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Evaluation Of Teaching And Mathematical Knowledge In Primary Teachers For The Teaching Of Probability

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Abstract

To provide information to transform teaching practice through improved primary school mathematics teacher training (6-12 years), a tool to assess the mathematical knowledge for teaching didactic probability has been designed. For its preparation the model of category analysis of the mathematical-didactic knowledge of the teacher was considered. This model includes four categories of fundamental knowledge: a) content knowledge, b) knowledge of the contents in relation to students, c) content knowledge in relation to education, and d) knowledge of curriculum and intra-and interdisciplinary connections. These categories can identify, classify, analyse and evaluate both the knowledge teachers need for teaching and the knowledge teachers actually implement when teaching a particular content, in this case the one related to the teaching of probability in elementary education. These skills have been being widely investigated in recent years in future teachers. Nevertheless, in Chile, there is very little data concerning current teachers. The analysis of the data will allow, first, to describe the strengths and weaknesses of the various categories of teacher knowledge involved in the teaching of probability, and secondly, relevant information to guide the initial and constant training of teachers in relation to the knowledge needed for the teaching of probability.

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1. Introduction

The incorporation of probability in the school curriculum has been gaining importance since the National Council of Teachers of Mathematics included “Data and Chance” as one of the five content areas in the Curriculum and

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Evaluation Standard for School Mathematics (NCTM, 1989). Since then, the study of probability has been introduced progressively in the global curriculum guidelines to promote in students the ability to gradually establish connections between mathematics and other areas of knowledge, as well as with daily life experiences (NCTM, 2000; CCSSI, 2010; MEC, 2007; Mineduc, 2012, among others). This curriculum innovation is the product of the new demands of today's society, which have reformed the knowledge of probability that citizens need to handle; and learn in school from an early age.

Chile is not an exception to this reality, therefore the Ministry of Education has incorporated the topic of "Data and Probability "in the *Bases Curriculares 2012* as a continuum in primary education (6-12 years), ranging from the understanding of ideas of chance and probability present in everyday situations, to the simple probability calculation through experimentation and application of Laplace's rule, and thus "respond to the need that all students start involving in issues related to probability" (Mineduc, 2012, p.5). Simultaneously the mismatches between the national and international curriculums may be reduced. From this perspective, and according to Stohl (2005), it is essential to have prepared teachers who can get their students to achieve the objectives laid down by the new curriculum, especially considering that there is a strong relationship between the teacher's knowledge and their students' results (Leinhardt & Smith, 1985; Ball & McDiarmid, 1990; Monk, 1994; Mullens, Murnane & Willett, 1996; Darling-Hammond & Young, 2002; Chambliss, Graeber & Clarke, 2003; Frome, Lasater, & Cooney, 2005). However, in countries like Chile international figures related to primary school teachers reveal severe shortcomings in the quality of education in general and particularly in mathematics (OECD, 2010). This fact demonstrates that the vast majority of teachers lack courses which can prepare them effectively, both in educational and didactic disciplines, from the initial stages of their training. In this context there is a need to focus on teacher training for primary education, particularly in teacher education, specifically in the development and strengthening of didactic and mathematical knowledge, since a teacher cannot teach what he does not know. This knowledge has been studied for a long time and is particularly relevant to the proposal made by Schulman (1986), who distinguishes three types of knowledge for teaching: a) content knowledge, b) pedagogical knowledge and c) pedagogical content knowledge or pedagogical and didactic content knowledge necessary to achieve effective teaching. Since then, several authors have focused their investigations to describe mathematical knowledge for teaching, highlighting the work of Ball, Lubienski & Mewborn (2001), who present a new proposal based on the ideas of Schulman: "Mathematical knowledge for Teaching" (MKT) defined as "the mathematical knowledge used by the teacher in the classroom to produce instruction and student growth" (Hill, Ball & Schilling, 2008). It is pivotal to mention that although models for teaching mathematical knowledge have reached their place in research and teacher education, they are still very general and do not allow a thorough analysis of the different types of knowledge that teachers should possess to achieve effective teaching of mathematics, and even more so in the case of probability. Hence, Godino (2009) performed an analysis of the main models for teaching knowledge (Schulman, 1986) and mathematical knowledge for teaching (Schoenfeld & Kilpatrick, 2008; Hill, Ball & Schilling, 2008), proposing a theoretical model about mathematical-didactic knowledge of the teacher; which integrates some of the categories of the previous models that complement and develop with elements from the onto-semiotic approach to teaching mathematics (Godino, 2002; Godino, Batanero & Font, 2007), thus providing new levels and analysis tools.

This study is developed under this perspective, and aims to analyse in depth the mathematical-didactic knowledge for teaching probability in primary education teachers so as to determine their strengths and weaknesses and finally give them special training to improve the teaching of this topic at school.

2. The didactic- mathematical knowledge of teachers for the teaching of probability

Godino, Batanero, Roa & Wilhelmi (2008) developed an integrative model for the mathematic-didactic knowledge for mathematics teachers which provides a group of tools to analyse, evaluate and develop the teacher's knowledge regarding these two aspects regularly, based on a system of categorization. Explicit categories of analysis are used for the epistemic and cognitive dimension areas from a pragmatic and anthropologic point of view of mathematics, where the mathematical object is seen as an emerging and intervening object in the teaching practices. Thus, this model uses the categories of objects and processes of the onto-semiotic approach to carry out

the analysis of both the mathematical activity and knowledge present in a suitable teaching of mathematics. Similarly, from this model a breakdown of mathematical and didactic knowledge is presented which consists of the following basic categories of knowledge. The first is common content knowledge, specialized and expanded, which allows, first, to analyse the knowledge common content of teachers to carry out the teaching and learning of probability, as well as the configurations of objects and processes required for the planning of the study process, and secondly, guides reflection, from the part of the teacher, on possible generalizations, customizations and connections about probability with more advanced topics. Second is knowledge of content in relation to students, which allows an analysis of the teacher's knowledge to perform a systematic reflection on student learning, i.e. how students learn and the difficulties and mistakes they may face in the teaching-learning process of teaching of probability. This requires that the teacher has a comprehensive knowledge of the subject matter (cognitive aspect) and their students (affective aspect). Next is content knowledge in relation to teaching, which allows analysis of the knowledge that teachers have about relationships that may exist between teaching and learning, and their ability to identify potential effects of classroom management methods (time, materials, teaching experience) about student learning. And, finally, knowledge of curriculum and intra and interdisciplinary connections, which permits an analysis the teacher's knowledge in relation to the treatment given to the study of probability in primary education, according to current curriculum guidelines (Mineduc, 2012), as well as the connections that the teacher can establish between this issue and other issues from both mathematics or other areas of knowledge. Through these four categories of analysis and the "Guide for the wording of slogans" (Godino, 2009), which guide the development of assessment items or proposed activities, it is possible to address some of the aspects related to the teaching and learning process that would lead to knowing the professional skills of teachers of mathematics by using task resolution.

From this perspective, this paper presents a questionnaire to assess didactic-mathematic knowledge of math teachers to teach probability in primary education, since there are no validated instruments that address the content of probability that the teacher should teach from first to sixth grade of primary education according to the current *bases curriculares* and existing curricula in Chile (Mineduc, 2012). The questionnaire also includes approaches from Principles and Standard for School Mathematics (NCTM, 2000) in relation to the teaching of probability, given the close relationship between the Chilean and American curriculums.

3. Methodology

The questionnaire was developed according to the theoretical model of didactic-mathematic knowledge of professor Godino (2009), which considers the four categories of fundamental knowledge necessary for a teacher to carry out the teaching and learning process. The instrument consists of ten situation problems and open questions, based not only on curriculum-related interests but also on direct personal experience and related literature, as in the case of some items that were drawn from the research of Cañizares (1997), who took them from Green (1983) and Fischbein & Gazit (1984) at the same time. The problem situations and questions on the questionnaire also consider some of the typical responses from the students who participated in this research. These items and answers have served as input to pose situation problems in this study. The method proposed by Godino (2009) has been chosen to build the instrument, which includes two phases: first a mathematical task that asks teachers to implement, through the solution of the task or situation, the most relevant issues and skills of probability to be evaluated and developed. Second, the items of evaluation or proposals of activities are formulated to address the various facets and levels of teacher knowledge desired to be evaluated and analysed. An important point to highlight regarding the construction of items is that the answers to the questions did not look obvious, i.e. they could not be answered solely from mathematical knowledge by people who are not experienced in primary school teaching. Thus, through the responses to such situations, it is likely to investigate the mathematic and didactic knowledge of the teacher related to the teaching of probability.

Once designed, the instrument was subjected to a validation process that considered two aspects. Content validity was first secured from the selection of content related to the study of probability in elementary education from the different curricular referents involved (Mineduc, 2012 and NCTM, 2000). Subsequently, to test the validity of the

items, i.e., if they really measure what are meant to measure, the instrument was subjected to the evaluation of expert opinion on the subject of evaluation of knowledge for teaching mathematics. In this validation process 10 experts from Chile and Spain made judgments about the questions by assessing the degree of adequacy of each of the items within the given dimension (content knowledge, knowledge content in relation to students, knowledge of content in relation to education and knowledge of the curriculum and intra- and inter- connections). Specifically, these experts have analysed three aspects in relation to each of the ten situations that make up the questionnaire: a) the degree of correspondence (if each particular item belongs in the dimension), b) formulation (opinion regarding the clarity and the language used in each item), and c) the relevance (the degree of relevance of the item regarding the dimension).

4. Results

Once the validation process of the questionnaire had been done, the final wording of the ten problem situations based on the opinions of experts was written. The following is an example (Figure 1) of one of the items forming the questionnaire:

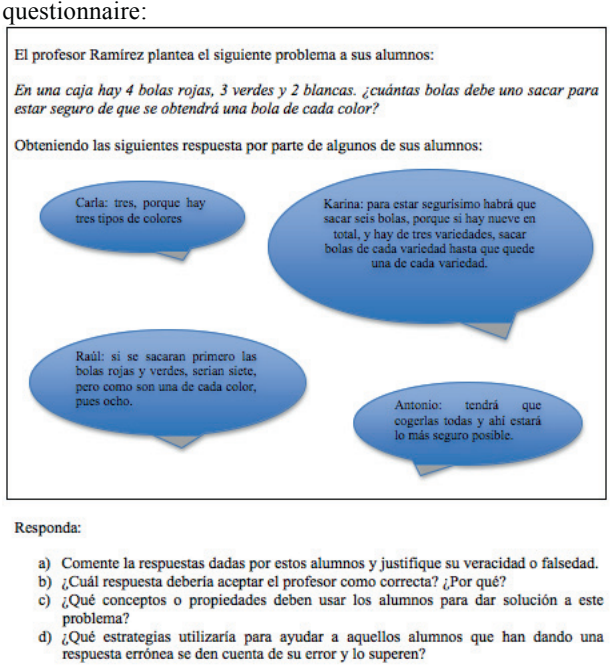

Figure 1: Example item from the questionanaire to assess didactic and mathematical knowledge for teaching probability in primary education.

Through this item it is possible to identify the mathematical knowledge necessary for the resolution of the given problem, which in this case corresponds to the understanding of the concept of insurance event, in addition to basic notions that allow enumerating combinatorial extraction possibilities. It is important to note that this problem and the student responses included have been taken from the research of Cañizares (1997), who in turn took it from Fischbein & Gazit (1984).

Moreover, this item can help identify the mathematical and didactic knowledge necessary for the resolution of the problem situation, which corresponds specifically to content knowledge in relation to students, knowledge of content (common, specialized and extended) and content knowledge in relation to teaching. Thus, through questions a), b) and c), evidence of content knowledge is sought, since the teacher should first solve the problem in order to identify the concepts and properties involved and then analyse each one of the answers given by the students justifying why it is right or wrong. On the other hand, the purpose of question d) is to demonstrate content

knowledge in relation to teaching since the teacher should plan strategies that help students realize their mistakes and thus allow them to understand the problem and give a correct solution.

5. Considerations

The application of the questionnaire to a group of 120 current primary education teachers, as indicated, will allow in-depth analysis of their mathematical knowledge and teaching about probability, and identify their weaknesses and training needs. From the data obtained sufficient evidence will arise so as to establish guidelines for the training needs of teachers of primary education teachers. These guidelines will be reflected in the design of a training course through realistic model (Melief, Tigchelaar & Korthaegen, in collaboration with van Rijswijk, 2010), as it is a teaching methodology focused on the reality that gives meaning to the dialectical relationship between theory and practice, which has emerged as an effective method for transforming beliefs about teaching practice and, ultimately, for methodological innovation of future math teachers (Alsina, 2007; 2010).

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