partial \hat{\pi}}{\partial w_i} = \frac{-4(\tilde{a} + w_{-i} - 2w_i)}{9}$$

which is non-positive for all w_i in the interval $[0, (\tilde{a}+w_{-i})/2]$ and negative in its interior. If w_i were outside that interval the supplier would make zero profits, so no other prices are relevant. Since the function is monotonous, it is quasi-concave. Supplier Uj's (marginal) profit as a function of w_i is

$$(\hat{\pi}_j - \bar{\pi}_j)(w_i) = \frac{w_i(\tilde{a} + w_{-i} - 2w_i)}{3}$$

and its first and second derivatives are

$$\frac{\partial(\hat{\pi}_j - \bar{\pi}_j)}{\partial w_i} = \frac{\tilde{a} + w_{-i} - 4w_i}{3}$$

$$\frac{\partial^2(\hat{\pi}_j - \bar{\pi}_j)}{\partial w_i^2} = \frac{-4}{3} < 0$$

so the function is concave, and thus quasi-concave. So, B_i^i is the boundary of a convex set. ■

Proof of Proposition 1. For list prices, using expressions (3.13) and (3.17) we have

$$w_i \leq \hat{w}_j^L \iff \frac{a\,[1 + 2\mu(1-2\gamma)]}{2 - 4\mu\gamma} \leq \frac{a\,[1 + 4\mu(1-2\gamma)]}{2 - 8\mu\gamma}$$

$$\begin{aligned} \iff [2 - 4\mu\gamma]\,[1 + 2\mu(1-2\gamma)] + [2 - 4\mu\gamma]\,2\mu(1-2\gamma) \\ \geq [2 - 4\mu\gamma]\,[1 + 2\mu(1-2\gamma)] - 4\mu\gamma\,[1 + 2\mu(1-2\gamma)] \end{aligned}$$

$$\iff \underbrace{[2-4\mu\gamma]\,2\mu(1-2\gamma)}_{>0\ \forall\gamma<1} \geq \underbrace{-4\mu\gamma\,[1+2\mu(1-2\gamma)]}_{<0\ \forall\gamma<1}.$$

So, for all $\gamma < 1$ collusion strictly increases list prices. Using (3.2) and (3.8) we see that transaction prices strictly increase in list prices, and fringe supply is strictly decreasing in list prices. In (3.9) we see that retailer quantities are increasing in list price, so collusion increases retailers' supply.

Using symmetry, we know that total supply falls with collusion if

$$2(q_j^F(w_j^L)+q_i(w_j^L)) \geq 2(\hat{q}_j^F(\hat{w}_j^L)+\hat{q}_i(\hat{w}_j^L))$$

$$\iff \gamma(1-w_j^L)+\frac{\beta}{4-\beta}[a(1-2\gamma)+2\gamma w_j^L] \geq \gamma(1-\hat{w}_j^L)+\frac{\beta}{4-\beta}[a(1-2\gamma)+2\gamma\hat{w}_j^L]$$

$$\iff \gamma(\hat{w}_j^L-w_j^L) \geq +\frac{\beta 2\gamma}{4-\beta}[w_j^L-\hat{w}_j^L].$$

But the left-hand side is positive and the right-hand side is negative for all $\beta, \gamma \in (0,1)$, so total quantity falls. The retail price is decreasing in total quantity, so it increases. Suppliers' profits increase because they are maximizing their joint profits.

For retailers' profits, let $\pi_j(w_j^L)$ be R_i's profit in competition and $\hat{\pi}_j(\hat{w}_j^L)$ in collusion. Be the symmetry of equilibrium list and transaction prices we have

$$\hat{\pi}_j(\hat{w}_j^L) \geq \pi_j(w_j^L) \iff \left[\frac{\tilde{a}(\hat{w}_j^L)-\hat{w}_i}{3}\right]^2 \geq \left[\frac{\tilde{a}(w_j^L)-w_i}{3}\right]^2$$

$$\iff (4-2\beta)\tilde{a}(\hat{w}_j^L) \geq (4-2\beta)\tilde{a}(w_j^L)$$

$$\iff \hat{w}_j^L \geq w_j^L$$

which we know is true with strict inequality for β, $\gamma \in (0,1)$. ■

Proof of Lemma 2. We need both profit functions to be quasi-concave. We have

$$\hat{\pi}_i(w_i) = \left[\frac{6a(1-\gamma)+w_{-i}(6+\gamma)-w_i(12-\gamma)}{18}\right]^2$$

$$\Rightarrow \frac{\partial\hat{\pi}_i}{\partial w_i} = \frac{-2(12-\gamma)}{324}\,[6a(1-\gamma)+w_{-i}(6+\gamma)-w_i(12-\gamma)] < 0$$

where the sign comes from the fact that w_i will never be so large that the profits are negative. So, R_i's profits are strictly decreasing and quasi-concave for all $\gamma \in (1,1)$. For U_j's profits, we have

$$(\hat{\pi}_j - \bar{\pi}_j)(w_i) = w_i \frac{\left(a(1-\gamma) + w_{-i}\left(\frac{6+\gamma}{6}\right) - w_i\left(\frac{12-\gamma}{6}\right)\right)}{3}$$
$$\Rightarrow \frac{\partial^2(\hat{\pi}_j - \bar{\pi}_j)}{\partial w_i^2} = \frac{-2(12-\gamma)}{18} < 0$$

so the function is also quasi-concave for all $\gamma \in (0,1)$. ■

Proof of Lemma 3. Taking advantage of the fact that all equilibrium price, quantity and profit functions are continuous in $\beta \in [0,1]$, we will evaluate them at $\beta = 1$ and $\beta = 0$ for the proofs. In the competitive equilibrium with public prices we have

$$\lim_{\beta \to 1} w_j^L = \frac{a\left[1 + \frac{4}{27}(1-2\gamma)\right]}{2 - \frac{8\gamma}{27}}$$
$$\iff \lim_{\beta \to 1} w_j^L = \frac{a(31-8\gamma)}{54-8\gamma}$$
$$\Rightarrow \lim_{\beta \to 1} q_j^F = \gamma\left(a - \lim_{\beta \to 1} w_j^L\right) = \frac{a\gamma 23}{54-8\gamma}$$
$$\Rightarrow \lim_{\beta \to 1} w_j^L q_j^F = \frac{a^2\gamma 23(31-8\gamma)}{4 \cdot (27-4\gamma)^2}$$

and

$$\lim_{\beta \to 1} w_i = \frac{9a(1-\gamma)}{(27-4\gamma)}$$
$$\lim_{\beta \to 1} q_i = \frac{6a(1-\gamma)}{(27-4\gamma)}.$$

Therefore

$$\lim_{\beta \to 1} \pi_i = \frac{54a^2(1-\gamma)^2}{(27-4\gamma)^2} + \frac{a^2\gamma 23(31-8\gamma)}{4 \cdot (27-4\gamma)^2}.$$

In the private prices model we have

$$\lim_{\beta \to 1} \tilde{w}_j^L = \frac{a(20-5\gamma)}{6(6-\gamma)}$$

$$\Rightarrow \lim_{\beta \to 1} \tilde{q}_j^L = \gamma \left(a - \frac{a(20-5\gamma)}{6(6-\gamma)} \right) = \frac{a\gamma(16-\gamma)}{6(6-\gamma)}$$

$$\Rightarrow \lim_{\beta \to 1} \tilde{w}_j^L \tilde{q}_j^L = \frac{a^2\gamma(16-\gamma)\cdot(20-5\gamma)}{36(6-\gamma)^2}$$

and

$$\lim_{\beta \to 1} \tilde{w}_i = \frac{2a(1-\gamma)}{(6-\gamma)}$$

$$\lim_{\beta \to 1} \tilde{q}_i = \frac{a(1-\gamma)(12-\gamma)}{9(6-\gamma)}$$

$$\lim_{\beta \to 1} \tilde{\pi}_i = \frac{a^2(1-\gamma)^2(12-\gamma)}{9(6-\gamma)^2} + \frac{a^2\gamma(16-\gamma)\cdot(20-5\gamma)}{4\cdot 9(6-\gamma)^2}.$$

Figure 1 shows that this difference is strictly positive for all $\gamma \in (0,1)$.[13] Regarding the limit as $\beta \to 0$, with no bargaining power the supplier earns zero from selling to the retailer. The best it can do is monopolize the fringe, earning $\gamma a^2/4$ whether prices are public (and competitive) or private. Therefore, in the limit with $\beta \to 0$, profits are the same for all $\gamma \in (0,1)$. ■

Proof of Proposition 2. Begin with list prices. Since all variables are simply scaled by a, it is sufficient to prove the results for $a = 1$. Using (3.13) and (4.3) we can solve

$$w_j^L(\beta,\gamma) - \tilde{w}_j^L(\beta,\gamma) = 0$$

for γ given β. This yields a quadratic equation on γ with two real solutions, which are functions of β

$$\gamma(\beta) = \begin{cases} 1 \\ \\ \frac{3(16(1-\beta)+3\beta^2)}{2(4(1-\beta)+\beta^2)} > 1 \ \forall \beta \in (0,1) \end{cases}$$

Both solutions involve $\gamma \geq 1$. We care about $\gamma \in (0,1)$, because at $\gamma = 1$ non-positive prices appear (even with $\beta > 0$), which has no economic interpretation. Therefore, for all β, $\gamma \in (0,1)$ either $w_j^L > \tilde{w}_l^L$ or $w_j^L < \tilde{w}_l^L$. Figure 2 shows the public competitive list price is always higher. So, $w_j^L > \tilde{w}_l^L$ for all β, $\gamma \in (0,1)$. Since fringe supply is a function of list prices only, this implies $q_j^F < \tilde{q}_j^F$ for all β, $\gamma \in (0,1)$. For transaction prices we use (3.14) and (4.2) to solve

[13] We used Maple 2021. All code is available upon request.

$$w_i(\beta, \gamma) - \tilde{w}_i(\beta, \gamma) = 0$$

for γ given β. The real solutions to this quadratic equation are 0 and 1, so we have $w_i > \tilde{w}_i$ or $w_i < \tilde{w}_i$ for all β, $\gamma \in (0, 1)$. Figure 3 shows that $w_i < \tilde{w}_i$ is the case. For retail supply, use (3.2) and (4.4) to solve

$$q_i(\beta, \gamma) - \tilde{q}_i(\beta, \gamma) = 0$$

for γ given β. The solutions are

$$\gamma(\beta) = \begin{cases} 0 \\ \\ 1 \\ \\ \frac{3(20-13\beta)}{4(2-\beta)} > 1 \ \forall \beta \in (0, 1) \end{cases}$$

Therefore, for all β, $\gamma \in (0, 1)$ either $q_i > \tilde{q}_i$ or $q_i < \tilde{q}_i$. Figure 4 shows that $q_i > \tilde{q}_i$ is the case. From (3.6) we know that $\pi_i = q_i^2$, proving the results for retailers' profits. For retail prices, we solve

$$p(\beta, \gamma) - \tilde{p}(\beta, \gamma) = 0$$

for γ given β. The solutions are

$$\gamma(\beta) = \begin{cases} 0 \\ \\ 1 \\ \\ \frac{3(4-\beta-2\beta^2)}{4(1-\beta)+\beta^2} > 1 \ \forall \beta \in (0, 1) \end{cases}$$

so one price is always higher than the other for all β, $\gamma \in (0, 1)$. Figure 5 shows the public competitive retail price is always higher. This in turn implies the public competitive total downstream supply is always smaller. For suppliers' profits, we solve

$$\pi_i(\beta, \gamma) - \tilde{\pi}_j(\beta, \gamma) = 0$$

for γ given β. The solutions are

$$\gamma(\beta) = \begin{cases} 0 \\ 1 \\ 1 \\ \frac{3(51\beta^4-172\beta^3+108\beta^2+\sqrt{3381\beta^8-28616\beta^7+102984\beta^6-206752\beta^5+254736\beta^4-198912\beta^3+97792\beta^2-28672\beta+4096}+96\beta-64)}{4(5\beta^2-12\beta+4)\beta(-2+\beta)} \\ \frac{-3(-51\beta^4+172\beta^3-108\beta^2+\sqrt{3381\beta^8-28616\beta^7+102984\beta^6-206752\beta^5+254736\beta^4-198912\beta^3+97792\beta^2-28672\beta+4096}-96\beta+64)}{4(5\beta^2-12\beta+4)\beta(-2+\beta)} \end{cases}$$

Only interior solutions interest us. Figure 6 shows that the fourth solution is greater than 1 for all $\beta \in (0,1)$. Figure 7 shows the fifth solution. It has a discontinuity at 0.4, which is the only solution of the polynomial in the denominator in $(0,1)$. For $\beta < 0.4$, the solution is always greater than 1. Figure 8 shows the behavior of the fifth solution for $\beta \in (0.5,1)$. We see that the solution is only between 0 and 1 in an interval (β', β''), $0 < \beta' < \beta'' < 1$.

So, for $\beta < \beta'$, either $\pi_j > \tilde{\pi}_j$ or $\pi_j < \tilde{\pi}_j$ is true for all values of γ, and the same is true if $\beta > \beta''$. For intermediate values of β, the value of γ at which the sign of the difference is reversed is a function of β. This defines a function $\gamma(\beta)$. The infimum (β') and supremum (β'') of this interval are, respectively, real solutions in $(0,1)$ of

$$\frac{-3(-51\beta^4+172\beta^3-108\beta^2-96\beta+64)}{4(5\beta^2-12\beta+4)\beta(-2+\beta)} - \frac{3\sqrt{3381\beta^8-28616\beta^7+102984\beta^6-206752\beta^5+254736\beta^4-198912\beta^3+97792\beta^2-28672\beta+4096}}{4(5\beta^2-12\beta+4)\beta(-2+\beta)} = 0$$

and

$$-1+\frac{-3(-51\beta^4+172\beta^3-108\beta^2-96\beta+64)}{4(5\beta^2-12\beta+4)\beta(-2+\beta)} - \frac{3\sqrt{3381\beta^8-28616\beta^7+102984\beta^6-206752\beta^5+254736\beta^4-198912\beta^3+97792\beta^2-28672\beta+4096}}{4(5\beta^2-12\beta+4)\beta(-2+\beta)} = 0.$$

The corresponding real solution to the first equation is $16/13 - 4\sqrt{3}/13 = \beta' \approx 0.6978$. The second expression is of too high order and has no simple analytic solution. However, we know that $\beta'' \in (0.6978, 1)$. Considering the fifth solution only over $V = \{\beta \in (0,1) : \gamma(\beta) \in (0,1)\}$ we have an injective and surjective function, getting $\bar{\beta}(\gamma)$ by taking the inverse of $\gamma(\beta)$ (we will use this procedure again later). By Lemma 3 we know that $\pi_j(\beta = 1, \gamma) > \tilde{\pi}_j(\beta = 1, \gamma)$ for all $\gamma \in (0,1)$, and Figure 9 shows that the limit of the profit difference when $\beta \to 0.5$ is negative for all $\gamma \in (0,1)$, completing the proof. ■

Proof of Proposition 3. *I.* From Propositions 1 and 2 we know that for all β, $\gamma \in (0,1)$

$$\left[\check{w}_j^L \geq w_j^L \wedge w_j^L \geq \tilde{w}_j^L\right] \Rightarrow \check{w}_j^L \geq \tilde{w}_j^L$$

$$\left[\check{q}_j^F \leq q_j^F \wedge q_j^F \leq \tilde{q}_j^F\right] \Rightarrow \check{q}_j^F \leq \tilde{q}_j^F$$

$$[\check{q}_i \geq q_i \wedge q_i \geq \tilde{q}_i] \Rightarrow \check{q}_i \geq \tilde{q}_i$$

$$[\check{p} \geq p \wedge p \geq \tilde{p}] \Rightarrow \check{p} \geq \tilde{p}$$

$$[\check{\pi}_i \geq \pi_i \wedge \pi_i \geq \tilde{\pi}_i] \Rightarrow \check{\pi}_i \geq \tilde{\pi}_i.$$

II. Using (3.18) and (4.2) we solve

$$\check{w}_i(\beta, \gamma) - \tilde{w}_i(\beta, \gamma) = 0$$

for β given γ. The are three solutions, all of which are independent of γ:

$$\beta = \begin{cases} 0 \\ 1 + \frac{\sqrt{105}}{15} > 1 \\ 1 - \frac{\sqrt{105}}{15} > 1 \in (0,1) \end{cases}$$

Therefore there is a single interior solution, $\beta^T = 1 - \sqrt{105}/15$. Making a continuity argument, we take limits as $\beta \to 1$.

$$\lim_{\beta \to 1} \hat{w}_i = \frac{9(1-\gamma)}{27 - 8\gamma}$$

and

$$\lim_{\beta \to 1} \tilde{w}_i = \frac{2(1-\gamma)}{6-\gamma}$$

with

$$\frac{9(1-\gamma)}{27-8\gamma} > \frac{2(1-\gamma)}{6-\gamma} \iff 16 > 9.$$

So, for $\beta > 1 - \sqrt{105}/15$, $\breve{w}_i > \tilde{w}_i$. Figure 10 plots the limit of $\hat{w}_i - \tilde{w}_i$ as $\beta \to 0.2$ for all $\gamma \in (0,1)$, showing that $\breve{w}_i < \tilde{w}_i$ for $\beta < 1 - \sqrt{105}/15$.

III. Now we solve

$$\breve{\pi}_j(\beta,\gamma) - \tilde{\pi}_j(\beta,\gamma) = 0$$

for γ given β. The solutions are

$$\gamma(\beta) = \begin{cases} 0 \\ \\ 1 \\ \\ 1 \\ \\ \frac{3(55\beta^3-212\beta^2+240\beta-64)}{2(5\beta^3-22\beta^2+28\beta-8)} \end{cases}$$

Figure 11 shows the fourth solution, marking $\gamma = 1$ in blue. We see that the solution is in $(0,1)$ in a small interval below 0.4, which is a solution of the polynomial in the denominator. Restricting the domain of $\gamma(\beta)$ to $V = \{\beta \in (0,1) : \gamma(\beta) \in (0,1)\}$, we get an injective and surjective function, so we can define $\underline{\beta}(\gamma)$ for all $\gamma \in (0,1)$. Figure 12 shows the difference between profits, where we can visualize $\underline{\beta}(\gamma)$.

IV. Follows from Proposition 1, by which $\hat{\pi}_j - \tilde{\pi}_j > \pi_j - \tilde{\pi}_j$ for all $\beta, \gamma \in (0,1)$. ■

Proof of Lemma 4. Inequality $\tilde{w}_j^L - \tilde{w}_i < w_j^L - w_i$ follows directly from Proposition 2. Solving

$$(\tilde{w}_j^L - \tilde{w}_i)(\beta,\gamma) - (\breve{w}_j^L - \breve{w}_i)(\beta,\gamma) = 0$$

for γ given β we get

$$\gamma(\beta) = \begin{cases} 1 \\ \\ \frac{-3(7\beta^2-40\beta+48)}{41\beta^2-74\beta+8} > 1 \ \forall \beta \in (0,1) \end{cases}$$

so the discount must always be larger in one equilibrium than in the other. Figure 13 shows it is always larger in the public collusive equilibrium. ■

Appendix B: Figures

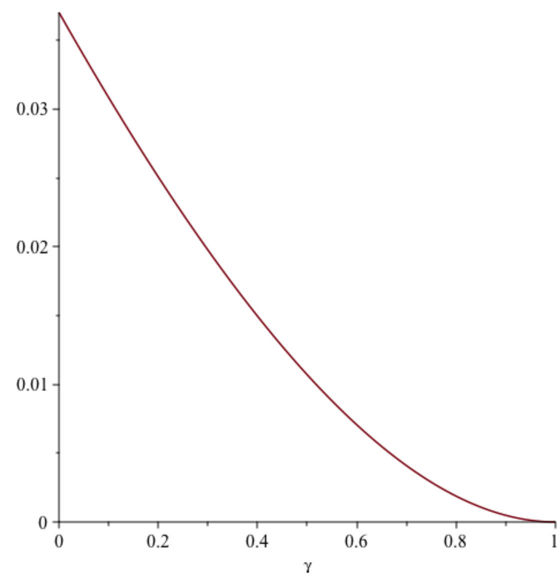

Figure 1: Difference between a supplier's competitive profits with public prices and profits with private prices in the limit $\beta \to 1$, as a function of $\gamma \in (0, 1)$.

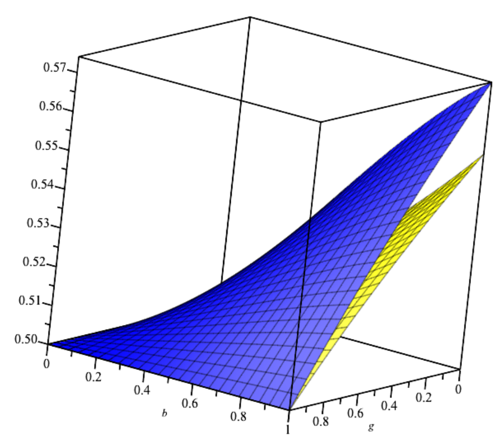

Figure 2: Proposition 2: Public competitive list price (blue) and private list price (yellow) as functions of β, $\gamma \in (0, 1)$. Parameter a normalized to 1. (Letters b and g represent β and γ.)

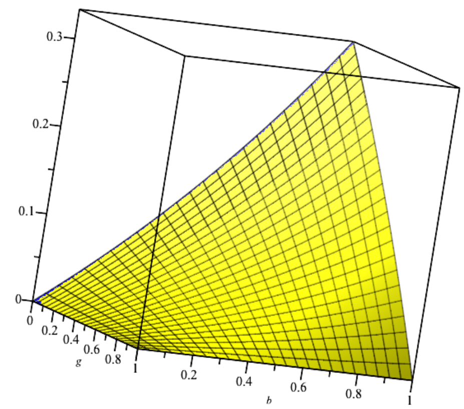

Figure 3: Proposition 2: Public competitive transaction price (blue) and private transaction price (yellow) as functions of β, $\gamma \in (0, 1)$. Parameter a normalized to 1. (Letters b and g represent β and γ.)

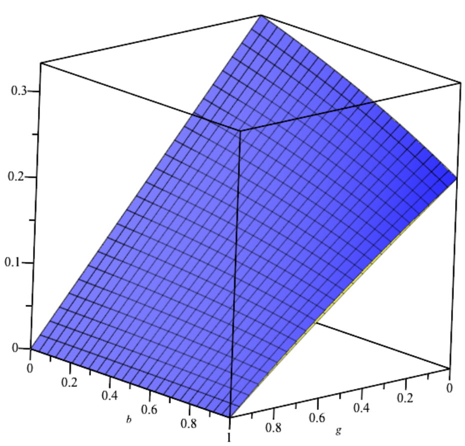

Figure 4: Proposition 2: Public competitive retail supply (blue) and private retail supply (yellow) as functions of β, $\gamma \in (0, 1)$. Parameter a normalized to 1. (Letters b and g represent β and γ.)

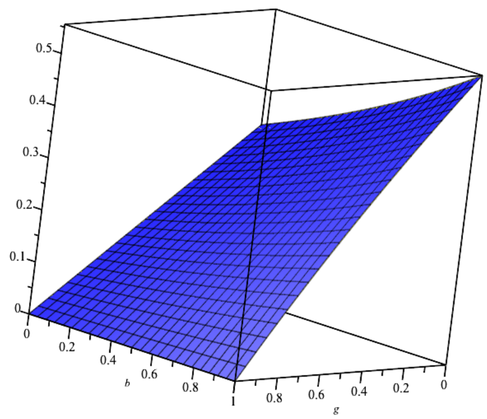

Figure 5: Proposition 2: Public competitive retail price (blue) and private retail price (yellow) as functions of β, $\gamma \in (0, 1)$. Parameter a normalized to 1. (Letters b and g represent β and γ.)

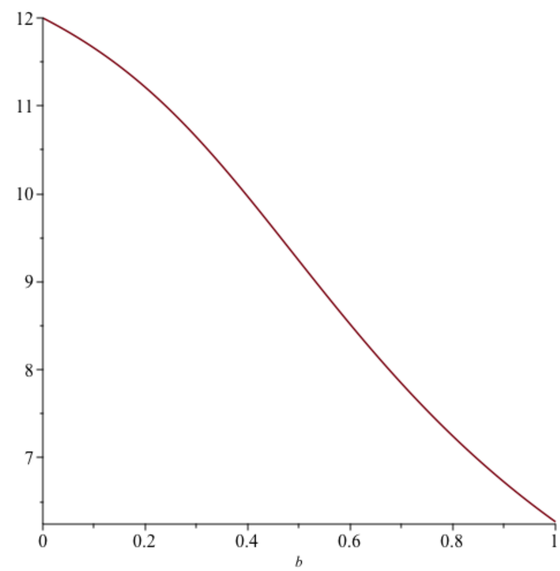

Figure 6: Proposition 2: Fourth solution to $\pi_j - \tilde{\pi}_j = 0$ for $\beta \in (0, 1)$. Parameter a normalized to 1. (Letter b represents β.)

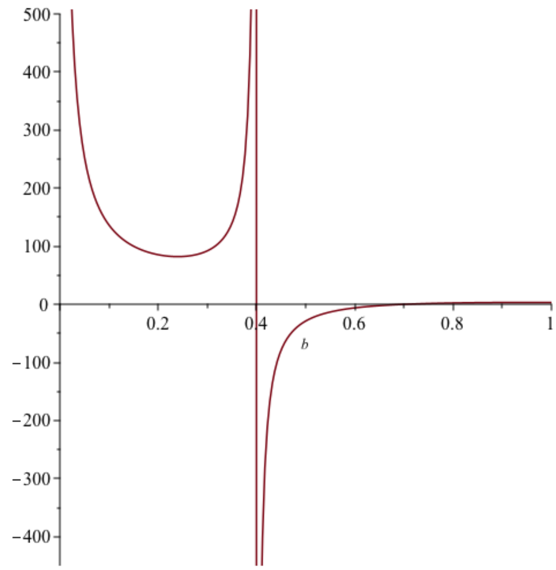

Figure 7: Proposition 2: Fifth solution of $\pi_j - \tilde{\pi}_j = 0$ for $\beta \in (0, 1)$. Parameter a normalized to 1. (Letter b represents β.)

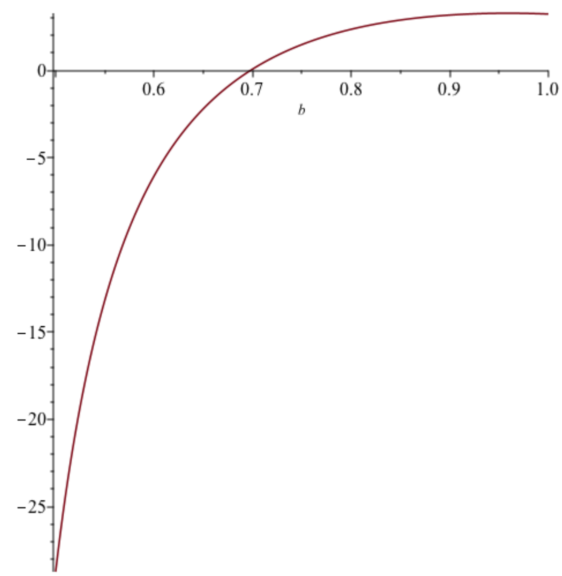

Figure 8: Proposition 2: Fifth solution of $\pi_j - \tilde{\pi}_j = 0$ for $\beta \in (1/2, 1)$. Parameter a normalized to 1. (Letter b represents β.)

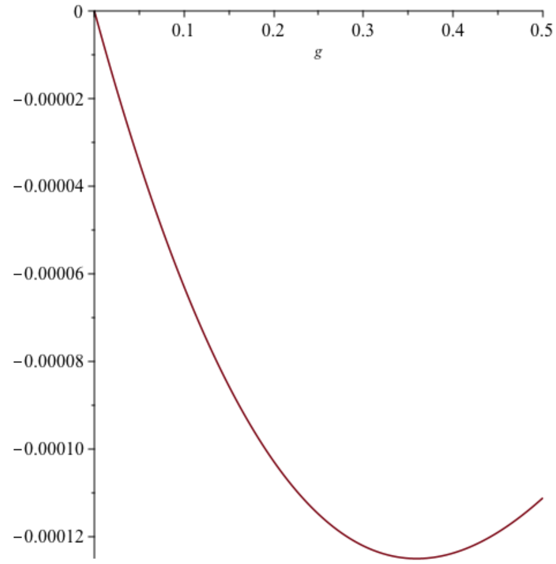

Figure 9: Proposition 2: limit $\pi_j(\beta,\gamma) - \tilde{\pi}_j(\beta,\gamma)$ as $\beta \to 0.5$. Parameter a normalized to 1. (Letter g represents γ.)

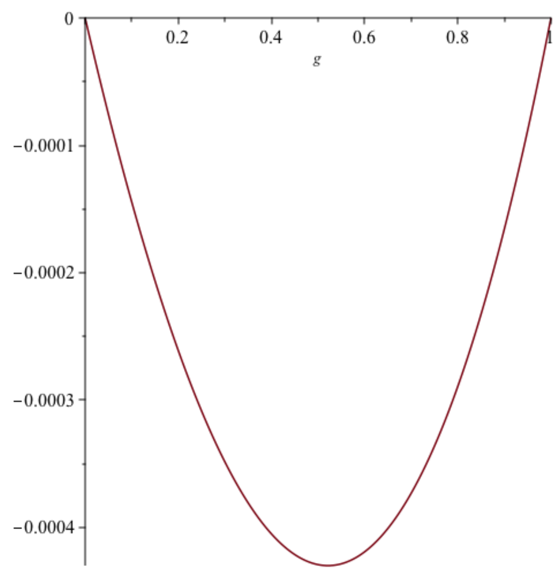

Figure 10: Proposition 3: $\lim_{\beta\to 0.2}(\breve{w}_i - \tilde{w}_i)$ for $\gamma \in (0,1)$. Parameter a normalized to 1. (Letter g represents γ.)

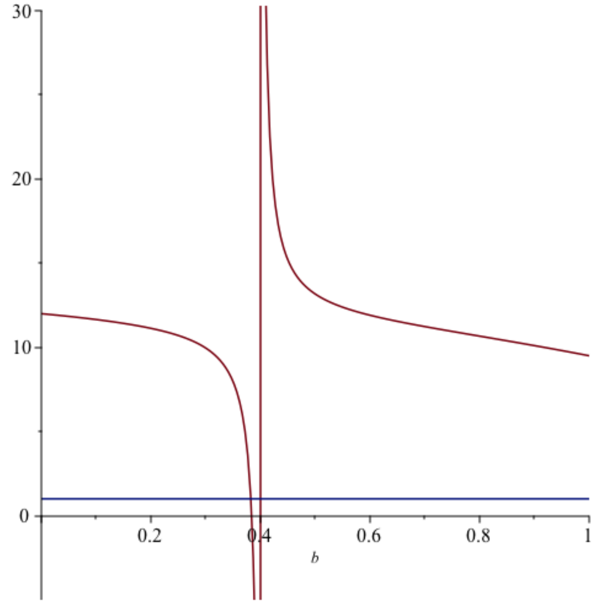

Figure 11: Proposition 3: Fourth solution of $\hat{\pi}_j - \tilde{\pi}_j = 0$ for $\beta \in (0, 1)$. Parameter a normalized to 1. (Letter b represents β.)

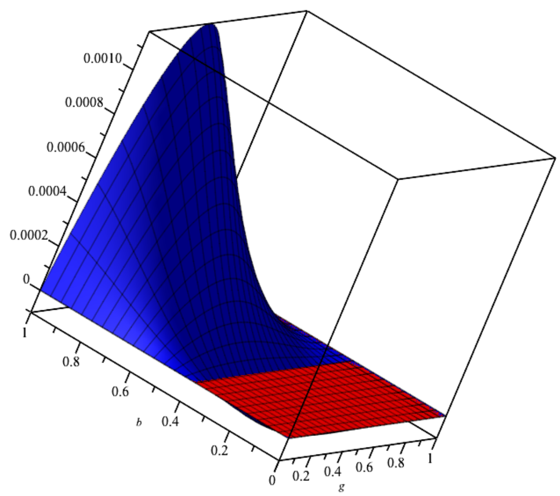

Figure 12: Proposition 3: Expression $\hat{\pi}_j - \tilde{\pi}_j$ (blue) as function of β, $\gamma \in (0, 1)$. Zero marked in red. Parameter a normalized to 1. (Letters b and g represent β and γ.)

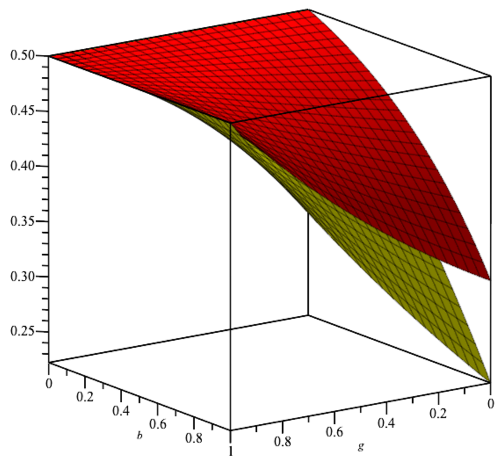

Figure 13: Lemma 4: Discount in the public collusive (red) and private (yellow) equilibria for β, $\gamma \in (0, 1)$. Parameter a normalized to 1. (Letters b and g represent β and γ.)

Appendix C: No Bilateral Monopoly

We lift Horn and Wolinsky (1988)'s bilateral monopoly assumption. To avoid the Bertrand paradox, we introduce increasing marginal costs for retailers. Their cost functions are symmetric and additively separatable by supplier, so in equilibrium retailers buy from both suppliers. The timing of the public and private games are the same. The equilibrium concept is subgame perfection. Because each supplier bargains with both retailers, we focus on symmetric Nash-in-Nash equilibria.

7.1 Cost Functions

Each retailer R_i, $i \in \{A, B\}$ can buy inputs from any of the two suppliers U_j, $j \in \{1, 2\}$. It buys l_{ij} from each supplier, using it to produce q_{ij}. In equilibrium, $l_{i,j} = q_{i,j}$ for all i, j. The cost for R_i of producing q_{i1} and q_{i2} is

$$C_i(q_{i_1}, q_{i,2}) = w_{i,1} \cdot q_{i1} + \kappa \frac{q_{i1}^2}{2} + w_{i,2} \cdot q_{i2} + \kappa \frac{q_{i2}^2}{2} \tag{7.1}$$

where κ measures the curvature of the cost function. This type of cost function comes from two assumptions. First, either decreasing returns to scale or some capacity constraint, so that the marginal cost is increasing.

Second, some degree of asset specificity in the relationship between retailers and their suppliers, as in Williamson (1985). There is asset specificity even in industrial markets for products usually considered commodities, like chicken breeding and cow milk.[14] This can be generated by different sanitary standards and procedures each supplier has, for example.

7.2 Public Equilibria Model

Downstream, both retailers compete à la Cournot, and their products are perfect substitutes. Each supplier has a local monopoly over a fringe, so the inverse demand function is

$$p = a - q_1^F - q_2^F - q_{A1} - q_{A2} - q_{B1} - q_{B2}$$

where $q_j^F = \gamma(a - w_j^F)$. Define

[14] These examples come from Chile.

$$\tilde{a} = a - q_1^F - q_2^F = a(1-2\gamma) + \gamma(w_1^L + w_2^L).$$

To ensure a positive price, we set $\gamma \in (0, 1/2)$ ($\gamma = 0$ is the Horn and Wolinsky (1988) model). Note that this assumption would not change any result in the main model. In the last stage, R_i solves

$$\max_{(q_{i,1},q_{i2})\in\mathbb{R}_+^2} \left[\tilde{a} - q_{A1} - q_{A2} - q_{B1} - q_{B2}\right](q_{i,1} + q_{i2}) - w_{i,1}\cdot q_{i1} - \frac{q_{i1}^2}{2} - w_{i,2}\cdot q_{i2} - \frac{q_{i2}^2}{2}. \tag{7.2}$$

Equilibrium quantities as functions of input prices are

$$\begin{aligned} q_{i,j} = &\frac{[14\kappa + 8\kappa^2 + \kappa^3]\left[(2\kappa+\kappa^2)\tilde{a} - (4\kappa+\kappa^2)w_{i,j} + (8+2\kappa)(w_{i,-j} - w_{i,j}) + \kappa(w_{-i,j} + w_{-i,-j})\right]}{[14\kappa + 8\kappa^2 + \kappa^3]^2 - 4\kappa^2} \\ &+ \frac{2\kappa\left[(2\kappa+\kappa^2)\tilde{a} - (4\kappa+\kappa^2)w_{i,-j} + (8+2\kappa)(w_{i,j} - w_{i,-j}) + \kappa(w_{-i,j} + w_{-i,-j})\right]}{[14\kappa + 8\kappa^2 + \kappa^3]^2 - 4\kappa^2}. \end{aligned} \tag{7.3}$$

7.3 Nash Bargaining

If bargaining between R_i and U_j fails, R_i can only use inputs from U_{-j}, and R_{-i} reacts optimally to this. Off-path quantities are

$$q_{i,-j}^{Off} = \frac{(2+\kappa)\tilde{a} - (4+\kappa)w_{i,-j} + w_{-i,j} + w_{-i,-j}}{6 + 6\kappa + \kappa^2}$$

$$q_{-i,j}^{Off} = \frac{\kappa\tilde{a} - \kappa w_{-i,j} + 2(w_{-i,-j} - w_{-i,j}))}{\kappa(4+\kappa)} - \frac{\kappa\cdot q_{i,-j}^{Off}}{\kappa(4+\kappa)}$$

$$q_{-i,-j}^{Off} = \frac{\kappa\tilde{a} - \kappa w_{-i,-j} + 2(w_{-i,j} - w_{-i,-j}))}{\kappa(4+\kappa)} - \frac{\kappa\cdot q_{i,-j}^{Off}}{\kappa(4+\kappa)}.$$

Claim C.1. *Letting* $\kappa = 1$, $q_{-i,j}^{Off} \geq q_{-i,j}$.

Proof. Claim C.1 is true if

$$\frac{3\tilde{a} - 13w_{-i,j} + 8w_{-i,j} + w_{i,j} + w_{i,-j}}{21} \geq \frac{2\tilde{a} - 8w_{-i,j} + 5w_{-i,-j} + w_{i,-j}}{13}$$

$$\frac{3\tilde{a} - 13w_{i,j} + 8w_{i,-j} + w_{-i,j} + w_{-i,-j}}{21} \geq 0$$

which is true because retailer supply is never negative. ■

As long as input prices are such that zero supply is not optimal, the inequality will be strict. Using the passive beliefs assumption, on and off-path profits for retailers and suppliers are

$$\hat{\pi}_i = [\tilde{a} - q_{i,1} - q_{i,2} - q_{-i,1} - q_{-i,2}]\,(q_{i,1} + q_{i,2}) - w_{i,1} \cdot q_{i,1} - \kappa \frac{q_{i,1}^2}{2} - w_{i,2} \cdot q_{i,2} - \kappa \frac{q_{i,2}^2}{2}$$

$$\bar{\pi}_i = \left[\tilde{a} - q_{i,-j}^{Off} - q_{-i,j}^{Off} - q_{-i,-j}^{Off}\right] q_{i,-j}^{Off} - w_{i,-j} \cdot q_{i,-j}^{Off} - \kappa \frac{(q_{i,-j}^{Off})^2}{2}$$

and

$$\hat{\pi}_j = w_{i,j} \cdot q_{i,j} + w_{-i,j} \cdot q_{-i,j} + w_i^L \gamma (a - w_j^L)$$

$$\bar{\pi}_j = w_{-i,j} \cdot q_{-i,j}^{Off} + w_i^L \gamma (a - w_j^L).$$

In solving the Nash bargaining problem, we face three barriers. First, there may exist values of κ for which a retailer's off-path profits are higher than its on-path profits. The reason is that total off-path quantities are smaller, so the retail price increases. This increase is large enough to compensate for the fall in quantity sold. Therefore, it is possible that Nash bargaining only works for values of κ that make it unprofitable to produce too much off-path. However, this increases the retail price even more, so it is not obvious that such a κ exists. Let W_κ^R be the set of transaction prices such that, given list prices and κ, $\hat{\pi}_i > \bar{\pi}_i$.

Second, as Claim C.1 shows, off-path quantities are larger than off-path ones. In particular $q_{-i,j}^{Off} > q_{-i,j}$. Therefore, given $w_{-i,j}$ and w_j^L, the smallest $w_{i,j}$ such that U_j and R_i can reach an agreement is larger than zero. However, it must be smaller than the transaction price U_j would set in a take-it-or-leave-it offer. Let W_κ^U be the set of transaction prices such that, given list prices and κ, $\hat{\pi}_j > \bar{\pi}_j$.

Third, and related, the solution to the Nash bargaining problem is increasingly complex in the number of parameters. Characterizing the set K such that for all $\kappa \in K$ Nash bargaining works is not straightforward.

Claim C.2. *With $\kappa = 1$, $W_1 = W_1^R \cap W_1^U \neq \emptyset$ and there exists a unique symmetric solution to the Nash bargaining problem.*

The Nash bargaining problem can be described by the payoff set $B_i^j = \{[\hat{\pi}_i - \bar{\pi}_i, \hat{\pi}_i - \bar{\pi}_i]\,|w_{i,j} \in W\}$ and the disagreement point $(0, 0)$. We look for a Nash-in-Nash equilibrium, in which negotiations are conducted simultaneously, with players assuming the rest of the transaction prices are in their equilibrium values. The bargaining problem's objective function is

$$\max_{w_{i,j}\in W_1} \left[\hat{\pi}_i - \bar{\pi}_i\right]^{(1-\beta)} \cdot \left[\hat{\pi}_j - \bar{\pi}_j\right]^{\beta}.$$

Since $\beta \in (0,1)$, both supplier and retailer earn positive profits bargaining. Thus, we can use a logarithmic transformation. We have four (symmetric) fist order conditions with the same functional form and parameters

$$\left(\frac{\beta}{(\hat{\pi}_j - \bar{\pi}_j)(w_{i,j},\cdot)} \frac{\partial(\hat{\pi}_j - \bar{\pi}_j)(w_{i,j},\cdot)}{\partial w_{i,j}} + \frac{\beta}{(\hat{\pi}_i - \bar{\pi}_i)(w_{i,j},\cdot)} \frac{\partial(\hat{\pi}_i - \bar{\pi}_i)(w_{i,j},\cdot)}{\partial w_{i,j}}\right)\Big|_{w_{i,j}=w_{i,-j}=w_{-i,j}=w_{-i,-j}} = 0.$$

The unique symmetric solution is

$$w_{i,j}(w_1^L, w_2^L) = \frac{367\beta\tilde{a}}{2600 - 765\beta} \quad \forall i,j.$$

We have proved Claim C.2. Letting $\eta \equiv 367\beta/(2600 - 765\beta)$, we see that η is strictly increasing in β and ranges from 0 to 0.2.

7.4 Public Competitive Equilibrium

Having found the real solution for all $a > 0$, $\beta \in (0,1)$ and $\gamma \in (0,1/2)$, we solve U_j's maximization problem in τ_0. Since profits from selling to the fringe only affect the negotiation through $\tilde{a}$, it is enough to solve

$$\max_{w_j^L \geq 0} (q_{1,j} \cdot w_{1,j} + q_{2,j} \cdot w_{2,j})(w_j^L, w_{-j}^L) + w_j^L \gamma (a - w_j^L).$$

Write the equilibrium transaction price as $\eta\tilde{a}$. Plugging $\eta\tilde{a}$ into (7.2) we get $q_{i,j} = \phi\tilde{a}$ for all i, j, with

$$\phi \equiv \frac{\left[16\kappa + 8\kappa^2 + \kappa^3\right](2\kappa + \kappa^2)(1-\eta)}{\left[14\kappa + 8\kappa^2 + \kappa^3\right]^2 - 4\kappa^2}.$$

Let $\Omega \equiv 2\eta\phi$. By the symmetry of $w_{i,j}$ and $q_{i,j}$ for all i and j, we can rewrite the maximization problem as

$$\max_{w_j^L \geq 0} \Omega \tilde{a}^2 + w_j^L \gamma (a - w_j^L).$$

The first and second-order conditions are

$$\frac{\partial \pi_j}{\partial w_j^L} = \gamma \left[2\Omega(a(1-2\gamma) + (w_j^L + w_{-j}^L)) + a - 2w_j^L \right] = 0$$

$$\frac{\partial^2 \pi_j}{\partial (w_j^L)^2} = \gamma \left[2\Omega\gamma - 2 \right] < 0$$

where the negative sign of the second-order condition comes from $\kappa = 1$ and $\eta \in (0, 1/5)$, which imposes a low upper bound on ϕ and Ω, and $\gamma \in (0, 1/2)$. The best response function is

$$w_j^L = \frac{a \left[1 + 2\Omega(1-2\gamma) \right]}{2(1-\Omega\gamma)} + \frac{2\Omega\gamma w_{-j}^L}{2(1-\Omega\gamma)}. \tag{7.4}$$

We can see that list prices are strategic complements, and the similarity to the model in Section 3. The symmetric equilibrium list price is

$$w_j^L = \frac{a \left[1 + 2\Omega(1-2\gamma) \right]}{2 - 4\Omega\gamma}. \tag{7.5}$$

7.5 Public Collusive Equilibrium

As in Section 3, finding the best collusive list prices means maximizing joint profits in τ_0. The cartel solves

$$\max_{(w_1^L, w_{2L}) \in \mathbb{R}_+^2} \pi_j + \pi_j = 2\Omega \tilde{a}^2 + w_1^L \gamma (a - w_1^L) + w_2^L \gamma (a - w_2^L).$$

First and second partial derivatives are

$$\frac{\partial (\pi_1 + \pi_2)}{\partial w_j^L} = 4\Omega [a(1-2\gamma) + \gamma(w_1^L + w_2^L)]\gamma + \gamma(a - 2w_j^L) = 0$$

$$\frac{\partial^2 (\pi_1 + \pi_2)}{\partial (w_j^L)^2} = -\gamma \left[2 - 4\Omega\gamma \right] < 0$$

$$\frac{\partial^2 (\pi_1 + \pi_2)}{\partial w_j^L \partial w_{-j}^L} = 4\Omega\gamma^2 > 0.$$

The Hessian matrix will be negative definite if both eigenvalues k are negative. The eigenvalues are

$$k = \frac{\partial^2(\pi_1 + \pi_2)}{\partial(w_j^L)^2} \pm \frac{\partial^2(\pi_1 + \pi_2)}{\partial w_j^L \partial w_{-j}^L}$$

so it is sufficient for the sum to be negative. The sum is -2γ, so the function is quasi-concave and the first-order conditions characterize the best collusive list prices. These are

$$\check{w}_j^L = \frac{a\left[1 + 4\Omega(1 - 2\gamma)\right]}{2 - 8\Omega\gamma}. \tag{7.6}$$

Since all input prices are scaled by a, all other equilibrium variables are scaled by a. Therefore, all comparisons can be made normalizing a to one.

7.6 Sustainability of Collusion

Assume both suppliers have a common discount factor $\delta \in (0, 1)$, and again consider the grim trigger strategy. Let π^C and π^M be a supplier's profits in competition and collusion, and $\pi^D(x)$ the profit of deviating to a list price x. The incentive compatibility constraint is

$$\frac{\pi^M}{1 - \delta} \geq \pi(x) + \frac{\delta \pi^C}{1 - \delta} \quad \forall x \geq 0 \tag{7.7}$$

so we only have to consider the best deviation list price. That price is given by the best response function, expression (7.4). Let w^D be this list price. The critical discount factor is given by

$$\underline{\delta} = \frac{\pi^D(w^D) - \pi^M}{\pi^D - \pi^C}. \tag{7.8}$$

Since the cheater is playing its best response, $\pi^D(w^D) \geq \pi^M > \pi^C$. Therefore, by (7.8) there exists $\underline{\delta} \in [0, 1)$ such that, for all $\delta \geq \underline{\delta}$ collusion is sustainable with a grim trigger strategy.

7.7 Welfare Effects of Collusion

Proposition C.1. *Collusion in list prices (i) raises list prices, (ii) raises transaction prices, (iii) reduces fringe supply, (iv) increases retail supply, (v) reduces total supply, (vi) increases the retail price, (vii) increases suppliers' profits and (viii) increases retailers' profits.*

Proof. Beginning with list prices

$$\hat{w}_{i,j} > w_{i,j} \iff \frac{[1+4\Omega(1-2\gamma)]}{2-8\Omega\gamma} > \frac{[1+2\Omega(1-2\gamma)]}{2-4\Omega\gamma}$$

$$\begin{aligned} \iff & [1+2\Omega(1-2\gamma)]\,[2-4\Omega\gamma] + 2\Omega(1-2\gamma)[2-4\Omega\gamma] \\ & \qquad > [1+2\Omega(1-2\gamma)]\,[2-4\Omega\gamma] - 4\Omega\gamma\,[1+2\Omega(1-2\gamma)] \end{aligned}$$

$$\iff \underbrace{2\Omega(1-2\gamma)[2-4\Omega\gamma]}_{>0} > \underbrace{-4\Omega\gamma\,[1+2\Omega(1-2\gamma)]}_{<0}.$$

Since $\tilde{a}$ is increasing in list prices, it is higher under collusion. Note that other equilibrium values can be written as

$$w_{i,j} = \tilde{a} \cdot \eta$$

$$q_{i,j} = \tilde{a} \cdot \phi$$

$$p = \tilde{a} \cdot (1-4\phi)$$

$$\pi_i = \tilde{a}^2 \left[2\phi - 8\phi^2 - 2\eta\phi - \kappa\phi^2\right].$$

So collusion raises transaction prices, retailer supply, the retail price and retailers' profits. Since each fringe's supply is strictly decreasing in the list price at which it buys, fringe supply falls. Since total supply decreases in the retail price, total supply falls. Both suppliers' profits increase because the public equilibria are symmetric and collusion maximizes joint profits. ■

7.8 Private Equilibrium

In the private equilibrium, the equations for prices and quantities in τ_2' are the same as in τ_2 the public equilibria. In τ_1' U_j's maximizes its profits with respect to $\tilde{w}_j^l$, given all transaction prices $\tilde{w}_{i,j}$. Note that we can additively separate (7.2) into a term that is multiplied by $\tilde{a}$ and one that is not. Rewriting we get

$$q_{i,j} = \lambda\tilde{a} + \Lambda_{i,j}.$$

With

$$\lambda \equiv \frac{\left[16\kappa + 8\kappa^2 + \kappa^3\right](2\kappa + \kappa^2)}{\left[14\kappa + 8\kappa^2 + \kappa^3\right]^2 - 4\kappa^2}$$

and Λ represents the remainder, containing all the terms with transaction prices

$$\Lambda_{i,j} \equiv \frac{\left[14\kappa + 8\kappa^2 + \kappa^3\right]\left[-(4\kappa + \kappa^2)w_{i,j} + (8 + 2\kappa)(w_{i,-j} + w_{i,j}) + \kappa(w_{-i,j} + w_{-i,-j})\right]}{\left[14\kappa + 8\kappa^2 + \kappa^3\right]^2 - 4\kappa^2} + \frac{2\kappa\left[-(4\kappa + \kappa^2)w_{i,-j} + (8 + 2\kappa)(w_{i,j} + w_{i,-j}) + \kappa(w_{-i,j} + w_{-i,-j})\right]}{\left[14\kappa + 8\kappa^2 + \kappa^3\right]^2 - 4\kappa^2}.$$

U_j's problem is

$$\max_{w_j^L \geq 0} \ \pi_j = w_{A,j} \cdot (\lambda\tilde{a} + \Lambda_{i,j}) + w_{B,j} \cdot (\lambda\tilde{a} + \Lambda_{i,j}) + w_j^L \gamma(a - w_j^L).$$

The first and second-order conditions are

$$\frac{\partial \pi_j}{\partial w_j^L} = \gamma(a - 2w_j^L) + \gamma\lambda(w_{A,j} + w_{B,j}) = 0$$

$$\frac{\partial^2 \pi_j}{\partial (w_j^L)^2} = -2\gamma < 0$$

so the function is concave. List prices as a function of transaction prices are

$$w_j^L = \frac{a + \lambda(w_{A,j} + w_{B,j})}{2}. \tag{7.9}$$

On and off-path profits are

$$\hat{\pi}_i = \left[\tilde{a}(w_{i,j}) - q_{i,1} - q_{i,2} - q_{-i,1} - q_{-i,2}\right](q_{i,1} + q_{i,2}) - w_{i,1} \cdot q_{i,1} - \kappa\frac{q_{i,1}^2}{2} - w_{i,2} \cdot q_{i,2} - \kappa\frac{q_{i,2}^2}{2}$$

$$\bar{\pi}_i = \left[\tilde{a}^{Off} - q_{i,-j}^{Off} - q_{-i,j}^{Off} - q_{-i,-j}^{Off}\right] q_{i,-j}^{Off} - w_{i,-j} \cdot q_{i,-j}^{Off} - \kappa\frac{(q_{i,-j}^{Off})^2}{2}$$

$$\hat{\pi}_j = w_{i,j} \cdot q_{i,j} + w_{-i,j} \cdot q_{-i,j} + w_i^L(w_{-i,j})\gamma(a - w_j^L(w_{-i,j}))$$

$$\bar{\pi}_j = w_{-i,j} \cdot q_{-i,j}^{Off} + w_i^{L,Off}\gamma(a - w_j^{L,Off}).$$

We use the same assumptions about off-path profits as in the baseline model: off-path list prices are set according to (7.9). Therefore, $\tilde{a} = \tilde{a}^{Off}$ and $w_j^L(w_{i,j}) = w_j^{L,Off}$. Let $\tilde{W}_\kappa^R$ be the set of transaction prices such that, given κ and all other transaction prices are at their equilibrium values, $\hat{\pi}_i > \bar{\pi}_i$. Let $\tilde{W}_\kappa^U$ be the analogous set such that $\hat{\pi}_j > \bar{\pi}_j$, and let $\tilde{W}_\kappa = \tilde{W}_\kappa^R \cap \tilde{W}_\kappa^U$.

Claim C.3. *With $\kappa = 1$, $\tilde{W}_\kappa \neq \emptyset$ and there exists a unique symmetric equilibrium for the Nash bargaining problem.*

The bargaining problem is described by the set of payoffs $\tilde{B}_i^j = \left\{ [\hat{\pi}_i - \bar{\pi}_i, \hat{\pi}_j - \bar{\pi}_j] \, | w_i \in \tilde{W}_1 \right\}$ and disagreement point $(0, 0)$. The bargaining problem's objective function is

$$\max_{w_{i,j} \in \tilde{W}} [\hat{\pi}_i - \bar{\pi}_i]^{(1-\beta)} \cdot [\hat{\pi}_j - \bar{\pi}_j]^{\beta}.$$

Since $\beta \in (0, 1)$, both supplier and retailer earn positive profits bargaining. Thus, we can use a logarithmic transformation. We have four (symmetric) fist order conditions with the same functional form and parameters

$$\left(\frac{\beta}{(\hat{\pi}_j - \bar{\pi}_j)(w_{i,j}, \cdot)} \frac{\partial (\hat{\pi}_j - \bar{\pi}_j)(w_{i,j}, \cdot)}{\partial w_{i,j}} + \frac{\beta}{(\hat{\pi}_i - \bar{\pi}_i)(w_{i,j}, \cdot)} \frac{\partial (\hat{\pi}_i - \bar{\pi}_i)(w_{i,j}, \cdot)}{\partial w_{i,j}} \right) \Bigg|_{w_{i,j} = w_{i,-j} = w_{-i,j} = w_{-i,-j}} = 0.$$

The unique symmetric solution is

$$\tilde{w}_{i,j} = \frac{33397 a \beta (\gamma - 1)}{7340 \beta \gamma + 69615 \beta + 4404 \gamma - 236600} \tag{7.10}$$

which completes the proof of Claim C.3.

7.9 Welfare Effects of Publishing List Prices

Proposition C.2. *Relative to the public competitive equilibrium, in the private equilibrium*

I. For all $\beta \in (0, 1), \gamma \in (0, 1/2)$, in the private prices equilibrium (i) list prices fall, (ii) retailer supply falls, (iii) fringe supply increases, (iv) total supply increases, (v) the retail price falls and (vi) retailers' profits fall.

II. There exists $\beta^T \in (0, 1)$ such that $w_{i,j} > \tilde{w}_{i,j}$ if, $\beta > \beta^T$, with the inequality reversed otherwise.

III. For all $\gamma \in (0, 1/2)$ there exists $\bar{\beta}(\gamma) \in (0, 1)$ such that $\pi_j > \tilde{\pi}_j$ if, given γ, $\beta > \bar{\beta}(\gamma)$, with the inequality reversed otherwise.

Proof. Beginning with list prices, solving

$$w_j^L(\beta, \gamma) - \tilde{w}_j^L(\beta, \gamma) = 0$$

for γ given β. We get the solutions

$$\gamma(\beta) = \begin{cases} 1 \\ \frac{455(229347\beta^2 - 1177280\beta + 1352000)}{17616\beta(283\beta^2 - 1216\beta + 1300)} > 1 \;\; \forall \beta \in (0,1) \end{cases}$$

so one list price is always higher than the other. Figure 14 shows that $w_j^L > \tilde{w}_j^L$ for all β, γ. This implies $q_j^F < \tilde{q}_j^F$ for all parameter values.

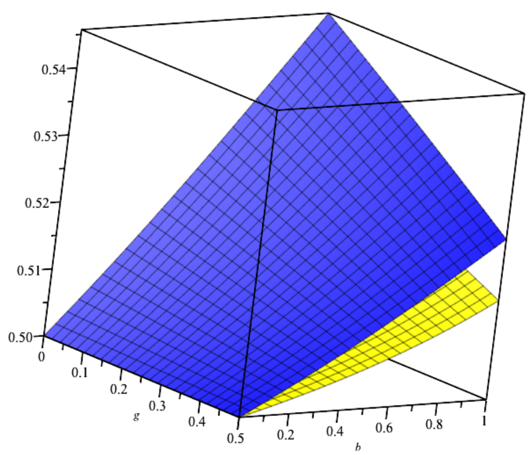

Figure 14: Proposition C.2: List prices in the public competitive (blue) and private (yellow) equilibria for all $\beta \in (0,1)$, $\gamma \in (0,1/2)$.

For transaction prices we solve

$$w_{i,j}(\beta,\gamma) - \tilde{w}_{i,j}(\beta,\gamma) = 0$$

for β. We get the solutions

$$\beta = \begin{cases} 0 \\ \frac{23095 - 5\sqrt{7743193}}{21782} \approx 0.42 \\ \frac{23095 + 5\sqrt{7743193}}{21782} > 1 \end{cases}$$

so there is a single, constant solution in $(0,1)$: $\beta^T \approx 0.42$. Figure 15 shows the difference $\tilde{w}_{i,j} - w_{i,j}$. Taking the limit as $\beta \to 1$ we get

$$\lim_{\beta \to 1} w_{i,j} - \tilde{w}_{i,j} = \frac{-336\gamma(\gamma - 1)}{(16\gamma - 175)(32\gamma - 455)}$$

which is positive for all $\gamma \in (0, 1/2)$. Therefore, $w_{i,j} > \tilde{w}_{i,j}$ if, and only if, $\beta > \beta^T$. For retailer supply, we solve

$$q_{i,j}(\beta, \gamma) - \tilde{q}_{i,j}(\beta, \gamma) = 0$$

for γ given β. We get the solutions

$$\gamma(\beta) = \begin{cases} 1 \\ 1 \\ \frac{7(22901549\beta^2 - 93474930\beta + 93605200)}{17616(283\beta^2 - 1216\beta + 1300)} > 1 \ \forall \beta \in (0,1) \end{cases}$$

so retailer supply is always larger in one equilibrium. Figure 16 shows $q_{i,j} > \tilde{q}_{i,j}$ for some parameter values, so it is true for all parameter values.

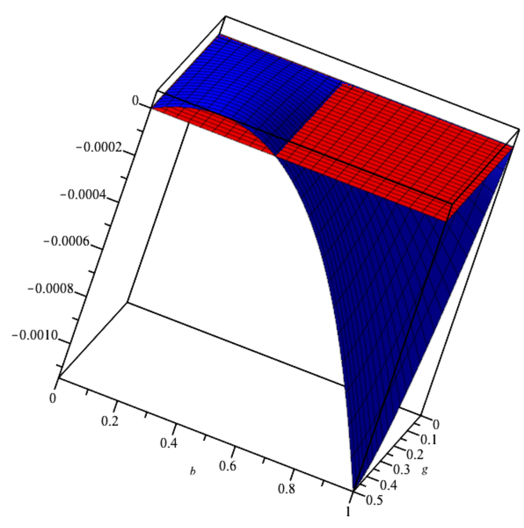

Figure 15: Proposition C.2: Difference $\tilde{w}_{i,j} - w_{i,j}$ (blue) and zero (red) for all $\beta \in (0,1)$, $\gamma \in (0, 1/2)$.

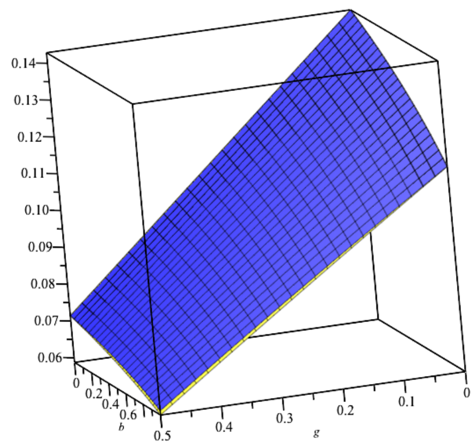

Figure 16: Proposition C.2: Retailer supply in the public competitive (blue) and private (yellow) equilibria for all $\beta \in (0,1)$, $\gamma \in (0,1/2)$.

For the retail price, solving

$$p(\beta,\gamma) - \tilde{p}(\beta,\gamma) = 0$$

for γ given β we get the solutions

$$\gamma(\beta) = \begin{cases} 1 \\ \\ 1 \\ \\ \frac{7(12746689\beta^2 - 161762680\beta + 240739200)}{(52848(283\beta^2 - 1216\beta + 1300)} > 1 \ \forall \beta \in (0,1) \end{cases}$$

so the retail price is always higher in one equilibrium. Figure 17 shows $p > \tilde{p}$ for some parameter values, so it is true for all parameter values. Since the private retail price is lower, private total supply is higher.

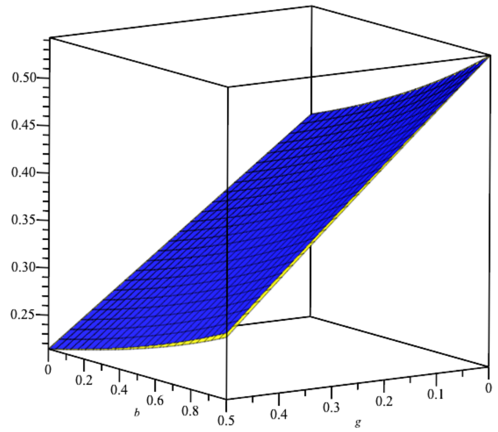

Figure 17: Proposition C.2: Retail price in the public competitive (blue) and private (yellow) equilibria for all $\beta \in (0, 1)$, $\gamma \in (0, 1/2)$.

For the suppliers' profits, solving

$$\pi_j(\beta, \gamma) - \tilde{\pi}_j(\beta, \gamma) = 0$$

for γ given β we get five solutions. The first three are 0, 1 and 1. The fourth is

$$s_4 = 7(590173249\beta^4 - 2373478080\beta^3 + 2208619100\beta^2 + 2(108787574860479319\beta^8 - 987748333975916310\beta^7 + 3582292747812927475\beta^6 - 6611800139174925000\beta^5 + 6587831050716002500\beta^4 - 3546981544563600000\beta^3 + 1051917698520000000\beta^2 - 175356455040000000\beta + 14806022400000000)^{0.5} + 386568000\beta - 243360000) \cdot f(\beta)$$

with

$$f(\beta) = \frac{1}{(17616\beta(29\beta^2 - 64\beta + 12)\beta(283\beta - 650))}.$$

The fourth solution is always greater than 1 (Figure 18). The fifth solution is

$$s_5 = -7(-590173249\beta^4 + 2373478080\beta^3 - 2208619100\beta^2 + 2(108787574860479319\beta^8 - 987748333975916310\beta^7 + 3582292747812927475\beta^6 - 6611800139174925000\beta^5 + 6587831050716002500\beta^4 - 3546981544563600000\beta^3 + 1051917698520000000\beta^2 - 175356455040000000\beta + 14806022400000000)^{0.5} - 386568000\beta + 243360000) \cdot f(\beta).$$

Figure 19 shows the fifth solution in $\beta \in [0.32, 0.335]$. This is the only interval in which $\gamma(\beta) \in (0, 1/2)$. Since the solution is strictly increasing, for all $\gamma \in (0, 1/2)$ there exists $\bar{\beta}(\gamma) \in (0, 1)$ such that $\pi_j > \tilde{\pi}_j$ if, and only if, $\beta > \bar{\beta}(\gamma)$ (Figure 20) shows that public competitive supplier profits are higher for high values of β).

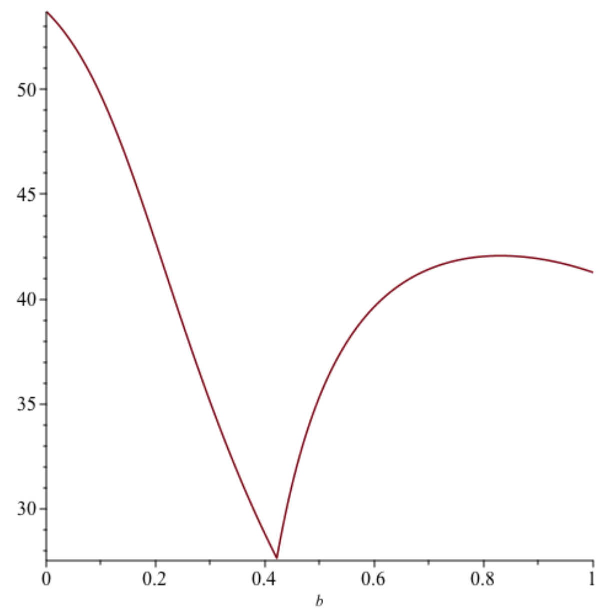

Figure 18: Proposition C.2: Fourth solution of $\pi_j - \tilde{\pi}_j$ for all $\beta \in (0, 1)$.

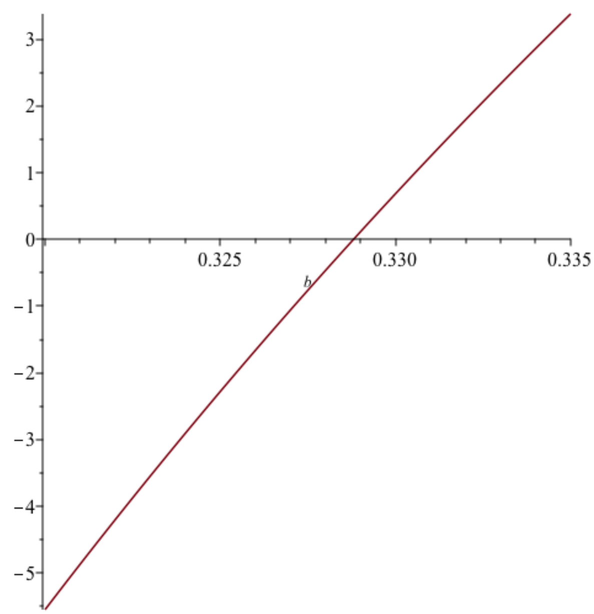

Figure 19: Proposition C.2: Fifth solution of $\pi_j - \tilde{\pi}_j$ for $\beta \in [0.32, 0.335]$.

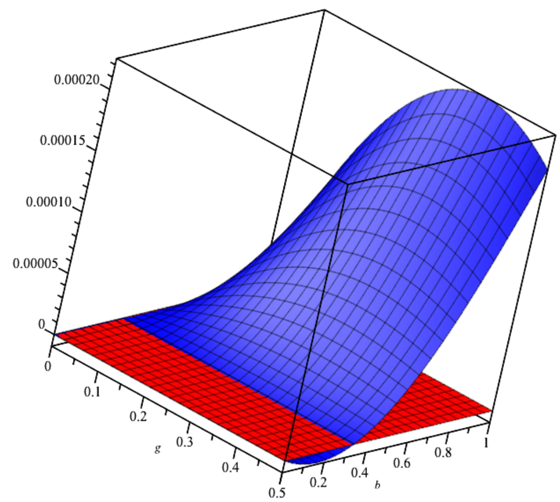

Figure 20: Proposition C.2: Difference $\pi_j - \tilde{\pi}_j$ (blue) and zero (red) for all $\beta \in (0,1)$, $\gamma \in (0, 1/2)$.

For retailers' profits, solving

$$\pi_i(\beta, \gamma) - \tilde{\pi}_i(\beta, \gamma) = 0$$

for γ given β we get five solutions. The first three are 0, 1 and 1. The fourth is

$s_4 = 7(40221149\beta^3 - 181727170\beta^2 + (2085691755769801\beta^6 - 20884893843503860\beta^5 + 79815791394547300\beta^4 - 141704855204952000\beta^3 + 111023236964640000\beta^2 - 29511650688000000\beta + 6580454400000000)^{0.5} + 169618800\beta + 81120000) \cdot h(\beta)$

with

$$h(\beta) = \frac{1}{(35232(283\beta^2 - 1216\beta + 1300)\beta)}.$$

The fourth solution is always greater than 1 (Figure 21). The fifth solution is

$s_5 = -7(-40221149\beta^3 + 181727170\beta^2 + (2085691755769801\beta^6 - 20884893843503860\beta^5 + 79815791394547300\beta^4 - 141704855204952000\beta^3 + 111023236964640000\beta^2 - 29511650688000000\beta + 6580454400000000)^{0.5} - 169618800\beta - 81120000) \cdot h(\beta)$

which is also always greater than 1 (Figure 22). So, in one equilibrium retailer profits are always higher than in the other. Figure 23 shows that $\pi_j > \tilde{\pi}_j$ for some parameter values, so it is true for all parameter values. This completes the proof. ■

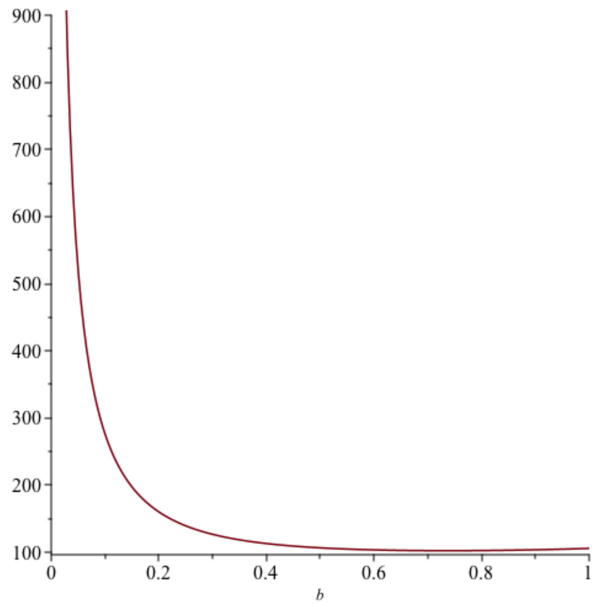

Figure 21: Proposition C.2: Fourth solution of $\pi_i - \tilde{\pi}_i$ for all $\beta \in (0, 1)$.

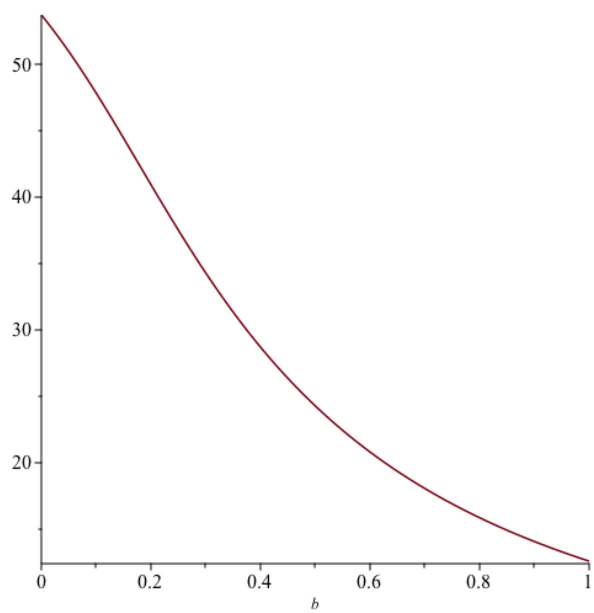

Figure 22: Proposition C.2: Fifth solution of $\pi_j - \tilde{\pi}_j$ for all $\beta \in (0, 1)$.

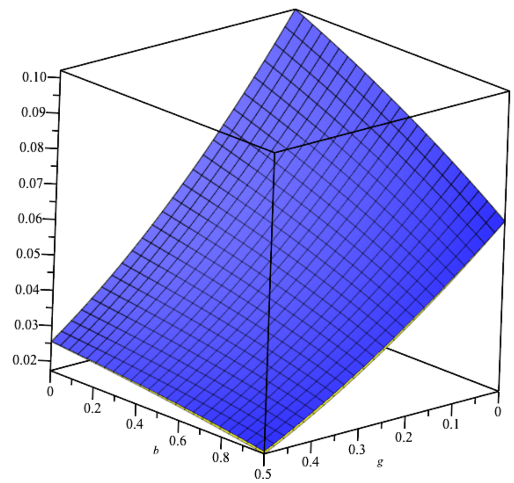

Figure 23: Proposition C.2: Retailer profit in the public competitive (blue) and private (yellow) equilibria for all $\beta \in (0,1)$, $\gamma \in (0,1/2)$.

As we can see, the optimal response effect is not as strong in this model: for high values of β the public competitive transaction price is higher than the private one.

Proposition C.3. *Relative to the public collusive equilibrium, in the private equilibrium*

I. For all $\beta \in (0,1), \gamma \in (0,1/2)$, in the private prices equilibrium (i) list prices fall, (ii) retailer supply falls, (iii) fringe supply increases, (iv) total supply increases, (v) the retail price falls and (vi) retailers' profits fall.

II. There exists $\beta_T \in (0,1)$ such that $\breve{w}_{i,j} > \tilde{w}_{i,j}$ if, $\beta > \beta_T$, with the inequality reversed otherwise.

III. For all $\gamma \in (0,1/2)$ there exists $\underline{\beta}(\gamma) \in (0,1)$ such that $\breve{\pi}_j > \tilde{\pi}_j$ if, given γ, $\beta > \underline{\beta}(\gamma)$, with the inequality reversed otherwise.

IV. For all γ, $\underline{\beta}(\gamma) < \bar{\beta}(\gamma)$, and $\beta_T < \beta^T$.

Proof. Part *I* follows directly from Propositions C.1 and C.2:

$$\left[\breve{w}_j^L \geq w_j^L \wedge w_j^L \geq \tilde{w}_j^L\right] \Rightarrow \breve{w}_j^L \geq \tilde{w}_j^L$$

$$\left[\breve{q}_j^F \leq q_j^F \wedge q_j^F \leq \tilde{q}_j^F\right] \Rightarrow \breve{q}_j^F \leq \tilde{q}_j^F$$

$$\left[\breve{q}_i \geq q_i \wedge q_i \geq \tilde{q}_i\right] \Rightarrow \breve{q}_i \geq \tilde{q}_i$$

$$\left[\breve{p} \geq p \wedge p \geq \tilde{p}\right] \Rightarrow \breve{p} \geq \tilde{p}$$

$$\Rightarrow \check{Q} \leq \tilde{Q}$$

$$[\check{\pi}_i \geq \pi_i \wedge \pi_i \geq \tilde{\pi}_i] \Rightarrow \check{\pi}_i \geq \tilde{\pi}_i.$$

II. For transaction prices solve

$$\check{w}_{i,j}(\beta, \gamma) - \tilde{w}_{i,j}(\beta, \gamma) = 0$$

for β. We get the solutions

$$\beta = \begin{cases} 0 \\ \\ \frac{56895 - 5\sqrt{97524105}}{51214} \approx 0.147 \\ \\ \frac{56895 + 5\sqrt{97524105}}{51214} > 1 \end{cases}$$

so there is a single, constant solution in $(0, 1)$: $\beta_T \approx 0.147$. By Propositions C.1 and C.2 we know $\check{w}_{i,j} > \tilde{w}_{i,j}$ if $\beta > \beta_T$. Figure 24 shows the difference $\tilde{w}_{i,j} - \check{w}_{i,j}$; we see that $\check{w}_{i,j} < \tilde{w}_{i,j}$ if $\beta < \beta_T$.

III. For suppliers' profits, solving

$$\check{\pi}_j(\beta, \gamma) - \tilde{\pi}_j(\beta, \gamma) = 0$$

for γ given β we get the solutions

$$\gamma(\beta) = \begin{cases} 1 \\ \\ 1 \\ \\ 1 \\ \\ \frac{91(197843017\beta^3 - 861294840\beta^2 + 1013313600\beta - 162240000)}{35232(8207\beta^3 - 36962\beta^2 + 44996\beta - 7800)} \end{cases}$$

Figure 25 shows the only subset of $\beta \in (0, 1)$ such that, according to the fourth solution, $\gamma(\beta) \in (0, 1/2)$. Since the function is strictly decreasing, we can create the function $\underline{\beta}(\gamma)$.

By Propositions C.1 and C.2 we know that, given γ, $\breve{\pi}_j > \tilde{\pi}_j$ if $\beta > \underline{\beta}(\gamma)$. Figure 26 shows that $\breve{\pi}_j < \tilde{\pi}_j$ if $\beta < \underline{\beta}(\gamma)$.

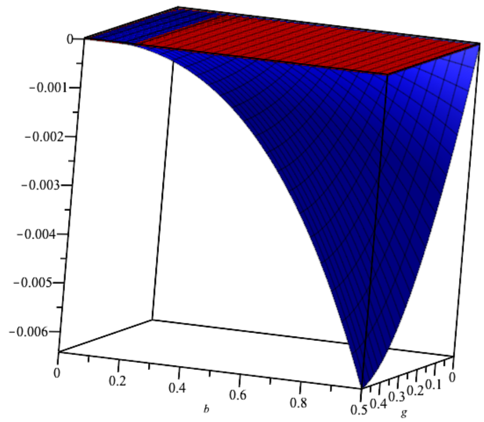

Figure 24: Proposition C.3: Difference $\tilde{w}_{i,j} - \breve{w}_{i,j}$ (blue) and zero (red) for all $\beta \in (0,1)$, $\gamma \in (0,1/2)$.

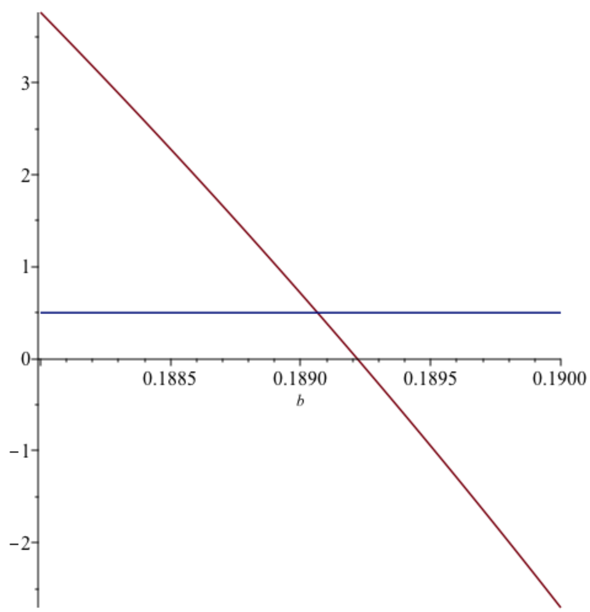

Figure 25: Proposition C.3: Fourth solution of $\breve{\pi}_j - \tilde{\pi}_j$ for $\beta \in [0.32, 0.335]$.

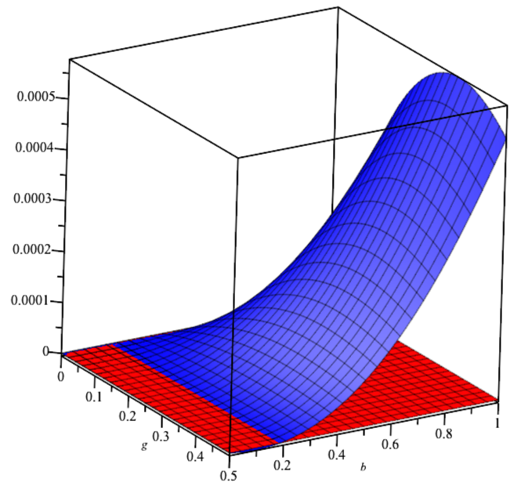

Figure 26: Proposition C.3: Difference $\breve{\pi}_j - \tilde{\pi}_j$ (blue) and zero (red) for all $\beta \in (0,1)$, $\gamma \in (0,1/2)$.

IV. From Proposition C.1, $\beta_T < \beta^T$ and, for every γ, $\underline{\beta}(\gamma) < \bar{\beta}(\gamma)$. ■

Propositions C.2 and C.3 are remarkably similar to Propositions 2 and 3, with the difference that the optimal response effect is not as strong, so $w_{i,j} > \tilde{w}_{i,j}$ for high values of β. Lemma C.1 proves that dispersion always falls when list prices cease to be published.

Lemma C.1. *For all $\beta \in (0,1)$, $\gamma \in (0,1/2)$, $\tilde{w}_j^L - \tilde{w}_{i,j} < w_j^L - w_{i,j}$ and $\tilde{w}_j^L - \tilde{w}_{i,j} < \breve{w}_j^L - \breve{w}_{i,j}$.*

Proof. First we solve

$$(w_j^L - w_{i,j})(\beta,\gamma) - (\breve{w}_j^L - \breve{w}_{i,j})(\beta,\gamma) = 0$$

for γ given β. We get the solutions

$$\gamma(\beta) = \begin{cases} 1 \\ \\ \frac{-5(153\beta - 530)}{734\beta} > 1 \;\; \forall \beta \in (0,1) \end{cases}$$

So dispersion is always larger in one equilibrium. Figure 27 shows that dispersion is larger in the public collusive equilibrium. Now solve

$$(w_j^L - w_{i,j})(\beta,\gamma) - (\tilde{w}_j^L - \tilde{w}_{i,j})(\beta,\gamma) = 0$$

for γ given β. We get the solutions

$$\gamma(\beta) = \begin{cases} 1 \\ \\ \frac{-455(229347\beta^2-1177280\beta+1352000)}{1468(72841\beta^2-147073\beta+39000)} \end{cases}$$

Figure 28 shows the second solution, and Figure 29 shows it is never in $(0, 1/2)$. Therefore, dispersion in one equilibrium is always larger. Figure 30 shows $w_j^L - w_{i,j} > \tilde{w}_j^L - \tilde{w}_{i,j}$ in all cases. By transitivity, $\check{w}_j^L - \check{w}_{i,j} > \tilde{w}_j^L - \tilde{w}_{i,j}$. ■

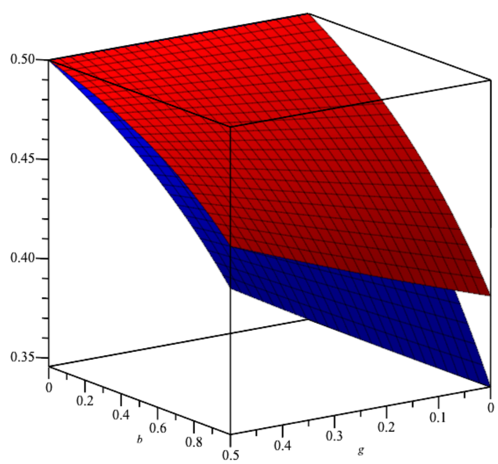

Figure 27: Lemma C.1: Wholesale price dispersion in the public competitive (blue) and public collusive (red) equilibria for all $\beta \in (0, 1)$, $\gamma \in (0, 1/2)$.

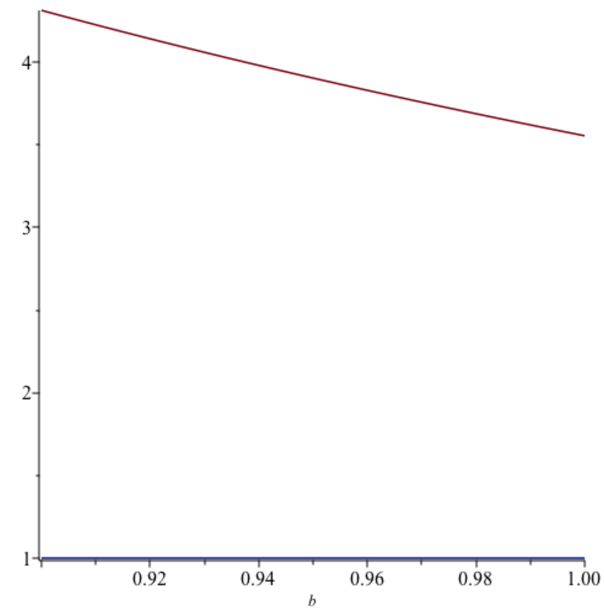

Figure 28: Lemma C.1: Second solution of $(w_j^L - w_{i,j}) - (\tilde{w}_j^L - \tilde{w}_{i,j})$ for all $\beta \in (0, 1)$.

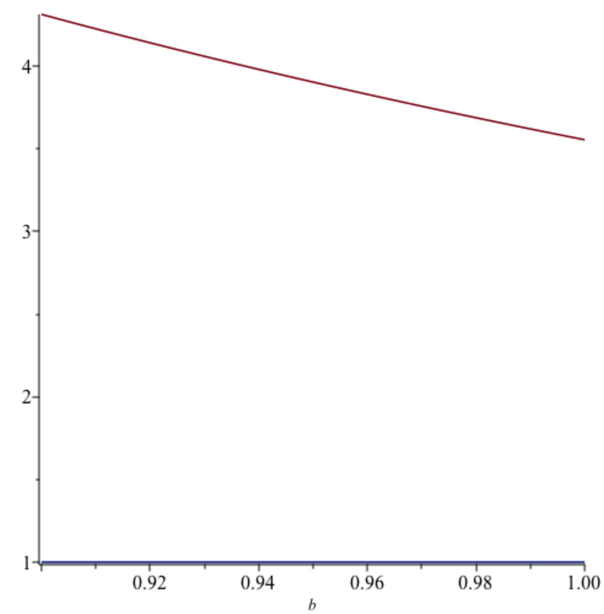

Figure 29: Lemma C.1: Second solution of $(w_j^L - w_{i,j}) - (\tilde{w}_j^L - \tilde{w}_{i,j})$ for $\beta \in [0.9, 1)$.

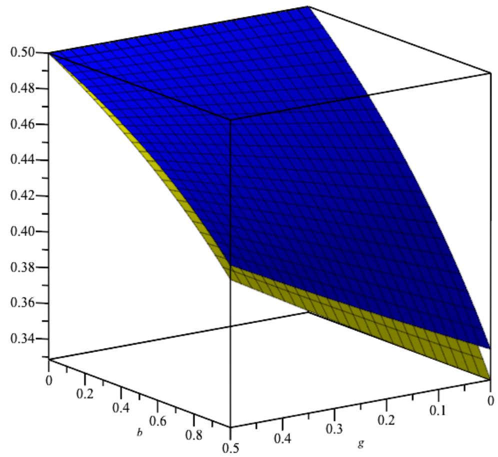

Figure 30: Lemma C.1: Wholesale price dispersion in the public competitive (blue) and private (yellow) equilibria for all $\beta \in (0,1)$, $\gamma \in (0,1/2)$.

Appendix D: Responsive Fringe

The model in this appendix differs from the baseline model in that the fringe's input demand/downstream supply function depends on the retail price. The rest of the model, including timing in the public and private settings, is the same.

Fringe input demand/downstream supply. Fringes sell at marginal cost. Their supply function solves

$$p = w_j^L + \frac{q_j^F}{\gamma} \quad j = 1, 2$$

with $\gamma \in (0, 1)$ to ensure all prices and quantities are non-negative.[15] Therefore

$$q_j^F(w_j^L) = \gamma(p - w_j^L) \quad j = 1, 2. \tag{7.11}$$

Note that γ indexes the efficiency of the fringe, but not necessarily its size, because all equilibrium variables may depend on γ. Therefore, it is not obvious that the fringes market share will converge to 100% as $\gamma \to +\infty$, and note that the industrial markets we study are characterized by a few large firms that serve most of the market. Inserting (7.11) in the inverse demand function, define

$$\tilde{a} = a + \gamma(w_1^L + w_2^L). \tag{7.12}$$

Therefore, the inverse demand function can be expressed as

$$p = \frac{\tilde{a} - q_A - q_B}{(1 + 2\gamma)}. \tag{7.13}$$

7.10 Public Competitive Equilibrium

Solving by backward induction, in τ_2 R_i solves

$$\max_{q_i \geq 0} \left[\frac{\tilde{a} - q_i - q_{-i} - w_i(1 + 2\gamma)}{1 + 2\gamma} \right] q_i$$

so

[15] In particular, $\gamma < 1$ ensures fringe supply is never negative when suppliers collude.

$$q_i = \frac{\tilde{a} + (1+2\gamma)(w_{-i} - 2w_i)}{3} \tag{7.14}$$

$$p = \frac{\tilde{a} + (1+2\gamma)(w_{-i} + w_i)}{3(1+2\gamma)} \tag{7.15}$$

$$\pi_i(w_i, w_{-i}) = \frac{\left[\tilde{a} + (1+2\gamma)(w_{-i} - 2w_i)\right]^2}{9(1+2\gamma)}. \tag{7.16}$$

Before solving the Nash bargaining problem, we have to define what the optimal response of all players is when bargaining between retailer R_i and supplier U_j breaks down. Consistent with the model in the main body, input prices will remain the same, but the other R_{-i}'s residual demand will change. Therefore, R_{-i} will change its supply, the retail price will change, and fringe input demand will change with it. So, if negotiations between U_j and R_i fail, R_{-i} downstream supply is

$$q_{-i}^{Off} = \frac{\tilde{a}^{Off} - (1+2\gamma)w_{-i}}{2}$$

$$\Rightarrow p^{Off} = \frac{\tilde{a}^{Off} + (1+2\gamma)w_{-i}}{2(1+2\gamma)}$$

$$\Rightarrow q_j^{F,Off} = \gamma\left(\frac{\tilde{a}^{Off} + (1+2\gamma)w_{-i}}{2(1+2\gamma)} - w_j^L\right)$$

With $\tilde{a}^{Off} = \tilde{a}$. Then, on and off-path profits for retailers and suppliers are

$$\hat{\pi}_i = \frac{\left[\tilde{a} + (1+2\gamma)(w_{-i} - 2w_i)\right]^2}{9(1+2\gamma)}$$

$$\bar{\pi}_i = 0$$

$$\hat{\pi}_j = w_i \cdot q_i + w_j^L\gamma\left(\frac{\tilde{a} + (1+2\gamma)(w_{-i} + w_i)}{3(1+2\gamma)} - w_j^L\right)$$

$$\bar{\pi}_j = w_j^L\gamma\left(\frac{\tilde{a} + (1+2\gamma)w_{-i}}{2(1+2\gamma)} - w_j^L\right).$$

Therefore $\hat{\pi}_i - \bar{\pi}_i = \hat{\pi}_i$, and $\hat{\pi}_j - \bar{\pi}_j$ is

$$\hat{\pi}_j - \bar{\pi}_j = q_i\left[w_i - \frac{\gamma w_j^L}{2(1+2\gamma)}\right].$$

That is, the transaction price has to be at least $\gamma w_j^L/(1+2\gamma)$ for bargaining to be profitable for the supplier. Since $\beta < 1$, w_i will always be smaller than the one U_j would make in a take-ti-or-leave-it offer in $\tau 1$, w_i^M. To find w_i^M we solve

$$\max_{w_i \geq 0} q_i \left[w_i - \frac{\gamma w_j^L}{2(1+2\gamma)} \right].$$

The first-order condition is

$$\frac{\tilde{a} + (1+2\gamma)(w_{-i} - 4w_i)}{3} - \frac{\gamma w_j^L}{3} = 0$$

$$\Rightarrow w_i^M = \frac{\tilde{a} + (1+2\gamma)w_{-i} - \gamma w_j^L}{4}.$$

We want to prove

$$w_i^M > \frac{\gamma w_j^L}{(1+2\gamma)} \iff \frac{\gamma w_{-j}^L + (1+2\gamma)w_{-i}}{2} + \frac{a}{2} + \gamma a > \gamma w_j^L.$$

But $p = a - Q$, so $p \leq a$, and $q_j^F \geq 0$, so $a \geq p \geq w_j^L$ for all j. Thus, there is a range with positive measure over which both parties can agree on a price. The Nash bargaining problem can then be described by the payoff set $B_i^j = \left\{ [\hat{\pi}_i - \bar{\pi}_i, \hat{\pi}_j - \bar{\pi}_j] \, | w_i \in \left[\gamma w_j^L/(2+4\gamma), w_i^M \right] \right\}$ and the disagreement point $(0, 0)$.

Lemma D.1. *The utility possibility set is convex.*

Proof. It is sufficient that both payoff functions are quasi-concave. Given all other prices (passive beliefs simplify the proof), retailer R_i's profit as a function of w_i is

$$\hat{\pi}_i(w_i) = \frac{[\tilde{a} + (1+2\gamma)(w_{-i} - 2w_i)]^2}{9(1+2\gamma)}$$

with $\tilde{a} > 0$ because list prices are non-negative. Its first derivative is

$$\frac{\partial \hat{\pi}}{\partial w_i} = \frac{-4(a + (1+2\gamma)(w_{-i} - 2w_i))}{9}$$

which is non-positive for all w_i in the interval $[0, (a + w_{-i}(1+2\gamma))/(2(1+2\gamma))]$ and negative in its interior. Since transaction prices cannot be negative, and a price above the upper bound implies a negative retailer supply, the first derivative is non-positive. Since the function is monotonous, it is quasi-concave. The first and second derivatives of $\hat{\pi}_j - \bar{\pi}_j$ are

$$\frac{\partial(\hat{\pi}_j - \bar{\pi}_j)}{\partial w_i} = \frac{a + (1+2\gamma)(w_{-i} - 4w_i)}{3} - \frac{\gamma w j^L}{3}$$

$$\frac{\partial^2(\hat{\pi}_j - \bar{\pi}_j)}{\partial w_i^2} = \frac{-4(1+2\gamma)}{3} < 0$$

so the function is quasi-concave. ■

By Lemma D.1 there is a unique solution to the Nash bargaining problem

$$\max_{w_i \geq \gamma w_j^L/(2+4\gamma)} \left[\left(\frac{\tilde{a} + (1+2\gamma)(w_{-i} - 2w_i)}{3} \right) \left(w_i - \frac{\gamma w_j^L}{(1+2\gamma)} \right) \right]^{\beta} \cdot \left[\frac{(\tilde{a} + (1+2\gamma)(w_{-i} - 2w_i))^2}{9(1+2\gamma)} \right]^{1-\beta}.$$

Since $\beta > 0$, $w_i > \gamma w_j^L/(1+2\gamma)$ and $\hat{\pi}_j - \bar{\pi}_j > 0$, so we can use the logarithmic transformation. The first-order condition is

$$\frac{-2(1+2\gamma)(2-\beta)}{\tilde{a} + (1+2\gamma)(w_{-i} - 2w_i)} + \frac{\beta}{w_i - \frac{\gamma w_j^L}{(1+2\gamma)}} = 0.$$

Solving for w_i we get

$$w_i = \frac{\beta\left(\tilde{a} + (1+2\gamma)w_{-i}\right) + (2-\beta)_j^L}{4(1+2\gamma)}.$$

So

$$w_i = \frac{\tilde{a}\beta(4-\beta) + \gamma(2-\beta)\left(4w_j^L + \beta w_{-j}^L\right)}{(16-\beta^2)(1+2\gamma)}.$$

Notice how both transaction prices are strictly increasing in each other, and strictly increasing in both list prices. Given the list price of the competition, in τ_0 upstream supplier U_j sets list price w_j^L to solve

$$\max_{w_j^L \in \mathbb{R}_+} \pi_j(w_j^L) = w_i(w_j^L, w_{-j}^L) \cdot q_i(w_j^L, w_{-j}^L) + w_j^L \gamma(p(w_j^L, w_{-j}^L) - w_j^L). \tag{7.17}$$

In doing so U_j considers two forces: the bargaining and fringe effects. The bargaining effect measures how the list price affects U_j's bargaining position, and the fringe effect how much the list price affects U_j's profits from selling to its fringe. The first-order condition of (7.17) is

$$\frac{\partial \pi_j}{\partial w_j^L} = \underbrace{\frac{\partial\left[w_i(w_j^L,\cdot)\cdot q_i(w_j^L,\cdot)\right]}{\partial w_j^L}}_{\text{bargaining effect}} + \underbrace{\gamma\left[(p-w_j^L)+w_j^L\frac{\partial(p-w_j^L)}{\partial w_j^L}\right]}_{\text{fringe effect}} = 0.$$

The second-order condition is

$$\frac{\partial^2 \pi_j}{\partial (w_j^L)^2} = -\frac{6\gamma\left[\left(\gamma+\frac{1}{3}\right)\beta^4-(16\gamma+\frac{32}{3})\beta^2-16\beta\gamma+128\gamma+\frac{256}{3}\right]}{((\beta^2-16)^2(1+2\gamma))} < 0$$

where the sign comes from $\beta,\gamma \in (0,1)$. Therefore, the function is concave, and the unique maximizer of (7.17) is a function of w_{-j}^L. This best response function is

$$w_j^L = \frac{(16+\beta^2-\beta^3)(a\beta+6\gamma w_{-j}^L+4a)}{(3\gamma+1)\beta^4+(-48\gamma-32)\beta^2-48\beta\gamma+384\gamma+256}. \tag{7.18}$$

So symmetric public competitive equilibrium list prices are

$$w_j^L = \frac{a(16+\beta^2-\beta^3)}{3\beta^3\gamma+\beta^3-6\beta^2\gamma-4\beta^2-30\beta\gamma-16\beta+72\gamma+64}. \tag{7.19}$$

Note that in list prices a is just a scale parameter so all prices and quantities are scaled by a. Therefore profits are scaled by a^2, and we can normalize a to 1 for simplicity. We call this the *public competitive equilibrium.*

7.11 Public Collusive Equilibrium

Now suppliers maximize joint profits, so the problem is

$$\begin{aligned}\max_{(w_1^L,w_2^L)\in\mathbb{R}_+^2} \pi_1+\pi_2 = {} & w_A(w_1^L,w_2^L)\cdot q_A(w_1^L,w_2^L)+w_1^L\gamma(p(w_1^L,w_2^L)-w_1^L) \\ & + w_B(w_1^L,w_2^L)\cdot q_B(w_1^L,w_2^L)+w_2^L\gamma(p(w_1^L,w_2^L)-w_2^L). \end{aligned} \tag{7.20}$$

Using (7.15), the first-order condition with respect to list price $\hat{w}_j^L$ is

$$\frac{\partial(\pi_1+\pi_2)}{\partial w_j^L} = \underbrace{\frac{\partial\left[w_i(w_j^L,\cdot)\cdot q_i(w_j^L,\cdot)\right]}{\partial w_j^L}}_{\text{bargaining effect}} + \underbrace{\gamma\left[(p-w_j^L)+w_j^L\frac{\partial(p-w_j^L)}{\partial w_j^L}\right]}_{\text{fringe effect}}$$
$$+\underbrace{\frac{\partial\left[w_{-i}(w_j^L,\cdot)\cdot q_{-i}(w_j^L,\cdot)\right]}{\partial w_j^L}+\gamma w_{-j}^L\cdot\frac{\partial p}{\partial w_j^L}}_{\text{externality}}=0$$

where the new terms reflect the externality over U_{-j} that was previously not internalized. Simplifying we get

$$\frac{\partial(\pi_1+\pi_2)}{\partial w_j^L} = \frac{-2((3\gamma w_j^L+a+w_j^L)\beta^4+(12\gamma w_{-j}^L+7a)\beta^3}{(16-\beta^2)^2(1+2\gamma)}$$
$$+\frac{((-24w_j^L-12w_{-j}^L)\gamma+4a-32w_j^L)\beta^2+(-96\gamma w_j^L-48a)\beta+(384w_j^L-192w_{-j}^L)\gamma-64a+256w_j^L)\gamma}{(16-\beta^2)^2(1+2\gamma)}=0.$$

The second partial derivatives are symmetric

$$\frac{\partial^2(\pi_1+\pi_2)}{\partial(w_j^L)^2} = -\frac{4\gamma(3\beta^2\gamma+\beta^2-12\beta\gamma-8\beta+12\gamma+16)(\beta+4)^2}{(\beta^2-16)^2(1+2\gamma)}<0$$

and

$$\frac{\partial^2(\pi_1+\pi_2)}{\partial w_j^L\partial w_{-j}^L} = -\frac{4\gamma(3\beta^2\gamma+\beta^2-12\beta\gamma-8\beta+12\gamma+16)(\beta+4)^2}{(\beta^2-16)^2(1+2\gamma)}<0.$$

The eigenvalues of the Hessian matrix are

$$k=\frac{\partial^2(\pi_1+\pi_2)}{\partial(w_j^L)^2}\pm\frac{\partial^2(\pi_1+\pi_2)}{\partial w_j^L\partial w_{-j}^L}.$$

So one is zero and the other is strictly negative. Therefore, the Hessian matrix is negative semidefinite and (7.20) is concave. Therefore, the first-order conditions find maximizers of (7.20). In particular, there is an equilibrium with symmetric collusive list prices

$$\check{w}_j^L=\frac{a(4+\beta+\beta^2)}{3\beta^2\gamma+\beta^2-12\beta\gamma-8\beta+12\gamma+16}. \tag{7.21}$$

Again, all prices and quantities are scaled by a, so we can normalize it to 1. We call this the *public collusive equilibrium.*

7.12 Sustainability of Collusion

Assume both suppliers have a common discount factor $\delta \in (0,1)$, and again consider the grim trigger strategy. Let π^C and π^M be a supplier's profits in competition and collusion, and $\pi^D(x)$ the profit of deviating to a list price x. The incentive compatibility constraint is

$$\frac{\pi^M}{1-\delta} \geq \pi(x) + \frac{\delta \pi^C}{1-\delta} \quad \forall x \geq 0 \tag{7.22}$$

so we only have to consider the best deviation list price. That price is given by the best response function, expression (7.18). Let w^D be this list price. The critical discount factor is given by

$$\underline{\delta} = \frac{\pi^D(w^D) - \pi^M}{\pi^D - \pi^C}. \tag{7.23}$$

Since the cheater is playing its best response, $\pi^D(w^D) \geq \pi^M > \pi^C$. Therefore, by (7.23) there exists $\underline{\delta} \in [0,1)$ such that, for all $\delta \geq \underline{\delta}$ collusion is sustainable with a grim trigger strategy.

7.13 Welfare Effects of Collusion

Proposition D.1. *For all β, γ, collusion (i) raises list prices, (ii) raises transaction prices, (iii) reduces fringe supply, (iv) increases retailer supply, (v) reduces total supply, (vi) increases retail prices, (vii) increases suppliers' profits and (viii) increases retailers' profits.*

Proof. Beginning with list prices, we solve

$$w_j^L(\beta,\gamma) - \check{w}_j^L(\beta,\gamma) = 0$$

for γ given β. The solution is

$$\gamma(\beta) = \frac{2\beta(2-\beta)}{3(\beta^2 - 2\beta - 1)} < 0 \;\; \forall \beta \in (0,1)$$

So either $w_j^L > \check{w}_j^L$ or $w_j^L < \check{w}_j^L$ for all parameters. Figure 31 shows that $w_j^L < \check{w}_j^L$ is true for some parameters, so it is true for all parameters.

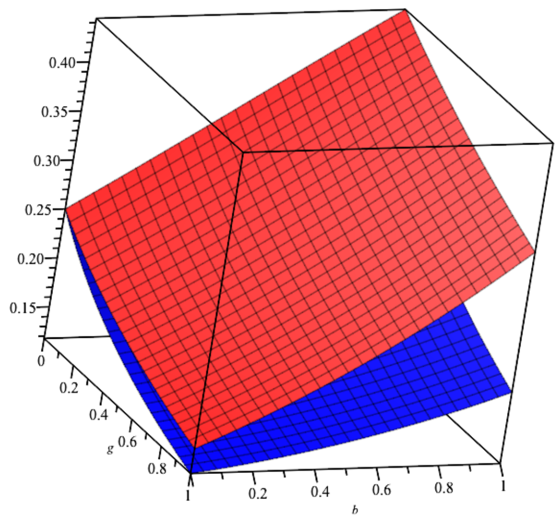

Figure 31: Proposition D.1: List prices in the public competitive (blue) and public collusive (red) equilibria for $\beta, \gamma \in (0, 1)$.

We know that transaction prices are strictly increasing in both list prices, so $\breve{w}_i > w_i$ for all β, γ. Using (7.15) we see that as list and transaction prices increase, retail prices must also increase (the increase in list prices raises $\tilde{a}$). Therefore, total downstream supply falls. By (7.14), retailer supply increases with list prices and, by the symmetry of equilibrium transaction prices, falls with transaction prices. We solve

$$q_i(\beta, \gamma) - \breve{q}_i(\beta, \gamma) = 0$$

for γ given β. The solutions are

$$\gamma(\beta) = \begin{cases} 0 \\ \\ \frac{2\beta(2-\beta)}{3(\beta^2-2\beta-1)} < 0 \;\; \forall \beta \in (0,1) \end{cases}$$

so one is always larger than the other. Figure 32 shows that $q_i < \breve{q}_i$ for all parameters. Since total supply falls while retailers supply increases, fringe supply must fall. The result for suppliers' profits follows from joint maximization. The results for retailers' profits come from $\pi_i = q_i^2/(1 + 2\gamma)$. ■

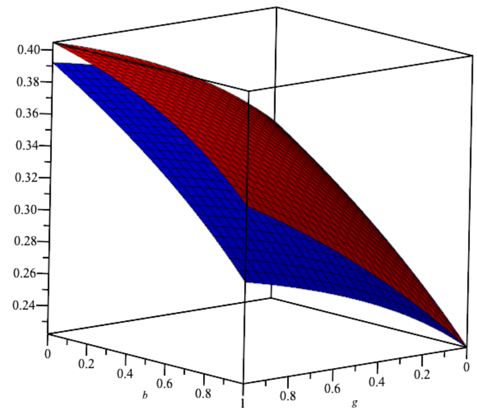

Figure 32: Proposition D.1: Retailer supply in the public competitive (blue) and public collusive (red) equilibria for $\beta, \gamma \in (0,1)$.

As we can see, Proposition 1 and Proposition D.1 have the same results.

7.14 Private Equilibrium

Given w_1^L, w_2^L, w_A and w_B, the equilibrium in τ_2' is the same as in the model with public prices. In τ_1', U_j knows $\tilde{w}_i$ and solves

$$\max_{w_j^L \in \mathbb{R}_+} \pi_j(w_j^L) = \gamma \cdot w_j^L \cdot \left[\frac{\Big(a + \gamma(w_j^L + w_{-j}^L) + (1+2\gamma)\,(w_{-i} + w_i)\Big)}{3(1+2\gamma)} - w_j^L \right] \\ + w_i \frac{\Big(a + \gamma(w_j^L + w_{-j}^L) + (1+2\gamma)\,(w_{-i} - 2w_i)\Big)}{3}.$$

The first and second-order conditions are

$$\frac{\partial \pi_j}{\partial w_j^L} = \gamma \left\{ \frac{w_i}{3} + \left[\frac{\Big(a + \gamma(w_j^L + w_{-j}^L) + (1+2\gamma)\,(w_{-i} + w_i)\Big)}{3(1+2\gamma)} - w_j^L \right] + w_j^L \left[\frac{\gamma}{3(1+2\gamma)} - 1 \right] \right\} = 0$$

$$\frac{\partial^2 \pi_j}{\partial (w_j^L)^2} = \frac{-2\gamma^2}{3(1+2\gamma)} - 2\gamma < 0$$

so the function is concave. The best response function is

$$w_j^L = \frac{a + (1+2\gamma)(2w_i + w_{-i})}{6+10\gamma} + \frac{\gamma w_{-j}^L}{6+10\gamma}$$

so equilibrium list prices, as functions of transaction prices, are

$$w_j^L = \frac{[6+10\gamma]\left[a + (1+2\gamma)(2w_i + w_{-i})\right] + \gamma\left[a + (1+2\gamma)(2w_{-i} + w_i)\right]}{[6+10\gamma]^2 - \gamma^2}. \tag{7.24}$$

As in the baseline model we assume that, if negotiations break down, U_j will set the off-path list price according to (7.24) using as the argument the last discussed transaction price. On and off-path profits are

$$\hat{\pi}_i = [p(w_i, w_{-i}) - w_i] \cdot q_i(w_i, w_{-i})$$

$$\bar{\pi}_i = 0$$

$$\hat{\pi}_j = q_i(w_i, w_{-i}) \cdot w_i + w_j^L(w_i, w_{-i}) \cdot \gamma \cdot \left(p(w_i, w_{-i}) - w_j^L(w_i, w_{-i})\right)$$

$$\bar{\pi}_j = w_j^{L,Off} \cdot \gamma \cdot \left(p^{Off} - w_j^{L,Off}\right)$$

with $w_j^{L,Off} = w_j^L(w_i, w_{-i})$. Since $\tilde{a}^{Off} = \tilde{a}$ and $p^{Off} > p$, this defines a minimum w_i such that an agreement can be reached

$$w_i \geq \frac{\gamma w_j^{L,Off}}{2(1+2\gamma)}. \tag{7.25}$$

An agreement is possible if the transaction price U_j would set in a take-it-or-leave-it offer is larger than the minimum price. Let $w^{Min} \geq 0$ be the smallest transaction price such that (7.25) obtains. In our solution to the Nash bargaining problem we show that a unique solution to the bargaining problem exists. The bargaining problem is described by the set of payoffs $\tilde{B}_i^j = \left\{[\hat{\pi}_i, \hat{\pi}_j - \bar{\pi}_j] \,|w_i \geq w_i^{Min}\right\}$ and disagreement point $(0,0)$. The problem is

$$\max_{w_i \geq w_i^{Min}} \left[\hat{\pi}_j - \bar{\pi}_j\right]^\beta \cdot \left[\hat{\pi}_i\right]^{1-\beta}.$$

Since $\beta > 0$, $w_i > w_i^{Min}$, and we can use the logarithmic transformation. Solving the symmetric first-order condition

$$\left(\frac{\beta}{(\hat{\pi}_j - \bar{\pi}_j)(w_i, w_{-i})} \frac{\partial(\hat{\pi}_j - \bar{\pi}_j)(w_i, w_{-i})}{\partial w_i} + \frac{\beta}{\hat{\pi}_i(w_i, w_{-i})} \frac{\partial \hat{\pi}_i(w_i, w_{-i})}{\partial w_i}\right)\Big|_{w_i = w_{-i}} = 0$$

we get the private equilibrium transaction price

$$\tilde{w}_i = \frac{(-297\beta - 55)\gamma^3 + (-558\beta - 74)\gamma^2 + (-348\beta - 24)\gamma - 72\beta}{(648\beta - 1650)\gamma^4 + (1512\beta - 4365)\gamma^3 + (1314\beta - 4266)\gamma^2 + (504\beta - 1824)\gamma + 72\beta - 288}. \tag{7.26}$$

Welfare Effects of Publishing List Prices

Proposition D.2. *Relative to the public competitive equilibrium, in the private equilibrium, for all β, $\gamma \in (0,1)$, (i) list prices are lower, (ii) transaction prices are lower, (iii) fringe supply is higher, (iv) retailer supply is lower, (v) the retail price is lower, (vi) total supply is higher, (vii) suppliers' profits are lower and (viii) retailers' profits are lower.*

Proof. Beginning with transaction prices, consider

$$w_i(\beta,\gamma) - \tilde{w}_i(\beta,\gamma) = \frac{(-2\beta^3 - \beta^2 + 24\beta + 8)\gamma - \beta^3 + 16\beta}{6(\gamma + \frac{1}{2})((\beta^3 - 2\beta^2 - 10\beta + 24)\gamma + \frac{(\beta+4)(\beta-4)^2}{3}}$$
$$+ \frac{(297\beta + 55)\gamma^3 + (558\beta + 74)\gamma^2 + (348\beta + 24)\gamma + 72\beta}{(648\beta - 1650)\gamma^4 + (1512\beta - 4365)\gamma^3 + (1314\beta - 4266)\gamma^2 + (504\beta - 1824)\gamma + 72\beta - 288}.$$

Multiplying each term by the denominator of the other, the resulting denominator is

$$\varphi_0 = \gamma(1+2\gamma)(243\beta^4\gamma^3 + 459\beta^4\gamma^2 - 291\beta^3\gamma^3 + 288\beta^4\gamma - 488\beta^3\gamma^2 - 639\beta^2\gamma^3 + 60\beta^4 - 268\beta^3\gamma -$$
$$946\beta^2\gamma^2 + 2526\beta\gamma^3 - 48\beta^3 - 416\beta^2\gamma + 5156\beta\gamma^2$$
$$- 2640\gamma^3 - 48\beta^2 + 3472\beta\gamma - 5312\gamma^2 + 768\beta - 3520\gamma - 768).$$

Taking limits as $\beta \to 0$ and $\beta \to 1$ we get

$$\lim_{\beta\to 0} \varphi_0 = -16\gamma(1+2\gamma)(165\gamma^3 + 332\gamma^2 + 220\gamma + 48)$$
$$\lim_{\beta\to 1} \varphi_0 = -3\gamma(1+2\gamma)(267^{\gamma}3 + 377\gamma^2 + 148\gamma + 12)$$

both of which are negative for all $\gamma \in (0,1)$. If the first partial derivative with respect to β is of constant sign for all β, $\gamma \in (0,1)$, then the numerator is never zero, and one transaction price is always higher than the other. The first partial derivative is

$$\frac{\partial \varphi_0}{\partial \beta} = \gamma(1+2\gamma)(972\beta^3\gamma^3 + 1836\beta^3\gamma^2 - 873\beta^2\gamma^3 + 1152\beta^3\gamma - 1464\beta^2\gamma^2 - 1278\beta\gamma^3 + 240\beta^3 - 804\beta^2\gamma - 1892\beta\gamma^2 + 2526\gamma^3 - 144\beta^2 - 832\beta\gamma + 5156\gamma^2 - 96\beta + 3472\gamma + 768)$$

which is positive for all β, $\gamma \in (0,1)$ (Figure 33 graphs the derivative). So, either $w_i > \tilde{w}_i$ for all parameter values, or the reverse is true. Figure 34 shows that $w_i > \tilde{w}_i$ is true for some parameter values, so it is true for all of them.

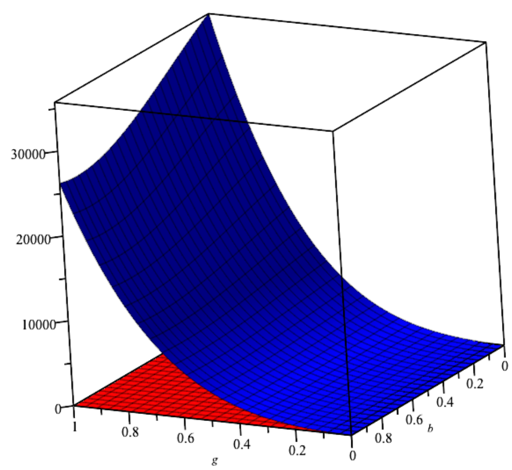

Figure 33: Proposition D.2: First partial derivative of φ_0 with respect to β for all β, $\gamma \in (0,1)$ (blue). Zero marked in red.

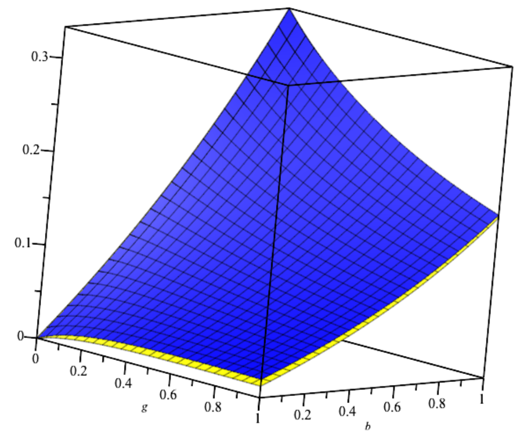

Figure 34: Proposition D.2: Transaction prices in the public competitive (blue) and private (yellow) equilibria for all β, $\gamma \in (0,1)$.

For list prices, consider the numerator of $w_j^L - \tilde{w}_j^L$

$$\begin{aligned}\varphi_1 = &(-135\gamma^3 - 297\gamma^2 - 210\gamma - 48)\beta^4 + (1101\gamma^3 + 2198\gamma^2 + 1444\gamma + 312)\beta^3 \\ &+ (-3375\gamma^3 - 6446\gamma^2 - 4096\gamma - 864)\beta^2 + (6420\gamma^3 + 12952\gamma^2 + 8672\gamma + 1920)\beta \\ &- 5280\gamma^3 - 10624\gamma^2 - 7040\gamma - 1536.\end{aligned}$$

Taking limits as $\beta \to 0$ and $\beta \to 1$ we get

$$\lim_{\beta \to 0} \varphi_1 = -5280\gamma^3 - 10624\gamma^2 - 7040\gamma - 1536$$

$$\lim_{\beta \to 1} \varphi_1 = -1269\gamma^3 - 2217\gamma^2 - 1230\gamma - 216$$

both of which are negative for all $\gamma \in (0,1)$. If the first partial derivative with respect to β is of constant sign for all β, $\gamma \in (0,1)$, then the numerator is never zero, and one list price is always higher than the other. The first partial derivative is

$$\frac{\partial \varphi_1}{\partial \beta} = 4(-135\gamma^3 - 297\gamma^2 - 210\gamma - 48)\beta^3 + 3(1101\gamma^3 + 2198\gamma^2 + 1444\gamma + 312)\beta^2$$
$$+ 2(-3375\gamma^3 - 6446\gamma^2 - 4096\gamma - 864)\beta + (6420\gamma^3 + 12952\gamma^2 + 8672\gamma + 1920)$$

which is positive for all β, $\gamma \in (0, 1)$ (Figure 35 graphs the derivative). Figure 36 shows that the public competitive list price is always higher. Since the retail price is strictly increasing in both list and transaction prices, the public competitive retail price is higher and total downstream supply is lower.

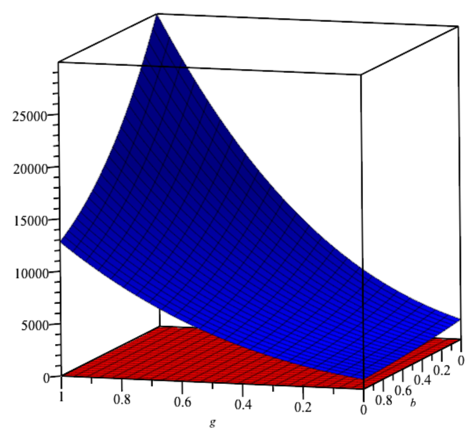

Figure 35: Proposition D.2: First partial derivative of φ_1 with respect to β for all β, $\gamma \in (0, 1)$ (blue). Zero marked in red.

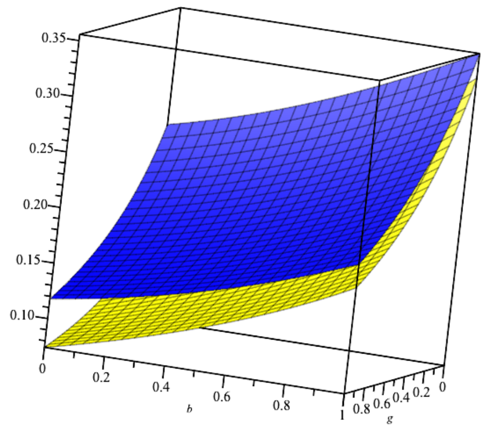

Figure 36: Proposition D.2: List prices in the public competitive (blue) and private (yellow) equilibria for all β, $\gamma \in (0, 1)$.

For retailer supply, taking the numerator of $q_i - \tilde{q}_i$ we get

$$\varphi_2 = (-171\beta^4+831\beta^3-2037\beta^2+3438\beta-2640)\gamma^4+(-351\beta^4+1628\beta^3-3982\beta^2+6916\beta-5312)\gamma^3 + (-236\beta^4 + 1052\beta^3 - 2592\beta^2 + 4624\beta - 3520)\gamma^2 + (-52\beta^4 + 224\beta^3 - 560\beta^2 + 1024\beta - 768)\gamma.$$

Taking limits as $\beta \to 0$ and $\beta \to 1$ we get

$$\lim_{\beta\to 0} \varphi_2 = -2640\gamma^4 - 5312\gamma^3 - 3520\gamma^2 - 768\gamma < 0$$
$$\lim_{\beta\to 1} \varphi_2 = -579\gamma^4 - 1101\gamma^3 - 672\gamma^2 - 132\gamma.$$

Since both are negative, the numerator will never be zero if the first partial derivative with respect to β has a constant sign. The derivative is

$$\frac{\partial\varphi_2}{\partial\beta} = (-684\beta^3 + 2493\beta^2 - 4074\beta + 3438)\gamma^4 + (-1404\beta^3 + 4884\beta^2 - 7964\beta + 6916)\gamma^3 + (-944\beta^3 + 3156\beta^2 - 5184\beta + 4624)\gamma^2 + (-208\beta^3 + 672\beta^2 - 1120\beta + 1024)\gamma$$

where the coefficient of each power of γ is a strictly positive function of β for all $\beta \in (0, 1)$. Therefore, one retailer supply must always be larger than the other. Figure 37 shows that $q_i > \tilde{q}_i$

for all parameter values. By (7.16) retailer profits can be rewritten as $q_i^2/(1+2\gamma)$, so retailer profits are also higher in the public competitive equilibrium. Since in the public competitive equilibrium total supply is lower, but retailer supply is higher, fringe supply must be lower.

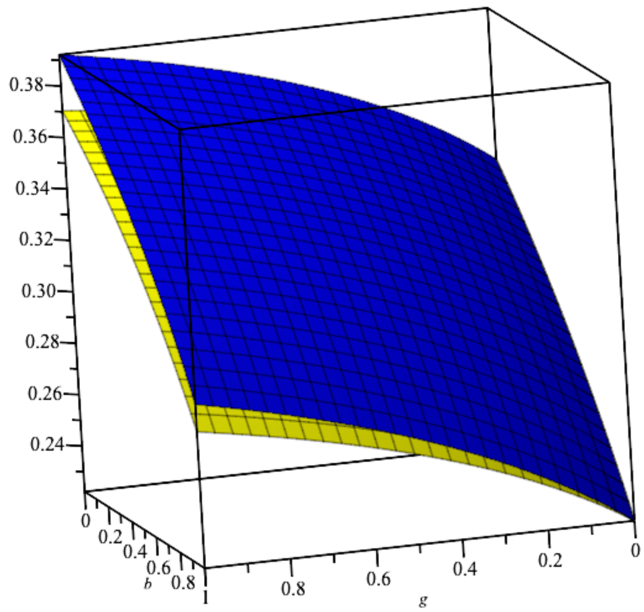

Figure 37: Proposition D.2: Retailer supply in the public competitive (blue) and private (yellow) equilibria for all β, $\gamma \in (0, 1)$.

For supplier profit take the numerator of $\pi_j - \tilde{\pi}_j$

$$
\begin{aligned}
\varphi_3 = {} & 505\gamma((-134261/4545\beta^6+320966/1515\beta^5-276827/505\beta^4+613064/909\beta^3-262208/1515\beta^2 \\
& -63184/101\beta+\beta^8-5146/1515\beta^7+542080/909)\gamma^7+(-1856083/13635\beta^6+13213754/13635\beta^5 \\
& -34753319/13635\beta^4+44643896/13635\beta^3-15043136/13635\beta^2-12169664/4545\beta+2181/505\beta^8 \\
& -64246/4545\beta^7+7318784/2727)\gamma^6+(-340394/13635\beta^7-10945678/40905\beta^6 \\
& +25858744/13635\beta^5-207379456/40905\beta^4+277572704/40905\beta^3 \\
& -12731008/4545\beta^2-197200448/40905\beta+35887/4545\beta^8+209007104/40905)\gamma^5+(35999/4545\beta^8 \\
& -198310/8181\beta^7-35714552/122715\beta^6+50506976/24543\beta^5-686029036/122715\beta^4+191038304/24543\beta^3 \\
& -462466816/122715\beta^2-577923584/122715\beta+130731008/24543)\gamma^4+(80445440/24543 \\
& +192496/40905\beta^8-571192/40905\beta^7-23222936/122715\beta^6+164070784/122715\beta^5-452810176/122715\beta^4 \\
& +654982912/122715\beta^3-360937984/122715\beta^2-65665024/24543\beta)\gamma^3+(145752064/122715 \\
& +67684/40905\beta^8-13000/2727\beta^7-9031232/122715\beta^6+63811936/122715\beta^5-178855168/122715\beta^4 \\
& +53695232/24543\beta^3-164706304/122715\beta^2-1060864/1215\beta)\gamma^2+(9568256/40905+13024/40905\beta^8 \\
& -2432/2727\beta^7-648512/40905\beta^6+4584704/40905\beta^5-1449472/4545\beta^4+20301824/40905\beta^3 \\
& -13631488/40905\beta^2-1212416/8181\beta)\gamma+(352(\beta-4)^2(\beta^6+58/11\beta^5-334/11\beta^4+800/11\beta^3-960/11\beta^2 \\
& +512/11))/13635).
\end{aligned}
$$

Taking the limits as $\beta \to 0$ and $\beta \to 1$

$$
\lim_{\beta\to 0}\varphi_3 = \frac{128\gamma(571725\gamma^7+2573010\gamma^6+4898604\gamma^5+5106680\gamma^4+3142400\gamma^3+1138688\gamma^2)}{243} + \frac{+128\gamma(224256\gamma+18432)}{243} > 0
$$

$$
\lim_{\beta\to 1}\varphi_3 = \frac{\gamma(1416351\gamma^7+6156189\gamma^6+11117729\gamma^5+10804975\gamma^4+6094288\gamma^3+1990916\gamma^2)}{27} + \frac{+\gamma(348000\gamma+25056)}{27} > 0.
$$

If the first partial derivative with respect to β is of constant sign for all γ, then the numerator will never be zero. The first partial derivative is

$$\frac{\partial\varphi_3}{\partial\beta} = 505\gamma((-268522/1515\beta^5 + 320966/303\beta^4 - 1107308/505\beta^3 + 613064/303\beta^2$$
$$- 524416/1515\beta - 63184/101 + 8\beta^7 - 36022/1515\beta^6)\gamma^7 + (-3712166/4545\beta^5 + 13213754/2727\beta^4$$
$$- 139013276/13635\beta^3 + 44643896/4545\beta^2 - 30086272/13635\beta - 12169664/4545 + 17448/505\beta^7$$
$$-449722/4545\beta^6)\gamma^6 + (-2382758/13635\beta^6 - 21891356/13635\beta^5 + 25858744/2727\beta^4 - 829517824/40905\beta^3$$
$$+277572704/13635\beta^2 - 25462016/4545\beta - 197200448/40905 + 287096/4545\beta^7)\gamma^5 + (287992/4545\beta^7$$
$$-1388170/8181\beta^6 - 71429104/40905\beta^5 + 252534880/24543\beta^4 - 2744116144/122715\beta^3 + 191038304/8181\beta^2$$
$$-924933632/122715\beta - 577923584/122715)\gamma^4 + (1539968/40905\beta^7 - 3998344/40905\beta^6 - 46445872/40905\beta^5$$
$$+ 164070784/24543\beta^4 - 1811240704/122715\beta^3 + 654982912/40905\beta^2 - 721875968/122715\beta$$
$$-65665024/24543)\gamma^3 + (541472/40905\beta^7 - 91000/2727\beta^6 - 18062464/40905\beta^5 + 63811936/24543\beta^4$$
$$-715420672/122715\beta^3 + 53695232/8181\beta^2 - 329412608/122715\beta - 1060864/1215)\gamma^2 + (104192/40905\beta^7$$
$$- 17024/2727\beta^6 - 1297024/13635\beta^5 + 4584704/8181\beta^4 - 5797888/4545\beta^3 + 20301824/13635\beta^2$$
$$-27262976/40905\beta - 1212416/8181)\gamma + 704(\beta-4)(\beta^6 + 58/11\beta^5 - 334/11\beta^4 + 800/11\beta^3 - 960/11\beta^2$$
$$+ 512/11)/13635 + (352(b-4)^2(6\beta^5 + 290/11\beta^4 - 1336/11\beta^3 + 2400/11\beta^2 - 1920/11\beta))/13635)$$

and all the coefficients of the powers of γ, including the constant, are strictly negative functions of β for all $\beta \in (0, 1)$. Therefore, either $\pi_j > \tilde{\pi}_j$ (or the reverse) is always true. Figure 38 shows that for some parameter values, $\pi_j > \tilde{\pi}_j$, so public competitive supplier profits are always higher.

■

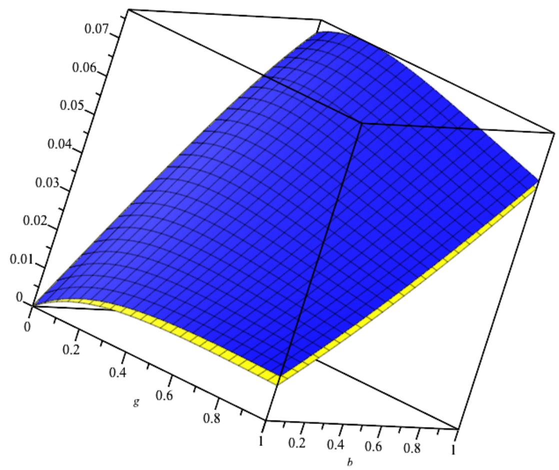

Figure 38: Proposition D.2: Supplier profits in the public competitive (blue) and private (yellow) equilibria for all β, $\gamma \in (0,1)$.

The results of Proposition D.2 are remarkably similar to those of Proposition 2 except for two points: public competitive transaction prices and supplier profits are always higher than private ones. This means that the optimal response effect is not so strong in this model.

Corollary D.1. *Relative to the public collusive equilibrium, in the private equilibrium, for all* β, $\gamma \in (0,1)$, *(i) list prices are lower, (ii) transaction prices are lower, (iii) fringe supply is higher, (iv) retailer supply is lower, (v) the retail price is lower, (vi) total supply is higher, (vii) suppliers' profits are lower and (viii) retailers' profits are lower.*

Proofs. Follows directly from Propositions D.1 and D.2:

$$\left[\check{w}_j^L \geq w_j^L \wedge w_j^L \geq \tilde{w}_j^L\right] \Rightarrow \check{w}_j^L \geq \tilde{w}_j^L$$

$$[\check{w}_i \geq w_i \wedge w_i \geq \tilde{w}_i] \Rightarrow \check{w}_i \geq \tilde{w}_i$$

$$\left[\check{q}_j^F \leq q_j^F \wedge q_j^F \leq \tilde{q}_j^F\right] \Rightarrow \check{q}_j^F \leq \tilde{q}_j^F$$

$$[\check{q}_i \geq q_i \wedge q_i \geq \tilde{q}_i] \Rightarrow \check{q}_i \geq \tilde{q}_i$$

$$[\check{p} \geq p \wedge p \geq \tilde{p}] \Rightarrow \check{p} \geq \tilde{p}$$

$$\Rightarrow \check{Q} \leq \tilde{Q}$$

$$[\check{\pi}_j \geq \pi_j \wedge \pi_j \geq \tilde{\pi}_j] \Rightarrow \check{\pi}_j \geq \tilde{\pi}_j$$

$$[\check{\pi}_i \geq \pi_i \wedge \pi_i \geq \tilde{\pi}_i] \Rightarrow \check{\pi}_i \geq \tilde{\pi}_i$$

which completes the proof. ■

Corollary D.1 is different from Proposition 3 in that public collusive transaction prices and supplier profits are always higher than private ones. The other difference that stems from Proposition D.2 and Corollary D.1, as opposed to Propositions 2 and 3, is that in this new model no indicator can qualitatively discriminate between Situations 1 and 2. Lemma D.2 shows that wholesale price dispersion always falls when list prices cease to be published.

Lemma D.2. *For all* β, $\gamma \in (0,1)$, $\tilde{w}_j^L - \tilde{w}_i < w_j^L - w_i$ *and* $\tilde{w}_j^L - \tilde{w}_i < \check{w}_j^L - \check{w}_i$.

Proof. First we solve

$$(\check{w}_j^L - \check{w}_i)(\beta,\gamma) - (w_j^L - w_i)(\beta,\gamma) = 0$$

for γ given β. The solutions are

$$\gamma(\beta) = \begin{cases} \frac{-2\beta(\beta-2)}{3(\beta^2-2\beta-1)} < 1 \;\; \forall \beta \in (0,1) \\ \\ \frac{4-\beta}{3(\beta-2)} < 0 \;\; \forall \beta \in (0,1) \end{cases}$$

so the dispersion in one equilibrium must always be larger than in the other. Figure 39 shows that dispersion in the public collusive equilibrium is always larger. The numerator of $(w_j^L - w_i) - (\tilde{w}_j^L - \tilde{w}_i)$ is

$$\varphi_4 = (513\beta^4 - 2493\beta^3 + 6111\beta^2 - 10314\beta + 7920)\gamma^4 + (1188\beta^4 - 5985\beta^3 + 15321\beta^2 - 27168\beta + 21216)\gamma^3$$
$$+ (1005\beta^4 - 5354\beta^3 + 14222\beta^2 - 26824\beta + 21184)\gamma^2 + (366\beta^4 - 2116\beta^3 + 5776\beta^2 - 11744\beta + 9344)\gamma$$
$$+ 48\beta^4 - 312\beta^3 + 864\beta^2 - 1920\beta + 1536.$$

And its limits as $\gamma \to 0$ and $\gamma \to 1$ are

$$\lim_{\gamma \to 0} \varphi_4 = 48\beta^4 - 312\beta^3 + 864\beta^2 - 1920\beta + 1536$$
$$\lim_{\gamma \to 1} \varphi_4 = 3120\beta^4 - 16260\beta^3 + 42294\beta^2 - 77970\beta + 61200$$

The first partial derivative with respect to γ is

$$\frac{\partial \varphi_4}{\partial \gamma} = 4(513\beta^4 - 2493\beta^3 + 6111\beta^2 - 10314\beta + 7920)\gamma^3 + 3(1188\beta^4 - 5985\beta^3 + 15321\beta^2 - 27168\beta + 21216)\gamma^2$$
$$+ 2(1005\beta^4 - 5354\beta^3 + 14222\beta^2 - 26824\beta + 21184)\gamma + 366\beta^4 - 2116\beta^3 + 5776\beta^2 - 11744\beta + 9344$$

which is strictly positive because all its terms are strictly positive. Therefore dispersion in one equilibrium is always higher than in the other. Figure 40 shows that dispersion is always higher in the public competitive equilibrium. By transitivity, dispersion is also higher in the public collusive equilibrium. ■

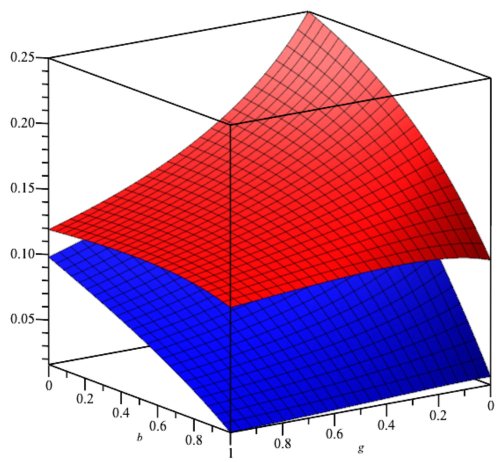

Figure 39: Lemma D.2: Wholesale price dispersion in the public competitive (blue) and public collusive (red) equilibria for all β, $\gamma \in (0, 1)$.

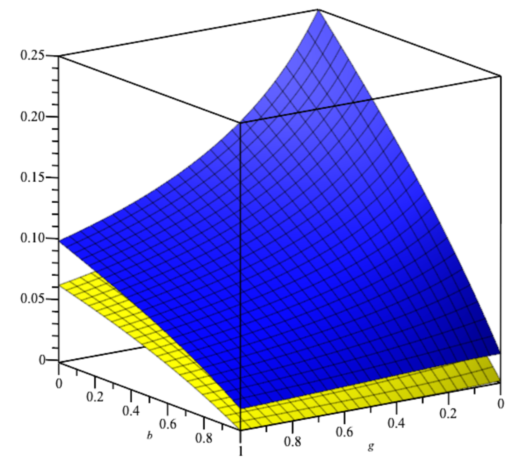

Figure 40: Lemma D.2: Wholesale price dispersion in the public competitive (blue) and private (yellow) equilibria for all β, $\gamma \in (0, 1)$.

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