International Journal of Modern Physics A
Vol. 25, No. 16 (2010) 3271–3306
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DOI: 10.1142/S0217751X10049293

BARE AND INDUCED LORENTZ AND CPT INVARIANCE VIOLATIONS IN QED

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Received 3 December 2009
Revised 18 March 2010

We consider QED in a constant axial vector background (Æther). Further Lorentz invariance violations (LIV) might occur owing to radiative corrections. The phenomenology of this model is studied, clarifying issues related to the various regularizations employed, with a particular emphasis on the induced photon mass. To this concern, it is shown that in the presence of LIV dimensional regularization may produce a radiatively induced finite photon mass. The possible physical role of the large momentum cutoff is elucidated and the finite temperature radiative corrections are evaluated. Finally, various experimental bounds on the parameters of the model are discussed.

Keywords: Lorentz and CPT invariance violation; photon mass; QED in constant axial vector background.

1. Introduction and Motivations

The bounds for the validity of fundamental laws in physics has attracted more and more attentions and interests in view of remarkable improvements in the experimental technique, both in laboratory research and in astrophysics.[1–5] Among others, some important investigations concern possible violations of space–time symmetries in vacuum due to the presence of constant backgrounds named Æthers.[6,7] In practice, the Lorentz and CPT Invariance Violation (LIV) in Quantum Electrodynamics

(QED)[8–14] has not yet been detected.[15–17] Nonetheless, it is not even excluded and it might occur,[1–5,18–21] in particular, spontaneous symmetry breaking[22–24] may cause LIV after condensation of massless axion-like fields[25–29] and/or of certain vector fields[30–33] (maybe, of gravitational origin[34,35]), as well as short distance space–time asymmetries may come from string[36–38] and quantum gravity effects[39–47] and noncommutative structure of the space–time.[48–51]

1.1. *Lorentz and gauge invariance violations*

In this paper, we shall investigate how a constant axial vector b_μ, a torsion-like background Æther, coupled to massive fermions may change electrodynamics, due to the vacuum polarization effects. In the last ten years, this issue has been treated quite extensively in the literature by means of different regularizations to deal with 1-loop ultraviolet divergences, with a rich variety of answers[52–69] following the effective field theory approach, to the lowest orders in the LIV parameters.[9,10,3] This means that, in fact, within this approach it is admittedly impossible to predict the actual values of the LIV structural constants, including their radiatively induced parts, leaving them to be eventually obtained only from experiments. In other words, it is just the empirical and phenomenological nature of latter constants that does explain why their determination really lies beyond any formal argument related to renormalization of divergent integrals in perturbation theory. For example, the spatial component of the Chern–Simons four-vector η_μ, which might constitute another constant Æther background,[8] is essentially set equal to zero by the absence of vacuum birefringence from distant radiogalaxies.[70] In the same manner, the spatial component of the axial vector constant Æther b_μ is severely constrained by the torsion pendulum experiments with polarized electrons[71] to be smaller than the benchmark value $m_e^2/M_{\text{Planck}} = 2 \times 10^{-17}$ eV. Hence, the only left narrow possibility is a temporal constant Æther, as described by the axial vector $b_\mu = (b, 0, 0, 0)$ and the Chern–Simons vector $\eta_\mu = (\eta, 0, 0, 0)$ — the metric tensor is $g_{\mu\nu} = \mathrm{diag}(+, -, -, -)$.

Of course, since the Chern–Simons Lagrangian and the photon mass terms are local functional of the gauge vector potential, one possibility[11–13,67,68] is to set the Chern–Simons four-vector η_μ as well as the photon mass m_γ exactly equal to zero by assumption, i.e. as a renormalization prescription, in such a manner to enforce the strong gauge invariance principle and the Ward identities, even in the occurrence of some possible Lorentz invariance violating effects. In such a circumstance, perhaps the simplest and most conservative one, the quantum photodynamics is governed by the usual Lorentz covariant and gauge invariant Maxwell Lagrangian, supplemented by the gauge fixing terms. In so doing, however, any possible LIV effect is necessarily confined inside the spinor matter and does not propagate, since the photodynamics with the minimal coupling interaction is dictated by the Lorentz and gauge invariances.

The situation drastically changes if we consider the Maxwell–Chern–Simons modifications of QED, as originally suggested by Carroll, Field and Jackiw in their seminal paper.[8] In fact, all these modifications of QED are weakly gauge invariant, in the sense that only the action, but not the Lagrangian, is invariant under the local gauge transformations. Moreover, the Maxwell–Chern–Simons free radiation field with a temporal Chern–Simons vector and a massless photon is inconsistent,[25–27,73] since it exhibits acausal and tachyonic behavior of the free photons. Thus, the only way to restore consistency is by means of a tiny but nonvanishing photon mass $m_\gamma \propto b$. In such a case, the strong experimental bounds on the photon mass[72] definitely endorse a rather stringent limit also on a temporal Æther background. In the recent literature,[62,63,74–77] a class of Lorentz invariance violating models with photon speed less than one has been considered and their possible phenomenological consequences have been discussed. These are models which break gauge invariance. Specifically, in this paper we show that even if one starts with a gauge invariant Maxwell Lagrangian minimally coupled with LIV fermions, then the 1-loop corrections might indeed radiatively generate a Chern–Simons term and a Proca mass term for photons. This entails that the 1-loop effective field theory is safely causal but no longer gauge invariant. In other words, a Lorentz invariance violation in the fermion sector may induce the breaking of gauge invariance of the photon sector.

There are several reasons why this latter possibility, in spite of appearing somewhat unlikely and unorthodox at a first sight, is actually much more natural and pregnant, once the Lorentz covariant framework is supposed to be abandoned because of some still unknown quantum effect occurring at very high energies and momenta. As a matter of fact, on the one hand, the Lorentz covariant quantization of the electromagnetic vector potential must be gauge invariant, in order to fulfill the principles of the special relativity. On the other hand, the covariant quantization of the Abelian Proca massive vector field is also perfectly consistent and safe, when coupled to a conserved electric current. Nevertheless, the photon vector field must be massless, i.e. gauge invariant, if the Einstein postulates with an inertial frame independent unit light velocity have to be filled. Thus, special relativity and gauge invariance are tightly related. On the contrary, if the axioms of the special relativity are somewhere abandoned, as we shall deal here with, then the LIV quantization of the electromagnetic field quite naturally drives towards a massive photon in order to guarantee causality and consistency, as we will discuss in the present paper. As a consequence, the very stringent limit on the photon mass will provide a rather severe bound upon the allowed LIV parameters.

Actually, in order to derive unambiguously any prediction in the LIV models, one has to put into the game some physically justified ideas about the high energy behavior. The key point of our approach is that the presence of a constant background Æther modifies the dispersion relations between energy and momentum, not only at low energies but even more at very high energies and momenta. This is because of a new phenomenon which is forbidden in the Lorentz covariant framework. Namely, high energy electrons, positrons and photons with certain polarizations become acausal, that means they will never be detected, so that they cannot appear as asymptotic states, if their momenta exceed the value $\Lambda_e \sim \frac{1}{2}m_e^2/b$.

Moreover, owing to the Æther background, high energy electrons and positrons with the other polarizations undergo bremßtrahlung in vacuum, while photons with the other polarization undergo electron positron pairs creation. Thus, the very high energy–momentum electrons, positrons and photons are not allowed to appear as physical, stable asymptotic states, on their mass shells. This special issue has recently been proposed[78] to provide a possible interpretation of the ATIC, PAMELA, HESS and Fermi data.

This feature suggests that the LIV modification of QED involving the constant background Æthers might be trusted at most as an effective field theory, in spite of being renormalizable, valid up to a very high energy–momentum scale, beyond which some new and more fundamental physics will enter into the game. Since a conservative value for this very high energy–momentum scale Λ_e will be found to be of the order $m_e^2/2b \sim 10^{26}$ eV, it is plausible that the would-be new physics will perhaps concern quantum gravity, noncommutative field theories and string theories, as originally suggested[36,79,39] long ago in the literature. Accordingly, a new fundamental length $\ell_e \sim \Lambda_e^{-1}$ appears below which the principles of special relativity could be modified by some new physical phenomenon.

1.2. *LIV radiative corrections*

The unavoidable existence of a large momentum cutoff $\Lambda_f \sim \frac{1}{2}m_f^2/b$ for the free Dirac field ψ_f of mass m_f in a constant Æther background was discovered by Kostelecký and Lenhert[80] and its role was further investigated by other authors.[61,64,81,82] Its very existence, which is of a kinematical and nonanalytic nature, will also provide an essential tool to understand the meaning and the role of the radiative corrections. In this paper, we shall consider the fermion loops involving electron positron virtual pairs and giving rise to ultraviolet divergent Feynman integrals. Actually, at variance with the covariant case, their regularization is a nontrivial task, because there are indeed several inequivalent ways to implement the well-known dimensional[61] and Pauli–Villars[62,63] regularizations in a LIV divergent Feynman integral. In particular, dimensional regularization with the 't Hooft–Veltman–Breitenlohner–Maison recipe to define the algebra of the γ_5 matrix in 2ω complex space–time dimensions[83–85] does not automatically guarantee gauge invariance of radiative corrections in a LIV model, as we shall see, in a manifest contrast to the Lorentz covariant case.

The necessary existence of the very high energy–momentum physical bound Λ_e in the LIV models does indeed provide a natural way to understand *à la* Wilson[86] the integration over the loops momenta. As a matter of fact, the domain $|\mathbf{p}| < \Lambda_e$ actually corresponds to the integration over physical modes, i.e. the large distance physics, while the integration over the outer complementary region in momentum space will mean the effective inclusion of the unknown short distance physics not directly accessible within the LIV model. In so doing, a divergent photon mass term arises, the finite part of which is arbitrary. Conversely, in the dimensional regularization method the above physical separation of the momentum space integration is not at all evident. Nonetheless, owing to the nontriviality of the γ_5 algebra in complex space–time dimensions, a finite induced Proca mass term for photon just appears from loop integration.

To this concern, it is very important to recall the close relationship between the Æther residual symmetry group and the induced Chern–Simons vector in the dimensional regularization. Actually, as thoroughly discussed in Ref. 61, there are two ways to implement dimensional regularization in the presence of a temporal Æther $b_\mu = (b, 0, 0, 0)$ in the fermionic sector. The first one, called $\overline{\mathrm{DR}}$, leads to a nonvanishing Chern–Simons vector, is compatible with the existence of the physical large momentum cutoff, and leaves the residual symmetry group $\mathrm{O}(2\omega - 1)$ as the maximal invariance group in momentum space. The other one, which has been denoted by $\widehat{\mathrm{DR}}$, leads conversely to a null Chern–Simons vector, i.e. to the strong gauge invariance, but at the price of a further violation of the Lorentz symmetry in the unphysical dimensions. In this case, the residual symmetry group is $\mathrm{O}(3) \times \mathrm{O}(2\omega - 3)$ and the existence of the physical large momentum cutoff is completely disregarded.

Hence, once Lorentz invariance is broken, the enforcement of the strong gauge invariance with $\eta_\mu = m_\gamma = 0$ appears to be somewhat unnatural already at the 1-loop approximation. As nicely shown in Ref. 61 it looks like a trick *ad hoc* involving a double violation of the Lorentz invariance even in the unphysical space–time. Thus, contrary to the first sight impression, a comparison between the two methods drives to a finite and nonvanishing result for both the radiatively generated Chern–Simons vector and the photon mass term, as the most natural and interesting option, at least in our opinion. These theoretically estimated finite values can be used to set a bound on the temporal vector Æther b_μ from the experimental data, notably from the limits on the photon mass.

It is worthwhile to stress again that the very high momentum region $|\mathbf{p}| > \Lambda_e \sim m_e^2/2b$ just corresponds to the physically inaccessible domain for the minimal LIV modification of quantum electrodynamics. Thus the main results of our analysis is that, once the Lorentz invariance is broken, both a Chern–Simons term and a photon mass are possible and natural, as further endorsed by the calculation of the finite temperature 1-loop effective action.

2. Consistency of Lorentz Invariance Violation

Before the calculation of the induced Lorentz invariance violations, let us examine the consistency of LIV in QED based on stability of its constituents — electrons, positrons and photons. To the lowest order it means that, within the range of validity of Lorentz covariant QED, electrons should not undergo a bremßtrahlung in vacuum $e^- \to e^-\gamma$, i.e. photon emission in the absence of external fields, and real photons should not annihilate in vacuum into real pairs $\gamma \to e^-e^+$. Of course, the latter processes are forbidden in covariant QED but might be allowed if Lorentz covariance is broken, that is, when a constant Æther background is there. We shall call the above processes vacuum decays of the fundamental particles, or simply decays, and the fundamental decaying particles will be denoted by $\tilde{e}^-$, $\tilde{e}^+$ and $\tilde{\gamma}$.

2.1. *LIV effective Lagrangian*

Here we will assume that the leading LIV effects are dominated by the softest interactions with an Æther background coupled to the CPT odd operators of canonical mass dimension equal to three. Hence, we can write in the most general Lagrange density

$$\mathcal{L} = \mathcal{L}_{\text{INV}} + \mathcal{L}_{\text{LIV}} + \mathcal{L}_{\text{AS}} \,, \tag{1}$$

$$\begin{aligned}\mathcal{L}_{\text{INV}} = &-\frac{1}{4}F^{\alpha\beta}(x)F_{\alpha\beta}(x) + \frac{1}{2}m_\gamma^2 A_\nu(x)A^\nu(x)\\ &+ \bar{\psi}(x)[\mathrm{i}\not{\partial} + e\not{A}(x) - m_e]\psi(x)\,,\end{aligned} \tag{2}$$

$$\mathcal{L}_{\text{LIV}} = \frac{1}{2}\eta_\alpha A_\beta(x)\tilde{F}^{\alpha\beta}(x) + b_\mu\bar{\psi}(x)\gamma_5\gamma^\mu\psi(x)\,, \tag{3}$$

$$\mathcal{L}_{\text{AS}} = A^\mu(x)\partial_\mu B(x) + \frac{1}{2}\varkappa B^2(x)\,, \tag{4}$$

where A_μ and $\psi(x)$ stand for the LIV photon and electron positron fields respectively, $e > 0$ is the positron charge, $\tilde{F}^{\alpha\beta}(x) = \frac{1}{2}\varepsilon^{\alpha\beta\rho\sigma}F_{\rho\sigma}(x)$ is the dual field strength, while B is the auxiliary Stückelberg scalar field. Note that we have included the Proca mass term for the photon in the Lorentz invariant Lagrangian $\mathcal{L}_{\text{INV}}$ because, as we shall discuss in the sequel, the latter is required by self-consistency, i.e. stability and causality, of such a LIV extension of QED. Moreover, as we shall see, it is generally radiatively induced by the LIV spinor Lagrangian $b_\mu\bar{\psi}(x)\gamma_5\gamma^\mu\psi(x)$. However, we will not elaborate here its dynamical origin: depending on whether it is generated by a Higgs mechanism or it is a fundamental Proca mass, quite different experimental bounds[72] can be applied to it.[a] The auxiliary Stückelberg Lagrangian $\mathcal{L}_{\text{AS}}$, which further violates gauge invariance beyond the photon mass term, has been necessarily introduced to provide, just owing to the so-called Stückelberg trick, the simultaneous occurrences of power counting renormalizability and perturbative unitarity.[b]

Hereafter, the LIV constant vectors η_μ and b_μ — the Æther — are supposed to be universal and proportional $\eta_\mu \propto b_\mu$, including both the classical background $\eta_\mu^{c\ell}$, $b_\mu^{c\ell}$ as well as the QED radiative corrections $\eta_\mu = \eta_\mu^{c\ell} + \Delta\eta_\mu$, $b_\mu = b_\mu^{c\ell} + \Delta b_\mu$.

As a matter of fact, if LIV manifests itself as a fundamental phenomenon in the large scale universe, or it is a result of condensation of axial vector/axion gradient fields, it is quite plausible that LIV is induced universally by different species of fermion fields coupled to the very same axial vector b^μ, albeit with different magnitudes depending upon flavors. Then both LIV vectors become[52–61] collinear, i.e.

[a] We also leave the room for the photon mass generation in a plasma-like medium.

[b] For an equivalent BRST treatment see for example Ref. 87.

$\eta_\mu \propto b_\mu$. At the meantime it has been found[25–27,61,73,80] that a consistent quantization of photons just requires the Chern–Simons vector to be spacelike, whereas for the consistency of the spinor free field theory a spacelike axial vector b^μ is generally not allowed, but for the pure spacelike case which, however, is severely bounded by the experimental data.[88,89]

As already mentioned, to generally provide the self-consistency of the present LIV model, one has to introduce[66] in the above Lagrangian (1) also a photon mass, for assembling both a bare and an induced masses $m_\gamma^2 = m_0^2 + \Delta m_\gamma^2$. It happens (see below) that the induced photon mass squared Δm_γ^2 is $\mathrm{O}(\alpha b^2)$ where $\alpha = e^2/4\pi$ is the fine structure constant. Thus, it turns out that a minimal breaking of the Lorentz and CPT symmetries in the spinor matter sector leads to the loss of gauge invariance in the photon sector.

As we shall better see below, all the above-mentioned new features will eventually result in the Lorentz covariance violating modifications of the classical equations for particle mass shells, i.e. LIV modified transverse photons ($\tilde{\gamma}$)

$$(k^2 - m_\gamma^2)^2 - (\eta \cdot k)^2 + \eta^2 k^2 = 0\,, \tag{5}$$

and LIV modified electrons and positrons ($\tilde{e}^\mp$)

$$(p^2 + b^2 - m_e^2)^2 - 4(b \cdot p)^2 + 4b^2 m_e^2 = 0\,. \tag{6}$$

Let us examine the possibilities: (i) $\tilde{e}^\pm \to \tilde{e}^\pm \tilde{\gamma}$, namely the triggered decay[c] of LIV modified electrons moving with emission of modified photons $\tilde{\gamma}$. In the Lorentz covariant QED it would correspond to the electron decay in its rest frame, which is impossible due to energy conservation. As well, one can also search for: (ii) $\tilde{\gamma} \to \tilde{e}^+ \tilde{e}^-$ photon decay with pair creation, a process which is forbidden for massless photons in conventional QED. Conversely, it turns out that both kinds of processes may occur in the Lorentz invariance violating quantum electrodynamics for high energy particles, as the physics is different in differently moving frames.

To be comprehensible in our analytical calculations and for the sake of description of the possible phenomenological consequences, we definitely assume the privileged class of rotation invariant LIV isotropic inertial frames — the Æther — to be concordant with the rest frame of Cosmic Microwave Background Radiation (CMBR). Some stability issues for spacelike anisotropic Æthers were insofar examined in Refs. 1–5, 61, 73, 80, 90.

2.2. *LIV free fields quantization*

The canonical quantization of the LIV spinor field has been thoroughly analyzed in Refs. 80 and 61. Here, for the sake of clarity and completeness, we aim to briefly

[c]It can be qualified as either a Cherenkov-type radiation from superluminal $\tilde{e}^\pm$, if LIV is dominating for fermions, or a bremßtrahlung of photons in an axial vector background, when fermions are not superluminal. In order to avoid any semantic ambiguity, we shall dub it LIV decay.

develop the canonical quantization of a free massive Proca vector field with additional LIV Chern–Simons term: namely, a rather new issue that will be called the canonical quantization of the Proca–Chern–Simons vector field. The classical Euler–Lagrange field equations that follows from the general Lagrangian (1) take the form

$$\partial_\lambda F^{\lambda\nu} + m_\gamma^2 A^\nu + \eta_\alpha \tilde{F}^{\alpha\nu} + \partial^\nu B + e\bar{\psi}\gamma^\nu\psi = 0\,, \tag{7}$$

$$\{\gamma^\mu[\mathrm{i}\partial_\mu - b_\mu\gamma_5 + eA_\mu(x)] - m_e\}\psi(x) = 0\,, \tag{8}$$

$$\partial_\nu A^\nu = \varkappa B\,. \tag{9}$$

After contraction of Eq. (7) with ∂_ν, we find

$$(\Box + \varkappa m_\gamma^2)B(x) = 0 \tag{10}$$

whence it follows that the auxiliary Stückelberg field is always a decoupled unphysical real scalar field, which is never involved in the electromagnetic interactions $\forall\varkappa \in \mathbb{R}$. Turning now to free field theory, i.e. $e = 0$, and momentum space

$$A^\nu(x) = \int \frac{\mathrm{d}^4k}{(2\pi)^{3/2}}\, \tilde{A}^\nu(k)\mathrm{e}^{-\mathrm{i}k\cdot x}\,, \quad B(x) = \int \frac{\mathrm{d}^4k}{(2\pi)^{3/2}}\, \tilde{B}(k)\mathrm{e}^{-\mathrm{i}k\cdot x}\,,$$

we obtain the momentum space free field equations for the LIV massive vector field and the auxiliary scalar field

$$\left\{g^{\lambda\nu}(k^2 - m_\gamma^2) - k^\lambda k^\nu + \mathrm{i}\varepsilon^{\lambda\nu\alpha\beta}\eta_\alpha k_\beta\right\}\tilde{A}_\lambda(k) + \mathrm{i}k^\nu\tilde{B}(k) = 0\,, \tag{11}$$

$$\mathrm{i}k^\lambda\tilde{A}_\lambda(k) + \varkappa\tilde{B}(k) = 0\,. \tag{12}$$

Contractions with k_ν and η_ν respectively yields

$$-m_\gamma^2 k\cdot\tilde{A}(k) + \mathrm{i}k^2\tilde{B}(k) = 0\,, \tag{13}$$

$$k\cdot\tilde{A}(k) = \mathrm{i}\varkappa\tilde{B}(k)\,, \tag{14}$$

$$\eta\cdot\tilde{A}(k)(k^2 - m_\gamma^2) - (\eta\cdot k)k\cdot\tilde{A}(k)(1 - \varkappa^{-1}) = 0\,. \tag{15}$$

Hence, for the specially suitable choice $\varkappa = 1$ that greatly simplifies the whole treatment,[d] we eventually get

$$\tilde{B}(k) + \mathrm{i}k\cdot\tilde{A}(k) = 0\,, \tag{16}$$

$$(k^2 - m_\gamma^2)\tilde{B}(k) = 0 = (k^2 - m_\gamma^2)\eta\cdot\tilde{A}(k)\,, \tag{17}$$

$$\left\{g^{\lambda\nu}(k^2 - m_\gamma^2) + \mathrm{i}\varepsilon^{\lambda\nu\alpha\beta}\eta_\alpha k_\beta\right\}\tilde{A}_\lambda(k) = 0\,. \tag{18}$$

Consider now the kinetic 4×4 Hermitian matrix $\mathbb{K}$ with matrix elements

$$K_{\lambda\nu} \equiv g_{\lambda\nu}(k^2 - m_\gamma^2) + \mathrm{i}\varepsilon_{\lambda\nu\alpha\beta}\eta^\alpha k^\beta \tag{19}$$

[d] In the covariant canonical quantization of the massless gauge field this choice corresponds to the well-known Feynman gauge, in which the equations of motion as well as the photon propagator take the simplest form.

which satisfies

$$K_{\lambda\nu} = K^{*}_{\nu\lambda}\,. \tag{20}$$

The Levi-Civita symbol in the four-dimensional Minkowski space–time is normalized according to

$$\varepsilon_{0123} = -\varepsilon^{0123} \equiv 1$$

in such a way that

$$\begin{aligned}\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\mu}{}^{\lambda\rho\sigma} &= -g^{\nu\lambda}g^{\alpha\rho}g^{\beta\sigma} - g^{\alpha\lambda}g^{\beta\rho}g^{\nu\sigma} - g^{\beta\lambda}g^{\nu\rho}g^{\alpha\sigma}\\ &\quad + g^{\nu\rho}g^{\alpha\lambda}g^{\beta\sigma} + g^{\alpha\rho}g^{\beta\lambda}g^{\nu\sigma} + g^{\beta\rho}g^{\nu\lambda}g^{\alpha\sigma}\,.\end{aligned}$$

Now, it turns out that we have

$$\begin{aligned}S^{\nu}{}_{\lambda} &\equiv \varepsilon^{\mu\nu\alpha\beta}\eta_\alpha k_\beta \varepsilon_{\mu\lambda\rho\sigma}\eta^\rho k^\sigma\\ &= \delta^{\nu}{}_{\lambda}\mathtt{D} + k^\nu k_\lambda \eta^2 + \eta^\nu\eta_\lambda k^2 - \eta\cdot k(\eta_\lambda k^\nu + \eta^\nu k_\lambda)\,,\end{aligned} \tag{21}$$

where

$$\mathtt{D} \equiv (\eta\cdot k)^2 - \eta^2 k^2 = \frac{1}{2}\,S^{\nu}{}_{\nu}$$

in such a manner that we find

$$S^{\nu}{}_{\lambda}\eta^\lambda = S^{\nu}{}_{\lambda}k^\lambda = 0\,,\quad S^{\mu\nu}S_{\nu\lambda} = \mathtt{D}S^{\mu}{}_{\lambda}\,,\quad S^{\nu}{}_{\nu} = 2\mathtt{D}\,,$$

while

$$S^{\mu\lambda}\varepsilon_{\lambda\nu\alpha\beta}\eta^\alpha k^\beta = \mathtt{D}\varepsilon^{\mu}{}_{\nu\alpha\beta}\eta^\alpha k^\beta\,. \tag{22}$$

Note that for a timelike and spatial isotropic Axion Æther $\eta^\mu = (\eta, 0, 0, 0)$ we find $\mathtt{D} = \eta^2\mathbf{k}^2 > 0$. Then, to our purpose, it is convenient to introduce the two orthonormal one-dimensional Hermitian projectors

$$\boldsymbol{\pi}^{\mu\nu}_{\pm} \equiv \frac{S^{\mu\nu}}{2\mathtt{D}} \pm \frac{\mathrm{i}}{2\sqrt{\mathtt{D}}}\,\varepsilon^{\mu\nu\alpha\beta}\eta_\alpha k_\beta = \left(\boldsymbol{\pi}^{\nu\mu}_{\pm}\right)^{*}, \tag{23}$$

which enjoy the properties $\forall k^\mu = (k_0, \mathbf{k})$:

$$\begin{aligned}\boldsymbol{\pi}^{\mu\nu}_{\pm}\eta_\nu = \boldsymbol{\pi}^{\mu\nu}_{\pm}k_\nu = 0\,, &\qquad g_{\mu\nu}\boldsymbol{\pi}^{\mu\nu}_{\pm} = 1\,,\\ \boldsymbol{\pi}^{\mu\lambda}_{\pm}\boldsymbol{\pi}_{\pm\lambda\nu} = \boldsymbol{\pi}^{\mu}_{\pm\nu}\,, &\qquad \boldsymbol{\pi}^{\mu\lambda}_{\pm}\boldsymbol{\pi}_{\mp\lambda\nu} = 0\,,\end{aligned} \tag{24}$$

$$\begin{aligned}\mathtt{D}\big(\boldsymbol{\pi}^{\mu\nu}_{+} + \boldsymbol{\pi}^{\mu\nu}_{-}\big) &= S^{\mu\nu}\,,\\ \sqrt{\mathtt{D}}\big(\boldsymbol{\pi}^{\mu\nu}_{+} - \boldsymbol{\pi}^{\mu\nu}_{-}\big) &= \mathrm{i}\varepsilon^{\mu\nu\alpha\beta}\eta_\alpha k_\beta\,.\end{aligned} \tag{25}$$

It follows therefrom that we can build up a pair of complex and spacelike chiral polarization vectors by means of a constant and spacelike four vector: for example $\epsilon_\nu = (0, 1, 1, 1)/\sqrt{3}$, in such a manner that we can set

$$\varepsilon^{\mu}_{\pm}(k) \equiv \left[\frac{\mathbf{k}^2 - (\epsilon\cdot k)^2}{2\mathbf{k}^2}\right]^{-1/2}\boldsymbol{\pi}^{\mu\nu}_{\pm}\epsilon_\nu\,, \tag{26}$$

which satisfy the orthogonality relations

$$-\frac{1}{2}g_{\mu\nu}\varepsilon_{\pm}^{\mu *}(k)\varepsilon_{\pm}^{\nu}(k)+\text{c.c.}=1\,,\quad g_{\mu\nu}\varepsilon_{\pm}^{\mu *}(k)\varepsilon_{\mp}^{\nu}(k)+\text{c.c.}=0 \tag{27}$$

as well as the closure relations

$$-\frac{1}{2}\left[\varepsilon_{+}^{\mu *}(k)\varepsilon_{+}^{\nu}(k)+\varepsilon_{-}^{\mu *}(k)\varepsilon_{-}^{\nu}(k)+\text{c.c.}\right]=\mathtt{D}^{-1}S^{\mu\nu}\,. \tag{28}$$

Now, we are ready to find the general solution of the free field equations (18) for the special choice $\varkappa = 1$ of the Stückelberg parameter. As a matter of fact, from the relationships (24) and (25), we readily obtain

$$\begin{aligned} K^{\mu}{}_{\nu}\varepsilon_{\pm}^{\nu}(k) &= \left[\delta^{\mu}{}_{\nu}(k^2-m_{\gamma}^2)+\sqrt{\mathtt{D}}(\boldsymbol{\pi}^{\mu}_{+\nu}-\boldsymbol{\pi}^{\mu}_{-\nu})\right]\varepsilon_{\pm}^{\mu}(k) \\ &= \left(k^2-m_{\gamma}^2\pm\sqrt{\mathtt{D}}\right)\varepsilon_{\pm}^{\mu}(k) \end{aligned} \tag{29}$$

which shows that the polarization vectors of positive and negative chirality respectively are solutions to the vector field equations if and only if

$$k_{\pm}^{\mu}=(\omega_{\mathbf{k},\pm},\mathbf{k})\,,\quad \omega_{\mathbf{k},\pm}=\sqrt{\mathbf{k}^2+m_{\gamma}^2\pm\eta|\mathbf{k}|}\,, \tag{30}$$

$$\varepsilon_{\pm}^{\mu}(\mathbf{k},\eta)=\varepsilon_{\pm}^{\mu}(k_{\pm})\,,\qquad (k_{\pm}^{0}=\omega_{\mathbf{k},\pm})\,. \tag{31}$$

Note that the above pair of LIV chiral polarizations do not coincide at all with the transverse elliptic polarizations of the conventional Maxwell plane waves. Nonetheless, if we introduce the further pair of orthonormal polarization four vectors, i.e. the temporal and longitudinal polarization real vectors respectively,

$$\varepsilon_{T}^{\mu}(k)\equiv\frac{k^{\mu}}{\sqrt{k^2}}\qquad\qquad (k^2>0)\,, \tag{32}$$

$$\varepsilon_{L}^{\mu}(k)\equiv(k^2\mathtt{D})^{-1/2}(k^2\eta^{\mu}-k^{\mu}\eta\cdot k)\quad (k^2>0) \tag{33}$$

which fulfill by construction

$$g_{\mu\nu}\varepsilon_{T}^{\mu}(k)\varepsilon_{T}^{\nu}(k)=1=-g_{\mu\nu}\varepsilon_{L}^{\mu}(k)\varepsilon_{L}^{\nu}(k)\,, \tag{34}$$

$$g_{\mu\nu}\varepsilon_{T}^{\mu}(k)\varepsilon_{L}^{\nu}(k)=g_{\mu\nu}\varepsilon_{T}^{\mu}(k)\varepsilon_{\pm}^{\nu}(k)=g_{\mu\nu}\varepsilon_{L}^{\mu}(k)\varepsilon_{\pm}^{\nu}(k)=0\,, \tag{35}$$

then we have at our disposal $\forall k^{\mu}$ with $k^2>0$ a complete orthonormal set of four polarization four vectors, namely

$$\varepsilon_{A}^{\mu}(k)=\begin{cases} \dfrac{k^{\mu}}{\sqrt{k^2}} & \text{for } A=T \\ \dfrac{(k^2\eta^{\mu}-k^{\mu}\eta\cdot k)}{\sqrt{k^2\mathtt{D}}} & \text{for } A=L \\ \varepsilon_{\pm}^{\mu}(k_{\pm}) & \text{for } A=\pm \end{cases}\qquad (k^2>0)\,. \tag{36}$$

It follows that if we introduce the 4×4 polarization matrix

$$g_{AB} = g^{AB} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (A, B = T, L, +, -)\,, \tag{37}$$

then we can write the full orthogonality and closure relations

$$\frac{1}{2}\, g_{\mu\nu} \varepsilon_A^{\mu *}(k) \varepsilon_B^{\nu}(k) + \text{c.c.} = g_{AB}\,, \quad \frac{1}{2}\, g^{AB} \varepsilon_A^{\mu *}(k) \varepsilon_B^{\nu}(k) + \text{c.c.} = g^{\mu\nu}\,. \tag{38}$$

It is very important to realize that the on mass shell polarization vector $\varepsilon_-^{\mu}(k_-)$ with $k_-^2 = m_\gamma^2 - \eta|\mathbf{k}| > 0$, is well defined if and only if the spatial momentum $\mathbf{k}$ stands below the momentum cutoff Λ_γ, i.e. inside the large momentum sphere

$$|\mathbf{k}| < \frac{m_\gamma^2}{\eta} \equiv \Lambda_\gamma\,. \tag{39}$$

Now, in order to implement the canonical quantization of the LIV massive vector field for the especially simple choice $\varkappa = 1$, it is convenient to introduce the plane waves according to

$$u_{\mathbf{k}A}^{\nu}(x) = \left[(2\pi)^3 2\omega_{\mathbf{k}A}\right]^{-1/2} \varepsilon_A^{\nu}(\mathbf{k}) \exp\{-\mathrm{i}\omega_{\mathbf{k}A} x^0 + \mathrm{i}\mathbf{k}\cdot\mathbf{x}\}\,, \tag{40}$$

where

$$\varepsilon_T^{\nu}(\mathbf{k}) = \frac{k^{\nu}}{m_\gamma}\,, \quad \varepsilon_L^{\nu}(\mathbf{k}) = \frac{m_\gamma^2 \eta^{\nu} - k^{\nu}(\eta\cdot k)}{m_\gamma |\mathbf{k}| \eta}\,, \tag{41}$$

for

$$k_0 = \omega_{\mathbf{k}T} = \omega_{\mathbf{k}L} = \sqrt{\mathbf{k}^2 + m_\gamma^2} \equiv \omega_{\mathbf{k}}\,, \tag{42}$$

$$\varepsilon_+^{\nu}(\mathbf{k}) \equiv \varepsilon_+^{\nu}(k_+)\,, \quad \varepsilon_-^{\nu}(\mathbf{k}) \equiv \varepsilon_-^{\nu}(k_-)\theta(k_-^2)\,. \tag{43}$$

It follows therefrom that the canonical quantization of the free LIV massive vector field for $\varkappa = 1$ takes the form

$$A^{\nu}(x) = \sum_{\mathbf{k},A} \left[a_{\mathbf{k}A} u_{\mathbf{k}A}^{\nu}(x) + a_{\mathbf{k}A}^{\dagger} u_{\mathbf{k}A}^{\nu *}(x)\right]\,, \tag{44}$$

where

$$\sum_{\mathbf{k},A} \equiv \sum_{A=T,L,\pm} \int \mathrm{d}\mathbf{k}\,, \tag{45}$$

whereas the canonical commutation relations holds true, viz.

$$\left[a_{\mathbf{k}A}, a_{\mathbf{p}B}^{\dagger}\right] = -\eta_{AB}\delta(\mathbf{k} - \mathbf{p})\,, \tag{46}$$

all the other commutators being equal to zero. According to Eq. (16), we obtain

$$B(x) = \frac{1}{\mathrm{i}} \int \mathrm{d}\mathbf{k}\, k_\nu \left[a_{\mathbf{k}T} u_{\mathbf{k}T}^{\nu}(x) - a_{\mathbf{k}T}^{\dagger} u_{\mathbf{k}T}^{\nu *}(x)\right]_{k_0 = \omega_{\mathbf{k}}} \tag{47}$$

in such a manner that the physical Hilbert space $\mathfrak{H}_{\mathrm{phys}}$ with positive semidefinite metric, for the LIV massive $\tilde{\gamma}$-photons, is selected out from the Fock space $\mathfrak{F}$ by means of the subsidiary condition

$$B^{(-)}(x)|\mathrm{phys}\rangle = 0\,, \quad \forall|\mathrm{phys}\rangle \in \mathfrak{H}_{\mathrm{phys}} \subset \mathfrak{F}\,, \tag{48}$$

which remains true even in the presence of the interaction with the spinor field, as described by the basic Lagrange density (1). On the other side, it turns out that the LIV massive $\tilde{\gamma}$-photons are described by the quantized field

$$V^{\nu}(x) = \int \mathrm{d}\mathbf{k} \sum_{A=L,\pm} \left[a_{\mathbf{k}A} u^{\nu}_{\mathbf{k}A}(x) + a^{\dagger}_{\mathbf{k}A} u^{\nu *}_{\mathbf{k}A}(x)\right] \tag{49}$$

with the standard nonvanishing canonical commutation relations

$$\left[a_{\mathbf{k}A}, a^{\dagger}_{\mathbf{p}B}\right] = \delta_{AB}\delta(\mathbf{k}-\mathbf{p})\,, \quad A,\ B = L,\ \pm\,,$$

all the other commutators being equal to zero. Note that the LIV massive $\tilde{\gamma}$-photon 1-particle states of definite spatial momentum $\mathbf{k}$ do exhibit three polarization states: one linear longitudinal polarization of real vector $\varepsilon^{\nu}_{L}(k)$ with dispersion relation $k^2 = m^2_{\gamma}$ and two chiral transverse states with complex vectors $\varepsilon^{\nu}_{\pm}(k_{\pm})$ and dispersion relations (5) and (30), the negative chirality states $\varepsilon^{\nu}_{-}(k_{-})$ being well defined only for $|\mathbf{k}| < \Lambda_{\gamma} \Leftrightarrow k^2_{-} > 0$.

If the Æther is timelike and isotropic $\eta_{\mu} = (\eta, 0, 0, 0) \propto b_{\mu} = (b, 0, 0, 0)$, then from Eqs. (6), (5) and (30) one can find the following dispersion relations for the photon chiral polarizations and for fermion normal modes respectively, namely

$$\omega^2_{\mathbf{k},\pm} = k^2 + m^2_{\gamma} \pm k\eta\,, \tag{50}$$

$$\omega^2_{\mathbf{p},\pm} = p^2 + m^2_{e} + b^2 \pm 2bp\,, \tag{51}$$

$$k_0 = \pm\sqrt{\left(k \pm \frac{1}{2}\eta\right)^2 + m^2_{\gamma} - \frac{1}{4}\,\eta^2} \equiv \pm\omega_{\mathbf{k},\pm}\,, \tag{52}$$

$$p_0 = \pm\sqrt{(p \pm b)^2 + m^2_{e}} \equiv \pm\omega_{\mathbf{p},\pm}\,, \tag{53}$$

in which we have set $k^{\mu} = (k_0, \mathbf{k})$, $p^{\mu} = (p_0, \mathbf{p})$ and $k = |\mathbf{k}|$, $p = |\mathbf{p}|$. The necessary condition for stability is $m^2_{\gamma} \geq \frac{1}{4}\eta^2$ but in fact it is not sufficient, as we shall see later on. Both dispersion relations exhibit a similar pattern of velocities, namely the group velocities for fermions are bounded by the conventional speed of light ($\hat{\mathbf{a}} = \mathbf{a}/|\mathbf{a}|$)

$$\mathbf{v}_{\pm} \equiv \nabla_{\mathbf{p}}\omega_{\mathbf{p},\pm} = \hat{\mathbf{p}}(p \pm b)\omega^{-1}_{\mathbf{p},\pm}\,, \quad |\mathbf{v}_{\pm}| < 1 \tag{54}$$

and the very same for the LIV modified photons with $b \to \eta/2$, $m^2_{e} \to m^2_{\gamma} - \frac{1}{4}\eta^2$,

$$\mathbf{u}_{\pm} \equiv \nabla_{\mathbf{k}}\omega_{\mathbf{k},\pm} = \hat{\mathbf{k}}(k \pm \eta/2)\omega^{-1}_{\mathbf{k},\pm}\,, \quad |\mathbf{u}_{\pm}| < 1\,. \tag{55}$$

Their unusual feature is the variation in magnitude and sign depending on wave vectors.

On the other side, the phase velocities for both particles in the LIV background are not bounded by the conventional speed of light when chiralities are chosen in (52) and (53) with negative signs,

$$w_{\gamma,-} \equiv k^{-1}\omega_{\mathbf{k},-} \begin{cases} \geq 1\,, & \text{for } k \leq \dfrac{m_\gamma^2}{\eta}\,, \\[2ex] < 1\,, & \text{for } k > \dfrac{m_\gamma^2}{\eta}\,; \end{cases} \tag{56}$$

$$w_{e,-} \equiv p^{-1}\omega_{\mathbf{p},-} \begin{cases} \geq 1\,, & \text{for } p \leq \dfrac{(m_e^2+b^2)}{2b}\,, \\[2ex] < 1\,, & \text{for } p > \dfrac{(m_e^2+b^2)}{2b}\,. \end{cases} \tag{57}$$

In other words, the 1-particle four-vectors k^μ and p^μ for negative chiralities 1-particle states do leave the causality (and further on stability) region when

$$k^\mu k_\mu < 0 \quad \text{for} \quad |\mathbf{k}| > \frac{m_\gamma^2}{\eta}\,, \qquad p^\mu p_\mu < 0 \quad \text{for} \quad |\mathbf{p}| > \frac{m_e^2}{2b}\,. \tag{58}$$

In spite of similarity, the two bounds are quite different phenomenologically: whereas the electron mass is estimated to be much larger than any possible background b, there is no evidence for a photon mass at a very stringent limit.[72,90] However, if the photon mass is strictly zero, then the phase propagation of photons with negative chiralities in the presence of Chern–Simons interaction is acausal and the latter ones become unstable for all values of wave vectors (see below).

Concerning the 1-particle fermion states, we see that the necessary causality requirement

$$g^{\mu\nu}p_\mu p_\nu = \omega_{\mathbf{p},\pm}^2 - p^2 \simeq m_e^2 \pm 2bp > 0 \tag{59}$$

is always fulfilled by the upper frequencies ($\omega_{\mathbf{p},+}$), while for the lower frequencies ($\omega_{\mathbf{p},-}$), it leads to the unavoidable physical ultraviolet cutoff for, e.g. $\tilde{e}^{\pm}$ particles

$$p < \frac{m_e^2}{2b} \equiv \Lambda_e \tag{60}$$

that involves negative chirality 1-particle states and positive chirality 1-antiparticle states. Now we are ready to face the nontrivial problem of the 1-loop radiative quantum corrections induced by the electromagnetic minimal coupling between LIV modified fermions and the Proca–Chern–Simons vector particles.

3. The 1-loop Photon Self-Energy

Our aim here is to compute the 1-loop induced parity even effective action. For the sake of simplicity, we refer to the classical spinor Lagrangian density involving only one species of massive fermion with mass m, viz.

$$\mathcal{L}_{\text{spinor}} = \bar{\psi}(x)(\mathrm{i}\not{\partial} + e\not{A}(x) - m - \not{b}\gamma_5)\psi(x)\,, \tag{61}$$

which leads to the momentum space four-dimensional Feynman propagator

$$S(p,b,m) = \frac{[p^2 + b^2 - m^2 + 2(b \cdot p + m\not{b})\gamma_5](\not{p} + m + \not{b}\gamma_5)}{4\mathrm{i}[(b \cdot p)^2 - m^2 b^2] - \mathrm{i}(p^2 + b^2 - m^2 + \mathrm{i}\varepsilon)^2} \,. \tag{62}$$

From the Feynman rules, the 1-loop photon self-energy[94,96–98] or vacuum polarization tensor, is formally determined to be

$$\Pi^{\mu\nu}(k;b,m) = -\mathrm{i}e^2 \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\, \mathrm{tr}\{\gamma^\mu S(p)\gamma^\nu S(p+k)\}\,. \tag{63}$$

However, the above formal expression does exhibit ultraviolet divergencies by superficial power counting, which have to be properly regularized. The general structure of the regularized 1-loop photon self-energy tensor reads as follows:

$$\mathrm{reg}\,\Pi^{\mu\nu} = \mathrm{reg}\,\Pi^{\mu\nu}_{\mathrm{even}} + \mathrm{reg}\,\Pi^{\mu\nu}_{\mathrm{odd}}\,. \tag{64}$$

The 1-loop parity odd part has been evaluated[61] for a vanishing external momentum and reads

$$\mathrm{reg}\,\Pi^{\mu\nu}_{\mathrm{odd}} = \frac{\mathrm{i}e^2}{2\pi^2}\,\epsilon^{\mu\nu\rho\sigma} b_\rho k_\sigma \quad (b^2 \ll m^2)\,, \tag{65}$$

which corresponds to the induced Chern–Simons coefficient

$$\Delta\eta_\mu = -\frac{2\alpha}{\pi} b_\mu \qquad \left(\alpha = \frac{e^2}{4\pi}\right). \tag{66}$$

The regularization independence as well as the physical meaning of the parity odd part of the polarization tensor has been discussed in Refs. 61, 64, 69. It has been proven that the dimensional regularization properly applied to the parity odd part of polarization tensor (63) with propagators (62) was able to reproduce the same induced Chern–Simons coefficient (66) as it was obtained with the physically motivated cutoff (58) from Sec. 2. This result is accounted for by the fact that the above Chern–Simons coefficient is finite and not screened by a power-like or logarithmic divergence. For the CPT and parity even part induced by LIV, the relationship between two regularizations is more subtle as the quadratic divergence in the Lorentz invariant part is dominant and screens the subdominant effects of LIV contributions. These subtleties will be considered below.

3.1. *Induced photon mass (IPM) in dimensional regularization*

The regularized expression of the 1-loop photon self-energy tensor is thereby given

$$\mathrm{i}\,\mathrm{reg}\,\Pi^{\mu\nu}(k;b,m,\mu) \equiv e^2\mu^{4-2\omega} \int \frac{\mathrm{d}^{2\omega} p}{(2\pi)^{2\omega}}\, \mathrm{tr}\{\gamma^\mu S(p)\gamma^\nu S(p+k)\}\,, \tag{67}$$

where dimensional regularization is employed to give a meaning to the loop integral, which appears by power counting to be superficially quadratically divergent in four dimensions. Notice that the Dirac matrices involved in the regularized loop integral (67) have now to be understood and treated according to the algebraically consistent general algorithm.[83–85]

Dimensional regularization is supposed to fulfill gauge invariance. This is certainly true for a Lorentz covariant field theory model. When Lorentz covariance is broken, there are at least two different ways of implementing dimensional regularization.[61] Both of them do indeed respect the gauge invariance for the parity odd part of the effective action, up to the 1-loop approximation, although leading to two different gauge invariant results. On the contrary, to keep the gauge invariance is much more subtle for the parity even part and, eventually, it does no longer hold true. The details of the calculation can be found in App. A.

Here below we proceed to the 1-loop radiatively induced generation of a photon mass and the corresponding breakdown of the gauge invariance owing to the presence of a constant temporal axial vector Æther background coupled with the fermionic matter. After lengthy calculations presented in App. B, we find the following result for the induced photon mass in a LIV background:

$$\mathrm{reg}\,\Delta\Pi^{\mu\nu}_{\mathrm{even}} = \frac{2\alpha}{3\pi}\; b^2 \bar{g}^{\mu\nu}\,. \tag{68}$$

The important meaning of the above result is that, in the presence of an explicit breaking of the Lorentz invariance in the fermionic sector, the gauge symmetry of the Abelian vector field may be lost, owing to anomalous radiative quantum corrections, so that an induced tiny Proca mass term for the $\tilde{\gamma}$-photon field does arise in the effective Lagrangian, that is

$$\frac{1}{2}\Delta m_\gamma^2 A_\mu A^\mu\,, \quad \Delta m_\gamma^2 = \frac{2\alpha}{3\pi}\, b^2\,. \tag{69}$$

This radiatively generated Proca mass term[e] for $\tilde{\gamma}$-photons accompanies the introduction of a general bare photon mass which is necessary to make the whole formulation of the LIV QED fully consistent.

3.2. *IPM with the physical UV cutoff*

It is instructive to repeat the calculation of the radiatively generated photon mass in the physical cutoff regularization. To this concern, we recall that the large ultraviolet cutoff Λ, for 1-particle states of negative helicity as well as 1-antiparticle states of positive helicity with momentum $\mathbf{p}$ and mass m, is provided by Eq. (60) that is $|\mathbf{p}| < \Lambda \simeq m^2/2b$, with $b = b_0$. In accordance with our previous notations

$$\mathrm{reg}\,\Pi^{\mu\nu}_{\mathrm{cov}}(k,m) = (k^2 g^{\mu\nu} - k^\mu k^\nu)\,\mathrm{reg}\,\Pi_{\mathrm{cov}}(k^2,m^2) + \mathrm{reg}\,\tilde{\Pi}^{\mu\nu}_{\mathrm{cov}}(\Lambda^2,m^2)\,, \tag{70}$$

[e]Notice that the radiatively generated photon mass term was already reported in our previous paper[66] but with erroneous sign, i.e. a tachyonic imaginary mass instead of a real Proca mass.

we have

$$\begin{aligned}
\operatorname{reg}\tilde{\Pi}^{\mu\nu}_{\text{cov}}(m,\Lambda) &= \frac{\mathrm{i}e^2}{4\pi^4}\int \mathrm{d}^4p\,\theta(\Lambda^2-\mathbf{p}^2)N_0^{\mu\nu}(p,m)D^2(p,m)\\
&= \frac{\mathrm{i}\alpha}{\pi^3}\int \mathrm{d}^4p\,\theta(\Lambda^2-\mathbf{p}^2)\frac{2p^\mu p^\nu-(p^2-m^2)g^{\mu\nu}}{(p^2-m^2+\mathrm{i}\varepsilon)^2}\\
&\equiv -g^{\mu\nu}m^2\operatorname{reg}\Pi_{\text{cov}}\left(\frac{\Lambda}{m}\right), \qquad (71)
\end{aligned}$$

$$\operatorname{reg}\Delta\Pi^{\mu\nu}_{\text{even}}(b,m,\Lambda) = 2I_0^{\mu\nu}(b,m)+J_0^{\mu\nu}(b,m)\equiv Ag^{\mu\nu}b^2+Bb^\mu b^\nu \qquad (72)$$

and suppose the constant vector breaking Lorentz symmetry to be purely temporal, that is $b^\mu=(b,0,0,0)$. Then, on the one side, we find

$$\operatorname{reg}\Pi_{\text{cov}} = \frac{\mathrm{i}\alpha}{2m^2\pi^3}\int \mathrm{d}^4p\,\theta(\Lambda^2-\mathbf{p}^2)\frac{p^2-2m^2}{(p^2-m^2+\mathrm{i}\varepsilon)^2}\,, \qquad (73)$$

$$\operatorname{reg}\Delta\Pi^{\imath\jmath}_{\text{even}}(b,m,\Lambda)\delta_{\imath\jmath} = -3Ab^2\,. \qquad (74)$$

On the other side, we obtain

$$\begin{aligned}
\operatorname{reg}\Pi_{\text{cov}} &= \frac{2\mathrm{i}\alpha}{m^2\pi^2}\int_0^\Lambda \mathrm{d}p\,p^2\int_{-\infty}^{\infty}\mathrm{d}p_0\left[\frac{1}{D}-\frac{m^2}{D^2}\right]\\
&= \frac{\alpha}{\pi m^2}\int_0^\Lambda \mathrm{d}p\left(\frac{2p^2}{\omega_p}+\frac{p^2m^2}{\omega_p^3}\right) \qquad (75)
\end{aligned}$$

with

$$D\equiv p_0^2-\mathbf{p}^2-m^2+\mathrm{i}\varepsilon=(p_0-\omega_p+\mathrm{i}\varepsilon)(p_0+\omega_p-\mathrm{i}\varepsilon)\quad \omega_p=\sqrt{p^2+m^2}\,. \qquad (76)$$

After setting $\epsilon\equiv m/\Lambda\simeq 2b/m$, we get

$$\begin{aligned}
\operatorname{reg}\Pi_{\text{cov}} &= \frac{\alpha}{\pi\epsilon^2}(1+\epsilon^2)^{-1/2}\\
&= \frac{\alpha}{\pi\epsilon^2}\left(1-\frac{1}{2}\epsilon^2+\frac{3}{8}\epsilon^4\right)\\
&\quad +\mathrm{O}(\epsilon^4)\quad \left[\epsilon=\frac{m}{\Lambda}\simeq\frac{2b}{m}\ll 1\right], \qquad (77)
\end{aligned}$$

which means that the divergent photon mass term arising from the covariant piece of the vacuum polarization tensor within the ultraviolet physical cutoff regularization becomes

$$\operatorname{reg}\tilde{\Pi}^{\mu\nu}_{\text{cov}}(m,\Lambda) = g^{\mu\nu}m^2\,\frac{\alpha}{\pi}\left(-\frac{1}{\epsilon^2}+\frac{1}{2}-\frac{3}{8}\epsilon^2\right)+\mathrm{O}(\epsilon^4)\,. \qquad (78)$$

Next, the piece of 1-loop vacuum polarization tensor which in principle violate Lorentz invariance can be obtained as follows. First, we have

$$\begin{aligned} -2\delta_{\imath\jmath}I_0^{\imath\jmath}(b,m) &= \frac{16\mathrm{i}e^2b^2}{(2\pi)^4}\int \mathrm{d}^4p\,\theta(\Lambda^2-\mathbf{p}^2)[2\mathbf{p}^2+3(p_0^2-\mathbf{p}^2-m^2)] \\ &\quad\times\frac{m^2-p_0^2-\mathbf{p}^2}{(p_0^2-\mathbf{p}^2-m^2+\mathrm{i}\varepsilon)^4} \\ &= \frac{e^2b^2}{\mathrm{i}\pi^3}\int_0^\Lambda \mathrm{d}p\int_{-\infty}^{\infty}\mathrm{d}p_0\left(\frac{16p^6}{D^4}+\frac{32p^4}{D^3}+\frac{12p^2}{D^2}\right). \end{aligned} \tag{79}$$

Moreover, we get

$$\delta_{\imath\jmath}J_0^{\imath\jmath}(b,m) = \frac{e^2b^2}{\mathrm{i}\pi^3}\int_0^\Lambda \mathrm{d}p\int_{-\infty}^{\infty}\mathrm{d}p_0\left(\frac{3p^2}{D^2}-\frac{8p^6}{D^4}\right), \tag{80}$$

so that we can finally write

$$A = -\frac{\mathrm{i}e^2}{3\pi^3}\int_0^\Lambda \mathrm{d}p\int_{-\infty}^{\infty}\mathrm{d}p_0\left(24\frac{p^6}{D^4}+32\frac{p^4}{D^3}+9\frac{p^2}{D^2}\right), \tag{81}$$

and by performing first the energy integration

$$A = \frac{e^2}{2\pi^2}\int_0^\Lambda \mathrm{d}p\left(5\frac{p^6}{\omega_p^7}-8\frac{p^4}{\omega_p^5}+3\frac{p^2}{\omega_p^3}\right) \tag{82}$$

and then momentum integration,[95] we finally obtain

$$A = \frac{2\alpha}{\pi}\cdot\frac{m^2}{\Lambda^2}\left(1+\frac{m^2}{\Lambda^2}\right)^{-\frac{5}{2}}. \tag{83}$$

Next we turn to the calculation of the coefficient B. To this concern, we find

$$g_{\mu\nu}\,\mathrm{reg}\,\Delta\Pi^{\mu\nu}_{\mathrm{even}}(b,m,\Lambda) = (4A+B)b^2 \tag{84}$$

and contraction with the metric tensor gives

$$g_{\mu\nu}[2I_0^{\mu\nu}(b,m)+J_0^{\mu\nu}(b,m)] = \frac{3e^2b^2}{2\pi^2}m^2\int_0^\Lambda \mathrm{d}p\left(3\frac{p^2}{\omega_p^5}-5\frac{p^4}{\omega_p^7}\right). \tag{85}$$

It follows that we have

$$4A+B = \frac{3e^2m^2}{2\pi^2}\int_0^\Lambda \mathrm{d}p\left(3\frac{p^2}{\omega_p^5}-5\frac{p^4}{\omega_p^7}\right) \tag{86}$$

and from Ref. 95 we eventually obtain

$$4A+B = \frac{3e^2}{2\pi^2}\left(\frac{\Lambda^3}{u^3}-\frac{\Lambda^5}{u^5}\right) \tag{87}$$

and consequently

$$B = -\frac{2\alpha}{\pi}\cdot\frac{m^2}{\Lambda^2}\left(1+\frac{m^2}{\Lambda^2}\right)^{-\frac{5}{2}} = -A\,. \tag{88}$$

Hence, the physical high momentum fermion cutoff leads to the following result for the vacuum polarization tensor in the limit of a null photon momentum:

$$\lim_{k\to 0} \operatorname{reg} \Delta\Pi^{\mu\nu}_{\text{even}}(k,b,m,\Lambda) = \frac{2\alpha}{\pi}\cdot\frac{m^2}{\Lambda^2}\left(1+\frac{m^2}{\Lambda^2}\right)^{-\frac{5}{2}}(b^2 g^{\mu\nu} - b^\mu b^\nu) \tag{89}$$

which corresponds to a *bona fide* vanishing contribution to the photon mass in the presence of a very large momentum cutoff for spinor matter.

To sum up, we see that in order to remove the previously obtained divergent photon mass term for $b^2 > 0$, one has to introduce the Proca mass Lorentz invariant counterterm

$$\mathcal{L}_{\text{ct}} = \frac{1}{2} Z_\gamma m^2 A_\mu A^\mu \tag{90}$$

in which

$$Z_\gamma = \frac{\alpha}{\pi}\left(\frac{1}{\epsilon^2} - F_\gamma\right) + \mathrm{O}(\epsilon^4)\,, \tag{91}$$

where $F_\gamma(\epsilon)$ is an arbitrary function analytic for $\epsilon \to 0$. A comparison with the result (68) fix the finite part of the counterterm to be given by

$$F_\gamma(\epsilon) = \frac{1}{2} - \frac{13}{24}\epsilon^2 + \mathrm{O}(\epsilon^4)\,. \tag{92}$$

3.3. *IPM at finite temperature*

According to Refs. 65, 64, 69, it is a consistency check to get the radiatively induced photon mass term at a finite temperature T. To this concern, let us perform the Wick rotation with the conventional substitutions

$$\begin{gathered} \frac{1}{(2\pi)^4}\int \mathrm{d}^4 p\, \theta(\Lambda^2 - \mathbf{p}^2) \to \frac{\mathrm{i}}{\beta}\sum_{n=-\infty}^{\infty}\int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\,\theta(\Lambda^2 - \mathbf{p}^2)\,, \\ p_0 \to \mathrm{i}\omega_n = \frac{\pi \mathrm{i}}{\beta}(2n+1) \quad n \in \mathbb{Z}\,, \end{gathered} \tag{93}$$

where $\beta = 1/kT$, k being the Boltzmann constant. Hence, we obtain

$$m^2 \Pi_\beta = \frac{4\alpha}{\pi\beta}\int_0^\Lambda \mathrm{d}p\, p^2 \sum_{n=-\infty}^{\infty}\left(\frac{1}{D_1} + \frac{m^2}{D_1^2}\right), \tag{94}$$

$$A_\beta = \frac{8\alpha}{3\pi\beta}\sum_{n=-\infty}^{\infty}\int_0^\Lambda \mathrm{d}p\, p^2\left(24\frac{p^4}{D_1^4} - 32\frac{p^2}{D_1^3} + \frac{9}{D_1^2}\right) \tag{95}$$

with

$$D_1 = \omega_n^2 + m^2 + p^2 = \frac{\pi^2}{\beta^2}(2n+1)^2 + p^2 + m^2\,. \tag{96}$$

Now, if we suitably set

$$D_z = \omega_n^2 + m^2 + zp^2 \equiv a + zp^2 \quad [az > 0]\,, \tag{97}$$

we can rewrite the above expression in the form

$$m^2\Pi_\beta = \frac{4\alpha}{\pi\beta}\sum_{n=-\infty}^{\infty}\left[\frac{\partial}{\partial a}\int_0^\Lambda \mathrm{d}p\, p^2 \ln D_1 - \lim_{z\to 1}\frac{\mathrm{d}}{\mathrm{d}z}\int_0^\Lambda \mathrm{d}p\,\frac{m^2}{D_z}\right], \tag{98}$$

$$A_\beta = -\frac{8\alpha}{3\pi\beta}\sum_{n=-\infty}^{\infty}\lim_{z\to 1}\left(9\frac{\mathrm{d}}{\mathrm{d}z} + 16\frac{\mathrm{d}^2}{\mathrm{d}z^2} + 4\frac{\mathrm{d}^3}{\mathrm{d}z^3}\right)\int_0^\Lambda \frac{\mathrm{d}p}{D_z}\,. \tag{99}$$

From Ref. 95 we get

$$\begin{aligned} A_\beta &= -\frac{8\alpha}{3\pi\beta}\sum_{n=-\infty}^{\infty}\lim_{z\to 1}\left(9\frac{\mathrm{d}}{\mathrm{d}z} + 16\frac{\mathrm{d}^2}{\mathrm{d}z^2} + 4\frac{\mathrm{d}^3}{\mathrm{d}z^3}\right) f(z)\,, \\ f(z) &= \int_0^\Lambda \frac{\mathrm{d}p}{D_z} = \frac{1}{\sqrt{(az)}}\,\mathrm{arctg}\left(\Lambda\sqrt{\frac{z}{a}}\right), \end{aligned} \tag{100}$$

$$\frac{\partial}{\partial a}\int_0^\Lambda \mathrm{d}p\, p^2 \ln D_1 = \Lambda - \sqrt{a}\,\mathrm{arctg}\,\frac{\Lambda}{\sqrt{a}}\,, \tag{101}$$

in such a manner that we eventually find for $a = \omega_n^2 + m^2$:

$$\begin{aligned} f(1) &= \frac{1}{\sqrt{a}}\,\mathrm{arctg}\,\frac{\Lambda}{\sqrt{a}} = \frac{1}{\sqrt{(\omega_n^2+m^2)}}\,\mathrm{arctg}\,\frac{\Lambda}{\sqrt{(\omega_n^2+m^2)}}\,, \\ \frac{\mathrm{d}f}{\mathrm{d}z} &= -\frac{1}{2z}\left[f(z) - \frac{\Lambda}{a+z\Lambda^2}\right], \\ \frac{\mathrm{d}^2 f}{\mathrm{d}z^2} &= \frac{3}{4z^2}\left[f(z) - \frac{\Lambda}{a+z\Lambda^2}\right] - \frac{1}{2z}\cdot\frac{\Lambda^3}{(a+z\Lambda^2)^2}\,, \\ \frac{\mathrm{d}^3 f}{\mathrm{d}z^3} &= -\frac{15}{8z^3}\left[f(z) - \frac{\Lambda}{a+z\Lambda^2}\right] + \frac{5a\Lambda^3 + (4+5z)\Lambda^5}{4z^2(a+z\Lambda^2)^3}\,. \end{aligned} \tag{102}$$

As a preliminary check of the above formulae, let us first reproduce the value of the integral which occurs in the calculation of the temperature dependent parity odd part for the photon polarization tensor. To this aim, consider the integral[64]

$$\begin{aligned} I_{\mathrm{odd}} &= \int_0^\Lambda \mathrm{d}p\, p^2\,\frac{3\omega_n^2 + 3m^2 - p^2}{(\omega_n^2+m^2+p^2)^3} \\ &= \lim_{z\to 1}\left(-3\frac{\mathrm{d}}{\mathrm{d}z} - 2\frac{\mathrm{d}^2}{\mathrm{d}z^2}\right)\int_0^\Lambda \frac{\mathrm{d}p}{D_z} \\ &= \frac{3}{2}\left[f(1) - \frac{\Lambda}{a+\Lambda^2}\right] - \frac{3}{2}\left[f(1) - \frac{\Lambda}{a+\Lambda^2}\right] + \frac{\Lambda^3}{(a+\Lambda^2)^2} \\ &= \frac{\Lambda^3}{(\omega_n^2+m^2+\Lambda^2)^2} = \frac{\Lambda^3(\beta/\pi)^4}{[(2n+1)^2 + (\beta/\pi)^2(\Lambda^2+m^2)]^2}\,, \end{aligned} \tag{103}$$

which is in perfect agreement with Eq. (11) of Ref. 64. Moreover, after setting

$$\eta \equiv \frac{\beta}{\pi}\sqrt{m^2+\Lambda^2}\,, \tag{104}$$

by taking the sum from Ref. 95, we have

$$\begin{aligned}
&\sum_{n=-\infty}^{\infty} \frac{2e^2\Lambda^3\beta^3/\pi^6}{[(2n+1)^2+(\beta/\pi)^2(\Lambda^2+m^2)]^2} \\
&\quad = \frac{e^2\Lambda^3\beta^3}{4\eta^3\pi^5}\left(2\tanh\frac{\pi\eta}{2} - \pi\eta\,\mathrm{sech}^2\,\frac{\pi\eta}{2}\right) \\
&\quad \approx \frac{\alpha}{\pi}\left[2\tanh\frac{\pi\eta}{2} - \pi\eta + \pi\eta\tanh^2\frac{\pi\eta}{2}\right] = \begin{cases} 0 & \text{for } \beta = 0\,, \\ \dfrac{2\alpha}{\pi} & \text{for } T = 0\,, \end{cases}
\end{aligned} \tag{105}$$

which is again in accordance with Eq. (11) of Ref. 65.

Turning back to the parity even part of vacuum polarization, we find for $a = \omega_n^2 + m^2$:

$$\begin{aligned}
m^2\Pi_\beta &= \frac{4\alpha}{\pi\beta}\sum_{n=-\infty}^{\infty}\left\{\Lambda - \sqrt{a}\,\mathrm{arctg}\,\frac{\Lambda}{\sqrt{a}}\right. \\
&\qquad \left. + \frac{1}{2}m^2\left[\frac{1}{\sqrt{a}}\mathrm{arctg}\frac{\Lambda}{\sqrt{a}} - \frac{\Lambda}{a+\Lambda^2}\right]\right\}\,,
\end{aligned} \tag{106}$$

$$\begin{aligned}
A_\beta &= \frac{8\alpha\Lambda^3}{3\pi\beta}\sum_{n=-\infty}^{\infty}\frac{3a-\Lambda^2}{(a+\Lambda^2)^3} \\
&= \frac{8\alpha\Lambda^3}{3\pi\beta}\sum_{n=-\infty}^{\infty}\left\{\frac{3}{(a+\Lambda^2)^2} - \frac{4\Lambda^2}{(a+\Lambda^2)^3}\right\} \\
&= \frac{8\alpha}{3\pi^2}\left\{3\left(\frac{\beta\Lambda}{\pi}\right)^3\Sigma_2 - 4\left(\frac{\beta\Lambda}{\pi}\right)^5\Sigma_3\right\}\,,
\end{aligned} \tag{107}$$

in such a manner that we come to the limiting values

$$\lim_{\beta\to 0} A_\beta = 0\,, \quad \lim_{\beta\to\infty} A_\beta = \frac{2\alpha}{\pi}\cdot\frac{m^2}{\Lambda^2}\left(1+\frac{m^2}{\Lambda^2}\right)^{-\frac{5}{2}}\,, \tag{108}$$

in perfect agreement with our previous calculation. On the other side, we have that the following integral representations hold true:

$$\Lambda - \sqrt{a}\mathrm{arctg}\frac{\Lambda}{\sqrt{a}} = \int_0^\Lambda \mathrm{d}y\;\frac{y^2}{y^2+a}\,, \tag{109}$$

$$\frac{1}{\sqrt{a}}\mathrm{arctg}\frac{\Lambda}{\sqrt{a}} = \int_0^\Lambda \frac{\mathrm{d}y}{y^2+a}\,. \tag{110}$$

Then we find, after setting $Y(y) = (\beta/\pi)\sqrt{m^2+y^2}$, $\eta = (\beta/\pi)\sqrt{m^2+\Lambda^2}$,

$$\begin{aligned} m^2\Pi_\beta &= \frac{4\alpha}{\pi\beta}\int_0^\Lambda \mathrm{d}y \sum_{n=-\infty}^{\infty}\left\{\frac{y^2+m^2/2}{y^2+\omega_n^2+m^2} - \frac{m^2/2}{\omega_n^2+m^2+\Lambda^2}\right\} \\ &= \frac{4\alpha\beta}{\pi^3}\int_0^\Lambda \mathrm{d}y \sum_{n=0}^{\infty}\left\{\frac{2y^2+m^2}{(2n+1)^2+Y^2} - \frac{m^2}{(2n+1)^2+\eta^2}\right\} \\ &= \frac{\alpha\beta}{\pi^2}\left\{\int_0^\Lambda \frac{\mathrm{d}y}{Y}\,(2y^2+m^2)\tanh\frac{\pi Y}{2} - \frac{m^2\Lambda}{\eta}\tanh\frac{\pi\eta}{2}\right\}. \end{aligned} \tag{111}$$

Hence we finally get

$$\Pi_\beta = \begin{cases} 0 & \text{for } \beta = 0\,, \\ \Pi_\infty & \text{for } T = 0\,, \end{cases} \tag{112}$$

where

$$\Pi_\infty = \left(\frac{\alpha}{\pi}\right)\frac{1}{\epsilon^2}(1+\epsilon^2)^{-1/2}\,, \quad \epsilon = \frac{m}{\Lambda}\,, \tag{113}$$

in full agreement once again with our previous equation (77).

3.4. *Discussion of the physical meaning*

Let us discuss the physical meaning of the above result. The large momentum physical bound (60) for spinor matter is necessarily provided by the fermion operator $\bar\psi\gamma^\mu\gamma_5\psi$, which is CPT odd and of mass dimension three, coupled to the Æther's temporal vector $b_\mu = (b,0,0,0)$.

This means that the LIV 1-particle states of momentum $\mathbf{p}$, for charged massive spinor fields, can be understood as a complete set of stable asymptotic states for each polarization, if and only if the momenta stay below the large energy–momentum physical bound Λ. For a universal flavor independent Æther, a conservative ultraviolet momentum cutoff for $\tilde{e}^{\mp}$ particles is expressed by

$$|\mathbf{p}| < \Lambda_e \simeq \frac{m_e^2}{2b} < 10^{26}\ \mathrm{eV}\,. \tag{114}$$

Once the existence of such a large physical ultraviolet cutoff has been acknowledged for spinor matter, then the radiatively induced parameters can be determined. In particular, the 1-loop radiatively induced Chern–Simons coefficient and $\tilde\gamma$-photon mass are given by[61,64,69]

$$\Delta\eta^\nu = -\frac{2\alpha}{\pi}b^\nu\,, \quad \Delta m_\gamma^2 = \frac{2\alpha}{3\pi}b^\mu b_\mu\,, \tag{115}$$

where α is the fine structure constant, as a result of $\overline{\mathrm{DR}}$-dimensional regularization. We remark that the above induced Chern–Simons coefficient turns out to be finite and it is not screened by any power-like or logarithmic divergence. It is thereby uniquely reproduced in the physical cutoff regularization as well as with the finite temperature method. For the parity even part induced by LIV, the relationship between the physically motivated regularizations is less definite, as the quadratic divergence in the Lorentz invariant part is dominant and screens the subleading LIV effects. In this case, the proper subtraction of the Lorentz invariant divergence is provided by the requirement of universality, i.e. coincidence with $\overline{\mathrm{DR}}$-dimensional regularization.

3.5. *LIV vacuum polarization at the leading order*

After a straightforward but tedious calculation, it is possible to prove that the k-dependent part of the Lorentz invariance violating $\mathrm{O}(b^2)$ correction to the 1-loop vacuum polarization tensor does actually fulfill transversality with respect to the external momentum k_μ, as well as to the Æther's vector b_μ. It is convenient to write[66] the 1-loop $\tilde{\gamma}$-photon self-energy, or LIV QED vacuum polarization tensor, up to the quadratic approximation in the Lorentz symmetry breaking four-vector b_μ. It consists of the sum of the Lorentz covariant part and of the radiatively generated Lorentz symmetry breaking part:

$$\begin{aligned} \operatorname{reg}\Pi^{\mu\nu}(b,k,m) &\approx \operatorname{reg}\Pi^{\mu\nu}_{\mathrm{cov}}(k,m) + \Delta\Pi^{\mu\nu}_{\mathrm{even}}(b,k,m) + \Delta\Pi^{\mu\nu}_{\mathrm{odd}}(b,k,m)\,, \\ \operatorname{reg}\Pi^{\mu\nu}_{\mathrm{cov}}(k,m) &= (k^2 g^{\mu\nu} - k^\mu k^\nu)\operatorname{reg}\Pi(k^2)\,, \end{aligned} \tag{116}$$

$$\begin{aligned} \Delta\Pi^{\mu\nu}_{\mathrm{even}}(b,k,m) &\approx \frac{\alpha}{\pi}\left\{\frac{2}{3}b^2 g^{\mu\nu} - A(b,k,m)S^{\mu\nu}\right\}, \\ \Delta\Pi^{\mu\nu}_{\mathrm{odd}}(b,k,m) &\approx 2\mathrm{i}\frac{\alpha}{\pi}\,\varepsilon^{\mu\nu\rho\sigma}b_\rho k_\sigma\,, \end{aligned} \tag{117}$$

where

$$S^{\mu\nu} = [(b\cdot k)^2 - b^2k^2]\bar{g}^{\mu\nu} - (b\cdot k)(b^\mu k^\nu + b^\nu k^\mu) + b^2 k^\mu k^\nu + k^2 b^\mu b^\nu\,, \tag{118}$$

whereas, using dimensional regularization with 2ω space–time dimensions,

$$\operatorname{reg}\Pi(k^2) = -\frac{\alpha}{3\pi}\left(\frac{1}{2-\omega} - \mathbf{C} - \ln\frac{m^2}{4\pi\mu^2} + \frac{k^2}{5m^2}\right) + O\left(\frac{k^2}{m^2}\right)^2. \tag{119}$$

The form factor $A(b,k,m)$ can be calculated by looking, e.g. at the coefficient of the tensor

$$(b\cdot k)(b^\mu k^\nu + b^\nu k^\mu)\,,$$

which can be readily extracted from the integral of Eq. (A.26) in the App. A. We find

$$\begin{aligned} &\frac{\alpha}{\pi} b\cdot k(b^\mu k^\nu + b^\nu k^\mu)A(b,k,m) \\ &\quad = -8\mathrm{i}e^2\int_p \frac{b\cdot(p+k)(p^2-m^2) + b\cdot p[(p+k)^2-m^2]}{(p^2-m^2+\mathrm{i}\varepsilon)^2[(p+k)^2-m^2+\mathrm{i}\varepsilon]^2} \\ &\qquad \times [b^\mu p^\nu + b^\nu p^\mu + b^\mu(p+k)^\nu + b^\nu(p+k)^\mu] \end{aligned}$$

$$
\begin{aligned}
&- 16\mathrm{i}e^2m^2 b\cdot k(b^\mu k^\nu + b^\nu k^\mu)I(2,2)\\
&= -\frac{4\alpha m^2}{\pi(k^2)^2} b\cdot k(b^\mu k^\nu + b^\nu k^\mu)\int_0^1 \frac{dx}{R^2}\,x(1-x)\,, \qquad (120)
\end{aligned}
$$

where $R \equiv x^2 - x + m^2/k^2$. From Ref. 95 we obtain

$$
\begin{aligned}
A(b,k,m) &= -\frac{4m^2}{(k^2)^2}\left\{\frac{2k^2}{4m^2-k^2} + \frac{2m^2-k^2}{4m^2-k^2}\int_0^1 \frac{dx}{R}\right\}\\
&= \frac{-8m^2}{k^2(4m^2-k^2)} - \frac{16m^2(2m^2-k^2)}{[-k^2(4m^2-k^2)]^{3/2}}\cdot \mathrm{Arcth}\sqrt{\frac{-k^2}{4m^2-k^2}}\,. \qquad (121)
\end{aligned}
$$

By performing the Wick rotation in turning to the Euclidean formulation, we get

$$
\begin{aligned}
A(b,k_E,m) &= \frac{8m^2}{k_E^2(4m^2+k_E^2)} - \frac{16m^2(2m^2+k_E^2)}{[k_E^2(4m^2+k_E^2)]^{3/2}}\cdot \mathrm{Arcth}\sqrt{\frac{k_E^2}{4m^2+k_E^2}}\\
&= \left\{(4m^2+2k_E^2)\mathrm{Arcth}\sqrt{\frac{k_E^2}{4m^2+k_E^2}} - \sqrt{k_E^2(4m^2+k_E^2)}\right\}\\
&\quad\times \frac{8m^2}{[k_E^2(4m^2+k_E^2)]^{3/2}}\,. \qquad (122)
\end{aligned}
$$

We notice that in the limit of a null external momentum, we find

$$
\lim_{k\to 0} k^2 A(b,k,m) = \frac{-2}{3m^2} \qquad (123)
$$

which entails that the order of magnitude of the Lorentz invariance violating corrections to the photon polarization tensor turns out to be vanishingly small, viz. $b^2/m^2 \simeq 4.5\times 10^{-42}$. It follows therefrom[f] that the momentum space effective kinetic operator takes the form

$$
\frac{1}{2}\,(-k^2g^{\mu\nu} + k^\mu k^\nu)[1-\Pi(0)] + \frac{\alpha}{3\pi}(b^2g^{\mu\nu} + m^{-2}S^{\mu\nu}) + \frac{\alpha}{\pi}\mathrm{i}\varepsilon^{\mu\nu\rho\sigma}b_\rho k_\sigma \qquad (124)
$$

which drives to the action and Lagrangian counterterm in the BPHZ subtraction scheme

$$
\mathcal{A}_{\mathrm{ct}} = \frac{1}{2}\,\Pi(0)\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\tilde{A}_\mu(k)(-k^2g^{\mu\nu} + k^\mu k^\nu)\tilde{A}_\nu(-k)\,, \qquad (125)
$$

$$
\mathcal{L}_{\mathrm{ct}} = -\frac{1}{4}\,(Z_3-1)F^{\mu\nu}(x)F_{\mu\nu}(x)\,, \qquad (126)
$$

$$
Z_3 = 1 + \Pi(0) = 1 + \frac{\alpha}{3\pi}\left(-\frac{1}{\epsilon} + \mathbf{C} + \ln\frac{m^2}{4\pi\mu^2}\right). \qquad (127)
$$

[f]According to our conventions, the 2-point proper vertex turns out to be $\Gamma^{\mu\nu} = -k^2g^{\mu\nu} + k^\mu k^\nu + \Pi^{\mu\nu}$.

Moreover, we retrieve[66] the finite lowest-order radiatively induced local effective Lagrangian for the LIV $\tilde{\gamma}$-photon:

$$\begin{aligned}
\mathcal{L}_{\text{eff}} = & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\left(1+\frac{2\alpha b^2}{3\pi m^2}\right)+\frac{\alpha b^2}{3\pi}A^\mu A_\mu \\
& +\frac{\alpha}{3\pi m^2}\, b_\nu b^\rho F^{\nu\lambda}F_{\rho\lambda}-\frac{\alpha}{2\pi}b_\lambda A_\mu \epsilon^{\lambda\mu\rho\sigma}F_{\rho\sigma} \\
= & -\frac{1}{4}\,(1+\varepsilon)F^{\mu\nu}F_{\mu\nu}+\frac{1}{2}\,\varepsilon m^2 A^\mu A_\mu \\
& +\varepsilon\frac{b^\lambda b^\nu}{2b^2}F_{\lambda\rho}F_\nu{}^\rho-\frac{\alpha}{\pi}b_\mu A_\nu \tilde{F}^{\mu\nu} \quad \left(\varepsilon=\frac{2\alpha b^2}{3\pi m^2}\right).
\end{aligned} \tag{128}$$

4. Summary of the Main Results

To sum up, we would like to list the main achievements of our tough analysis and make few more comments on consistency requirements and estimates for the LIV vector components.

We recall that a temporal Æther b^μ is actually required for a consistent fermion quantization.[61,73,80] However, in the lack of a photon mass and/or a bare CS vector of different direction, it appears that a temporal Æther $b^\mu = (b,0,0,0)$ just leads[25–27,73] to the instability of the photon dynamics, that means imaginary energies for the soft photons. On the other hand, a spacelike vector b^μ causes problems for fermion quantization[61,80] and thereby for the very meaning of the radiative corrections.

To avoid this mismatch with classically massless photons, one can adopt the induced LIV for *light-like* axial vectors b^μ. In particular, for a light-like universal axial vector $b^\mu = (\pm b, \mathbf{b})$ with $b = |\mathbf{b}|$, we find the dispersion relations for the LIV 1-particle states of a fermion species f that read

$$p^0_+ + b = \pm\sqrt{(\mathbf{p}+\mathbf{b})^2+m_f^2}\,, \quad p^0_- - b = \pm\sqrt{(\mathbf{p}-\mathbf{b})^2+m_f^2}\,. \tag{129}$$

Now, it turns out that the requirement $p^2_\pm > 0$ for the LIV free 1-particle spinor physical states only drives to the high momenta cutoff $|\mathbf{p}| \leq m_e^2/4b$. However, one has to keep in mind that the room for the existence of a universal privileged spatial direction, such as the Æther vector $\mathbf{b}$, is essentially excluded by the torsion pendulum experiments with polarized electrons[71] which yield $|\mathbf{b}| \leq 5\times 10^{-21}$ eV, that should be compared to the benchmark value $m_e^2/M_{\text{Planck}} = 2\times 10^{-17}$ eV.

Thus, the only alternative to implement the LIV, with an essential contribution from the fermion sector, consists in supplementing the photon dynamics with a finite Lorentz invariant photon mass term so that

$$m_\gamma^2 = \mu_\gamma^2 + \Delta m_\gamma^2 = \mu_\gamma^2 + \frac{2\alpha}{3\pi}\sum_f q_f^2 b_f^2 \tag{130}$$

for fermions with different charges q_f and LIV four vectors b_f, where μ_γ is a classical photon mass. For a universal temporal vector $b_f^\mu \equiv b^\mu = (b, 0, 0, 0)$, one finds the dispersion law

$$k_0^2 = \left(|\mathbf{k}| \pm \frac{8\alpha}{\pi} b\right)^2 + m_\gamma^2 - b^2 \left\{ \left(\frac{8\alpha}{\pi}\right)^2 + O\left(\frac{b|\mathbf{k}|}{m_e^2}\right) \right\}. \tag{131}$$

Hence, if $m_\gamma \geq 8\alpha b/\pi$, then the photon energy remains real for any wave-vector $\mathbf{k}$ and LIV QED happens to be consistent and the longitudinal polarization appears, due to the presence of a tiny photon mass m_γ. Then, the up-to-date very stringent experimental bound on the photon mass,[72] $m_\gamma < 6 \times 10^{-17}$ eV, does produce the limit $b < 3 \times 10^{-15}$ eV. Note that in Ref. 90 it is claimed that a closer bound on a birefringent photon mass comes from the last five years results on the oldest light of the universe, the Wilkinson Microwave Anisotropy Probe data, viz. $m_\gamma < 3 \times 10^{-19}$ eV, which entails the benchmark valued bound $b < 2 \times 10^{-17}$ eV and in turn $\Lambda_e \sim 10^{28}$ eV $\sim M_{\text{Planck}}$.

If the only source for LIV in QED is universally induced by fermions, one can compare these bounds with the following ones, which are required to align the threshold of fermion UV instability to the existing experimental data from LEP.[91,92] To fulfill it one has to provide roughly $\mu m_e/2b > 100$ GeV or $\mu > 4 \times 10^5 b$. It gives a more stringent estimation $b < 10^{-22}$ eV.

Even the first bound entails the very large cutoff $\Lambda_e \simeq m_e^2/2b \simeq 10^{26}$ eV, whereas the second, more stringent bound leads to the cutoff larger than the Planck mass scale. It means that only the leading order in the LIV vector expansion makes any practical sense.

There are no better bounds on b_μ coming from the UHECR's data on the speed of light for photons. This is because the increase of the speed of light depends quadratically on components of b_μ. Thus, for example, the data cited in Refs. 1–5, 9, 10, 15–17 do imply much less severe bounds on $|\mathbf{b}|$ or b_0 than those mentioned above. In particular, the above reported field theoretical constraints for internal consistency do actually lead to much more stringent limits on the temporal Æther $b^\mu = (b, 0, 0, 0)$ than the most recent UHECR's data.

For the Lorentz invariance violating modification of QED, as discussed in the present paper, the generic bounds on LIV and CPT breaking parameters within the quantum gravity phenomenology are less efficient to compete with the laboratory estimations. They are dominating over other LIV effects in the high energy astrophysics.

An interesting bound on the deviations of the speed of light is given in Ref. 99, where space–time fluctuations are addressed to produce modifications of the speed of light. However, their estimation does not yield a better bound for a LIV vector b_μ, yielding, at best, $b_0 < 10^{-12}$ eV, which is certainly less stringent as compared to the bounds discussed above.

5. Conclusions and Final Discussion

In this paper, the minimal model involving a Lorentz and CPT invariance violating modification of QED has been investigated thoroughly. We have shown explicitly that the introduction of a single LIV term $b_\mu\bar{\psi}(x)\gamma^\mu\gamma_5\psi(x)$ into the fermionic matter Lagrangian does indeed give rise to a profound modification of the whole theoretical model, in spite of the presently allowed very small value for the temporal axial vector Æther background $b_\mu = (b, 0, 0, 0)$ with $b < 3 \times 10^{-15}$ eV. Actually, the important lesson we have learned from the present investigation is that a tiny but nonvanishing breaking of the Lorentz and CPT symmetries does unavoidably produce some drastic changes into the model, even of a nonperturbative nature. As a paradigmatic example, it turns out that, above the threshold $\Lambda_e \sim m_e^2/2b$, electrons and positrons must decay by emitting tachyon-like photons and becoming left- or right-handed polarized. Since they are massive, their chiralities mix and therefore in a while they become again vector-like and again decay. Thus, finally, high energy electrons and positrons will be washed out from the asymptotic Hilbert space and will not contribute at all to the imaginary part of the vacuum polarization operator. Of course, this might appear a little bit embarrassing, as it hurts the long standing intuition and conventional wisdom, as developed in the Lorentz invariant quantum field theory. When one restores the full polarization operator with the help of the dispersion relations, one should keep thereby only momenta lower than the threshold.

In this paper, which is essentially focused on the calculation of the 1-loop vacuum polarization tensor, we acknowledge that a nonvanishing Chern–Simons vector and tiny photon mass are truly advocated and welcome for: (i) the propagation of some LIV effect with a speed less than one is allowed; (ii) the photodynamics is safely free from any acausal tachyonic effects; (iii) the radiative corrections perfectly match with the existence of the physical large momentum cutoff $\Lambda_e \lesssim M_{\text{Planck}}$, beyond which quantum gravity might jeopardize the Einstein relativity principles; (iv) the resulting effective quantum theory does fulfill locality, causality and unitarity up to the physical large momentum cutoff Λ_e that represents the intrinsic limit of the validity of the whole approach. In conclusion, to the aim of seriously approaching any possible Lorentz invariance violation for QED, one has to dismiss many of the well-established achievements as dictated by the application of the original Einstein's critical analysis about space–time causality to the realm of the quantum field theory.

Acknowledgments

This work was supported by Grants INFN IS-PI13 and INFN-MICINN Collaboration. The work of A. A. Andrianov was supported by CUR Generalitat de Catalunya under project 2009SGR502, by Grant FPA2007-66665, by the Consolider-Ingenio 2010 Program CPAN (CSD2007-00042), and by Grant RFBR 09-02-00073 and Program RNP2009-1575. The work of J. Alfaro was partially supported by Fondecyt

1060646. The work of M. Cambiaso was partially supported by projects DGAPA-UNAM-IN109107 and CONACYT # 55310. We are pleased to thank Dr. Fabrizio Sgrignuoli for his careful check of our calculations. P. Giacconi and R. Soldati wish to acknowledge the support of the Istituto Nazionale di Fisica Nucleare, Iniziativa Specifica PI13, that contributed to the successful completion of this project.

Appendix A

In this appendix, we shall develop the technical details that are involved in the evaluation of the 1-loop parity even part of the vacuum polarization tensor in the constant temporal Æther b_μ. The trace in Eq. (67) amounts to be

$$\begin{aligned}\mathrm{tr}\,\{&\bar{\gamma}^\mu(p^2+b^2-m^2+2b\cdot p\gamma_5+2mb_\alpha\bar{\gamma}^\alpha\gamma_5)(\gamma^\beta p_\beta+m+b_\beta\bar{\gamma}^\beta\gamma_5)\\&\times\bar{\gamma}^\nu[(p+k)^2+b^2-m^2+2b\cdot(p+k)\gamma_5+2mb_\lambda\bar{\gamma}^\lambda\gamma_5]\\&\times[\gamma^\sigma(p+k)_\sigma+m+b_\sigma\bar{\gamma}^\sigma\gamma_5]\}\,,\end{aligned}\tag{A.1}$$

where we have taken into account that the external indices μ, ν as well as the four-vector b_α are physical, i.e. $\mu,\ \nu,\ \alpha=0,1,2,3$ so that, consequently, the corresponding matrices $\bar{\gamma}^\mu,\bar{\gamma}^\nu$ are physical and contraction of b_α with a γ matrix always involves some $\bar{\gamma}$-matrix, whereas the loop momenta and any γ matrix contracted with it are 2ω dimensional, i.e. $\not{p}=p_\alpha\gamma^\alpha=\bar{p}_\alpha\bar{\gamma}^\alpha+\hat{p}_\alpha\hat{\gamma}^\alpha$, see for instance Refs. 83–85.

The general structure of the photon self-energy tensor has already been presented in Eq. (64). Here, we are interested in the Maxwell–Chern–Simons parity even term: the only nonvanishing contributions to such a term are given by the traces of the products of six, four and two gamma matrices, since the traces of the products of three and five gamma matrices do indeed vanish in 2ω-dimensions. A straightforward computation gives, up to the overall factor $\mathrm{tr}\,\mathbb{I}=2^\omega\equiv 4$ and keeping in mind that $\hat{g}^{\alpha\beta}p_\alpha p_\beta=-\hat{p}^2$,

(a) six gamma matrices:

$$\begin{aligned}&+4m^2\{b^2[p\cdot(p+k)+b^2]-2b\cdot(p+k)b\cdot p\}\bar{g}^{\mu\nu}+8m^2b\cdot kb^\nu p^\mu\\&\quad+8m^2b\cdot p(b^\mu k^\nu+b^\nu p^\mu+b^\mu p^\nu)-4m^2b^2(2p^\mu p^\nu+k^\mu p^\nu+k^\nu p^\mu)\\&\quad-8m^2(p^2+p\cdot k)b^\mu b^\nu-8m^2\hat{p}^2(2b^\mu b^\nu-b^2\bar{g}^{\mu\nu})\,;\end{aligned}\tag{A.2}$$

(b) four gamma matrices:

$$\begin{aligned}&-8b\cdot pb\cdot(p+k)\hat{p}^2\bar{g}^{\mu\nu}+\{2p^\mu p^\nu+k^\mu p^\nu+k^\nu p^\mu-p\cdot(p+k)\bar{g}^{\mu\nu}\}\\&\quad\times\{(p^2+b^2-m^2)[(p+k)^2+b^2-m^2]+4b\cdot pb\cdot(p+k)\}\\&\quad+(2b^\mu b^\nu-b^2\bar{g}^{\mu\nu})\{(p^2+b^2-m^2)[(p+k)^2+b^2-m^2]\\&\quad+4(b\cdot p)[b\cdot(p+k)]+2m^2[p^2+(p+k)^2+2b^2]\}\end{aligned}$$

$$
\begin{aligned}
&+\{2b\cdot(p+k)(p^2+b^2-m^2)+2b\cdot p[(p+k)^2+b^2-m^2]\}\\
&\times[2b\cdot p\bar{g}^{\mu\nu}-2(b^\mu p^\nu+b^\nu p^\mu)+b\cdot k\bar{g}^{\mu\nu}-b^\mu k^\nu-b^\nu k^\mu]\\
&+4m^2 b\cdot p[4b\cdot p\bar{g}^{\mu\nu}-2(b^\mu p^\nu+b^\nu p^\mu)-2b^\mu k^\nu]\\
&+4m^2 b\cdot k[4b\cdot p\bar{g}^{\mu\nu}-2b^\nu p^\mu+b\cdot k\bar{g}^{\mu\nu}-b^\mu k^\nu-b^\nu k^\mu]\\
&-2m^2b^2[p^2+(p+k)^2+2b^2-2m^2]\bar{g}^{\mu\nu}\,,
\end{aligned}
\tag{A.3}
$$

(c) two gamma matrices:

$$
m^2\{(p^2+b^2-m^2)[(p+k)^2+b^2-m^2]-4b\cdot pb\cdot(p+k)\}\bar{g}^{\mu\nu}\,. \tag{A.4}
$$

Putting altogether we find

$$
\begin{aligned}
&\operatorname{reg}\Pi^{\mu\nu}_{\mathrm{even}}(k,b,m,\mu)\\
&=4\mathrm{i}e^2\mu^{4-2\omega}(2\pi)^{-2\omega}\int \mathrm{d}^{2\omega}p\,N^{\mu\nu}(p,k,b,m)D(p,b,m)D(p+k,b,m)\,,
\end{aligned}
\tag{A.5}
$$

where the scalar propagator reads

$$
D(p,b,m)=\{(p^2+b^2-m^2+i\varepsilon)^2-4[(b\cdot p)^2-b^2m^2]\}^{-1}\,, \tag{A.6}
$$

so that

$$
\begin{aligned}
&[D(p,b,m)D(p+k,b,m)]^{-1}\\
&\quad=\{(p^2+b^2-m^2+\mathrm{i}\varepsilon)^2-4[(b\cdot p)^2-b^2m^2]\}\\
&\qquad\times\{[(p+k)^2+b^2-m^2+\mathrm{i}\varepsilon]^2-4[(b\cdot p+b\cdot k)^2-b^2m^2]\}\,.
\end{aligned}
\tag{A.7}
$$

Furthermore, we have

$$
\begin{aligned}
N^{\mu\nu}(p,k,b,m)=&-8b\cdot pb\cdot(p+k)\hat{p}^2\bar{g}^{\mu\nu}\\
&+\{2p^\mu p^\nu+k^\mu p^\nu+k^\nu p^\mu-\bar{g}^{\mu\nu}(p^2-m^2+k\cdot p)\}\\
&\times\{(p^2+b^2-m^2)[(p+k)^2+b^2-m^2]+4b\cdot pb\cdot(p+k)\}\\
&+(2b^\mu b^\nu-b^2\bar{g}^{\mu\nu})\{(p^2+b^2-m^2)[(p+k)^2+b^2-m^2]\\
&+4b\cdot pb\cdot(p+k)+2m^2[p^2+(p+k)^2+2b^2-4\hat{p}^2]\}\\
&+\{2b\cdot(p+k)(p^2+b^2-m^2)+2b\cdot p[(p+k)^2+b^2-m^2]\}\\
&\times[2b\cdot p\bar{g}^{\mu\nu}-2(b^\mu p^\nu+b^\nu p^\mu)+b\cdot k\bar{g}^{\mu\nu}-b^\mu k^\nu-b^\nu k^\mu]\\
&+2m^2\bar{g}^{\mu\nu}[2(b\cdot k)^2-b^2(k^2-2m^2)]-4m^2b\cdot k(b^\mu k^\nu+b^\nu k^\mu)\\
&-4m^2b^2(2p^\mu p^\nu+k^\mu p^\nu+k^\nu p^\mu)-8m^2(p^2+p\cdot k)b^\mu b^\nu\,.
\end{aligned}
\tag{A.8}
$$

Now we can expand the polarization tensor in powers of the Lorentz symmetry breaking axial vector b_λ. To this aim, let us first define the following quantities:

$$\begin{aligned} D(p,0,m) &\equiv D(p,m) = (p^2 - m^2 + i\varepsilon)^{-2}\,, \\ N^{\mu\nu}(p,k,0,m) &\equiv N_0^{\mu\nu}(p,k,m) = \{2p^\mu p^\nu + k^\mu p^\nu + k^\nu p^\mu \\ &\qquad - [p\cdot(p+k) - m^2]\bar{g}^{\mu\nu}\}(p^2 - m^2)[(p+k)^2 - m^2]\,, \end{aligned} \tag{A.9}$$

$$\frac{\partial}{\partial b_\lambda} D(p,b,m) = -4[D(p,b,m)]^2\{b^\lambda(p^2 + m^2 + b^2) - 2p^\lambda b\cdot p\}\,, \tag{A.10}$$

$$\begin{aligned} \frac{\partial^2}{\partial b_\kappa \partial b_\lambda} D(p,b,m) &= 32[D(p,b,m)]^3\{b^\lambda(p^2 + m^2 + b^2) - 2p^\lambda b\cdot p\} \\ &\quad \times \{b^\kappa(p^2 + m^2 + b^2) - 2p^\kappa b\cdot p\} - 4[D(p,b,m)]^2 \\ &\quad \times \{g^{\kappa\lambda}(p^2 + b^2 + m^2) + 2b^\lambda b^\kappa - 2p^\lambda p^\kappa\}\,, \end{aligned} \tag{A.11}$$

$$\begin{aligned} D''(p,b,m) &\equiv \frac{1}{2} b_\kappa b_\lambda \lim_{b\to 0} \frac{\partial^2}{\partial b_\kappa \partial b_\lambda} D(p;b,m) \\ &= -2[D(p,m)]^2\{b^2(p^2 + m^2) - 2(b\cdot p)^2\}\,, \end{aligned} \tag{A.12}$$

$$\begin{aligned} N_2^{\mu\nu}(p,k,b,m) &\equiv \frac{1}{2} b_\kappa b_\lambda \lim_{b\to 0} \frac{\partial^2}{\partial b_\kappa \partial b_\lambda} N^{\mu\nu}(p,k,b,m) \\ &= -8 b\cdot p b\cdot(p+k)\hat{p}^2 \bar{g}^{\mu\nu} \\ &\quad + \{2p^\mu p^\nu + k^\mu p^\nu + k^\nu p^\mu - \bar{g}^{\mu\nu}(p^2 - m^2 + k\cdot p)\} \\ &\quad \times \{b^2[(p+k)^2 + p^2 - 2m^2] + 4b\cdot p b\cdot(p+k)\} \\ &\quad + (2b^\mu b^\nu - b^2\bar{g}^{\mu\nu}) \\ &\quad \times \{(p^2 - m^2)[(p+k)^2 - m^2] + 2m^2[p^2 + (p+k)^2 - 4\hat{p}^2]\} \\ &\quad + \{2b\cdot(p+k)(p^2 - m^2) + 2b\cdot p[(p+k)^2 - m^2]\} \\ &\quad \times [2b\cdot p\bar{g}^{\mu\nu} - 2(b^\mu p^\nu + b^\nu p^\mu) + b\cdot k\bar{g}^{\mu\nu} - b^\mu k^\nu - b^\nu k^\mu] \\ &\quad + 2m^2\bar{g}^{\mu\nu}[2(b\cdot k)^2 - b^2(k^2 - 2m^2)] - 4m^2 b\cdot k(b^\mu k^\nu + b^\nu k^\mu) \\ &\quad - 4m^2 b^2(2p^\mu p^\nu + k^\mu p^\nu + k^\nu p^\mu) - 8m^2(p^2 + p\cdot k)b^\mu b^\nu\,. \end{aligned} \tag{A.13}$$

It follows that we can write, up to the lowest-order approximation,

$$\operatorname{reg}\Pi^{\mu\nu}_{\text{even}}(k,b,m,\mu) = \operatorname{reg}\{\Pi^{\mu\nu}_{\text{cov}}(k,m,\mu) + \Delta\Pi^{\mu\nu}_{\text{even}}(k,b,m,\mu)\} + \mathrm{O}(b^2)^2\,, \tag{A.14}$$

in which we have set

$$\begin{aligned} \operatorname{reg}\Pi^{\mu\nu}_{\text{cov}}(k,m,\mu) &= 4\mathrm{i}e^2 \int_p N_0^{\mu\nu}(p,k,m) D(p,m) D(p+k,m) \\ &= (k^2 g^{\mu\nu} - k^\mu k^\nu)\operatorname{reg}\Pi(k,m)\,, \end{aligned} \tag{A.15}$$

where

$$\int_p \equiv \mu^{2\epsilon}(2\pi)^{-2\omega}\int \mathrm{d}^{2\omega}p\,, \quad \epsilon = 2-\omega\,. \tag{A.16}$$

As it is well known (see, e.g. Ref. 94) the gauge invariant part of the dimensionally regularized Lorentz invariant polarization function is provided by

$$\begin{aligned}\operatorname{reg}\Pi(k,m) &= -\frac{8e^2\mu^{2\epsilon}}{(4\pi)^\omega}\int_0^1 \mathrm{d}x\, x(1-x)\Gamma(2-\omega)[m^2 - x(1-x)k^2]^{\omega-2}\\ &= -\frac{\alpha}{3\pi}\left\{\frac{1}{\epsilon} - \mathbf{C} + \ln\frac{4\pi\mu^2}{m^2} - \int_0^1 \frac{\mathrm{d}x}{R}(4x^4 - 8x^3 + 3x^2)\right\}\\ &\quad + \text{irrelevant for } \epsilon \to 0\,,\end{aligned} \tag{A.17}$$

with $R = x^2 - x + m^2/k^2$. Note that we have

$$\int_0^1 \frac{\mathrm{d}x}{R}(4x^4 - 8x^3 + 3x^2) \overset{k\to 0}{\sim} -\frac{k^2}{5m^2} \tag{A.18}$$

so that the well-known Uehling–Serber result is recovered

$$\operatorname{reg}\Pi(k,m) - \operatorname{reg}\Pi(0;m^2) = \frac{-\alpha k^2}{15\pi m^2} + \mathrm{O}\left(\frac{k^2}{m^2}\right)^2. \tag{A.19}$$

The covariant divergent part can be removed from vacuum polarization by adding the usual local counterterm

$$\mathcal{L}_{ct} = -\frac{1}{4}(Z_3 - 1)F^{\mu\nu}F_{\mu\nu}\,, \tag{A.20}$$

where the lowest-order photon wave function renormalization constant in the BPHZ on the mass shell subtraction scheme[93] is given by

$$Z_3 = 1 - \frac{\alpha}{3\pi}\left(\frac{1}{\epsilon} - \mathbf{C} + \ln\frac{4\pi\mu^2}{m^2}\right) + \mathrm{O}(\epsilon)\,, \tag{A.21}$$

whence we can immediately segregate the divergent part of the $\mathrm{O}(\alpha)$ shift in the electric charge[94]

$$Z_3 - 1 = \operatorname{reg}\Pi(0;m^2) + \mathrm{O}(\alpha^2) \approx \frac{-\alpha}{3\pi\epsilon}\,, \tag{A.22}$$

so that we eventually come to the well known (see, e.g. Refs. 93 and 94) finite expression for the polarization invariant function in the BPHZ on the mass shell subtraction scheme

$$\hat{\Pi}(k,m) = \frac{\alpha}{3\pi}\left\{\frac{4}{3} - \int_0^1 \frac{\mathrm{d}x}{R}\left[4x^3 - x^2\left(3 - \frac{4m^2}{k^2}\right)\right]\right\}. \tag{A.23}$$

The first correction to the even part of the vacuum polarization tensor (formally) violating Lorentz and CPT invariance reads

$$
\begin{aligned}
&\operatorname{reg} \Delta\Pi^{\mu\nu}_{\text{even}}(k,b,m,\mu) \\
&\quad = 4\mathrm{i}e^2\mu^{2\epsilon}\int \mathrm{d}^{2\omega}p(2\pi)^{-2\omega}\{N_0^{\mu\nu}(p;k,m)D''(p;b,m)D(p+k;m) \\
&\qquad + N_0^{\mu\nu}(p;k,m)D(p;m)D''(p+k;b,m) \\
&\qquad + N_2^{\mu\nu}(p;k,b,m)D(p;m)D(p+k;m)\} \equiv 2I^{\mu\nu} + J^{\mu\nu}\,.
\end{aligned}
\tag{A.24}
$$

According to Eq. (A.13), we have

$$
\begin{aligned}
I_{\mu\nu} &= -8\mathrm{i}e^2\int_p \left\{2p^\mu p^\nu + k^\mu p^\nu + k^\nu p^\mu - [p\cdot(p+k) - m^2]\bar{g}^{\mu\nu}\right\} \\
&\qquad \times \frac{b^2(p^2-m^2) - 2(b\cdot p)^2 + 2b^2m^2}{(p^2-m^2+i\varepsilon)^3[(p+k)^2-m^2+i\varepsilon]}\,,
\end{aligned}
\tag{A.25}
$$

$$
\begin{aligned}
J^{\mu\nu}(k;b,m) &= \int_p \frac{-16\mathrm{i}e^2}{(p^2-m^2+i\varepsilon)^2[(p+k)^2-m^2+i\varepsilon]^2} \\
&\qquad \times \Bigg(2b\cdot pb\cdot(p+k)\hat{p}^2\bar{g}^{\mu\nu} \\
&\qquad - \frac{1}{4}\{2p^\mu p^\nu + k^\mu p^\nu + k^\nu p^\mu - \bar{g}^{\mu\nu}(p^2-m^2+p\cdot k)\} \\
&\qquad \times \{b^2[(p+k)^2+p^2-2m^2] + 4b\cdot pb\cdot(p+k)\} \\
&\qquad - \frac{1}{4}\,(2b^\mu b^\nu - b^2\bar{g}^{\mu\nu})\{(p^2-m^2)[(p+k)^2-m^2] \\
&\qquad + 2m^2[p^2+(p+k)^2-4\hat{p}^2]\} \\
&\qquad - \frac{1}{2}\,\{b\cdot(p+k)(p^2-m^2) + b\cdot p[(p+k)^2-m^2]\} \\
&\qquad \times [2b\cdot p\bar{g}^{\mu\nu} - 2(b^\mu p^\nu + b^\nu p^\mu) + b\cdot k\bar{g}^{\mu\nu} - b^\mu k^\nu - b^\nu k^\mu] \\
&\qquad - \frac{1}{2}\,m^2\bar{g}^{\mu\nu}[2(b\cdot k)^2 - b^2(k^2-2m^2)] + m^2 b\cdot k(b^\mu k^\nu + b^\nu k^\mu) \\
&\qquad + m^2b^2(2p^\mu p^\nu + k^\mu p^\nu + k^\nu p^\mu) + 2m^2(p^2+p\cdot k)b^\mu b^\nu\Bigg)\,.
\end{aligned}
\tag{A.26}
$$

Appendix B

In this appendix, we present the details of the calculation that leads to the appearance of the induced photon mass at 1-loop using the so-called $\overline{\mathrm{DR}}$ version of dimensional regularization. Actually, it turns out that a possible nonvanishing photon mass might be radiatively generated to the lowest order, within the LIV modification of QED and using dimensional regularization to deal with divergencies, if and only if

$$\mathrm{reg}\,\Delta\Pi^{\mu\nu}_{\mathrm{even}}(k=0,b,m,\mu) \equiv \mathrm{reg}\,\Delta\Pi^{\mu\nu}_{\mathrm{even}}(b,m,\mu) \neq 0\,. \tag{B.1}$$

Now, we have

$$\begin{aligned} I^{\mu\nu}(0;b,m) \equiv I_0^{\mu\nu}(b,m) = -8\mathrm{i}e^2 \int_p [2p^\mu p^\nu - (p^2-m^2)\bar{g}^{\mu\nu}] \\ \times \frac{b^2(p^2-m^2) - 2(b\cdot p)^2 + 2b^2m^2}{(p^2-m^2+\mathrm{i}\varepsilon)^4}\,, \end{aligned} \tag{B.2}$$

so that can we rewrite the above expression in terms of the basic integrals:

$$\begin{aligned} I_0^{\mu\nu}(b,m) = -16\mathrm{i}e^2 \Big\{ b^2 I^{\mu\nu}(3,0) - 2b_\rho b_\sigma I^{\mu\nu\rho\sigma}(4,0) + 2b^2m^2 I^{\mu\nu}(4,0) \\ - \frac{1}{2}\,\bar{g}^{\mu\nu} b^2 I(2,0) + \bar{g}^{\mu\nu} b_\rho b_\sigma I^{\rho\sigma}(3,0) - b^2m^2\bar{g}^{\mu\nu} I(3,0) \Big\}\,, \end{aligned} \tag{B.3}$$

where the basic covariant integral of generic tensor rank is defined by

$$I^{\mu\nu\cdots\rho\sigma}(n,0) \equiv \int_p \frac{p^\mu p^\nu \cdots p^\rho p^\sigma}{(p^2-m^2+\mathrm{i}\varepsilon)^n}\,. \tag{B.4}$$

In a quite similar way, we obtain

$$\begin{aligned} J^{\mu\nu}(0;b,m) \equiv J_0^{\mu\nu}(b,m) = 4\mathrm{i}e^2 \int_p (p^2-m^2+\mathrm{i}\varepsilon)^{-4} \\ \times \{-8(b\cdot p)^2\hat{p}^2\bar{g}^{\mu\nu} + m^2\bar{g}^{\mu\nu}[2b^2(p^2-m^2) + 4(b\cdot p)^2] \\ + (2p^\mu p^\nu - p^2\bar{g}^{\mu\nu})[2b^2(p^2-m^2) + 4(b\cdot p)^2] \\ + (2b^\mu b^\nu - b^2\bar{g}^{\mu\nu})[(p^2-m^2)^2 + 2m^2(2p^2-4\hat{p}^2)] \\ + 8b\cdot p(p^2-m^2)(b\cdot p\bar{g}^{\mu\nu} - b^\mu p^\nu - b^\nu p^\mu) \\ + 4b^2m^4\bar{g}^{\mu\nu} - 8m^2b^2p^\mu p^\nu - 8m^2p^2b^\mu b^\nu\} \end{aligned} \tag{B.5}$$

and, in terms of the basic integrals, we can eventually write

$$\begin{aligned} J_0^{\mu\nu}(b,m) = 16\mathrm{i}e^2\bar{g}^{\mu\nu} \Big\{ 2b_\rho b_\sigma \hat{g}_{\lambda\kappa} I^{\lambda\kappa\rho\sigma}(4,0) \\ - \frac{3}{4}\,b^2 I(2,0) + b_\rho b_\sigma I^{\rho\sigma}(3,0) - b^2m^2 I(3,0) \Big\} \\ - 16\mathrm{i}e^2 \Big\{ -b^2 I^{\mu\nu}(3,0) - 2b_\rho b_\sigma I^{\mu\nu\rho\sigma}(4,0) + 2b^2m^2 I^{\mu\nu}(4,0) \\ - \frac{1}{2}\,b^\mu b^\nu I(2,0) + 2b^\mu b_\rho I^{\nu\rho}(3,0) + 2b^\nu b_\rho I^{\mu\rho}(3,0) \Big\} \\ + \text{evanescent terms for } \omega \to 2\,. \end{aligned} \tag{B.6}$$

To sum up, the lowest-order Lorentz noninvariant mass term can be reduced to the following combination of the basic integrals:

$$\begin{aligned}
\mathrm{reg}\,\Delta\Pi^{\mu\nu}_{\text{even}}(b,m,\mu) &= 2I^{\mu\nu}_0(b,m) + J^{\mu\nu}_0(b,m) \\
&= 16\mathrm{i}e^2\bar{g}^{\mu\nu}\Big\{2b_\rho b_\sigma \hat{g}_{\lambda\kappa} I^{\lambda\kappa\rho\sigma}(4,0) \\
&\quad + \frac{1}{4}\,b^2 I(2,0) - b_\rho b_\sigma I^{\rho\sigma}(3,0) + b^2 m^2 I(3,0)\Big\} \\
&\quad - 16\mathrm{i}e^2\Big\{b^2 I^{\mu\nu}(3,0) - 6b_\rho b_\sigma I^{\mu\nu\rho\sigma}(4,0) + 6b^2 m^2 I^{\mu\nu}(4,0) \\
&\quad - \frac{1}{2}\,b^\mu b^\nu I(2,0) + 2b^\mu b_\rho I^{\nu\rho}(3,0) + 2b^\nu b_\rho I^{\mu\rho}(3,0)\Big\} \\
&\quad + \text{evanescent terms for } \omega \to 2\,.
\end{aligned} \tag{B.7}$$

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