PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
INSTITUTO DE ECONOMÍA

ESSAYS ON ECONOMICS OF THE FAMILY

ALEJANDRO CARLOS SIERRA CAMPODÓNICO

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Ph.D. in Economics

Advisors:
ALEXANDRE JANIAK
JEANNE LAFORTUNE

Santiago de Chile, June 2023

PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
INSTITUTO DE ECONOMÍA

ESSAYS ON ECONOMICS OF THE FAMILY

ALEJANDRO CARLOS SIERRA CAMPODÓNICO

Members of the Committee:
ALEXANDRE JANIAK
JEANNE LAFORTUNE
EMILIO DEPETRIS-CHAUVIN
MARION GOUSSÉ

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Ph.D. in Economics

Santiago de Chile, June 2023

For my family's ceaseless belief, specially my mother, my partner's unyielding support, and the invaluable camaraderie of friends and fellow Ph.D. students, I dedicate this thesis to you.

ACKNOWLEDGEMENTS

I want to begin by thanking my parents, Alejandro and Gloria. Their unconditional love has been the basis of everything. To my father, for his love and empathy, and to my mother for her love, empathy, strength, patience and support. Without you, none of this would have been possible.

I cannot fail to mention my sister Titi, always there with her sympathy and support.

To my partner, Carmen Maura, I thank you for your unconditionality, for your constant support, your patience, affection and for being my companion in this journey and those to come. Thank you for making me a better human being.

To my childhood friends, the ”Larrys”, your support and good vibes were always there when I needed it most. Special thanks to Felipe and Piña, without you this road would have been much harder.

To my fellow PhD students, Sebastián Castillo, Sebastián Monroy, Francisca Torrealba, Matías Borquez and Jonathan Rojas, thank you for putting up with those sleepless nights of study, for the advice and for making this stage something memorable. Special thanks also to Sebastián Monroy and his beautiful family, who have always welcomed and supported us.

Many thanks to my guiding teachers: Alexandre Janiak for his trust, his hard work, dedication, patience, guiding me, advising me. Jeanne Lafortune for her patience, hard work and dedication. They are an inspiration to become a better economist.

I would like to thank Emilio Depetris-Chuvin and Marion Goussé for being in my degree committee.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

INTRODUCTION

Economics of the family covers mainly two significant strands of research: what happens inside the marriage and who marries whom (Lundberg and Pollak, 1993; Browning et al., 2014). There is vast literature on how individuals choose a partner and the associated implications on economic outputs and welfare (Becker, 1973; Boulier and Rosenzweig, 1984; Pencavel, 1998; Tertilt, 2005; Field and Ambrus, 2008; Chiappori et al., 2018a). There are two approaches in the marriage market literature, one that emphasizes search such as search frictions (Mortensen, 1988), and another that ignores frictions assuming that it is relatively easy to find a partner (Gale and Shapley, 1962; Shapley and Shubik, 1971; Roth and Sotomayor, 1992). In each approach, it is possible to distinguish between a context of "transfers," in which individuals can "compensate or reward" for their attributes and a context of "non-transfers," in which the couple must accept the observable characteristics of the other. For example, when people get together, it is not possible to divide the surplus obtained by getting married. From an economic perspective, with or without transfers, the marriage decision depends on the outside option and the marriage's utility (Weiss and Willis, 1997).

When matched individuals produce a good that they can consume together, the literature shows that having this common good affects the decision to stay in the relationship or not (Browning et al., 2013; Couprie, 2007; Goussé et al., 2017). This thesis uses this logic to evaluate the role of: (i) different search methods; and (ii) the use of violence by a partner .

The first chapter comes from the idea that in recent decades have seen an increase in homogamy in the United States and the arrival of new internet-based search methods for relationships, but no study has explored the potential link between those two changes. Using a novel database where existing couples report their search method, I document that more attractive individuals (in terms of education or age) are less likely to use alternative search mechanisms, like internet. I then build a model with heterogeneous agents and search frictions in which the search method is endogenously determined. The model indicates that the arrival of cheaper search methods leads to higher assortative matching, although it could be Pareto improving for some values of the search costs. I then test these predictions and find supportive evidence in the data.

The second chapter (co-authored with Alexandre Janiak and Jeanne Lafortune) is inspired by the fact that easier divorce has often been argued as a way to reduce intimate partner violence (IPV) but empirical evidence on this topic is mixed. In this chapter we propose a theoretical model where violence can be used either as a retention mechanism or in response to exogenous cues. When the model transitions from mutual consent to unilateral divorce, the incidence of cue-trigger violence decreases but that of coercive violence increases. The positive impact on violence occurs more strongly when transfers between spouses are not possible or when violence is more costly to the inflicting partner. The model thus predicts that when the cost of violence is higher or when it is more culturally allowed for spouses to transfer resources, divorce liberalization should lead to decreases in violence while the opposite would be true in more traditional settings. This reconciles the variety of estimates provided in the literature.

1. LOVE ME TINDER: POLARIZING EFFECTS OF ALTERNATIVE SEARCH METHODS IN DATING MARKET

1.1. Introduction

In recent decades there have been significant demographic changes in American households. The demand for college education has been increasing, especially for women. Assortative mating has also increased (Greenwood et al., 2016). This phenomenon has received much attention in the literature because partner choice has potential consequences for inequality[1], labor supply, human capital, fertility, aging, immigration, and other economic variables (Chiappori, 2020).

While a lot of attention has been given to who people match with, less focus has been placed on the way individuals match, despite a very big technological transformation in that market. This has often been interpreted as implying that preferences for spousal attributes have changed. In the 1990s, online methods began gaining popularity[2] and rapid growth worldwide (Bailey, 2010; Orr, 2004; Paumgarten, 2011). Rosenfeld and Thomas (2012) document a considerable increase in the number of couples that formed online at the expense of meeting through friends and family over this period. Bellou (2015) shows that the advent of higher-speed internet positively impacts marriage rates and that online methods are displacing traditional methods such as family, friends, or neighbors. Online dating is also a stronger predictor of transition to marriage for heterosexual couples (Rosenfeld, 2017).

Why are people using online methods to search for a spouse, and what is the consequence of this on patterns of matching? This paper tries to study how different search methods can impact marriage markets, both theoretically and empirically.

[1] Greenwood et al. (2014) shows that if the match patterns of the 2000s were like those of the 1960s, there would be a significant reduction in inequality.

[2] In the 1990s, several milestones benefited the spread of online methods: in 1995, the online dating site match.com was created. In 1998, e-mail became a massive and daily tool for Americans, and also, the movie "You've Got Mail" with Tom Hanks and Meg Ryan was released, bringing to popular culture the possibility of finding a partner through the internet (Team, 2021).

I use a database to motivate the model, that identifies the method couples have used to meet each other, I first show that more attractive (more educated and younger) individuals are less likely to use alternative search mechanisms. This would suggest that internet-based search methods are negatively selected.

Then I build a model with an endogenous selection of search methods, heterogeneous agents, and search frictions. I show that if one of the search method experiences a fall in its cost, equilibrium in the model is such that the least attractive individuals select with a higher probability the search method that is the least costly. More attractive individuals continue to employ the more expensive search strategy. That is, markets tend to polarize in terms of characteristics (i.e., attractiveness). Moreover, the fall in costs, although polarizing in nature, is a Paretian improvement in the welfare of the agents under some circumstances.

In the model, agents look for a partner in a world with search frictions and non-transferable utility and differ on their characteristics where these characteristics are supermodular in the utility of each partner. They must also select the method they use to search for a partner where they can use traditional or non-traditional methods, where search must be done exclusively. Agents can choose between two search methods, traditional and non-traditional, which incur usage fees. Individuals prefer to match with an attractive individual, but this is particularly strong for attractive individuals. They prefer any match to singlehood. I solve for an equilibrium that is robust to single and couple deviation showing that it is unique. I then show that if one method is cheaper than another, the likely outcome is that the least attractive individuals search using the cheaper method while the most attractive individuals stay in the expensive search method. This leads to perfect assortative mating. This matches my motivating stylized fact which is that least attractive people are using new search methods more intensively.

I then realize a comparative statics exercise where I lower the price of the alternative method and show that in addition to having perfect assortative matching (despite search frictions), the model also indicates that this change can be Pareto improving for some cost interval. Agents who are less attractive find a spouse with a higher probability and pay lower costs while more attractive individuals are happier because they are more likely to match with a better spouse.

The model's results are tested with the data on search methods used by existing American couples, I divide the search methods into traditional (high costs) and non-traditional methods including internet-based applications. I first show that more attractive individuals who find a match through traditional methods have a higher probability of matching with someone who is like them. This suggests that the capacity to segregate depends on the search method as suggested by the model. Furthermore, I show that this is stronger when internet reduces significantly the cost of alternative search methods. Thus, as in the model, markets become more polarized when the alternative method becomes less expensive. In addition, people who met through the traditional method have on average a better quality of relationship which matches the model prediction.

Using Norwegian data, Kirkebøen et al. (2022) constructs a causal identification strategy showing that colleges are local marriage markets. Going to an educational establishment increases the probability of marrying someone from there. Also, for Norway, Eika et al. (2019) shows that colleges are more likely to marry each other than in a counterfactual where people meet randomly. In the Danish context, Nielsen and Svarer (2009) finds similar results. Studying at a ranked college influences the quality of the potential partner. Kaufmann et al. (2013) shows that in Chile, people who attend top universities have a better quality partner, measured in education, concerning others. Furthermore, the effect is more substantial for women. The theoretical model predicts that people with better characteristics (more educated) tend to search more in the traditional method, which could be interpreted as colleges. These people would be willing to use a more expensive method to find better partners. This paper also contributes to the small literature that has studied internet-based search methods

In the case of online methods, the evidence for assortative mating is mixed. Hitsch et al. (2010), using data from online platforms, find that there is sorting on education, age, income and race for people who search on these platforms and posits that search frictions are essential in explaining educational matching patterns. Lin and Lundquist (2013) show that white people who search for partners in online methods are more likely to interact with people with the same race and less education than with people with similar education but different race. Individuals who use the internet are less assortatively matched, which is consistent with my model. In South Korea, Lee (2016) shows a weak sorting in income and geographical proximity. However, this increases in the case of education with other demographic variables. In all these studies, only one database was analyzed where agents were searching for partners in the online method, but there is no information to compare with searches in other methods. I, on the other hand, study the selection of the means of matching, both theoretically and empirically.

Other fields have looked at the selection of search strategy. Rosenfeld et al. (2019), using the same data as in this paper, finds that online matchmakers are more likely to marry people of different education, race, but similar ages. My model is in line with these results. We think that age is a particularly strong measure of "quality". Rosenfeld (2018) investigates hetero and homosexual dating app usage, noting higher activity among gay men and hetero women. However, it neglects individual traits' influence, a factor my model, considering personal and potential partners' education, incorporates. Thomas (2020) indicates that internet dating promotes more diverse relationships in terms of education, race, and religion, although the study is largely descriptive. Srikanth (2019) advances Rosenfeld and Thomas (2012) approach by categorizing matchmaking methods. She illustrates how the internet is supplanting social circles, particularly for the LGBTQ+ community. Contrary to her findings, my analysis exploits the changing patterns of traditional meeting venues, like universities and bars, in classifying search methods and I propose a theoretical model that explains why agents choose certain search methods.

The theoretical model predicts that people with better characteristics (more educated) tend to search more in the traditional method, which could be interpreted as colleges. These people would be willing to use a more expensive method to find better partners.

This links this paper to the literature that has worked at universities and their role in matching paths. An alternative explanation for why markets have become polarized is that marriage now offers higher or more complementary returns to spousal education than before. The fact has been discussed theoretically and empirically before (Chiappori et al., 2009; Goldin and Katz, 2008; Lafortune, 2013; Oreopoulos and Salvanes, 2011). Enrollment in university careers has been growing for women, approaching and, in many cases, surpassing those of men. The opposite is true in the labor market because the participation rate of women is generally lower than that of men, and so are salaries. Therefore, the high female participation rate does not seem to have high returns in the labor market so this difference may be compensated in the marriage market. The puzzle between women's choice to be educated and their benefit in the labor market does not take into account that there may be different search methods for finding a partner, which is what this model highlights. Another explanation that may explain assortative matching in marriages is a combination of technological changes (which helped increase productivity in household production) that increased the value of children's education, allowing more educated women to devote more time to child rearing (Choo and Siow, 2006; Lafortune et al., 2022). My work seeks to explain another mechanism by which assortative matching occurs, which is due to the difference in costs between different search methods, which due to the advent of the internet the cost of a low media, this generated an increase in sorting.

1.2. Motivating fact

I start by asking if there is a difference between individuals who look for partners online and those who do it through more traditional methods. Most large-scale surveys do not include this type of question. I solve this by relying on a smaller data set.

Specifically, I employ a dataset called "How Couples Meet and Stay Together 2017" (HCMST2017), which is a cross-sectional survey from 2017 with information on how people met, the respondent's and the partner's education, income, self-reported satisfaction with the relationship[3].

HCMST2017 is nationally representative cross-sectional data from 3,510 survey respondents. The public data include codes of the open-text answers for how couples meet. Collection of data done by telephone and subjects without Internet access at home are given Internet access. The survey divides the sample into the following categories:

- S_1: Married.
- S_2: Boyfriend, girlfriend or romantic partner (no sexual partner)[4].
- S_3: Single.

For individuals who are married or in another form of relationship, the survey asked about the method through which the respondent met his or her current partner. I classify the different methods as shows in the following table:

[3]The data is available at https://data.stanford.edu/hcmst2017

[4]In the conducted survey, individuals sharing a bond with their partner sans sexual relations were categorized as having a "romantic partner." This term typically denotes someone with whom emotional intimacy and affection is shared, often coupled with sexual attraction and exclusivity, with relationships being voluntary, reciprocal, and founded on mutual respect and understanding.

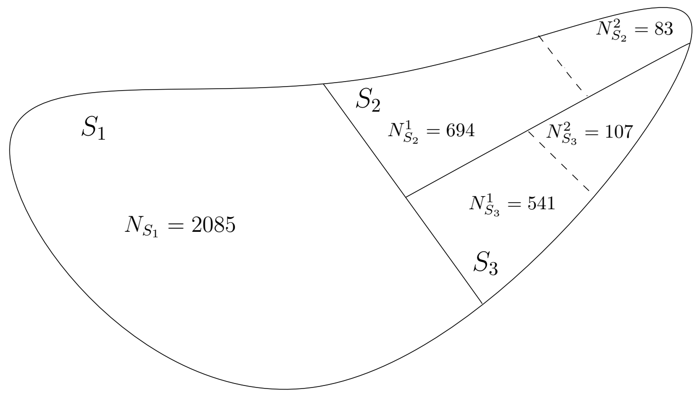


Figure 1.1. Subsamples of the different categories. $N^1_{S_2}$: Boyfriend/girlfriend, $N^2_{S_2}$: Romantic partner, $N^1_{S_3}$: Had a partner, $N^2_{S_3}$: Never had a partner.

Random	**Institution**	**Social Circle**	**Online**
Public	School	Singles Event (Non-internet)	Met Through The Internet
Vacation	College	Set Up On Blind Date	
Bar/Restaurant	Military	Family	
Volunteering Organization	Church	Friends	
Customer-client	Work	Neighbors	
Party		Others	

Table 1.1. Classification of the different ways to find a partner.

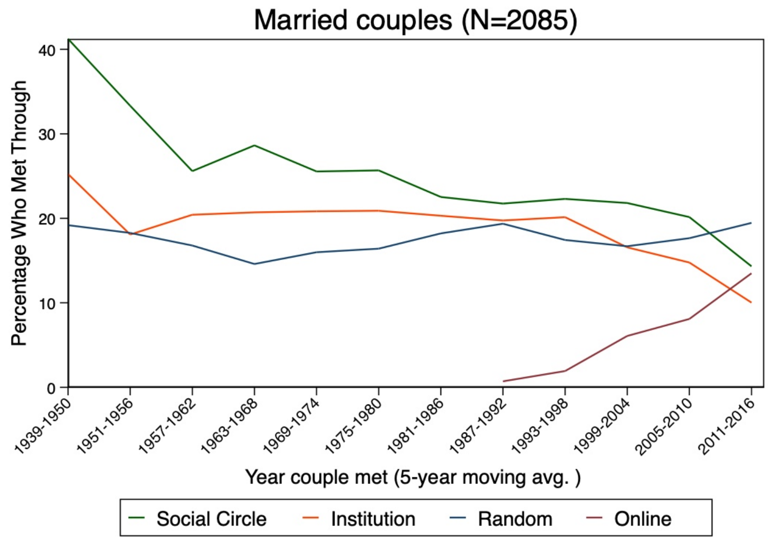


Figure 1.2. Average percentage of how they met along with the year they met of married and heterosexual. Data smoothed with a moving average of five years. Percentages do not add 100% because more than one category can apply.

In the following figure I present married people, heterosexuals and people in a relationship at the time of the survey will be considered.

Some respondent provide more than one method, in that case, I restricted only to people who met through a single method.

It can be seen in figure 1.3 that searching online has increased. The Internet became massive and accessible to households in more or less 1995 (Rosenfeld and Thomas (2012)) in the United States, which would explain the significant increase in online searches for dating, notable from about 1990. The social circle, as a way to connect with partners, has had a monotonic decline from the 1970s to 2015. It also is the most common way to find a partner. From about 1940 to 1960, matching through institutions had been stable. After 2000 went into steep decline. Those who match trough random methods have a relatively stable behavior, unlike the other ways of looking for a partner. In addition, married people, heterosexuals and people in a relationship at the time of the survey will be considered. The following figure shows how people met their partners through the different search methods:

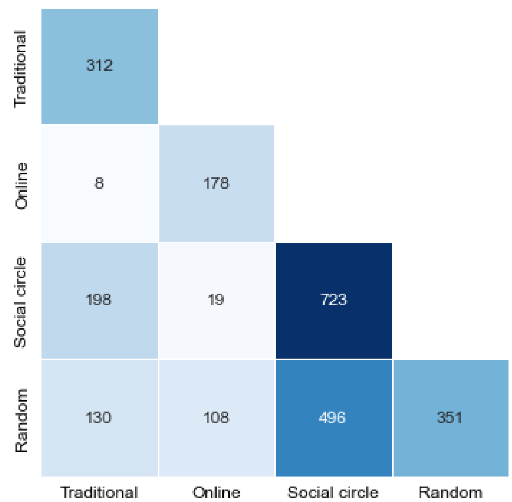


Figure 1.3. Number of people using different methods to find a partner

Of the sample of heterosexual persons, 312 reported that they found their current partner through traditional methods, 178 through the Internet, 723 through social circles and 361 through random. Those who searched outside of diagnosis found their partner by more than one method.

In the theoretical model, agents will be able to hoose choose between two methods: T and w. We will call method T “traditional” and w as alternative or “non-traditional” methods for finding a mate. We will define search through **Institution** as T; if people search through **Random**, **Social** or **Online**, they will be considered to search in w. I will assume that if any individual searches by the traditional method and by other methods, it will be assumed that he/she searches by the T method.

Rosenfeld et al. (2019), using the same data, shows that for heterosexual couples in the U.S., meeting online has become the most popular way to meet, surpassing friends for the first time around 2013. The more traditional way of meeting, such as through family, church and school, has been in decline since 1940. Data show that heterosexual couples in the U.S. are now much more likely to meet online than any other way.

If we analyze that the methods have a certain efficiency to make the match, in the case of searching for partners through random or social circle methods, these technologies are relatively static. On the other hand, online methods have had an exponential growth due to the emergence of the Internet. Then, the cost of using the w method falls in relative terms because of the emergence of these new search technologies.

Thomas (2020) suggests that the more traditional methods through which people find romantic partners are highly segregated on one social dimension, the resulting couples will also tend to be highly segregated on that dimension. Despite general trends of declining religious identification and attendance, religion may continue to act as a filter for many men and women seeking partners with similar values and cultural orientations (**?**). There is also evidence that universities can be local markets for finding a partner (Kirkebøen et al., 2022). For example, education is a form of premarital investment that agents use to improve their position in the marriage market, and that marriage market conditions may influence the amount of education investment they make (Lafortune, 2013). It could then be argued that it is more costly to access traditional methods because pre-investment is required, unlike the other methods.

The database allows me, in addition, to measure the way the individual has met their most recent partner. I then construct a database from HCMST2017. A can be seen in table 1.2.

	N	Mean	Std. dev.	Min.	Max.
$\mathbb{P}(T)$	2637	0.29	0.452	0	1
Relationship quality	2788	0.600	0.489	0	1
Some college$_{\text{surveyed}}$	2788	0.4824	0.500	0	1
Some college$_{\text{partner}}$	2788	0.497	0.500	0	1
Year Couple Met	2757	1993.867	16.951	1939	2017
Age when met	2757	26.254	11.43	0	84
$female$	2788	0.501	0.500	0	1
Income	2788	92308.960	65185.480	2500	274999.500

Table 1.2. Summarize statistics of married, not married and heterosexual who found a partner only by one method T or w, but where social circle is included

In the definition of the data $\mathbb{P}(T)$ represents whether the couple met only through the T method ($\mathbb{P}(T) = 1$), and therefore, when $\mathbb{P}(T) = 0$ the person found a partner by the w method. In addition, the variable was constructed, leaving out the category of social circle. It is worth noting that on average people are more likely to meet through the w method but that the dispersion is relatively large. The relationship quality variable is self-reported and takes values from 1 to 5, where 1 represent a very poor relationship and 5 excellent. For simplicity of analysis, transform this variable to a variable between 0 and 1, where 1 represents that the relationship is excellent and zero the other categories. On average, the quality is good, which may be related to the sample being studied. Some college$_{\text{surveyed}}$ (Some college$_{\text{partner}}$) represents whether the respondent (partner) has a university degree. In both categories, approximately half have a university degree, together with the fact that the variance is relatively high (close to the average). “Year Couple Met’ has an average of approximately 1994. The variable ”Age when met” is in what age the individual met with his partner. On average, the age when met is close to 26, and the highest value is approximately 81. There are 50,2% (mean) of females in the sample. On average, the income[5] of the household is close to 100,000 dollars per year, and the coefficient of variation of this variable is close to 7%.

To study the relationship between search methods with the characteristics of the surveyed and couple, I start by estimating the following equation using a linear probability model (LPM):

$$\mathbb{P}(T)_i = \beta_0 + \beta_1 \text{Some college}_{\text{surveyed},i} + \boldsymbol{\gamma^1} \boldsymbol{X}_i + \boldsymbol{\theta^1} \boldsymbol{M}_t + \varepsilon_i \tag{1.1}$$

where $\mathbb{P}(T)$ is a binary variable that indicates if the person i met her partner in year t through method T; $\boldsymbol{X}_i$ is a vector of individual control variables; $\boldsymbol{M}_t$ are year couple met fixed effects for the year the couple met; and ε_i is an error term.

5 Originally this variable was in categories, so the median of each type is taken to obtain a continuous variable.

Sample	(1) Married $\mathbb{P}(T)$	(2) Married & not Married $\mathbb{P}(T)$	(3) Married $\mathbb{P}(T)$	(4) Married & not Married $\mathbb{P}(T)$
Some college$_{surveyed}$	0.159***	0.116***	0.173***	0.141***
	(0.024)	(0.021)	(0.022)	(0.019)
$\ln(\text{Age when met})$	-0.380***	-0.347***	-0.376***	-0.344***
	(0.041)	(0.038)	(0.041)	(0.038)
$female$	0.018	0.023	0.018	0.021
	(0.022)	(0.019)	(0.022)	(0.019)
$\ln(income)$	0.023	0.038***		
	(0.015)	(0.012)		
Year Couple Met	✓	✓	✓	✓
N	1,718	2,137	1,718	2,137
R^2	0.161	0.158	0.160	0.154

Dependent variable: Method used to find a partner. Regressions (1) correspond to a sample with married and heterosexual couples with income and others controls, (2) with married, not married and heterosexual couples with income and others controls. Regression (3) correspond to a sample with married and heterosexual couples, (4) with married, not married and heterosexual couples with education of the surveyed with others controls. All regressions are weighted.

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 1.3. Regression estimates: Education and others controls with different search methods.

Table 1.3 shows the results of running specification (1.1). On average, more educated and younger people are more likely to meet through the T method. And there is no significant difference in gender and current income. Since education and youth are characteristics sought on the marriage market, this would suggest that more attractive individuals are more likely to find a spouse through traditional methods.

The magnitude for estimation (2) and (4) are such that someone with some college is approximately $13pp$ likely to have found her spouse through traditional methods than someone with less education of the same age and who met their spouse in the same year. One more year of age reduces that probability by $40pp$.

1.3. Model

In this section, I introduce a general equilibrium model where potential partners search for each other using two search methods: a traditional method (T) and an alternative one (w). Having shown that the use of alternative search methods have increased in recent years but those tend to be employed by less attractive individuals, I elaborate a model that could explain those facts.

This paper fits in an extensive literature that studies how individuals choose a partner and the associated implications on economic outputs and welfare (Becker, 1973; Boulier and Rosenzweig, 1984; Pencavel, 1998; Tertilt, 2005; Field and Ambrus, 2008; Chiappori et al., 2018a; Chiappori, 2020). Models in this literature have been done from a partial equilibrium (Grossbard, 1993) or general equilibrium perspective (Gersbach and Haller, 2017). The former assumes that couples are producers and sellers of activities that benefit their partner. The latter analyzes couples' interactions collectively and the efficiency of decisions restricted to the market conditions they face. My model has a general equilibrium perspective, given that agents endogenously choose the search method and that has implications for the proportion of agents who will stay in the traditional or non-traditional method.

While many of the models proposed assume frictionless matching (Gale and Shapley, 1962; Shapley and Shubik, 1971; Roth and Sotomayor, 1992; Chiappori et al., 2018b), I emphasize a potential role for search frictions a lá Mortensen (1988), i.e, each individual has a probability of meeting his or her potential partner and decides whether to stay together or to continue searching. In my model, the search follows an urn-ball process[6] that consists of the most attractive agents receiving multiple applications and only keeping the most attractive ones, and from that pool of potential matches, they only choose one at random. As Chiappori (2020) states, assuming search frictions depends on whether the market size is relevant. Consider analyzing a small community, where the absence of

[6]In the matching and search literature, Albrecht et al. (2004) develops the match function based on the urn-ball process.

friction could be plausible. This could also apply to large populations when the focus is on specific feature matching. The stability of the match is also important when analyzing these models for models with transfers (Lauermann and Nöldeke, 2015) and without transfers (Atakan, 2006). Relationships with higher surplus are more stable (Browning et al., 2013; Couprie, 2007; Goussé et al., 2017).

I also assume that utility is not transferable as in Gale and Shapley (1962); Roth and Sotomayor (1992); Dagsvik (2000); Hitsch et al. (2010); Agarwal (2015); Menzel (2015). As this simplifies the analysis, this is because my model is static, so transfers would not play an essential role in this context since individuals make an expectation of their benefits of finding someone in the different search methods without having the possibility of negotiating with the partner.

I also assume that individuals in a relationship derive benefits that depend on their own characteristics and that of their peers. As it is done usually, I assume supermodularity (Siow, 2015b; Chiappori et al., 2012; Hiller, 2018; Chiappori, 2020). This is a necessary condition for generating assortative matching in the model.

Education is one of the variables most studied in the assortative mating literature, and it is a fact that the relationship between marriage and education has changed. Goldin (2021) shows that it is increasingly common for American women to be more educated and less married. Regarding education, Bailey et al. (2013) shows that less educated women marry younger; on the other hand, more educated people marry later. Greenwood et al. (2014) argues that sorting has been increasing considerably in recent decades, although other authors (Siow, 2015a) argue that, correcting for the change in the distribution of female education, this increase is attenuated. Chiappori et al. (2020) suggests that sorting has increased, but it is not homogeneous, and that the increase is more pronounced for the more educated. Lafortune et al. (2022) using U.S. census data, shows that thanks to a combination of technological changes, more educated women tend to spend more time raising their children. The mechanism given in my model is the difference between the costs of the different search methods.

1.3.1. Description of the economy

The economy is composed of types of agents, x and y, in equal quantities. I assume a discrete number of agents for the moment, but I will consider later the limiting case where these quantities tend to infinity. This limiting case is similar to the assumption of a continuous mass of agents, which will simplify the analysis. Agents of one type want to be matched with agents of the other type. Agents differ in addition in their characteristics, being able to be A or B. There is an equal number of agents with characteristics A and B within each type of agents. Once an agent of type x is matched with an agent of type y, they each obtain marital utility $\Omega\left(x_k, y_l\right)$ with $l, k \in \{A, B\}$ depending on the characteristics of each agent in the couple. If an agent is single, e.g. she is not matched to an agent of the other type, she receives utility b. I assume that:

$$\Omega\left(x_A, y_A\right) > \Omega\left(x_A, y_B\right) = \Omega\left(x_B, y_A\right) > \Omega\left(x_B, y_B\right) > b > 0. \tag{1.2}$$

From these inequalities, one can infer that agents prefer to be matched to someone from the pool of agents with characteristic A rather than someone with B. Moreover, a better characteristic for one of the matched agents does not only improve the marital utility received by the other agent but it also increases the utility that this agent gets. Finally, agents of any type prefer to be matched than being single.

Agents have to engage in a search process to find a partner. Each agent chooses between two search methods, T and w, each associated with a separate market, with potentially a different pull of agents to be matched to. The composition of each pool is determined endogenously and depends on the entry decision of each agent makes. Agents can only opt for one of the two methods. If an agent chooses the method T, she pays a cost $c_T > 0$, while she pays $c_w > 0$ if she chooses w, with $c_T \geq c_w$. Figure 1.4 illustrates how agents of types x and y interact in the search process depending on their characteristic (A or B) and the search method they choose.

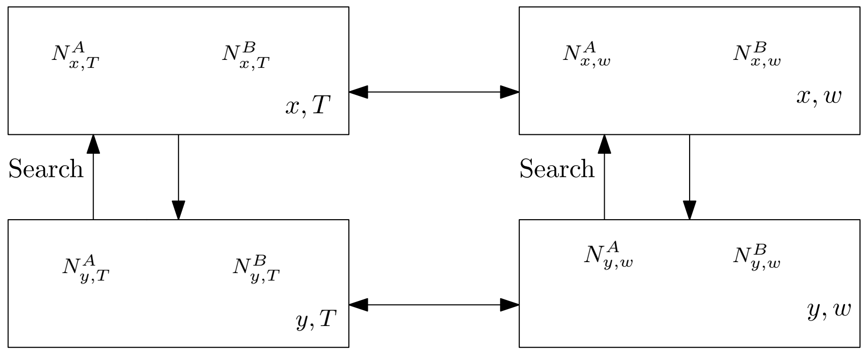


Figure 1.4. Representation of the search.

I denote by $N^A_{x,T}$ the number of agents of type x with characteristic A that are using the method T, while a number $N^B_{x,T}$ of agents of the same type use the same method within those associated with the characteristic B. Among the agents of type x using the method w, there is a number $N^A_{x,w}$ of agents with the characteristic A and $N^B_{x,w}$ with the characteristic B. I use a symmetric notation for the agents of type y with characteristics A and B who opt for the methods T or w.

Once a search method is chosen, each agent of type x randomly chooses an agent of type y who has chose the same method and proposes to be matched to her.[7] Each agent of type y has an equal probability to be picked in the selected pool. Because of the random nature of the process, a given agent of type y may receive more than one proposal. She can also be unlucky and receive no proposal: in this case she becomes single. Then, the agent of type y decides whether to accept or not the proposal: she compares all the options she has in hand from agents of type x and accepts the one delivering the larger marital utility. In case of a tie, she will simply pick one randomly among the proposals delivering the highest marital utility. Agents of type x whose proposal is rejected become single.[8].

[7]The assumption that x makes the proposal and not y is without loss of generality. I show in the appendix that the matching probabilities would be the same under the alternative assumption.

[8]For more details see the appendix section 1.7

1.3.2. Equilibrium

Let $p_{x_k,l}$ be the probability that an agent of type x with characteristic $k \in \{A, B\}$ is matched to an agent of type $l \in \{y_A, y_B\}$ using method T and $g_{x_k,l}$ be the same probability when using method w. I show in the appendix that the probabilities are symmetric in the case of an agent of type y.

I focus on the limiting case where the total number of agents of type x and y tends to infinity. Even though I consider a very large number of agents, the relative number of agents of each type is always equal to one given that they are in equal quantities. I denote by $\eta^k_{x,T}$ the fraction of agents with characteristic k among the agents of type x who are using the method T. I only study symmetric equilibria. I show in the appendix that the matching probabilities are the following[9]:

$$p_{x_A,A} = \left(1 - e^{-\eta^A_{x,T}}\right) \tag{1.3}$$

$$p_{x_A,B} = \frac{(1 - \eta^A_{x,T})}{\eta^A_{x,T}} \left(1 - e^{-\eta^A_{x,T}}\right) \tag{1.4}$$

$$p_{x_B,A} = e^{-\eta^A_{x,T}} \eta^A_{x,T} \frac{\left(1 - e^{-(1-\eta^A_{x,T})}\right)}{1 - \eta^A_{x,T}} \tag{1.5}$$

$$p_{x_B,B} = e^{-\eta^A_{x,T}} \left(1 - e^{-(1-\eta^A_{x,T})}\right) \tag{1.6}$$

The term $e^{-\eta^A_{x,T}}$ found in probabilities (1.5) and (1.6) represents the probability event in which the agent does not receive any proposal of type A. The intuition behind this term is that type B agents will only be able to be with someone if and only if their potential

[9]It should be noted that assuming a well-behaved matching function (see Petrongolo and Pissarides (2001)), probabilities with slopes of the same sign are obtained for each event. For more details see the appendix section 1.10 and subsection 1.10.11

partner is not approached by a type A agent. And the probabilities for w:

$$g_{x_A,A} = \left(1 - e^{-\eta^A_{x,w}}\right) \tag{1.7}$$

$$g_{x_A,B} = \frac{(1-\eta^A_{x,w})}{\eta^A_{x,w}}\left(1 - e^{-\eta^A_{x,w}}\right) \tag{1.8}$$

$$g_{x_B,A} = e^{-\eta^A_{x,w}}\eta^A_{x,w}\frac{\left(1 - e^{-(1-\eta^A_{x,w})}\right)}{1-\eta^A_{x,w}} \tag{1.9}$$

$$g_{x_B,B} = e^{-\eta^A_{x,w}}\left(1 - e^{-(1-\eta^A_{x,w})}\right) \tag{1.10}$$

where $\eta^A_{x,w}$ is the proportion of agents of type x_k that are in the method w for the total number of agents in the same method. The higher the proportion of agents of type x_k are in method w, the more attractive it becomes but the more expensive it is to find someone. Agents pay a cost $c_T > 0$ if they search T, then the utility expected for agent's is:

$$S_T(x_A) = p_{x_A,A}\Omega(x_A, y_A) + p_{x_A,B}\Omega(x_A, y_B) + b\left(1 - p_{x_A,A} - p_{x_A,B}\right) - c_T \tag{1.11}$$

$$S_T(x_B) = p_{x_B,A}\Omega(x_B, y_A) + p_{x_B,B}\Omega(x_B, y_B) + b\left(1 - p_{x_B,A} - p_{x_B,B}\right) - c_T \tag{1.12}$$

Moreover, the expected utility if the agents look in w is:

$$S_w(x_A) = g_{x_A,A}\Omega(x_A, y_A) + g_{x_A,B}\Omega(x_A, y_B) + b\left(1 - g_{x_A,A} - g_{x_A,B}\right) - c_w \tag{1.13}$$

$$S_w(x_B) = g_{x_B,A}\Omega(x_B, y_A) + g_{x_B,B}\Omega(x_B, y_B) + b\left(1 - g_{x_B,A} - g_{x_B,B}\right) - c_w \tag{1.14}$$

Given expressions (1.11), (1.13) and (1.12), (1.14), it is possible to determine the method chosen by agents of each type by comparing the expected utility they would get by choosing each method. I show that the following types of equilibria exist:

(i) **Separating Equilibria**

- $S_{(A,T)\times(B,w)}$: All A's in T and all B's in w.
- $S_{(A,w)\times(B,T)}$: All A's in w and all B's in t.

(ii) **Pooling Equilibria**

- P_T: All in T.

- P_w: All in w.
- $P_{T,w}$: Pure Pooling Equilibria.

(iii) **Partially Pooling Equilibria**

- $S_{(B,T)\times((A,T),(A,w))}$: All B in T and some A on T and w.
- $S_{(B,w)\times((A,T),(A,w))}$: All B in w and some A on T and w.
- $S_{(A,T)\times((B,T),(B,w))}$: All A in T and some B on T and w.
- $S_{(A,w)\times((B,T),(B,w))}$: All A in w and some B on T and w.

I now determine the range of values of the participation costs c_T and c_w consistent with the possible equilibria above.

1.3.3. Characterization according to the participation costs

Given that it is the difference between the two participation costs that matters for the choice of the search method (rather than their specific levels), I consider the cost of using method T as fixed and vary the cost c_w to determine which type of equilibrium exists under its possible values. I denote by $c \equiv c_T - c_w \geq 0$. Hence, a decrease in c_w implies an increase in c. I only focus on symmetric equilibria and make the following assumptions about marital utility:

(i) **Supermodularity**: $\Omega(x_A, y_A) + \Omega(x_B, y_B) > 2\Omega(x_A, y_B)$

(ii) By property of positive real numbers we know that:

$$\Omega(x_A, y_A) = k_1\Omega(x_A, y_B), \quad \Omega(x_A, y_B) = k_2\Omega(x_B, y_B), \quad \Omega(x_B, y_B) = k_3 b$$

$$\Omega(x_A, y_A) = k_1 k_2 k_3 b, \quad \Omega(x_A, y_B) = k_2 k_3 b, \quad \Omega(x_B, y_B) = k_3 b$$

with $k_1, k_2, k_3 > 1$. Then I assume that:

$$k_1 \geq 2.17 \tag{1.15}$$

It's important to note that with the coefficient k_1, the utility derived from a union of type A agents is perceived as more than double compared to a union between type A and B agents. With the assumptions presented, I submit the following proposition:

PROPOSITION 1.1. *If individuals' utilities comply with supermodularity* (1)*,* (1.2)*, and* (1.15)*, then, a decrease in the costs of the non-traditional method will polarize the marriage markets.*

PROOF. See Appendix 1.9. In which all possible equilibria are analyzed. $\square$

In the case of symmetric equilibria and the same ratio between characteristics and agents, by decreasing the costs of the non-traditional method and refining the equilibrium by pairwise deviations, the marriage market tends to polarize in characteristics. As shown in appendices 1.9 and 1.10, the equilibria that hold for $c > 0$ are P_T, P_w, $S_{(A,T)\times((B,T),(B,w))}$, $S_{(B,w)\times((A,T),(A,w))}$ and $S_{(A,T)\times(B,w)}$. In the case of P_T, P_w and $S_{(B,w)\times((A,T),(A,w))}$, it can be shown that it does not hold up to deviations of pairs for some intervals of c[10]. This result can be explained intuitively. If one of the search method experiences a fall in its cost, equilibrium in the model is such that the least attractive individuals select a higher fraction of the search method that is the least costly. More attractive individuals continue to employ the more expensive search strategy. That is, markets tend to polarize in terms of characteristics.

[10] For the case of P_w for $c > c_0$, the equilibrium does not survive if refined with pairwise deviations. For $c < c_0$ there will exist two equilibria P_T and P_w, we will only study the P_T equilibrium for that interval, since P_w is the trivial equilibrium since it is the cheapest method. For more details see the appendix 1.9.10 and 1.10.10

Defining the following constants:

$$c_0 = be^{-\frac{1}{2}}\left(1-e^{-\frac{1}{2}}\right)\Bigg(k_3(k_2+1)-2\Bigg) - b\left(1-e^{-1}\right)(k_3-1)$$

$$c_1 = be^{-\frac{1}{2}}\left(1-e^{-\frac{1}{2}}\right)\Bigg(k_3(k_2+1)-2\Bigg)$$

$$c_2 = b\Bigg(k_3\left(e^{-1}(k_2+1)-1\right)+1-2e^{-1}\Bigg)$$

$$c_3 = bk_1k_2k_3\Bigg[(1-e^{-1})-(1-e^{-\frac{1}{2}})\Bigg] + b\Bigg[2\left(1-e^{-\frac{1}{2}}\right)-\left(1-e^{-1}\right)\Bigg] + bk_2k_3(1-e^{-\frac{1}{2}})$$

$$c_4 = bk_2k_3\left(1-e^{-1}\right)(k_1-1) + be^{-1}\left(1-k_2k_3\right)$$

With assumptions (1.2) and (1.15) it is possible to show that $c_3 > c_1 > c_0$ and $c_3 > c_2$. If supermodularity holds then $c_4 > c_2$. Now it is possible to represent the equilibria that hold at the different costs c:

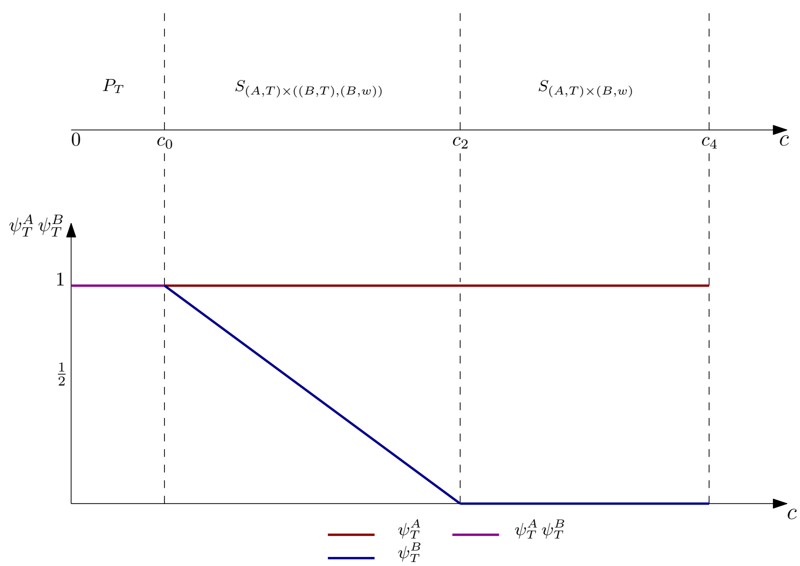


Figure 1.5. Costs that sustain separate and pooling equilibria. With $\psi_T^A \equiv \frac{N_{x,T}^A}{N_x^A}$, $\psi_T^B \equiv \frac{N_{x,T}^B}{N_x^B}$

Figure 1.5 shows that agents begin to change search methods as method w becomes cheaper (c increases, leaving c_T constant). Type x_B agents are the first to go because they are less likely to find a mate. If $c > c_2$, then the markets are polarized. In method T, there will only be agents of type A, and in w, only of type B. Since the equilibria are defined for different costs, the probabilities of different agents finding a partner in the T method can also be illustrated:

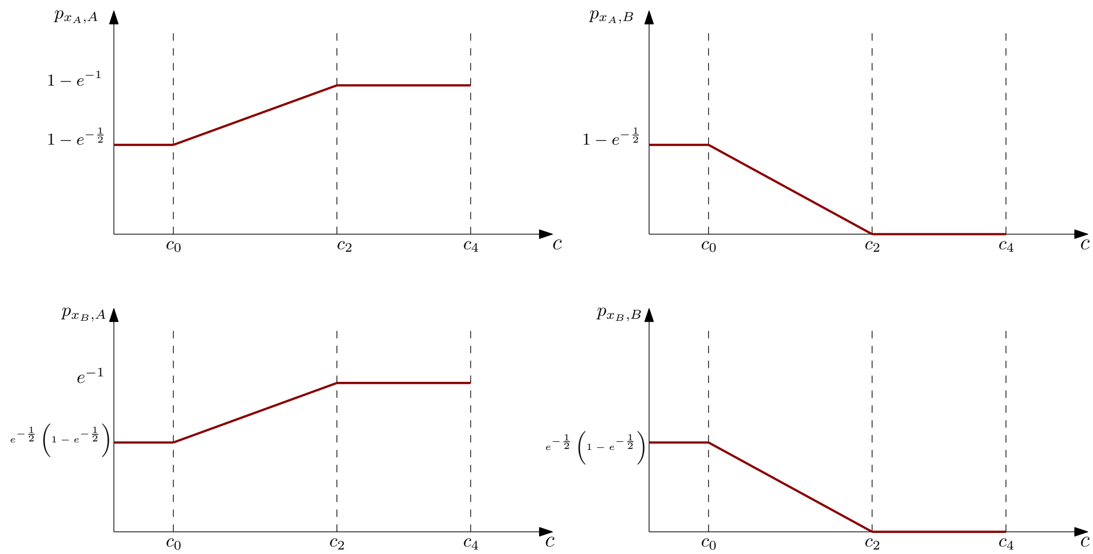


Figure 1.6. Probabilities associated with the search method T.

Figure 1.6 shows how the probabilities of finding a partner change for agents x_A and x_B. As c increases (method w becomes cheaper, with c_T constant), agents of type A increase the probability of finding someone of the same type because those of type B emigrate. In the same way, finding a type person is less likely if the costs of w are cheaper in relative terms. Competing with type A results is less attractive. And similarly, if the agents search in w, the probabilities as a function of c are:

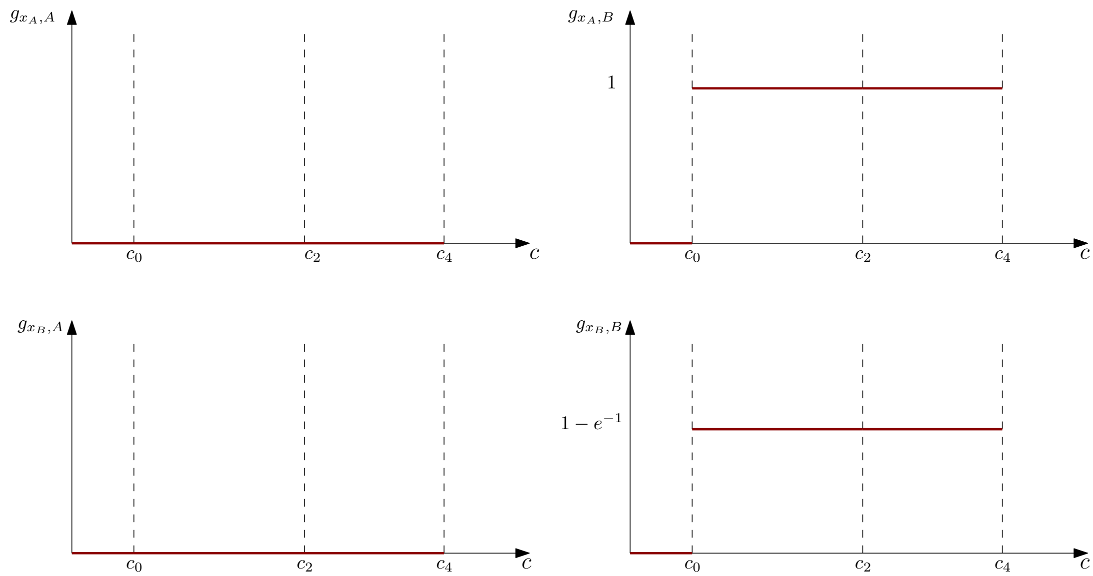


Figure 1.7. Probabilities associated with the search method w.

When $g_{x_A,B} = 1$, as it appears in Figure 1.7, it is the case that agent A can deviate and will undoubtedly find type B agents who will accept. Then in the case when $g_{x_B,B} = 1 - e^{-1}$, it is when there are already "many" agents of type B in w, so the probability of finding only type B will not change from the migration from T to w. There will be no agents of type A in w, so the probability of finding a match with that type will be zero.

Probabilities characterize the expected utility. With the expressions given by (1.11), (1.12), (1.13) and (1.14), it is possible to compare the agents' preferences for the various methods. Note that $c = c_T - c_w$ and that only the probabilities depend on that constant. For agents, it is necessary to occupy their due costs. The expected utility of searching both methods for agent A is represented by:

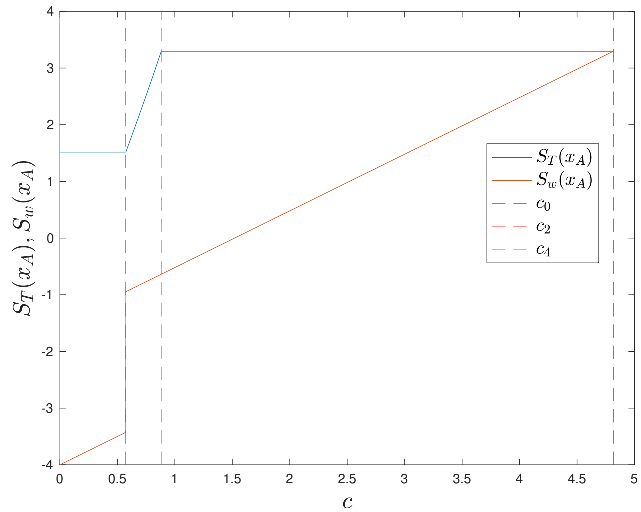


Figure 1.8. Agent A expected utilities with both methods.

Agent A will strictly prefer method T over w if $c \leq c_4$. Even though method w becomes cheaper and cheaper, agents A will stay in T and prefer agent B to migrate to w. Figure 1.9 shows that agent B will prefer method T if w is relatively expensive. When costs begin to fall, there is a trade-off between being in T and having a greater probability of joining type A (given that there are fewer agents of type B) or going to w and having a relatively high probability of finding a type B.

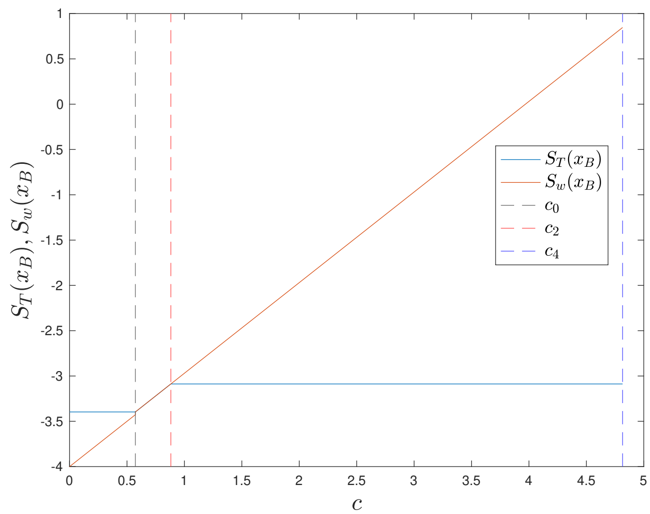


Figure 1.9. Agent B expected utilities with both methods.

From the results obtained, the following propositions can be derived

PROPOSITION 1.2. *Attractive individuals will be more likely to match with an attractive individual when they search in traditional methods*

PROOF. Decreasing method cost w, high-quality agents stay with traditional methods while others migrate, thereby raising the probability of encountering type A couples in traditional venues. □

Figure 1.6 shows that as the cost of the methods w decreases (which mechanically is the same as c increases), the agents with the best characteristics will remain in the traditional methods, and the others will tend to migrate. Therefore, it is clear that the probability of finding type A couples in the traditional methods increases.

PROPOSITION 1.3. *A lowering of the costs of non-traditional methods will lead to polarization.*

PROOF. This is illustrated in the following figures 1.8 and 1.9. □

For relatively low c_w costs, it can be seen that type B agents tend to migrate to non-traditional methods, and type A agents remain in traditional methods. This polarization occurs because, for type B agents, who are less likely to match with a type A, it becomes increasingly attractive to go w. These benefits type A agents, who increase their utility with each type A agent that migrates. Thus, markets become polarized.

PROPOSITION 1.4. *Searching through traditional methods leads to more stable relationships.*

PROOF. This is illustrated in the figure 1.8. □

Type A agents are the ones to look for in traditional methods, which have a higher utility. So these agents would be expected to have a more stable relationship since they benefit more from that union than other agents[11].

1.3.4. Welfare

Suppose an economy where only the method T exists. This implies that the expected utility of both agents will be constant and equal to:

$$S_T(x_A) = \left(1 - e^{-\frac{1}{2}}\right)\left(\Omega\left(x_A, y_A\right) + \Omega\left(x_A, y_B\right) - 2b\right) + b - c_T \tag{1.16}$$

$$S_T(x_B) = e^{-\frac{1}{2}}\left(1 - e^{-\frac{1}{2}}\right)\left(\Omega(y_B, x_A) + \Omega(y_B, x_B) - 2b\right) + b - c_T \tag{1.17}$$

Assuming that $c_2 > c_1$ then $k_2 \geq \frac{e}{1-e^{\frac{1}{2}}}$ and $c_4 > c_3$. If $c > c_1 + b$ then agent B is better. off with the existence of w.

Agent A is better off if $\bar{c} < c < c_4$ with the existence of w.

Both agents benefit from the existence of the new search method. In the case of agent B, it is better even with relatively high w prices. On the other hand, agent A will benefit only from relatively low prices of w, i.e., ensuring that agents of type B exit the market.

[11] I can sequentially model the problem and qualitatively we can obtain similar results. See appendix 1.8

1.4. Additional empirical tests

I will test additional prediction steaming from the proposal of the previous section. To test the proposition 1.2, I use the following regression:

$$\text{Some College}_{partner,i} = \beta_0 + \beta_1 \text{Some college}_{\text{surveyed},i} + \beta_2 \mathbb{P}(T)_i +$$
$$\beta_3 \mathbb{P}(T)_i \cdot \text{Some college}_{\text{surveyed},i} + \boldsymbol{\gamma^1 X}_i + \boldsymbol{\theta^1 M}_t + \varepsilon_i \tag{1.18}$$

where Some College$_{partner,i}$ represents whether individual i's partner has a university education (takes the value of one) or not; Some college$_{\text{surveyed},i}$ represents whether individual i has a university education; $\mathbb{P}(T)_i$ represents whether individual i met his or her partner by the traditional method; $\boldsymbol{X}_i$ is a vector of individual control variables; $\boldsymbol{M}_t$ are year couple met fixed effects for the year the couple met; and ε_i is an error term.

Note that β_3 measures the interaction between individual i finding his or her partner through the traditional method and having a university education. If this coefficient is positive, it means that the traditional method reinforces the finding of a college-educated partner if individual i has at least a college degree.

Result are presented in Table 1.4 where the first column represents the estimation of regression (1.18), for a sample of married hetrosexuals after 1998; the second column is the estimation of the married and unmarried sample after 1998; and the following columns are equivalent to the previous ones but without restriction of the year in which they met. To study the relationship between the education of the couple and the surveyed, I estimated the following LPM:

Sample	(1) Married after 1998 Some college$_{partner}$	(2) Married & not Married after 1998 Some college$_{partner}$	(3) Married Some college$_{partner}$	(4) Married & not Married Some college$_{partner}$
Some college$_{surveyed}$	0.339***	0.326***	0.336***	0.325***
	(0.051)	(0.039)	(0.031)	(0.027)
$\mathbb{P}(T)$	0.040	0.010	0.051	0.027
	(0.076)	(0.053)	(0.037)	(0.032)
$\mathbb{P}(T)$·Some college$_{surveyed}$	0.074	0.122*	0.043	0.081*
	(0.085)	(0.065)	(0.046)	(0.042)
ln(Age when met)	-0.037	0.055	0.042	0.064**
	(0.051)	(0.041)	(0.034)	(0.029)
$female$	-0.102***	-0.068**	-0.059***	-0.053***
	(0.036)	(0.029)	(0.022)	(0.020)
$\ln(income)$	0.173***	0.120***	0.158***	0.134***
	(0.031)	(0.017)	(0.017)	(0.013)
Year Couple Met	✓	✓	✓	✓
N	548	919	1,717	2,135
R^2	0.353	0.297	0.301	0.279

Dependent variable: Education of partner. Regressions (1) correspond to a sample with married and heterosexual couples with Year Couple Met $\geq$ 1998, (2) with married, not married and heterosexual couples with Year Couple Met $\geq$ 1998. Regression (3) correspond to a sample with married and heterosexual couples, (4) with married, not married and heterosexual couples with education of the surveyed with others controls. All regressions are weighted.

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 1.4. Regression estimates: Search method with Education partner.

Those results suggest that more educated people, on average, have more educated partners. This has been shown before in the literature. The test of the proposition 1.2 relies on the interaction between $\mathbb{P}(T)$ and the educational attainment of the surveyed. I also see that women have less educated partners, and there is a positive correlation between income and the partner's education.

The magnitude for estimation (2) are such that someone with some college is $33pp$ likely to have a spouse with the same college and this is reinforced by approximately $12pp$ if it is through the T method. For the samples that are close to the Internet mass date, the reinforcement effect of the T method is significant and positive. For the full sample it is significant but only when including cohabiting partners. Therefore, proposition 1.2 states: "Attractive individuals will be more likely to match with an attractive individual when they search in traditional methods" is seen in the data.

To test prediction 1.3, the following regression will be performed:

$$\text{Some College}_{partner,i} = \beta_0 + \beta_1 \text{Some college}_{\text{surveyed},i} + \beta_2 \mathbb{P}(T)_i + \beta_3 \mathbf{1}_{\text{Year Couple Met} \geq \bar{y},i}$$
$$\beta_3 \mathbb{P}(T)_i \cdot \text{Some college}_{\text{surveyed},i} + \beta_4 \text{Some college}_{\text{surveyed},i} \cdot \mathbf{1}_{\text{Year Couple Met} \geq \bar{y},i} +$$
$$\beta_5 \mathbb{P}(T)_i \cdot \mathbf{1}_{\text{Year Couple Met} \geq \bar{y},i} + \beta_6 \mathbb{P}(T)_i \cdot \text{Some college}_{\text{surveyed},i} \cdot \mathbf{1}_{Y-\delta \leq \text{Year Couple Met}_i \leq Y+\delta} +$$
$$\boldsymbol{\gamma}^{\mathbf{1}} \boldsymbol{X}_i + \boldsymbol{\theta}^{\mathbf{1}} \boldsymbol{M}_i + \varepsilon_i \tag{1.19}$$

where the variable $\mathbf{1}_{Y-\delta \leq \text{Year Couple Met}_i \leq Y+\delta}$ takes the value of one if individual i met his or her partner between the years $[Y-\delta, Y+\delta]$, $\delta \in \mathbb{N}_0$, and takes the value of zero otherwise. To test hypothesis 1.3, we will take advantage of the fact that in the 1990s the use of the Internet became massive, so we will compare the β_6 of the estimation (1.19) by changing the different years in which the couples met. This is similar to equation estimated before, but I allow that the coefficient of the interaction (β_6) is allowes to differ over time. This is represented by the following figures:

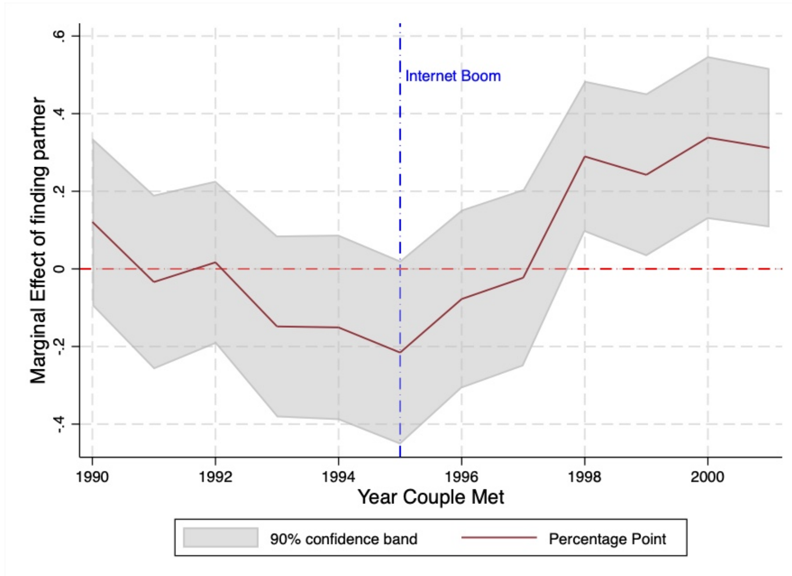


Figure 1.10. Marginal effect of finding partner (β_6) coefficient for the sample of married with $\delta = 2$.

As can be seen in Figures 1.10 and 1.11, there is practically no effect of matching through traditional methods before the internet boom. After 1997 the coefficients on the interaction of $\mathbb{P}(T)$ and education tends to be positive and significant at 10%, which is maintained if the couples meet around 2000. This is stronger for married couples, although the effect for the sample that includes unmarried people is maintained[12].

[12]In the appendix the coefficient $beta_6$ is calculated for $\delta = 0$, $\delta = 1$ and $\delta = 3$. For all estimates it holds that the coefficient is significant at 10% near the late 1990s. For more details see the appendix section 1.6.

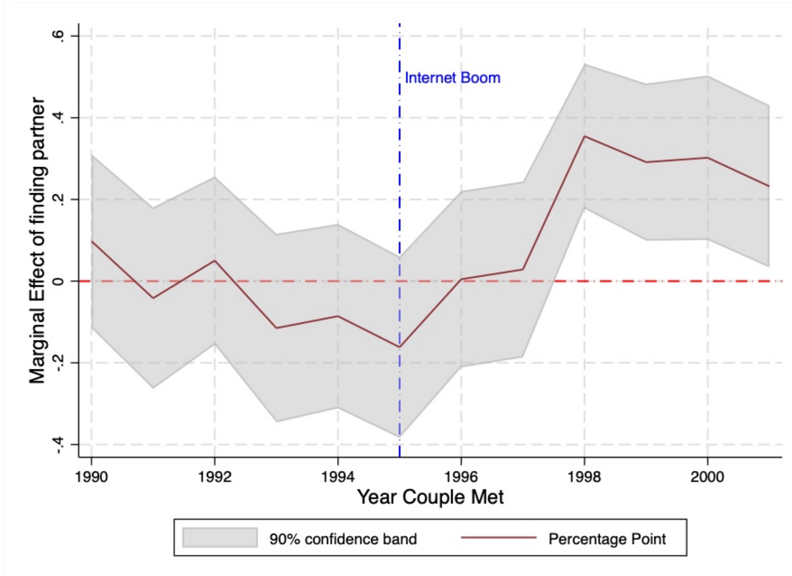


Figure 1.11. Change in β_6 coefficient for the sample of married. and not married with $\delta = 2$.

The internet boom can be interpreted as the fall of cost of non-traditional methods. These results thus suggest as in the model that decreasing the cost of searching for a spouse trough alternative methods leads to more homogamy in the marriage market.

To test the third prediction (1.4), I estimate of quality the following regression:

$$Quality_i = \beta_0 + \beta_1 \text{Some college}_{\text{surveyed},i} + \beta_2 \mathbb{P}(T)_i +$$
$$\beta_3 \mathbb{P}(T)_i \cdot \text{Some college}_{\text{surveyed},i} + \boldsymbol{\gamma}^{\mathbf{1}} \boldsymbol{X}_i + \boldsymbol{\theta}^{\mathbf{1}} \boldsymbol{M}_t + \varepsilon_i \tag{1.20}$$

where $Quality_i$ represents the quality of the relationship self-reported by individual i. All other variables are equivalent to the previous estimates.

	(1)	(2)	(3)	(4)
Sample	Married	Married & not Married	Married	Married & not Married
	$Quality$	$Quality$	$Quality$	$Quality$
Some college$_{surveyed}$	-0.000	0.026	0.029	0.049*
	(0.032)	(0.028)	(0.028)	(0.025)
$\mathbb{P}(T)$	0.095**	0.093***	0.094**	0.100***
	(0.037)	(0.035)	(0.038)	(0.035)
$\ln$(Age when met)	-0.018	-0.021	-0.017	-0.020
	(0.034)	(0.029)	(0.035)	(0.029)
$female$	-0.076***	-0.059***	-0.070***	-0.059***
	(0.024)	(0.022)	(0.023)	(0.021)
$\ln(income)$	0.032*	0.037**		
	(0.018)	(0.015)		
Year Couple Met	✓	✓	✓	✓
N	1,718	2,137	1,718	2,137
R^2	0.071	0.081	0.067	0.076

Dependent variable: Quality relationship self-reported. Regressions (1) correspond to a sample with married and heterosexual couples with income and others controls, (2) with married, not married and heterosexual couples withwith income and others controls. Regression (3) correspond to a sample with married and heterosexual couples, with married, not married and heterosexual couples with education of the surveyed without income. All regressions are weighted.

*** p<0.01, ** p<0.05, * p<0.1

Table 1.5. Regression estimates: Quality relationship with education surveyed and search methods.

The theoretical model mechanistically predicts that people who search in method T would have higher utility than those who search in w. It is clear that if people are found using the T method, the quality of the self-reported relationship is higher. It is striking that there is no correlation between the respondent's education and the self-reported relationship quality. The magnitude are such that searching through traditional methods increase the probability of finding one's relationship to be excellent by 0.1. Suppose it associates high relationship quality with future stability. In that case, people who search by traditional methods are more likely to have a more stable relationship, so predicition 1.4 is confirmed by the data.

1.5. Conclusion

Sorting between couples is a topic that has been widely discussed in the family economics literature, but not much attention has been paid to how couples meet. This paper tries to study how different search methods can impact marriage markets, both theoretically and empirically.

I built a theoretical model that posits the following predictions: (i) Attractive individuals will be more likely to match with an attractive individual when they search in traditional methods; (ii) A lowering of the costs of non-traditional methods will lead to polarization; (iii) Searching through traditional methods leads to more stable relationships.

All hypotheses are tested empirically. The massiveness of the Internet has generated a polarization in the U.S. marriage market. More stable relationships tend to be met through traditional methods. People who meet through non-traditional methods tend to be less educated and have less educated partners and have worse relationships.

My work seeks to explain another mechanism by which assortative matching occurs, which is due to the difference in costs between different search methods, which due to the advent of the internet the cost of a low media, this generated an increase in sorting.

Although these results help us to see another explanation for assortative matching, the topic requires further exploration, such as studying this behavior dynamically. The policy implications are that it may provide a mechanism for why couples segregate, which may contribute to an increase in inequality.

1.6. First Appendix: Effect of technological change and polarization of the marriage market

In this section we present a robustness check for the estimation of the following regression:

$$\text{Some College}_{partner,i} = \beta_0 + \beta_1 \text{Some college}_{\text{surveyed},i} + \beta_2 \mathbb{P}(T)_i + \beta_3 \mathbf{1}_{\text{Year Couple Met} \geq \bar{y},i}$$
$$\beta_3 \mathbb{P}(T)_i \cdot \text{Some college}_{\text{surveyed},i} + \beta_4 \text{Some college}_{\text{surveyed},i} \cdot \mathbf{1}_{\text{Year Couple Met} \geq \bar{y},i} +$$
$$\beta_5 \mathbb{P}(T)_i \cdot \mathbf{1}_{\text{Year Couple Met} \geq \bar{y},i} + \beta_6 \mathbb{P}(T)_i \cdot \text{Some college}_{\text{surveyed},i} \cdot \mathbf{1}_{Y-\delta \leq \text{Year Couple Met}_i \leq Y+\delta} +$$
$$\boldsymbol{\gamma^1 X}_i + \boldsymbol{\theta^1 M}_i + \varepsilon_i$$

where the variable $\mathbf{1}_{Y-\delta \leq \text{Year Couple Met}_i \leq Y+\delta}$ takes the value of one if individual i met his or her partner between the years $[Y-\delta, Y+\delta]$, $\delta \in \mathbb{N}_0$, and takes the value of zero otherwise.

If the coefficient $\beta_6 > 0$ of regression (1.19), it is interpreted as the polarization occurring in the marriage market. On average, individuals who are more educated will have more educated partners if they seek through traditional methods and meet close to the time when non-traditional methods lowered their costs. The internet boom is interpreted in this model as lowering the costs of alternative search mechanisms. Figures 1, 2, 3 and 4 show that at the end of the 1990s the coefficient β_6 is positive and significant at 10%. Regardless of the sample and the value of δ, there is a polarization of the market, which according to the theoretical model is attributed to a decrease in the costs of non-traditional methods, due to the massiveness of the Internet.

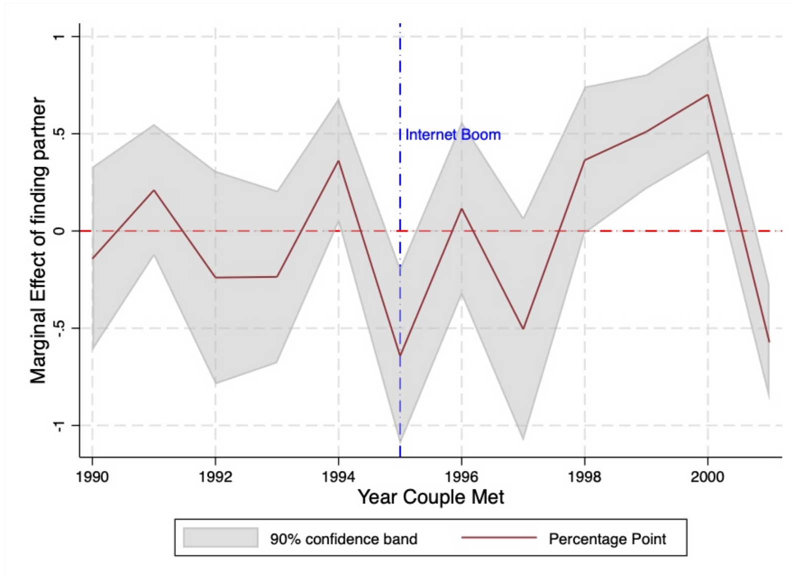


Figure 1.12. Change in β_6 coefficient for the sample of married with $\delta = 0$.

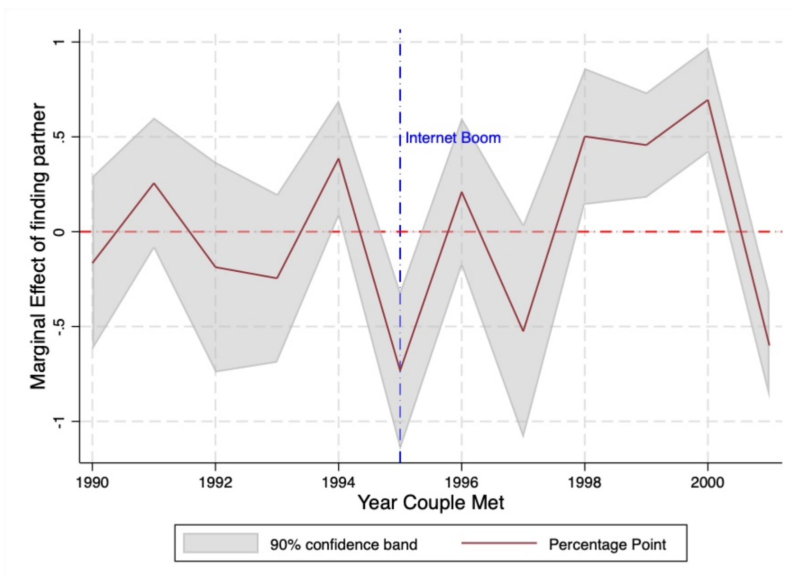


Figure 1.13. Change in β_6 coefficient for the sample of married and not married with $\delta = 0$.

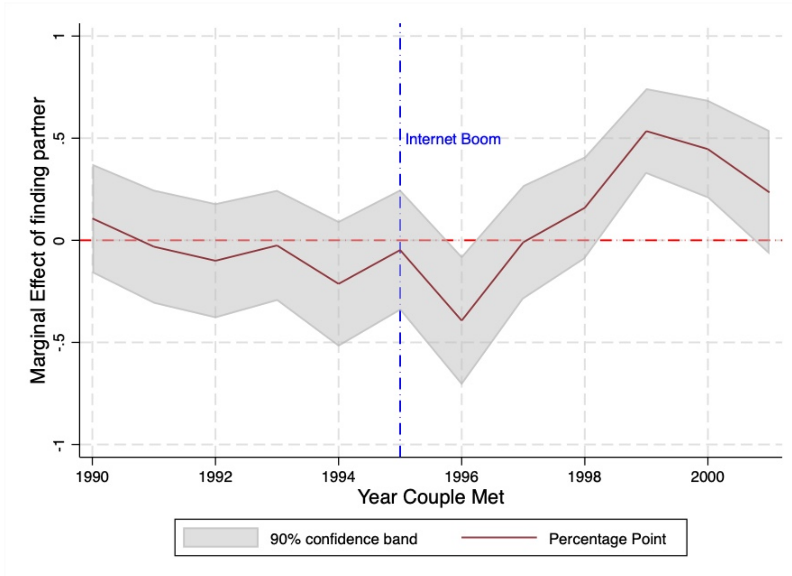


Figure 1.14. Change in β_6 coefficient for the sample of married with $\delta = 1$.

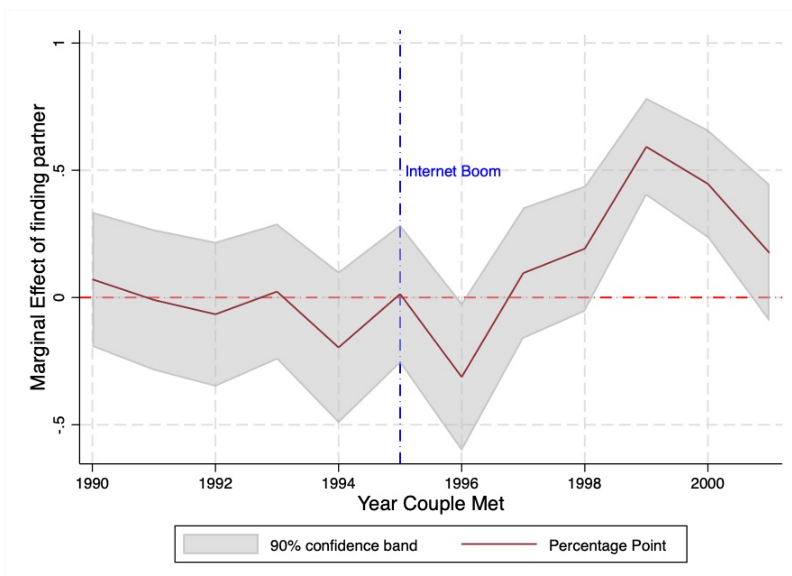


Figure 1.15. Change in β_6 coefficient for the sample of married and not married with $\delta = 1$.

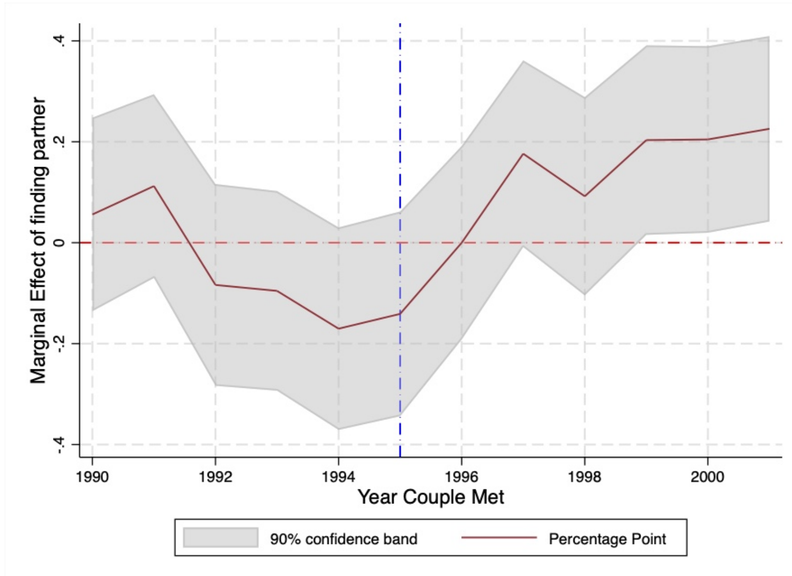


Figure 1.16. Change in β_6 coefficient for the sample of married with $\delta = 3$.

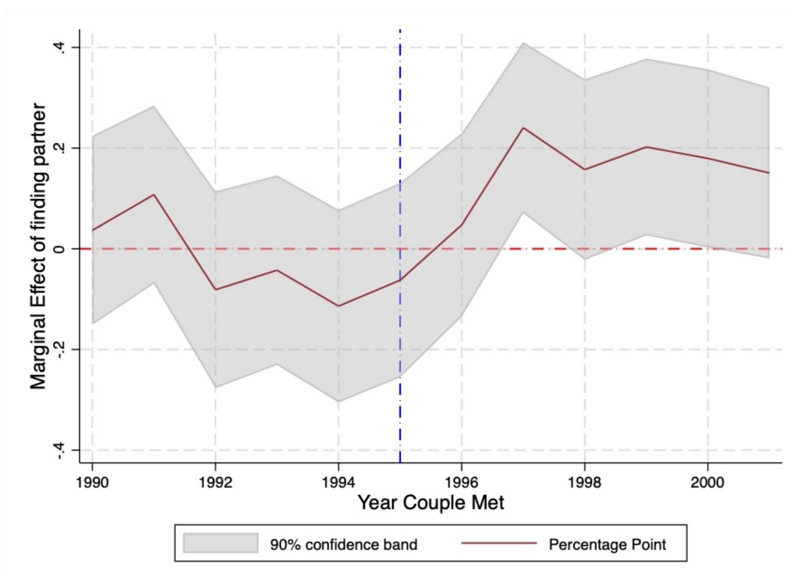


Figure 1.17. Change in β_6 coefficient for the sample of married. and not married with $\delta = 3$.

1.7. First Appendix: Microeconomic Foundations

For simplicity, we will study how agent x looks for an agent y with its different characteristics. By symmetry, the same will happen for the agents y when they look for an x.

Let us assume that agents of type y have the same probability of receiving proposals. Given the utility they generate, they always choose x_A over x_B. The same person of type y_k, $k \in \{A, B\}$ can receive many proposals; among them, only one will be chosen randomly.

Following (Albrecht et al., 2004)[13], if the number of competitors that an agent of type x_A has is i (all of type x_A), then the probability that a unique agent of type y_A accepts the competitors is:

$$\phi(i; N^A_{x,T} - 1) = \begin{pmatrix} N^A_{x,T} - 1 \\ i \end{pmatrix} \left(\frac{1}{N^A_{y,T} + N^B_{y,T}}\right)^i \left(1 - \frac{1}{N^A_{y,T} + N^B_{y,T}}\right)^{N^A_{x,T} - i - 1} \tag{1.21}$$

with $\begin{pmatrix} N^A_{x,T} - 1 \\ i \end{pmatrix} = \frac{(N^A_{x,T}-1)!}{i!(N^A_{x,T}-i-1)!}$

Let define $\eta^A_{y,T} = \frac{N^A_{y,T}}{N^A_{y,T}+N^B_{y,T}}$ the proportion of y_A on method T and $p_{x_A,A}$ the probability that an agent x_A matches an agent of type y_A where:

$$p_{x_A,A} = \left(1 - \sum_{i=0}^{N^A_{x,T}-1} \phi(i; N^A_{x,T} - 1)\frac{i}{i+1}\right) \eta^A_{y,T} \tag{1.22}$$

where $\sum_{i=0}^{N^A_{x,T}-1} \phi(i; N^A_{x,T}-1)\frac{i}{i+1}$ is the probability of all possible i competitors has success on match and $\eta^B_{y,T} = \frac{N^B_{y,T}}{N^A_{y,T}+N^B_{y,T}}$ is the proportion of y_B on method T. With the above, it is possible to propose the following tree of probabilities associated with x_A:

[13]The authors micro found an urn-ball process in which firms receive multiple job applications. In this specification, the logic is similar, but different types of agents can send the applications.

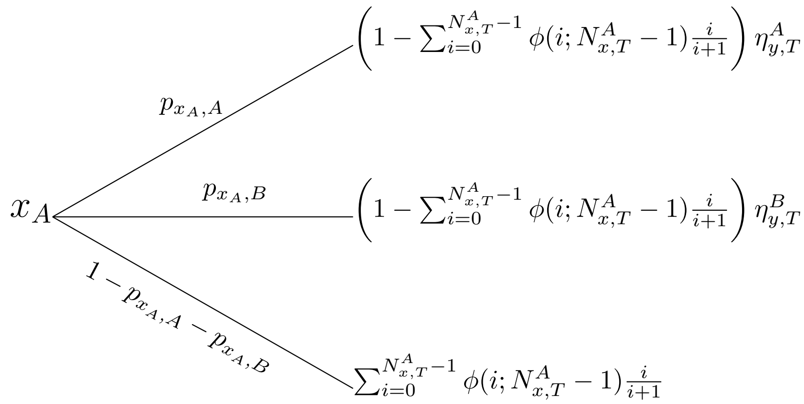


Figure B1. Probability tree of an agent of type x_A.

Let $p_{x_B,A}$ be the probability that an agent of type x_B has a match with agent y_A. Remember that agents x_B, if they compete with a x_A, then with probability 1 x_B, will be rejected. Then the probability is given by:

$$p_{x_B,A}=\left(1-\frac{1}{N_{y,T}^A+N_{y,T}^B}\right)^{N_{x,T}^A}\left(1-\sum_{i=0}^{N_{x,T}^B-1}\phi(i;N_{x,T}^B-1)\frac{i}{i+1}\right)\eta_{y,T}^A \tag{1.23}$$

where $\left(1-\frac{1}{N_{y,T}^A+N_{y,T}^B}\right)^{N_{x,T}^A}$ represents the probability of an agent of a type y_A no longer receives any proposals from an individual of type x_A. The tree of probabilities associated with x_B is:

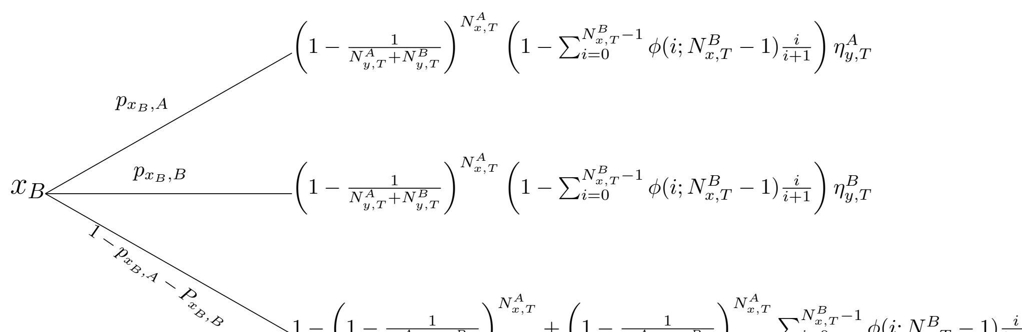


Figure B2. Probability tree of an agent of type x_B.

Since an agent of type x was arbitrarily chosen to construct the probabilities, the above also applies to an agent of type y. Then

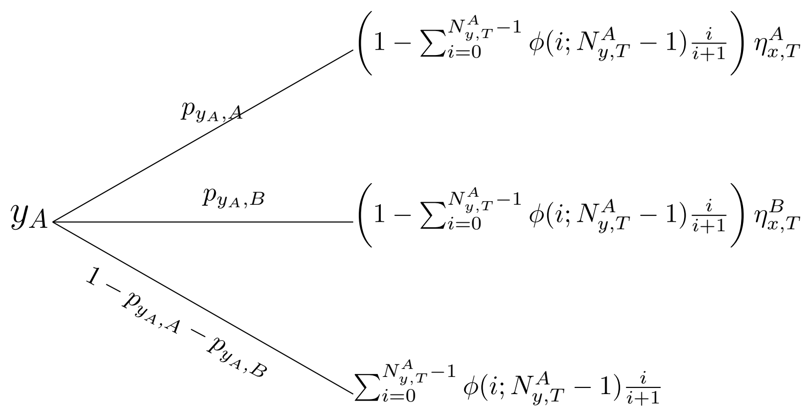


Figure B3. Probability tree of an agent of type y_A.

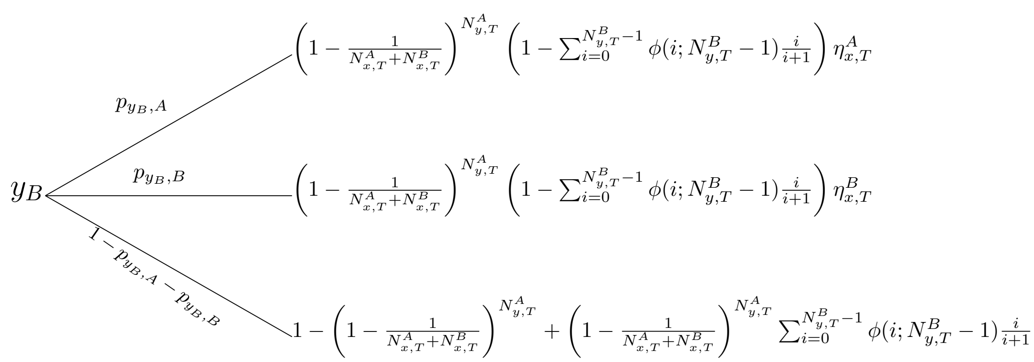


Figure B4. Probability tree of an agent of type y_B.

1.7.1. Binomial to Poisson

Let $\lambda_{A,T}=\frac{N^A_{x,T}}{N^A_{y,T}+N^B_{y,T}}\Rightarrow\frac{\lambda_{A,T}}{N^A_{x,T}}=\frac{1}{N^A_{y,T}+N^B_{y,T}}$ then by (1.21) we have:

$$\phi(i;N^A_{x,T})=\begin{pmatrix}N^A_{x,t}\\i\end{pmatrix}\left(\frac{\lambda_{A,T}}{N^A_{x,T}}\right)^i\left(1-\frac{\lambda_{A,T}}{N^A_{x,T}}\right)^{N^A_{y,T}-i}$$

$$\Rightarrow\frac{N^A_{x,T}!}{i!\left(N^A_{x,T}-i\right)!}\left(\frac{\lambda_{A,T}}{N^A_{x,T}}\right)^i\left(1-\frac{\lambda_{A,T}}{N^A_{x,T}}\right)^{N^A_{x,T}-i}$$

Let's make $\lim_{N^A_{x,T}\to\infty}$ then:

$$\frac{\lambda^i_{A,T}}{i!}\lim_{N^A_{x,T}\to\infty}\frac{N^A_{x,T}!}{\left(N^A_{x,T}-i\right)!}\left(\frac{1}{N^A_{x,T}}\right)^i\left(1-\frac{\lambda_{A,T}}{N^A_{x,T}}\right)^{N^A_{x,T}-i}$$
$$\Rightarrow\frac{\lambda^i_{A,T}}{i!}\lim_{N^A_{x,T}\to\infty}\underbrace{\frac{N^A_{x,T}!}{\left(N^A_{x,T}-i\right)!}\left(\frac{1}{N^A_{x,T}}\right)^i}_{P_1}\underbrace{\left(1-\frac{\lambda_{A,T}}{N^A_{x,T}}\right)^{N^A_{x,T}}}_{P_2}\underbrace{\left(1-\frac{\lambda_{A,T}}{N^A_{x,T}}\right)^{-i}}_{P_3}$$

We are going to solve the expression by parts. With respect to P_1 we have:

$$\lim_{N^A_{x,T}\to\infty}\frac{N^A_{x,T}!}{\left(N^A_{x,T}-i\right)!}\left(\frac{1}{N^A_{x,T}}\right)^i$$
$$\Rightarrow\lim_{N^A_{x,T}\to\infty}\frac{N^A_{x,T}\left(N^A_{x,T}-1\right)\left(N^A_{x,T}-2\right)\cdots\left(N^A_{x,T}-i\right)\left(N^A_{x,T}-i-i\right)\cdots 1}{\left(N^A_{x,T}-i\right)\left(N^A_{x,T}-i-i\right)\cdots 1}\left(\frac{1}{N^A_{x,T}}\right)^i$$
$$\Rightarrow\lim_{N^A_{x,T}\to\infty}\frac{N^A_{x,T}\left(N^A_{x,T}-1\right)\left(N^A_{x,T}-2\right)\cdots\left(N^A_{x,T}-i+1\right)}{\left(N^A_{x,T}\right)^i}$$
$$\Rightarrow\lim_{N^A_{x,T}\to\infty}\frac{N^A_{x,T}}{N^A_{x,T}}\frac{\left(N^A_{x,T}-1\right)}{N^A_{x,T}}\frac{\left(N^A_{x,T}-2\right)}{N^A_{x,T}}\cdots\frac{\left(N^A_{x,T}-i+1\right)}{N^A_{x,T}}=1 \tag{1.24}$$

Now let's work with P_2[14]

$$\lim_{N^A_{x,T}\to\infty}\left(1-\frac{\lambda_{A,T}}{N^A_{x,T}}\right)^{N^A_{x,T}}$$

with $u=-\frac{N^A_{x,T}}{\lambda_{A,T}}$ we have:

$$\lim_{u\to\infty}\left(1+\frac{1}{u}\right)^{-u\lambda_{A,T}}=e^{-\lambda_{A,T}} \tag{1.25}$$

And finally P_3 is equivalent to:

$$\lim_{N^A_{x,T}\to\infty}\left(1-\frac{\lambda_{A,T}}{N^A_{x,T}}\right)^{-i}=1 \tag{1.26}$$

[14]We know that $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$

Therefore, putting together the expressions (1.24), (1.25) and (1.26) we have:

$$\lim_{N_{x,T}^A \to \infty} \phi(i; N_{x,T}^A) = \frac{\lambda_A^i}{i!} e^{-\lambda_{A,T}} \tag{1.27}$$

Let us analyze the probabilities for agents x of type A.

$$\begin{aligned}
& p_{x_A,A} = \left(1 - \sum_{i=0}^{N_{x,T}^A - 1} \phi(i; N_{x,T}^A - 1) \frac{i}{i+1}\right) \eta_{y,T}^A \\
\Rightarrow & p_{x_A,A} = \lim_{N_{x,T}^A \to \infty} \left(1 - \sum_{i=0}^{N_{x,T}^A - 1} \frac{\lambda_A^i}{i!} e^{-\lambda_{A,T}} \frac{i}{i+1}\right) \eta_{y,T}^A \\
\Rightarrow & p_{x_A,A} = \eta_{y,T}^A - \eta_{y,T}^A \lim_{N_{x,T}^A \to \infty} \sum_{i=0}^{N_{x,T}^A - 1} \frac{\lambda_A^i}{i!} e^{-\lambda_{A,T}} \left(1 - \frac{1}{i+1}\right) \\
\Rightarrow & p_{x_A,A} = \eta_{y,T}^A - \eta_{y,T}^A e^{-\lambda_{A,T}} \left(\lim_{N_{x,T}^A \to \infty} \sum_{i=0}^{N_{x,T}^A - 1} \frac{\lambda_A^i}{i!} - \frac{1}{\lambda_{A,T}} \lim_{N_{x,T}^A \to \infty} \sum_{i=0}^{N_{x,T}^A - 1} \frac{\lambda_A^{i+1}}{(i+1)!} \right) \\
\Rightarrow & p_{x_A,A} = \eta_{y,T}^A - \eta_{y,T}^A e^{-\lambda_{A,T}} \left(e^{\lambda_{A,T}} - \frac{1}{\lambda_{A,T}} \left(e^{\lambda_{A,T}} - 1\right) \right) \\
\Rightarrow & p_{x_A,A} = \frac{\eta_{y,T}^A}{\lambda_{A,T}} \left(1 - e^{-\lambda_{A,T}}\right)
\end{aligned} \tag{1.28}$$

In the same way, it is possible to obtain the probability of finding a person of type B being a type x of type A:

$$p_{x_A,B} = \frac{\eta_{y,T}^B}{\lambda_{A,T}} \left(1 - e^{-\lambda_{A,T}}\right) \tag{1.29}$$

Therefore:

$$1 - p_{x_A,A} - p_{x_A,B} = 1 - \frac{1}{\lambda_{A,T}} \left(1 - e^{-\lambda_{A,T}}\right) \tag{1.30}$$

Now analyzing the case that people are type B. Let's start by parts:

$$\lim_{N_{x,T}^A \to \infty} \left(1 - \frac{1}{N_{y,T}^A + N_{y,T}^B}\right)^{N_{x,T}^A}$$

Assuming that the constant $\lambda_{A,T} = \frac{N^A_{x,T}}{N^A_{y,T}+N^B_{y,T}}$ it holds, then:

$$\begin{aligned}
&\lim_{N^A_{x,T}\to\infty}\left(1-\frac{\lambda_{A,T}}{N^A_{x,T}}\right)^{N^A_{x,T}} \\
\Rightarrow &\lim_{N^A_{x,T}\to\infty}\left(1+\frac{-\lambda_{A,T}}{N^A_{x,T}}\right)^{N^A_{x,T}\frac{-\lambda_{A,T}}{-\lambda_{A,T}}} = e^{-\lambda_{A,T}} \qquad (1.31)
\end{aligned}$$

So by symmetry we have:

$$p_{x_B,A} = e^{-\lambda_{A,T}}\frac{\eta^A_{y,T}}{\lambda_{B,T}}\left(1-e^{-\lambda_{B,T}}\right) \qquad (1.32)$$

$$p_{x_B,B} = e^{-\lambda_{A,T}}\frac{\eta^B_{y,T}}{\lambda_{B,T}}\left(1-e^{-\lambda_{B,T}}\right) \qquad (1.33)$$

$$1-p_{x_B,A}-p_{x_B,B} = e^{-\lambda_{A,T}}\frac{1}{\lambda_{B,T}}\left(1-e^{-\lambda_{B,T}}\right) \qquad (1.34)$$

with $\lambda_{B,T} = \frac{N^B_{x,T}}{N^A_{y,T}+N^B_{y,T}}$. Let's rewrite convenient $\lambda_{A,T}$:

$$\lambda_{A,T} = \frac{N^A_{x,T}}{N^A_{y,T}+N^B_{y,T}}\frac{N^A_{x,T}+N^B_{x,T}}{N^A_{x,T}+N^B_{x,T}} = \eta^A_{x,T}\theta^T_{x,y}, \quad \theta^T_{x,y} \equiv \frac{N^A_{x,T}+N^B_{x,T}}{N^A_{y,T}+N^B_{y,T}}$$

We can rewrite the probabilities of the agents on T:

$$\begin{aligned}
p_{x_A,A} &= \frac{\eta^A_{y,T}}{\eta^A_{x,T}\theta^T_{x,y}}\left(1-e^{-\eta^A_{x,T}\theta^T_{x,y}}\right) \\
p_{x_A,B} &= \frac{\eta^B_{y,T}}{\eta^A_{x,T}\theta^T_{x,y}}\left(1-e^{-\eta^A_{x,T}\theta^T_{x,y}}\right) \\
p_{x_B,A} &= e^{-\eta^A_{x,T}\theta^T_{x,y}}\frac{\eta^A_{y,T}}{\eta^B_{x,T}\theta^T_{x,y}}\left(1-e^{-\eta^B_{x,T}\theta^T_{x,y}}\right) \\
p_{x_B,B} &= e^{-\eta^A_{x,T}\theta^T_{x,y}}\frac{\eta^B_{y,T}}{\eta^B_{x,T}\theta^T_{x,y}}\left(1-e^{-\eta^B_{x,T}\theta^T_{x,y}}\right)
\end{aligned}$$

In the case that if w is searched, the probabilities are:

$$g_{x_A,A} = \frac{\eta^A_{y,w}}{\eta^A_{x,w}\theta^w_{x,y}}\left(1 - e^{-\eta^A_{x,w}\theta^w_{x,y}}\right)$$
$$g_{x_A,B} = \frac{\eta^B_{y,w}}{\eta^A_{x,w}\theta^w_{x,y}}\left(1 - e^{-\eta^A_{x,w}\theta^w_{x,y}}\right)$$
$$g_{x_B,A} = e^{-\eta^A_{x,w}\theta^w_{x,y}}\frac{\eta^A_{y,w}}{\eta^B_{x,w}\theta^w_{x,y}}\left(1 - e^{-\eta^B_{x,w}\theta^w_{x,y}}\right)$$
$$g_{x_B,B} = e^{-\eta^A_{x,w}\theta^w_{x,y}}\frac{\eta^B_{y,w}}{\eta^B_{x,w}\theta^w_{x,y}}\left(1 - e^{-\eta^B_{x,w}\theta^w_{x,y}}\right)$$

Analyzing the symmetric equilibrium then we have:

$$\theta^T_{x,y} = \theta^w_{x,y} = 1, \quad \eta^A_{x,T} = \eta^A_{y,T}, \quad \eta^B_{x,T} = \eta^B_{y,T}, \quad \eta^A_{x,w} = \eta^A_{y,w}, \quad \eta^B_{x,w} = \eta^B_{y,w} \tag{1.35}$$

With (1.35), the probabilities associated with type A and B searching T are:

$$p_{x_A,A} = \left(1 - e^{-\eta^A_{x,T}}\right)$$
$$p_{x_A,B} = \frac{(1-\eta^A_{x,T})}{\eta^A_{x,T}}\left(1 - e^{-\eta^A_{x,T}}\right), \quad \eta^A_{x,T} + \eta^B_{x,T} = 1$$
$$p_{x_B,A} = e^{-\eta^A_{x,T}}\eta^A_{x,T}\frac{\left(1 - e^{-(1-\eta^A_{x,T})}\right)}{1-\eta^A_{x,T}}, \quad \eta^A_{x,T} + \eta^B_{x,T} = 1$$
$$p_{x_B,B} = e^{-\eta^A_{x,T}}\left(1 - e^{-(1-\eta^A_{x,T})}\right)$$

In te case of search on w for:

$$g_{x_A,A} = \left(1 - e^{-\eta^A_{x,w}}\right)$$
$$g_{x_A,B} = \frac{(1-\eta^A_{x,w})}{\eta^A_{x,w}}\left(1 - e^{-\eta^A_{x,w}}\right), \quad \eta^A_{x,w} + \eta^B_{x,w} = 1$$
$$g_{x_B,A} = e^{-\eta^A_{x,w}}\eta^A_{x,w}\frac{\left(1 - e^{-(1-\eta^A_{x,w})}\right)}{1-\eta^A_{x,w}}, \quad \eta^A_{x,w} + \eta^B_{x,w} = 1$$
$$g_{x_B,B} = e^{-\eta^A_{x,w}}\left(1 - e^{-(1-\eta^A_{x,w})}\right)$$

1.8. First Appendix: Sequential Game

Assume the following sequential game, where the agents are assumed to act symmetrically. Agents of type x_A choose first where to look for a partner, via T or w. Then agents of type x_B choose in which method to search. The following figure shows the sequence of the game: Note that the decisions in both branches of type B agents depend on c_0. Note

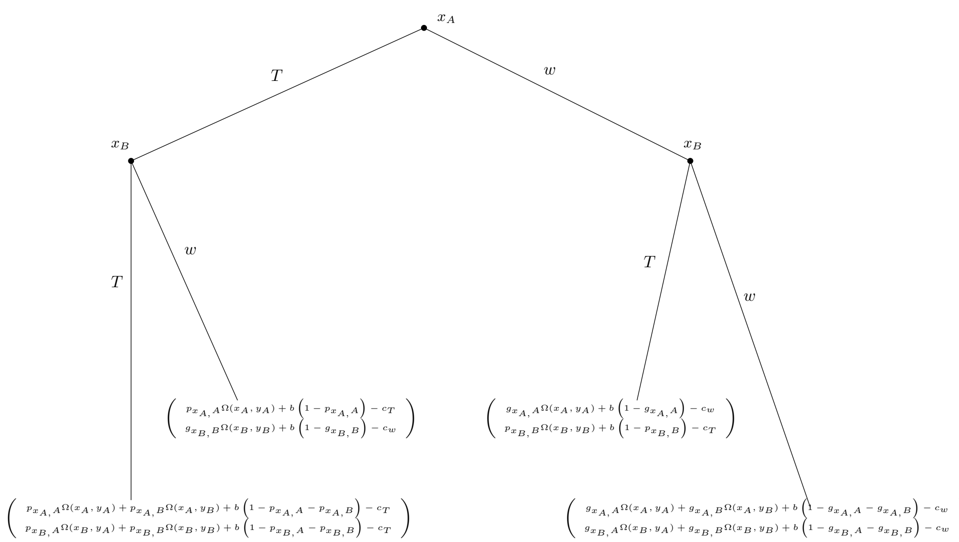


Figure C1. Sequential game, where type A agents play first.

that the decisions in both branches of type B agents depends on c_0. In the branch where type A agents choose T, then separating equilibria will exist if $c > c_0$ then type B agents will choose to search for a partner via T. Which is in line with the model that the selection is simultaneous, but the difference is that in the original model there is a range of c for which type B agents migrate to the w method.

1.9. First Appendix: Separating and pooling equilibriums

Let be an economy where the agents are in the same proportions both in terms of type and characteristics. In addition, it will be assumed that supermodularity is satisfied and that the following constants exist:

$$c_0 = be^{-\frac{1}{2}}\left(1-e^{-\frac{1}{2}}\right)\Bigg(k_3(k_2+1)-2\Bigg) - b\left(1-e^{-1}\right)(k_3-1)$$

$$c_1 = be^{-\frac{1}{2}}\left(1-e^{-\frac{1}{2}}\right)\Bigg(k_3(k_2+1)-2\Bigg)$$

$$c_2 = b\Bigg(k_3\left(e^{-1}(k_2+1)-1\right)+1-2e^{-1}\Bigg)$$

$$c_3 = bk_1k_2k_3\Bigg[(1-e^{-1})-(1-e^{-\frac{1}{2}})\Bigg] + b\Bigg[2\left(1-e^{-\frac{1}{2}}\right)-\left(1-e^{-1}\right)\Bigg] + bk_2k_3(1-e^{-\frac{1}{2}})$$

$$c_4 = bk_2k_3\left(1-e^{-1}\right)(k_1-1) + be^{-1}\left(1-k_2k_3\right)$$

where:

$$k_1 \geq \frac{1+e}{e-1}, \quad k_2(k_1-2)+1>0, \quad c_2>c_1 \Rightarrow c_3>c_2 \quad c_3>c_1>c_0 \quad c_4>c_2 \tag{1.36}$$

Next, the following equilibria will be studied, where only the symmetrical equilibria will be analyzed:

(i) **Separating Equilibria**

- $S_{(A,T)\times(B,w)}$: All A's in T and all B's in w.
- $S_{(A,w)\times(B,T)}$: All A's in w and all B's in T.

(ii) **Pooling Equilibria**

- P_T: All in T.
- P_w: All in w.
- $P_{T,w}$: Pure Pooling Equilibria.

(iii) **Partially Pooling Equilibria**

- $S_{(B,T)\times((A,T),(A,w))}$: All B on T and some A on T and w.

- $S_{(B,w)\times((A,T),(A,w))}$: All B on w and some A on T and w.
- $S_{(A,T)\times((B,T),(B,w))}$: All A on T and some B on T and w.
- $S_{(A,w)\times((B,T),(B,w))}$: All A on w and some B on T and w.

1.9.1. $S_{(A,T)\times(B,w)}$: All A's in T and all B's in w

For this equilibrium to hold, it must be true that:

$$S_T(x_A) > S_w(x_A) \quad S_T(x_B) < S_w(x_B) \Rightarrow \eta^A_{x,T} = 1, \quad \eta^B_{x,w} = 1$$

Since the agents are in the same proportion in terms of characteristics and types and Replacing in the functional form of the probabilities given by (1.3), (1.4), (1.5), (1.5), (1.6), (1.7), (1.8), (1.9), (1.9) and (1.6) we have:

$$\begin{aligned}
&p_{x_A,A} = 1 - e^{-1} \\
&p_{x_A,B} = 0 \\
&g_{x_A,A} = 0 \\
&g_{x_A,B} = 1 \\
&p_{x_B,A} = e^{-1} \\
&p_{x_B,B} = 0 \\
&g_{x_B,A} = 0 \\
&g_{x_B,B} = 1 - e^{-1}
\end{aligned} \tag{1.37}$$

The expected utility of agent x_A, defined in (1.11) and (1.13) are:

$$S_T(x_A) = b\left(1 - e^{-1}\right) k_1 k_2 k_3 + e^{-1} b - c_T \tag{1.38}$$

$$S_w(x_A) = b k_2 k_3 - c_w \tag{1.39}$$

If it is fulfilled that (1.36) then:

$$S_T(x_A) > S_w(x_A)$$

The expected utility of agent x_B, defined in (1.12) and (1.14) are:

$$S_T(x_B) = be^{-1}k_2k_3 + b(1 - e^{-1}) - c_T \tag{1.40}$$

$$S_w(x_B) = b(1 - e^{-1})k_3 + e^{-1}b - c_w \tag{1.41}$$

If it is fulfilled that (1.36) then:

$$S_T(x_B) < S_w(x_B)$$

Therefore, the equilibrium hold.

1.9.2. $S_{(B,T)\times(A,w)}$: All B's in T and all A's in w

For this equilibrium to hold, it must be true that:

$$S_T(x_A) < S_w(x_A) \quad S_T(x_B) > S_w(x_B)$$

This equilibrium does not hold if the assumptions (1.36) are met. This is because it requires that the inequalities of the expected utilities of the agents' search methods be contrary to the previous equilibrium, presented in section . Note that this equilibrium is similar to $S_{(A,T)\times(B,w)}$ in 1.9.2. Then not exists $c > 0$ to hold the equilibrium.

1.9.3. P_T: All in T

For this equilibrium to hold, it must be true that:

$$S_T(x_A) > S_w(x_A) \quad S_T(x_B) > S_w(x_B) \Rightarrow \eta^A_{x,T} = \frac{1}{2}, \quad \eta^B_{x,T} = \frac{1}{2}$$

Since the agents are in the same proportion in terms of characteristics and types and Replacing in the functional form of the probabilities given by (1.3), (1.4), (1.5), (1.5), (1.6), (1.7), (1.8), (1.9), (1.9) and (1.6) we have:

$$p_{x_A,A} = 1 - e^{-\frac{1}{2}}$$

$$p_{x_A,B} = 1 - e^{-\frac{1}{2}}$$

$$g_{x_A,A} = 0$$

$$g_{x_A,B} = 0$$

$$p_{x_B,A} = e^{-\frac{1}{2}}\left(1 - e^{-\frac{1}{2}}\right)$$

$$p_{x_B,B} = e^{-\frac{1}{2}}\left(1 - e^{-\frac{1}{2}}\right)$$

$$g_{x_B,A} = 0$$

$$g_{x_B,B} = 0$$

The expected utility of agent x_A, defined in (1.11) and (1.13) are:

$$S_T(x_A) = b\left(1 - e^{-\frac{1}{2}}\right)(k_2k_3(k_1+1) - 2) + b - c_T \tag{1.42}$$

$$S_w(x_A) = b - c_w \tag{1.43}$$

If it is fulfilled that (1.36) then:

$$S_T(x_A) > S_w(x_A)$$

The expected utility of agent x_B, defined in (1.12) and (1.14) are:

$$S_T(x_B) = be^{-\frac{1}{2}}\left(1 - e^{-\frac{1}{2}}\right)\left(k_3(k_2+1) - 2\right) + b - c_T \tag{1.44}$$

$$S_w(x_B) = b - c_w \tag{1.45}$$

If it is fulfilled that (1.36) then:

$$S_T(x_B) > S_w(x_B)$$

Therefore, equilibrium hold.

1.9.4. P_w: All in w.

In this equilibrium, given that $c_T > c_w$, this equilibrium will always hold.

1.9.5. $S_{(B,T)\times((A,T),(A,w))}$: All B on T and some A on T and w

For this equilibrium to hold, it must be true that:

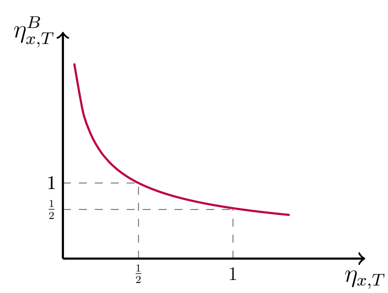
$$S_T(x_A) = S_w(x_A) \quad S_T(x_B) > S_w(x_B)$$

Since the proportions of the agents are equivalent, it is possible to have the following expression[15]:

$$\begin{aligned} &\eta_x^B = \eta_{x,T}^B \eta_{x,T} + \eta_{x,w}^B (1 - \eta_{x,T}), \quad \eta_{x,w}^B = 0 \\ \Rightarrow &\eta_x^B = \eta_{x,T}^B \eta_{x,T} \end{aligned} \tag{1.46}$$

With (**??**) and for the equilibrium $S_{(B,T)\times((A,T),(A,w))}$ we have:

$$\begin{aligned} &\frac{1}{2} = \eta_{x,T}^B \eta_{x,T}, \quad \eta_{x,T}^B \in [\frac{1}{2}, 1] \quad \eta_{x,T} \in [\frac{1}{2}, 1] \\ &\eta_{x,T}^B + \eta_{x,T}^A = 1 \end{aligned} \tag{1.47}$$



[15]Note that $N_x^B = N_{x,T}^B + N_{x,w}^B \Rightarrow \frac{N_x^B}{N_x} = \frac{N_{x,T}^B}{N_{x,T}} \frac{N_{x,T}}{N_x} + \frac{N_{x,w}^B}{N_{x,w}} \frac{N_{x,w}}{N_x} \Rightarrow \eta_x^B = \eta_{x,T}^B \eta_{x,T} + \eta_{x,w}^B \eta_{x,w}$

So the probabilities for type x_A are:

$$p_{x_A,A} = \left(1 - e^{-(1-\eta^B_{x,T})}\right)$$

$$p_{x_A,B} = \left(1 - e^{-(1-\eta^B_{x,T})}\right)\frac{\eta^B_{x,T}}{(1-\eta^B_{x,T})}$$

$$g_{x_A,A} = \left(1 - e^{-1}\right)$$

$$g_{x_A,B} = 0$$

And for type x_B are:

$$p_{x_B,A} = e^{-(1-\eta^B_{x,T})}\left(1 - e^{-\eta^B_{x,T}}\right)\frac{(1-\eta^B_{x,T})}{\eta^B_{x,T}}$$

$$p_{x_B,B} = e^{-(1-\eta^B_{x,T})}\left(1 - e^{-\eta^B_{x,T}}\right)$$

$$g_{x_B,A} = e^{-1}$$

$$g_{x_B,B} = 0$$

The expected utility of agent x_A, defined in (1.11) and (1.13) are:

$$S_T(x_A) = bk_2k_3\frac{\left(1 - e^{-(1-\eta^B_{x,T})}\right)}{(1-\eta^B_{x,T})}\left(k_1(1-\eta^B_{x,T}) + \eta^B_{x,T}\right) + b\left(1 - \frac{(1-e^{-(1-\eta^B_{x,T})})}{(1-\eta^B_{x,T})}\right) - c_T$$

$$S_w(x_A) = b\left[k_1k_2k_3\left(1-e^{-1}\right) + e^{-1}\right] - c_w$$

If it is fulfilled that (1.36) then:

$$S_T(x_A) \neq S_w(x_A)$$

Therefore, $\nexists\, c > 0$ to hold the equilibrium.

1.9.6. $S_{(B,w)\times((A,T),(A,w))}$: All B on w and some A on T and w

For this equilibrium to hold, it must be true that:

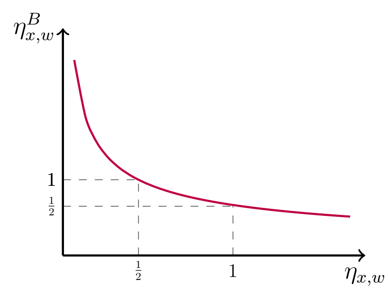
$$S_T(x_A) = S_w(x_A) \quad S_T(x_B) < S_w(x_B)$$

As in the equilibrium $S_{(B,T)\times((A,T),(A,w))}$ studied in 1.9.5 it is possible to have the following:

$$\eta_x^B = \eta_{x,T}^B \eta_{x,T} + \eta_{x,w}^B (1 - \eta_{x,T}), \quad \eta_{x,T}^B = 0$$
$$\eta_x^B = \eta_{x,w}^B \eta_{x,w}$$

For the equilibrium $S_{(B,w)\times((A,T),(A,w))}$ we have:

$$\frac{1}{2} = \eta_{x,w}^B \eta_{x,w}, \quad \eta_{x,w}^B \in [\frac{1}{2}, 1] \quad \eta_{x,w} \in [\frac{1}{2}, 1] \tag{1.48}$$
$$\eta_{x,w}^B + \eta_{x,w}^A = 1 \tag{1.49}$$



So the probabilities for type x_A are:

$$p_{x_A,A} = \left(1 - e^{-1}\right), \quad \eta_{y,T}^A = 1$$

$$p_{x_A,B} = 0$$

$$g_{x_A,A} = \left(1 - e^{-(1-\eta_{x,w}^B)}\right)$$

$$g_{x_A,B} = \left(1 - e^{-(1-\eta_{x,w}^B)}\right) \frac{\eta_{y,w}^B}{(1 - \eta_{x,w}^B)}$$

In the case of agent of type x_B:

$$p_{x_B,A} = e^{-1}$$

$$\Rightarrow p_{x_B,B} = 0$$

$$g_{x_B,A} = e^{-(1-\eta^B_{x,w})}\left(\frac{\left(1-e^{-\eta^B_{x,w}}\right)}{\eta^B_{x,w}}(1-\eta^B_{x,w})\right)$$

$$g_{x_B,B} = e^{-(1-\eta^B_{x,w})}\left(1-e^{-\eta^B_{x,w}}\right)$$

The expected utility of agent x_A, defined in (1.11) and (1.13) are:

$$S_T(x_A) = (1-e^{-1})k_1k_2k_3b + e^{-1}b - c_T$$

$$S_w(x_A) = bk_2k_3\left(1-e^{-(1-\eta^B_{x,w})}\right)\left(k_1 + \frac{\eta^B_{x,w}}{(1-\eta^B_{x,w})}\right) + b\left(1 - \frac{(1-e^{-(1-\eta^B_{x,w})})}{(1-\eta^B_{x,w})}\right) - c_w$$

If it is fulfilled that (1.36) then:

$$S_T(x_A) = S_w(x_A)$$

Let's study what happens with agent x_B.

$$S_T(x_B) = b\left(e^{-1}k_2k_3 + (1-e^{-1})\right) - c_T$$

$$S_w(x_B) = bk_3e^{-(1-\eta^B_{x,w})}\frac{\left(1-e^{-\eta^B_{x,w}}\right)}{\eta^B_{x,w}}\left(k_2(1-\eta^B_{x,w}) + \eta^B_{x,w}\right) + b\left(1 - e^{-(1-\eta^B_{x,w})}\frac{\left(1-e^{-\eta^B_{x,w}}\right)}{\eta^B_{x,w}}\right) - c_w$$

If it is fulfilled that (1.36) then:

$$S_w(x_B) > S_T(x_B)$$

The equilibrium hold if $0 < c < c_3$. Note that if $c = 0 \Rightarrow \eta^B_{x,w} = 0$, and by equation (1.48) we have $\eta_{x,w} = 0$ that have non sense, because the equilibrium hold all type B on w. Then we need a restriction, ie, $\eta^B_{x,w} \in [\frac{1}{2}, 1]$.

1.9.7. $S_{(A,w)\times((B,T),(B,w))}$: All A on w and some B on T and w

For this equilibrium to hold, it must be true that:

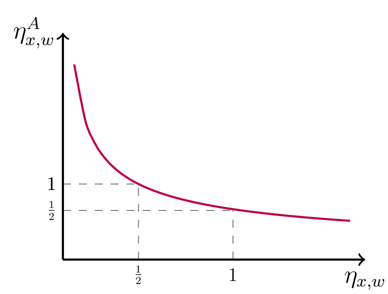
$$S_T(x_A) < S_w(x_A) \quad S_T(x_B) = S_w(x_B)$$

Since the proportions of the agents are equivalent, it is possible to have the following expression[16]:

$$\begin{aligned} &\eta^A_x = \eta^A_{x,T}\eta_{x,T} + \eta^A_{x,w}(1-\eta_{x,T}) \quad \eta^A_{x,T} = 0 \\ &\eta^A_x = \eta^A_{x,w}\eta_{x,w} \end{aligned} \tag{1.50}$$

For the equilibrium $S_{(A,w)\times((B,T),(B,w))}$ we have:

$$\begin{aligned} &\frac{1}{2} = \eta^A_{x,w}\eta_{x,w}, \quad \eta^A_{x,w} \in [\frac{1}{2}, 1] \quad \eta_{x,w} \in [\frac{1}{2}, 1] \\ &\eta^B_{x,w} + \eta^A_{x,w} = 1 \end{aligned} \tag{1.51}$$



[16]Nothe that $N^A_x = N^A_{x,T} + N^A_{x,w} \Rightarrow \frac{N^A_x}{N_x} = \frac{N^A_{x,T}}{N_x}\frac{N_{x,T}}{N_{x,T}} + \frac{N^A_{x,w}}{N_x}\frac{N_{x,w}}{N_{x,w}}$

So the probabilities for type x_A are:

$$
\begin{aligned}
&p_{x_A,A} = 0, \quad \eta^A_{y,T} = 0 \\
&p_{x_A,B} = 1 \\
&g_{x_A,A} = \left(1 - e^{-\eta^A_{x,w}}\right) \\
&g_{x_A,B} = \left(1 - e^{-\eta^A_{x,w}}\right) \frac{(1 - \eta^A_{x,w})}{\eta^A_{x,w}}
\end{aligned}
$$

And for the case of agent B:

$$
\begin{aligned}
&p_{x_B,A} = 0 \\
&p_{x_B,B} = (1 - e^{-1}) \\
&g_{x_B,A} = e^{-\eta^A_{x,w}} \left(\frac{\left(1 - e^{-(1-\eta^A_{x,w})}\right)}{(1 - \eta^A_{x,w})} \eta^A_{x,w} \right) \\
&g_{x_B,B} = e^{-\eta^A_{x,w}} \left(1 - e^{-(1-\eta^A_{x,w})}\right)
\end{aligned}
$$

If it is fulfilled that (1.36) then:

$$
S_T(x_B) \neq S_w(x_B) \tag{1.52}
$$

Therefore $\nexists\, c > 0$ to hold the equilibrium.

1.9.8. $S_{(A,T)\times((B,T),(B,w))}$: All A on T and some B on T and w

For this equilibrium to hold, it must be true that:

$$
S_T(x_A) > S_w(x_A) \quad S_T(x_B) = S_w(x_B)
$$

As in the equilibrium $S_{(A,w)\times((B,T),(B,w))}$ studied in 1.9.7 it is possible to have the following:

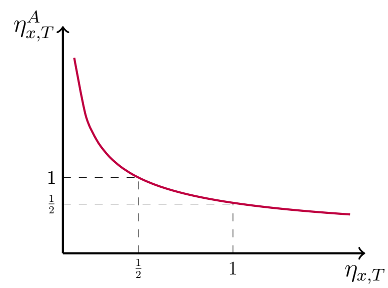
$$\eta_x^A = \eta_{x,T}^A \eta_{x,T} + \eta_{x,w}^A (1 - \eta_{x,T}), \quad \eta_{x,w}^A = 0 \tag{1.53}$$

$$\Rightarrow \eta_x^A = \eta_{x,T}^A \eta_{x,T} \tag{1.54}$$

With (**??**) and for the equilibrium $S_{(A,T)\times((B,T),(B,w))}$ we have:

$$\frac{1}{2} = \eta_{x,T}^A \eta_{x,T}, \quad \eta_{x,T}^A \in [\frac{1}{2}, 1] \quad \eta_{x,T} \in [\frac{1}{2}, 1] \tag{1.55}$$

$$\eta_{x,T}^A + \eta_{x,T}^B = 1$$



So the probabilities for type A are:

$$p_{x_A,A} = \left(1 - e^{-\eta_{x,T}^A}\right)$$

$$p_{x_A,B} = \left(1 - e^{-\eta_{x,T}^A}\right) \frac{(1 - \eta_{y,T}^A)}{\eta_{x,T}^A}$$

$$g_{x_A,A} = 0$$

$$g_{x_A,B} = 1$$

So the probabilities for type B are:

$$
\begin{aligned}
p_{x_B,A} &= e^{-\eta^A_{x,T}} \frac{\left(1-e^{-(1-\eta^A_{x,T})}\right)}{(1-\eta^A_{x,T})} \eta^A_{x,T} \\
p_{x_B,B} &= e^{-\eta^A_{x,T}} \left(1-e^{-(1-\eta^A_{x,T})}\right) \\
g_{x_B,A} &= 0 \\
g_{x_B,B} &= \left(1-e^{-1}\right)
\end{aligned}
$$

Then it must be fulfilled that:

$$
\begin{aligned}
S_T(x_B) &= be^{-\eta^A_{x,T}} \frac{\left(1-e^{-(1-\eta^A_{x,T})}\right)}{(1-\eta^A_{x,T})} \left[k_3 \left(k_2 \eta^A_{x,T} + (1-\eta^A_{x,T})\right) - 1\right] + b - c_T \\
S_w(x_B) &= b\left(\left(1-e^{-1}\right) k_3 + e^{-1}\right) - c_w
\end{aligned}
$$

If it is fulfilled that (1.36) then:

$$S_T(x_B) = S_w(x_B)$$

Let us study the utilities for agent x_A, assuming symmetry:

$$S_T(x_A) = b\frac{\left(1-e^{-\eta^A_{x,T}}\right)}{\eta^A_{x,T}} \left(k_2 k_3 \left(k_1 \eta^A_{x,T} + \left(1-\eta^A_{x,T}\right)\right) - 1\right) + b - c_T$$

$$S_w(x_A) = bk_2k_3$$

If it is fulfilled that (1.36) then:

$$S_T(x_A) > S_w(x_A)$$

Therefore, equilibrium hold.

1.9.9. $P_{T,w}$: Pure Pooling Equilibria

It must be fulfilled that:

$$S_T(x_A) = S_w(x_A) \quad S_T(x_B) = S_w(x_B)$$

It is easy to show that the only way this equilibrium can be sustained is that $c_T = c_w$. Therefore, $\nexists\, c > 0$ to hold the equilibrium.

1.9.10. Multiple Equilibrium Refinement

With the assumptions given by (1.15) we have to:

$$c_4 > c_3 > c_2 > c_1 > c_0 > 0 \tag{1.56}$$

We are going to study multiple equilibria, which are:

- **Case 1**: $S_{(A,T)\times((B,T),(B,w))}$ and P_T
- **Case 2**: $S_{(B,w)\times((A,T),(A,w))}$ and $S_{(A,T)\times(B,w)}$.

1.9.10.1. Case 1

We know in the equilibrium P_T that:

$$\begin{aligned}
&S_T(x_A) > S_w(x_A), \quad S_T(x_B) > S_w(x_B)\\
&S_T(x_A) = b\left(1 - e^{-\frac{1}{2}}\right)\Big[k_2k_3(k_1+1) - 2\Big] + b - c, \quad S_w(x_A) = b\\
&S_T(x_B) = be^{-\frac{1}{2}}\left(1 - e^{-\frac{1}{2}}\right)\Big[k_3(k_2+1) - 2\Big] + b - c, \quad S_w(x_B) = b
\end{aligned}$$

And the probabilities are:

$$\begin{aligned}
&p_{x_A,A} = 1 - e^{-\frac{1}{2}}, \quad p_{x_A,B} = 1 - e^{-\frac{1}{2}}, \quad g_{x_A,A} = 0, \quad g_{x_A,B} = 0\\
&p_{x_B,A} = e^{-\frac{1}{2}}\left(1 - e^{-\frac{1}{2}}\right), \quad p_{x_B,B} = e^{-\frac{1}{2}}\left(1 - e^{-\frac{1}{2}}\right), \quad g_{x_B,A} = 0, \quad g_{x_B,B} = 0
\end{aligned}$$

The equilibrium holds if $0 \leq c \leq c_1$.

In the case of $S_{(A,T)\times((B,T),(B,w))}$ we have:

$$S_T(x_A) > S_w(x_A), \quad S_T(x_B) = S_w(x_B)$$

$$S_T(x_A) = b\frac{\left(1 - e^{-\eta^A_{x,T}}\right)}{\eta^A_{x,T}}\left[k_2 k_3\left(k_1\eta^A_{x,T} + (1-\eta^A_{x,T})\right) - 1\right] + b - c, \quad S_w(x_A) = bk_2k_3$$

$$S_T(x_B) = be^{-\eta^A_{x,T}}\frac{\left(1 - e^{-\eta^A_{x,T}}\right)}{\eta^A_{x,T}}\left[k_3\left(k_2\eta^A_{x,T} + (1-\eta^A_{x,T})\right) - 1\right] + b - c, \quad S_w(x_B) = b\left((1-e^{-1})k_2 \right.$$

If $\eta^A_{x,T} = \dfrac{1}{2} \Rightarrow S_T(x_B) = e^{-\frac{1}{2}}\left(1 - e^{-\frac{1}{2}}\right)\left[k_3(k_2+1) - 2\right] + b - c_0 = b\left((1-e^{-1})k_3 + e^{-1}\right)$

$$(1.57)$$

And the probabilities are:

$$p_{x_A,A} = \left(1 - e^{-\eta^A_{x,T}}\right), \quad p_{x_A,B} = \left(1 - e^{-\eta^A_{x,T}}\right)\frac{(1-\eta^A_{y,T})}{\eta^A_{x,T}}, \quad g_{x_A,A} = 0, \quad g_{x_A,B} = 1$$

$$p_{x_B,A} = e^{-\eta^A_{x,T}}\frac{\left(1 - e^{-(1-\eta^A_{x,T})}\right)}{(1-\eta^A_{x,T})}\eta^A_{x,T}, \quad p_{x_B,B} = e^{-\eta^A_{x,T}}\left(1 - e^{-(1-\eta^A_{x,T})}\right), \quad g_{x_B,A} = 0$$

$$g_{x_B,B} = \left(1 - e^{-1}\right)$$

The equilibrium holds if $c_0 \leq c \leq c_2$. And we know $c_2 > c_1 > c_0 > 0$. Suppose that in equilibrium P_T (symmetric) a pair of agents B deviates to w. In this case $g_{x_B,B} = 1$, then in the case of agent B we have:

$$S_T(x_B) > S_w(x_B)$$

$$S_T(x_B) = be^{-\frac{1}{2}}\left(1 - e^{-\frac{1}{2}}\right)\left[k_3(k_2+1) - 2\right] + b - c, \quad S_w(x_B) = bk_3$$

Let's study the deviation:

$$be^{-\frac{1}{2}}\left(1 - e^{-\frac{1}{2}}\right)\left[k_3(k_2+1) - 2\right] + b - c > bk_3$$

$$\Rightarrow be^{-\frac{1}{2}}\left(1 - e^{-\frac{1}{2}}\right)\left[k_3(k_2+1) - 2\right] + b - bk_3 \equiv c_{d_1} > c \quad (1.58)$$

We know that $c_1 > c_0 > c_d$. Therefore, the deviation is sustainable if and only if $c < c_{d_1}$.

1.9.10.2. Case 2

We know in the equilibrium $S_{(A,T)\times(B,w)}$ that:

$$S_T(x_A) > S_w(x_A), \quad S_T(x_B) < S_w(x_B)$$

$$S_T(x_A) = b(1-e^{-1})k_1k_2k_3 + be^{-1} - c, \quad S_w(x_A) = bk_2k_3$$

$$S_T(x_B) = bk_2k_3e^{-1} + b(1-e^{-1}) - c, \quad S_w(x_B) = bk_3(1-e^{-1}) + be^{-1}$$

And the probabilities are:

$$p_{x_A,A} = 1-e^{-1}, \quad p_{x_A,B} = 0, \quad g_{x_A,A} = 0, \quad g_{x_A,B} = 1$$

$$p_{x_B,A} = e^{-1}, \quad p_{x_B,B} = 0, \quad g_{x_B,A} = 0, \quad g_{x_B,B} = 1-e^{-1}$$

The equilibrium holds if $c_2 \leq c \leq c_4$. In the case of $S_{(B,w)\times((A,T),(A,w))}$ we have:

$$S_T(x_A) = S_w(x_A), \quad S_T(x_B) < S_w(x_B)$$

$$S_T(x_A) = b(1-e^{-1})k_1k_2k_3 + be^{-1} - c, \quad S_w(x_A) = b\frac{\left(1-e^{-(1-\eta^B_{x,w})}\right)}{(1-\eta^B_{x,w})}\left(k_2k_3\left[k_1 - \eta^B_{x,w}(k_1-1)\right] - 1\right) + b$$

$$S_T(x_B) = b\left(e^{-1}k_2k_3 + (1-e^{-1})\right) - c$$

$$S_w(x_B) = be^{-(1-\eta^B_{x,w})}\frac{\left(1-e^{-\eta^B_{x,w}}\right)}{\eta^B_{x,w}}\left(k_2k_3(1-\eta^B_{x,w}) + k_3\eta^B_{x,w} - 1\right) + b$$

And the probabilities are:

$$p_{x_A,A} = \left(1-e^{-1}\right), \quad p_{x_A,B} = 0, \quad g_{x_A,A} = \left(1-e^{-(1-\eta^B_{x,w})}\right), \quad g_{x_A,B} = \left(1-e^{-(1-\eta^B_{x,w})}\right)\frac{\eta^B_{x,w}}{(1-\eta^B_{x,w})}$$

$$p_{x_B,A} = e^{-1}, \quad p_{x_B,B} = 0, \quad g_{x_B,A} = e^{-(1-\eta^B_{x,w})}\left(\frac{\left(1-e^{-\eta^B_{x,w}}\right)}{\eta^B_{x,w}}(1-\eta^B_{x,w})\right)$$

$$g_{x_B,B} = e^{-(1-\eta^B_{x,w})}\left(1-e^{-\eta^B_{x,w}}\right)$$

The equilibrium holds if $c_3 \leq c \leq c_4$.

Suppose that in the symmetric equilibrium $S_{(B,w)\times((A,T),(A,w))}$ the agents that are in w

decide to go to T. Then we have the following:

$$S_T^d(x_A) = b\left(1 - e^{-1}\right)k_1k_2k_3 + e^{-1}b - c$$

Let's compare the benefits

$$\begin{aligned}
&S_T^d(x_A) > S_w(x_A)\Big|_{\eta^B_{x,w}=1} \\
\Rightarrow &b\left(1 - e^{-1}\right)k_1k_2k_3 + e^{-1}b - c > bk_2k_3 + b \\
\Rightarrow &c_{d_2} = c_4 > c
\end{aligned}$$

So if the cost $c < c_4$ the agents of type A have incentives to deviate to T.

1.10. First Appendix: Matching function

Assume a matching function on T for agents x with characteristics A with agents of type y with characteristics A:

$$M_1\left(N^A_{x,T}, N^A_{y,T}, N^B_{y,T}\right) = A_1 N^A_{x,T} \frac{N^A_{y,T}}{N^A_{y,T} + N^B_{y,T}} \tag{1.59}$$

Function (1.59) is a constant return to scale. It is increasing in $N^A_{x,T}$ and $N^A_{y,T}$ and decreasing in $N^B_{y,T}$. $A_1 \in (0,1)$ represents the efficiency of the match between individuals x_A and y_A. I will define the probability that an agent x_A encounters a pair of type y_A as:

$$\tilde{p}_{x_A,A} = \frac{M_1\left(N^A_{x,T}, N^A_{y,T}, N^B_{y,T}\right)}{N^A_{x,T}} = A_1 \eta^A_{y,T} \tag{1.60}$$

Note that $\frac{d\tilde{p}_{x_A,A}}{d\eta^A_{x,T}} > 0$, $\lim_{\eta^A_{x,T} \to 0} \tilde{p}_{x_A,A} = 0$ and $\lim_{\eta^A_{x,T} \to 1} \tilde{p}_{x_A,A} = A_1$. Let the following functions be:

$$M_2\left(N^A_{x,T}, N^A_{y,T}, N^B_{y,T}\right) = A_2 N^A_{x,T} \frac{N^B_{y,T}}{N^A_{y,T} + N^B_{y,T}} \tag{1.61}$$

$$M_3\left(N^A_{x,T}, N^A_{y,T}, N^B_{y,T}\right) = A_3 N^B_{x,T} \frac{N^A_{y,T}}{N^A_{y,T} + N^B_{y,T}} \tag{1.62}$$

$$M_4\left(N^A_{x,T}, N^A_{y,T}, N^B_{y,T}\right) = A_4 N^B_{x,T} \frac{N^B_{y,T}}{N^A_{y,T} + N^B_{y,T}} \tag{1.63}$$

where expression (1.61) represents the matching function of the individuals for x_A and y_B, expression (1.62) is for x_B and y_A, and finally (1.63) represents the match between x_B and y_B. Also assume that $A_1 > A_2 > A_3 > A_4$. With the above, the following probabilities are represented:

$$\tilde{p}_{x_A,B} = \frac{M_2\left(N^A_{x,T}, N^A_{y,T}, N^B_{y,T}\right)}{N^A_{x,T}} = A_2\left(1 - \eta^A_{y,T}\right) \tag{1.64}$$

$$\tilde{p}_{x_B,A} = \frac{M_3\left(N^A_{x,T}, N^A_{y,T}, N^B_{y,T}\right)}{N^B_{x,T}} = A_3 \eta^A_{y,T} \tag{1.65}$$

$$\tilde{p}_{x_B,B} = \frac{M_4\left(N^A_{x,T}, N^A_{y,T}, N^B_{y,T}\right)}{N^B_{x,T}} = A_4\left(1 - \eta^A_{y,T}\right) \tag{1.66}$$

Only symmetric equilibria will be studied, then $\eta^A_{y,T} = \eta^A_{x,T}$. Individual A's expected utility from using method T is:

$$S_T(x_A) = \tilde{p}_{x_A,A}\Omega(x_A, y_A) + \tilde{p}_{x_A,B}\Omega(x_A, y_B) + b\,(1 - \tilde{p}_{x_A,A} - \tilde{p}_{x_A,B}) - c_T$$
$$\Rightarrow S_T(x_A) = b\eta^A_{x,T}\Bigg(k_2k_3\,(Ak_1 - A_2) + A_2 - A_1\Bigg) + A_2bk_2k_3 + b\,(1 - A_2) - c_T \tag{1.67}$$

$$S_T(x_B) = \tilde{p}_{x_B,A}\Omega(x_B, y_A) + \tilde{p}_{x_B,B}\Omega(x_B, y_B) + b\,(1 - \tilde{p}_{x_B,A} - \tilde{p}_{x_B,B}) - c_T$$
$$\Rightarrow S_T(x_B) = b\eta^A_{x,T}\Bigg(k_3\,(A_3k_2 - A_4) - A_3 + A_4\Bigg) + bA_4k_3 + b(1 - A_4) - c_T \tag{1.68}$$

Moreover, the expected utility if the agents look at w is:

$$S_w(x_A) = b\eta^A_{x,w}\Bigg(k_2k_3\,(Ak_1 - A_2) + A_2 - A_1\Bigg) + A_2bk_2k_3 + b\,(1 - A_2) - c_w \tag{1.69}$$
$$S_w(x_B) = b\eta^A_{x,w}\Bigg(k_3\,(A_3k_2 - A_4) - A_3 + A_4\Bigg) + bA_4k_3 + b(1 - A_4) - c_w \tag{1.70}$$

We will make the following assumptions:

(i) $\Omega(x_A, y_A) > \Omega(x_A, y_B) = \Omega(x_B, y_A) > \Omega(x_B, y_B) > b > 0$

(ii) $\Omega(x_A, y_A) + \Omega(x_B, y_B) > 2\Omega(x_A, y_B)$

(iii) By property of positive real numbers we know that:

$$\Omega(x_A, y_A) = k_1\Omega(x_A, y_B), \quad \Omega(x_A, y_B) = k_2\Omega(x_B, y_B), \quad \Omega(x_B, y_B) = k_3b$$
$$\Omega(x_A, y_A) = k_1k_2k_3b, \quad \Omega(x_A, y_B) = k_2k_3b, \quad \Omega(x_B, y_B) = k_3b$$

By assumption **??**, we have that $k_1, k_2, k_3 > 1$.

Next, the following equilibria will be studied, where only the symmetrical equilibria will be analyzed:

(i) **Separating Equilibria**

- $S_{(A,T)\times(B,w)}$: All A's in T and all B's in w.
- $S_{(A,w)\times(B,T)}$: All A's in w and all B's in T.

(ii) **Pooling Equilibria**

- P_T: All in T.
- P_w: All in w.
- $P_{T,w}$: Pure Pooling Equilibria.

(iii) **Partially Pooling Equilibria**

- $S_{(B,T)\times((A,T),(A,w))}$: All B on T and some A on T and w.
- $S_{(B,w)\times((A,T),(A,w))}$: All B on w and some A on T and w.
- $S_{(A,T)\times((B,T),(B,w))}$: All A on T and some B on T and w.
- $S_{(A,w)\times((B,T),(B,w))}$: All A on w and some B on T and w.

1.10.1. $S_{(A,T)\times(B,w)}$: All A's in T and all B's in w

For this equilibrium to hold, it must be true that:

$$S_T(x_A) > S_w(x_A) \quad S_T(x_B) < S_w(x_B)$$

For the equilibrium $S_{(A,T)\times(B,w)}$ we have:

$$\eta^A_{x,T} = 1, \quad \eta^A_{x,w} = 0$$

The expected utility of agent x_A, defined in (1.67) and (1.69) are:

$$S_T(x_A) = b\Big(k_2k_3\left(A_1k_1 - A_2\right) + A_2 - A_1\Big) + A_2bk_2k_3 + b\left(1 - A_2\right) - c_T$$

$$S_w(x_A) = A_2bk_2k_3 + b\left(1 - A_2\right) - c_w$$

$$S_T(x_A) > S_w(x_A)$$

$$\Rightarrow b\Big(k_2k_3\left(Ak_1 - A_2\right) + A_2 - A_1\Big) - \underbrace{\left(c_T - c_w\right)}_{\equiv c} > 0$$

$$\Rightarrow \bar{c}_4 \equiv b\Big(k_2k_3\left(A_1k_1 - A_2\right) + A_2 - A_1\Big) > c \tag{1.71}$$

The expected utility of agent x_B, defined in (1.68) and (1.70) are:

$$S_T(x_B) = b\Bigg(k_3\left(A_3k_2 - A_4\right) - A_3 + A_4\Bigg) + bA_4k_3 + b(1 - A_4) - c_T$$

$$S_w(x_B) = bA_4k_3 + b(1 - A_4) - c_w$$

$$S_w(x_B) > S_T(x_B)$$

$$\Rightarrow bA_4k_3 + b(1 - A_4) - c_w > b\Bigg(k_3\left(A_3k_2 - A_4\right) - A_3 + A_4\Bigg) + bA_4k_3 + b(1 - A_4) - c_T$$

$$\Rightarrow c > b\Bigg(k_3\left(A_3k_2 - A_4\right) - A_3 + A_4\Bigg) \equiv \overline{c}_2$$

By construction $\overline{c}_4 > \overline{c}_2$. Therefore if $\overline{c}_2 < c < \overline{c}_4$ then the equilibrium hold.

1.10.2. $S_{(A,w)\times(B,T)}$: All A's in w and all B's in T

From the results found in 1.10.1 $\nexists c > 0$ to support the equilibrium.

1.10.3. P_T: All A in T

For this equilibrium to hold, it must be true that:

$$S_T(x_A) > S_w(x_A) \quad S_T(x_B) > S_w(x_B)$$

For the characteristics of this equilibrium, it is satisfied that:

$$\eta^A_{x,T} = \frac{1}{2} \quad \eta^A_{x,w} = 0$$

The expected utility of agent x_A, defined in (1.67) and (1.69) are:

$$S_T(x_A) = b\frac{1}{2}\left(k_2k_3\left(Ak_1 - A_2\right) + A_2 - A_1\right) + A_2bk_2k_3 + b\left(1 - A_2\right) - c_T$$
$$S_w(x_A) = A_2bk_2k_3 + b\left(1 - A_2\right) - c_w$$

$$S_T(x_A) > S_w(x_A)$$
$$\Rightarrow b\frac{1}{2}\left(k_2k_3\left(Ak_1 - A_2\right) + A_2 - A_1\right) + A_2bk_2k_3 + b\left(1 - A_2\right) - c_T > A_2bk_2k_3 + b\left(1 - A_2\right) - c_w$$
$$\Rightarrow \overline{c}_0' \equiv b\frac{1}{2}\left(k_2k_3\left(Ak_1 - A_2\right) + A_2 - A_1\right) > c$$

The expected utility of agent x_B, defined in (1.68) and (1.70) are:

$$S_T(x_B) = b\frac{1}{2}\left(k_3\left(A_3k_2 - A_4\right) - A_3 + A_4\right) + bA_4k_3 + b(1 - A_4) - c_T$$
$$S_w(x_B) = bA_4k_3 + b(1 - A_4) - c_w$$

$$S_T(x_B) > S_w(x_B)$$
$$\Rightarrow b\frac{1}{2}\left(k_3\left(A_3k_2 - A_4\right) - A_3 + A_4\right) + bA_4k_3 + b(1 - A_4) - c_T > bA_4k_3 + b(1 - A_4) - c_w$$
$$\Rightarrow \overline{c}_0 \equiv b\frac{1}{2}\left(k_3\left(A_3k_2 - A_4\right) - A_3 + A_4\right) > c$$

By construction $\overline{c}_0' > \overline{c}_0$. Therefore, if $0 \leq c < \overline{c}_0$ then the equilibrium hold.

1.10.4. P_w: All A in w

For this equilibrium to hold, it must be true that:

$$S_T(x_A) < S_w(x_A) \quad S_T(x_B) < S_w(x_B)$$

For the characteristics of this equilibrium, it is satisfied that:

$$\eta^A_{x,T} = 0 \quad \eta^A_{x,w} = \frac{1}{2}$$

The expected utility of agent x_A, defined in (1.67) and (1.69) are:

$$S_T(x_A) = A_2bk_2k_3 + b\,(1 - A_2) - c_T$$
$$S_w(x_A) = b\frac{1}{2}\Big(k_2k_3\,(Ak_1 - A_2) + A_2 - A_1\Big) + A_2bk_2k_3 + b\,(1 - A_2) - c_w$$

$$S_w(x_A) > S_T(x_A)$$
$$\Rightarrow A_2bk_2k_3 + b\,(1 - A_2) - c_T < b\frac{1}{2}\Big(k_2k_3\,(Ak_1 - A_2) + A_2 - A_1\Big) + A_2bk_2k_3 + b\,(1 - A_2) - c_w$$
$$\Rightarrow -\,b\frac{1}{2}\Big(k_2k_3\,(Ak_1 - A_2) + A_2 - A_1\Big) < c$$

The expected utility of agent x_B, defined in (1.68) and (1.70) are:

$$S_T(x_B) = bA_4k_3 + b(1 - A_4) - c_T$$
$$S_w(x_B) = b\frac{1}{2}\Big(k_3\,(A_3k_2 - A_4) - A_3 + A_4\Big) + bA_4k_3 + b(1 - A_4) - c_w$$

$$S_T(x_B) < S_w(x_B)$$
$$\Rightarrow b\frac{1}{2}\Big(k_3\,(A_3k_2 - A_4) - A_3 + A_4\Big) + bA_4k_3 + b(1 - A_4) - c_w > bA_4k_3 + b(1 - A_4) - c_T$$
$$\Rightarrow -\,b\frac{1}{2}\Big(k_3\,(A_3k_2 - A_4) - A_3 + A_4\Big) < c$$

Therefore, $\forall c > 0$ hold the equilibrium.

1.10.5. $S_{(B,T)\times((A,T),(A,w))}$: All B on T and some A on T and w

For this equilibrium to hold, it must be true that:

$$S_T(x_A) = S_w(x_A) \quad S_T(x_B) > S_w(x_B)$$

Since the proportions of the agents are equivalent, it is possible to have the following expression:

$$\begin{aligned}
&\eta_x^B = \eta_{x,T}^B \eta_{x,T} + \eta_{x,w}^B (1 - \eta_{x,T}), \quad \eta_{x,w}^B = 0 \\
\Rightarrow &\eta_x^B = \eta_{x,T}^B \eta_{x,T}
\end{aligned}$$

With (**??**) and for the equilibrium $S_{(B,T)\times((A,T),(A,w))}$ we have:

$$\begin{aligned}
&\frac{1}{2} = \eta_{x,T}^B \eta_{x,T}, \quad \eta_{x,T}^B \in [\frac{1}{2}, 1] \quad \eta_{x,T} \in [\frac{1}{2}, 1] \\
&\eta_{x,T}^B + \eta_{x,T}^A = 1
\end{aligned}$$

The expected utility of agent x_A, defined in (1.67) and (1.69) are:

$$\begin{aligned}
S_T(x_A) &= b\eta_{x,T}^A \Bigg(k_2 k_3 \left(Ak_1 - A_2 \right) + A_2 - A_1 \Bigg) + A_2 b k_2 k_3 + b \left(1 - A_2 \right) - c_T \\
S_w(x_A) &= b \Bigg(k_2 k_3 \left(Ak_1 - A_2 \right) + A_2 - A_1 \Bigg) + A_2 b k_2 k_3 + b \left(1 - A_2 \right) - c_w
\end{aligned}$$

$$\begin{aligned}
&S_T(x_A) = S_w(x_A) \\
\Rightarrow &b\eta_{x,T}^A \Bigg(k_2 k_3 \left(Ak_1 - A_2 \right) + A_2 - A_1 \Bigg) - c_T = b \Bigg(k_2 k_3 \left(Ak_1 - A_2 \right) + A_2 - A_1 \Bigg) - c_w \\
\Rightarrow &b \Bigg(k_2 k_3 \left(Ak_1 - A_2 \right) + A_2 - A_1 \Bigg) \left(\eta_{x,T}^A - 1 \right) = c
\end{aligned}$$

Therefore, $\nexists c > 0$ that hold the equilibrium.

1.10.6. $S_{(B,w)\times((A,T),(A,w))}$: All B on w and some A on T and w

For this equilibrium to hold, it must be true that:

$$S_T(x_A) = S_w(x_A) \quad S_T(x_B) < S_w(x_B)$$

As in the equilibrium $S_{(B,T)\times((A,T),(A,w))}$ studied in. 1.10.5 it is possible to have the following:

$$\eta_x^B = \eta_{x,T}^B \eta_{x,T} + \eta_{x,w}^B (1 - \eta_{x,T}), \quad \eta_{x,T}^B = 0$$
$$\eta_x^B = \eta_{x,w}^B \eta_{x,w}$$

With (**??**) and for the equilibrium $S_{(B,w)\times((A,T),(A,w))}$ we have:

$$\frac{1}{2} = \eta_{x,w}^B \eta_{x,w}, \quad \eta_{x,w}^B \in [\frac{1}{2}, 1] \quad \eta_{x,w} \in [\frac{1}{2}, 1]$$
$$\eta_{x,w}^B + \eta_{x,w}^A = 1$$

The expected utility of agent x_A, defined in (1.67) and (1.69) are:

$$S_T(x_A) = b\bigg(k_2k_3\left(Ak_1 - A_2\right) + A_2 - A_1\bigg) + A_2bk_2k_3 + b\left(1 - A_2\right) - c_T$$
$$S_w(x_A) = b\eta_{x,w}^A\bigg(k_2k_3\left(Ak_1 - A_2\right) + A_2 - A_1\bigg) + A_2bk_2k_3 + b\left(1 - A_2\right) - c_w$$

$$S_w(x_A) = S_T(x_A)$$
$$\Rightarrow b\eta_{x,w}^A\bigg(k_2k_3\left(Ak_1 - A_2\right) + A_2 - A_1\bigg) - c_w = b\bigg(k_2k_3\left(Ak_1 - A_2\right) + A_2 - A_1\bigg) - c_T$$
$$\Rightarrow b\bigg(k_2k_3\left(Ak_1 - A_2\right) + A_2 - A_1\bigg)\left(1 - \eta_{x,w}^A\right) = c$$

This is supported by $\forall \eta^A_{x,T} \in [0,1]$. The expected utility of agent x_B, defined in (1.68) and (1.70) are:

$$S_T(x_B) = b\Big(k_3\left(A_3k_2 - A_4\right) - A_3 + A_4\Big) + bA_4k_3 + b(1-A_4) - c_T$$
$$S_w(x_B) = b\eta^A_{x,w}\Big(k_3\left(A_3k_2 - A_4\right) - A_3 + A_4\Big) + bA_4k_3 + b(1-A_4) - c_w$$

$$\begin{aligned}
&S_w(x_B) > S_T(x_B)\\
\Rightarrow &b\eta^A_{x,w}\Big(k_3\left(A_3k_2 - A_4\right) - A_3 + A_4\Big) - c_w > b\Big(k_3\left(A_3k_2 - A_4\right) - A_3 + A_4\Big) - c_T\\
\Rightarrow &c > b\Big(k_3\left(A_3k_2 - A_4\right) - A_3 + A_4\Big)\left(1 - \eta^A_{x,w}\right)
\end{aligned}$$

Because of the restrictions present in this equilibrium $\eta^A_{x,w} \in [0, \frac{1}{2}]$. Therefore, if $\overline{c}_3 \leq c \leq \overline{c}_4$ then the equilibrium hold. With:

$$\overline{c}_3 \equiv \frac{b}{2}\Big(k_2k_3\left(Ak_1 - A_2\right) + A_2 - A_1\Big)$$

1.10.7. $S_{(A,w)\times((B,T),(B,w))}$: All A on w and some B on T and w

For this equilibrium to hold, it must be true that:

$$S_w(x_A) > S_T(x_A) \quad S_T(x_B) = S_w(x_B)$$

Since the proportions of the agents are equivalent, it is possible to have the following expression:

$$\begin{aligned}
\eta^A_x &= \eta^A_{x,T}\eta_{x,T} + \eta^A_{x,w}(1-\eta_{x,T}) \quad \eta^A_{x,T} = 0\\
\eta^A_x &= \eta^A_{x,w}\eta_{x,w}
\end{aligned}$$

With (**??**) and for the equilibrium $S_{(A,w)\times((B,T),(B,w))}$ we have:

$$\begin{aligned}
&\frac{1}{2} = \eta^A_{x,w}\eta_{x,w}, \quad \eta^A_{x,w} \in [\frac{1}{2}, 1] \quad \eta_{x,w} \in [\frac{1}{2}, 1]\\
&\eta^B_{x,w} + \eta^A_{x,w} = 1
\end{aligned}$$

The expected utility of agent x_A, defined in (1.67) and (1.69) are:

$$S_T(x_A) = A_2bk_2k_3 + b\,(1 - A_2) - c_T$$

$$S_w(x_A) = b\eta^A_{x,w}\Big(k_2k_3\,(Ak_1 - A_2) + A_2 - A_1\Big) + A_2bk_2k_3 + b\,(1 - A_2) - c_w$$

$$S_w(x_A) > S_T(x_A)$$

$$\Rightarrow b\eta^A_{x,w}\Big(k_2k_3\,(Ak_1 - A_2) + A_2 - A_1\Big) + A_2bk_2k_3 + b\,(1 - A_2) - c_w > A_2bk_2k_3 + b\,(1 - A_2) - c_T$$

$$\Rightarrow c > -b\eta^A_{x,w}\Big(k_2k_3\,(Ak_1 - A_2) + A_2 - A_1\Big)$$

This is supported by $\forall \eta^A_{x,w} \in [0,1]$. The expected utility of agent x_B, defined in (1.68) and (1.70) are:

$$S_T(x_B) = bA_4k_3 + b(1 - A_4) - c_T$$

$$S_w(x_B) = b\eta^A_{x,w}\Big(k_3\,(A_3k_2 - A_4) - A_3 + A_4\Big) + bA_4k_3 + b(1 - A_4) - c_w$$

$$S_w(x_B) = S_T(x_B)$$

$$\Rightarrow b\eta^A_{x,w}\Big(k_3\,(A_3k_2 - A_4) - A_3 + A_4\Big) + bA_4k_3 + b(1 - A_4) - c_w = bA_4k_3 + b(1 - A_4) - c_T$$

$$\Rightarrow c = -b\eta^A_{x,w}\Big(k_3\,(A_3k_2 - A_4) - A_3 + A_4\Big)$$

Therefore, $\nexists c > 0$ that hold the equilibrium.

1.10.8. $S_{(A,T)\times((B,T),(B,w))}$: All A on T and some B on T and w

For this equilibrium to hold, it must be true that:

$$S_T(x_A) > S_w(x_A) \quad S_T(x_B) = S_w(x_B)$$

As in the equilibrium $S_{(A,w)\times((B,T),(B,w))}$ studied in 1.10.7 it is possible to have the following:

$$\eta^A_x = \eta^A_{x,T}\eta_{x,T} + \eta^A_{x,w}(1-\eta_{x,T}), \quad \eta^A_{x,w} = 0 \tag{1.72}$$

$$\Rightarrow \eta^A_x = \eta^A_{x,T}\eta_{x,T} \tag{1.73}$$

For the equilibrium $S_{(A,T)\times((B,T),(B,w))}$ we have:

$$\frac{1}{2} = \eta^A_{x,T}\eta_{x,T}, \quad \eta^A_{x,T} \in [\frac{1}{2},1] \quad \eta_{x,T} \in [\frac{1}{2},1] \tag{1.74}$$

$$\eta^A_{x,T} + \eta^B_{x,T} = 1$$

The expected utility of agent x_A, defined in (1.67) and (1.69) are:

$$S_T(x_A) = b\eta^A_{x,T}\Bigg(k_2k_3\left(Ak_1 - A_2\right) + A_2 - A_1\Bigg) + A_2bk_2k_3 + b\left(1-A_2\right) - c_T$$

$$S_w(x_A) = A_2bk_2k_3 + b\left(1-A_2\right) - c_w$$

$$S_T(x_A) > S_w(x_A)$$

$$\Rightarrow b\eta^A_{x,T}\Bigg(k_2k_3\left(Ak_1 - A_2\right) + A_2 - A_1\Bigg) + A_2bk_2k_3 + b\left(1-A_2\right) - c_T > A_2bk_2k_3 + b\left(1-A_2\right) - c_w$$

$$\Rightarrow c < b\eta^A_{x,T}\Bigg(k_2k_3\left(Ak_1 - A_2\right) + A_2 - A_1\Bigg)$$

This is supported by $\forall\eta^A_{x,T} \in [\frac{1}{2},1]$. The expected utility of agent x_B, defined in (1.68) and (1.70) are:

$$S_T(x_B) = b\eta^A_{x,T}\Bigg(k_3\left(A_3k_2 - A_4\right) - A_3 + A_4\Bigg) + bA_4k_3 + b(1-A_4) - c_T$$

$$S_w(x_B) = bA_4k_3 + b(1-A_4) - c_w$$

$$S_w(x_B) = S_T(x_B)$$

$$\Rightarrow bA_4k_3 + b(1-A_4) - c_w = b\eta^A_{x,T}\Bigg(k_3\left(A_3k_2 - A_4\right) - A_3 + A_4\Bigg) + bA_4k_3 + b(1-A_4) - c_T$$

$$\Rightarrow c = b\eta^A_{x,w}\Bigg(k_3\left(A_3k_2 - A_4\right) - A_3 + A_4\Bigg)$$

Therefore, if $\overline{c}_0 \leq c < \overline{c}_2$ then the equilibrium hold.

1.10.9. $P_{T,w}$: Pure Pooling Equilibria

For this equilibrium to hold, it must be true that:

$$S_T(x_A) = S_w(x_A) \quad S_T(x_B) = S_w(x_B)$$

The expected utility of agent x_A, defined in (1.67) and (1.69) are:

$$S_T(x_A) = b\eta_{x,T}^A \Bigg(k_2 k_3 \left(Ak_1 - A_2 \right) + A_2 - A_1 \Bigg) + A_2 b k_2 k_3 + b \left(1 - A_2 \right) - c_T$$

$$S_w(x_A) = b\eta_{x,w}^A \Bigg(k_2 k_3 \left(Ak_1 - A_2 \right) + A_2 - A_1 \Bigg) + A_2 b k_2 k_3 + b \left(1 - A_2 \right) - c_w$$

$$\begin{aligned}
&S_T(x_A) = S_w(x_A) \\
\Rightarrow & b\eta_{x,T}^A \Bigg(k_2 k_3 \left(Ak_1 - A_2 \right) + A_2 - A_1 \Bigg) - c_T = b\eta_{x,w}^A \Bigg(k_2 k_3 \left(Ak_1 - A_2 \right) + A_2 - A_1 \Bigg) - c_w \\
\Rightarrow & c = b \left(\eta_{x,T}^A - \eta_{x,w}^A \right) \Bigg(k_2 k_3 \left(Ak_1 - A_2 \right) + A_2 - A_1 \Bigg)
\end{aligned}$$

Therefore, $\nexists c > 0$ that hold the equilibrium.

1.10.10. Multiple Equilibrium Refinement

The equilibrium to be refined are:

- $S_{(B,w)\times((A,T),(A,w))}$
- $S_{(A,T)\times(B,w)}$

Suppose that in the symmetric equilibrium $S_{(B,w)\times((A,T),(A,w))}$ the agents that are in w decide to go to T. Then we have the following:

$$S_T^d(x_A) = b \Bigg(k_2 k_3 \left(Ak_1 - A_2 \right) + A_2 - A_1 \Bigg) + A_2 b k_2 k_3 + b \left(1 - A_2 \right) - c_T$$

Let's compare the benefits

$$
\begin{aligned}
&S_T^d(x_A) > S_w(x_A)\Big|_{\eta_{x,w}^A=0} \\
\Rightarrow &b\Bigg(k_2k_3\left(Ak_1 - A_2\right) + A_2 - A_1\Bigg) - c_T > -c_w \\
\Rightarrow &c_{d_2} = c_4 > c
\end{aligned}
$$

So if the cost $c < c_4$ the agents of type A have incentives to deviate to T.

1.10.11. Comparison between the probabilities

The probability that an agent x of characteristic A encounters an agent y of type A, in the microfounded case and the other are:

$$
\begin{aligned}
p_{x_A,A} &= \left(1 - e^{-\eta_{x,T}^A}\right) \\
\Rightarrow p'_{x_A,A} &= e^{-\eta_{x,T}^A} > 0
\end{aligned}
$$

$$
\begin{aligned}
\tilde{p}_{x_A,A} &= A_1\eta_{y,T}^A \\
\Rightarrow \tilde{p}'_{x_A,A} &= A_1 > 0
\end{aligned}
$$

Both probabilities are increasing in the proportion of agents of type x_A in the traditional method. Let's study the other cases:

$$
\begin{aligned}
p_{x_A,B} &= \frac{(1-\eta^A_{x,T})}{\eta^A_{x,T}}\left(1-e^{-\eta^A_{x,T}}\right) \\
\Rightarrow p'_{x_A,B} &= \frac{e^{-\eta^A_{x,T}}}{(\eta^A_{x,T})^2}\left((\eta^A_{x,T})^2-\eta^A_{x,T}-1+e^{\eta^A_{x,T}}\right)<0, \quad \forall \eta^A_{x,T}\in[0,1]
\end{aligned}
$$

$$
\begin{aligned}
\tilde{p}_{x_A,B} &= A_2\left(1-\eta^A_{x,T}\right) \\
\Rightarrow \tilde{p}'_{x_A,B} &= -A_2<0
\end{aligned}
$$

$$
\begin{aligned}
p_{x_B,A} &= e^{-\eta^A_{x,T}}\eta^A_{x,T}\frac{\left(1-e^{-(1-\eta^A_{x,T})}\right)}{1-\eta^A_{x,T}} \\
\Rightarrow p'_{x_B,A} &= \frac{e^{-1-\eta^A_{x,T}}}{\left(\eta^A_{x,T}-1\right)^2}\left(-e^{\eta^A_{x,T}}+e\left(1-\eta^A_{x,T}+(\eta^A_{x,T})^2\right)\right)>0, \quad \forall \eta^A_{x,T}\in[0,1]
\end{aligned}
$$

$$
\begin{aligned}
\tilde{p}_{x_B,A} &= A_3\eta^A_{x,T} \\
\Rightarrow \tilde{p}'_{x_B,A} &= A_3>0
\end{aligned}
$$

$$
\begin{aligned}
p_{x_B,B} &= e^{-\eta^A_{x,T}}\left(1-e^{-(1-\eta^A_{x,T})}\right) \\
p'_{x_B,B} &= -e^{-\eta^A_{x,T}}<0
\end{aligned}
$$

$$
\begin{aligned}
\tilde{p}_{x_B,B} &= A_4\left(1-\eta^A_{x,T}\right) \\
\Rightarrow \tilde{p}'_{x_B,B} &= -A_4<0
\end{aligned}
$$

For both ways of modeling the problem, the probabilities have a relatively similar performance.

2. YOU ARE NOTHING WITHOUT ME: DOMESTIC VIOLENCE AND DIVORCE

2.1. Introduction

Intimate Partner Violence (IPV) is a pressing global public health issue, with an estimated 26% of ever-married or partnered women aged 15 years and older worldwide having experienced physical and/or sexual violence at least once in their lifetime (WHO, 2021). Women subjected to abuse are more likely to report a range of health issues, spanning physical, mental, and reproductive health, as well as negative consequences on their children (Campbell, 2002; Carrell and Hoekstra, 2010; Aizer, 2011; Carlson et al., 2019; McGarry and Hinsliff-Smith, 2023). IPV can also have detrimental economic repercussions for women worldwide, such as unstable employment, reduced income, and decreased productivity (Farmer and Tiefenthaler, 2004; Lloyd, 1997; Borchers et al., 2016). Efforts to mitigate violence against women by governments and international organizations often involve legislative changes aimed at promoting female empowerment (UN, 2012).

Divorce laws can be a method to empower women (Guarnieri and Rainer, 2018); access to unilateral divorce can allow women to exit abusive relationships. However, the empirical literature presents an ambiguous impact of simplified divorce procedures - shifting from mutual consent to unilateral divorce legislation - on instances of IPV. The existing ambiguity is typically interpreted through the lens of bargaining or backlash theories. The former suggests a reduction in violence owing to the increase in a woman's bargaining power facilitated by unilateral divorce. Conversely, the latter proposes that violence might escalate as a result of men feeling their traditional roles is under threat due to this legal change. To the best of our knowledge, there is no theoretical framework that can reconcile the results presented by both theories.

Our paper introduces a theoretical model that provides a potential explanation for why easing access to the divorce may sometimes lead to an increase or a decrease in IPV. We establish a model featuring two-period agents who face a decision on whether to dissolve their marriage or not. In the first period, the agents decide whether or not to marry and in the second period, married agents, can employ coercive violence to retain partners in case of a divorce decision. Additionally, certain agents are subject to engage in irrational violence -cue-trigger violence- during their marriage, and alterations to divorce laws assist in terminating some of these unions.

In our model, during the first period, each agent derives utility from a household production function which depends on their characteristics and that of their partner as well as a random shock, which may be updated in the subsequent period. Then, during the second period, after having updated their match-specific shock, agents engage in a sequential game. Therefore, agents can decide whether to initiate a divorce and whether to resort to coercive violence. Once decisions are taken, they may also be subject to cues that lead one partner to exert "exogenous" violence. We then add in addition the possibility for spouses to transfer resources once the shock has been observed.

In our theoretical framework, agents' decisions may change depending on the context in which they find themselves. Under bilateral divorce, both coercive violence and transfers are not likely to emerge in equilibrium. This is because both methods serve as retention strategies for couples contemplating divorce, thus, they become redundant. However, cue-trigger violence will arise more often as there will be more existing relationships. Under unilateral divorce, there can be incentives to employ coercive violence under certain circumstances, and transfers may sometimes replace this violence. Since there are fewer relationships surviving the second period, cue-triggered violence is lower than in a setting of mutual consent divorce because there are fewer men that irrationally exert violence that have a cohabiting partner.

Our model thus predicts that divorce liberalization could lead to either an increase or a decrease in violence depending on the context. This matches the empirical evidence available in the literature. In the United States, Stevenson and Wolfers (2006) finds that switching from mutual consent to unilateral divorce led to a 30% drop in IPV. Similarly, Brassiolo (2016) investigated the same relationship in Spain, examining the nationwide effect of changes in divorce legislation on IPV. Interestingly, the findings aligned with those from the United States context, indicating a similar influence of legislative change on IPV. Conversely, García-Ramos (2021) estimates the impact of a change in divorce laws in Mexico on IPV. The study reveals that while there is no immediate impact on violence against women following the legislative change, a long-term reduction in IPV is observed when transitioning from bilateral to unilateral divorce. This article attributes the contradictory results, when compared to those discovered in the United States and Spain, to the unique context of a developing country. We provide a broader explanation for why the results may differ: it is not the level of development that matters but rather whether transfers are culturally acceptable or whether violence is sufficiently condemned publicly that would explain the difference.[1]

A similar divide exists in studies that look at economic conditions of men and women and their impact on IPV. In general, the economic literature explains that an increase in women's bargaining power will decrease violence against women. (Manser and Brown, 1980; McElroy and Horney, 1981; Chiappori et al., 2002; Aizer, 2010; Voena, 2015; Anderberg et al., 2016). These studies have principally studied developed economies. However, the backlash model, which is prominently referenced in sociological studies (Woods, 1985; Roe, 1998; Macmillan and Gartner, 1999; Halperin-Kaddari and Freeman, 2016; Flood et al., 2021), suggests the opposite. This framework suggests that women gain bargaining power, and they generate a negative reaction from their spouses who use violence to regain their position of power. Tauchen et al. (1991) model instrumental violence, arguing that it falls when men earn more than women and showing that, in that case, both benefit. One of the main differences with previous work is that we include different types of violence, along with the possibility that agents can divorce. Therefore, it is relevant whether agents have access to unilateral or bilateral divorce.

Empirical evidence consistent with this has been found mostly in developing countries. In India, Eswaran and Malhotra (2011) shows that women who have to work outside the

[1] One recent study finds that the implementation of the *Khul* in Egypt, a form of divorce liberalization, led to a decrease in IPV (Corradini and Buccione, 2023).

household are more likely to receive violence. Erten and Keskin (2021) examine the influence of labor market dynamics on IPV in Cambodia and discover that a more substantial participation of women in the workforce corresponds to a higher likelihood of experiencing violence. On the other hand, if the woman's income is higher than the man's, then domestic violence occurs. Bobonis et al. (2013) finds that conditional cash transfers to women in Mexico lead to more threats of violence towards them, although less violence is being exerted. In addition, Bloch and Rao (2002) suggests that husbands resort to violence as a method to extract monetary resources from the families of their wives. In contrast to earlier studies, our concentration lies in understanding the influence of the legal structure surrounding the ease of divorce on the application of purposeful violence.

Our model can also be used to explain this dichotomy. In setting where violence is more socially acceptable and where it is more accepted for husbands to make transfers to their spouses, an increase in facilities for accessing a unilateral divorce reduces violence since marginal relationships would disappear. On the other hand, in settings where violence is "cheaper" for men or where men cannot show their weakness by transferring resources to their spouse, a decrease in the costs of divorce for women has an increase in coercive violence.

Distinct from prior research, in this paper, we propose that agents might strategically employ coercive violence and transfers as methods to hinder the occurrence of divorces. This is based on the sociological literature which argues that IPV reduces the victim's self-esteem significantly, which leads them to believe that they would fare poorly outside the relationship (Ergin et al., 2006; Tariq, 2013; Bahadir-Yilmaz and Oz, 2019; de Piñar-Prats et al., 2022). But violence in our model is not uni-faceted but can also arise for other reasons. Some literature portrays IPV as a cue-trigger phenomenon. With the use of experimental data, Angelucci (2008) demonstrates that a mix of financial transfers and investment in human capital decreases alcohol consumption, subsequently leading to a reduction in aggressive behavior towards partners. Card and Dahl (2011) propose that the risk of IPV is influenced by the emotional ups and downs related to the results of soccer matches. The authors show that the propensity for partner violence increases in the face of unexpected defeats, especially during important matches.

In sum, our research aims to introduce an alternative mechanism to clarify the outcomes observed in empirical studies, by incorporating two distinct forms of violence and the concept of transfers within the context of both bilateral and unilateral divorce. Adopting a strategic outlook, agents may resort to using coercive violence and/or transfers as mechanisms to retain their partners in instances where the legislation inhibits such retention. A portion of these marriages may engage in cue-trigger violence spontaneously. However, when the legal framework permits, some individuals will find the means to extricate themselves from these violent relationships.

The rest of this paper is organized as follows. Section 2.2 introduces the two-period model, where the incidence of mutual consent and unilateral divorce and their effects on violence and transfers are studied. Section 2.3 concludes.

2.2. Model

In this section, we present a two-period model, where agents decide to marry or separate. Specifically, we assume that, in the first period, agents meet a partner with probability $p \in (0, 1)$ and decide together whether to marry or stay single. In the second period, married agents have the option to divorce. This decision may require the agreement of the partner, depending on the institutional context: the economy can be characterized by either a unilateral or a bilateral divorce regime. Alternatively, agents, who were single in the first period, find a potential partner with a probability p in the second period.

In the second period, married individuals potentially have access to two technologies that can influence their relationship status. A fraction of agents have access to the first technology, which, if they choose to use it, lowers the value of being a single partner. We label this technology as *coercive violence*. The other technology allows spouses to transfer utility between them.

Finally, *cue-trigger* violence may occur for exogenous reasons.

We first describe an economy without violence and characterize its equilibrium in Section 2.2.1. The decisions taken by the agents are thus only about marriage and divorce in this economy. The two types of violence are then introduced in this context in Sections 2.2.2 and 2.2.3 below.

2.2.1. A benchmark economy without violence

The economy is populated by two types of agents, i and j, who live for two periods and can be either single or married with an agent of the other type. Both types of agents discount the future by a factor $\beta \in (0, 1)$.

In the first period, a fraction p of agents is matched with an agent of the other type and both have to decide whether to marry or stay single. Marriage occurs only if it is beneficial to both agents. A fraction $(1 - p)$ is not matched and necessarily stays single.

Single agents of type $k = \{i, j\}$ receive flow utility $b_k > 0$ each period, while the flow utility received by married agents includes a random component equal to $z_k \in [0, B]$, where z_k comes from a cumulative distribution function $F(z_k)$, as well as a marital surplus $\Omega(x, y)$ determined by the characteristics of both agents, which we denote by $x \in [\underline{x}, \overline{x}]$ in the case of type-i agents and $y \in [\underline{y}, \overline{y}]$ in the case of agents of the other type. We assume that the function $\Omega(x, y)$ is strictly increasing and concave in both arguments and $\Omega : [\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}] \to A \subset \mathbb{R}_+$.

In the second period, each married agent of type $k = \{i, j\}$ may experience a change in their random component z_k with probability $\lambda_k \in (0, 1)$. Then, agents take part in the following sequential game.[2] They first witness the change in their respective z_k values. Then, agents of type i may choose to divorce, given the observed z_k's. Finally, the agents of type j decide if they terminate the relationship.

2.2.1.1. Equilibrium period-two marriage and divorce decisions

We consider two types of divorce regimes. Under unilateral divorce, separation occurs if any of the two agents in a marriage choose to separate, while both agents' agreement is necessary under bilateral divorce. The value of an agent of type k, who decides whether to separate in the second period is the following in the unilateral divorce regime:

$$V_k^U(z_{k,2}, z_{-k,2}, x, y) = \max_{\text{Single,Married}} \left\{ b_k, [z_{k,2} + \Omega(x, y)]\, D_{-k}(z_{k,2}, z_{-k,2}, x, y) + (1 - D_{-k}(\cdot))b_k \right\} \tag{2.1}$$

where by $V_k^U(z_k, z_{-k}, x, y)$ is the utility of agent of type k in the second period when paired with an agent of type $-k$.[3] The first option in the brackets in the right-hand side of equation (2.1) is the flow utility b_k of being single while the second option refers to the utility from staying in the marriage, which is equal to the sum of the random utility component and the marital surplus. However, the latter is obtained only if the partner

[2] In the benchmark economy without violence, the sequential game could be a simultaneous game. The sequential aspect of the game becomes relevant once violence is introduced further below.

[3] The notation $-k$ represents an agent that is other than k.

chooses to stay in the relationship (when $D_{-k}(\cdot) = 1$). If the partner wishes to end the relationship (when $D_{-k}(\cdot) = 0$), then the payoff is the same as in the case of being single, in the unilateral divorce regime. Notice that the maximization problem in equation (2.1) also represents the decision of singles when they meet a potential partner of type $-k$.

In the last period, there exists a threshold $z^*_{i,2}(x, y)$ such that agent i is indifferent between marriage and divorce. It is defined as follows:

$$D_j(z^*_{i,2}, z_j, x, y) \left(z^*_{i,2} + \Omega(x, y) - b_i\right) = 0 \tag{2.2}$$

An agent of type i chooses to stay in the marriage as long as the random component $z_{i,2}$ faced by the agent in the second period is above the threshold $z^*_{i,2}(x, y)$.

Notice that the decision of j is independent of $z_{i,2}$. Hence it follows that

$$z^*_{i,2} = b_i - \Omega(x, y). \tag{2.3}$$

One can characterize the problem of agents of type j similarly. The decisions of both types of agent can be represented graphically as in Figure C1, depending to the random component $z_{k,2}$ each one faces in the second period. In the region where $z_{i,2} \geq z^*_{i,2}$ and $z_{j,2} \geq z^*_{j,2}$, that is, the areas in grey in Figure C1, relationships are sustained (if previously married) or initiated (if previously single).

Under a bilateral divorce regime, the value of an agent of type k is given by

$$\begin{aligned} &V^B_k(z_{k,2}, z_{-k,2}, x, y) = \\ &\quad \max\left\{b_k \left(1 - D_{-k}(z_{k,2}, z_{-k,2}, x, y)\right) + D_{-k}(\cdot) \left[z_{k,2} + \Omega(x, y)\right], \left[z_{k,2} + \Omega(x, y)\right]\right\}, \end{aligned} \tag{2.4}$$

where $V^B_k(z_k, z_{-k}, x, y)$ is the utility of agent of type k in the second period who is paired with an agent of type $-k$. Notice that equation (2.4) differs from equation (2.1) because now divorce needs the agreement of both agents.

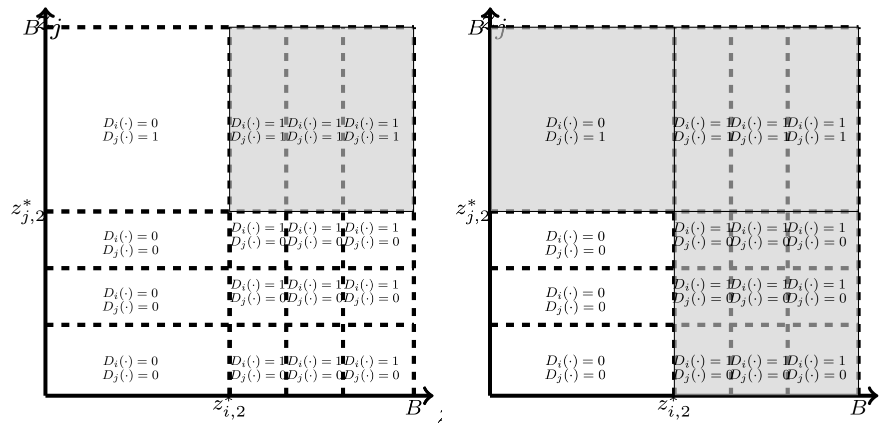


Figure C1. **Left**: Representation of relation with unilateral consent in the second period. Areas shaded in grey denote enduring marital unions, whereas uncolored areas characterize divorce. **Right**: Representation of relation with mutual consent in the second period. Areas shaded in grey denote enduring marital unions, whereas uncolored areas refer to divorce situations.

Without loss of generality, suppose that $k = i$, then we have that

$$V_k^B(z_{i,2}, z_{j,2}, x, y) = \begin{cases} z_{i,2} + \Omega(x,y) & \text{if } D_j(\cdot) = 1 \\ \max\left\{b_i, z_{i,2} + \Omega(x,y)\right\} & \text{if } D_j(\cdot) = 0 \end{cases} \tag{2.5}$$

According to the first case in equation (2.5), if an agent of type j wants to continue the relationship, then the paired agent of type i has no choice but to continue as well. On the other hand, if the agent of type j wants to divorce in the second period—as can be seen in the second case in equation (2.5), then it is solely up to agent i to decide whether to continue or end the relationship.

One can characterize the decisions of agents of type j similarly. The following figure thus depicts the decisions of both types of agent, according to the random utility component they face:

One can conclude from the comparison of Figures C1 (**Left**) and C1 (**Right**) that, under a bilateral divorce regime, one sees less divorce given a set of marriages than under unilateral divorce.

2.2.1.2. Equilibrium period-one marriages

Given the equilibrium decisions of period 2, one can obtain the equilibrium decisions of period 1. An agent, who is single in the first period, receives a flow utility b_k and is paired with someone else in the second period with probability p. Let $G(x)$ and $G(y)$ be the distributions of characteristics among singles of types i and j respectively and let $N(x,y)$ be the mass of marriages involving characteristics x and y. Given that only singles search for a partner, the utility of a single of type i in the first period is given by

$$S(x) = p\beta \int\int\int V_i^U(z'_{i,2}, z'_{j,2}, x, y)dF(z'_i)dF(z'_j)dG(y) + b_i\left(1 + (1-p)\beta\right) \quad (2.6)$$

Given that the random component of marriage utility may change between period 1 and 2 with probability λ_i and λ_j for each agent in a marriage, the value of being married in period 1 for agent of type k can be written as follows:

$$M_k^r(z_{i,1}, z_{j,1}, z^*_{i,2}, z^*_{j,2}, x, y) = (z_{k,1} + \Omega(x,y)) + \beta\mathbb{E}_{z_{i,2},z_{j,2}}\left[V_k^r(z_{i,2}, z_{j,2}, x, y)\right] \quad (2.7)$$

depending on the divorce regime $r \in \{U, B\}$. Properties of the values M_k^r are given in the following lemma:

Lemma 2.1. *The function $M_k^r(\cdot)$ is monotonic in $z_{k,1}$ and is differentiable in almost all of its domain. The marginal utility of agent k is higher if the other agent is more likely to stay in the relationship.*

PROOF. See Appendix 2.6.1. □

The set of paired agents in period 1 have to choose between marriage and singlehood. Their value is given by the following expression:

$$V_{1,i}(\cdot) = \max_{\text{Single,Married}}\left\{S(x), M_{i,1}^r(z_i, z_j, x, y)D_{j,1}(z_i, z_j, x, y) + (1 - D_{j,1}(\cdot))S(x)\right\} \quad (2.8)$$

By Lemma 2.1, there exists a unique threshold $z_{i,1}^{r*}$ such that an agent is indifferent between marriage and singlehood, for each divorce regime $r = U, B$. This threshold is given by the intersection of the two functions $M^r(z_{i,1}, \cdot)$ and $S(x)$, which are depicted in Figure C2.

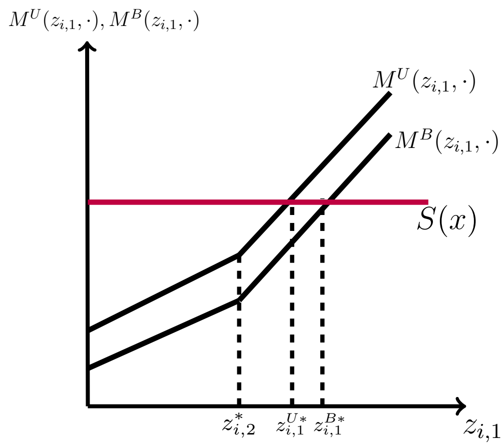


Figure C2. The function $M^r(z_{i,1}, \cdot)$ represented over $z_{i,1}$.

Figure C2 identifies a change in the slope of the M^r values around the cutoff $z_{i,2}^*$. It is due to marriage being less likely to survive in the second period when below the threshold $z_{i,2}^*$: a marginal increase in $z_{i,1}$ implies a lower increase in the value of marriage from a present value perspective when divorce is likely to happen in the second period, as explained in Lemma 2.1. Additionally, the $S(x)$ locus crosses the $M^r(\cdot)$ locus only once, as displayed in Figure C2, if the probability p is small enough, which also implies that $z_{i,1}^{*U} > z_{i,2}^*$. In the case where the value of being single for both types of agents are equal and the probabilities that $z_{k,2}$ changes between period 1 and 2 is the same for both agents, then $M_k^U(\cdot) > M_k^B(\cdot)$ if the following property holds:

$$\frac{(1-\lambda_k F(z_{k,2}^*))}{(1-F(z_{k,2}^*))\lambda_k}\mathbb{E}\left(b_k - (z_k + \Omega(x,y))|z_k < z_{k,2}^*\right) > \mathbb{E}\left(z_k + \Omega(x,y) - b_k|z_k \geq z_{k,2}^*\right) \quad (2.9)$$

When agents are symmetric (that is, when $b_i = b_j$ and $\lambda_i = \lambda_j$), a preference emerges for unilateral over bilateral divorce during the initial period. This preference arises due to the agents assigning a greater weight to lower values of z_k in comparison to the higher ones, as evidenced by the inequality $\frac{(1-\lambda_k F(z_{k,2}^*))}{(1-F(z_{k,2}^*))\lambda_k} > 1$. In situations where the potential of low z_k

is deemed more significant, agents show an inclination towards a unilateral divorce regime to evade the potential of low z_k and obtain b_k. Conversely, when the agents perceive the expected value of z_k as high, they tend to favor a bilateral divorce regime[4].

Lemma 2.2. *If p is sufficiently large, individuals will be "pickeir" in their choice of relationship in the first period when to remain married. There exists a range p such that $z_{i,1}^{U*} > z_{i,2}^{*}$*

PROOF. See Appendix 2.6.1. □

2.2.2. Introducing coercive violence

Consider now a scenario where agent i possesses a coercive technology in the second period. This technology serves to diminish the outside option (b_j) available to agent j. For instance, in the context of a couple where one member desires to leave the relationship while the other does not, the latter can pay a fixed cost of $a > 0$ which reduces their utility by $v > 0$ and $\xi > 0$ if they were to leave the relationship. If they stay in the relationship this technology reduces bu ξ their utility. It is worth noting that the technology in question cannot be utilized to locate a partner, as the value of being single remains unchanged and is represented by equation (2.6). The timing of the use of the technology is the following:

(i) Both agents observe their respective $z_i \sim F(z_i)$ and $z_j \sim F(z_j)$.
(ii) Partner i decides and pays a cost $a > 0$ whether violence will be applied which lowers the partner's outside option by $v + \xi$ in the case where the partner decides to separate. If the relationship holds, the utility falls by ξ.
(iii) The partner also decides whether to stay in the relationship.
(iv) Agents of type j decide whether to divorce or not.
(v) Payments are given to the respective agents.

[4]For more details of equation (2.9) see Appendix 2.6.1

The decision of agent i in the last period, if he is previously married in the context of unilateral divorce is:

$$V_{2,i}^{U,M}(\cdot) = \max_{\text{Single,Married},T_i(\cdot)} \left\{ b_i, \left[z_{i,2} + \Omega(x,y)\right] D_j(z_{i,2}, z_{j,2}, x, y) + (1 - D_j(\cdot))b_i - aT_i(z_{i,2}, z_{j,2}, x, y) \right\} \tag{2.10}$$

where $V_{2,i}^{U,M}(\cdot) = V_{2,i}^{U,M}(z_{i,2}, z_{j,2}, x, y)$. In the context outlined in Equation (2.10), the decision-making process is initiated by agents of type i. These agents are aware of their corresponding $z_{i,2}$ values, as well as those of their partners. The decision involves two key components: the determination to remain within the relationship, and the decision to deploy coercive violence. Importantly, these decisions take into account the anticipated actions of their partners concerning potential divorce outcomes.

In the case of agent j we have:

$$\begin{aligned} V_{2,j}^{U,M}(\cdot) = \max_{\text{Single,Married}} &\left\{ b_j - T_i(\cdot)(v+\xi), \left[z_{j,2} + \Omega(x,y) - T_i(\cdot)\xi\right] D_i(z_{i,2}, z_{j,2}, x, y) \right. \\ &\left. + (1 - D_i(\cdot))\Big(b_j - (v+\xi)T_i(\cdot)\Big) \right\} \end{aligned} \tag{2.11}$$

where $V_{2,j}^{U,M}(\cdot) = V_{2,j}^{U,M}(z_{i,2}, z_{j,2}, x, y)$ and $T_i(\cdot) = T_i(z_{i,2}, z_{j,2}, x, y)$. When considering agents of type j, their role is to make the terminal decision in this sequential game. Unlike their type i counterparts, these agents are not required to anticipate any reciprocal actions. The resolution of the sequential game yields the subsequent lemma, derived from the unique characteristics of these type j agents:

Lemma 2.3. *For given values of $z_{i,2}$ and $z_{j,2}$, a unique Nash equilibrium emerges characterized by the application of coercive violence.*

PROOF. See Appendix 2.4. $\square$

The only Nash equilibria in this game is one where violence is:

$$T_i(\cdot) = \begin{cases} 1, & z^*_{j,2} - v \leq z_{j,2} < z^*_{j,2} \quad z_{i,2} \geq z^*_{i,2} + a \\ 0, & \text{Other cases} \end{cases} \tag{2.12}$$

That is violence will be exerted when she would like to leave without violence but will stay if violence is exerted. It is also only employed when the partner's i desire to stay in the relationship is high enough to be worth the cost. As was done in the previous subsection 2.2.1, we can characterize the decisions of the last period in figure **??**.

The range $z_{i,2} \in [z^*_{i,2}, z^*_{i,2} + a]$ and $z_{j,2} \in [z^*_{j,2} - v, z^*_{j,2}]$ represents a category of marital unions that would not endure in the absence of technological access by agent i. Consequently, relative to the circumstances presented in subsection 2.2.1, there are more marriages than when violence was not exerted but those added relationships all include victims of intrafamily violence.

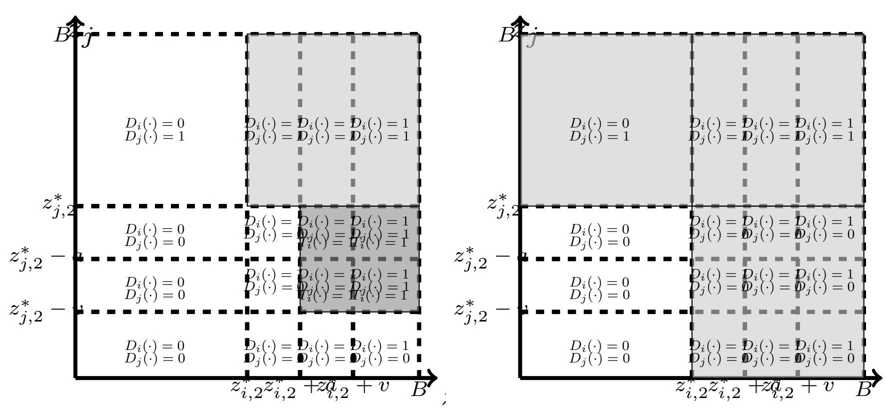


Figure C3. **Left**: Unilateral divorce with agents of type i with access a coercive violence. **Right**: Bilateral divorce with agents of type i with access a coercive violence.

As in the previous subsection 2.2.1, if the agents were married in the first period, the z_k can be updated by a Markov process. Then the value of being married for agent k in the first period is:

$$M_k^U\left(z_{k,1}, z_{-k,1}, z_{k,2}^*, z_{-k,2}^*, x, y|T_i(\cdot)\right) = (z_{k,1} + \Omega(x,y)) + \beta\mathbb{E}_{z_{i,2},z_{j,2}}\left[V_{2,k}^{U,M}(z_{k,2}, z_{-k,2}, x, y)\right] \tag{2.13}$$

Since equation (2.13) is similar to that of the canonical case represented by (2.7), it is intuitive to infer that this function is increasing in $z_{k,1}$. In addition, the use of violence by agent i is in a situation where j wants a divorce. So:

Lemma 2.4. *Agent i is strictly better off if he has access to the technology and agent j is strictly worse off when receiving violence.*

PROOF. See Apendix 2.6.1. □

The intuition of lemma 2.4 is that agent i withholds agent j with violence. This is because j desires a divorce but agent i wants the relationship to be maintained. If lemmas 2.1, 2.2, 2.3 and 2.4 are fulfilled, then we can propose the following proposition:

PROPOSITION 2.1. *If the agent faces a sufficiently low a and there is unilateral divorce then the divorce rate is reduced conditional on the number of marriages, but the marriage rate may fall.*

PROOF. See Apendix 2.6.1. □

When agents of type i use coercive violence, it incurs a cost, a. As per Figure C3 (**Left**), these agents use violence when $z_{i,2} \geq z_{i,2}^* + a$ and they're coupled with agents of type j with $z_{j,2}$ in $[z_{j,2}^* - v, z_{j,2}^*]$ range. Coercive violence, which reduces the outside option by v, serves as a means of retaining partners. Consequently, fewer dissolutions occur. In the first period, agents of type j, anticipating potential violence due to decreasing costs, may opt out of marriage to avoid potential domestic violence.

If divorce is only available under mutual consent, agents of type i have the following utility:

$$V_{i,2}^{B,M} = \max_{\text{Single,Married},T_i(\cdot)} \Big\{ b_i \left(1 - D_j(\cdot)\right) + D_j(\cdot) \left[z_{i,2} + \Omega(x,y) - aT_i(\cdot)\right],$$
$$z_{i,2} + \Omega(x,y) - aT_i(\cdot) \Big\} \tag{2.14}$$

In the case of agents of type j, the utility is:

$$V_{j,2}^{B,M} = \max_{\text{Single,Married}} \Big\{ \left(b_j - (v+\xi)T_i(\cdot)\right)\left(1 - D_j(\cdot)\right) + D_j(\cdot) \left[z_{i,2} + \Omega(x,y) - T_i(\cdot)\xi\right],$$
$$z_{i,2} + \Omega(x,y) - T_i(\cdot)\xi \Big\} \tag{2.15}$$

In that case, the only Nash equilibria will be that no violence will be exerted[5]

PROPOSITION 2.2. *When there is a transition from mutual consent to unilateral divorce, there will be an increase in divorce and an increase in violence. This increase will be larger this lower the act of exerting violence for the prolent partner.*

PROOF. See Appendix 2.6.1. □

When there is a transition from mutual to unilateral divorce, we observe that unions where $z_{i,2} < z_{i,2}^* + a$ and $z_{j,2} < z_{j,2}^*$ will now result in divorce while they stayed married previously. Furthermore, while there was no violence in mutual consent divorce, violence can now be used to coerce the partner to stay. Violence will be exerted wherever $z_{i,2} \geq z_{i,2}^* + a$ and $z_{j,2}^* \in [z_{j,2}^* - v, z_{j,2}^*]$. As "$a$" falls, the range of $z_{i,2}$ where violence will be exerted will increase.

2.2.2.1. Coercive violence with transfers

The timing of actions established in the prior subsection is retained, with the added flexibility that agents can now choose between executing transfers and/or applying coercive violence. The transfers made by type i agents are designed to induce indifference in

[5] For more details see the proof of the proposition 2.2 in the appendix 2.6.1.

type j agents between continuing in the relationship or opting for divorce. These transfers are denoted by $s = s^N \equiv b_j - (z_{j,2} + \Omega(x,y))$, a result of Nash bargaining, under the presumption that type i agents possess maximal bargaining power[6]. In the context of mutual consent, the value of agent i, if he is previously married, in the second period is:

$$V_{i,2}^{B,M} = \max_{\text{Single,Married},T_i(\cdot),s} \Big\{ b_i \left(1 - D_j(\cdot)\right) + D_j(\cdot) \left[z_{i,2} + \Omega(x,y) - aT_i(\cdot) - s\right], \\ z_{i,2} + \Omega(x,y) - aT_i(\cdot) - s \Big\} \tag{2.16}$$

where $V_{2,i}^{B,M}(\cdot) = V_{2,i}^{B,M}(z_{i,2}, z_{j,2}, x, y)$ and $T_i(\cdot) = T_i(z_{i,2}, z_{j,2}, x, y)$. In the case of agent j we have:

$$V_{j,2}^{B,M} = \max_{\text{Single,Married}} \Big\{ \left(b_j - (v+\xi)T_i(\cdot)\right)\left(1 - D_j(\cdot)\right) + D_j(\cdot)\left[z_{i,2} + \Omega(x,y) - T_i(\cdot)\xi + s\right], \\ z_{i,2} + \Omega(x,y) - T_i(\cdot)\xi + s \Big\} \tag{2.17}$$

In the current model, the values of s are contingent on the actions taken by agent i. As in the preceding section, this problem can be tackled through a sequential game, where agent i selects between using violence, making Pareto-optimal transfers, continuing the relationship, or terminating it. Subsequently, agent j decides whether to continue the relationship or end it. However, since divorce requires mutual consent, both agents must agree to dissolve the relationship. The result of the sequential game can be summarized in the following figure:

Note that in Figure C5, there are no incentives for agent i to use violence and/or transfers. This is because the rupture of the relationship occurs if both agents agree.

In an economy with unilateral divorce, it's assumed that type-i agents can employ coercive violence and/or conduct transfers as delineated by equation 2.20. In this context,

[6] $s = 0$ represents the decision that agents of type i decide not to make transfers. For more details see Appendix 2.5.

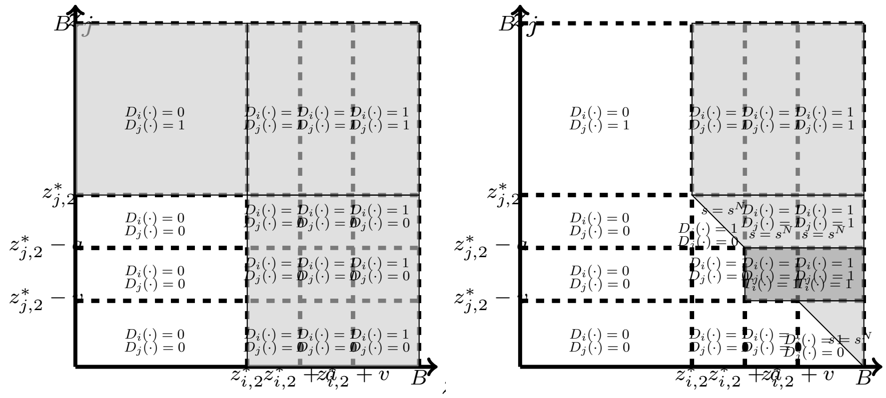


Figure C5. **Left**: Bilateral divorce with agents of type i having cost in the use of violence. **Right**: Unilateral divorce with agents of type i having a cost in the use of violence when $a < v$.

the utility of agents of type i can be expressed as:

$$V_{2,i}^{U,M}(\cdot) = \max_{\text{Single,Married},T(\cdot),s} \left\{ b_i, \left[z_{i,2} + \Omega(x,y) - s\right] D_j(z_{i,2}, z_{j,2}, x, y) + (1 - D_j(\cdot))b_i - aT_i(\cdot) \right\} \tag{2.18}$$

In the case of agent j we have:

$$V_{2,j}^{U,M}(\cdot) = \max_{\text{Single,Married}} \Big\{ b_j - T_i(\cdot)(v+\xi), \left[z_{j,2} + \Omega(x,y) + s - T_i(\cdot)\xi\right] D_i(\cdot) + (1 - D_i(\cdot))\Big(b_j - (v+\xi)T_i(\cdot)\Big) \Big\} \tag{2.19}$$

In the given economy, the solution can be obtained by a sequential game where agent i has to make a decision between staying in the relationship or divorcing, while considering options such as using violence or making transfers, given that divorce is unilateral, i.e., only one agent's decision is sufficient for divorce. Following the agent's i decision, agents of type j must then decide whether to divorce or not. The result of the sequential game can be summarized in the following figure:

PROPOSITION 2.3. *When transitioning from bilateral to unilateral divorce, coercive violence increase when agents of type i have access to transfers if $a < v$. However, in comparison to an economy where only coercive violence is accessible, the presence of both mechanisms in a unilateral divorce setting tends to decrease violence.*

PROOF. See Apendix 2.6.1. □

In comparing two economies where type i agents have the capacity to exercise coercive violence and transfers, the influence of access to unilateral versus mutual constent has different implications. In a bilateral divorce setting, the need for mutual consent to divorce negates the incentive for type i agents to either employ coercive violence or make transfers. The former is a retention strategy against type j agents, while the latter could potentially be substituted by coercive violence.

However, in the context of unilateral divorce, where only one party's agreement to separation is required, both coercive violence and transfers are applied as strategies to deter type j agents from seeking a divorce. When $a < v$, there exist certain scenarios where it becomes more beneficial for type i agents to utilize violence as opposed to transfers.

Assuming that type i agents have maximum bargaining power, transfers are made such that type j agents become indifferent between choosing divorce or staying in the relationship. We further assume that they opt to stay. The asymmetry in the cost of coercive violence, where type i agents pay a cost of a to exert this form of violence and reduce the outside options for type j agents by v (where $v > a$), creates conditions in which certain segments of $z_{i,2}$ and $z_{j,2}$ may prefer violence over transfers.

Suppose a type i agent, with a marital utility of $z^*_{i,2} + \Omega(x, y) + v$, is married to a type j agent, whose marital utility equals $z^*_{j,2} + \Omega(x, y) - v$. For the type j agent to decline divorce, the type i agent must transfer $s = v$ to retain them in the marriage. However, if the same type i agent has access to coercive violence, they could incur a cost of a, reducing their utility to $z^*_{i,2} + \Omega(x, y) + v - a$, and simultaneously decrease the outside option for the type j agent by $v + \xi$ and the utility by[7] ξ. This makes violence a more favorable choice if $v > a$.

[7]The decisions made by agents of type j are not affected by ξ

This explanation also sheds light on the triangular pattern in the areas where marriages persist with transfers. Coercive violence, while less attractive in terms of utility (when available, as this is a sequential game, so for $z_{j,2} < z^*_{j,2} - v$ violence will not occur, only transfers), for $z_{j,2}$ values closer to the indifference threshold, type i agents only need to make "smaller" transfers for the relationship to sustain. Paying a for the use of coercive violence does not prove beneficial in terms of utility.

2.2.3. Introducing coercive and cue-trigger violence

Assume now that agents i may exert violence for exogenous reasons. The timing is as followed:

(i) Agents receive their respective $z_k \sim F(z_k)$ and know that of their partner.

(ii) Type i agents make the decision to persist in the relationship, and whether to exert coercive violence for which they pay a cost a but lower the outside option of their partner by $v + \xi$. If their partner decides to stay, their utility decreases by ξ.

(iii) Agents of type j decide whether to divorce or not.

(iv) With probability $d \in (0, 1)$, type i agents who are married, use cue-trigger (pay a cost $a > 0$) violence, and reducing type j agents' utility by $v + \xi$. If they had previously used violence they do not pay the cost again for both agents.

(v) Payments are given to the respective agents.

Drawing parallels to the analysis in subsection 2.2.2, the fulfillment of Lemma 2.3 extends to the instance of combined coercive and cue-trigger violence. Type i agents will opt for coercive violence if the condition $z_{i,2} \geq z^*_{i,2} + a$ is met (similar to the threshold identified in the case of purely coercive violence), conditional that $z_{j,2}$ lies within the range $[z^*_{j,2} - v(1-d), z^*_{j,2} + d(v+\xi)]$.

Compared with the case when cue-trigger violence is not available, an increased prevalence of intrafamilial violence is observed, attributable to marriages where type j agents incur an additional ξ cost associated with the manifestation of cue-trigger violence. It is critical to note that this ξ cost is borne by type j agents on a singular occasion, regardless of the frequency of violent episodes they encounter.

Therefore, when both forms of violence coexist, type j agents impose more stringent criteria to remain in the relationship ($z_{i,2} \geq z^*_{i,2} + d(\xi + v)$) as compared to the scenario devoid of cue-trigger violence. As a consequence, the sphere of influence for coercive violence to retain partners is amplified.

Agent's j decision to remain in marriage will be different since if they divorce, they get either b_j or $b_j - v - \xi$ if they have been victims of coercive violence and $z_{j,2} + \Omega(x, y) - dv - \xi$ if they stay married. They will want to divorce more often than before. Agent i's utility in marriage will also be reduced by da which will imply they will prefer to initiate divorce when their $z_{i,2}$ is lower than $z^*_{i,2} + da$.

If bilateral divorce is instead in place, we will again find that coercive violence will never be used. However, the decision to remain married will change for both, due the possibility of cue-trigger violence. Agents will be less likely to want to remain in the relationship than when there is no such violence.

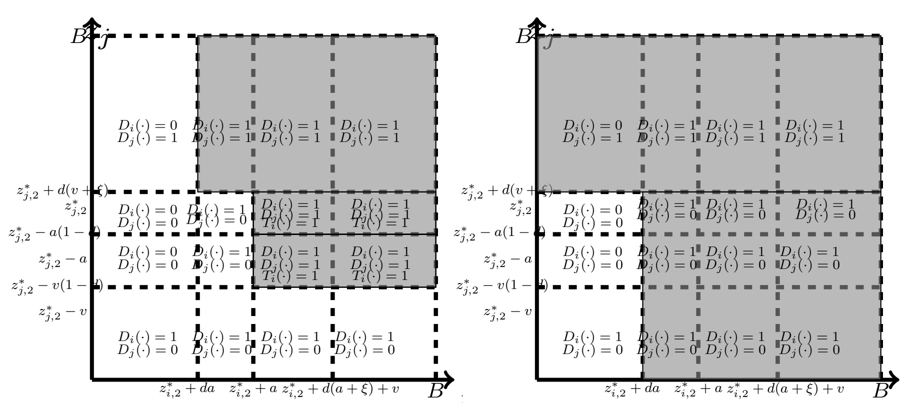


Figure C6. **Left**: Unilateral divorce with cue-trigger and coercive violence when $a < v$. **Right**: Bilateral divorce with cue-trigger and coercive violence.

PROPOSITION 2.4. *When cue-trigger violence ocurrs with probability d, a transition from mutual consent to unilateral divorce will lead to more divorces but may decrease the incidence of violence if d is large enough. This effect will be increasing in a.*

PROOF. See Appendix 2.4 □

2.2.3.1. Coercive and cue-trigger violence with transfers

Preserving the timing established in the preceding subsection, type i agents now have the capacity to enact transfers that render type j agents ambivalent between maintaining the marital status or opting for divorce. Under specific intervals, these transfers can effectively substitute for violence. It is noteworthy that if $d \to 0$, the resolution of this sequential game mirrors the solution deduced in subsection (2.2.2.1). It is worth noting that the use of coercive violence by type i's agent while diminishing the outside option of type j agents by $v + \xi$ and their utility by ξ within marriage.

Comparing two scenarios — one where type i agents lack access to cue-trigger violence in a unilateral divorce context, and another where they do have access (as depicted in Figures C5 and C7, respectively) — one observes a reduction in the frequency of coercive violence in the latter. In essence, as the proportion of agents with access to cue-trigger violence augments, the incentive for these agents to employ coercive violence correspondingly diminishes.

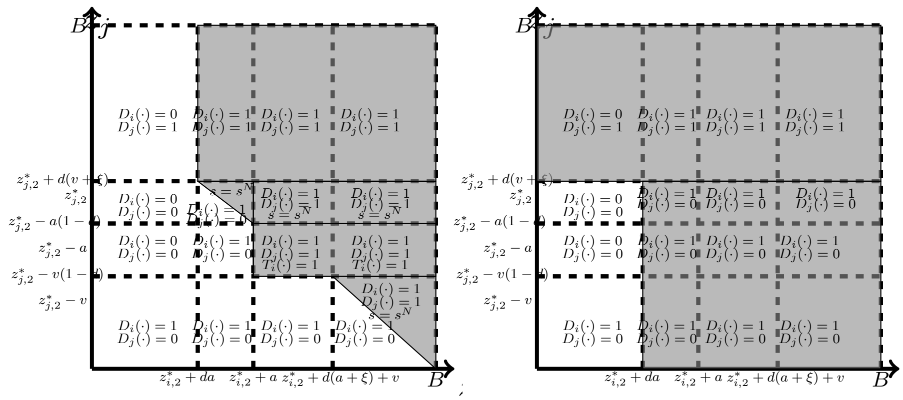


Figure C7. **Left**: Unilateral divorce with cue-trigger and coercive violence and transfers. $v > a$. **Right**: Bilateral divorce with cue-trigger and coercive violence.

2.3. Conclusion

Globally, one in four men believe it is justifiable to hit women in certain contexts (UN, 2023). Governments can mitigate violence against women by enhancing their empowerment. There is a consensus of empowerment that granting women access to unilateral divorce. However, the academic literature presents conflicting forecasts regarding its impact on IPV. The economic theory, largely anchored in bargaining models, suggests that transitioning from a bilateral to a unilateral divorce framework augments women's bargaining power, thereby potentially reducing violence. Conversely, sociological literature, rooted in backlash theory, posits that such a shift may actually increase violence, as men feeling their roles threatened might employ violence as a strategic tool to prevent women from exiting the relationship. The causal evidence is also ambiguous.

We propose a model in which the transition from mutual agreement to individual decision divorce laws could either escalate or mitigate violence. This is attributed to the existence of two forms of violence: coercive and cue-trigger.

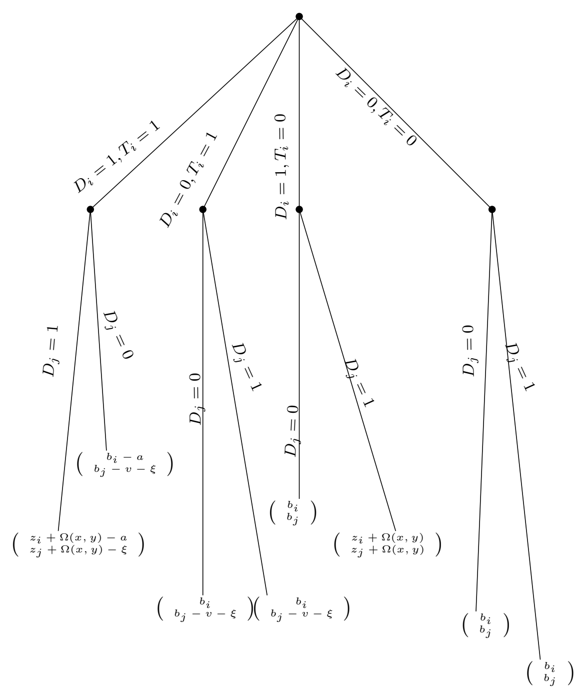
In scenarios where cue-trigger violence (is exogenous) is a rarity, the shift in legal norms could result in heightened violence. This occurs as there would be couples eager to divorce, but violence may hinder this process. Conversely, in situations where cue-trigger violence is prevalent, the legal changes would lead to a reduction in violence. This is due to the fact that the new laws would facilitate the dissolution of violent relationships.

For further research, it is possible to empirically test the model by studying the surveys used in the United States and Mexico. In the case of Mexico, there are questions about the opinions of male and female roles in society, it is possible to use some of these questions to make a proxy for the cost of a.

In formulating policies that promote women's empowerment, it's crucial to understand the predominant nature of violence. By escalating the penalties associated with violent acts, we can deter the employment of strategic violence. Furthermore, liberalizing divorce laws doesn't necessarily imply a decrease in violence levels. Therefore, both strategies should be considered for enhancing women's societal status and reducing gender-based violence.

2.4. Second Appendix: Sequential game with coercive and cue-trigger violence

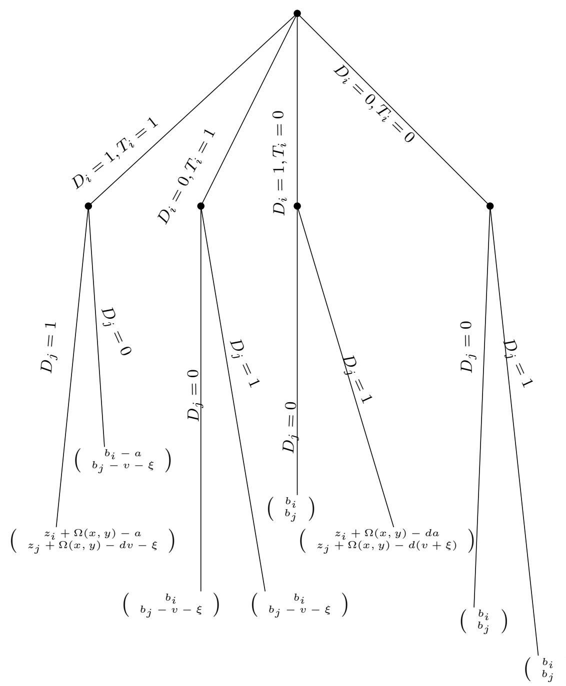
The following figure represents the sequential game:



Appendix Figure A1. Sequential game: Player i plays first and then player j.

The way to solve the game represented by figure A1 is by backward induction. Agent j will choose to remain married if and only if $z_{j,2} \geq b_j - \Omega(x,y) - v$ and is at the node where agent i wants to marry and occupies violence; $z_{j,2} \geq b_j - \Omega(x,y)$ and is at the node where agent i wants to marry and does not occupy violence. Therefore, if $b_j - \Omega(x,y) - v \leq z_{j,2} \leq b_j - \Omega(x,y)$ and $z_{i,2} \geq b_i - \Omega(x,y) + a$ then exist a unique pure strategy Nash Equilibrium to this game, that is violence with marriage. Note that it is equivalent to that imposed by the expression (2.12).

When adding cue-trigger violence, the structure of the game remains the same as in Figure A1, but the payouts change. This is reflected in Figure A2. The resolution of this game is equivalent to the previous one, only the areas where divorces and violent marriages occur are re-scaled.



Appendix Figure A2. Sequential game: Player i plays first and then player j with coercive and cue-trigger violence.

2.5. Endogenous Transfers

In this section, we introduce the assumption of exogenous transfers and adopt an axiomatic approach based on the work of Nash (Nash, 1950,9) to determine the optimal transfer from agent i to agent j in a context of unilateral divorce. Specifically, we seek to identify the conditions under which agent i has incentives to use transfers or violence when only agent i can make transfers. The transfer to be made by agent i is given by:

$$s^N =_s (z_{i,2} + \Omega(x,y) - s - b_i)^{\alpha} (z_{j,2} + \Omega(x,y) + s - b_j)^{1-\alpha} \tag{2.20}$$

where $\alpha \in [0,1]$ represents the bargaining power of agent i, $z_{j,2} \in [0, z_j^{*1}]$ and $z_{i,2} \in [z_i^{*4}, B]$. The solution for expression (2.20) is:

$$\alpha \ln \left(z_{i,2} + \Omega(x,y) - s - b_i\right) + (1-\alpha) \ln \left(z_{j,2} + \Omega(x,y) + s - b_j\right)$$

$$\Rightarrow [s] : \frac{-\alpha}{z_{i,2} + \Omega(x,y) - s^N - b_i} + \frac{1-\alpha}{z_{j,2} + \Omega(x,y) + s^N - b_j} = 0$$

$$s^N = (1-\alpha)\left(z_{i,2} + \Omega(x,y) - b_i\right) + \alpha\left(b_j - (z_{j,2} + \Omega(x,y))\right) \tag{2.21}$$

We will assume that the bargaining power of agents of type i is maximum, i.e. $\alpha = 1$.

2.6. Second Appendix: Proofs, and comparison of different systems of divorce

2.6.1. Omitted Proofs of Results

PROOF. Lemma 2.1. The equation (2.7) for $r = U$ is equivalent to:

$$\begin{aligned}
&M_i^U(z_{i,1}, z_{j,1}, z_{i,2}^*, z_{j,2}^*, x, y) = \\
&(z_{i,1} + \Omega(x,y)) + \beta(1-\lambda_i)(1-\lambda_j) \max_{\text{Single,Married}} \left\{ b_i, \left[z_{i,1} + \Omega(x,y)\right] D_j(z_{j,2}) + (1 - D_j(z_{j,2}))b_i \right\} \\
&+ \beta(1-\lambda_j)\lambda_i \int \max_{\text{Single,Married}} \left\{ b_i, \left[z'_{i,2} + \Omega(x,y)\right] D_j(z_{j,2}) + (1 - D_j(z_{j,2}))b_i \right\} dF(z'_{i,2}) \\
&+ \beta(1-\lambda_i)\lambda_j \int \max_{\text{Single,Married}} \left\{ b_i, \left[z_{i,2} + \Omega(x,y)\right] D_j(z'_{j,2}) + (1 - D_j(z'_{j,2}))b_i \right\} dF(z'_{j,2}) \\
&+ \beta\lambda_i\lambda_j \int\int \max_{\text{Single,Married}} \left\{ b_i, \left[z'_{i,2} + \Omega(x,y)\right] D_j(z'_{j,2}) + (1 - D_j(z'_{j,2}))b_i \right\} dF(z'_{i,2}) dF(z'_{j,2})
\end{aligned} \tag{2.22}$$

Equation (2.22) can be rewritten as:

$$\Rightarrow M^U(\cdot) = (z_{i,1} + \Omega(x,y)) \left[1 + \beta(1-\lambda_i)(1-\lambda_j)\mathbf{1}_{z_{i,1}>z^*_{i,2}}\mathbf{1}_{z_{j,1}>z^*_{j,2}} + \beta(1-\lambda_i)\lambda_j\left(1 - F(z^*_{j,2})\right)\mathbf{1}_{z_{i,1}>z^*_{i,2}}\right]$$

$$+ \beta b_i \Bigg[(1-\lambda_i)(1-\lambda_j)\left(\mathbf{1}_{z_{i,1}\le z^*_{i,2}} + \mathbf{1}_{z_{i,1}>z^*_{i,2}}\mathbf{1}_{z_{j,1}\le z^*_{j,2}}\right) + (1-\lambda_j)\lambda_i\left(F(z^*_{i,2}) + (1-F(z^*_{i,2}))\mathbf{1}_{z_{j,1}\le z^*_{j,2}}\right)$$

$$+ (1-\lambda_i)\lambda_j\left(F(z^*_{j,2}) + (1-F(z^*_{j,2}))\mathbf{1}_{z_{i,1}\le z^*_{i,2}}\right) + \lambda_i\lambda_j\left(F(z^*_{i,2}) + F(z^*_{j,2})(1-F(z^*_{i,2}))\right)\Bigg] +$$

$$\beta\left(\mathbb{E}\left(z_i | z_i \ge z^*_{i,2}\right) + \Omega(x,y)\right)\left[(1-\lambda_j)\lambda_i\left(1-F(z^*_{i,2})\right)\mathbf{1}_{z_{j,1}>z^*_{j,2}} + \lambda_i\lambda_j\left(1-F(z^*_{i,2})\right)\left(1-F(z^*_{j,2})\right)\right]$$

Taking the partial derivative of the above expression with respect to $z_{i,1}$ we obtain the following:

(i) If $z_{i,1} \le z^*_{i,2}$:

$$\frac{\partial M^U(z_{i,1}, z_{j,1}, z^*_{i,2}, z^*_{j,2}, x, y)}{\partial z_{i,1}} = 1 \tag{2.23}$$

(ii) If $z_{i,1} > z^*_{i,2}$ and $z_{j,1} > z^*_{j,2}$:

$$\frac{\partial M^U(z_{i,1}, z_{j,1}, z^*_{i,2}, z^*_{j,2}, x, y)}{\partial z_{i,1}} = 1 + \beta(1-\lambda_i)\left(1 - \lambda_j F(z^*_{j,2})\right) > 0 \tag{2.24}$$

(iii) If $z_{i,1} > z^*_{i,2}$ and $z_{j,1} \le z^*_{j,2}$:

$$\frac{\partial M^U(z_{i,1}, z_{j,1}, z^*_{i,2}, z^*_{j,2}, x, y)}{\partial z_{i,1}} = 1 + \beta(1-\lambda_i)\lambda_j\left(1 - F(z^*_{j,2})\right) > 0 \tag{2.25}$$

It is also true that:

$$\left.\frac{\partial M^U(z_{i,1}, z_{j,1}, z^*_{i,2}, z^*_{j,2}, x, y)}{\partial z_{i,1}}\right|_{z_{i,1}>z^*_{i,2}} > \left.\frac{\partial M^U(z_{i,1}, z_{j,1}, z^*_{i,2}, z^*_{j,2}, x, y)}{\partial z_{i,1}}\right|_{z_{i,1}\le z^*_{i,2}}$$

In the case of $M_i^B(\cdot)$ we have:

$$M_i^B(\cdot) = (z_{i,1} + \Omega(x,y)) \Big[1 + \beta(1-\lambda_i)(1-\lambda_j) \left(\mathbf{1}_{z_{i,1} > z^*_{i,2}} + \mathbf{1}_{z_{i,1} \leq z^*_{i,2}} \mathbf{1}_{z_{j,1} > z^*_{j,2}} \right) +$$

$$\beta(1-\lambda_i)\lambda_j \left(\mathbf{1}_{z_{i,1} > z^*_{i,2}} + \mathbf{1}_{z_{i,1} \leq z^*_{i,2}} (1 - F(z^*_{j,2})) \right) \Big] + \beta b_i \Big[(1-\lambda_i)(1-\lambda_j) \mathbf{1}_{z_{i,1} \leq z^*_{i,2}} \mathbf{1}_{z_{j,1} \leq z^*_{j,2}} +$$

$$(1-\lambda_i)\lambda_j \mathbf{1}_{z_{i,1} \leq z^*_{i,2}} F(z^*_{j,2}) + \lambda_i(1-\lambda_j) \mathbf{1}_{z_{j,1} \leq z^*_{j,2}} F(z^*_{i,2}) + \lambda_i \lambda_j F(z^*_{i,2}) F(z^*_{j,2}) \Big] +$$

$$\beta \lambda_i F(z^*_{i,2}) \left(\mathbb{E}\left(z_i | z_i < z^*_{i,2} \right) + \Omega(x,y) \right) \Big[(1-\lambda_j) \mathbf{1}_{z_{j,1} > z^*_{j,2}} + \lambda_j (1 - F(z^*_{j,2})) \Big]$$

$$\beta \lambda_i (1 - F(z^*_{i,2})) \left(\mathbb{E}\left(z_i | z_i \geq z^*_{i,2} \right) + \Omega(x,y) \right) \tag{2.26}$$

Obtaining the partial derivative of the aforementioned equation with respect to $z_{i,1}$ yields the following result:

(i) If $z_{i,1} \leq z^*_{i,2}$ and $z_{j,1} > z^*_{j,2}$:

$$\frac{\partial M^B(z_{i,1}, z_{j,1}, z^*_{i,2}, z^*_{j,2}, x, y)}{\partial z_{i,1}} = 1 + \beta(1-\lambda_i)\left((1-\lambda_j) + \lambda_j (1 - F(z^*_{j,2})) \right) > 0 \tag{2.27}$$

(ii) If $z_{i,1} \leq z^*_{i,2}$ and $z_{j,1} \leq z^*_{j,2}$:

$$\frac{\partial M^B(z_{i,1}, z_{j,1}, z^*_{i,2}, z^*_{j,2}, x, y)}{\partial z_{i,1}} = 1 + \beta(1-\lambda_i)\lambda_j (1 - F(z^*_{j,2})) > 0 \tag{2.28}$$

(iii) If $z_{i,1} > z^*_{i,2}$:

$$\frac{\partial M^B(z_{i,1}, z_{j,1}, z^*_{i,2}, z^*_{j,2}, x, y)}{\partial z_{i,1}} = 1 + \beta(1-\lambda_i) > 0 \tag{2.29}$$

It also holds that:

$$\left. \frac{\partial M^B(z_{i,1}, z_{j,1}, z^*_{i,2}, z^*_{j,2}, x, y)}{\partial z_{i,1}} \right|_{z_{i,1} > z^*_{i,2}} > \left. \frac{\partial M^B(z_{i,1}, z_{j,1}, z^*_{i,2}, z^*_{j,2}, x, y)}{\partial z_{i,1}} \right|_{z_{i,1} \leq z^*_{i,2}}$$

□

PROOF. Lemma 2.2. We evaluate the utility of being married for individual i for the case of unilateral divorce (2.7), without loss of generality, $z_{i,1} = z^*_{i,2}$:

$$M_i^U\left(z_{i,1} = z^*_{i,2}, \cdot\right) = b_i + \beta b_i +$$
$$\beta(1 - F(z^*_{i,2}))\lambda_i \mathbb{E}\left(z_{i,2} + \Omega(x,y) - b_i | z_{i,2} \geq z^*_{i,2}\right)\left((1-\lambda_j)\mathbf{1}_{z_{j,1}\geq z^*_{j,2}} + \lambda_j(1 - F(z^*_{j,2}))\right)$$

In the case of bilateral divorce, we have:

$$M_i^B(z_{i,1} = z^*_{i,2}, \cdot) = b_i + \beta b_i + \beta\lambda_i(1 - F(z^*_{i,2}))\left(\mathbb{E}\left(z_i | z_i \geq z^*_{i,2}\right) + \Omega(x,y) - b_i\right)$$
$$\beta\lambda_i F(z^*_{i,2})\left(\mathbb{E}\left(z_i | z_i < z^*_{i,2}\right) + \Omega(x,y) - b_i\right)\left[(1-\lambda_j)\mathbf{1}_{z_{j,1}\geq z^*_{j,2}} + \lambda_j(1 - F(z^*_{j,2}))\right]$$
$$- \beta b_i \lambda_i F(z^*_{i,2})(1-\lambda_j)\mathbf{1}_{z_{j,1}<z^*_{j,2}}$$

Let's study what conditions must be met for the following inequality to be satisfied:

$$M_i^U\left(z_{i,1} = z^*_{i,2}, z_{j,1} = z^*_{j,2}, \cdot\right) > M_i^B(z_{i,1} = z^*_{i,2}, z_{j,1} = z^*_{j,2}, \cdot)$$
$$\Rightarrow \frac{F(z^*_{i,2})}{1 - F(z^*_{i,2})}\frac{1 - \lambda_j F(z^*_{j,2})}{\lambda_j F(z^*_{j,2})} > \frac{\mathbb{E}\left(z_i + \Omega(x,y) - b_i | z_i \geq z^*_{i,2}\right)}{\mathbb{E}\left(b_i - (z_i + \Omega(x,y)) | z_i < z^*_{i,2}\right)} \tag{2.30}$$

$$M_j^U\left(z_{j,1} = z^*_{j,2}, z_{i,1} = z^*_{i,2}, \cdot\right) > M_j^B(z_{j,1} = z^*_{j,2}, z_{i,1} = z^*_{i,2}, \cdot)$$
$$\Rightarrow \frac{F(z^*_{j,2})}{1 - F(z^*_{j,2})}\frac{1 - \lambda_i F(z^*_{i,2})}{\lambda_i F(z^*_{i,2})} > \frac{\mathbb{E}\left(z_j + \Omega(x,y) - b_j | z_j \geq z^*_{j,2}\right)}{\mathbb{E}\left(b_j - (z_j + \Omega(x,y)) | z_j < z^*_{j,2}\right)} \tag{2.31}$$

Then $M_i^U\left(z_{i,1} = z^*_{i,2}, \cdot\right) > M_i^B\left(z_{i,1} = z^*_{i,2}, \cdot\right)$. The value of being single, represented by equation (2.6), is:

$$S(x) = b_i + \beta b_i + p\beta\left(\int\left(1 - F(z^*_{i,2})\right)\left(1 - F(z^*_{j,2})\right)\mathbb{E}\left(z_{i,2} + \Omega(x,y) - b_i | z_{i,2} \geq z^*_{i,2}\right)dG(y)\right)$$

So if

$$S(x) > M_i^U\left(z_{i,1} = z^*_{i,2}, \cdot\right)$$
$$\Rightarrow p > \frac{(1 - F(z^*_{i,2}))\lambda_i \mathbb{E}\left(z_{i,2} + \Omega(x,y) - b_i | z_{i,2} \geq z^*_{i,2}\right)\left((1-\lambda_j)\mathbf{1}_{z_{j,1}\leq z^*_{j,2}} + \lambda_j(1 - F(z^*_{j,2}))\right)}{\left(\int\left(1 - F(z^*_{i,2})\right)\left(1 - F(z^*_{j,2})\right)\mathbb{E}\left(z_{i,2} + \Omega(x,y) - b_i | z_{i,2} \geq z^*_{i,2}\right)dG(y)\right)} \equiv \bar{p_i}$$

Since $M_i^U(\cdot)$ and $M_i^B(\cdot)$ are monotonic in $z_{i,1}$ then if $p \in [\min\{p_i, p_j\}, 1]$ implies $z^{*B} > z_{i.1}^{*U} > z_{i,2}^*$. $\square$

PROOF. Lemma 2.4. We evaluate the role of being married for type i agents who do not have access to violence, evaluated in $z_{i,1} = z^*_{i,2} + a$ and $z_{j,1} \in [z^*_{j,2} - v, z^*_{j,2}]$. Then:

$$M_i(\cdot) = b_i(1+\beta) + a + \beta\lambda_i\lambda_j\left(1 - F(z^*_{i,2})\right)\left(1 - F(z^*_{j,2})\right)\mathbb{E}\left(z_{i,2} + \Omega(x,y) - b_i | z_{i,2} \geq z^*_{i,2}\right)$$
$$+ \beta(1-\lambda_i)\lambda_j\left(1 - F(z^*_{j,2})\right)(b+a)$$

In the case that agent i has access to violence is:

$$M_i(\cdot|T_i(\cdot)) = b_i(1+\beta) + a + \beta(1-\lambda_i)\lambda_j\left(1 - F(z^*_{j,2})\right)(b_i + a)$$
$$+ \beta\lambda_i(1-\lambda_j)\left(1 - F(z^*_{i,2} + a)\right)\mathbb{E}\left(z_{i,2} + \Omega(x,y) - b_i - a | z_{i,2} \geq z^*_{i,2} + a\right) +$$
$$\beta\lambda_i\lambda_j\left(1 - F(z^*_{i,2} + a)\right)\left(F(z^*_{j,2}) - F(z^*_{j,2} - v)\right)\mathbb{E}\left(z_{i,2} + \Omega(x,y) - b_i - a | z_{i,2} \geq z^*_{i,2} + a\right)$$
$$+ \beta\lambda_i\lambda_j\left(1 - F(z^*_{i,2})\right)\left(1 - F(z^*_{j,2})\right)\mathbb{E}\left(z_{i,2} + \Omega(x,y) - b_i | z_{i,2} \geq z^*_{i,2}\right)$$

Therefore, $M_i(\cdot|T_i(\cdot)) > M_i(\cdot)$. Given by lemma 2.1, the function is monotone, then the above will be true for all larger $z^*_{i,2} + a$.

In the case of agent j who do not have access to violence, evaluated in $z_{i,1} = z^*_{i,2} + a$ and $z_{j,1} = z^*_{j,2} - v$ we have:

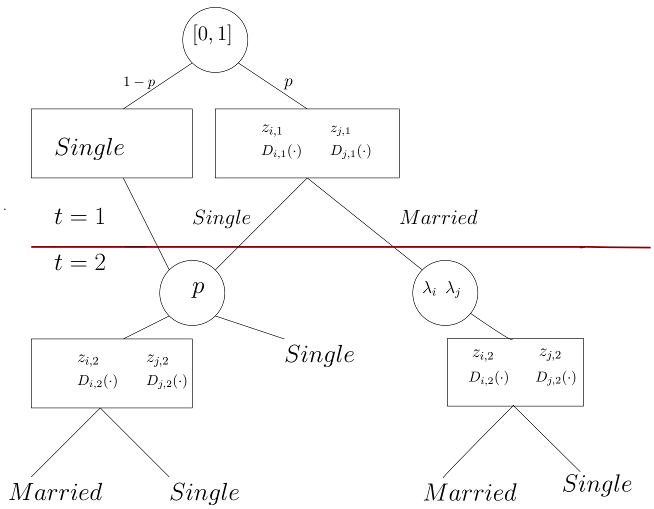
$$M_j(\cdot) = b_j - v + \beta b_j +$$
$$\beta\lambda_j\left(1 - F(z^*_{j,2} - v)\right)\mathbb{E}\left(z_{j,2} + \Omega(x,y) - b_i | z_{j,2} \geq z^*_{j,2} - v\right)\left(\lambda_i\left(1 - F(z^*_{i,2} - v) + (1-\lambda_i)\right)\right)$$

In the case that agent j suffers violence, its utility is:

$$M_j(\cdot|T_i(\cdot)) = b_j - v + \beta b_j +$$
$$\beta\lambda_j\left(1 - F(z^*_{j,2})\right)\mathbb{E}\left(z_{j,2} + \Omega(x,y) - b_i | z_{j,2} \geq z^*_{j,2}\right)\left(\lambda_i\left(1 - F(z^*_{i,2}) + (1-\lambda_i)\right)\right)$$

Therefore, $M_i(\cdot|T_j(\cdot)) < M_j(\cdot)$. Given by lemma 2.1, the function is monotone in $z_{j,2}$, then the above will be true $\forall z_{j,2} \in [z^*_{j,2} - v, z^*_{j,2}]$ □

PROOF. Proposition 2.1. The timing of the model is represented by the following figure:



Appendix Figure C1. Timing of the model.

There is a mass of agents of mass 1, who with probability $1-p$ are born to single and with probability p can choose in the same period ($t = 1$) whether to be single and marry. By Lemma 2.1 it is guaranteed that there exists a unique $z_{k,1}$ such that in the first period agent, k solves equation (2.8).

Let's first analyze what happens in the first period with single and married people. Note that Lemma 2.2 must be satisfied in order for $z^*_{i,1} > z^*_{i,2}$. The mass of singles will be:

$$Single = (1-p) + p\Bigg(1 - \left(1 - F(z^*_{i,1})\right)\left(1 - F(z^*_{j,1})\right)\Bigg) Single_{t=2} \tag{2.32}$$

where $Single$ represents the mass of singles in the economy and $Single_{t=2}$ is the mass of singles at $t = 2$. And married people are:

$$Married = p\left(1 - F(z^*_{i,1})\right)\left(1 - F(z^*_{j,1})\right) Married_{t=2} \tag{2.33}$$

where $Married$ represents the mass of married in the economy and $Married_{t=2}$ is the mass of married at $t = 2$. In the second period, there is a probability λ_k with which the random variable is updated. Let us analyze the mass of married couples in the second period:

$$Married_{t=2} = Married_{t=2}\Big|Married_{t=1} + Married_{t=2}\Big|Single_{t=1} \tag{2.34}$$

$$Married_{t=2}\Big|Married_{t=1} = (1-\lambda_i)(1-\lambda_j) + (1-\lambda_j)\lambda_i\left(1-F(z^*_{i,2})\right) +$$

$$(1-\lambda_i)\lambda_j\left(1-F(z^*_{j,2})\right) + \lambda_i\lambda_j\left(1-F(z^*_{i,2})\right)\left(1-F(z^*_{j,2})\right) \tag{2.35}$$

$$Married_{t=2}\Big|Single_{t=1} = (1-\lambda_i)(1-\lambda_j)\left(F(z^*_{i,1})-F(z^*_{i,2})\right)\left(F(z^*_{j,1})-F(z^*_{j,2})\right)$$

$$+(1-\lambda_j)\lambda_i\left(F(z^*_{j,1})-F(z^*_{j,2})\right)\left(1-F(z^*_{i,2})\right) + \lambda_i\lambda_j\left(1-F(z^*_{i,2})\right)\left(1-F(z^*_{j,2})\right)$$

$$+(1-\lambda_i)\lambda_j\left(F(z^*_{i,1})-F(z^*_{i,2})\right)\left(1-F(z^*_{j,2})\right) \tag{2.36}$$

where $Married_{t=2}\Big|Married_{t=1}$ represents those married in period two if they were married in period one; $Married_{t=2}\Big|Single_{t=1}$ represents those married in period two if they were single in period one.

With appendix 2.4 and lemma 2.4 we guarantee that $T_i(\cdot)$ is defined according to equation (2.12) and that $z^*_{i,1}$ increases in the case that the agent has access to violence and that $z^*_{j,1}$ falls. Deriving equation (2.33) with respect to $z_{i,1}$ is:

$$\frac{\partial Married}{\partial z^*_{i,1}} = p\left(1-F(z^*_{j,1})\right)\left(\frac{\partial Married_{t=2}}{\partial z^*_{i,1}} - \frac{dF(z^*_{i,1})}{dz^*_{i,1}}Married_{t=2}\right) \tag{2.37}$$

$$\frac{\partial Married_{t=2}}{\partial z^*_{i,1}} = (1-\lambda_i)\frac{dF(z^*_{i,1})}{dz^*_{i,1}}\Bigg((1-\lambda_j)\left(F(z^*_{j,1})-F(z^*_{j,2})\right) + \lambda_j\left(1-F(z^*_{j,2})\right)\Bigg) > 0$$

It is not clear the sign of (2.37). Now let us derive expression (2.37) with respect to $z^*_{j,1}$:

$$\frac{\partial^2 Married}{\partial z^*_{j,1}\partial z^*_{i,1}} = p\left(1-F(z^*_{j,2})\right)\left(\frac{\partial^2 Married_{t=2}}{\partial z^*_{j,1}\partial z^*_{i,1}} - \frac{dF(z^*_{i,1})}{dz^*_{i,1}}\frac{\partial Married_{t=2}}{\partial z^*_{j,1}}\right)$$
$$- p\frac{dF(z^*_{j,1})}{dz^*_{j,1}}\left(\frac{\partial Married_{t=2}}{\partial z^*_{i,1}} - \frac{dF(z^*_{i,1})}{dz^*_{i,1}} Married_{t=2}\right) \tag{2.38}$$

$$\frac{\partial^2 Married_{t=2}}{\partial z^*_{j,1}\partial z^*_{i,1}} = (1-\lambda_i)\frac{dF(z^*_{j,1})}{dz^*_{j,1}}\frac{dF(z^*_{i,1})}{dz^*_{i,1}}(1-2\lambda_j) \lesseqgtr 0$$

$$\frac{\partial Married_{t=2}}{\partial z^*_{j,1}} = (1-\lambda_j)\frac{dF(z^*_{j,1})}{dz^*_{j,1}}\left((1-\lambda_i)\left(F(z^*_{i,1})-F(z^*_{i,2})\right)+\lambda_i\right) > 0$$

Therefore, it is not clear the sign of the expression (2.38). This implies that it is not possible to know if married individuals are increasing. But divorces increase, because they occur in the second period and depend on $z^*_{i,2}$ and $z^*_{j,2}$. □

PROOF. Proposition 2.2. Taking the result of appendix 2.4, we know that there is only one solution in the sequential game represented by Figure A1. The above represents agents in the second period, who have access to unilateral divorce, and type i agents have access to coercive violence. Remember that agent i first decides whether to stay in the relationship and whether to apply coercive violence or not. Let us solve the game, by backward induction, assuming the following intervals where z_j could occur:

- If $z_{j,2} \in [0, z^*_{j,2} - v[$ then we have:
  - Agent j will decide the divorce for that interval of $z_{j,2}$. Therefore, agents of type i will remain single for all possible values of $z_{i,2}$, because both types of agents are in a unilateral divorce context.
- In the case where $z_{j,2}$ falls within the range of $[z^*_{j,2} - v, z^*_{j,2}[$, we have the following cases for agents of type i:
  - If agents of type i receive a $z_{i,2} \in [z^*_{i,2} + a, B]$ have incentive to use coercive violence. Recall that this technology is used as a method of retention for j-type agents. If agents of type i receive a $z_{i,2} \in [0, z^*_{i,2} + a[$, then they will

not use violence (very costly for the z received). Note that if a¿B-z, there will be no incentive to use coercive violence for any agent type i agents.

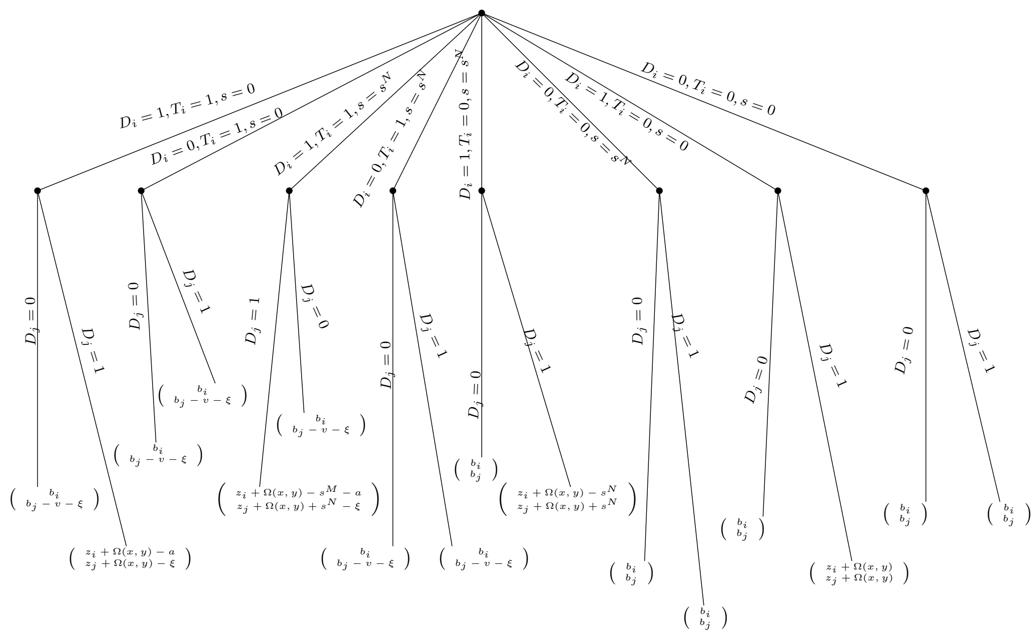
- If the value of $z_{j,2}$ falls within the interval $[z^*_{j,2}, B]$, then we have the following:
  - Under these conditions, type i agents lack incentive to apply coercive violence, as type j agents prefer to maintain the relationship, negating the necessity for a coercive retention strategy by type i agents. Thus, if $z_{i,2}$ resides within the range $[z^*_{i,2}, B]$, the marriage persists, while divorce occurs for all other $z_{i,2}$ intervals.

Therefore, the result of the sequential game in the context of unilateral divorce and with access to coercive violence is represented by Figure **??**.

In a bilateral divorce framework, a mutual agreement to divorce is required. Let's propose a sequential game, similar to the previous scenario, where type-i agents decide whether to remain married and whether to apply coercive violence, followed by agents of type$j$ decision on whether to stay married. It is apparent that if type-i agents opt to stay, the marriage continues. Hence, coercive violence will never be employed by agents of type i, rendering the cost of violence (a) irrelevant. This outcome can be depicted in Figure **??**.

□

PROOF. Proposition 2.3. Assume that, during the second period, agents make a decision to either stay in the relationship or leave. Agents of type i have the ability to deploy coercive violence, serving as a retention mechanism for type-j agents who wish to dissolve the marriage, and also to provide transfers. The latter can substitute for coercive violence as it ensures type-j agents' indifference between remaining married and being single. Figure C2 presents a sequential game, set in a unilateral divorce scenario. Here, type-i agents first determine whether to remain married and whether to resort to coercive violence and/or transfers toward type-j agents. Subsequently, type-j agents decide on whether to pursue divorce.



Appendix Figure C2. Sequential game: Player i plays first and then player j in the context of unilateral divorce.

We know that agents of type i have maximum bargaining power, then the transfers they will make to agents of type i have the following form:

$$s^N = b_j - (z_{j,2} + \Omega(x, y))$$

We will solve the game via backward induction, considering the following potential ranges for $z_{j,2}$:

- If agents of type j receive a $z_{j,2}$ between the intervals $[0, z^*_{j,2} - v[$ then we have the following:
  - In this scenario, coercive violence will not persist, as type-i agents will only employ this mechanism if type-j agents' z_j falls within the interval $[z^*_{j,2} - v, z^*_{j,2}[$. As the transfers made by type-i agents make type-j agents indifferent between staying in the relationship or not (with our assumption being that they stay), the decision to make transfers is determined by the

following:

$$
\begin{aligned}
& z_{i,2} + \Omega(x,y) - s^N \geq b_i \\
\Rightarrow & z_{i,2} + \Omega(x,y) - (b_j - (z_{j,2} + \Omega(x,y))) \geq b_i \\
\Rightarrow & z_{i,2} + z_{j,2} \geq z^*_{i,2} + z^*_{j,2} \qquad (2.39)
\end{aligned}
$$

It is important to note that if $z_{j,2} = 0$ then $z_{i,2} \geq z^*_{i,2} + z^*_{j,2}$. And if $z_{j,2} \to z^*_{j,2} - v$ then $z_{i,2} \geq z^*_{i,2} + v$. Note that if agents of type j receive a $z_j = 0$, then agents of type i, in order to have incentive to transfer, must have $z_{i,2} \geq z^*_{i,2} + z^*_{j,2}$. And if agents of type j receive $z^*_{j,2} - v$ then agents of type i must have $z_{i,2} \geq z^*_{i,2} + v$.

- Let's assume that $v > a$. If $z_{j,2} \in [z^*_{j,2} - v, z^*_{j,2} - a[$ then we have:
  - For type i agents to prefer the use of violence over transfers, the following must be satisfied:

$$
\begin{aligned}
& z_{i,2} + \Omega(x,y) - a \geq z_{i,2} + \Omega(x,y) - s \\
\Rightarrow & s \geq a, \quad s \in]a, v] \qquad (2.40)
\end{aligned}
$$

Therefore, it suffices that $z_{i,2} \geq z^*_{i,2} + a$ for agents of type i to prefer to use violence rather than transfer.

Let's study the case in which $z_{i,2} \in [z^*_{i,2}, z^*_{i,2} + a[$. In this case the agents of type i prefer to use transfers rather than violence, then if $z_{j,2} = z^*_{j,2} - v$ then, for the equation (2.39) we have: $z_{i,2} \geq z^*_{i,2} + v$; if $z_{j,2} = z^*_{j,2} - v \Rightarrow z_{i,2} \geq z^*_{i,2} + a$. Remember that we assume that $v > a$ then no transfers will be made in this case.

- Let's assume that $v < a$. If $z_{j,2} \in [z^*_{j,2} - a, z^*_{j,2} - v[$ then we have:
  - Let us compare the decision to use coercive violence and transfers.

$$
s \geq a
$$

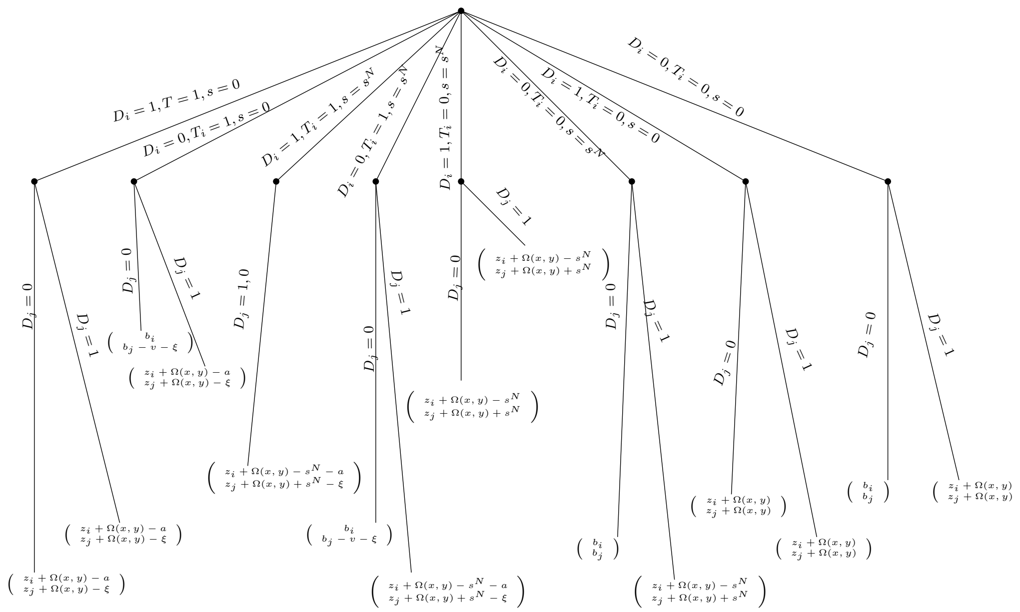
but $s \in]v, a]$ then transfers are more attractive to type i agents than coercive violence.

- Let's assume that $v > a$. If $z_{j,2} \in [z^*_{j,2} - a, z^*_{j,2}[$ then we have:
  - Transfers will now be between $s \in [0, v[$, , then agents of type i will have no incentive to use coercive violence and will use transfers. For the equation (2.39) we have: if $z_{j,2} = z^*_{j,2} - a$ then $z_{i,2} \geq z^*_{i,2} + a$; if $z_{j,2} = z^*_{j,2}$ then $z_{i,2} = z^*_{i,2}$.
- Let's assume that $v > a$. If $z_{j,2} \in [z^*_{j,2} - v, z^*_{j,2}[$ then we have only transfers.
- If $z_{j,2} \geq z^*_{j,2}$ then we have:
  - In case the agents of type i receive a $z_{i,2} \in [z^*_{i,2}, B]$, then they will continue with the relationship, otherwise, they will divorce.

Assuming $a < v$, the situation is depicted in Figure C3. Alternatively, if $a > v$, the solution is given by the following:

Appendix Figure C3. Unilateral divorce with agents of type i having a cost in the use of violence when $a < v$

Now suppose we are in an economy where agents have access to coercive violence and transfers and divorce is bilateral. The sequential game is represented by the following figure:



Appendix Figure C4. Sequential game: Player i plays first and then player j in the context of bilateral divorce.

In this setting, alongside the advantage of making the first move, due to the bilateral nature of the divorce, type i agents lack incentives to employ either coercive violence or transfers. This is because a divorce can only proceed if both parties agree, thereby eliminating the need for any mechanism to retain type j agents within the relationship. The result of the sequential game is represented by Figure C5. $\square$

REFERENCES

Agarwal, N. (2015). An empirical model of the medical match. *American Economic Review*, 105(7):1939–78.

Aizer, A. (2010). The gender wage gap and domestic violence. *American Economic Review*, 100(4):1847–59.

Aizer, A. (2011). Poverty, violence, and health the impact of domestic violence during pregnancy on newborn health. *Journal of Human resources*, 46(3):518–538.

Albrecht, J., Tan, S., Gautier, P., and Vroman, S. (2004). Matching with multiple applications revisited. *Economics Letters*, 84(3):311–314.

Anderberg, D., Rainer, H., Wadsworth, J., and Wilson, T. (2016). Unemployment and domestic violence: Theory and evidence. *The Economic Journal*, 126(597):1947–1979.

Angelucci, M. (2008). Love on the rocks: Domestic violence and alcohol abuse in rural mexico. *The BE Journal of Economic Analysis & Policy*, 8(1).

Atakan, A. E. (2006). Assortative matching with explicit search costs. *Econometrica*, 74(3):667–680.

Bahadir-Yilmaz, E. and Oz, F. (2019). Experiences and perceptions of abused turkish women regarding violence against women. *Community mental health journal*, 55(4):686–692.

Bailey, C. M. (2010). Recent trends: Online dating. *Retrieved October*, 1:2010.

Bailey, M. J., Guldi, M. E., and Hershbein, B. J. (2013). Is there a case for a" second demographic transition"? three distinctive features of the post-1960 us fertility decline. Technical report, National Bureau of Economic Research.

Becker, G. S. (1973). A theory of marriage: Part i. *Journal of Political economy*, 81(4):813–846.

Bellou, A. (2015). The impact of internet diffusion on marriage rates: evidence from the broadband market. *Journal of Population Economics*, 28(2):265–297.

Bloch, F. and Rao, V. (2002). Terror as a bargaining instrument: A case study of dowry violence in rural india. *American Economic Review*, 92(4):1029–1043.

Bobonis, G. J., González-Brenes, M., and Castro, R. (2013). Public transfers and domestic violence: The roles of private information and spousal control. *American Economic Journal: Economic Policy*, 5(1):179–205.

Borchers, A., Lee, R. C., Martsolf, D. S., and Maler, J. (2016). Employment maintenance and intimate partner violence. *Workplace Health & Safety*, 64(10):469–478.

Boulier, B. L. and Rosenzweig, M. R. (1984). Schooling, search, and spouse selection: Testing economic theories of marriage and household behavior. *Journal of political Economy*, 92(4):712–732.

Brassiolo, P. (2016). Domestic violence and divorce law: When divorce threats become credible. *Journal of Labor Economics*, 34(2):443–477.

Browning, M., Chiappori, Pierre-André, and Weiss, Y. (2014). *Economics of the Family*. Cambridge University Press.

Browning, M., Chiappori, P.-A., and Lewbel, A. (2013). Estimating consumption economies of scale, adult equivalence scales, and household bargaining power. *Review of Economic Studies*, 80(4):1267–1303.

Campbell, J. C. (2002). Health consequences of intimate partner violence. *The Lancet*, 359(9314):1331–1336.

Card, D. and Dahl, G. B. (2011). Family violence and football: The effect of unexpected emotional cues on violent behavior. *The quarterly journal of economics*, 126(1):103–143.

Carlson, J., Voith, L., Brown, J. C., and Holmes, M. (2019). Viewing children's exposure to intimate partner violence through a developmental, social-ecological, and survivor lens: The current state of the field, challenges, and future directions. *Violence against women*, 25(1):6–28.

Carrell, S. E. and Hoekstra, M. L. (2010). Externalities in the classroom: How children exposed to domestic violence affect everyone's kids. *American Economic Journal: Applied Economics*, 2(1):211–228.

Chiappori, P.-A. (2020). The theory and empirics of the marriage market. *Annual Review of Economics*, 12:547–578.

Chiappori, P.-A., Dias, M. C., and Meghir, C. (2018a). The marriage market, labor supply, and education choice. *Journal of Political Economy*, 126(S1):S26–S72.

Chiappori, P.-A., Dias, M. C., and Meghir, C. (2018b). The marriage market, labor supply, and education choice. *Journal of Political Economy*, 126(S1):S26–S72.

Chiappori, P.-A., Dias, M. C., and Meghir, C. (2020). Changes in assortative matching: Theory and evidence for the us. Technical report, National Bureau of Economic Research.

Chiappori, P.-A., Fortin, B., and Lacroix, G. (2002). Marriage market, divorce legislation, and household labor supply. *Journal of political Economy*, 110(1):37–72.

Chiappori, P.-A., Iyigun, M., and Weiss, Y. (2009). Investment in schooling and the marriage market. *American Economic Review*, 99(5):1689–1713.

Chiappori, P.-A., Oreffice, S., and Quintana-Domeque, C. (2012). Fatter attraction: anthropometric and socioeconomic matching on the marriage market. *Journal of Political Economy*, 120(4):659–695.

Choo, E. and Siow, A. (2006). Who marries whom and why. *Journal of political Economy*, 114(1):175–201.

Corradini, V. and Buccione, G. (2023). Unilateral divorce rights, domestic violence and women's agency: Evidence from the egyptian khul reform. *Journal of Development Economics*, 160:102947.

Couprie, H. (2007). Time allocation within the family: Welfare implications of life in a couple. *The Economic Journal*, 117(516):287–305.

Dagsvik, J. K. (2000). Aggregation in matching markets. *International Economic Review*, 41(1):27–58.

de Piñar-Prats, A., Fernández-Alcántara, M., and Pérez-Marfil, M. N. (2022). Needs and support perceived by women for coping with the experience of intimate partner violence in andalusia (spain): A qualitative study. *Journal of Interpersonal Violence*, 37(15-16):NP14289–NP14309. PMID: 33866832.

Eika, L., Mogstad, M., and Zafar, B. (2019). Educational assortative mating and household income inequality. *Journal of Political Economy*, 127(6):2795–2835.

Ergin, N., Bayram, N., Alper, Z., Selimoglu, K., and Bilgel, N. (2006). Domestic violence: A tragedy behind the doors. *Women & health*, 42(2):35–51.

Erten, B. and Keskin, P. (2021). Trade-offs? the impact of wto accession on intimate partner violence in cambodia. *The Review of Economics and Statistics*, pages 1–40.

Eswaran, M. and Malhotra, N. (2011). Domestic violence and women's autonomy in developing countries: theory and evidence. *Canadian Journal of Economics/Revue canadienne d'économique*, 44(4):1222–1263.

Farmer, A. and Tiefenthaler, J. (2004). The employment effects of domestic violence. In *Accounting for worker well-being*. Emerald Group Publishing Limited.

Field, E. and Ambrus, A. (2008). Early marriage, age of menarche, and female schooling attainment in bangladesh. *Journal of political Economy*, 116(5):881–930.

Flood, M., Dragiewicz, M., and Pease, B. (2021). Resistance and backlash to gender equality. *Australian Journal of Social Issues*, 56(3):393–408.

Gale, D. and Shapley, L. S. (1962). College admissions and the stability of marriage. *The American Mathematical Monthly*, 69(1):9–15.

García-Ramos, A. (2021). Divorce laws and intimate partner violence: Evidence from mexico. *Journal of Development Economics*, 150:102623.

Gersbach, H. and Haller, H. (2017). *Groups and markets: general equilibrium with multi-member households*. Springer.

Goldin, C. (2021). *Career and Family: Women's Century-Long Journey toward Equity*. Princeton University Press.

Goldin, C. and Katz, L. F. (2008). Transitions: Career and family life cycles of the educational elite. *American Economic Review*, 98(2):363–69.

Goussé, M., Jacquemet, N., and Robin, J.-M. (2017). Marriage, labor supply, and home production. *Econometrica*, 85(6):1873–1919.

Greenwood, J., Guner, N., Kocharkov, G., and Santos, C. (2014). Marry your like: Assortative mating and income inequality. *The American Economic Review*, 104(5):348–353.

Copyright - Copyright American Economic Association May 2014; Características del documento - ; Equations; Tables; Última actualización - 2021-09-09; CODEN - AENRAA; SubjectsTermNotLitGenreText - United States–US.

Greenwood, J., Guner, N., Kocharkov, G., and Santos, C. (2016). Technology and the changing family: A unified model of marriage, divorce, educational attainment, and married female labor-force participation. *American Economic Journal: Macroeconomics*, 8(1):1–41.

Grossbard, S. (1993). On the economics of marriage-a theory of marriage, labor and divorce.

Guarnieri, E. and Rainer, H. (2018). Female empowerment and male backlash.

Halperin-Kaddari, R. and Freeman, M. A. (2016). Backlash goes global: Men's groups, patriarchal family policy, and the false promise of gender-neutral laws. *Canadian Journal of Women and the Law*, 28(1):182–210.

Hiller, T. (2018). On the stability of couples. *Games*, 9(3):48.

Hitsch, G. J., Hortaçsu, A., and Ariely, D. (2010). Matching and sorting in online dating. *American Economic Review*, 100(1):130–63.

Kaufmann, K. M., Messner, M., and Solis, A. (2013). Returns to elite higher education in the marriage market: Evidence from chile. *Available at SSRN 2313369*.

Kirkebøen, L., Leuven, E., and Mogstad, M. (2022). College as a marriage market. Technical report, National Bureau of Economic Research.

Lafortune, J. (2013). Making yourself attractive: Pre-marital investments and the returns to education in the marriage market. *American Economic Journal: Applied Economics*, 5(2):151–78.

Lafortune, J., Salisbury, L., and Siow, A. (2022). The changing identities of american wives and mothers. Technical report, Journal of Economic Literature.

Lauermann, S. and Nöldeke, G. (2015). Existence of steady-state equilibria in matching models with search frictions. *Economics Letters*, 131:1–4.

Lee, S. (2016). Effect of online dating on assortative mating: Evidence from south korea. *Journal of Applied Econometrics*, 31(6):1120–1139.

Lin, K.-H. and Lundquist, J. (2013). Mate selection in cyberspace: The intersection of race, gender, and education. *American Journal of Sociology*, 119(1):183–215.

Lloyd, S. (1997). The effects of domestic violence on women's employment. *Law & Policy*, 19(2):139–167.

Lundberg, S. and Pollak, R. A. (1993). Separate spheres bargaining and the marriage market. *Journal of political Economy*, 101(6):988–1010.

Macmillan, R. and Gartner, R. (1999). When she brings home the bacon: Labor-force participation and the risk of spousal violence against women. *Journal of Marriage and the Family*, pages 947–958.

Manser, M. and Brown, M. (1980). Marriage and household decision-making: A bargaining analysis. *International economic review*, pages 31–44.

McElroy, M. B. and Horney, M. J. (1981). Nash-bargained household decisions: Toward a generalization of the theory of demand. *International economic review*, pages 333–349.

McGarry, J. and Hinsliff-Smith, K. (2023). Intimate partner violence. In *Gender-Based Violence: A Comprehensive Guide*, pages 81–90. Springer.

Menzel, K. (2015). Large matching markets as two-sided demand systems. *Econometrica*, 83(3):897–941.

Mortensen, D. T. (1988). Matching: Finding a partner for life or otherwise. *American Journal of Sociology*, 94:S215–S240.

Nash, J. (1950). The bargaining problem. *Econometrica: Journal of the econometric society*, pages 155–162.

Nash, J. (1953). Two-person cooperative games. *Econometrica: Journal of the Econometric Society*, pages 128–140.

Nielsen, H. S. and Svarer, M. (2009). Educational homogamy how much is opportunities? *Journal of Human Resources*, 44(4):1066–1086.

Oreopoulos, P. and Salvanes, K. G. (2011). Priceless: The nonpecuniary benefits of schooling. *Journal of Economic perspectives*, 25(1):159–84.

Orr, A. (2004). *Meeting, mating, and cheating: Sex, love, and the new world of online dating*. Reuters Books.

Paumgarten, N. (2011). Looking for someone: Sex, love and loneliness on the internet. *The New Yorker*.

Pencavel, J. (1998). Assortative mating by schooling and the work behavior of wives and husbands. *The American Economic Review*, 88(2):326–329.

Petrongolo, B. and Pissarides, C. A. (2001). Looking into the black box: A survey of the matching function. *Journal of Economic literature*, 39(2):390–431.

Roe, M. J. (1998). Backlash. *Columbia Law Review*, pages 217–241.

Rosenfeld, M. (2018). Are tinder and dating apps changing dating and mating in the usa? In *Families and technology*, pages 103–117. Springer.

Rosenfeld, M. J. (2017). Marriage, choice, and couplehood in the age of the internet. *Sociological Science*, 4:490–510.

Rosenfeld, M. J. and Thomas, R. J. (2012). Searching for a mate: The rise of the internet as a social intermediary. *American Sociological Review*, 77(4):523–547.

Rosenfeld, M. J., Thomas, R. J., and Hausen, S. (2019). Disintermediating your friends: How online dating in the united states displaces other ways of meeting. *Proceedings of the National Academy of Sciences*, 116(36):17753–17758.

Roth, A. E. and Sotomayor, M. (1992). Two-sided matching. *Handbook of game theory with economic applications*, 1:485–541.

Shapley, L. S. and Shubik, M. (1971). The assignment game i: The core. *International Journal of game theory*, 1(1):111–130.

Siow, A. (2015a). Assortative mating in the marriage market.

Siow, A. (2015b). Testing becker's theory of positive assortative matching. *Journal of Labor Economics*, 33(2):409–441.

Srikanth, H. (2019). Love at first byte: An economic analysis of the internet dating apocalypse.

Stevenson, B. and Wolfers, J. (2006). Bargaining in the shadow of the law: Divorce laws and family distress. *The Quarterly Journal of Economics*, 121(1):267–288.

Tariq, Q. (2013). Impact of intimate partner violence on self esteem of women in pakistan. *American Journal of Humanities and Social Sciences*, 1(1):25–30.

Tauchen, H. V., Witte, A. D., and Long, S. K. (1991). Domestic violence: A nonrandom affair. *International Economic Review*, pages 491–511.

Team, E. E. (2021). The history of online dating 1959-2016 (us): Infographic.

Tertilt, M. (2005). Polygyny, fertility, and savings. *Journal of Political Economy*, 113(6):1341–1371.

Thomas, R. J. (2020). Online exogamy reconsidered: Estimating the internet's effects on racial, educational, religious, political and age assortative mating. *Social Forces*, 98(3):1257–1286.

UN (2012). Handbook for national action plans on violence against women.

UN (2023). Breaking down gender biases shifting social norms towards gender equality.

Voena, A. (2015). Yours, mine, and ours: Do divorce laws affect the intertemporal behavior of married couples? *American Economic Review*, 105(8):2295–2332.

Weiss, Y. and Willis, R. J. (1997). Match quality, new information, and marital dissolution. *Journal of labor Economics*, 15(1, Part 2):S293–S329.

WHO (2021). *Violence against women prevalence estimates, 2018: global, regional and national prevalence estimates for intimate partner violence against women and global and regional prevalence estimates for non-partner sexual violence against women*. WHO.

Woods, L. (1985). Mediation: A backlash to women's progress on family law issues. *Clearinghouse Rev.*, 19:431.