

THE HORIZONTAL BRANCH OF NGC 1851: CONSTRAINTS FROM ITS RR LYRAE VARIABLES

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ABSTRACT

We use the pulsational properties of the RR Lyrae variables in the globular cluster NGC 1851 to obtain detailed constraints of the various sub-stellar populations present along its horizontal branch. On the basis of detailed synthetic horizontal branch modeling, we find that minor helium variations ($Y \sim 0.248\text{--}0.280$) are able to reproduce the observed periods and amplitudes of the RR Lyrae variables, as well as the frequency of fundamental and first-overtone RR Lyrae stars. Comparison of number ratios among the blue and red horizontal branch components and the two observed subgiant branches also suggest that the RR Lyrae variables originated from the progeny of the bright subgiant branch. The RR Lyrae variables with slightly enhanced helium ($Y \sim 0.270\text{--}0.280$) have longer periods at a given amplitude, as is seen with Oosterhoff II (OoII) RR Lyrae variables, whereas the RR Lyrae variables with $Y \sim 0.248\text{--}0.270$ have shorter periods, exhibiting properties of Oosterhoff I (OoI) variables. This correlation does suggest that the pulsational properties of RR Lyrae stars can be very useful for tracing the various subpopulations and can provide suitable constraints on the multiple population phenomenon. It appears to be of great interest to explore whether this conclusion can be generalized to other globular clusters hosting multiple populations.

Key words: Galaxy: stellar content – globular clusters: general – globular clusters: individual (NGC 1851) – stars: horizontal-branch – stars: Population II – stars: variables: RR Lyrae

Online-only material: color figures

1. INTRODUCTION

Especially following the detection of two distinct subgiant branches (SGBs) in the color–magnitude diagram (CMD) of NGC 1851 (Milone et al. 2008), attempts to piece together the formation history of this cluster have become alluring. One promising explanation of the split between the bright SGB (SGBb) and faint SGB (SGBf) is that the two subpopulations differ in age by about 1 Gyr, and this scenario has been discussed in a number of studies (Milone et al. 2008; Carretta et al. 2011a, 2011b; Gratton et al. 2012a). Another valid explanation is that the SGB splitting is due to differing C+N+O contents and that the two SGBs are nearly coeval (Cassisi et al. 2008; Ventura et al. 2009). The horizontal branch (HB) of NGC 1851 is also bimodal, with both a prominent red HB clump and a blue tail. From the morphology of the HB and the main sequence (MS), strong helium variations within the cluster do not seem likely (Salaris et al. 2008; D’Antona et al. 2009), and recent spectroscopy of the blue HB stars suggests minor helium enhancements (Gratton et al. 2012a). Lastly, the red giant branch (RGB) is known to harbor different populations (Grundahl et al. 1999; Calamida et al. 2007; Lee et al. 2009; Han et al. 2009).

The stellar distribution along the HB of globular clusters (GCs) is commonly used to understand their formation and evolution (Gratton et al. 2010; Dotter et al. 2010), and previous papers dealing with the modeling of the HB of NGC 1851 (Salaris et al. 2008; Gratton et al. 2012a) used synthetic HB models to obtain scenarios of the formation of this cluster’s bimodal HB. In this paper, we will perform a more detailed investigation of the portion of the HB dealing with the instability

strip (IS) by discussing the case of the RR Lyrae period distribution in NGC 1851.

Period distributions of RR Lyrae stars have been shown to place strong constraints in the framework of canonical HB evolution. For example, the problem of the peaked distribution of the RR Lyrae periods in M3 (Castellani & Tornambe 1981; Rood & Crocker 1989) has challenged model predictions, as nicely described by Catelan (2004). Recent studies have come to explain both the M3 period distribution and HB morphology as the consequence of a range of initial He together with a uniform total RGB mass loss (with a very small spread; Caloi & D’Antona 2008), or a suitable bimodal mass-loss efficiency along the RGB but a single initial He abundance (Castellani et al. 2005).

The RR Lyrae properties of NGC 1851 have been used to describe this cluster as “truly an unusual Oosterhoff type I object” (Downes et al. 2004). This is largely because NGC 1851 is at the extreme end of the OoI-type clusters, with its RR Lyrae variables having not only a longer than average period for its Oosterhoff class, but also a ratio of first-overtone RR Lyrae (RR1) to fundamental mode RR Lyrae (RR0) stars more in line with OoII-type GCs (Walker 1998). The suggestion has also been made that the RR Lyrae variables can be divided into two subgroups based on their Ca *uvby* photometry (Lee et al. 2009), although they acknowledge that their sample is small, and the apparent bi-modality may merely reflect a calcium metallicity spread in the variables. Further suggestions that this cluster may be different than other Galactic GCs come from its phase-space distribution, which indicates that it may be associated with the Canis Major dwarf (Frinchaboy et al. 2004; Martin et al. 2004,

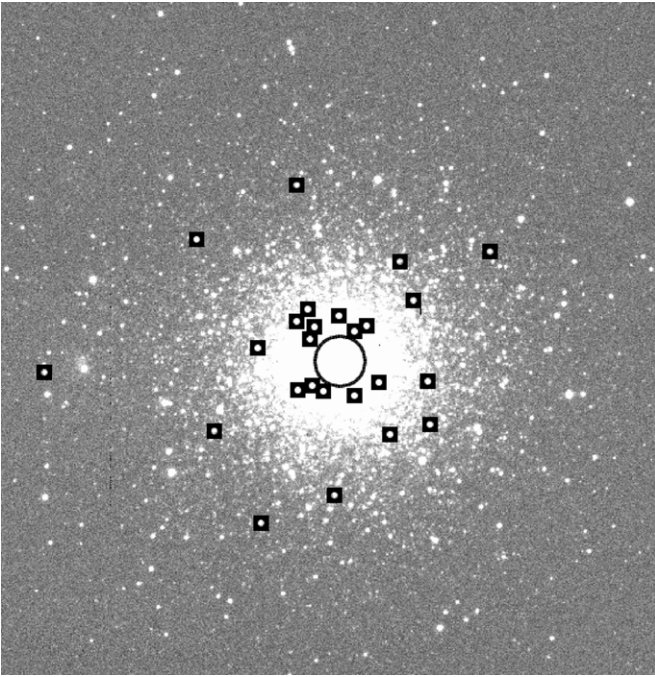


Figure 1. Image of NGC 1851 showing the 25 RR Lyrae variables used in our analysis. The central 40'' is designated by a circle. This region is known to harbor RR Lyrae variables, but due to the severe crowding, no calibrated photometry or amplitudes exist for the RR Lyrae stars in this region.

however see also López-Corredoira et al. 2007, who show that the signatures of the Canis Major dwarf can be fully accounted for by Galactic models without new substructures).

Reproducing theoretically both the morphology of its unusual CMD as well as the period distribution of its RR Lyrae variables is an important step in piecing together the formation scenario of NGC 1851.

2. RR LYRAE OBSERVATIONS

2.1. Sample and Completeness

The most complete study of the RR Lyrae variables in NGC 1851 was carried out by Walker (1998). He presents 33 variables in a 13.6 arcmin² area centered on this cluster in the *B*, *V*, and *I* passbands, 30 of which are RR Lyrae stars. Recently, Sumerel et al. (2004) discovered 19 additional variables and Downes et al. (2004) reported eleven variables, all within 40'' of the cluster center. There is overlap in these two samples, as described by the 2011 update of NGC 1851 in the Clement et al. (2001) catalog, and most of the new discoveries are RR Lyrae stars, although the classification for a handful of these stars is still uncertain. Neither Sumerel et al. (2004) nor Downes et al. (2004) provide calibrated mean magnitudes or amplitudes for their variables, making it difficult to use these stars in our analysis. As there is no indication that the Walker (1998) sample is incomplete at distances greater than 40'' from the cluster center, we limit our sample of RR Lyrae stars with which to compare our HB models to this outer region. Of the 29 RR Lyrae stars studied by Walker (1998) that are greater than 40'' from the center, 25 have both unblended magnitudes and well-determined periods. Thus, our sample of variables is 25/29 or 86% complete (outside the inner core).

The position on the sky of our sample of variables is shown in Figure 1, and the central 40'' is designated by a circle. Light curves for these stars are presented by Walker (1998) in the

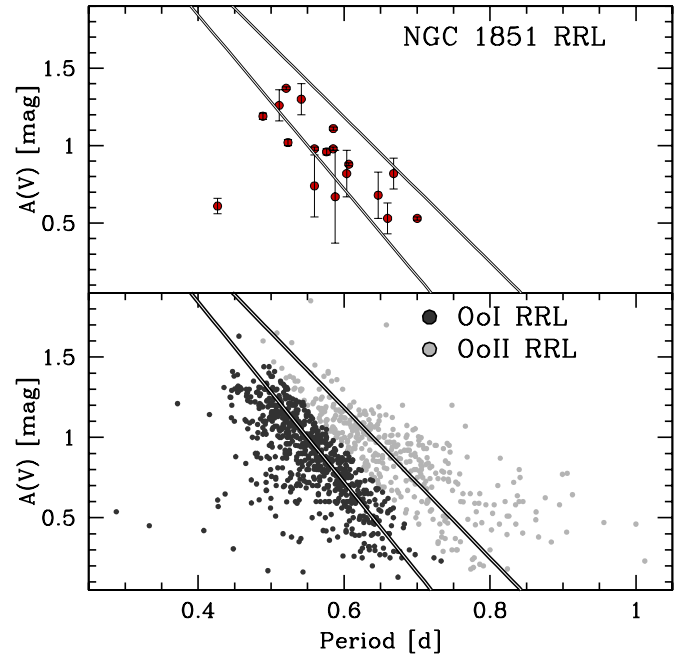


Figure 2. Top: period–amplitude diagram for our sample of RR0 Lyrae variables in NGC 1851. Bottom: period–amplitude relation for 1097 RR0 Lyrae variables in 40 Galactic GCs. Our division of OoI- and OoII-type RR Lyrae variables are shown by dark and light circles, respectively. The lines derived by Clement & Shelton (1999a) Oosterhoff I and Oosterhoff II RR0 are overplotted.

(A color version of this figure is available in the online journal.)

BVI passbands, so for these 25 RR Lyrae variables, robust mean magnitudes, periods and amplitudes are available. The edges of the IS, judged by the measured colors of variables near the strip boundaries, were also determined by Walker (1998), as well as the RR1–RR0 boundary.

2.2. Period–Amplitude Diagram

Clement & Shelton (1999a) show that the position of an RR Lyrae variable in the period–amplitude (PA) diagram is not a function of metal abundance, but rather of Oosterhoff type, and derive PA relations for OoI- and OoII-type RR Lyrae stars empirically. More recently, Cacciari et al. (2005) study the PA plane of three typical OoI-type GCs, three typical OoII-type GCs, and three intermediate types and find that there is a unique PA relation independent of metallicity for RR0 variables in OoI-, OoII-, and intermediate-type clusters. The periods and *V*-amplitude of our sample of RR0 Lyrae variables are shown in Figure 2, and the PA relation of typical OoI and OoII-type systems is overplotted. We note that although many of the RR Lyrae stars in NGC 1851 have periods and amplitudes that cause them to fall near the OoI PA relation, there are a number of stars following the OoII PA relation.

It is well known that the Blazhko effect, or other effects such as a rapidly changing period, can cause scatter in the PA plane (Clement & Shelton 1999a). The Blazhko effect causes the amplitude of light variation to vary over timescales longer than the basic pulsation period. The Walker (1998) RR Lyrae variables were observed over an ample time frame (126 total frames observed over 15 nights during a 1.5 year time span), so determining the amplitudes using the average light curves of the RR Lyrae stars is straightforward. Nevertheless, a visual determination of the change in amplitudes in each RR0 is obtained and shown as an error bar in Figure 2.

Table 1
MW GCs with a Population of RR Lyrae Variables

Name1	Name2	[Fe/H]	HB Type	Oo-Type	N_{RR0}	Reference
NGC 362	...	-1.30	-0.87	OoI	21	Székelly et al. (2007)
NGC 3201	...	-1.51	+0.08	OoI	58	Piersimoni et al. (2002); Layden & Sarajedini (2003)
NGC 4590	M68	-2.27	+0.17	OoII	12	Walker (1994)
IC 4499	...	-1.62	+0.11	OoI	62	Kunder et al. (2011)
Ruprecht 106	...	-1.78	-0.82	OoI	12	Kaluzny et al. (1995)
NGC 5053	...	-2.30	+0.52	OoII	6	Nemec (2004); Arellano Ferro et al. (2010)
NGC 6934	...	-1.56	+0.25	OoI	51	Kaluzny et al. (2001)
NGC 6981	M72	-1.48	+0.14	OoI	20	Dickens & Flinn (1972); Bramich et al. (2011)
NGC 7006	...	-1.46	-0.28	OoI	43	Wehlau et al. (1999)
NGC 7078	M15	-2.33	+0.67	OoII	50	Corwin et al. (2008)
NGC 5139	wCen	-1.64	+0.89	OoII	84	Kaluzny et al. (2004); Benko et al. (2006)
NGC 5272	M3	-1.50	+0.18	OoI	152	Corwin & Carney (2001)
NGC 5466	...	-1.70	+0.58	OoII	12	Corwin et al. (1999); Arellano Ferro et al. (2008)
NGC 6229	...	-1.43	+0.24	OoI	29	Borissova et al. (2001)
NGC 6426	...	-2.15	+0.58	OoII	8	Papadakis et al. (2000)
NGC 6441	...	-0.44	-0.73	OoIII	25	Pritzl et al. (2003)
NGC 6388	...	-0.45	-0.69	OoIII	4	Pritzl et al. (2002)
NGC 6266	M62	-1.18	+0.55	OoI	74	Contreras et al. (2010)
NGC 6333	M9	-1.79	+0.87	OoII	6	Clement & Shelton (1999b)
NGC 6341	M92	-2.35	+0.91	OoII	11	Kopacki (2001)
NGC 6362	...	-1.07	-0.58	OoI	19	Olech et al. (2001)
NGC 6626	M28	-1.46	+0.90	OoI	8	Wehlau & Butterworth (1990)
NGC 5904	M5	-1.33	+0.31	OoI	52	Szeidl et al. (2011)
NGC 5986	...	-1.63	+0.97	OoII	6	Alves et al. (2001)
NGC 6121	M4	-1.18	-0.06	OoI	15	Cacciari (1979); Sturch (1977)
NGC 6171	M107	-1.03	-0.73	OoI	15	Clement & Shelton (1997); Dickens (1970)
NGC 6712	...	-1.02	-0.62	OoI	7	Sandage (1966)
NGC 6723	...	-1.10	-0.08	OoI	15	Menzies (1974)
NGC 6864	M75	-1.29	-0.07	OoI	8	Corwin et al. (2003)
NGC 5024	M53	-2.06	+0.81	OoII	22	Arellano Ferro et al. (2011)
NGC 7089	M2	-1.66	+0.92	OoII	18	Lee & Carney (1999); Lázaro et al. (2006)
NGC 2419	...	-2.20	+0.86	OoII	28	Di Criscienzo (2011)
NGC 1851	...	-1.18	-0.32	OoI	19	Walker (1998)
NGC 5286	...	-1.70	+0.80	OoII	27	Zorotovic et al. (2010)
NGC 2808	...	-1.18	-0.49	OoI	6	Kunder et al. (2012)
NGC 6101	...	-1.98	...	OoII	3	Cohen et al. (2011)
NGC 4147	...	-1.78	+0.55	OoI	5	Stetson (2005)
NGC 6809	M55	-1.93	...	OoI	4	Olech et al. (1999)
NGC 6715	M54	-1.44	+0.54	OoI	52	Sollima et al. (2010)
NGC 1904	M79	-1.58	...	OoII	6	Kains et al. (2012)

The change from the average light curve amplitude to the maximum Blazhko amplitude ranges from ~ 0.1 – 0.3 mag, and photometric uncertainties lead to amplitude uncertainties of ~ 0.01 – 0.05 mag. We conclude that even when taking amplitude variations into account, the variables are both OoI and OoII type.

In comparison, Figure 2 shows the periods and V -amplitudes of 1097 RR0 Lyrae variables in 39 Galactic GCs. The data for this diagram come from Table 1, where each cluster is listed along with the number of RR0 Lyrae stars within the cluster that have well-determined periods and V -amplitudes. For completeness, the [Fe/H] from Carretta et al. (2009), HB type, and Oosterhoff type for each GC are also given. This sample of RR0 Lyrae variables is divided by their position in the PA plane following the lines that Clement & Shelton (1999a) derived for Oosterhoff I and Oosterhoff II RR0 stars (see Figure 2). Here, an OoI RR Lyrae variable is defined by

$$A(V) > -5.1453 P + 4.02 \quad (1)$$

and an OoII-type RR Lyrae variable by

$$A(V) < -5.1453 P + 4.02, \quad (2)$$

where $A(V)$ is the V -amplitude and P is the period. We define an Oosterhoff ratio for each GC, which is simply the number of OoI-type RR0 Lyrae stars compared to the total number of RR0 Lyrae stars in the GC, OoI_{RR0}/Tot_{RR0} .

We note that Cacciari et al. (2005) showed that for the M3 RR0 Lyrae stars, a quadratic PA relation is a closer fit than a linear one. However, their relation does not approximate high amplitude RR0 Lyrae variables well ($A_V > 1.5$ mag), largely because such variables are absent in M3. As our comprehensive sample includes a handful of such stars (one OoI- and three OoII-type RR Lyrae), the Cacciari et al. (2005) relation is not used here.

Figure 3 shows the histogram of the Oosterhoff ratio of the GCs in our sample. Most of these clusters lack a complete sample of RR Lyrae variables and at least some of the RR Lyrae amplitudes are likely affected by the Blazhko effect or some other light curve “noise.” But even with these caveats, it is clear from the figure that the Oosterhoff ratio splits the GCs into two groups; the OoI-type clusters have RR Lyrae variables with shorter periods for a given amplitude and hence have larger Oosterhoff ratios (with respect to the OoII-type clusters). Furthermore, there is an absence of clusters falling in

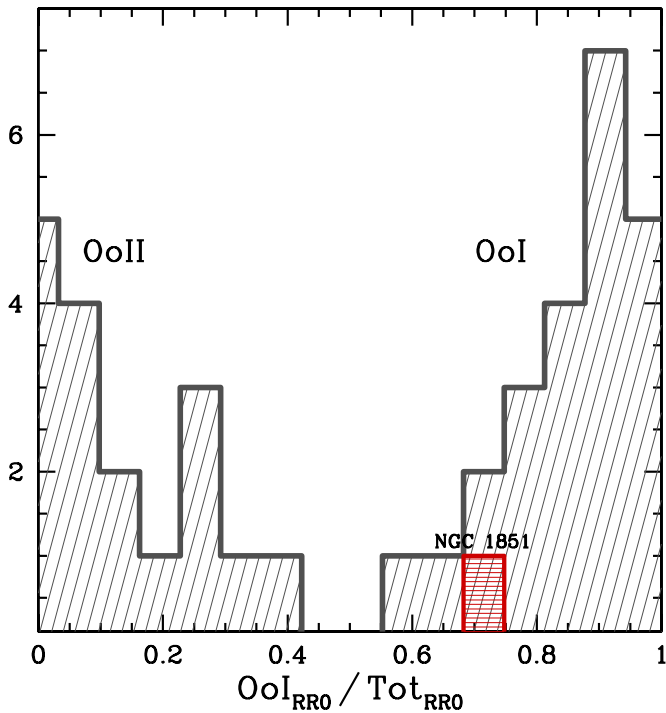


Figure 3. Histogram of the ratio of OoI stars in the Milky Way GCs. The Oosterhoff ratio of the NGC 1851 RR0 Lyrae stars is highlighted.

(A color version of this figure is available in the online journal.)

the “gap.” We therefore believe that our Oosterhoff ratio is useful to distinguish between OoI- and OoII-type GCs. Moreover, this ratio may be used to evaluate the degree for which a GC is a typical OoI- or OoII-type cluster.

The majority of OoI-type GCs have a RR Lyrae star population in which $\text{OoI}_{\text{RR0}}/\text{Tot}_{\text{RR0}} > 0.8$ (i.e., 80% of the variables can be defined by Equation (1)). In contrast, the Oosterhoff ratio of NGC 1851 is 0.74. Equations (1) and (2) are defined somewhat arbitrarily, and therefore we experimented with a variety of cuts (where we modified the zero points and slopes) to distinguish between OoI- and OoII-type stars. The Oosterhoff ratio of NGC 1851 varied between ~ 0.63 – 0.75 , a percentage that is smaller than 83% of the rest of the OoI-type clusters. This indicates that NGC 1851 contains variables with more OoII-like periods and V-amplitudes than the majority of the other Milky Way OoI-type GCs.

The other three OoI GCs that have comparatively small Oosterhoff ratios are NGC 362 (0.71), NGC 4590 (0.58), and NGC 6362 (0.67). It has been noted that the RR Lyrae variables of NGC 362 and NGC 1851 are remarkably similar in the PA diagram, suggesting similar masses and luminosities. This is also seen here, as they have very similar Oosterhoff ratios. Recently, it was found that like NGC 1851, NGC 362 also has a split subgiant branch, although the SGBf component includes only a few percent of the total number of SGB stars of the cluster (just $\sim 2\%$ – 3% ; Piotto et al. 2012).

New photometry of the NGC 1851 RR Lyrae variables, in which periods and amplitudes of the inner RR Lyrae stars have been obtained, suggests that NGC 1851 has a slightly higher Oosterhoff ratio of ~ 0.80 (P. Amigo et al. 2012, in preparation). We therefore suspect that the Oosterhoff ratio of NGC 1851, while still lower than the majority of the OoI-type clusters, is not as abnormal as the GCs NGC 362, NGC 4590, and NGC 6362, discussed above.

2.3. First-overtone versus the Fundamental Mode

The number ratio of first-overtone RR Lyrae stars to total RR Lyrae stars, N_1/N_{tot} , is usually given to quantify the frequency of the different RR Lyrae pulsators. In general, OoII-type GCs have about two to four times the frequency of RR1 stars as OoI-type GCs. This is thought to be related to the transition temperature between the IS for first-overtone pulsation and fundamental one. For example, moving the transition from RR0 to RR1 variables toward lower temperatures (i.e., transforming fundamental in first-overtone pulsators) has the twofold effect of increasing the periods of the RR0 Lyrae population, as well as increasing the relative number of first overtones. This is discussed in detail by Castellani et al. (2003).

NGC 1851 has a large N_1/N_{tot} with respect to most OoI-type GCs. This ratio strongly depends on the completeness of the sample, and because NGC 1851 is too crowded for ground-based observations to resolve, the Walker (1998) sample is incomplete at distances close to the core. Assuming the Walker (1998) RR Lyrae sample is complete at distances larger than $40''$ (see Figure 1), $N_1/N_{\text{tot}} = 0.27$, one of the largest for OoI-type GCs. For example, from the Castellani et al. (2003) compilation of 32 clusters with 12 or more pulsators and well-recognized period and pulsation modes, only 3 of the 17 OoI GCs have an N_1/N_{tot} greater than 0.27. Recent studies have discovered RR Lyrae stars closer than $40''$ from the center. Downes et al. (2004) find $N_1/N_{\text{tot}} = 0.54$, and using Clement’s catalog, the value of this ratio is ~ 0.40 . These ratios are somewhat uncertain, however, because of the lack of mode identification for some of these newly discovered pulsators. The periods listed in Sumerel et al. (2004) are “tentative” and the periods derived by Downes et al. (2004) do not cover a full period for any of their objects. Furthermore, because these stars lack amplitudes, their position in a PA diagram also cannot be used as a diagnostic to identify fundamentals and first overtones. Although there is no consensus on the value of N_1/N_{tot} yet, it is clear that this ratio is at least 0.27, and likely even larger. An N_1/N_{tot} larger than 0.27 is also consistent with the results from P. Amigo et al. (2012, in preparation).

3. SYNTHETIC HB MODELING

Previous synthetic HB models for NGC 1851 have been presented by Catelan et al. (1998), and more recently by Salaris et al. (2008), Han et al. (2009), and Gratton et al. (2012a). Salaris et al. (2008) compared their simulations to the *Hubble Space Telescope* observations by Milone et al. (2008) and found two satisfactory scenarios to reproduce the CMD of HB stars. In both of these models, the blue HB, red HB, and variable stars are predicted to come from the SGBf, and the stars from the SGBb are confined to the red portion of the observed sequence. This inference was based on the number ratio $\text{SGBf}/\text{SGBb} = 55:45$ determined in Milone et al. (2008). Salaris et al. (2008) also found that the initial He abundance of HB stars had to be relatively uniform to reproduce the CMD derived by Milone et al. (2008). In these data, the measured magnitudes and colors of the RR Lyrae population are at random phases; therefore, the portion of the observed HB crossing the IS could not be used for detailed constraints on the models. However, they did verify that recent theoretical pulsation models of RR Lyrae stars (Di Criscienzo et al. 2004a) predict an IS for NGC 1851 too red by ~ 0.03 – 0.04 mag in ($F606W$ – $F814W$) compared to the Milone et al. (2008) data.

Han et al. (2009), on the other hand, found from their *UVI* photometry that the RR Lyrae variables could come from both the SGBb and the SGBf. They construct two population models for NGC 1851. In the first, the second generation (SG) population is more enhanced in metallicity but not in helium ($\Delta Z = 0.0004$ and $\Delta \text{age} = 0.1$ Gyr), and in the second, both metal and helium abundances are enhanced ($\Delta Z = 0.0004$, $\Delta Y = 0.05$, and $\Delta \text{age} = 0.1$ Gyr). They find that their ΔZ -only model is in conflict with the observed CMDs of NGC 1851, but that their $\Delta Z + \Delta Y$ model is in good agreement with the observations from the MS to the HB. In this model, the RR Lyrae variables exhibit different He abundances. Because their RR Lyrae variable sample was found at a random phase of pulsation, the RR Lyrae colors could not be used as a stringent constraint in their models and the RR Lyrae star periods were not discussed.

More recently, Gratton et al. (2012a) considered the revised SGBb/SGBf ratio determined by Milone et al. (2009) and introduced new spectroscopic constraints; they find a small difference in the iron content between the SGBb and SGBf, and argue that an age spread of ~ 1.5 Gyr is the most viable explanation for the splitting SGB. They also find that the RHB stars separate into two groups depending on their O and Na abundances, and that the BHB stars are slightly helium enriched as compared to the RHB stars. Hence, to satisfy these constraints, the HB is modeled with four different components, with the IS originating from the SGBb. That each SGB hosts multiple generations of stars is shown from spectroscopy of stars on the double SGB (Lardo et al. 2012), making it likely that multiple components may be needed to model the HB.

Our own synthetic HB calculations described below are aimed at answering the following question: what is the most straightforward way to reproduce the RR Lyrae IS of NGC 1851—and in particular the pulsational properties of its RR Lyrae variables? We seek to provide a simple and attractive explanation for the cluster HB and IS morphology, keeping the number of free parameters to a minimum, yet still reproducing the RR Lyrae star properties that make this cluster stand out as having an unusual Oosterhoff type.

3.1. Synthetic HB models

The HB evolutionary tracks used here are from the BaSTI stellar library (Pietrinferni et al. 2004, 2006, 2009) and have already been described by Salaris et al. (2008) and Cassisi et al. (2008). They are also the same ones that Gratton et al. (2012a) employed. Briefly, evolutionary tracks are for a normal α -enhanced ($[\alpha/\text{Fe}] = 0.4$) metal mixture, with $[\text{Fe}/\text{H}] = -1.31$ dex and $Y = 0.248$. The HB tracks were interpolated among the models with $Y = 0.248$ and additional BaSTI models with $Y = 0.300$, to determine HB tracks for intermediate values of Y , at the same iron content. Similarly, to determine HB tracks with a milder CNO enhancement, an interpolation between the reference set and the models with the CNO sum enhanced by 0.3 dex (Pietrinferni et al. 2009) is used for a portion of the synthesis, as in Gratton et al. (2012a). Hence, the $[\text{Fe}/\text{H}]$, α -enhancement and CNO enhancement is consistent with spectroscopic results from Carretta et al. (2011b) and Gratton et al. (2012a). We wish to remind the reader that as long as the CNO sum is unchanged, the effect of the observed CNONa anticorrelations (overimposed to a standard α -enhanced metal mixture) on the evolutionary tracks and isochrones is negligible, and standard α -enhanced models are adequate to represent the whole cluster population. Only an enhancement of the CNO

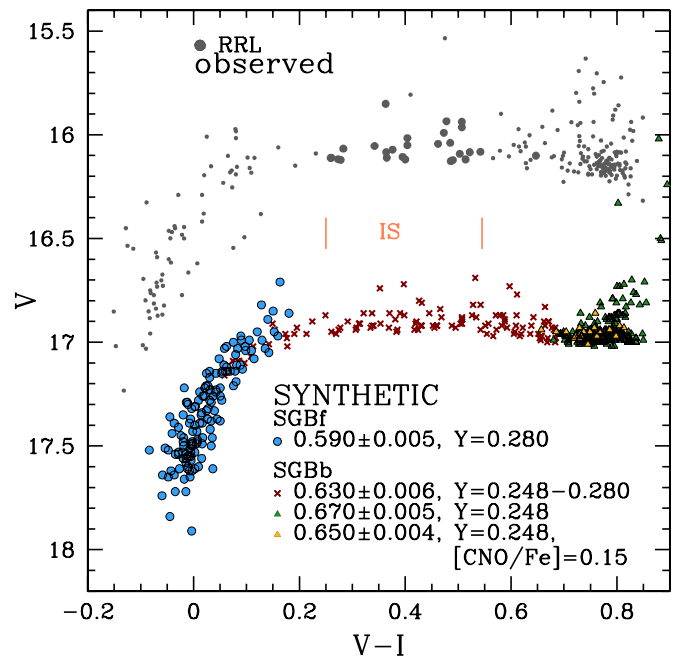


Figure 4. Qualitative comparison of the observed HB and the synthetic HB, assuming four separate HB components. The RR Lyrae instability strip is marked, and corresponds to that found by the Walker (1998) study of RR Lyrae variables in NGC 1851. The Walker (1998) RR Lyrae variables are designated by large circles.

(A color version of this figure is available in the online journal.)

sum requires the calculation of appropriate models. On the other hand, CNONa anticorrelations, even at constant CNO, affect the bolometric corrections of filters like *B* and *U* (Sbordone et al. 2011) at low effective temperatures, but not longer wavelength filters.

The four HB components described by Gratton et al. (2012a) are used as starting points for our calculations. Objects from our synthetic HB that fall within the observed IS from Walker (1998) are considered RR Lyrae variables (this region is labeled in Figure 4) and their period is calculated from the pulsation equation given by Di Criscienzo et al. (2004a). The intensity mean magnitudes and colors given by Walker (1998) are used as a comparison to the synthetic HB, because the static magnitudes and colors from stellar evolution models are better represented by intensity-averaged quantities (Di Criscienzo et al. 2004a). Although the Walker (1998) observations include the *BVI* passbands, we employ only the *V* and *I* magnitudes, because—as discussed before—they are not affected by the observed CNONa abundance anticorrelations.

In addition to the observed *V* and (*V* − *I*) distribution of the HB stars in the CMD, and the observed (*B*:*V*:*R*) (blue:variable:red HB) ratio of (*B*:*V*:*R*) = (33 ± 8:10 ± 5:56 ± 11) (in line with the results by Catelan et al. 1998; Saviane et al. 1998), we impose as a further constraint on our simulation the observed distribution of the RR Lyrae periods.

As in Gratton et al. (2012a), we adopted $E(B - V) = 0.02$ mag (Walker 1998) and fixed the apparent distance modulus to $(m - M)_V = 15.56$ mag by matching the observed mean magnitude of the RHB with our synthetic counterpart. The (*B*:*V*:*R*) ratio of our “best-fit” simulation is (27:9:64), consistent with the observed value. For reasons that will become clear in the discussion that follows, we consider a preliminary reference age of 11 Gyr for the progenitors of the RHB stars. This implies, for the assumed metallicity and a “normal” $Y = 0.248$, an initial

mass of $0.86 M_{\odot}$ for the stars at the tip of the RGB. The HB components are described below.

1. As in Gratton et al. (2012a), the majority of the RHB population is modeled with normal CNO abundance, a normal $Y = 0.248$, and a Gaussian mass distribution with $\langle M \rangle = 0.67 \pm 0.005 M_{\odot}$. This corresponds to a total mean mass loss $\Delta M = 0.19 M_{\odot}$ along the RGB, for the assumed 11 Gyr age.
2. A smaller RHB subpopulation, which is Ba-rich, makes up $\sim 10\%$ of the HB population. It is modeled, as in Gratton et al. (2012a), employing a Gaussian mass distribution with $\langle M \rangle = 0.65 \pm 0.004 M_{\odot}$ (corresponding to $\Delta M \sim 0.21 M_{\odot}$), a 0.15 dex enhanced CNO abundance, and normal $Y = 0.248$. If we assume that the mean total mass loss has to be constant among all cluster RGB stars—and equal to $\Delta M = 0.19 M_{\odot}$ as determined for the rest of the RHB component—the mean value of the mass for this HB subpopulation implies an age ~ 1 Gyr older for the progenitors of this HB component.
3. The horizontal part of the BHB, including the RR Lyrae IS, makes up $\sim 10\%$ of the cluster stellar content. This component is the focus here, and is the only one that is modified from Gratton et al. (2012a). In particular, instead of adopting a constant He abundance $Y = 0.265$, the helium content for stars between the blue end of the red clump and the beginning of the BHB blue tail has a continuous distribution between $Y = 0.248$ and 0.280 . A simple, flat probability distribution for Y and constant $\Delta M = 0.19 M_{\odot}$ (for an age of 11 Gyr) with a 1σ Gaussian spread of $0.005 M_{\odot}$ —as for the RHB stars—for all RGB progenitors provide a good match to the observed RR Lyrae periods, as discussed below.
The mean He abundance in the IS is $\langle Y \rangle = 0.271$, close to the constant abundance $Y = 0.265$ employed by Gratton et al. (2012a) for this component, and the mean mass is $\langle M \rangle = 0.634 M_{\odot}$. It is worth noting that the observed HB distribution of these stars is well matched by both this simulation and the simulation by Gratton et al. (2012a). This spread in He content is necessary to reproduce the observed period distribution.
4. The blue tail of the BHB population makes up $\sim 20\%$ of the HB stars. As in Gratton et al. (2012a), this component is modeled with normal CNO abundance and $Y = 0.28$. The Gaussian mass distribution has $\langle M \rangle = 0.59 \pm 0.005 M_{\odot}$, which for an age of 11 Gyr would correspond to a mean $\Delta M = 0.22 M_{\odot}$. If the mean total mass loss is instead fixed at $\Delta M = 0.19 M_{\odot}$, then this value of $\langle M \rangle$ implies that the progenitors of the BHB blue tail stars are ~ 1.5 Gyr older than the RHB component with normal CNO and Y . Note that the constraint on the progenitor Y is weaker for BHB stars (see discussion in Gratton et al. 2012a) and a small spread of the order of 0.01 may be present.

These results from the HB synthetic modeling can be interpreted in terms of the progeny of the SGBb and SGBf subpopulations (the ratios $\sim 2/3$ and $\sim 1/3$ of the total SGB population, respectively, are adopted as determined by Milone et al. 2009) as follows.

1. The sum of the fraction of stars in the blue tail of the BHB and in the mildly CNO-enhanced Ba-rich RHB component is $\sim 35\%$ of the total HB population. If we consider as a reference the CMD of 11 Gyr old SGB stars with “normal” Y and CNO abundance, then the progenitors of these two

HB components will be distributed along a fainter SGB than the reference one. In the case of the progenitors of the BHB component, this is an age effect, for a change of Y does not have a major effect on the SGB luminosity. For the Ba-rich RHB progenitors, the reason is the slightly higher age and the mildly enhanced CNO abundance, which both act in the direction of producing a fainter SGB. As a result, both the Ba-rich RHB stars and the blue tail HB progenitors display an approximately coincident SGB, which we tentatively identify as the SGBf in the cluster CMD.

2. The sum of the fraction of stars in the horizontal BHB (including the IS) and the RHB stars with normal composition amounts to $\sim 65\%$ of the total HB population. We identify their progenitors as the stars harbored by the SGBb in the cluster CMD.

To conclude this section, it is worth noting that the exact value of the assumed reference age (11 Gyr) is not critical. Had a different age been assumed, i.e., 10 or 12 Gyr, the previous conclusions would still be valid. The only difference is that all values of ΔM would need to be shifted downward (or upward) by $\sim 0.02 M_{\odot}$ —to keep the mass distribution along the HB unchanged—but the interpretation of the results would be identical. Finally, the different chemical composition—and small age differences—assigned to the SGBb and SGBf populations does not substantially affect SGB and RGB timescales; as a consequence, the number ratio SGBb/SGBf will be approximately equal to the number ratio of their HB progeny.

3.2. Comparison with Observations

This synthetic HB model is shown compared to the observed one in Figure 4, where an observational scatter of $\sigma_{V,I} = 0.01$ mag is assumed. The four components are highlighted for clarity and the RR Lyrae region is labeled. Our full synthetic HB model reproduces two peculiarities in the CMD of NGC 1851 pointed out by Brocato et al. (1999), namely the clump of stars near the red edge of the HB and the slightly tilted HB ($\Delta_V^{\text{tilt}} \sim 0.1$ mag). Features such as these are also present in NGC 6362 (which has an RR Lyrae population with properties very similar to NGC 1851), and in the extreme cases of NGC 6388 and NGC 6441 ($\Delta_V^{\text{tilt}} \sim 0.5$ mag).

The focus here concerns the component that includes the IS, which comes from the SGBb. As Walker (1998) mentioned, the ZAHB is very clearly defined and is not horizontal, being slightly brighter at bluer colors. This is reproduced in our synthetic HB model by stars that range in helium abundance from $Y = 0.25$ – 0.28 , and range in mass from 0.61 to $0.65 M_{\odot}$.

The theoretical periods and pulsation amplitudes from the RR Lyrae variables in our synthetic HB model are compared to the observed periods and amplitudes in Figure 5. Here, the observed periods come from 28 RR0 and 18 RR1 variables as determined by Walker (1998) and Sumerel et al. (2004) to encompass all the data available in the literature (see the 2011 update of NGC 1851 in Clement et al. 2001). We also compare the theoretical periods with 27 RR0 and 18 RR1 variables as determined by P. Amigo et al. (2012, in preparation). These authors derive periods for the recently identified inner RR Lyrae variables based on light curves with ~ 200 – 300 points in each of the B -, V -, and I -passbands. The observed amplitudes come exclusively from the Walker (1998) RR0 Lyrae sample, as Sumerel et al. (2004) present instrumental magnitudes only. The amplitudes from the P. Amigo et al. (2012, in preparation) sample are not used, because unlike when determining periods, amplitudes can be

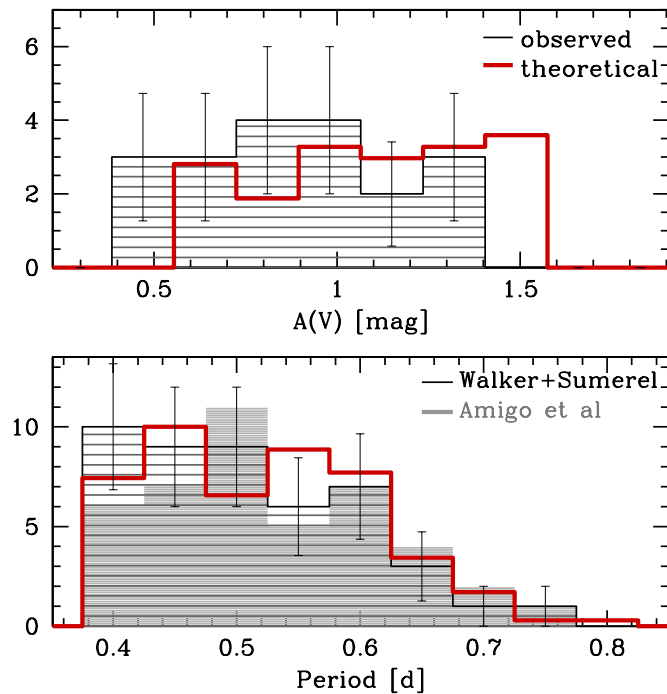


Figure 5. Comparison between the theoretical periods and amplitudes from our synthetic HB vs. the observed periods and amplitudes. A K-S test indicates that statistical significance is detected between the observed and theoretical periods and amplitudes.

(A color version of this figure is available in the online journal.)

affected by crowding and blending issues (e.g., Majaess et al. 2012) and we do not have a feel for how/if blending affects their (preliminary) amplitude determinations.

Marconi et al. (2011) provided a detailed comparison between the impact of the He abundance on the pulsation properties of RR Lyrae stars and concluded that He content marginally affects the pulsation behavior of RR Lyrae stars. They noted that the increase in the average pulsational period associated with the He increase is only due to the brighter luminosities that characterize He-enhanced evolutionary models. As a consequence, from a theoretical point of view, the impact of an He-enhancement on the pulsation properties of RR Lyrae stars can be directly taken into account when adopting evolutionary tracks for the appropriate He abundance and pulsational model predictions obtained for a canonical He abundance.

The periods of the RR1 variables are fundamentalized via $\log P_0 \sim \log P + 0.127$, where P_0 is the fundamental mode period, and the theoretical periods are calculated for all HB stars falling within the observed IS using the Di Criscienzo et al. (2004a) RR Lyrae pulsation models. The mean observed RR0 Lyrae period in NGC 1851 is $\langle P_0 \rangle = 0.571$ (Catelan 2009a), and from our model $\langle P_0 \rangle = 0.569 \pm 0.006$ days is calculated, where the uncertainty is the error in the mean. The number of RR Lyrae stars in the simulations is ~ 5 times larger than the number of observed RR Lyrae stars. In this way, in the synthetic HB model, the effect of statistical fluctuations in the number of objects at a given magnitude and color is minimized.

The observed minimum fundamentalized period of the first overtones and the minimum fundamentalized period from the synthetic HB is very similar, $P_F \sim 0.34$ days. It has been shown that the minimum fundamentalized period is a robust observable to constrain the evolutionary properties of RR Lyrae stars (Bono et al. 1995), so it is especially encouraging that the observed and theoretical values agree.

We have performed a Kolmogorov–Smirnov (K-S) test to establish whether one can reject the null hypothesis that the two samples of periods (observed and synthetic) come from the same distribution. From a comparison between the combined Walker (1998) and Sumerel et al. (2004) sample and the synthetic RR Lyrae stars, the K-S test returns a probability of $P = 0.86$, well above the default threshold $P_{th} = 0.05$ below which one rejects the null hypothesis. When the Amigo et al. period distribution is compared with our theoretical one, if we assume that both samples are drawn from the same parent population, the samples will differ by this amount or more 94% of the time. On this basis, we find that the synthetic periods from our simulated HB and the observed periods agree well with each other.

Theoretical pulsation amplitudes are also determined from the Di Criscienzo et al. (2004a) relations, although three points should be taken into account.

1. Theoretical amplitudes depend on the mixing length, l/H_p , (where l/H_p is the pressure scale height), which is uncertain and may change from the blue to the red edge of the IS (Marconi et al. 2003). Pulsation amplitudes are affected by l/H_p in the sense that a larger value of l/H_p translates to smaller pulsation amplitudes as a consequence of the larger efficiency of convective motions, and hence larger quenching to the pulsation mechanism provided by convection.
2. The synthetic pulsational amplitude–period predictions provided by Di Criscienzo et al. (2004a) show a large scatter, of the order of ~ 0.2 mag (standard deviation) in the V-amplitude at a fixed period (see Figure 6 of their paper, where the dashed lines represent the standard deviation). Since our predictions of the V-amplitude in our synthetic HBs are based on an $A(V)$ –period relation obtained by an interpolation of the data provided by Di Criscienzo et al. (2004a), we do not expect great accuracy in our $A(V)$ estimates. In addition, Di Criscienzo et al. (2004a) have shown that for $P < 0.68$ days, pulsation model predictions for the V-amplitude are still more affected by a change in the adopted mixing length value (see the lower panel of the quoted figure): for increasing mixing length values, they predict a significant decrease in the dependence of the V-amplitude on the pulsation period.
3. When dealing with pulsating structures, the static magnitudes differ from the measured magnitudes, which are mean quantities averaged over the pulsation cycle. In finding theoretical amplitudes, a correction between static and intensity-averaged magnitudes is required. The discrepancy between static and mean values is a function of the pulsation amplitude, and the corrections adopted here come from Bono et al. (1995).

When using an $l/H_p = 2.0$, the Walker (1998) V-amplitude distribution is similar to that observed, although the theoretical amplitudes appear to be ~ 0.1 mag larger. This is not completely surprising considering the scatter in the Di Criscienzo et al. (2004a) V-amplitude relation as well as the magnitude corrections discussed above. Extrapolating linearly between $l/H_p = 1.5$ and 2.0, an increase in the mixing length of 0.1 would cause a decrease of the theoretical amplitudes by ~ 0.08 mag. Such a decrease would provide a satisfactory agreement between theory and observations. A 0.1 change in l/H_p is well within the uncertainties in the mixing length calibration, and not nearly large enough to affect the predicted pulsation periods (Bono & Stellingwerf 1994; Di Criscienzo et al. 2004b; Marconi & Degl’Innocenti 2007). An $l/H_p = 2.0$ was also used by Bono

et al. (2007) to derive a visual distance modulus from non-linear convective models of RR Lyrae stars to NGC 1851 of $(m - M)_V = 15.52 \pm 0.11$ mag, which is similar to the distance modulus adopted here. In contrast, when using an $l/H_p = 1.5$, Bono et al. (2007) find a distance modulus of $(m - M)_V = 15.40 \pm 0.12$. Therefore, we conclude that using a larger value of l/H_p , ($l/H_p \sim 2.0$) provides a consistent comparison between our synthetic HB and pulsational predictions.

In general, the RR Lyrae variables with $Y < 0.27$ fall in the OoI area of the PA diagram, whereas the RR Lyrae variables with $Y > 0.27$ fall close to the OoII line. Assuming that the PA diagram can be effectively used to classify RR Lyrae stars into an Oosterhoff type, this means that He and Oosterhoff type are correlated in this cluster. This is not completely unexpected, as an increase in He makes RR Lyrae variables brighter and, as a consequence, higher helium abundance makes their pulsational period longer (Bono et al. 1997; Marconi 2009).

In general, for a given total mass, the HB stars with $Y < 0.27$ are redder than those with $Y > 0.27$. The red part of the IS, where the fundamental mode RR Lyrae stars reside, is consequently more populated, and a smaller N_1/N_{tot} is obtained. Our synthetic HB yields $N_1/N_{\text{tot}} \sim 0.1$ for the stars with $Y < 0.27$. This is a ratio that is seen for the majority of the OoI-type GCs. In contrast, the blue part of the HB contains more stars with higher helium abundances, and as the first-overtone RR Lyrae variables reside in the blue part of the IS, a larger N_1/N_{tot} is obtained. Our synthetic HB yields $N_1/N_{\text{tot}} \sim 0.45$ for stars with $Y > 0.27$, a ratio more in line with OoII-type GCs. The observed ratio of first overtone to total RR Lyrae variables for NGC 1851 in our sample is $N_1/N_{\text{tot}} \sim 0.30$ and is easily explained (and reproduced with our synthetic HB) by the spread in Y along the RR Lyrae IS.

We find that simulations using a constant helium for the portion of the HB containing the IS (as in Gratton et al. 2012a) do not fit the constraints given by the NGC 1851 RR Lyrae variables as well. For example, adopting $Y = 0.265$ results in an $N_1/N_{\text{tot}} = 0.11$ (versus the observed $N_1/N_{\text{tot}} = 0.28$).

We note that Milone et al. (2008) provide an upper limit to a possible dispersion in helium abundance of $\Delta Y = 0.026$ between the two SGBs in NGC 1851, a value close to the spread assumed in the synthetic HB presented here. Other estimates of the He spread in NGC 1851 are slightly larger, i.e., $\Delta Y = 0.04$ (Ventura et al. 2009), $\Delta Y = 0.05$ (Han et al. 2009), or $\Delta Y = 0.048$ (Gratton et al. 2010).

4. DISCUSSION AND CONCLUSIONS

The population distribution of the stars along the HB has been modeled assuming the presence of (at least) four populations with differing helium contents. In our simulations, the only parameter we vary is the initial He abundance of the HB progenitors, keeping the same total RGB mass loss for all components. Both the RR Lyrae period distribution and the number ratio of first-overtone RR Lyrae to total RR Lyrae stars, N_1/N_{tot} , provide constraints pertaining to the component of the HB containing the IS. It is straightforward to reproduce the observed distribution of RR Lyrae stars inside the IS with minor He variations ($Y \sim 0.248\text{--}0.280$) and from an HB subpopulation corresponding to the progeny of a fraction of the SGBb stellar population.

Therefore, the IS of NGC 1851 belongs to an SG of stars. That an SG exists within the SGBb component is in agreement with recent spectroscopy of the SGB stars, showing that

each SGB hosts multiple subgenerations of stars (Lardo et al. 2012). D’Antona & Caloi (2008) have also postulated that the longer periods of the NGC 1851 RR Lyrae may indicate that these variables may belong to the SG, and our synthetic HB strengthens this notion. Chemical anomalies in GCs suggest that self-enrichment is a common feature among GCs. The quasi-constancy of heavy metals in most GCs leads to the assumption that abundance variations are not affected or are scarcely affected by supernova ejecta, but involves formation of an SG of stars from matter processed into the first generation (FG) stars.

The SG will most probably show a spread in He (D’Antona & Caloi 2008) because the self-enriched material may come from different progenitors that have different chemical peculiarities, or may be diluted in different fractions with matter from the FG. We remark that such a helium spread is an essential ingredient in order to reproduce the pulsation properties of the RR Lyrae population as a whole. Simulations using constant He across the IS give synthetic period distributions that do not match the observed one as well and result in the observed N_1/N_{tot} being lower than what is observed. Actually, one can note that variations of He in 19 GCs have also been deduced by Bragaglia et al. (2010) from 1400 RGB stars with abundance determinations. As discussed in Gratton et al. (2010), a star-to-star spread in the He abundance may explain many aspects of the HBs of GCs.

It is worth pointing out that there have been suggestions of problems in the late stages of HB evolution in current HB tracks (Catelan 2009b; Valcarce & Catelan 2008). Moreover, Catelan (2009b) shows that in the case of NGC 1851, overluminous stars on the blue HB could be interpreted by an underestimate of the luminosity evolution along the HB rather than in terms of a moderate level of helium enrichment. Here, we do not attempt to resolve this ambiguity for NGC 1851; rather, we assume that the evolutionary tracks adopted represent the HB evolution accurately, and remind the reader that our comparisons are ultimately subject to both theoretical and observational uncertainties.

The pulsation periods and amplitudes from the RR Lyrae variables resulting from variations in He along the IS have different characteristics. The RR Lyrae variables with a “normal” helium have periods and amplitudes, as well as an N_1/N_{tot} ratio, that are in line with OoI-type GCs. In contrast, the RR Lyrae variables with slightly enhanced He ($0.27 < Y < 0.28$) have longer periods and a higher ratio of N_1/N_{tot} , indicative of RR Lyrae variables in OoII-type GCs. In the absence of spectroscopy of the RR Lyrae variables in NGC 1851, the synthetic horizontal part of the HB and RR Lyrae IS presented here is the simplest one that reproduces the available observations with the smallest amount of free parameters. New observations of the RR Lyrae variables may require more complex modeling, however, and would be particularly interesting.

Oosterhoff I clusters tend to be more metal-rich and host fainter RR Lyrae variables than OoII clusters (Caputo et al. 2000). As the metallicity has an effect on the absolute magnitude of an RR Lyrae, it has been difficult to disentangle whether the metallicity difference alone is affecting the brightness differences, or whether there are differences in the intrinsic magnitudes of RR Lyrae variables in OoI and OoII GCs caused by something other than just metallicity (like evolution or helium). In this cluster, where an internal spread in $[\text{Fe}/\text{H}]$ is small at most, our results indicate that a difference in helium abundance in the RR Lyrae variables is affecting where in the PA diagram the RR Lyrae star falls.

Our model consists of a BHB that is He-enriched ($Y \sim 0.28$) yet older than the RHB ($Y = 0.248$). This can be explained if the cluster was formed by a process such as a merger with populations that differ in He and age. Such a scenario has already been discussed by, e.g., Carretta et al. (2011a) and Bekki & Yong (2012). Hence, the BHB would not be an SG of stars originating from the same population as the RHB (the SGBb).

Since this paper was submitted, results from an intermediate resolution spectroscopic analysis of the two SGBs by Gratton et al. (2012b) indicate that the $[\text{Fe}/\text{H}]$ difference between the SGBb and SGBf is ~ 0.07 dex, the SGBf being more metal-rich. The RR Lyrae stars in our scenario are the progeny of SGBb; hence, this metallicity difference does not affect our results. We find that for BHB stars hotter than the IS, a 0.1 dex increase in $[\text{Fe}/\text{H}]$ at fixed $(V - I)$ changes the HB masses (at fixed Y) by $\sim 0.01 M_{\odot}$, and M_V changes by ~ 0.01 mag. Therefore, the effect of such an $[\text{Fe}/\text{H}]$ difference between the two SGBs has a negligible effect on our HB modeling.

We have shown that a spread in He reproduces the pulsational properties of the RR Lyrae sample as a whole, indicating the presence of an SG of stars in NGC 1851. Our analysis therefore demonstrates that RR Lyrae properties in a given GC can provide suitable constraints on the multiple population phenomenon in that GC. It is worth carrying out more studies of this kind to investigate further this connection with the occurrence of the multiple population phenomenon, especially in GCs with a sizable population of RR Lyrae stars and in which the stellar chemical patterns are well known.

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