Comment on "Impossibility of distant indirect measurement of the quantum Zeno effect"

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In the paper by Hotta and Morikawa [Phys. Rev. A **69**, 052114 (2004)] the nonexistence of the quantum Zeno effect caused by indirect measurements has been claimed. It is shown here that the pertinent proof is incorrect and that the claim is unfounded.

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The authors of the quoted paper [1] claim nonexistence of the impediment of the evolution of a quantum system by its indirect measurement, known as the quantum Zeno effect (QZE). They attempt to prove that with distant indirect measurements the final survival probability of the object is not affected by repeated measurements. We show that this proof is faulty, and consequently, the above claim is unfounded.

The proof involves the subdivision of the system's Hilbert space $\mathcal{H}_{\mathcal{Z}}=\mathcal{H}_{\mathcal{C}}\oplus\mathcal{H}_{\mathcal{W}}$ in a core-zone supspace $\mathcal{H}_{\mathcal{C}}$ and a wave-zone subspace $\mathcal{H}_{\mathcal{W}}$. By definition, throughout the time evolution a wave-zone state remains in $\mathcal{H}_{\mathcal{W}}$, see Eq. (7) of Ref. [1]. This is formally expressed by the vanishing of the operator product $\hat{P}_{\mathcal{C}}\hat{U}_{+}(t)\hat{P}_{\mathcal{W}}$, where $\hat{U}_{+}(t)$ represents the unitary time-evolution operator and

$$\hat{P}_{\mathcal{C}}=\sum_{C}|C\rangle\langle C|, \quad \hat{P}_{\mathcal{W}}=\sum_{W}|W\rangle\langle W| \tag{1}$$

are the projectors into the core and wave zones, respectively. However, a core-zone state $|C\rangle\in\mathcal{H}_{\mathcal{C}}$ is said to decay, with finite probability, into a wave-zone state [see Eqs. (19)–(21) of Ref. [1]]. Such an evolution requires the operator product $\hat{P}_{\mathcal{W}}\hat{U}_{+}(t)\hat{P}_{\mathcal{C}}$ to be *nonvanishing*.

We prove here that, in contrast to this assumption, this operator product exactly vanishes,

$$\hat{P}_{\mathcal{W}}\hat{U}_{+}(t)\hat{P}_{\mathcal{C}}\equiv 0. \tag{2}$$

For the proof we start with the explicit expression for the unitary time-evolution operator

$$\hat{U}_{+}(t)=\mathcal{T}\exp\left[-\frac{i}{\hbar}\int_{0}^{t}dt'\,\hat{H}(t')\right], \tag{3}$$

where $\mathcal{T}$ indicates the proper time ordering. Equation (3) may be expanded in a series of powers of the Hamiltonian $\hat{H}(t)$ as

$$\hat{U}_{+}(t)=\sum_{n=0}^{\infty}\left(-\frac{i}{\hbar}\right)^{n}\int_{0}^{t}dt_{n}\ldots\int_{0}^{t_2}dt_{1}\hat{H}(t_{n})\ldots\hat{H}(t_{1}). \tag{4}$$

According to Eq. (7) of Ref. [1] the relation $\hat{U}_{+}(t)|W\rangle\in\mathcal{H}_{\mathcal{W}}$ holds, which upon insertion of Eq. (4) can be shown to imply $\hat{H}(t)|W\rangle\in\mathcal{H}_{\mathcal{W}}$. From this relation it follows that consequently $\langle W|\hat{H}^{\dagger}(t)=\langle W|\hat{H}(t)\in\mathcal{H}_{\mathcal{W}}^{*}$, where $\mathcal{H}_{\mathcal{W}}^{*}$ denotes the dual Hilbert space of $\mathcal{H}_{\mathcal{W}}$, and thus one concludes that the following relation holds:

$$\langle W|\hat{U}_{+}(t)=\sum_{W'}\langle W|\hat{U}_{+}(t)|W'\rangle\langle W'|\in\mathcal{H}_{\mathcal{W}}^{*}. \tag{5}$$

This relation, being absent in Ref. [1], is the central ingredient that allows us to prove now Eq. (2).

Equation (5) [together with Eq. (7) of Ref. [1]] can now be used to prove that the commutator $[\hat{P}_{\mathcal{W}},\hat{U}_{+}(t)]$ vanishes;

$$\begin{aligned}[\hat{P}_{\mathcal{W}},\hat{U}_{+}(t)]&=\sum_{W}[|W\rangle\langle W|\hat{U}_{+}(t)-\hat{U}_{+}(t)|W\rangle\langle W|]\\&=\sum_{W,W'}[|W\rangle\langle W|\hat{U}_{+}(t)|W'\rangle\langle W'|-|W'\rangle\\&\quad\times\langle W'|\hat{U}_{+}(t)|W\rangle\langle W|]\equiv 0.\end{aligned} \tag{6}$$

Of course, then also

$$[\hat{P}_{\mathcal{W}},\hat{U}_{+}(t)]\hat{P}_{\mathcal{W}}\equiv 0, \tag{7}$$

which allows us to perform one last step by formally inserting $\hat{P}_{\mathcal{W}}=\hat{1}-\hat{P}_{\mathcal{C}}$ to yield

$$[\hat{P}_{\mathcal{W}},\hat{U}_{+}(t)]-\hat{P}_{\mathcal{W}}\hat{U}_{+}(t)\hat{P}_{\mathcal{C}}+\hat{U}_{+}(t)\hat{P}_{\mathcal{W}}\hat{P}_{\mathcal{C}}\equiv 0. \tag{8}$$

Since both the first term, see Eq. (6), and the third term (cf. $\hat{P}_{\mathcal{W}}\hat{P}_{\mathcal{C}}\equiv 0$) are zero, also the second term necessarily vanishes,

$$\hat{P}_{\mathcal{W}}\hat{U}_{+}(t)\hat{P}_{\mathcal{C}}\equiv 0, \tag{9}$$

and thus Eq. (2) has been proved.

This equality proves that *no transition* occurs *from core zone to wave zone* either. The subspaces are completely separated, and the probability of finding that the observed state belongs to $\mathcal{H}_{\mathcal{W}}$ [see Eq. (20) of Ref. [1]] vanishes: $p_1=0$. In terms of measurements, the system and meter are disconnected, and the QZE is excluded on trivial grounds.

The correct analysis requires consideration of the formal structure of $\hat{U}_{+}(t)$ in terms of the Hamiltonian $\hat{H}(t)$, whose result is the above Eq. (5). Rather than the time-evolution operator's unitarity, $U_{+}^{\dagger}(t)=U_{-}(t)$, its left-hand operation warrants time reversal and the correct result, for $\hat{H}(t)$ being an arbitrary time-dependent Hamiltonian, including the special cases of a constant or an even function of time.

[1] M. Hotta and M. Morikawa, Phys. Rev. A **69**, 052114 (2004).