A Milk Collection Problem with Blending

Germán Paredes-Belmar[1], Vladimir Marianov[2], Andrés Bronfman[3], Carlos Obreque[4], Armin Lüer-Villagra[5]

Abstract

A milk collection problem with blending is introduced. A firm collects milk from farmers, and each farm produces one out of three possible qualities of milk. The revenue increases with quality, and there is a minimum requirement at the plant for each quality. Different qualities of milk can be blended in the trucks, reducing revenues, but also transportation costs, resulting in higher profit. A mixed integer-programming model, a new cut, and a branch-and-cut algorithm are proposed to solve medium-sized instances. A three-stage heuristic is designed for large instances. Computational experience for test instances and a large-sized real case is presented.

Keywords: Routing, milk collection, different qualities of milk.

[1] Graduate Program, Department of Electrical Engineering, Pontificia Universidad Católica de Chile, Santiago, Chile and Engineering Sciences Department, Universidad Andres Bello, Santiago, Chile. german.paredes@unab.cl
[2] **Corresponding Author**. Department of Electrical Engineering, Pontificia Universidad Católica de Chile, Av. Vicuña Mackenna 4860, Santiago, Chile. Tel.: +562 23544974, Fax: +562 25522563. marianov@ing.puc.cl
[3] Graduate Program, Department of Electrical Engineering, Pontificia Universidad Católica de Chile, Santiago, Chile and Engineering Sciences Department, Universidad Andres Bello, Santiago, Chile. abronfman@unab.cl
[4] Department of Industrial Engineering, Universidad del Bío-Bío, Concepción, Chile. cobreque@ubiobio.cl
[5] Engineering Sciences Department, Universidad Andres Bello, Santiago, Chile. armin.luer@unab.cl

1. Introduction

The cost of collecting milk from producers in the milk production supply chain has a significant impact on profit (Rojas and Lusa, 2005; Lahrichi et al., 2013). Milk producers are frequently scattered over extended rural areas, sometimes far from processing plants, making transportation cost a relevant component of total cost. It is also common for small producers to organize themselves into cooperatives, able to obtain better commercial terms with, usually, a single buyer (FAO, 2012). Each cooperative sells the milk produced by its members to the buyer, or firm, who performs the collection process. This arrangement is convenient for the cooperative members, but poses some challenges to the buyer, who must collect milk from all farms in the cooperative, although some may be located far from the plant. Moreover, milk produced by different farmers can have different qualities or grades, used for different final products. Currently, firms address the differences in quality by either using separate trucks for collecting different qualities, or using tanks with separate compartments for different qualities. Both solutions are expensive, and particularly if some farmers produce small quantities of milk, as in the real case in this study.

A different approach, consisting of mixing or blending different qualities of milk in some of the trucks, is also possible. Blending degrades the quality of part of the collected milk, as the blended product is classified as its lowest quality component, which reduces the firm's revenue. However, the savings in transportation cost exceed the reduction in revenue and ultimately increase profit.

Blending is a common practice. GLH Incorporated (2014), in a report for the United States' Food and Drug Administration (FDA), explicitly declares that "It is a common practice in some states, including two large milk production states, that bulk milk pickup tankers pick up milk from both Grade 'A' and non-Grade 'A' milk producers on the same tanker. Then, these loads are delivered to non-Grade 'A' processing facilities." New York regulations allow commingled milk in trucks (New York SDAM, 2003). A plant in the south of Chile, which is this paper's case study, also uses blending. The common use of blending makes this a relevant practice that, to the best of the authors' knowledge, has never been analyzed in the literature.

Note that blending does not violate any regulations, as long as the resulting milk is correctly classified at the processing facility. Therefore, this procedure could be extended to wherever the grading of milk or dairy products is performed by bacterial limits and somatic cell count, or by fat content. Examples of countries with such regulations, among others, are the United States (Chite, 1991; U.S. DHHS, 2009), Canada (CDIC, 2005), Bolivia (UNIA, 2011), Mexico (INIFAP, 2011), Panama (Pinzón, 2015), and India (MHFW, 2015). This procedure also applies when the industry imposes such a classification, as in the case of the Murray Goulburn Co-operative in New South Wales and Sydney, Australia (Devondale Murray Goulburn, 2014).

The blending of different products or product qualities in the same vehicle also applies to other industries. Nema and Gupta (1999) analyze the blending of hazardous materials with different degrees of risk in the same truck. Bing et al. (2014) solve a waste collection problem, in which collection is either performed after separating some recyclables at collection points, or by loading different classes of waste in the same truck and classifying them at the processing site. These are also possible applications for this study's approach.

Finally, yet importantly, Sethanan and Pitakaso (2016) state that the mixing of raw milk from different collection centers in the same compartment would be a valuable extension of their research, as they do not use blending, and this would "add to the ability of (their) technique to model real world problems."

An optimization of the blending procedure is proposed, as this is of practical relevance, and has not been previously addressed. This study's contributions are several. First, the Milk Collection Problem with Blending (MB) is introduced. For each truck in a heterogeneous fleet, the MB solution indicates what farmers each truck must visit, and the route it must follow, to deliver all produced milk to the plant. This also specifies whether it is more convenient to perform blending in the trucks or at the plant. The objective is to maximize the firm's profits.

Second, a mixed integer formulation for the problem is proposed, as well as a branch-and-cut algorithm, using a new cut and known cuts to solve medium-sized instances optimally.

Third, a heuristic procedure is designed to solve large instances. This heuristic partitions a set of farms into clusters, each with a fewer number of farms. Solving the problem by clustering is non-trivial, as there are milk quotas to be fulfilled and a given truck fleet; therefore, trucks and milk quotas must be assigned to clusters. A mathematical programming formulation is proposed that i) allocates milk requirements to each cluster, so as to ensure the fulfillment of the global minimum requirements for each milk quality at the plant, and ii) finds the best allocation of trucks to each cluster, according to the available fleet. Finally, the collection problem is solved for each cluster using the branch-and-cut algorithm.

Test instances of up to 100 nodes are solved, and a real instance is solved that includes 500 farms. The solutions obtained using this new approach are then compared to the solutions currently implemented by the firm, and with the solutions obtained by collecting each quality of milk separately, using the vehicle routing problem (VRP) for each. Managerial insight is provided. Finally, solutions for trucks with and without compartments, and with and without blending are compared, which demonstrates that blending dominates all solutions.

Note that the problem is NP-Hard, as for one quality of milk, it reduces the VRP, which is NP-Hard (Irnich et al., 2014; Toth and Vigo, 2002).

The remainder of the paper is organized as follows: Section 2 presents the literature review. Section 3 describes the milk collection problem with blending. Section 4 details the development of the mixed integer programming (MIP) model, the valid cuts, and their separation algorithms. Section 5 illustrates a procedure for solving large instances. Section 6 is devoted to numerical experience, with the test instances, the actual case, and the full heuristic. Different alternative approaches are compared in this section, including the use of compartments. Section 7 concludes.

2. Literature review

The literature includes different variants of the milk collection problem, as well as a number of solution methods, none of which include blending. Sankaran and Ubgade (1994) were the first to address this problem as a special case of the VRP. They designed collection routes to minimize transportation costs, using a Decision Support System (DSS) to solve a 70-farm case in Etah, India, obtaining yearly savings of USD $15,000. Igbaria et al. (1996), Prasertsri and Kilmer (2004) and Butler et al. (2005) solved similar real problems using a DSS tool.

Butler et al. (1997) solve a milk collection problem in which some producers must be visited twice, while the milk from the rest of the farms is collected only once a day. Basnet et al. (1999) propose an exact model and a heuristic procedure for assigning trucks to predefined routes in New Zealand, the goal of which is to minimize the time at which the last truck delivers its load to the plant. Hoff and Løkketangen (2007) solve a real case in Norway, using the model defined by Chao (2002) for a truck and trailer routing problem. Claassen and Hendriks (2007) address a goat milk collection problem in the Netherlands. The producers are visited according to individual frequencies, rather than daily. The authors minimize the deviations, whether surplus or deficit, between collection and production. Dayarian et al. (2013) and Dayarian et al. (2015a) solve a milk collection and distribution problem over multiple periods, considering significant seasonal production variations over a tactical time horizon. Dayarian et al. (2015b) introduce a routing problem with multiple depots, a heterogeneous fleet, and time windows, motivated by a real milk collection problem in Canada. They solve an integer programming formulation using column generation and a branch-and-price algorithm.

Other authors examine the collection of different qualities of milk. Dooley et al. (2005), in a New Zealand application, classify the milk into two qualities. Each quality of milk is collected separately, and the transportation cost is minimized. Caramia and Guerriero (2010) address a problem with four qualities of milk, which are not allowed to be blended. They use trucks with compartments and apply a local search heuristic, which first allocates producers to truck compartments to minimize the number of trucks. The routing problem is solved in the second stage, with the goal of minimizing travel distance. Lahrichi et al. (2013) addressed a case in Canada, in which trucks with two compartments start their route at a single depot in multiple periods and collect three qualities of milk, which are not allowed to be blended. The milk is transported to a set of plants, and the trucks travel back to the depots, at a minimum travel cost. The authors solve the problem using a tabu search heuristic. Sethanan and Pitakaso (2016) determine the milk collection routes for a set of milk collection centers. They consider the use of different qualities of milk without blending, as they use trucks with compartments. The article suggests, as a future work, the blending of milk from different collection points in the same compartment. Masson et al. (2015) study an annual dairy transportation problem, inspired by a

Canadian milk collection problem. They generalize the problem proposed by Lahrichi et al. (2013), which considers variations on a basis of daily demand.

Literature also handles the collection of fresh agricultural products, in terms of related problems. A review of this literature is provided in the work of Shukla and Jharkharia (2013). The trucks in that case are not usually equipped to maintain a product's freshness; therefore, delivery time constraints are required. Related articles study agricultural problems using trucks with compartments to transport different types of products and avoid mixing them (Fallahi et al., 2008; Henke et al., 2015; Lahyani et al., 2015; Mendoza et al., 2010).

3. Milk collection with blending

Farms in this study's version of the problem produce three qualities of milk with decreasing revenues; quality A is better than B, which is better than C. Quotas for each quality must be satisfied at the plant. Blending of the different qualities of milk is allowed in the trucks along the route to save transportation costs, and at the plant to satisfy its minimum requirements. Blending makes the problem non-separable by quality of milk. In the numerical tests, all routes end at the plant, but they can begin anywhere, and the trucks are heterogeneous. For example, Figure 1 illustrates a five-node network, in which the node marked "0" is the plant. In this example, the routes start and end at the plant, and the number beside each arc is the transportation cost. The quota for each quality of milk is 200 liters, and the unit revenues per liter of milk are 1.0, 0.7, and 0.3 monetary units (MU) for milk A, B, and C, respectively. The capacity of each truck is 220 liters.

If no blending occurs, the problem becomes a separable VRP for each milk quality. The solution requires three trucks. The transportation cost is 280, following the routes 0-1-2-4-2-0 (milk A), 0-2-0 (milk B), and 0-3-0 (milk C), while the revenue is 420, with a total profit of 140. If blending is allowed, the MB requires the same three trucks. The optimal routes are 0-1-0 (milk A), 0-2-4-2-0 (milk B, resulting from the blending of milks B and A), and 0-3-0 (milk C) at a cost of 220 and a revenue of 414. The total profit is 194, or 39% higher than the profit obtained without blending. The revenue in the latter case is reduced by 6 units, while costs are reduced by 60 units.

Note that at the beginning of each season, or with a frequency that depends on the regulations (e.g., FDA, 2015), each farm's production line is inspected and a grade or quality, and consequently, a unit price, is assigned to that producer's milk for that season. The farmer is paid for the volume of milk he delivers, at the price set during the last inspection. Both volume and price are known *a priori*. Consequently, the total payment is a fixed amount, not possible to optimize, which does not depend on whether or not it is blended. Hence, payment is not included in the objective. The plant bears the cost of the reduction in revenue because of blending.

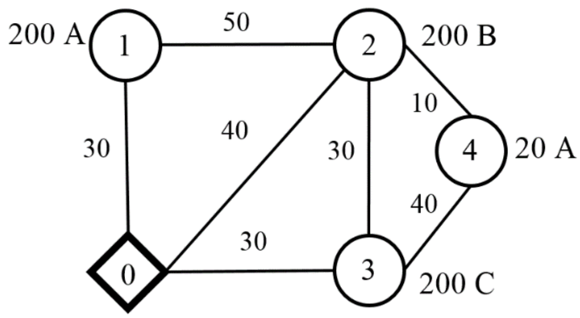


Figure 1. Network for comparing MB and VRP solutions

The model allows for the blending of different qualities of milk at the plant, as this is sometimes profitable, as noted in the following example. Producers 1, 2, and 3 in Figure 2 each produce 100 liters of milk B, and producers 4 and 5 each produce 200 liters of milk A. The quotas at the plant are 350 liters of milk B and 200 liters of milk A. The capacity of each truck is 500 liters, and two trucks are required for collecting all the milk. In Figure 2a), blending at the plant is not allowed. Truck 1 follows the route shown as a continuous line, and truck 2 follows the dotted route; truck 1 collects 500 liters of milk B, while truck 2 collects 200 liters of milk A. The revenue is 550 units, the cost is 330 units, and the profit is 220 units. Figure 2b displays the optimal solution when blending at the plant is allowed, with truck 1 collecting 300 liters of milk B, and truck 2 collecting 400 liters of milk A. At the plant, 50 liters of milk A are blended with milk B to satisfy the milk B quota. The revenue is 595 units, and the cost is 320 units. The final profit is 275 units, which is 55 units higher than the solution illustrated in Figure 2a.

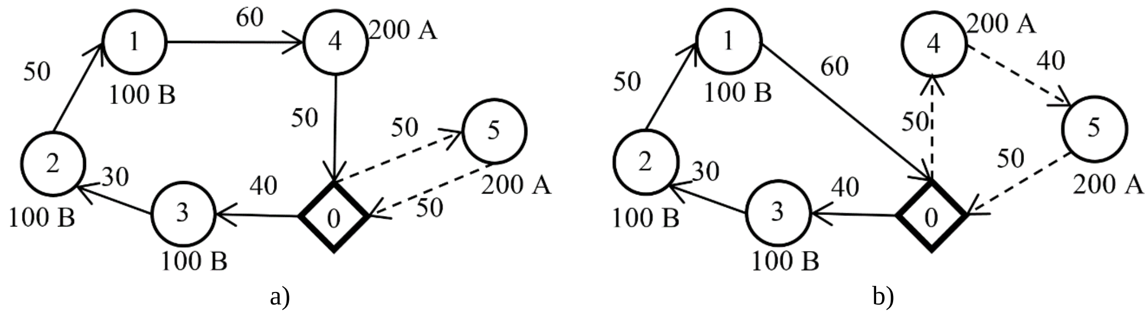


Figure 2. Effects of blending at the plant

Note that to satisfy the quotas for the three qualities of milk, the following relations must hold:

$$q_A \geq P_A \tag{1}$$

$$\left(q_A - P_A\right) + q_B \geq P_B \tag{2}$$

$$\left(q_A - P_A\right) + \left(q_B - P_B\right) + q_C \geq P_C \tag{3}$$

where q_A, q_B, and q_C are the total production of milk A, B, and C, respectively, and P_A, P_B, and P_C are the minimum quotas at the plant for milk A, B, and C, respectively.

It can be observed that blending can also be applied to trucks with compartments; that is, each compartment in such a truck can carry blended milk.

4. Mixed integer model

Let $G(N, A)$ be a complete graph, with N as the set of nodes representing producers, and A as the set of arcs, or roads. N_0 is defined as $N \cup \{0\}$, where 0 is the node that identifies the plant location. A^0 defines the set of arcs connecting the plant with the producers. K is the set of trucks, and T is the set of qualities of milk. N^t identifies the producers of milk quality $t \in T$; D^t = {milk quality r| blend of r and t results in r. Includes $r = t$}. IT is the set of ordered pairs (i, t) of producer i and milk quality t, as each client produces only one quality of milk. However, it is easy to generalize the model by making one copy of each farm for each quality of milk it produces, with all copies sited in the same location. Q^k is the capacity of truck k; q_i^t is the amount of milk t produced by farm i; c_{ij}^k the travel cost of truck k over the arc $(i, j) \in A \cup A^0$; α^t is the revenue per unit of milk quality t; and P^t is the quota for milk quality t at the plant.

The notation from the work of Yaman (2006) is used for the heterogeneous truck fleet. Let $K_i = \{k \in K: t \in T, q_i^t \leq Q^k \ \ \forall t\}$ be the set of trucks that can visit producer i and, for each arc $(i, j) \in A \cup A^0$, let the truck set $K_{ij} = \{k \in K: q_i^t + q_j^t \leq Q^k \ \ \forall t\}$ be those trucks that can travel from producer i to producer j, collecting all milk from both producers without exceeding truck capacity. Node 0_k is the node at which truck k starts its trip, which ends at the plant. Finally, the set $AK = \{(i, j, k): (i, j) \in A \cup A^0, k \in K_{ij}\}$ is defined.

Decision Variables

$$x_{ij}^k = \begin{cases} 1 & \text{If truck } k \text{ travels directly from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases}$$

$$y_i^{kt} = \begin{cases} 1 & \text{If truck } k \text{ loads milk type } t \text{ from producer } i \\ 0 & \text{otherwise} \end{cases}$$

$$z^{kt} = \begin{cases} 1 & \text{If truck } k \text{ delivers milk type } t \text{ to the plant} \\ 0 & \text{otherwise} \end{cases}$$

$w^{kt} =$ Volume of milk quality t that truck k delivers to the plant.

$v^{tr} =$ Volume of milk of quality t delivered to the plant, blended for its use as milk of quality r.

The formulation of the MB problem is as follows:

$$Z = Max \sum_{t \in T} \sum_{r \in T} \alpha^r v^{tr} - \sum_{(i,j,k) \in AK} c_{ij}^k x_{ij}^k \qquad (4)$$

Subject to

$$\sum_{t \in T} \sum_{i \in N:(i,t) \in IT} q_i^t y_i^{kt} \leq Q^k \qquad \forall k \in K \qquad (5)$$

$$\sum_{k \in K_i} y_i^{kt} = 1 \qquad \forall i \in N, t \in T : (i,t) \in IT \qquad (6)$$

$$\sum_{j:(0_k, j, k) \in AK} x_{0_k j}^k \leq 1 \qquad \forall k \in K \qquad (7)$$

$$\sum_{i:(i,j,k)\in AK} x_{ij}^k = \sum_{h:(j,h,k)\in AK} x_{jh}^k \qquad \forall k \in K_j, j \in N_0 \qquad (8)$$

$$\sum_{p:(p,i,k)\in AK} x_{pi}^k = y_i^{kt} \qquad \forall k \in K_i, i \in N, t \in T : (i,t) \in IT \qquad (9)$$

$$z^{kt} \leq 1 - \sum_{\substack{r\in D^t : r\neq t, \\ (i,r)\in IT}} y_i^{kr} \qquad \forall k \in K_i, i \in N, t \in T \qquad (10)$$

$$\sum_{t\in T} z^{kt} \leq 1 \qquad \forall k \in K \qquad (11)$$

$$w^{kt} \leq z^{kt} Q^k \qquad \forall k \in K, t \in T \qquad (12)$$

$$w^{kt} \leq \sum_{r:t\in D^r} \sum_{h\in N^r} q_h^r y_h^{kr} \qquad \forall k \in K, t \in T \qquad (13)$$

$$\sum_{k\in K} \sum_{t\in T} w^{kt} = \sum_{(i,t)\in IT} q_i^t \qquad (14)$$

$$\sum_{r\in D^t} v^{tr} = \sum_{k\in K} w^{kt} \qquad \forall t \in T \qquad (15)$$

$$\sum_{t\in T} v^{tr} \geq P^r \qquad \forall r \in D^t \qquad (16)$$

$$y_i^{kt} + y_i^{kr} \leq 1 \qquad \forall (t,r) \in PM; (i,t), (j,t) \in IT \qquad (17)$$

$$\sum_{i\in S} \sum_{j\in S} x_{ij}^k \leq |S| - 1 \qquad \forall S \subseteq N, k \in K \qquad (18)$$

$$y_i^{kt}, z^{kt} \in \{0,1\} \qquad \forall i \in N, k \in K_i, t \in T : (i,t) \in IT \qquad (19)$$

$$x_{ij}^k \in \{0,1\} \qquad \forall (i,j,k) \in AK \qquad (20)$$

$$w^{kt}, v^{tr} \geq 0 \qquad \forall k \in K; t, r \in T, r \in D^t \qquad (21)$$

Objective (4) maximizes profit, which is the revenue from the milk received at the plant, minus the transportation cost. Recall that the price paid to the farmers is not included in the objective because it is constant. Constraints (5) limit the capacity of each truck. Constraints (6) require collection of the milk of each farm by exactly one truck. Constraint (7) imposes one route at most for each truck, which must start at some node 0_k for each truck k. Constraints (8) are flow balance equations for each node and each truck. The route of truck k, by virtue of constraints (9), must stop at node i, if it collects the milk from that node. Constraints (10) avoid a truck loading milk of a quality lesser than t if it is delivering milk quality t to the plant. Constraints (11) require a truck k to contain only one quality of milk, possibly blended. The relationship between continuous variables w^{kt}, or volume of milk t, and the binary variables z^{kt}, or the assignment of milk quality t to truck k, is set by constraints (12). Constraints (13) measure the volume of milk A, B or C arriving at the plant in truck k.

Note that constraints (10) - (13) set the blending rules, and different rules could be included by changing these constraints. Constraint (14) forces the transportation of all the produced milk to the plant. Constraints (15) balance the amount of each quality of milk arriving at the plant, and the amount of remaining milk of each quality after blending at the plant. Constraints (16) enforce the plant's quotas. Constraints (17) avoid prohibited blends. Constraints (18) prevent unwanted sub-tours for each truck. Finally, constraints (19) - (21) set the variables' domain.

The agreement between the firm and the cooperative stipulates collection of all produced milk, for this study's real case. However, visiting some low-production, distant farms could be more expensive than the profit obtained from collecting their milk. In this case, if the agreements between the members of the cooperative allow for this practice, these farms could be paid for their milk, but not visited. This case is a variant of the Price Collecting Routing Problem (Balas, 1989; Tang and Wang, 2006). This study's model can be easily modified to solve this problem, replacing constraints (6) and (14) with the following constraints:

$$\sum_{k \in K_i} y_i^{kt} \leq 1 \qquad \forall i \in N, t \in T : (i,t) \in IT \qquad (6')$$

$$\sum_{k \in K} \sum_{t \in T} w^{kt} = \sum_{k \in K} \sum_{(i,t) \in IT} q_i^t y_i^{kt} \qquad (14')$$

It is also easy to modify the model to represent a situation in which trucks have compartments, each of which can carry blended milk. Each compartment in this modified model is treated as if it was a single truck. However, all the "trucks," representing compartments of the same actual truck, are forced to travel together. The following constraints replace the original constraints in the model:

$$\sum_{j:(0_k, j, k) \in AK} x_{0_k j}^k \leq |C| \qquad \forall k \in K \qquad (7')$$

$$\sum_{p:(p,i,k) \in AK} x_{pi}^k \geq y_i^{kt} \qquad \forall k \in K_i, i \in N, t \in T : (i,t) \in IT \qquad (9')$$

Constraint (7') now states that the number of compartments that depart from node 0_k is at most the highest number |C| of compartments in any truck, and constraint (9'), as opposed to requiring exactly one truck visiting farm *i*, allows all compartments belonging to the same truck visiting that node, if one or more of its compartments collects milk there. Finally, the following constraint is added:

$$x_{ij}^{k} = x_{ij}^{s} \qquad \forall (i,j) \in A, (k,s) \in CM \qquad (22)$$

where CM is the set of compartments k and s in the same truck. The constraint (22) forces all compartments in the set CM to follow the same route.

The production of a farm P can exceed the compartment size C. In that case, we make $\lceil P/C \rceil$ copies of the farm, located at the same point. For example, if $P = 3.5C$, there will be four copies of the farm, with three of them producing C, and one producing $0.5C$.

This model is solved using the same cuts as in the case with no compartments.

4.1. Valid Inequalities

The following cuts are used in the branch-and-cut procedure.

Proposition 1

The cut

$$x_{ij}^{k} \le \sum_{h \ne i} x_{jh}^{k} \qquad \forall (i,j) \in A \cup A^{0} : i, j \ne \{0\}, k \in K \qquad (23)$$

is valid for the MB.

Proof: Dror et al. (1994). Note that constraints (5)–(9) and (18) are a model for a capacitated VRP, for each truck. The cut in Proposition 1, proven to be valid for the VRP by Dror et al. (1994), works together with these constraints, uses only the VRP variables, and performs exactly the same function as in that problem. ■

Proposition 2

$$\sum_{i \in S} x_{ij}^{k} \le \sum_{h \in S^{c}, m \in S : h \ne j} x_{hm}^{k} \qquad \forall S \subseteq N, j \in S^{c}, k \in K \qquad (24)$$

is a valid cut for the MB.

Proof. This inequality follows from the balance constraints (8): for a set $S \subseteq N$, if a truck k uses an arc (i, j) with $i \in S$ and $j \in S^c \subseteq N \backslash S$, there must be another arc (h, m) with $m \in S$; otherwise, there is no continuity in the route. It must hold that $h \neq j$, since otherwise, there will be a sub-tour including node j and nodes belonging to S.

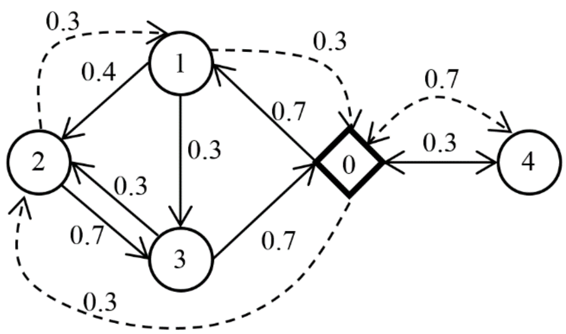


Figure 3. Sub-graph of a non-integer solution that shows the use of cut (24)

Cut (24) is a tighter extension of the connectivity constraint in the work of Drexl (2014). Figure 3 displays part of a non-integer solution of the problem with two routes, in which the route for the truck $k = 1$ is the continuous line, and the route for truck $k = 2$ is the dashed line.

Truck routes originate at the plant. The reader can be convinced that the connectivity constraint from the work of Drexl (2014) holds for any set S. However, cut (24) does not. For example, with $S = \{1, 2\}$, $i = 1$, $j = 3$, $k = 1$, $x_{1,3}^{1} + x_{2,3}^{1} = 0.3 + 0.7 \not\leq x_{0,1}^{1} = 0.7$. ■

Proposition 3

The following "multi-star inequality"

$$\sum_{t \in T} \sum_{\substack{i \in S^c \\ j \in S}} \left(Q^k - q_i^t\right) x_{ij}^k \geq \sum_{a \in S} \sum_{t \in T} q_a^t y_a^{kt} + \sum_{t \in T} \sum_{\substack{h \in S \\ m \in S^c}} q_m^t x_{hm}^k \qquad \forall S \subseteq N, k \in K \tag{25}$$

is valid for the MB.

Proof: Yaman (2006). Each vehicle k traveling from node $i \in S^c$ to node $j \in S$ must have enough capacity to carry the production of incoming node i, the production of visited nodes in set S, and the production of the subsequent node m.■

Proposition 4

The following inequality

$$\sum_{i \in S^c, j \in S} x_{ij}^k \geq \sum_{t \in T} y_h^{kt} \qquad \forall S \subseteq N, h \in S, k \in K \tag{26}$$

is a valid cut for the MB.

Proof: Toth and Vigo (2002). If a farmer $h \in S$ with milk $t \in T$ must be visited by a vehicle k, such that $y_h^{kt} = 1$, then the route of vehicle k must visit the set S, from a node $i \in S^c$ to a node $j \in S$, that is, $x_{ij}^k = 1$.■

Constraints (18) cannot be directly used in the model, as there is an exponential number of them. These constraints, and the integrality constraints (19) - (20), are relaxed to solve this relaxed problem. If the solution found by the relaxed formulation is not a feasible integer optimum, a constraint (18) is added, as well as cuts (23) - (26) as required, and solve again. Separation algorithms are used for each cut to find what constraints and cuts to add at every iteration. Once new cuts are no longer found, the procedure continues with a branch and bound step, after which the separation algorithms are applied again, and so on. The branch and bound used is the standard procedure in the CPLEX software package, using standard options.

4.2 Separation algorithms

Once a solution is found, its *supporting graph* $G_S^k(N^k, A_S^k)$ is built for each truck $k \in K$, where $A_S^k = \{(i, j) \in A \cup A^0 \colon x_{ij}^k > 0\}$; that is, the graph includes all arcs for which the associated decision variables are nonzero in the solution. Three independent separation algorithms exist, corresponding to different cuts.

4.2.1 Separation algorithm for (23)

This algorithm searches for arcs (i, j) of the supporting graph, such that $i \neq 0$ and $j \neq 0$, that is, the arc does not begin or end at the plant and, for each such arc, verifies the violation of cut (23). Note that cut (23) could have been added to the main model, generating $|K|\cdot|N|^2$ new constraints; however, this would increase the solution time. The order of this algorithm is $O(|K|\cdot|N|^2)$.

4.2.2 Separation algorithm for (24) and (25)

This algorithm analyzes all arcs belonging to the supporting graph and not connected to node 0, or the plant. For each arc (i, j) in the sub-graph corresponding to truck k of the supporting graph, the set S of nodes connected to j, or the head of the arc, is found. After a node is added to the set, the violation of cut (24) is checked and the cut is added as required. If a cut is added, the process continues with the next truck k + 1. The order of this algorithm is $O(|K|\cdot|N|^4)$.

For cut (25), the sets S that are just found are used, as well as the *greedy randomized algorithm* proposed in the works of Augerat et al. (1995) and Baldacci et al. (2004). This procedure starts from a known set S. Let $t \in S^c$ be a node, such that $\sum_{j\in S} x_{tj}^k = \max_{i\in N\setminus S}\left(\sum_{j\in S} x_{ij}^k\right)$. If cut (25) is violated for $S' = S \cup \{t\}$ in the supporting graph, then a cut is added to the problem. This procedure is repeated while there are remaining nodes to add to S'. The order of this algorithm is $O(|K|\cdot|N|^3)$.

4.2.3 Separation algorithm for (26) and (18)

A set S' is built, starting from node 0. All nodes connected to S' through an arc $(i, j) \in A_S^k$ (i.e., arcs corresponding to truck k) are added one by one until no more connected nodes are found. Then, a set S in S'^c is built using the same procedure, starting with any node $l \notin S'^c$. A cut is added for each node $h \in S$. The process is repeated for each truck k. The order of this algorithm is $O(|K|\cdot|N|^2)$.

Finally, the same sets S are analyzed for the violation of the constraint (18).

5. A three-stage heuristic for large instances

Solving an instance larger than 100 farms using the branch-and-cut method would be time consuming and, given the available tools, impractical. Some trial runs were conducted to solve the problem, using both VRP, or separate trucks for different qualities of milk, and the formulation presented in Section 3. Instances of up to 80 – 100 nodes and five to ten vehicles could be solved in both cases.

A three-stage heuristic (TSH) is proposed that divides the large instances into smaller sub-problems, and solves each sub-problem separately.

A set of farms is partitioned into clusters in the first stage. Cluster formation is the subject of an extensive body of literature; for a comprehensive review, see the work of Xu and Tian (2015). Any known method could be used for the problem at hand, provided the amount of milk in each cluster can fill the smallest truck available, and the maximum number of farms in each cluster does not exceed 100, which is the maximum number of farms the blending procedure can manage at this time.

As a clustering procedure, *k*-means (MacQueen, 1967; Žalik, 2008) is a fast, efficient method. *K*-means locates *k* virtual "mean" points and allocates each farm to its closest mean, considering distances over the road network, in such a way as to minimize the sum of the farm-mean distances. Each group of farms around a virtual mean, or cluster, provides shape to an area of the partition. The value of *k* is chosen heuristically, so that each cluster results in a manageable instance for the method in Section 3. Note that as the number of clusters increases, the sub-problems become easier, but the quality of the global solution decreases. Further details on the implementation of *k*-means can be found in the work of Hartigan and Wong (1979).

The *k*-means is a general procedure that can be applied to any large instance. However, this does not explicitly use any information that may be available regarding the geographical region, such as the existence of natural barriers that vehicles cannot cross as highways, rivers, or mountains. As information exists in this case study regarding these natural and fabricated barriers, it is used to perform an *ad-hoc* heuristic ("geographical") clustering, which leads to improved results. The use of such a method must be evaluated on a case-by-case basis. In this study's real case, this geographical partition works more efficiently than *k*-means, as further discussed in the Section regarding computational results.

The second stage consists of assigning milk quotas to each cluster, which guarantees *a priori* the global quota of each quality of milk required at the plant, and allocating trucks to each cluster. These tasks are performed using a Mixed Integer Programming (MIP) formulation.

Let *C* be the set of clusters or areas, and *ASIG* the set of producer-cluster pairs (*i*, *c*). Let $\underline{T}$ be the lower bound of the number of trucks per cluster. This lower bound is computed as the minimum number of trucks required to collect all the milk in the cluster, if all the milk had the same quality. Let also $\overline{T}$ be the upper bound of the number of trucks per area. This bound is computed as the number of trucks required to collect all the milk without the use of blending, which is solving a VRP for every milk quality in the cluster. The variables are:

$$\hat{y}_{ck} = \begin{cases} 1 & \text{If truck } k \text{ is allocated to cluster } c \\ 0 & \text{otherwise} \end{cases}$$

u^{trk} = Volume of milk of quality *t* delivered to the plant by truck *k*, blended for its use as milk of quality *r*, used to satisfy the minimum milk volume requirements at the plant.

u_{+}^{trk} = Volume of milk of quality *t* delivered to the plant by truck *k*, blended for its use as milk of quality *r*, not used to satisfy the minimum milk volume requirements at the plant (surplus).

The formulation of the model that assigns trucks and allocates milk quotas to each cluster is as follows:

$$Z = Max \sum_{t \in T} \sum_{r \in T} \sum_{k \in K} \alpha^{r} \left(u^{trk} + u_{+}^{trk} \right) \tag{27}$$

Subject to

(5) - (6), (10) - (15), and (18)

$$\sum_{r \in T} \left(u^{trk} + u_{+}^{trk} \right) = w^{kt} \qquad \forall t \in T, k \in K \tag{28}$$

$$\sum_{t \in T: r \in D^t} u^{trk} = P^r \qquad \forall r \in T \qquad (29)$$

$$\sum_{c \in C} \hat{y}_{ck} = 1 \qquad \forall k \in K \qquad (30)$$

$$\underline{T} \le \sum_{k \in K} \hat{y}_{ck} \le \overline{T} \qquad \forall c \in C \qquad (31)$$

$$\sum_{\substack{i \in N: \\ (i,t) \in IT \wedge \\ (i,c) \in ASIG}} q_i^t \le \sum_{k \in K} Q_k \hat{y}_{ck} \qquad \forall c \in C \qquad (32)$$

$$\sum_{\substack{t \in T: \\ (i,t) \in IT}} y_i^{kt} \le \sum_{\substack{c \in C: \\ (i,c) \in ASIG}} \hat{y}_{ck} \qquad \forall i \in N, k \in K \qquad (33)$$

$$u^{trk}, u_+^{trk} \ge 0 \qquad \forall k \in K; t, r \in T : r \in D^t \qquad (34)$$

$$\hat{y}_{ck} \in \{0,1\} \qquad \forall c \in C, k \in K \qquad (35)$$

The model maximizes the revenue from the milk received at the plant (36). The constraints (5) - (6), (10) - (15), and (18) guarantee truck capacity feasibility, and measures the volumes of milk delivered to the plant. Constraints (28) and (29) require all the milk collected by every truck being delivered to the plant, and used to satisfy the minimum requirements, and possibly some extra amount. Constraints (30) force the allocation of every truck to an area. Constraints (31) allocate the correct number of trucks to every area, while constraints (32) assure that the allocated truck capacity is sufficient. Constraints (33) relate the decision variables, forcing that if a truck visits a producer, it is assigned to the area in which the producer is located. Finally, (34)-(35) state the domain of decision variables.

The third stage of the heuristic involves the branch-and-cut method from Section 4.

6. Computational Results

6.1 Test instances

The model is first applied to 40 test instances, which range from 23 to 101 nodes. Nine of these instances (eil22.vrp–eil101.vrp, att48.vrp) belong to the TSPLib set (Reinelt, 1991). The instances (a32.vrp-a80.vrp) are taken from the work of Augerat et al. (1995). The instances (tai75A.vrp – tai75D.vrp and c50.vrp-c75.vrp) belong to the work of Taillard (1999). Instances f45.vrp and f72.vrp are taken from the work of Fisher (1994). In all instances, the coordinates and the production (or demand) of the nodes are known.

The branch-and-cut method is used with CPLEX Version 12.5 and AMPL version 20130109. All experiments were run on a PC Intel i7-2600, 3.4 GHz, 16GB RAM, and Ubuntu Server 12.04 LTS.

Three heterogeneous trucks are used in these instances, and the total capacity is sufficient to transport all the milk. The networks are symmetric; that is, $c_{ij} = c_{ji}$. The transportation cost on each arc $(i, j) \in A$ is equal to the Euclidean distance d_{ij} between the end nodes of the arc, that is, $c_{ij} = d_{ij}$. Distances have been approximated to their closest integer value. The income per liter is 1.0, 0.7, and 0.3 units for qualities A, B, and C, respectively. The milk produced at each farm is equal to the demand in each node of the test instances, multiplied by a scale factor f, which is used for all test results to have the same order of magnitude.

Qualities are assigned to farms using the following rule, which makes a cyclic assignment, that is: node 1 produces milk A, node 2 produces milk B, node 3 produces milk C, node 4 produces milk A, and so on.

The quotas of milk are defined arbitrarily and, in the test instances, are the same for all qualities of milk. Equations (1) – (3) are followed to fix the minimum amounts.

Table 1 displays the results. *Net* indicates the instance. The notation Q[*,*;*] indicates the capacity for each truck in thousands of liters. P[*,*,*] is the quota of milk A, B, and C, respectively, in thousands of liters. $|N_0|$ is the number of nodes in the network, including the plant. Each truck starts and ends its route at the plant. Z is the optimal profit, in monetary units. *VA*, *VB*, and *VC* are the amounts of milk of each quality (liters) after delivery and blending at the plant. *#It* represents the number of iterations of the separation algorithms, T is the CPU time in seconds, *TB* indicates if there are blends in the trucks, and *PB* indicates if there are blends at the plant.

Note that blending is used in more than half of the cases, improving profit over the alternative of a separate collection. Running times are reasonable, and these are the summation of the times taken by all four processors of the computer. Therefore, clock time is approximately one quarter of the times shown in the Table.

Net, Q, P	$\|N_0\|$	*f*	*Z*	*VA*	*VB*	*VC*	*T*	*TB*	*PB*
eil22.vrp, Q = [10;15;20], P=[6;5;4]	22	1	15,947	9,800	7,200	5,500	12	no	no
eil23.vrp, Q = [6;7;8], P= [1; 1.5; 2]	23	1	7,207	6,100	2,009	2,080	6	no	no
eil30.vrp, Q = [5;5.5;6], P=[2.2;2.4;2.6]	30	1	7,117	2,400	6,000	4,350	99	yes	no
eil31.vrp, Q = [55;50;40], P=[10;5;8]	31	1	60,080	28,300	34,800	27,200	66	no	no
a32.vrp, Q= [10;15;20], P=[12;10;8]	32	1	26,660	16,200	10,000	14,800	23	no	yes
eil33.vrp, Q=[20;20;15], P=[8;7;6]	33	1	20,409	11,200	11,690	6,480	58	no	no
a33.vrp, Q = [15;20;25], P = [15;8;6]	33	100	29,417	17,600	11,400	15,600	62	no	no
a34.vrp, Q = [20;20;25], P = [10;12;14]	34	100	30,496	15,900	16,000	14,000	40	no	yes
a36.vrp, Q = [20;15;15], P = [10;12;14]	36	100	29,233	16,000	14,200	14,000	110	no	yes
a37.vrp, Q = [20;15;10], P = [10;8;6]	37	100	24,837	10,000	16,200	14,500	45	yes	no
a38.vrp, Q = [20;20;10], P = [10;15;15]	38	100	28,596	10,000	20,000	18,100	570	yes	no
a39.vrp, Q = [20;20;20], P = [10;12;14]	39	100	30,808	14,600	17,300	16,600	110	no	no
a44.vrp, Q = [25;20;15], P = [20;16;12]	44	100	38,771	23,400	16,000	17,600	101	yes	yes
a45.vrp, Q = [25;20;20], P = [20;18;18]	45	100	40,282	23,300	18,000	18,000	136	yes	no
f45.vrp, Q=[20;15;10], P=[5;10;5]	45	5	23,705	9,760	18,020	8,320	80	yes	no
a46.vrp, Q = [30;25;20], P = [16;17;18]	46	100	40,696	22,400	19,900	18,000	66	no	no
a48.vrp, Q = [30;25;20]; P = [20;20;20]	48	100	39,800	20,100	20,000	22,500	230	yes	yes
att48.vrp, Q= [50,45,40], P=[38,34,30]	48	1	17,452	40,000	37,500	40,000	284	yes	no
c50.vrp, Q=[35;30;30], P=[15;30;26]	51	100	49,803	21,700	30,000	26,000	84	yes	yes
eil51.vrp, Q= [25;30;35], P=[22;23;24]	51	1	50,128	22,400	29,600	25,700	154	no	no
a53.vrp, Q = [30;30;30], P = [20;20;20]	53	30	46,662	23,490	24,870	23,250	183	no	no
a54.vrp, Q = [15;15;15], P = [5;5;5]	54	50	22,414	12,600	11,650	9,200	304	no	no
a55.vrp, Q = [15;15;20], P = [5;10;15]	55	50	24,694	11,900	12,250	17,800	270	no	no
a60.vrp, Q = [20;10;20], P = [8;12;16]	60	50	25,041	11,800	13,650	16,000	3,565	yes	yes
a61.vrp, Q = [35;35;35]; P = [30;20;10]	61	100	60,644	30,400	34,600	23,500	561	no	no
a62.vrp, Q = [15;15;15], P = [10;11;12]	62	50	22,917	12,500	11,000	13,150	1,022	yes	no
a63.vrp, Q = [20;20;20], P = [5;10;20]	63	50	24,447	10,050	13,600	20,000	2,930	yes	yes
a64.vrp, Q = [20;20;20], P = [5;10;20]	64	50	24,100	11,750	10,650	20,000	5,395	yes	yes
a65.vrp, Q = [15;15;15], P = [10;12;14]	65	50	28,046	14,350	15,000	14,500	478	yes	no
a69.vrp, Q = [20;20;20], P = [10;15;15]	69	50	25,822	11,750	15,500	15,000	1,552	yes	no
f71.vrp, Q=[50;50;50], P=[25;20;10]	72	1	72,864	26,865	49,998	37,977	3,483	yes	no
f72.vrp, Q=[60;60;60], P=[20;30;40]	72	1	72,072	26,861	47,979	40,000	3,659	yes	yes
c75.vrp, Q=[40;50;50], P=[40;45;50]	76	100	86,677	40,000	46,400	50,000	13,760	yes	yes
tai75A.vrp, Q=[25;30;35], P=[10;15;20]	76	5	65,477	21,980	15,000	31,800	4,806	yes	yes
tai75B.vrp, Q=[30;30;30], P=[20;25;25]	76	5	48,238	24,530	25,000	25,000	15,056	yes	yes
tai75C.vrp, Q=[15;20;25], P=[5;10;15]	76	5	25,906	13,085	10,000	24,515	4,537	yes	yes
tai75D.vrp, Q=[20;35;30], P=[15;15;20]	76	5	65,477	20,935	19,880	30,060	2,645	no	no
eil76.vrp, Q=[45;48;51], P=[45;43;40]	76	1	91,461	46,800	46,700	42,900	1,700	no	no
a80.vrp, Q = [20;20;20], P = [16;10;16]	80	50	29,977	16,250	14,650	16,200	5,626	no	no
eil101.vrp, Q=[50;55;60], P=[47;48;49]	101	1	96,115	48,800	48,000	49,000	66,843	yes	yes

Table 1. Results for test instances

The optimal solution was found for all test instances.

6.2 Real Case

This study's methodology is applied to a real case, and its performance is compared with the VRP and with the result of the current procedure used by the firm, located in the south of Chile. The firm uses trucks with no compartments. An expert planner designs the routes by hand, and there is occasional heuristic blending of small amounts of milk in the trucks. The firm collects milk from 500 farms spread across a geographical region of approximately 9,600 square kilometers. The average daily production of the farms ranges from 57 to 25,000 liters, as illustrated in Figure 4. Note that 53.2% of farmers produce less than 2,326 liters.

Of these 500 farms, 313 produce milk of quality A – in short, milk A; 159 produce milk B; and 28 produce milk C. The volume of milk is 1,435,168 liters of milk A, 268,564 liters of milk B, and 74,475 liters of milk C. The quotas for milk A, B, and C at the plant are 1,250,000, 300,000, and 100,000 liters, respectively. The current criterion is maximizing the amount of milk A. The revenue, in monetary units per liter of milk, is $1.5 \cdot 10^{-2}$, $1.05 \cdot 10^{-2}$, and $4.5 \cdot 10^{-3}$ for milk A, B, and C, respectively.

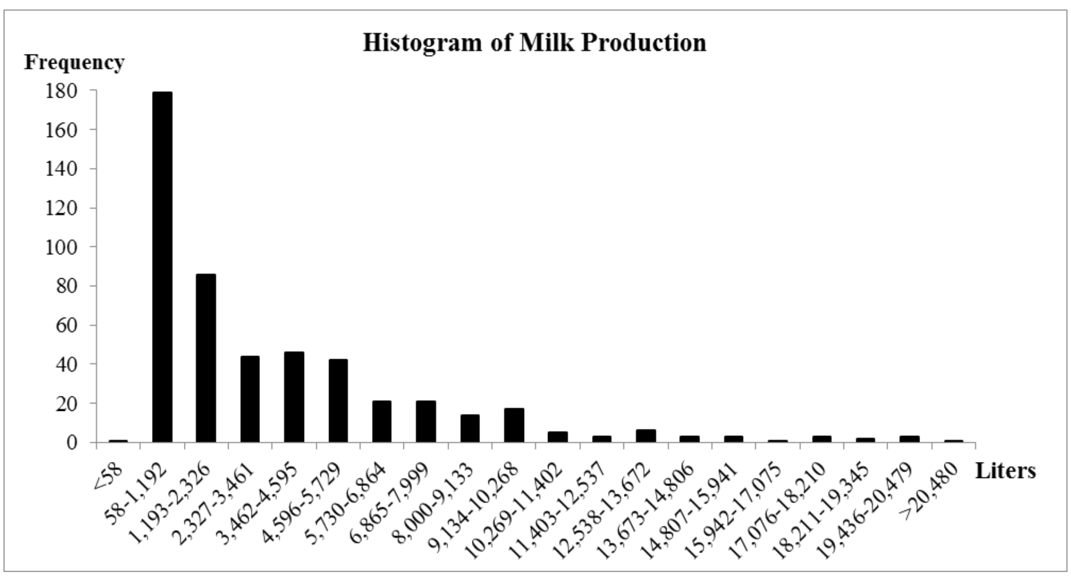


Figure 4. Production histogram of actual farms

The fleet is composed of 100 trucks, whose capacities are as follows: 15 15,000-liter trucks; 20 20,000-liter trucks; 15 25,000-liter trucks; and 50 30,000-liter trucks.

As this is too large to use the procedure described in Section 4, the region is partitioned into areas, or clusters. First, this study takes advantage of such geographical barriers as rivers, lakes, mountains, and highways to form the clusters. The road network is tree-like, rather than meshed. Consequently, the network distance between producers can be long, even if the Euclidean distance is short. Figure 5 displays an example. The dotted line in Figure 5a notes the 19 km route from farm 66 to farm 140, separated by a Euclidean distance of 2 km. The solid lines are rivers. The dotted line in Figure 5b indicates the shortest route between farms 71 and 181, partially using the solid line, which is a highway. The route distance is 21 km, while the Euclidean distance is 4 km. Different trucks will likely visit these pairs of farms.

Using these barriers as boundaries, the region was divided into 25 areas, containing clusters of between 3 and 39 farms each, as displayed in Figure 6 ("geographical" partition).

a) b)

Figure 5. Natural barriers in the area

Once the set of farms was partitioned into clusters, the model in Section 5 was used to assign trucks and allocate milk requirements to clusters, or the **second stage of the heuristic**. This stage takes 5,103 seconds of CPU time.

The collection problem was then optimally solved for each cluster, using the branch-and-cut algorithm, or the **third stage of the heuristic**. Table 2 notes the results of the three-stage heuristic, and compares it with VRP, or the optimized routes for trucks collecting each quality of milk separately, solved for each cluster independently, and the firm's current procedure.

Figure 6. Partition of the region into 25 independent areas. A diamond denotes the plant location.

The VRP model presented by Irnich et al. (2014) was employed, using the objective (1) with the same cuts used in MB. The MB solution in this case does not require blending at the plant, while VRP requires large amounts of milk being blended at the plant to satisfy the quotas. Using the current procedure, 627 liters of milk B are used to complete the milk C quota. Note that although the MB requires more blending in the trucks, which seems counterintuitive, the profit is significantly higher.

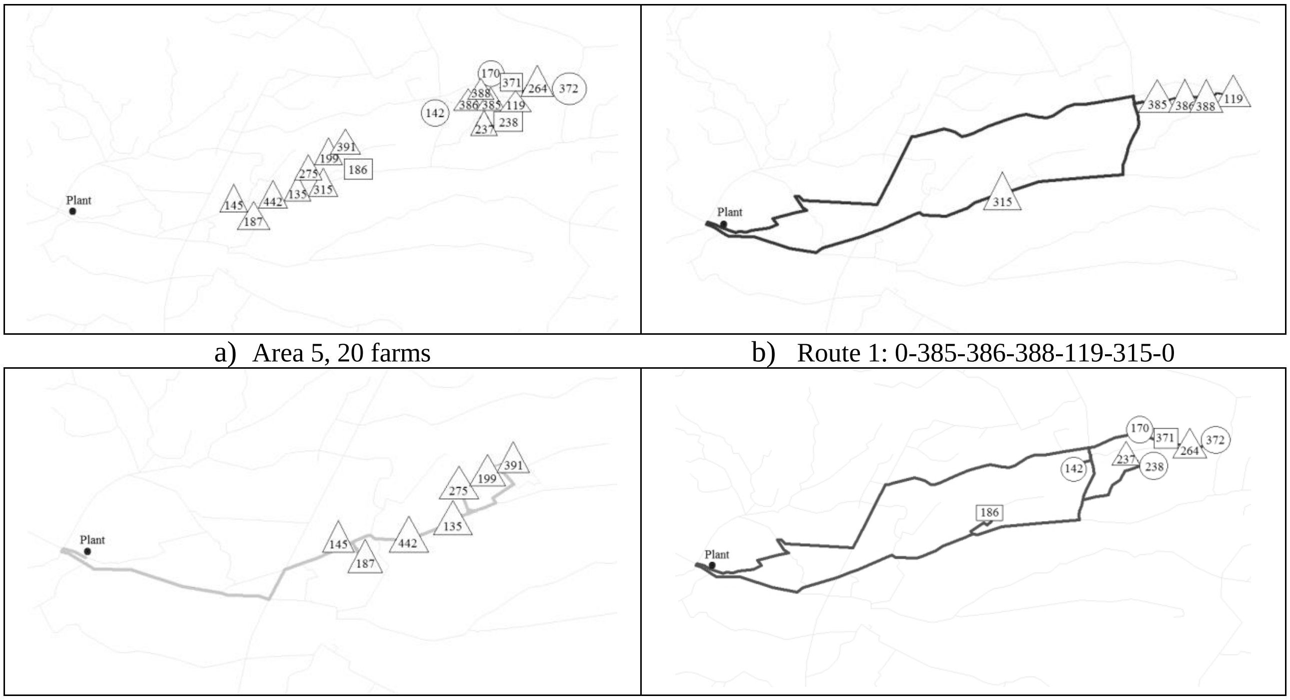
The reported CPU time is the sum of all CPU times required to solve the 25 clusters. As a 4-core computer was used, the clock time was 1.65 hours. Note that all the milk must be collected daily from all farms belonging to the cooperative. However, the routes' programming does not need to be performed on a daily basis, as the production volume in each farm changes slightly; this only occurs in exceptional situations owing to meteorological variations, cattle nutrition changes, or cattle diseases (Dayarian et al., 2015b). Rather, the production changes follow a seasonal cycle.

	MB	**VRP**	**Current procedure**
Trucks	81	99	100
Revenue [MU]	23,266	24,273	22,804
Costs [MU]	10,093	12,319	18,659
Profit [MU]	**13,173**	**11,954**	**4,145**
Milk A [l]	1,278,815	1,435,168	1,051,791
Milk B [l]	306,060	268,564	627,043
Milk C [l]	193,332	74,475	99,373
A → B [l]	0	31,436	-
A → C [l]	0	25,525	-
B → C [l]	0	-	627
CPU time	23,836	6,241	-

Table 2. Results for the real case for a region partitioned into 25 clusters. Profit denoted in boldface.

The VRP solution improves the profit over the firm’s current procedure, but requires 18 more routes compared to the MB. The current 100 routes would decrease to 81 if the MB were applied. MB decreases the transportation costs to roughly one-half of the current transportation costs, and leads to a profit that is 3.2 times that of the current procedure. The use of the VRP leads to a profit that is 2.8 times the current profit.

Figure 7a illustrates area 5 of the real case and its collection routes (Figures 7b, 7c, 7d) as an example of how the MB uses blending. The two first routes collect 49,762 liters of milk A, while the third route collects a blend of 10,455 liters of milk A, B, and C, resulting in 10,455 liters of C milk. The transportation costs of routes 1, 2, and 3 are 83, 49, and 105 monetary units, respectively. The total cost is 237 monetary units. The profit in this area is 556 monetary units. The VRP requires four routes, with at least two vehicles collecting milk A, one collecting milk B, and one collecting milk C. The total amount of milk collected, before blending, is 52,551 liters of milk A, 2,285 liters of milk B, and 5,381 liters of milk C. The transportation costs for the VRP solution are 361 monetary units, the revenue 836, and the profit 475 monetary units.



a) Area 5, 20 farms
b) Route 1: 0-385-386-388-119-315-0
c) Route 2: 0-135-199-391-275-442-145-187-0
d) Route 3: 0-186-238-237-142-170-264-372-371-0

Figure 7. Area 5 and collection routes

The effect of using different partitions is also explored. A second partition was defined, with 13 larger areas, by aggregating some of the 25 areas in the “geographical” partition. Contiguous areas were aggregated for, as the natural barriers were the easiest to cross. Third and fourth partitions were also constructed, with 19 and 25 areas using *k*-means.

Table 3 shows the results using different partitions.

As expected, the trucks’ utilization is more efficient for larger areas, and fewer vehicles are required to collect the milk. Among other factors, the use of greater or fewer areas depends on the amount of time available to solve the problem. The geographical partition provides superior results as it uses more information than the *k*-means. Increasing milk quotas for the highest quality milk also has an effect on revenue and CPU time. Table 4 displays the results of two runs for the 25-area geographical partition, in which the required amounts of milk A were 1,200,000 and 1,300,000 liters, while the requirements for milk B and C were the same as previously, or 300,000 and 100,000 liters.

# of Areas	25 (geographical)	25 (*k*-means)	19 (*k*-means)	13 (geographical)
Trucks	81	88	72	72
Revenue [UM]	23,266	23,652	23,205	23,719
Costs [UM]	10,093	10,776	9,256	9,564
Profit [UM]	**13,173**	**12,876**	**13,949**	**14,155**
A [l]	1,278,815	1,315,653	1,262,638	1,330,337
B [l]	306,060	315,702	324,208	295,110
C [l]	193,332	146,852	191,361	152,760
A→B [l]	0	0	0	4,890
A→C [l]	0	0	0	0
B→C [l]	0	0	0	0
CPU Time [s]	**23,836**	**18,491**	**61,142**	**57,962**

Table 3. Different partitions. Profits and CPU time are indicated in boldface.

Milk A	1,200,000	1,300,000
Trucks	77	84
Revenue [UM]	21,904	23,572
Costs [UM]	9,946	10,794
Profit [UM]	**13,348**	**12,778**
A [l]	1,235,361	1,397,757
B [l]	217,204	213,699
C [l]	325,642	166,571
A→B [l]	82,796	86,301
A→C [l]	0	0
B→C [l]	0	0
CPU Time [s]	24,537	35,267

Table 4. Tests with different milk quotas. Profits are indicated in boldface.

The solutions in Table 4 indicate that the profit could decrease for increasing requirements of milk A because this necessarily incurs a higher cost of transportation and increases the number of required trucks.

6.3 Prize-collecting version

Some of the clusters in the real case were used to demonstrate that implementing a prize-collecting version of the problem, that is, replacing constraints (6) and (14) by (6’) and (14’), increases the profit. Table 5 displays how the optional collection, or the prize-collecting version, dominates the implementation in which all farms must be visited.

		Optional visit			Must visit all		
Net	Q, P [Ml]	Profit	Revenue	Cost	Profit	Revenue	Cost
Cluster 16	Q = [30;15;15]; P = [20;5;2]	**319**	625	306	**187**	662	475
	Q = [30;15;15]; P = [20;10;10]	**256**	561	305	**180**	650	470
	Q = [30;15;15]; P = [43;0;11]	**151**	702	551	**141**	705	564
Cluster 2	Q = [30;30;20;15]; P = [74;0;0]	**389**	1,115	726	**252**	1,212	960
	Q = [30;30;20;15]; P = [74;9;0]	**284**	1,208	924	**252**	1,212	960
	Q = [30;30;20;15]; P = [30;30;23]	**105**	875	770	**104**	876	772
Cluster 20	Q = [30;30;30]; P = [45;0;0]	**553**	868	315	**470**	894	424
	Q = [30;30;30]; P = [57;0;6]	**470**	894	424	**470**	894	424
	Q = [30;30;30]; P = [30;10;10]	**403**	718	315	**386**	807	421

Table 5. Results for three clusters using the prize-collecting version

6.4 Blending and the use of trucks with compartments

Tests were performed on some of the case study’s original clusters (clusters 4, 7, 10, 11, 16, 17, 23, and 24), as shown in Table 6, to demonstrate that blending is convenient even with compartmentalized trucks. Different settings are compared in these tests: single-compartment trucks using VRP (VRP), single-compartment trucks with blending (MB), and different compartment-sized multiple-compartment trucks without (TC) and with (MBTC) blending. Truck capacities are noted in the first column. It is considered that all compartments have the same size in each test, and two compartment capacities (CC) were attempted: 10,000 and 15,000 liters. The figures in bold font in the table indicate the best solutions found for the case.

An analysis of the results in Table 6 yields the following conclusions:

- **Blending versus compartments**: Generally, the solutions using blending and those using compartments do not dominate one another, although the compartment solutions in the instances tested lead to a higher profit in most cases; an exception is case 2, with cluster 24. The dominance of each of these solutions depends on the size of the compartments and the milk volume from each farm. However, while not illustrated in the table, the use of compartments generally requires larger numbers of trucks, and trucks with compartments are more expensive.

- **Blending versus not blending**: Solutions with blending dominate solutions without blending, whether with or without compartments.

Truck Capacities [liters]	Solutions	Cluster 4	Cluster 7	Cluster 10	Cluster 11	Cluster 16	Cluster 17	Cluster 23	Cluster 24
30,000 30,000 30,000	VRP	307.56	-136.22	-	-95	-	581.10	**298.83**	162.6
	MB	336.74	-56.99	523.56	-5.83	206.67	581.10	**298.83**	255.94
	TC (CC = 10,000)	**365.89**	**101.22**	535.98	56.68	302.56	**665.54**	**298.83**	286.87
	TC (CC = 15,000)	360.27	-6.42	535.12	52.19	249.13	607.11	138.69	**291.05**
	MBTC (CC = 10,000)	**365.89**	**101.22**	**564.95**	**124.56**	**364.00**	**665.54**	**298.83**	286.87
	MBTC (CC = 15,000)	361.88	47.98	535.12	52.19	342.87	607.11	138.69	**291.05**
30,000 20,000 10,000	VRP	307.56	-136.22	-	-95	-	-	**159.22**	162.6
	MB	337.74	-56.99	523.56	-5.83	206.67	490.98	**159.22**	**255.94**
	TC (CC = 10,000)	**363.10**	**101.22**	-	56.68	-	-	**159.22**	171.6
	MBTC (CC = 10,000)	**363.10**	**101.22**	**528.32**	**124.56**	**276.12**	**494.96**	**159.22**	229.86
30,000 30,000	VRP	-	-	-	-	-	-	**298.83**	-
	MB	336.74	-56.99	371.65	-5.83	197.71	524.93	**298.83**	255.94
	TC (CC = 10,000)	**365.89**	**101.22**	-	56.68	-	-	**298.83**	286.87
	TC (CC = 15,000)	360.27	-6.42	-	52.19	-	-	-	**291.05**
	MBTC (CC = 10,000)	**365.89**	**101.22**	**564.95**	**124.56**	**364.00**	**543.67**	**298.83**	286.87
	MBTC (CC = 15,000)	361.88	47.98	511.14	52.19	342.87	-	-	**291.05**

Table 6. Profits for different truck configurations

- **Solution feasibility**: Some solutions using the VRP are infeasible, as the truck fleet is limited. The use of compartments improves the situation, but infeasible problems still exist. However, in all tested instances, blending makes solutions fully feasible. Conversely, note that if the size of the compartments is inadequate, the solution can again be infeasible even when blending is used. This is especially important when the farms and amounts of milk produced change over time.

- **Compartment size**: A smaller compartment size in the instances tested results in improved solutions in most clusters, except for cluster 24. Note that in a majority of the clusters analyzed, the farms produce between 1,000 and 3,000 liters; see also Figure 4.
- **General conclusion**: Blending improves solutions in all cases in terms of feasibility, profit, and efficiency in the use of trucks.

7. Conclusions

This study introduced the MB, motivated by the milk collection procedure currently used in several places. Small amounts of milk of different qualities are blended to reduce transportation costs. The procedure prescribes a set of farms to be visited by each vehicle, the route, and the blending pattern, and defines whether there is a need for blending at the plant to satisfy the quotas for each quality of milk. This model can be useful for other kinds of products, in which the mixing of different qualities of products in the same truck changes the truck's status.

The MIP model is solved using a branch-and-cut method, for which a model is proposed using both known cuts and a new cut. Polynomial separation algorithms are defined, and the procedure is tested on known test instances of up to 101 nodes. A three-stage heuristic procedure is then designed to solve the real case, involving 500 farms scattered over a large region. The region under study is partitioned in the first stage. A mathematical model is used in the second stage designed to set each partition's requirements and select the truck fleet allocated to each partition. The real case is solved in the third stage, obtaining favorable results in comparison with the current collection system. As the problem is new, there are no further efficient ad-hoc heuristic methods. This study's goal, in any case, was to design a heuristic that would provide favorable results within the time limits, given how frequently the problem needs to be solved, rather than aiming for faster times. Time is not critical, as routing does not change daily, but in a 4-core desktop computer, the solution takes 1.65 hours.

Different approaches are also compared as follows: single compartment trucks with and without blending, multiple-compartment trucks with and without blending, and optional and mandatory collection; several instances are solved for these comparisons. The results indicate that, in the instances that were solved, blending always dominates unblended milk collection. Additionally, in most cases, using multiple-compartments trucks is better than using single-compartment trucks with blending. However, if multiple-compartment trucks are available, blending in the compartments always dominates. Furthermore, blending enables feasibility in all cases, and selecting an incorrect compartment size can make the problem infeasible. Finally, using the actual single-compartment truck fleet and blending, significantly improves profit for the actual case, primarily due to the savings in transportation cost.

If the farms in the compartment case exceed the size of a compartment, neither the size of the farm subdivisions nor the truck compartments' size are optimized, although different compartment sizes may lead to different results. This is a compelling future extension.

Other many possible extensions of this work include the location of milk collection points to accumulate milk from small and distant farmers. Additionally, another extension involves the consideration of the random nature of each producer's daily milk availability. Other extensions of interest involve speeding up solutions by using heuristics that solve the whole problem at once by not requiring subroutines, and the analysis of different blending rules that are less conservative than the current rules used at the actual firm.

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