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Taxation and Investment: Lessons from the Microeconomic Structure.

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Taxation and investment: lessons from the microeconomic structure

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Abstract

This paper addresses the role of taxes over the firm's investment decisions. We propose a framework that considers microeconomic decisions to evaluate aggregate effects of taxes over the economy's investment dynamics.

We obtain strong results and policy recommendations. First, and contrary to other studies, we conclude that the tax system significantly affects the long run level of capital stock in the economy. Second, we indicate that the "first-best" policy is to tax *retired* profits rather than current profits. Third, when the "first-best" policy is not available, because it does not provide enough revenues, we should implement a tax system that minimize the distortion over the relative price of investment. This tax system is a mixture of zero investment subsidies (credit to investment and depreciation allowances) and positive, but small, corporate tax.

We simulate the effects of those tax policies in an economy with heterogenous firms and, under a reasonable set of parameters, we find that the effect of a tax in current profits over the long run level of capital stock, may be as large as 43 per cent of capital stock, when we compare an economy with no distortions versus an economy with a 15 per cent corporate tax rate.

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INTRODUCTION

The determinants of investment is a topic of considerable relevance among economist and government authorities due to its impacts in different areas of economic interest. One of these areas, which is one of the principal concerns of the economic authorities, is its effect in the country's growth rate (and thus in per capita output) as investment might be one of the engines behind larger growth rates.

This paper focuses in obtaining policy lessons that can be applied in the case of Chile¹. Even though there is some economic literature studying investment in Chile, we pursue this topic because the study of investment in Chile has mainly focused in empirical research that uses aggregate data. However, this analysis makes quite difficult to understand the behavior of firms, which are the ones making the decisions about capital investments. Firms in Chile, as in the rest of the world, seem to be quite heterogenous and thus decisions differ across firms. Camhi, Engel and Micco (1997) focused in the labor heterogeneity across firms in Chile and showed that aggregated movements in labor were due to job creation, job destruction plus reallocation of employment across incumbent firms. This case shows that focusing in aggregated data to describe economic behavior implies an over-simplification of the problem that eliminates important microeconomic information.

It should be said that the line of research that uses aggregate data has been mainly pursued due to the scarcity of other types of data sets. Recently, Bustos, Engel and Galetovic (1998) -BEG- provided a different econometric analysis. They studied firms' investment problem and tested their conclusion by using an annual panel of firms, with data ranging from 1985 to 1995. A characteristic of the data set is that it included only firms classified as "Sociedades Anonimas Abiertas"² which size, in general, is considerable larger than the medium firm size of the economy. This selection problem of the data set do not allow to extend the results over the Chilean economy though.

¹however the analysis is quite general and thus the result might be extended to other countries with a economic structure similar to the one here considered.

²Firms which stock is traded in the Chilean stock market

This paper, rather than using aggregated (macro) data or rather than extending the analysis of BEG, will provide simulations for the Chilean economy under different assumptions. To simulate, we build the aggregated effects in the economy starting from the microeconomic decisions of the firms. Thus we first solve firms' problems and later we aggregate these decisions.

The microeconomic specification, and its posterior aggregation, allows us to understand precisely the dynamics of investment and the effects of fiscal policy in this decisions. Additionally to these qualitative results, it allows us to obtain some quantitative results. These quantitative results rely in one hand in the specification used to model the economy but also, in the set of parameters chosen to produce the simulation. The attractiveness of this last property, is that we might borrow those parameters from microeconomic studies which is more reliable than the one obtained from the use of aggregated data.

A second characteristic of this paper is that it answers questions concerning tax policy such as: Is there an effect over investment if we tax firms' profits? if yes, Is there a tax system that minimize those distortions? Are those effects significant? What are the qualitative and quantitative effects of depreciation allowances (and/or investment credits) over investment decisions? We try to answer those questions to illustrate the importance of removing "distortions" that affect the microeconomic level (each individual firm, in our case). In fact, we will show that we may obtain large effects at the aggregate (macroeconomic) level.

The way we proceed is the following. First, we assume there is a large number of firms in the economy and we solve each firm's decision problem. This property allow us to work with heterogeneous firms. The heterogeneity among firms will depend in shocks affecting the productivity of each them. Thus, even when the firms might have similar technologies (production functions), each firm has an individual shock that alters the production function. We will show that this effect might produce considerable differences in investment decisions at the firm level.

To solve each firm's problem³, we choose some functional forms. First, we assume a

³each firm solves the allocation of investment over time

quadratic investment cost, as in the "adjustment cost" literature. Second, we assume a general specification of the fiscal policy that can be specialized to (1) the case of corporate taxes which base are the firms' current profits, (2) the case of corporate taxes which base are *retired* profits, (3) the existence of depreciation allowances or, (4) investment tax credits. Variations in tax policies will produce variation in investment decision at the firm level.

Later, we will aggregate the firms' decisions to obtain the aggregate allocations of the economy. To do so, we will obtain the stationary distribution of firms -meaning, we will obtain the long run fraction of firms with a given capital stock and, we will repeat the exercise for any possible level of capital stock.

To validate the output of the model, we obtain the empirical distribution of firms across investment decisions and we compare the moments of the distribution obtained from the simulation of the model with the moments of the actual distribution. Hence, if the model reproduces the observed empirical distribution of firms, we can be confident of the output of the analysis and policy recommendations obtained from it.

The analysis provides some interesting qualitative conclusions. First corporate taxes, which base are the firms' (current) profits, affect negatively the long run capital stock. This result is quite intuitive: an increase in the tax rate decreases the rental rate of return from capital stock (marginal benefit of investment), and it provides less incentives to accumulate capital stock.

The second conclusion is that corporate taxes, with *retired profits* as base, do not distort the optimal investment decision. We show that this type of tax is similar to a mixture between corporate taxes based in current profits plus investment tax credits (or alternatively depreciation allowances), where both type of taxes have the same magnitude. The effect of the policy is that as above the marginal benefit from capital is decreased but additionally, the marginal cost of investment is also negatively affected. These two effects have opposite impacts in investment that offset each other. This is a very important result because it allow to tax the capital stock (and collect revenue) without affecting its dynamic accumulation.

We also provide quantitative measure of the effect of taxes in any of those systems. These quantitative results show that the tax policy produce quite important effects. In fact if we

consider the introduction of a corporate tax (current profits) equal to 20 per cent, the long run level of capital stock decrease in almost 50 per cent compared to the case with no tax.

What is the optimal tax policy conditional in a given level of government revenue collection? Even when we indicate that the "first-best" tax policy is to implement a *retired profits* tax system, this system might not provide enough revenues as required by the fiscal budget. Thus we characterize a "second-best" policy (the one providing enough revenues).

The "second-best" policy is chosen among the set of possible policies that provide a given level of tax revenue. The chosen policy is the one providing larger capital accumulation (and thus larger investment) and it corresponds to the one with *the smallest possible corporate tax rate*. This result may seem obvious, but it is not. In fact for a given government revenue, we have a set of policies in which as we increase the corporate tax rate we might also increase the investment tax credit or the depreciation allowances at the same time (thus we hold constant total revenue). It might be tempting to choose those policies that provide larger investment tax credits, as a way of providing incentives to capital accumulation, however these policies should have associated a larger corporate tax which affect negatively capital accumulation. Thus the main policy lesson is that if the *retired profit* tax system is not available, because it does not provide enough revenues, we should choose a tax system that *minimize distortions over the relative price of investment*.

The paper is developed in the following way. Section 2 describes a very simple firm's problem that highlights the main intuition of this paper. Section 3 presents a more realistic economic environment and builds the aggregate behavior of the economy, starting from the firm's microeconomic structure (it states the firm's problem, indicates the information structure and explains the way the economy is aggregated). Section 4 presents a simulation of this economy under different tax regimes and discusses the results. Finally section 5 concludes.

A FIRST APPROXIMATION TO THE EFFECTS OF TAXES ON INVESTMENT DECISIONS

In this section, we will specify a simple problem for a representative firm that chooses the path of its investment in capital stock. This case will provide us with the main intuition behind this paper.

We will assume that the representative firm has a time horizon ranging from $t=0$ to infinity. At each period of time, the firm is endowed with a given level of capital stock, k_t . This capital stock is used to obtain corporate profits which are denoted as $\pi(k_t)$, where $\pi' > 0$, $\pi'' < 0$. Corporate profits are not "economic" profits as they only deduct variable costs - e.g. labor cost or the cost of other variable inputs. Hence they might be defined simply as capital compensation. The government taxes corporate profits at the rate τ_t , that might vary over time.

After-tax corporate profits might be used to invest in future capital stock (reinvestment of profits) or might be retired from the firm by the firm's owners. If the profits are reinvested, the firm faces adjustment cost. To model this characteristic, we will assume the firm has an increasing and concave adjustment cost function, $c(I_t)$, where I_t is investment t . The investment costs face a subsidy from the government equal to a rate σ_t of total investment cost. Furthermore capital stock depreciation, from the corporate tax perspective, is analogous to variable cost and thus they decrease tax payments. Hence another effect of investment over firm's tax payments is depreciation allowances. We will assume that the present value of depreciation allowances of current investment is the rate z_t of the total investment cost.

Finally, the capital stock of the firm follows the law of motion: $k_{t+1} = I_t + (1 - \delta)k_t$, where δ is depreciation rate. We will assume that the firm might borrow or lend at the capital market interest rate, r_t . Thus the firm's problem is:

$$V_0(k_0) = \max_{I_t} \sum_{t=0}^{\infty} R_t [(1 - \tau_t)\pi(k_t) - (1 - \sigma_t - z_t)C(I_t)]$$

s.a

$$k_{t+1} = I_t - \delta k_t \quad (1)$$

Where $R_t = (\frac{1}{1+r_0})(\frac{1}{1+r_1})..(\frac{1}{1+r_t})$ and the associated first order condition with respect to investment at time t is:

$$R_{t+1}(1 - \tau_{t+1})\partial\pi(k_{t+1})/\partial k_{t+1} = R_t(1 - \sigma_t - z_t)\partial C(I_t/\partial I_t) \quad (2)$$

$$\Rightarrow \frac{\partial C(I_t)/\partial I_t}{\partial\pi(k_{t+1})/\partial k_{t+1}} = \frac{1}{1 + r_{t+1}} \frac{1 - \tau_{t+1}}{1 - \sigma_t - z_t} \quad (3)$$

Condition (2) indicates that the firm chooses investment such that it equates the marginal benefit of investment (the left hand side) with its marginal cost (right hand side). The marginal benefit is the present value of the increase in future profits while the marginal cost is the present value of the decrements in retired resources (profits) from the firm, as those resources are used instead to invest in capital stock.

Condition (3) rewrites condition (2) in a very intuitive way. It states an equality between the marginal rate of transformation of current retirement of profits versus future retirement of profits (left hand side) with its ratio of prices (right hand side). In fact, investment is a way of transforming current profits into future profits, throughout the accumulation of capital stock. To clarify the point, note that a marginal increase in investment produces a lower retirement of profits from the firm, equal to $C'(I_t)$ while, its associated marginal benefit is the increase in future profits, which is equal to $\pi'(k_{t+1})$.

The right hand side of (3) shows that the capital market allows to transform a unit of future income into current income at the rate $\frac{1}{r_{t+1}}$. However this price, which is the price of investment, is modified by the fiscal tax system. Larger investment credits (or depreciation allowances) decrease investment prices and thus makes cheaper to transform current in future profits. In the same way, a larger future corporate tax rate provides smaller profits for each unit of current profit sacrificed and thus makes more expensive to invest. Hence we may conclude that as the tax system varies, the price of investment varies. *Thus two simple conclusions emerge. First, a larger future tax in corporate profits, τ_{t+1} , will decrease investment⁴. The intuition is quite clear in this case, as investment price rises. Second,*

⁴simple comparative statics show this result

larger subsidies to investment -investment tax credits or depreciation allowances- provide incentives to increase investment because its price is smaller.

This quite simple framework is shown in figure 1. The figure presents the firm's decision in the plane current profit retirement, d_t , and future profit retirement, d_{t+1} . Point A shows the maximum current retirement of profits, $\tilde{d}_t = (1 - \tau_t)\pi(k_t)$ ⁵ while point B is the maximum future retirement of profits⁶, $\tilde{d}_{t+1} = (1 - \tau_{t+1})\pi(k_{t+1})$. Those points define the budget constraint. Notice that the marginal rate of transformation is strictly convex to the origin due to the properties of π'' , C'' . It follows that a unique equilibrium (d_t^*, d_{t+1}^*) exists. Total investment at time t is defined as $I_t^* = (1 - \tau_t)\pi(k_t) - d_t^*$.

Figure 1 illustrates the effect of increasing future corporate tax rate, e.g the increase of τ_{t+1} , holding constant τ_t ⁷. In this case, as we indicated above, the relative price of substituting d_t by d_{t+1} (investment price) is smaller and thus the budget constraint moves inwards. It should be noticed that current after-tax profits are not affected, though. Hence the budget constraint moves, but not in parallel. The new equilibrium is (d_t^{**}, d_{t+1}^{**}) , where $d_t^{**} < d_t^*$ which implies an increase in investment. Conversely, an increase in σ_t or z_t produces the opposite effects because the relative price of investment rises.

Figure 2 shows an additional exercise, e.g. an increase in τ_t while holding constant τ_{t+1} . In this case, the relative price of investment do not vary while the maximum amount of current retired profits is lowered. Thus point A moves to the left in the graph while the slope of the budget remains the same. Thus the budget constraint moves inwards in parallel, producing a negative income effect. In this case, investment decreases because for any initial level of capital stock, the firm has less "income" to spent in investment however, there is no negative substitution effect in investment. This case is similar to the Chamley (1986) result stating that the optimal capital tax rate should be zero in the long run if there is an asymptotic steady state and markets are complete. The intuition is that we will not distort investment decisions in the long run and taxing the initial level of capital stock is similar

⁵Associated with zero future retirement of profits

⁶Associated with zero current retirement of profits

⁷investment credits and depreciation allowances are also held constant.

to a lump-sum tax, producing a simple income effect.

[Insert figure 1 and 2]

Finally, an obvious extension of this framework is to notice that fixing investment subsidies and future corporate tax rate at the same rate, e.g. $\tau = \sigma + z, \forall t$, relative prices are not distorted and thus we do not have effect of the tax system over optimal dynamics investment decisions⁸. Note that in this case we might write after-tax profit as in:

$$(1 - \tau_t)\pi(k_t) - (1 - \sigma_t - z_t)C(I_t) = (1 - \tau_t)[\pi(k_t) - C(I_t)] \quad (4)$$

Where $\pi(k_t) - C(I_t)$ are the *retired* profits from the firms. *This is an special case in which firms pay taxes over its retirement of profits only -profits are not taxed if they are reinvested- and, in which optimal investment decisions are not distorted by taxes.* This is one of the main conclusions of the paper.

Next section will extend this framework to allow heterogeneity across firms. This variation of the model will allow us to explain difference in investment and size of firms. We will argue that similar conclusions, to the above stated, will hold. Simulations will provide us with approximations to the quantitative effects of the tax system.

THE ECONOMIC ENVIRONMENT

The firm's problem

In our economy, there will be a large number of firms that "live" between time $t=0$ to infinity. Each firm will face a similar -but not equal- problem that we will describe next.

As in the last section, each firm is endowed at the beginning of each period of time t with a given level of capital stock, k_{it} , where $i \in \Omega$ indexes firms and Ω is the set of firms in the economy.. Similarly to above, capital stock is used to obtain corporate profits. Capital compensation is not homogeneous across firms in the economy. In fact, we will

⁸Up to the lump-sum income effect

assume that each firm in the economy is affected by an individual shock, w , that modifies corporate profits. This shock has three possible states of nature, e.g. $w \in (w^1, w^2, w^3)$ where $w^1 < w^2 < w^3$. We will denote corporate profits, after uncertainty is resolved, by $w_t \pi(k_t)$, where w_t indicates the firm's shock at t . We will assume that the government taxes this capital compensation at the rate τ_t .

The after-tax corporate profits can be used (1) to invest in capital stock accumulation or (2) to pay dividends to the firm's owners. As in the adjustment cost literature, we assume that the firm faces a convex cost for its investment. We will also assume that the adjustment cost function is negatively affected by its initial level of capital stock, k_t . Let $C = C(I_t, k_t)$ be the adjustment cost function. As usual the rationality of these assumption deals with "internal" or "external" adjustment costs of the firm. The main effect of this assumption is an incentive to smooth investment over time. Later in the paper, we will modify this assumption as a way to see how the model behaves under different specifications. The properties of the adjustment cost function are $C_I, C_{II} > 0, C_k < 0, C_{kk} > 0$.

In addition to the capital compensation tax rate, we will assume that the government provides (1) an investment tax credit, σ_t , as a fraction of investment costs of the firm and, (2) depreciation allowances. We will denote by $D_t(s)$ the date t depreciation allowance for each unit of capital of age s . Hence, the corporate income tax paid by the firm at time t is:

$$Tax_t = \tau_t [w_t \pi(k_t) - \sum_{s=0}^{\infty} D_t(s) C(I_{t-s}, k_{t-s})] - \sigma_t C(I_t, k_t)$$

This equation might be quite general and might be consistent with different tax systems, which will be analyzed later.

To finally present the problem of a firm, we will specify the law of motion of capital stock and some additional notation. Since we have heterogeneity across firms, we may assume that depreciation rate face of firms, δ_i , differ across firms. Thus the law of motion capital is $k_{it+1} = I_{it} - \delta_i k_{it}$.

As additional notation, we will define the story of shocks of firm i at time t :

$$w_i^t = (w_{i0}, w_{i1}, \dots, w_{it})$$

Notice that in the above definition when time is used as a superscript, it indicates the story of shocks until t while when t is used as subscript, it indicates the current shock. Also, let $Prob(w_i^t|w_i^0)$ be the probability of occurrence of shock w_i^t evaluated at $t=0$ and conditional in the shock occurred at that moment and $R_t = (\frac{1}{1+r_1})\dots(\frac{1}{1+r_t})$ be the discount factor used by the firm -which depends in the interest rate, r_t , obtained from the capital market.

Using this notation, we may define the problem of a firm endowed with some initial level of capital stock, k_{i0} , and a given initial shock w_{i0} , as in:

$$\begin{aligned} V_{i0}(k_{i0}, w_{i0}) &= \max_{I_{it}(w_i^t)} \sum_{t=0}^{\infty} \sum_{w_i^t} R_t Prob(w_i^t|w_{i0}) [(1 - \tau_t)s(w_i^t)\pi(k_{it}) - (1 - \sigma_t)C(I_{it}(w_i^t), k_{it})] \dots \\ &\dots + \sum_{t=0}^{\infty} R_t \tau_t \sum_{w_i^t} Prob(w_i^t|w_{i0}) \sum_{s=0}^{\infty} D_{it}(s) C(I_{i,t-s}, k_{i,t-s}) \end{aligned} \quad (5)$$

s.a

$$\begin{aligned} k_{it+1} &= I_{it}(w_i^t) - \delta_i k_{it} \dots \forall w_i^t \\ I_{it}(w_i^t) &\in \kappa \end{aligned}$$

A couple of remarks of this problem are the followings. First, this problem is a version of problem (1), stated last section, that includes uncertainty. Second, we index the investment decision by the story of shocks (conditional in the initial shock which is known). Intuitively the indexation matters because a firm with a larger number of positive shock will be able to spend a larger amount in investment. Third, we assume that the investment decision must belong to a set of values that we denote κ . This set will play an important role. Suppose we want to discuss the case of irreversibility in investment. In that case, κ will only include positive values. Another case to consider might be liquidity constraints in investment. If we assume that firms cannot borrow to invest, the maximum possible investment of the firm is

the total amount of after-tax corporate profits occurred in the period. This last property will impose an upper-bound to the set κ .

To simplify the problem, notice that we might write the depreciation allowances as in ⁹:

$$\begin{aligned} \sum_{t=0}^{\infty} R_t \tau_t \sum_{w_i^t} \text{Prob}(w_i^t) \sum_{s=0}^{\infty} D_{it}(s) C(I_{i,t-s}, k_{i,t-s}) &= \sum_{t=-\infty}^0 R_t C(I_{i,t}, k_{i,t}) \sum_{s=-t}^{\infty} \frac{R_{t+s}}{R_t} \tau_{t+s} D_{it+s}(s) \\ &+ \sum_{t=0}^{\infty} R_t z_t \sum_{w_i^t} \text{Prob}(w_i^t) C(I_{i,t}(w_i^t), k_{i,t}) \end{aligned}$$

Where z_t was defined as $z_t = \sum_{s=0}^{\infty} \frac{R_{t+s}}{R_t} D_{t+s}(s) \tau_{t+s}$, which is the present value of contributions to the profits of the firm (value of the firm) of current and anticipated depreciation allowances, for each dollar spent in capital stock accumulation at time t .

The expression indicates that total depreciation allowances of the firm might be decomposed in two terms: the first term corresponds to investment made in the past while the second corresponds to depreciation allowances due to current and future investments. Notice that since the first term correspond to past investments, it does not affect current and future investment decisions and thus it will be neglected in the future.

This property of depreciation allowances plus the separability over time of the profit function allow to write the firm's problem in a dynamic programming setup, as in (this

⁹Note that we may change the integration limits as in:

$$\begin{aligned} \sum_{t=0}^{\infty} R_t \tau_t \sum_{s=0}^{\infty} D_{it}(s) C(I_{i,t-s}, k_{i,t-s}) &= \sum_{t=-\infty}^{\infty} \sum_{s=\max(0,-t)}^{\infty} R_{t+s} \tau_{t+s} D_{it+s}(s) C(I_{i,t}, k_{i,t}) \\ &= \sum_{t=-\infty}^{\infty} R_t C(I_{i,t}, k_{i,t}) \sum_{s=\max(0,-t)}^{\infty} \frac{R_{t+s}}{R_t} \tau_{t+s} D_{it+s}(s) \end{aligned}$$

Decomposing the second summation, we have:

$$\begin{aligned} \sum_{t=0}^{\infty} R_t \tau_t \sum_{s=0}^{\infty} D_{it}(s) C(I_{i,t-s}, k_{i,t-s}) &= \sum_{t=-\infty}^0 R_t C(I_{i,t}, k_{i,t}) \sum_{s=-t}^{\infty} \frac{R_{t+s}}{R_t} \tau_{t+s} D_{it+s}(s) \\ &+ \sum_{t=0}^{\infty} R_t C(I_{i,t}, k_{i,t}) \sum_{s=0}^{\infty} \frac{R_{t+s}}{R_t} \tau_{t+s} D_{it+s}(s) \end{aligned}$$

program is just another way of writing problem 5):

$$\begin{aligned}
V(k_{it}, w_t^k) &= \max_{k_{it+1}(w_t^k)} [(1 - \tau_t)s(w_t^k)\pi(k_{it}) - (1 - \sigma_t - z_t)C(I_{it}(w_t^k), k_{it})] + \dots \\
&\dots + \frac{1}{1 + r_t} \sum_{j=1}^3 Prob(w_{t+1}^j | w_t^k) V(k_{it+1}, w_{t+1}^j) \\
&\quad s.a \\
k_{it+1}(w_t^k) &= I_{it}(w_t^k) - \delta_i k_{it}, \\
k_{it+1}(w_t^k) &\in \Gamma
\end{aligned} \tag{6}$$

Where $V(k_{it}, w_t^k)$ is the value function¹⁰ conditional in capital endowment and shock at time t¹¹. The problem shows that the marginal cost of investment is not only affected by investment tax credits but also, by depreciation allowances over those investment costs. Note that we replace investment by future capital stock as decision variable, hence the set Γ is analogous to κ .

A useful property of this problem, as in any dynamic programming problem, is the solution being function only of the state variables. Hence, the solution for a single firm i at time t might be written as $k_{it+1} = g(k_{it}, w_t^k)$, where g is a single-valued function. This solution is known in the literature as policy function (see Stockey, Lucas and Prescott, 1989 or Ljungqvist and Sargent, 2001). To solve the dynamic problem we will use numerical methods later in the paper.

The aggregate economy

The above procedure solved the problem of each of the firms in the economy. We turn next to characterize the long run aggregate behavior of the economy. To simplify the notation, we will eliminate the time index and we will use a prime to indicate future period of time -thus primes indicate next period variables rather than derivatives.

The heterogeneity across firms determine different investment rates and capital stocks

¹⁰ maximum welfare attainable given preferences, technology, information and the state variables

¹¹ k_{it}, w_t^k are the state variables

across firms. In fact notice that from the firms' solution, $k' = g(k, w)$, firms with the same initial level of capital may have different future levels of capital stock (and hence different investment rates). This result depends in the shock. If ex-ante equal firms have a different shocks, the policy function determines different levels of capital stock in the future. To characterize the long run behavior of the economy, we will determine the distribution of capital stock across firms. This distribution will, in fact, determine the long run level of capital stock. For instance, a distribution of firms that is skewed to low levels of capital stock will produce a low long run level of aggregated capital stock -the economy will have too few firms holding a large levels of capital but a large fraction holding a low level of capital. The contrary will hold if the distribution is skewed to larger levels of capital stock.

To characterize the distribution of firms, we define:

$$\lambda_t(k, w) = Prob(k_t = k, w_t = w)$$

This function indicates the fraction of firms holding k as capital stock and having a shock w . Varying k and w over the support of capital stock and shocks will define a distribution function that, as usual, integrates to one. To characterize the dynamic behavior of the economy, we will determine the evolution of the distribution function over time. Note that the distribution function in the next period of time is:

$$\lambda_{t+1}(k_{t+1} = k', w_{t+1} = w') = \sum_{k_t} \sum_{w_t} Prob(k_{t+1} = k', w_{t+1} = w', k_t = k, w_t = s)$$

Where the equality follows from the properties of joint distributions. Using the joint distributions of current and future capital stock and of current and future shock, (k', w', k, w) , is useful because using Bayes' rule, we get:

$$\begin{aligned} \lambda_{t+1}(k_{t+1} = k', w_{t+1} = w') &= \sum_{k_t} \sum_{w_t} Prob(k_{t+1} = k', w_{t+1} = w' | k_t = k, w_t = s) Prob(k_t = k, w_t = w) \\ &= \sum_{k_t} \sum_{w_t} Prob(k_{t+1} = k' | k_t = k, w_t = w) Prob(w_{t+1} = w' | w_t = s) Prob(k_t = k, w_t = w) \end{aligned}$$

By definition $\lambda(k, w) = Prob(k_t = k, w_t = w)$, thus:

$$\lambda_{t+1}(k', w') = \sum_{k_t} \sum_{w_t} 1(k', k, w) Prob(w_{t+1} = w' | w_t = w) \lambda_t(k, w)$$

Where $1(k', k, w)$ is an indicator function equal to one if $k' = g(k, w)$ and zero otherwise. This indicator function replaces $Prob(k_{t+1} = k' | k_t = k, w_t = w)$ because the policy function is single-valued, meaning that there is a unique future level of capital stock for a combination (k, w) . Hence conditional in (k, w) the probability of choosing k' is zero if the future level of capital is different from the one dictated by the policy function and one if we take the "correct" k' . Using this property, we may finally write the evolution of the distribution function as:

$$\lambda_{t+1}(k', w') = \sum_{k'=g(k,w)} \sum_{w_t} \lambda_t(k, w) Prob(w_{t+1} = w' | w_t = w) \quad (7)$$

This expression is quite intuitive. It indicates that the "t+1" fraction of firms with capital stock and shock (k', w') will depend in how it evolves the distribution at t. In fact, if we take a given group at t, e.g. the fraction $\lambda_t(k, w)$, they will evolve to the group $\lambda_{t+1}(k', w')$ if: (1) the future shock is w' , conditional in the current shock being w and, (2) the future capital stock -the firm choose- is k' . This last property depends in the policy function. Finally, the summation indicates that we sum over all initial distribution of firms.

A property of the distribution function is that whenever $Prob(w' | w) > 0, \forall (w, w')$, the distribution function converges asymptotically to a long run distribution¹², $\lambda(k, s)$. This long run distribution can be used to obtain the long run level of capital stock (and thus the long run level of investment) in this economy. In fact the long run level of capital, k_t , is defined as:

$$k_t = \sum_{k,w} \lambda(k, w) g(k, w) \quad (8)$$

¹²Stokey, Lucas and Prescott, 1989; Ljungqvist and Sargent, 2001

Where $g(k, w)$ is the policy function which indicates the optimal decision level of capital stock if the state variables are (k, w) and, $\lambda(k, w)$ is the fraction of households with the state variables (k, w) .

Another aggregate variable of interest is the long run level of government revenue, which is defined as:

$$rev_t = \sum_{k,w} \lambda(k, w) [\tau_t s(w) \pi(g(k, w)) - (1 - \sigma_t - z_t) C(g(k, w)(1 + \delta), g(k, w))] \quad (9)$$

This expression indicates that government revenue depends tax policy; in each firm's optimal decision, $g(k, w)$; in the fraction of firms choosing a particular solution and; in the shock $s(w)$ that affects long run profits.

The economy is completely characterized by equations (6) to (9). We will next simulate its behavior under different assumptions.

SIMULATIONS

The structure

We turn now to simulate the economy. We use those simulations because the solutions to the individual's problem might be highly non-linear and thus very difficult to solve. The simulation, through the use of computer capacities, helps us in the task.

To simulate the economy we will assume some functional forms. First, the production function and the cost function will be $\pi(k_t) = k_t^{0.7}$, $C(I_t, k_t) = C_0(I_t + \frac{I_t^2}{2k_t})$, $C_0 > 0$. Additionally, we will assume that the interest rate is fixed at 5 percent, while the depreciation rate will be set at 5 percent also. The production function here assumed is similar to assume a capital share equal to 70 percent and the adjustment cost function follows the specification commonly used in the literature.

The shock are going to be specified as $w_1 < 0, w_2 = 1, w_3 > 1$. We specify $w_1 < 0$ because this is similar to assume negative corporate profits for the firm in the "bad" state of nature. The case $w_2 = 0$ implies a completely neutral shock to the production function

while $w_3 > 1$ indicates a state of nature where the production function is positively affected. Finally, we need to specify the transition matrix, e.g. the matrix containing the probability of being in a future state of nature, conditional in the current state of nature. In other words, the transition matrix provides the probabilities, for a firm, of having a positive, neutral or negative shock conditional in the current state of nature. Cerda, S. and Zurita, F. (2001) provide a similar transition matrix for the case of the Chilean economy in the period 1960-2000. To calculate this matrix, they basically calculate these probabilities by using the growth rate of the Chilean GDP. For instance, they calculated the probability of a boom (positive shock) if the current state was a boom, using data of GDP between 1960 and 2000, and they repeated the calculation for each possible combination of future state and current state. We will base our calculations in their transition matrix. Thus we will assume that the transition matrix is:

future shock				
Negative	Neutral	Positive		
0.568	0.146	0.286	Negative	current shock
0.182	0.364	0.454	Neutral	
0.1	0.26	0.65	Positive	

To simulate, we form a grid of 267 capital stock points starting at zero at a distance of 0.03 among them. We will evaluate each of those 267 capital stock points and we will determine which one is the optimal decision for the firm. Notice that we assume that capital stock is always positive. This is similar to assume irreversibility in investment (investment must be always positive).

We will solve two sequential problems. First, we solve the problem for each firm in the economy through the use of the Bellman equation -equation 6 above stated¹³

¹³To solve the firms' Bellman equation, we will index by i the capital stock (in our grid of 267 points) and by h the shock of the firms. Thus for each pair (i, h) , $i \in [1, \dots, m]$ and $h \in [1, \dots, n]$ -where $m=267$ and $n=3$ -,

Later and using the solutions of the above problem, we solve the stationary distribution of firms across capital stock -equation 7¹⁴.

We describe next the empirical observations and the results when we used different tax systems.

The empirical observations and the results

The empirical distribution of firms across investment decisions was obtained from the 1990-1996 versions of the ENIA¹⁵. This survey provides data about a panel of firms in the industrial sector of the Chilean economy. Among the data, it is possible to find the flow of investment at each year for each firm, measured in millions of Chilean pesos. Figures 3 to 9 plots the distribution of firms across investment for all firms with positive investment rate and they show the associated descriptive statics¹⁶.

[Insert figures 3 to 9]

we will solve the following Bellman equation:

$$\begin{aligned} v(k(i), w(h)) &= \max_{k'} (1 - \tau_t) w(h) \pi(k(i)) - (1 - \sigma_t - z_t) C(k' + \delta k(i), k(i)) \dots \\ &+ \frac{1}{1 + r} \sum_{j=1}^m \text{Prob}(w(j) | w(h)) v(k', w') \end{aligned}$$

To solve this problem we iterate in the Bellman equation until we converge to the unique value function. To do so, we guess an initial value function and we replace this value in the right hand side. We solve the maximization problem and we compute the optimal value of the problem (the left hand side of the equation) which becomes our new value function. If the initial guess was correct, we stop and we had solved the problem. If the initial guess is different from the new value function, we used this last function as initial value for the next iteration. We iterate until convergence is acquired. In that case, the optimal capital stock is the one that solved the last iteration. To solve this problem we use the computer program Matlab 6.0. The computer problem is available from the author by request.

¹⁴Similarly to the Bellman equation case, to solve for the stationary distribution, we start with any initial distribution and we iterate in equation 7 until we converge to the stationary distribution.

¹⁵”Encuesta Nacional Industrial Anual” elaborated by the INE

¹⁶The distribution that includes firms with no positive investment is similar

As it is possible to conclude from the figures, *firms do choose different levels of investment* because the distribution is clearly not degenerated. Firms concentrate at low level of investment, even though there is a significant fraction of firms with larger investment. In addition, *the distribution is quite stable across time* indicating that there are similar incentives to invest in this time period.

We initially simulate the model with no distortion, e.g corporate taxes, investment tax credits and depreciation allowances are set equal to zero. Figure 10 to 12 show the optimal decision rules for firms with negative, neutral and positive shocks¹⁷. Not surprisingly, for any initial levels of capital stock firms with positive or neutral shocks accumulate larger capital. This is because the shocks act like an income effect that allows larger capital stock accumulation. The optimal policy function, in all the three cases, is quite concave. This is due to the convexity of the adjustment cost function that provides incentives to accumulate larger capital stock for smaller levels of investments.

[Insert figures 10 to 12]

Figure 13 plots the stationary distribution of firms across capital stock¹⁸.

The distribution seems to match quite well the observed distribution, as seen in the figure in 13 when compared to figures 3 to 9. Columns 1 to 7 of table 1 present the descriptive statistics of the distribution obtained from the ENIA for each year, where the moments are measured as fraction of the mean of the distribution (we use this measure as a way of normalization). As we observe, the moments of the distribution almost do not change over time. The seventh column shows the mean of the moments in the period 1990-96, which are quite similar to the moments in any of the distribution measured per year. Column 8 presents the moment of the distribution obtained when we simulate our economy. The moments seems to be in line with the "true" moments. In fact, the simulated standard deviation and the skewness of the distributions are quite similar to the "true" ones. Even

¹⁷This is the result of solving the Bellman equation -equation 6- when the tax system is absent.

¹⁸Which is analogous to investment decisions since they should differ only by the depreciation rate in the long run.

though the "simulated" kurtosis has the same sign (positive) as the "true" kurtosis, its magnitude is very large. Thus the results are in general satisfactory with the exception of the kurtosis.

[Insert figure 13 and table 1]

We will next introduce a tax over current firm profits. We will simulate this case by assuming $\tau = 0.15; \sigma = z = 0$ ¹⁹. Figure 14 shows the stationary distribution of firms in this case, while the eighth column of table 1 presents the moments of the simulated distribution. Comparing figure 13 and 14, we observe that the distribution shifts to the left and therefore the introduction of the corporate tax in current profits decreases the per firm level of capital stock in the long run. The results show that the average level of capital stock in the economy drops by almost 43 per cent²⁰. A second conclusion, which emerges from the moments of the simulated distribution is that the size of kurtosis decreases, similar to the "true" distribution²¹. This result is quite interesting because this tax system is similar to the one applied in Chile during the period and it clearly provides a simulation that approximates to the empirical observation.

[Insert figure 14]

Why does the long run capital stock drop in this case? It will be easy to obtain intuition from the first order condition of the individual firms. As in section 2, we may write the marginal rate of transformation being equal to the ratio of prices as in:

$$\frac{\partial C(I_t)/\partial I_t}{E_t[\partial \pi(k_{t+1})/\partial k_{t+1} - \partial C/\partial k_{t+1}]} = \frac{1}{1 + r_{t+1}} \frac{1 - \tau_{t+1}}{1 - \sigma_t - z_t} \quad (10)$$

Where $E_t \frac{\partial \pi(k_{t+1})}{\partial k_{t+1}} = \sum_{j=1}^3 \text{Prob}(w(j)|w)w(j) \frac{\partial \pi(k_{t+1})}{\partial k_{t+1}}$ is the expected marginal benefit of investment. In the long run equilibrium, comparative statics in this equation yields:

¹⁹similar to the current case in Chile

²⁰From an initial level of 1.62 to 0.94 when $\tau = 0.15; \sigma = z = 0$

²¹Similarly, the skewness decreases its size and approaches the size of the true skewness.

$$\frac{\partial k^*}{\partial \tau} = \frac{E_t \frac{\partial \pi}{\partial k} - \frac{\partial C}{\partial k}}{D} < 0 \quad (11)$$

Where $D < 0$ by second order conditions, $C_k < 0$ by properties of the adjustment cost function and k^* is the long run level of capital stock. The intuition is that as we increase the tax rate, the marginal return from the capital stock is lowered²². As a consequence, firms will accumulate less capital stock.

Consider next the case: $\tau = (\sigma + z) = 0.15, \forall t$. Figure 15 shows the stationary distribution in this case, while the 9th column of table 1 presents its moments. Notice that quite surprisingly the distribution is exactly the same as the one obtained with no distortions and thus, the economy accumulates capital stock in the same way as in an economy with no distortions. Why? When $\tau = (\sigma + z), \forall t$ the relative price of investment is not distorted and therefore the investment decision is not varied. This is the same result indicated in section 2 when we assumed $\tau = (\sigma + z), \forall t$. In that section we indicated that the relative price of investment is not distorted and thus the investment decision is not distorted either, up to the income effect produced by the sum-lump tax in the initial level of capital. This last effect is not important in the long run²³ though.

[Insert figure 15]

How do we interpret this case? This tax system is analogous to implement *a simple tax over retired profits of the firms*, as shown in equation (4). It follows that taxing retired profits will not affect the optimal capital stock accumulation.

In conclusion, taxing firm's profits might have important effects over the long run level of capital stock because the tax impacts the marginal return of the capital stock. However, it is possible to implement a tax system that do not affect investment decision. This system will tax retired profits, rather than current profits. This result follows from the fact that

²²the change in the marginal return from capital stock is given by the change in expected profits, $E_t \frac{\partial \pi}{\partial k}$, and the change in the adjustment cost function, $\frac{\partial C}{\partial k}$

²³In the long run, the initial level of capital stock completely depreciates and thus, taxing the initial level of capital stock does not matter to determine the long run level of capital stock

this system does not distort the relative price of investment and it is similar to implement jointly a tax in current profits and a subsidy in investment through direct investment credits or through the use of depreciation allowances.

The "second-best" tax system

The above result indicated that there exists a tax system that minimize distortions in investment decisions. However, when deriving that policy we did not restrict our attention to the set of policies providing a required level of government revenue. This last dimension of the problem is quite important though, because obviously the government impose a tax system to collect revenues. This subsection will focus in solving the investment problem, but restricting the attention to tax systems providing a given level of revenues required by the government.

To derive our tax system, we will consider a large combination of policies. We will consider any tax system that might have $\tau \in [0, 0.01, \dots, 0.2]$ and $(\sigma + z) \in [0, 0.01, \dots, 0.2]$. Thus we will evaluate 441 different tax systems. Figure 16 shows a large variation in the long run level of capital among the different tax possible systems. The capital stock ranges from 0.8 when $\tau = 0.20, \sigma + z = 0$ to 3.2 when $\tau = 0, \sigma + z = 0.2$. An obvious qualification is that the first policy will provide positive revenues to the government while the second will provide negative revenues to the government.

[Insert figure 16]

We next restrict our attention to tax policies that provide a given level (range) of revenues. Figure 17 illustrates the combination of policies that provide a collection of revenues between 2.0 and 2.5 units of output per firm, where the graph normalizes to zero any policy that provides a different revenue. The figure shows that there is a set of policies providing the required level of revenues and each policy has associated a different level of capital stock in the long run. The associated long run capital stock for each policy can be seen in table 2. The "second-best" policy is the one providing the largest level of capital stock in the long run, among this set of policies. Two conclusions emerge from figure 17 and table 2. First,

the "first-best" policy is not available, meaning that it does not provide the required level of government revenue. Second, the maximum level of long run capital stock is obtained when $\tau = 0.12, \sigma + z = 0$, which is the tax system with lowest level corporate tax rate and investment subsidies.

[Insert figure 17 and table 2]

Why do we obtain the minimum level of corporate tax rate possible as "second-best" tax policy? From equation (10), we know that the relative price of investment depends in the tax system. The fourth column of table 2 shows this relative price for each possible tax policy. The relative price is the smallest when $\tau = 0.12$ and $(\sigma + z) = 0$. As we move to other tax policies, the relative price increases and thus investment will face less incentives. *Thus the "second-best" policy is the tax system that minimizes the distortion in the relative price of investment.*

Figure 18 and table 3 show the same exercise for a smaller required revenue. The result reaffirms our conclusion, meaning that the policy with the smallest corporate tax rate is the one that maximizes the long run level of capital stock, since it has the smallest relative price of investment.

[Insert figure 18 and table 3]

CONCLUSION

We propose a framework that considers microeconomic decisions to evaluate aggregate investment behavior in the economy. We validate the model by replicating the distribution of firms, in the Chilean economy, across investment decisions.

We obtain strong results and policy recommendations. First, and contrary to other studies, we conclude that the tax system significantly affect the long run level of capital stock when it taxes current firms profits. Second, we indicate that the "first-best" policy is to tax *retired* profits, rather than current profits. Third when the "first-best" policy is not available, because it does not provide enough revenues, we should implement a tax system

that minimize the distortion over the relative price of investment. This tax system is a mixture of zero subsidies (credit to investment and depreciation allowances) and positive but small corporate tax.

This study should be extended to include liquidity constraints in firms. This last factor might be quite important when considering a economy with a large fraction of firms of small size, as those firms are not able to provide the collateral required to borrow from the banking system. This might affect the stationary distribution and thus the long run level stock of capital because it imposes an upper-bound in each firm's investment decision.

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Figure 1

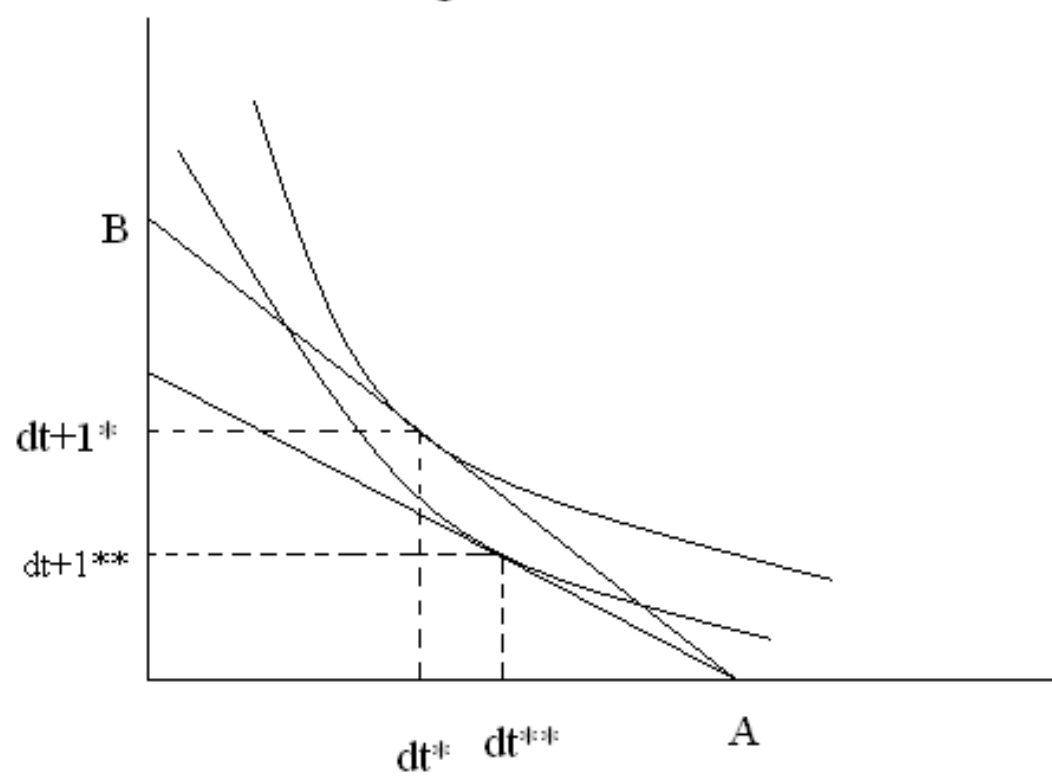
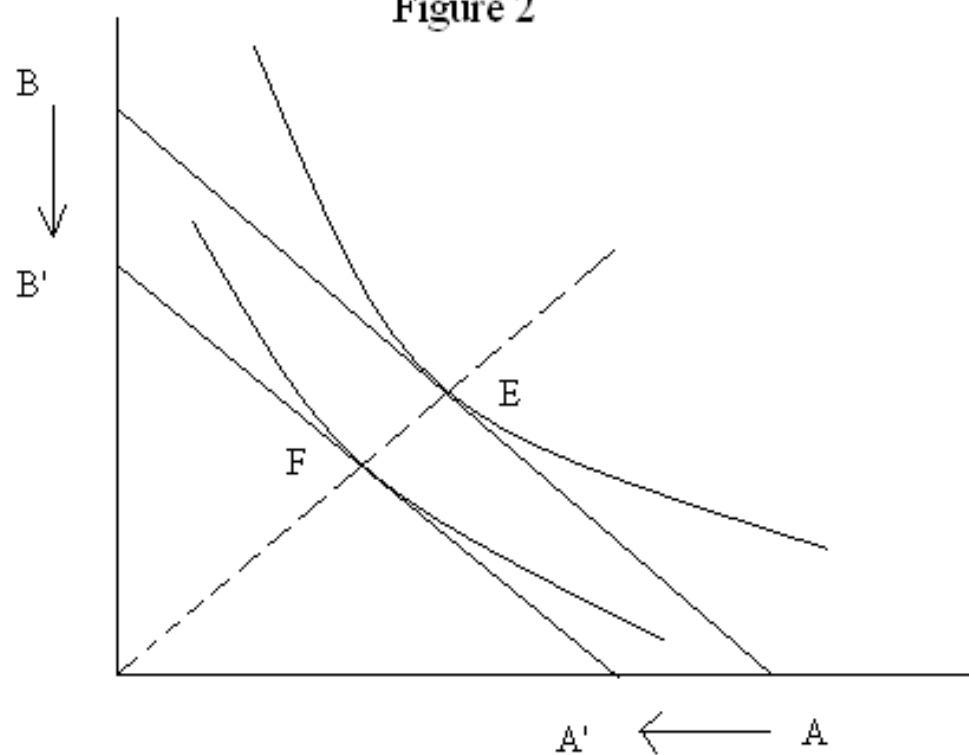
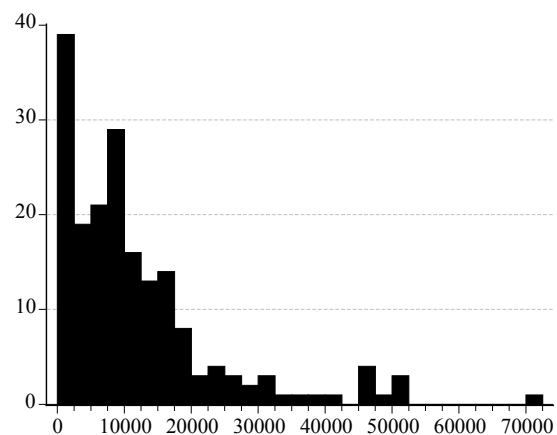


Figure 2

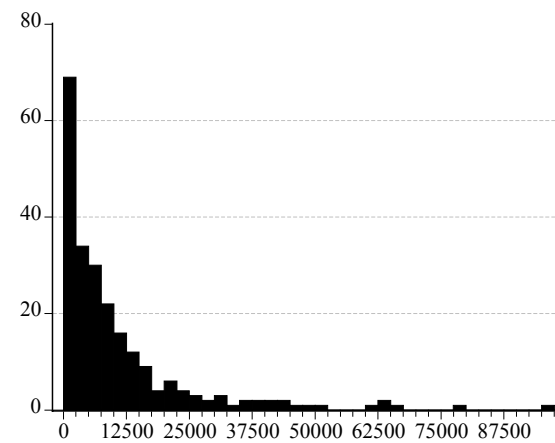


Figures 3 to 6



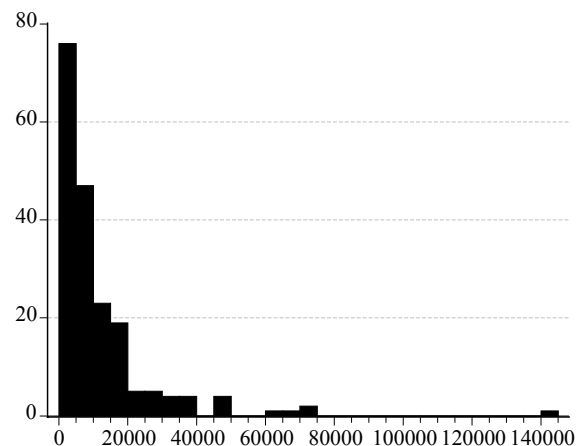
■ flow of investment, 1990

Mean	11790.99
Median	8300.000
Maximum	70000.00
Minimum	10.00000
Std. Dev.	12065.61
Skewness	1.942123
Kurtosis	7.297581
Jarque-Bera	261.4616
Probability	0.000000



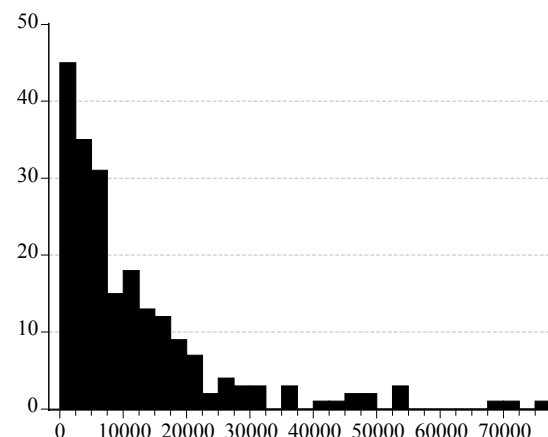
■ flow of investment, 1991

Mean	10892.23
Median	6200.000
Maximum	96000.00
Minimum	10.00000
Std. Dev.	14478.23
Skewness	2.723097
Kurtosis	11.96830
Jarque-Bera	1064.216
Probability	0.000000



■ flow of investment, 1992

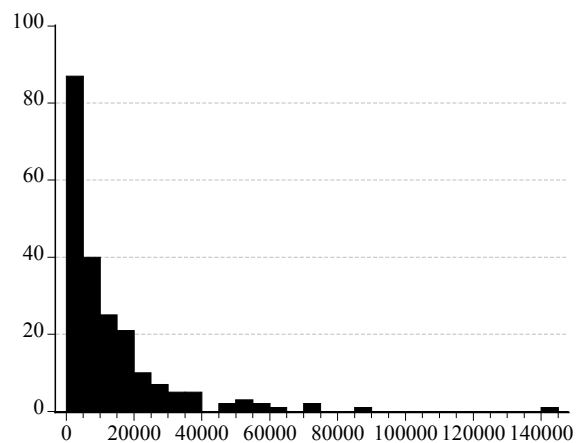
Mean	11676.38
Median	6964.500
Maximum	143700.0
Minimum	9.000000
Std. Dev.	16182.36
Skewness	4.012970
Kurtosis	27.16979
Jarque-Bera	5188.754
Probability	0.000000



■ flow of investment, 1993

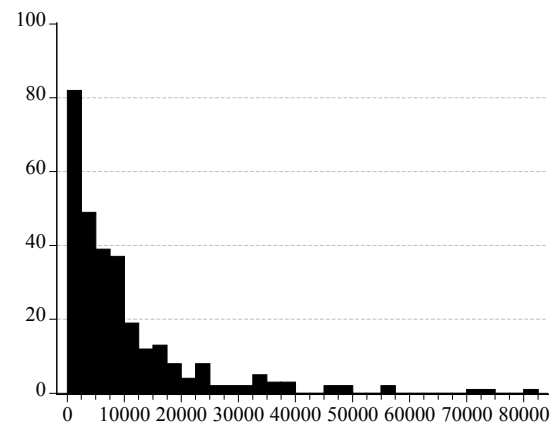
Mean	11747.67
Median	6795.000
Maximum	76538.00
Minimum	15.00000
Std. Dev.	13360.88
Skewness	2.312891
Kurtosis	9.153840
Jarque-Bera	523.5306
Probability	0.000000

Figures 7 to 9



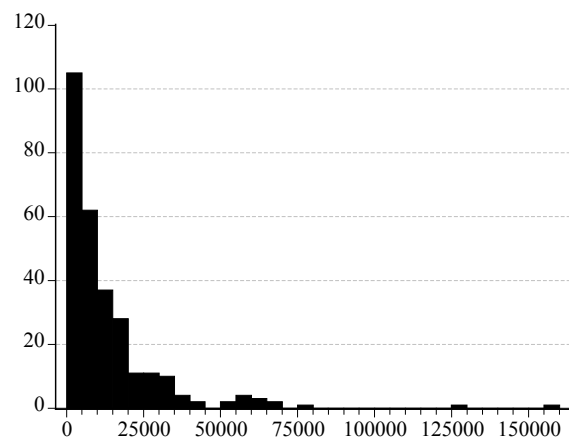
■ flow of investment, 1994

Mean	12777.95
Median	6825.000
Maximum	144200.0
Minimum	13.00000
Std. Dev.	17278.66
Skewness	3.441419
Kurtosis	20.63592
Jarque-Bera	3165.858
Probability	0.000000



■ flow of investment, 1995

Mean	9921.596
Median	5643.000
Maximum	82100.00
Minimum	15.00000
Std. Dev.	12309.19
Skewness	2.709437
Kurtosis	12.21152
Jarque-Bera	1413.426
Probability	0.000000



■ flow of investment, 1996

Mean	13204.22
Median	7650.000
Maximum	158544.0
Minimum	10.00000
Std. Dev.	17833.85
Skewness	3.777456
Kurtosis	24.40938
Jarque-Bera	6099.351
Probability	0.000000

Figure 10: Optimal capital accumulation, negative shock

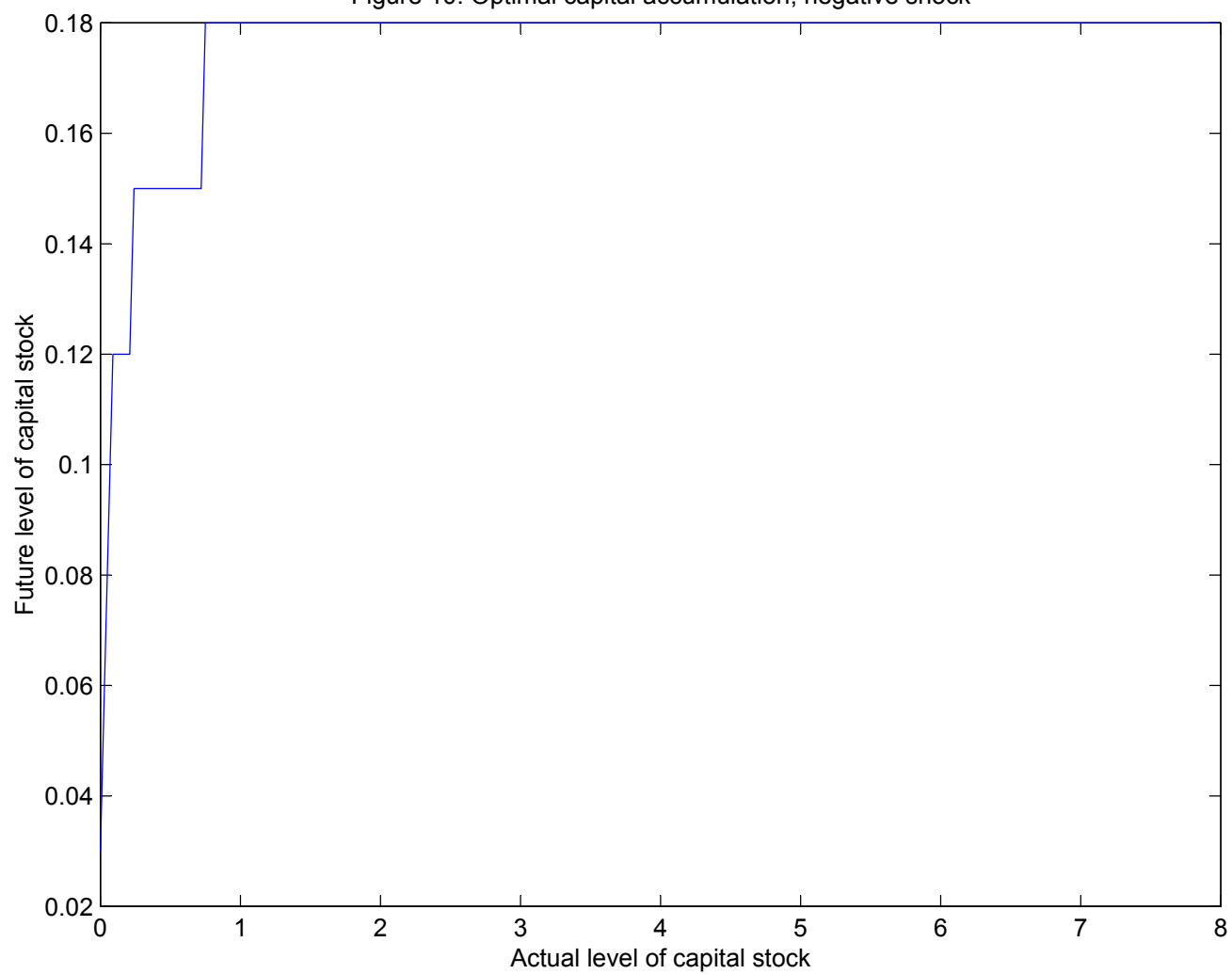


Figure 11: Optimal capital accumulation, neutral shock

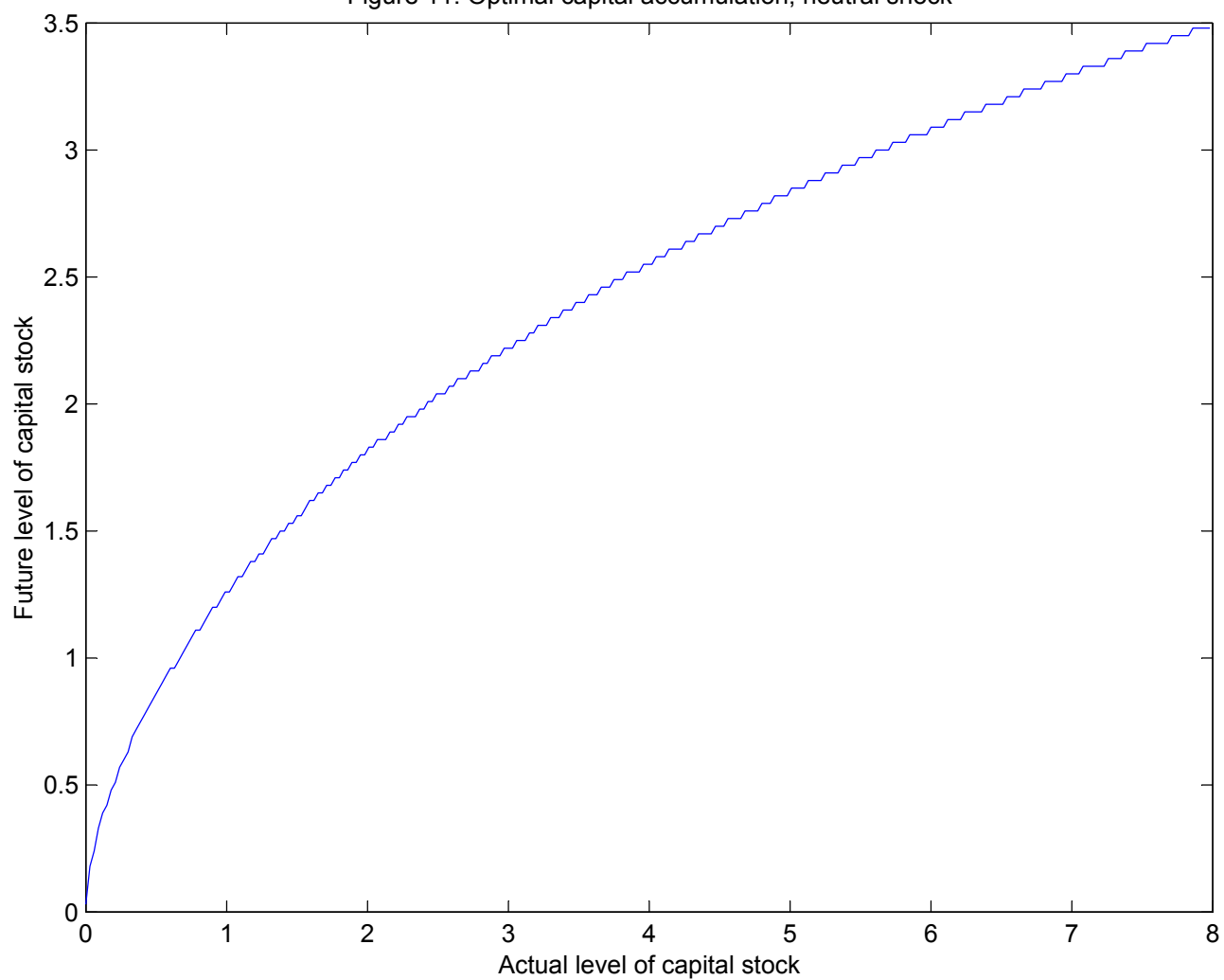


Figure 12: Optimal capital accumulation, positive shock

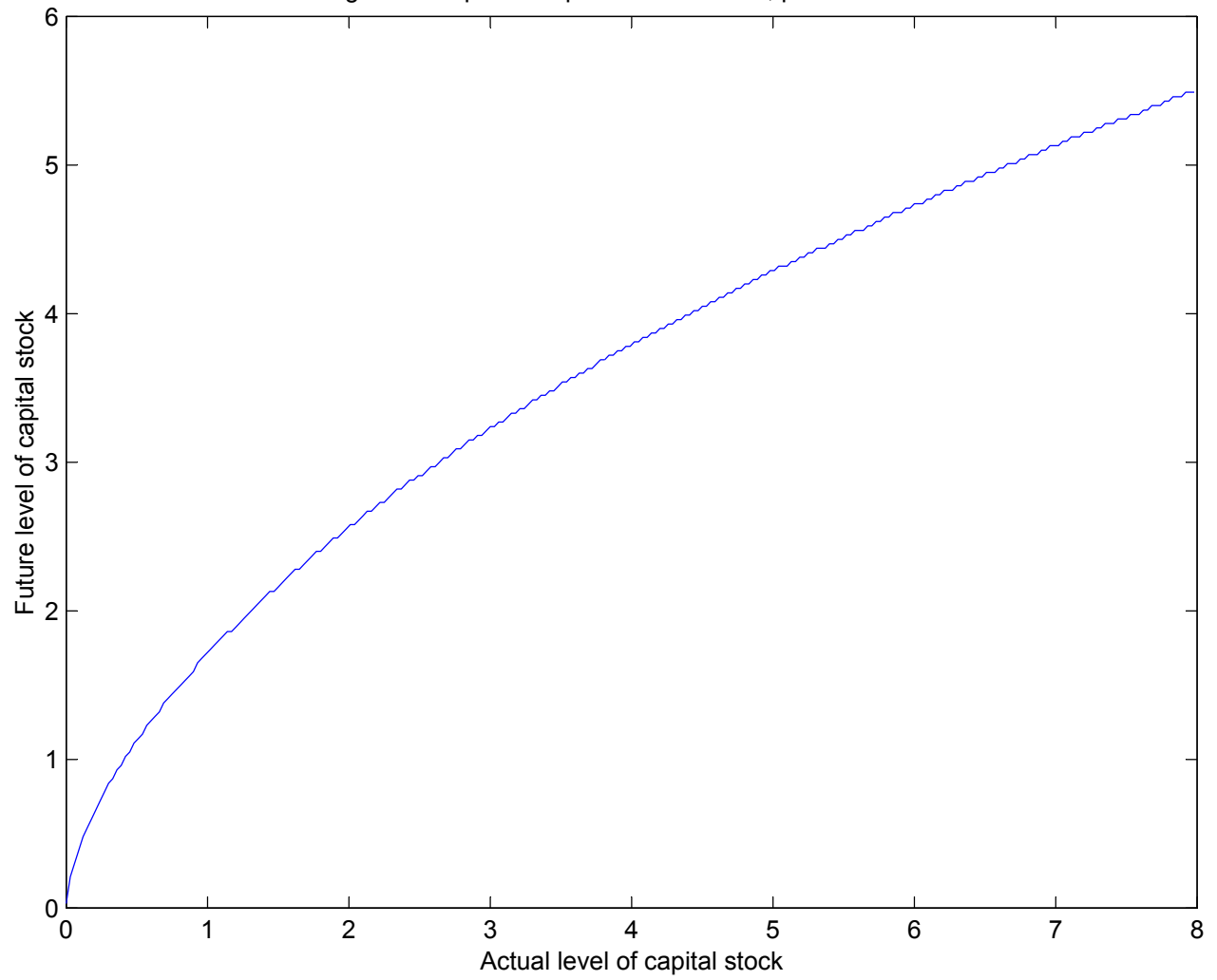


Figure 13: Stationary distribution, $\tau=\sigma=z=0$

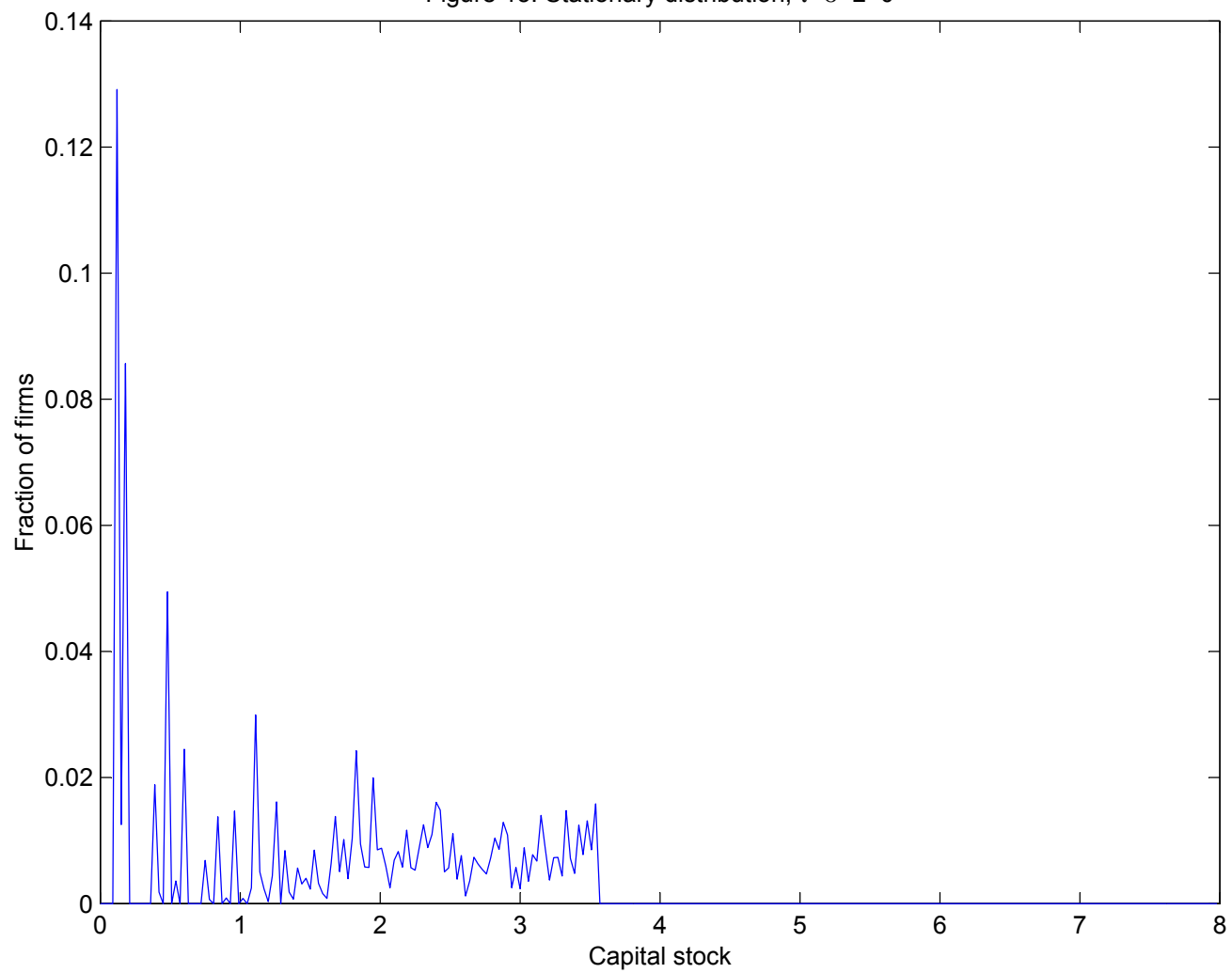


Table 1

<u>Observed distribution</u>									<u>Simulated distribution</u>		
	1990	1991	1992	1993	1994	1995	1996	Mean (1990-96)	$\tau=\sigma+z=0$	$\tau=15\%; \sigma+z=0$	$\tau=\sigma+z=20\%$
Std. Dev.	102.3%	132.9%	138.6%	113.7%	135.2%	124.1%	135.1%	126%	71.0%	71.2%	71%
Skewness	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.5%	1.9%	5.5%
Kurtosis	0.1%	0.1%	0.2%	0.1%	0.2%	0.1%	0.2%	0.1%	175%	34%	175%
Observations	187	232	192	212	212	297					

The source of the data is the 1990-96 versions of the ENIA. The moments are measured as fraction of the mean of the distribution. The column "mean" corresponds to the mean values obtained in the period 1990-1996. The three last columns correspond to the distribution obtained through simulation exercises.

Figure 14: Stationary distribution, $\tau=0.15; \sigma=z=0$

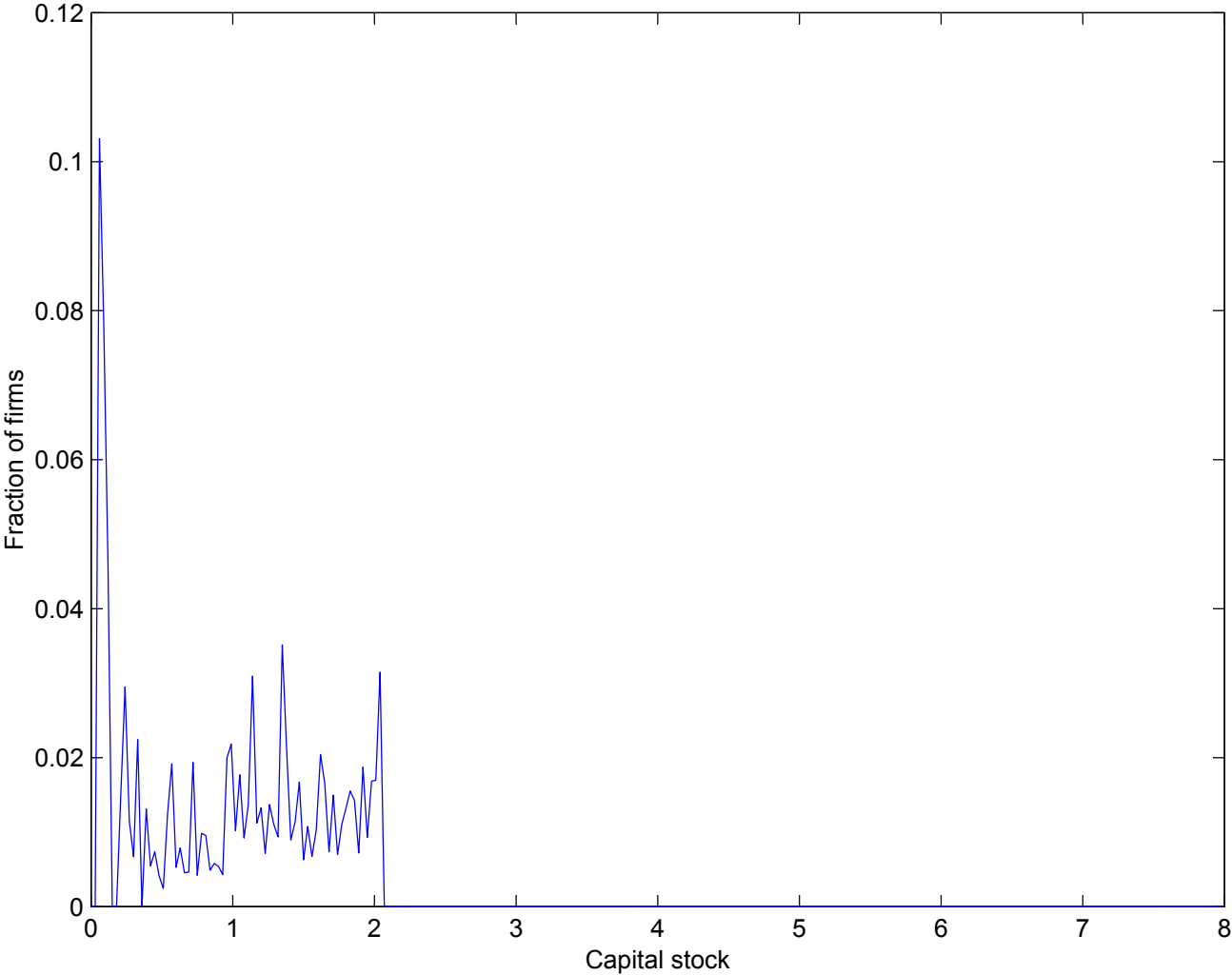


Figure 15: Stationary distribution, $\tau=\sigma=z=0.15$

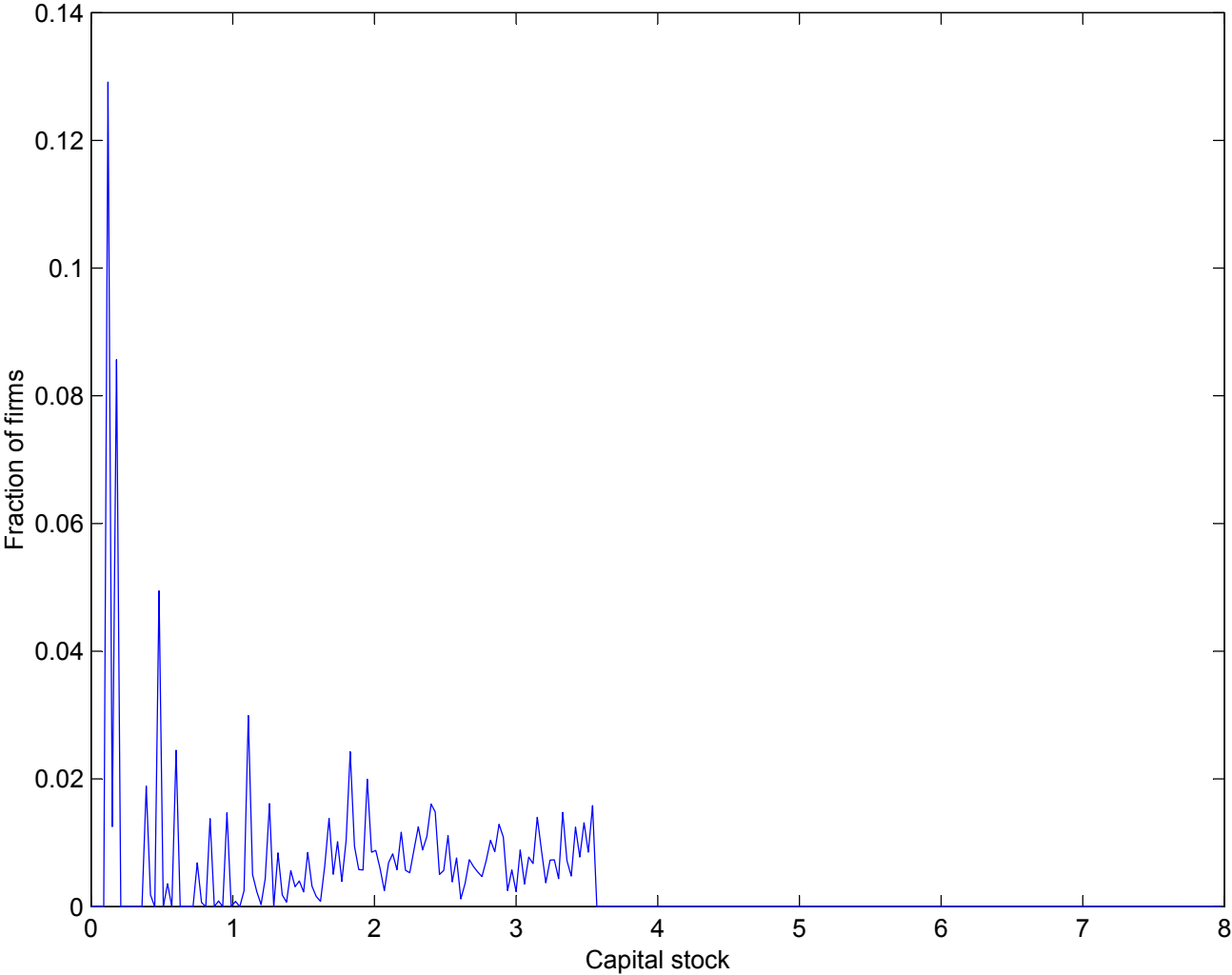


figure 16: Long run capital stock, any REV

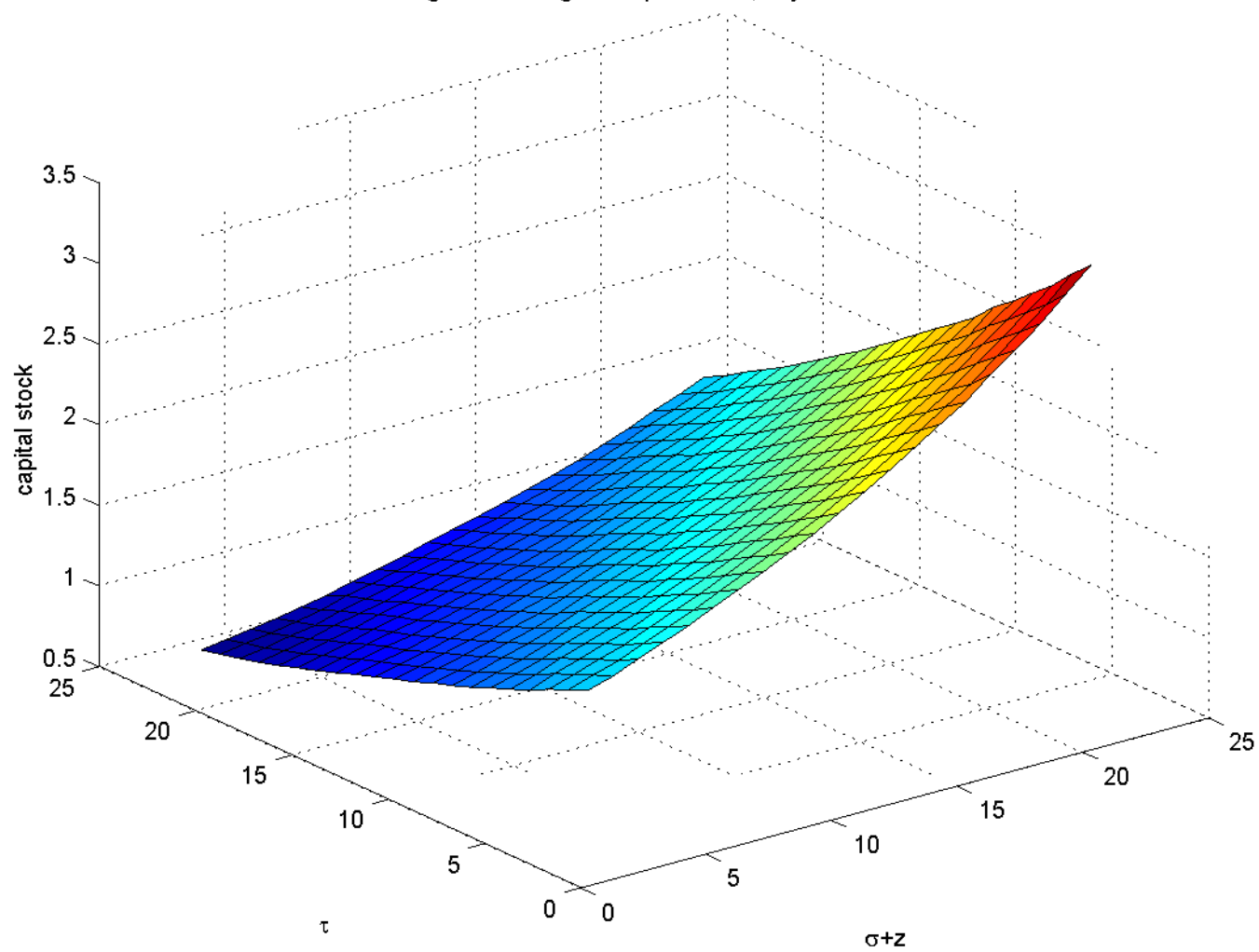


figure 17: Long run capital stock, $2 < \text{REV} < 2.5$

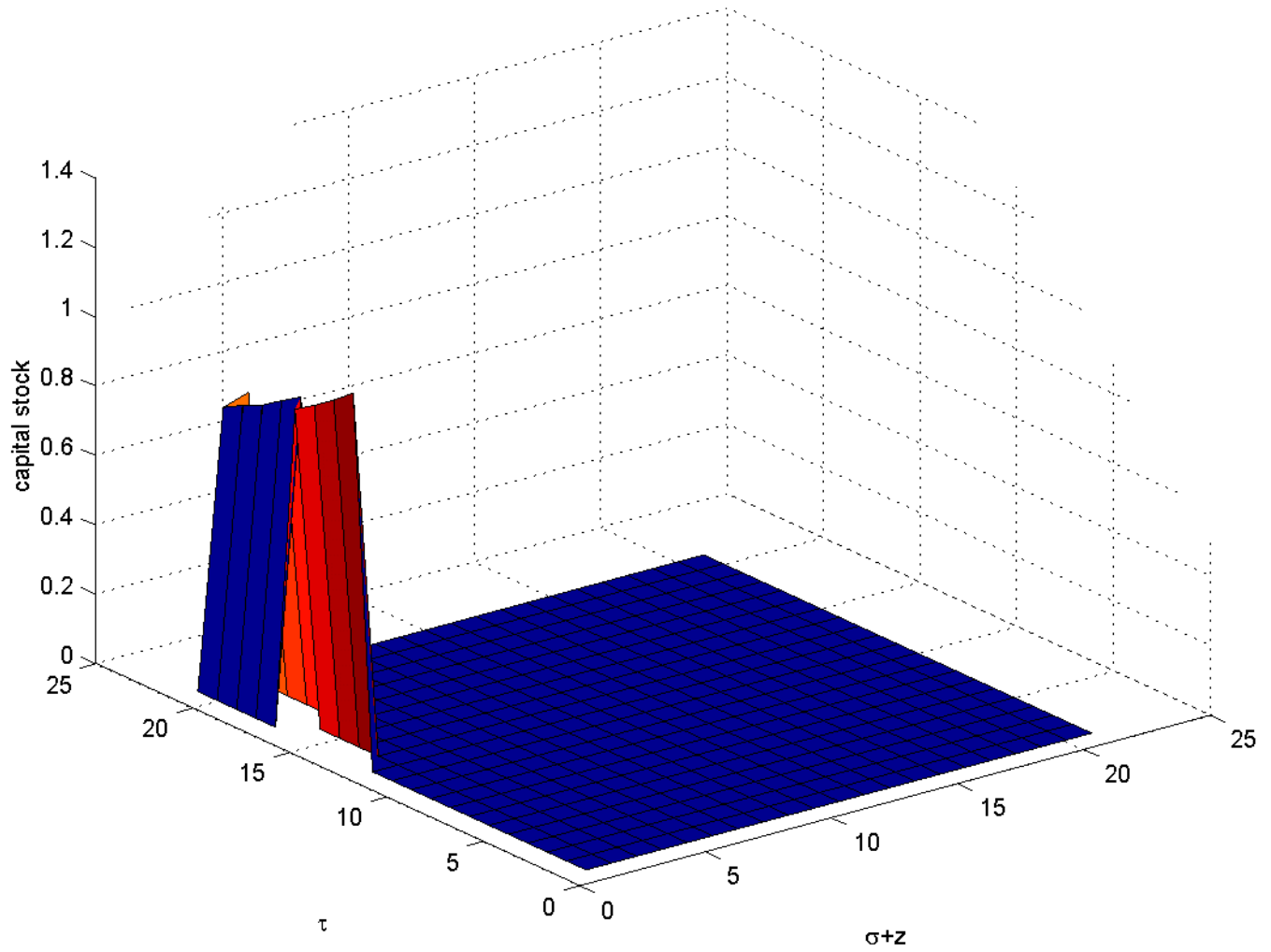


figure 18: Long run capital stock, $0.5 < \text{REV} < 1$

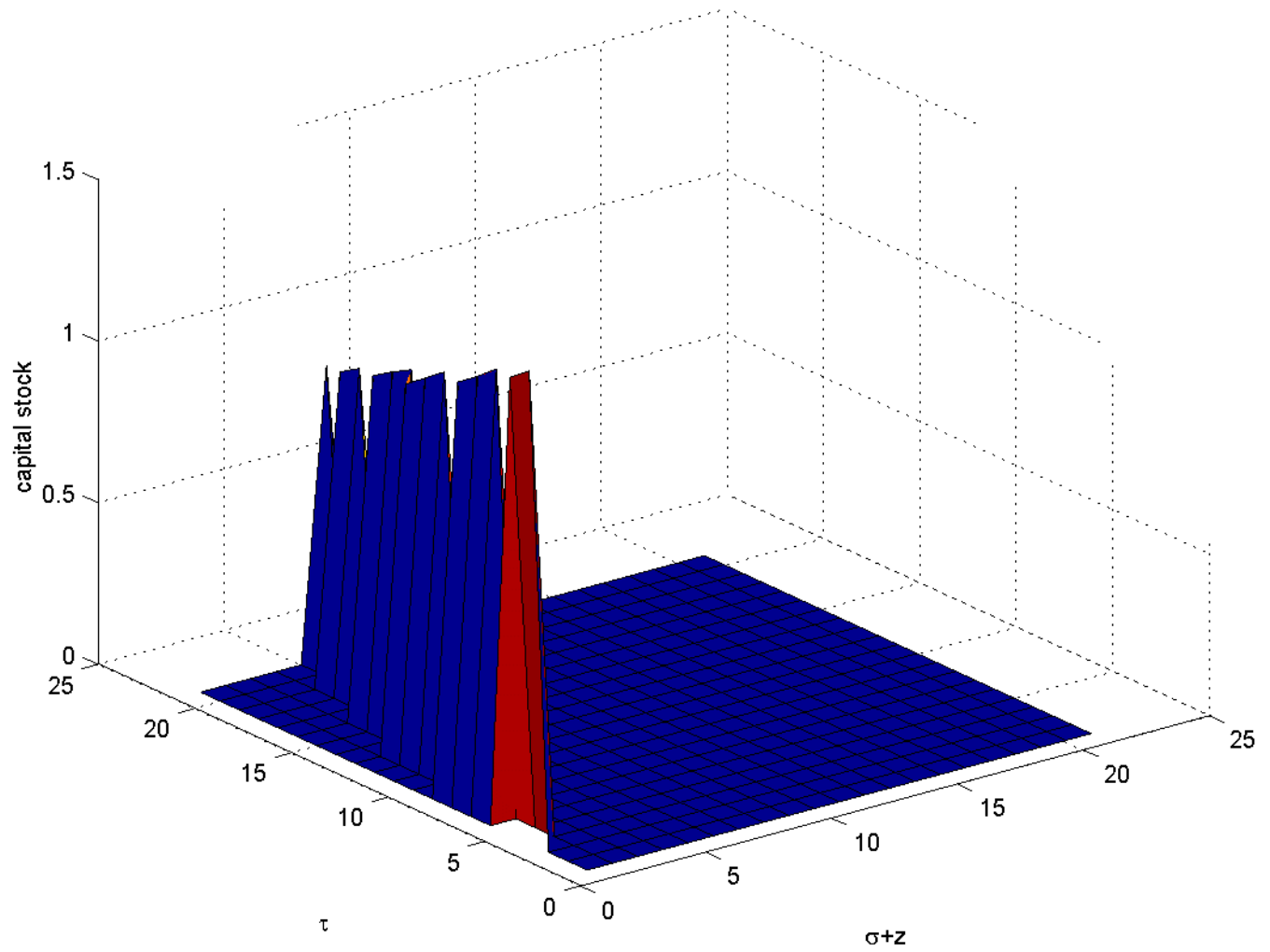


Table 2

Long Run Capital Stock, 2<Rev<2.5			
Z+ Inv. Credit %	Corporate tax %	Capital Stock	Relative price Inv.
0	12	1.0647	1.1932
1	16	0.9308	1.2375
2	20	0.8219	1.2863

Table 3

Long Run Capital Stock, 0.5<Rev<1			
Z+ Inv. Credit %	corporate tax %	capital stock	relative price inv.
0	4	1.412	1.0938
1	8	1.2667	1.1299
2	12	1.1297	1.1693
3	15	1.0522	1.1982
4	18	0.9597	1.2293
5	20	0.9039	1.2469