PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
SCHOOL OF ENGINEERING

A REAL OPTIONS APPROACH FOR JOINT OVERHAUL AND REPLACEMENT STRATEGIES WITH MEAN REVERTING PRICES

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Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Master of Science in Engineering

Advisor:
ALEJANDRO MAC CAWLEY VERGARA

Santiago de Chile, March 2017

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Gratefully to my family

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ABSTRACT

Due to its significant impact on economic performance, an effective equipment overhaul and replacement strategy is a key aspect of physical asset management in capital-intensive industries, such as the mining industry. Classical approaches suggest periodic interventions based on the physical condition of the equipment, considering factors such as availability and operational costs. These fixed models generally ignore two important aspects: first, the flexibility of the decision to overhaul or replace, which may be re-evaluated within a given period, and second, the uncertainty around economic factors that may affect future maintenance decisions, such as the product price. This work improves on classical models by considering the effect of integrated price uncertainty in the definition of joint overhaul and replacement strategy, using a real options approach and a mean reversion binomial model to represent the uncertainty in price. More specifically, we develop a real options model and use a backwards recursion algorithm to determine an optimal intervention policy that maximizes expected profits. We then present a numerical study of the mining industry to validate the effectiveness of the proposed methodology. Results show that the option-based decision model economically outperforms the classical periodic strategy approach by 10.5%, offering evidence that a new approach to equipment overhaul and replacement strategy is needed.

Keywords: Joint Replacement and Overhaul policy, Real options, Mean Reversion

1. INTRODUCTION

1.1. Background

Production systems face challenges in the definition of strategic decisions due to variability and uncertainty in the market environment and unexpected failures of production equipment which deteriorate progressively with time and use. This variability results in that optimal decisions or action plans can change over time. The increasing capacity and complexity of the industrial equipment has generated interest in developing and implementing proactive strategies that improve equipment reliability and maximize the profit considering the uncertainty of external variables.

Price variability has a direct impact on the profitability in different industries. Prices vary for different reasons such as movements in the exchange rate, supply busts, technological development and changes in demand (Hu et al., 2012). The commodity industry is particularly sensitive to the variation in price. In addition to the inherent uncertainty of the price, commodity prices usually present mean reverting behaviors that impose another challenge in the generation of profit. For example, the copper price has dropped steadily since 2011 from US$ 4 to US$ 2.14 per pound, with losses in 2015 in the copper industry that reaches US$ 27 thousand million (PwC, 2016). In addition, changes in a production level to respond to price changes are usually slow due to the magnitude of the infrastructure investments involved, so reactions at the operational level may respond more adequately to price fluctuations.

Deterioration of production equipment is one of the most challenging problems in the commodity industry. Typically, equipment deteriorates with time and usage, which may result in failures that can results in high maintenance costs and huge production losses. In order to control this deterioration two actions are possible: (i) to perform an extensive maintenance, known overhaul, in which the equipment is brought to an improved condition, and (ii) to replace the equipment (Zhang and Jardine, 1998). Performing an overhaul is less expensive than the replacement, but it has less impact. However, it is possible that a large number of overhauls and large replacement cycles may result in poor performance because of the low availability and high operating costs. As an example, the replacement cost of a mining truck can reach between US$1 million to US$ 4 million, depending on the truck capacity (Comisión Chilena del Cobre, 2015).

Availability is one of the key performance indicators for productive equipment and has high impact on the production rate in a productive system (Kim and Thomas, 2013). The impact of equipment availability on the benefits mainly depends on the equipment capacity and the system complexity. Commodity industries usually use equipment with high production capacity and unexpected failures and decreases in the equipment availability have significant effects on the system profitability. Similarly, little variations in the commodity price can have huge impact in the benefits. A maintenance strategy that can react properly to variations in the commodity price can control availability levels that can take advantage of rises in price and avoid important investment when the price goes down.

The definition of joint overhaul and replacement strategies is usually performed considering the minimization of the total cost over the equipment useful life (Kim and Thomas, 2013). The major limitation of this approach is that it defines periodic maintenance interventions and replacements and does not consider the option to adapt and update the policies when new market information is available. To add flexibility into these decisions to respond proactively to price changes can maximize benefits by taking advantage of price increases and avoiding losses when the price goes down. Options that provide flexibility in maintenance strategies are, for example, to reduce the probability of failure of equipment in scenarios where the price increases more than expected or delay a replacement decision if the price goes down.

1.2. Maintenance decisions

Normally, maintenance includes actions oriented to two main objectives: (1) to extend the equipment useful life and (2) improve the system reliability by reducing the failure risk (Wang and Pham, 2013). Depending on the objective and the criteria, there are different maintenance models in the literature. Some typical objectives are to minimize operation and maintenance costs, to maximize availability, to maximize profit and to minimize failures per unit of time. According to the criteria, models in the literature can be classified into different categories: age-replacement policy, block replacement policy, preventive maintenance and replacement policy, limit failures policy, among others (Wang, 2002).

Models in the literature typically consider three types of maintenance: (1) minimal repair, in which the system is restored into the condition it was just before the failure, (2) perfect maintenance or replacement, in which the system returns to the condition ”as good as new”, and (3) imperfect maintenance, intervention that brings the system into a condition better than before, in between ”as good as new” and ”as bad as old” (Ouali et al., 2011). There are different ways to model the effect of an imperfect maintenance in a system. Pham and Wang (Pham and Wang, 1996) and Doyen and Gaudoin (Doyen and Gaudoin, 2004) present surveys of imperfect maintenance models.

Maintenance activities are usually classified into two categories: corrective maintenance, applied once the system has failed, and preventive maintenance, in which the equipment is maintained while it is operative. Preventive maintenance seeks to reduce the probability of failure or the state of deterioration. Normally, this maintenances are time-based, performed after defined time intervals, or condition-based, following a predetermined criteria (Wu and Zuo, 2010). Preventive maintenance models have been widely studied in the literature (Wang, 2002). Since the early work of Barlow and Hunter (Barlow and Hunter, 1960), models have integrated different factors in order to represent reality more accurately.

1.3. Basic definitions

There is a basic terminology used in the maintenance literature. Definitions of the terms used in this work are shown below: (for an extended list see Endrenyi et al (Endrenyi et al., 2001)):

- *Failure:* Mechanical breakdown, deterioration beyond a threshold level, appearance of defects in the system performance or decrease in the system performance below a critical level.
- *Failure rate:* rate of occurrence of events that prevent a device to accomplish a required function.
- *Availability:* probability that a system will be able to operate within the tolerances at a given instant of time.
- *Repair:* restoration wherein a failed device is returned to operable condition.
- *Minimal repair:* repair with minimal effort that brings the equipment into the same operative condition it had just before the failure
- *Corrective maintenance:* maintenance implemented once the equipment has failed
- *Preventive maintenance:* maintenance implemented while the equipment is still operative
- *Imperfect maintenance:* maintenance in which the equipment is brought into an intermediate condition better than before but no as good as new
- *Overhaul:* intensive imperfect maintenance requiring a major effort, bringing the equipment into a condition of considerable improvement compared with the condition before the intervention
- *Replacement:* renewal in which the equipment is removed and a new one is put in place. Also referred as perfect maintenance.

1.4. Periodic overhaul and replacement policy

Periodic preventive maintenance policies have been studied widely in the literature (Wang, 2002). The problem is to balance the operation and maintenance costs with the capital costs required to apply certain preventive policy. Equipment deteriorates with time and use, which can result in an increase in its failure rate resulting in an increase in production loses and operation and maintenance costs. A maintenance-intensive policy can control the failure rate, but it also raises the capital costs for the company.

An early model within the periodic maintenance policy category is the periodic replacements with minimal repair at failures model, in which the unit is replaced after fixed time intervals kT $(k = 1, 2 \ldots)$ and failures in between replacements are corrected by minimal repairs (Barlow and Hunter, 1960). With the posterior integration of the concept of imperfect maintenance, several variations and extensions of the model had been studied. In these models, preventive maintenances are applied at fixed intervals, with minimal repairs when necessary, until the equipment is replaced. Different models can be formulated depending on the type of preventive maintenance (minimum, imperfect, perfect), type of replacement (preventive or corrective and cost structure.

An extension of the periodic maintenance model is to consider the preventive maintenance as overhauls that extend the equipment useful life in a replacement cycle. Zhang and Jardine (Zhang and Jardine, 1998) propose a model in which overhauls are implemented after fixed time intervals T, failures are corrected by minimal repairs and the equipment is replaced after $(N - 1)$ overhauls in the period NT. Decision variables are the number of overhauls within a replacement cycle and the time between them. The objective is to minimize the total cost per unit of time:

$$C(N, T) = \frac{C_r + C_o(N - 1) + C_m \hat{H}(NT)}{NT} \tag{1.1}$$

Where C_r, C_o and C_m are the replacement, overhaul and minimal repair costs. $\hat{H}(NT)$ is the number of equipment failures in the interval NT.

Extensions of the periodic overhaul and replacement model have included external factors beyond the internal costs and failure rate parameters. One of these extensions is to considerate guarantee contracts. Pascual and Ortega (Pascual and Ortega, 2006) propose a model in which the equipment owner can negotiate an improved warranty contract by having the option to improve their overhaul and replacement policy. Similarly, Chien (Chien, 2008) studies the effects on optimal policy by considering a warranty contract with free replacement and imperfect maintenance.

Within the vast literature on equipment maintenance and replacement policies, most of the models consider deterministic scenarios and their objective is to minimize the average discounted costs (Kim and Thomas, 2013). In practice, many of the factors affecting maintenance policies are uncertain and have high variability, which imposes a limitation on these models. The works that have considered uncertainty have focused on modeling stochastically internal equipment parameters as failure times, operating and maintenance costs, failure types, among others (Abdel-Hameed, 2013).

More recently, maintenance studies have considered different types of external uncertainties. Mardin and Arai (Mardin and Arai, 2011) propose a model based on dynamic systems, where the overhaul and replacement policies are affected by the technological improvement of the equipment in the replacement. Nguyen et al. (Nguyen et al., 2011) consider a model in which the emergence of new technologies in the future applying a binomial model. Finally, Richardson et al. (Richardson et al., 2013) propose a replacement model with real options in which the uncertainty is given by the waiting time in which the replacement is carried out.

1.4.1. Imperfect maintenance models

An overhaul is basically an intensive imperfect maintenance in which the equipment condition is improved into a better than before but not as good as new condition. There are several different ways to model this situation in the literature. Malik (Malik, 1979) introduces the concept of virtual age where the age of the system is reduced into a prior age after the overhaul. A limitation of this model is that it does not alter the equipment failure rate. Nakagawa (Nakagawa, 1979) proposes a model in which the overhaul is a minimal repair with probability p and a perfect maintenance with probability $(1 - p)$. This model tends to overestimate the overhaul impact in the equipment later ages if probability p is considered constant. Kijima (Kijima and Nakagawa, 1992) use a model in which the age reduction is proportional to the age in which the overhaul is implemented considering a restoration factor.

In this work, we use the failure rate reduction model proposed by Zhang and Jardine (Zhang and Jardine, 1998). In this model, the equipment failure rate is directly improved after an overhaul by an improvement degree p. Let's call $\lambda_{k-1}(t)$ the failure rate just before the overhaul, $\lambda_k(t)$ the failure rate just after the overhaul, T the overhaul interval and $p \in (0, 1)$ the improvement degree. The failure rate after the overhaul is between the rate it had in the last overhaul time and the previous rate:

$$\lambda_k(t) = p\lambda_{k-1}(t - T) + (1 - p)\lambda_{k-1}(t) \tag{1.2}$$

Note that if the improvement factor $p = 0$ then $\lambda_k(t) = \lambda_{k-1}(t)$ which corresponds to a minimal repair, in which the failure rate is not altered. On the other hand, if $p = 1$ then $\lambda_k(t) = \lambda_{k-1}(t-T)$ which returns the failure rate to the condition of the previous overhaul period.

1.5. Real options and flexibility value

Nowadays, markets are characterized by rapid changes and the ability to adapt and respond to them is essential for business success. A significant limitation of the maintenance models presented in Sec. 1.4 is that the models cannot review and update the optimal policies once there is new market information available of uncertain variables. The real options tool has been widely applied to evaluate problems that involves uncertain variables and flexibility in the project decisions.

A real option is the chance to make changes in a project development or investment when previously unknown information is available. Ford et al. (Ford et al., 2004) define real options as ”options are strategies that include a right, without an obligation, to take specific actions in the future, at some cost, and contingent on how conditions, initially uncertain, evolve.” Another definition is proposed by Mun (Mun, 2006):

> a systematic approach and integrated solution using financial theory, economic analysis, management science, decision sciences, statistics, and econometric modelling in applying options theory in valuing real

> physical assets, as opposed to financial assets, in a dynamic and uncertain business environment where business decisions are flexible in the context of strategic capital investment opportunities, and project capital expenditures.

This methodology originally used for financial option valuation has been extended to the application of real assets and investment projects. The concept of real options is used to differentiate them from financial, stock and commodity options. The real options methodology allows for analyse the value generated by the flexibility of implementing critical decisions during a project life time that can not be captured by using a traditional discounted cash flow analysis or sensitivity analysis (Cobb and Charnes, 2007).

There are different types of real options that can be classified according to the decision nature. Trigeorgis (Trigeorgis, 1993) classifies real options in investment decisions into the option to defer, to abandon, to alter scale, to switch, to grow and time-to-build option. Literature in real options have focused on investment decisions such as mining, oil and gas and energy projects (Savolainen, 2016). However, the broad field of application of real options includes natural resources, competition and business strategy, manufacturing, real estate, R & D, public good, mergers and acquisitions, corporate governance, interest rates, inventory, labour, venture capital, advertising, legal and environmental development (Zeng et al., 2011).

Real options literature in manufacturing systems have been classified based on the type of flexibility in basic, system and aggregate levels (Sethi and Sethi, 1990). The basic level includes machine flexibility, material handling system flexibility and operation flexibility. Maintenance decisions such as overhaul and replacement implementation can be considered within this flexibility level. The system level considers decisions with process, product, volume and expansion flexibilities. Finally, the aggregate level is concerned about flexibility at the plant level and includes program, production and market flexibility. For a survey on manufacturing flexibility and real options see (Bengtsson, 2001).

1.5.1. Valuation methods

There are three approaches used typically in the literature for real options valuation: Black-Scholes closed form equations, Monte Carlo simulations and Binomial lattices. This section presents a brief description, advantages and disadvantages of each approach.

1.5.1.1. Black-Scholes option pricing model

The model consists in a closed form equation originally used for valuate preumium value of financial options. The Black-Scholes formula is widely used for its simplicity and straightforward application on single real options such as the option to expand, abandon or delay an investment (Mun, 2006).

The major limitations of the Black-Scholes model is practical applicability. There are several assumptions in the model, such as that the option can only be exercised in its maturity date that complicate the practical use in real problems.

1.5.1.2. Monte Carlo simulation

Monte Carlo simulation is a method first used by Boyle (Boyle, 1977) to value real options and consists basically in the simulation of large numbers of possible price paths and calculate the value of the option for each path. The value of the option is the mean of the total discounted present value of the flexibility for each simulation. This approximation method is general and can be applied into a wide range of situations and, unlike the Black-Scholes and binomial lattices, can be extended to more complex problems with several uncertainty sources. However, Monte Carlo simulation is computationally expensive due to the evaluation of the option value for each simulated path and convergence could be slow and time consuming. The method is usually applied to solve for complex real options when the alternative methods are less attractive (Wang and De Neufville, 2005).

In contrast to the Black-Scholes model and Binomial lattices, Monte Carlo simulations provide an option value but there are not insights that allows for a better understanding between the variables and the option value. In comparison, the binomial model provides a solution in a tree structure that allows to map out the different options implementation and the Black-Scholes involves a closed-form analytic solution that shows the relationship between variables and volatility. This lack of insights makes the method works like a ”black box” that does not allow further analysis (Bastian-Pinto et al., 2010).

1.5.1.3. Binomial lattice

The binomial model for option valuation proposed by Cox et al. (Cox et al., 1979) consists in a discrete-time approach for model the evolution of price over time. The model consists in the construction of a binomial tree that starts with the current price x and grows and decay with a fixed rate u and d, respectively (see Fig. 1.1). Then the option value is calculated using a backward induction from the last interval nodes to the initial. The up and down movements at each node are calculated to be $u = e^{\sigma\sqrt{\triangle t}}$ and $d = e^{-\sigma\sqrt{\triangle t}}$, and the up and down probabilities are $p = \frac{e^{r\triangle t}-d}{u-d}$ and $1 - p$. This model is an approximation of a geometric Brownian motion and converges weakly as $\triangle t$ goes to zero.

The use of the binomial model is very extended in the literature of real options valuation due to its flexibility and straightforward implementation (Hahn and Dyer, 2008). Other advantage is that the decision process is mapped out in an optimal decision tree allowing a transparent and intuitive analysis to better support critical decisions. The recombinant nature of the binomial tree, which is basically the fact that up and down movements are equivalent to down and up movements, reduces the computational complexity in comparison with a binary tree in which there are 2^n nodes instead of $n + 1$ in the case of the recombinant tree. On the other hand, for computational efficiency the model is limited to problems with few uncertainty sources and bounded time horizons (Savolainen, 2016).

1.5.2. Modeling commodity prices

There are different approaches to model the stochastic commodity prices behavior. These approaches can be classified into two main categories: (i) structural models, where a partial equilibrium price is derived from a model of supply and demand and (ii) reduced form models, based on a diffusion process in which the price is usually modeled by partial

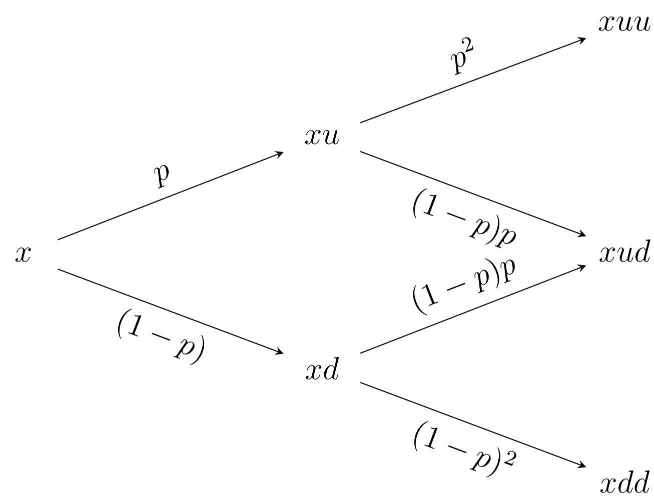

FIGURE 1.1. Recombinant binomial tree.

differential equations. Due to the simplicity and greater applicability of reduced-form models, these have been widely applied in the literature to model commodity prices (Ribeiro, 2004).

A reduced-form model widely used to model stochastic processes is the Geometric Brownian motion which is basically a random path process with a drift, whose formulation ensures non-negativity of prices. Brennan and Schwartz (Brennan and Schwartz, 1985) use this approach to model commodity prices where the price S_t is given by:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW_t \tag{1.3}$$

Where, μ is the drift parameter, σ is the volatility and dW_t is the standard Weiner disturbance term.

Different extensions of the Geometric Brownian motion model have been made to add other factors such as seasonality, stochastic interest rates and stochastic jumps. However, most commodity price models use processes with mean reversion, since they consider the microeconomic fact that prices tend to a long term mean due to variations in supply and demand (Hahn and Dyer, 2008). In these models the logarithm of the commodity price follows an Ornstein-Uhlenbeck process with mean reversion:

$$dY_t = \kappa(\bar{Y} - Y_t)dt + \sigma dz \tag{1.4}$$

Where $Y_t = \log(S_t)$ is the logarithm of the commodity price S_t, κ is the mean reversion coefficient, $\bar{Y}$ is the logarithm of the long term average price, σ is the volatility and dz is the Wiener standard process. The use of the logarithm of the price instead of the price is based on the assumption that commodity prices are lognormally distributed (Brandão and Dyer, 2005).

In addition to the economic argument that supports the mean reverting behavior of commodity prices, empirical studies of historical data have shown that mean reverting models can accurately model the commodity price behavior (Schwartz, 1997) (Bessembinder et al., 1995). In terms of real options valuation, several authors have stated that to use the geometric Brownian process to model prices that are indeed mean reverting can result in an overestimation of the value of flexibility in the net present value of a project (Hahn and Dyer, 2008).

1.5.2.1. Mean reverting binomial model

As discussed in Sec. 1.5.2, mean reverting models are usually used to represent the evolution of commodity prices. For this reason, an extension of the binomial model presented in Sec. 1.5.1.3 is proposed by Hahn and Dyer (Hahn and Dyer, 2008) to cope with the necessity of model more general distributions of prices. The model maintains the tree structure with fixed up and down rates but the nodes probabilities depends on the node value reflecting the local drift of the mean reversion. Considering the mean reverting movement shown in Eq. 1.4 the up and down probabilities are given by:

$$p_t = \max\left(0, \min\left(1, \frac{1}{2} + \frac{\kappa(\bar{Y} - Y_t)(\triangle t)}{2\sigma}\right)\right) \quad \text{(probability of up move)} \tag{1.5}$$

$$1 - p_t \text{ (probability of down move)}$$

A main feature of the problem studied in this work is the interdependence between decisions over time. The equipment condition represented by its failure rate at any state will depend on all the maintenance decisions taken since the initial period. The value generated by implementing, for example, an overhaul and then nothing is not equivalent to the value of doing nothing and then implement an overhaul. This path-dependence of the decisions breaks the recombinant structure of the binomial tree, increasing the number of nodes to 2^n for each decision period. The non-recombinant binomial tree is shown in Fig. 2.1. The main consequence of path dependent problems is the increase in the computational complexity, which limits the number of variables and the time horizon (Wang and De Neufville, 2005).

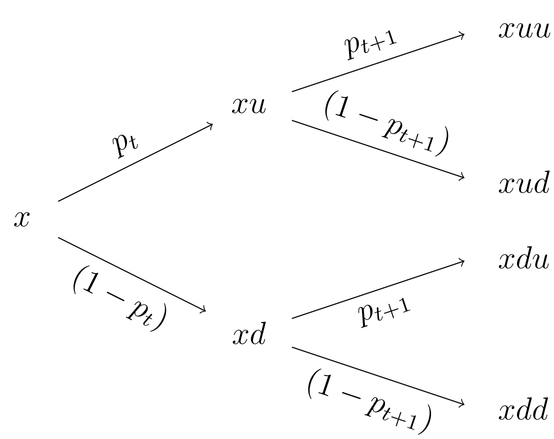

FIGURE 1.2. Non-recombinant binomial tree.

1.6. Objectives

The general objective of this work is to analyze the value of flexibility in joint overhaul and replacement decisions considering the external price uncertainty using a real options approach. In order to achieve this objective, the specific objectives are: (1) establish the value of flexibility using a mean reverting binomial model and compare with the periodic model, (2) analyze how optimal overhaul and replacement decisions change depending on the external price and (3) determine which are the variables that have significant impact on the flexibility value.

1.7. Hypothesis

The research hypothesis is:

(i) There is flexibility value in the definition of overhaul and replacement strategies in a commodity price uncertain environment using a real options approach

(ii) There are differences in the optimal decisions between the flexible binomial model and the periodic model

(iii) There is a relation between the maintenance decisions and the commodity price scenario

1.8. Thesis outline

The thesis is organized as a research paper presentation. In Chapter 1 an introductory section is presented. This chapter presents the problem background, problem statement, theoretical framework and the work objectives. Chapter 2 is the paper presentation itself. Finally, general conclusions and future work suggestions are presented in Chapter 3.

2. A REAL OPTIONS APPROACH FOR JOINT OVERHAUL AND REPLACEMENT STRATEGIES WITH MEAN REVERTING PRICES

2.1. Introduction

In asset, intensive and resource-based industries, such as gas, petrochemicals, energy, forestry and mining, investment and maintenance policies can significantly affect firms profitability. As a result, maintenance policies are a part of strategic and operational decision-making in these industries, and maintenance costs can account for a significant portion of their operational costs. Breakdowns and downtimes also impact plant capacity, product quality, and production costs, as well as health, safety and the environment (Parida and Kumar, 2006). In addition, commodity-based industries face significant price volatility and cyclical fluctuations; the rapid (and often unexpected) transitions between price booms and price slumps are challenging issues for decision makers in these resource-based industries to deal with. Cashin et al. (2002).

Due to price uncertainty and the impact of maintenance decisions on the operating costs, one might expect managers to devote significant attention to these decisions. But in practice, most firms base their equipment maintenance strategies on deterministic price scenarios and aim only to minimize the present value of lifetime equipment costs (Kim and Thomas, 2013). Also, most existing maintenance models focus on studying and modeling equipment failure and repair processes, with little or no attention paid to the effect of product prices on maintenance policies. Even in the case of power plants, where price is the determinant factor in the production process, maintenance decisions consider it a production loss cost rather than part of the income or profit function (Carazas and Souza, 2010).

Alsyouf (2007) suggests an explanation for managers failure to take product prices into account in the analysis of maintenance decisions. Using the Swedish paper-mill industry as a case, he defines profitability as the product of productivity and price recovery (Sumanth, 1998). Since price recovery is directly related to the product market price, and in the case of commodity industries, prices cannot be controlled by the firm; maintenance policies is the preferred mechanism to improve the productivity and profitability of the firm. With improvements in profitability which can go up to a as much as 12.5%, managers mainly focus their attention on maintenance policies rather than considering the product price in their decision process.

While real options methodology has not been extensively used in the maintenance context, it has received attention in operations research because it allows for the integration of both operational and financial considerations within a stylized, but representative, modeling setting (Ding et al., 2007). The methodology offers a two-stage framework for a firms decision to do nothing, overhaul equipment, or replace equipment. In the first stage, this decision is made in the presence of price uncertainty; in the second stage, after observing prices, the firm exercises its option to do nothing, overhaul or replace, and failures are corrected by minimal repairs.

Our primary contribution is the integration of existing Periodic Maintenance (PM) models with Real Options (RO) methodology, allowing managers to account for price variability in the maintenance decision process. Combining these two methodologies extends the scope of the existing models by introducing the following improvements: (i) accounting for uncertainty in price, (ii) considering a flexible, rather than a periodic, policy, (iii) allowing for the creation of contingency plans for different price scenarios. We compare the standard PM model with a version that uses real options, analysing the differences between the maintenance decisions each prescribes. We determine that the addition of flexibility generates value. Finally, we perform a sensitivity analysis to examine which variables significantly affect the real-options decision process.

The rest of this paper is organized as follows. We first present a literature review, focusing on real options applications, uncertainty in maintenance models and resolution methods. Section 2.3 describes two models: the conventional PM model and the proposed RO model. In section 2.4, we present the mean reversion binomial model. We then proceed to present a numerical case study with information from the mining industry, which validates our results and presents a sensitivity analysis. In section 2.7, we discuss the implications of our results and present possible extensions of the model and future work. Finally, Sec. 2.8 concludes.

2.2. Literature review

Maintenance activities are usually classified into two categories: corrective maintenance, which is applied once a system has failed, and preventive maintenance, in which the equipment is maintained while still operative. Preventive maintenance aims to reduce the probability of failure or deterioration of the equipment. Normally, this maintenance is either time-based (performed at defined time intervals), or condition-based (based on predetermined criteria for the condition of the equipment) (Wu and Zuo, 2010). Mathematical models for maintenance activity vary based on the type of maintenance (minimal, imperfect or perfect), replacement type (preventive or corrective) and cost structure (Wang, 2002). Most of these models account for uncertainty in the equipment failure process, but in commodity industries such as mining, forestry, oil and gas, commodity price volatility introduces an additional source of uncertainty.

Real options account for the possibility of making changes in any given project or investment after previously unknown information is revealed. Mun (2006) defines real options methodology as: a systematic approach and integrated solution, using financial theory and economic analysis, in applying options theory in valuing real physical assets in a dynamic and uncertain business environments. This methodology, initially used to value financial derivatives, has been extended to the valuation of flexibility in highly uncertain environments (Gunther McGrath and Nerkar, 2004). Usually, this flexibility includes the option to expand, contract or abandon a project (Dixit and Pindyck, 1994). The wide range of potential applications for real options includes natural resources investments, R & D projects, operations management, market competition and revenue management (Zeng et al., 2011). Real options research has focused primarily on investment decisions, but operations research (OR) applications have also proven to be useful (Crasselt and Lohmann, 2016). Examples of real options applications at an operational level include production planning, machine flexibility, material handling system flexibility and operational flexibility (For a survey on real options in OR see Zeng et al. (2011)).

Real options has been used to model operational planning in volatile price environments. Dalal and Alghalith (2009) study production decisions under production and price uncertainty. Song (2009) considers the problem of production and preventive maintenance control in a system subject to multiple uncertainties, such as random customer demands, machine failure, repair and stochastic processing times. Lim (2013) present a joint optimal pricing and order quantity model, in which demand is modeled as a function of external price and cost is modeled as a function of quantity. But little attention has been paid to the use of real options from a maintenance perspective (Jin and Ni, 2013). Maintenance strategy literature has traditionally considered uncertainty by stochastically modeling internal equipment parameters such as failure times, operational and maintenance costs, and failures conditions, among others (Abdel-Hameed, 2013). However, some recent contributions have considered external uncertainties that affect maintenance and replacement decisions. Mardin and Arai (2011) consider a system dynamics model for overhaul and replacement policies subject to the emergence of new technologies that compete with currently used equipment. Nguyen et al. (2011) propose solving for the optimal overhaul and replacement policy by modeling profit flows and new technology purchase prices as stochastic processes. Richardson et al. (2013) study the problem of determining when to order replacements for equipment with long and uncertain delivery lead times using simulations in a real options framework. No previous work has considered the effects of commodity price uncertainty in the definition of joint equipment overhaul and replacement strategies.

Andersson (2007) studied almost 300 different commodities over a time period of 36 years (1970-2006) and found clear evidence that commodity prices are mean-reverting, in contrast to other financial assets. The economic argument to support mean-reverting behavior is that high commodity prices stimulate investment to increase production as well as the development of alternative products, resulting in price decreases. The opposite occurs when prices are low; as a result, the price tends to revert to a long term mean. Other empirical studies also support the idea that mean-reverting models can accurately model the evolution of commodity prices (Schwartz, 1997; Bessembinder et al., 1995). It is common that when determining the value of real options in operations problems, which usually involve interdependence between decisions across time periods, the preferred modelling approach is a discrete one(Bengtsson, 2001). The preferred method to model mean-reverting processes in discrete time are multinomial lattices (Cox et al., 1979) and Monte Carlo simulation (Boyle, 1977). However, for path-dependent options, these approaches present significant limitations due to computational complexity and practical implementation constraints (Lander and Pinches, 1998). The advantages of a binomial model over alternative valuation methods include its versatility, easy implementation and precision. Moreover, in contrast to other methods that resemble a black box, binomial models visually map out decision processes in the optimal decision tree, allowing for further analysis (Bastian-Pinto et al., 2010). Extensions of this model include the use of multi-factor mean-reverting processes (Wang and Dyer, 2010), non-censored binomial mean-reverting processes (Bastian-Pinto et al., 2010) and binomial models with changing volatility (Haahtela, 2010).

There is an ample range of studies that use binomial trees to represent mean-reverting processes in the financial and commodity sectors. In the area of commodity prices, Slade (2001) applies a mean-reverting process to metal prices in order to value options in a mining operation. Hahn and Dyer (2008) consider an oil and gas switching option which requires a binomial model of two correlated one-factor commodity price models. Bastian-Pinto et al. (2010) extend the model presented by Hahn and Dyer and evaluate the option to expand considering bio-fuels prices. In the financial arena, Hull and White (1994) uses trees to represent and value interest rate derivatives and Jaillet et al. (2004) use multinomial trees to value swing options. To represent the mean-reverting behavior of commodity prices, we use the model proposed by Hahn and Dyer (2008) and analyze how these price changes affect maintenance and replacement decisions.

One challenge in relying on real options to formulate operational problems is the latters path dependency. A maintenance decision made in any given period depends not only on the prevailing commodity price, but also on previous maintenance decisions, which directly affect the equipment failure rate. This augments the recombining structure of the binomial tree originally presented by Cox et al. (1979); instead of having N+1 nodes at time N, the non-recombinant tree has 2^N nodes. Backwards deduction heuristics have been used to solve an optimal decision tree using the binomial lattice approach (Bertocchi et al., 2006; Brandão and Dyer, 2005; Li and Kouvelis, 1999; Li et al., 2009; Messina and Bosetti, 2006). To determine the optimal maintenance policy and the value of flexibility in the option tree, we will start at the end of the tree and work backwards with a dynamic programming approach according to a risk-neutral valuation and risk-neutral probabilities (Huchzermeier and Loch, 2001).

2.3. Model formulation

Our model is inspired by Zhang and Jardine (1998), who aim to determine the optimal overhaul and replacement policy for a single asset. We add two characteristics to their model. First, we include the opportunity for the decision maker to choose between three possible actions at any given time: minimal repair at failure, overhaul or replacement. Second, we maximize profit instead of minimizing cost, subject to a mean-reverting binomial process of product prices. Each maintenance action will have a different outcome, which is modeled as a reduction in the failure rate, cost and profit loss due to equipment unavailability. Product prices are modelled and determined externally through a mean reversion binomial model.

2.3.1. Failure rate reduction model

Lets consider $\lambda_{k-1}(t)$ as the equipment failure rate just before time $(k-1)$ and $\lambda_k(t)$ as the failure rate just after the overhaul. T corresponds to the time since the last overhaul and $p \in (0,1)$ a constant denoting the degree of improvement of the overhaul. For any time t after an overhaul, the failure rate can be expressed as:

$$\lambda_k(t) = p\lambda_{k-1}(t-T) + (1-p)\lambda_{k-1}(t) \tag{2.1}$$

Note that if the improvement factor $p = 0$ then $\lambda_k(t) = \lambda_{k-1}(t)$ which corresponds to a minimal repair, in which the failure rate is not altered. On the other hand, if $p = 1$ then

$\lambda_k(t) = \lambda_{k-1}(t - T)$ which returns the failure rate to the condition of the equipment at the previous overhaul period.

2.3.2. Failure cost model

The discounted cost due to minimal repairs in a time interval $(0, t)$ depends on the minimal repair cost C_m and the cost of the lost production while the equipment was in maintenance (T_m):

$$(C_m + xT_mP) \int_0^t \lambda(t)e^{-\theta t}dt \tag{2.2}$$

Where $\lambda(t)$ corresponds the failure rate, $e^{-\theta t}$ is the discount factor, x is the commodity price and P is the equipment productivity.

2.3.3. Periodic Maintenance model

As a base case, we will use a standard model of periodic overhaul and replacement interventions which minimizes the total discounted cost in an infinite time horizon. The decision variables are: the number of overhauls implemented in a replacement cycle (N) and the time between overhauls (T). The replacement is performed in the period $t = NT$. The discounted cost function for the first replacement cycle can be expressed as:

$$C_1(N,T) = \sum_{i=1}^{N-1}(C_o + xT_oP)e^{-i\theta T} + (C_m + xT_mP)\int_0^{NT} \hat{\lambda}(t)e^{-\theta t}dt + (C_r + xT_rP)e^{-\theta NT}$$

Where $\hat{\lambda}(t)$ is the effective failure rate at time t due to the periodic policy (N, T). The discounted cost function for n production cycles is:

$$C_{\text{n cycles}}(N,T) = C_1 + C_2e^{-\theta NT} + \ldots + C_ne^{-(n-1)\theta NT}$$

Since $C_1 = C_2 = \ldots = C_n$, in a infinite repetition the total discounted cost can be expressed as:

$$\min_{N,T} \frac{C_1}{(1 - e^{-\theta NT})} \tag{2.3}$$

2.3.4. Real Options Model

If we consider a finite time horizon, which can be divided into T time intervals of equal length the profit in each period depends on price and the equipment performance. So if we relax the condition for a fixed policy, as was the base case, and all other assumptions remain unchanged. The main difference between the Real Option (RO) model and the Periodic Maintenance (PM) model is that the commodity price varies at each period T. Since Andersson (2007) shows that commodity prices follow a mean reverting process, we model the logarithm of the commodity price x_t as a one-factor OrnsteinUhlenbeck process (Schwartz and Smith, 2000), defined by the folowing equation:

$$dY_t = \kappa(\bar{Y} - Y_t)dt + \sigma dz \tag{2.4}$$

Where $Y_t = \log(x_t)$ is the logarithm of the commodity price, κ is the mean reversion coefficient, $\bar{Y}$ is the logarithm of the long term average price, σ is the volatility and dz is the Wiener standard process. For notation we consider $\hat{\lambda}_{k-1}(t)$ as the failure rate after an intervention (overhaul or replacement) and $\hat{\lambda}_k(t)$ the failure rate immediately after it.

For each period (T) there are three possible decisions or options:

- *Overhaul* : The failure rate is reduced by a factor $p \in (0, 1)$ at a cost C_o. The profit in period t given this option is:

$$\pi_t = x_t P - (C_m + x_t T_m P) \int_t^{t+1} \hat{\lambda}(t) e^{-\theta t} dt - C_o - x_t T_o P$$

- *Replacement*: The equipment is replaced at a cost C_r. The failure rate immediately after replacement is the original rate and the profit in period t given this option is:

$$\pi_t = x_t P - (C_m + x_t T_m P) \int_t^{t+1} \hat{\lambda}(t) e^{-\theta t} dt - C_r - x_t T_r P$$

- *Do nothing*: The deterioration of the component continues and the profit in period t given this option is:

$$\pi_t = x_t P - (C_m + x_t T_m P) \int_t^{t+1} \hat{\lambda}(t) e^{-\theta t} dt$$

In order to determine the best maintenance policy, the decision maker will maximize expected profit from interval t to the final interval T, which is given by the following recursive equation:

$$V_t(x_t, R^*_{t-1}) = \max_{R_t} \pi_t + \mathbb{E}\{V_{t+1}(x_{t+1}, R^*_t) e^{-\theta}\} \tag{2.5}$$

where R^*_{t-1} corresponds to the real option selected at time interval $t-1$, R^*_t is the real option selected in the current time interval and $\mathbb{E}\{V_{t+1}(x_{t+1}, R^*_t)e^{-\theta}\}$ is the expected value for the next period. So $V_t(x_t, R^*_{t-1})$ is the total expected value from interval t to the final interval.

2.4. Mean reversion binomial model

In order to implement the real option model, we will use a binomial lattice process which allows us to model the price fluctuations in discrete intervals and adds the flexibility of real options. In order to model the commodity price mean reversion binomial process, we will use the one proposed by Hahn and Dyer (Hahn and Dyer, 2008). They propose a binomial sequence of n periods of length $\triangle t$ in a time horizon T. The objective is to find a binomial sequence that converges to a general differential equation of the form: $dY_t = \mu(Y,t)dt + \sigma(Y,t)dz$, where $\mu(Y,t)$ and $\sigma(Y,t)$ is the instantaneous drift and standard deviation functions. Note that, in contrast with the general binomial model, both the drift and deviation are functions of time. As we assume a simple mean reverting process, $\mu(Y,t) = \kappa(\bar{Y} - Y_t)$ and $\sigma(Y,t) = \sigma$. So the up and down transition probabilities which converge to the mean reverting process can be modelled by the following expressions:

$$p_t = \max\left(0, \min\left(1, \frac{1}{2} + \frac{\kappa(\bar{Y} - Y_t)(\triangle t)}{2\sigma}\right)\right) \text{ (Price up probability)} \quad (2.6)$$

$$1 - p_t \text{ (Price down probability)}$$

As it can be observed, price probabilities are bounded to values between 0 and 1 and commodity prices follows over time, a mean reverting pattern of increase and decrease in each node given by $u = e^{\sigma\sqrt{(\triangle t)}}$ and $d = e^{-\sigma\sqrt{(\triangle t)}}$, respectively.

An important source of complexity for this problem is the fact that maintenance decisions are path dependent, since the failure rates and overhaul effects are dependent on the previous maintenance decisions. The replacement decision is the only one that resets the system to its initial state. The current commodity price is also dependent on the path. In the case of a recombinant decision trees, the path followed to reach a given node is not affected by history. Fig. 2.1 shows how the path dependency of a binomial tree. In this case there are 2^t nodes at each period instead of $t + 1$ in the recombinant case, which significantly increases the computational complexity of the problem.

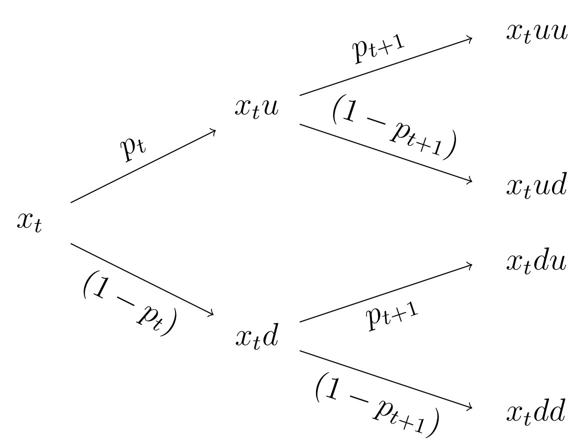

FIGURE 2.1. Non-recombinant binomial tree.

To construct the decision tree, we start by initialize the root ($t = 0$) from which two states of nature nodes follow: the up and down price states. From each state of nature follows the decision nodes, which account for the three real options possibilities: do nothing, overhaul or replacement. For each decision node we determine the profit, obtaining all the possible decision paths.

Once the decision tree is generated, to determine the optimal maintenance decisions we can conveniently solve by working backwards recursively from the end of the tree (Rubinstein, 1994). For each state of the nature node at time t, the algorithm selects the child node which maximizes the profit in time t plus the expected value of the previous nodes:

$$V_t(x_t, R_t) = \max_{R_t} \left\{ \pi_t(x_t, R_t) + \frac{\mathbb{E}[V_{t+1}(x_{t+1}, R_{t+1})]}{(1+r)} \right\} \tag{2.7}$$

where,

$$\mathbb{E}[V_{t+1}(x_{t+1}, R_{t+1})] = P_t^u \cdot V_{t+1}(u \cdot p_t) + P_t^d \cdot V_{t+1}(d \cdot p_t)$$

The expected NPV for the RO model is obtained by taking the pondered sum of all the optimal path in the tree, which corresponds to the value of the initial node V_0.

Algorithm 1 Solution algorithm

for all node **in** state of the nature level **in** $t = T$ **do**
  select the highest profit π_t **in** node children and save as node value V_t
end for
for $t = (T-1)$ to 1 **do**
  for all node **in** decision level **do**
    for all node **in** children nodes **do**
      compute NPV as the sum of node value pondered by node probability
      $\text{NPV}_t = \text{NPV}_t + P_{t+1}V_{t+1}$
    end for
    compute node NPV $\text{NPV}_t = \text{NPV}_t + \pi_t$
  end for
  for all node **in** state of the nature level **do**
    select the highest NPV in node children and save as node value V_t
  end for
end for

The PM and RO models considers the following assumptions:

(i) A single equipment in an intensive productive system.
(ii) n decision periods in a fixed time horizon of evaluation $T = n\Delta t$.
(iii) Equipment is subjected to three types of actions: minimal repair, imperfect overhaul, and replacement; each action has its own associated costs $(C_m < C_o < C_r)$ and implementation times $(T_m < T_o < T_r)$.
(iv) Equipment has an exponential failure rate $\lambda(t) = e^{\beta_0 + \beta_1 t}$.
(v) An overhaul improves the equipment in a fixed degree $(0 < p < 1)$ affecting directly its failure rate.
(vi) A minimal repair is implemented after every equipment failure and does not modify the failure rate.
(vii) The overhaul and replacement options are available for each decision period.

(viii) The commodity price is uncertain and presents a mean reverting behaviour.

(ix) The long-term mean commodity price is $x = \bar{x}$.

(x) Price volatility (σ) and mean reversion parameters (κ) are constants.

2.5. Numerical case study

We now present an application of our model to a mining industry case, which uses similar parameters to those used by Pascual et al. (2016) specifically, a production unit with a single critical repairable component which operates continuously over $T = 8$ decision periods. The failure rate is modeled by $\lambda(t) = e^{\beta_0+\beta_1 t}$. The overhaul improvement degree is $p = 0.7$. The minimal repair, overhaul and replacement costs are $C_m = 6$, $C_o = 45$ and $C_r = 100$, respectively. The discount rate is $\theta = 0.08$ and the failure rate parameters are $\beta_0 = 3$ and $\beta_1 = 0.18$. The long term price, volatility and mean reverting parameter are $x_0 = 3$, $\sigma = 0.1$ and $\kappa = 0.15$, respectively.

To solve the RO model, the algorithms were coded using Python (v2.7) and the ETE 3 (Huerta-Cepas et al., 2016) toolkit for tree-like structure analysis and visualization. The code was executed using using a laptop with 8GB of RAM with a 2.5GHz dual-core Intel Core i5.

2.6. Results and sensitivity analysis

In this section we present the results obtained using the binomial lattice model. We compare the optimal decisions and expected present value under the RO model with those of the PM model to show the impact of incorporating commodity prices into maintenance decisions.

Our results show a 10.5% difference in Net Present Value (NPV) between the RO model and the PM model (N,T) 219 vs. 198, respectively (see Table 2.1). This difference of 10.5%, is a result of the flexibility to adjust maintenance policies based on price, which is embedded in the real options model. It is also known as the value of flexibility (Kulatilaka, 1995).

TABLE 2.1. Net Present Value (NPV) difference between the PM and RO models

	PM (N,T)	RO	Diff
NPV	198	218.8	10.5%

TABLE 2.2. Comparison of the High Prices Policy (HPP), Low Prices Policy (LPP) with the optimal PM policy.

	HPP	LPP	PM policy
Overhaul periods	4	3,6	2,6
Replacement period	3,6,8	5,8	4,8
Percentage of paths (%)	25	75	-
Average equipment availability (%)	87	83	85

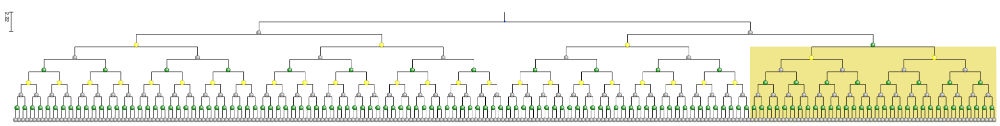

FIGURE 2.2. Optimal decision tree for the case study. Right and left nodes denote up and down price movements respectively. Do nothing, overhaul and replacement options are in colors gray, yellow and green, respectively.

To compare how optimal decisions change when price uncertainty is taken into account, we define two policies that depend on the price scenario: (i) High Prices Policy (HPP) corresponds to the optimal policy paths which have over a 25% of the times, a increase in the product price; and (ii) Low Prices Policy (LPP) corresponds to the optimal policy paths which have over a 25% of the times, a decrease in the product price. A visualization of optimal decisions in the binomial tree is presented in Fig. 2.2 (orange shows a group of HPP). Table 2.2 presents a comparison of the actions taken under HPP, LPP and the PM policies.

The PM model suggests that overhauls should optimally be implemented every two periods, in a replacement cycle of $NT = 4$ periods ($(N, T) = (2, 2)$). Table 2.3 shows the distribution of the different decisions along the different decision periods. We can observe that in the last three periods, the optimal decisions for all the paths are the same, so there is no difference for any price path. The main difference is in the previous periods where the replacement decision is moved from $t = 8$ in the PM model to $t = 7$ in the RO model.

TABLE 2.3. Portion of paths per decision per decision period.

Option	portion of paths (%)							
Do nothing	100	75	75	25	75	100	0	100
Overhaul	0	0	25	0	0	0	0	0
Replacement	0	25	0	75	25	0	100	0
Decision period	1	2	3	4	5	6	7	8

TABLE 2.4. Cost structure sensitivity analysis

	$C_m = 4$					$C_m = 5$					$C_m = 6$				
	PM policy		NPV			PM policy		NPV			PM policy		NPV		
C_r/C_o	N	T	RO	PM	ΔV	N	T	RO	PM	ΔV	N	T	RO	PM	ΔV
2	1	3	477	455	4.8%	1	3	345	312	10.6%	2	2	212	169	25.4%
2.5			453	436	3.9%	2	2	312	293	6.5%			172	150	14.7%
3	2	3	434	417	4.1%	3	2	293	273	7.3%	3	2	153	131	16.8%
3.5			414	397	4.3%			273	254	7.5%			133	111	19.8%

We can observe a relationship between prevailing prices and the levels of equipment availability. HPP scenarios have an average equipment availability of 87%, compared to an availability of 85% in the PM model. When prices are high, the flexible model seeks to increase equipment availability in order to take advantage of them. When prices are low, on the other hand, the decision maker sacrifices equipment availability in order to avoid replacement and overhaul costs, resulting in an average equipment availability of 83%. Thus, the RO approach provides a contingency plan, with two different policies to be applied depending on the commodity price.

The NPV and the value of flexibility is significantly affected by the the replacement and overhaul cost ratio (C_r/C_o) and the minimal repair cost (C_m) parameters. The effect of these parameters over the NPV of the PM and RO model is presented in Table 2.4. The NPV of both PM and RO models is reduced as the replacement and overhaul cost ratio increases, which comes from the direct effect of the replacement cost over the income. In the case of the flexibility value, it increases as the minimal repair cost C_m increases, and its increment depends on the corresponding PM policy interval. However, the value of flexibility differences remain within the same order of magnitude with an average value of $\Delta V = 4.3\%, 8\%$ and 19.2% for $C_m = 4, 5, 6$ respectively.

TABLE 2.5. Aging parameter sensitivity analysis

	PM policy		NPV		
β_1	N	T	RO	PM	ΔV
0.04	2	3	502	443	13%
0.06			444	396	12%
0.08			400	347	15%
0.14	2	2	288	268	7%
0.16			250	233	7%
0.18			218	198	10.5%
0.2			194	160	21%
0.22	1	2	168	149	12.8%
0.24			142	126	12.7%
0.26			116	103	12.6%
0.28			88	78	12.8%

Performing a sensitivity analysis on the aging parameter shows that it also has a significant impact on NPV and flexibility value. For an exponential failure rate, the aging parameter is defined as β_1 in the equation $\lambda_0(t) = e^{\beta_0+\beta_1 t}$. Table 2.5 shows the optimal PM policy and NPV for different values of β_1. As with the cost structure, different values of β_1 result in different optimal policies for the PM and RO models. For both PM and RO models, an increase in the aging parameter results in a NPV decrease and the flexibility value depends on the PM policy. For the PM policies $(N, T) = (2, 3), (2, 2)$ and $(1, 2)$ we obtained average NPV differences between the PM and RO models of $\Delta V = 13.6\%, 11.5\%$ and 12.7%.

Since the RO model reacts to price changes and prices follow a mean-reverting process, it is important to analyze the effect of the volatility of prices over the flexibility value. Fig 2.3 shows the impact of varying the volatility of the commodity price from 10% to 27% on the flexibility value. Over this range, the flexibility value for the mean-reverting model increases by 13% (from 20.7 to 23.4) and the NPV increases by 40% (from 218 to 307). When volatility is higher, prices rise and fall more erratically, increasing the value of postponing or changing maintenance decisions. As a result, including commodity price considerations in the definition of overhaul and replacement policy is more valuable in highly uncertain environments.

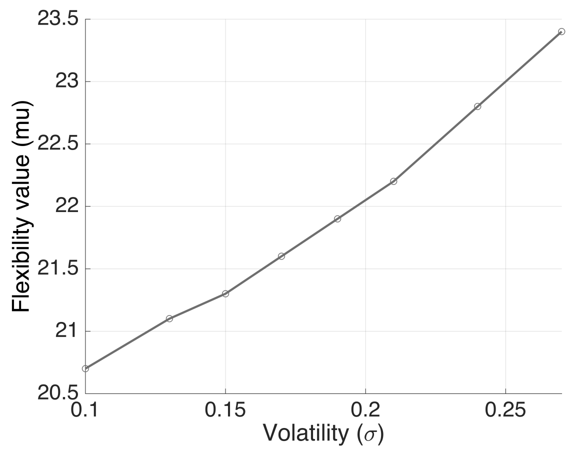

FIGURE 2.3. Effect of the commodity price volatility (σ) on the value of flexibility.

2.7. Discussion

Equipment overhaul and replacement strategy has a significant impact on the profitability of a production system. Typically, these strategies consist of a fixed calendar of periodic interventions based solely on minimizing total equipment costs; this approach ignores price changes or the possibility of revising and updating maintenance policies. We have shown that using real options to introduce the possibility of re-evaluating overhaul and replacement decisions and the flexibility to respond to price changes adds significant value, increasing NPV by as much as 10.5%.

The finding that a flexible maintenance policy adds value is consistent with previous work that applies flexible models to other external variables, such as technological change or replacement lead times (Richardson et al., 2013; Mardin and Arai, 2011). When prices are high, the strategy calls for more overhauls and replacements, decreasing the equipment failure rate and increasing productivity, which allows the firm to take advantage of the higher prices for its product. In practice, the RO model allows decision makers to consider a set of policies that can be considered a contingency maintenance plan based on the observed commodity price.

The relationship between minimal repair, overhaul and replacement costs has a significant effect on optimal maintenance policies. Specifically, the ratio between replacement and overhaul costs determines the strategies recommended by both the PM and RO models. Results show that a higher cost ratio tends to lengthen replacement cycles for both models, encouraging more overhauls to control aging. The value of flexibility is significantly affected by the minimal repair cost and the equipment aging parameter. These variables have a similar effect since option value increases along with how important and irreversible is the decision in question (Dixit and Pindyck, 1994).

In the proposed model, price variability and mean-reverting behavior have an impact on the value of flexibility. The volatility of the commodity price is directly proportional to this option value. This finding results from the fact that flexibility is more valuable in more volatile environments, and is consistent with previous works that have applied the binomial model to model commodity prices such as oil and gas and metals (Hahn and Dyer, 2008).

The proposed model assumes that maintenance decisions are integrated and the implementation of a flexible strategy does not incur additional costs. In reality, maintenance on asset intensive industries is normally performed in an owner-client relationship through maintenance contracts (Pascual et al., 2016). This situation may lead to additional costs (flexibility costs) associated with changes in maintenance decisions, which are not considered in this model. In addition, production chains and equipment fleets in asset intensive industries usually work together. The inclusion of more than one productive asset in the proposed model may present relevant new insights. For industries with more complex price and volatility patterns, a binomial model might not accurately represent reality and a flexible model may be less attractive.

2.8. Conclusion

We have developed and implemented a model that defines an optimal equipment overhaul and replacement policy in a volatile price environment by integrating a real options methodology with a traditional periodic maintenance model. We consider a single asset and adopt a discrete approach with a mean-reverting binomial process to model price behavior. Decision makers can choose to implement an overhaul or a replacement in every time period, and failures are corrected by minimal repairs.

Our results show that the economic benefits from of the RO approach with exceed those of the PM models typically used by managers, due to the addition of flexibility in their decision process. This added flexibility value stems from two factors: the opportunity to account for price volatility and the possibility of reevaluating maintenance decisions after they are made. Sensitivity analysis shows that the optimal decisions suggested by the RO model change based on the commodity price scenario, which provides managers a maintenance contingency plan.

The proposed methodology does not consider the impact of any costs incurred from the process of changing maintenance policies. Adding such costs would reduce the flexibility value, and further research is required to determine the potential effect of this factor. Finally, the non-recombinant nature of the binomial model imposes a limitation on our work, since the number of price scenarios increases exponentially as the time horizon lengthens. A possible extension of the proposed model is to consider algorithms that may reduce computational complexity. Stochastic programming is an alternative that can reduce the number of scenarios, reducing computational complexity in comparison with the binomial model (Hu et al., 2012). This extension would allow for extensions of the proposed model to more complex scenarios that could include multiple equipment suppliers, warranty contracts or other sources of uncertainty, such as technology improvements or maintenance with incomplete information.

3. GENERAL CONCLUSIONS AND FUTURE RESEARCH

Our contribution was to analyze the value of flexibility in the definition of overhaul and equipment replacement strategies that considers the external price factor in a commodity industry using a real options approach. We compared the value generated by a single asset in a capital intensive industry with a typical periodic maintenance policy with a flexible model using real options in a defined time horizon. To model the evolution of the commodity price we use a binomial model with mean reversion. The model is particularly useful for supporting flexible maintenance decisions in industries where the price of the product presents high volatility and high maintenance and replacement costs such as the mining industry, forestall, energy or gas.

The obtained results show that the model with real options significantly outperforms the periodic maintenance model based on the minimization of the total costs. We showed in the optimum binomial decision tree how the price scenarios affect maintenance policies allowing to delay or anticipate decisions depending on the commodity price in order to maximize the profit. In particular, there is a relationship between the availability of equipment and the price level in which for high prices paths there is an increase of overhaul and replacement implementations. This graphical tool allows to support the decisions of the companies and can provide a set of different policies to apply according to the observed price.

The main parameters that affect the value of flexibility are the equipment cost structure and the aging parameter. The use of a real options approach becomes more attractive for assets in which the ratio of overhaul and replacement costs is higher and for equipment that have a high rate of deterioration over time. For the price of the commodity, variations in price volatility showed that as the price presents greater volatility the value of flexibility increases proportionally. Further research is needed to better understand the cost of considering flexible maintenance policies rather than periodic ones. The model presented considers that the decisions are direct and there are no intermediaries but normally the supplier companies establish maintenance contracts in a relationship that may include guarantee contracts. Including flexibility in the definition of these contracts can result in a cost increase for both parties and even an increase in operational planning effort within the company. Considering this cost would reduce the value of using a real options model for maintenance decisions.

One of the limitations discussed in this work was the computational complexity that arises from the path dependent nature of the problem. Future work may include algorithms that allow to reduce the possible scenarios to evaluate reducing the complexity of resolution. This reduction of complexity will allow to extend the scope of the model by allowing to include relevant industry factors. A possible extension is to consider that the production is carried out by a fleet of equipment instead of a single asset, or consider that the purchase of a new equipment when replacing can be made to more than one equipment supplier. Another interesting extension is to include other sources of uncertainty. A relevant uncertainty for maintenance decisions is to consider the emergence of new technology in the future and to analyze the impact on profitability by the costs reductions and the increased efficiency.

The presented work provides an option-based methodology to analyze the value of flexibility in the definition of equipment overhaul and replacement policies considering the commodity price uncertainty. The results show that the possibility to re-evaluate the decisions and the addition of flexibility in overhaul and replacement decisions considering the uncertainty of the price adds significant value. Further analysis is required to include more complexities of the overhaul and replacement decision process.

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