

Quadratic sequences of powers and Mohanty's conjecture

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Abstract

We prove under the Bombieri–Lang conjecture for surfaces that there is an absolute bound on the length of sequences of integer squares with constant second differences, for sequences which are not formed by the squares of integers in arithmetic progression. This answers a question proposed in 2010 by Browkin and Brzezinski, and independently by Gonzalez-Jimenez and Xarles. We also show that under the Bombieri–Lang conjecture for surfaces, for every $k \geq 3$ there is an absolute bound on the length of sequences formed by k th powers with constant second differences. This gives a conditional result on one of Mohanty's conjectures on arithmetic progressions in Mordell's elliptic curves $y^2 = x^3 + b$. Moreover, we obtain an unconditional result regarding infinite families of such arithmetic progressions. We also study the case of hyperelliptic curves of the form $y^2 = x^k + b$. These results are proved by unconditionally finding all curves of genus zero or one on certain surfaces of general type. Moreover, we prove the unconditional analogues of these arithmetic results for function fields by finding all the curves of low genus on these surfaces..

Keywords

Curves of low genus, Quadratic sequences, Mohanty's conjecture, Function field arithmetic.