PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
SCHOOL OF ENGINEERING

MEDIUM TERM POWER PORTFOLIO OPTIMIZATION CONSIDERING LOCATIONAL ELECTRICITY PRICES AND RISK AVERSION FOR A POWER PRODUCER IN A DEREGULATED ELECTRICITY MARKET

ÁLVARO HUGO LORCA GÁLVEZ

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Master of Science in Engineering

Advisor:
JOSÉ PEDRO PRINA PACHECO

Santiago de Chile, June 2011

PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
SCHOOL OF ENGINEERING

MEDIUM TERM POWER PORTFOLIO OPTIMIZATION CONSIDERING LOCATIONAL ELECTRICITY PRICES AND RISK AVERSION FOR A POWER PRODUCER IN A DEREGULATED ELECTRICITY MARKET

ÁLVARO HUGO LORCA GÁLVEZ

Members of the Committee:
JOSÉ PEDRO PRINA PACHECO
JORGE RAFAEL VERA ANDREO
VÍCTOR MANUEL ALBORNOZ SANHUEZA
GONZALO ERNESTO PIZARRO PUCCIO

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Master of Science in Engineering

Santiago de Chile, June 2011

Dedicado a Alejandra

ACKNOWLEDGEMENTS

The development of this thesis has meant a great effort for me. It has been a long journey with many moments of joy and also with moments of frustration. I am grateful for everything life has shown me through it. I learned much about engineering and math, but more importantly, I learned about myself.

I feel grateful for many persons that have helped me directly or indirectly in the completion of this project.

First of all I thank my advisor José Prina. There is no doubt that he was an excellent supervisor and I am grateful for the disposition and support he has shown. I also thank him for the opportunity he presented me for working on a project in parallel to the initial development of this thesis, which was a great contribution to increase my understanding of the electric industry.

I also thank the other members of the committee: Professors Gonzalo Pizarro, Víctor Albornoz and Jorge Vera. For their time and for their comments. I thank Professor Víctor Albornoz for inviting me to participate in one of his classes and in the Operations Seminar held in October 2010 at the Universidad Técnica Federico Santa María. I am particularly grateful to Professor Jorge Vera, who first motivated me to follow the Mathematical Engineering Diploma, then to enter in the Operations Research area and later to continue with doctoral studies. It is straightforward to notice what an excellent academic and teacher he is.

I also thank Professor Sergio Gutiérrez for the motivation on doing research and for his honest advice when I had to make important decisions. And Professor Rolando Rebolledo, for motivating me to follow an academic career with his example of an excellent academic, teacher and person.

I am grateful with my colleagues at Pricing UC for everything I have learned with them, and with Professor Juan Carlos Ferrer for the working opportunity.

I also thank many friends for the good times spent together. Constantino Baboun, Gabriel Delgado, Carlos Pérez, Fabián Cádiz, Patricio Gallardo, Cristián Paris, Estéban Ávila, Sergio Carrasco, José Miguel, Nicolás, Sergio, José Luis, and my cousins. I am also grateful for everything I have learned with Ángela.

I feel fortunate for the family I have. I thank my mother for her unique capability of understanding and supporting, my father for the example of working energetically for what one wants, and my brother for having motivated me to study engineering.

Finally, I feel remarkably grateful for my wife Alejandra. In simple words, her presence illuminates my life.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

This thesis describes the development and implementation of a medium term power portfolio optimization model for a power producer, considering generation and commercial aspects. The methodology developed consists of three steps: 1) the modeling of the multivariate stochastic evolution of locational electricity prices, 2) the construction of a scenario tree that represents the locational electricity prices model, and 3) the formulation of a stochastic optimization model that incorporates the above mentioned scenario tree. With this methodology a power producer holding thermal generating units in more than one location in a deregulated electricity market may maximize expected benefits in the medium term while keeping a limited risk exposure. Central to the model is the possibility of trading electricity forward contracts and contracts for difference in different locations. The model also recommends amounts of electricity transactions in locational spot markets and power production in generating units. The stochastic programming model developed has a deterministic linear programming equivalent problem that is solved using available commercial software. From the results obtained it can be concluded that the correlation between locational electricity prices in deregulated electricity markets is relevant for power producers holding generating units in those locations since it affects the relation between expected profit and risk.

Keywords: conditional value at risk, deregulated electricity markets, electricity price models, integrated risk management, locational electricity prices, power portfolio optimization, scenario tree construction methods, stochastic programming.

RESUMEN

Esta tesis describe el desarrollo e implementación de un modelo de optimización del portafolio energético de un productor de electricidad en el mediano plazo, considerando aspectos de generación y comerciales. La metodología desarrollada consta de tres etapas: 1) la modelación del comportamiento estocástico multivariado de precios locales de electricidad, 2) la generación de un árbol de escenarios que representa al modelo de precios locales de electricidad, y 3) la formulación de un modelo de optimización estocástica que incorpora el árbol de escenarios mencionado. Con esta metodología, un productor de electricidad que dispone de unidades generadoras térmicas en más de una localidad en un mercado eléctrico desregulado puede maximizar su beneficio esperado en el mediano plazo manteniendo una exposición al riesgo limitada. Central al modelo es la posibilidad de transar contratos futuros y contratos por diferencia sobre electricidad en distintas localidades. Adicionalmente, el modelo recomienda cantidades a ser transadas en los mercados spot locales de electricidad y niveles de generación de las unidades generadoras. El problema de optimización estocástica desarrollado es expresado como un problema de optimización lineal determinístico equivalente que se ha resuelto utilizando software comercial disponible. De los resultados obtenidos se puede concluir que en mercados eléctricos desregulados la correlación entre los precios de electricidad en distintas localidades es relevante para productores de electricidad que disponen de unidades generadoras en dichas localidades debido a que afecta la relación entre beneficio esperado y riesgo.

Palabras clave: gestión integrada del riesgo, mercados eléctricos desregulados, métodos de generación de árboles de escenarios, modelos de precios de electricidad, optimización de portafolio energético, precios locales de electricidad, programación estocástica, valor condicional en riesgo.

I. INTRODUCTION

The electric industry is extremely relevant in today's world. In 2003 the financial volume of the world's physical consumption of electricity was more than 1,000,000 [million US$], equivalent to approximately 2 percent of the world's gross domestic product in that year. In 2007 the world generated 18.8 million GWh of electricity, and the US Energy Information Administration has estimated that the world electricity generation will increase by 87 percent from 2007 to 2035.

Many crucial industrial and home activities depend on electricity. This makes electric systems fundamental drivers of the economy. Based on the nature of the activities, electric power systems can be decomposed into generation, transmission and distribution systems. Generation consists of the process of transforming some type of energy into electrical energy; the transmission system provides the medium for transportation of electricity; and the distribution system is in charge of providing power from the transmission system to final customers (including protection and control equipment) at an adequate voltage level.

With respect to the wholesale trading of electricity, in contrast to other commodity markets electricity markets are characterized by the non-storability of electricity (in practical terms), which implies that an exact matching of supply and demand must occur at all times. This leads to high volatility of electricity spot prices. Another particular property of electricity markets derives from the necessity of the transmission network, which prevents the development of a global market and frequently gives rise to locational characteristics.

Electricity markets can be divided into regulated and deregulated markets. In regulated electricity markets energy generation of available units is scheduled in advance by a central planner, generally according to the minimization of the total operation cost subject to several technical and security constraints. In contrast, in deregulated electricity markets (or competitive electricity markets) energy production of generating units is determined through an auction process that involves supply bids by generators and demand bids by consumers, matching, as a result, supply and demand. Bidding involves submitting supply and demand bids to the power exchange a day ahead of physical trading (or less in the intra-day markets). This means that generating companies submit information about electricity amounts they are willing to produce in a certain range of time (for instance a certain hour) for a minimum payment. Market clearing prices are determined by the balance between supply and demand.

Particular aspects of deregulated electricity markets vary from market to market, but according to Burger et al. (2007) in general they can be decomposed into:

- **Forward and futures market.** This is the market where electricity products to be delivered later than in the next day are traded.
- **Day ahead market.** In this market, electricity products to be delivered in the next day are traded. These are the most common spot products and can be traded through power exchanges (pools) or through bilateral contracts (particular agreements).
- **Intra-day market.** Intra-day electricity products are delivered on the same day and can also be traded through power exchanges or bilateral contracts.
- **Balancing (or real-time) and reserve market.** These markets serve the purpose of balancing imbalances between supply and demand in real time.

Since the transmission of electricity is affected by power losses and, possibly, congestion, electricity supply and demand conditions may be highly local. This means that electricity prices depend on the location in which electricity is delivered and hence "spatial risk" (related to unknown differences between locational electricity prices) is a relevant risk for power producers holding generating units in more than one electricity price location. Slight differences exist between markets that use locational electricity pricing methods. However, in general terms the locational marginal price (LMP) of a particular node (or location, or zone) is calculated, under locational marginal pricing, as a marginal price for energy plus a marginal price for losses plus a marginal price for congestion. Decomposing locational prices into these components has several advantages. One of the most important is that the price of energy at a particular location gives a correct signal to investors in generation and transmission assets. For instance, if the price at a location is consistently high, there may be several factors interacting such as lack of supply, congestion that prevents bringing power from other locations, and power losses. This may drive the attention of investors and after investing in increasing capacity, these conditions would be improved and electricity consumers may benefit from lower electricity prices. Deregulated electricity markets that use locational marginal pricing systems differ in their mechanisms. For instance, the Nord Pool presents 6 different regions that differ in price only when congestion occurs, but when congestion is not present the 6 regions have identic prices. Another example is the New York system, where 11 zones exist with prices that are correlated but different. More information about locational marginal pricing can be found in Shahidehpour et al. (2002), and the web sites of the New York Independent System Operator (http://www.nyiso.com/public/index.jsp, last visit 10 July 2010) and of the New England Independent System Operator (http://www.iso-ne.com, last visit 10 July 2010).

In addition to "spatial risk", power producers in deregulated electricity markets are exposed to multiple risks associated with uncertain factors such as electricity spot prices, fuel prices, water inflows and the failure of generating units. In order for some of these risks to be managed many standardized energy derivatives are usually available. An energy derivative is a contract that is derived from an underlying energy related commodity. According to Eydeland & Wolyniec (2003) the most frequently used risk management tools are futures contracts, forward contracts, swaps, and options, which are typical derivatives in financial markets, but that assume particular properties in electricity markets. Forward contracts are agreements to buy or sell a commodity at a future time, with a price to be paid at delivery specified at the moment of signing the contract. For example, a forward contract on electricity may state that a certain power producer must provide another company with constant 50 [MW] power on the peak hours (for instance 6.00-22.00) of all the weekdays of November 2011 and that the payment per unit of energy delivered will be 40.1 [US$/MWh]. Such a contract would alleviate the risk of low electricity prices: if the power producer decided to try to sell that electricity through the spot market, then it would risk low spot price cases. Other relevant electricity derivatives are contracts for difference, which are equivalent to electricity forward contracts on a locational price spread (or equivalently on the price difference between two locations over a specified period).

For a power producer, deciding which contracts to sign is a complex decision making problem that involves significant uncertainty. The fundamental problem is to balance risk and profit appropriately. Accordingly, the development of risk management tools that can effectively support this decision making process is essential for power producers.

The motivation for this thesis comes from the fact that medium term risk generated by the stochastic fluctuations of locational electricity prices in deregulated electricity markets is highly relevant for power producers owning electricity generators in more than one location of the same market. This thesis deals with the the management of such risk by developing a methodology for recommending contractual positions that maximize expected profit while appropriately balancing risk exposure. The effects of the correlation between different locational electricity prices is of particular interest since it is expected to affect the relation between expected profit and risk.

This thesis describes the development, implementation and test of a medium term power portfolio stochastic optimization problem (using techniques from stochastic programming). Using the methodology presented, a power producer holding generating units in more than one location within a deregulated electricity market, can effectively maximize its expected profit while appropriately balancing risk exposure. The model is able to recommend contractual positions to be entered in, and its most relevant contribution is the consideration of more than one locational electricity spot price process through an approximating multivariate scenario tree that accounts for the correlation between these processes and is generated solving a nonlinear programming problem. The model is centered in the possibility of entering into electricity forwards and contracts for difference through the development of a stochastic programming problem with recourse from which the deterministic equivalent corresponds to a linear programming problem solved using commercial software (CPLEX). From the results obtained it can be concluded that locational electricity price aspects are relevant for power producers holding generating units in more than one location of a deregulated electricity market. Detailed results for a case study may be seen in chapter 6 and the conclusions obtained may be seen in chapter 7.

As stated above, the methodology used in this work includes stochastic programming techniques. Stochastic programming deals with optimization problems where part of the data and/or parameters are uncertain. According to Ruszczyński & Shapiro (2003), in many decision making problems, ignoring uncertainty may lead to inferior or wrong decisions. Hence, stochastic programming models are particularly important in areas where uncertainty is highly relevant such as finance, energy, transportation and logistics, production and inventory management, water resources management, and telecommunications. Stochastic programming methodologies allow to incorporate into the decision making process information about probability distributions of events of interest. An adequate modeling of uncertainty together with appropriate objectives and constraints are fundamental aspects for a successful application of stochastic programming to actual problems.

Many stochastic programming problems are two stage decision making models, where a decision must be made now and, after some uncertain event is revealed, another decision must be made. Extending this idea to a multistage setting we have multistage stochastic programming models, in which decision variables and constraints are divided into sets corresponding to stages $t = 0, ..., T$. If ξ_t represents the information that becomes known in stage t, and x_t represents the decisions to be taken in stage t, then we have the following sequence of actions: decision x_0 (with initial information ξ_0), observation of ξ_1, decision x_1, observation of ξ_2, decision x_2, etc. The objective is to design the decision process in such a way that, for instance, an expected total cost is minimized. An important condition in this multistage process is that every vector of decisions x_t may depend only on the information available at time t (on $(\xi_0, \xi_1, ..., \xi_t)$), but not on future information (which gives rise to "nonanticipativity constraints"). In order to represent the information process $\{\xi_0, \xi_1, ...\}$, a scenario tree may be used, in which case the stochastic program has an equivalent deterministic problem that may be solved using standard optimization techniques.

The main objective of this work is to model and solve a risk averse power portfolio optimization problem that incorporates the stochastic behavior of locational electricity prices. In order to do this, the following specific objectives have been considered:

1. To jointly model the stochastic evolution through time of locational electricity prices.
2. To develop a methodology for generating computationally tractable scenario trees that approximate the locational electricity prices model.
3. To develop an appropriate optimization model (incorporating a scenario tree) and to solve it for different risk aversion levels of the power producer.

The remainder of this document is organized as follows. Chapter 2 contains a literature review of different methodologies that have been (could be) used by generating companies in deregulated electricity markets for managing generation and contractual aspects. Chapter 3 presents various electricity price models. Chapter 4 describes scenario tree construction methods, and describes its application to the specific context of this work. Chapter 5 presents the optimization model developed. Chapter 6 summarizes the data, implementation and results for a particular application of the optimization model, using an electricity prices model presented in chapter 3 and a scenario tree construction method developed in chapter 4. Chapter 7 draws some conclusions and presents future work perspectives.

II. LITERATURE REVIEW

This chapter will begin presenting articles related to electricity generation scheduling, then it will present articles concerned with contractual aspects, and finally it will present articles that deal with the coordination of generation scheduling and contractual involvement for generating companies in deregulated electricity markets.

An adequate scheduling of the operation of generating units is highly important for generating companies in deregulated electricity markets. In this context, the unit commitment problem consists of determining an hourly schedule of power generating units and the generating level of each unit in order to minimize the system operation cost while satisfying electricity demand, over a time horizon of typically 24 hours. This problem involves operation constraints of generating units, an electricity demand constraint (balance between supply and demand), and it may involve a nonlinear cost function of electricity generation. The cost function and operation constraints depend on the type of generating units. For thermal generating units the generation cost functions are generally assumed to be quadratic (for instance Shiina & Birge (2004)) and may also incorporate start up and shut down costs. Operation constraints (may) include power output limits, minimum up time and minimum down time constraints, and ramp-up and ramp-down limits (maximum output difference allowed for a unit in subsequent hours).

Carpentier et al. (1996) deal with the unit commitment problem for 50 thermal generating units in a time horizon of one day with hourly time periods. Electricity demand and unit failures are assumed to be stochastic variables represented using a scenario tree. Operational aspects considered include minimum off times and start-up costs of generating units. The problem is solved using a price decomposition method in which at each iteration a stochastic dynamic subproblem has to be solved for each unit. Simulation is used to demonstrate the benefits of this approach compared to a classical deterministic alternative.

Takriti et al. (1996) deal with a very similar unit commitment problem, but in a hydrothermal context. Hourly time periods are used over a time horizon of one day. Electricity demand and unit failures are assumed to be stochastic variables represented using a scenario tree. Operation details include minimum on and off times and a quadratic cost function of generating units. The problem is solved using Lagrangian Relaxation by relaxing nonanticipativity constraints typical of stochastic programming problems, and then relaxing electricity demand satisfaction constraints, in order to obtain a sub problem for every generating unit that is solved using dynamic programming.

Shiina & Birge (2004) deal with the unit commitment problem in a thermal context. A time horizon of one day with hourly time periods is considered. Electricity demand is assumed to be stochastic, represented through a scenario tree. Operational details include minimum up and down times, start up costs and a quadratic cost function. The resulting mixed integer quadratic programming problem is solved using a Dantzig-Wolfe decomposition scheme. A test case with 20 units and 8 scenarios is presented.

An important extension of the unit commitment problem is what has been called *price based unit commitment* (for instance by Shahidehpour et al. (2002) and Denton et al. (2003)), which consists of finding an operation schedule that maximizes the revenues from energy selling for a given set of generating units, taking into account electricity spot prices. This is a highly complex decision making problem due to the economical and technical constraints of generating units, outlined above, and due to the uncertainty present in electricity markets. An important uncertain component related to the classical unit commitment problem is electricity demand, but the most important uncertain component related to the price based unit commitment problem is precisely the electricity spot price process.

Arroyo & Conejo (2000) present a detailed formulation of a thermal unit scheduling problem that considers a spot market in which energy and spinning reserve may be sold. A time horizon of one day with hourly time periods is considered. The approach is completely deterministic, but the main contribution of this work is to develop a highly rigorous modeling of the operational aspects of the generating unit. The resulting problem is a mixed integer linear programming problem solved using commercial software.

Ni & Luh (2000) deal with the price based unit commitment problem in a hydrothermal context. Hourly time periods are used over a time horizon of one day. Electricity spot and reserve spinning prices are modeled using a Markov chain. The model incorporates operational details and constraints over spot market transactions and reserve market transactions. A component that enhances the selling of electricity when its price variance is higher is considered in the objective function since according to the authors generating companies prefer to sell more electricity when its price variance is high in order to benefit from eventual spikes. The problem is solved using Lagrangian Relaxation, by relaxing the spot and reserve markets transactions constraints in order to obtain one subproblem for each generating unit. A test case consistent of 10 thermal generating units and a pumped-storage system with 4 generating units is presented.

Takriti et al. (2000) deal with the price based unit commitment problem in a thermal context, considering different fuels. A time horizon of one week separated into hourly time periods is used. Electricity price and demand are assumed to be stochastic, represented through a scenario tree. Operational constraints include minimum up and down times. The main contribution is considering detailed fuel constraints by taking into account several fuels with different prices, heat rates, and supplies. The problem is solved with Lagrangian Relaxation by decomposing it into a first stage unit commitment problem and a second stage fuel allocation linear program. A test case consistent of 33 thermal units is presented.

Shrestha et al. (2004) deal with the price based unit commitment problem in a thermal context. A time horizon of one day with hourly time periods is used. Electricity price is assumed to be stochastic, following a Markov chain with three possible values at each time period. Operational constraints include minimum up and down times. The problem is solved with Lagrangian Relaxation by relaxing demand constraints and separating the problem into one subproblem for each generating unit. A test case is presented consistent of 11 generating units. The results show that the technique is particularly effective when price uncertainty is high, in contrast with using a deterministic technique, and that participating in an hour-ahead power market may enhance benefits significantly.

Conejo et al. (2004) present a mean-variance scheduling model for one thermal generating unit. A one day time horizon with hourly time periods is used. The objective function consists of expected profit plus profit variance multiplied by a risk aversion factor. The correlation matrix of the different hourly electricity prices is used in order to determine profit variance. Operational details include a quadratic cost function, minimum up and down times, and ramp up and down limits. The resulting mixed integer quadratic programming problem is solved using commercial software.

Fleten & Kristoffersen (2007) develop a model for optimizing the day-ahead bidding strategies of a hydropower producer. A time horizon of one day with hourly time periods is considered. Electricity price is assumed to be stochastic, represented through a scenario tree. The problem is formulated as a mixed-integer linear program. A case study consistent of a generation plant with two reservoirs is analyzed. The authors conclude that the stochastic approach presented is more robust with respect to changes in the data than deterministic approaches that only consider the expected value of electricity price.

An issue that has very important implications with respect to the generation planning of a generating company is the management of contracts for the purpose of balancing risk exposure and expected profit. Generating companies in deregulated electricity markets are exposed to multiple risks related to the different aspects of the generation industry, such as the procurement of fuel, water inflows, low electricity prices, high fuel prices, and forced unit outages. In order for some of these risks to be managed many standardized energy derivatives are usually available. An energy derivative is a contract that is derived from an underlying energy related commodity. According to Eydeland & Wolyniec (2003) the most frequently used risk management tools are futures contracts, forward contracts, swaps and options.

Forward contracts are agreements to buy or sell a commodity at a future time, with a price to be paid at delivery specified at the moment of signing the contracts. Kaye et al. (1990), Tanlapco et al. (2002), Byström (2003) and Niu et al. (2005) show how important forward and futures contracts are in electricity markets. Kaye et al. (1990) demonstrate using simulation that forward contracts offer participants an opportunity to reduce their risk exposure. Tanlapco et al. (2002) argument the hedging superiority of using futures contracts on electricity over futures contracts on other related commodities such as crude oil, for generating companies in a particular deregulated electricity market. Byström (2003) show how electricity futures can be used for short term hedging in the Nordic Power Exchange. Niu et al. (2005) provide a model used to analyze the effect of different forward contract levels of generating companies on real-time market prices.

Besides forwards and futures contracts many other contracts that can serve the purpose of risk management are generally available for generating companies in deregulated electricity markets. Marckhoff & Wimschulte (2009) present an introduction to contracts for difference, which are electricity forward contracts on a locational price spread, or equivalently, on the price difference between two zones over a specified time period. Carmona & Durrleman (2003) deal with *spread options* and remark the importance they have in energy markets. Spread options are generalizations of the typical call and put options and also depend on more than one underlying asset. For instance, a spread option may be an option written on the difference between electricity spot price and natural gas spot price.

Liu & Wu (2006, 2007) face the problem of energy allocation between spot markets and bilateral contracts for a generating company, by formulating and solving a portfolio optimization problem that consists of maximizing an utility function dependent on the expected value and the variance of the portfolio return (mean-variance approach). The problem results in a quadratic programming problem. These papers focus on selecting the appropriate portfolio of the involvement in different risky locational spot markets, and various bilateral contracts, taking into account the generating capacity of the company using a quadratic cost curve and the correlation between the returns of these trading alternatives.

Vehviläinen & Keppo (2005) present a general framework for selecting power portfolios composed of electricity derivatives. The problem formulated consists of optimizing an utility function of the total wealth at the end of certain time horizon, where this wealth function depends on the values of the electricity prices and the instruments considered, and the proportions held of each. The problem is solved by first approximating the utility function with a Taylor series around the initial portfolio, obtaining a quadratic programming problem estimated using simulation. Weekly time periods are used, and a time horizon of one year is considered.

Börger et al. (2007) deal with joint asset return distributions in energy markets and analyze how the class of multivariate generalized hyperbolic distributions perform in adjusting the joint return distributions of electricity, oil, natural gas, coal and CO_2 allowances. The importance of this work relies on the accurate representation of the dependence structure of the returns of these different commodities.

So far this chapter has presented references on generation planning and references on contract management in a relatively independent manner, but the main concern of this thesis is precisely the relation between these two problems for a generating company. According to Fleten et al. (1997) the physical resources of a generation company must be coordinated in concert with its financial resources in order to mitigate risk factors, and according to Cabero et al. (2005) in addition to financial instruments specifically designed to manage risk, production resources can be used for risk management to a certain extent. Some authors have named this combined perspective as *power portfolio optimization* (for instance Sen et al. (2006), Xu et al. (2006)) and others as *integrated risk management* (for instance Mo et al. (2001), Cabero et al. (2005)), depending on the particular emphasis. Cabero et al. (2010) categorize the different approaches for incorporating the risk management perspective to the operation planning process of a generation company in the following criteria:

- The planning horizon.
- The detail of generation aspects (operational constraints, aggregation of reservoirs, etc.).
- The richness of the electricity market model.
- The sources of uncertainty/risk.
- The risk measure used.

- The risk management decisions considered.
- The technique used to obtain numerical results.

In particular, according to Xu et al. (2006) the different studies on medium term power portfolio optimization and risk management can be categorized into the following approaches:

- Portfolio evaluation based on existing financial models.
- Markowitz mean-variance based methods.
- Multistage stochastic programming models.

Denton et al. (2003) present an overview of the market risks faced by generating companies in deregulated electricity markets, and categorize them as i) short term (less than one month), centered in operational risks; ii) medium term (one month to one year), centered in trading risks; and iii) long term (more than one year), centered in asset valuation risks. One of the contributions of their work is to develop a real options model for the valuation of a thermal generator. This method roughly consists of maximizing the net profit from operating the energy asset over a specified operating period, given a set of price scenarios for fuel and electricity, and considering the physical constraints on the operation of the asset, They use a variant of the price based unit commitment problem to obtain the value of the asset over its operating period.

Blaesig & Haubrich (2005) present an overview of different risks related to generation planning and contracts in deregulated electricity markets, focusing on the importance of developing an integrated risk management process in which risks are considered simultaneously with generation and trading planning instead of separating generation and trading planning from the management of risks.

Fleten et al. (1997) develop one of the first stochastic programming models for the coordination of physical generation resources with contracts of financial nature. This power portfolio optimization model is developed for a medium term hydropower generation scheduling problem in which forward contracts on electricity can be traded and must be coordinated with generation. One reservoir is considered and the stochastic variables modeled are electricity prices, prices on forward contracts and water inflow to reservoirs, represented in a scenario tree with four time periods of varying length and four outcomes per time period, over a time horizon of 85 weeks. Risk is accounted for by penalizing low financial performance in every time period. More recent publications present extensions of this work (Fleten & Wallace (1998) and Fleten et al. (2002)), with results presented for a case having 7 hydro plants with 11 reservoirs.

Mo et al. (2001) present a model similar to the model in Fleten et al. (1997), where hydropower generation scheduling has to be coordinated with future contracts on electricity. Water inflows and electricity prices are assumed to be random variables represented over a time horizon of two years, with weekly time periods for decisions but three long periods for water inflows. The resulting large stochastic dynamic optimization problem is solved using a combination of stochastic dynamic programming and stochastic dual dynamic programming. Kristiansen (2004) presents an application of this model to Norway's second largest generation company by the year 2004.

Yu (2003) presents a mean-variance approach to power portfolio optimization, considering thermal generating units and forward contracts, with the objective of minimizing the variance of profit subject to a lower bound on expected profit. A time horizon of one day or one week may be considered, separated into hourly time periods. Operation details of generating units are considered. The main contribution is to incorporate geographically separated markets into the analysis, focusing on the transactions of electricity products with these different markets. The resulting problem is a mixed integer programming problem solved using heuristics for a test case involving 2 generating units.

Paravan et al. (2004) develop a model similar to the one presented by Fleten et al. (1997), but the main contribution lies in the incorporation of thermal generating units, highlighting the need for the consideration of their technical constraints, and in providing a general scheme of the power portfolio of a hydrothermal generation company. Water inflows and electricity prices are assumed to be random, and the importance of their correlation is stressed. Risk is accounted for by using the conditional value at risk measure and the contracts considered are forwards and futures on electricity. The model is solved using Lagrangian Relaxation but only results for a very small test case are presented.

Shrestha et al. (2005) also present a similar model to the one presented by Fleten et al. (1997), where hydropower generation and forward contracts are considered. Six time periods are used within a time horizon of one year. Water inflows and electricity prices are assumed to be completely uncorrelated variables, represented in a scenario tree with 3 outcomes in every time period. Two different objective functions are considered: first, in one case, the maximization of revenue in a risk neutral setting is solved as a linear program; then, the minimization of revenue variance is solved as a nonlinear program. A case study is presented considering only one power plant.

Cabero et al. (2005) present an *integrated risk management model* for a hydrothermal generation company in a time horizon of one year with monthly time periods. The main contribution of this work is to consider fuel price, electricity demand, water inflows, and electricity price as stochastic variables, obtaining electricity price and the company generation on every scenario by combining the other three stochastic factors through the use of a market equilibrium model. Positions on electricity forward contracts are the main recommendation of the model and risk exposure is considered by imposing a limit on conditional value at risk. According to the authors the main drawbacks of this work are that a) the interaction between the electricity forward market and the electricity spot market is not taken into account (since generation decisions are obtained from the market equilibrium model), and b) the equilibrium scenarios are obtained assuming perfect future information of the generation companies instead of using a nonanticipativity approach. Another drawback is that the scenario tree that represents the stochastic variables lacks accuracy due to the high number of stochastic variables considered.

Xu et al. (2006) deal with a power portfolio optimization problem that considers thermal, pumped storage and hydro units on the generation side, and forwards and options on electricity on the contracts side. Electricity spot prices are assumed to be stochastic, and time periods of 16 and 8 hours are considered over a time horizon of a month to a year. Risk aversion is incorporated by penalizing semi-variances of spot market transactions in the objective function. The problem is solved by a Lagrangian Relaxation method, relaxing the load obligation constraints for decoupling the problem into financial market subproblems and generation unit subproblems. Some drawbacks of this model are a) load obligation constraints are considered in an expected form only and in order to obtain feasibility for each simulated trajectory, heuristics must be used, and b) electricity prices are assumed to be independent across time periods. Results for an example incorporating 3 thermal units, 1 pumped storage unit and 1 hydro unit with fixed scheduling are presented over a one year horizon.

Sen et al. (2006) face a power portfolio optimization problem for thermal generating companies that incorporates the possibility of signing forward contracts on electricity and natural gas. Electricity demand and electricity spot prices are assumed to be stochastic, and natural gas forward prices are assumed to be perfectly correlated with electricity forward prices. In the model formulation, forward and spot market transactions are decided over monthly time periods, with generation decisions for every month and scenario, having time periods of 16 and 8 hours. A time horizon of one year is assumed and risk exposure is considered in two different ways: a) from month to month by limiting the extent to which the portfolio is allowed to change, and b) from day to day by setting a limit on the daily loss within the generation side of the model. The resulting problem is a mixed integer linear program solved using a nested column generation method by decomposing it into a financial problem (the one with monthly time periods) and several generation subproblems (the ones with shorter time periods). A test case is presented with about 50 generating plants, aggregating them into groups based on fuel type.

Conejo et al. (2008) develop a power portfolio optimization model for a thermal generating company with focus on electricity futures decisions. Electricity spot prices are considered as stochastic variables over a one year time horizon with 72 time periods composed of 6 blocks of hours for each month, where each block consists of a particular set of hours of every week (for instance one block may consist of Monday peak prices). Technical aspects of generating units are only considered in a simplified way, by setting restrictions for every generator on the amount of electricity produced in peak hours (high price hours) with respect to the electricity produced in off-peak hours (low price hours). Conditional value at risk is penalized in the objective function in order to account for risk aversion. The problem is solved for a case study incorporating 6 generating units.

Pineda et al. (2008) present an analysis of the impact of forced outages of generating units over forward contracting decisions, in a thermal context. A three month time horizon is considered with time periods of approximately a week. One of the main contributions of this work is to combine stochastic electricity prices with stochastic forced unit outages, both represented within a scenario tree. Operational details of generating units are not considered due to the medium term time horizon. Conditional value at risk is penalized in the objective function in order to account for risk aversion. A case study of one unit is presented. The authors conclude that high forced outage rates imply the convenience of high levels of forward contracting in order to hedge the risk of buying electricity when generating units are forced out.

Garcés & Conejo (2010) develop a model for the scheduling of thermal generating units and forward contracts involvement. The time horizon considered is one week with 42 time periods of varying lengths. Electricity price is assumed to be stochastic, represented through a scenario tree. Operation details of generating units are considered. Conditional value at risk is penalized in the objective function in order to account for risk aversion. A test case with 7 generating units is presented. The main contribution of this work is to effectively provide a framework for coordinating short term forward contracting decisions with detailed operational aspects of thermal generating units.

Cabero et al. (2010) present an extension to the model presented by Cabero et al. (2005) to the case of a hydrothermal generation company in an oligopolistic electricity market, taking the generation company as a price maker (instead of a price taker), with the consideration of generation decisions and forward involvement on electricity and fuels. In this model the generation and contract decisions of every generating company are obtained by formulating an optimization problem and using a Benders decomposition approach for solving it. In particular, the decisions of the generation company of interest may be obtained. In the case study presented, the model has been solved for an instance equivalent to the size of the Spanish wholesale electricity market, over a time horizon of one year with monthly time periods.

The model developed in this thesis is relatively similar to the ones developed by Conejo et al. (2008), Sen et al. (2006), Xu et al. (2006) and Cabero et al. (2005), but in contrast to their work, the main contribution of this work is to consider a generating company that holds generators in more than one location of a deregulated electricity market, with each location having a particular locational electricity price process. Using the methodology presented in this thesis, a power producer holding generating units in more than one location of a deregulated electricity market can be supported in the decision making process of entering into appropriate contractual levels, in order to effectively maximize its expected revenues while appropriately balancing risk exposure.

Yu (2003) and Liu & Wu (2006, 2007) also consider more than one location in their models, but the model presented in this thesis is substantially different from theirs. This thesis presents a multistage stochastic programming approach to power portfolio optimization, where decisions are taken subsequently depending on the evolution of stochastic factors, whereas in their models only statistical properties (mean, variance, correlations) of stochastic variables are considered instead of considering the dynamic behavior of stochastic components. Furthermore, the model developed in this thesis uses the conditional value at risk of profit for risk assessment whereas in their models only profit variance is used for risk assessment.

The next chapter will present an essential topic for generation planning and the management of contracts in deregulated electricity markets: electricity price models.

III. ELECTRICITY PRICE MODELS

In deregulated electricity markets energy is traded both on power exchanges (or power pools) and through bilateral contracts. In a power exchange, buyers and sellers take part in an auction by submitting their bids in terms of prices and quantities of energy for every time period of the next operating day (or other time horizon) and the spot prices for every time period are derived from the intersection between the aggregated supply and demand curves. Market mechanisms like these (among others), the physical characteristics of electricity, and the patterns and periodicities that characterize demand contribute to the particular features present in electricity prices.

Electricity is a commodity with very limited storability, which restrict greatly the arbitrage possibilities on electricity and thus spot prices are highly dependent on temporal and local supply and demand conditions (Lucia & Schwartz (2000)). On one hand, the non-storability of electricity makes its price strongly dependent on its determinants in every precise moment. And on the other hand, transportation constraints for electricity consist of capacity limits and transportation losses over large distances, which lead to high relevance of the particular local supply and demand conditions. Besides, demand of electricity is weather and business cycle dependent, and completely price inelastic in the short term. All these aspects contribute to making the operation of power systems a complicated task that requires a constant balance between production and consumption.

According to Karakatsani & Bunn (2008), price formation in spot electricity markets is a complex process with substantial modeling challenges, due to a convolution of factors including: i) instantaneous nature, ii) the shape of the supply function, iii) the exercise of market power (which results from oligopolistic structures, agent's asymmetries and inelastic demand in the short term), iv) complex market designs due to real-time balancing, and v) substantial agents learning through daily-repeated auctions but subject to frequent regulatory interventions and market structure changes.

The main features of electricity prices in deregulated markets, according to Meyer-Brandis & Tankov (2008), are described next.

- **Multiple seasonality.** Multiple seasonalities exist in electricity prices: daily, weekly and yearly periodicities exist. In addition, there is a "calendar effect", for instance, holidays usually have a special behavior.
- **Mean reversion.** Contrary to stock prices, electricity prices generally tend to exhibit a stationary behavior (see Meyer-Brandis & Tankov (2008) on Nord Pool prices). This means that the property of mean reversion is present (at least towards a trend which may exhibit slow stochastic variations).
- **Extreme volatility.** The limited storability of electricity generates extreme price volatility because supply and demand shocks cannot be smoothed by inventories (Deng (2000)).
- **Jumps and spikes.** A noticeable property of electricity spot prices is the presence of spikes, which are rapid upward price moves followed by a quick return to about the same price level. Spikes can be caused by limited supply (for instance, by outages), sudden demand increases, etc. Sometimes the series also present normal jumps (without returning to about the same price level immediately), and jumps can even be downward.
- **Non-Normal distribution of returns.** Electricity spot returns are generally non-Normal (spot return equals spot price difference from one period to the next divided by spot price in the first time period). This is revealed by positive skewness and kurtosis significantly greater than 3 (Normal distribution kurtosis), and it is mainly explained by the presence of spikes in the price series (Meyer-Brandis & Tankov (2008)).

An appropriate modeling of electricity prices is highly important for several activities such as management (portfolios, investment, risk), trading, and derivative pricing (with electricity as underlying asset). Other important reasons for modeling electricity prices are regulatory policy making and competition analysis in electricity markets (for instance, if market power is applied).

How to know if a particular model is appropriate? Of course it depends on the objective: if a yearly production plan is to be determined, then the behavior of electricity from one hour to the next is not important and it may be more appropriate to consider a price model of daily averages or even monthly averages (in such a case, simplifications can be made on the hourly variations, if necessary, for instance, a common alternative is to assume constant price in certain hour groups of the same time period). Hence, one important question is: what features need to be taken into account? In this regard, besides the particular objective that the model pursues, the history of the price and other relevant data, such as historical electricity demand or fuels prices, need to be studied carefully so as to understand what features are relevant for the particular purpose of the model, since it should capture the features appropriately. This leads to the following question: how to verify that these features are appropriately captured by a given model? According to Geman (2005) two aspects are fundamental in commodity price modeling: a) the empirical statistical properties of the price data (mean, variance, etc.) should be consistent with the model, and b) the "dynamics" of the price data should be consistent with the model: aspects such as the similarities of the trajectories and the changes from one time period to another should be verified. Finally, the model considered should be parsimonious and robust: an adequate level of model complexity should be considered according to the available data, for instance, if there is very few data a model with many parameters should be hardly trusted (that would not be a parsimonious model), and the changes in the parameters induced by changes in the data should be smooth (robust model).

The rest of this chapter contains a review of the main electricity price models found in the literature for deregulated electricity markets (although the structure of many of these models may be appropriate, sometimes with slight modifications, for fuel prices and other commodities with an uncertain evolution over time). Section 3.1 contains a presentation of the different electricity price modeling approaches, section 3.2 presents a review of specific electricity price models developed by various authors, and section 3.3 presents extensions to some of the models presented in 3.2.

3.1 Modeling approaches

A diffusion is a continuous time stochastic process that corresponds to the solution of a stochastic differential equation. Diffusion models are used in biology, stochastic control, physics, finance, and many more areas.

Itô diffusions are the solutions $\{X_t\}_t$ of stochastic differential equations of the form:

$$dX_t = b(t, X_t)dt + \sigma(t, X_t)dW_t,$$

where $\{W_t\}_t$ is a standard Brownian motion, the continuous time stochastic process on which diffusions are generally based. This process has independent increments and for any $s < t$, it satisfies that $W_t - W_s$ is Normal with mean 0 and standard deviation $\sqrt{t-s}$. Because Brownian motion is nowhere differentiable and integrals with respect to Brownian motion differentials are not defined under Riemann or Lebesgue integration theory, the rules of standard calculus are not applicable with diffusions. Yet, the necessary mathematics have been developed under the name of stochastic calculus. In particular, an integral with respect to Brownian motion differences exists (which gives meaning to the expression "dX_t") and there is an analogue to the Fundamental Theorem of Calculus called Ito's Lemma.

One of the most important properties of diffusions is the Markov property. In simple words it means that all the information relevant for the future behavior of the process is contained in the present state. This property is essential for most of the theoretical properties of diffusions.

A particular type of diffusions, important for electricity price modeling, are *jump-diffusions*. They present sudden jumps in their trajectories, and in order to preserve the Markovian property these processes must have a continuous time Markov chain as the determinant of the jump instants. For an introduction to stochastic differential equations see Oksendal (2005).

Continuous time processes are obviously appropriate for variables that depend continuously on time, but they can also be used to model discrete time variables by observing their states at particular instants of interest. The main benefit of using continuous time processes (when there is no need for continuous time) is making use of their theoretical properties, but if a continuous time process does not capture the necessary aspects it might be more appropriate to use a discrete time process.

Regarding discrete time models of electricity prices, the most used time series are the ARMA (autoregressive moving average) processes. An ARMA(p,q) process $\{Y_t\}_t$ is given by:

$$Y_t - \phi_1 Y_{t-1} - \phi_2 Y_{t-2} - ... - \phi_p Y_{t-p} = c + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + ... + \theta_q \epsilon_{t-q}.$$

In this model, Y_t depends linearly on its last p values (autoregressive components), and on ϵ_t and its last q values (moving average components). $\{\epsilon_t\}_t$ is a "white noise", with ϵ_t being independent normal random variables with mean 0 and standard deviation σ. In particular, a discretized mean reverting diffusion with fixed time steps is an ARMA$(1, 0)$ model.

An ARIMA (autoregressive integrated moving average) model for a series $\{X_t\}_t$ corresponds to an ARMA model for the transformed series defined by $Y_t = (1-B)^d X_t$, where B is the "backshift operator" defined by $BX_t = X_{t-1}$, which in turn means $(1 - B)X_t = X_t - X_{t-1}$, and $(1 - B)^2 X_t = (1 - B)(1 - B)X_t$, etc. An ARIMA model for $\{X_t\}_t$ is an ARMA model for the series $\{X_t\}_t$ differentiated d times. This model is one alternative for dealing with trends, since with successive differentiation a stationary behavior may be obtained.

A SARIMA (seasonal ARIMA) model for a series $\{X_t\}_t$ corresponds to a "special ARIMA" model:

$$(1 - \Phi_1(B^s)^1 - ... - \Phi_p(B^s)^p)(1 - \phi_1 B^1 - ... - \phi_p B^p)(1 - B)^d (1 - B^s)^D X_t$$

$$= (1 + \Theta_1(B^s)^1 + ... + \Theta_q(B^s)^q)(1 + \theta_1 B^1 + ... + \theta_q B^q)\epsilon_t,$$

where $B^s X_t = X_{t-s}$. This model corresponds to an ARMA series with many zero-valued parameters (forced to be zero before estimating their values), and it is specially adequate for seasonal time series.

Other very important time series model in commodity price modeling is the GARCH (generalized autoregressive conditional heteroskedasticity) model. For a detailed presentations of time series models, see Brockwell & Davis (1996) and Hamilton (1994).

A very important category of electricity price models correspond to structural models (also named equilibrium or fundamental models), in which the price formation in electricity markets is emulated by balancing supply and demand, by assuming a competitive equilibrium model for the electricity market. According to Vehviläinen & Pyykkönen (2005) statistical models work better for short time intervals as the static model structure and the relatively small set of parameters are not able to capture the longer-term dynamic characteristics of spot prices. In contrast, an important advantage of structural models is that they can adapt to market changes. For example, statistical models will not be useful if a new power plant that affects the market equilibrium is introduced, but structural models can predict the new price behavior by considering the effects of the new power plant. Yet, a disadvantage of structural models is that they typically require a comprehensive data set that is difficult to collect and maintain. Burger et al. (2007) and Eydeland & Wolyniec (2003) present a detailed review of the main fundamentals affecting electricity prices in energy markets (demand, plant outages, fuel prices, water inflow, weather, etc.). Structural models are based on the interactions between (some of) these factors.

In structural models one obtains a model for the joint behavior of electricity prices and the factors that contribute to their formation, so they are one particular form of multivariate models. Multivariate models are very important in risk management when there is more than one risk factor that evolves in time. For instance, a hydro power generator should be interested in the behavior of both water inflows and electricity prices. Furthermore, multivariate models are important for modeling the joint behavior of locational electricity prices present in deregulated electricity markets.

3.2 Models reviewed

This section presents several models of the time evolution of electricity prices, developed by various authors.

3.2.1 Mean reverting diffusion

In Lucia & Schwartz (2000) the following model for the daily average spot electricity price of the Nord Pool is presented:

$$\log S(t) = f(t) + X(t),$$
$$f(t) = a + bt + c\cos(\frac{2\pi t}{365}) + d\sin(\frac{2\pi t}{365}),$$
$$dX(t) = -\frac{X(t)}{\lambda}dt + \sigma dW(t).$$

Where $S(t)$ is the average spot price of electricity on day t (for discrete t), $f(t)$ is a deterministic trend function that includes a yearly seasonal component where a, b, c, d are parameters, and X is an Ornstein-Uhlenbeck stochastic process, that accounts for the stochastic behavior of electricity prices. X is a process based on a Brownian motion W with volatility σ. X tends to revert to the value 0 with a "rate of mean reversion" given by $1/\lambda$.

This model was fitted using data from Nord Pool and applied to the valuation of forward and futures contracts on electricity. The main contribution of this work is explaining the sinusoidal shape of the forward and futures curve on electricity.

A bivariate version of this model is presented in 3.3.1, and an N dimensional version is presented in appendix A, including parameter estimation, statistical properties and how to simulate trajectories of the process.

An interesting observation is that the model assumes continuous time but observations only for discrete values of time have the "average spot price" interpretation. One could use an equivalent discrete model directly, but it is common to use continuous time models so as to make use of their theoretical properties.

3.2.2 Mean reverting jump-diffusion

Consider the following extension to the model presented in subsection 3.2.1:

$$\log S(t) = f(t) + X(t),$$
$$dX(t) = -\frac{X(t)}{\lambda}dt + \sigma dW(t) + J(t)dq(t).$$

Where $S(t)$ is the average spot price of electricity on day t, $f(t)$ is a deterministic component, and X is a mean reverting jump-diffusion. The jump intensity γ is determined by the Poisson process $q(t)$ that has independent increments and satisfies: $P(dq(t) = 0) = 1-\gamma dt$, $P(dq(t) = 1) = \gamma dt$. This is equivalent to approximating the increments (on dt) of a Poisson process by a distribution with the outcomes 0 and 1 by matching its mean value, which is justified by the "order property" that states that $P(N(t+dt)-N(t) > 1) = o(dt)$ for a Poisson process $\{N(t)\}_{t\geq 0}$. On the other hand, $J(t)$ determines the jump size, and may have any appropriate distribution. Using a normal distribution may ease the parameter estimation procedure, but it may be unrealistic in most cases to have negative jumps and hence reasonable alternatives are, for instance, lognormal and Pareto distributions. A bivariate version of this model is presented in section 3.3.2.

This model has been widely used for electricity price modeling. Its importance relies in the frequent appearance of jumps in electricity spot prices due to unpredictable events such as forced generator outages or sudden increases in electricity demand. In particular, Weron & Misiorek (2008) used it for modeling the California Power Exchange and the Nord Pool electricity spot prices, using a Normal distribution for jumps.

3.2.3 Sum Ornstein-Uhlenbeck diffusion

Meyer-Brandis & Tankov (2008) present a model in which the logarithm of the price is modeled as a sum of a deterministic component and two other factors. In this model the first factor ($Y(t)$) corresponds to a "base signal" with a slow rate of mean reversion, and the second factor ($Z(t)$) represents spikes, with a high rate of mean reversion. The model is:

$$\log S(t) = f(t) + Y(t) + Z(t),$$

$$dY(t) = -\frac{Y(t)}{\lambda}dt + \sigma dW(t),$$
$$dZ(t) = -\frac{Z(t)}{\mu}dt + dL(t).$$

Where $f(t)$ is a deterministic component, Y is a standard Ornstein-Uhlenbeck process driven by a Brownian motion, and Z is the component responsible for spikes, a special Ornstein-Uhlenbeck process driven by a Lévy process L that determines the time periods in which spikes occur and their sizes. In fact, $dL(t) = J(t)dq(t)$, similar to the jump component shown in 3.2.2.

The importance of this model relies in the frequent appearance of spikes in electricity spot prices, instead of simple jumps. Notice the difference with the pure jump model in which, after jumps, prices only return to normal states by the property of mean reversion. If "spiky paths" are to be represented in a jump diffusion, the rate of mean reversion must be very big (and such a rate of mean reversion may be unappropriate in "normal levels"). The essential difference between this model and typical jump-diffusion models is that the component Z that generates jumps has a big enough rate of mean reversion such that the process returns quickly to the normal values, therefore producing spiky paths.

The authors applied the model using data from various sources, including the European Power Exchange and the Nord Pool, and the parameter estimation process developed by the authors involves first filtering out the spiky component of the series. A bivariate version of this model is presented in 3.3.2, and in 3.2.10 another approach to modeling spikes is presented.

3.2.4 Mean reverting diffusion with two types of jumps and an external factor

The following model is presented in Deng (2000):

$$d\begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} = \begin{pmatrix} \kappa_1(t)(\theta_1(t) - X(t)) \\ \kappa_2(t)(\theta_2(t) - Y(t)) \end{pmatrix} dt + \begin{pmatrix} \sigma_1(t) & 0 \\ 0 & \sigma_2(t) \end{pmatrix} d\begin{pmatrix} W_1(t) \\ W_2(t) \end{pmatrix}$$
$$+ d\overrightarrow{Z}_1(t) + d\overrightarrow{Z}_2(t);$$
$$E[W_1(t)W_2(t)] = \rho\, dt.$$

Where $X(t)$ denotes the electricity price logarithm, $Y(t)$ is other factor that may be the electricity demand or the logarithm of the price of a relevant fuel (for instance natural gas), the $\theta_i(t)$'s are the mean reverting levels (similar to the deterministic components $f(t)$ in the models shown above), the $\kappa_i(t)$'s are the rates of mean reversion, the $\sigma_i(t)$'s are the volatilities, the $W_i(t)$'s are correlated Brownian motions, and the $\overrightarrow{Z}_i(t)$ processes are two-dimensional jump components that determine the intensity, sizes and directions of each type of jump. Note that in this model all the parameters are shown as time-dependent (yet deterministic), which may account for seasonal components: for instance this may enhance having higher volatilities in warm seasons.

The contribution of this model, compared to the models presented above, is to incorporate an external factor and to account for two types of jumps. The author applies this model to the valuation of forwards and options on electricity at the Electricity Reliability Council of Texas (ERCOT).

3.2.5 Mean reverting diffusion with two types of jumps, stochastic volatility and an external factor

An important property used in commodity price modeling is stochastic volatility. Deng (2000) also presents an extension to the model presented in 3.2.4, incorporating stochastic volatility. The model is:

$$d\begin{pmatrix} X(t) \\ V(t) \\ Y(t) \end{pmatrix} = \begin{pmatrix} \kappa_1(t)(\theta_1(t) - X(t)) \\ \kappa_V(t)(\theta_V(t) - V(t)) \\ \kappa_2(t)(\theta_2(t) - Y(t)) \end{pmatrix} dt + \begin{pmatrix} \sqrt{V(t)} & 0 & 0 \\ 0 & \sigma_2(t) & 0 \\ 0 & 0 & \sigma_3(t) \end{pmatrix} d\begin{pmatrix} W_1(t) \\ W_2(t) \\ W_3(t) \end{pmatrix}$$

$$+ d\overrightarrow{Z}_1(t) + d\overrightarrow{Z}_2(V(t), t);$$

$$E[W_i(t)W_j(t)] = \rho_{ij}\, dt; i, j = 1, 2, 3.$$

The definition of the components is similar to the model shown in 3.2.4, but the difference is that now the volatility of $X(t)$ is $\sqrt{V(t)}$, where $V(t)$ is a stochastic process that follows a mean reverting diffusion itself, with its stochastic shocks correlated with the shocks of $X(t)$ and $Y(t)$. Note that this time the second jump component $\overrightarrow{Z}_2(V(t), t)$ depends on time and also on $V(t)$. This is because the intensity of type 2 jumps is intended to depend on $V(t)$ (this may represent that in more volatile periods the jump frequency is higher). Electricity prices not only present high volatilities, but in many cases also present an unpredictable time evolution of volatility. Therefore the main contribution of this work is assuming volatility as a stochastic process itself. This model was also applied to the valuation of forwards and options on electricity.

3.2.6 ARIMA model

In Contreras et al. (2003), an ARIMA model is presented for forecasting next-day hourly electricity prices. The model presented is formulated as follows:

$$\phi(B)p_t = \theta(B)\epsilon_t.$$

Where p_t is electricity price or its logarithm at hour $t = 1, 2, ...$, $\{\epsilon_t\}_t$ is a "white noise", and $\phi(B)$ and $\theta(B)$ are polynomial functions of the backshift operator B that extend the traditional ARIMA model by incorporating different factors that account for, for instance, seasonalities by including generalized daily or weekly differentiations (instead of simple differentiations such as $(1 - B)^d$, for example, a daily differentiation is $(1 - B^{24})$ and a weekly differentiation is $(1 - B^{168})$).

For example, one model presented is:

$$\begin{gathered}(1 - \phi_1 B^1 - \phi_2 B^2 - \phi_3 B^3 - \phi_4 B^4 - \phi_5 B^5) \\ \times(1 - \phi_{23} B^{23} - \phi_{24} B^{24} - \phi_{47} B^{47} - \phi_{48} B^{48} - \phi_{72} B^{72} - \phi_{96} B^{96} - \phi_{120} B^{120} - \phi_{144} B^{144}) \\ \times(1 - \phi_{168} B^{168} - \phi_{336} B^{336} - \phi_{504} B^{504}) \log p_t \\ = c + (1 - \theta_1 B^1 - \theta_2 B^2) \times (1 - \theta_{24} B^{24}) \times (1 - \theta_{168} B^{168} - \theta_{336} B^{336} - \theta_{504} B^{504})\epsilon_t,\end{gathered}$$

where, on the left side of the equal sign, the first factor accounts for information of the past five hours, the second factor for information related to the past days, and the third factor for the past weeks; and, on the right side, similarly but for the noise term (c is a constant). This is a generalization of differentiating the data and then fitting an ARMA model.

The contribution with respect to the approaches presented above is that a highly accurate model is obtained due to the fact that the this model can successively take into account many of the past prices. Test cases are presented fitting data from the Spanish and the Californian electricity markets and the statistical methodology for selecting a model and fitting the parameters is described.

3.2.7 External factor model

Zhou et al. (2009) use the information of electricity demand and scheduled generator outages to model electricity prices. When a generator is scheduled for an outage, this has a similar effect to an increase in electricity demand, and this motivates considering the variable $DO(t)$, defined as the sum of electricity demand and scheduled generator outages, which has a strong correlation with electricity price.

In their work, an ARMA model is fitted to $DO(t)$, and the electricity price model presented is:

$$p_t = \alpha + \beta DO(t) + \epsilon_t,$$

where p_t is electricity price at time period t, ϵ_t is the error term, and α and β are parameters.

The main contribution is the construction of a simple model for the relation between electricity spot price and an external variable, $DO(t)$, and the use of a complex statistical model (an ARMA model) for this external variable. The model is fitted with data from the Midwest Independent System Operator (MISO).

3.2.8 Dynamic regression model and transfer function model

Nogales et al. (2002) present the following "dynamic regression model":

$$\log p_t = c + w^d(B) \log d_t + w^p(B) \log p_t + \epsilon_t,$$

where p_t is electricity price at hour t, c is a constant, d_t is electricity demand at hour t, $w^d(B)$ and $w^p(B)$ are polynomials on the backshift operator B (similar to $\phi(B)$ in 3.2.6), and $\{\epsilon_t\}_t$ is white noise.

The authors also present the following transfer function model:

$$\log p_t = c + w^d(B) \log d_t + N_t,$$

with an analog definition of the components, and with a stochastic component N_t that follows an ARMA model.

The contribution of these models is to incorporate an external factor in an ARMA based model. Both models are fitted using data from the Spanish and the Californian electricity markets. According to the authors both alternatives are highly accurate and efficient price forecasting tools.

3.2.9 The SMaPS model

Burger et al. (2004) present a model called "SMaPS" (Spot Market Price Simulation model) given by:

$$\log S_t = f(t, \frac{L_t}{v_t}) + X_t + Y_t,$$

where S_t is electricity price at hour t, L_t is the total system load, X_t accounts for short-term price variations that are not explained by the load, Y_t accounts for long-term price variations, the deterministic function v_t specifies the expected relative availability of generators in the system, and f is a deterministic function called the "merit order curve" that establishes a relationship between electricity prices and load.

In this model, L_t, X_t and Y_t are stochastic factors assumed to be independent, and L_t is modeled as:

$$L_t = \widehat{L}_t + l_t,$$

where $\widehat{L}_t$ is a deterministic load forecast and l_t follows a SARIMA model with a 24-hour seasonality. The process X_t follows a SARIMA process as well, and Y_t follows a (discretized) Brownian Motion.

The main contribution of this model with respect to the models presented above is to use two stochastic processes to model the evolution of electricity prices, where one of this components accounts for the long term behavior and the other for the short term variations, producing paths that may capture both aspects. The model is fitted using data from the European Energy Exchange and is used for pricing electricity derivatives.

3.2.10 Regime switching model I

In Huisman & Mahieu (2003) a model that accounts for spikes in the price process is presented. In this model, the price process follows a mean reverting diffusion, most of the time, until a jump is generated, and then at the next time period (having daily time periods) a jump in the opposite direction is generated so as to make the process return to the "normal regime". This is achieved using "regime switching" that basically consists of having a set of different regimes, each of them represented by a particular behavior of the stochastic process (that may be completely different from the behavior in the other regimes), and there is a discrete time Markov chain that determines the regime at every particular time period.

A simple version of the model is given by:

$$d[\log S(t)] = \mu_{r_t} dt + \sigma_{r_t} dW(t).$$

Where $\{r_t\}_t$ is a discrete time Markov chain with state space $\{-1, 0, 1\}$ and transition probabilities given by:

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & p_{00} & 1 - p_{00} \\ 1 & 0 & 0 \end{pmatrix},$$

with $r_t = 0$ representing the normal regime, $r_t = 1$ the initial jump regime (upward jump), and $r_t = -1$ the subsequent reversal to normal levels (downward jump). In the normal regime, the logarithm of the price behaves like a diffusion with drift μ_0. In the jump regimes, having $\mu_{-1} = -\mu_1$ and $\sigma_1 = \sigma_{-1}$ enhances the behavior of having an upward jump immediately followed by a similar downward jump, returning the process to normal levels, because once a lognormal upward jump is generated it is immediately followed by a similar lognormal jump in the opposite direction. Since time periods are days, one-day price spikes are represented. Slight modifications could produce behaviors such as more time in the “high price state”, different jump behaviors and/or mean reversion.

This model and some variants have been tested using data from electricity markets in California, UK, Germany, and the Netherlands. The authors conclude that typical mean reversion jump diffusions lead to a mistaken mean reverting behavior, whereas their model leads to a better specification of electricity price dynamics. Higgs & Worthington (2008) apply a very similar model to the Australian electricity market.

This is a particular case of a regime switching model. The next section presents another regime switching approach for representing spikes. Readers interested in the theory of regime switching models can find more details in Hamilton (1994).

3.2.11 Regime switching model II

Schindlmayr (2005) presents a model in which a daily 24-dimensional vector process is used instead of using a 24-hour seasonality. The hourly prices are modeled as a decomposition into a daily average and an hourly profile:

$$\log S_t^i = s_t + h_t^i,$$

where S_t^i is the price on day t and hour i, $i = 1, ..., 24$, $s_t = \frac{1}{24}\sum_{i=1}^{24} \log S_t^i$ is the daily average of the logarithm of electricity prices, and $h_t = (h_t^1, ..., h_t^{24})$ is the hourly profile, that satisfies $\sum_{i=1}^{24} h_t^i = 0$.

The daily average price process s_t is modeled as the sum of a seasonal component and an AR(1) model with regime switching. The seasonal component is determined using a regression model, as follows:

$$s_t = \sum_{d=1}^{N_{DT}} 1_{J_d^{DT}}(t)[\beta_d^A + \beta_d^B \cos(\frac{2\pi t}{365}) + \beta_d^C \sin(\frac{2\pi t}{365}) + \beta_d^D t] + y_t,$$

where y_t is a random component, $1_{J_d^{DT}}(t)$ is an indicator function that takes the value 1 if $t \in J_d^{DT}$, and 0 otherwise, and the sets J_d^{DT} define a partition into day types (for instance business and non-business days), $d = 1, ..., N_{DT}$. The random component y_t is divided into business (J_B) and non-business (J_H) days:

$$y_t = 1_{J_B}(t)y_t^B + 1_{J_H}(t)y_t^H,$$

and each of the components y_t^B, y_t^H is modeled as an AR(1) model with regime switching so as to account for spikes. Suppose that z_t represents either y_t^B or y_t^H. The time-series model is:

$$z_t - \mu_{r_t} = \phi_{r_t}(z_{t-1} - \mu_{r_{t-1}}) + \sigma_{r_t}\epsilon_t,$$

where $r_t \in \{1, 2\}$ denotes the regime at time t and ϵ_t has a standard Normal distribution. State 1 represents the normal behavior of electricity prices, and state 2 represents spiky periods.

The hourly profile process is modeled as:

$$h_t = \begin{pmatrix} h_t^1 \\ ... \\ h_t^{24} \end{pmatrix} = \widehat{h}_t + \triangle h_t,$$

where $\widehat{h}_t$ is a (24-dimensional) deterministic seasonal component (obtained similarly to the seasonal component of the daily process shown above), and $\triangle h_t$ is a stochastic vector, divided into business and non-business days (as it was the case for y_t):

$$\triangle h_t = 1_{J_B}(t)\triangle h_t^B + 1_{J_H}(t)\triangle h_t^H.$$

Now, let x_t represent either $\triangle h_t^B$ or $\triangle h_t^H$. The vector process x_t is decomposed into independent factor loads by Principal Component Analysis. This decomposition process consists of finding matrices W and P such that $W = xP$, where $P = (p_1, ..., p_{24})$ is an orthogonal 24x24 matrix, with $p_1, ..., p_{24}$ the principal component factors of x_t (after column-wise normalization to standard deviation 1) and W is a matrix that contains the independent factor loads. Let $w_t^i = W_{ti}$ be the time series of the i^{th} factor load. These factor loads are modeled as independent ARMA processes. The model for x can then be obtained as $x = WP^{-1} = WP^T$.

One of the contributions of this model is to present a highly detailed approach to modeling the daily average deterministic component separating days into groups and then using a different stochastic component for business and non-business days. Regime switching is used for modeling the stochastic component of daily averages, in order to capture the spiky behavior of electricity prices. Instead of having an upward and a downward jump, such as the model presented in 3.2.10, only two regimes are used, one representing "normal levels" and another representing "high price levels". Another contribution is to model intra-day variations using a stochastic vector process, in order to take into account the correlations of electricity prices in different hours of a same day. The model has been tested using data from the European Energy Exchange.

3.2.12 Regime switching model III

Haldrup & Nielsen (2006) analyze the zonal prices of the Nord Pool market. In this market, when no bottlenecks or congestions exist in the physical interconnections between zones, all zonal prices are identical, whereas different zonal prices appear in situations with transmission capacity constraints (congestion). The model presented is a "regime switching multiplicative seasonal autoregressive fractional integrated moving average" model:

$$A_{s_t}(B)(1 - a_{s_t}B^{24})(1 - B)^{d_{s_t}}(y_t - \mu_{s_t}) = \epsilon_{s_t,t}.$$

Where y_t is the log price at a given zone, after correcting it with a deterministic seasonality component, $s_t \in \{0, 1\}$ denotes the regime, determined by a Markov chain, with 0 being the non-congestion state in which the different zonal prices are identical, and 1 being the congestion state in which zonal prices are different. $A_{s_t}(B)$ is an eight order polynomial in the backshift operator B that accounts for intraday effects, $(1 - a_{s_t}B^{24})$ is a "daily quasi-difference filter" that accounts for daily seasonality, μ_{s_t} is a mean, and $\epsilon_{s_t,t}$ is the error term with variance $\sigma^2_{s_t}$. $(1 - B)^{d_{s_t}}$ is the component that accounts for long memory by having d_{s_t} fractional. And the same model is used for the difference of the logarithm of prices at the two different zones (which means that the same model is used for the price at any location and for relating locational prices).

Since in the non-congestion state (state 0) both prices are identical, if y_t denotes the difference of the logarithm of prices of the two different zones, then $A_0(B) = 1$, $a_0 = 0$, $d_0 = 0$, $\mu_0 = 0$ and $\sigma_0^2 = 0$ (and hence $\epsilon_{0,t} = 0$), generating $y_t = 0$ constantly in the non-congestion state, and hence making zonal prices equal in that state.

The fundamental contribution of this model is that it captures a particular joint behavior of the locational electricity spot prices of the Nord Pool: zonal prices are equal when there is no congestion and depart from each other when there is congestion. Another important contribution is to use a long memory model that takes into account the price variations at distant past time periods, since this property was tested and found to be relevant in the market analyzed. The model was fitted using data from various zones of the Nord Pool.

3.2.13 Supply-demand equilibrium model

In Kanamura & Ohashi (2007) the demand and supply curves are modeled independently and an equilibrium price is obtained. On one hand, the inelastic demand in day t is modeled as:

$$D_t = \overline{D}_t + X_t,$$

where $\overline{D}_t$ is a deterministic component, and X_t is a mean reverting diffusion:

$$dX_t = (\mu - \lambda X_t)dt + \sigma dW_t.$$

On the other hand, the supply curve is modeled with a "hockey stick function" that corresponds to a line with a moderate slope, followed by a parabolic curve and then by a highly sloped line. This relationship is given by:

$$P_t = f(S_t) = \epsilon_t + \begin{cases} \alpha_1 + \beta_1 S_t & \text{if } S_t < z - s \\ a + bS_t + cS_t^2 & \text{if } z - s \leq S_t \leq z + s \\ \alpha_2 + \beta_2 S_t & \text{if } S_t > z + s \end{cases}$$

Where S_t is supply, P_t is electricity price and ϵ_t is an error term. Electricity price is obtained from the intersection of supply and demand, as the point where $S_t = D_t$, and hence, $P_t = f(D_t)$. The "hockey stick" shape of the supply curve function directly captures the relationship between electricity demand and price spikes because of the high price slope for large supply values, more precisely for $S_t > z + s$.

The model was tested using data from the PJM market. This model corresponds to the category of structural models: electricity price is obtained as a byproduct of modeling the supply and demand curves, finding the intersection of supply and demand, instead of using a statistical model for electricity prices directly as it was done in the other models presented above. For the interested reader, a review of several alternatives for supply and demand modeling and forecasting in deregulated electricity markets may be seen in Bunn (2000).

3.2.14 Stochastic factor model

Another structural approach to electricity price modeling is presented in Vehviläinen & Pyykkönen (2005), where the factors affecting the spot price of the Nordic Market are modeled as stochastic factors that follow statistical processes and these are combined to form the spot price by assuming a market equilibrium model. According to the authors, the processes for fundamental factors are more stable in form and more accurately represented by statistical models than the more complicated spot price process. Monthly time periods are assumed and the model is best suited for medium term or long term analysis.

The model is built based on the features presented in the Nordic Market, in which production is mostly hydroelectric. The spot price process is basically built taking into account the most relevant stochastic climate factors, temperature and precipitation, since the hydrological situation is the main determinant of market conditions. The climate factors are assumed to follow the following relations:

$$x_{\triangle Temp}^{t+1} = c_{TempSer} x_{\triangle Temp}^{t} + \sigma_{\triangle Temp}^{t} \epsilon_{\triangle Temp}^{t},$$

$$x_{Temp}^{t} = c_{TempAve}^{t} + x_{\triangle Temp}^{t};$$

$$x_{\triangle Precip}^{t+1} = c_{PrecipSer} x_{\triangle Precip}^{t} + \sigma_{\triangle Precip}^{t} \epsilon_{\triangle Precip}^{t},$$

$$x_{Precip}^{t+1} = c_{PrecipAve}^{t} + x_{\triangle Precip}^{t},$$

where x^t_{Temp} is temperature at time t, $x^t_{\triangle Temp}$ is the deviation of temperature from its average temperature at time t: $c^t_{TempAve}$. And the component $\epsilon^t_{\triangle Temp}$ follows a Normal distribution with mean 0 and variance 1 (analogously for precipitation). $\epsilon^t_{\triangle Temp}$ is assumed to be correlated with $\epsilon^t_{\triangle Precip}$ with correlation factor $\rho^t_{\triangle Temp \triangle Precip}$. Parameters $c_{TempSer}$ and $c_{PrecipSer}$ are the autocorrelation parameters of the processes $x^t_{\triangle Temp}$ and $x^t_{\triangle Precip}$, respectively. This temperature-precipitation model corresponds to a bivariate AR(1) model of their deviations from their average values.

Once temperature and precipitation are modeled, a series of steps are developed for obtaining the supply and demand curves for electricity. Hydro-power production depends on the water levels in reservoirs, which are in turn filled by hydro-inflow, and hydro-inflow results from precipitation and the melting of snow-pack. Generation of snow-pack is a consequence of precipitation and temperature (at certain temperatures precipitation freezes, incrementing the snow-pack). Electricity demand is assumed to be driven by temperature, and its uncertain behavior is captured with an additional stochastic factor. Having modeled the dynamics of the supply and demand curves for electricity, the electricity spot price is obtained intersecting both curves.

The contribution of this approach is to incorporate climate factors into the modeling of the electricity demand and supply dynamics, presenting a series of steps that provide the relations between these and other relevant factors such as the characteristics of generating units. Another important aspect to remark is that this model is intended to be used for medium term or long term purposes, such as the valuation of contracts or a power plant. The model is fitted using data from the Nord Pool and it is applied to the valuation of an exotic derivative instrument that depends on the dynamics of temperature and electricity prices, taking advantage of the joint modeling of several factors that this approach provides.

3.2.15 Hidden Markov model

A pair of stochastic processes (S, Y) is a hidden Markov model if S (the state process) is a Markov process and Y (the observable process) is a partial observation of S. Yu & Sheblé (2006) propose a model of electricity markets as partially observable Markov processes driven by underlying economic forces, which are combined assuming certain relations in order to determine electricity spot price.

The article includes a model in which the state process corresponds to electricity demand having electricity price modeled as:

$$P_t = f(t) + X(s_t, t).$$

Where P_t is electricity price at hour t, $f(t)$ is a deterministic component, $s_t = 1, 2, 3$ is the demand state, with certain transition probabilities (Markov chain), and $X(s_t, t)$ is a component that accounts for low, normal or high prices, with a distribution depending on the demand state s_t. State 1 corresponds to low demand, 2 to medium demand, and 3 to high demand, with $X(s_t, t)$ having a different discrete distribution in each of the states (similarly to a regime-switching model).

The authors also include a model of higher complexity in which the state process consists of electricity demand, demand for ancillary services and the actual generation capacity. Other factors could be considered as well, such as the congestion of transmission networks.

The contribution of this approach is that a Markov chain is used to model the factors that contribute to the formation of electricity prices, being this Markov chain not necessarily observable and presenting states with particular meanings. This approach combines structural and regime switching models. The model has been tested with data from the New York Independent System Operator.

3.2.16 Forward curve models

The modeling of forward electricity price curves may be useful for medium and long term risk management applications of generating companies and other agents participating in electricity markets. As a byproduct of modeling the evolution of the prices for forward contracts, a spot price model is obtained. Denote $f(t, T)$ the forward price at time t for delivery at T (or in the time period $[T, T + dt]$). Then, under certain assumptions, the spot price is given by $S(t) = f(t, t)$. Audet et al. (2004) present the following model for the electricity forward curve:

$$df(t,T) = f(t,T)e^{-\alpha(T-t)}\sigma(T)dB_T(t); \quad t \in [0,T], T \in [0,\tau].$$

Where $B_T(t)$ is a Brownian motion that has the following correlation structure:

$$E[dB_{T_1}(t), dB_{T_2}(t)] = e^{-\rho|T_1-T_2|}dt,$$

and $[0,\tau]$ is the time horizon in which the model is valid. This model has been applied using data from the Nord Pool.

Similarly, in Clewlow & Strickland (1999) a multi-factor model is proposed for the forward curve process of oil and natural gas:

$$df(t,T) = f(t,T)\sum_{i=1}^{n}\sigma_i(t,T)dB_i(t); \quad t \in [0,T], T \in [0,\tau].$$

In this model, the $B_i(t)$ are independent Brownian motions. According to the authors, the model can be integrated (using Ito's lemma) to obtain:

$$f(t,T) = f(0,T)\exp(\sum_{i=1}^{n}\{-1/2\int_0^t \sigma_i(u,T)^2 du + \int_0^t \sigma_i(u,T)dB_i(u)\}),$$

and hence the associated spot price process corresponds to:

$$S(t) = f(t,t) = f(0,t)\exp(\sum_{i=1}^{n}\{-1/2\int_0^t \sigma_i(u,t)^2 du + \int_0^t \sigma_i(u,t)dB_i(u)\}).$$

The model has been tested using data from the New York Mercantile Exchange.

If the reader is further interested in the modeling of electricity forward curve dynamics another alternative may be seen in Borak & Weron (2008) where a semi-parametric factor model is applied to the electricity forward curve dynamics of the Nord Pool.

Forward contracts on electricity are fundamental for risk management purposes in electricity markets, and the importance of these models relies in the necessity of understanding the unpredictable behavior of electricity forward curve dynamics. The contribution of these models with respect to electricity spot price modeling is that spot prices are obtained as a byproduct of modeling the dynamics of the forward curve.

3.2.17 Neural networks and more on electricity price forecasting

Neural networks are non-parametric methods that have gained popularity because of its capability of approximating any non-linear function and because of its good performance in forecasting. For the interested reader, an introduction to artificial neural networks applied to electricity price and load forecasting is presented in Shahidehpour et al. (2002). Hong & Hsiao (2001) present an application of recurrent neural networks to price forecasting.

Despite the proven good forecasting performance of these methods in electricity markets they are of low interest in this thesis because they do not model uncertainty explicitly and because the models obtained for forecasting are generally hard or impossible to interpret.

Further on electricity price forecasting, a comparison of 12 electricity price models is given in Weron & Misiorek (2008) for the purpose of day-ahead electricity price forecasting. The models studied in this work include an AR model with electricity demand as external factor, a simple version of a regime switching model, a mean reverting jump diffusion, and some semi-parametric extensions of these models that do not assume specific distributions for the innovations. The models were applied and tested using data from the California Power Exchange and Nord Pool. The main conclusions of this work are that electricity demand is an important external factor to take into account for day-ahead electricity price forecasting, and that semi-parametric models tend to perform better than parametric models in the same task.

3.3 Model extensions

The models presented in this section are bivariate extensions of the models presented in 3.2.1, 3.2.2 and 3.2.3. These models are intended to represent the behavior of two locational electricicty prices, which is central in this thesis.

3.3.1 Bivariate mean reverting diffusion

The following model is a bivariate extension of the model presented in section 3.2.1:

$$\log S_i(t) = f_i(t) + X_i(t); i = 1, 2;$$
$$dX_i(t) = -\frac{X_i(t)}{\lambda_i} dt + \sigma_i dW_i(t); i = 1, 2;$$
$$E[dW_1(t) dW_2(t)] = \rho\, dt.$$

Where $S_i(t)$ accounts for the ith price at time t, f_i accounts for a deterministic component and X is a bivariate Ornstein-Uhlenbeck process (with mean reversion to 0 in each component). The $W_i(t)$ are correlated Brownian motions.

This model can be used to represent not only two correlated electricity price processes but also an electricity price process together with a fuel price process, for instance. An N dimensional version of this model is presented in appendix A, including parameter estimation, statistical properties and how to simulate trajectories of the process.

3.3.2 Bivariate mean reverting jump-diffusion

Consider the following extension to the model presented in 3.2.2:

$$\log S_i(t) = f_i(t) + X_i(t), i = 1, 2;$$
$$dX_i(t) = -\frac{X_i(t)}{\lambda_i} dt + \sigma_i dW_i(t) + J_i(t) dq_i(t), i = 1, 2;$$
$$E[dW_1(t) dW_2(t)] = \rho\, dt.$$

Where $dq_i(t)$ accounts for the jump times in the ith price component and $J_i(t)$ for the jump sizes. There are three jump intensities: γ_1, γ_2 and γ_{12}, where $P[dq_1(t) = 1, dq_2(t) = 0] = \gamma_1 dt$, $P[dq_1(t) = 0, dq_2(t) = 1] = \gamma_2 dt$, $P[dq_1(t) = 1, dq_2(t) = 1] = \gamma_{12} dt$, and $P[dq_1(t) = 0, dq_2(t) = 0] = 1 - (\gamma_1 + \gamma_2 + \gamma_{12})dt$.

The jump sizes $J_1(t)$ and $J_2(t)$ may follow any appropriate bivariate joint distribution (if only one jump occurs at one time period, only one marginal distribution matters, but if both jumps occur at the same time period then the joint distribution matters). Using a bivariate normal distribution may ease parameter estimation, but it may be unrealistic. Other alternative is using a bivariate log normal distribution.

3.3.3 Bivariate sum Ornstein-Uhlenbeck diffusion

The model presented in 3.2.3 may be extended as:

$$\log S_i(t) = f_i(t) + Y_i(t) + Z_i(t), i = 1, 2;$$
$$dY_i(t) = -\frac{Y_i(t)}{\lambda_i} dt + \sigma_i dW_i(t), i = 1, 2;$$
$$dZ_i(t) = -\frac{Z_i(t)}{\mu_i} dt + dL_i(t), i = 1, 2;$$
$$E[dW_1(t) dW_2(t)] = \rho\, dt.$$

Where $dL_i(t) = J_i(t) dq_i(t)$ as in the model shown in section 3.3.2. Notice that similarly to the model presented in 3.2.3, having a separate mean reverting component for jumps permits the modeling of price spikes (joint or separate as in 3.3.2).

The next chapter will present an introduction to scenario tree construction methods and will also present two different applications for approximating the bivariate electricity price model presented in 3.3.1, using scenario trees.

IV. SCENARIO TREE CONSTRUCTION APPLICATIONS FOR A BIVARIATE STOCHASTIC PROCESS

In decision making models it is often necessary to represent uncertainties in a form suitable for computation, and in this context, scenario trees have proven to be important means for representing time evolving uncertainties in stochastic programming.

A scenario tree represents a finite set of discrete outcomes over a certain time horizon, with associated probabilities. The scenarios of the tree describe possible future trajectories of the parameters considered as uncertain and the tree itself represents the unfolding of information in time. As an example, a simple scenario tree may be seen in figure 4.1. This tree has a time horizon of two time periods and three possible scenarios, corresponding to the node sequences 0-1-3 for scenario $s = 1$, 0-1-4 for scenario $s = 2$ and 0-2-5 for scenario $s = 3$. The $z_n, n = 0, ..., 5$, are the (vector) values taken by the uncertain components at every node with z_0 deterministic, and the p_{mn} are the transition probabilities from one node at a certain time period to another node at the next time period (in this case: $p_{01} + p_{02} = 1, p_{13}+p_{14} = 1, p_{25} = 1$). Another way to represent the information contained in a scenario tree is to present the values taken by the uncertain components at each scenario and time period, and probabilities of the whole scenarios. In this case, if y_t^s represents the values taken by the uncertain components in scenario s and time period t then $y_0^1 = y_0^2 = y_0^3 = z_0, y_1^1 = y_1^2 = z_1, y_1^3 = z_2, y_2^1 = z_3, y_2^2 = z_4, y_2^3 = z_5$, and if q_s represents the probability of scenario s then $q_1 = p_{01}p_{13}, q_2 = p_{01}p_{14}, q_3 = p_{02}p_{25}$.

Scenario trees should be complex enough in order to represent adequately the underlying stochastic process, but small enough in order for the stochastic program to be computationally tractable. This is one of the main difficulties of scenario tree construction methods: to build adequate scenario trees that are also small enough. At the same time, the stochastic process to be approximated by a scenario tree cannot be too complex (in order for its main characteristics to be represented using a reasonable scenario tree) but has to be a reasonable representation of the actual process that it models.

The remainder of this chapter is organized as follows. In section 4.1 an overview of scenario tree construction methods is presented. Section 4.2 deals with scenario tree

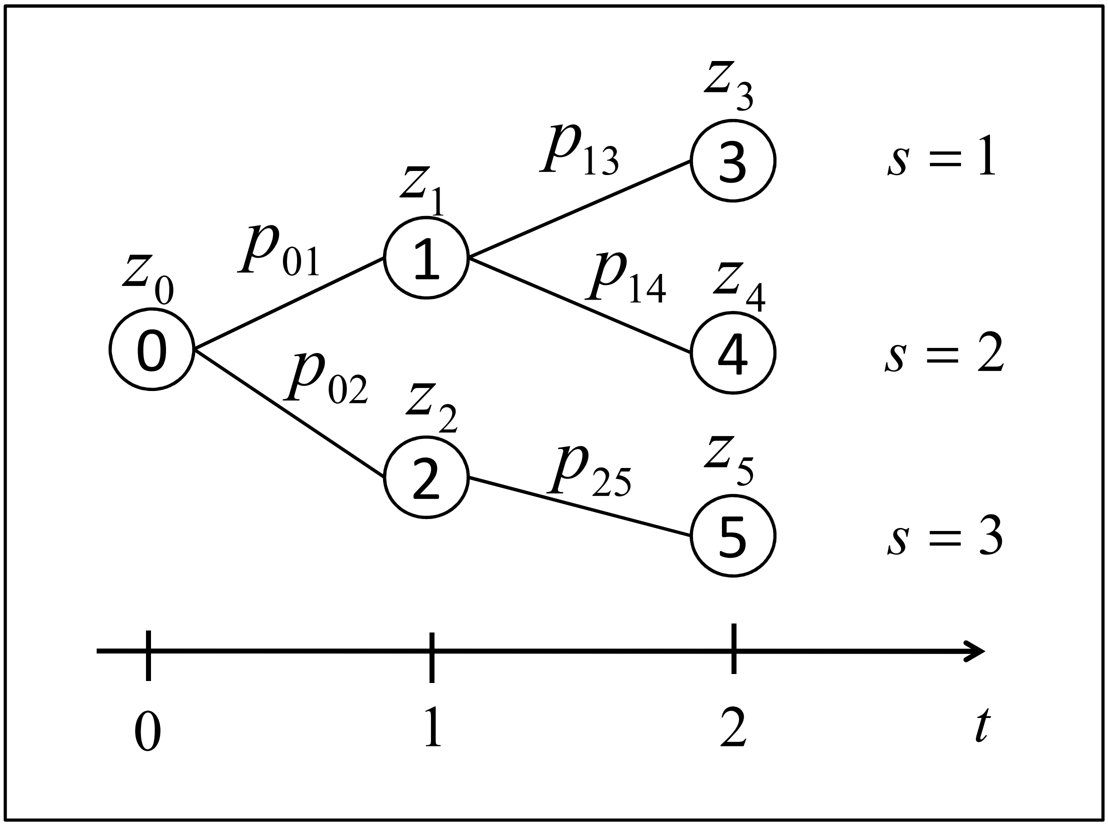


FIGURE 4.1. Scenario tree example

evaluation. Section 4.3 presents a framework for the scenario tree construction methods presented in sections 4.4 and 4.5. Finally, the scenario tree construction methods presented in sections 4.4 and 4.5 are analyzed in section 4.6 and one of them is selected for the application presented in chapter 6.

4.1 Overview of scenario tree construction methods and related methods

The main categories of scenario tree construction methods according to Kaut & Wallace (2003) are described in what follows.

- **Sampling methods** At every node of the scenario tree, several values are sampled for the stochastic process that the tree is intended to represent, say $\{X_t\}_t$. This is done by sampling directly from the distribution of X_t (at time t), or by evolving the process by an explicit formula $X_t = f(X_0, ..., X_{t-1}, \epsilon)$, sampling from ϵ.
  If a random vector is to be sampled, we generally have to sample every marginal separately and combine them afterwards. One problem of doing so is that the size of the tree grows exponentially with the dimension of the random vector. Another problem is how to get correlated random vectors. An approach for this problem is to find the principal components and sample those. Another approach is to use sampling methods in which marginal distributions and a correlation matrix are specified and multivariate vectors that satisfy these properties are sampled.
- **Target/moment matching methods** These methods have as a common element the construction of discrete distributions that satisfy certain statistical properties. These statistical properties generally correspond to the main moments of the marginals (mean, variance, skewness, kurtosis), auto-correlations of the marginals, and correlations between the components of the random vector (other statistical properties such as quantiles and minimum and maximum values can also be considered). An advantage of these methods is that the marginal distributions do not have to be known: they may be described by the statistical properties under consideration. The means for constructing the discrete distributions, and the tree itself, involves numerical procedures that require solving equations or using non-linear programming methodologies. Examples of such methods can be found in Hoyland & Wallace (2001) and Hoyland et al. (2001). The method presented in section 4.6 corresponds to this category.
- **Path-based methods** Independent scenarios are generated by evolving the stochastic process under consideration, in order to obtain a set of paths, called a fan. Once a fan is obtained, it is transformed into a scenario tree by clustering the scenarios in all but the last period. The clustering process is usually based on probability metric based approximations. Examples of such methods can be

found in Dupacová et al. (2000), Eichhorn et al. (2009) and Gröwe-Kuska et al. (2003). The method presented in section 4.5 corresponds to this category.

- **Optimal discretization** Pflug (2000) presents a method that tries to find an adequate scenario tree by minimizing an error term in the objective function of the optimization model from which stochastic parameters are approximated by the scenario tree. This method works only for univariate processes and constructs the whole multi-period scenario tree at once.

The following methods described are related to scenario tree construction methods.

- **Scenario reduction** These are methods for decreasing the size of a scenario tree. Basically, this method finds a subset of scenarios, of given cardinality, such that the probability distribution of the scenarios obtained is closest to the initial distribution in terms of some probability metric. Examples of such methods are described in Eichhorn et al. (2009) and Gröwe-Kuska et al. (2003).
- **Internal sampling methods** These methods consist of solving stochastic programming problems by sampling scenarios during the solution procedure, instead of using a pre-generated scenario tree. An example of such methods is described in Higle & Sen (1999).
- **Iterative methods in stochastic programming** These methods consist of solving the stochastic programming problem with a given scenario tree, then add or remove some scenarios, then solving the problem again, and repeating these steps iteratively. An example of such methods is described in Dempster & Thompson (1999).

4.2 Testing a scenario tree construction method

According to Kaut & Wallace (2003), when using scenario trees within stochastic programs, a scenario tree should be judged by the quality of the decisions it generates. The most important concern is not how well the distribution is approximated by the scenario tree but how good a decision it allows to obtain (the solution of the optimization problem where the scenario tree is used). Yet, one would expect that as the quality of the approximation of the distributions improves then the quality of the decisions also improves.

In order to evaluate the quality of the approximation of the corresponding distributions a reasonable alternative is to compare the performance of the statistical properties of the scenario trees obtained, which means comparing if they are similar to the statistical properties of the actual process that is approximated by the scenario trees. These statistical properties should include the mean, the variance, etc. This is the criteria that will be used to evaluate alternative scenario tree construction methods in section 4.6.4.

From another point of view, a very important requirement a scenario tree construction method must satisfy is stability: if several instances of generated trees are used for solving the optimization problem, approximately the same value of the objective function should be obtained (Kaut & Wallace (2003)).

Let $\{X_t\}$ be the stochastic process under consideration and let K be the number of scenario trees generated. Let $\{X_t^k\}$ be the stochastic process that corresponds to scenario tree $k = 1, ..., K$ (approximating $\{X_t\}$), let x_t^k be the optimal solution obtained for the approximating stochastic program by using $\{X_t^k\}$, let $F[x_t^k; \{X_t^k\}]$ be the optimal value (of the objective function) for the approximating stochastic program by using $\{X_t^k\}$, and let $F[x_t^k; \{X_t\}]$ be the value of the objective function for the (original) stochastic program and the solution x_t^k. Here it is assumed that $F[x_t^k; \{X_t\}]$ exists, but it may not exist in some cases because the approximating stochastic program may change the structure of the solutions. Additionally, even if $F[x_t^k; \{X_t\}]$ exists, it may be impossible to evaluate it, yet, in some cases it may be evaluated (or approximated), for instance, using simulation.

An "*in-sample stability*" is defined as:

$$F[x_t^k; \{X_t^k\}] \approx F[x_t^l; \{X_t^l\}]; \forall k, l;$$

and an "*out-of-sample stability*", as:

$$F[x_t^k; \{X_t\}] \approx F[x_t^l; \{X_t\}]; \forall k, l.$$

It is possible to see that in-sample stability means that the optimal value of the objective function in each of the K approximating problems is similar, and an out-of-sample stability means that the value of the objective function of the original problem evaluated at the optimal solution of the approximating problems is similar. Since $F[x_t^k; \{X_t\}]$ may not exist or may not be possible to calculate, it may not be possible to test for out-of-sample stability, although, according to Kaut & Wallace (2003) in most practical applications either both stabilities will be present or none. Section 6.3.5 presents some results related to in-sample stability, for the application developed in this thesis.

4.3 Implementation framework

This section presents a framework for the scenario tree construction methods applied in sections 4.4 and 4.5. The two methodologies implemented are a path-based method (based on Gröwe-Kuska et al. (2003)) and a target/moment matching method (based on Hoyland & Wallace (2001)). These two methodologies have been chosen for several reasons. First, both methods have been widely applied satisfactorily and widely analyzed theoretically. Second, these two methods are different in nature: the path-based method is based in discrete algorithms for minimizing a metric for probability distributions, whereas the target/moment matching method is based on minimizing, using nonlinear programming, the difference between certain relevant statistical properties of the scenario tree and of the underlying stochastic process to be approximated. Third, both methods are adequate for the type of scenario tree needed as input for the model presented in chapter 5.

The methods implemented are used for building scenario trees approximating a discrete version of the bivariate stochastic process presented in section 3.3.1, for a case consistent of two locations, defined by:

$$\log P_i(t) = f_i(t) + X_i(t), i = 1, 2;$$

$$dX_i(t) = -\frac{X_i(t)}{\lambda_i}dt + \sigma_i dW_i(t), i = 1, 2;$$

$$E[dW_1(t)dW_2(t)] = \rho dt.$$

This is a continuous-time process, but a discrete-time version of it is used and only the values at fixed time steps have an interpretation. $P_i(t)$ represents electricity price at location $i = 1, 2$ and month $t = 1, ..., T$; $f_i(t)$ is a deterministic component; and the $X(t)$ process is a bivariate mean reverting diffusion (also called bivariate Ornstein-Uhlenbeck process), with guiding correlated Brownian motions $W_i(t)$, $i = 1, 2$.

The discrete-time version used of this model corresponds to a bivariate auto regressive process. The reason for presenting the model in a continuous-time form is the possibility of taking advantage of certain theoretical properties of the process. More details about this model can be found in appendix A.

The data used for fitting the parameters of the model consists of historical locational electricity spot prices obtained from the web site of the NYISO (`http://www.nyiso.com/public/index.jsp`, last visit 10 July 2010), for two zones, from 1 May 2005 until 31 January 2009, from which monthly average prices are obtained. (The daily average prices may be seen in figure 6.1.)

A scenario tree approximating this process in a time span $\{1, ..., T\}$ is to be generated. Assume that the approximating scenario tree to be constructed is $\{\overline{P}_i^s(t) : i = 1, 2; t = 1, ..., T; s = 1, ..., S\}$, with corresponding probabilities $\{p_s : s = 1, ..., S\}$, where S is the number of scenarios of the tree. The number of time periods used is $T = 6$. The number of bundles at each time period will be called $N(t)$, where by "bundle" it is meant one set of possible trajectories that are identical until the corresponding time period. Since at time $t = 0$ the scenario tree has only one outcome (the initial condition) then $N(0) = 1$, and $N(T) = S$ (the number of scenarios) since at time $t = T$ all the possible trajectories have unfolded, . For both methods implemented the next structure of bundles will be used: $N(1) = 4, N(2) = 8, N(3) = 16, N(4) = 32, N(5) = 64, N(6) = 128$ (for example, there could be 4 ramifications at time $t = 1$ and 2 ramifications for all the following time periods). This could be chosen differently, but has been chosen as such so as to have sufficiently detailed information at time $t = 1$ and to represent simply the unfolding of information in the subsequent time periods.

There are many alternatives for both methods implemented, some of them discussed in section 4.6. The results shown are just one possibility for each.

4.4 Implementation of the path-based method

The method presented in this section is based in Gröwe-Kuska et al. (2003). This method consists of successively bundling similar scenarios according to a probability metric based approximation.

4.4.1 Description of the methodology

Let Y and Z be n-dimensional stochastic processes over a time horizon $\{1, ..., T\}$ with a finite set of outcomes (scenarios, trajectories) $\{Y^i\}_{i=1}^N$ with probabilities $\{p^i\}_{i=1}^N$, and $\{Z^j\}_{j=1}^M$ with probabilities $\{q^j\}_{j=1}^M$, respectively. Let $Y^i(t)$ and $Z^j(t)$ be the outcomes of scenario i of Y and of scenario j of Z at time t, respectively. Let P_Y and P_Z be the probability distributions of Y and Z, respectively. For probability distributions with finitely many scenarios, the Kantorovich distance D_K between P_Y and P_Z is defined as the optimal value of a linear transportation problem given by:

$$D_K(P_Y, P_Z) = \min\{\sum_{i=1}^{N}\sum_{j=1}^{M} \eta_{ij} C_T(Y^i, Z^j) : \eta_{ij} \geq 0, \sum_{i=1}^{N} \eta_{ij} = q^j, \sum_{j=1}^{M} \eta_{ij} = p^i,$$

$$\forall i = 1, ..., N, j = 1, ..., M\}$$

Where $C_t(Y^i, Z^j) = \sum_{\tau=1}^{t} |Y^i(\tau) - Z^j(\tau)|$, $t = 1, ..., T$, and $|.|$ denotes a norm in $\Re^n$, which means that $C_T(Y^i, Z^j)$ measures the distance between scenarios Y^i and Z^j over the whole time horizon. The idea of the method is to find a scenario tree Z that satisfies certain properties (such as being small enough) and that approximates scenario tree Y such that $D_K(P_Y, P_Z)$ is as minimal as possible.

A first necessary tool is scenario reduction, then scenario tree construction may be built upon it.

a) Scenario reduction

Assume now that P_Z corresponds to a reduced probability distribution of Y, or equivalently, that Z consists of a set of scenarios $\{Y^i\}_{i \in \{1,...,N\} \setminus J}$ where J denotes an index set of deleted scenarios. If J is given explicitly, an important question is: what are the probabilities of the remaining $\{1, ..., N\} \setminus J$ scenarios that make the Kantorovich distance between P_Y and P_Z minimal? Fortunately the answer is known (Gröwe-Kuska et al. (2003)) and these new probabilities are explicitly given by:

$$q^j = p^j + \sum_{i \in J(j)} p^i, j \in \{1, ..., N\} \setminus J,$$

where $J(j) = \{i \in J : j = j(i)\}$, with $j(i) \in arg \min_{j \notin J} C_T(Y^i, Y^j)$, $\forall i \in J$. This redistribution rule is interpreted as follows: the probability of a preserved scenario j corresponds to its former probability p^j plus the probabilities of all the eliminated scenarios that had scenario j as their closest scenario (from the set of preserved scenarios).

And now, a much more difficult question is: how to find an index set J with fixed cardinality h that minimizes the Kantorovich distance between P_Y and P_Z? It can be shown that this problem is equivalent to the following combinatorial problem:

$$\min\{\sum_{i \in J} p^i \min_{j \notin J} C_T(Y^i, Y^j) : J \subset \{1, ..., N\}, \#J = h\}.$$

This problem represents a set-covering problem that can be formulated as a binary integer program and it is NP-hard. Because of the difficulty, the problem can be handled through heuristics. One natural idea is to solve it by a “greedy” heuristic, reducing scenarios one by one, and this is called “backward reduction” (Gröwe-Kuska et al. (2003)). The following algorithm corresponds to a variant of “backward reduction” that, according to the authors, has provided more accurate solutions. The scenario reduction algorithm is presented in what follows.

- Step 0. Compute distances of scenario pairs: $c_{kj} = C_T(Y^k, Y^j); k, j \in S$.

- Step 1. Compute $c_{ll}^1 = \min_{j \in S, j \neq l} c_{lj}$, for $l \in S$, the shortest distance of one scenario to l, and compute $z_l^1 = p^l c_{ll}^1, l \in S$. Choose $l_1 \in arg \min_{l \notin S} z_l^1$. Set $J^1 = \{l_1\}$. l_1 is the first scenario eliminated.
- Step $i = 2, ..., h$. Compute $c_{kl}^i = \min_{j \notin J^{i-1}, j \neq l} c_{kj}$, for $l \notin J^{i-1}, k \in J^{i-1} \bigcup \{l\}$; and $z_l^i = \sum_{k \in J^{i-1} \bigcup \{l\}} p^k c_{kl}^i$, for $l \notin J^{i-1}$. Choose $l_i \in arg \min_{l \notin J^{i-1}} z_l^i$. Set $J^i = J^{i-1} \bigcup \{l_i\}$. l_i is the i-th scenario eliminated.
- Final step. $J = J^K$ is the set of deleted scenarios, $I = S \setminus J$ is the set of preserved scenarios. Compute optimal probabilities $\{q(s)\}_{s \in I}$ according to the rule: $q^j = p^j + \sum_{\{i \in J(j)\}} p^i$, for $j \in I$, where $J(j) = \{i \in J : j \in arg \min_{k \in I} c_{ik}\}$.

b) Scenario tree construction

Once a scenario reduction algorithm is implemented, a recursive cluster analysis method may be implemented to bundle similar scenarios at all stages. The algorithm presented here falls under such category.

In Gröwe-Kuska et al. (2003), the authors present a scenario construction method based on given tolerances of the Kantorovich distance between the resulting and the original scenario tree, but here, a modification is presented by considering a fixed number of nodes at each time step. This is chosen so as to build a tree with a known size of bundles at each time period.

Take a fixed N_t, $t = 1, ..., T$, with $N_t \leq N_{t+1}$, and $N_0 = 1$, where N_t represents the total number of nodes (bundles) that the approximating scenario tree must have at time t. By having a fixed number of nodes at each time period, the degree of precision of the unfolding of information from time to time is controlled, as well as the size of the resulting scenario tree through time.

The scenario tree construction algorithm is presented in what follows.

- Step $t = T$.

Using the scenario reduction algorithm presented, determine an index set of preserved scenarios $I_T \subset I_{T+1} := \{1, .., N\}$ and optimal probabilities q_T^s, for $s \in I_T$, with $\#I_T = N_T$. Set $Y_{app}^s = Y^s$ for $s \in I_T$.

- Steps $t = T-1, ..., 1$.

  Reduction of scenarios in $\{0, ..., t\}$**.** Consider for $s \in I_{t+1}$ that $Z^{s,t} = \{Y^s(\tau)\}_{\tau=0}^t$ is the trajectory of scenario s only considered from 0 to t. For the set of scenarios $\{Z^{s,t}\}_{s \in I_{t+1}}$ with probabilities q_{t+1}^s ($s \in I_{t+1}$), determine, using the scenario reduction algorithm, an index set of preserved scenarios $I_t \subset I_{t+1}$ and optimal probabilities q_t^s, for $s \in I_t$, such that $\#I_t = N_t$.

  Scenario bundling. For each $j \in J_t := I_{t+1} - I_t$ select an index $j^* \in arg\min_{i \in I_t} c_t[Z^{i,t}, Z^{j,t}]$, add q_t^j to $q_t^{j^*}$ (i.e. redefine $q_t^{j^*} \leftarrow q_t^{j^*} + q_t^j$), and "bundle" scenario j with j^* (in $0, ..., t$), by redefining $Y^{j^*}(\tau) \leftarrow Y_{app}^j(\tau)$ for $\tau = 0, ..., t$.

- Step $t = 0$.

  Set $I := I_T$. Set $q^s := q_T^s$, for $s \in I$. The scenario tree obtained is $Z = \{Y_{app}^s\}_{s \in I}$, with probabilities given by q^s, $s \in I$, respectively.

4.4.2 Implementation and results

The scenario tree Y to be approximated consists of a *fan* of 500 simulations of the process presented in section 4.3, each with probability $\frac{1}{500}$. A *fan* is a scenario tree where scenarios are only bundled together at $t = 0$ and in $t = 1, ..., T$ each follows a particular trajectory. The simulations may be seen in figure 4.2.

The norm used for scenario reduction is the l^2 norm, or in other words, Euclidean distance (square root of the sum in both dimensions of the squared differences). The bivariate trajectories of the scenario tree obtained may be seen in figure 4.3.

The performance of the scenario tree obtained may be seen in figures 4.4, 4.5, 4.6, 4.7, 4.8, where the theoretical statistical properties are plotted together with the statistical properties computed from the tree. The equations for obtaining the theoretical statistical properties are presented in appendix A.

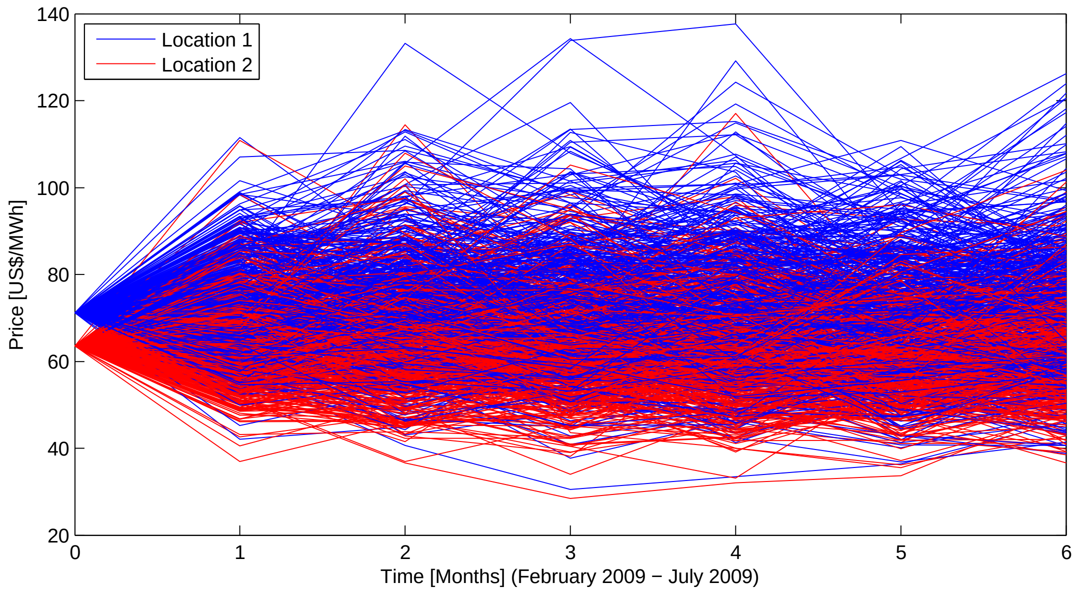


FIGURE 4.2. Simulations for the electricity prices process (500)

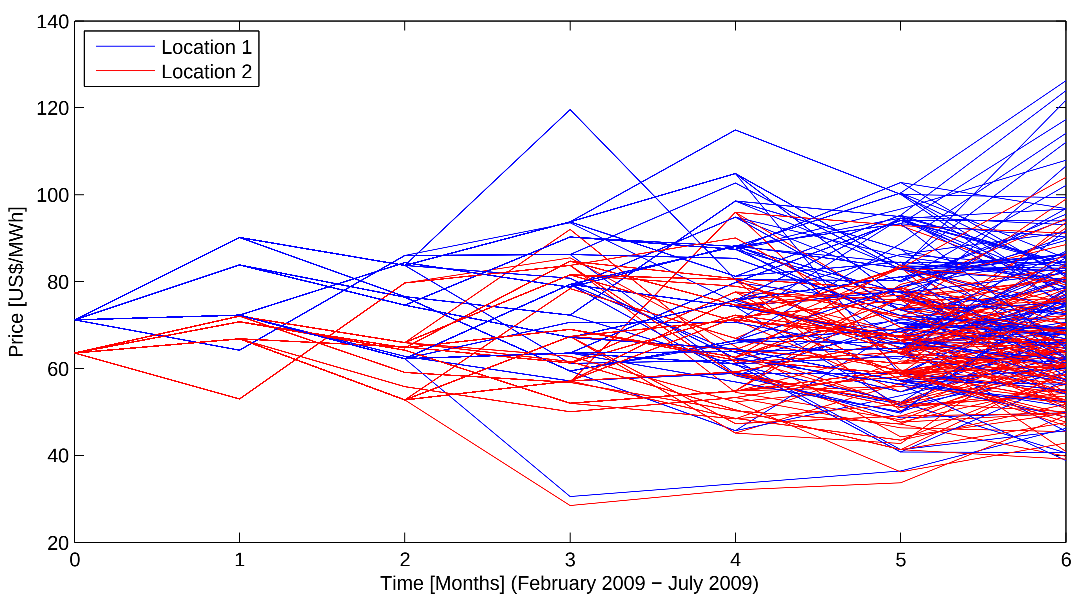


FIGURE 4.3. Scenario tree obtained using the path-based method presented

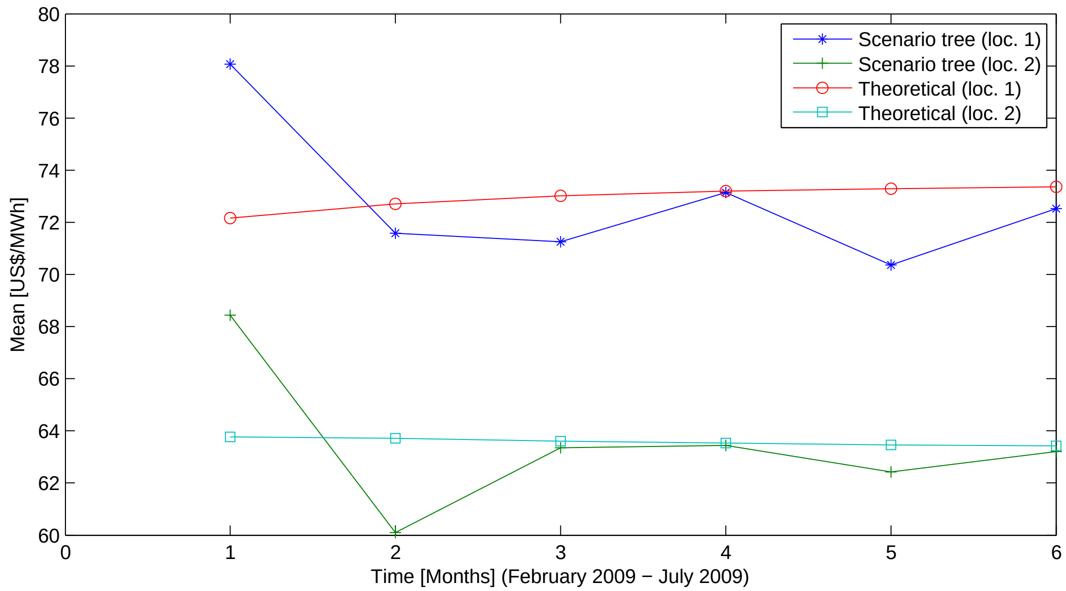


FIGURE 4.4. Mean performance for the scenario tree generated using the path-based method

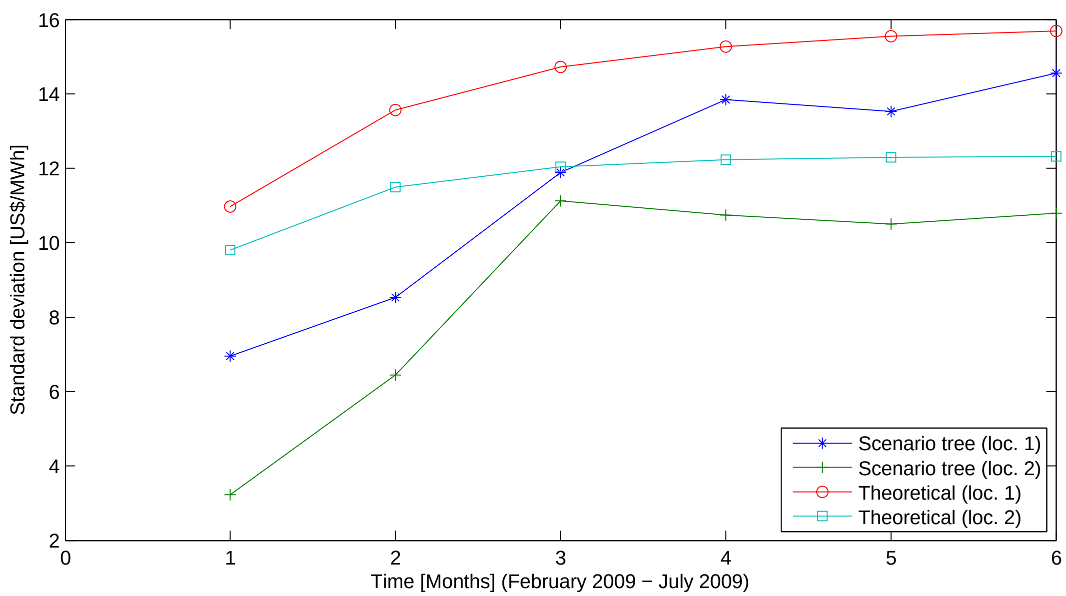


FIGURE 4.5. Standard deviation performance for the scenario tree generated using the path-based method

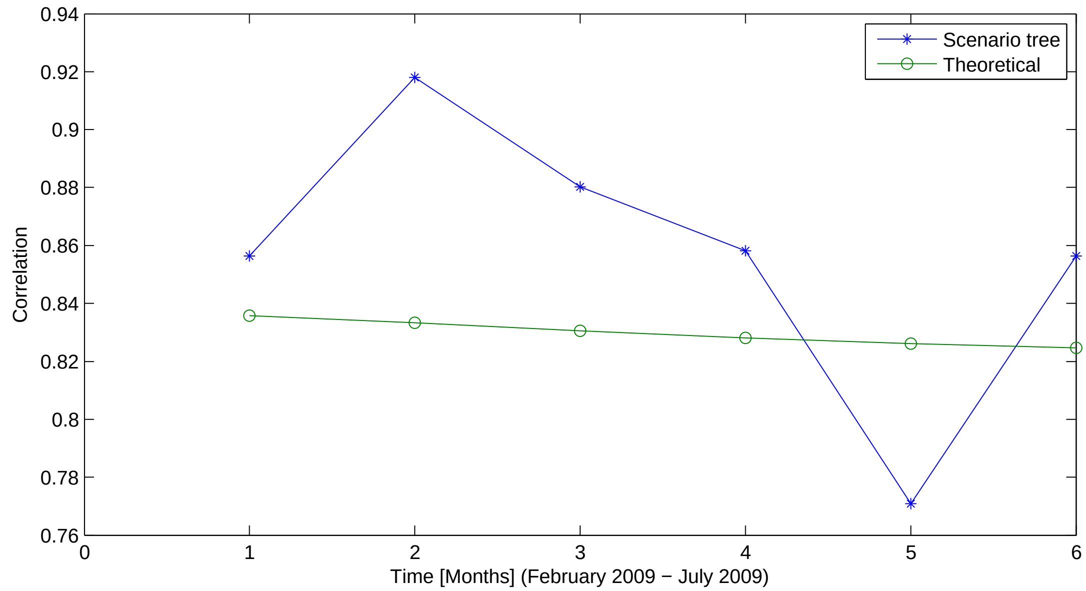


FIGURE 4.6. Correlation performance for the scenario tree generated using the path-based method

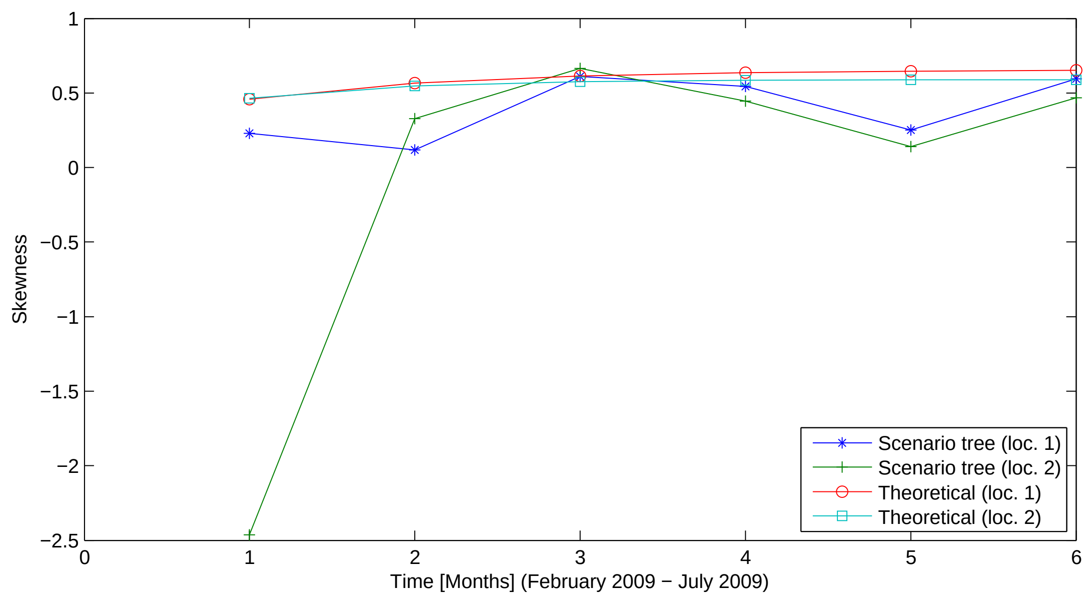


FIGURE 4.7. Skewness performance for the scenario tree generated using the path-based method

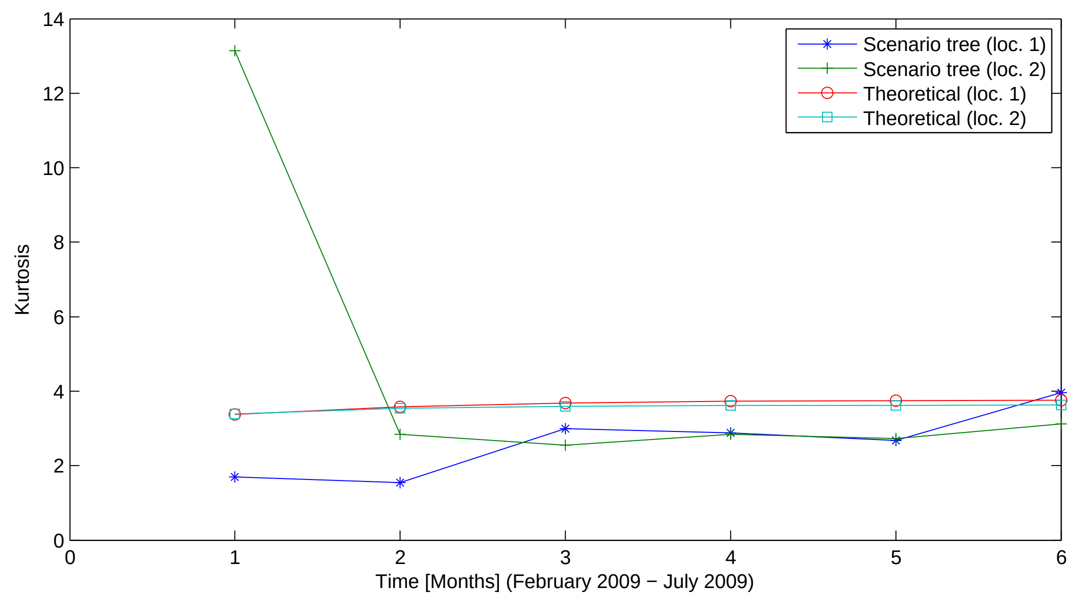


FIGURE 4.8. Kurtosis performance for the scenario tree generated using the path-based method

4.5 Implementation of the moment matching method

The methodology implemented is based in Hoyland & Wallace (2001). The fundamental idea presented in the paper is to match as closely as possible certain relevant statistical properties (such as mean, standard deviation, etc.) of the approximating scenario tree with the same properties of the actual original process.

This methodology will be applied to the Ornstein-Uhlenbeck component of the model presented in section 4.3 (the "$\{X(t)\}$ process"), and with a scenario tree for that component it will be easy to obtain a reasonable price process scenario tree.

4.5.1 Description of the methodology

Because of the symmetry of the Ornstein-Uhlenbeck component (the "$\{X(t)\}$ process"), a scenario tree $\overline{X} = \{\overline{X}_i^s(t) : i = 1, 2; t = 1, ..., T; s = 1, ..., S\}$, with probabilities $\{q_s : s = 1, ..., S\}$, approximating X is built, and then the scenario tree approximating P

is obtained as:

$$\overline{P}_i^s(t) = \exp[f_i(t) + \overline{X}_i^s(t)], i = 1, 2; t = 1, ..., T; s = 1, ..., S;$$

keeping the probability of every escenario: $p_s = q_s$, where $\{p_s : s = 1, ..., S\}$ are the associated probabilities of $\overline{P}$.

The scenario tree for X starts from $X(0) = \log S(0) - f(0)$ and selects 4 possible outcomes for $X(1)$ (each with a probability determined jointly), which means that all scenarios $s = 1, ..., S$ are bundled into four groups at time $t = 1$. Then, from each of the four outcomes at $t = 1$, two possible outcomes are considered for $X(2)$ (each with a selected probability) (there are 8 bundles at time $t = 2$), and subsequently, there are two ramifications until $t = T$. This means that $S = 4 \times 2^{T-1} = 2^{T+1}$.

At every time step, the possible outcomes (values for $\overline{X}$) and their associated probabilities are chosen so as to minimize a measure of distance between the relevant statistical properties of X (for that time step) and the same properties of $\overline{X}$. The statistical properties considered as relevant are: mean, standard deviation, covariance (between $X_1(t)$ and $X_2(t)$), auto-covariance (covariance between $X_i(t-1)$ and $X_i(t)$), skewness and kurtosis. All of these statistical properties are analitically known for the stochastic process X as functions of both time and the parameters that define the process, and the formulas for obtaining them are presented in appendix A.

The algorithm is presented in (a), (b), (c).

a) Step $t = 1$

Assume that a vector $x = (x_1, ..., x_8)$ contains the four possible outcomes for $X(1)$, and the probabilities are contained in a vector $p = (p_1, ..., p_4)$. With (x_1, x_2) as the first outcome, with probability p_1, being x_1 the value in dimension $i = 1$, and x_2 the value in dimension $i = 2$; (x_3, x_4) the second outcome, with probability p_2; (x_5, x_6) the third outcome, with probability p_3; and (x_7, x_8) the fourth outcome, with probability p_4.

With these vectors, the above mentioned relevant statistical properties at $t = 1$ can be computed for the scenario tree. The mean in dimension 1 is: $\overline{m}_1(1, x, p) = x_1p_1 + x_3p_2 + x_5p_3 + x_7p_4$ (for simplicity $\overline{m}_1(1)$). In dimension 2, the mean is: $\overline{m}_2(1, x, p) = x_2p_1+x_4p_2+x_6p_3+x_8p_4$ (for simplicity $\overline{m}_2(1)$). The standard deviations can be calculated as:

$$\overline{sd}_1(1, x, p) = \sqrt{[x_1 - \overline{m}_1(1)]^2p_1 + [x_3 - \overline{m}_1(1)]^2p_2 + [x_5 - \overline{m}_1(1)]^2p_3 + [x_7 - \overline{m}_1(1)]^2p_4},$$

$$\overline{sd}_2(1, x, p) = \sqrt{[x_2 - \overline{m}_2(1)]^2p_1 + [x_4 - \overline{m}_2(1)]^2p_2 + [x_6 - \overline{m}_2(1)]^2p_3 + [x_8 - \overline{m}_2(1)]^2p_4}.$$

The covariance is:

$$\begin{aligned}\overline{c}(1, x, p) = [x_1 - \overline{m}_1(1)][x_2 - \overline{m}_2(1)]p_1 + [x_3 - \overline{m}_1(1)][x_4 - \overline{m}_2(1)]p_2 \\ +[x_5 - \overline{m}_1(1)][x_6 - \overline{m}_2(1)]p_3 + [x_7 - \overline{m}_1(1)][x_8 - \overline{m}_2(1)]p_4.\end{aligned}$$

The auto-covariances are not necessary in this time step because they equal the respective variances since $X_i(0)$ is deterministic. The skewness and kurtosis in dimension 1 are:

$$\overline{sk}_1(1, x, p) = ([x_1-\overline{m}_1(1)]^3p_1+[x_3-\overline{m}_1(1)]^3p_2+[x_5-\overline{m}_1(1)]^3p_3+[x_7-\overline{m}_1(1)]^3p_4)/[\overline{sd}_1(1)]^3,$$

$$\overline{k}_1(1, x, p) = ([x_1-\overline{m}_1(1)]^4p_1+[x_3-\overline{m}_1(1)]^4p_2+[x_5-\overline{m}_1(1)]^4p_3+[x_7-\overline{m}_1(1)]^4p_4)/[\overline{sd}_1(1)]^4.$$

And analogously in dimension 2.

Let $J(1)$ be the set of relevant statistical properties, $\alpha_j(1)$, the theoretical values of these statistical properties (that is, the actual values for the Ornstein-Uhlenbeck component), and $g_j(1, x, p)$ the values of these properties for the outcomes represented by x and p (as expressed above). Then, x and p are chosen as a solution (hopefully the optimal solution) of the following nonlinear optimization problem (with appropriately chosen weights

$w_j(1)$):

$$\begin{array}{lll} Min_{\{x,p\}} & G(1) = \sum_{j \in J(1)} w_j(1) |g_j(1,x,p) - \alpha_j(1)|^2 & (1) \\ s.t. & p_1 + p_2 + p_3 + p_4 = 1; & (2.1) \\ & p_k \geq 0, k = 1,2,3,4. & (2.2) \end{array}$$

The problem as stated above is sufficient. Yet, a variety of constraints can be added in order to obtain better results and solution time (depending on the method used to solve the problem). The constraints that can be added are the following ones:

$$\begin{array}{ll} p_1 = p_4; & (3.1) \\ p_2 = p_3; & (3.2) \\ \frac{x_1+x_7}{2} = m_1(1); & (4.1) \\ \frac{x_3+x_5}{2} = m_1(1); & (4.2) \\ \frac{x_2+x_8}{2} = m_2(1); & (4.3) \\ \frac{x_4+x_6}{2} = m_2(1); & (4.4) \\ \underline{x}_k \leq x_k \leq \overline{x}_k, k = 1, ..., 8; & (5) \\ g_j(1,x,p) = \alpha_j(1), j \in J^\star(1). & (6) \end{array}$$

Where constraints (3) and (4) enforce that the tree built is symmetric with respect to its mean, in both dimensions. With these constraints, it is unnecessary to have the mean and the skewness as parts of the objective function's relevant statistical properties. Besides, instead of using them as constraints for the optimization problem, many variables could be eliminated using these relations (having $g_j(1,x,p)$ reformulated accordingly), thus possibly reducing computation time. This is the advantage of using this methodology for the Ornstein-Uhlenbeck instead of using it directly for the price process.

Constraints (5) are upper and lower bounds for the x_k that may help to obtain scenario trees without extreme values.

Constraints (6) assure that some of the properties of the tree are forced to be equal to the theoretical statistical properties, with $J^\star(1) \subset J(1)$.

A critical issue is the starting solution for the nonlinear programming method used to solve the problem. A good alternative is to solve the problem using many different initial solutions (that can be simulated according to the distribution of $X(1)$), optimizing, and then keeping the best solution found.

b) Steps $t = 2, ..., T^\star$

The tree has been built in time periods $\{1, ..., t-1\}$. There are 2^t outcomes at time $t-1$, each with a corresponding probability. The number of outcomes to be determined for time t are 2^{t+1}. This is obtained analogously to Step $t = 1$, minimizing the sum of the squares of the difference between the theoretical statistical properties of the stochastic process X at time t and the resulting statistical properties of the tree at time t.

Again, vectors x and p for the outcomes and the associated conditional probabilities are considered, the expressions $g_j(t, x, p)$ represent the resulting statistical properties of the tree at time t, $\alpha_j(t)$ their respective theoretical values, $w_j(t)$ are appropriately chosen weights, and $J(t)$ is the set of statistical properties considered.

The expression:

$$\min_{\{x,p\}} G(t) = \sum_{j \in J(t)} w_j(t) |g_j(t, x, p) - \alpha_j(t)|^2$$

is obtained with x and p subject to analogous constraints to the ones presented in step $t = 1$. The presence of auto-covariance in the set of statistical properties considered ensures that the values obtained for time period $t-1$ (at Step $t-1$) relate appropriately to the values obtained at time period t (in this Step).

c) Steps $t = T^\star + 1, ..., T$ (if $T > T^\star$)

At some point in time the number of variables in steps $t = 2, ..., T^\star$ becomes too big for computational tractability. If this is the case it is necessary to go through the steps presented here.

In these steps, instead of solving for all the outcomes at time t at once, the outcomes are obtained by parts. This is similar to what has been called in Hoyland & Wallace (2001) "scenario tree aggregation", which basically corresponds to constructing various scenario trees and then combining them in order to have one big, more representative, scenario tree.

Instead of using all the information over time periods $0, 1, ..., t-1$, of the scenario tree already built, only the information of the scenario tree over time periods $t - T^\star, t - T^\star + 1, ..., t-2, t-1$ is used, and the outcomes at time t are obtained by solving a number of problems that equals the number of outcomes of the tree at time $t-T^\star$. For instance, if $t = 6$ and $T^\star = 5$ then at step $t = 6$ instead of using the information of time periods $0, 1, ..., 5$, only the information over time periods $1, ..., 5$ is used. In this example, the outcomes at time $t = 6$ are obtained as four separate components (because there are four sub-trees that begin at time $t = 1$, since there are four outcomes at $t = 1$). Following the example, if $t = 7$, the information over $2, ..., 6$ is used and the outcomes are obtained as eight separate components (because there are eight outcomes at $t = 2$).

The means for obtaining the outcomes and the associated conditional probabilities for each part of the outcomes at time t is exactly the same as the one used in steps $t = 2, ..., T^\star$, but using the appropriate information as mentioned in the above paragraph. Yet, note that now the theoretical statistical properties $\alpha_j(t)$ are determined based on the sub-tree that relates to the outcomes to be obtained in the respective tree section.

4.5.2 Implementation and results

The methodology described was implemented and tested. The value of parameter $T^\star$ used was 6, which means that $T^\star = T$ and the steps presented in 4.5.1 (c) were not used (although they are important when limited computational time is available and T is relatively big). The method was implemented in MATLAB, solving the non-linear optimization problems with the "fmincon" function of the Optimization Toolbox, using an "active-set algorithm". The tree structure used is depicted in figure 4.9.

Symmetry constraints (of the type (4.1)-(4.4)) have been used in all steps, favoring computational tractability and obtaining better results (this forces a perfect match for the X scenario tree in all the odd moments). And also in all steps, the standard deviation (in both dimensions), the covariance (between dimensions), and the auto-covariances of order

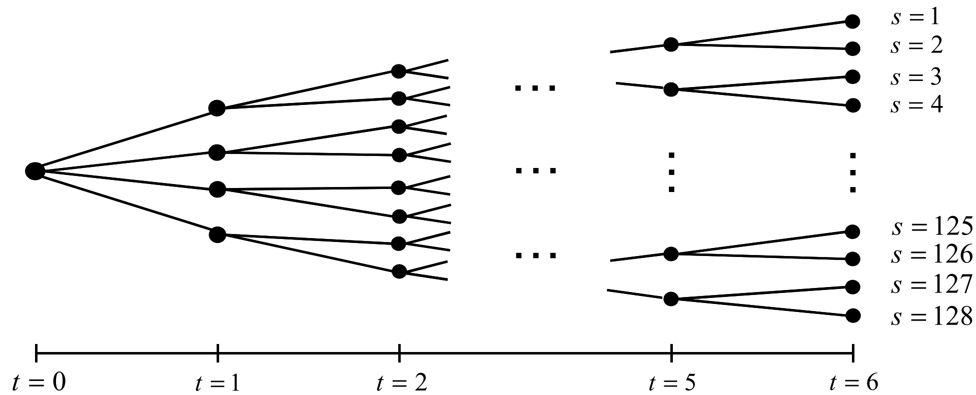


FIGURE 4.9. Scenario tree structure

1 (in both dimensions), have been included as constraints. Finally, lower and upper limits have been used for x and p in all steps.

Each optimization problem is solved starting from initial solutions sampled from the conditional distributions at each time step (Normal distributions). They are solved as many times as necessary (starting from different sampled initial conditions) until certain tolerances have been satisfied for all statistical properties. After a certain number of initial solutions have been considered, if the tolerances have not been satisfied, the best solution is kept.

A perfect match is obtained for the properties of the Ornstein-Uhlenbeck component (the $\{X_t\}$ process) with the scenario tree constructed. The scenario tree obtained for the Ornstein-Uhlenbeck component may be seen in figure 4.10, and the scenario tree for the price process (the $\{P_t\}$ process) in figure 4.11.

The performance of the price scenario tree obtained (for the $\{P_t\}$ process) may be seen in figures 4.12, 4.13, 4.14, 4.15, 4.16, where the theoretical statistical properties are plotted together with the statistical properties obtained by the tree. The equations for obtaining the theoretical statistical properties are presented in appendix A.

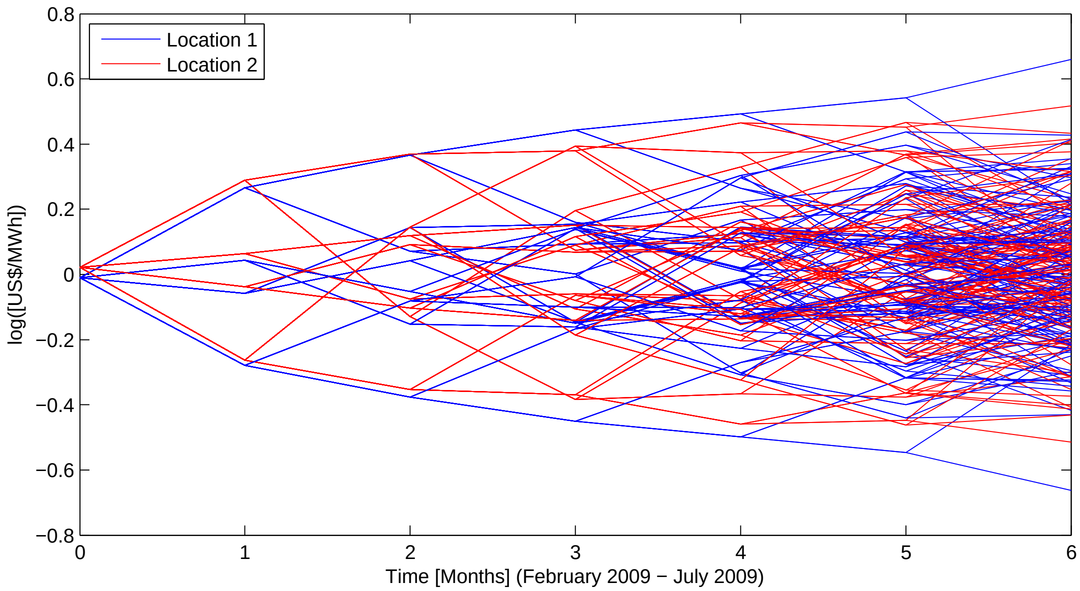


FIGURE 4.10. Scenario tree for $\{X_t\}$

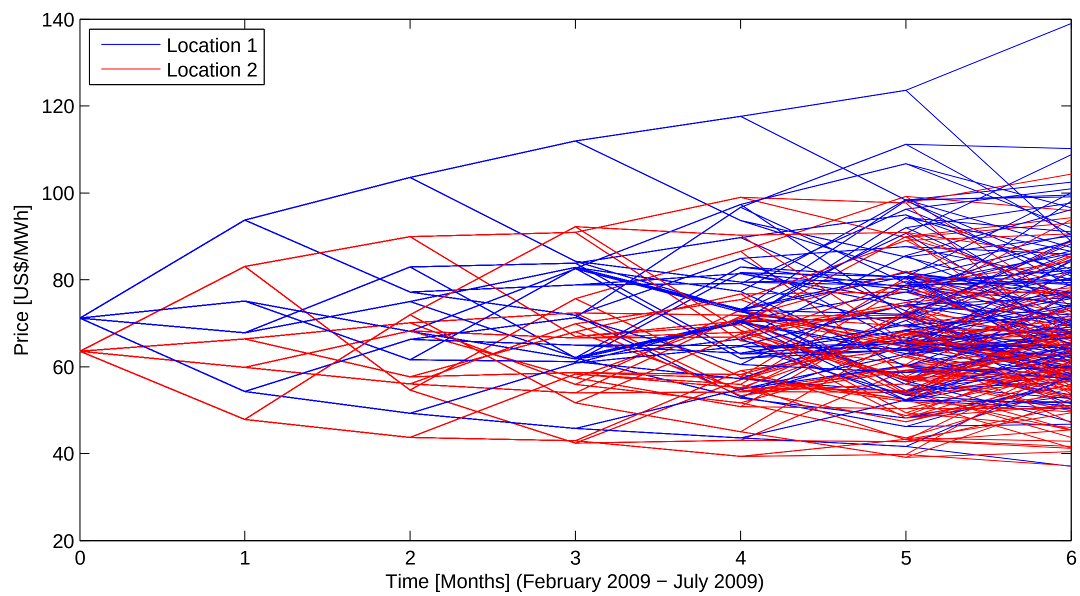


FIGURE 4.11. Scenario tree obtained using the moment matching method

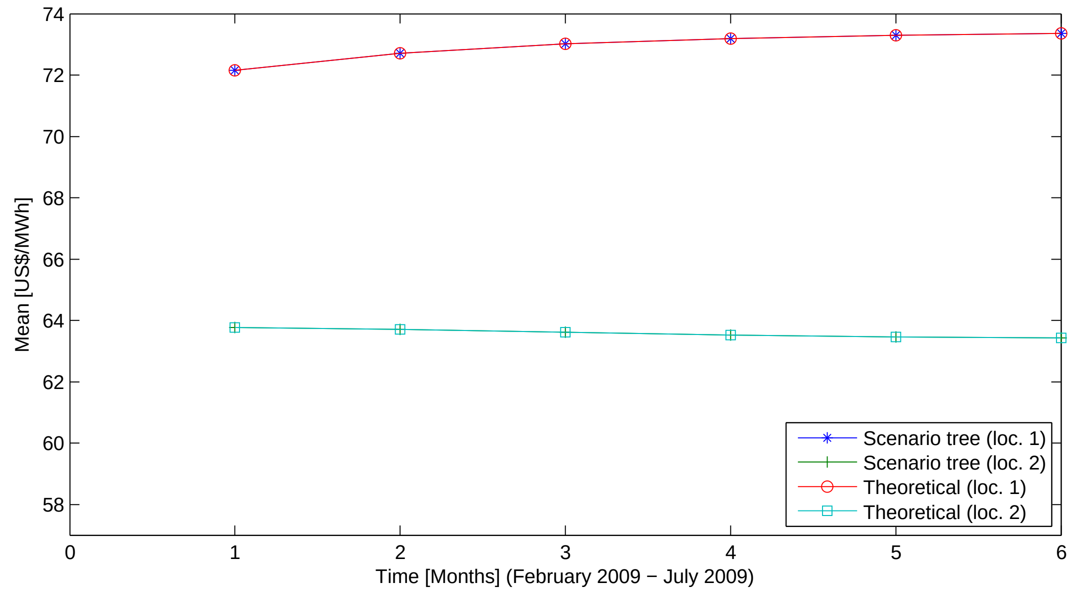


FIGURE 4.12. Mean performance for the scenario tree generated using the moment matching method

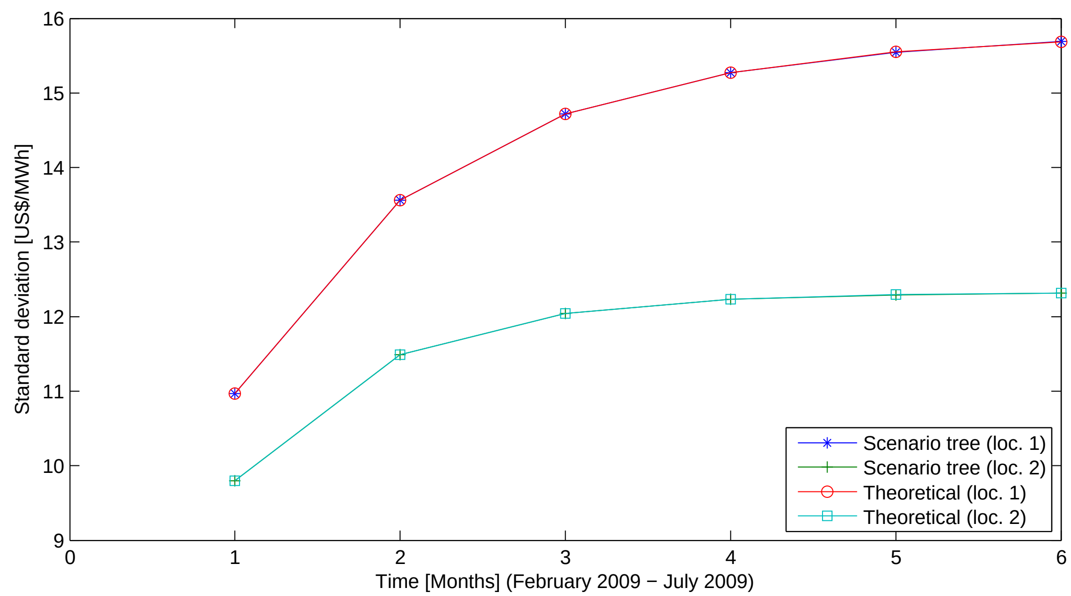


FIGURE 4.13. Standard deviation performance for the scenario tree generated using the moment matching method

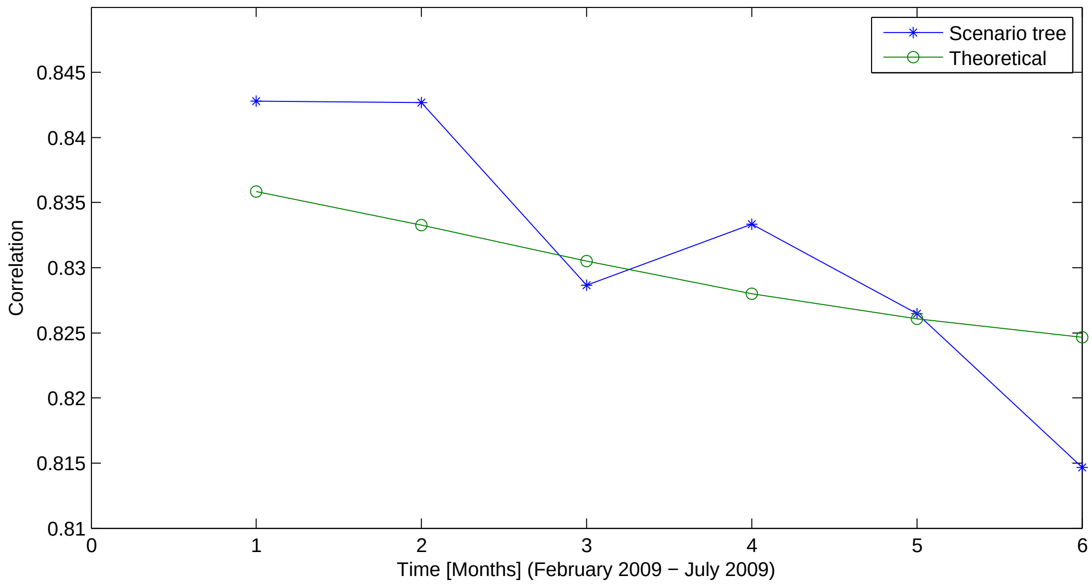


FIGURE 4.14. Correlation performance for the scenario tree generated using the moment matching method

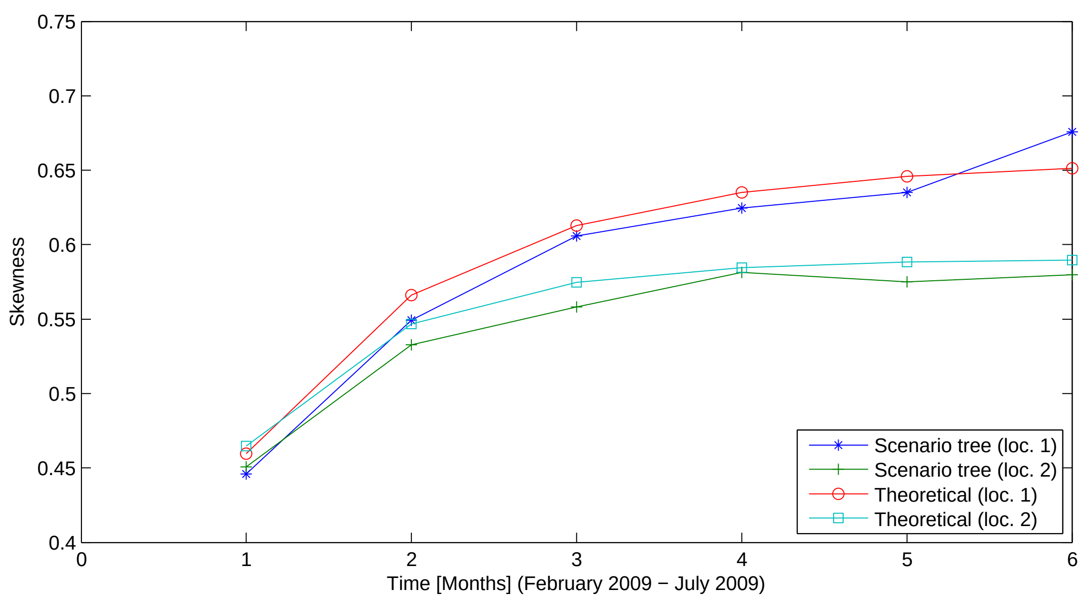


FIGURE 4.15. Skewness performance for the scenario tree generated using the moment matching method

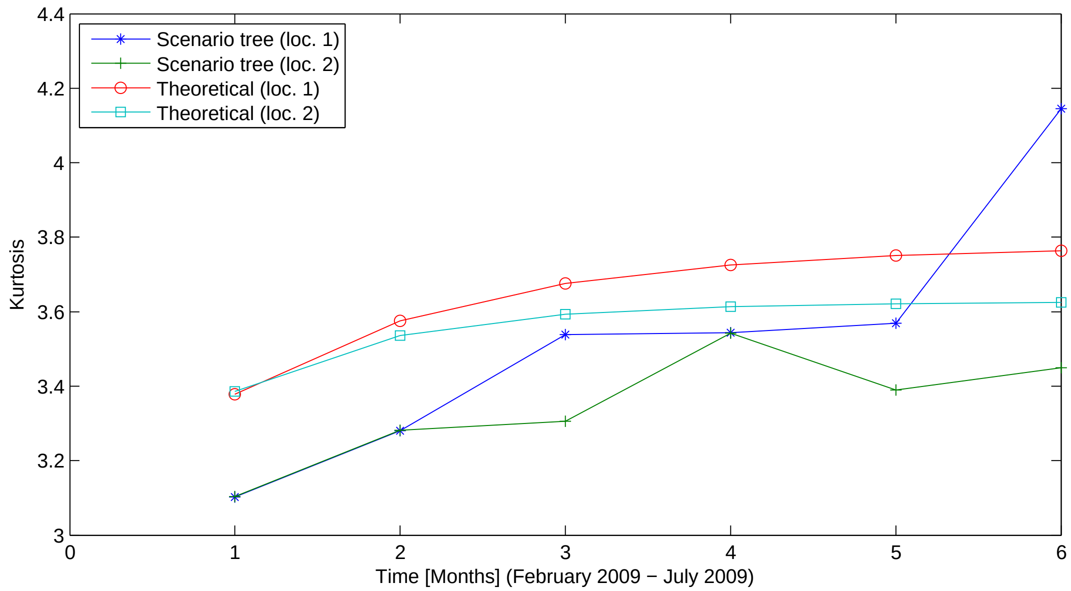


FIGURE 4.16. Kurtosis performance for the scenario tree generated using the moment matching method

4.6 Discussion

It must not be forgotten that the scenario trees obtained depend upon the simulations for the path-based method implemented and the sampled initial solutions for the moment matching method implemented. This means that the scenario trees are stochastic and different scenario trees are obtained every time each method is executed. For this reason it is important to check the stability of the optimal values (and solutions) of stochastic programs that use these methodologies.

4.6.1 Variants

The path-based method presented is directly implemented for the price process (the P process) whereas the moment matching method presented is implemented for the Ornstein-Uhlenbeck component (the X process) (and with the scenario tree for the Ornstein-Uhlenbeck component a scenario tree for the price process is obtained). For both methods the implementation could have been different, however for the path-based method the results for the price process did not seem to improve by using the method of first building a scenario tree for the Ornstein-Uhlenbeck component, whereas for the moment matching method the performance of the statistical properties and the computing time improved considerably when doing this.

Another aspect that could have been selected differently for both methods is the structure of bundles (or nodes) of the tree (the $N(t)$ component). The number of bundles at each time period was respectively given by 4-8-16-32-64-128, with the moment matching method forced to have a number of ramifications at each time period given by 4-2-2-2-2-2 and a similar structure of ramifications for the path-based method. This structure was chosen with the objective of representing sufficiently well the distribution of the process in the first time period (4 possible outcomes) and in the following time periods (with more than 4 possible outcomes), hence successively representing the unfolding of information appropriately. With more ramifications at each different time period the scenario tree may become too big for the computational tractability of an optimization problem. And with less ramifications, it may become too small for the process to be adequately represented by the scenario tree.

Since the most time consuming part of the path-based method presented is the initial scenario reduction (step $t = T$ of the scenario tree construction algorithm), an alternative for this method is to reduce scenarios by groups in that part (for instance, from 1000 scenarios to 900, then to 800, etc.), instead of reducing all scenarios at once. This may prove to be useful when the initial fan to be used consists of too many scenarios. Another important variant for this method is to use a different scenario reduction algorithm (such as "forward selection", discussed in Gröwe-Kuska et al. (2003)).

An important variant for the moment matching method is to obtain the whole scenario tree by solving at once one big optimization problem that incorporates all time periods and their respective statistical properties (instead of solving successively for the different time periods). Yet, this alternative results intractable for the case implemented and unnecessary because a perfect match is generally obtained for the Ornstein-Uhlenbeck component with the method as shown. Another variant, useful for big T, is to use steps (c) (having $T^\star < T$), which may decrease the computational burden of solving the problem, keeping good results. Finally, the set of statistical properties considered in the optimization process could also include quantiles, auto-covariances of order higher than 1, or other statistical properties.

4.6.2 Advantages and disadvantages of the path-based method

The main advantage of the path-based method implemented is that it can be used for transforming any scenario tree into a more simple scenario tree. This in turn implies that a scenario tree can be obtained for any stochastic process that can be simulated, by approximating a fan generated with a representative number of simulated trajectories.

A disadvantage of the method is that taking too many initial simulated trajectories may make the initial reduction of scenarios too slow (step $t = T$ of the scenario tree construction algorithm). Another disadvantage is that fundamental statistical properties of the distribution of the process at different time periods may not be well replicated by the scenario tree.

4.6.3 Advantages and disadvantages of the moment matching method

The fundamental advantage of the moment matching method is that the statistical properties considered as relevant will generally be well replicated in the scenario tree, or if it is not the case, at least the statistical properties given more relevance through choosing bigger weights in the objective function should be well replicated.

One very important advantage of the particular implementation presented is that the symmetry of the Ornstein-Uhlenbeck component is exploited, which helps obtaining a perfect match for the statistical properties of the Ornstein-Uhlenbeck component. Another advantage is that lower and upper limits for the conditional probabilities and the outcomes may be forced using constraints.

An important disadvantage of the moment matching method as presented here is that it may not be computationally feasible to construct scenario trees for big values of T. For bigger T it is a good option to use steps (c) (having $T^{\star} < T$), but in such a case it is necessary to know the theoretical conditional statistical properties of the process conditional on certain past value. For the process approximated here they were known analitically, but it might not be the case for other processes. In such a case it could also be a good option to change the structure of bundles ($N(t)$).

4.6.4 Method selection

The performance of the statistical properties of the scenario tree generated using the moment matching method is much better than the performance of the statistical properties of the scenario tree generated using the path-based approach. Table 4.1 presents a comparison between the sums of the errors of the statistical properties of the scenario trees obtained (with respect to the theoretical statistical properties of the original process). The difference is quite significant, favoring the moment matching method, and therefore, the moment matching method is chosen for building the scenario tree to be used as an input for the stochastic programming model presented in chapter 5.

TABLE 4.1. Sums of errors of statistical properties from the scenario tree construction methods

Statistical property / Method	Path-based	Moment matching
Mean (loc. 1)	12.6130	0.00008
Mean (loc. 2)	9.9167	0.00004
Standard deviation (loc. 1)	16.4728	0.0132
Standard deviation (loc. 2)	17.3523	0.0083
Correlation	0.1227	0.0164
Skewness (loc. 1)	1.2208	0.0836
Skewness (loc. 2)	3.9436	0.0711
Kurtosis (loc. 1)	6.4916	1.4536
Kurtosis (loc. 2)	13.6621	1.3029

The next chapter develops a power portfolio optimization model for a power producer, assuming a multivariate locational electricity price process represented through a scenario tree. Then, chapter 6 presents a particular application of the optimization model involving the moment matching method implemented.

V. POWER PORTFOLIO OPTIMIZATION MODEL

For power producers in deregulated electricity markets the scheduling of the operation of generating units and the planning of contractual decisions are two fundamental aspects that must be coordinated in order to maximize benefits, subject to several operational, technical, and financial constraints, while also taking into account the diverse operational and financial risk factors present in the business, such as electricity and fuel prices, forced unit outages, and water inflows.

This chapter presents a medium term power portfolio optimization model for a power producer in a deregulated electricity market, considering generation and commercial aspects. A central aspect in this model is the possibility of trading electricity forward contracts and contracts for difference (CfDs) in different locations. The model also recommends electricity transactions in locational spot markets and power production of generating units. Locational electricity prices are modeled as stochastic variables using a scenario tree and risk is measured explicitly through conditional value at risk (CVaR).

The model presented in this chapter is relatively similar to the ones developed by Conejo et al. (2008), Sen et al. (2006), Xu et al. (2006) and Cabero et al. (2005), but in contrast to their work, the main contribution of this model is to consider a generating company that holds generators in more than one location of a deregulated electricity market, with each location having a locational electricity price process correlated to the other locational electricity price processes, and these processes explicitly modeled.

The model presented can be a valuable decision making tool for a power producer holding generating units in more than one location of a deregulated electricity market that has to decide appropriate contractual involvement levels in order to effectively maximize its expected profit while appropriately balancing risk exposure.

The remaining of this chapter is organized as follows: 5.1 presents the notation necessary for the mathematical formulation of the model, 5.2 presents a detailed description of the problem and 5.3 presents the mathematical formulation of the optimization model.

An application of the model using data from the New York Independent System Operator (NYISO) is presented in the next chapter.

5.1 Notation

The notation used throughout this chapter is stated in this section.

5.1.1 Sets

- $\{1, ..., L\}$: set of locations.
- $\{1, ..., I\}$: set of generating units.
- $\{1, ..., S\}$: set of scenarios.
- $\{1, ..., T\}$: set of time periods.
- $\{1, ..., K\}$: set of hour groups.
- $\{1, ..., N^f\}$: set of blocks in each buying or selling forward contract.
- $\{1, ..., N^{CfD}\}$: set of blocks in each CfD contract.
- B_{st}: set of scenarios bundled to scenario s at time period t.
- μ^{High}: set of high-price hour groups.

5.1.2 Scenario dependent parameters

- P^E_{lst}: average spot price [US$/MWh] of electricity at location l during time period t under scenario s.

5.1.3 Scenario independent parameters

- α: confidence level of conditional value at risk.
- γ: risk aversion parameter.
- ϕ_{tk}: duration [hours] of hour group k in time period t.
- p_s: probability of scenario s.
- β_{ltk}: electricity price aggregation factor for hour group k in time period t at location l.
- C_{it}: variable production cost of unit i [US$/MWh] at time period t.
- ψ_i: location of unit i.

- $\underline{x}_{it}$: minimum power output [MW] of unit i at time period t.
- $\overline{x}_{it}$: maximum power output [MW] of unit i at time period t.
- H_{ik}: minimum percentage of electricity generated per hour during high-price hour group k that must be generated per hour in its associated low-price hour group, in every time period, for generating unit i.
- μ_k: low-price hour group associated to high-price hour group k.
- D_{ltk}: electricity [MWh] commitment at location l during time period t and hour group k, as a result of existent contractual positions at the beginning of the time horizon. Positive if electricity is delivered, negative if electricity is received.
- P^D_{ltk}: price of electricity [US$/MWh] for D_{ltk}.
- $\overline{f}^B_{ltkb}$: maximum amount of energy [MWh] that can be bought at location l through buying block b of forward contract spanning hour group k of time period t.
- $\overline{f}^S_{ltkb}$: maximum amount of energy [MWh] that can be sold at location l through selling block b of forward contract spanning hour group k of time period t.
- C^{fB}_{ltkb}: electricity price [US$/MWh] of buying block b of forward contract spanning hour group k of time period t at location l.
- C^{fS}_{ltkb}: electricity price [US$/MWh] of selling block b of forward contract spanning hour group k of time period t at location l.
- $\overline{g}_{lmtkb}$: maximum amount of energy [MWh] that can be contracted through block b of CfD contract spanning hour group k of time period t and equivalent to selling electricity at location l and buying at location m.
- C^{CfD}_{lmtkb}: electricity price [US$/MWh] for block b of CfD contract spanning hour group k of time period t and equivalent to selling electricity at location l and buying at location m.

5.1.4 Variables

- x_{istk}: energy [MWh] generated by unit i during hour group k of time period t, under scenario s.

- e_{lstk}: energy [MWh] traded at the pool at location l during hour group k of time period t, under scenario s. Positive if electricity is sold, negative if electricity is bought.
- f^B_{ltkb}: energy [MWh] bought at location l through buying block b of forward contract with delivery in hour group k of time period t.
- f^S_{ltkb}: energy [MWh] sold at location l through selling block b of forward contract with delivery in hour group k of time period t.
- g_{lmtkb}: energy [MWh] sold at location l and bought at location m through block b of CfD contract spanning hour group k of time period t.
- π_s: profit [US$] obtained under scenario s.
- ξ: value at risk [US$].
- η_s: variable associated to scenario s, used to compute conditional value at risk.

5.2 Problem description

This section presents a detailed description of the different aspects of the model developed.

5.2.1 Decision framework

A time horizon of 6 to 12 months (or weeks) may be considered. Contractual decisions are taken at the beginning of the horizon (as "here-and-now" decisions) affecting the whole time horizon, and spot market trading as well as generation decisions are made throughout the horizon (as "wait-and-see" decisions). It is assumed that day-ahead spot market transactions are made with "perfect information", which means that the spot price in every time period is known in the moment of carrying out spot market transactions. Figure 5.1 presents a scheme of the decisions of the model.

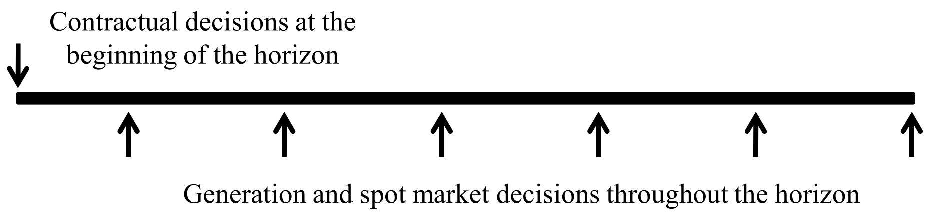


FIGURE 5.1. Model decisions scheme

5.2.2 Locational electricity prices

Because of energy losses on the electricity grid and transmission congestion, electricity price differences over locations are highly relevant. In the model presented, locational electricity prices are incorporated explicitly. The model considers different locations, assuming that in each of them there is a liquid spot market for electricity, and with each of them having a different electricity spot price. Spot prices at different locations are correlated and this is explicitly modeled.

5.2.3 Scenario tree and hour groups

Electricity spot prices at the different locations considered form a stochastic vector process represented through a scenario tree that approximates it, containing the stochastic behavior of electricity prices through time at all relevant locations. Each scenario of the scenario tree is a plausible trajectory of the price vector over the whole time horizon.

The price considered in every time period is an aggregation of the price at all hours of the corresponding time period (an average or similar). It is assumed that the electricity price is constant over certain hour groups, which means that the price at a particular hour of a particular time period (in a particular scenario and a particular location) is determined as the aggregated price determined by the scenario tree (P^E_{lst}) multiplied by a price aggregation factor associated to the corresponding hour group (β_{ltk}). Electricity prices should be relatively stable in each hour group (the hour groups considered may be related to the load duration curve, since demand is highly correlated to price). For example, the hour groups considered could be peak and off-peak in a simple case with two hour groups, or working day peak and off peak, Saturday, and Sunday, in a more complex case with 4 hour groups. (The decomposition into hour groups used in the examples in the next chapter is explained in 6.1.1.)

Note that the number of dimensions, scenarios, and time periods of the stochastic price vector process should be moderate in order to capture the essential properties of this process in a reasonably sized scenario tree so as to guarantee the computational tractability of the problem. Yet, the number of scenarios cannot be so small in order to ensure that the properties of the price process are satisfactorily represented, and the number of time periods cannot be too small either because otherwise decisions could be too short sighted. Consequently, there is a tradeoff between the computational ease of solving the problem and the accurate representation of the stochastic price vector process and practical validation of the model.

5.2.4 Generating units

It is assumed that the power producer holds a set of thermal generating units in each of the locations considered. Each of these generating units is characterized by a variable production cost, minimum and maximum power outputs, and restrictions on the electricity produced during different hour groups of every specific time period, among other constraints.

The variable production cost depends on the time period, in order to represent fuel price forecasts or fuel contracts held. Fuel price could also be represented as a stochastic process within the scenario tree, but the difficulty of solving the problem would increase considerably, and since electricity prices are much more volatile than fuel price in the time horizon considered, it is more appropriate to capture electricity price randomness stochastically. It is interesting to mention that there are also alternatives for hedging fuel costs, which are not considered in this work.

The minimum and maximum power outputs also depend on time, in order to represent possible scheduled unit outages. If a unit is to present an outage during a whole time period, then the minimum and maximum power outputs are both 0 in that time period. If it is to present an outage only part of that time period, then the minimum and maximum power outputs must be set appropriately, and if there is no outage in that time period, then the minimum and maximum power outputs are the usual values. It is assumed that the outage periods are known at the beginning of the horizon and therefore the model ignores forced unit outages.

Restrictions on the electricity produced during different hour groups of every time period are considered based on the fact that since electricity production is modeled in a medium term manner, some short term operation restrictions, such as ramping limits, have an influence over it. Short term ramping limits are associated to restrictions over power output changes of generating units. It is assumed in this model that these limits result in minimum levels of power outputs in low-price hour groups (non-peak hours of the week) such that the electricity produced in these hour groups is at least a certain percentage of the electricity generated during their associated high-price hour groups (peak hours of the week). This restrictions are based on the model presented by Conejo et al. (2008).

It is also assumed that the power producer is a price taker, which means that its production does not influence electricity spot price. This is a reasonable assumption if the generating capacity of the power producer is small enough compared to the total production of the electricity market under consideration.

5.2.5 Contracts

Power producers trade electricity through organized electricity pools (spot markets), derivative markets, and through bilateral contracts. Some of the electricity derivatives that exist in many electricity markets are forward and futures contracts, options, cap contracts, and contracts for difference.

The contracts considered in this model are forwards and contracts for difference, which are explained next. For an introduction to derivatives in energy markets see Eydeland & Wolyniec (2003) and Burger et al. (2007).

a) Forwards

In traditional financial markets, a forward contract is an agreement in which a unit of a good is to be bought by the holder of the contract, that has a “long position” on the contract, at a given delivery date and price, both determined at the moment of entering the contract. As typical examples of hedging applications with forward contracts, the producer of a particular good may enter into short positions of forward contracts (selling the product) in order to hedge the risk of low price of the good at the moment of delivery, and a buyer of the good may enter into long positions (buying the product) in order to hedge the risk of high price at delivery.

In electricity markets, a forward contract on electricity determines a price and a time span in which electricity is delivered according to certain contract arrangements. For example, a forward contract may state that a certain company must provide the holder of the long position in the forward contract with constant 50 [MW] power on the peak hours (for instance 6.00-22.00) of all the working days of November 2011 and that the payment per unit of electricity delivered will be 40.1 [US$/MWh].

In the present model, following Conejo et al. (2008), it is assumed that forward selling contracts (short positions) consist of blocks of electricity (with constant power) spanning a specified period of time with associated decreasing prices, and that forward buying contracts (long positions) consist of blocks of electricity (of constant power) spanning a specified period of time with associated increasing prices. Hence, if a producer buys electricity through forward contracts, the electricity price increases with the amount of electricity bought (in blocks), and if a producer sells electricity through forward contracts, the electricity price decreases with the amount of electricity sold (in blocks). This assumption is intended to represent the limited market liquidity usually present in forward contract markets of electric energy, although in possible actual applications of the model the contracts and blocks considered could represent the existent forward contract alternatives available at the moment of applying the model.

It is also assumed that forward contracts have time spans of exactly one time period, with delivery periods matching the hours of one hour group, although it would be straightforward to consider forward contracts with longer time spans and delivery on hours associated to any set of hour groups.

The different blocks of forward contracts may also represent bilateral contracts. In case the user of the model has the possibility of signing certain bilateral contracts of buying or selling electricity with clauses similar to forward contracts, these bilateral contracts could be easily incorporated into the model through a forward contract with one block and the particular specifications. Appendix E.1 presents a model variation of the model that considers actual available contracts.

b) Contracts for difference

An important price risk in electricity markets is created by price variations over different locations, called "spatial risk" by Marckhoff & Wimschulte (2009). This risk can be hedged through contracts for difference (CfDs) and other instruments (which may or may not be present depending on the electricity market) such as financial transmission rights (FTRs), physical transmission rights (PTRs) and transmission congestion contracts (TCCs).

CfDs are electricity forward contracts on a locational price spread (or equivalently, on the price difference between two zones over a specified period). Consider two locations "A" and "B" and associated electricity prices S_t^A and S_t^B at time period t, respectively. Suppose t_1 is the initial time period of a CfD contract and t_2 is the final time period (the contract is signed before t_1). Suppose the CfD contract is over the price spread between A and B. Then, the payoff for the holder on a long position of the contract is given (in this thesis) by:

$$\sum_{t=t_1}^{t_2}(S_t^A - S_t^B).$$

In this work it is assumed that the price or cost of the contract is "similar" to its expected payoff. The price or cost of these contracts are assumed to be slightly more inconvenient than expected payoffs in order to represent their lack of liquidity. If the price or cost of the contract is equal to its expected payoff then risk neutral power producers would be indifferent of wether to enter into contracts for difference or not. In the model presented, CfDs are treated similarly to forward contracts: as the quantity of contracts to be bought increases, the price to be paid increases by block. It is also assumed that contracts for difference have time spans of exactly one time period, with delivery on exactly one hour group.

Notice that CfD payoffs can be replicated with forward contracts on electricity at both locations involved (short position on location A, long position on location B). Nevertheless, the low liquidity of CfDs and forwards encourages the incorporation of both types of contracts in the same model. CfDs are widely used, for instance, in the Nord Pool but are not present in all electricity markets. Anyway, in case of non-existence of these contracts, they can be replicated through bilateral contracts, assuming there are market agents willing to take the opposite positions.

For more information on CfDs, incorporating their applicability, pricing and usage in the Nord Pool market, see Marckhoff & Wimschulte (2009).

5.2.6 Risk modeling

Conditional value at risk is used for risk measurement and is incorporated into the objective function multiplied by a risk aversion parameter. This will allow to obtain a tradeoff curve between expected profit and risk by varying the risk aversion parameter.

The definition of CVaR that will be adopted in this work is:

$$CVaR_\alpha = E[\pi | \pi \leq VaR_\alpha],$$

where π is profit, α is the confidence level (usually between 0.9 and 0.99) and VaR (value at risk) is defined as the α quantile of π:

$$VaR_\alpha = \max\{x : P(\pi \leq x) \leq 1 - \alpha\}.$$

For instance, if $\alpha = 0.9$ then CVaR is the expected value of profit given that profit is below its 10% quantile, or in other words that CVaR is the average of profit over its left 10% distribution tail. The concept is depicted in figure 5.2. Several advantages of using CVaR as risk measure for power portfolio optimization problems have been described in Denton et al. (2003).

See appendix C for a thorough explanation of how CVaR is incorporated into the optimization model (because of its length this is presented in an appendix).

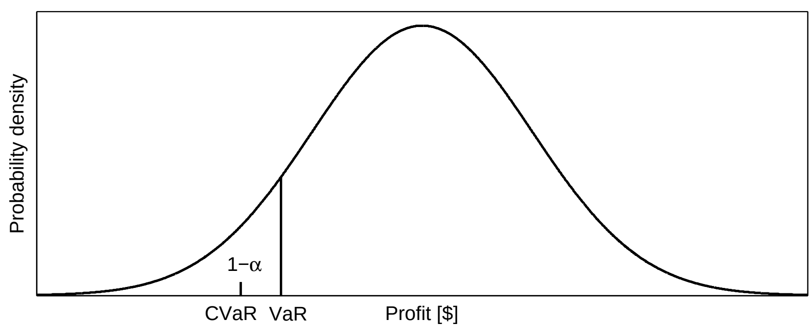


FIGURE 5.2. CVaR concept

5.2.7 Model usage

The model presented in this chapter is intended to be used on a weekly or monthly basis. At the beginning of each time period decisions pertaining to the rest of the time horizon (for instance the following 6 months) and involving the contracts mentioned above can be made using the recommendations of the model. Decisions regarding spot market transactions and generation planning should be made using shorter time models, based on the contractual involvement recommended by the model, and at the beginning of the next time period contractual decisions spanning the next upcoming periods (for instance the next 6 months) could be made again with assistance of the model, incorporating the new information. As a result of the process, contractual positions are updated at the beginning of every time period.

5.3 Mathematical formulation

The problem described above is formulated as a stochastic programming problem with recourse (see Birge & Louveaux (1997)), and given a particular scenario tree this problem is formulated as an equivalent linear program. The optimization problem is:

$$\max_{\{x_{istk}, e_{lstk}, f^B_{ltkb}, f^S_{ltkb}, g_{lmtkb}, \pi_s, \xi, \eta_s\}} \sum_{s=1}^{S} p_s \pi_s + \gamma(\xi - \tfrac{1}{1-\alpha} \sum_{s=1}^{S} p_s \eta_s) \quad (1)$$

$$\begin{aligned}
s.t. \quad \pi_s = & \sum_{l=1}^{L} \sum_{t=1}^{T} \sum_{k=1}^{K} e_{lstk} \beta_{ltk} P^E_{lst} - \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{k=1}^{K} x_{istk} C_{it} \\
& + \sum_{l=1}^{L} \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{b=1}^{N^f} (C^{fS}_{ltkb} f^S_{ltkb} - C^{fB}_{ltkb} f^B_{ltkb}) \\
& + \sum_{l=1}^{L} \sum_{\{m=1,\dots,L: m \neq l\}} \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{b=1}^{N^{CfD}} C^{CfD}_{lmtkb} g_{lmtkb} \\
& + \sum_{l=1}^{L} \sum_{t=1}^{T} \sum_{k=1}^{K} D_{ltk} P^D_{ltk}, \forall s = 1, \dots, S; \quad (2)
\end{aligned}$$

$$\phi_{tk} \underline{x}_{it} \leq x_{istk} \leq \phi_{tk} \overline{x}_{it}, \forall i = 1, \dots, I, \forall s = 1, \dots, S, \forall t = 1, \dots, T, \forall k = 1, \dots, K; \quad (3.1)$$

$$0 \leq f^B_{ltkb} \leq \overline{f}^B_{ltkb}, \forall l = 1, \dots, L, \forall t = 1, \dots, T, \forall k = 1, \dots, K, \forall b = 1, \dots, N^f; \quad (3.2)$$

$$0 \leq f^S_{ltkb} \leq \overline{f}^S_{ltkb}, \forall l = 1, \dots, L, \forall t = 1, \dots, T, \forall k = 1, \dots, K, \forall b = 1, \dots, N^f; \quad (3.3)$$

$$\begin{aligned}
0 \leq g_{lmtkb} \leq \overline{g}_{lmtkb}, & \forall l = 1, \dots, L, \forall m = 1, \dots, L : m \neq l, \forall t = 1, \dots, T \\
& \forall k = 1, \dots, K, \forall b = 1, \dots, N^{CfD}; \quad (3.4)
\end{aligned}$$

$$\begin{aligned}
& \sum_{\{i=1,\dots,I: \psi_i = l\}} x_{istk} + \sum_{b=1}^{N^f} f^B_{ltkb} + \sum_{\{m=1,\dots,L: m \neq l\}} \sum_{b=1}^{N^{CfD}} g_{mltkb} \\
& \quad = e_{lstk} + D_{ltk} + \sum_{b=1}^{N^f} f^S_{ltkb} + \sum_{\{m=1,\dots,L: m \neq l\}} \sum_{b=1}^{N^{CfD}} g_{lmtkb}, \\
& \quad \forall l = 1, \dots, L, \forall s = 1, \dots, S, \forall t = 1, \dots, T, \forall k = 1, \dots, K; \quad (4)
\end{aligned}$$

$$H_{ik} \frac{x_{istk}}{\phi_{tk}} \leq \frac{x_{ist\mu_k}}{\phi_{t\mu_k}}, \forall i = 1, \dots, I, \forall s = 1, \dots, S, \forall t = 1, \dots, T, \forall k \in \mu^{High}; \quad (5)$$

$$\eta_s \geq \xi - \pi_s, \forall s = 1, \dots, S; \quad (6.1)$$

$$\eta_s \geq 0, \forall s = 1, \dots, S; \quad (6.2)$$

$$x_{is_1tk} = x_{is_2tk}, \forall i = 1, \dots, I, \forall s_1 = 1, \dots, S, \forall t = 1, \dots, T, \forall s_2 \in B_{s_1t},$$

$$\forall k = 1, ..., K; \tag{7.1}$$

$$e_{ls_1tk} = e_{ls_2tk}, \forall l = 1, ..., L, \forall s_1 = 1, ..., S, \forall t = 1, ..., T, \forall s_2 \in B_{s_1t},$$
$$\forall k = 1, ..., K. \tag{7.2}$$

Problem (1)-(7.2) is described below.

5.3.1 Objective function

The objective function corresponds to the expected total profit of the power producer plus an approximation of CVaR multiplied by the risk aversion parameter γ. A detailed explanation of the CVaR expression is given in appendix C.

5.3.2 Constraints

Constraints (2) express that π_s is the total profit of the power producer under scenario s. This is composed by the revenues obtained from trading in electricity spot markets, the costs of producing electricity, the revenues of selling electricity through forwards contracts, the costs of buying electricity through forward contracts, the revenues obtained from transactions in contracts for difference, and the revenues obtained from existent contractual positions.

Constraints (3.1) state the minimum and maximum electricity production possible for each unit during each hour group of each time period. (3.2)-(3.4) enforce bounds on forwards and contracts for difference positions for each block.

Constraints (4) represent electricity balance in each location for every hour group and time period: the electricity produced plus the electricity bought through forwards and CfDs equals the electricity traded in the spot market (positive when selling, negative when buying) plus the electricity traded through existent contractual positions, plus the electricity sold through forwards and CfDs.

Constraints (5) enforce electricity production relations between low-price hour groups and high-price hour groups.

Constraints (6.1)-(6.2) are necessary for CVaR definition (see appendix C).

Constraints (7.1)-(7.2) are the non-anticipativity constraints for the variables that depend on scenarios: electricity production and electricity spot market trading. These constraints state that if in $\{1, ..., t\}$ two scenarios s_1 and s_2 have the same history (they are indistinguishable in that time span), then the same decisions must be taken in t under both scenarios.

5.3.3 Results

For a particular value of this risk aversion parameter the model obtains a set of contractual involvement decisions in the beginning of the time horizon ("here and now" decisions), and a set of generation and spot market trading decisions in future time periods that depend on the evolution of locational electricity prices ("wait and see" decisions). Solving the model for different values of the risk aversion parameter, γ, provides the efficient frontier between expected profit and risk for the power producer.

5.3.4 Problem size

The number of continuous variables of the linear program presented equals:

$$ISTK + LSTK + 2LTKN^f + 2\frac{L(L-1)}{2}TKN^{CfD} + 2S + 1.$$

The number of constraints equals:

$$S + ISTK + 2LTKN^f + 2\frac{L(L-1)}{2}TKN^{CfD} + LSTK$$

$$+IST\#\mu^{High} + 2S + (I + L)K \sum_{t=1}^{T} \sum_{s_1=1}^{S} \#B_{s_1,t}.$$

The next chapter presents an application of the model using data from the NYISO.

VI. APPLICATION

This chapter presents an application of the power portfolio optimization model presented in chapter 5. The case studied consists of a power producer that holds two thermal generating units, each of them in a different location of a deregulated electricity market and under a different locational electricity price process. The power producer's problem is to optimize its contractual, spot market and generation decisions in order to maximize its expected benefit given a degree of risk aversion, over a time horizon of six months. The tradeoff curve between profit and risk is obtained by solving the model for several values of the risk aversion parameter.

In order to implement the model, three steps can be mentioned. In a first step, a stochastic vector process is used to model the two locational electricity prices jointly (using one of the models from chapter 3). In a second step, a scenario tree that represents this process is generated (using one of the methods presented in chapter 4). In the third step, the model presented in chapter 5 is solved using the scenario tree generated.

The remaining of this chapter is organized as follows: 6.1 presents the data set used, 6.2 presents the details of the implementation, and 6.3 presents the results obtained. The notation used in this chapter is the same given in section 5.1.

6.1 Data and model parameters

6.1.1 Electricity prices and hour groups

Historical data of locational electricity spot prices is obtained from the NYISO web site (http://www.nyiso.com/public/index.jsp, last visit 10 July 2010). Hourly electricity prices for two zones, from 1 May 2005 until 31 January 2009, are used. It is assumed that the power producer holds one generator in each location and wants to optimize its contractual decisions concerning the six next months, from February 2009 until July 2009. The historical daily average prices for the two zones may be seen in figure 6.1.

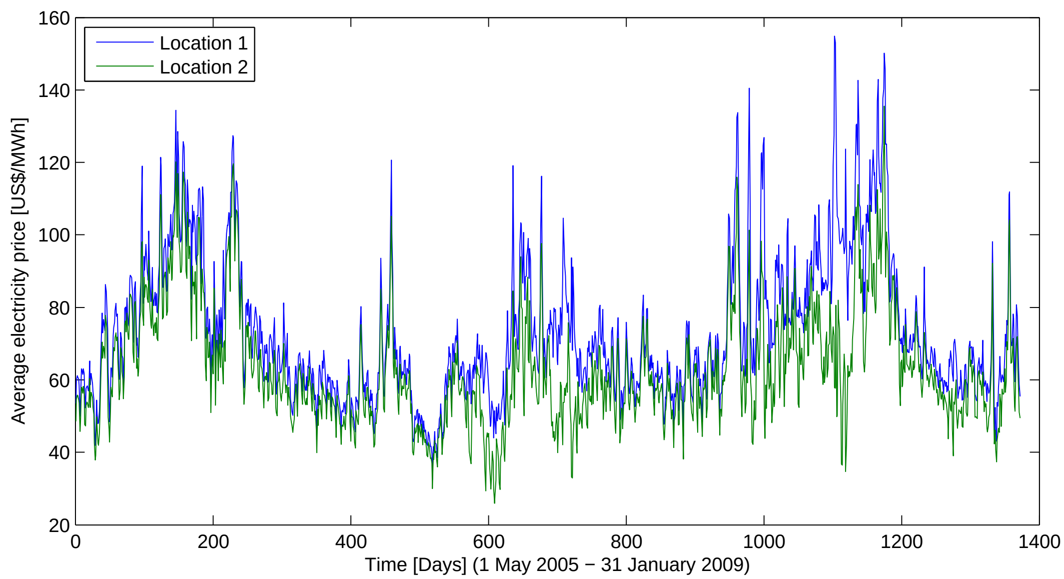


FIGURE 6.1. Daily average electricity prices

Using the daily average electricity prices, "adjusted" monthly average prices are obtained by first calculating the average prices of each type of day (Monday, Tuesday...) and then averaging these 7 values, for each month. The adjusted average electricity prices may be seen in figure 6.2. The idea of using these adjusted averages instead of usual averages is to eliminate the effects of having more days of a certain type in a given month.

These adjusted monthly average electricity prices are used for fitting the parameters of a stochastic model with which a scenario tree is generated with the methodology described in section 4.5, thus obtaining the adjusted monthly average electricity price at location l, scenario s, and month t, P^E_{lst}, and the respective probability p_s of each scenario s.

A number of $K = 6$ different hour groups are used to represent price variations within each month. Each of this groups consists of a set of hours of every week, since intra weekly seasonalities are evident for electricity prices. This means that the model assumes the same price in every hour of a given time period and hour group. The hour groups selected are shown in table 6.2, and explained next: hour group 1 represents off-peak (low price) week day hours, consisting of hour 23 (from 23.00 until 24.00) of Sunday, hours 0-6 of Monday

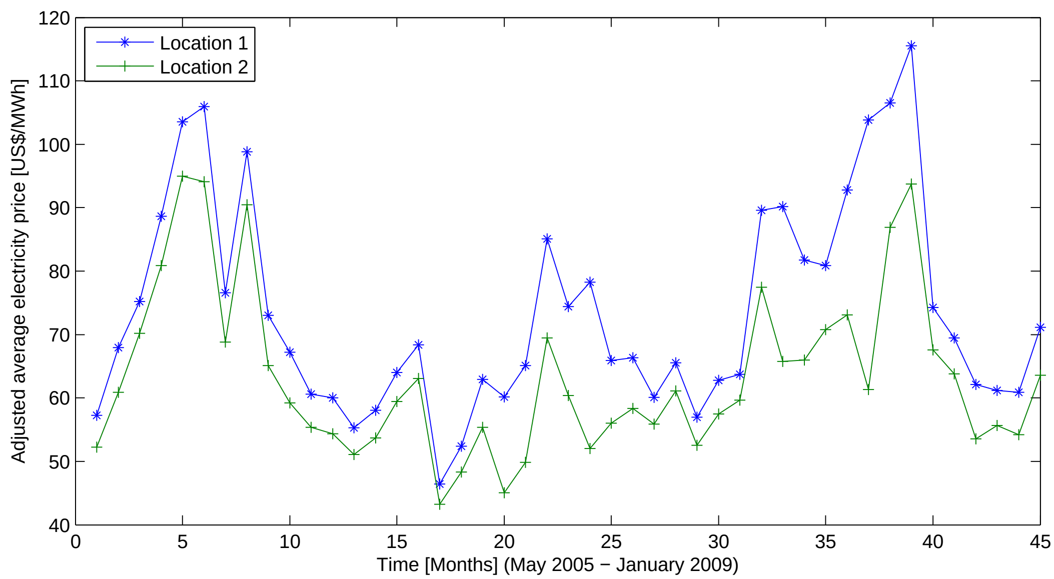


FIGURE 6.2. Adjusted monthly average electricity prices

through Friday and hour 23 of Monday through Thursday; hour group 2 represents on-peak (high price) week day hours, consisting of hours 7-22 of Monday through Friday; hour group 3 represents off-peak Saturday prices, consisting of hour 23 of Friday and hours 0-8 of Saturday; hour group 4 represents on-peak Saturday prices, consisting of hours 10-22 of Saturday; group 5 represents off-peak Sunday prices, consisting of hour 23 of Saturday and hours 0-8 of Sunday; and group 6 represents on-peak Sunday prices, consisting of hours 7-22 of Sunday.

The distribution of hour groups is chosen so as to represent intra month variations such that $\beta_{ltk}P^E_{lst}$ is a good representative of electricity price in hour group k of month t, at location l, under scenario s, where β_{ltk} is the price aggregation factor for location l, month t and hour group k. If these factors were not dependent on time, and hence constant for the whole year (so that $\beta_{ltk} = \beta_{lk}$) then they would take the values presented in table 6.2, and intra-week price variations would be approximated as presented in figure 6.3. Yet, intra week price relations generally depend on the month because electricity consumption

TABLE 6.1. Hour groups distribution

Day	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
Monday	0-6, 23	07-22				
Tuesday	0-6, 23	07-22				
Wednesday	0-6, 23	07-22				
Thursday	0-6, 23	07-22				
Friday	0-6	7-22	23			
Saturday			0-8	10-22	23	
Sunday	23				0-8	7-22
Total hours	**40**	**80**	**10**	**14**	**10**	**14**

at different moments of the week depends on normal life cycles, weather, etc., and hence, price aggregation factors β_{ltk} are determined dependent on the particular month t.

TABLE 6.2. Electricity price aggregation factors (β_{ltk}), using 6 groups with fixed factors for the whole year

Location	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
$l = 1$	0.7704	1.1499	0.7860	1.0843	0.7537	1.0441
$l = 2$	0.7743	1.1623	0.7746	1.0517	0.7380	1.0139

With these hour groups, high-price hour groups correspond to groups 2, 4 and 6, and the low-price hour group associated to each of them are groups 1, 3 and 5, respectively. Hence: $\mu^{High} = \{2, 4, 6\}$ and $\mu_2 = 1, \mu_4 = 3, \mu_6 = 5$.

Finally, the duration ϕ_{tk} of each hour group k in each of the next 6 months t, is summarized in table 6.3.

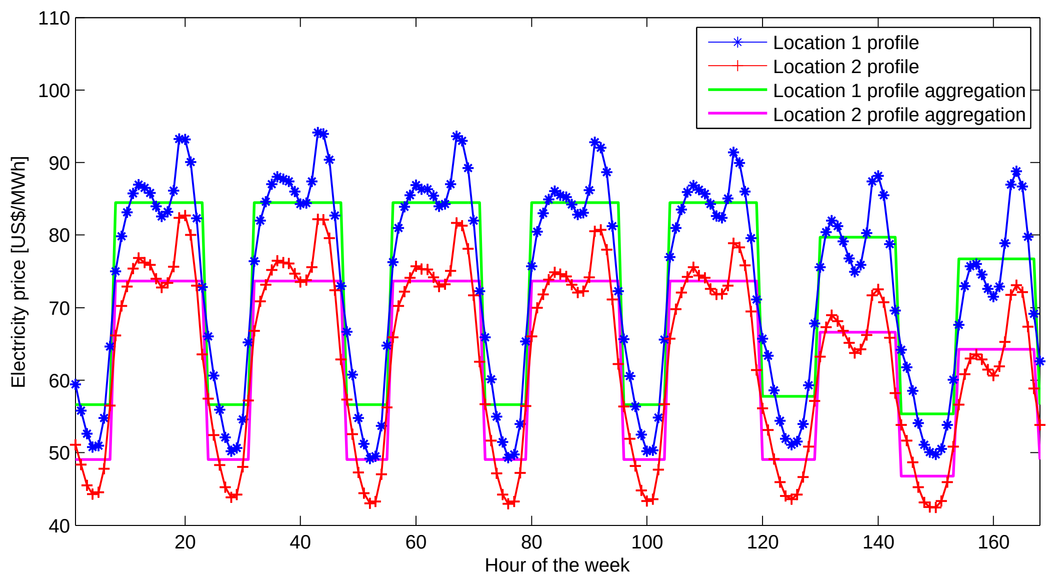


FIGURE 6.3. Electricity prices aggregation into 6 hour groups with fixed factors for the whole year

TABLE 6.3. Duration [hours] of hour groups at each month (ϕ_{tk})

Month	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$
February ($t=1$)	160	320	40	56	40	56
March ($t=2$)	177	352	40	56	49	70
April ($t=3$)	176	352	40	56	40	56
May ($t=4$)	168	336	50	70	50	70
June ($t=5$)	176	352	40	56	40	56
July ($t=6$)	183	368	41	56	40	56

6.1.2 Generating units

One unit is considered at each of the two locations considered, with both units having the exact same properties. The location, generating cost, minimum and maximum generating outputs, and the relation factor between per-hour generation at high-price hour groups and their respective low-price hour groups are shown in table 6.4.

TABLE 6.4. Generating units parameters

Unit	ψ_i (location)	$C_{it}[US\$/MWh]$	$\underline{x}_{it}[MWh/h]$	$\overline{x}_{it}[MWh/h]$	H_{ik}
$i = 1$	1	60	0	300	0.3
$i = 2$	2	60	0	300	0.3

6.1.3 Contracts

There is a number $N^f = 3$ of different blocks of buying and selling contracts. The upper bound of the quantity of electricity that can be bought or sold through forward contracts at location l, month t and hour group k, through block b, are given by $\overline{f}^B_{ltkb} = f^{liq}\phi_{tk}$ (buying) and $\overline{f}^S_{ltkb} = f^{liq}\phi_{tk}$ (selling), where $f^{liq} = 100[MWh/h]$ is a "forwards liquidity parameter". The prices per unit of electricity contracted through buying or selling block b forward contracts at location l, for delivery in hour group k of month t, are given, respectively, by:

$$C^{fB}_{ltkb} = (1 + bC)\sum_{s=1}^{S} p_s \beta_{ltk} P^E_{lst},$$

$$C^{fS}_{ltkb} = (1 - bC)\sum_{s=1}^{S} p_s \beta_{ltk} P^E_{lst},$$

where $C = 0.5\% = 0.005$. These expressions are close to the expected value of electricity price in the corresponding time span, but for buying contracts the price is a little higher (and increasing through blocks 1, 2, 3) and for selling contracts a little lower (and decreasing through blocks 1, 2, 3). Hence, with these prices, for a power producer that only wishes to maximize its expected profit (risk neutral case), forward contracts are not convenient.

The basic assumption behind this price-liquidity selection is that the power producer does not have enough market power with respect to entities that may want to hold the opposite forward contract positions. Also, having block 2 more inconvenient than block 1, and block 3 more inconvenient than block 2, represents the limited market liquidity that may exist. (In appendix E.1 a model that considers actual available forward contracts is presented, which may be more appropriate for real applications, instead of using an approach that is only intended to provide an approximation to real markets.)

With contracts for difference liquidity conditions and prices are chosen in a similar manner. There is a number $N^{CfD} = 3$ of different blocks for selling in location 1 and buying at 2, and for the opposite contracts (selling in location 2 and buying at location 1) the same. The upper bound for selling at location l and buying at location m, at hour group k of month t, through CfD block b, is given by $\overline{g}_{lmtkb} = g^{liq}\phi_{tk}$, where $g^{liq} = 100[MWh/h]$ is a "CfDs liquidity parameter". The price per unit of electricity contracted through such a contract is given by:

$$C^{CfD}_{lmtkb} = \begin{cases} (1+bC)\sum_{s=1}^{S} p_s[\beta_{ltk}P^E_{lst} - \beta_{mtk}P^E_{mst}] & \text{if } \sum_{s=1}^{S} p_s[\beta_{ltk}P^E_{lst} - \beta_{mtk}P^E_{mst}] \geq 0 \\ (1-bC)\sum_{s=1}^{S} p_s[\beta_{ltk}P^E_{lst} - \beta_{mtk}P^E_{mst}] & \text{if } \sum_{s=1}^{S} p_s[\beta_{ltk}P^E_{lst} - \beta_{mtk}P^E_{mst}] < 0 \end{cases}$$

where $C = 0.5\% = 0.005$ (as in the forwards case). These expressions are close to the difference of the expected values of electricity price in the corresponding time span, but making the contracts slightly more inconvenient (with increasing inconvenience through blocks 1, 2, 3) for pure expected profit maximizers (risk neutral power producers), similarly to what was adopted in the forwards case.

6.1.4 Existent contractual positions

Existent contractual positions at location l and hour group k of time period t are assumed to be given by $D_{ltk} = D^{perhour}\phi_{tk}$, with $D^{perhour} = 150[MWh/h]$. This may represent bilateral contracts already assumed, forward contracts already assumed, CfD contracts already assumed, etc. The payoff to be received per unit of electricity of existent contractual positions is assumed to be given by the expected electricity price at the particular location, time period, and hour group: $P^D_{ltk} = \sum_{s=1}^{S} p_s\beta_{ltk}P^E_{lst}$.

6.1.5 Confidence level of CVaR

The confidence level of conditional value at risk (CVaR) is assumed to be $\alpha = 0.9$, which is a generally adopted value. This means that the profit's distribution tail considered in CVaR weights 10% of the total probability.

6.2 Implementation

This section summarizes the three steps developed for solving the problem. In a first step a stochastic model is fitted to the adjusted monthly average electricity prices presented in section 6.1.1. In a second step a scenario tree that represents that stochastic model is generated. In a third step, the optimization model is solved.

6.2.1 Electricity prices model

The following model is used to represent the stochastic evolution of the adjusted monthly average electricity prices for the two locations over time:

$$\log P_i(t) = f_i(t) + X_i(t); i = 1, 2;$$

$$dX_i(t) = -\frac{X_i(t)}{\lambda_i}dt + \sigma_i dW_i(t); i = 1, 2;$$

$$E[dW_1(t)dW_2(t)] = \rho dt.$$

This model is a bivariate mean reverting diffusion and has been presented in chapter 3. This is a continuous-time process, but only the values at fixed time steps have an interpretation. $P_i(t)$ represents the adjusted monthly average of electricity price at location $i = 1, 2$ and month $t = 1, ..., 6$; $f_i(t)$ is a deterministic component; and the $X(t)$ process is a bivariate mean reverting diffusion, also called bivariate Ornstein-Uhlenbeck process, with guiding correlated Brownian motions $W_i(t)$, $i = 1, 2$.

This model is complex enough so as to capture the fundamental properties of the two locational monthly average prices: autocorrelation (correlation between price at different time periods), mean reversion, higher volatility at higher prices, and very importantly in this thesis, correlation between the price variations of the two locations. This model is also simple enough in order to be well represented in a reasonably sized scenario tree.

Using the data shown in section 6.1.1, the parameters obtained for this model are shown in table 6.5. The parameters for the deterministic component have been obtained using a standard linear regression, and the parameters for the Ornstein-Uhlenbeck component have been obtained using a maximum likelihood based method. Parameter estimation for this model is thoroughly explained in appendix A. Fixed values are chosen for the deterministic component: $f_i(t) = a_i$, because of the data set used (in general cases it is more accurate to at least use annual seasonalities, but the data was so noisy that no reasonable annual seasonalities could be captured).

TABLE 6.5. Bivariate log mean reverting diffusion parameters fitted to the adjusted monthly average electricity prices and initial price conditions

Location	$P_i(0)$	$f_i(t) = a_i$	λ_i	σ_i	ρ
$i = 1$	71.1323	4.2741	2.8582	0.1782	0.8382
$i = 2$	63.5603	4.1303	2.0150	0.1919	

The data used for the adjusted monthly average electricity prices together with a simulated future trajectory may be seen in figure 6.4.

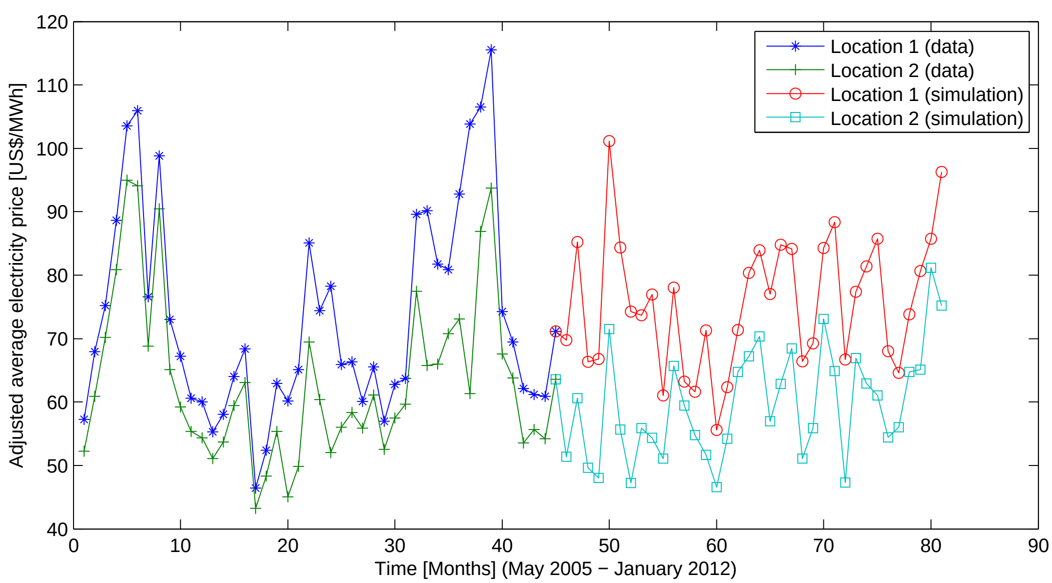


FIGURE 6.4. Data and one simulation for the adjusted monthly average electricity prices

6.2.2 Scenario tree generation

The detailed methodology for generating the scenario tree is presented in section 4.5. Summarizing, the method consists of generating a scenario tree for the Ornstein-Uhlenbeck component (the "X" process), since this process has a symmetric distribution at each location and month. If $\overline{X}_i^s(t)$ is the value of the approximating scenario tree of the Ornstein-Uhlenbeck component (the X process) at location i and time period t under scenario s, with associated probability $p(s)$ of scenario s, then its associated value in the approximating scenario tree of the price process is $\overline{P}_i^s(t) = exp(f_i(t) + \overline{X}_i^s(t))$, with associated probability $p(s)$ of scenario s. An example of a scenario tree generated for the Ornstein-Uhlenbeck component may be seen in figure 6.5, and its associated price scenario tree in figure 6.6.

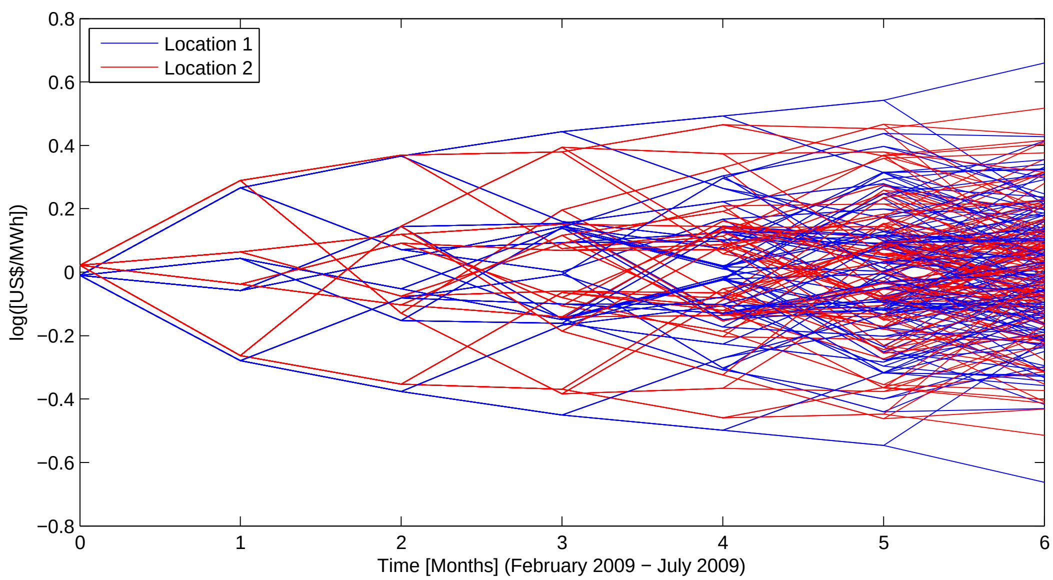


FIGURE 6.5. One Ornstein-Uhlenbeck component ("X process") scenario tree

The methodology for generating the Ornstein-Uhlenbeck component (the "X process") scenario tree consists in solving a nonlinear optimization problem consistent of minimizing a weighted sum of the squares of the distances between the relevant statistical properties of the scenario tree and the theoretical statistical properties of the process. The adjusted

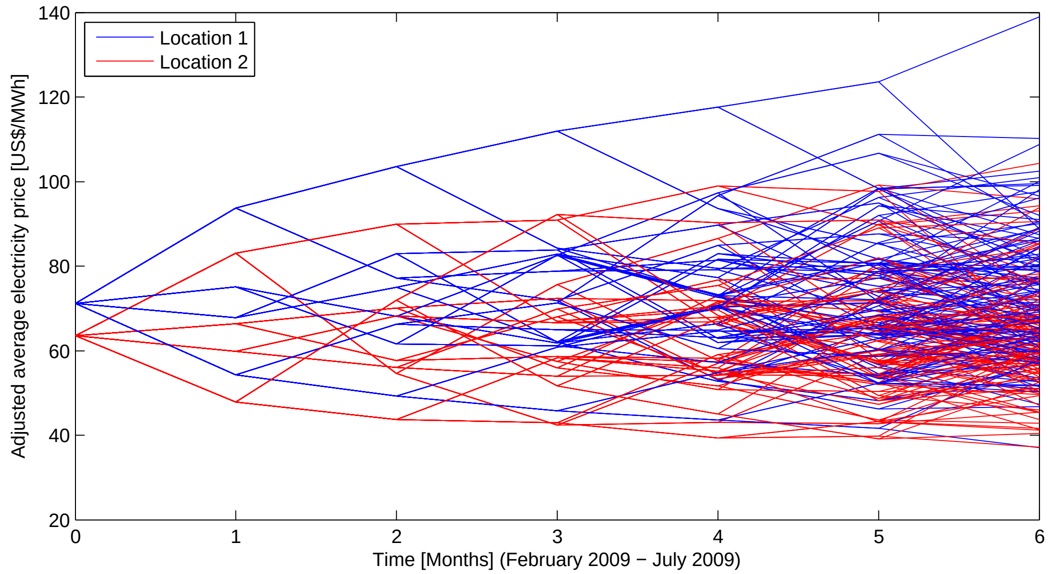


FIGURE 6.6. One adjusted monthly average electricity prices scenario tree

average electricity price scenario tree obtained has $S = 128$ scenarios. The details and result evaluation of the method and its application is presented in section 4.5.

6.2.3 Optimization

The linear programming problem presented in chapter 5, using the scenario tree presented in subsection 6.2.2 and the parameters shown in section 6.1, has been implemented in MATLAB and solved using CPLEX 12 with the help of YALMIP (that has been used to facilitate the communication between MATLAB and CPLEX) (if the reader is interested in YALMIP refer to the web site `http://users.isy.liu.se/johanl/yalmip/`, last visit 10 July 2010). The linear programming problem solved has approximately 10^5 variables.

6.3 Results

This section presents various results obtained from the application of the optimization model.

6.3.1 Basic results

Solving the problem for different values of the risk aversion parameter γ allows to obtain the efficient frontier relating optimal expected profit and risk. The values used for γ are 0 (risk neutral case), 0.01, 0.02, 0.04, 0.08, 0.16, 0.32, 0.64 and 1.28. Figure 6.7 presents the change in expected profit, profit standard deviation and CVaR as the risk aversion parameter changes.

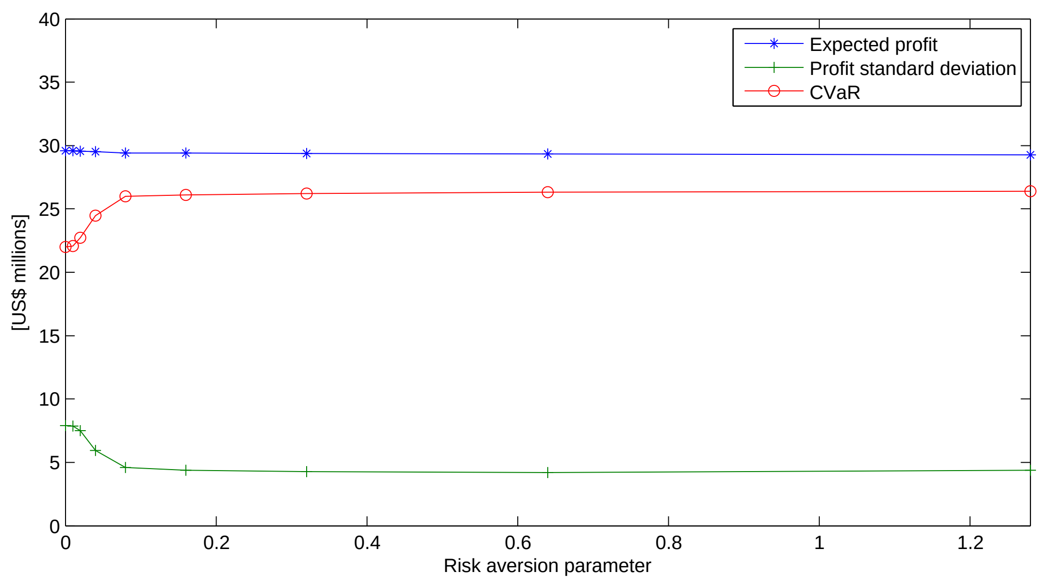


FIGURE 6.7. Expected profit, profit standard deviation and CVaR as functions of the risk aversion parameter

The efficient frontier between expected profit and risk is shown in figure 6.8 using negative CVaR as risk metric. Notice that CVaR as defined in this thesis is the average profit given that profit is below its α quantile, therefore as CVaR increases, risk decreases, and that is the reason for plotting negative CVaR vs expected profit. Another important property of profit is standard deviation. The relation obtained between expected profit and profit standard deviation is shown in figure 6.9.

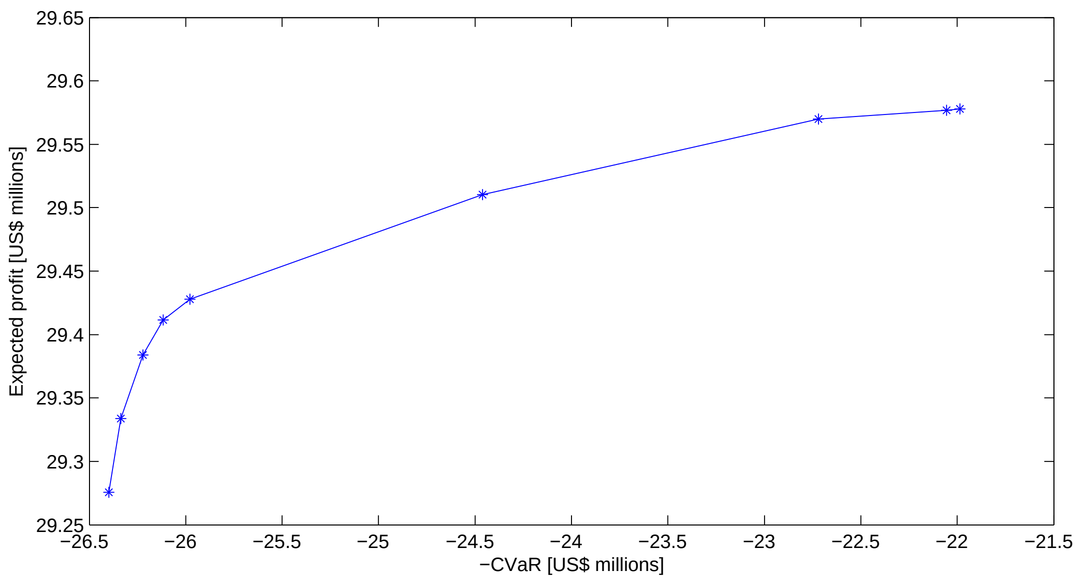


FIGURE 6.8. Negative CVaR vs expected profit

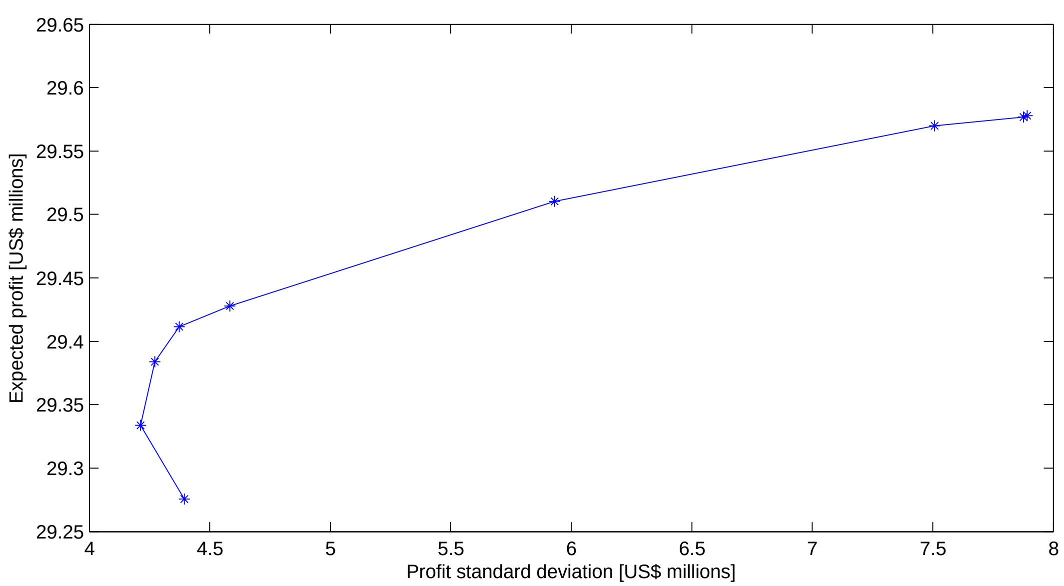
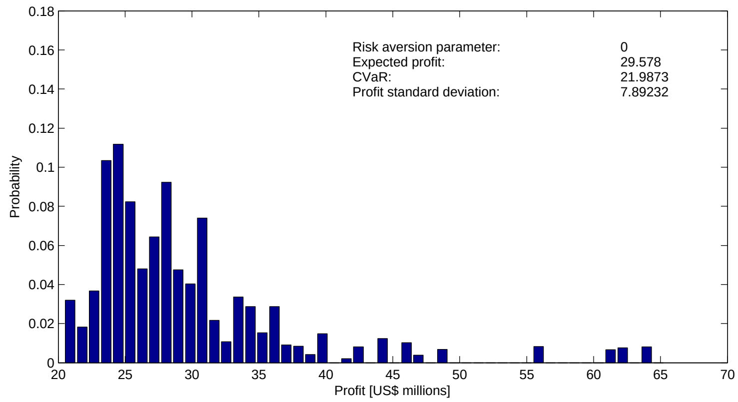
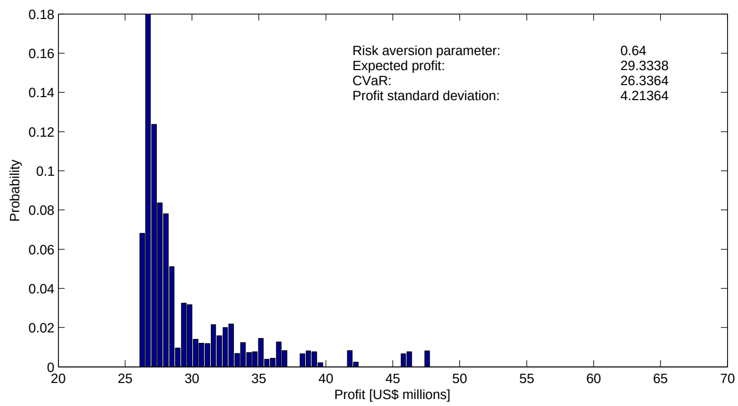


FIGURE 6.9. Profit standard deviation vs expected profit

Histograms for profit distribution in the risk neutral case and in a risk averse case are shown in figure 6.10. In this figure it can be seen how higher risk aversion concentrates profit distribution, thus reducing risk exposure.



(a) Risk aversion parameter: 0.



(b) Risk aversion parameter: 0.64.

FIGURE 6.10. Profit histogram in a risk neutral case and in a risk averse case.

6.3.2 Value of using stochastic programming

Modeling a problem using stochastic programming increases the complexity of the model and the computational difficulty of solving the problem, compared to using a deterministic approach, but since it can lead to better solutions, for instance with higher expected profit and/or lower risk, its use is justified. In this subsection a measure that will be called the value of using stochastic programming (VSP) will be calculated, and it measures how much better is the objective function in the risk neutral case using stochastic programming compared to solving the problem ignoring the stochastic components of the problem by replacing them with their expected values. The expected profit, CVaR, and profit standard deviation of the expected value solution (solution of the problem considering average values) and of the stochastic programming solution (solution considering probability distributions) are presented in table 6.6. Expected profit, CVaR, and profit standard deviation of the expected value solution are evaluated with respect to the scenario tree of the stochastic programming problem, after calculating profit under every scenario with the expected value solution.

TABLE 6.6. Stochastic programming solution compared to expected value solution (values in [million US$])

Solution	Expected profit	CVaR	Profit std. deviation
Stochastic prog.	29.5780	21.9873	7.8923
Expected value	25.4380	-7.6508	19.827

The VSP obtained is:

$$VSP = (29.578 - 25.438) \text{ [million US\$]} = 4.14 \text{ [million US\$]},$$

$$VSP(\%) = \frac{29.578 - 25.438}{25.438} = 16.28\%.$$

This means that it is convenient to face the problem using stochastic programming instead of ignoring the uncertainty of electricity prices, since the decisions recommended by the stochastic programming approach are better than those recommended by assuming the expected values of uncertain parameters. Also notice that the CVaR of the stochastic programming solution is much bigger than the CVaR of the expected value solution and the profit standard deviation of the stochastic programming solution is much smaller than the profit standard deviation of the expected value solution, which means that ignoring uncertainty in this case also leads to riskier solutions, highlighting the convenience of the stochastic programming approach.

For the interested reader, Birge & Louveaux (1997) present a similar measure to the one presented here, called the value of the stochastic solution (VSS). The measure used here (VSP) is a simplification of the VSS.

6.3.3 Value of the multi-location analysis

One alternative for facing the power portfolio problem is to simply solve one optimization problem for each location, instead of solving for both locations in an integrated manner. Let the solution obtained by solving for each location separately be called the single -location solution, that depends on the risk aversion parameters used at both locations and that does not account for CfDs (because they need information on both locations in order to be considered).

Table 6.7 presents expected profit and CVaR for the single-location solution varying both risk aversion parameters. Figures 6.11 and 6.12 show efficient frontier curves between CVaR and expected profit obtained by the single-location solution by fixing the risk aversion parameter at one location and varying it at the other location, and obtained for the multi-location solution in the base case and in a case without allowing the use of CfD contracts. Notice that for the multi-location solution, in the risk neutral case, expected profit is 29.5780 [million US$] with a CVaR of 21.9873 [million US$] (for both the base case and the case without allowing the use of CfD contracts).

TABLE 6.7. Expected profit [million US$] (CVaR [million US$]) for the single-location solution varying the risk aversion parameters at both locations

Loc. 1 / Loc. 2	0	0.04	0.08	0.64
0	29.58 (21.99)	29.57 (22.17)	29.52 (23.07)	29.48 (23.71)
0.04	29.51 (23.85)	29.51 (24.03)	29.46 (24.77)	29.42 (25.28)
0.08	29.44 (25.06)	29.43 (25.25)	29.38 (25.99)	29.35 (26.20)
0.64	29.42 (25.35)	29.41 (25.52)	29.36 (26.18)	29.32 (26.27)

From these results it can be inferred that it is relevant to consider a multi-location analysis instead of a separate analysis for each location, since higher total expected profits can be obtained when keeping the risk level fixed (and lower levels of risk may be obtained for fixed levels of total expected profit).

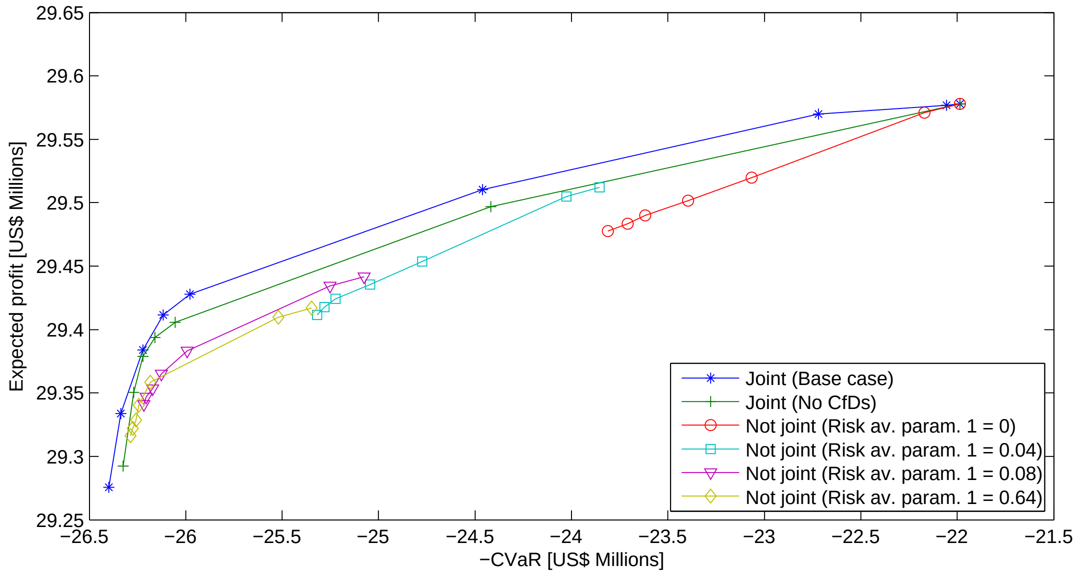


FIGURE 6.11. Negative CVaR vs expected profit curves for the single-location problem using various fixed risk aversion parameters at location 1 and varying the risk aversion parameter at location 2, together with multi-location solution curves

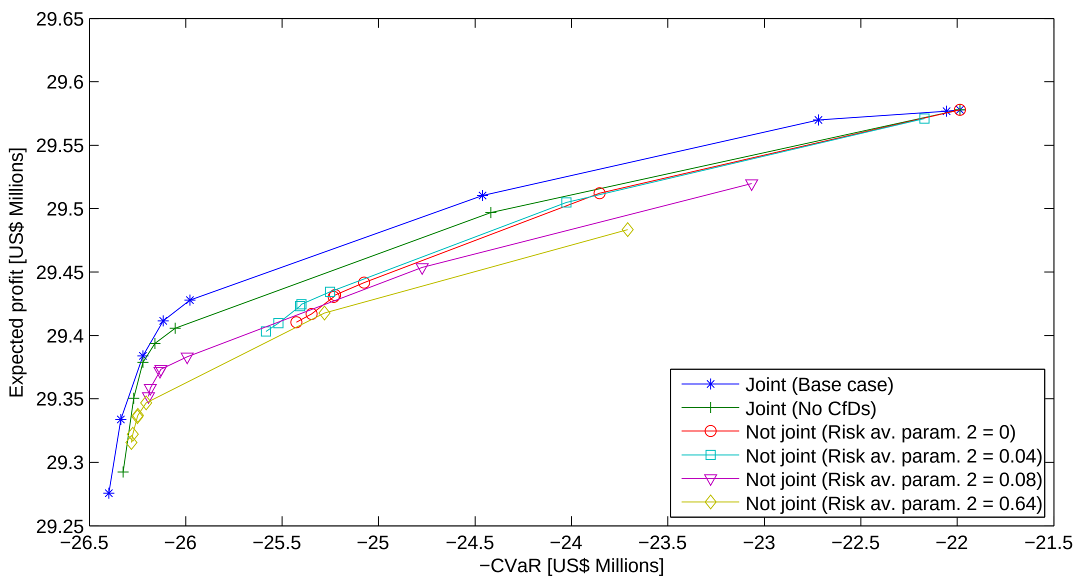


FIGURE 6.12. Negative CVaR vs expected profit curves for the single-location problem using various fixed risk aversion parameters at location 2 and varying the risk aversion parameter at location 1, together with multi-location solution curves

6.3.4 Allowing contracts to be taken at any time period

The model presented considers contractual decisions (forwards and CfDs) as "here-and-now" decisions because these are the recommendations of the model for the initial time period (time period "0"), whereas the generation and spot market decisions are subject to the unfolding of information and taken throughout the time horizon, as "wait-and-see" decisions. Yet, besides having contractual decisions as "here-and-now" decisions, one could also consider in the model contractual decisions taken over time, thus having variables such as f^S_{lstukb}: the electricity sold through a block b forward contract at location l and month t (decision taken at time period t), under scenario s, for delivery in hour group k of time period u (with $u > t$). Hence, in such a case contractual decisions would be taken at a certain time period, under a certain scenario, for delivery in a future time period. Notice that the number of variables and constraints of such an optimization problem largely increases (non-anticipativity constraints must also be considered for these decisions). The model extension allowing contracts to be taken at any time period is shown in appendix E.2.

Figures 6.13 and 6.14 compare the base case solution (contractual decisions only as "here-and-now" decisions) and the case allowing contracts to be taken at any time period (as both "here-and-now" and "wait-and-see' decisions). As it can be expected, the solution allowing contracts to be taken at any time period is better in terms of the efficient frontier between expected profit and CVaR, since for a fixed level of CVaR higher expected profits may be obtained in that case, but an unexpected result is obtained for the comparison of the relation between profit standard deviation and expected profit, which is not contradictory since minimizing CVaR is not equivalent to minimizing profit standard deviation.

6.3.5 Stability and average results

The results shown in subsections 6.3.1 through 6.3.4 have been obtained using one particular scenario tree generated. Yet, scenario trees are stochastic, as the solutions obtained when building a scenario tree is a realization of a method in which stochastically generated initial solutions are used. Therefore, there is an inherent instability in the scenario

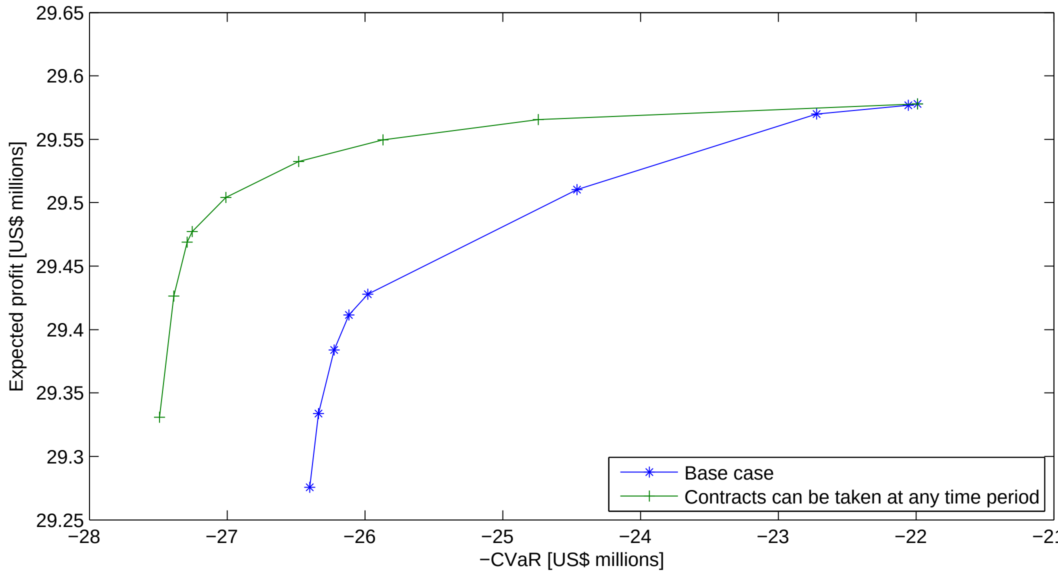


FIGURE 6.13. Negative CVaR vs expected profit for the base case and for the case allowing contracts to be taken at any time period

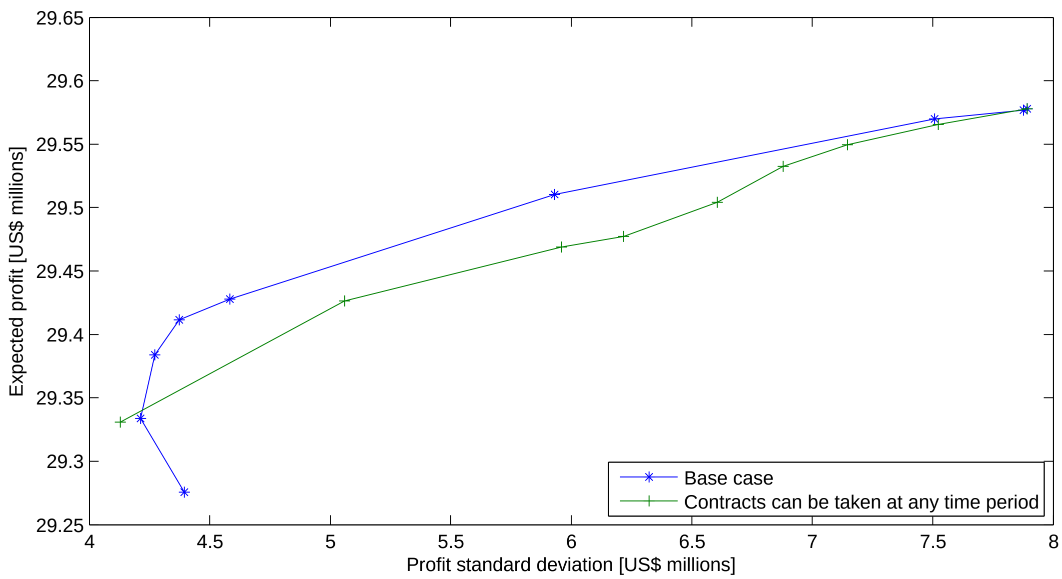


FIGURE 6.14. Profit standard deviation vs expected profit for the base case and for the case allowing contracts to be taken at any time period

trees generated. The efficient frontier between expected profit and negative CVaR may be seen in figure 6.15 and the relation between expected profit and profit standard deviation in figure 6.16, for 30 scenario tree instances, together with their average curves. Figure 6.17 shows the change in the aggregated profit distribution (for the 30 tree instances) as the risk aversion parameter changes. The justification for averaging results may be seen in appendix D.

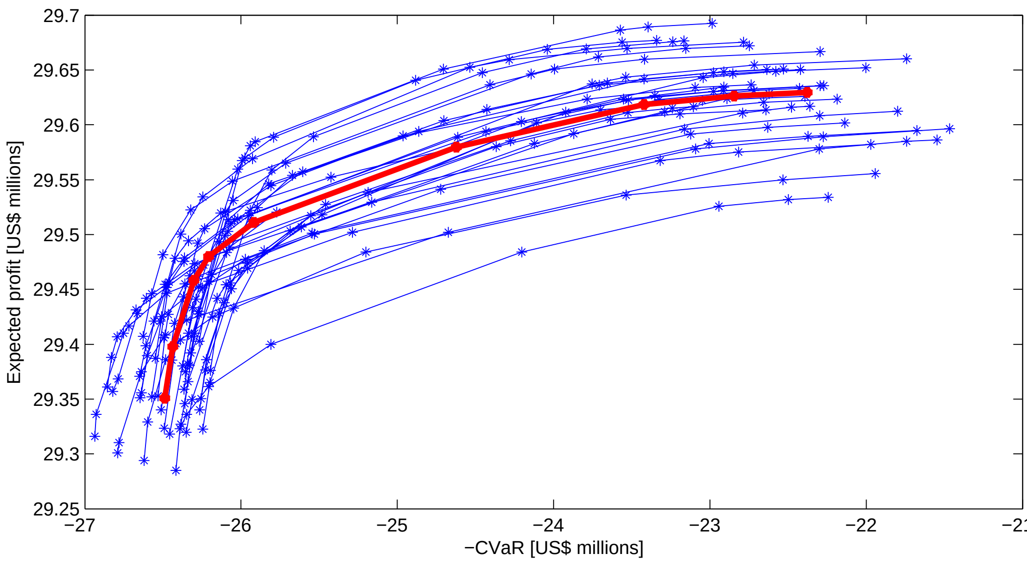


FIGURE 6.15. Negative CVaR vs expected profit (for 30 scenario tree instances and average curve)

6.3.6 Changing the correlation parameter

One of the main objectives of this thesis is to characterize the effects of the correlation parameter over the relationship between expected profit and risk. In this subsection, the sensibility of the relation between expected profit and risk with respect to changes in the correlation parameter is analyzed. (The results for other parameters being changed are shown in appendix C.)

Since the results obtained depend on the particular scenario tree generated, and hence may be unstable, as it has been seen in 6.3.5, characterizing the change in the results with

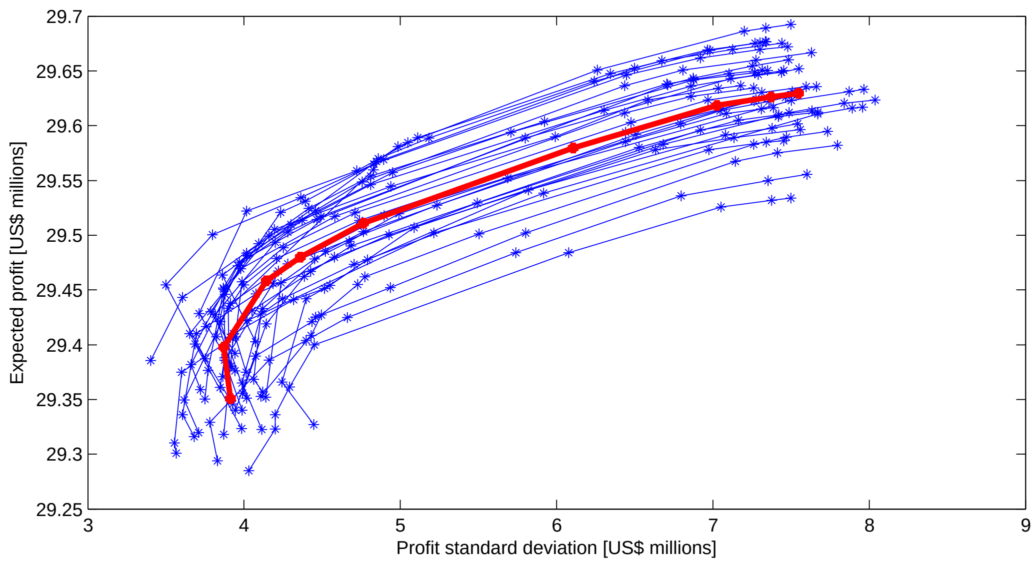
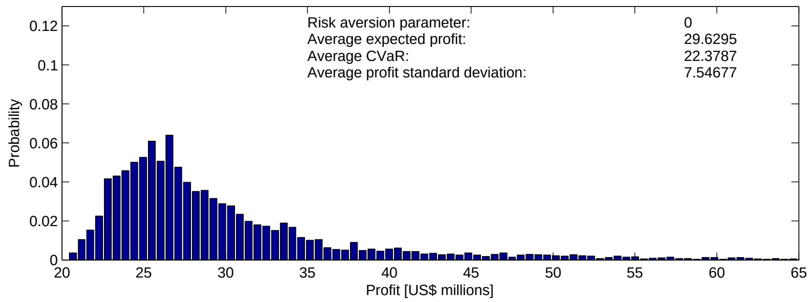


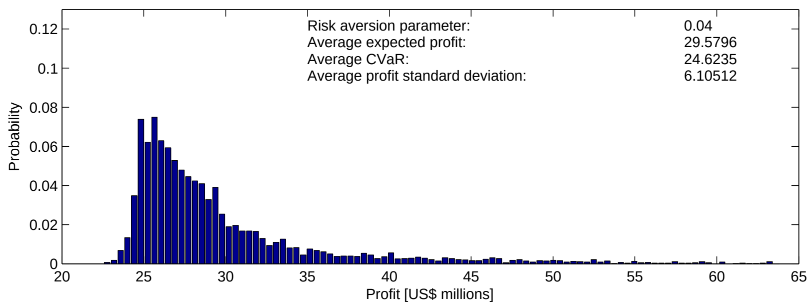
FIGURE 6.16. Profit standard deviation vs expected profit (for 30 scenario tree instances and average curve)

respect to changes in the correlation parameter ρ is not as simple as generating one scenario tree for each of several values of ρ and then obtaining for each of them the relation between expected profit and risk. When this is done, not much is seen. Instead, for each $\rho = 0, 0.2, 0.4, 0.6, 0.8, 0.9999$ a number of 30 scenario trees have been generated independently and for each of these values of ρ average curves have been obtained.

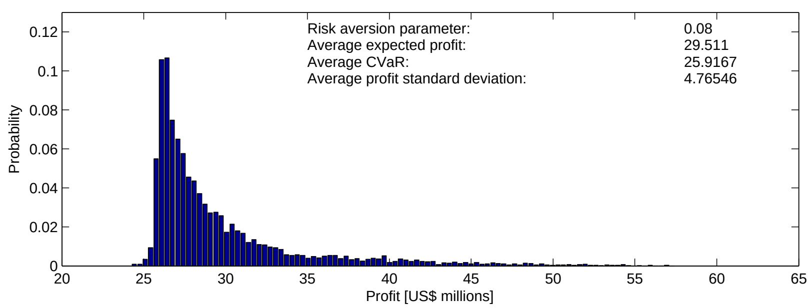
Appendix D justifies that in the actual original problem (considering the original price process instead of a scenario tree that approximates it), the expected profit in the risk neutral case does not depend on ρ. Based on this fact, the average curves obtained for each value of ρ are shifted so as to obtain their risk neutral points equal to their average over the 180 scenario trees generated. These adjusted average efficient frontier curves between expected profit and risk may be seen in figures 6.18 and 6.19 for the six correlation values mentioned. In this figures it may be seen how much more inconvenient it is to have high values of the correlation parameter, since for a fixed level of CVaR the expected profit obtained for high values of the correlation parameter is lower than the expected profit obtained for low values of the correlation parameter.



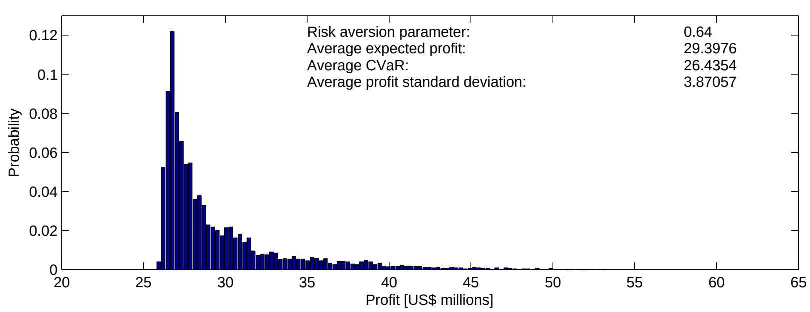
(a) Risk aversion parameter: 0.



(b) Risk aversion parameter: 0.04.



(c) Risk aversion parameter: 0.08.



(d) Risk aversion parameter: 0.64.

FIGURE 6.17. Aggregated profit histogram for different risk aversion parameters.

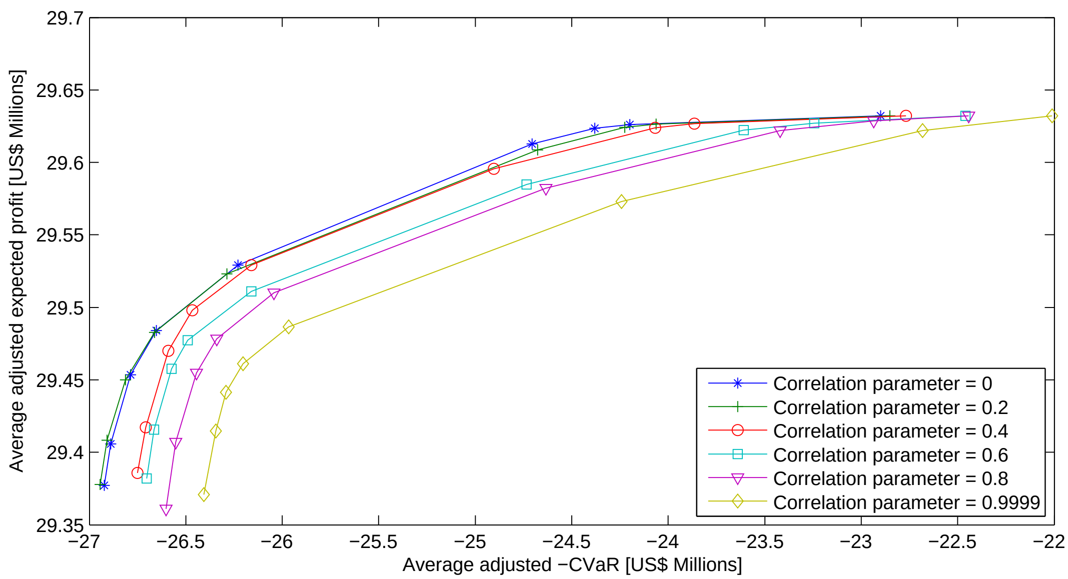


FIGURE 6.18. Adjusted average negative CVaR vs adjusted average expected profit for different values of the correlation parameter ρ

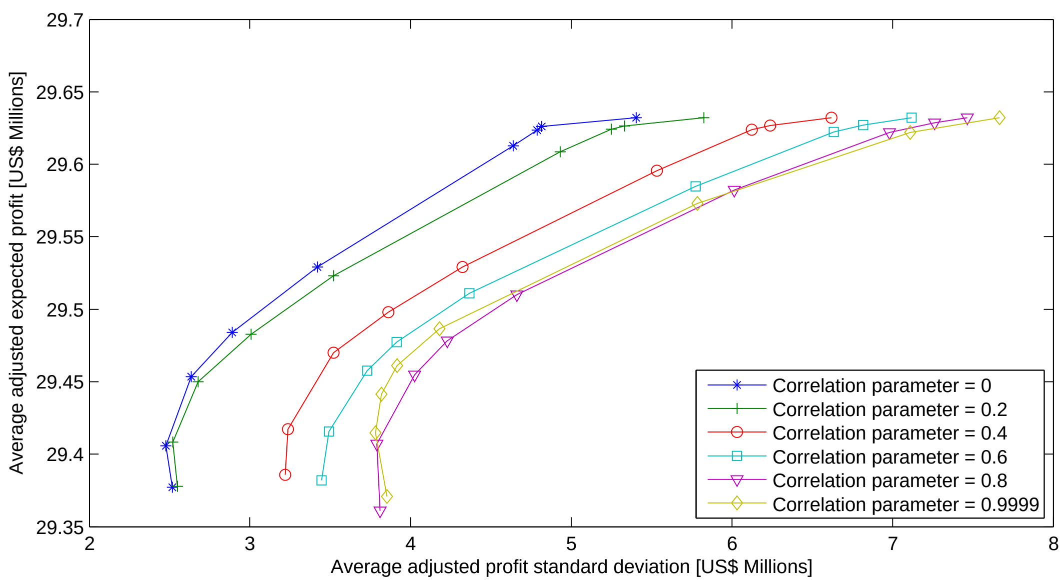


FIGURE 6.19. Adjusted average profit standard deviation vs adjusted average expected profit for different values of the correlation parameter ρ

VII. CONCLUSION

In this thesis, a medium term power portfolio optimization problem considering locational electricity prices and risk aversion for a power producer in a deregulated electricity market has been modeled and solved using stochastic programming techniques. The methodology consisted of three steps: 1) modeling the evolution of locational electricity prices, 2) generating a scenario tree representative of the evolution of locational electricity prices, and 3) developing, solving and testing a stochastic optimization problem. The main contribution is to consider a generating company that holds generators in more than one location of a deregulated electricity market, with each location having a particular locational electricity price process. The model is centered in the possibility of transacting electricity forward contracts and contracts for difference in the different locations, and also recommends electricity transactions in locational spot markets and power production in generating units. Using the methodology presented, a power producer holding thermal generating units in more than one location of a deregulated electricity market may maximize expected profit in the medium term while keeping a regulated risk exposure.

The current work has extended the knowledge of methodologies that support decision making for power producers considering contracts management and generation planning in an integrated manner. It is important to remark that ignoring uncertainty in these decision making problems, and others that present such remarkable risks, may lead to highly inconvenient solutions. The consideration of uncertainty within optimization models is of upmost importance in many problems and stochastic programming has proven to be an effective approach in this matter. Furthermore, one fundamental benefit of incorporating uncertainty within optimization models is the possibility of assessing risk, from which this thesis is only a particular example.

From the results obtained it can be concluded that the relation between locational electricity prices is relevant for power producers holding thermal generating units in more than one location of a deregulated electricity market. Changing the correlation parameter of the model presented for locational electricity prices significantly affected the efficient frontier between expected profit and risk. As the correlation parameter decreases, so does risk, for fixed values of expected profit, as is intuitively expected from the reduction of risk associated to the diversification of a portfolio known from financial theory. In the particular application presented, changing the correlation parameter from 0 to 1 increased the profit coefficient of variation (profit standard deviation divided by expected profit) in about 50% when keeping expected profit constant: for the risk neutral case, with an expected profit of 29.64 [million US$], changing the correlation parameter from 0 to 1 increased the profit coefficient of variation from 17.54% to 26.32%, and for a highly risk averse case with an expected profit of 29.38 [million US$], changing the correlation parameter from 0 to 1 increased the profit coefficient of variation from 8.51% to 13.27% (figure 6.19 depicts these relations).

The current work could be extended in several directions. One future work perspective is to increase the size of the problem: fundamentally to consider a longer time horizon. In order to do this the scenario tree construction method developed has to be modified to obtain a tractable nonlinear programming problem (it may be sufficient to change the bundle structure of the tree), and specific optimization algorithms may have to be implemented so as to solve the resulting equivalent deterministic optimization problem.

Another future work possibility is to incorporate details from technical generation aspects, in order to obtain a similar model in a shorter time horizon (for instance one month) and with shorter time periods (for instance days). In such a case it would also be convenient to use another locational electricity prices model and to appropriately modify the scenario tree construction method implemented. Doing this would be useful for better understanding the effects of the relation between locational electricity prices over the efficient frontier between expected profit and risk, and over the decisions regarding electricity generation and contracts planning.

Additionally, several extensions and variants of the power portfolio optimization model presented could be developed. One alternative is to incorporate fuel prices as stochastic variables correlated to locational electricity prices. In that case fuel derivatives and contracts could be included into the decisions addressed by the model (such as take-or-pay contracts on natural gas or swing options). For instance, it is straightforward to implement a case with one electricity price process (one location) and a related fuel price (such as natural gas), using the same stochastic process and the same scenario tree construction method presented in the application of this thesis. Other related extension is to incorporate more electricity derivatives (besides forwards and contracts for difference) such as options on electricity and spread options on electricity and fuel prices.

As a final comment, this work has presented a particular application of operations research in the electric industry. The high complexity of the decision making problems present in this industry makes the discipline well suited for supporting those decisions, and the high relevance of this industry for society makes the discipline not only important, but essential, for making better decisions within it.

REFERENCES

Arroyo, J. & Conejo, A. (2000), 'Optimal response of a thermal unit to an electricity spot market', *IEEE Transactions on Power Systems* **15**(3), 1098–1104.

Audet, N., Heiskanen, P., Keppo, J. & Vehviläinen, I. (2004), Modeling electricity forward curve dynamics in the nordic market, *in* D. Bunn, ed., 'Modelling Prices in Competitive Electricity Markets', Vol. 1, John Wiley & Sons, Chichester, pp. 251–265.

Birge, R. & Louveaux, F. (1997), *Introduction to Stochastic Programming*, Springer, New York.

Blaesig, B. & Haubrich, H. (2005), Methods of risk management in the generation and trading planning, *in* 'The IEEE Russia Power Tech Conference', St. Petersburg, Russia, pp. 1–7.

Borak, S. & Weron, R. (2008), 'A semiparametric factor model for electricity forward curve dynamics', *The Journal of Energy Markets* **1**(3), 3–16.

Börger, R., Cartea, A., Kiesel, R. & Schindlmayr, G. (2007), 'Cross-commodity analysis and applications to risk management', *Journal of Futures Markets* **29**(3), 197–217.

Brockwell, J. & Davis, R. (1996), *Introduction to Time Series and Forecasting*, Springer, New York.

Bunn, D. (2000), 'Forecasting loads and prices in competitive power markets', *Proceedings of the IEEE* **88**(2), 163–169.

Burger, M., Bernhard, K., Müller, A. & Schindlmayr, G. (2004), 'A spot market model for pricing derivatives in electricity markets', *Quantitative Finance* **4**(1), 109–122.

Burger, M., Graeber, B. & Schindlmayr, G. (2007), *Managing Energy Risk: An Integrated View on Power and Other Energy Markets*, John Wiley & Sons, Chichester.

Byström, H. (2003), 'The hedging performance of electricity futures on the nordic power exchange', *Applied Economics* **35**(1), 1–11.

Cabero, J., Baíllo, A., Cerisola, S., Ventosa, M., García-Alcalde, A., Perán, F. & no, G. R. (2005), 'A medium-term integrated risk management model for a hydrothermal generation company', *IEEE Transactions on Power Systems* **20**(3), 1379–1388.

Cabero, J., Ventosa, M., Cerisola, S. & Baíllo, A. (2010), 'Modeling risk management in oligopolistic electricity markets', *IEEE Transactions on Power Systems* **25**(1), 263–271.

Carmona, R. & Durrleman, V. (2003), 'Pricing and hedging spread options', *Society for Industrial and Applied Mathematics* **45**(4), 627–685.

Carpentier, P., Cohen, G., Culioli, J. & Renaud, A. (1996), 'Stochastic optimization of unit commitment: a new decomposition framework', *IEEE Transactions on Power Systems* **11**(2), 1067–1073.

Clewlow, L. & Strickland, C. (1999), A multi-factor model for energy derivatives. Available online: http://www.erasmusenergy.com/articles/103/1/A-Multi-Factor-Model-for-Energy-Derivatives/Page1.html.

Conejo, A., García-Bertrand, R., Carrión, M., Caballero, A. & de Andrés, A. (2008), 'Optimal involvement in futures markets of a power producer', *IEEE Transactions on Power Systems* **23**(2), 703–711.

Conejo, A., Nogales, F., Arroyo, J. & García-Bertrand, R. (2004), 'Risk-constrained self-scheduling of a thermal power producer', *IEEE Transactions on Power Systems* **19**(3), 1569–1574.

Contreras, J., Espínola, R., Nogales, F. & Conejo, A. (2003), 'Arima models to predict next-day electricity prices', *IEEE Transactions on Power Systems* **18**(3), 1014–1020.

Dempster, M. & Thompson, R. (1999), 'Evpi-based importance sampling solution procedures for multistage stochastic linear programmes on parallel mimd architectures', *Annals of Operations Research* **90**(1), 161–184.

Deng, S. (2000), Pricing electricity derivatives under alternative stochastic spot price models, *in* 'The 33rd Annual Hawaii International Conference on System Sciences', Hawaii, pp. 1–10.

Denton, M., Palmer, A., Masiello, R. & Skantze, P. (2003), 'Managing market risk in energy', *IEEE Transactions on Power Systems* **18**(2), 494–502.

Dupacová, J., Consigli, G. & Wallace, S. (2000), 'Scenarios for multistage stochastic programs', *Annals of Operations Research* **100**(1-4), 25–53.

Eichhorn, A., Heitsch, H. & Römisch, W. (2009), Scenario tree approximation and risk aversion strategies for stochastic optimization of electricity production and trading, *in* J. Kallrath & P. Pardalos, eds, 'Optimization in the Energy Industry', Vol. 1, Springer, Berlin, pp. 321–346.

Eydeland, A. & Wolyniec, K. (2003), *Energy and Power Risk Management: New Developments in Modeling, Pricing and Hedging*, John Wiley & Sons, New Jersey.

Fleten, S. & Kristoffersen, T. (2007), 'Stochastic programming for optimizing bidding strategies of a nordic hydropower producer', *European Journal of Operations Research* **181**(2), 916–928.

Fleten, S. & Wallace, S. (1998), Power scheduling with forward contracts, *in* 'Proceedings of the Nordic MPS', Molde, pp. 1–26.

Fleten, S., Wallace, S. & Ziemba, W. (1997), Portfolio management in a deregulated hydropower based electricity market. Available online: http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.51.8760.

Fleten, S., Wallace, S. & Ziemba, W. (2002), Hedging electricity portfolios via stochastic programming, *in* C. Greengard & A. Ruszczyński, eds, 'Decision Making Under Uncertainty: Energy and Power', Vol. 1, Springer-Verlag, New York, pp. 71–93.

Garcés, L. & Conejo, A. (2010), 'Weekly self-scheduling, forward contracting, and offering strategy for a producer', *IEEE Transactions on Power Systems* **25**(2), 657–666.

Geman, H. (2005), *Commodities and Commodity Derivatives: Modelling and Pricing for Agriculturals, Metals and Energy*, John Wiley & Sons, Chichester.

Gröwe-Kuska, N., Heitsch, H. & Römisch, W. (2003), Scenario reduction and scenario tree construction for power management problems, *in* 'The IEEE Bologna Power Tech Conference', Bologna, Italy, pp. 1–7.

Haldrup, N. & Nielsen, M. (2006), 'A regime switching long memory model for electricity prices', *Journal of Econometrics* **135**(1-2), 349–376.

Hamilton, J. (1994), *Time Series Analysis*, Princeton University Press, New Jersey.

Higgs, H. & Worthington, A. (2008), 'Stochastic price modeling of high volatility, mean-reverting, spike-prone commodities: The australian wholesale spot electricity market', *Energy Economics* **30**(6), 3172–3185.

Higle, J. & Sen, S. (1999), 'Stochastic decomposition: An algorithm for two-stage linear programs with recourse', *Mathematics of Operations Research* **16**(3), 650–669.

Hong, Y. & Hsiao, C. (2001), 'Locational marginal price forecasting in deregulated electric markets using a recurrent neural network', *IEEE Power Engineering Society Winter Meeting* **2**, 539–544.

Hoyland, K., Kaut, M. & Wallace, S. (2001), 'A heuristic for moment-matching scenario generation', *Computational Optimization and Applications* **24**(2-3), 169–185.

Hoyland, K. & Wallace, S. (2001), 'Generating scenario trees for multistage decision problems', *Management Science* **47**(2), 295–307.

Huisman, R. & Mahieu, R. (2003), 'Regime jumps in electricity prices', *Energy Economics* **25**(5), 425–434.

Kanamura, T. & Ohashi, K. (2007), 'A structural model for electricity prices with spikes: Measurement of spike risk and optimal policies for hydropower plant operation', *Energy Economics* **29**(5), 1010–1032.

Karakatsani, N. & Bunn, D. (2008), 'Intra-day and regime-switching dynamics in electricity price formation', *Energy Economics* **30**(4), 1776–1797.

Kaut, M. & Wallace, S. (2003), Evaluation of scenario-generation methods for stochastic programming. Available online: http://edoc.hu-berlin.de/series/speps/2003-14/PDF/14.pdf.

Kaye, R., Outhred, H. & Bannister, C. (1990), 'Forward contracts for the operation of an electricity industry', *IEEE Transactions on Power Systems* **5**(1), 46–52.

Kristiansen, T. (2004), 'Financial risk management in the electric power industry using stochastic optimization', *Advanced Modeling and Optimization* **6**(2), 17–24.

Liu, M. & Wu, F. (2006), 'Managing price risk in a multimarket environment', *IEEE Transactions on Power Systems* **21**(4), 1512–1519.

Liu, M. & Wu, F. (2007), 'Portfolio optimization in electricity markets', *Electric Power Systems Research* **77**(8), 1000–1009.

Lucia, J. & Schwartz, E. (2000), 'Electricity prices and power derivatives: Evidence from the nordic power exchange', *Review of Derivatives Research* **5**(1), 5–50.

Marckhoff, J. & Wimschulte, J. (2009), 'Locational price spreads and the pricing of contracts for difference: Evidence from the nordic market', *Energy Economics* **31**(2), 257–268.

Meyer-Brandis, T. & Tankov, P. (2008), 'Multi-factor jump-diffusion models of electricity prices', *International Journal of Theoretical and Applied Finance* **11**(5), 503–528.

Mo, B., Gjelsvik, A. & Grundt, A. (2001), 'Integrated risk management of hydro power scheduling and contract management', *IEEE Transactions on Power Systems* **16**(2), 216–221.

Ni, E. & Luh, P. (2000), Optimal integrated bidding and hydrothermal scheduling with risk management and self-scheduling requirements, *in* 'The 3rd World Congress on Intelligent Control and Automation', P. R. China, pp. 2023–2028.

Niu, H., Baldick, R. & Zhu, G. (2005), 'Supply function equilibrium bidding strategies with fixed forward contracts', *IEEE Transactions on Power Systems* **20**(4), 1859–1867.

Nogales, F., Contreras, J., Conejo, A. & Espínola, R. (2002), 'Forecasting next-day electricity prices by time series models', *IEEE Transactions on Power Systems* **17**(2), 342–347.

Oksendal, B. (2005), *Stochastic Differential Equations: An Introduction with Applications*, Springer, Berlin.

Paravan, D., Sheblé, G. & Golob, R. (2004), Price and volume risk management for power producers, *in* 'The 8th Internacional Conference on Probabilistic Methods Applied to Power Systems', Iowa, pp. 699–704.

Pflug, G. (2000), 'Scenario tree generation for multiperiod financial optimization by optimal discretization', *Mathematical Programming* **89**(2), 251–271.

Pineda, S., Conejo, A. & Carrión, M. (2008), 'Impact of unit failure on forward contracting', *IEEE Transactions on Power Systems* **23**(4), 1768–1775.

Ruszczyński, A. & Shapiro, A. (2003), *Stochastic Programming (Handbooks in Operations Research and Management Science, vol. 10)*, Elsevier, Amsterdam.

Schindlmayr, G. (2005), A regime-switching model for electricity spot prices. Available online: http://symposium.fbv.uni-karlsruhe.de.

Sen, S., Yu, L. & Genc, T. (2006), 'A stochastic programming approach to power portfolio optimization', *Operations Research* **54**(1), 55–72.

Shahidehpour, M., Yamin, H. & Li, Z. (2002), *Market Operations in Electric Power Systems: Forecasting, Scheduling, and Risk Management*, John Wiley & Sons, New York.

Shiina, T. & Birge, J. (2004), 'Stochastic unit commitment problem', *International Transactions in Operations Research* **11**, 19–32.

Shrestha, G., Kai, S. & Goel, L. (2004), 'An efficient stochastic self-scheduling technique for power producers in the deregulated power market', *Electric Power Systems Research* **71**(1), 91–98.

Shrestha, G., Pokharel, K., Lie, T. T. & Fleten, S. (2005), 'Medium term power planning with bilateral contracts', *IEEE Transactions on Power Systems* **20**(2), 627–633.

Takriti, S., Birge, J. & Long, E. (1996), 'A stochastic model for the unit commitment problem', *IEEE Transactions on Power Systems* **11**(3), 1497–1508.

Takriti, S., Krasenbrink, B. & Wu, L. (2000), 'Incorporating fuel constraints and electricity spot prices into the stochastic unit commitment problem', *Operations Research* **48**(2), 268–280.

Tanlapco, E., Lawarrée, J. & Liu, C. (2002), 'Forward contracts for the operation of an electricity industry', *IEEE Transactions on Power Systems* **17**(3), 577–582.

Vehviläinen, I. & Keppo, J. (2005), 'Managing electricity market price risk', *European Journal of Operations Research* **145**(1), 136–147.

Vehviläinen, I. & Pyykkönen, T. (2005), 'Stochastic factor model for electricity spot price - the case of the nordic market', *Energy Economics* **27**(2), 351–367.

Weron, R. & Misiorek, A. (2008), 'Forecasting spot electricity prices: A comparison of parametric and semiparametric time series models', *International Journal of Forecasting* **24**(4), 744–763.

Xu, J., Luh, P., White, F., Ni, E. & Kasiviswanathan, K. (2006), 'Power portfolio optimization in deregulated electricity markets with risk management', *IEEE Transactions* *on Power Systems* **21**(4), 1653–1662.

Yu, W. & Sheblé, G. (2006), 'Modeling electricity markets with hidden markov model', *Electric Power Systems Research* **76**(6-7), 445–451.

Yu, Z. (2003), 'A spatial mean-variance mip model for energy market risk analysis', *Energy Economics* **25**(3), 255–268.

Zhou, Q., Tesfatsion, L. & Liu, C. (2009), Scenario generation for price forecasting in restructured wholesale power markets, *in* 'Power Systems Conference and Exposition', Seattle, pp. 1–8.

A. MULTIVARIATE MEAN REVERTING DIFFUSION

This appendix presents the technical details of the electricity price model used in the application of this thesis.

Consider the model:

$$\log P_i(t) = f_i(t) + X_i(t), \forall i = 1, ..., N;$$
$$dX_i(t) = -\frac{X_i(t)}{\lambda_i}dt + \sigma_i dW_i(t), \forall i = 1, ..., N;$$
$$E[dW_i(t)dW_j(t)] = \rho_{ij}dt, \forall i, j = 1, ..., N,$$

where $P_i(t)$ represents (electricity) price (or any other component) at time t, $f_i(t)$ is a deterministic component, and the $X(t)$ process is a multivariate mean reverting diffusion (or multivariate Ornstein-Uhlenbeck process), with guiding correlated Brownian motions $W_i(t)$, $i = 1, ..., N$. This model has been presented with $N = 1$ (section 3.2.1) and $N = 2$ (sections 3.3.1 and 6.2.1).

A.1 Statistical properties

The solution of the stochastic differential equation for $X_i(t)$ is given by (see Oksendal (2005), Meyer-Brandis & Tankov (2008)):

$$X_i(t) = X_i(s)\exp\left(-\frac{t-s}{\lambda_i}\right) + \sigma_i \exp\left(-\frac{t}{\lambda_i}\right)\int_s^t \exp\left(\frac{r}{\lambda_i}\right)dW_i(r), s < t,$$

where:

$$\int_s^t \exp\left(\frac{r}{\lambda_i}\right)dW_i(r) \sim \mathcal{N}\{0; \sqrt{\int_s^t [\exp\left(\frac{r}{\lambda_i}\right)]^2 dr}\},$$

with $\sqrt{\int_s^t [\exp(\frac{r}{\lambda_i})]^2 dr} = \sqrt{\frac{\lambda_i[\exp(\frac{2t}{\lambda_i}) - \exp(\frac{2s}{\lambda_i})]}{2}}$, and:

$$corr\left[\int_s^t \exp\left(\frac{r}{\lambda_i}\right)dW_i(r); \int_s^t \exp\left(\frac{r}{\lambda_j}\right)dW_j(r)\right] = \frac{E[\int_s^t \exp(\frac{r}{\lambda_i})dW_i(r)\int_s^t \exp(\frac{r}{\lambda_j})dW_j(r)]}{\sqrt{\int_s^t[\exp(\frac{r}{\lambda_i})]^2 dr}\sqrt{\int_s^t[\exp(\frac{r}{\lambda_j})]^2 dr}}$$

$$= \rho_{ij} \frac{\int_s^t \exp(\frac{r}{\lambda_i}) \exp(\frac{r}{\lambda_j}) dr}{\sqrt{\int_s^t [\exp(\frac{r}{\lambda_i})]^2 dr} \sqrt{\int_s^t [\exp(\frac{r}{\lambda_j})]^2 dr}}$$

$$= \rho_{ij} \frac{\frac{\lambda_i \lambda_j}{\lambda_i + \lambda_j} [1 - \exp(-\frac{(t-s)(\lambda_i + \lambda_j)}{\lambda_i \lambda_j})]}{\sqrt{\frac{\lambda_i}{2} [1 - \exp(-\frac{2(t-s)}{\lambda_i})]} \sqrt{\frac{\lambda_j}{2} [1 - \exp(-\frac{2(t-s)}{\lambda_j})]}}.$$

Therefore, for $s < t$:

$$X_i(t)|X_i(s) \sim \mathcal{N}\{X_i(s) \exp -(\frac{t-s}{\lambda_i}); \sigma_i \sqrt{\frac{\lambda_i}{2} [1 - \exp(-\frac{2(t-s)}{\lambda_i})]}\},$$

with:

$$corr[X_i(t)|X_i(s); X_j(t)|X_j(s)] = \rho_{ij} \frac{\frac{\lambda_i \lambda_j}{\lambda_i + \lambda_j} [1 - \exp(-\frac{(t-s)(\lambda_i + \lambda_j)}{\lambda_i \lambda_j})]}{\sqrt{\frac{\lambda_i}{2} [1 - \exp(-\frac{2(t-s)}{\lambda_i})]} \sqrt{\frac{\lambda_j}{2} [1 - \exp(-\frac{2(t-s)}{\lambda_j})]}}.$$

and:

$$Cov[X_i(s); X_i(t)] = Cov[X_i(s); X_i(s) \exp(-\frac{t-s}{\lambda_i}) + \sigma_i \exp(-\frac{t}{\lambda_i}) \int_s^t \exp(\frac{r}{\lambda_i}) dW_i(r)]$$

$$= \exp(-\frac{t-s}{\lambda_i}) Var[X_i(s)] + \sigma_i \exp(-\frac{t}{\lambda_i}) Cov[X_i(s); \int_s^t \exp(\frac{r}{\lambda_i}) dW_i(r)]$$

$$= \exp(-\frac{t-s}{\lambda_i}) Var[X_i(s)].$$

Since $\log P_i(t) = f_i(t) + X_i(t)$:

$$P_i(t)|P_i(s) \sim log\mathcal{N}\{f_i(t)[\log P_i(s) - f_i(s)] \exp -(\frac{t-s}{\lambda_i}); \sigma_i \sqrt{\frac{\lambda_i}{2} [1 - \exp(-\frac{2(t-s)}{\lambda_i})]}\},$$

with:

$$corr[\log P_i(t)|P(s); \log P_j(t)|P(s)] = \rho_{ij} \frac{\frac{\lambda_i \lambda_j}{\lambda_i + \lambda_j} [1 - \exp(-\frac{(t-s)(\lambda_i + \lambda_j)}{\lambda_i \lambda_j})]}{\sqrt{\frac{\lambda_i}{2} [1 - \exp(-\frac{2(t-s)}{\lambda_i})]} \sqrt{\frac{\lambda_j}{2} [1 - \exp(-\frac{2(t-s)}{\lambda_j})]}}.$$

And because of the log-normal distribution properties, in particular, for $t > 0$:

$$E[P_i(t)] = \exp[\mu_i(t) + \frac{\sigma_i^2(t)}{2}],$$

$$\sqrt{Var[P_i(t)]} = \exp\left[\mu_i(t) + \frac{\sigma_i^2(t)}{2}\right]\sqrt{\exp\sigma_i^2(t) - 1},$$

$$Skewness[P_i(t)] = \frac{E[(P_i(t) - E[P_i(t)])^3]}{\sqrt{Var[P_i(t)]}^3} = [\exp\sigma_i^2(t) + 2]\sqrt{\exp\sigma_i^2(t) - 1},$$

$$Kurtosis[P_i(t)] = \frac{E[(P_i(t) - E[P_i(t)])^4]}{\sqrt{Var[P_i(t)]}^4} = \exp 4\sigma_i^2(t) + 2\exp 3\sigma_i^2(t) + 3\exp 2\sigma_i^2(t) - 3,$$

$$Cov[P_i(t); P_j(t)] = \exp\left[\mu_i(t) + \mu_j(t) + \frac{\sigma_i^2(t) + \sigma_j^2(t)}{2}\right][\exp(\rho_{ij}(t)\sigma_i(t)\sigma_j(t)) - 1].$$

Assuming $P_i(0)$ known and with:

$$\mu_i(t) = f_i(t) + [\log P_i(0) - f_i(0)]\exp(-\frac{t}{\lambda_i}),$$

$$\sigma_i(t) = \sigma_i\sqrt{\frac{\lambda_i}{2}[1 - \exp(-\frac{2t}{\lambda_i})]},$$

$$\rho_{ij}(t) = \rho_{ij}\frac{\frac{\lambda_i\lambda_j}{\lambda_i+\lambda_j}[1 - \exp(-\frac{t(\lambda_i+\lambda_j)}{\lambda_i\lambda_j})]}{\sqrt{\frac{\lambda_i}{2}[1 - \exp(-\frac{2t}{\lambda_i})]}\sqrt{\frac{\lambda_j}{2}[1 - \exp(-\frac{2t}{\lambda_j})]}}.$$

A.2 Simulating trajectories

From the solution for X shown above, if time is discretized into $\{t_k\}_{k=0}^{n}$ and $\triangle t_k = t_k - t_{k-1}$, then, with $X_i(t_0) = \log P_i(t_0) - f_i(t_0)$ known:

$$X_i(t_k) = X_i(t_{k-1})\exp(-\frac{\triangle t_k}{\lambda_i}) + \sigma_i\sqrt{\frac{\lambda_i}{2}[1 - \exp(-\frac{2\triangle t_k}{\lambda_i})]}\epsilon_k^i, i = 1, ..., N; k = 1, ..., n.$$

Where $\epsilon_k^i \sim \mathcal{N}\{0; 1\}$ with ϵ_k^i independent from ϵ_l^j for $k \neq l$ and:

$$corr[\epsilon_k^i; \epsilon_k^j] = \rho_{ij}\frac{\frac{\lambda_i\lambda_j}{\lambda_i+\lambda_j}[1 - \exp(-\frac{\triangle t_k(\lambda_i+\lambda_j)}{\lambda_i\lambda_j})]}{\sqrt{\frac{\lambda_i}{2}[1 - \exp(-\frac{2\triangle t_k}{\lambda_i})]}\sqrt{\frac{\lambda_j}{2}[1 - \exp(-\frac{2\triangle t_k}{\lambda_j})]}}.$$

With simulated trajectories for X, simulated trajectories for P can be obtained as: $P_i(t_k) = \exp[f_i(t_k) + X_i(t_k)]$.

A.3 Parameter estimation for the deterministic component

A particular form must be chosen for the $f_i(t)$ component. For instance, $f_i(t) = a_i + b_i t + c_i \cos\left(\frac{2\pi t}{365}\right) + d_i \sin\left(\frac{2\pi t}{365}\right)$ in order to contain a drift and an annual seasonality (if time is measured in days). The parameters can then be obtained by means of a linear regression using the data for $\log P_i(t)$.

A.4 Parameter estimation for the Ornstein-Uhlenbeck component

Once $f_i(t)$ has been estimated, historical data for $X_i(t)$ may be obtained from the historical data of $P_i(t)$ as $X_i(t) = \log P_i(t) - f_i(t)$. Assume the instants of the historical data are given by $\{t_k\}_{k=1}^n$, with $\triangle t_k = t_k - t_{k-1} = \triangle t$ (constant) (historical data equally spaced). Now notice that the model for X in a discrete-time approximation is:

$$X_i(t_k) = \phi_i X_i(t_{k-1}) + \tilde{\sigma}_i \epsilon_k^i,$$

with

$$\phi_i = \exp\left(-\frac{\triangle t}{\lambda_i}\right),$$

$$\tilde{\sigma}_i = \sigma_i \sqrt{\frac{\lambda_i}{2}\left[1 - \exp\left(-\frac{2\triangle t}{\lambda_i}\right)\right]},$$

and $\epsilon_k^i \sim \mathcal{N}\{0; 1\}$, with ϵ_k^i independent from ϵ_l^j for $k \neq l$, and

$$corr[\epsilon_k^i; \epsilon_k^j] = \rho_{ij} \frac{\frac{\lambda_i \lambda_j}{\lambda_i + \lambda_j}\left[1 - \exp\left(-\frac{\triangle t_k(\lambda_i + \lambda_j)}{\lambda_i \lambda_j}\right)\right]}{\sqrt{\frac{\lambda_i}{2}\left[1 - \exp\left(-\frac{2\triangle t_k}{\lambda_i}\right)\right]}\sqrt{\frac{\lambda_j}{2}\left[1 - \exp\left(-\frac{2\triangle t_k}{\lambda_j}\right)\right]}}.$$

This corresponds to a multivariate AR(1) model (multivariate autoregressive model of order 1) (see Brockwell & Davis (1996)), from which ϕ_i can be estimated as:

$$\phi_i = corr[X_i(t_k); X_i(t_{k-1})] = \frac{\sum_{k=2}^n [X_i(t_{k-1}) - \mu_i][X_i(t_k) - \mu_i]}{w_i^2},$$

with:

$$\mu_i = \frac{\sum_{k=1}^n X_i(t_k)}{n}, w_i = \sqrt{\frac{\sum_{k=1}^n [X_i(t_k) - \mu_i]^2}{n-1}}.$$

And then:

$$\lambda_i = -\frac{\triangle t}{\log \phi_i}.$$

Now, as the model may be written as: $X_i(t_k) - \phi_i X_i(t_{k-1}) = \tilde{\sigma}_i \epsilon_k^i$, then $\tilde{\sigma}_i$ and ρ_{ij} can be estimated as:

$$\tilde{\sigma}_i = \sqrt{\frac{\sum_{k=2}^{n}[Z_i(t_k) - \delta_i]^2}{n-1}},$$

$$\rho_{ij} = \frac{\sqrt{\frac{\lambda_i}{2}[1 - \exp{(-\frac{2\triangle t}{\lambda_i})}]}\sqrt{\frac{\lambda_j}{2}[1 - \exp{(-\frac{2\triangle t}{\lambda_j})}]}}{\frac{\lambda_i\lambda_j}{\lambda_i+\lambda_j}[1 - \exp{(-\frac{\triangle t(\lambda_i+\lambda_j)}{\lambda_i\lambda_j})}]} corr(Z_i(t_k); Z_j(t_k)),$$

with:

$$Z_i(t_k) = X_i(t_k) - \phi_i X_i(t_{k-1}),$$

$$\delta_i = \frac{\sum_{k=2}^{n} Z_i(t_k)}{n-1},$$

$$corr(Z_i(t_k); Z_j(t_k)) = \frac{\sum_{k=2}^{n}[Z_i(t_k) - \delta_i][Z_j(t_k) - \delta_j]}{\sqrt{\frac{\sum_{k=2}^{n}[Z_i(t_k)-\delta_i]^2}{n-2}}\sqrt{\frac{\sum_{k=2}^{n}[Z_j(t_k)-\delta_j]^2}{n-2}}}.$$

Finally:

$$\sigma_i = \frac{\tilde{\sigma}_i}{\sqrt{\frac{\lambda_i}{2}[1 - \exp{(-\frac{2\triangle t}{\lambda_i})}]}}.$$

Observation: if the historical data are not equally spaced (i.e. $\triangle t_k$ is not constant) then the parameters of the Ornstein-Uhlenbeck component may be estimated by maximum likelihood through numerical optimization.

B. CONDITIONAL VALUE AT RISK

This appendix is intended to explain conditional value at risk (CVaR) as a risk measure and to explain how to model it within an optimization problem.

A very common risk measure is value at risk (VaR). If π is the profit distribution, then VaR at a confidence level α, VaR_α, is defined in this thesis as:

$$VaR_\alpha = \max\{x : P[\pi \leq x] \leq 1 - \alpha\}$$

(i.e., VaR_α is the α quantile of the profit distribution).

Because of VaR's inability to distinguish between a possible loss that is slightly or far greater than VaR, and because it violates the financial idea of diversification (it lacks sub-additivity), CVaR is a risk metric widely used instead that over passes these drawbacks. CVaR at a confidence level α, $CVaR_\alpha$, is defined in this thesis as:

$$CVaR_\alpha = E[\pi | \pi \leq VaR_\alpha].$$

Which means that $CVaR_\alpha$ is the expected value of profit given that profit is below VaR_α.

Sections B.1 and B.2 present two different formulations of CVaR that may be used within optimization models. The formulation used in the power portfolio optimization model developed in chapter 5 is the one presented in section B.1.

B.1 Formulation 1

Assume there is a finite number of scenarios, $1, ..., S$, each with a fixed associated probability p_s and a fixed profit π_s. An approximation of CVaR can be computed as the solution to the following optimization problem (Conejo et al. (2008)):

$$\begin{array}{lll} Max_{\{\xi,\eta_s\}} & \xi - \frac{1}{1-\alpha}\sum_{s=1}^{S} p_s \eta_s & (1) \\ s.t. & \eta_s \geq \xi - \pi_s; \forall s = 1, ..., S. & (2) \\ & \eta_s \geq 0; \forall s = 1, ..., S. & (3) \end{array}$$

This is explained in what follows. Consider $\pi_{(s)}$ such that the $\pi_{(s)}$ variables are an increasing ordering of the π_s variables, i.e., $\pi_{(1)} = \min\{\pi_s\}_{s=1}^S, ..., \pi_{(S)} = \max\{\pi_s\}_{s=1}^S$. And, with $p_{(s)}$ and $\eta_{(s)}$ the respective ones according to $\pi_{(s)}$, i.e., if $\pi_{(s)} = \pi_u$ then $p_{(s)} = p_u$ and $\eta_{(s)} = \eta_u$. Now note that for every fixed ξ then η_s necessarily takes the value $\max(0, \xi - \eta_s)$, because there is a maximization over a negative pondering of the η_s variables, and therefore it can be assumed that $\eta_s = \max(0, \xi - \pi_s)$, for every s, in order to maximize hereafter only over ξ.

Now, if $\xi < \pi_{(1)}$, then $\xi - \frac{1}{1-\alpha}\sum_{s=1}^S p_s\eta_s = \xi - \frac{1}{1-\alpha}\sum_{s=1}^S p_s \times 0 = \xi$, that is increasing in ξ, hence in the optimal solution $\xi \geq \pi_{(1)}$; and if $\xi > \pi_{(S)}$, then $\xi - \frac{1}{1-\alpha}\sum_{s=1}^S p_s\eta_s = \xi - \frac{1}{1-\alpha}\xi + \frac{1}{1-\alpha}\sum_{s=1}^S p_s\pi_s$, which is decreasing in ξ because $\frac{1}{1-\alpha} > 1$, and hence in the optimal solution $\xi \leq \pi_{(S)}$.

Take n such that $p_{(1)} + ... + p_{(n)} \leq 1 - \alpha < p_{(1)} + ... + p_{(n+1)}$. Suppose that $\xi = \pi_{(j)}$ for certain j. Then:

$$\Omega_j := \xi - \frac{1}{1-\alpha}\sum_{s=1}^S p_s\eta_s = \pi_{(j)} - \frac{1}{1-\alpha}\sum_{s=1}^j p_{(s)}(\pi_{(j)} - \pi_{(s)})$$

$$= \pi_{(j)}(1 - \frac{1}{1-\alpha}\sum_{s=1}^j p_{(s)}) + \frac{1}{1-\alpha}\sum_{s=1}^j p_{(s)}\pi_{(s)},$$

which implies that:

$$\Omega_{j+1} - \Omega j = (\pi_{(j+1)} - \pi_{(j)})(1 - \frac{1}{1-\alpha}\sum_{s=1}^j p_{(s)}).$$

Notice that in this last expression $\pi_{(j+1)} - \pi_{(j)} \geq 0$ for every j, $1 - \frac{1}{1-\alpha}\sum_{s=1}^j p_{(s)} \geq 0$ when $j < n$ and $1 - \frac{1}{1-\alpha}\sum_{s=1}^j p_{(s)} \leq 0$ when $j \leq n+1$. This means that Ωj is increasing in j for $j < n$ and decreasing in j for $j > n$. Therefore, $\xi = \pi_{(n)}$, which is exactly VaR_α, is an optimal solution.

All of these steps mean that the optimal value is: $\Omega_n = \pi_{(n)}(1 - \frac{1}{1-\alpha}\sum_{s=1}^n p_{(s)}) + \frac{1}{1-\alpha}\sum_{s=1}^n p_{(s)}\pi_{(s)}$. This is an approximation of CVaR because $\sum_{s=1}^n p_{(s)}$ is approximately

$1-\alpha$, and hence, $\pi_n(1-\frac{1}{1-\alpha}\sum_{s=1}^{n}p_{(s)})$ is approximately 0 and $\frac{1}{1-\alpha}\sum_{s=1}^{n}p_{(s)}\pi_{(s)}$ is approximately $E[\pi|\pi \leq VaR_\alpha]$ (because $\pi_{(s)} \leq \xi = \pi_{(n)} = VaR_\alpha, \forall s = 1,...,n$, in the optimal solution).

Observation: for using this formulation within an utility maximization optimization problem, an objective function that is composed of the expected profit plus a risk aversion parameter multiplied by CVaR is used. By solving the problem for different values of the risk aversion parameter, a tradeoff curve between expected profit and CVaR is obtained. Another way of incorporating the CVaR metric into the optimization problem is maximizing expected profit subject to a lower bound on CVaR.

B.2 Formulation 2

An alternative formulation, developed by the author, is shown in what follows. It is an exact formulation of CVaR but it uses S binary variables and $S+1$ linear variables, and has much more computationally complicated constraints. In this case, CVaR will be presented within a set of constraints, with CVaR explicitly constrained using a lower bound. By changing the lower bound on CVaR a tradeoff curve between expected profit and CVaR is obtained.

The variables necessary for the formulation are the binary variables $u_s, s = 1,...,S$, and the linear variables ξ and $d_s, s = 1,...,S$. The set of constraints that determine CVaR correspond to:

$$\sum_{s=1}^{S} u_s \geq 1; \qquad (1)$$

$$1-\alpha-\max_{\{s=1,...,S\}} p_s \leq \sum_{s=1}^{S} p_s u_s \leq 1-\alpha; \qquad (2)$$

$$m_1 u_s \leq \pi_s - \xi \leq M_1(1-u_s); \forall s = 1,...,S; \qquad (3)$$

$$m_2 u_s \leq d_s \leq M_2 u_s; \forall s = 1,...,S; \qquad (4)$$

$$m_2(1-u_s) \leq d_s - \pi_s \leq M_2(1-u_s); \forall s = 1,...,S; \qquad (5)$$

$$\sum_{s=1}^{S} p_s d_s \geq \underline{\Omega} \sum_{s=1}^{S} p_s u_s. \qquad (6)$$

The variable ξ is VaR, and the variable u_s takes the value 1 if $\pi_s < \xi$ and the value 0 otherwise. This is enforced by constraints (3), with m_1 sufficiently low (or negatively high) and M_1 sufficiently high.

The variable d_s takes the value π_s if $u_s = 1$ and 0 otherwise. This is enforced by constraints (4) and (5), with m_2 sufficiently low (or negatively high) and M_2 sufficiently high.

Constraints (2) ensure that the sum of the probabilities of the scenarios below ξ is close to $1 - \alpha$. This in turn forces ξ to be VaR. Constraint (1) is necessary to ensure that the sum of those probabilities remains positive.

Finally, constraint (6) is equivalent to having $\underline{\Omega}$ as a lower bound to CVaR.

B.3 Discussion

Formulation 1 is a linear formulation consisting of $S+1$ variables whereas formulation 2 is a mixed integer-linear formulation with S binary variables and $S + 1$ linear variables that may be much harder computationally. Another drawback of Formulation 2 is that an appropriate selection of the parameters M_1, M_2, m_1, m_2 may be difficult (in order to have a reasonable timing for solving the resulting problem). The drawback of formulation 1 is that only an approximation of CVaR is obtained, but if the number of scenarios is big enough and $\max\{p_s\}_{s=1}^{S}$ is a small number, then it is assured that the approximation is close (in the application of this thesis the approximation is extremely close).

Finally, Formulation 1 allows incorporating CVaR into the objective function using a risk aversion parameter, which seems more convenient to the author than incorporating a constraint on CVaR using a lower bound, because taking any value of the risk aversion parameter assures feasibility of the problem, whereas a lower bound on CVaR makes the problem feasible only within certain ranges.

C. CHANGING PARAMETERS

This appendix presents the results obtained for the application developed in chapter 6 when using different values of certain parameters.

C.1 Changing CVaR confidence (α)

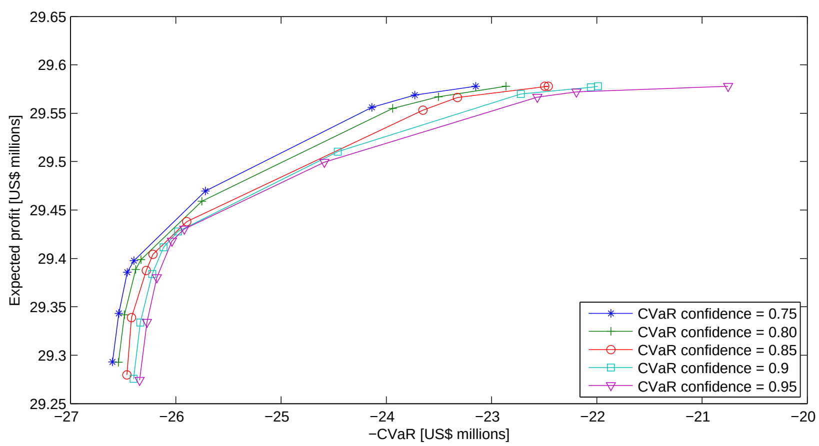


FIGURE C.1. Negative CVaR vs expected profit for different CVaR confidence values

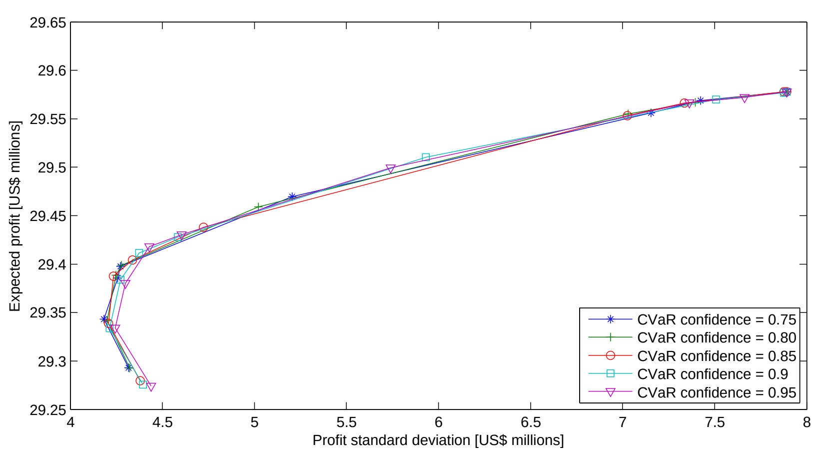


FIGURE C.2. Profit standard deviation vs expected profit for different CVaR confidence values

C.2 Changing the price of forward and CfD contracts (C)

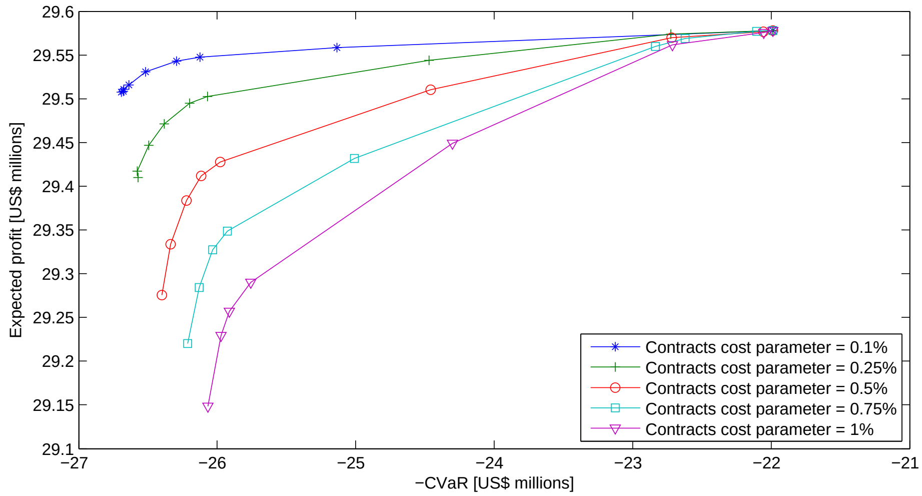


FIGURE C.3. Negative CVaR vs expected profit for different contracts cost parameter values

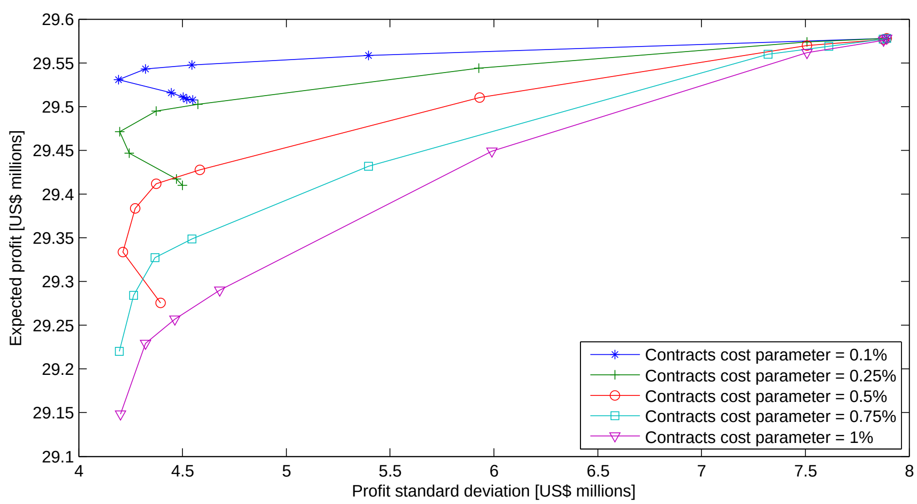


FIGURE C.4. Profit standard deviation vs expected profit for different contracts cost parameter values

C.3 Changing the liquidity of forward contracts (f^{liq})

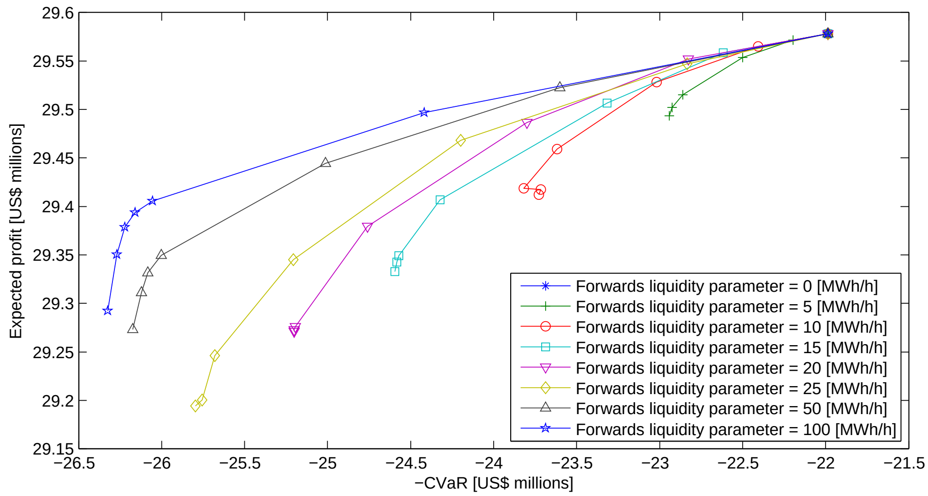


FIGURE C.5. Negative CVaR vs expected profit for different forwards liquidity parameter values and value 0 [US$/MWh] for the CfDs liquidity parameter

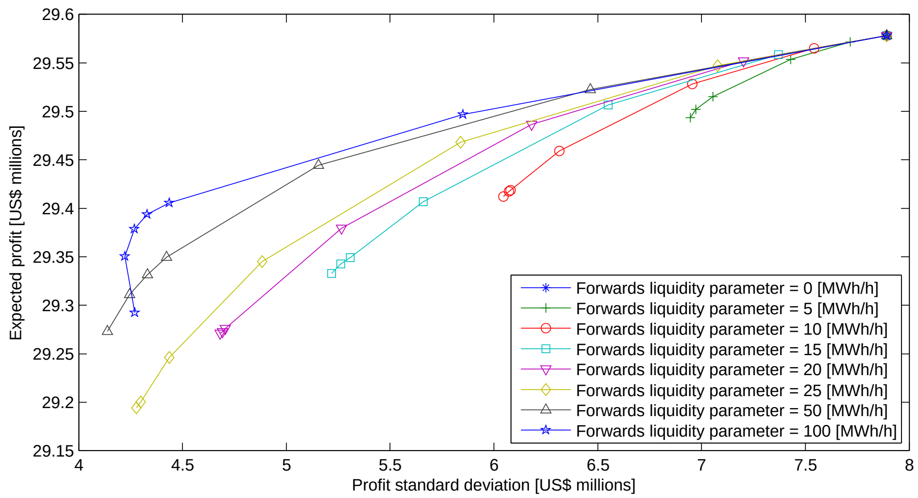


FIGURE C.6. Profit standard deviation vs expected profit for different forwards liquidity parameter values and value 0 [US$/MWh] for the CfDs liquidity parameter

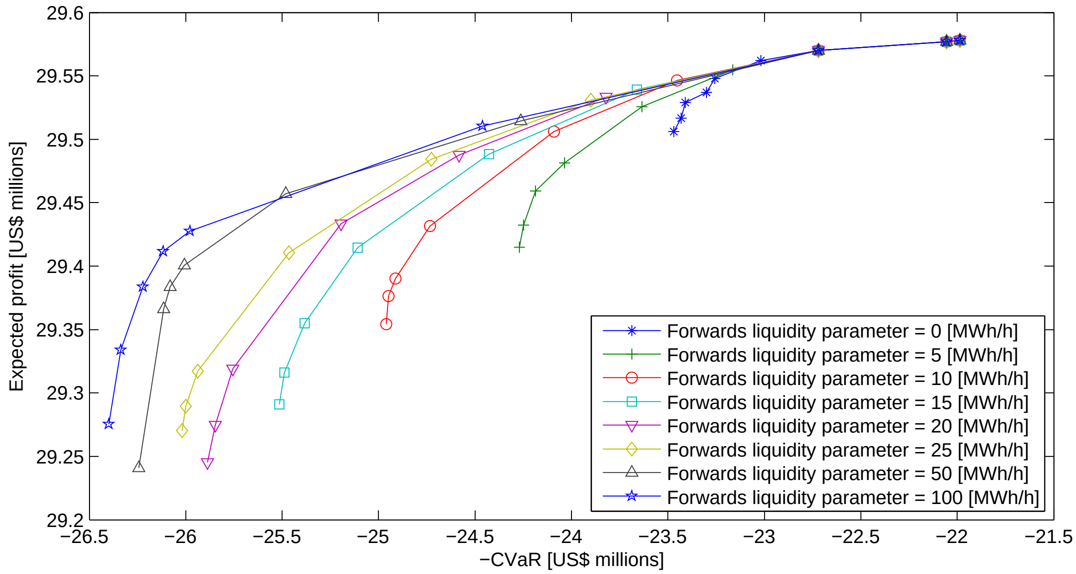


FIGURE C.7. Negative CVaR vs expected profit for different forwards liquidity parameter values and value 100 [US$/MWh] for the CfDs liquidity parameter

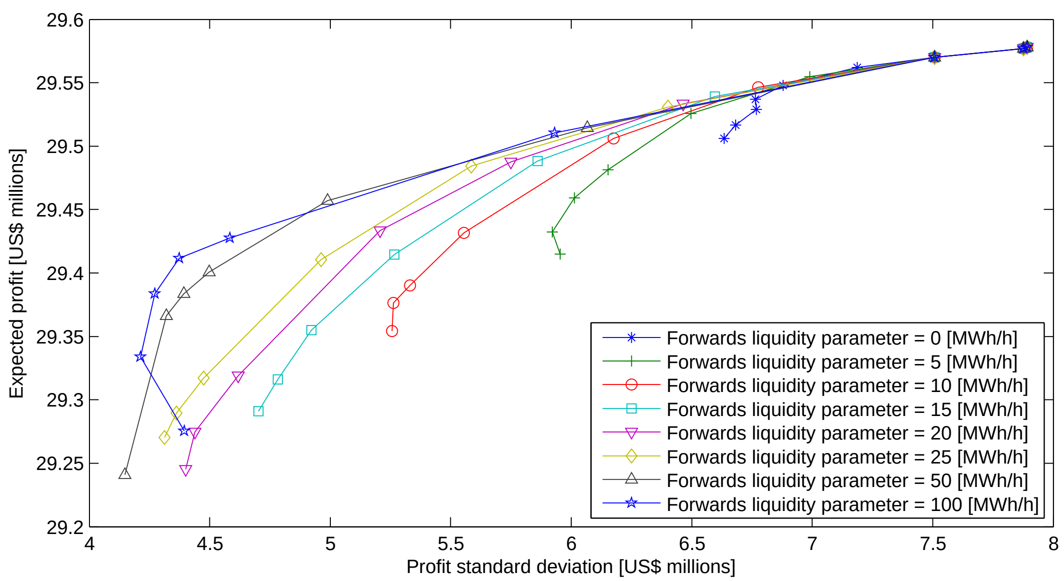


FIGURE C.8. Profit standard deviation vs expected profit for different forwards liquidity parameter values and value 100 [US$/MWh] for the CfDs liquidity parameter

C.4 Changing the liquidity of CfD contracts (g^{liq})

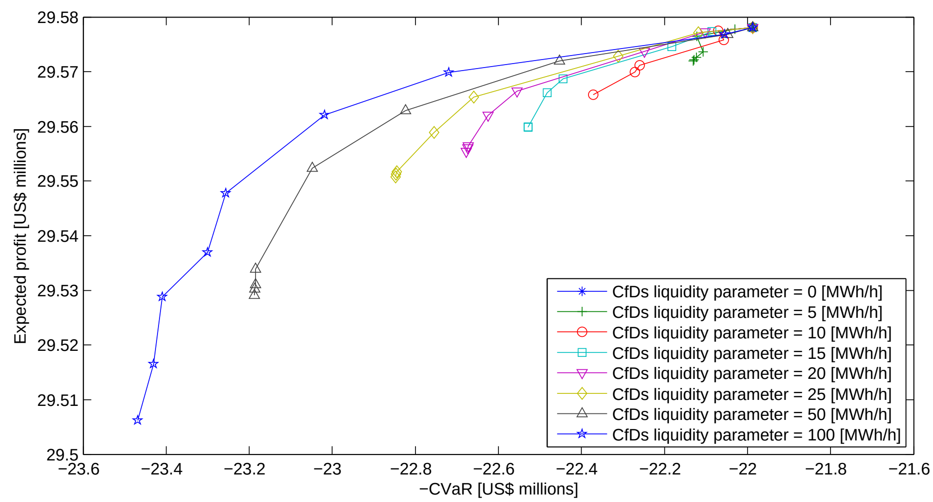


FIGURE C.9. Negative CVaR vs expected profit for different CfDs liquidity parameter values and value 0 [US$/MWh] for the forwards liquidity parameter

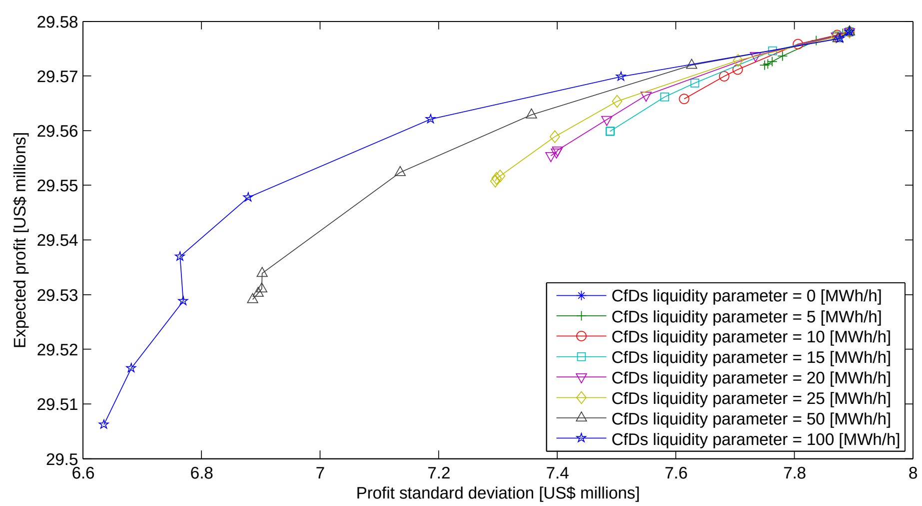


FIGURE C.10. Profit standard deviation vs expected profit for different CfDs liquidity parameter values and value 0 [US$/MWh] for the forwards liquidity parameter

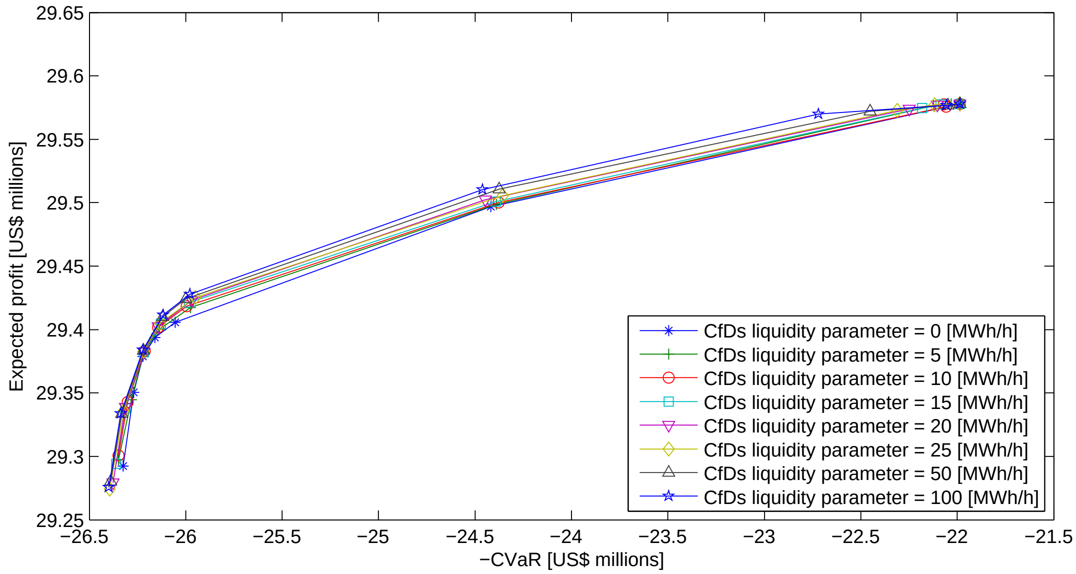


FIGURE C.11. Negative CVaR vs expected profit for different CfDs liquidity parameter values and value 100 [US$/MWh] for the forwards liquidity parameter

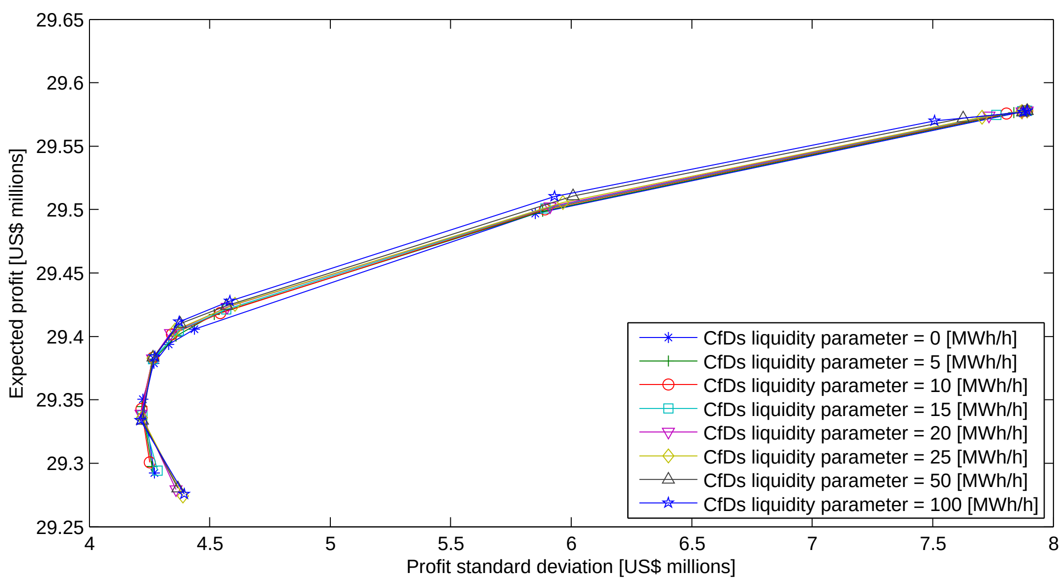


FIGURE C.12. Profit standard deviation vs expected profit for different CfDs liquidity parameter values and value 100 [US$/MWh] for the forwards liquidity parameter

C.5 Changing existent contractual positions ($D^{perhour}$)

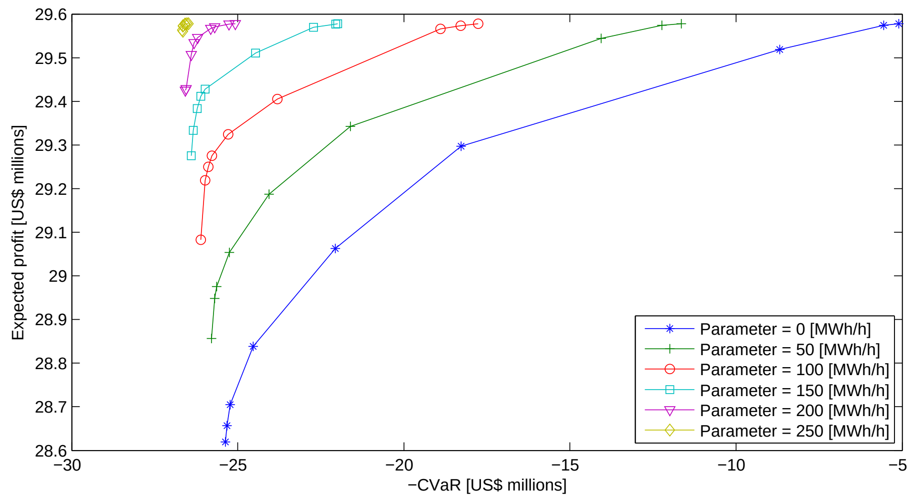


FIGURE C.13. Negative CVaR vs expected profit for different values of $D^{perhour}$ (the existent contractual positions parameter)

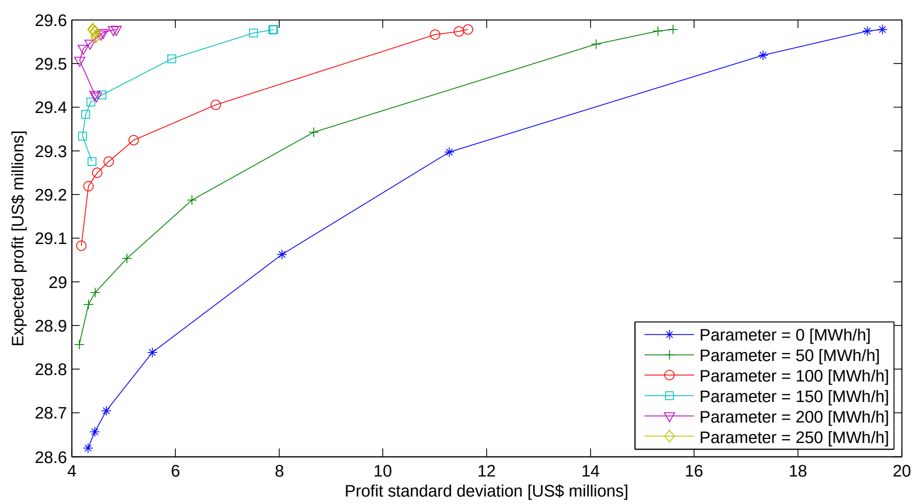


FIGURE C.14. Profit standard deviation vs expected profit for different values of $D^{perhour}$ (the existent contractual positions parameter)

C.6 Changing the maximum possible output of generators ($\overline{x}_{it}$)

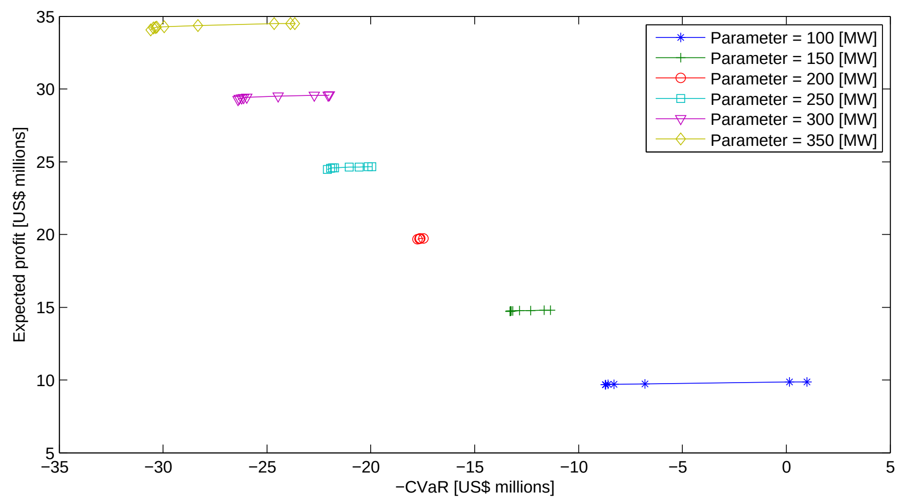


FIGURE C.15. Negative CVaR vs expected profit for different maximum generating capacities of generators

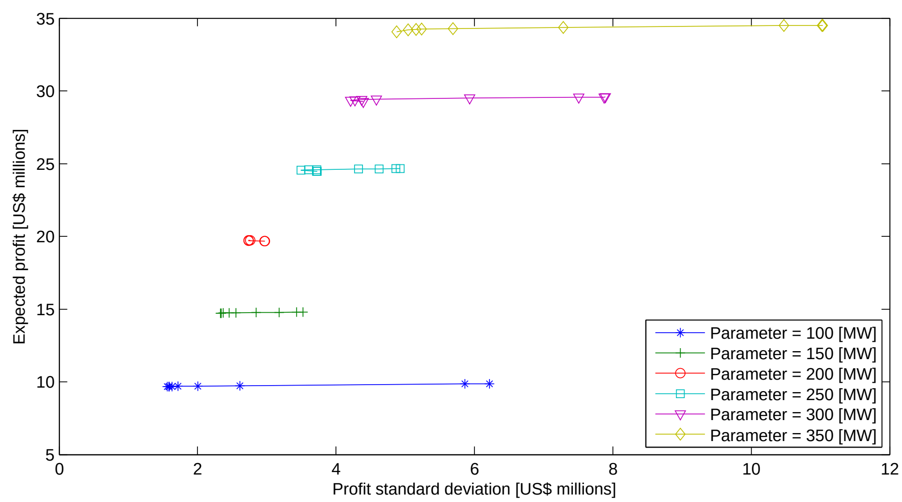


FIGURE C.16. Profit standard deviation vs expected profit for different maximum generating capacities of generators

C.7 Changing the cost of generation (C_{it})

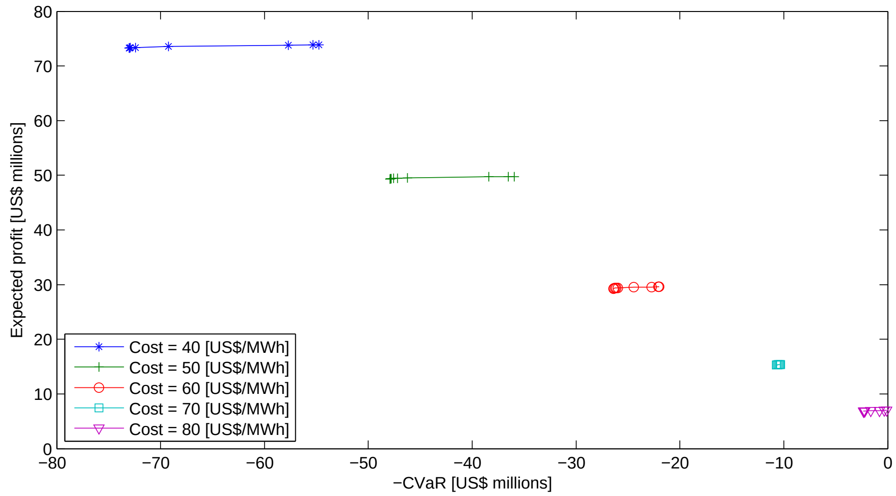


FIGURE C.17. Negative CVaR vs expected profit for different generation costs

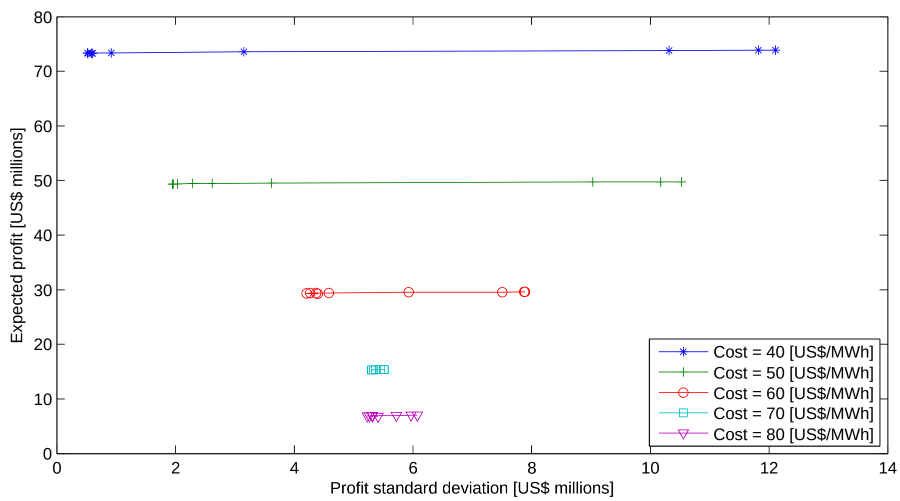


FIGURE C.18. Profit standard deviation vs expected profit for different generation costs

C.8 Changing the relationship between on-peak and off-peak generation (H_{ik})

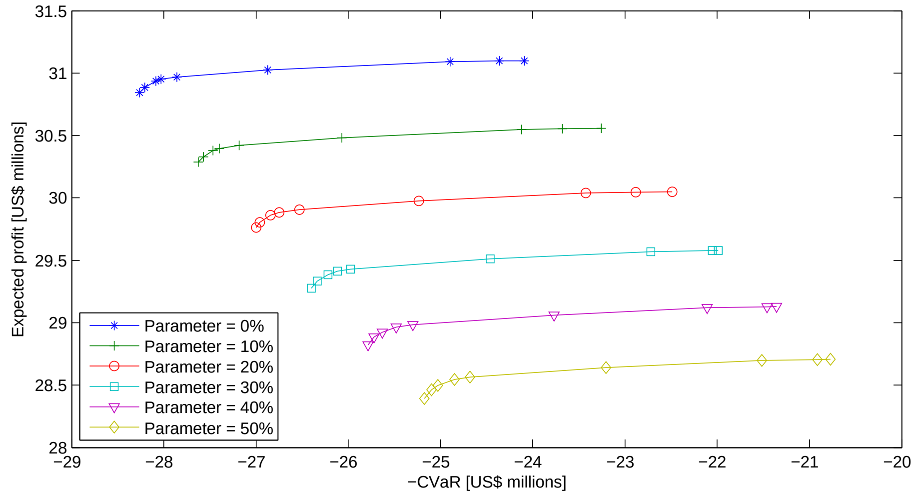


FIGURE C.19. Negative CVaR vs expected profit for different values of H_{ik}

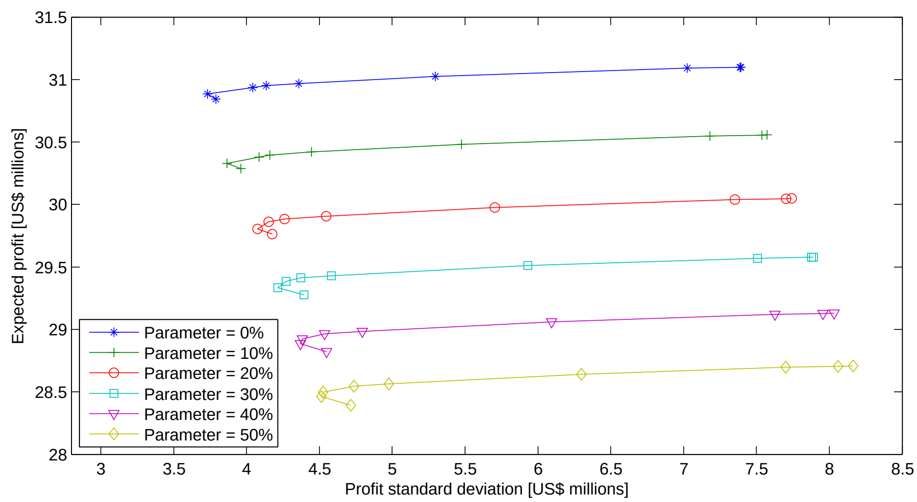


FIGURE C.20. Profit standard deviation vs expected profit for different values of H_{ik}

D. AVERAGING RESULTS

The scenario trees generated with the methodology presented in section 4.5 (and subsection 6.2.2) are stochastic, since they depend upon sampled initial solutions. Averaging any type of observable (such as expected profit or a decision variable) from the results obtained from many trees approximates the expected value of such observable on the probability space from which the scenario trees are obtained by the method. This means that the average of an observable from many generated scenario trees can be seen as an approximate expected value of such observable given the particular method being used, or an approximate expected value of such observable on the probability space that corresponds to the particular scenario tree generation method used.

Section D.1 presents a result used in subsection 6.3.5 as aid for the analysis of the effects of the correlation parameter ρ.

D.1 In the risk neutral case there is no dependency on the correlation parameter

Assume $\gamma = 0$ and for simplicity that there are no contractual decisions involved (not even existent contractual positions). Also assume that there are two locations and there is one generating unit in every location. The actual original problem (considering the original price process instead of a scenario tree that approximates it) corresponds (for this particular case) to:

$$\max_{\{x_{ltk}(\omega)\}} \mathbf{E}[\textstyle\sum_{l=1}^{L} \sum_{t=1}^{T} \sum_{k=1}^{K} x_{ltk}(\omega)(\beta_{ltk} P_{lt}^{E}(\omega) - C_{lt})] \tag{1}$$

$$s.t. \quad \phi_{tk}\underline{x}_{lt} \leq x_{ltk}(\omega) \leq \phi_{tk}\overline{x}_{l}t, \forall l = 1,...,L, \forall\omega \in \Omega, \forall t = 1,...,T, \forall k = 1,...,K; \tag{2.1}$$

$$H_{lk}\frac{x_{ltk}(\omega}{\phi_{tk}} \leq \frac{x_{lt\mu_k}(\omega)}{\phi_{t\mu_k}}, \forall l = 1,...,L, \forall\omega \in \Omega, \forall t = 1,...,T, \forall k \in \mu^{High}. \tag{2.2}$$

Where ω represents one particular trajectory of the bivariate electricity price process (process with a respective probability space that gives meaning to $\mathbf{E}$ in the objective function),

$P^E_{lt}(\omega)$ represents the adjusted monthly average electricity price at location l and time period t under trajectory ω, and $x_{ltk}(\omega)$ is the decision variable that corresponds to the generation in the unit located in location l, in hour group k of time period t, under trajectory ω.

Since the marginal distribution of electricity price at a certain location does not depend on ρ, and the above problem only depends on these marginal distributions, then it can be separated into L problems, each one for a specific location, because: i) $\mathbf{E}$ is a linear functional, ii) the objective function components related to $x_{ltk}(\omega)$ only depend on the marginal distribution of $P^E_{lt}(\omega)$ (with l fixed), and iii) the set of constraints for $x_{ltk}(\omega)$ does not depend on ω. Therefore, the optimal value (optimal expected profit) does not depend on ρ.

This demonstration can be extended to the general problem so as to assume that in the risk neutral case the expected profit in the actual original problem does not depend on ρ.

E. POWER PORTFOLIO OPTIMIZATION MODEL VARIATIONS

This appendix presents two variations of the power portfolio optimization model shown in chapter 5.

E.1 Model with specific available contracts

The model presented in chapter 5 assumes certain liquidity conditions and a particular cost curve of forward contracts and contracts for difference (that intend to approximate contractual availability conditions), and it is assumed that the delivery of these contracts occurs in one particular hour group for each. In practice, however, a power producer may know exactly what forward contracts, contracts for difference, and bilateral contracts are available, and their respective time spans for delivery. In this section, a variation of the model using the set of specific available contracts that exist is presented.

Besides the notation of chapter 5 (section 5.1), consider (replacing some components):

- N^{fB}: number of available buying forward contracts.
- N^{fS}: number of available selling forward contracts.
- N^{CfD}: number of available contracts for difference.
- $\mathcal{F}_b^{B,delivery}$: set of time period-hour group pairs (t,k) corresponding to the delivery time span of buying forward contract b.
- $\mathcal{F}_b^{S,delivery}$: set of time period-hour group pairs (t,k) corresponding to the delivery time span of selling forward contract b.
- $\mathcal{G}_b^{delivery}$: set of time period-hour group pairs (t,k) corresponding to the delivery time span of contract for difference b.
- l_b^{fB}: location of buying forward contract b.
- l_b^{fS}: location of selling forward contract b.
- $l_b^{CfD,1}$: location in which electricity is delivered (sold) for contract for difference b.
- $l_b^{CfD,2}$: location in which electricity is received (bought) for contract for difference b.

- $\overline{f}^B_{tkb}$: electricity [MWh] level of buying forward contract b at hour group k of time period t.
- $\overline{f}^S_{tkb}$: electricity [MWh] level of selling forward contract b at hour group k of time period t.
- $\overline{g}_{tkb}$: electricity [MWh] level of contract for difference b at hour group k of time period t.
- C^{fB}_b: electricity price [US$/MWh] for buying forward contract b.
- C^{fS}_b: electricity price [US$/MWh] for buying forward contract b.
- C^{CfD}_b: electricity price [US$/MWh] for contract for difference b.
- f^B_b: binary variable, 1 if buying forward contract b is signed, 0 if not.
- f^S_b: binary variable, 1 if selling forward contract b is signed, 0 if not.
- g_b: binary variable, 1 if buying contract for difference b is signed, 0 if not.

The mathematical formulation of this problem is the following mixed-integer program:

$$\max_{\{x_{istk},e_{lstk},f^B_b,f^S_b,g_b,\pi_s,\xi,\eta_s\}} \sum_{s=1}^S p_s\pi_s + \gamma(\xi - \tfrac{1}{1-\alpha}\sum_{s=1}^S p_s\eta_s) \tag{1}$$

$$\begin{aligned}
s.t. \quad \pi_s = & \sum_{l=1}^L \sum t = 1^T \sum_{k=1}^K e_{lstk}\beta_{ltk}P^E_{lst} - \sum_{i=1}^I \sum_{t=1}^T \sum_{k=1}^K x_{istk}C_{it} \\
& + \sum_{b=1}^{N^{fS}} \sum_{\{(t,k)\in\mathcal{F}^{S,delivery}_b\}} C^{fS}_b \overline{f}^S_{tkb} f^S_b - \sum_{b=1}^{N^{fB}} \sum_{\{(t,k)\in\mathcal{F}^{B,delivery}_b\}} C^{fB}_b \overline{f}^B_{tkb} f^B_b \\
& + \sum_{b=1}^{N^{CfD}} \sum_{\{(t,k)\in\mathcal{G}^{delivery}_b\}} C^{CfD}_b \overline{g}_{tkb} g_b \\
& + \sum_{l=1}^L \sum_{t=1}^T \sum_{k=1}^K D_{ltk}P^D_{ltk}, \forall s = 1,...,S; \qquad (2)
\end{aligned}$$

$$\phi_{tk}\underline{x}_{it} \leq x_{istk} \leq \phi_{tk}\overline{x}_{it}, \forall i = 1,...,I, \forall s = 1,...,S, \forall t = 1,...,T, \forall k = 1,...,K; \tag{3.1}$$

$$f^B_b \in \{0,1\}, \forall b = 1,...,N^{fB}; \tag{3.2}$$

$$f^S_b \in \{0,1\}, \forall b = 1,...,N^{fS}; \tag{3.3}$$

$$g_b \in \{0,1\}, \forall b = 1,...,N^{CfD}; \tag{3.4}$$

$$\begin{aligned}
& \sum_{\{i=1,...,I:\psi_i=l\}} x_{istk} + \sum_{\{b=1,...,N^{fB}:l^{fB}_b=l,(t,k)\in\mathcal{F}^{B,delivery}_b\}} \overline{f}^B_{tkb} f^B_b \\
& \quad + \sum_{\{b=1,...,N^{CfD}:l^{CfD,2}_b=l,(t,k)\in\mathcal{G}^{delivery}_b\}} \overline{g}_{tkb} g_b \\
& \quad = e_{lstk} + D_{ltk} + \sum_{\{b=1,...,N^{fS}:l^{fS}_b=l,(t,k)\in\mathcal{F}^{S,delivery}_b\}} \overline{f}^S_{tkb} f^B_b \\
& \quad + \sum_{\{b=1,...,N^{CfD}:l^{CfD,1}_b=l,(t,k)\in\mathcal{G}^{delivery}_b\}} \overline{g}_{tkb} g_b,
\end{aligned}$$

$$\forall l=1,...,L, \forall s=1,...,S, \forall t=1,...,T, \forall k=1,...,K. \tag{4}$$

$$H_{ik}\frac{x_{istk}}{\phi_{tk}} \leq \frac{x_{ist\mu_k}}{\phi_{t\mu_k}}, \forall i=1,...,I, \forall s=1,...,S, \forall t=1,...,T, \forall k \in \mu^{High}; \tag{5}$$

$$\eta_s \geq \xi - \pi_s, \forall s=1,...,S; \tag{6.1}$$

$$\eta_s \geq 0, \forall s=1,...,S; \tag{6.2}$$

$$x_{is_1tk} = x_{is_2tk}, \forall i=1,...,I, \forall s_1=1,...,S, \forall t=1,...,T, \forall s_2 \in B_{s_1t}, \forall k=1,...,K; \tag{7.1}$$

$$e_{ls_1tk} = e_{ls_2tk}, \forall l=1,...,L, \forall s_1=1,...,S, \forall t=1,...,T, \forall s_2 \in B_{s_1t}, \forall k=1,...,K. \tag{7.2}$$

E.2 Model allowing contracts to be taken at any time period

In the model presented in chapter 5 forward and CfD contracts can only be signed previously to the initial time period (in time period “0”). In this section, an extension of the model incorporating the possibility of signing forward and CfD contracts at later time periods is presented. This variant has been implemented and the results are compared to the model of chapter 5 in subsection 6.3.4. The computational time used for solving this problem is about 20 times the computational time used for solving the original problem (using the data presented in chapter 6).

Besides the notation of chapter 5 (section 5.1), consider (replacing some components):

- $\overline{f}^{B}_{lukb}$: maximum electricity [MWh] that can be bought at location l through buying block b of forward contracts spanning hour group k of time period u.
- $\overline{f}^{S}_{lukb}$: maximum electricity [MWh] that can be sold at location l through selling block b of forward contracts spanning hour group k of time period u.
- C^{fB}_{lstukb}: electricity price [US$/MWh] under scenario s at time period t and location l, for buying block b of forward contract spanning hour group k of time period u.
- C^{fS}_{lstukb}: electricity price [US$/MWh] under scenario s at time period t and location l, for selling block b of forward contract spanning hour group k of time period u.

- $\overline{g}_{lmukb}$: maximum electricity [MWh] that can be contracted through block b of CfD contracts spanning hour group k of time period u and equivalent to selling electricity at location l and buying at location m.
- $C^{CfD}_{lmstukb}$: electricity price [US$/MWh] under scenario s at time period t, for block b of CfD contract spanning hour group k of time period u and equivalent to selling electricity at location l and buying at location m.
- f^{B}_{lstukb}: electricity [MWh] bought under scenario s at time period t and location l through buying block b of forward contract with delivery in hour group k of time period u.
- f^{S}_{lstukb}: electricity [MWh] sold under scenario s at time period t and location l through selling block b of forward contract with delivery in hour group k of time period u.
- $g_{lmstukb}$: electricity [MWh] contracted under scenario s at time period t, through block b of CfD contract spanning hour group k of time period u and equivalent to selling electricity at location l and buying at location m.

The mathematical formulation of this problem is:

$$\max_{\{x_{istk}, e_{lstk}, f^B_{ltkb}, f^S_{ltkb}, g_{lmtkb}, \pi_s, \xi, \eta_s\}} \sum_{s=1}^{S} p_s \pi_s + \gamma(\xi - \tfrac{1}{1-\alpha} \sum_{s=1}^{S} p_s \eta_s) \quad (1)$$

$$\begin{aligned}
s.t. \quad \pi_s = {} & \sum_{l=1}^{L} \sum t = 1^T \sum_{k=1}^{K} e_{lstk} \beta_{ltk} P^E_{lst} - \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{k=1}^{K} x_{istk} C_{it} \\
& + \sum_{l=1}^{L} \sum_{t=0}^{T-1} \sum_{u=t+1}^{T} \sum_{k=1}^{K} \sum_{b=1}^{Nf} (C^{fS}_{lstukb} f^S_{lstukb} - C^{fB}_{lstukb} f^B_{lstukb}) \\
& + \sum_{l=1}^{L} \sum_{\{m=1,\dots,L: m \neq l\}} \sum_{t=0}^{T-1} \sum_{u=t+1}^{T} \sum_{k=1}^{K} \sum_{b=1}^{N^{CfD}} C^{CfD}_{lmstukb} g_{lmstukb} \\
& + \sum_{l=1}^{L} \sum_{t=1}^{T} \sum_{k=1}^{K} D_{ltk} P^D_{ltk}, \forall s = 1, \dots, S; \quad (2)
\end{aligned}$$

$$\phi_{tk} \underline{x}_{it} \leq x_{istk} \leq \phi_{tk} \overline{x}_{it}, \forall i = 1, \dots, I, \forall s = 1, \dots, S, \forall t = 1, \dots, T, \forall k = 1, \dots, K; \quad (3.1)$$

$$\begin{aligned}
& 0 \leq f^B_{lstukb} \leq \overline{f}^B_{lukb}, \forall l = 1, \dots, L, \forall s = 1, \dots, S, \forall t = 0, \dots, T-1, \\
& \quad \forall u = t+1, \dots, T, \forall k = 1, \dots, K, \forall b = 1, \dots, N^f; \quad (3.2)
\end{aligned}$$

$$\begin{aligned}
& 0 \leq f^S_{lstukb} \leq \overline{f}^S_{lukb}, \forall l = 1, \dots, L, \forall s = 1, \dots, S, \forall t = 0, \dots, T-1, \\
& \quad \forall u = t+1, \dots, T, \forall k = 1, \dots, K, \forall b = 1, \dots, N^f; \quad (3.3)
\end{aligned}$$

$$0 \leq g_{lmstukb} \leq \overline{g}_{lmukb}, \forall l = 1, ..., L, \forall m = 1, ..., L : m \neq l, \forall s = 1, ..., S,$$
$$\forall t = 0, ..., T-1, \forall u = t+1, ..., T, \forall k = 1, ..., K, \forall b = 1, ..., N^{CfD}; \quad (3.4)$$

$$\sum_{\{i=1,...,I:\psi_i=l\}} x_{istk} + \sum_{r=0}^{t-1} \sum_{b=1}^{N^f} f^B_{lsrtkb}$$
$$+ \sum_{v=1}^{S} \sum_{r=0}^{t-1} \sum_{\{m=1,...,L:m\neq l\}} \sum_{b=1}^{N^{CfD}} g_{mlvrtkb}$$
$$= e_{lstk} + D_{ltk} + \sum_{b=1}^{N^f} f^S_{ltkb} + \sum_{\{m=1,...,L:m\neq l\}} \sum_{b=1}^{N^{CfD}} g_{lmstukb},$$
$$\forall l = 1, ..., L, \forall s = 1, ..., S, \forall t = 1, ..., T, \forall k = 1, ..., K. \quad (4)$$

$$H_{ik} \frac{x_{istk}}{\phi_{tk}} \leq \frac{x_{ist\mu_k}}{\phi_{t\mu_k}}, \forall i = 1, ..., I, \forall s = 1, ..., S, \forall t = 1, ..., T, \forall k \in \mu^{High}; \quad (5)$$

$$\eta_s \geq \xi - \pi_s, \forall s = 1, ..., S; \quad (6.1)$$

$$\eta_s \geq 0, \forall s = 1, ..., S; \quad (6.2)$$

$$x_{is_1tk} = x_{is_2tk}, \forall i = 1, ..., I, \forall s_1 = 1, ..., S, \forall t = 1, ..., T, \forall s_2 \in B_{s_1t},$$
$$\forall k = 1, ..., K; \quad (7.1)$$

$$e_{ls_1tk} = e_{ls_2tk}, \forall l = 1, ..., L, \forall s_1 = 1, ..., S, \forall t = 1, ..., T, \forall s_2 \in B_{s_1t},$$
$$\forall k = 1, ..., K. \quad (7.2)$$

$$f^B_{lstukb} = f^B_{lvtukb}, \forall l = 1, ..., L, \forall s = 1, ..., S, \forall t = 0, ..., T-1, \forall v \in B_{st},$$
$$\forall u = t+1, ..., T, \forall k = 1, ..., K, \forall b = 1, ..., N^f; \quad (7.3)$$

$$f^S_{lstukb} = f^S_{lvtukb}, \forall l = 1, ..., L, \forall s = 1, ..., S, \forall t = 0, ..., T-1, \forall v \in B_{st},$$
$$\forall u = t+1, ..., T, \forall k = 1, ..., K, \forall b = 1, ..., N^f; \quad (7.4)$$

$$g^B_{lmstukb} = g^B_{lmvtukb}, \forall l = 1, ..., L, \forall m = 1, ..., L : m \neq l, \forall s = 1, ..., S,$$
$$\forall t = 0, ..., T-1, \forall v \in B_{st}, \forall u = t+1, ..., T, \forall k = 1, ..., K, \forall b = 1, ..., N^f. \quad (7.5)$$