Self similarity and sample length dependence of conductance in quasiperiodic lateral magnetic superlattices

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We study the transport of electrons in a Fibonacci magnetic superlattice produced on a two-dimensional electron gas modulated by parallel magnetic-field stripes arranged in a Fibonacci sequence. Both the transmission coefficient and conductance exhibit self similarity and the six-circle property. The presence of extended states yields a finite conductivity at infinite length, that may be detected as an abrupt change in the conductance as the Fermi energy is varied, much as a metal-insulator transition. This is a unique feature of transport in this kind of structure, arising from its inherent two-dimensional nature.

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I. INTRODUCTION

The discovery of quasicrystals in 1984 (Ref. 1) has attracted a great amount of experimental and theoretical attention to quasiperiodic systems.[2] It has been shown that a one-dimensional (1D) quasiperiodic array of electric barriers is characterized by its self-similar energy spectrum and critical (neither extended nor localized) states. Recent advances in semiconductor and nanotechnologies have permitted the realization of a uniform magnetic field at nanometer scales, by creating magnetic dots or depositing ferromagnetic or superconducting material patterns on heterostructures.[3] The energy spectrum and transport properties of a two-dimensional electron gas (2DEG) modulated by a regular array of nanoscale magnetic-field inhomogeneities have been investigated both theoretically and experimentally.[4]

In this paper we discuss electron motion in a 2DEG subject to the field of a nearby quasiperiodic array of parallel magnetic stripes. This case differs from electric or dielectric modulation in that 2DEG electron tunneling through magnetic barriers is inherently a two-dimensional process.[4] The effective potential experienced by the electron is dependent on the wave vector perpendicular to the tunneling direction. As we show below, since for a quasiperiodic magnetic pattern this potential is still quasiperiodic for any given transverse wave vector, both the transmission and conductance display quasiperiodic properties. Our main finding, however, is that the presence of extended states somewhere in the spectrum produces a residual conductivity at infinite length, which is lost as the incident energy decreases, much in the manner of a metal-insulator transition.

II. FORMULATON

We consider a 2DEG under an inhomogeneous perpendicular magnetic field produced by two types of magnetic blocks P and Q arranged in a Fibonacci sequence [Fig. 1(a)]. The magnetic field is assumed to be uniform along the y direction and to vary along the x direction. Throughout this work we use the Landau gauge $\mathbf{A}=(0, A(x),0)$. For magnetic block P/Q of width $L_{P/Q}=d_{P/Q}+l_{P/Q}$, we assume for simplicity a magnetic profile $B(x)=B_{P/Q}l_B[\delta(x)-\delta(x-d_{P/Q})]$ [Fig. 1(b)]. Its corresponding vector potential can be chosen as $A(x)=B_{P/Q}l_B[\theta(x)-\theta(x-d_{P/Q})]$ [Fig. 1(c)], where $l_B=\sqrt{\hbar/eB_P}$ and $\theta(x)$ is the Heaviside step function. By introducing in addition the cyclotron frequency $\omega_c=eB_P/m^*$ (m^* is the effective mass of electrons), all quantities below are transformed into dimensionless units. For GaAs and an estimated $B_P=0.1T$, then $l_B=81.3$ nm, $\hbar\omega_c=0.17$ mev and $l_B\omega_c=1.4$ m/sec. Writing the wave function in the form $e^{iqy}\psi(x)$ (q is the wave number associated with the spatial degree of freedom in the direction of the stripes),

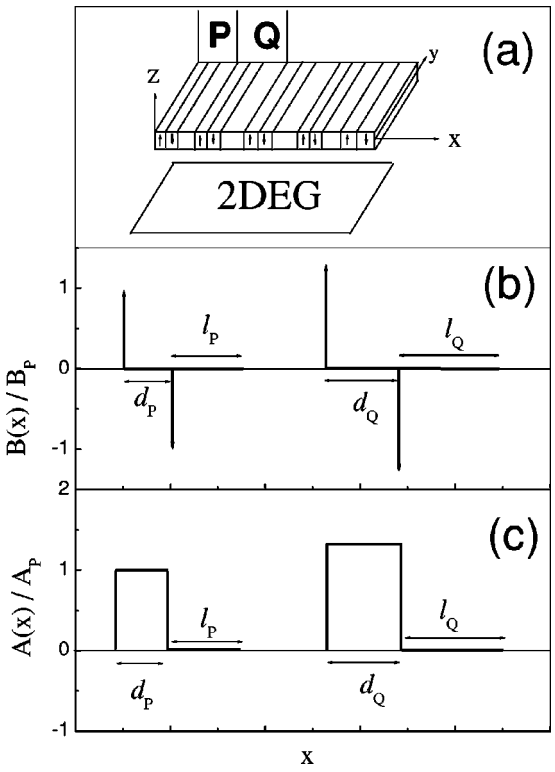

FIG. 1. The Fibonacci magnetic superlattice (a), showing the magnetic profile $B(x)$ of building blocks P and Q (b), and the corresponding y component of the vector potential $A(x)$(c).

one obtains the following 1D Schrödinger equation governing the motion of 2DEG electrons in the presence of the magnetic modulation:

$$\left(\frac{d^2}{dx^2}-f_j(x)[f_j(x)+2q]\right)\psi(x)=2(q^2/2-E)\psi(x). \quad (1)$$

Here $f_j(x)$ is an oscillating function arising from the Fibonacci sequence S_j constructed from the vector potentials A_P and A_Q, and $V(x,q)=f_j(x)[f_j(x)+2q]/2$ can be considered as an effective q-dependent potential for motion along the tunneling direction. The dependence on q of the quantity $V(x,q)$ implies that this problem is inherently two dimensional. Here and in what follows, we assume $B_P \geqslant B_Q$. Then, in the units chosen, the function $f_j(x)$ is a sequence of barriers of height $r=B_Q/B_P \leqslant 1$. For a given q, electron tunneling through the magnetic structure will be equivalent to electron motion in a 1D Fibonacci electric potential with square barriers ($q \geqslant -r/2$), square barriers and wells ($-1/2<q<-r/2$), or square wells ($q \leqslant -1/2$).

Matching wave functions at the edges of the magnetic block Q yields the following transfer matrix M_Q for an electron propagating through such block:

$$M_Q=\begin{pmatrix}[\cos(k_Q d_Q)-i\mu_Q^+\sin(k_Q d_Q)]e^{-ik_0 l_Q} & -i\mu_Q^-\sin(k_Q d_Q)e^{ik_0 l_Q}\\ i\mu_Q^-\sin(k_Q d_Q)e^{-ik_0 l_Q} & [\cos(k_Q d_Q)+i\mu_Q^+\sin(k_Q d_Q)]e^{ik_0 l_Q}\end{pmatrix}, \quad (2)$$

where $k_Q=\sqrt{2E-q^2-r(r+2q)}$, $k_0=\sqrt{2E-q^2}$, and $\mu_Q^{\pm}=\frac{1}{2}(k_Q/k_0 \pm k_0/k_Q)$. The transfer matrix M_P can be obtained by the replacements $Q \to P$ and $r \to 1$ in the above Eq. (2).

A Fibonacci multilayer system S_j has F_j layers, where F_j is a Fibonacci number satisfying the recursion relation $F_{j+1}=F_j+F_{j-1}(j \geqslant 1)$, with $F_0=F_1=1$. Then $M_{j+1}=M_j M_{j-1}(j \leqslant 1)$, with initial condition $M_0=M_Q, M_1=M_P$, which yields a trace map $x_{j+1}=2x_j x_{j-1}-x_{j-2}$ and a constant of motion $I=x_{j+1}^2+x_j^2+x_{j-1}^2-2x_{j+1}x_j x_{j-1}-1$,[2] where $x_j=TrM_j/2$. The constant of motion I characterizes the extent of quasiperiodicity of the Fibonacci system.

Now we consider a simple case, i.e., $r=1$, $d_P=d_Q=d$, which is likely to be the easiest to realize experimentally. Then one has $k_P=k_Q=k$ and the constant of motion,

$$I=\{(k^2-k_0^2)\sin(kd)\sin[k_0(l_Q-l_P)]/2kk_0\}^2. \quad (3)$$

For the case $l_P=l_Q, I=0$, which corresponds to a purely periodic magnetic superlattice. According to Eq. (3) I is also dependent on the normal wave vector q through k_0 and k. It is in general positive definite for most incident energies E if $l_P \neq l_Q$. One thus expects the quasiperiodic self similarity to appear in the energy spectra, transmission and, possibly, the conductance. In terms of the matrix M_j the transmission coefficient becomes

$$T(E,q)=4/[\mathrm{Tr}(M_j^t M_j)+2], \quad (4)$$

where the superscript t denotes the transpose of a matrix. The conductance g is calculated from the Landauer-Büttiker formula[5] by averaging the electron flow over half the Fermi surface,

$$g=\int_{-\pi/2}^{\pi/2} T(E,\sqrt{2E}\sin\theta)\cos\theta d\theta. \quad (5)$$

Here θ is the angle between the velocity of incidence and the tunneling axis x, E is the incident energy, and the conductance is expressed in units of the quantity $e^2 m^* v l_y/\hbar^2$, where v is the velocity of the incident electrons and l_y the width of the sample.

III. RESULTS AND DISCUSSION

We show in Fig. 2 typical transmission spectra for different transverse wave numbers and Fibonacci sequences S_9, S_{12}, and S_{15}. The magnetic structure parameters are chosen as $r=1$, $d_P=d_Q=1$, $l_P=1$, $l_Q=2$. From the left to right column, q is -0.7, 0.0, and 0.7, which correspond to the equivalent Fibonacci electric superlattices constructed from two square-well, two low square-barrier, and two high square-barrier blocks, respectively. From the top to bottom row the transmission spectra are for S_9, S_{12}, and S_{15}. respectively. As clearly shown in Fig. 2, the transmission spectra for S_9, S_{12}, and S_{15} are self similar, i.e., the transmission peak clusters and the transmission gaps for different Fibonacci sequences are arranged in a very similar way, re-

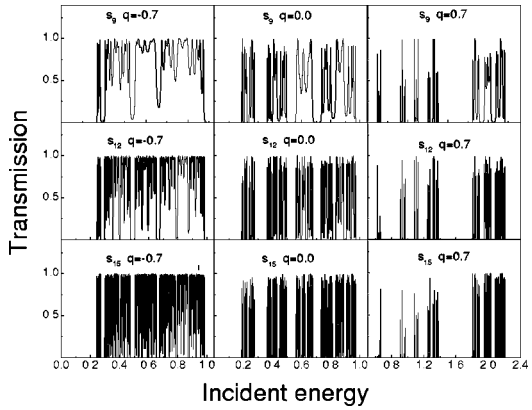

FIG. 2. Transmission spectra of Fibonacci magnetic superlattices S_9, S_{12}, S_{15} (from top to bottom) for $q=-0.7$ (left column), $q=0.0$ (middle column), and $q=0.7$ (right column). The magnetic structure parameters are $r=1$, $d_P=d_Q=1$, $l_P=1$, $l_Q=2$.

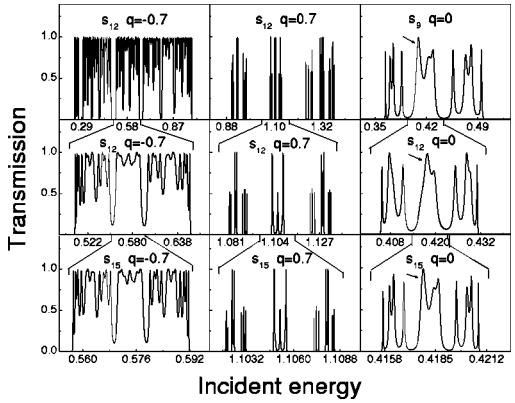

FIG. 3. Transmission spectra of Fibonacci magnetic superlattices S_{12}, S_{15} for $q=-0.7$ (left column), $q=0.7$ (middle column) and S_9, S_{12}, S_{15} for $q=0.0$ (right column). The magnetic structure parameters is the same as in Fig. 2.

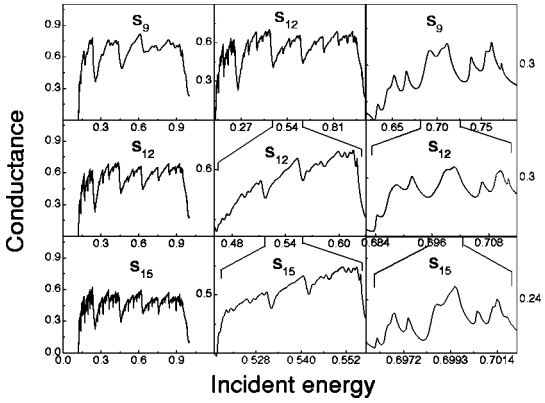

FIG. 4. Conductance of Fibonacci magnetic superlattices S_9, S_{12}, S_{15}. The magnetic structure parameters are the same as in Fig. 2, except $B_P=B_Q=2$ for the right column, set to discern the subtle structure.

gardless of the value of the transverse wave number q. In fact, the self similar transmission spectra are the refection of the self similarity in the corresponding energy spectra (not shown).

Figure 3 illustrates the self-similarity of the transmission spectra more clearly. The first and second rows show that, regardless of the value of q, the transmission bands are tribranching hierarchically in a self similar way. It is the self similarity between the whole and the local spectra. Also one can readily observe the similarity between the transmission spectra of S_{12} and S_{15} at quite different scales. The third column shows in more detail this scaling property at $q=0$ as the length is increased. We notice in these data that the evolving structure has a six-circle symmetry, arising from the property $M_{j+6}=M_j$. In fact, the scale change of the incident energy E between the spectra for S_9, S_{12}, and S_{15} is given by the scaling index of the renormalization-group transformation of the six-circle map $[1+4(1+I)^2]^{1/2}+2(1+I)$.[2] The self similarities of the transmission spectra arise from the self similar energy spectra (not shown) of this special structure. Close inspection of the data in the third column shows in addition that there are states with transmission coefficient equal to unity, that persist as the length is increased (arrows in the figure).[6,7] These exotic extended states play a crucial role in the unusual length dependence of the conductance described below.

Numerical results for the conductance for structures S_9, S_{12}, and S_{15} are shown in Fig. 4. The first column shows the development of further fine structure each time the Fibonacci number increases, keeping the position of the main dips at each step roughly unchanged. Although according to Eq. (5) the conductance is an average of transmission coefficients over half the Fermi surface, self-similarity is still present, as made evident in the next two columns. In the center column we repeat the central panel of the first column in order to show how a change of scale produces a similar spectra in S_{12}, while an increase in length (S_{15}) gives more detailed structure, also similar to the whole pattern. The column on the right illustrates the six-circle scaling property of the conductance.

Close inspection of Fig. 4 also shows that as the length of the sample increases, the conductance decreases. To find out what governs this behavior we have calculated the conductance at incident energies $E=0.125$, 0.30, 0.45, and 0.60 for different sample lengths l_x, and plotted them in the upper ($r=0.5$) and bottom panels ($r=1.0$) of Fig. 5. We find that in all cases the conductance dependence in l_x is bounded from below by a power-law decrease.[8] In order to investigate the possibility of a residual conductivity at infinite length, we include in the figures a fit using the function $g=g_0 e^{\beta l_x^{-\alpha}} \sim g_0(1+\beta l_x^{-\alpha})$. We notice that in the upper panels the residual conductivity g_0 rises abruptly by four orders of magnitude when increasing the energy from 0.30 to 0.45. Closer study of this range shows that the rise occurs between 0.43 and 0.45. We interpret the change as the capturing of one or more conducting channels by the convolution (5), arising possibly from exotic extended states. The bottom panels, showing the special case $r=1$, permit a check of this ansatz. Since the conductance is a convolution over a range of wave numbers q, if at a particular value of this quantity an extended state is present, it will contribute to transport at infi-

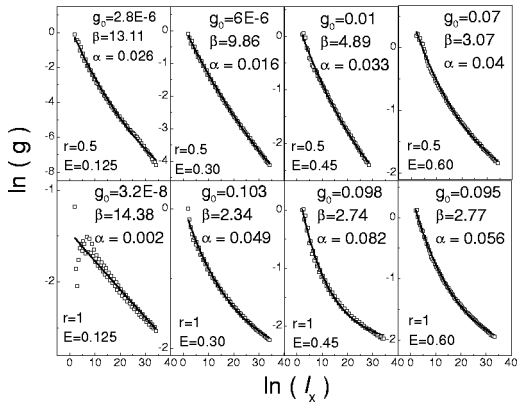

FIG. 5. Length dependence of the conductance. $B_P=2B_Q$ (upper panels) and $B_P=B_Q$ (bottom panels), and the other parameters are the same as in Fig. 2. The solid lines are an exponential fit described in the text.

nite length. As may be easily checked, when $r=1$ the effective potential in Eq. (2) vanishes everywhere at wave number $q_c=-1/2$, a state captured by the convolution (2) at energies $E>0.125$. The bottom panels show that a drop by several orders of magnitude occurs when this energy is approached from above, confirming that the loss of an extended state is indeed reflected in a large change in the conductance.

This behavior is different from either the usual 1D (Ref. 8) or the 2D (Ref. 9) Fibonaci systems. We attribute it as a manifestation of the presence of exotic extended states within the spectrum of a Fibonacci structure. Since the conductance is a convolution over a range of wave numbers q, if at a particular value of this quantity an extended state is present, it will contribute to transport at infinite length. When such conducting channels are present, one may write $g \sim g_0 + g_c$, exhibiting the contributions from the critical and the extended states. Since g_0 has no dependence on l_x and $g_c \propto g_\alpha l_x^{-\alpha}$,[9] the conductance g may be approximated by the function $g_0 e^{\beta l_x^{-\alpha}}$, where $\beta \propto g_\alpha / g_0$. It can be expected $\beta > 1$ due to the predominant weight of the critical channels.

The self similarities and the length dependence of the transmission and conductance of a Fibonacci magnetic superlattice reported above are robust with regard to changes in the particular shape of the magnetic barriers, and the choice of vector potential.[10] This makes an experimental verification of the properties found very plausible. As our results suggest, a 2DEG subject to the inhomogeneous magnetic field of a Fibonacci or other quasiperiodic sequence of magnetic stripes deposited on a nearby parallel surface should exhibit self similarity and an unusual length dependence in the conductance perpendicular to the stripes. Extended states along the direction perpendicular to the stripes may contribute or not to the bulk conductance depending on the energy of the incoming electrons. A possible experimental test of this finding is to measure the conductance as the Fermi energy is varied by means of a gate voltage. The loss of extended states within the energy range available for transport would reveal a drop akin to a metal-insulator transition.

IV. CONCLUSION

In summary, we have discussed the quasiperiodic behavior of electrons in a Fibonaci lateral magnetic superlattice. We have shown that its transmission and conductance possess both the self similarity and six-circle properties found in other kinds of quasiperiodic systems. Moreover, interesting scaling properties of conductance with respect to the sample size in the tunneling direction have been found, exhibiting the presence of exotic extended states.

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