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DOCTORADO EN ECONOMÍA

ESSAYS IN INCOME TAX EVASION

by

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March, 2022

Santiago, Chile

To my family, classmates, and friends

Ackowledgments

I am thankful to all my committee: Nicolás, Martín, Marcelo, and Eduardo, however, each one deserves specific thanks words. Firstly, I am thankful to Eduardo Zilberman, who accepted to be part of my committee and gave me a series of advice on my theoretical paper; much of these are fundamentals in my constant proofreading processes and was very helpful to clarify the exposition of the results. Second, I am grateful to Nicolás Figueroa, who accepted being my advisor without knowing me and constantly supported me to take every step during this process. Also, he taught me how to think as a theoretical economist, continually requiring me to revisit every equation to understand every term well and explain it in simple words. Third, I am beholden to Marcelo Bérgolo, who taught me how to think as an empiric public economist and supported me during the empirical paper process. Moreover, he has acted as a role model showing me how to develop high-quality research from South America. Finally, but not least, I am totally indebted to Martín Besfamille. Martin accepted me as his teaching assistant without knowing me, only intending to teach me public economics. I learned from him not only all my knowledge about public economics and how to teach it, but also, and more importantly, I discovered the field that is passionate about me and that I decided to devote my career. All that will come is majority motivated and produced by his inspiration.

Also, I want to thank all my Phd's classmates. Some of them arrived when I was mainly alone in the student room and gave me the necessary company to get over the difficulties of this process. With them, I have learned how to establish friendships in this environment. Part of this thank needs to be to Romina, my friend and coauthor. Romi, without your help and company, my JMP would not be what it is now. Besides that, thanks to my University friends for their support and constant company; your help and words in the dark times were fundamental to finding the ways to arrive here. In this line, some relevant professors deserve recognition, thanks to Eugenio Giolito and Carlos Garcia for motivating me to study more and research, creating the ambition to hold a PhD and be a professor.

Finally, my family deserves it all. They encouraged me to follow my dreams, and they supported me during this journey. I hope to reward all the love they have given me following their learnings, being a good man, and helping all that needs it.

Abstract

Governments recurrently come back to ask the best way to increase tax collection, based on constant necessities to increase its spending. Generally, increasing tax collection is associated with a tax raise, mainly motivated by the agent's mechanical responses, i.e., assuming that taxpayers do not adjust their taxable income. However, the behavioral response (the adjustment in the agent's taxable income, for instance) produces a variety of consequences in different markets. Indeed, a rise in taxes changes the evasion's gains altering the incentives to participate in occupations with higher evasion facilities. This thesis is devoted to studying this issue throughout two chapters. Each of these chapters inquires over the consequences of the tax policy in contexts where tax evasion and occupational decision coexist.

The first chapter studies the causal effect of income tax evasion on self-employment decisions. We develop a theoretical model to disentangle the mechanisms behind this effect when the marginal tax rate changes. Then, we obtain a proxy of tax evasion using a consumption-based approach at the household level using data for Chile. To identify the causal effect of income tax evasion, we use two advantages from the Chilean setting. First, it establishes equal marginal tax rates across self-employed and wage-earners isolating the evasion channel. Secondly, we exploit a marginal tax reform that affects agents given their pre-reform taxable income. We obtain two behavioral parameters using a difference-in-difference approach. Firstly, the elasticity of evasion to marginal tax rate equals 1.4. Also, we find that an increase of 1 percentage point in the evasion rate raises the probability of being self-employed by 6.1 percentage points, with a semi-elasticity of 0.16. We also show that the evasion channel explains 99.73% of the effect of taxes on self-employment decisions. Finally, we document that the deadweight loss associated with the tax reform is between 2.82 - 3.01% depending on the tax compliance policy. Moreover, we theoretically and empirically

demonstrate that not considering evasion in this measure produces a biased estimation of the tax effect on welfare.

The second chapter incorporates occupational decisions into the hierarchical model of Sanchez and Sobel (1993) to investigate distortions in tax policies design: the audit function, a linear marginal tax rate, and the IRS budget. In this economy, evasion is only possible in the self-employment sector. The optimal audit is efficient below a cut-off level, and above this level, are equal to zero. This result is held under two extensions: the audit cost is monotonically non-decreasing in the self-employment wage, and the fine rate rises in self-employment wage but is bounded from above. The marginal tax rate is smaller than one, indicating that not considering occupational decisions produces an upward bias on taxes. The optimal IRS budget does not allow auditing the entire self-employment sector, but it is larger than the result from a cost-benefit analysis. Finally, differential taxation is optimal if the marginal tax rate in the self-employment sector is higher than the dependent sector. This result produces that the distortions in the optimal allocation of agents increase compared to an environment with one marginal tax rate.

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Chapter 1

Tax Evasion and Self-Employment Decisions: Evidence From an Income Tax Reform in Chile

with Romina Safojan

1.1 Introduction

Self-employment decisions have multiple motivations, significantly affecting the labor market characteristics and public policies responses. Particularly, the decision to become self-employed depends on the tax policy. Indeed, Bosch and de Boer (2019) show that labor supply, investment and income declarations of self-employed are highly responsive to payroll and income taxes. These behavioral responses may distort the agent's optimal employment choice (Wen and Gordon, 2014). Besides tax incentives, other facts can distort the occupational decision, evasion for instance. Slemrod (2007) and Kleven et al. (2011) find that tax evasion comes mainly from self-reported income, evidencing that the self-employed are more prone to evasion than other occupation types. Thus, higher tax evasion opportunities in the self-employment than in the wage-earner sector might incentivize workers to choose the former.[1] However, still little is known about tax evasion motivations

[1]Evidence from a theoretical point of view is provided by Kesselman (1989) and Watson (1985) in a general equilibrium model, and Castillo (2021) in a tax policy design model. Empirically, Bárány (2019), Gentry and

to become self-employed. This study tries to solve this gap aiming to disentangle how tax evasion decisions affect workers' choice of self-employment and how this may affect the social welfare that results from income tax policies.

In contexts where the tax scheme levies differently self-employed and wage-earners, two channels affect the occupational decision by evasion incentives. One channel is the pure evasion effect, named the evasion channel. In contrast, the difference in taxes between occupations produced by the tax scheme and the income report (with evasion) gives the differential taxation effect. Thus, the second channel captures the impact of evasion on tax paid and its difference with another occupation.

The literature faces multiple empirical challenges to isolate the evasion channel in self-employment occupational decisions. First, it is challenging to measure income tax evasion owing to self-employed workers' self-reporting their income, making it hard to obtain how much income each worker evades. Second, in terms of identification, since workers may anticipate their evasion behavior, tax evasion and occupational decisions are simultaneously determined, leading to an endogeneity issue for the empirical estimation. Third, isolating the evasion channel from the differential taxation channel is problematic because it depends on the tax scheme. This paper overcomes these issues from different edges.

We focus on the Chilean economy, where the self-employment sector constitutes a significant local labor market component. Studying Chile provides us three main advantages for isolating the evasion effect on self-employment decisions: (i) the Chilean income tax scheme levies with equal marginal taxes self-employment and wage-earner sectors dropping the differential taxation; (ii) the availability of an expenditure and income survey at the individual and household levels allows us to estimate tax evasion following a consumption-based approach (Pissarides and Weber, 1989; Hurst, Li and Pugsley, 2014, among others); and (iii) an income tax reform implemented in 2013 that differently affected the financial incentive of self-employed to evade, gives us a quasi-experimental variation in the tax rate to overcome any endogeneity issues concerning tax evasion and self-employment decisions.

The first step is to build an occupational choice model with tax evasion, to identify the mechanisms behind taxes' effect on self-employment decisions. This model assumes that the economy has two occupations: self-employed and wage-earners, with both sectors facing the same tax sched-

Hubbard (2000), Bruce (2000), Schuetze (2000), Cullen and Gordon (2007), and Fossen and Steiner (2009) explore the effects of income tax variations between occupations on self-employment.

ule, characterized by two income brackets. Two channels explain the occupational decision. The first channel is related to taxable income, showing the direct effect of tax rate changes on self-employment taxable income net of evasion. The second channel is the effect of evasion on the occupational decision, which captures the impact of evasion, isolating it from the differential taxation effect. We show that the effect of tax changes on self-employment decisions is a weighted sum of the taxable income and evasion channel, where the weights are the corresponding elasticities. Therefore, this model explains the mechanisms behind the tax effect on the occupational decision and the behavioral parameters to assess the relevance of the evasion channel.

Then, to measure tax evasion, we use a pooled cross-section of the individual and household survey data of Chile for the period 2007-2017, the *Encuesta de Presupuestos Familiares* (EPF), administered by the Instituto Nacional de Estadísticas (INE). This survey jointly measures individual and households' income, expenditures, and employment in Chile. These facts provide a remarkable opportunity to estimate the extent of household tax evasion and individual occupation. We estimate tax evasion following the methodology proposed by Hurst, Li and Pugsley (2014) that compares households' Engel curves between self-employed and wage-earners.[2] Among other assumptions, this methodology assumes that only self-employed households can evade, this is validated in multiple studies (See, for instance, Cabral, Gemmell and Alinaghi, 2019; Kukk, Paulus and Staehr, 2020; Nygård, Slemrod and Thoresen, 2019, among others.). Unlike Hurst, Li and Pugsley (2014), we incorporate the possibility of having heterogeneity in income tax evasion among self-employed households improving this methodology.[3] We find that the evasion rate in the self-employment sector is 10.89 percent of the taxable income, on average, for 2007-2017, being the intensive (the average evasion rate for evaders) and extensive (the percentage of self-employed that evade taxes) margins of 12.87 and 84.71 percent, respectively. Also, we show that not considering differences in tax evasion across self-employed households would introduce measurement errors to this estimation, underestimating misreporting in Chile.

To identify how the tax evasion behavior determines the decision of being self-employed, we

[2]We define a household as self-employed if its principal is self-employed. As a sensitivity analysis, we consider an alternative definition that indicates whether the head or the couple (if married or cohabiting) is self-employed, or we restrict the sample to male-headed households without a self-employed couple.

[3]The assumption over the same tax evasion rate across agents has the pitfall that it fails to capture workers' heterogeneous behavior across the income distribution that significantly affects occupational choices (see Albarea et al., 2019; Waseem, 2018; Engstrom and Hagen, 2017).

exploit the exogenous variation in marginal income tax rates driven by a tax reform implemented in Chile in 2013. This reform reduced the marginal tax rates in different proportions across all income brackets except from the lowest one. Individuals in this bracket conform our control group, because they are exempted from being taxed. Although the tax scheme equally levies self-employed and wage-earner in Chile (the Second Category Tax), self-employment provides an alternative to reduce tax burdens through sheltering income since agents in this sector self-report their income. Being the tax scheme equal in both sectors, we eliminate the differential taxation effect of the income tax changes on the occupational choices and isolate the reform's effect through tax evasion.[4] This tax scheme's characteristic establishes the perfect quasi-experimental setting to identify the evasion channel in occupational choices using the policy tax change as the instrument for tax evasion rate.

Thus, first, we estimate the impact of the income tax policy change on the evasion rate using a Difference-in-Difference (DiD) approach. The tax reform produces a significant fall in the tax evasion rate by -0.0206 percentage points. This effect means an elasticity of evasion to the marginal tax rate of 1.4. This result evidence the relevance of income taxes for the evasion decisions since agents' evasion behavior overreacts to tax changes. Comparing the results obtained using the alternative self-employed household definition shows that households with a self-employed couple are less sensitive to tax changes.

Second, we estimate how tax evasion affects self-employment decisions using the policy change as an instrument for evasion following an Instrumented Difference-in-Differences (IV-DiD) approach (see, for instance, Hudson, Hull and Liebersohn, 2017; De Chaisemartin and d'Haultfoeuille, 2018). We obtain that an increase in 1 percentage point in the evasion rate increases the probability of being self-employed by 6.1 percentage points. This significant effect shows that evasion is a critical determinant in self-employment decisions. Finally, to normalize this effect, we compute the associate semi-elasticity, obtaining that an increase of 1 percent in the evasion rate increases the self-employment rate by 0.16 percentage points.

The last step in this analysis is to obtain the effect of the tax change on self-employment decisions to assess the relevance of the evasion channel on it. We use the reduced form of the IV-DiD to obtain that the semi-elasticity of self-employment decision to the marginal tax changes reaches 0.223. Lastly, we use the elasticity of evasion and the self-employment decisions semi-elasticity to

[4]The only possible source of difference is for wage-earners that are also firm-owner taxpayers. However, we drop those cases to avoid any inconsistencies.

compute the evasion channel. We obtain that the evasion channel gives 99.73% of the tax effect on self-employment decisions. This result reinforces the relevance of evasion in occupational decisions.

These important results push us to study the welfare effect of tax changes in this framework. We take advantage of the income tax policy change to identify the effect of tax rate changes on social welfare by estimating the deadweight loss. Following Chetty (2009*a*) and Gorodnichenko, Martinez-Vazquez and Sabirianova Peter (2009), and based on our theoretical model, we develop a model that captures the welfare effect due to tax changes. This model encompasses the mechanisms and incentives behind the tax change effects showing the relevance of different elements that our setting has: evasion, occupational decisions, and the tax compliance policy. We show that omitting evasion produces a smaller welfare effect estimation because the metric does not incorporate the effect of evasion on wage-earner welfare. Also, if occupational decisions are not included, the evasion effect in self-employment decisions is not included in the welfare effect, producing bigger estimations. We demonstrate that none of these problems are solved with an extension of Feldstein (1999) that includes the extensive margin. Finally, we obtain the welfare effect due to tax reform, identifying the behavioral parameters to measure the deadweight loss (DWL).

We compute a DWL between 2.82% and 3.01% of the average taxable income, depending on whether the tax compliance policy is the strongest or null, respectively. Additionally, we demonstrate the relevance of considering evasion in the DWL measure, showing that: (i) not including evasion produces the tax compliance policy is irrelevant for the DWL; (ii) not considering evasion produces an upward bias in the DWL estimation; and (iii) using the elasticity of taxable income instead of a welfare effect measure produces an upward bias and a downward bias, depending on whether consider or not the occupational decision, respectively.

This paper contributes to four strands of literature. First, this study contributes to the literature exploring the motivations to be self-employed, providing empirical and theoretical evidence on tax evasion's impact on these decisions and the channels that explain this. Concerning self-employment decisions, some studies have explored the effects of income tax variation between wage-earner and self-employed on self-employment (Gentry and Hubbard, 2000; Bruce, 2002; Schuetze, 2000; Cullen and Gordon, 2007; Fossen and Steiner, 2009; Bosch and de Boer, 2019; Wen and Gordon, 2014) estimating a combination of direct and indirect effects. For instance, Bruce (2000) finds a negative effect of tax rate differential on the decision to become self-employed. However, it is impossible to determine if the effect is explicitly associated with tax evasion. Also, Parker (2003) does not find a significant impact of evasion on self-employment choice, which could be associated with the strong assumption in his tax evasion model. Thus, they cannot isolate the evasion channel in self-employment decisions, contrary to our study that takes advantage of the Chilean income tax scheme's particularity, where income taxes are equal across occupational sectors.

Closer to our study, in a cross-country analysis for developed economies, Bárány (2019) shows that self-employment provides an opportunity for income tax evasion that partially drives the choice to become self-employed. The model estimates that the difference in utility generated by the tax rate differential between self-employed and wage-earners within industry-occupation increases the magnitude of the misreporting rate. Unlike this paper, we estimate tax evasion at the household level, allowing us to quantify the effects of income tax rates on tax evasion and self-employment choice at the micro level. Additionally, this strand of literature does not explore the welfare effect of tax changes using the micro level estimations of the tax effect on self-employment decisions, in this study we also provide this. Therefore, we contribute to this literature by providing a reliable estimation of the evasion's relevance in self-employment decisions and an estimation of the tax reform effect on social welfare based on the tax evasion effect on self-employment decisions.

A second contribution relies on the literature that tries to estimate the welfare effects of tax reform. Starting from the seminal paper of Feldstein (1999), this literature focus on finding a behavioral parameter that captures the welfare effect associated with tax changes. Recently, some studies pointed out that the elasticity of taxable income is not enough to estimate the deadweight loss and capture the effect of tax changes on welfare (Chetty, 2009*a*,*b*; Gorodnichenko, Martinez-Vazquez and Sabirianova Peter, 2009; Kleven, 2018; Doerrenberg, Peichl and Siegloch, 2017). We are part of these studies that explain the relevance of including evasion and shed light on the importance of incorporating the occupational decisions and tax compliance policy. Our main contributions are twofold. First, we estimate a more flexible model with two marginal tax rates (instead of one), including occupational decisions and tax compliance policy. Second, we show the relevance of evasion's effect on the DWL, demonstrating that not considering occupational decisions significantly bias this measure, and the DWL is sensitive to the tax compliance policy. In this sense, we give more elements to analyze and capture better welfare implications due to tax reforms.

The third contribution of our paper is the measurement of tax evasion. The seminal paper of

Pissarides and Weber (1989) proposes a consumption-based method that compares Engel curves to identify a household's income misreporting rate. Hurst, Li and Pugsley (2014), among others, extend this approach and study the identification implications comparing different types of surveys and using income metrics to capture the permanent consumption hypothesis. Furthermore, other papers analyze the implications of different individual/household characteristics (Cabral, Gemmell and Alinaghi, 2019; Kukk, Paulus and Staehr, 2020; Nygård, Slemrod and Thoresen, 2019, among others). While this literature finds heterogeneity among self-employed households, none of them allows for differential evasion rates for each household. We further extend the consumption-based approach allowing heterogeneity in self-employed household evasion behavior, evidencing differences in evasion across the income distribution.

Finally, we contribute to the literature that studies agents' behavioral responses to tax changes in developing countries using a quasi-experimental variation to isolate tax effects. This strand of literature is still novel and mainly focuses on firms' behavior (for a recent survey, see Pomeranz and Vila-Belda, 2019). For agents' responses, most papers focus on individual response to personal income taxation, focusing on high-income taxpayers (Tortarolo, Cruces and Castillo, 2020; Jouste et al., 2021; Bergolo, Burdin, De Rosa, Giaccobasso, Leites and Rueda, 2021), wealth (Londoño-Vélez and Ávila-Mahecha, 2021) or the income distribution (Bergolo, Burdin, De Rosa, Giaccobasso and Leites, 2021). Regarding evasion, the evidence focuses on the effect of audits on firms' behavior (Pomeranz, 2015; Carrillo, Pomeranz and Singha, 2017). Therefore, no evidence exist either on the effect of evasion on self-employment decisions or the effect of taxes on agents' evasion decisions. In this sense, this paper provides new relevant evidence over the sensitivity of tax evasion to taxes and the effect of evasion in self-employment decisions in a developing country. Developing economies have large self-employment sectors and low tax administration capacity, so these estimations are crucial for public policy.

The study continues as follows. Section 1.2 introduces a tax evasion and occupational choice theoretical model that formalizes the channels of these decisions. Section 1.3 describes the Chilean tax system's main relevant characteristics and the income tax reform of 2013, as the exogenous policy change in marginal tax rates exploited to identify the effect of interest. Section 1.4 details relevant definitions and the database characteristics. Section 1.5 explains the tax evasion measurement procedure. Section 1.6 shows the effect of tax changes in evasion and evasion on self-employment decision. Section 1.7 measures the deadweight loss in this setting, and Section 1.8 concludes.

1.2 Theoretical Model

This section presents and fully characterizes a two-stage theoretical model to disentangle the mechanisms behind the occupational and evasion decisions when income tax rates change. For this reason, our exposure focuses on determining the channels involved in these decisions, to estimate them empirically, rather than theoretically explaining each choice.

Additionally, we fully characterize the agents' decisions at the extensive (occupational choice) and intensive (evasion) margins, shed light on tax changes' economic effect. Thus, we show that two separated channels affect occupational decisions: taxable income and evasion.

1.2.1 Basic Framework

Consider an economy with a continuum of risk-neutral agents with only two alternative occupations, namely wage-earners and self-employed, indexed by w and s, respectively.[5] While wage-earners pay their taxes through a third-party (i.e., their employers), self-employed workers self-report their income, making them possible to misreport and evade taxes.[6] In this economy, agents face the same tax schedule in both sectors. This assumption is consistent with the Chilean tax scheme. Under these assumptions, workers choose an occupation based on their productivity and the possibility of misreporting (if self-employed).

Agent's taxable income equals $z = z^i$, where i indicates the employment sector such that $i = \{s, w\}$. We assume that z^i has a distribution F_i with support $[\underline{z}^i, \overline{z}^i]$.[7] Let us define the

[5]For simplicity, we assume risk-neutral agents. This assumption allows us to isolate the effect of uncertainty from audit probability rather than evasion incentives. Other papers that make this assumption are Chetty (2009*a*), and Gorodnichenko, Martinez-Vazquez and Sabirianova Peter (2009)

[6]Kleven et al. (2011) and Slemrod (2007) show that evasion mainly comes from income self-declaration. Wage-earners assumption omits the possibility that firms and workers collude to misreport the agent's income. Kleven, Kreiner and Saez (2016) formalize this assumption and demonstrate that third-party reports helps tax enforcement policies. However, Best (2014); Bjørneby, Alstadsæter and Telle (2021); Bergolo, Burdin, De Rosa, Giaccobasso and Leites (2021) demonstrated that in some cases is possible that third-party report do not solve the evasion problem entirely.

[7]Implicitly, we assume that agent's productivity equals to $n = \{n^s, n^w\}$, being an independent draw from a distribution function with support $[\underline{n}^i, \overline{n}^i]$. Also, we assume that only exist one consumption good, with normalized price equal to one, produced in a competitive market with a linear technology that uses only labor. These assumptions produce $n^i = w^i$, that is, agent's productivity equalizes the sector's wage. Thus, we can map the distribution of n^i

reported income x^i in sector $i = \{s, w\}$. Wage-earners cannot evade, then $x^w = z^w$.[8] On the other hand, in the self-employment sector, reported income can differ from the taxable income, such that $x^s = z^s - e$, where e is the evaded income. To make the model tractable, workers are "naive" or "myopic" since they do not anticipate its behavioral response of z^i to taxes. When we obtain the behavioral parameters to estimate tax effect on agents' decisions, we drop down this assumption and allow those behavioral responses exist.

In each occupation, workers pay a tax liability $T(x^i)$. Without loss of generality, we assume that the tax scheme comprises two income brackets with the corresponding marginal tax rates t_1 and t_2, where $t_2 > t_1$. The threshold that separates tax brackets is exogenously determined and equal to A. In order to obtain a close form solution, we assume that $A = \overline{z}/2$, so $\overline{z}^s = \overline{z}^w$. Thus, if $x^i < A$, agents' tax liability is equal to $T(x^i) = t_1 x^i$, otherwise $T(x^i) = t_1 A + t_2(x^i - A)$ for $i = \{s, w\}$.

1.2.2 Agents Decisions: Evasion and Occupational Choice

Agents make two decisions to maximize their utility. First, they decide the sector at which to work $i \in \{s, w\}$ (extensive margin of labor supply).[9] If agents choose sector w, their utility equals consumption. Assuming no savings, agents' indirect utility takes the form $U^w(z^w) = z^w - T(z^w)$. If agents choose sector s, their utility equals consumption, which is determined by their expected after-tax income. In this sector, agents must decide x^s, the second decision to maximize their utility. Self-employed agents face an audit probability $\rho(x^s)$ determined by their income declaration x^s. Thus, workers expected after-tax income equals $z^s - T(x^s) - \rho(x^s)\pi\,[T(z^s) - T(x^s)]$, where π is the fine rate. If audited, the government detects if they evaded with probability one and charges a fine $\pi > 1$ on the evaded income taxes. For a closed form solution, we assume a specific form of the audit function, $\rho(z^s - e) = 1 - (z^s - e)/\overline{z}$, so $\rho' = -1/\overline{z} < 0$. Therefore, their indirect utility is given by

$$U^s(z^s) = \max_{x^s} \quad \{z^s - T(x^s) - \rho(x^s)\pi\,[T(z^s) - T(x^s)]\}$$

to $z^i = w^i l(n^i)$, with l being agent's labor supply.

[8] This model does not include tax deductions, producing that taxable income is equal to reported income in the wage-earner sector.

[9] Conditional on working, the decision is between sectors, without considering the inactivity option.

We solve this model by backward induction. First, we characterize the optimal evasion for self-employed agents. Afterward, we characterize a threshold function to determine the taxable income level for which agents decide to work as self-employed.[10] The following Lemma characterizes the optimal evasion decision.

Lemma 1. *The optimal evasion decision* e^* *is characterized as follows*

$$e^*(z^s,\rho,\pi,t_1,t_2) = \begin{cases} \dfrac{z^s}{2} - A\left(1-\dfrac{1}{\pi}\right) & \text{if } \underline{z}^s \le z^s \le A \\ \dfrac{z^s}{2} - A\left(1-\dfrac{1}{\pi}\right) - \dfrac{(z^s-A)}{2}\dfrac{(t_2-t_1)}{t_1} & \text{if } A < z^s < Z_1 \\ z^s - A & \text{if } Z_1 \le z^s < Z_2 \\ \dfrac{z^s}{2} - A\left(1-\dfrac{1}{\pi}\right) & \text{if } Z_2 \le z^s \le \overline{z}^s \end{cases} \tag{1.1}$$

where $Z_1 = A\left[1+\dfrac{t_1(2-\pi)}{\pi t_2}\right]$ *and* $Z_2 = \dfrac{\overline{z}^s}{\pi}$.

Proof.

See Appendix 1.9.1. ■

For simplicity, we will refer to each section as e_i with $i = \{1,2,3,4\}$, so e_1 corresponds to e^* for $\underline{z}^s \le z^s \le A$ and so forth. Note that evasion depends on the marginal tax rates only when the taxpayer declares an income lower than A and its taxable income is higher than this threshold (section e_2). Hence, changes in the marginal tax rate only affect evasion when agents evade through jumping into a lower tax bracket. Moreover, the optimal tax evasion in e_2 is decreasing in t_2, because the expected penalty increases.[11] For e_3, agents bunch at the threshold level A as evidenced by Saez (2010) and Kleven (2016).

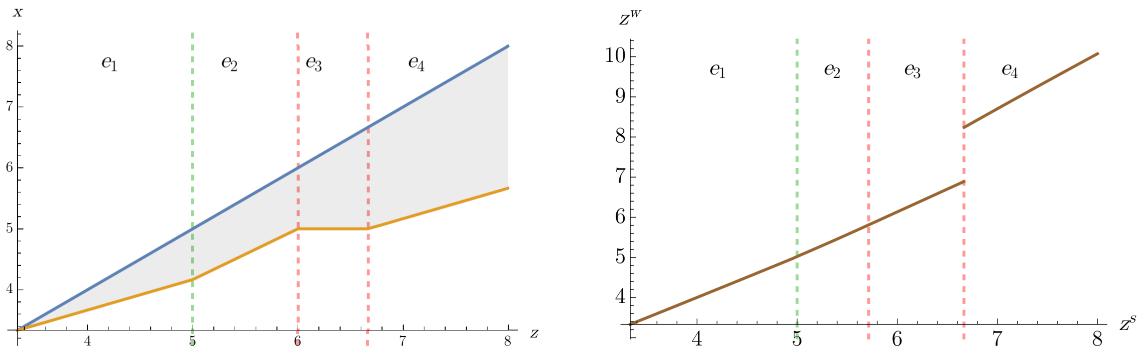
Figure 1.1a shows the difference between the true (blue line) and observed (brown line) reports. Evasion, the grey zone in 1.1a, is increasing in self-employment income with a smaller slope in e_2, indicating that a high enough t_2 can discourage evasion via increasing expected penalties.[12] This is evidenced in Lemma 3 (in the Appendix) that shows that the larger the marginal taxes difference, the lower the evasion. Also, the bunching observed in e_3 increases evasion in e_4, displaying the regressivity produced by tax evasion in this framework.

[10] Since we assume that $\rho' < 0$, the self-employed's utility is concave and there exists an interior solution (Yitzhaki, 1974)

[11] See Lemma 3 in Appendix 1.9.1.

[12] Notice that, e^* is decreasing in z^s this section if $t_1 < \frac{t_2}{2}$.

Figure 1.1: Evasion Across Self-employment Income and the Threshold Function



(a) True vs. Observed Income declaration

(b) Threshold Function

Note: The figures assume the following parameters values: $\overline{z}^s = 10$, $\underline{z}^s = 0$, $A = \overline{z}^s/2$, $t_1 = 0.15$, $t_2 = 0.25$, $\pi = 1.5$ *and* $\rho(z^s - e) = 1 - {}^{(z^s - e)}/_{\overline{z}^s}$*. The dashed green line represents threshold* A *and the dashed red lines* Z_1 *and* Z_2*. In Panel (a), the grey area represents the evasion amount.*

Given the optimal evasion amount e^* and the taxable income z^s, we define a threshold function $H(z^s, e^*)$ in the extensive margin which gives the taxable income level that equalizes the indirect utilities in both sectors (i.e., $U^w(H(z^s, e^*)) = U^s(z^s, e^*)$). Hence, for each potential self-employed's taxable income z^s, we can find a threshold level $H(\cdot)$ that determines the minimum taxable income in the wage-earner sector to work as a wage-earner, such that $z^w > H(z^s, e^*)$.[13] The following Lemma formalizes the optimal threshold function.

Lemma 2. *Given the optimal evasion* e^**, the minimum income in the wage-earner sector to work as a wage-earner is defined by*

$$H(z^s, e^*) = \begin{cases} z^s + e_1 \dfrac{t_1(1-\rho(z^s - e_1)\pi)}{1-t_1} & \text{if } z^w \leq A \text{ and } z^s \leq A \\ \dfrac{z^s(1-\rho(z^s-e_2)\pi t_2)}{1-t_2} - \dfrac{(z^s-e_2)t_1 + A(t_2-t_1))(1-\rho(z^s-e_2)\pi)}{1-t_2} & \text{if } z^w > A \text{ and } A < z^s < Z_1 \\ \dfrac{z^s(1-\rho(z^s-e_3)\pi t_2)}{1-t_2} - \dfrac{(z^s-e_3)t_1 + A(t_2-t_1))(1-\rho(z^s-e_3)\pi)}{1-t_2} & \text{if } z^w > A \text{ and } Z_1 \leq z^s < Z_2 \\ \dfrac{z^s - (t_1 A + t_2(z^s - e_4 - A)) - \rho(z^s-e_4)\pi t_2 e_4}{1-t_2} - A\dfrac{t_2-t_1}{1-t_2} & \text{if } z^w > A \text{ and } Z_2 < z^s < \overline{z}^s \end{cases} \tag{1.2}$$

Proof.

See Appendix 1.9.1 ■

[13] Notice that the indirect utility in the wage-earner sector is increasing in self-employment income and equals the indirect utility in the self-employed sector if $e^* = 0$. Also, evasion increases the self-employed's indirect utility. Hence, at least for some agents, the indirect utility is higher in the self-employment than in the wage-earner sector, producing the existence of the threshold level. For formal proof in a context of a unique marginal tax rate, see Castillo (2021).

For simplicity, we define corresponding sections as $H_i(z^s, e_i)$, such that $H_1(z^s, e_1)$ for $z^w \leq A$ and $z^s \leq A$ and so forth. This function characterizes one threshold level for each section of the optimal evasion function e^*. Only $H_1(z^s, e_1)$ does not depend on both marginal tax rates. However, from the optimal evasion characterization, marginal tax changes only affect evasion in section e_2, or in $H_2(z^s, e_2)$. In the other sections, the effect of evasion comes from mechanical responses to taxes or income changes.

Figure 1.1b shows the threshold function using the same parameters as for the optimal evasion. All agents situated above the dark brown line decide to work in w, otherwise in s. This function evidences a jump at Z_2, that can be explained from the change in the tax bracket: for $z^s > Z_2$ the marginal tax rate faced is t_2, otherwise is t_1. In this section, the gains from evasion rise as shown in Figure 1.1a, increasing the wage-earner sector's wage requirement for choosing this sector.

1.2.3 Tax Change Effect on Occupational Decision

We derive the threshold function's partial derivative to explicitly show the mechanisms behind changes in the marginal tax rates into occupational decisions, and to display the behavioral parameters relevant to the empirical analysis. Considering that the Chilean tax scheme splits taxpayers into those levied with zero marginal tax and those with a positive one, we focus on the effect of t_2 on the threshold function.[14] Since the threshold function comprises four sections, we show only a general characterization of a lower-bound estimation for the partial derivative. Therefore, the effects behind the movements in each sector are gathered in the following expression

$$
\begin{aligned}
\frac{\partial H(z^s, e^*)}{\partial t_2} &= \frac{\partial z^S}{\partial t_2} \frac{\partial H(z^s, e^*)}{\partial z^s} + \frac{\partial e^*}{\partial t_2} \frac{\partial H(z^s, e^*)}{\partial e} \\
&= \underbrace{\frac{\partial z^s}{\partial t_2} \frac{t_2}{z^s}}_{\varepsilon_{z^s, t_2}} \underbrace{\frac{\partial H(z^s, e^*)}{\partial z^s} z^s}_{\eta_{H, z^s}} \frac{1}{t_2} + \underbrace{\frac{\partial e^*}{\partial t_2} \frac{t_2}{e}}_{\varepsilon_{e, t_2}} \underbrace{\frac{\partial H(z^s, e^*)}{\partial e} e}_{\eta_{H, e}} \frac{1}{t_2}
\end{aligned}
$$

where ε_{z^s, t_2} is the elasticity of the taxable income in the self-employed sector, ε_{e, t_2} is the elasticity of evasion, and η_{H, z^s} and $\eta_{H, e}$ are the semi-elasticities of the threshold function concerning the taxable income and to evasion, respectively. The following Proposition formalizes the lower-bound effect of tax changes on the threshold function.

[14] Although in the Chilean tax scheme some taxpayers face a zero marginal tax rate, we do not impose $t_1 = 0$ to avoid problems in variables definitions.

Proposition 1. *Given the optimal evasion e^* and the threshold function $H(z^s, e^*)$, the effect of changes on t_2 in the threshold function can be summarized as*

$$\frac{\partial H(z^s, e^*)}{\partial t_2} t_2 = \eta_{H,t_2} = \varepsilon_{z^s,t_2}\eta_{H,z^s} + \varepsilon_{e,t_2}\eta_{H,e}$$

Proof.

See Appendix 1.9.1 ■

Note that we express the partial derivative as a semi-elasticity, η_{H,t_2}, that captures the effect on the probability of being a self-employed worker given by a one percent change in t_2. The semi-elasticity to income captures the income differential effect, evidencing how attractive is the option of being self-employed because of income differential. On the other hand, the semi-elasticity to evasion captures the evasion incentives in the self-employment sector, reflecting the gains from hiding income to the tax authority. Thus, this equation shows that the effect of tax changes is captured by a weighted sum of the effects of taxable income and evasion over the threshold function.

We conclude that when the marginal income tax changes, the movements in the threshold function are associated with two channels:

1. *Taxable Income Channel:* It relies on the taxable income differences across sectors, net of evasion, due to tax changes. When taxes fall, taxable income rises.[15] The increase in self-employment income makes this sector more attractive, increasing the optimal threshold wage. Thus, we expect a negative effect since $\varepsilon_{z^s,t_2} < 0$ and $\eta_{H,z^s} > 0$.

2. *Evasion Channel:* Represents the direct evasion effect due to tax changes. If taxes fall, the gains for evading also fall, decreasing the evasion rate and making less attractive for agents to participate in the sector where evasion is possible. Thus, the threshold function decreases. Therefore, we expect a positive effect since $\varepsilon_{e,t_2} > 0$ and $\eta_{H,e} > 0$.

Proposition 1 shows that the differential tax channel disappears in a framework where wage-

[15] We assume that the income effect does not exist or is smaller than the substitution effect, producing that substitution effect prevails.

earners and self-employed face the same tax scheme as in the Chilean context.[16] This fact allows us to isolate the evasion channel when marginal tax rates change empirically. Also, because of the Chilean setting, any tax change should affect self-employment decision through evasion because wage-earners and self-employed face the same tax scheme. We argue and demonstrate in Section 1.6 that the evasion channel is a critical determinant of the occupational decisions in this framework, driving the effect of taxes on self-employment decisions.

1.3 Income Tax Reform in Chile

This section describes the Chilean income tax system's characteristics, explains the income tax reform exploited for the analysis, and describes the workforce evolution for the period studied. Our identification strategy relies on the capacity to isolate the evasion channel from the differential taxation effect, taking advantage of the Chilean tax schedule's specific structure and a quasi-exogenous variation in the tax structure across taxable income tax brackets.

1.3.1 Chilean Income Tax System

The Chilean workers' income is levied under a global tax, named Complementary Global Tax (CGT) which encompasses capital gains/earnings from the firm's ownership and labor income. This is a progressive marginal tax with a rate from 0 to 40% collected once a year in May. This tax is implicitly declared every month or with every income realization, but in April, taxpayers must submit an income declaration to the Chilean IRS (*Servicios de Impuestos Internos* in Spanish). The IRS compares the tax declaration with the monthly information and classifies each taxpayer into the corresponding income bracket. Since this tax category includes labor and capital earnings, it comprises two different categories: the First Category Tax (FCT) and the Second Category Tax (SCT). The FCT levies capital and/or firm's gains and produce a tax credit over the CGT, producing that high-income taxpayer have incentives to retain some gains and re-invest others.[17]

Since we want to focus on workers instead of firms' owners, we only consider changes in the SCT, eliminating self-declared entrepreneurs from our data. This restriction eliminates potential prob-

[16] The differential tax channel means the effect in self-employment decisions motivated by a different tax scheme between occupations.

[17] See Flores et al. (2020) and Fairfield and Jorratt De Luis (2016) for a detailed explanation.

lems surging from changes in incentives due to tax differential between FCT and SCT.[18] Moreover, entrepreneurs may have different motivations and skills to the wage-earner or self-employment sector, implying low probabilities of a transition between these two groups of occupational choices (see Perry et al., 2007). Thus, we focus only on the transition between wage-earner and self-employment.

Before explaining the SCT, clarify the distinction of taxpayers' status. Wage-earners are workers who face a third-party income report, having or not a long-term contract, but having an income and payroll tax declaration through their employers. Opposite to this, self-employed workers fill an invoice for their employers. This document is like a tax declaration, is filled at the IRS services (web page mainly) and brings information about its gross labor income. The taxpayer should declare whom, she or her employer, withheld a percentage of the gross income to pay income and payroll taxes.[19] Thus, self-employed paid taxes on their own. Also, a worker could have income from wage-earner and self-employment occupations and have two types of declarations. This means the transit cost from one occupation to another is zero, and the definition of each one depends on the primary source of income or the self-declaration.

The SCT scheme is progressive, composed by eight income brackets given by a stable monetary unit, the UTM (Monthly Taxable Unit).[20] Under this tax scheme, self-employed and wage-earners are levied with the same marginal tax rates producing that do not exist differential tax incentives between both occupations. The marginal tax rate (MTR) goes from 0 to 40 percent. In Table 1.1 we show these rates for each income bracket in 2013 (there are the same now), also in Figure 1.6 we show the same visually and compares MTR for each income bracket before and after-tax changes (explained below). The taxable income comprises the gross income minus the amount paid in health insurance and pensions, thus, taxable income equals 83 percent of the gross income. For self-employed workers, taxable income is equal to the income filled in its declaration. Some deductions exist for workers but are related to tax incentives for voluntary retirement savings.

[18]Differences in marginal tax rates between FCT and SCT create incentives to change the labor status of a worker and declare some income as capital gains from a small business or firm.

[19]Before 2020, a taxpayer could decide if the IRS retains its payroll taxes or not. If she prefers to pay its payroll tax on her own, the IRS retains 10 percent of the gross income. After 2020, this decision does not exist, and the retention raises 0.75 percentage points by year to reach 17 percent, 10 percent for labor taxes, and 7 percent for payroll taxes. In Chile, health insurance has a mandatory payroll tax of 7 percent and pensions 10 percent of the gross income, or taxable income if applied, for wage-earners.

[20]The UTM (*Unidad Tributaria Mensual* in Spanish) is updated using the CPI. This produces stable thresholds for each income bracket across years.

1.3.2 2013 Income Tax Reform

The *Educational Reform* introduced in 2013 aimed at increasing the resource available for the educational system.[21] As a part of this reform, the SCT was reduced, affecting the marginal tax rates of six over eight of the income brackets. In Table 1.1, we show the marginal tax rate before and after the policy and the changes in the marginal tax rates by income bracket, and in Figure 1.6, we represent it graphically.

Table 1.1: Income Tax Rates and UTM bounds for 2012 and 2013

Bracket	UTM 2012/3	Tax Rate 2012	Tax Rate 2013	Δ%
1	0 to 13.5	0%	0%	0%
2	13.51 to 30	5%	4%	- 20%
3	30.01 to 50	10%	8%	- 20%
4	50.01 to 70	15%	13.50%	- 10%
5	70.01 to 90	25%	23%	- 8%
6	90.01 to 120	32%	30.40%	- 5%
7	120.01 to 150	37%	35.50%	- 4.05%
8	150.01 or more	40%	40%	0%

Note: UTM, Monthly Tributary Unit. Source: Own elaboration based on the IRS.

Table 1.1 shows that the policy reform did not affect the first income bracket, which maintained their marginal tax rate equal to zero. The reform only affected agents who faced a positive marginal tax rate, and the magnitude of the tax changes differs by income bracket, introducing heterogeneity for different taxable income levels from the second bracket up to the eighth. This implies that tax incentives for evading did not change for the income bracket one but it was for other brackets. Also, the tax reform did not produce a tax differential between self-employed and wage-earners, both occupations still are levied under the same tax scheme.

[21] The reform increased the financing for the educational system in Chile. It consisted of three main pillars: 1) an increase in the provision of preschool education to reach 60 percent of the most economically vulnerable households; 2) an increase in the government subsidies to primary and secondary education in 21 percent, and 3) an increase in the aid and scholarships to students to attend tertiary education.

Although falling in the marginal tax rate is small in percentage point, normalizing it allows appreciates the significant effect that the policy has in some brackets. The tax falls go from 20 percent for income brackets two and third to 5 percent for top brackets. This means a progressive policy since lower-income workers had a higher fall in the marginal tax rate. These facts produce two potential effects on agents' decisions. First, it would imply a falling in evasion due to diminishing the marginal gains from evading. Second, since evasion is less attractive, incentives to participate in the wage-earner sector increases. Considering that most Chilean workers belong to the first income bracket, these changes only affected less than half of the workforce.[22]

This policy aims to increase tax collection to finance education, but the marginal tax rates in the SCT were reduced between the second to seventh income brackets, and the average tax was reduced in the eighth bracket. Two elements explain this fact. First, since tax collection from the SCT is half of FCT, a reduction in the marginal tax rates in the SCT does not produce a more prominent distortion in tax collection.[23] Second, the policy includes raises in the FCT from 20 percent to 25 percent in 4 years and a fall in tax evasion in 0.5 GDP points. These policies were the only tax incentives that affected wage-earners and self-employed during the period studied (2006/7 – 2016/7).

The policy attracted high interest from different policymakers and policy parties, however, it did not produce that taxpayers anticipated it. This can be seen in Google search, where any topic related to this policy was not among the ten most searching topics, the same happens for 2014.[24] This evidences that the majority of taxpayers did not anticipate this reform.

1.3.3 Workforce Evolution across the Period

The Chilean workforce is composed mainly of wage-earners, who represent three-quarters of the whole workers. The other relevant groups are employers, self-employed and non-remunerated family members. The self-employment sector is the second biggest group, representing 22% of the workforce in 2016/7. The evolution of both groups is relatively stable across years (See Table 1.13),

[22]See Figure 1.2 to a graphical exposure of this fact in the sample.

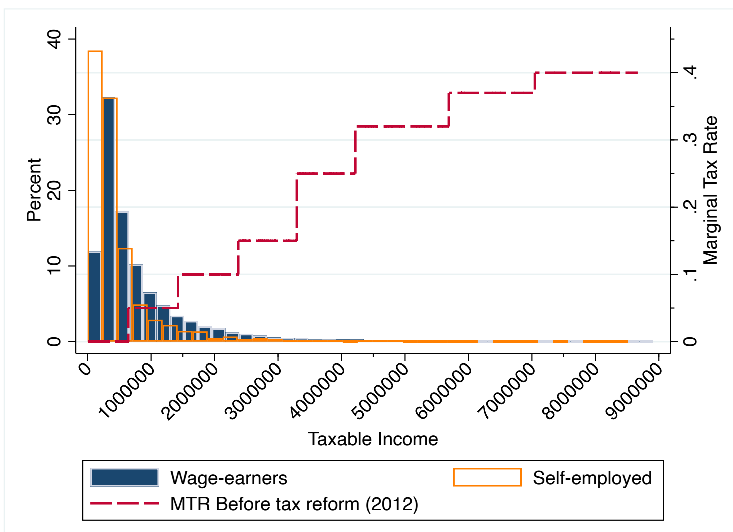
[23]Tax collection from the SCT represents 20.3 percent of the income tax collection or 7.9 percent of the whole tax collection in 2020. The principal source of tax collection is the VAT (49.32 percent) and for the income tax collection is the FCT (56.25 percent). This result shows that SCT produces a small revenue but is the third revenue source behind VAT and FCT.

[24]Figure 1.7 shows the searching interest across 2013 and 2014 for items Educational Reform and Tax Reform compared with the tenth most searching item in the corresponding year.

but the conformation across deciles or income brackets do not follow a clear pattern.

Focusing on the household head employment type, Table 1.13 displays the distribution of self-employed and wage-earners workers across the income brackets. Two important facts emerge from this table. First, the self-employed and wage-earners distribution is homogeneous across the period. Independently of the occupation type, most workers are excluded from paying taxes given their taxable income (i.e., they belong to the first income bracket). However, as Figure 1.2 shows, a higher share of self-employed workers falls in income bracket one compared to wage-earners. Second, the percentage of self-employed increased from 2006/7 to 2012/3 but decreased in the last year, being the income tax policy change of 2013 a potential explanation for the trend change. Also, the self-employment rate in the second bracket increases across years, this is the first bracket with a positive marginal tax rate. The 2013 reform might partially explain this fact. Additionally, Figure 1.2 shows that only a small group of workers face a tax rate bigger than 10 percent (from the third bracket onward).

Figure 1.2: Occupation Densities and SCT



Note: Left y-axis shows the percentage of agents in the pre-reform period, and the right y-axis shows the marginal tax rate. The estimation is made without sample weights. Source: Own elaboration based on the EPF and Chilean IRS data.

Concerning the evolution across income deciles, Table 1.14 displays the percentage of self-employed by this income desegregation. The general conclusion is that a general pattern does not exist, like desegregation by income brackets. In this sense, we do not see a monotonic behavior across deciles and years. For instance, the sixth decile shows an inverted U-shape, but the second decile is the opposite. Additionally, in any survey release, we cannot see a monotonic decrease in self-employed participation across deciles. For example, in 2006/7, decile sixth shows a more significant rate than fifth and seventh; and in release 2016/7, the same happens for deciles eight, seventh and ninth.

1.4 Relevant Definitions and Database

The data source for this study is the survey *Encuesta de Presupuestos Familiares* (EPF) microdata from Chile, collected by the *Instituto Nacional de Estadísticas* (INE). This cross-sectional data is a socio-economic survey applied to households that includes data about its members' main job employment characteristics and expenses and incomes for a given period. For consistency in expenditures and income measures, we use the last three releases of the survey (i.e., 2006/7, 2011/2, and 2016/7).[25] The survey covers Gran Santiago and the regional capital cities, with some metropolitan areas (60 communes).

The survey's frequency and collection periods allow us to estimate the relationship between tax evasion and employment choice for two main reasons. First, the Educational Reform implementation was between the collection of the EPFs 2011/2 and 2016/7. Second, although short, the spanned period allows us to control trends in workers' employment decisions. Considering these characteristics, we can assess the effect of policy changes in marginal tax rates given by this exogenous variation.

We use the *Encuesta de Presupuestos Familiares* because it provides the two relevant outcomes in this study: agent's occupation and evasion. This survey captures the household income and expenses, asking about the agent's occupation and income and its expenditures in specific periods. Since our primary method to capture evasion is comparing food expenditure and income through an Engel curve, we need an accurate measure of both variables to capture the evasion measure (explained in Section 1.5).

The critical variables that we use across this study are expenditures, incomes, and agent's

[25]While the first release of this survey is 1956/7, there are some differences between the oldest releases about geographical coverage, the methodology for measuring expenditures and income, and other relevant characteristics.

occupations. From the agent's self-declaration, we determine the occupation type of each one: self-employed, wage-earner, entrepreneur, domestic worker, and non-remunerated family member. Thus, a self-employed or wage-earner worker is an agent whom self-declared participating in this occupation. In our empirical analysis, we consider only these occupations (self-employed and wage-earners). All variables are self-declared by agents, however, expenditures variables need to present evidence, for example, a bill, to make a consistent record about household spending.

Regarding expenditure variables, the survey measures acquired consumption as the final consumption expenditure.[26] The data collection time frame consists of 15 days in which all household members aged 15 years and older register their expenditures, including recurrent expenditures (water, electricity, contributions, rental, and others) based on the last paid consumption. We consider two measures of expenditures in the analysis: total family expenditures and food expenditures. The first refers to total household outlays, including spending on non-durables, durables, education, health, transportation, recreation activities, and other expenses. Food expenditures are only expenses on food and non-alcoholic beverages. We use food expenditures as the primary expenditure variable since patterns of expenditures could be different for self-employed and wage-earners, and restricting it to food expenditures would mitigate this concern.

The primary income measure used is after-tax total family income, including income as wage-earner and self-employed, property income, and transfers (pension and financial income). To test the robustness of our estimations to this measure, we use the household's total taxable income from wage-earner and self-employment work.

Concerning the tax measures, we construct a measure of tax liability based on agent's taxable income. We construct a measure of taxable income by reverse tax engineering from the after-tax income declaration (for the year 2007) or using the declared labor income (for the remaining years), obtaining a before-tax income measure.[27] We recover before-tax income using the after-tax income by imputing tax liability obtained using the yearly corresponding tax scheme. Then, based on the taxable income measure, we assign each taxpayer to the corresponding income bracket in each year and compute tax variables for each of them.

We restrict the sample to households with a head between the ages of 20 to 59 years (inclusive)

[26]Final consumption expenditure corresponds to goods and services consumption acquired and used by the household. We adjusted the variables by CPI (Base=2017).

[27]Only in EPF VI, year 2007, declared labor income is net of taxes.

who report being self-employed or wage-earners in the main occupation. Domestic workers are excluded as mainly being employed informally and, also, entrepreneurs are excluded since their might face capital taxation. Also, we drop households with no reported income or expenditures or with null head's income. Hence, our sample is based on agents who declare an occupation (self-employed or wage-earners) and a positive income. In this sense, we focus on agents that belong to the labor force and make an occupational decision, taking away the informality issue.[28]

Table 1.2 shows descriptive statistics of the sample population. Most of the workers aged 20 to 59 years are male. The percentage of the sample population that never attended school/has preschool education is very low. Most of the workers have (complete or incomplete) high school or higher education, increasing the average educational level across the period. The working population comprises mainly wage-earners (more than 75 percent) with a significant share of self-employed workers (21 percent on average). Regarding households' characteristics, they have four members, on average, and the percentage of single-parent families has been increasing across the period from 26 percent in 2006/7 to 31 percent in 2016/7. A high share of total household reported income is destined for food consumption, representing approximately 18.4 percent, on average, of total expenditures.

[28]We refer to informality as an agent who declares positive income but does not declare an occupation. We assume that agents who avoid either labor or payroll taxes but declare an occupation and positive income are formals that evade taxes.

Table 1.2: Summary Statistics about Sample Population of occupied workers in age-group 20-59 years

VARIABLES	2006/7	2011/2	2016/7
Male	0.60	0.56	0.56
	(0.49)	(0.50)	(0.50)
Age-Group			
20-29 years	0.25	0.25	0.25
	(0.43)	(0.43)	(0.43)
30-39 years	0.28	0.25	0.27
	(0.45)	(0.44)	(0.44)
40-49 years	0.28	0.27	0.25
	(0.45)	(0.44)	(0.43)
50-59 years	0.19	0.23	0.24
	(0.39)	(0.42)	(0.42)
Education Level			
Never Attended/ Preschool	0.00	0.00	0.00
	(0.06)	(0.07)	(0.05)
Primary School	0.14	0.14	0.10
	(0.35)	(0.35)	(0.31)
High School	0.50	0.47	0.43
	(0.50)	(0.50)	(0.50)
Higher Education	0.35	0.38	0.46
	(0.48)	(0.49)	(0.50)
Occupation Type			
Employer	0.02	0.02	0.02
	(0.15)	(0.15)	(0.14)
Self-employed	0.21	0.20	0.22
	(0.41)	(0.40)	(0.41)
Wage-earner	0.76	0.77	0.76
	(0.43)	(0.42)	(0.43)
Non-remunerated family member	0.01	0.00	0.00
	(0.08)	(0.06)	(0.05)
Household Size	3.92	3.88	3.62
	(1.69)	(1.68)	(1.63)
Single-parent family	0.26	0.28	0.31
	(0.44)	(0.45)	(0.46)
Total Household Income	990,479	1,173,078	1,323,117
	(1,120,389)	(1,440,891)	(1,460,529)
Total Household expenditure	992,721	1,061,964	1,226,869
	(1,076,354)	(1,082,477)	(1,124,849)
Household Food Expenditure	183,131	193,532	226,910
	(124,384)	(144,336)	(164,214)

Note: Expenditure variables measure the effective expenditures in consumption of goods and services acquired and used by the households. Income and expenditure variables are measured in Current Chilean Pesos. Standard Deviations in parentheses. Source: Own elaboration based on the EPF data (2006-2017).

1.5 Measuring Tax Evasion

1.5.1 Empirical Strategy

Measuring tax evasion is not straightforward.[29] Following Pissarides and Weber (1989), most of the empirical literature on tax evasion measurement focuses on comparing self-employed and wage-earner to measure tax evasion. We follow this methodology by comparing Engel curves for self-employed and wage-earners in Chile. Specifically, we follow the paper of Hurst, Li and Pugsley (2014), where the authors quantify the extent to which the self-employed under-report their income in the U.S. using household surveys through Engel curves. We extend this paper by incorporating heterogeneity in the evasion measure.

Engel curves describe the relationship between workers' income and expenditure. By comparing differences in the Engel curves of wage-earners and self-employed, we infer the actual income of the last group and, thus, the self-employed reporting gap based on their reported expenditures. The measurement relies on four main assumptions:

1. All income groups report expenditure on food correctly,
2. Employees report income correctly,
3. Self-employed workers under-report their income,
4. Conditioning on observable characteristics, wage-earners and self-employed workers have the same preferences on food consumption.

Although this methodology is not out of critiques (see, for instance, Engstrom and Hagen, 2017; Kukk and Staehr, 2017), this is a widely used method which assumptions seem to be verified for Chile's workforce.

First, total food expenditures density functions are similar between wage-earners and self-employed (see Figure 1.8 in Appendix), ratifying assumption one. Regarding assumptions two and three, compared to self-employed workers, wage-earners cannot evade taxes since a third party reports their salary.[30] The reported income comparison reflects that household and household

[29] See Schneider and Enste (2002); Alm (2012); and Slemrod and Weber (2012) for detailed discussion about the many approaches for measuring tax evasion.

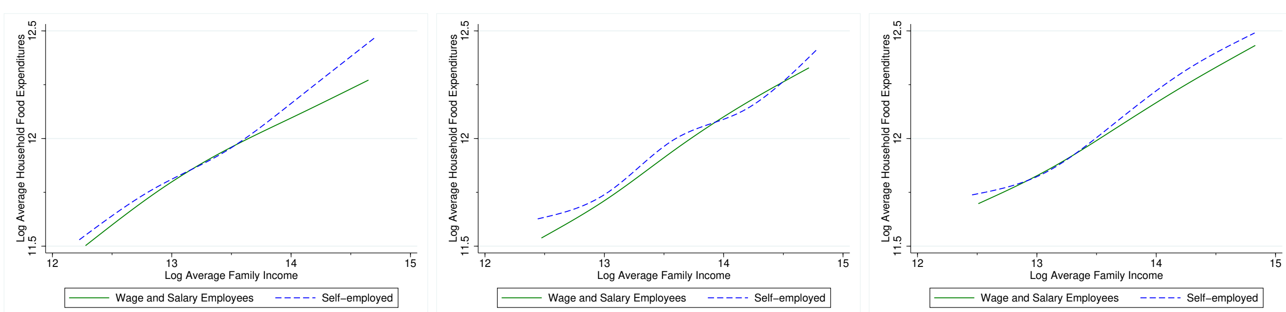
[30] This assumption is confirmed in the literature. See Slemrod (2019) for a general discussion, Kleven, Kreiner and Saez (2016) for a theoretical explanation about the collusion difficulties between agents and firms, and Kleven et al. (2011) for an empirical demonstration that evasion is low in the wage-earner sector.

head personal income reports differ by occupation, reflecting misreporting, giving support to this assumption.

We consider only two kinds of household: self-employed and wage-earners and salary (hereafter referred to as wage-earners, indistinctly). Self-employed households are households where the head reports being self-employed in primary employment. As robustness, we use two alternative definitions. The first definition is according to which a household is self-employed if the head or the couple is self-employed; we denominate this as mix definition. This definition accounts for the possibility that the joint household income could finance expenditures within the household, and tax evasion decisions could take place even when only one member of the couple is self-employed (Hashimzade, Myles and Yousefi, 2020). The second definition is as in Hurst, Li and Pugsley (2014), restricting the sample to households with a male head and without a self-employed couple; we denominate this as HLP definition.

As a first approximation of the Engel curves, Figure 1.3 shows the non-parametric estimates of the total food expenditure Engel curves, estimated separately for the wage-earners (green line) and self-employed (dashed blue line) households and each data wave.[31] While both curves have a positive slope, wage-earners register a linear relationship between total household income and food expenditure. This pattern is more variable across the income distribution of self-employed, being the differences more evident in the tails. Moreover, the curves display higher food expenditures for a given income level, on average, for self-employed relative to wage-earner households.

Figure 1.3: Non-parametric Estimates of Total Food Expenditure Engel Curves



(a) EPF VI: 2006/7 **(b)** EPF VII: 2011/2 **(c)** EPF VIII: 2016/7

Note: Non-parametric estimates of the natural logarithm of total food expenditure and household income were obtained after controlling for a set of covariates. Source: Own elaboration based on the EPF data.

[31] Figure 1.9 in the Appendix shows the Engel Curves for each survey releases with mix household definition.

1.5.2 Regression Model

Hurst, Li and Pugsley (2014) assume that all self-employed households have the same misreporting rate of their current income. However, the misreporting rate may depend on different household characteristics and, thus, be different across household types.[32] We extend their approach by including in the regression model interaction terms between the self-employed household indicator and a set of household head characteristics (education level, age, and gender), introducing heterogeneity in the measure of tax evasion at the household level.

Hence, combining a first-order approximation of the Engel curves for wage-earners and self-employed households, we estimate the following regression model in the pooled cross-section considering the years $t = \{2006/7, 2011/2, 2016/7\}$:

$$\ln c_{ikt} = \alpha + \beta \ln y_{ikt} + \gamma_{0i} D_{it} + \gamma'_{1i} D_{it} \times Z_{it} + \Gamma' X_{ikt} + \delta_t + \varepsilon_{ikt} \tag{1.3}$$

where k is the type of occupation of the household i, whether it is wage-earner ($k = W$) or self-employed ($k = S$). $\ln c_{ikt}$ is the natural logarithm of the final food consumption expenditures of household i of type k in year t. $\ln y_{ikt}$ is the natural logarithm of the current household income, being β the income elasticity that is common across occupations by assumption. D_{it} is a dummy variable that indicates whether the household i is self-employed (wage-earner, otherwise) in year t. This variable is also interacted by indicator variables of household head characteristics Z_{it}, such that γ'_{1i} is a vector of coefficients (one for each interaction term) that captures the difference in income between self-employed households i with specific characteristics and the average wage-earner household with equal consumption, ceteris paribus. Vector Z_{it} includes age-group indicators (7 groups), education level indicators (secondary and superior), and male indicator.

The vector X_{ikt} includes a set of demographic controls, allowing a higher precision in the identification of the transitory income. This vector includes an indicator of whether the household head is male; whether the household head has a couple; a series of indicators of the household head's couple employment status; household head education-group dummies; a series of household size dummies; an indicator of whether there is at least one dependent child (i.e., under 15 years old) in the household, and dwelling tenancy status indicator as a proxy of household's wealth. ε_{ik}

[32] The empirical evidence explains the heterogeneity in evasion behavior (Alstadsæter, Johannesen and Zucman, 2018, among others).

represents the transitory income's unpredictable components that are unobserved determinants of household consumption, and δ_t is a year fixed effect.

Under the assumption that households can borrow and lend in asset markets, a more precise specification of the Engel curve would require using the permanent income instead of a combined measure with a transitory component as the determinant of household consumption. However, this would require counting with panel data to disaggregate the income in its permanent and transitory components. Provided this data limitation, the Engel curve assumes that current income is a good proxy of permanent income, as derived in Hurst, Li and Pugsley (2014). According to the authors, it is reasonable to assume that all households accurately report current income, y_{ikt}, which is a measure of permanent income plus a transitory component. Unlike Hurst, Li and Pugsley (2014), we assume in equation (1.3) that self-employed households can misreport their income differently. Therefore, the linear combination of γ_i coefficients will be equal to $-\beta(\ln y_{iSt} - \ln y_{iWt})$, where the difference in permanent incomes between occupations gives a measure of the self-employed household reported income, $\ln \kappa_{iS}$. The critical assumption to estimate κ is that the Engel curve parameters across wage-earner and self-employed households are equal.

This method allows the possibility of having negative misreporting rates for some households, i.e., $1 - \kappa_{iSt} < 0$.[33] We interpret these values as self-employed households honestly reporting their income and consuming less than a wage-earner worker with the same characteristics. Therefore, we replace negative misreporting rates with a zero value, evidencing that these workers do not evade, and identifying the extensive (i.e., the decision of evading taxes) and intensive (i.e., the amount of taxes evaded) margins in tax evasion behavior.

A potential endogeneity concern arises from estimating the Engel curve in equation (1.3) since wealthier households might have higher expenditure and, at the same time, more spending may require a higher income. Another source of endogeneity may be associated with a taste for consumption that may lead to working extended hours or self-selecting into jobs with higher marginal returns. Moreover, household income self-reports may be subject to Classical Measurement Error. To mitigate these issues for the identification of tax evasion, we follow Hurst, Li and Pugsley (2014) using an instrumental variable approach to estimate the Engel curves using the household head's industry sector indicators as instruments for total household income. In this approach, the exclu-

[33] Negative misreporting rate ranges from 14 to 16 percent of the sample in the pool specification.

sion restriction means that evader agents have a higher income than the reported allowing them to consume more than wage-earners with the same reported income. To capture the actual agent's income well, it is necessary to instrument income to clean the bias that evasion produces.

1.5.3 Estimated Tax Evasion Rates

In Table 1.3 we present the estimation of the tax evasion rate (i.e., the percentage of non-declared income) and extensive margin of evasion (i.e., the percentage of self-employed who evade taxes) among self-employed households.[34] Since the percentages are conditional on being a self-employed household, we interpret these results as an estimation of evasion behavior. These estimations assume that households with negative predicted misreported income are, in fact, not evaders and misreported income for these households is null.[35] In Table 1.11 in Appendix 1.9.1 we report the second stage coefficient estimates for the income variable and the mean self-employed linear combination of the interaction terms and the misreporting measure using the Engel curve in equation (1.3) following an IV approach. These results show the robustness of our estimation to the alternative definitions of self-employed households.

[34] In Table 1.15, we also provide the the intensive margin (i.e., the average evasion rate among evaders).

[35] Since there are no significant differences between predicted residuals for evader self-employed households and the whole sample, we do not expect any bias from taking this assumption.

Table 1.3: Estimation of Tax Evasion: Evasion Rate and Percentage of Evaders

	EPF Survey			
	2007	2013	2017	Average
Panel A: Main Specification				
Evasion Rate	11.44%	9.98%	11.26%	10.89%
% of Evaders	83.16%	84.84%	86.13%	84.71%
Panel B: Household Head Mix				
Evasion Rate	6.89%	6.36%	7.43%	6.89%
% of Evaders	45.09%	46.06%	49.88%	47.01%
Panel C: Household Head Occupation following HLP				
Evasion Rate	11.90%	9.93%	13.31%	11.71%
% of Evaders	77.43%	76.33%	81.56%	78.44%

Note: Evasion Rate refers to the mean misreporting rates over self-employed households. % of Evaders indicates the percentage of self-employed households with a positive misreporting rate. The means of the estimated coefficients were calculated after replacing negative misreporting rates with zero. Source: Own elaboration based on the EPF data.

The main results of our estimation are in Panel A. We find that the average evasion in the self-employment sector is 10.89 percent, 12.87 percent and 84.71 percent the average evasion for evaders and % of Evaders, respectively.[36] The U-shape at the evasion rate in Panel A, with a substantial raise after the tax reduction in 2013, would indicate a changing behavior of evaders at the margin to formalize their work because they evade less at the margin. We explore these explanations formally using the empirical model in Section 1.6.[37]

Comparing Panel A with Panel B, a stricter self-employed household definition in Panel A produces higher misreporting rates at two margins of adjustment: Evasion Rate and % of Evaders. In all Panels, the evasion rate displays a U-shape. The difference of our primary specification with Panel B would reflect the differential behavior of households in which the couple is self-employed

[36]The average evasion for evader is displayed in Table 1.15 in the Appendix.
[37]In Table 1.12 in the Appendix we show the results in the cross-section estimation.

and the household head a wage-earner. The comparison between panels A and C reflects the effect of focusing only on male heads without a self-employed couple. We find similar results.

In addition, we compare the evasion rate for evaders, presented in Table 1.15, with misreporting rate obtained from using HLP's method. The results show a downward bias in the estimate of the misreporting rate using the HLP's method in column (3) of Table 1.11, which could be explained by the increased explanatory power given by allowing heterogeneity in the misreporting.

The *Evasion Rate* is the measure that we use in the empirical estimation since it represents the self-employment sector and not only evaders. This measure allows us to compare better the effect of evasion on self-employment decisions by incorporating the non-evasion decisions that might affect them. Hence, to avoid some bias and to provide a complete meaning of the evasion incentives, we incorporate the zero-evasion decision and use the *Evasion Rate* measure.

1.6 Tax Evasion Effect on Self-employment Decisions

Our theoretical model presented in Section 1.2 evidence that tax evasion effect on self-employment decisions comes from two channels: taxable income and evasion. While the first channel relates to taxable income in the self-employment sector, the second channel encompasses the direct effect of evasion on occupational decisions. This section estimates the second channel taking advantage of the Chilean tax system that eliminates the differential tax effect on the taxable income: wage-earner and self-employed are levied with the same tax scheme. Furthermore, we exploit a quasi-experimental tax rate variation due to the 2013 Chilean tax reform to address endogeneity concerns in the causal relation between self-employment and income tax evasion.

1.6.1 Identification Strategy

Difference-in-Difference (DID) Approach

As we show in Section 1.2, self-employment and evasion decisions are simultaneous. This is because agents anticipate how much income to hide to the tax authority in self-employment, producing that evasion behavior alters the comparison between occupations.[38] We use the Chilean setting and the 2013 tax reform to deal with this problem because of two main benefits. First,

[38] As more extensive evasion is, the larger the incentive to participate in occupations where it exists.

the reform affected both sectors equally, dropping out the differential taxation problem. Second, the reform changed the marginal tax rates differently across income brackets (See Table 1.1). Specifically, the first income bracket, which faces a marginal tax rate equal to zero, did not suffer a tax change, while the marginal tax rate fell for the rest of the income brackets.[39] This produced that agents in the first income bracket maintained their incentives to evade, while evasion incentives fell for self-employed that paid taxes because of the decrease in the marginal tax they faced. Thus, the quasi-experimental tax rate variation affected the agents' evasion incentives differently without directly changing their self-employment decisions. We use the different intensities in the treatment to identify the causal effect of evasion on self-employment decisions.

Given the explanation above, the policy reform setting easily translates to a difference-in-differences (DiD) research design. We follow this approach exploiting the differential treatment intensity in the tax reform. Our strategy follows the literature that begins with the seminal paper of Feldstein (1995) that exploits a cross-sectional variation generated by the 1986 tax reform in the US. In our case, we use the cross-sectional variation produced by the 2013 tax reform in a pooled cross-section.

Assigning Treatment Status

The policy reform created two groups, depending on income brackets, separating the sample between agents affected by tax falling (treated) and those who maintained the same marginal tax rate (controls). Hence, our comparison relies on the differential tax effect between agents who faced a positive marginal tax rate and those who did not, before and after the policy. This DiD comparison enables us to isolate the evasion channel and control for factors that simultaneously affect both groups, like economic or sectorial fluctuation or demand-side effects.

The treatment status depends on two facts in our setting: the agent's income and the income brackets. These elements might have some empirical issues. Related to agents' income, mean reversion and tax effects on taxable income might produce problems in the treatment stratus. Since we use a pooled cross-section sample, mean reversion is not a problem, and the tax effect on taxable income will be treated later because it can invalidate our empirical strategy. Also, being treated relies on the income bracket that an agent belongs to, producing that using income brackets

[39] As shown in Table 1.1, agents who faced a positive tax liability suffered an average fell of 5.8% in their marginal tax rates, going between 4% to 20% depending on the income bracket.

across the period to assign treatment status appears reasonable. However, either the number or the bracket's threshold might change. This is not a problem in the Chilean context since the brackets threshold depends on the UTM (a stable monetary unit), allowing us to follow each bracket across the period studied.

Therefore, a treated agent is an agent who belongs to an income bracket bigger than the first. In other words, an agent is treated if it belongs to the second income bracket up to the eighth in each cross-section. Using the bracket threshold in UTM, we assign the treatment status in each cross-section, obtaining a characterization of treated and controls across the period studied.

Estimating Equations

In a first step, we exploit the treatment intensity to identify the evasion behavior. To do that, we estimate the tax reform's effect on the evasion decisions using the following DiD specification.

$$Evasion_{ht} = \alpha_1 + \delta_{1t} + \phi_1 Policy_t T_h + \sigma_{11} T_h + \sigma_{12} Policy_t + X_{ht}\gamma_1 + \nu_{1ht} \tag{1.4}$$

$Evasion_{ht}$, is the share of tax evasion on total reported income of household head h in year t, being $t = \{2006/7, 2011/2, 2016/7\}$. T_h is a dummy indicating whether the household head h belongs to the treatment group; $Policy_t$ is a dummy indicating if the year corresponds with the new taxation regime (i.e., 2016/7). α is a constant, δ_t is a year fixed effect, X_{ht} is a vector of household and individual socio-economic characteristics that includes a male indicator, age-group dummies, education-group dummies, marital status, couple's occupation type, household size dummies, owner dwelling indicator, single-parent family indicator, a child under 15 years in the family indicator, 1-digit industry-codes dummies, and agent's gross income. We correct for inconsistent answers that can potentially bias our estimates by restricting the sample dropping the 10 percent lower- and upper-tails of the food expenditure distribution, since some households report low (high) food expenditures but high (low) head income.[40] To address some heteroscedasticity problems, we use robust standard errors.

Equation (1.4) measures the effect of tax policy on the tax evasion rate, given by the coefficient ϕ_1. This coefficient provides a measure of the evasion changes that are necessary to obtain the elas-

[40] These inconsistencies might be associated with evasion behaviors that cannot be captured with our data.

ticity of evasion to the marginal tax rate $\varepsilon_{e,t}$, one component of the evasion channel in Proposition 1. The relevance of identifying evasion channel is to assess the importance of evasion behaviors in tax changes' effect on self-employment decisions.

In a second step, we focus on obtaining the critical relation of interest: evasion incentives effects on self-employment decisions. The way to capture that relation is by estimating the evasion rate's effect on self-employment decisions using the DiD set up to solve the empirical problems. Particularly, we estimate the following regression

$$SE_{ht} = \alpha_2 + \delta_{2t} + \beta_1 Evasion_{ht} + \sigma_{12} T_h + \sigma_{22} Policy_t + X_{ht}\gamma_2 + u_{2ht} \tag{1.5}$$

where SE_{ht} is a dummy variable that indicates if household head h in year t is self-employed (or wage-earner, otherwise).

As we mentioned and explained in Section 1.2, agents anticipate their evasion decisions producing that self-employment and evasion decisions are simultaneous. Thus, by using the variation in the evasion rate given by the tax reform, we solve the endogeneity problem in the causal relation of evasion on self-employment decisions. Specifically, we instrument $Evasion_{ht}$ with the DiD interaction $Policy_t T_h$ to estimate β_1 that identifies the effect of tax evasion on self-employment decisions. By using this coefficient, we compute occupational decisions semi-elasticity to evasion, $\eta_{H,e}$ in Proposition 1, and joining this parameter with $\varepsilon_{e,t}$ we can estimate the evasion channel $\varepsilon_{e,t}\eta_{H,e}$.

The Instrumented Difference-in-Differences (IV-DID) approach (see Hudson, Hull and Liebersohn, 2017; De Chaisemartin and d'Haultfoeuille, 2018) solves the endogeneity problem under the assumption that the individual unobserved effects do not correlate with the assignment into the marginal tax rate policy change.[41] We trust the validity of this assumption since workers were assigned into income brackets based on their taxable income, predetermining their treatment assignment. Under this assumption, the policy change becomes a valid instrument of tax evasion.

Note that, intuitively, β_1 in the IV-DiD model relates the tax policy effects on evasion with the evasion effect on the self-employment decisions. We can interpret that coefficient as a Wald estimator, i.e., the ratio between the reduced form and the first stage (equation (1.4)), where the

[41] Other papers that follow a similar empirical approach are Duflo (2001) and Sigurdsson (2019).

reduced form is

$$SE_{ht} = \alpha_3 + \delta_{3t} + \psi_1 Policy_t T_h + \sigma_{13} T_h + \sigma_{23} Policy_t + X_{ht}\gamma_3 + u_{3ht} \tag{1.6}$$

Notice that ψ_1 captures the change in the probability of being self-employed due to tax reform. Hence, we can construct the occupational decisions semi-elasticity to tax changes $\eta_{H,t}$, the behavioral parameter that captures the whole effect of taxes on self-employment decisions defined in Proposition 1. Using that, we are able to assess the relevance of evasion channel in the whole effect of taxes on self-employment decision by $\frac{\varepsilon_{e,t}\eta_{H,e}}{\eta_{H,t}}$.

Validity of Identification Assumptions

For the valid identification of the effect of tax evasion on self-employment choice, the policy change instrument needs to satisfy Instrumental Variable (IV) and DiD assumptions.

Concerning DiD, parallel trend assumption is basic to ensure that treated and controls are well defined. We look for a similar behavior between groups in the pre-policy period, which indicates any change in groups' behavior is attributable to the policy, producing that controls are a useful counterfactual measure to capture the policy effects by comparing them with treated.[42] For the evasion rate, Figure 1.12 shows the evolution of controls (blue line) and treated (red line) in a non-parametric (Panel (a)) and parametric (Panel (b)) estimations. In both Panels, both groups moved similarly before the reform, evidencing a decrease in the evasion rate. Thus, the figure shows compelling evidence of non-differential pre-trends in the evasion rate. On the other hand, Figure 1.13 shows the evolution in self-employment rate across years between controls (blue line) and treated (red line) in a non-parametric (Panel (a)) and parametric (Panel (b)) estimations. Similar to the evasion rate case, the self-employment rate follows the same trend in both groups before tax reform. This indicates a non-differential pre-trend in the self-employed rate.

Another important assumption is that the treatment has no compositional effects, i.e., control and treatment groups' differences should be unaffected by the policy. To verify the compositional effect, we use a DiD approach similar to equation (1.4), using each control variable (those in vector X) as a dependent variable and controlling only for year fix effects. The relevant coefficient

[42] Since we use three surveys, we only have three points to verify the parallel trends assumption. Specifically, we only have two years, or releases, before the policy.

is $Policy \times Treatment$ because it captures any difference between controls and treated in the dependent variable due to tax reform. Figure 1.14 shows these coefficients. Almost all control variables did not change because of the tax reform, and only three over seventeen variables are statistically different from zero. However, the difference is small enough to argue that the effects are not economically significant. Thus, the reform did not modify the groups' composition.

Concerning the IV identification requirements, the instrument's relevance is tested by equation (1.4). This is verified in Table 1.4 that shows an F-statistic larger than 10 in each specification and the relevant coefficient is always significant at 1%.

While the tax reform generated quasi-exogenous variation on the occupational decisions, it could have affected the taxpayers' reported income that determines the exposure to treatment for agents. Hence, the exclusion restrictions deserve more attention in our setting. In our regression model, the exclusion restriction relies on the potential self-selection of workers into treatment or control groups based on their tax misreporting behavior or adjustments in the intensive labor margin. To address this potential issue, we follow the traditional procedure in the elasticity of taxable income literature (see, for instance Gruber and Saez, 2002; Kleven and Schultz, 2014; Saez, Slemrod and Giertz, 2012; Weber, 2014) using the pre-treatment data to obtain a predicted income variable without the behavioral responses. Concretely, we estimate the following regression model

$$x_{ht} = \alpha_4 + \lambda_t + X_{ht}\varphi_1 + Industry_{ht}\varphi_2 + \nu_{ht}$$

where x_{ht} is the reported income for the household head h in year t and $Industry_{ht}$ is a vector that contains a set of the household head's industry-sector of activity dummies. We regress this equation for years $t = \{2006/7, 2011/2\}$ and obtain a declared income prediction for 2017. We use this prediction to assign agents in 2017 to the corresponding income bracket, getting a predicted treatment variable.

Finally, we provide a sensitivity analysis to our IV-DiD estimation, following an IV approach in which we use the net-of-marginal-tax rate as an instrument for the tax evasion rate. With this approach, we can obtain a semi-elasticity that indicates the sensitivity of evasion to changes in the net-of-marginal-tax rate. This analysis aims to provide a sensitivity analysis of the evasion effect on self-employment decisions because we exploit the net-of-marginal-tax rate instead of the tax reform.

Also, as a consequence, we can complement our analysis over the relevance of the marginal tax rate in evasion behavior, providing the semi-elasticity to the net-of-marginal-tax rate. Specifically, the empirical model is the following

$$Evasion_{ht} = \alpha_5 + \delta_{4t} + \phi_2 \ln(1 - MTR) + X_{ht}\gamma_4 + \nu_{4ht} \tag{1.7}$$

$$SE_{ht} = \alpha_6 + \delta_{5t} + \beta_2 Evasion_{ht} + X_{ht}\gamma_5 + u_{5ht} \tag{1.8}$$

Similarly to our main specification, we use the predicted income to obtain a predicted net-of-marginal-tax rate to avoid self-selection problems.

1.6.2 Estimation Results

Tax Evasion Response to Policy Tax Change

This section presents the effect of the marginal tax rate (MTR) change due to tax reform on the evasion behavior represented by the evasion rate. Our primary unit is the head of each household, producing that our sample is composed of heads of households that are wage-earners and self-employed.

Table 1.4 reports the baseline model (equation (1.4)) and the sensitivity analysis (equation (1.7)). The pooled OLS estimation results of equation (1.4) are in columns (1) and (2), considering the treatment variable (POLS treatment) and its prediction (POLS PT), respectively. Column (3) shows the net-of-marginal-tax rate estimated effect (POLS MTR). Then, in column (4), we show the estimation using the predicted net-of-marginal-tax rate (POLS PMTR). Results in column (2) correspond to our preferred specification. In the final row, we provide the computation of the corresponding elasticity. To obtain this parameter, we interpret the DiD estimation as the evasion rate change, so we divide this coefficient by the average evasion rate in the pre-reform period to obtain the percentage change in evasion. We obtain the same variation for taxes obtaining the difference before and after the reform for the MTR between treated and controls, and dividing it by the MTR in the pre-reform period. Finally, we obtain the elasticity of evasion to MTR by dividing the percentage change in evasion by the percentage change in the MTR. This behavioral parameter is part of the evasion channel described in Proposition 1, so it is necessary to measure the relevance of evasion in the whole effect of taxes on self-employment decisions.

Table 1.4: Effect of Tax Reform on Tax evasion

	(1)	(2)	(3)	(4)
Dep. Vble: Evasion Rate	POLS treatment	POLS PT	POLS MTR	POLS PMTR
Policy	0.0073**	0.0082**		
	(0.003)	(0.004)		
Treatment	0.0344***	0.0276***		
	(0.003)	(0.003)		
Policy × Treatment[a]	-0.0104***	-0.0206***		
	(0.004)	(0.004)		
Ln Mg Net-of-Tax Rate			-0.2620***	-0.2163***
			(0.021)	(0.021)
Observations	15,063	15,063	15,063	15,063
R-squared	0.212	0.208	0.215	0.208
F-statistic	33.71	33.86	35.03	33.98
Covariates	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Delta Tax[b]	-0.016	-0.016		
Mean Evasion[c]	0.026	0.026		
Mean Tax[d]	0.028	0.028		
Elasticity ($^a/_c/^b/_d$)	0.701	1.395		

*Notes: Columns (1) and (2) report POLS estimates of the tax policy change and tax policy change based on predicted treatment, and columns (3) and (4) of the effects of the net-of-marginal-tax rate and its prediction, respectively, using the household sampling weights. Covariates include: male indicator, age-group dummies, education-group dummies, marital status, couple's occupation type, household size dummies, owner dwelling indicator, single-parent family indicator, a child under 15 years in the family indicator, 1-digit industry-codes dummies, and agent's gross income (in Ln). Mean Evasion and Mean Tax are average weights for the whole sample. Robust standard errors in parentheses. Significance level: *** p<0.01, ** p<0.05, * p<0.1.*

The effects of the tax policy on the evasion rate are shown in columns (1) and (2). The tax reform reduced the tax evasion rate by -0.0104 percentage points. Addressing potential endogeneity concerns using the predicted treatment, the magnitude of the effect becomes considerably larger, obtaining a -0.0206 percentage points reduction in the evasion rate. These results indicate that behavioral components might downward bias the policy effect. As we mentioned, this coefficient represents a change in the evasion rate, and we use this to obtain the elasticity of evasion to MTR equal to 1.39 (last row in column (2)). Evasion overreacts to changes in MTR since an increase of 1 percent in the MTR increases the evasion rate by 1.4 percent. Therefore, this behavioral parameter highlights the relevance of the MTR as an economic incentive to tax evasion. Finally, this result is robust to instrumenting gross income with its prediction or using the predicted income (see Table 1.16 and 1.17).

The impact of MTR on evasion is significant, and as we mentioned, it is evidence of the relevance of tax incentives in evasion behavior. Kleven et al. (2011) also find a positive elasticity, but with a smaller magnitude, 0.16 before audits and 0.085 after audits. This paper obtains the evasion elasticity using a bunching approach instead of an exact measure of evasion like us. This difference produces that our approximation captures the tax effect in the middle of income brackets, giving a complete result of the evasion behavior. However, the difference also might be explained by the lower tax administration capacity in our environment.

Results in column (3) show that an increase in the net-of-marginal-tax rate of one percent point significantly decreases the tax evasion rate by -0.2620 percentage points. Using the predicted net-of-marginal-tax rate in column (4), the coefficient falls to -0.2163. The magnitude of these results reinforces the relevance of taxes in evasion behaviors. These results are robust to instrumenting gross income with its prediction or using the predicted income (see Table 1.16 and 1.17).

The analysis over the elasticity of evasion to the MTR is robust to alternatives measures based on household head's and the couple's occupation type (mix household definition), or a more homogeneous sample following HLP (see tables 1.18 and 1.19 in Appendix). However, the elasticities magnitude sheds light on differences depending on the self-employed definition. Specifically, comparing the estimation results in the main specification with the mix and HLP definitions evidence that households with a self-employed couple and with woman heads have an evasion less sensitive to tax changes. This result reinforces the idea that tax evasion is also a household strategic decision.

Tax Evasion effect on Self-employment Decisions

This section shows the causal effect of tax evasion on the decision to be self-employed. Like before, our primary unit is the head of each household, producing that our sample is composed of heads of households that are wage-earners and self-employed.

Table 1.5 reports results of equation (1.5) and the sensitivity analysis of equation (1.8). Column

(1) displays the pooled OLS estimate (POLS). The IV-DiD estimates of tax evasion as a determinant of self-employment are exposed in column (2), where we use the tax policy change as an instrument for workers' tax evasion (IV-DiD), and in column (3), where we use the predicted treatment to exploit the tax policy change (IV-DiD PT). The remaining columns show the IV estimates related to the sensitivity analysis, so in columns (4) we use the net-of-marginal-tax rate as an instrument (IV-DID MTR) and column (5) the predicted net-of-marginal-tax rate (IV-DiD PMTR). Our preferred results are displayed in column (3). All the estimations are robust to instrumenting gross income with its prediction or using the predicted income (see Table 1.20 and 1.21).

Table 1.5: Effect of Tax Evasion on Self-employment

	(1)	(2)	(3)	(4)	(5)
Dep. Vble: Self-employment	POLS	IV-DiD	IV-DiD PT	IV-DiD MTR	IV-DiD PMTR
Evasion Rate^a	2.9797***	6.5101***	6.1695***	7.0437***	7.0003***
	(0.059)	(2.045)	(1.108)	(0.532)	(0.630)
Policy		0.0034	0.0031		
		(0.012)	(0.013)		
Treatment		0.0030	0.0157		
		(0.064)	(0.027)		
Observations	15,063	15,063	15,063	15,063	15,063
R-squared	0.493	0.114	0.183	-0.010	0.001
F-statistic	248.4	67.97	71.84	64.50	64.73
F-stat. Weak Ident.		7.65	23.63	162.46	101.49
Covariates	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓
Mean Evasion^b	0.026	0.026	0.026		
Semi-elasticity ($a \times b$)	0.077	0.169	0.160		

*Notes: Column (1) reports POLS estimates, in columns (2) and (3) evasion is instrumented using the tax policy change and its prediction, and in columns (4) and (5) the net-of-marginal-tax rate, using the household sampling weights. Covariates include: male indicator, age-group dummies, education-group dummies, marital status, couple's occupation type, household size dummies, owner dwelling indicator, single-parent family indicator, a child under 15 years in the family indicator, 1-digit industry-codes dummies, and agent's gross income (in ln). Mean Evasion is the average weights in the whole sample. Robust standard errors in parentheses. Significance level: *** p<0.01, ** p<0.05, * p<0.1.*

The effect of the evasion rate on self-employment decision using the tax policy change as an instrument is shown in columns (2) and (3). When we use the predicted treatment to exploit the tax policy change as an instrument for tax evasion, the estimated marginal effect is not significantly different (see columns (2) and (3)). The preferred estimation (column (3)) indicates that an increase in 1 percentage point in the evasion rate increases the probability of being self-employed by 6.17 percentage points. Comparing this result with column (1), we observe a negative bias in the estimated coefficient by OLS. This bias could be explained, for instance, by workers' unobserved ability, according to which high ability workers tend to evade less and be wage-earners.

The estimation result in column (5) indicates that an increase of 1 percent in tax evasion produces a raise in the self-employment rate, on average, of 7 percentage points. The comparison of columns (4) and (5) evidence that the evasion effect is robust to using the predicted measure of net-of-marginal-tax rate. Also, the comparison between column (3) and column (5) shows the robustness of our main results.

The magnitude of the evasion effect is significant, indicating that agents tend to overreact to changes in tax evasion incentives. Since the change in the evasion rate reflects both the intensive (how much income shelters each household) and extensive (whether a household evades or not) margins, the estimated coefficient will capture both effects. Recall that the institutional background allows us to isolate the pure evasion channel from the differential taxation, producing that the significant impact represents only the incentives through evasion. Although it is difficult to compare the magnitude of this effect with previous findings in the literature such as Wen and Gordon (2014); Bosch and de Boer (2019) and Bárány (2019) since these studies do not isolate evasion channel, these results are consistent with our findings.

To understand more the magnitude of the effect of tax evasion on self-employment decision, we compute the semi-elasticity of the occupational decision to evasion, $\eta_{H,e}$ in Proposition 1. This estimation shows that an increase of 1 percent in evasion rate increases the probability of being self-employed by 0.16 (last row in column (3)) percentage points. Hence, although the coefficient is significant, the semi-elasticity shows a moderate effect of tax evasion.

The relevance of evasion incentive in self-employment decision is robust to the alternative evasion measure and sample composition restrictions (see Tables 1.22 and 1.23 in Appendix). However, using the self-employed household mix definition, all coefficients are significantly higher than the main specification. This fact might arise from a different evasion behavior in self-employed households driven by households with self-employed couples. On the other hand, the estimated elasticity is slightly higher in the HLP definition, showing that households with woman heads respond fewer than others to evasion incentives. This result is interesting considering those households' (potential) vulnerability, giving evidence of a (possible) heterogeneity response depending on it.

Then, as the tax policy change corresponds to a fall in marginal tax rates, this policy generated a significant fall in tax evasion combined with a reduction in self-employment. Results in Table 1.4 and 1.5 highlight the relevance of evasion incentives as a determinant for self-employment decisions.

Quantifying the Magnitude of Evasion Channel

The final step to measure the relevance of evasion for self-employment decisions is to determine the size of the evasion channel in the whole effect of taxes on self-employment decisions. As we showed in Proposition 1, the effect of taxes on self-employment decisions in an environment where taxpayers face the same tax scheme is summarized with the following equation

$$\eta_{H,t_2} = \underbrace{\varepsilon_{z^s,t}\eta_{H,z^s}}_{\text{Taxable Income Channel}} + \underbrace{\varepsilon_{e,t}\eta_{H,e}}_{\text{Evasion Channel}}$$

The evasion channel is composed by two terms, $\varepsilon_{e,t}$ and $\eta_{H,e}$. The first behavioral parameter is obtained from the effect of the marginal tax policy change on the evasion rate in Section 1.6.2. The second parameter is obtained in the last section by the effect of evasion on the probability of being self-employed. Therefore, using the reduced form of the IV-DiD (equation (1.6)) we recover occupational decisions semi-elasticity to tax, $\eta_{H,t}$, and we assess the importance of the evasion channel on it. Table 1.6 present the result of the reduced form. Columns (1) and (2) consider the treatment variable (POL treatment) and its prediction (POLS PT), respectively. Results in column (2) correspond to our main specification. In the final row, we provide the computation of the corresponding semi-elasticity for the DiD estimations.

Table 1.6: Effect of Tax Reform on Self-employment

	(1)	(2)
Dep. Vble: Self-employment	POLS treatment	POLS PT
Policy	0.0439***	0.0539***
	(0.014)	(0.014)
Treatment	0.2271***	0.1857***
	(0.014)	(0.014)
Policy × Treatment[a]	-0.0691***	-0.1271***
	(0.017)	(0.018)
Observations	15,063	15,063
R-squared	0.243	0.236
F-statistic	108.6	106.2
Covariates	✓	✓
Year FE	✓	✓
Delta Tax[b]	-0.016	-0.016
Mean Tax[c]	0.028	0.028
Semi-elasticity $((a/b) \times c)$	0.121	0.223

*Notes: Columns (1) and (2) report POLS estimates of the tax policy change and tax policy change based on predicted treatment, using the household sampling weights. Covariates include: male indicator, age-group dummies, education-group dummies, marital status, couple's occupation type, household size dummies, owner dwelling indicator, single-parent family indicator, a child under 15 years in the family indicator, 1-digit industry-codes dummies, and agent's gross income (in Ln). Mean Tax is the weighted average for the whole sample and Delta Tax is the weighted MTR difference in the whole sample. Robust standard errors in parentheses. Significance level: *** p<0.01, ** p<0.05, * p<0.1.*

This table shows that the policy reform decreases the self-employment rate in -0.127 percentage points (column (2)). This result aligns with our theory: the falling in the MTRs, because of the tax policy changes produces a decrease in the probability of being self-employed, implying a positive relation between the MTR and self-employment decision.

Using the DiD coefficient in column (2), we obtain that the occupational decision semi-elasticity to MTR, $\eta_{H,t}$, reaches 0.223. This means increasing one percent in the marginal tax rate increases the probability of being self-employed by 0.223 percentage points. With this behavioral parameter, we can compute how relevant evasion channel is. From Table 1.4 we obtain $\varepsilon_{e,t} = 1.39$ and from

Table 1.5, $\eta_{H,e} = 0.16$. By using both parameters, the evasion channel reaches 0.222. Therefore, the 99.73% of the effect of taxes on self-employment decisions is explained by the evasion channel.

We demonstrate that the whole effect of taxes on the occupational decision is due to the evasion channel. In an environment where different occupations face the same marginal tax rate, as in the Chilean case, any tax change affects self-employment decisions through evasion incentives. Evasion is relevant in occupational decisions and is the mechanism that agents use to adjust their behavior in front of tax changes. Hence, any tax reform should consider the effects of evasion on occupational choice since it is the primary adjustment mechanism.

1.7 Welfare Effects

The relevance of tax changes on evasion and evasion on self-employment decisions motivates us to study the welfare effect of tax reforms. Feldstein (1999) demonstrates that the elasticity of taxable income (ETI) is sufficient to capture deadweight loss (DWL), connecting income tax with commodity taxation. However, in subsequent papers, some authors demonstrate that this is not always true in the presence of evasion (Chetty, 2009*a*,*b*; Kleven, 2018). Particularly, Gorodnichenko, Martinez-Vazquez and Sabirianova Peter (2009) show that a DWL measure that incorporates evasion into welfare effects is smaller than using the ETI. Thus, a DWL measure based only on the use of the ETI gives a misperception over the magnitude of tax policy effects on welfare.

Although other papers elaborate novel models incorporating evasion, no model combines tax evasion, self-employment decisions, and tax compliance policy. This is important since an agent's evasion and occupational decisions depend on taxes and tax compliance policy. Thus, tax reform affects welfare through its effect on taxable income, but also through evasion (intensive margin) and occupational decisions (extensive margin). Besides that, tax compliance policy changes agents' expected penalties, modifying their responses. For these reasons, it is relevant to incorporate evasion in the DWL measure and introduce self-employment decisions and tax compliance policy into the model to estimate welfare effects correctly.

We follow a sufficient statistic approach (see Chetty, 2009*a*,*b*; Kleven, 2018, for a detailed description), where we first theoretically develop a measure of welfare effects that connects agents' behavioral parameters with the effect of taxes on welfare. Later, we connect this metric with DWL based on the ETI, firstly proposed by Feldstein (1999). We show how we compute each relevant parameter, and present the results of our estimations.

1.7.1 Theoretical Derivation

The sufficient statistic approach assumes that social welfare corresponds to the sum of the worker's utilities at the optimum plus the tax revenues in the wage-earner and self-employment sectors. In this sense, tax revenue is given back to taxpayers as a lump-sum transfer. To simplify the notation, we define U^i and R^i as the agent's indirect utility function and tax revenue in sector $i = \{s, w\}$, the characterization of each ones is the same as in Section 1.2. Thus, the social welfare function (SWF) can be written as

$$
\begin{aligned}
W = \Bigg\{ & \int_{\underline{z}^s}^{A} \int_{H_1(z^s,e_1)}^{A} U^w(C)dF(z^w)dF(z^s) + \int_{\underline{z}^s}^{A} \int_{\underline{z}^w}^{H_1(z^s,e_1)} U^s(C)dF(z^w)dF(z^s) + \int_{A}^{Z_1} \int_{H_2(z^s,e_2)}^{\overline{z}^w} U^w(C)dF(z^w)dF(z^s) \\
& + \int_{A}^{Z_1} \int_{A}^{H_2(z^s,e_2)} U^s(C)dF(z^w)dF(z^s) + \int_{Z_1}^{Z_2} \int_{H_3(z^s,e_3)}^{\overline{z}^w} U^w(C)dF(z^w)dF(z^s) + \int_{Z_1}^{Z_2} \int_{A}^{H_3(z^s,e_3)} U^s(C)dF(z^w)dF(z^s) \\
& + \int_{Z_2}^{\overline{z}^s} \int_{H_4(z^s,e_4)}^{\overline{z}^w} U^w(C)dF(z^w)dF(z^s) + \int_{Z_2}^{\overline{z}^s} \int_{A}^{H_4(z^s,e_4)} U^s(C)dF(z^w)dF(z^s) \Bigg\} + \int_{\underline{z}^s}^{A} \int_{H_1(z^s,e_1)}^{A} R^w(z^w)dF(z^w)dF(z^s) \\
& + \int_{\underline{z}^s}^{A} \int_{\underline{z}^w}^{H_1(z^s,e_1)} R^s(z^s,e_1)dF(z^w)dF(z^s) + \int_{A}^{Z_1} \int_{H_2(z^s,e_2)}^{\overline{z}^w} R^w(z^w)dF(z^w)dF(z^s) + \int_{A}^{Z_1} \int_{A}^{H_2(z^s,e_2)} R^s(z^s,e_2)dF(z^w)dF(z^s) \\
& + \int_{Z_1}^{Z_2} \int_{H_3(z^s,e_3)}^{\overline{z}^w} R^w(z^w)dF(z^w)dF(z^s) + \int_{Z_1}^{Z_2} \int_{A}^{H_3(z^s,e_3)} R^s(z^s,e_3)dF(z^w)dF(z^s) + \int_{Z_2}^{\overline{z}^s} \int_{H_4(z^s,e_4)}^{\overline{z}^w} R^w(z^w)dF(z^w)dF(z^s) \\
& + \int_{Z_2}^{\overline{z}^s} \int_{A}^{H_4(z^s,e_4)} R^s(z^s,e_4)dF(z^w)dF(z^s)
\end{aligned}
$$

Given that we have a quasi-linear utility function, the worker will always allocate the lump-sum rebate to consumption and, thus, we define $R^w(z^w) = T(z^w)$, $R^s(z^s, e) = T(z^s - e) + \rho(z^s - e)\pi\left[T(z^s) - T(z^s - e)\right]$ and $U^i(C) = z^i - R^i(z^i)$ with $i = \{s, w\}$. The term into curly brackets represents agents' utility at the optimum.[43]

As in Section 1.2.3, we compute the effect of the marginal tax (t_2) on the social welfare function.

[43] Recall that H is the threshold function that represents the minimum wage-earner income induces a worker to go to that sector.

Tax changes produce two different effects: behavioral and mechanical. The behavioral effect relies on the agent's decision respond to tax movements, which in this case are taxable income ($\frac{\partial z^i}{\partial t_2}$) and evasion ($\frac{\partial e}{\partial t_2}$). Hereafter we assume that taxable income changes with taxes because, as we pointed out previously, behind that parameter are the labor supply and, also, to follow the traditional literature in DWL estimation (Feldstein, 1999). Since agents' utilities are at the optimum, behavioral responses are zero. The mechanical effect is the marginal tax effect on tax paid to assume income reporting does not change (or its declaration does not change) by tax changes. Mechanical responses in agents' utility and lump-sum transfer are the same, thus cancel out. Hence, the effect of taxes on welfare consists only of behavioral responses in the lump-sum transfer part of the SWF. The effect of taxes on the SWF is summarized in the following equation

$$\frac{\partial W}{\partial t} = \hat{\varepsilon}_{z^w,t}\hat{z}^w + \hat{\varepsilon}_{x,t}\hat{x}^s\hat{\rho}(z^s - e)\pi + \hat{\varepsilon}_{e,t}\hat{e}\hat{\rho}(z^s - e)\pi - \hat{\eta}_{H,t}\frac{\hat{H}(z^s, e^*) - \hat{z}^s}{t}$$

where we replace t_2 for t to simplify the exposition, and all terms represent weighted average variables.[44] This equation shows that tax effect on welfare is given by the following elements: (i) the effect in the wage-earner sector through the elasticity of taxable income ($\hat{\varepsilon}_{z^w,t}$) times the taxable income ($\hat{z}^w$); (ii) the effect in the self-employment sector that is compassed by the tax compliance policy ($\hat{\rho}\pi$) multiplied by the sum of the elasticity of reported income ($\hat{\varepsilon}_{x,t}$) times reported income ($\hat{x}^s$) and the elasticity of evasion ($\hat{\varepsilon}_{e,t}$) times evasion ($\hat{e}$); (iii) the effect in the extensive margin that is given by the occupational decision's semi-elasticity ($\hat{\eta}_{H,t}$) times the difference between the threshold function ($\hat{H}$) and self-employment taxable income ($\hat{z}^s$) divided by the tax rate t. Interestingly, the extensive margin effect has a negative sign indicating two facts: agents can increase their utility by choosing a sector where they will pay fewer taxes, and by doing this, reducing tax revenue.

Following Gorodnichenko, Martinez-Vazquez and Sabirianova Peter (2009) we obtain the income measure of welfare change ($\varepsilon_W = \frac{1}{\hat{z}}\frac{\partial W}{\partial t}$), where $\hat{z}$ is the weighted average taxable income in the population.

$$\varepsilon_W = \hat{\varepsilon}_{z^w,t}\frac{\hat{z}^w}{\hat{z}} + \hat{\varepsilon}_{x,t}\frac{\hat{x}^s}{\hat{z}}\hat{\rho}(z^s - e)\pi + \hat{\varepsilon}_{e,t}\frac{\hat{e}}{\hat{z}}\hat{\rho}(z^s - e)\pi - \hat{\eta}_{H,t}\frac{\hat{H}(z^s, e^*) - \hat{z}^s}{t\hat{z}} \tag{1.9}$$

Equation 1.9 shows the impact of tax changes in welfare in a money metric. The welfare effect

[44] The procedure to obtain this equation is in Appendix 1.9.2

is composed of a weighted sum of behavioral parameters, where the weights depend on the margin (intensive or extensive) and the sector. Also, this equation highlights the relevance of evasion, self-employment decisions, and the tax compliance policy for this measure. Concerning evasion, the third term shows its relevance. Since $\hat{\varepsilon}_{e,t} > 0$, not considering evasion introduce a downward bias in the estimation. This effect is explained because agents can change their evasion decisions because of tax reform. Essentially, evasion is tax revenue losses, and, although self-employed agents' utility increases by an increase in consumption, this raising is canceling out because of evasion effect on the lump-sum transfer. This action implies that revenue falls, and the area that welfare considers (the tax revenue part) goes down.[45] Notice that, the same happen if the tax compliance policy is null, i.e., $\hat{\rho}(z^s - e)\pi = 0$. In the same line, the last term shows the relevance of the extensive margin. Since $\hat{\eta}_{H,t} > 0$ and $\hat{H} > \hat{z}^s$, it clear that not considering the extensive margin produce an upward bias in the welfare effect estimation. This is because the measure does not consider a way to adjust for tax changes, moving to another occupation to increase the agent's utility.

The next step is to obtain ε_W as a function of the elasticity of taxable income $\varepsilon_{z,t}$ for the whole sample. We express the ETI as a function of the welfare effects due to tax changes. We compute the ETI including the extensive and intensive evasion margin, showing that it is the weighted sum of taxable income elasticity in both sectors minus the effect over the extensive margin.

$$\varepsilon_{z,t} = \hat{\varepsilon}_{z^w,t}\frac{\hat{z}^w}{\hat{z}} + \hat{\varepsilon}_{x,t}\frac{\hat{x}}{\hat{z}} - \hat{\eta}_{H,t}\frac{\hat{H}(z^s, e^*) - \hat{x}}{\hat{z}} \tag{1.10}$$

For simplicity we replace t_2 by t.[46] Note that equation (1.10) is an extension of the ETI used in Feldstein (1999) which incorporates the extensive margin. However, it differs from Feldstein, where the ETI can approximate the DWL. In this case, we theoretically compute the behavioral parameters and the weights that must be used to link ETI to DWL. Still, this equation allows us to compare the effect of including evasion into the DWL measure when the extensive margin is considered.

To obtain the formula that captures the welfare effect in our setting, we replace equation (1.9)

[45] Another way to see this is assuming that the welfare effect is given by the behavioral effect of taxes on the lump-sum transfer. Evasion falling lump-sum transfer, implying that the subsidy goes down. If we assume that not all agents evade, particularly wage-earners, not considering evasion means that not considering the decrease in the transfer for this group by income tax evasion.

[46] In Appendix 1.9.2 we show the derivation of equation (1.10).

into equation (1.10) obtaining

$$\varepsilon_{z,t} = \varepsilon_W + (1 - \hat{\rho}(z^s - e)\pi)\hat{\varepsilon}_{x,t}\frac{\hat{x}}{\hat{z}} - \hat{\rho}(z^s - e)\pi\hat{\varepsilon}_{e,t}\frac{\hat{e}}{\hat{z}} + \hat{\eta}_{H,t}\left[\hat{H}(z^s, e^*) - \hat{x}\right]\frac{1-t}{t\hat{z}} - \hat{\eta}_{H,t}\frac{\hat{e}}{t\hat{z}} \tag{1.11}$$

where, from Proposition 1

$$\eta_{H,t} = \varepsilon_{z^s,t}\eta_{H,z^s} + \varepsilon_{e,t}\eta_{H,e}$$

We use equation (1.11) in the DWL formula (explained in the next section) and refer this to our measure. It is clearly that we need estimating four behavioral parameters: $\varepsilon_{z^w,t}$, $\varepsilon_{x,t}$, $\varepsilon_{e,t}$ and $\eta_{H,t}$.

Equation (1.11) highlights the relevance of considering evasion in the DWL measure. Note that, if we incorporate self-employment decisions in Feldstein's model, we should obtain $\varepsilon_W = \varepsilon_{z,t}$, where $\varepsilon_{z,t}$ comes from equation (1.10). This equality produces a bias in the DWL measure since it does not consider relevant elements, particularly the last two terms in equation (1.11), which rely on the evasion gains effect in the extensive margin. This means that equation (1.10) incorporates occupational decisions but not the gains from evaded taxes that modify self-employment incentives. Regarding evasion, not considering it produce an upward bias in the measure because $\varepsilon_{e,t} = 0$. Not considering evasion is the same as a non-existent tax compliance policy, producing that the measure does not consider the deterring effect of audits and penalties in the evasion behavior. Finally, and in contrast to past studies (Chetty, 2009*a*; Gorodnichenko, Martinez-Vazquez and Sabirianova Peter, 2009), equation (1.11) also shows the sensitivity of the DWL measure to the tax compliance policy.

In summary, extending Feldstein's model, including self-employment decisions, does not solve the bias in the use of the ETI since it does not capture the whole effect of tax changes in the DWL measure. Particularly the tax evasion effect in the self-employment decisions. More relevant for our work, not including evasion produce a systematic downward bias in equations (1.9) and (1.11), ending up in a smaller welfare effect than what taxes produce. The source of this biased comes from omitting the effect of evasion in the lump-sum transfer and its effect on wage-earners consumption. [47] Therefore, as Chetty (2009*a*) said, the ETI is not enough to capture the DWL, in our case, due to the decrease in wage-earners consumption because of evasion. Hence evasion is relevant, but it also is necessary to include the extensive margin and the tax compliance policy.

[47] For self-employed, this effect cancels out with the increase in after-tax income due to evasion.

1.7.2 DWL Formula

To calculate the DWL, we use the formula proposed in Feldstein (1999). This study connects labor with commodity taxes, assuming that labor taxes are equal to taxes on an aggregate consumption good. By doing that, it is possible to approximate the DWL by Hargerber's triangle in commodity taxation. The DWL measure in this context corresponds to

$$DWL = -0.5 \times \frac{t^2}{1-t} \times \varepsilon_{z,t}$$

where t is the average tax rate in the pre-reform period.[48]

As we mentioned, we replace $\varepsilon_{z,t}$ by equation (1.11) and compare DWL using our derivation with the ETI obtained from data. Also, since our measure crucially depends on the tax enforcement policy, $\rho(z^s - e)\pi$, we evaluate the extreme scenarios. The worst-case scenario means that the enforcement policy does not exist and agents can freely evade, this is $\rho(z^s - e)\pi \to 0$. The opposite case, the best-case scenario, means that the enforcement policy eliminates all evasion incentives, so $\rho(z^s - e)\pi \to 1$.

1.7.3 Parameters Estimation

Computing the DWL formula presented before requires the estimation of some behavioral parameters: $\varepsilon_{z^w,t}$, $\varepsilon_{x,t}$, $\varepsilon_{e,t}$ and $\eta_{H,t}$. Additionally, we estimate the ETI for the whole sample to compare the DWL using our measure versus using it directly. We can separate these parameters into two groups: elasticities and semi-elasticities. Given that both groups represent different things, we discuss the methodology to estimate them separately.

Elasticities Estimation

The elasticities that we need to estimate belong to two different groups: income and evasion. As we mentioned, $\varepsilon_{e,t}$ is a result of the first stage in the IV-DiD model explained in Section 1.6. Table 1.4 presents the estimation for $\varepsilon_{e,t}$ that reaches 1.395. The second group of elasticities are

[48]In Appendix 1.9.2 we provide an extended discussion about estimating the DWL using a triangle or a trapezoid based on the same ideas as in Feldstein (1999). We are working on the implication of using a triangle or a trapezoid to calculate the DWL, motivated by the Chilean setting where the initial period presents a positive average MTR and moved to another with a small one.

ETIs in different samples: wage-earners ($\varepsilon_{z^w,t}$), self-employed ($\varepsilon_{x,t}$) and the whole sample ($\varepsilon_{z,t}$).

The ETI's estimation has been widely discussed and studied in the literature.[49] We can group the different approaches into three groups: Share Analysis, DiD approach, and IV-DiD method. The difference between the DiD and the IV-DiD approaches relies on whether explicitly using an MTR variable (traditionally the net-of-tax rate) instrumenting it with tax reform in a certain group or directly exploiting the tax reform. In both cases, we obtain a Wald estimator that compares the percentage changes in taxable income versus the percentage change in the MTR. Also, the specific empirical model depends on data and the exogenous variation (if it exists).

The main challenge in the ETI estimation relies on the capacity to isolate the effect of tax changes in the taxable income. Since agents respond to different incentives adjusting their income, the relation between the MTR and taxable income is endogenous. For instance, we have the mean reversion problem and heterogenous shocks across the income distribution with micro panel data, and with a cross-section, the issues of controlling for base year income or income trends and the compositional effect appears. In both cases, it is necessary to have an exogenous variation to control for different trends in variables, groups, or income. However, Weber (2014) shows that micro panel data issues are solved with income trends controls, but the result is susceptible to the base year, and with a pooled cross-section, it is not always possible to account for some trends or matching data to control for a base period. All these issues produce that obtaining an unbiased ETI estimation is complicated.

Instead of these methods, we provide a more transparent and straightforward way to estimate the ETIs, the share analysis. Following Saez, Slemrod and Giertz (2012) and Saez (2017) we take advantage of the tax reform to calculate the change in income participation for agents who face a tax change. This method is a Wald estimation that divides the log difference between income share and MTR for treated. The following equation shows this method

$$\varepsilon_{y,t} = \frac{ln(p_0) - ln(p_1)}{ln(MTR_0)–ln(MTR_1)}$$

where p is the share of income accruing for the treated group and MTR the marginal tax rate for this group, weighted by income weights. The subscripts indicate the period before and after the tax reform. The identification assumption of this methodology relies on, without the reform, the

[49]See Saez, Slemrod and Giertz (2012) for a survey of it.

treated income participation remains constant between $t = 0$ and $t = 1$. This assumption means that tax reform is the only source of variation for the income share of treated agents, and, in this sense, $\varepsilon_{y,t}$ is an unbiased estimation of the percentage change in taxable income as a response to a percentage change in the MTR. Finally, in our setting, treated agents, represent a small group in the sample (see Table 1.13 and Figure 1.2), thus the shape analysis appears as good to estimate the ETI (Saez, Slemrod and Giertz, 2012).

Table 1.7 shows the income shares and the MTR for the years 2007 and 2017, the period that we use to obtain the ETIs. We compare 2007 with 2017 to avoid some problems with the proximity of the 2013 tax reform. In this sense, we estimate long-run ETIs. Also, to compare similar taxpayers, we only use the first three brackets producing that controls are agents who belong to the first income bracket (not face tax changes) and treated agents are those in income brackets two and three. With this approach, we keep with the 94.72% of the sample, 97.31% of self-employed and 93.92% of wage-earners, dropping a small part of the sample representing high-income taxpayers. Finally, each column represents different taxable income and samples, being z^w the taxable income in the wage-earner sector, x^s the reported income in the self-employed sector, and z the reported income in the whole sample.[50]

Table 1.7: Elasticity of Taxable Income

	Sample		
Variable	$y = z^w$	$y = x^s$	$y = z$
Share of Taxable Income 2007	60.31%	46.25%	57.74%
Share of Taxable Income 2017	68.45%	47.98%	65.15%
MTR 2007	5.81%	5.80%	5.81%
MTR 2017	4.66%	4.53%	4.64%
$\varepsilon_{y,t}$	-0.57	-0.15	-0.54

Note: All estimation are made restricting the sample to the first three income brackets. Share income is the share of taxable income accruing by income bracket two and three. MTR is calculate using income weights.

[50] Recall that, since self-employed worker can evade, their income report differs from their taxable income, so $x^s = z^s - e$, but $x^w = z^w$. Also, $z = x^s$ if sector s and $z = z^w$ if sector w

In the last row of Table 1.7, we present the ETI estimations. The first column indicates that a one percent increase in the MTR for wage-earners decreases their taxable income by -0.57 percent. For the self-employed, column two indicates that an increase of one percent in the MTR for the self-employment sector decreases the reported income by -0.15 percent. Finally, column three shows that an increase of one percent in the MTR decreases the reported income by -0.54 in the whole sample.

Results in Table 1.7 get the expected sign, indicating that an increase in the MTR produces agents reporting less income. However, contrary to earlier contributions (Kleven and Schultz, 2014; Bosch and de Boer, 2019), this Table also shows that self-employed respond fewer to tax changes than wage-earners. This fact might be explained due to an adjustment through evasion in this sector.[51] However, the magnitude of obtained elasticities is similar as Saez (2017) for 1-0.1% of the income distribution, and Saez, Slemrod and Giertz (2012) for the top 1 % comparing 1981 vs. 1984 and 1992 vs. 1993, both using the share analysis.

Semi-elasticity Estimation

The semi-elasticity that we need is $\eta_{H,t}$, which captures the effect of tax changes in the self-employment decisions. Specifically, it shows how the probability of being self-employed changes due to a one percent change in the marginal tax rate. In Proposition 1 we characterize $\eta_{H,t}$ as the sum of two channels: taxable income and evasion. However, to provide a clear and simple estimation, instead of estimating each channel separately, we use the estimation from the reduced form of the IV-DiD provided in Section 1.6. Table 1.6 shows that $\eta_{H,t} = 0.223$.

1.7.4 Welfare Estimations

Estimating the DWL requires other parameters than those shown in the two preceding sections. Additionally to those parameters, equation (1.11) depends on terms that are the mean of some variables. Table 1.8 presents those results that are the average value before the policy.

[51] Note that $x^s = z^s - e$, producing that tax change effect on evasion downward the effect on self-employment taxable income.

Table 1.8: DWL Parameters

Variables	
Mean Reported Taxable Income ($\hat{z}$)	\$ 714,376
Mean WE Income ($\hat{z}^w$)	\$ 791,045
Mean Self-employment Income ($\hat{z}^s$)	\$ 539,688
Mean Self-employment Report ($\hat{x}^s$)	\$ 473,177
Threshold proxy ($\hat{H}(z^s, e^*)$)	\$ 863,492
Evasion rate	10.750%
Evasion ($\hat{e}$)	\$ 58,015
Tax 2012 (simple average)	23.43%

Note: All monetary units are in 2017 Chilean pesos. Source: Own elaboration based on EPF and Chilean IRS data.

In Table 1.9 we present the primary DWL estimations. Panel A and B show the DWL using ε_W from equation (1.9) and $\varepsilon_{z,t}$ from equation (1.11), comparing different tax compliance policy ($\rho\pi$) scenarios. While in Panel A we assume a non-existent tax compliance policy, in Panel B, we assume the strongest policy. Panel C uses the $\varepsilon_{z,t}$ from data, which result is shown in the third column of Table 1.7. This panel provides the comparison with the methodology proposed in Feldstein (1999). Finally, Panel D provides the result if we extend Feldstein methodology incorporating the extensive margin, thus $\varepsilon_{z,t}$ comes from equation (1.10). We show our preferred estimation, using the simple average MTR for 2012.

Table 1.9: DWL Estimation

Behavioral Parameters	
$\varepsilon_{e,t}$	1.40
$\varepsilon_{z^w,t}$	-0.57
$\varepsilon_{x^s,t}$	-0.15
$\eta_{H,t}$	0.22
Estimations	
Panel A: Worst-case Scenario	
$\rho \times \pi \to 0$	
ε_W	-1.06
$\varepsilon_{z,t}$	-0.84
DWL	3.01%
Panel B: Best-case Scenario	
$\rho \times \pi \to 1$	
ε_W	-1.05
$\varepsilon_{z,t}$	-0.79
DWL	2.82%
Panel C: Using ETI (Feldstein approach)	
$\varepsilon_{z,t}$	-0.54
DWL	1.92%
Panel D: Feldstein with Extensive Margin	
$\varepsilon_{z,t}$	-0.85
DWL	3.06%

The estimated DWL is between 2.81% and 3.01% of the average taxable income, depending on the tax compliance policy. This range evidence that tax compliance policy is not trivial to capture welfare effects of tax reforms. Moreover, as weaker the tax compliance policy is, as more significant the DWL.[52] This means that with a more vigorous policy, it is more difficult for agents to avoid

[52]Recall that a non-existent tax compliance policy is comparable to not considering evasion in the DWL measure.

its tax liability, and tax policies produce less distortion since agents react only through taxable income, not evading taxes. Therefore, evasion increases the DWL due to increased tax revenue losses.

Panel C and D provide two relevant comparisons, the Feldstein approach and the Feldstein approach with extensive margin, respectively. Comparing both estimations shows that including the extensive margin produces more significant measures. This result is reinforced by comparing Panel C with Panel A or B, where always our estimation is bigger than following the traditional Feldstein approach. This result goes in the opposite direction that the bias found by Gorodnichenko, Martinez-Vazquez and Sabirianova Peter (2009). This is partially explained by the inclusion of evasion and occupational decision simultaneously but highlights the relevance of using a metric that incorporates the welfare effect instead of the ETI from data.

To explain better the different direction bias in comparison with Gorodnichenko, Martinez-Vazquez and Sabirianova Peter (2009), we compare Panel D with Panel A or B. Panel A is similar to not considering evasion, as we discuss in the theoretical derivation of the DWL measure, thus the identical result is expected. However, as tax compliance policy increases, the relevance of evasion appears, and the DWL estimation diverges. In this sense, we recover the result obtained in Gorodnichenko, Martinez-Vazquez and Sabirianova Peter (2009), where their upper-bound estimation is similar to the Feldstein approach, but the lowe-bound is smaller. However, we get a similar result with a different model that exploits the tax compliance policy instead of transfer cost in evasion. As we showed, tax compliance policy is relevant in agents' decisions, an example of that is Kleven et al. (2011) where different elasticities of evasion are obtained before and after audits. Hence, including tax compliance policy appears reasonable and provides valuable results that highlight the relevance of evasion in the DWL measure, in concordance with the literature.

Finally, it is relevant to point out that Panel D is also a structural estimation since we theoretically obtain the form of the ETI. Unlike the Feldstein spirit, we construct a measure incorporating the extensive margin recovering the result obtained in past studies. But, if we believe that the ETI is a well proxy of welfare effect, the bias goes in the opposite direction than in Gorodnichenko, Martinez-Vazquez and Sabirianova Peter (2009), opening an exciting question to inquire about.

To study better the consequences of including evasion and occupational decision into the DWL measure, we provide a comparative statics and two comparisons with Panel C and D in Table 1.10.

In order to provide a more precise comparison with Table 1.9, we follow the numeration of Panels from this. Thus, In Panel E, we show the estimation assuming that evasion is not considered. This means that we assume that $\varepsilon_{e,t}$ does not exist. In the same line, Panel F assumes that the extensive margin is not considered, thus the economy only has one sector, the self-employment, and ε_{z^w} and $\eta_{H,t}$ not exist. In both Panels, we also provide ratios that compare the estimations of each Panel with Feldstein's approaches.

Table 1.10: Comparative Statics of the DWL Estimations

	Comparative Statics
Panel E: without Evasion	
DWL	2.74%
DWL/DWL_C	142.66%
DWL/DWL_D	89.60%
Panel F: without Extensive Margin	
DWL	0.35%
DWL/DWL_C	18.47%
DWL/DWL_D	11.60%

Note: DWL_C represent the estimation in Panel C and DWL_D represent the estimation in Panel D.

In Panel E, we show the DWL estimation if evasion is not considered. The first result is that both tax compliance policy scenarios collapse to the same outcome, leading that without evasion, tax enforcement policy is not relevant for the DWL. This estimation is smaller than our tax policy scenarios, evidencing that the evasion effect in the welfare effect (ε_W) dominates the effect in the DWL. In other words, the effect of not considering the effect of evasion in the lump-sum transfer to wage-earners is more significant than the tax compliance policy effect. On the other hand, the comparison with the Feldstein approaches shows that the estimation is larger than if we use the ETI but smaller than if we incorporate the extensive margin. The first result shows that not considering the occupational decision produces a smaller DWL measure because it does not consider what is happening in the wage-earner sector. The second result goes in line with Gorodnichenko, Martinez-Vazquez and Sabirianova Peter (2009) evidencing that the Feldstein approach produces an upward bias when we compare similar measures, however in this case, this is motivated by the effect of not considering tax evasion gains as we discuss for equation (1.11).

Panel F excludes the extensive margin from the analysis. Again, both tax scenarios collapse to the exact estimation, and the point estimate is smaller than our estimation in Panel A and B in Table 1.9. However, in this case, the measure is smaller than each Feldstein approach, either including or not the extensive margin. This fact implies that compared with the closer estimation, Panel C in Table 1.9, including evasion produces a smaller DWL estimation. This result reinforces one explanation given before, not including into the analysis what is happening in the wage-earner sector produces a downward bias. The wage-earner sector is the biggest one in our setting (see Table 1.13) and is an adjustment channel to tax change: if taxes fall, one response for self-employed workers is moving to the wage-earner sector. Thus, not incorporating them into the analyzes produce a partial measure of the whole effect.

Therefore, we show that comparing our measure with the corresponding Feldstein approach produces smaller estimations, indicating that does not use the evasion in the DWL measure induces an upward bias. We show that not considering evasion in our setting produces a smaller estimation to reinforce this conclusion. This means that not considering evasion always makes biased estimation because it does not consider the effect of evasion in wage-earners lump-sum transfer and the tax evasion gains effect in self-employment decisions. Moreover, not considering the extensive margin, or the self-employment decisions, also gives smaller estimations. Hence, including evasion continuously improves the DWL measure, capturing agents' mechanism to respond to tax changes.

1.8 Conclusion

This study investigates the relationship between income tax evasion and the decision to be self-employed. First, we develop a theoretical occupational choice model that demonstrates the channels involved in this relationship. Solving this model by backward induction, we obtain the optimal evasion and the threshold function. The optimal evasion captures two empirical facts:

it depends on the tax schedule and is optimum for some agents bunching at an income bracket threshold. The threshold function is the minimum wage-earner income to being a wage-earner worker and represents our model's occupational decision. We find that tax change's effect on the self-employment decisions can be separated into two channels: taxable income and evasion. The taxable income channel relies on the mechanical effect produced by tax changes on income net-of-evasion, and the evasion channel is the effect that the marginal tax change has on the evasion rate.

Using a Chilean survey, the *Encuesta de Presupuestos Familiares*, we obtain the evasion rate at a household level. This measure represents the amount of income that each household misreported to the Chilean IRS. To obtain that measure, we follow a consumption-based approach (Pissarides and Weber, 1989; Hurst, Li and Pugsley, 2014) that compares Engel curves between self-employed and wage-earner households. Instead of following the traditional literature, an improvement in the methodology is proposed, capturing the heterogeneity in evasion behavior across agent distribution. We document that, the evasion rate in Chile between 2007-2017 in the self-employment sector is 10.89% on average, being the intensive margin (the average evasion for evaders) of 12.87% and the extensive margin (the percentage of self-employed who evade) of 84.71%. Besides that, we show that not considering heterogeneity in agents evasion would underestimate misreporting in Chile.

Taking advantage of the Chilean tax schedule, we isolate the evasion channel since wage-earners and self-employed workers face the same tax schedule. Also, we exploit a marginal tax policy change as an exogenous variation to tax evasion that enables us identifying the causal effects of the policy change in marginal tax on evasion and self-employment, and lastly, the causal impact that evasion has on the occupational decision.

To capture the effect of evasion incentives in self-employment decisions, we follow an IV-DiD approach. This methodology exploits the comparison between controls (agents who do not face a tax change) and treated (agents who face a tax change) due to tax policy changes to solve the endogeneity between evasion and self-employment decisions. For this reason, we observe a DiD approach in the first stage and reduced form, and the IV in the second stage. Also, we provide a sensitivity analysis that uses the net-of-marginal-tax rate as an instrument for the evasion rate.

From the first stage, we estimate the effect of the marginal tax rate (MTR) on the evasion rate through the elasticity of evasion, obtaining a positive and significant effect. First, we document that the tax reform produces a fall in the evasion rate by -0.020 percentage points. Interpreting this estimation as the change in the evasion rate, we compute the associated elasticity to the MTR, showing that an increase in one percent in the MTR increases the evasion rate by 1.4 percent. This result highlights the relevance of taxes in evasion behavior since evasion overreacts to changes in the MTR.

Using an IV-DiD estimation method, we obtain the causal effect of evasion on the occupational decision, finding a positive impact. The estimation results indicate that an increase in evasion incentives ends up increasing the self-employment rate significantly. An increase in one percentage point in the evasion rate increases the probability of being self-employed by 6.1 percentage points. This estimation is large, evidencing that evasion incentives are crucial in self-employment decisions. Using the IV-DiD coefficient, we estimate a semi-elasticity of the occupational decision to evasion equal to 0.16. This means that an increase in one percent in the evasion rate increases the probability of being self-employed by 0.16 percentage points. Considering that taxes fall by 5.8% and the falling in self-employment rate for treated is 0.92 percentage points, the whole effect in self-employment rate can be explained by the evasion effect. This fact reinforces the result that evasion incentives are crucial in self-employment decisions.

With the first stage and IV-DiD, we quantify the magnitude of the evasion channel in the effect of the tax change on the occupational decision. To do that, we use the result of the reduced form of the IV-DiD estimating that the tax reform reduces the probability of being self-employed by -0.1271 percentage points. Since this estimation is the change in the probability of being self-employed, we compute that the semi-elasticity of self-employment decisions to the MTR reaches 0.223. Moreover, using the elasticity of evasion to the MTR and the semi-elasticity of self-employment decisions to evasion, we obtain that the evasion channel explains the 99.73% of the MTR effect on self-employment decisions. This result shows that, in a context where wage-earners and self-employed face the same tax scheme, the channel to adjust occupational decisions when a tax reform exists is evasion.

Given the relevance of evasion incentives in self-employment decisions, we ask about the welfare implication of tax changes in a framework with both elements. We follow a sufficient statistic approach to study that, including the model's extensive and intensive evasion margins and obtaining the deadweight loss (DWL). We get that the tax reform produces a DWL between 2.82% (with a non-existent tax compliance policy) and 3.01% (with the strongest tax compliance policy) of the average taxable income. Also, we assess the relevance of including evasion in the DWL measure showing three main results. First, the tax compliance policy is relevant only if evasion is considered, so any model that includes it must consider at least the effect of audits and fines. Secondly, if evasion is not considered, the DWL measure presents an upward bias over-estimating the impact of tax changes on welfare. Finally, if the elasticity of taxable income is used instead of the welfare effect measure, always exists a bias: positive if we consider the extensive margin and negative if not.

To conclude, throughout this study, we demonstrate, characterize and identify the effect of evasion on self-employed decisions. We can isolate the evasion channel from other determinants showing that tax evasion is a meaningful driver of the self-employment decision. This result allows us to infer that reducing tax evasion possibilities in self-employment occupations could reduce tax evasion. Moreover, the evidence indicates that a reduction in marginal tax rates could be relevant as a tax compliance policy. This fact is crucial for policy analysis, even more considering the welfare implications that evasion and self-employment decisions have. Finally, the relevance of evasion incentives on self-employment decisions and their effects on welfare highlights the relevance to consider the whole picture in the labor market when tax changes are planned and using taxes as a labor market policy.

1.9 Appendix

1.9.1 Tax Evasion Measure

Table 1.11 reports the second stage coefficient estimates for the income variable and the mean self-employed linear combination of the interaction terms and the misreporting measure using the Engel curve in equation (1.3) following an IV approach. In this table, we compare the estimations using Hurst-Li-Pugsley (HLP) method (Hurst, Li and Pugsley, 2014) in the first row of each panel with our estimation at the mean. Our primary definition of household head type considers that a household is self-employed if the head is self-employed (Panel A).[53] The other two panels show the robustness of the estimation results. In Panel B, we replace the definition of self-employed household with the alternative definition depending on household head and its couple occupations. In Panel C, we employ the definition of self-employed household given by the household head's occupation but restricting the sample as (Hurst, Li and Pugsley, 2014) to male-headed households removing households for which the couple (if married or cohabited) is self-employed.

[53]Estimation in column (3) is a raw prediction without replacing negative misreporting as zero value.

Table 1.11: Estimation of Tax Evasion

	Income	Self-employed	Misreport
	(1)	(2)	(3)
Panel A: Main Specification			
HLP	0.347	0.047	12.57%
	(0.026)	(0.017)	
2007	0.348	0.012	3.27%
	(0.025)	(0.038)	
2012	0.348	0.009	2.65%
	(0.025)	(0.033)	
2017	0.348	0.010	2.94%
	(0.025)	(0.036)	
Panel B: Household Head Mix			
HLP	0.343	0.019	5.26%
	(0.025)	(0.017)	
2007	0.344	0.005	1.45%
	(0.025)	(0.032)	
2012	0.344	0.004	1.27%
	(0.025)	(0.031)	
2017	0.344	0.006	1.72%
	(0.025)	(0.033)	
Panel C: Household Head Occupation following HLP			
HLP	0.371	0.055	13.77%
	(0.038)	(0.025)	
2007	0.371	0.014	3.66%
	(0.037)	(0.045)	
2012	0.371	0.010	2.76%
	(0.037)	(0.036)	
2017	0.371	0.015	3.95%
	(0.037)	(0.046)	

Note: The first row in each panel reports the coefficient estimates using the Hurst-Li-Pugsley method. Columns (1)-(3) report the mean of the estimated coefficients obtained using the IV method for estimating equation (1.3), estimations using household sampling weights. Standard errors are in parenthesis. Panel A and C definition considers only the self-employment occupation of the household head. Unlike Panel A, Panel C restricts to male-headed households and excludes households with self-employed couples. Panel B uses the self-employed household definition that indicates if either the household head or the couple are self-employed. Source: Own elaboration based on the EPF data.

Comparing estimations in column (1) show that the income elasticity is similar between years and methods. This result is robust to the household head occupation type alternative definition and restricted sample (Panels B and C, respectively). However, comparing the other columns in Panel A across methods indicates a potential bias in misreporting estimation generated by ignoring the heterogeneous behavior across self-employed workers.

Unlike the HLP method, the estimation results are more robust when we consider the primary definition of self-employed household (Panel A) or restrict the sample to a more homogeneous sub-population (Panel C). In Table 1.12, we show the result in each cross-section estimation that allows us made this comparison. Considering only the household head occupation for determining household's type increases the misreport estimation. This is consistent with the fact that using the definition based only on household head type, a significant share of the households that could have evaded because the couple is an evader is considered a not evader household. This also explains the similarity in misreport measures between Panels A and C, which is independent of the sample. Also, this comparison shows that mix evader households have a smaller income misreport than households with a self-employed head.

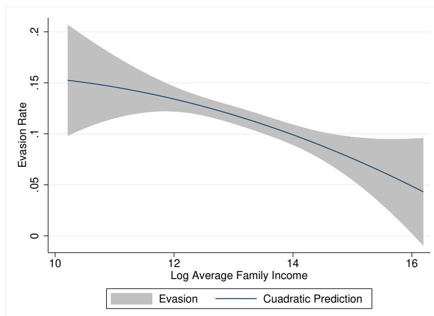
Table 1.12: Cross-Section Estimation of Tax Evasion

	HLP method			Own calculations		
	Income	Self-employed	Misreport	Income	Self-employed	Misreport
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Main Specification						
2007	0.321	0.057	16.21%	0.316	0.015	4.64%
	(0.040)	(0.022)	(0.054)	(0.039)	(0.044)	
2012	0.405	0.045	10.53%	0.413	0.009	2.23%
	(0.046)	(0.032)	(0.074)	(0.045)	(0.056)	
2017	0.323	0.029	8.62%	0.326	0.006	1.83%
	(0.045)	(0.034)	(0.094)	(0.044)	(0.050)	
Panel B: Household Head Mix						
2007	0.315	0.014	4.34%	0.312	0.006	1.84%
	(0.039)	(0.022)	(0.065)	(0.039)	(0.037)	
2012	0.402	0.017	4.09%	0.405	0.003	0.66%
	(0.045)	(0.033)	(0.066)	(0.046)	(0.054)	
2017	0.321	0.016	5.00%	0.324	0.006	1.70%
	(0.045)	(0.034)	(0.101)	(0.044)	(0.056)	
Panel C: Household Head Occupation following HLP						
2007	0.356	0.060	15.57%	0.354	0.016	4.51%
	(0.051)	(0.028)	(0.061)	(0.051)	(0.042)	
2012	0.400	0.027	6.42%	0.407	0.005	1.21%
	(0.064)	(0.043)	(0.097)	(0.064)	(0.059)	
2017	0.362	0.048	12.48%	0.371	0.012	3.24%
	(0.077)	(0.057)	(0.137)	(0.073)	(0.083)	

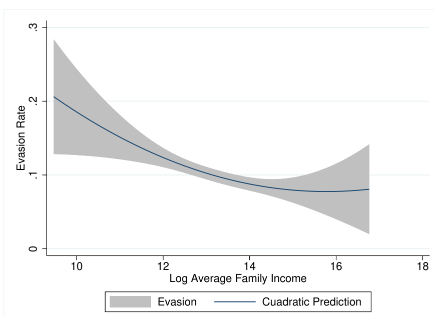
Finally, Figure 1.4 shows the predicted estimation of income misreport from estimating equation (1.3) using the IV approach to account for potential endogeneity of income. The figure displays the percentage of misreporting (i.e., tax evasion) by household's income level and the corresponding confidence interval.

The evasion pattern is similar across years, displaying a positive convexity in 2006/7 and 2016/7, more pronounced at the beginning of the period, and a negative convexity in 2011/2.[54] After the tax reform implemented in 2013, the evasion curve is inverted. The dispersion is more prominent in the tails of the income distribution. The heterogeneity in average evasion rate across the income distribution might reflect a different tax evasion behavior of households depending on their income level in response to tax changes, as we hypothesize.[55]

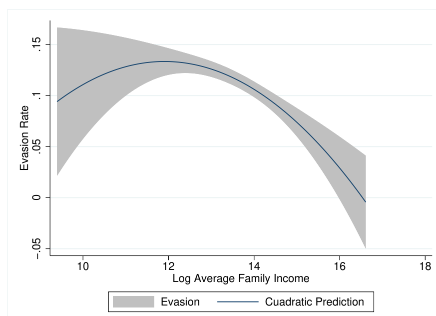
Figure 1.4: Predicted Evasion from the IV Regression Model



(a) 2006/7



(b) 2011/2



(c) 2016/7

Note: Non-parametric estimation results of the relationship between the evasion rate and the logarithm of the household. Predicted tax evasion is obtained from the IV estimation of equation (1.3). Source: Own elaboration based on the EPF data.

[54]The U-shape pattern is similar to the one obtained by Alstadsæter, Johannesen and Zucman (2018).

[55]Considering the alternative definition of self-employed household these curves change mainly at the tails (see Figure 1.10 and 1.11, respectively).

Agents Decisions

We made several specific assumption regarding some parameters in this model in order to obtain a reduced form results.[56] First, we assume that $z^i \in [0, \overline{Z}]$. Second, we impose that $A = \overline{z}^i/2$. Third, we impose that $\rho' < 0$, in specific we model the audit function as $\rho(z^S - e) = 1 - (z^s - e)/\overline{z}^i$, so $\rho' = -1/\overline{z}^i < 0$. All of these assumptions are made to characterize the optimal evasion function.

Optimal Evasion: Proof of Lemma 1

Since agents face a tax scheme with two tax brackets, we have four possible scenarios depending on the position of the agent's income and the income declaration across the income distribution. To simplify the result exposition, we name each solution with the corresponding number to each scenario. The first case to analyze is when $z^s \leq A$ and, consequently, $z^s - e < A$. In this case, self-employed agent maximize the following

$$\max_e U^s(C) = z^s - t_1(z^s - e) - \rho(z^s - e)\pi \left[t_1 z^s - t_1(z^s - e)\right]$$

The first order condition (FOC) produce an optimal evasion equals to

$$e^* = -\frac{(1 - \rho(z^s - e)\pi)}{\rho'(z^s - e)\pi}$$

By replacing with the parameter characterization yields

$$e^* = e_1 = -\frac{z^s}{2} - A\left(1 - \frac{1}{\pi}\right)$$

The second case is when $z^s > A$ but $z^s - e < A$. In this case, self-employed agents solve the following problem

$$\max_e U^s(C) = z^s - t_1(z^s - e) - \rho(z^s - e)\pi \left[t_1 A + t_2(z^s - A) - t_1(z^s - e)\right]$$

[56] These assumptions do not distort the principal results and are taken only to simplify the economic exposure of mechanism and relevant results.

The optimal evasion is

$$e^* = z^s - \frac{(1-\rho(z^s-e)\pi)}{\rho'(z^s-e)\pi} - \frac{t_1 A + t_2(z^s-A)}{t_1}$$

In this case, the existence condition for the solution is $z^S - e^* < A$. By replacing the audit function with its characterization into the evasion we obtain

$$e^* = e_2 = \frac{z^s}{2} - A\left(1 - \frac{1}{\pi}\right) - \frac{(z^s-A)}{2}\frac{(t_2-t_1)}{t_1}$$

Therefore, we will look for the conditions to the existence of this solution, i.e., the conditions to the above inequality hold.

$$\begin{aligned}
z^s - e^* &= \frac{z^s}{2} + A\left(1 - \frac{1}{\pi}\right) + \frac{(z^s-A)}{2}\frac{(t_2-t_1)}{t_1} < A = \frac{\overline{Z}}{2} \\
z^s &< A\left[1 + \frac{t_1(2-\pi)}{\pi t_2}\right] \equiv Z_1
\end{aligned}$$

Therefore, e_2 is the solution for $z^s \in (A, Z_1)$.

The third case is when $z^s > A$ and $z^s - e \geq A$. In this case, we made a Lagrangian and explore the condition using the Lagrange multiplier. Thus, the problem that agents solve is the following

$$\begin{aligned}
&\max_e U^s(C) = z^s - [t_1 A + t_2(z^s - A)] - \rho(z^s - e)\pi\,[t_1 A + t_2(z^s - A) - t_1 A - t_2(z^s - e - A)] \\
&\quad s.t \quad z^s - e \geq A \qquad (\lambda)
\end{aligned}$$

For a straightforward exposition, we first suppose that $\lambda = 0$, so $z^s - e > A$, finding the optimal evasion and characterizing the section where its solution holds. The optimal evasion from the FOC is

$$e^* = -\frac{(1-\rho(z^s-e)\pi)}{\rho'(z^s-e)\pi}$$

In this case, we also ask over the condition for the existence of this solution. By replacing the audit function with its characterization we obtain

$$e^* = e_4 = \frac{z^s}{2} - A\left(1 - \frac{1}{\pi}\right)$$

The critical requirement is $z^s - e^* > A$, this implies

$$\frac{z^s}{2} + A\left(1 - \frac{1}{\pi}\right) > \frac{\overline{Z}}{2} = A$$
$$z^s > \frac{\overline{Z}}{\pi} \equiv Z_2$$

Therefore, e_4 is the solution for $z^s \in (Z_2, \overline{Z}]$. Notice that, if $\pi < 2$ we have $Z_2 > A$. Thus, we impose $\pi \in (1, 2)$. With this assumption, it is sufficient to impose that $\pi < \frac{2t_2 - t_1(2-\pi)}{t_2}$ to guarantee that $Z_2 > Z_1$. We made this assumption.

Now, we proceed to demonstrate that, if we assume $\lambda \neq 0$, the result from the FOC is $\lambda > 0$. This result implies that, in section $A < z^s < Z_2$ the optimal evasion is given by $e^* = e_3 = z_s - A$. From the FOC we have

$$\lambda = t_2(1 - \rho(z^s - e)\pi) + \rho'(z^s - e)\pi t_2 e$$

By replacing the audit function with its characterization and $e = e_3 = z^s - A$ we obtain

$$\lambda = t_2\left[\frac{\overline{Z} - z^s\pi}{\overline{Z}}\right] = t_2\left[1 - \frac{z^s}{Z_2}\right]$$

Since $Z_2 > z^S$, $\lambda > 0$ yielding that the solution for $A < z^s < Z_2$ is e_3. e_3 is the solution in $z^s \in [Z_1, Z_2)$ because for $A \leq z^s < Z_1$ the solution e_2 brings a larger utility.

Finally, we provide a characterization of the effect of tax changes on evasion, separating the effect of t_1, t_2, and $\triangle t = t_2 - t_1$, in the following lemma.

Lemma 3. *The effect of tax changes, defining by movements in t_1, t_2 and $\triangle t = t_2 - t_1$, on evasion are*

1. *When t_1 raise, e^* also raise. This evidence a positive relation between evasion and t_1.*

2. *When t_2 raise, e^* fall. This evidence a negative relation between evasion and t_2. Moreover, the*

effect of t_2 is a normalization of the semi-elasticity of evasion regarding t_1 equals to $-\frac{\eta_{e_2,t_1}}{t_2}$. This means how many times one percent change in t_1 moves evasion in terms of t_2.

3. *When $\triangle t$ raise, e^* falls. This means that as bigger the difference between t_2 and t_1 as smaller the evasion incentives.*

In Lemma 3, we show that an increase either in t_2 or $\triangle t = t_2 - t_1$ falling evasion, but an increase in t_1 raise evasion. This fact is explained due to the marginal tax rate that agents really face. At e_2 agents face t_1, and an increase in this rate increases evasion; however, if t_2 or $\triangle t$ raise, the expected penalties raise discouraging evasion.

Threshold Function: Proof of Lemma 2

Since we are comparing two indirect utilities ($U^w(H(z^s, e^*)) = U^s(z^s, e^*)$) in different possibles scenarios, we have more than four cases to inspect for (specifically, we should analyze eight cases). We denominate each solution with a number associate to the evasion section that is optimum in each case.

The first case is $z^w, z^s \leq A$, so $z^s - e_1 < A$.

$$\begin{aligned} H(z^s)(1-t_1) &= z^s - t_1(z^s - e_1) - \rho(z^s - e_1)\pi\left[t_1 z^s - t_1(z^s - e_1)\right] \\ H(z^s) &= \frac{z^s - t_1(z^s - e_1) - \rho(z^s - e_1)\pi\left[t_1 e_1)\right]}{1-t_1} \equiv H_1(z^s, e_1) \end{aligned}$$

The second case is $z^w > A$ and $z^s > A$. In this section, we have three possibles options. The fist option is $z^s < Z_1$.

$$\begin{aligned} H(z^s) - t_1 A - t_2(H(z^s) - A) &= z^s - t_1(z^s - e_2) - \rho(z^s - e_2)\pi\left[t_1 A + t_2(z^s - A) - t_1(z^s - e_2)\right] \\ H(z^s) &= \frac{z^s(1 - \rho(z^s - e_2)\pi t_2)}{1-t_2} - \frac{(z^s - e_2)t_1 + A(t_2 - t_1))(1 - \rho(z^s - e_2)\pi)}{1-t_2} \equiv H_2(z^s, e_2) \end{aligned}$$

The second option is $Z_1 \leq z^s < Z_2$.

$$\begin{aligned} H(z^s) - t_1 A - t_2(H(z^s) - A) &= z^s - t_1(z^s - e_3) - \rho(z^s - e_3)\pi\left[t_1 A + t_2(z^s - A) - t_1(z^s - e_3)\right] \\ H(z^s) &= \frac{z^s(1 - \rho(z^s - e_3)\pi t_2)}{1-t_2} - \frac{(z^s - e_3)t_1 + A(t_2 - t_1))(1 - \rho(z^s - e_3)\pi)}{1-t_2} \equiv H_3(z^s, e_3) \end{aligned}$$

The last option is $Z_1 \leq z^s \leq \overline{z}^s$.

$$H(z^s) - t_1 A - t_2(H(z^s) - A) = z^s - t_1 A - t_2(z^s - e_4) - \rho(z^s - e_4)\pi\left[t_1 A + t_2(z^s - A) - t_1 A - t_2(z^s - e_4)\right]$$
$$H(z^s) = \frac{z^s - t_1 A - t_2(z^s - e_4) - \rho(z^s - e_4)\pi t_2 e_4}{1 - t_2} - \frac{A(t_2 - t_1)}{1 - t_2} \equiv H_4(z^s, e_4)$$

Tax Effects on the Threshold Function: Proof of Proposition 1

To simplify the exposure of each effect, we show the partial derivative in each section separately and try to standardize the parameters in each one. From Lemma 2 it is clear that $\dfrac{\partial H_1(z^s, e_1)}{\partial t_2} = 0$, thus we focus in the others sections. The effect of t_2 change on $H_2(z^s, e_2)$ is

$$\frac{\partial H_2(z^s, e_2)}{\partial t_2} = \frac{H_2(z^s, e_2) - A}{1 - t_2} + \frac{\partial z^s}{\partial t_2}\frac{\partial H_2(z^s, e_2)}{\partial z^s} + \frac{\partial e_2}{\partial t_2}\frac{\partial H_2(z^s, e_2)}{\partial e_2} - \frac{\rho(z^s - e_2)\pi(z^s - A)}{1 - t_2}$$

The structural parameters that define the effect of t_2 on the threshold function are: $\frac{\partial z^s}{\partial t_2}\frac{\partial H_2(z^s,e_2)}{\partial z^s}$ and $\frac{\partial e_2}{\partial t_2}\frac{\partial H_2(z^s,e_2)}{\partial e_2}$. This allow us to capture a lower-bound of tax effects. Now, we obtain the effect of t_2 on $H_3(z^s, e_3)$.

$$\frac{\partial H_3(z^s, e_3)}{\partial t_2} = \frac{H_3(z^s, e_3) - A}{1 - t_2} + \frac{\partial z^s}{\partial t_2}\frac{\partial H_3(z^s, e_3)}{\partial z^s} + \frac{\partial e_3}{\partial t_2}\frac{\partial H_3(z^s, e_3)}{\partial e_3} - \frac{\rho(z^s - e_3)\pi(z^s - A)}{1 - t_2}$$

Similar as before, the structural parameters that capture the effect of t_2 on the threshold function are $\frac{\partial z^s}{\partial t_2}\frac{\partial H_3(z^s,e_3)}{\partial z^s}$ and $\frac{\partial e_3}{\partial t_2}\frac{\partial H_3(z^s,e_3)}{\partial e_3}$. Finally, we obtain the effect of t_2 on $H_4(z^s, e_4)$.

$$\frac{\partial H_4(z^s, e_4)}{\partial t_2} = \frac{H_4(z^s, e_4) - A - (z^s - e_4 - A) - \rho(z^s - e_4)\pi e_4}{1 - t_2} + \frac{\partial z^s}{\partial t_2}\frac{\partial H_4(z^s, e_4)}{\partial z^s} + \frac{\partial e_4}{\partial t_2}\frac{\partial H_4(z^s, e_4)}{\partial e_4}$$

Similar as before, we can summarize the effect of t_2 on the threshold function with two structural parameters: $\frac{\partial z^s}{\partial t_2}\frac{\partial H_4(z^s,e_4)}{\partial z^s}$ and $\frac{\partial e_4}{\partial t_2}\frac{\partial H_4(z^s,e_4)}{\partial e_4}$.

From the above analysis is clear that the whole effect of t_2 on the threshold function can be summarized by $\frac{\partial z^s}{\partial t_2}\frac{\partial H(z^s,e^*)}{\partial z^s}$ and $\frac{\partial e}{\partial t_2}\frac{\partial H(z^s,e^*)}{\partial e}$. To obtain a clearer intuition, we made some changes

$$
\begin{aligned}
\frac{\partial H(z^s,e^*)}{\partial t_2} &= \frac{\partial z^S}{\partial t_2}\frac{\partial H(z^s,e^*)}{\partial z^s} + \frac{\partial e^*}{\partial t_2}\frac{\partial H(z^s,e^*)}{\partial e} \\
&= \underbrace{\frac{\partial z^s}{\partial t_2}\frac{t_2}{z^s}}_{\varepsilon_{z^s,t_2}}\underbrace{\frac{\partial H(z^s,e^*)}{\partial z^s}z^s}_{\eta_{H,z^s}}\frac{1}{t_2} + \underbrace{\frac{\partial e^*}{\partial t_2}\frac{t_2}{e}}_{\varepsilon_{e,t_2}}\underbrace{\frac{\partial H(z^s,e^*)}{\partial e}e}_{\eta_{H,e}}\frac{1}{t_2}
\end{aligned}
$$

1.9.2 DWL Derivation

Theoretical Formula

The effect of tax changes on welfare is given by

$$
\begin{aligned}
\frac{\partial W}{\partial t_2} = &\Bigg\{\left(\frac{\partial Z_1}{\partial t_2}\right)\int_{H_2(Z_1,e_2)}^{\overline{z}^w} z^w dF(z^w)f(Z_2) + \left(\frac{\partial Z_1}{\partial t_2}\right)\int_{A}^{H_2(Z_1,e_2)} Z_1 dF(z^w)f(Z_1) - \left(\frac{\partial Z_1}{\partial t_2}\right)\int_{H_3(Z_1,e_3)}^{\overline{z}^w} z^w dF(z^w)f(Z_1) \\
&- \left(\frac{\partial Z_1}{\partial t_2}\right)\int_{A}^{H_3(Z_1,e_3)} Z_1 dF(z^w)f(Z_1)\Bigg\} - \int_{A}^{Z_1}\left(\frac{\partial H_2(z^s,e_2)}{\partial t_2}\right)R^w(H_2)f(H_2)dF(z^s) + \int_{A}^{Z_1}\int_{H_2(z^s,e_2)}^{\overline{z}^w}\left[\frac{\partial R^w(z^w)}{\partial z^w}\frac{\partial z^w}{\partial t_2}\right]dF(z^w)dF(z^s) \\
&+ \int_{A}^{Z_1}\left(\frac{\partial H_2(z^s,e_2)}{\partial t_2}\right)R^s(z^s,e_2)f(H_2)dF(z^s) + \int_{A}^{Z_1}\int_{A}^{H_2(z^s,e_2)}\left[\frac{\partial R^s(z^s,e_2)}{\partial z^s}\frac{\partial z^s}{\partial t_2} + \frac{\partial R^s(z^s,e_2)}{\partial e_2}\frac{\partial e_2}{\partial t_2}\right]dF(z^w)dF(z^s) \\
&- \left(\frac{\partial H_3(z^s,e_3)}{\partial t_2}\right)R^w(H_3)f(H_3)dF(z^s) + \int_{Z_1}^{Z_2}\int_{H_3(z^s,e_3)}^{\overline{z}^w}\left[\frac{\partial R^w(z^w)}{\partial z^w}\frac{\partial z^w}{\partial t_2}\right]dF(z^w)dF(z^s) + \int_{Z_1}^{Z_2}\left(\frac{\partial H_3(z^s,e_3)}{\partial t_2}\right)R^s(z^s,e_3)f(H_3)dF(z^s) \\
&+ \int_{Z_1}^{Z_2}\int_{A}^{H_3(z^s,e_3)}\left[\frac{\partial R^s(z^s,e_3)}{\partial z^s}\frac{\partial z^s}{\partial t_2} + \frac{\partial R^s(z^s,e_3)}{\partial e_3}\frac{\partial e_3}{\partial t_2}\right]dF(z^w)dF(z^s) - \int_{Z_2}^{\overline{z}^w}\left(\frac{\partial H_4(z^s,e_4)}{\partial t_2}\right)R^w(H_4)f(H_4)dF(z^s) \\
&+ \int_{Z_2}^{\overline{z}^s}\int_{H_4(z^s,e_4)}^{\overline{z}^w}\left[\frac{\partial R^w(z^w)}{\partial z^w}\frac{\partial z^w}{\partial t_2}\right]dF(z^w)dF(z^s) + \int_{Z_2}^{\overline{z}^s}\left(\frac{\partial H_4(z^s,e_4)}{\partial t_2}\right)R^s(z^s,e_4)f(H_4)dF(z^s) \\
&+ \int_{Z_2}^{\overline{z}^s}\int_{A}^{H_4(z^s,e_4)}\left[\frac{\partial R^s(z^s,e_4)}{\partial z^s}\frac{\partial z^s}{\partial t_2} + \frac{\partial R^s(z^s,e_4)}{\partial e_4}\frac{\partial e_4}{\partial t_2}\right]dF(z^w)dF(z^s)
\end{aligned}
$$

Since we use a specific parametrization in this model, we assumed that all terms in $\{\cdot\}$ cancel between them. This assumption means that movement around Z_1 cancels out because people who move are equal and have the same income at the margin.

Notice that, because of the threshold function definition

$$
R^s(z^s,e) - R^w(z^w) = -\left(H(z^s,e) - z^s\right)
$$

Before simplifying further the expression of $\frac{\partial W}{\partial t_2}$, we obtain the derivatives over R^i. First, $R^w(z^w) = T(z^w)$, so $\frac{\partial R^w(z^w)}{\partial z^w} = T'(z^w)$ which, in this case, is equal to t_2 since this is the only rate that we change.

Looking at the marginal effects over $R^s(z^s, e_2)$.

$$\frac{\partial R^s(z^s,e_2)}{\partial z^s} = \underbrace{t_1 + \rho'(z^s - e_2)\pi\left[t_1 A + t_2(z^s - A) - t_1(z^s - e_2)\right] - \rho(z^s - e_2)\pi t_1}_{\frac{\partial U^s(z^s,e_2)}{\partial e_2}=0} + \rho(z^s - e_2)\pi t_2$$

$$\frac{\partial R^s(z^s,e_2)}{\partial z^s} = \rho(z^s - e_2)\pi t_2 \tag{1.12}$$

$$\frac{\partial R^s(z^s,e_2)}{\partial e_2} = \underbrace{-t_1 + \rho'(z^s - e_2)(-1)\pi\left[t_1 A + t_2(z^s - A) - t_1(z^s - e_2)\right] + \rho(z^s - e_2)\pi t_1}_{-\frac{\partial U^s(z^s,e_2)}{\partial e_2}=0} = 0 \tag{1.13}$$

Now, looking at the marginal effects over $R^s(z^s, e_3)$.

$$\frac{\partial R^s(z^s,e_3)}{\partial z^s} = t_1 + \rho'(z^s - e_3)\pi\left[t_1 A + t_2(z^s - A) - t_1(z^s - e_3)\right] - \rho(z^s - e_3)\pi t_1$$

but, $\rho(z^s - e_3) = 1/2$, so $\rho'(z^s - e_3) = 0$ yielding

$$\frac{\partial R^s(z^s,e_3)}{\partial z^s} = t_1(1 - \rho(z^s - e_3)\pi) + \rho(z^s - e_3)\pi t_2 \tag{1.14}$$

Recall that e_3 does not depend on t_2, then $\frac{\partial e_3}{\partial t_2} = 0$.

Last, the marginal effect over $R^s(z^s, e_4)$ is given by

$$\frac{\partial R^s(z^s,e_4)}{\partial z^s} = t_2 + \rho'(z^s - e_4)\pi t_2 e_4$$

From the FOC of e_4 we find that $t_2 + \rho'(z^s - e_4)\pi t_2 e_4 = \rho(z^s - e_4)\pi t_2$, then

$$\frac{\partial R^s(z^s,e_4)}{\partial z^s} = \rho(z^s - e_4)\pi t_2 \tag{1.15}$$

Similarly, e_4 does not depend on t_2 producing that the effect over revenue is irrelevant.

Replacing the expressions in equations (1.12) - (1.15) into $\frac{\partial W}{\partial t_2}$, doing algebra and replacing $\frac{\partial z^s}{\partial t_2} = \frac{\partial x^s}{\partial t_2} + \frac{\partial e_2}{\partial t_2}$ for $z^s < Z_1$ and $\frac{\partial z^s}{\partial t_2} = \frac{\partial x^s}{\partial t_2}$ for $z^s \geq Z_1$, yields

$$
\begin{aligned}
\frac{\partial W}{\partial t_2} = & -\int_A^{Z_1} \underbrace{\left(\frac{\partial H_2(z^s,e_2)}{\partial t_2} t_2\right)}_{\eta_{H_2,t_2}} \frac{[H_2(z^s,e_2)-z^s]}{t_2} f(H_2) dF(z^s) + \int_A^{Z_1} \int_{H_2(z^s,e_2)}^{\overline{z}^w} \underbrace{\left[\frac{t_2}{z^w}\frac{\partial z^w}{\partial t_2}\right]}_{\varepsilon_{z^w,t_2}} z^w dF(z^w) dF(z^s) \\
& + \int_A^{Z_1} \int_A^{H_2(z^s,e_2)} \rho(z^s-e_2)\pi x^s \underbrace{\left[\frac{t_2}{x^s}\frac{\partial x^s}{\partial t_2}\right]}_{\varepsilon_{x,t_2}} dF(z^w) dF(z^s) + \int_A^{Z_1} \int_A^{H_2(z^s,e_2)} \rho(z^s-e_2)\pi e_2 \underbrace{\left[\frac{t_2}{e_2}\frac{\partial z^s}{\partial t_2}\right]}_{\varepsilon_{e_2,t_2}} dF(z^w) dF(z^s) \\
& - \int_{Z_1}^{Z_2} \underbrace{\left(\frac{\partial H_3(z^s,e_3)}{\partial t_2} t_2\right)}_{\eta_{H_3,t_2}} \frac{[H_3(z^s,e_3)-z^s]}{t_2} f(H_3) dF(z^s) + \int_{Z_1}^{Z_2} \int_{H_3(z^s,e_3)}^{\overline{z}^w} \underbrace{\left[\frac{t_2}{z^w}\frac{\partial z^w}{\partial t_2}\right]}_{\varepsilon_{z^w,t_2}} z^w dF(z^w) dF(z^s) \\
& + \int_{Z_1}^{Z_2} \int_A^{H_3(z^s,e_3)} [t_1(1-\rho(z^s-e_3)\pi)] \frac{\partial x^s}{\partial t_2} dF(z^w) dF(z^s) + \int_{Z_1}^{Z_2} \int_A^{H_3(z^s,e_3)} \rho(z^s-e_3)\pi x^s \underbrace{\left[\frac{t_2}{x^s}\frac{\partial z^s}{\partial t_2}\right]}_{\varepsilon_{x,t_2}} dF(z^w) dF(z^s) \\
& - \int_{Z_2}^{\overline{z}^w} \underbrace{\left(\frac{\partial H_4(z^s,e_4)}{\partial t_2} t_2\right)}_{\eta_{H_4,t_2}} \frac{[H_4(z^s,e_4)-z^s]}{t_2} f(H_4) dF(z^s) + \int_{Z_2}^{\overline{z}^s} \int_{H_4(z^s,e_4)}^{\overline{z}^w} \underbrace{\left[\frac{t_2}{z^w}\frac{\partial z^w}{\partial t_2}\right]}_{\varepsilon_{z^w,t_2}} z^w dF(z^w) dF(z^s) \\
& + \int_{Z_2}^{\overline{z}^s} \int_A^{H_4(z^s,e_4)} \rho(z^s,e_4)\pi x^s \underbrace{\left[\frac{t_2}{x^s}\frac{\partial x^s}{\partial t_2}\right]}_{\varepsilon_{x,t_2}} dF(z^w) dF(z^s)
\end{aligned}
$$

By Collapsing all terms and not considering $(1-\rho(z^s-e_3)\pi)t_1\frac{\partial x^s}{\partial t_2}$ for simplicity, we obtain

$$
\frac{\partial W}{\partial t} = \hat{\varepsilon}_{z^w,t}\hat{z}^w + \hat{\varepsilon}_{x,t}\hat{x}^s\hat{\rho}(z^s-e)\pi + \hat{\varepsilon}_{e,t}\hat{e}\hat{\rho}(z^s-e)\pi - \hat{\eta}_{H,t}\frac{\hat{H}(z^s,e^*)-\hat{z}^s}{t}
$$

Taxable Income Elasticity Derivation

To obtain $\varepsilon_{z,t}$, we need to define the average taxable income in this context. Since we model the occupational choice, this elasticity is not a weighted sum between taxable incomes elasticities because we need to capture the effect in the extensive margin. The average taxable income is

$$
\begin{aligned}
\hat{z} = & \int_{\underline{z}^s}^{A}\int_{H_1(z^s,e_1)}^{A} z^w dF(z^w)dF(z^s) + \int_{\underline{z}^s}^{A}\int_{\underline{z}^w}^{H_1(z^s,e_1)} x^s dF(z^w)dF(z^s) + \int_{A}^{Z_1}\int_{H_2(z^s,e_2)}^{\overline{z}^w} z^w dF(z^w)dF(z^s) \\
& + \int_{A}^{Z_1}\int_{A}^{H_2(z^s,e_2)} x^s dF(z^w)dF(z^s) + \int_{Z_1}^{Z_2}\int_{H_3(z^s,e_3)}^{\overline{z}^w} z^w dF(z^w)dF(z^s) + \int_{Z_1}^{Z_2}\int_{A}^{H_3(z^s,e_3)} x^s dF(z^w)dF(z^s) \\
& + \int_{Z_2}^{\overline{z}^s}\int_{H_4(z^s,e_4)}^{\overline{z}^d} z^w dF(z^w)dF(z^s) + \int_{Z_2}^{\overline{z}^s}\int_{A}^{H_4(z^s,e_3)} x^s dF(z^w)dF(z^s)
\end{aligned}
$$

By deriving the above expression with respect to t_2, making all the assumptions made, and multiplying by t_2, we obtain an expression that depends on elasticities and semi-elasticities.

$$
\begin{aligned}
\frac{\partial \hat{z}}{\partial t_2} t_2 = & -\int_{A}^{Z_1} \underbrace{\left(\frac{\partial H_2(z^s,e_2)}{\partial t_2} t_2\right)}_{\eta_{H_2,t_2}} \left[H_2(z^s,e_2) - x^s(Z_1)\right] f(H_2) dF(z^s) + \int_{A}^{Z_1}\int_{H_2(z^s,e_2)}^{\overline{z}^w} \underbrace{\left(\frac{\partial z^w}{\partial t_2}\frac{t_2}{z^w}\right)}_{\varepsilon_{z^w,t_2}} z^w dF(z^w)dF(z^s) \\
& + \int_{A}^{Z_1}\int_{A}^{H_2(z^s,e_2)} \underbrace{\left(\frac{\partial x^s}{\partial t_2}\frac{t_2}{x^s}\right)}_{\varepsilon_{x,t_2}} x^s dF(z^w)dF(z^s) - \int_{Z_1}^{Z_2} \underbrace{\left(\frac{\partial H_3(z^s,e_3)}{\partial t_2} t_2\right)}_{\eta_{H_3,t_2}} \left[H_3(z^s,e_3) - x^s(Z_1)\right] f(H_3) dF(z^s) \\
& + \int_{Z_1}^{Z_2}\int_{H_3(z^s,e_3)}^{\overline{z}^w} \underbrace{\left(\frac{\partial z^w}{\partial t_2}\frac{t_2}{z^w}\right)}_{\varepsilon_{z^w,t_2}} z^w dF(z^w)dF(z^s) + \int_{Z_1}^{Z_2}\int_{A}^{H_3(z^s,e_3)} \underbrace{\left(\frac{\partial x^s}{\partial t_2}\frac{t_2}{x^s}\right)}_{\varepsilon_{x,t_2}} x^s dF(z^w)dF(z^s) \\
& - \int_{Z_2}^{\overline{z}^w} \underbrace{\left(\frac{\partial H_4(z^s,e_4)}{\partial t_2} t_2\right)}_{\eta_{H_4,t_2}} \left[H_4(z^s,e_4) - x^s(Z_1)\right] f(H_4) dF(z^s) + \int_{Z_2}^{\overline{z}^s}\int_{H_4(z^s,e_4)}^{\overline{z}^w} \underbrace{\left(\frac{\partial z^w}{\partial t_2}\frac{t_2}{z^w}\right)}_{\varepsilon_{z^w,t_2}} z^w dF(z^w)dF(z^s) \\
& + \int_{Z_2}^{\overline{z}^s}\int_{A}^{H_4(z^s,e_4)} \underbrace{\left(\frac{\partial x^s}{\partial t_2}\frac{t_2}{x^s}\right)}_{\varepsilon_{x,t_2}} x^s dF(z^w)dF(z^s)
\end{aligned}
$$

By collapsing all terms, using the same notation for weighted average variables and dividing by $\hat{z}$ we get

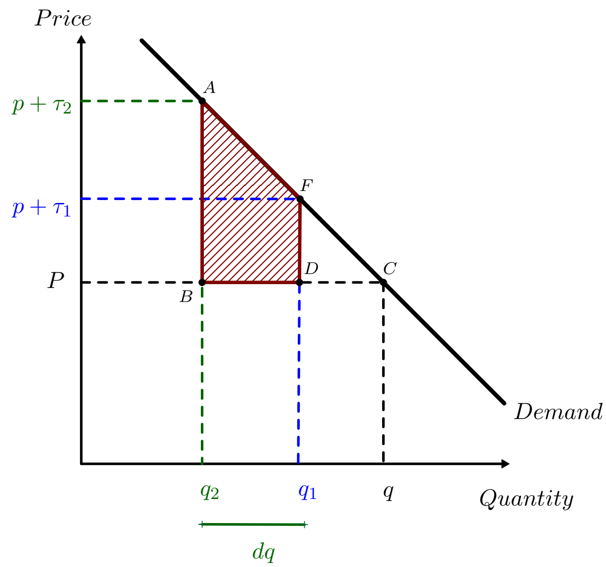
$$
\frac{\partial \hat{z}}{\partial t_2}\frac{t_2}{\hat{z}} = \varepsilon_{z,t} = \hat{\varepsilon}_{z^w,t}\frac{\hat{z}^w}{\hat{z}} + \hat{\varepsilon}_{x,t}\frac{\hat{x}^s}{\hat{z}} - \hat{\eta}_{H,t}\frac{\hat{H}(z^s,e^*) - \hat{x}^s}{\hat{z}}
$$

Derivation of DWL Formulas

Feldstein (1999) show two relevant things in this line: (i) the elasticity of taxable income

$\varepsilon_{z,t}$ (equation (1.10)) is enough to capture the DWL, and (ii) the DWL can be rough using the Harberger's triangle, or a variation of this. We follow this line but connecting the elasticity of taxable income with our measure of the welfare effect using equation (1.11). The Figure 1.5 shows the basic concepts behind Feldstein's derivation.

Figure 1.5: Deadweight Loss



The most popular formula is the Feldstein's triangle. In the figure, the DWL is capture by the triangle ABC. Applying this measure to our model yields

$$DWL = -0.5 \times \frac{t^2}{1-t} \times \varepsilon_{z,t}$$

We denominate this formula as DWL_1.

The policy that we use implies a change from a situation with a positive tax rate to another positive rate. This means that the initial situation had taxes. This scenario is quite different from the context where Harberger's triangle is used because (potentially) we are moving from a situation far from the social equilibrium. For this reason, we examine this effect using a trapezoid instead of a triangle. The second measure of the DWL that we use comes from Feldstein (1999) where the author evaluates an increase in the top-bracket marginal tax rate with the following formula

$$DWL = -0.5 \times \frac{(t_{t+1})^2 - (t_t)^1}{1 - t_t} \times \varepsilon_{z,t}$$

where subscript $t+1$ refers to the final situation and t the initial situation. We denominate this formula as DWL_2.

The final formula comes from our development of the trapezoid that captures the DWL. This approximation comes from the difference between our context and the evaluation made in Feldstein (1999), where we use tax changes across tax bracket and Feldstein evaluate a change in the top bracket. We estimate the trapezoid ABDF in Figure 1.5. Although in Figure 1.5 the tax changes imply an increase and the policy was a decrease, as we show, the trapezoid captures the tax difference allowing us to incorporate the direction of it. Our derivation is

The effect of tax changes is capture by the trapezoid ABDF instead of the triangle ABC in figure 1.5. We estimate the trapezoid's area as the sum of a triangle and a square area; this is

$$\begin{aligned} DWL &= -dq \times \tau_1 + \frac{(\tau_2 - \tau_1)(-dq)}{2} \\ &= (-dq)\frac{(2\tau_1 + \tau_2 - \tau_1)}{2} \\ &= (-dq)\frac{\tau_1 + \tau_2}{2} \end{aligned}$$

where τ is the commodity tax rate that is applied to a consumption good. Replacing $dq = dC$ yields

$$\begin{aligned} DWL &= -0.5 \times dC \times (\tau_1 - \tau_2) \\ &= -0.5 \times \frac{dC}{(1+\tau)} \frac{(1+\tau)}{C} \times \frac{\overbrace{d(1+\tau)}^{=d\tau=(\tau_2-\tau_1)}}{(1+\tau)} C \times (\tau_1 + \tau_2) \\ &= -0.5 \times \varepsilon_{C,\tau} \times \frac{C}{(1+\tau)}(\tau_2^2 - \tau_1^2) \end{aligned}$$

From Feldstein (1999) we know $\varepsilon_{C,\tau} C = -\varepsilon_{z,1-t} TI$, $(1+\tau)^{-1} = 1 - t$ and $\tau = \dfrac{t}{1-t}$. By using these result yields

$$
\begin{aligned}
DWL &= 0.5 \times \varepsilon_{z,1-t} TI \times (1-t) \left[\left(\frac{t_2}{1-t_2} \right)^2 - \left(\frac{t_1}{1-t_1} \right)^2 \right] \\
&= 0.5 \times \varepsilon_{z,1-t} TI \times \left[\frac{t_2^2}{1-t_2} - \frac{t_1^2}{1-t_1} \right]
\end{aligned}
$$

where t_1 is the average marginal tax rate in the initial situation and t_2 when taxes change. Using the above equation, applied to our derivation, the third DWL formula, DWL_3, that we estimate is the following

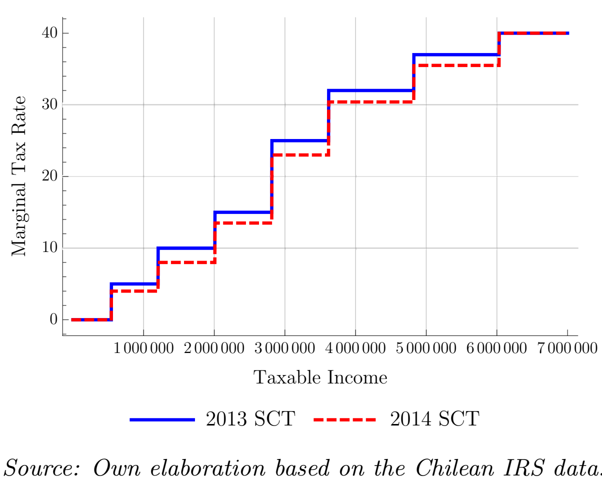
$$
DWL = -0.5 \times \left[\frac{t_{t+1}^2}{1-t_{t+1}} - \frac{t_t^2}{1-t_t} \right] \times \varepsilon_{z,t}
$$

where t_t is the average marginal tax rate in the initial situation and t_{t+1} when taxes change. We denominate this formula as DWL_3.

The two trapezoid formulae capture the direction of tax changes, either falling or increasing. This is showing in the term in bracket. This difference from the triangle potentially improves the measure since considers not only the direction of tax change, but also the magnitude of the effect. In this sense, the trapezoid incorporates a weight of the policy.

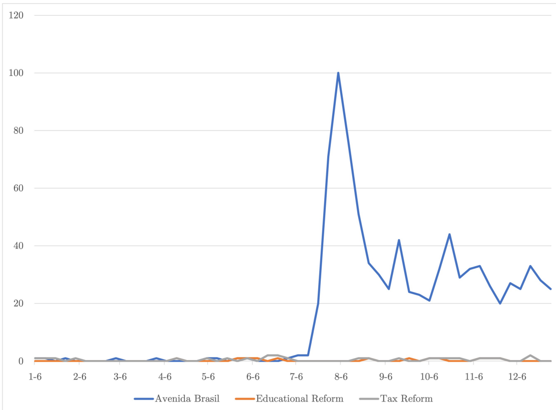
1.9.3 Tables and Figures

Figure 1.6: Second Category Tax Scheme

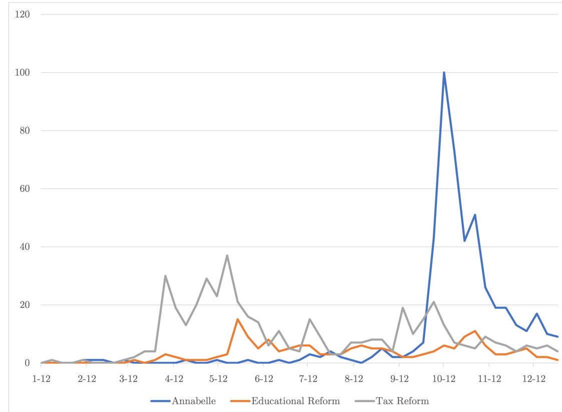


Source: Own elaboration based on the Chilean IRS data.

Figure 1.7: Google Trends Comparison: Educational and Tax reform



(a) 2013



(b) 2014

Note: The numbers in the y-axis represent the search interest relative to the maximum value in the specified region and period list. The value 100 indicates the top popularity of the term, 50 implies half popularity, and 0 means there was not enough data for this term. Numbers in the x-axis show the week for the respective year. Source: Own elaboration based on the Google Trends data.

Table 1.13: Percentage of workers by income bracket and occupation type

Bracket	2006/7		2011/2		2016/7	
	Self-employed	Wage-earner	Self-employed	Wage-earner	Self-employed	Wage-earner
1	82.10%	66.24%	77.77%	62.18%	72.13%	57.24%
2	11.06%	20.71%	14.64%	22.75%	17.33%	25.54%
3	3.89%	7.28%	4.00%	8.86%	5.84%	11.18%
4	1.14%	2.93%	1.97%	3.03%	1.71%	2.98
5	0.71%	1.09%	0.62%	1.44%	1.35%	1.57%
6	0.53%	0.67%	0.46%	1.11%	0.96%	1.01%
7	0.42%	0.44%	0.12%	0.49%	0.47%	0.20%
8	0.14%	0.64%	0.42%	0.14%	0.21%	0.27%
Total sample	24.86%	75.14%	21.88%	78.12%	22.78%	77.22%

Note: Estimations for household heads, calculated using the household sampling weights. Source: Own elaboration based on the EPF data.

Table 1.14: Percentage of Self-employed workers by Household Income Decile

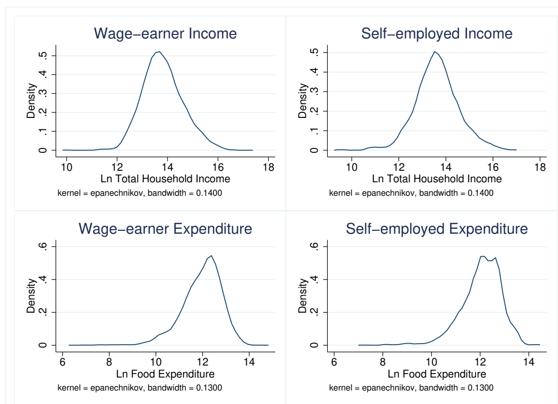
	Self-employed (%)		
Income decile	2006/07	2011/12	2016/17
1	42.86	35.99	39.66
2	27.66	20.18	26.94
3	26.16	22.65	24.45
4	22.84	21.26	17.85
5	20.41	19.09	19.14
6	21.69	22.48	19.62
7	20.84	21.44	17.60
8	18.46	17.90	19.78
9	16.25	13.61	15.07
10	16.17	14.65	15.29
Total	24.86	21.88	22.78

Note: Self-employment rates computed based on the employment type of the household heads considering only remunerative jobs. Source: Own elaboration based on the EPF data (2006-2017).

Figure 1.8: Income and Expenditure Density Function Comparison

(a) EPF VI: 2006/7

(b) EPF VII: 2011/2

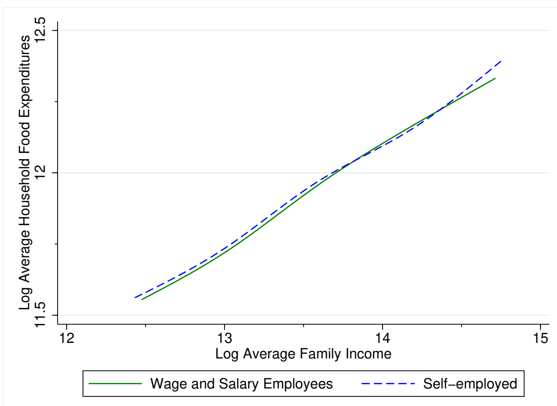


(c) EPF VIII: 2016/7

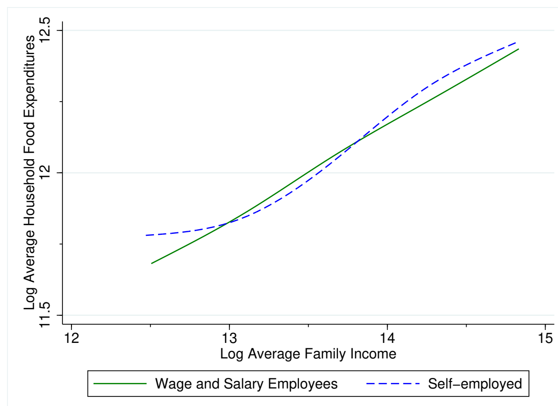
Note: Variables are expressed in natural logarithm. Densities are calculated for the household heads using the household sampling weight. Source: Own elaboration based on the EPF data.

Figure 1.9: Non-parametric Estimates of Total Food Expenditure Engel Curves using Mix Self-employed Household Definition

(a) 2007



(b) 2012



(c) 2017

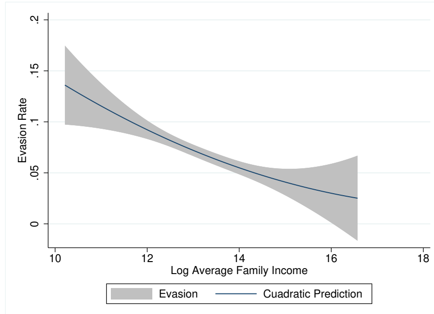
Note: Non-parametric estimation results of the natural logarithm of total food expenditure and household income were obtained after controlling for a set of covariates. Source: Own elaboration based on the EPF data.

Table 1.15: Estimation of Tax Evasion: Tax Evasion Rate vs Intensive

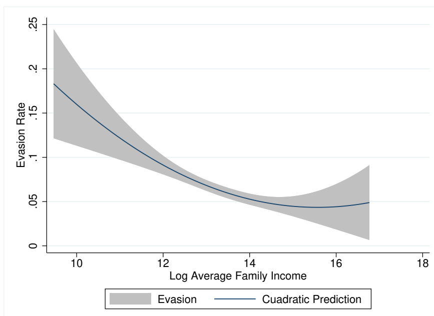
	EPF Survey			
	2007	2013	2017	Average
Panel A: Main Specification				
Evasion Rate	11.44%	9.98%	11.26%	10.89%
Evasion Rate for Evaders	13.76%	11.76%	13.08%	12.87%
Panel B: Household Head Mix				
Evasion Rate	6.89%	6.36%	7.43%	6.89%
Evasion Rate for Evaders	15.28%	13.82%	14.89%	14.66%
Panel C: Household Head Occupation following HLP				
Evasion Rate	11.90%	9.93%	13.31%	11.71%
Evasion Rate for Evaders	16.75%	14.39%	17.37%	16.17%

Note: Evasion Rate refers to the mean misreporting rates over self-employed households. Evasion Rate for Evaders is the percentage of misreporting among self-employed evader households. The means of the estimated coefficients were calculated after replacing negative misreporting rates with zero. Source: Own elaboration based on the EPF data.

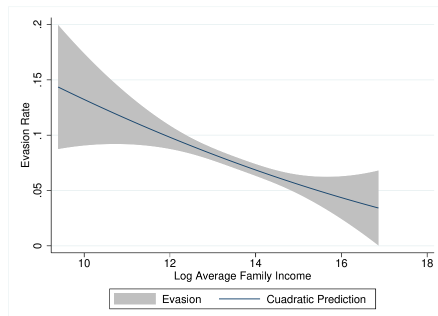
Figure 1.10: Predicted Evasion from the IV Model using Mix Household Definition



(a) 2006/7



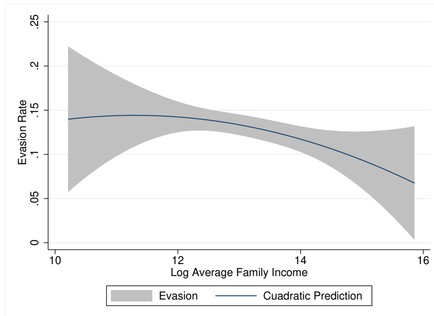
(b) 2011/2



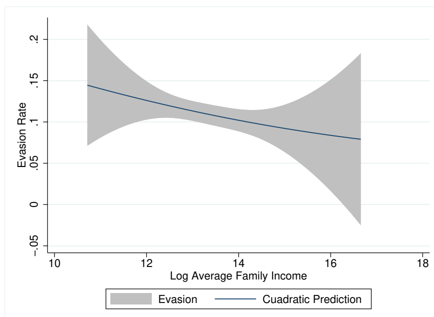
(c) 2016/7

Note: Non-parametric estimation results of the relationship between the evasion rate and the logarithm of the household. Predicted tax evasion is obtained from IV estimation of equation (1.3) changing the definition of self-employed by the occupation declared by the head of the household. Source: Own elaboration based on the EPF data.

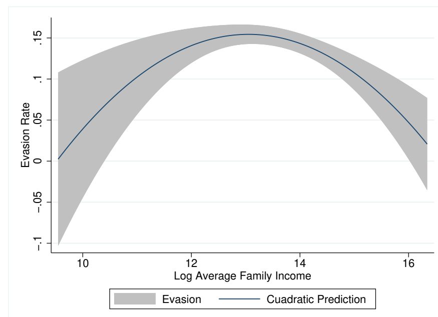
Figure 1.11: Predicted Evasion from the IV Model using the same Household Definition as HLP



(a) 2007



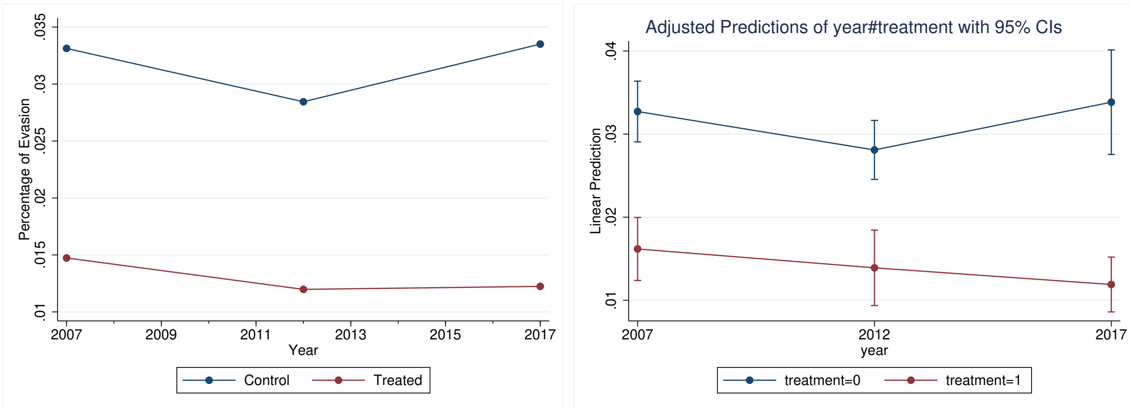
(b) 2012



(c) 2017

Note: Non-parametric estimation results of the relationship between the evasion rate and the logarithm of the household. Predicted tax evasion is obtained from IV estimation of equation (1.3) changing the definition of self-employed by the occupation declared by the head of the household. Source: Own elaboration based on the EPF data.

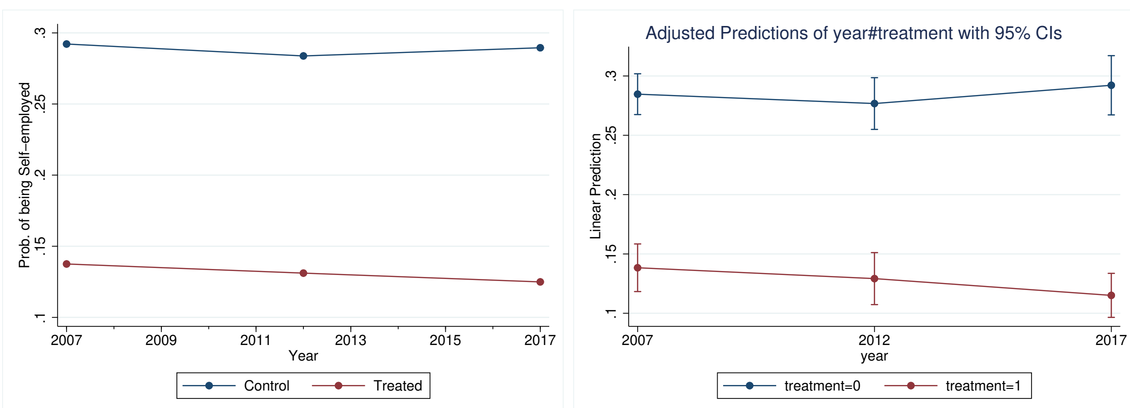
Figure 1.12: Parallel Trend Assumption on Tax Evasion Outcome



(a) Non-Parametric **(b)** Parametric

Note: Non-parametric estimation plots a linear fit of evasion rate estimation by the IV method. Parametric estimation is the predicted value from a regression. Source: Own elaboration based on the EPF data.

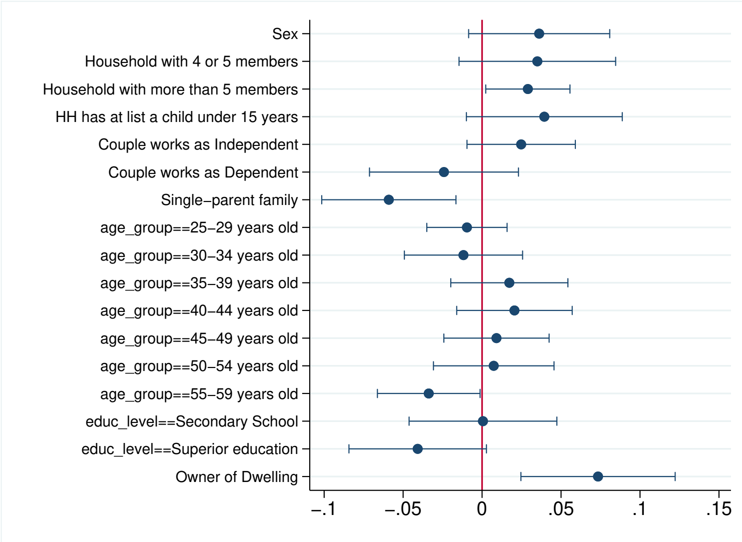
Figure 1.13: Parallel Trend Assumption on Self-employment Outcome



(a) Non-Parametric **(b)** Parametric

Note: Non-parametric estimation plots a linear fit of evasion rate estimation by the IV method. Parametric estimation is the predicted value from a regression. Source: Own elaboration based on the EPF data.

Figure 1.14: Compositional Effect



Note:Parametric estimation results of the Difference-in-Difference using each variable as a dependent variable. Source: Own elaboration based on the EPF data.

Table 1.16: Effect of Tax Reform on Tax evasion instrumenting Income

	(1)	(2)	(3)	(4)
Dep. Vble: Evasion Rate	POLS treatment	POLS PT	POLS MTR	POLS PMTR
Policy	0.0075**	0.0130***		
	(0.003)	(0.003)		
Treatment	0.0402***	0.0351***		
	(0.004)	(0.004)		
Policy × Treatment[a]	-0.0104***	-0.0266***		
	(0.004)	(0.004)		
Ln Mg Net-of-Tax Rate			-0.2919***	-0.2815***
			(0.025)	(0.025)
Observations	14,987	14,987	14,987	14,987
R-squared	0.208	0.202	0.212	0.202
F-statistic	33.70	33.20	35.03	34.37
Covariates	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Delta Tax[b]	-0.016	-0.016		
Mean Evasion[c]	0.026	0.026		
Mean Tax[d]	0.028	0.028		
Elasticity ($^a/_c/^b/_d$)	0.707	1.800		

*Notes: Columns (1) and (2) report POLS estimates of the tax policy change and tax policy change based on predicted treatment, and columns (3) and (4) of the effects of the net-of-marginal-tax rate and its prediction, respectively, using the household sampling weights. Covariates include: male indicator, age-group dummies, education-group dummies, marital status, couple's occupation type, household size dummies, owner dwelling indicator, single-parent family indicator, a child under 15 years in the family indicator, 1-digit industry-codes dummies, and agent's gross income (in Ln). Mean Evasion and Mean Tax are average weights for the whole sample. Robust standard errors in parentheses. Significance level: *** p<0.01, ** p<0.05, * p<0.1.*

Table 1.17: Effect of Tax Reform on Tax evasion using Predicted Income

	(1)	(2)	(3)	(4)
Dep. Vble.:Evasion Rate	POLS treatment	POLS PT	POLS MTR	POLS PMTR
Policy	0.0138***	0.0105***		
	(0.003)	(0.003)		
Treatment	0.0228***	0.0277***		
	(0.003)	(0.004)		
Policy × Treatment^a	-0.0256***	-0.0170***		
	(0.004)	(0.004)		
Ln Mg Net-of-Tax Rate			-0.1116***	-0.2233***
			(0.017)	(0.024)
Observations	15,061	15,061	15,061	15,061
R-squared	0.193	0.193	0.190	0.194
F-statistic	31.65	33.01	32.29	32.94
Covariates	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Delta Tax^b	-0.016	-0.016		
Mean Evasion^c	0.026	0.026		
Mean Tax^d	0.028	0.028		
Elasticity ($^a/_c/^b/_d$)	1.730	1.151		

*Notes: Columns (1) and (2) report POLS estimates of the tax policy change and tax policy change based on predicted treatment, and columns (3) and (4) of the effects of the net-of-marginal-tax rate and its prediction, respectively, using the household sampling weights. Covariates include: male indicator, age-group dummies, education-group dummies, marital status, couple's occupation type, household size dummies, owner dwelling indicator, single-parent family indicator, a child under 15 years in the family indicator, 1-digit industry-codes dummies, and agent's gross income (in Ln). Mean Evasion and Mean Tax are average weights for the whole sample. Robust standard errors in parentheses. Significance level: *** p<0.01, ** p<0.05, * p<0.1.*

Table 1.18: Effect of Tax Reform on Tax evasion using HH Mix Definition

	(1)	(2)	(3)	(4)
Dep. Vble.: Evasion Rate	POLS treatment	POLS PT	POLS MTR	POLS PMTR
Policy	0.0042**	0.0050**		
	(0.002)	(0.002)		
Treatment	0.0202***	0.0165***		
	(0.003)	(0.002)		
Policy × Treatment[a]	-0.0049*	-0.0103***		
	(0.003)	(0.003)		
Ln Mg Net-of-Tax Rate			-0.1708***	-0.1280***
			(0.015)	(0.016)
Observations	15,063	15,063	15,063	15,063
R-squared	0.301	0.299	0.304	0.299
F-statistic	59.29	60.45	61.58	60.59
Covariates	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Delta Tax[b]	-0.016	-0.016		
Mean Evasion[c]	0.022	0.022		
Mean Tax[d]	0.028	0.028		
Elasticity ($^a/_c/^b/_d$)	0.392	0.823		

*Notes: Columns (1) and (2) report POLS estimates of the tax policy change and tax policy change based on predicted treatment, and columns (3) and (4) of the effects of the net-of-marginal-tax rate and its prediction, respectively, using the household sampling weights. Covariates include: male indicator, age-group dummies, education-group dummies, marital status, couple's occupation type, household size dummies, owner dwelling indicator, single-parent family indicator, a child under 15 years in the family indicator, 1-digit industry-codes dummies, and agent's gross income (in Ln). Mean Evasion and Mean Tax are average weights for the whole sample. Robust standard errors in parentheses. Significance level: *** p<0.01, ** p<0.05, * p<0.1.*

Table 1.19: Effect of Tax Reform on Tax evasion using HH HLP Definition

	(1)	(2)	(3)	(4)
Dep. Vble.: Evasion Rate	POLS treatment	POLS PT	POLS MTR	POLS PMTR
Policy	0.0146**	0.0200***		
	(0.006)	(0.006)		
Treatment	0.0293***	0.0212***		
	(0.006)	(0.005)		
Policy × Treatment[a]	-0.0131*	-0.0284***		
	(0.008)	(0.008)		
Ln Mg Net-of-Tax Rate			-0.2390***	-0.1802***
			(0.036)	(0.037)
Observations	7,200	7,200	7,200	7,200
R-squared	0.217	0.216	0.220	0.215
F-statistic	26.34	27.07	27.28	26.75
Covariates	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Delta Tax[b]	-0.016	-0.016		
Mean Evasion[c]	0.030	0.030		
Mean Tax[d]	0.028	0.028		
Elasticity ($^a/_c/^b/_d$)	0.766	1.660		

*Notes: Columns (1) and (2) report POLS estimates of the tax policy change and tax policy change based on predicted treatment, and columns (3) and (4) of the effects of the net-of-marginal-tax rate and its prediction, respectively, using the household sampling weights. Covariates include: male indicator, age-group dummies, education-group dummies, marital status, couple's occupation type, household size dummies, owner dwelling indicator, single-parent family indicator, a child under 15 years in the family indicator, 1-digit industry-codes dummies, and agent's gross income (in Ln). Mean Evasion and Mean Tax are average weights for the whole sample. Robust standard errors in parentheses. Significance level: *** p<0.01, ** p<0.05, * p<0.1.*

Table 1.20: Effect of Tax Evasion on Self-employment instrumenting Income

	(1)	(2)	(3)	(4)	(5)
Dep. Vble.: Self-employment	POLS	IV-DiD	IV-DiD PT	IV-DiD MTR	IV-DiD PMTR
Evasion Rate^{a}	2.9477***	6.7902***	6.4431***	7.3514***	7.3296***
	(0.064)	(2.072)	(0.838)	(0.606)	(0.608)
Policy		0.0061	0.0063		
		(0.012)	(0.012)		
Treatment		0.0081	0.0285		
		(0.081)	(0.028)		
Observations	14,987	14,987	14,987	14,987	14,987
R-squared	0.490	0.055	0.131	-0.083	-0.077
F-statistic	240	64.57	68.39	60.45	60.85
F-stat. Weak Ident.		4.02	21.62	64.29	59.92
Covariates	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓
Mean Evasion^{b}	0.026	0.026	0.026		
Semi-elasticity ($a \times b$)	0.076	0.176	0.167		

*Notes: Column (1) reports POLS estimates, in columns (2) and (3) evasion is instrumented using the tax policy change and its prediction, and in columns (4) and (5) the net-of-marginal-tax rate, using the household sampling weights. Covariates include: male indicator, age-group dummies, education-group dummies, marital status, couple's occupation type, household size dummies, owner dwelling indicator, single-parent family indicator, a child under 15 years in the family indicator, 1-digit industry-codes dummies, and agent's gross income (in ln). Mean Evasion is the average weights in the whole sample. Robust standard errors in parentheses. Significance level: *** p<0.01, ** p<0.05, * p<0.1.*

Table 1.21: Effect of Tax Evasion on Self-employment using Predicted Income

	(1)	(2)	(3)	(4)	(5)
Dep. Vble.: Self-employment	POLS	IV-DiD	IV-DiD PT	IV-DiD MTR	IV-DiD PMTR
Evasion Rate[a]	3.0580***	7.3612***	5.7736***	6.5663***	7.1789***
	(0.057)	(0.943)	(1.143)	(0.946)	(0.665)
Policy		0.0057	0.0095		
		(0.013)	(0.011)		
Treatment		-0.0209	0.0361		
		(0.021)	(0.031)		
Observations	15,061	15,061	15,061	15,061	15,061
R-squared	0.483	-0.083	0.258	0.107	-0.036
F-statistic	251.1	58.71	75.65	68.47	62.44
F-stat. Weak Ident.		39.67	17.54	41.29	82.42
Covariates	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓
Mean Evasion[b]	0.026	0.026	0.026		
Semi-elasticity ($a \times b$)	0.079	0.191	0.150		

*Notes: Column (1) reports POLS estimates, in columns (2) and (3) evasion is instrumented using the tax policy change and its prediction, and in columns (4) and (5) the net-of-marginal-tax rate, using the household sampling weights. Covariates include: male indicator, age-group dummies, education-group dummies, marital status, couple's occupation type, household size dummies, owner dwelling indicator, single-parent family indicator, a child under 15 years in the family indicator, 1-digit industry-codes dummies, and agent's gross income (in ln). Mean Evasion is the average weights in the whole sample. Robust standard errors in parentheses. Significance level: *** p<0.01, ** p<0.05, * p<0.1.*

Table 1.22: Effect of Tax Evasion on Self-employment using HH Mix

	(1)	(2)	(3)	(4)	(5)
Dep. Vble.: Self-employment	POLS	IV-DiD	IV-DiD PT	IV-DiD MTR	IV-DiD PMTR
Evasion Rate[a]	2.3342***	13.6500*	12.3003***	10.8059***	11.8331***
	(0.070)	(6.983)	(3.377)	(1.000)	(1.390)
Policy		-0.0107	-0.0072		
		(0.021)	(0.018)		
Treatment		-0.0496	-0.0169		
		(0.132)	(0.053)		
Observations	15,063	15,063	15,063	15,063	15,063
R-squared	0.333	-2.269	-1.686	-1.126	-1.501
F-statistic	175.4	18.71	22.13	28.22	24.28
F-stat. Weak Ident.		3.42	11.76	122.27	67.06
Covariates	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓
Mean Evasion[b]	0.022	0.022	0.022		
Semi-elasticity ($a \times b$)	0.051	0.301	0.271		

*Notes: Column (1) reports POLS estimates, in columns (2) and (3) evasion is instrumented using the tax policy change and its prediction, and in columns (4) and (5) the net-of-marginal-tax rate, using the household sampling weights. Covariates include: male indicator, age-group dummies, education-group dummies, marital status, couple's occupation type, household size dummies, owner dwelling indicator, single-parent family indicator, a child under 15 years in the family indicator, 1-digit industry-codes dummies, and agent's gross income (in ln). Mean Evasion is the average weights in the whole sample. Robust standard errors in parentheses. Significance level: *** p<0.01, ** p<0.05, * p<0.1.*

Table 1.23: Effect of Tax Evasion on Self-employment using HH HLP

	(1)	(2)	(3)	(4)	(5)
Dep. Vble.: Self-employment	POLS	IV-DiD	IV-DiD PT	IV-DiD MTR	IV-DiD PMTR
Evasion Rate^a	3.0536***	6.0708**	5.3846***	8.3492***	8.0558***
	(0.107)	(2.783)	(1.358)	(1.140)	(1.427)
Policy		-0.0230	-0.0175		
		(0.021)	(0.016)		
Treatment		0.0578	0.0739***		
		(0.067)	(0.023)		
Observations	7,200	7,200	7,200	7,200	7,200
R-squared	0.516	0.196	0.327	-0.475	-0.368
F-statistic	138.5	46.06	51.87	31.06	32.20
F-stat. Weak Ident.		3.20	11.43	43.68	23.83
Covariates	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓
Mean Evasion^b	0.030	0.030	0.030		
Semi-elasticity ($a \times b$)	0.092	0.182	0.162		

*Notes: Column (1) reports POLS estimates, in columns (2) and (3) evasion is instrumented using the tax policy change and its prediction, and in columns (4) and (5) the net-of-marginal-tax rate, using the household sampling weights. Covariates include: male indicator, age-group dummies, education-group dummies, marital status, couple's occupation type, household size dummies, owner dwelling indicator, single-parent family indicator, a child under 15 years in the family indicator, 1-digit industry-codes dummies, and agent's gross income (in ln). Mean Evasion is the average weights in the whole sample. Robust standard errors in parentheses. Significance level: *** p<0.01, ** p<0.05, * p<0.1.*

Chapter 2

Tax Policies Design in a Hierarchical Two-Side Model with Occupational Decision

2.1 Introduction

Increased tax revenue has continuously been an important issue for governments because of increasing social expenditures and paying debts or saving for future spending. One option to raise tax collection is fighting against tax evasion since it increases revenue through effectiveness instead of changing the tax policy, which bureaucratically takes more time. For example, after the economic crisis of 2008, developed countries have focused on the fight against tax evasion and in understanding its mechanisms aiming to increase tax revenues (Slemrod, 2019). Therefore, enhancing the effectiveness of enforcement policies appears to be an excellent option to expand expenditure and maintain the institutional background at the same time.

Despite the importance of the increase tax collection, less attention has been paid to distortions made by those policies in more complex and realistic environments. As a matter of fact, some policies in the spirit of increasing tax revenue and improving income inequality end up in distortions over occupational decisions. For instance, in an environment with two possible occupations, a progressive tax schedule may induce an increase in evasion for risk-averse agents (Watson, 1985), and having high taxes changes resources to industries where evasion exists (Kesselman, 1989). Those facts come from the incentives to move into an occupation where evasion is more accessible than others and reflects the analysis's complexity.

Occupational decisions appear as a critical issue in distortions made by tax policies. Empirically speaking, some studies reinforce this fact. Firstly, Kleven et al. (2011) and Slemrod (2007) evidence that evasion in the third-party income reports is low, and this problem arises mainly in the income self-report occupations. Additionally, Bárány (2019) shows that high tax rates in the dependent employment sector produce high rates of self-employment. From a theoretical point of view, Kuchumova (2017) demonstrates that the tax collection agency should audit occupations with a minor level of information income reports, as this is an occupation with a self-declaration of their incomes. Bearing in mind these results, taking into account occupational decisions into tax policy models appears to be reasonable.

The current study incorporates the occupational decision into a hierarchical model formerly developed by Sanchez and Sobel (1993) to investigate possible distortions in the tax policies design: audits, the marginal tax rate and the budget for the IRS. As reported by Melumad and Mookherjee (1989), hierarchical models are useful to make normative analysis. Therefore, this paper focuses on the normative implications of incorporating occupational decisions into a tax policy model. Simultaneously, it intends to investigate further into other relevant issues, namely the forces behind the optimal policies given tax evasion and occupational choice, the possible distortion in the optimal allocation of agents driven by tax evasion or the institutional background, and what the mechanisms behind the possible distortions are.

Throughout the current study, a three-stage game is built in which the government, the tax authority (henceforth the IRS) and a continuum of risk-neutral agents interact. This is, each agent chooses between two possible occupations, being an employee or working as a self-employed. Additionally, each agent has one productivity in each occupation, which comes from independent distribution functions. In the dependent sector, agents cannot collude with their employer to declare less income, avoid taxes and share the gains. On the other hand, in the self-employment sector agents self-report their income, being able to misreport income and evade taxes. In the first stage, the government maximizes a social welfare function and commits to a linear marginal tax rate, a level of public goods, and the IRS budget. In the second stage, the IRS determines an audit function that seeks to maximize the expected tax collection subject to its budget constraint. Finally, in the third stage, agents maximize their utility choosing an occupation. In the case of deciding to work as a self-employed, it also chooses its income declaration. The agents make both decisions observing their productivity (which is private information), the audit function, the marginal tax rate, and the public goods provision.

Since productivity is private information, the revelation principle is used to solve the IRS's problem. Therefore, this study focuses on finding a direct incentive-compatible mechanism composed of a direct audit function and effective taxes, which are the tax liability that incentivizes agents to reveal their real income. The audit function takes a cut-off form, as in Sanchez and Sobel (1993), however, in this case the threshold level relies on the dependent productivity distribution as well. This solution suggests that, if the IRS has an insufficient budget to audit the entire self-employment sector, the audit level is efficient until the threshold wage, and after this, they do not face an audit. Thus, this result produces that effective tax is the same for all agents who do not face an audit. This fact generates a distortion in the occupational decision because agents with self-employment incomes higher than the threshold wage have the incentive to work as self-employed when they are more productive in the dependent sector. Finally, it is demonstrated that this kind of result holds for two extensions: a) if the audit cost can depend on the self-employment wage, it must be monotonically non-decreasing on self-employment income and b) if the fine rate is increasing in the self-employment wage, it must be bounded from above.

The optimal public good provision is equal as in the first-best solution as agents are risk-neutral. As is well explained in Boadway and Keen (1993), if agents have linear utility for consumption (or are risk-neutral on it), any change through tax evasion, and the consequent increase in the after-tax income, does not produce an impact on the optimal public good provision. These results come from the non-alteration of the marginal rate of substitution between the public good and private consumption. However, for the current study, this result reinforces the idea that tax evasion produces a distortion in the distribution due to the fact that some agents, those who face an audit and those who are dependent workers, increase the burden to finance public good in comparison to a situation without tax evasion.

The optimal marginal tax rate is smaller than one and can be characterized by two effects: welfare and revenue. This result suggests two critical highlights. First, if the government does not consider the occupational decision, taxes have an upward bias. This conclusion comes from the comparison with Sanchez and Sobel (1993), who obtain a marginal tax rate equal to one. Second, the optimal tax must consider, at the same time, the impact on tax revenue and social welfare. It is also demonstrated during this study that differential taxation, i.e., one marginal tax rate in each occupation, is possible but it implies a bigger marginal tax rate in the self-employed sector. This result produces an even more significant distortion in the allocation of agents than the scenario with only one marginal tax rate.

The optimal IRS's budget is insufficient to audit the entire self-employed sector and three effects can characterize this: behavioral, mechanical, and welfare. Even though the budget cannot allow audit all self-employed workers, this level is higher than the result from a cost-benefit analysis. This fact comes from the necessity to take into account the agents tax burden (Slemrod and Yitzhaki, 1987). Finally, this result shows that, in determining the size of the tax collection agency, the government must consider the effect of welfare and revenue (the mechanical effect) and the implication on the occupational decision made by agents (the behavioral effect).

This study draws upon the classic literature on evasion, beginning with the contribution of Allingham and Sandmo (1972) and Yitzhaki (1974). The literature is faced with the question on the relation between optimal taxation, optimal audit scheme and occupational choices before this work. In this line, the seminal paper is Sandmo (1981), which focuses on the impact of tax evasion in the labor market in an economy with two fixed sectors. Pestieau and Possen (1991) concentrate on the differences in risk-aversion among taxpayers who choose between a riskless work and a risky entrepreneurial activity in which evasion is possible. These authors find that the tax rate decreases with audit costs and increases with the concern for equality, and the audit scheme takes only extreme values. Similarly, Boadway, Marchand and Pestieau (1991) investigate the consequences of occupational choice in a linear income tax, where the occupational choice is between entrepreneurship and wage-earning. They claim that optimal tax rates depend on three effects: efficiency, equity and insurance, and heterogeneity in attitude toward risk. Furthermore, Parker (1999) empirically studies linear taxation in the presence of occupational choice adding the possibility of unemployment. He calibrates the model using UK's economy, finding that the optimal tax solution implies higher marginal rates and redistribution, and the self-employment sector faces large marginal tax rates but fewer effective tax rates than third-party employees do.

Although some studies incorporate occupational decisions there has not been any studies that focus on the implications of not considering them. Ergo, this paper's main contribution is to contrast the effect of not considering occupational decision into a model of optimal policy design. What is more, this study seeks to shed light into the consequences for the tax policy design taking into account, at the same time, occupational decision and income tax evasion. The interaction between occupational decision and income tax evasion allows a better understanding of the possible distortions behind the non-optimal policies. To explain further, tax evasion may come from the permission of institutional background instead of purely willingness from agents (Kleven et al., 2011). This fact could be partially explained by not considering both issues tax evasion and occupational choice simultaneously in the tax policy design.

Also, this paper contributes to the literatures of optimal tax policies design. In this line, the hierarchical model developed by Sanchez and Sobel (1993) is used as a benchmark. They make a hierarchical tax policy model where the tax collection agency maximizes tax revenue and interacts with a government in an economy with only one sector and a continuum of risk-neutral agents. These authors' paper shows that the audit scheme takes only extreme values. The IRS audits with the intensity to deter the evasion below some threshold, and above this threshold, audit equals zero. In addition, they show that the budget is insufficient to audit all taxpayers. This fact is given by the agents tax burden's effect, as is well explained in Slemrod and Yitzhaki (1987). Finally, they find that the optimal marginal rate is equal to one, and all the redistribution is through the public goods provision and a subsidy equal for all agents. Some of these results change by including occupational decisions, evidencing the relevance of incorporating this element in the design of tax policies.

The rest of the current paper is organized as follows. Section 2.2 presents the model, the requirement for equilibrium, and provides the first-best solution and the decentralization of it. Section 2.3 solves the IRS's problem, ensuring a second-stage equilibrium, and providing the characterization of the optimal audit policy. Section 2.4 solves the government's problem, obtaining the optimal tax schedule, the IRS budget, and the optimal provision of public goods. Section 2.5 briefly discusses some issues and provides a comparative static over the tax rate and the budget for the IRS. Finally, section 2.6 provides the conclusions of this study.

2.2 Model, Benchmark and Equilibrium Concepts

2.2.1 Model

The economy has three agents: the government, the IRS, and a continuum of risk-neutral agents of mass one, which interact in a hierarchical tax administration model. Also, the economy has two sectors, dependent and self-employment, indexed by $i = \{d, s\}$. Since the IRS perfectly knows in what sector each agent works, it is impossible to evade taxes in the dependent sector. However, in the self-employment sector tax evasion is possible, and the income verification is costly. Let us assume a homogeneous consumption good, which production is linear, uses only labor as an input, and this price is normalized to one. Those facts produce that each sector's wage rate is equal to one, and the effective wage is the same as the agent's productivity. In addition, it is implicitly assumed that the government has no resources (the technology, knowledge, expertise among other) to perform the audit process efficiently. What has been previously mentioned produces the necessity of the existence of a specialized agency to look after the tax compliance. Yet, as it is well explained in Melumad, Mookherjee and Reichelstein (1995), it is possible to attain the same second best performance level as one without an IRS in this setting.

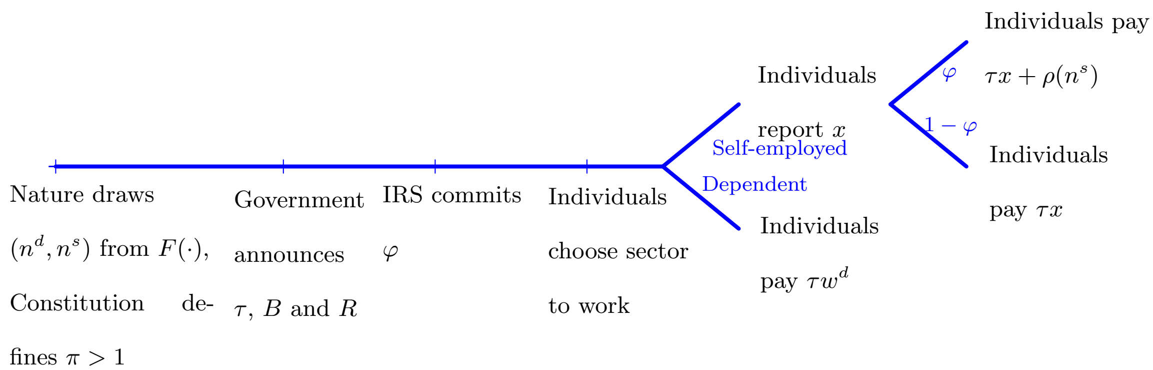
One interesting way to analyze income tax evasion in an occupational choice model is through differential taxation. Gomes, Lozachmeur and Pavan (2017) find the optimal differential taxation into an occupational decision model quite similar to the model developed in this paper. Given that, a natural question is about the possibility of analyzing distortions in tax policy directly through differential taxation. Appendix 2.7.1 faces this question and demonstrates that it is not trivial to follow this way.

The timing of the game is the following: in the first stage, the government chooses a linear marginal tax rate, τ, a budget for the IRS, B, and a level of public goods provision, R. The government cannot impose a marginal tax higher than one.[1] This assumption is not restrictive, afterwards, it will be shown that its holds at the optimum. The IRS selects an audit function in the second stage, φ. Finally, in the third stage, the agents decide the sector to work endogenously, after seeing the tax policy decisions chosen by the government and the IRS. If they decide to work as self-employed, choose the IRS's income report. A graphic exposition of the above explanation is

[1] Mirrlees (1971) obtains this as a result of an optimal non-linear tax schedule.

illustrated in Figure 2.1.

Figure 2.1: Timing of the Game



Agents

Let us assume a continuum of risk-neutral agents of mass one. Nature randomly chooses a pair of productivities $n = (n^d, n^s)$, each term represents the agent's productivity in the dependent and self-employment sector respectively, and is drawn independently from a distribution function. Formally, n^i comes from a cumulative distribution function F^i, twice continuously differentiable, with support $[\underline{n}^i, \overline{n}^i]$ for $i = \{d, s\}$. This function is common knowledge in the model.

The agents decide their occupation considering the government's tax schedule and the audit function committed by the IRS. When it comes to choosing an occupation, the agents compare their utility in each sector and choose which one gives them the highest utility. In each sector, the agents offer an inelastic labor supply equal to one.[2] The agents' utility function is separable between consumption, C, and public goods, R. Since agents are risk-neutral, their utility can be written as a quasi-linear function in consumption. In particular, let ϕ be the benefit for a certain level of public goods, which are increasing and concave (i.e., $\phi' > 0$ and $\phi'' < 0$). The utility for each agent, independently of its occupation, is

$$u(C, R) = C + \phi(R)$$

In the dependent sector, firms declare to the IRS the agent's income.[3] This fact produces that

[2] Implicitly, it has been taken into consideration the fact that effective wage is equal to productivity and write w or n indifferently.

[3] Collusion between firms and workers to underreport wages and divide the evaded amount is impossible. This

consumption in the dependent sector is equal to the after-tax income, i.e., $C(w^d) = w^d - \tau w^d$. Consequently, the utility in the dependent sector is

$$U^d(w^d) = w^d(1-\tau) + \phi(R)$$

In the self-employment sector, an agent makes an income declaration equals to x. Formally $x : [\underline{n}^s, \overline{n}^s] \to \mathbb{R}$. The IRS does not reward over-reporting implying that any declaration above real productivity is a dominated strategy. Hence, the income declaration will not exceed the real income, $x \leq w^s$. Agents will be audit with probability φ. If the IRS audits a taxpayer, it will be immediately discovered whether they evaded taxes and must pay the penalty ρ, which is a fine rate $\pi > 1$ over the evaded taxes. The penalty takes the following form

$$\rho(w^s) = \begin{cases} \pi\left(\tau w^s - \tau x\right) & \text{if } x < w^s \\ 0 & \text{if } x \geq w^s \end{cases}$$

It can be defined $U^s(w^s)$ as the utility with the optimal income declaration. Formally, this utility level is given by the following

$$U^s(w^s) = \max_{x \leq w^s} w^s - \tau x - \varphi(x) \max\left\{\pi\left(\tau w^s - \tau x\right), 0\right\} + \phi(R)$$

Let us denote by $x(w^s)$ the solution to this problem. Notice that, in this case, the expected consumption is equal to the expected after-tax income, $C(w^s) = w^s - \tau x - \varphi(x) \max\left\{\pi\left(\tau w^s - \tau x\right), 0\right\}$. Anticipating the optimal declaration in the self-employment sector, each agent chooses the sector in which it works. Now, $\mathcal{N}^i$ can be defined as the set of agents that decide to work in the sector i. Formally $\mathcal{N}^i(w^i) = \{(w^d, w^s) \mid U^i(w^i) \geq U^j(w^j)\}$ with $j \neq i$, and $i, j = d, s$.

IRS

The IRS chooses the audit function to maximize the expected tax collection, taking as given the marginal tax rate and its budget, which are determined by the government, and the fine rate. Formally, define the audit function as $\varphi : [\underline{w}^s, \overline{w}^s] \to [0, 1]$, and the cost of auditing an agent c, that

fact implies that firms always declare each agent's real productivity and tax evasion does not exist.

is linear and constant. The IRS has a budget B to finance the cost of its enforcement policy.[4] If the IRS does not use its entire budget, the excess must be returned to the government as a transfer along with the tax collection. The problem of the IRS is as follows

$$\max_{\varphi:[\underline{w}^s,\overline{w}^s]\to[0,1]} \int\limits_{\mathcal{N}^d(w^d)} \tau w^d dF^d(w^d)dF^s(w^s) \; + \int\limits_{\mathcal{N}^s(w^s)} \left[\tau x(w^s) + \varphi(x(w^s))\rho(w^s)\right] dF^d(w^d)dF^s(w^s)$$

$s.t.$

$$B \geq \int\limits_{\mathcal{N}^s(w^s)} c \cdot \varphi(x(w^s))dF^d(w^d)dF^s(w^s)$$

$$x(w^s) \in arg \max_{\tilde{x}(w^s)} \{U^s(w^s)\}$$

Government

The government chooses the marginal tax rate, $\tau \in [0, 1]$, a public goods provision, R, and the IRS budget, B, to maximize a social welfare function (SWF). It anticipates the optimal audit scheme obtained by the IRS, the decision made by agents, and uses the known information, i.e., the distribution of productivities and the form of utility in each sector. Let us define G as the social welfare function, and assume that $G' > 0$ and $G'' \leq 0$.

The government has a budget constraint that states the expected tax collection and transfers must go to finance a provision of public goods and the IRS budget. The cost of one unit of a public

[4] It is assumed that the IRS bears no other costs, like the operation cost.

good is one. The problem of the government is as follows

$$\max_{\tau,R,B} \int\limits_{\mathcal{N}^d(w^d)} G\left(U^d(w^d)\right) dF^d(w^d)dF^s(w^s) + \int\limits_{\mathcal{N}^s(w^s)} G\left(U^s(w^s)\right) dF^d(w^d)dF^s(w^s)$$

$s.t.$

$$\int\limits_{\mathcal{N}^d(w^d)} \tau w^d dF^d(w^d)dF^s(w^s) + \int\limits_{\mathcal{N}^s(w^s)} \left[\tau x(w^s) + \varphi(x(w^s))\rho(w^s)\right] dF^d(w^d)dF^s(w^s)$$

$$+ \left(B - \int\limits_{\mathcal{N}^s(w^s)} c \cdot \varphi(x(w^s))dF^d(w^d)dF^s(w^s)\right) \geq B + R$$

$$x(w^s) \in arg\max_{\tilde{x}(w^s)}\{U^s(w^s)\}$$

φ solves the problem of the IRS

In this case, we assume that G depends on the agent's utility instead of their sum. This assumption is made to capture the government concern for each possible income realization, obtaining a generalized social weight (Saez and Stantcheva, 2016).

2.2.2 Third Stage Equilibrium

Any equilibrium in the third stage requires a definition of the optimal income declaration and a characterization over the optimal occupational choices. Using the definition of $x(w^s)$ as the optimal income declaration implies that, at the equilibrium, any agent in the self-employment sector declares $x(w^s)$ and has a utility equals to $U^s(w^s)$. Hereafter, wages instead of productivity to clarify the notation and the results are used.

Definition 1 (Occupational Choice Rule)**.** *Define $\mathcal{W}^i(w^i)$ as the occupation set, which is the result of occupation decisions made by agents, in the following form*

$$\mathcal{W}^s(w^s) = \left\{(w^d, w^s) \in [\underline{w}^d, \overline{w}^d] \times [\underline{w}^s, \overline{w}^s] \;\mid\; U^s(w^s) \geq U^d(w^d)\right\}$$
$$\mathcal{W}^d(w^d) = \left\{(w^d, w^s) \in [\underline{w}^d, \overline{w}^d] \times [\underline{w}^s, \overline{w}^s] \;\mid\; U^s(w^s) < U^d(w^d)\right\}$$

This rule establishes, implicitly, a threshold function that defines a critical productivity/wage limit in the self-employment sector. This is up to what wage level some agents prefer to work as a self-employed.

Lemma 1 (Threshold Function)**.** *There exists a threshold function, $\kappa : [\underline{w}^s, \overline{w}^s] \rightarrow \mathbb{R}_+$, such that*

$$(w^d, w^s) \in \mathcal{W}^s(w^s) \Leftrightarrow w^d \leq \kappa(w^s)$$

Proof. See Appendix 2.7.4 ■

Considering the above analysis, a requirement for a third-stage equilibrium must contain two elements: 1) an optimal agents' income declaration, and 2) its occupational decision should reflect the sector in which they obtain the largest utility from. The formal third stage equilibrium is as follows.

Definition 2. *A third stage equilibrium is given by:*

1. *Each agent in the self-employment sector chooses $x(w^s) \in arg\max U^s(w^s)$.*
2. *The occupational decisions made for the agents is optimal and satisfy conditions in Definition 1.*
3. *The threshold function exists and holds with Lemma 1.*

2.2.3 Second Stage Equilibrium

In the second stage, the IRS maximizes the expected tax collection choosing an audit function anticipating the optimal decision made by the agents. Agents in the self-employment sector can hide information from the IRS and optimally choosing $x(w^s) \leq w^s$. Therefore, it is necessary to use a method that makes agents declare their real wage; this method is a mechanism design approach. Considering the revelation principle (Myerson, 1979, 1981), finding a direct incentive-compatible (IC) mechanism $\mathcal{M} : \{\alpha, \mathcal{T}\}$ is without loss of generality.[5] Being α the probability to audit an agent and $\mathcal{T}$ the effective tax paid by an agent, namely $\mathcal{T}(w^s) = \tau x(w^s)$, the direct IC mechanism

[5] In the dependent sector, agents cannot misreport their income; implying that it is not necessary to define an IC mechanism there.

is as follows

$$\mathcal{M} = \begin{cases} \mathcal{T}: & [\underline{w}^s, \overline{w}^s] \to [0, \overline{w}^s] \\ \alpha: & [\underline{w}^s, \overline{w}^s] \to [0, 1] \end{cases}$$

A direct IC mechanism in this setting produces that the optimal income declaration of every self-employed agent is equal to the real income earned ($x(w^s) = w^s$). Thus, in equilibrium, evasion does not exist.

The mechanism needs to be composed of an audit function and a tax liability, the effective taxes, which together produce that agents reveal their real income. Unless the mechanism follows the implementable requirements (declared below) some self-employed workers may have the incentive to cheat the IRS and hide part of their, or their whole, income. This approximation produces a simplification of the problem and allowing seeing the tax policy implication to incorporate the occupational choice.

Let us define $V^s(w^s, \mathcal{M})$ as the indirect utility in the self-employment sector given a mechanism $\mathcal{M}$. Any implementable IC mechanism has the following characteristics.

Lemma 2. *The IC mechanism $\mathcal{M}$ is implementable if and only if*

1. *$\alpha(w^s)$ is non-increasing in w^s*

2. $$\mathcal{T}(w^s) = \frac{w^s(1 - \tau\alpha(w^s)\pi)}{1 - \pi\alpha(w^s)} - \frac{\int_{j=\underline{w}^s}^{w^s} (1 - \alpha(j)\pi\tau)\, dj}{1 - \pi\alpha(w^s)} - \frac{V^s(\underline{w}^s)}{1 - \pi\alpha(w^s)}$$

Proof. See Appendix 2.7.5 ■

In the case that the audit function takes the value of $1/\pi$, the agent always reveals its real productivity.[6] This result was first formally established by Scotchmer (1987), and it allows redefining the support for audit function in the mechanism as $\alpha(w^s) \in [0, 1/\pi]$.[7]

[6] To see this, note that when $\varphi(w^s) = 1/\pi$ the utility in the self-employed sector is $U^s(w^s) = w^s - \tau w^s$, implying that $x(w^s) = w^s$.

[7] Additionally, condition 1 in Lemma 2 gives a restriction on marginal taxes which is the same as the assumption made early:

$$\begin{aligned} 1 - \alpha(w^s)\pi\tau &\geq 0 \\ \frac{1}{\alpha(w^s)\pi} &\geq \tau \end{aligned}$$

The upper limit of the audit function is $1/\pi$ and the marginal tax is limited from above with a decreasing function

Lemma 2 establishes the requirements for any mechanism to be implementable. Keeping in mind these requirements, it is possible to redefine the occupational choice rule for the second stage equilibrium and the threshold function that results from this.

Definition 3 (Occupational Choice Rule in an IC Mechanism)**.** *For a direct IC mechanism, it can be defined $\mathcal{W}^i(\mathcal{M})$ as the occupational set which is the result of agents occupational decisions by*

$$\mathcal{W}^s(\mathcal{M}) = \Big\{(w^d, w^s) \in [\underline{w}^d, \overline{w}^d] \times [\underline{w}^s, \overline{w}^s] \;\mid\; V^s(w^s, \mathcal{M}) \geq U^d(w^d)\Big\}$$
$$\mathcal{W}^d(\mathcal{M}) = \Big\{(w^d, w^s) \in [\underline{w}^d, \overline{w}^d] \times [\underline{w}^s, \overline{w}^s] \;\mid\; V^s(w^s, \mathcal{M}) < U^d(w^d)\Big\}$$

Lemma 3 (IC Threshold Function)**.** *For any direct IC mechanism $\mathcal{M}$, there exists a threshold function $\kappa : [\underline{w}^s, \overline{w}^s] \rightarrow \mathbb{R}_+$, that establishes the maximum wage to work as a self-employed*

$$w^d \leq \kappa(w^s)$$

Moreover, in equilibrium the threshold function is characterized by

$$\kappa'(w^s) = \frac{V^s_{w^s}(w^s, \mathcal{M})}{U^d_{\kappa(w^s)}(\kappa(w^s))}$$

where X_i denotes to the derivative of variable X with respect to i.

Proof. See Appendix 2.7.6 ■

The second stage equilibrium establishes the conditions over the IRS's choice and the allocation of agents in equilibrium. Furthermore, by definition, any equilibrium that fulfills the following conditions also holds with Definition 2 and, consequently, is a third stage equilibrium.

Definition 4. *The second stage equilibrium is given by:*

1. *The IRS maximizes the expected tax collection in the set of implementable direct IC mechanisms, which follow the conditions in Lemma 2.*

2. *The occupational decisions made by agents is optimal and satisfies the conditions in Definition 3.*

in the audit. This fact implies that the most restrictive limit is when the audits take it upper limit, in this case it is $1 \geq \tau$. The preceding implies that the marginal tax never exceeds one.

3. The threshold function exists, and holds with Lemma 3.

2.2.4 Full-Information Solution

A central element to compare distortions made by evasion and occupational choices is obtain the fist best solution. This solution is the result of a centralized problem where the government has complete information about agents and the economy. This solution gives the benchmark in this work.

The government decides the sector in which each agent will work as well as their consumption level. In this setting, the productivity of each agent is used instead of their wages. Recall that, the utility of each agent, independently of the sector in which they work, is $u(C_i(n^i), R) = C_i(n^i) + \phi(R)$. Therefore, the government must choose a consumption function, $C : [\underline{n}^d, \overline{n}^d] \times [\underline{n}^s, \overline{n}^s] \to \mathbb{R}_+$, a public good provision R, and an occupational choice function, $Z : [\underline{n}^d, \overline{n}^d] \times [\underline{n}^s, \overline{n}^s] \to \{d, s\}$. The occupational choice function determines the sector where each agent will optimally work. The problem that the government solves is as follows

$$\max_{\mathcal{Z}_d, \mathcal{Z}_s, C_d(n^d,n^s), C_s(n^d,n^s), R} \int\limits_{\mathcal{Z}_d} \left[G\left(u(C_d(n^d,n^s),R)\right)\right] dF^d(n^d)dF^s(n^s) + \int\limits_{\mathcal{Z}_s} \left[G\left(u(C_s(n^d,n^s),R)\right)\right] dF^d(n^d)dF^s(n^s)$$

s.t.

$$\int\limits_{\mathcal{Z}_d} n^d dF^d(n^d)dF^s(n^s) + \int\limits_{\mathcal{Z}_s} n^s dF^d(n^d)dF^s(n^s) \geq \int\limits_{\mathcal{Z}_d} C_d(n^d,n^s)dF^d(n^d)dF^s(n^s) + \int\limits_{\mathcal{Z}_s} C_s(n^d,n^s)dF^d(n^d)dF^s(n^s) + R$$

where Z_i is the set of agents on the work side i and C_i is the consumption function for agents in sector i, with $i = \{d, s\}$. The government has a resource constraint, which states that production in both sectors must be equal to the consumption and spending on public goods. The following proposition formalizes the government's solution to the centralized problem.

Proposition 1. *The solution to this problem is* $\{\mathcal{Z}_d^*, \mathcal{Z}_s^*,\ R^*, C_i^*(n^i,n^j)\}$*, with* $i \neq j$ *and* $i, j = \{d, s\}$*, and takes the following form*

$$
\begin{aligned}
\mathcal{Z}_d^* &= \{d \mid \forall (n^d, n^s) \in \mathcal{Z}_d^* \;\Rightarrow\; n^s < n^d\} \\
\mathcal{Z}_s^* &= \{s \mid \forall (n^d, n^s) \in \mathcal{Z}_s^* \;\Rightarrow\; n^s \geq n^d\} \\
1 &= \int\limits_{\mathcal{Z}_d^*} MRS(R^*, C_d^*) dF^d(n^d) dF^s(n^s) + \int\limits_{\mathcal{Z}_s^*} MRS(R^*, C_s^*) dF^d(n^d) dF^s(n^s) \\
R^* &= \int\limits_{\mathcal{Z}_d^*} \left(n^d - C_d^*(n^d, n^s)\right) dF^d(n^d) dF^s(n^s) + \int\limits_{\mathcal{Z}_s^*} \left(n^s - C_s^*(n^d, n^s)\right) dF^d(n^d) dF^s(n^s) \\
\frac{dG\left(u(C_d(n^d, n^s), R)\right)}{du(C_d(n^d, n^s), R)} &= \frac{dG\left(u(C_s(n^d, n^s), R)\right)}{du(C_s(n^d, n^s), R)}
\end{aligned}
$$

where MRS is the marginal rate of substitution and

$$
MRS(R^* C_i^*) = \left.\frac{du/dR}{du/dC_i}\right|_{R^*, C_i^*}
$$

Proof. See Appendix 2.7.2 ■

The result in Proposition 1 shows the first best solution for this setting. The first and second equation reflect the occupational choice rule and show that each agent works in their most productive occupation. After that, equation three reflects the optimal provision of public goods, which tells us that the marginal cost of providing the public good is equal to the sum among agents of the marginal rate of substitution between the public goods and the consumption good. This result is known as the Bowen-Lindahl-Samuelson (henceforth BLS) rule (Samuelson, 1954, 1955). The last two equations show the characterization for the optimal consumption bundle.

2.2.5 Decentralized Solution

The first best can be decentralized showing that the central issue in this context is evading taxes. One can assume that the government knows each agent's real income in each sector and, therefore, evasion is impossible. In this framework, the government maximizes the social welfare function subject to a resource constraint, but now it considers wages instead of productivities. The government does not choose a consumption function, it chooses a lump-sum taxes that induces the agent to decide the sector to work in and a level of consumption. Formally, the government solves the following problem

$$\max_{T_d(w^d),T_s(w^s),R} \int\limits_{\mathcal{N}_d(w^d)} G\left(U^d(w^d,R)\right) dF^d(w^d)dF^s(w^s) + \int\limits_{\mathcal{N}_s(w^s)} G\left(U^s(w^s,R)\right) dF^d(w^d)dF^s(w^s)$$

$s.t$

$$\int\limits_{\mathcal{N}_d(w^d)} T_d(w^d)dF^d(w^d)dF^s(w^s) + \int\limits_{\mathcal{N}_s(w^s)} T_s(w^s)dF^d(w^d)dF^s(w^s) \geq R$$

$$U^i(w^i,R) = w^i - T_i(w^i) + \phi(R), \text{ for } i = d, s$$

where $\mathcal{N}_i$ is the set of taxpayers who choose work in sector $i = \{d, s\}$. The first constraint is the resource constraint that shows that the tax revenue must be at least the same as the expenditure in public goods provision. The second constraint is the definition of utility in each sector.

Proposition 2. *The solution to this problem is $\{T_i^*(w^i), \mathcal{N}_i^*, R^*\}$, with $i = \{d, s\}$, and takes the following form*

$$\mathcal{N}_d^* = \{w^s \mid \forall (w^d, w^s) \in \mathcal{N}_d^* \;\Rightarrow\; w^s < w^d\}$$

$$\mathcal{N}_s^* = \{w^s \mid \forall (w^d, w^s) \in \mathcal{N}_s^* \;\Rightarrow\; w^s \geq w^d\}$$

$$1 = \int\limits_{\mathcal{N}_d^*} MRS(R^*, C_d^*)dF^d(w^d)dF^s(w^s) + \int\limits_{\mathcal{N}_s^*} MRS(R^*, C_s^*)dF^d(w^d)dF^s(w^s)$$

$$R^* = \int\limits_{\mathcal{N}_d^*} T_d^*(w^d)dF^d(w^d)dF^s(w^s) + \int\limits_{\mathcal{N}_s^*} T_s^*(w^s)dF^d(w^d)dF^s(w^s)$$

$$C_i^* = w^i - T_i^*(w^i), \; for\; i = s, d$$

$$\frac{dG\left(U^d(C_d(w^d), R)\right)}{dU^d(C_d(w^d), R)} = \frac{dG\left(U^s(C_s(w^s), R)\right)}{dU^s(C_s(w^s), R)}$$

where MRS is the marginal rate of substitution between the public good and the private con-

sumption, and

$$MRS(R^*, C_i^*) = \left.\frac{du/dR}{du/dC_i}\right|_{R^*, C_i^*}$$

Proof. See Appendix 2.7.3 ■

Proposition 2 shows that it is possible to attains the first best in an environment with lump-sum taxes. This result comes from the second welfare theorem and identifies that the crucial issue in this framework is tax evasion in the presence of occupational choices. Therefore, not only can evasion distort taxes but also the optimal and efficient allocation of workers.

2.3 IRS Problem

The IRS's problem involves finding a direct IC mechanism to incentive agents in the self-employment sector to reveal their real wage. The IRS finds a direct audit function that maximizes the expected collection and meets conditions in Lemma 2. Formally, the problem of the IRS is as follows

$$\max_{\alpha(w^s)} \int_{\mathcal{W}^d(\mathcal{M})} \tau w^d dF^d(w^d) dF^s(w^s) + \int_{\mathcal{W}^s(\mathcal{M})} \left[\tau x(w^s) + \alpha(w^s)\pi\left(\tau w^s - \tau x(w^s)\right)\right] dF^d(w^d) dF^s(w^s)$$

s.t.

$$B \geq \int_{\mathcal{W}^s(\mathcal{M})} c \cdot \alpha(w^s) dF^d(w^d) dF^s(w^s)$$

$\alpha(w^s)$ non-increasing in w^s

$$\mathcal{T}(w^s) = \frac{w^s(1 - \tau\alpha(w^s)\pi)}{1 - \pi\alpha(w^s)} - \frac{\int_{j=\underline{w}^s}^{w^s} (1 - \alpha(j)\pi\tau)\, dj}{1 - \pi\alpha(w^s)} - \frac{V^s(\underline{w}^s)}{1 - \pi\alpha(w^s)} = \tau x(w^s)$$

$$U^d(\kappa(w^s)) = V^s(w^s, \mathcal{M})$$

The first restriction is the budget constraints, which asserts that the IRS audits until equalizing the expected audit cost with its budget. The second and third conditions are the implementability requirements for the mechanism $\mathcal{M}$. Finally, the fourth condition is the threshold function's definition, which defines the occupational choice rule.

Some procedures to simplify the problem are made before solving it. The threshold function is weakly-increasing in the self-employment income because $\kappa'(w^s) > 0$. In addition, since at the equilibrium by the direct IC mechanism agents do not misreport their wages, the expected penalty is zero. Using both facts, the objective function and the budget constrain are rewritten as

$$\max_{\alpha(w^s)} \int_{\underline{w}^s}^{\overline{w}^s} \int_{\kappa(w^s)}^{\overline{w}^d} \tau w^d dF^d(w^d) dF^s(w^s) + \int_{\underline{w}^s}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} \mathcal{T}(w^s) dF^d(w^d) dF^s(w^s)$$

$$B \geq \int_{\underline{w}^s}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} c \cdot \alpha(w^s) dF^d(w^d) dF^s(w^s)$$

The second term in the objective function does not rely on productivity in the dependent sector, and the same occurs in the budget constraint. This evidence implies that it is possible to simplify both expressions taking into consideration only the mass of agents with productivity until $\kappa(w^s)$.

To solve this problem, firstly one has to replace the audit function using the IC condition, $dV^s(w^s,\mathcal{M})/dw^s = V^s_{w^s}(w^s,\mathcal{M}) = 1 - \pi\alpha(w^s)\tau$. Secondly, it can be used the fact that $\mathcal{T}(w^s) = w^s - V^s(w^s,\mathcal{M})$. With those changes it is possible to formulate the problem of the IRS using the indirect utility in the self-employment sector, $V^s(w^s,\mathcal{M})$, as the state variable, and its derivation to productivity, $V^s_{w^s}(w^s,\mathcal{M})$, as the control variable. This specification suggests that the condition over the audit scheme is replaced for a condition over the marginal indirect utility. More specifically, the marginal indirect utility must be non-decreasing and its support is $[1-\tau, 1]$. The Lagrangian of this problem is

$$\begin{aligned} \mathcal{L}(V^s(w^s,\mathcal{M}); V^s_{w^s}(w^s,\mathcal{M})) = &\int_{\underline{w}^s}^{\overline{w}^s} \int_{\kappa(w^s)}^{\overline{w}^d} \tau w^d dF^d(w^d) dF^s(w^s) \\ &+ \int_{\underline{w}^s}^{\overline{w}^s} \left[w^s - V^s(w^s,\mathcal{M})\right] F^d(\kappa(w^s)) f^s(w^s) dw^s + p(w^s) V^s_{w^s}(w^s,\mathcal{M}) \\ &- \mu \left(\int_{\underline{w}^s}^{\overline{w}^s} c \cdot \left[\frac{1 - V^s_{w^s}(w^s,\mathcal{M})}{\pi\tau} \right] F^d(\kappa(w^s)) f^s(w^s) dw^s - B \right) \end{aligned}$$

where $p(w^s)$ is the adjoint function associated with the state variable, and μ the Lagrange multiplier associated with the budget constraint. The necessary conditions to solve this problem are the following

1. $\dfrac{d\mathcal{L}^*(V^s(w^s,\mathcal{M});V^s_{w^s}(w^s,\mathcal{M}))}{dV^s_{w^s}(w^s,\mathcal{M})} = p(w^s) + \mu \left(\int\limits_{\underline{w}^s}^{\overline{w}^s} \left[\dfrac{c}{\pi\tau}\right] F^d(\kappa(w^s)) f^s(w^s) dw^s \right)$

2. $\dfrac{dp(w^s)}{dw^s} = -\dfrac{d\mathcal{L}^*(V^s(w^s,\mathcal{M});V^s_{w^s}(w^s,\mathcal{M}))}{dV^s(w^s,\mathcal{M})} = \int\limits_{\underline{w}^s}^{\overline{w}^s} F^d(\kappa(w^s)) f^s(w^s) dw^s$

3. $V^s(\overline{w}^s)$ is free, implying that $p(\overline{w}^s) = 0$.

4. $\mu \geq 0$, $\mu \left(\int\limits_{\underline{w}^s}^{\overline{w}^s} c \cdot \left[\dfrac{1 - V^s_{w^s}(w^s,\mathcal{M})}{\pi\tau}\right] F^d(\kappa(w^s)) f^s(w^s) dw^s - B \right) = 0$

The first necessary condition does not depend on the control variable, showing that the solution is linear on it. this implies that the control variable takes only its support's extreme values at the optimum. The second condition is positive, which indicates that the adjoint function is increasing, and the third condition suggests that it comes from the negatives to zero. Those three conditions reflect that the budget constraint is crucial for identifying the solution. In the case that $\mu = 0$, or the budget is enough to audit the entire self-employment sector, the solution should be auditing all self-employed workers. Finally, the solution shows that the IRS always uses its entire budget.

When the solution does not imply full auditing, the candidate for a solution is the wage that equalizes the budget constraint. The solution to this problem is defined as w^*, this level separates the audit function between its extreme values. Below the wage w^*, the IRS audits efficiently, or $\alpha(w^s) = {}^1\!/\!_\pi$. This level is efficient in the sense that deters evasion for every indirect mechanism.[8] The following equation characterizes the wage level that solves this problem.

$$\int\limits_{\underline{w}^s}^{w^*} F^d(\kappa(w^s)) f^s(w^s) dw^s = \frac{B\pi}{c} \tag{2.1}$$

The first necessary condition guarantees the existence of a solution since the adjoint function exists, and the second term is always non-negative. Also, since the budget constraint is strictly increasing, the solution must be unique.

[8] An indirect mechanism refers to a mechanism that does not produce an income declaration with the same support as the agent's income.

The audit function indicates that, below w^*, the IRS audits efficiently and, after the threshold wage, audits are equal to zero. The audit function takes the same form as in Sanchez and Sobel (1993) and fulfills the condition over audits in Lemma 2. Furthermore, for agents with productivity below the cut-off wage, the effective tax is equal to the tax liability designed by the government. In this sense, for those agents the effective tax takes the form as a poll tax. However, for the rest of taxpayers the effective tax is different, agents with productivity bigger than w^* pay taxes equal as the agent with the cut-off wage because they do not face an audit. The following proposition formalizes these results as the direct IC mechanism for this problem.

Proposition 3. *The direct IC mechanism $\mathcal{M}$ which solves this problem is*

$$\alpha(w^s) = \begin{cases} 1/\pi & \text{if} \quad \underline{w}^s \leq w^s < w^* \\ 0 & \text{if} \quad w^* \leq w^s \leq \overline{w}^s \end{cases} \tag{2.2}$$

$$\mathcal{T}(w^s) = \begin{cases} \tau w^s & \text{if} \quad \underline{w}^s \leq w^s < w^* \\ \tau w^* & \text{if} \quad w^* \leq w^s \leq \overline{w}^s \end{cases} \tag{2.3}$$

where the threshold w^* is defined by

$$\int_{\underline{w}^s}^{w^*} F^d(\kappa(w^s)) f^s(w^s) dw^s = \frac{B\pi}{c}$$

Corollary 1. *If the IRS has not audit cost (i.e., $c = 0$) the direct IC mechanism $\mathcal{M}$ is*

$$\alpha(w^s) = 1/\pi$$
$$\mathcal{T}(w^s) = \tau w^s$$

Proof. See Appendix 2.7.7 ■

Figure 2.2: IRS Problem's Results

(a) Audit **(b)** Effective Taxes **(c)** Occupational Decisions

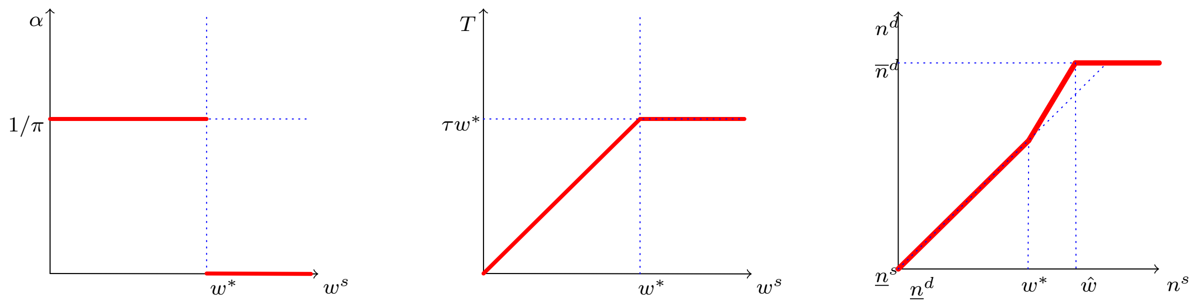


Figure 2.2 shows the results from Proposition 3. Panel (a) shows that all agents with an income higher than the threshold level face no audit. This fact produces that, to encourage truth-telling, those agents face a zero effective marginal tax rate, or pay the same taxes, as seen in Panel (b). Finally, the results on audit and effective taxes produce a distortion in the occupational choice for self-employed workers with wage higher than the cut-off. Distortions increase the incentive for being self-employed because their after-tax income increases, this arise mainly from the audit schedule. Therefore, if the solution implies audit the entire self-employed sector, the distortion will vanish.

The IRS use audit for twofold: to enforce tax compliance and as a threat to avoid income misreport. In a normative sense, and in line with Kuchumova (2017), an audit should be stronger in those income levels where it is likely to find evaders. As it is shown in Almunia and Lopez-Rodriguez (2018) for firms, agents make an strategical income declaration to avoid more forceful audits and take advantage of this. Hence, not only audits must be used as a tool for tax compliance but also as a threat to strategic behavior, which results in tax evasion. In this case, the agent's incentive is to under-report income to pay fewer taxes. For this reason, the audit must be higher at the low and middle-wage levels if the IRS cannot audit the entire self-employed sector.[9]

A characterization of the threshold function is necessary to give a complete result of the second stage equilibrium, and to understand the distortion shown in Figure 2.2 Panel (c) better. The following equation partially characterizes the threshold function, but for a complete result, it is

[9]This result can change with an upward incentive scheme, in other words, if the agent has the incentive to declare larger productivity. If low-productivity agents have the motivation to report larger productivity, or income, Bigio and Zilberman (2011) and Zilberman (2016) show that the audit schedule is increasing in the agent's productivity, producing the opposite result to the one shown in the equation 2.2.

necessary to solve the government problem.

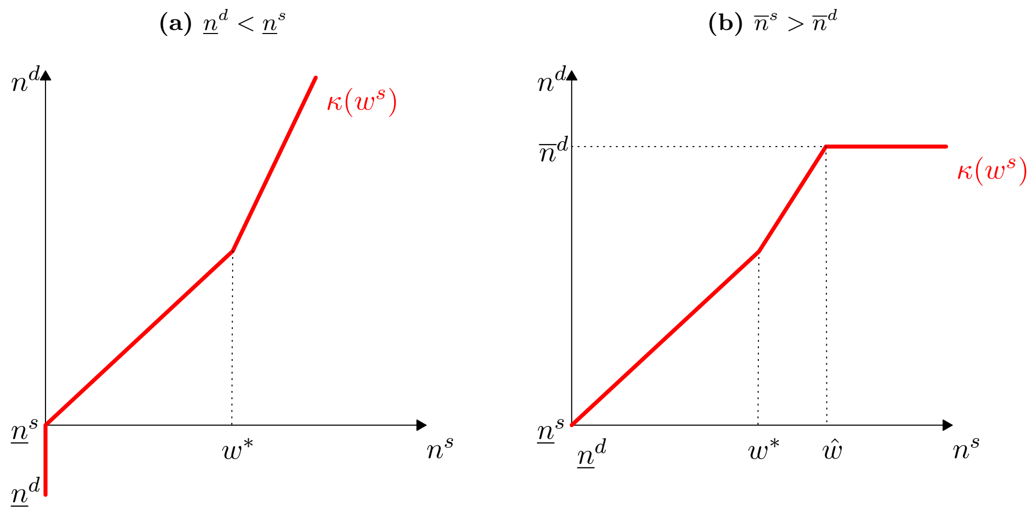
$$\kappa'(w^s) = \begin{cases} 1 & \text{if } \underline{w}^s \leq w^s < w^* \\ \\ \dfrac{1}{1-\tau} & \text{if } w^* \leq w^s \leq \overline{w}^s \end{cases}$$

The possibility of evading taxes makes that the best course of action for the IRS is to audit efficiently until some level, defined as w^*. This level depends on three exogenous variables for the IRS: the audit cost, the fine rate, and its budget. If those variables' conjugation implies that it is not optimal to audit all self-employment sector, distortions in occupational decisions appear and play only for agents with wages higher than the cut-off solution. These distortions produce that more agents decide to become self-employed as larger is the marginal tax rate. The explanation for this fact is simple: since the IRS must incentive taxpayers to tell their real wages, the effective tax for agents who do not face audits is a poll tax. This produce an increase in the utility for work as a self-employed, provoking distortions in occupational decisions.

2.4 Government Problem

Before dealing with the government's problem, some relevant aspects of the productivities distribution in this step are clarified. Bearing in mind the assumption of independence in the distribution function, there exist four feasible scenarios related to the possible non-common support in the productivities distribution. The cases are: either one of the lower bound is smaller than the other ($\underline{n}^d < \underline{n}^s$ or vice versa) or one upper bound is larger than the other ($\overline{n}^s > \overline{n}^d$ or vice versa). This paper focuses on the latter case, as is shown in Figure 2.3 Panel (b). The first case, shown in Figure 2.3 Panel (a), only produces that the threshold function takes the value of the lower bound self-employed productivity, which does not depend on taxes or the budget for the IRS. Whereas, the second case produce the existence of a limit level since the threshold function derivative is zero, and potentially, impact on the optimal results.

Figure 2.3: Productivity Distribution Cases



Let us define the threshold which solve the IRS's problem as w^* and the point where the threshold function takes the upper bound of the dependent distribution as $\hat{w}$.[10] Given this, the threshold function takes the following form

$$\kappa(w^s) = \begin{cases} w^s & \text{if } w^s < w^* \\ \dfrac{w^s - \tau w^*}{1-\tau} & \text{if } w^* \leq w^s < \hat{w} \\ \overline{w}^d & \text{if } \hat{w} \leq w^s \end{cases} \tag{2.4}$$

This function only depends on taxes in the zone where the IRS does not audit. Thus, any change in the IRS's budget affects the threshold function through the cut-off wage w^* in this zone. Both instruments, taxes and IRS's budget, modify not only the shape of the threshold function but also the level where it takes the maximum (i.e., $\hat{w}$). Any change in these intruments produces two effects: 1) distortions in the incentives to be in some occupation, and 2) changes in the mass of agents that, regardless of their dependent productivity, always prefer being self-employed. More specifically, Figure 2.3 Panel (b) shows that all agents with productivity larger than $\hat{w}$ always prefer being self-employed, even if they have the most significant productivity in the dependent sector.

With those specifications and incorporating the results of the IRS's problem to ensure a direct IC mechanism, the government's problem is as follows

[10] In order to clarify the another case, Appendix 2.7.13 shows the solution for $\overline{n}^d > \overline{n}^s$.

$$
\begin{aligned}
&\max_{\tau,R,B} \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d) dF^s(w^s) \\
&\int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d) dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d) dF^s(w^s) \\
&s.t \\
&\tau \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) + \tau \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) + \tau \int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right) dF^s(w^s) + \tau w^* \int_{w^*}^{\hat{w}} F^d\left(\kappa(w^s)\right) dF^s(w^s) \\
&+ \tau w^* \int_{\hat{w}}^{\overline{w}^s} F^d\left(\kappa(w^s)\right) dF^s(w^s) \geq B + R \\
&w^* \text{ solves } \int_{\underline{w}^s}^{w^*} F^d(\kappa(w^s)) f^s(w^s) dw^s = \frac{B\pi}{c} \\
&\alpha(w^s) = \begin{cases} 1/\pi & \text{if } \quad \underline{w}^s \leq w^s < w^* \\ 0 & \text{if } \quad w^* \leq w^s \leq \overline{w}^s \end{cases} \\
&\mathcal{T}(w^s) = \begin{cases} \tau w^s & \text{if } \quad \underline{w}^s \leq w^s < w^* \\ \tau w^* & \text{if } \quad w^* \leq w^s \leq \overline{w}^s \end{cases} \\
&U^d(w^s) \geq 0,\ V^s(w^s,\mathcal{M}) \geq 0
\end{aligned}
$$

The government faces five restrictions. First, the budget constraint consists of a balanced budget rule, which means that the expected tax collection must be at least equal to the expenses in the public goods provision and the IRS's budget. In the government budget constraint, the first two terms reflect the dependent sector, the third term represents the self-employed, who are audited efficiently, and the other terms reflect those agents who do not face audits. There exist three restrictions that come from the IRS's problem: 1) the definition of the threshold w^*, 2) the definition of audit function and 3) effective taxes. Finally, the fifth restriction is the limited liability, which states that any agent has a non-negative utility.

The government chooses the marginal tax rate, the IRS's budget and the provision of public goods, to maximize the social welfare function subject to its budget constraint. The procedure is to use the budget constraint and incorporate the rest of restrictions into the problem. Let δ be the Lagrange multiplier for the government budget constraint, the Lagrangian for this problem is

$$\mathcal{L} = \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s, \mathcal{M})\right) dF^d(w^d) dF^s(w^s)$$
$$+ \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s, \mathcal{M})\right) dF^d(w^d) dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s, \mathcal{M})\right) dF^d(w^d) dF^s(w^s) - \delta \left\{ B + R - \tau \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) \right.$$
$$\left. - \tau \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) - \tau \int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau w^* \int_{w^*}^{\hat{w}} F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau w^* \int_{\hat{w}}^{\overline{w}^s} F^d\left(\kappa(w^s)\right) dF^s(w^s) \right\}$$

2.4.1 Optimal Public Goods Provision

To describe the optimal BLS rule is followed Section 2.2.4, using the notation $MRS(RC)$ for the marginal rate of substitution (MRS) between the public good and the private consumption good. The following proposition formalizes the optimal BLS rule in this model.

Proposition 4. *The Bowen-Lindahl-Samuelson rule is*

$$\phi'(R) = \int_{\underline{w}^s}^{\overline{w}^s} \int_{\kappa(w^s)}^{\overline{w}^d} MRS(RC) dF^d(w^d) dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \int_{\underline{w}^d}^{\kappa(w^s)} MRS(RC) dF^d(w^d) dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} MRS(RC) dF^d(w^d) dF^s(w^s)$$
$$+ \int_{w^*}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} MRS(RC) dF^d(w^d) dF^s(w^s) = 1 \tag{2.5}$$

Proof. See Appendix 2.7.8 ■

It is important to highlight that the BLS rule is the same as the first best, and this result comes from the agent's utility. In particular, as is well established in Boadway and Keen (1993), since consumption is linear in the agent's utility, evasion does not distort the optimal provision of public goods. This is due to the fact that all agents have the same MRS, implying that variations in consumption due to evasion does not generate distortion on it.

This result implies two relevant conclusions for this study. Firstly, the government only has two instruments to improve social welfare: taxes and the IRS budget. Secondly, giving the audit solution, some agents pay fewer taxes than in the case without evasion but enjoy the same amount of public good. This difference produces a worsened and unequal distribution, because some taxpayers raise their utilities, and others bear, percentage-wise, more burden to finance the public goods.

2.4.2 Optimal Linear Tax Rate

In order to give a better interpretation of the marginal tax rate, $g(w)$ can be defined as the social value of consumption for an agent with productivity w expressed in terms of the public funds (Saez, 2001; Saez and Stantcheva, 2016). Formally, $g(w) = {}^{(G' \times U'_C)}/_{\delta}$. By definition, the sum of g among agents is equal to one. This expression indicates the importance of an increase in agent's consumption concerning public funds.

Proposition 5. *The optimal marginal tax rate is characterized by*

$$\begin{aligned}\frac{\tau}{1-\tau}\int_{w^*}^{\hat{w}}(w^s-w^*)\kappa_\tau(w^s)f^d(\kappa(w^s))dF^s(w^s) = &\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d}(1-g(w^d))w^d dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d}(1-g(w^d))w^d dF^d(w^d)dF^s(w^s)\\ &+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)}(1-g(w^s))w^s dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}(1-g(w^s))w^* dF^d(w^d)dF^s(w^s)\\ &+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}(1-g(w^s))w^* dF^d(w^d)dF^s(w^s)\end{aligned} \tag{2.6}$$

Proof. See Appendix 2.7.9. ∎

The marginal tax formula can be decomposed into two effects: welfare and revenue. The welfare effect shows the impact of changes in the agent's utility through the effect of taxes on consumption. When taxes rise, all agents face a more significant tax liability than before, therefore, consume less. However, the reduction in their consumption depends on each consumer's wage. This effect must be measured in public funds terms, and the term $g(w)w$ captures it. The revenue effect has two components. First, when taxes rise, increase the marginal tax liability, which is equal to the taxpayer's wage. Second, an increase in taxes produces that some agents change their occupation and pay a different tax level. When taxes rise, a taxpayer with a former wage $\kappa(w^s)$ bigger than w^*, moves from the dependent to the self-employment sector and pays τw^*. The term which captures this effect is $\tau(\kappa(w^s) - w^*)\kappa_\tau(w^s)f^d(\kappa(w^s))$. In equilibrium, the sum of those effects must be zero. Figure 2.4 shows the forces behind the marginal tax formula.

Figure 2.4: Effect of Raising Tax Rate

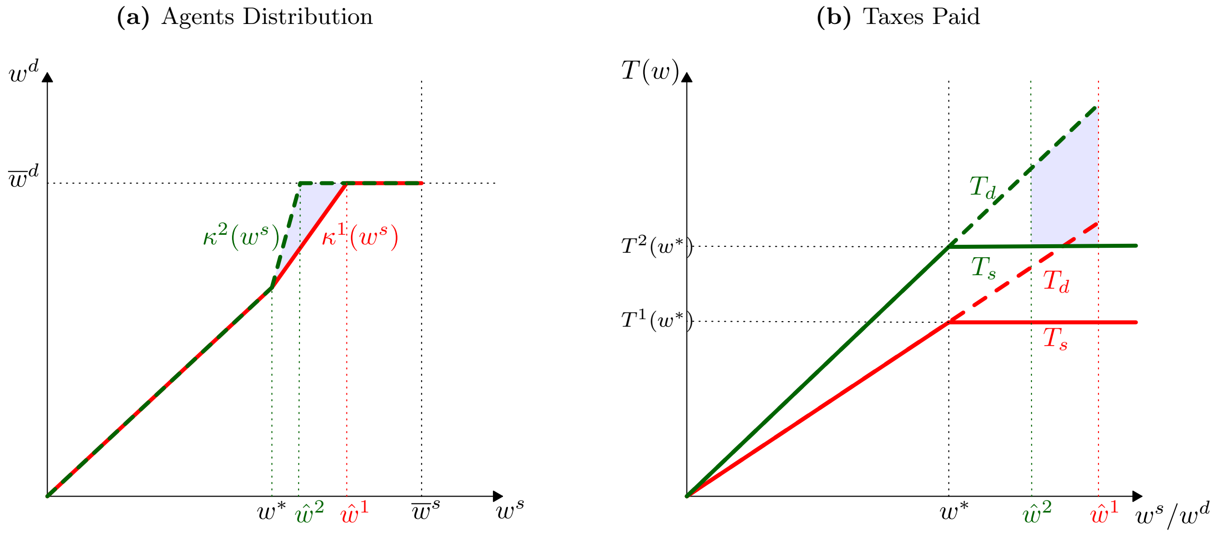


Figure 2.4 Panel (a) shows the consequences of an increase in the marginal tax in the threshold function and occupation distribution. The red line indicates the initial situation, whereas the dashed green line illustrates a raise in the marginal tax. All agents with a productivity tuple situated behind the red (green) line (dashed line) are self-employed. An increase in taxes produces that agents with productivity between w^* and $\hat{w}^1$ change their occupational decision, this is depicted as the blue zone. However, w^* does not change because the IRS's budget does not change.[11] Those facts provide an increase in the mass of self-employed agents. Additionally, an increase in the marginal tax rate produces that the productivity level $\hat{w}$ falls, from $\hat{w}^1$ to $\hat{w}^2$. This change produces the mass of agents that always prefer to be self-employed increase. These effects capture two pieces of evidence. Firstly, a collective of agents changes their occupation, changing the proportion of agents

[11]To portray this fact much more efficiently, let us differentiate the definition of the threshold level with respect to τ and w^* and equalize to zero:

$$\int_{\underline{w}^s}^{w^*} \left(\frac{\partial \kappa}{\partial w^*} dw^* + \frac{\partial \kappa}{\partial \tau} d\tau \right) f^d(\kappa(w^s)) dF^s(w^s) + F^d(w^*) f^s(w^*) dw^* dw^s = 0$$

By dividing both side by $d\tau$ obtaining:

$$\int_{\underline{w}^s}^{w^*} \left(\frac{\partial \kappa}{\partial w^*} \frac{dw^*}{d\tau} + \frac{\partial \kappa}{\partial \tau} \right) f^d(\kappa(w^s)) dF^s(w^s) + F^d(w^*) f^s(w^*) \frac{dw^*}{d\tau} dw^s = 0$$

The only way to obtain $\frac{dw^*}{d\tau} = 0$ is if $\frac{d\kappa}{d\tau} = 0$. This hold from the IRS's solution, which says that $\kappa(w^s) = w^s$ for $w^s < w^*$ and $\kappa(w^*) = w^*$.

in each occupation. Secondly, those agents who change their occupation face new taxes, producing an impact on tax revenue and an impact on the agents' utility.

Figure 2.4 Panel (b) shows the consequences of an increase in marginal tax in the taxes paid by agents and in tax revenue. Let us keep in mind that the red line represents the initial situation, and the green line refers to the tax rise. Moreover, it is also shown those taxes paid by the two occupations. The solid line is for the taxes paid in the self-employment sector and the dashed line in the dependent one. When taxes rise, all agents pay higher taxes than before, this is represented by the difference between the red and green lines, increasing tax revenue. However, some agents change their occupation and face different taxes. The agents who change their occupation are those who do not face audits, hence, bear taxes equal to τw^* instead of τw^d. This change produces a revenue loss for the government, depicted by the blue zone. Moreover, the more significant the increase in taxes, the bigger the revenue loss is. Those forces, the increase in revenue versus welfare and revenue losses, produce the equilibrium in the tax rate.

The inclusion of occupational choices produce taxes smaller than one due to losses in welfare and tax revenue. This result is opposed to Sanchez and Sobel (1993), who obtain a tax rate that is equal to one, showing that not considering occupational decisions induces an upward bias in the tax rate, and induces agents to work as self-employed. Furthermore, this finding evidences the forces behind the Laffer curve. If total revenue gains are more significant than welfare and revenue losses by occupational changes, the marginal tax will increase. In the opposite case, the marginal tax will decrease. For this reason, it is relevant to consider different occupations, and also different possibilities to evade.

The relationship between the marginal tax rate and the threshold function may explain the empirical relation between tax rates and income tax evasion. If the marginal tax falls, the threshold function will take smaller values, producing a reduction in the incentive to evade taxes. Empirically, Bárány (2019), and Berger et al. (2016) show a positive relationship between tax rates and evasion rates. This relationship indicates that to deter evasion, the government must reduce incentives to evade through falling taxes. This mechanism is similar to previous findings in the general equilibrium literature (Watson, 1985; Kesselman, 1989), and reinforces the importance of taxes as a tool in discouraging tax evasion.

2.4.3 Optimal IRS Budget

Changes in the IRS's budget produce changes in the agency's ability to audit more (less) agents, changing the threshold wage and the threshold function's upper level. Moreover, those changes produce a modification in the self-employed tax paid. For this reason, a modification in the IRS's budget induce changes in occupation decisions, but also in tax revenue. Those facts are into the forces behind the optimal budget for the IRS.

Proposition 6. *The optimal budget for the IRS is characterized by*

$$\tau\int_{w^*}^{\hat{w}}(w^*-\kappa(w^s))\frac{dw^*}{dB}\kappa'_B f^d(\kappa(w^s))dF^s(w^s)+\tau\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)+\tau\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)$$
$$=1+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}g(w^s)\tau\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}g(w^s)\tau\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s) \tag{2.7}$$

Proof. See Appendix 2.7.10. ■

It is possible to decompose this equation into three effects: behavioral, mechanical and welfare. To start with, the behavioral effect relates to the capacity to audit more (less) taxpayers, producing that a mass of agents changes their occupational decisions. As for the mechanical effect, this one reflects the impact on agents' taxes and the cost of changing the budget, which is equal to one. Since the threshold level changes, the taxpayer who does not face audit pays more taxes than before, and this effect is purely mechanical. This change produces that their consumption falls in the amount that budget affects taxes, producing a welfare loss, which must be measure in public fund terms. In equilibrium, the sum of the three effects must be zero, and this provides the characterization in Proposition 6.

This equation shows three relevant results. First, similar to Sanchez and Sobel (1993), the IRS always has a budget smaller than that which allows auditing the entire self-employment sector. This is because the marginal cost of increasing the budget is always one and the marginal gain is zero when the IRS audit the entire self-employment sector. Second, in determining the optimal budget, it is necessary to consider the effect on welfare. This result produces a distance from the cost-benefit analysis. Finally, as established in Slemrod and Yitzhaki (1987), the government must consider the tax burden when deciding the IRS's budget. The increase in the budget increases distortions in the tax burden by the effect of taxes on it.

2.5 Discussion

Although the equilibrium in this model is already shown, some questions are still open. Those questions are related to differential taxation, different assumptions over elements in the IRS's problem, and comparative statics. Firstly, this section looks into the optimality to impose two different linear tax rates, one for each occupation. This extension allows contemplating the possibility of recovering efficiency in the allocation of workers. Regarding the audit result, the problem of achieving the same result with non-linear audit costs and imposing an increasing fine rate is resolved. Finally, the effect on changing the audit costs on the tax rate and the IRS's budget is shown.

2.5.1 Differential Taxation

Differential taxation allows to seek efficiency in the allocation of agents, even if the audit schedule produces incentives to be self-employed. However, because of the linear marginal tax rate, it is difficult to recover the first best as every action taken to get rid of the incentive in the high-income agents through tax rate ends up affecting the rest of them. Also, it is critical for the design of both marginal tax rates that each of one of them meet with the IC requirement, in specific both marginal tax rates cannot be higher than one. By using differential taxation, the threshold function takes the following form

$$\kappa(w^s) = \begin{cases} \dfrac{w^s(1-t_s)}{1-t_d} & \text{if } w^s < w^* \\ \dfrac{w^s - t_s w^*}{1-t_d} & \text{if } w^* \leq w^s < \hat{w} \\ \overline{w}^d & \text{if } \hat{w} \leq w^s \end{cases}$$

In the above formulation, t_s and t_d are the tax rate in the self-employment and the dependent sector, respectively. In addition, the budget constraint change, now takes the following form

$$t_d \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) + t_d \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) + t_s \int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right) dF^s(w^s) + t_s w^* \int_{w^*}^{\hat{w}} F^d\left(\kappa(w^s)\right) dF^s(w^s)$$
$$+ t_s w^* \int_{\hat{w}}^{\overline{w}^s} F^d\left(\kappa(w^s)\right) dF^s(w^s) \geq B + R$$

Although this specification changes the threshold function, it does not alter the form of the optimal audit. This result comes from the necessary conditions for a solution to the IRS's problem, because differential taxation only changes the applicable tax rate for the determination of the threshold. This is to say, either audit the entire self-employment sector or until the level determined by the budget.[12] For this reason, the same optimal audit pattern is valid for this specification.

The government solves a Lagrangian similarly as in Section 2.4, choosing both tax rates and the budget for the IRS. As it has been previously explained, the public goods provision is the same as in the first best. This result is maintained in this framework because choosing two different tax rates does not alter the reason that support it, risk-neutrality. Also, since the focus is only in differential taxation, the optimal budget for the IRS is not obtained.

Proposition 7. *In a hierarchical model with occupational choices where the IRS maximizes expected tax collection and the government maximizes the social welfare, differential taxation implies a higher marginal tax rate in the sector where evasion is possible.*

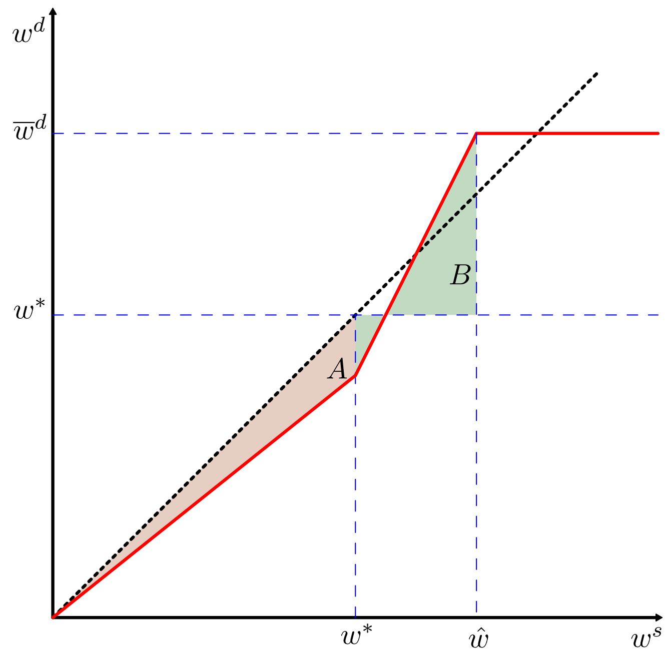
Proof. See Appendix 2.7.11. ■

Figure 2.5 shows the basic idea behind Proposition 7. The red line represents the threshold function and the black dotted line represents the same wage in both sectors. Zone A represent the losses from agents who decide to be a dependent worker, but paid higher taxes and declared higher income as a self-employed worker. In contrast, zone B shows the increase in tax collection from agents who paid higher taxes in the dependent sector because they paid a constant tax in the self-employment sector. Differential taxation is possible only if zone A is smaller than zone B. Thus, differential taxation happens if the increase in tax collection from agents above w^* is bigger than the losses from agents below it. When the difference in the marginal taxes rise, the threshold

[12] To see this, note that in the Lagrangian of the IRS's problem the marginal indirect utility is used as a control variable, thus conditions 2 and 3 do not change. Only condition 1 changes, using t_s instead of τ, and this does not change the form of the solution.

function fall, increasing A and decreasing B; thus, the government do not have the incentive to separate both marginal taxes a lot. Consequently, differential taxation aims to solve tax collection losses due to zero audits rather than improve the allocation of agents, ending up distorting the entire workers' occupational decision.

Figure 2.5: Differential Taxation



A higher tax in the self-employment sector goes in the opposite direction as the traditional literature in optimal taxation, which recommends smaller taxes for self-employed workers (Mirrlees, 1971, 1976). Although the differences in policy recommendations, the meaning of both arguments is the same: using tax incentives to vanish the motivation to evade. When occupational decisions are incorporated into the optimal taxation model, a rise in self-employment tax rate simultaneously vanishes the incentive to evade and increase welfare due to an increase in after-tax income for agents that move to the dependent sector. For this reason, both arguments go in the same way but with different recommendations.

The possibility of differential taxation produces a distortion in the allocation of agents, resulting in a larger distance from the first best than the former analysis. Ergo, this solution produces a larger distortion in the workers' allocation, as one with only one marginal tax rate for both occupations. For this reason, in this model, differential taxation is useful for increasing revenue rather than increasing the efficiency in allocating workers. Moreover, if the government decides to impose differential taxation, distortion will be higher than if it has only one marginal tax rate.

Corollary 2. *It is not possible in a hierarchical model with occupational choices and linear tax schemes to recover the efficiency of workers' allocation.*

This result is essential for policymakers to design tax and audit policies. The possible solution for the audit scheme's inefficiencies through differential taxation allows the government to improve economic efficiency and raise revenue. However, Corollary 2 reflects that it is impossible to reconcile incentives for taxpayers to tell the truth with the incentive to choose their occupations efficiently. Hence, the government needs to find other policies to improve efficiency and raise revenue at the same time.

2.5.2 Audit Results

Audit Costs

It is assumed that the audit cost depends on the agent's productivity in the self-employment sector and it is known for the IRS when the agent reveals its type. The audit cost is defined as $c(w^s)$. From Section 2.3, the condition that determines the form of the solution is the first necessary condition from the Lagrangian of the IRS's problem. In this condition, only the audit cost changes because neither the control variable changes nor does the state variable. The new first necessary condition is

$$\frac{d\mathcal{L}}{dV^s_{w^s}} = p(w^s) + \mu \left(\int_{\underline{w}^s}^{\overline{w}^s} \left[\frac{c(w^s)}{\pi\tau} \right] F^d(\kappa(w^s)) f^s(w^s) dw^s \right)$$

A condition similar to a single-crossing is found, in the sense that the above function should crosses one time the zero line. To obtain this condition, let us derive the equation above with regard to productivity in the self-employment sector and use two facts. First, by the linear income tax, the second derivative of tax is equal to zero. Second, the derivative of the adjunct function is positive. The following equation establishes the condition over the threshold function assuming that the solution of the IRS is not obtained.

$$-\frac{c'(w)}{c(w)} > \frac{\pi\tau}{\mu c(w)} + \frac{f^d(k(w))k'(w)}{F^d(k(w))} + \frac{f^{s\prime}(w)}{f^s(w)}$$

where w is the level that equalizes the derivative ${d\mathcal{L}}/{dV^s_{w^s}}$ to zero. The threshold condition is fulfilled with a positive marginal cost, or a strictly increasing cost because the right side is positive by definition.[13] Hence, if the audit cost is monotone non-decreasing in the self-employment income, the threshold solution is maintained.

An exciting example is if the IRS has countervailing incentives as ones developed in Lewis and Sappington (1989). One case is assuming that the IRS has fixed costs that decrease with an increasing marginal cost. This form is the same as in Lewis-Sappington's model. In this case, the IRS audits only agents in the middle of the productivity distribution because at the extreme levels the cost is higher than in the middle. Zilberman (2016) reaches a similar conclusion, but he uses a participation constraint that depends on the productivity of agents, simiar as Jullien (2000). This fact produces the countervailing incentives for the agent's declaration, and the IRS must choose a non-increasing audit scheme. Both examples indicate the possibility of obtaining a different solution than the traditional threshold one.

Fine Rate

Another interesting result is obtained when the fine rate is not linear. This issue regards the intensity of the penalty or the intention to deter high tax evasion levels. Two options are analyzed: 1) a fine rate increasing in the evaded amount, and 2) a fine rate rising in the reported income. These specifications may allow the IRS to audit more taxpayers at less cost. This effect comes from the efficient audit form because a higher fine rate replaces the audit's deter spirit, allowing fewer levels and, consequently, fewer costs. For this reason, it is reasonable to take this kind of fines into consideration.

Given the direct mechanism, evasion does not exist in equilibrium and any instrument that depends on an evaded level vanishes. Thus, a fine rate that depends on the amount of evaded taxes collapses to a linear fine rate in equilibrium. This situation is the same as the one proposed in the

[13] Let us assume distribution functions with a positive second derivative.

model and resulting in an equal optimal audit as one presented earlier.

As for a fine rate that depends on the reported wage, the result is different. In this case, a similar analysis that one realized with audit cost is necessary. Now, the fine rate depends on the self-employment wage, and the first necessary condition became in

$$\frac{d\mathcal{L}}{dV^s_{w^s}} = p(w^s) + \mu \left(\int_{\underline{w}^s}^{\overline{w}^s} \left[\frac{c}{\pi(w^s)\tau} \right] F^d(\kappa(w^s)) f^s(w^s) dw^s \right)$$

Deriving this condition with respect to the self-employment wage allows us to obtain the condition to maintain the single-crossing.

$$\frac{\pi'(w)}{\pi(w)} > \frac{\pi(w^s)\tau}{\mu c} + \frac{f^d(k(w))k'(w)}{F^d(k(w))} + \frac{f^{s\prime}(w)}{f^s(w)}$$

As it is stated before, w is the wage level that equalizes the ${d\mathcal{L}}/{dV^s_{w^s}}$ to zero. This expression shows that the growth in the fine rate must not be higher than the right side. In other words, to obtain the threshold form in the audit function, the marginal fine rate must be bounded from above. This result allows the imposition of an increasing fine rate but restricts the form that it raises. Therefore, it is possible to impose an increasing fine rate to put more penalties in higher incomes; and consequently, audit more agents and improve the IRS's effectiveness.

2.5.3 Comparative Statics

The effects are decomposed into direct and indirect. The former is the effect on taxes and the latter is the effect on the threshold level. Also, it is possible to separate the effect on the impact on the tax collection and the impact on social welfare. The derivation of the effect on taxes is in Appendix 2.7.12 and the effect in the budget is in Appendix 2.7.12.

The overall effect of changes in audit cost on the tax rate depends on the size and impact on agents who face no audits. When the audit costs decrease, the threshold level raises, producing that all agents who do not face an audit pay higher taxes than the others. However, if marginal tax raises, the group of agents with productivity below $\hat{w}$ will have the incentive to work as self-employed, pay fewer taxes than the dependent workers, and increase their welfare. Both effects make the overall impact depend crucially on what happens with those agents. Furthermore, the result relies on which force is more prominent. If the increase in revenue is more significant than the effect on welfare, the overall impact of audit cost on taxes is positive. This result is very intuitive, this is to say, if the government faces an increase in audit cost, taxes only increase when the revenue increases more significantly than the welfare loss. This conclusion is similar to the Laffer curve analysis.

When it comes to the budget for the IRS, the situation is similar. When the budget rises, it is feasible to audit more agents, increasing the threshold level. As the mass of agents who face zero audit decreases, it is possible to impose higher tax rates because fewer agents have the incentive to change their occupation. Nevertheless, this impacts on welfare through a smaller after-tax income and a decrease in agents consumption. The welfare for some taxpayers in the self-employment sector decreases, and as taxes rise, the entire welfare falls. Those effects produce that the entire effect in the budget is not apparent and depends on which force is more prominent.

2.6 Conclusion

By and large, this paper investigates the distortions produced by the introduction of occupational choices in a hierarchical model similar to Sanchez and Sobel (1993). In this sense, these authors' model is extended, incorporating the possibility for agents to choose between two occupations: dependent and self-employed. Income tax evasion is possible only in the self-employment sector because agents self-report their income. The focus is on distortion in the audit function, decided by the IRS, and in a linear marginal tax rate, the budget for the IRS, and public good provision, decided by the government.

The IRS maximizes expected tax collection subject to a budget constraint, with a constant audit cost, and considers that it must find a direct incentive-compatible mechanism. Such mechanism is composed of a direct audit and an effective tax, which is the tax paid by a self-employed agent who optimally declares its real income. It is of great importance to recall that the direct incentive-compatible mechanism has the intention to encourage the self-employment sector to reveal its real income. The optimal audits follow a threshold form. Before a cut-off wage, the audit level is efficient, due to the fact that it deters evasion in every indirect mechanism, and above this level, audits are equal to zero. This form states that the IRS must keep all efforts, using the audit to eliminate the incentive to under-declare income, which is the agent's incentive modeled in this paper. This audit pattern provokes that effective tax is equals for agents with a wage higher than the cut-off wage level. Afterward, it is shown that this result holds in two realistic extensions. First, if the audit cost depends on the self-self-employment reported income, the critical condition must be monotonously non-decreasing in the self-employed reported income. Second, the fine rate could be increased in the self-employment reported income, but its growth must be bounded from above.

The government maximizes social welfare subject to a resource constraint anticipating the incentive-compatible requirements and the IRS's solution. Because of risk-neutral agents, the optimal public good provision is the same as in the first best, producing that the government only has two instruments to attain its objectives: taxes and the IRS budget. The marginal tax rate is smaller than one and is composed of two effects: welfare and revenue. In regard to the optimal IRS's budget, it is lower than the level that allows auditing all self-employed but is higher than the result from a cost-benefit analysis. The optimal IRS's budget can be characterized through three effects: behavioral, mechanical, and welfare. If the audit cost raises, both instruments, i.e. the marginal tax rate and the IRS's budget, will face undefined changes. The crucial fact is if revenue increases (decreases) are bigger/smaller than welfare losses (gains). In this setting, differential taxation is optimal only if the marginal tax in the self-employment sector is higher than in the dependent sector. This result goes in the opposite direction than the traditional optimal taxation literature.

The under-budgeted situation gives the result of the audit. On the one hand, if the IRS has sufficient budget to audit the entire self-employment sector, it will do so. However, on the other hand, audits introduce distortion in the occupational decision, this means that agents do not work in the sector where they are more productive purely because of the institutional policy. Hence, agents work in an occupation that increases its utility because the institutional policy allows so. Moreover, although differential taxation is optimal, this instrument does not solve the distortion in agents' allocation. Differential taxation increases this problem because it makes marginal tax in the self-employment sector be higher than in the dependent sector.

The comparison with the results over the tax rate seen in Sanchez and Sobel (1993) conveys that occupational decision produces a fall in the marginal tax rate, for the forces introduced by occupational decisions. Hence, this study shows that not considering occupational decisions induce an upward bias in taxes. The upward bias in taxes may increase the distortion in workers' allocation from the audit result because they introduce more incentives to work as a self-employed in those agents who face smaller audits.

The two previously mentioned results - this is to say a marginal tax rate smaller than one and the distortion in the allocation of agents - show the relevance of taking into account occupational decisions made by agents in any audit or taxation model. Since the problems rely on the optimal policies, it is necessary to inquire over new instruments, mechanisms and strategies to increase enforcement policies' effectiveness. Even when the government chooses its tax administration policy optimally, this paper demonstrates that distortions exist, and it is necessary to incorporate other course of actions to solve them.

2.7 Appendix

2.7.1 Occupational Choice Model vs. Evasion Model

The following section demonstrates that although it is possible to nest a model of occupational choice akin to the one developed in Gomes, Lozachmeur and Pavan (2017) with a hierarchical model similar to one built by Sanchez and Sobel (1993), the solution diverges in some cases. In order to do this, the same model as in Gomes, Lozachmeur and Pavan (2017) is used along with the solution in the audit scheme obtained in Sanchez and Sobel (1993), and finally, it is also shown how the solution diverges.

There exist two sectors in economy, self-employed and dependent. Each agent has an ability tuple (n_d, n_s), which represents the productivity in the dependent and the self-employed sector, respectively. Agents choose the labor supply h_i with $i = \{d, s\}$, implying an effective labor in sector i equals to $h_i n_i$. Income for work in sector i is $y_i = w_i h_i n_i$, which w_i is the wage rate in sector $i = \{d, s\}$. The government can impose two different tax schemes, one for each sector. Let T_i be the income tax schedule in sector i. Agent's utility is

$$u_i(n_i) = w_i h_i n_i - \psi(h_i) - T_i(w_i h_i n_i) \text{ for } i = \{d, s\}$$

where ψ is the convex cost of labor. To nest this frame with a hierarchical model with tax evasion, make the following assumption. In the dependent sector, labor supply is inelastic and equals to one, this is $h_d = \overline{h}_d = 1$. The first-order conditions with respect to the labor supply in the self-employed sector and the derivatives with respect to productivity in each sector respectively are

$$\begin{array}{llcl} \langle h_s \rangle & w_s n_s \left[1 - T_s'(w_s h_s n_s)\right] & = & \psi'(h_s) \\ \langle n_s \rangle & w_s h_s \left[1 - T_s'(w_s h_s n_s)\right] & = & u_s'(n_s) \\ \langle n_d \rangle & w_d \left[1 - T_d'(w_d n_d)\right] & = & u_d'(n_d) \end{array}$$

The derivatives regarding productivity in each sector are obtained by simplifying the threshold function explained below. The threshold function reflects the ability level until any agent decides to work in the self-employed sector. Let $c : [\underline{n}_s, \overline{n}_s] \to [\underline{n}_d, \overline{n}_d]$ be the threshold function. By definition, this function holds with $u_s(n_s) = u_d(c(n_s))$ implying the following condition

$$c'(n_s) = \frac{u_s'(n_s)}{u_d'(c(n_s))} = \frac{w_s h_s \left[1 - T_s'(w_s h_s n_s)\right]}{w_d \left[1 - T_d'(w_d c(n_s))\right]} \tag{2.8}$$

This function is the same threshold function obtained in Gomes, Lozachmeur and Pavan (2017). Therefore, this as a result of a model with differential taxation and no evasion.

For the model with tax evasion, one can assume that in the self-employed and dependent sectors, the labor supply is inelastic and equal to one. Let x_s be the income report in the self-employed sector. This assumption produces migration to a model with evasion. In this case, the government also decides the audit strategy and imposes only one income tax schedule for all agents, T. Let ρ be the monitory strategy that depends on the declared income in the self-employed sector. If the government discovers an evader agent, they must pay a linear fine on the evaded amount with a penalty rate $\pi > 1$. The agent's utility in each sector is

$$\begin{aligned} u_d(n_d) &= w_d n_d - T(w_d n_d) - \psi(\overline{h}_d) \\ u_s(n_s) &= w_s n_s - T(x_s) - \rho(x_s)\pi\left[T(w_s n_s) - T(x_s)\right] - \psi(\overline{h}_s) \end{aligned}$$

In the case with evasion, the government chooses the audit probability and the tax schedule, and in the model without evasion, it decides two tax schedules, one for each occupation. In each formulation, the government chooses two instruments; hence, in this sense, the problem is similar.

In the model with evasion, only the derivatives with respect to each sector's ability are obtained. These derivatives are as follow

$$\begin{aligned} \langle n_s \rangle \quad w_s\left[1 - \rho(x_s)\pi T_s'(w_s n_s)\right] &= u_s'(n_s) \\ \langle n_d \rangle \quad w_d\left[1 - T_d'(w_d n_d)\right] &= u_d'(n_d) \end{aligned}$$

The threshold function in this case is

$$c'(n_s) = \frac{u_s'(n_s)}{u_d'(c(n_s))} = \frac{w_s\left[1 - \rho(x_s)\pi T_s'(w_s n_s)\right]}{w_d\left[1 - T_d'(w_d c(n_s))\right]} \tag{2.9}$$

For a clear comparison, the same wage rate is assumed in each model. The critical fact is to compare equation 2.8 with equation 2.9. The dependent sector has the same condition in each model; however, since self-employed agents face a fine and an audit probability, the self-employed sector's situation is different. Suppose that the taxes in the model without evasion are optimal for a particular distribution of ability. Now, let us proceed to find the conditions under which both threshold functions will be the same.

First, the dependent sector is the same in both models, so one can define the same tax schedule for each one, implying $T = T_b$. Also, it is assumed that c is equal in both models. With those, the numerator in equation 2.8 and equation 2.9 must be the same. This equality happens only if the following condition hold.

$$\rho(x_s) = \frac{1 - h_s\left[1 - T_s'(w_s h_s n_s)\right]}{\pi T_b'(w_s n_s)}$$

where h_s is the labor supply in the model without evasion. In the case that the above condition are met, both problems produce the same results and are correctly nested. However, if the audit strategy takes other forms, both solutions are not the same. Suppose the following audit scheme.

$$\rho(x_s) = \begin{cases} 1/\pi & \text{if } n_s < n^* \\ 0 & \text{if } n^* \leq n_s \end{cases}$$

where n^* is the threshold for the audit scheme. This strategy, which is optimal under the hierarchical model by Sanchez and Sobel (1993), shows that not always the same solution between a model with an occupational decision and one with tax evasion is obtained. Hence, it is not trivial to assume it is possible to nest both models and achieve similar results. In addition, the entire problem is different, principally for two facts. First, the government faces a cost for audit, which produces more expenditure and, consequently, a more significant revenue than in the absence of evasion. Second, the existence of evasion evidences how imperative is for the government to consider more incentive-compatible conditions to incentivize agents to declare their real type.

2.7.2 Proof of Proposition 1

The first two expressions in Proposition 1 are the occupational choice rules and states that each agent works in their more productive sector. To prove them, suppose that a mass of agents greater than zero is assigned to work in their less productive sector. This mass of agents has $w^s > w^d$, but works in the sector d. From the resource constraint, redirecting all these agents to the sector s produces and increases economic resources, which can be assigned to increase the consumption of some agents or increase the provision of public goods, resulting in a social welfare increase. This means that the initial solution cannot be optimal because there exists a reallocation of agents that provide higher social welfare. Therefore, each agent works in the sector with bigger productivity.

The fifth expression comes from the maximization of the government problem with respect to the consumption in each sector. The first-order condition (henceforth FOC) over consumption is

$$\frac{dG(u(C_i(n^i), R))}{du(C_i(n^i), R)} f^d(n^d) f^s(n^s) - \gamma f^d(n^d) f^s(n^s) = 0, \qquad \text{for } i = d, s$$

where γ is the Lagrange multiplier for the resource constraint. Assuming an interior solution and by using γ to equalize both conditions and the fifth expression is obtained.

The third expression comes from the maximization of the social welfare function with respect to public goods. The first-order condition with respect to the public good is

$$\int\limits_{\mathcal{Z}_d^*} \frac{dG}{du} \times \frac{d\phi(R)}{dR} dF^d(n^d) dF^s(n^s) + \int\limits_{\mathcal{Z}_s^*} \frac{dG}{du} \times \frac{d\phi(R)}{dR} dF^d(n^d) dF^s(n^s) - \gamma$$

Both sides can be divided by γ and replaced by the FOC from consumption in each sector.

$$\int_{\mathcal{Z}_d^*} \frac{\frac{dG}{du} \times \frac{d\phi(R)}{dR}}{\frac{dG}{du} \times \frac{du}{dC_d}} dF^d(n^d)dF^s(n^s) + \int_{\mathcal{Z}_s^*} \frac{\frac{dG}{du} \times \frac{d\phi(R)}{dR}}{\frac{dG}{du} \times \frac{du}{dC_s}} dF^d(n^d)dF^s(n^s) = 1$$

It can be defined $MRS(R^*C_i^*)$ with $i = \{d, s\}$, as the marginal rate of substitution between public good and private consumption, evaluated at the optimum. It is critical to note that, the MRS is equal for all agents, because of $du/dC_i = 1$ and $du/dR = \phi'(R)$. This condition gives us the following, which is the third expression.

$$\phi'(R) = \int_{\mathcal{Z}_d^*} MRS(R^*C_d^*)dF^d(n^d)dF^s(n^s) + \int_{\mathcal{Z}_s^*} MRS(R^*C_s^*)dF^d(n^d)dF^s(n^s) = 1$$

Finally, the fourth expression shows the resources needed to finance the optimal provision of public goods R^* and comes from the resource constraint.

2.7.3 Proof of Proposition 2

Before a more structured proof, it is possible to demonstrate the decentralization of the first-best invoking the second welfare theorem. In this demonstration, it is sufficient to note that the lump-sum taxes are the transfer vector in a quasi-equilibrium with transfers.[14]

The structured proof begins deriving the social welfare function concerning public goods. Let η be the Lagrange multiplier for resource constraint. The first-order condition, assuming an interior solution, is

$$\int_{\mathcal{N}_d} \frac{dG}{dU^i} \times \frac{d\phi(R)}{dR} dF^d(w^d)dF^s(w^s) + \int_{\mathcal{N}_s} \frac{dG}{dU^i} \times \frac{d\phi(R)}{dR} dF^d(w^d)dF^s(w^s) = \eta$$

and the first-order condition (FOC) for the consumption is

$$\frac{dG\left(U^i(C_i^*(w^i), R^*)\right)}{dU^i(C_i^*(w^i), R^*)} f^d(w^d)f^s(w^s) - \eta f^d(w^d)f^s(w^s) = 0$$

With what it is expressed above, it is possible to obtain the third expression.

$$\int_{\mathcal{N}_d^*} MRS(R^*, C_d^*)dF^d(w^d)dF^s(w^s) + \int_{\mathcal{N}_s^*} MRS(R^*, C_s^*)dF^d(w^d)dF^s(w^s) = 1$$

[14] For a complete explanation of this see Mas-Colell et al. (1995) Chapter 16.

Now, by using the FOC of consumption yields

$$\frac{dG\left(U^d(C_d(w^d),R)\right)}{dU^d(C_d(w^d),R)} = \frac{dG\left(U^s(C_s(w^s),R)\right)}{dU^s(C_s(w^s),R)}$$

The definition of consumption in each sector is $C_i = w^i - T_i^*(w^i)$, and the resource constraint require that

$$R^* = \int\limits_{\mathcal{N}_d^*} T_d^*(w^d)dF^d(w^d)dF^s(w^s) + \int\limits_{\mathcal{N}_s^*} T_s^*(w^s)dF^d(w^d)dF^s(w^s)$$

Hence, the optimal lump-sum transfer must finance the public good and equalize the marginal social welfare in both sectors for all wages. Finally, from the optimal lump-sum transfer, and the fact that $G' \geq 0$ and $G'' < 0$, agents prefer to work in the more productive sector.

2.7.4 Proof of Lemma 1

Suppose there exist two agents, A and B. Agent A has a bigger productivity in the dependent sector but the same in the self-employed one in comparison with agent B. This is $w_A^d > w_B^d$ and $w_A^s = w_B^s$. This produces that both agents have the same utility in the self-employed sector. However, agent A has a more considerable utility in the dependent sector than agent B. What has been mentioned before comes from the assumption that marginal tax cannot be higher than one, which produces that $dU^d/dw^d = 1 - \tau \geq 0$, or the utility in the dependent sector is increasing in the dependent productivity. Let us assume that the difference in agent B's utilities is that such agent decided to work as a dependent worker. Hence, since agent A has more utility in the dependent sector than agent B, agent A also works in the dependent sector. Therefore, every agent with more significant productivity in the dependent sector, and the same productivity in the self-employed sector, than agent B, makes the same occupational decision.

Now, let us assume that there is also an agent C, who is less productive in the dependent sector and has the same productivity in the self-employed sector than agent B. This is $w_B^d > w_C^d$ and $w_B^s = w_C^s$. Agent C has productivities such that the utility in both sectors is equal, i.e., $U^d(w_C^d) = U^s(w_C^s)$. By Definition 1, agent C works as a self-employed; therefore, every agent with less productivity in the dependent sector and the same in the self-employed one than agent C, makes the same occupational decision than such agent. This is, as all those agents have a strictly bigger utility in the self-employed sector than in the dependent one. Because utility in the dependent sector is increasing in the dependent productivity, and agent C has equal utility in both sectors, every agent with less productivity in the dependent sector has a strictly bigger productivity in the self-employed sector.

The last two arguments show that for the same productivity in the self-employed sector there is a level of productivity in the dependent sector, which equalizes utility in both sectors and determines a switch in the occupational decision. Formally, this switching level can be named as κ, or in the example $\kappa(w^s) = w^d_C$. Hence, every agent with $w^d \leq \kappa(w^s)$, works in the self-employed sector. This result is also accurate for every possible self-employed productivity. Therefore, the threshold function exists and is valid for every potential productivity in the self-employed sector.

2.7.5 Proof of Lemma 2

To obtain an incentive-compatible mechanism is necessary to analyze two restrictions on a self-employed agent, participation and incentive compatibility. In this case, the participation constraint is the limited liability condition, which refers to the non-negative utility in each sector.

$$U^s(w^s) = w^s - \mathcal{T}(w^s) - \alpha(w^s)\pi\left[\tau w^s - \mathcal{T}(w^s)\right] \geq 0$$
$$U^d(w^d) = w^d - \tau w^d \geq 0$$

The incentive-compatible restriction can be replaced by the following

$$V(w^s, \mathcal{M}) = \max_x \{w^s - \mathcal{T}(x) - \alpha(x)\pi\left(\tau w^s - \mathcal{T}(x)\right)\}$$

By using the envelop theorem, one can derive with respect to w^s the above expression.

$$\frac{\partial V(w^s, \mathcal{M})}{\partial w^s} = 1 - \alpha(x)\pi\tau - \frac{dx}{dw^s}\left[\mathcal{T}'(x) + \alpha'(x)\pi\left(\tau w^s - \mathcal{T}(x)\right) - \alpha(x)\pi\mathcal{T}'(x)\right]$$

The term in brackets is zero from optimal income declaration.

$$\frac{\partial V(w^s, \mathcal{M})}{\partial x} = \mathcal{T}'(x) + \alpha'(x)\pi\left(\tau w^s - \mathcal{T}(x)\right) - \alpha(x)\pi\mathcal{T}'(x) = 0$$

As truth-telling is seeking, let us impose this condition, i.e., $x = w^s$, in the derivative and obtain

$$\frac{\partial V(w^s, \mathcal{M})}{\partial w^s} = 1 - \alpha(w^s)\pi\tau$$

Now, by using the fundamental theorem of calculus, rewrite the indirect utility for self-employed agents and use their definition to obtain the following equality.

$$\int\limits_{j=\underline{w}^s}^{w^s} (1 - \alpha(j)\pi\tau) + V(\underline{w}^s) = V^s(w^s) = w^s - \mathcal{T}(w^s) - \alpha(w^s)\pi\left(\tau w^s - \mathcal{T}(w^s)\right)$$

By doing algebra the effective tax formula is obtained

$$\mathcal{T}(w^s) = \frac{w^s(1-\tau\alpha(w^s)\pi)}{1-\pi\alpha(w^s)} - \frac{\int\limits_{j=\underline{w}^s}^{w^s}(1-\alpha(j)\pi\tau)\,dj}{1-\pi\alpha(w^s)} - \frac{V^s(\underline{w}^s)}{1-\pi\alpha(w^s)}$$

Now, in order to demonstrate the first statement, Guesnerie and Laffont (1984) are followed. First, obtain the first-order condition for the optimal income declaration from the utility in the self-employed agent.

$$\frac{\partial V(w^s,\mathcal{M})}{\partial x} = -\mathcal{T}'(x) - \alpha'(x)\pi(\tau w^s - \mathcal{T}(x)) + \alpha(x)\pi\mathcal{T}'(x) = 0$$

Second, it can be obtained the second-order condition (SOC), which must be zero or negative from the quasi-concavity of the utility.

$$\frac{\partial^s V(w^s,\mathcal{M})}{\partial x^2} = -\mathcal{T}''(x) - \alpha''(x)\pi(\tau w^s - \mathcal{T}(x)) + 2\alpha'(x)\pi\mathcal{T}'(x) + \alpha(x)\pi\mathcal{T}''(x) \leq 0$$

Third, the FOC can be derived with respect to w^s and assumed truth-telling.

$$\frac{\partial^s V(w^s,\mathcal{M})}{\partial x \partial w^s} = -\alpha'(w^s)\pi\tau + [-\mathcal{T}''(w^s) - \alpha''(w^s)\pi(\tau w^s - \mathcal{T}(w^s)) + 2\alpha'(w^s)\pi\mathcal{T}'(w^s) + \alpha(w^s)\pi\mathcal{T}''(w^s)] = 0$$

The term in brackets is negative or zero from the SOC. This fact implies that the first term must be positive or zero. The marginal tax is positive by assumption; hence, marginal audit must be negative or zero. Therefore, the audit must be non-increasing in the self-employed wage.

2.7.6 Proof of Lemma 3

The first part of this Lemma is the same as Lemma 1 because the form of the utility and the central assumption does not change. Hence, the existence of the threshold also holds in this case.

The second part comes from deriving the equality $V^s(w^s,\mathcal{M}) = U^d(\kappa(w^s))$ with respect to w^s.

$$\begin{aligned} V^s_{w^s}(w^s,\mathcal{M}) =& U^d_{\kappa(w^s)}(\kappa(w^s))\kappa'(w^s) \\ \kappa'(w^s) =& \frac{V^s_{w^s}(w^s,\mathcal{M})}{U^d_{\kappa}(\kappa(w^s))} \end{aligned}$$

where the expression X_i refers to the derivative of X with respect to i.

2.7.7 Proof of Corollary 1

If the audit cost is zero ($c = 0$), the first necessary condition for an optimum depends only on the adjoint function, and it does not rely on the audit cost. From the second necessary condition, the adjoint function is increasing, and from the third necessary condition, it comes from the negative numbers to zero. Those facts imply that the first necessary condition is equal to zero only when the productivity in the self-employed sector takes the upper bound value, i.e., $w^* = \overline{w}^s$. This result shows that the IRS audit all the taxpayers in the self-employed sector, and consequently, all agents in this industry have effective taxes equal to the taxes designed by the government.

Derivation of the Optimal Government Policies

2.7.8 Optimal BLS Rule

The optimal BLS rule is obtained from the government's problem deriving the Lagrangian with respect to the public goods. The Lagrange of the government's problem is

$$\mathcal{L} = \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d)dF^s(w^s)$$
$$+ \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d)dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d)dF^s(w^s) - \delta \left\{ B + R - \tau \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) \right.$$
$$\left. -\tau \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau \int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau w^* \int_{w^*}^{\hat{w}} F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau w^* \int_{\hat{w}}^{\overline{w}^s} F^d\left(\kappa(w^s)\right) dF^s(w^s) \right\}$$

The first-order condition with respect to R is

$$\int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} G'(U^d(w^d))\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} G'\left(U^d(w^d)\right) \phi'(R)dF^d(w^d)dF^s(w^s)$$
$$+ \int_{\underline{w}^s}^{w^*} \int_{\underline{w}^d}^{\kappa(w^s)} G'\left(V^s(w^s,\mathcal{M})\right) \phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} G'\left(V^s(w^s,\mathcal{M})\right) \phi'(R)dF^d(w^d)dF^s(w^s)$$
$$+ \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} G'\left(V^s(w^s,\mathcal{M})\right) \phi'(R)dF^d(w^d)dF^s(w^s) - \delta = 0$$

By doing algebra yields

$$\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d}\frac{G'(U^d(w^d))}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d}\frac{G'\left(U^d(w^d)\right)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s)$$
$$+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{G'\left(V^s(w^s,\mathcal{M})\right)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{G'\left(V^s(w^s,\mathcal{M})\right)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s)$$
$$+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{G'\left(V^s(w^s,\mathcal{M})\right)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s)=1$$

The derivative of the Lagrangian with respect to each agent's consumption in each sector are

$$G'(U^d(w^d))f^d(w^d)f^s(w^s)-\delta f^d(w^d)f^s(w^s)=0$$
$$G'\left(V^s(w^s,\mathcal{M})\right)f^d(w^d)f^s(w^s)-\delta f^d(w^d)f^s(w^s)=0$$

where the definition of consumption, $C(w)=w-\tau w$, is used to replaces taxes for consumption. By using the previous equations to replace the Lagrange multiplier in the FOC of the public good yields

$$\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d}\phi'(R)dF^d(w^d)dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d}\phi'(R)dF^d(w^d)dF^s(w^s)+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)}\phi'(R)dF^d(w^d)dF^s(w^s)$$
$$+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}\phi'(R)dF^d(w^d)dF^s(w^s)+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}\phi'(R)dF^d(w^d)dF^s(w^s)=1$$

Finally, and in order to obtain the same expression as the first-best, use the fact that the marginal utility of consumption is equal to one. Hence, the marginal rate of substitution is only expressed as $\phi'(R)$. The final expression is

$$\phi'(R)=\int_{\underline{w}^s}^{\overline{w}^s}\int_{\kappa(w^s)}^{\overline{w}^d}MRS(RC)dF^d(w^d)dF^s(w^s)+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)}MRS(RC)dF^d(w^d)dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}MRS(RC)dF^d(w^d)dF^s(w^s)$$
$$+\int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}MRS(RC)dF^d(w^d)dF^s(w^s)=1$$

2.7.9 Optimal Marginal Tax Rate

The optimal marginal tax rate solves the Lagrangian in the government's problem. The Lagrangian of the government's problem is

$$
\begin{aligned}
\mathcal{L} = & \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d)dF^s(w^s) \\
& + \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d)dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d)dF^s(w^s) - \delta\Bigg\{ B + R - \tau\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) \\
& - \tau\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau w^*\int_{w^*}^{\hat{w}} F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau w^*\int_{\hat{w}}^{\overline{w}^s} F^d\left(\kappa(w^s)\right) dF^s(w^s)\Bigg\}
\end{aligned}
$$

The first-order condition with respect to τ is

$$
\begin{aligned}
& \int_{\underline{w}^s}^{w^*}(-\kappa'_\tau)G\left(U^d(\kappa(w^s))\right) f^d(\kappa(w^s))dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} G'_{U^d}U^d_C\frac{dC_d}{d\tau}dF^d(w^d)dF^s(w^s) + \hat{w}'_\tau\int_{\kappa(\hat{w})}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d)f^s(\hat{w}) \\
& + \int_{w^*}^{\hat{w}}(-\kappa'_\tau)G\left(U^d(\kappa(w^s))\right) f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} G'_{U^d}U^d_C\frac{dC_d}{d\tau}dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\kappa'_\tau G\left(V^s(w^s,\mathcal{M})\right) f^d(\kappa(w^s))dF^s(w^s) \\
& + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s}V^s_C\frac{dC_s}{d\tau}dF^d(w^d)dF^s(w^s) + \hat{w}'_\tau\int_{\underline{w}^d}^{\kappa(\hat{w})} G\left(V^s(\hat{w},\mathcal{M})\right) dF^d(w^d)f^s(\hat{w}) + \int_{w^*}^{\hat{w}}\kappa'_\tau G\left(V^s(w^s,\mathcal{M})\right) f^d(\kappa(w^s))dF^s(w^s) \\
& + \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s}V^s_C\frac{dC_s}{d\tau}dF^d(w^d)dF^s(w^s) + (-\hat{w}'_\tau)\int_{\underline{w}^d}^{\kappa(\hat{w})} G\left(V^s(\hat{w},\mathcal{M})\right) dF^d(w^d)f^s(\hat{w}) + \int_{\hat{w}}^{\overline{w}^s}\kappa'_\tau G\left(V^s(w^s,\mathcal{M})\right) f^d(\kappa(w^s))dF^s(w^s) \\
& + \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s}V^s_C\frac{dC_s}{d\tau}dF^d(w^d)dF^s(w^s) - \delta\Bigg\{ -\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{\underline{w}^s}^{w^*}(-\kappa'_\tau)\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s) \\
& - \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau\hat{w}'_\tau\int_{\kappa(\hat{w})}^{\overline{w}^d} w^d dF^d(w^d)f^s(\hat{w}) - \tau\int_{w^*}^{\hat{w}}(-\kappa'_\tau)\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s) - \int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right) dF^s(w^s) \\
& - \tau\int_{\underline{w}^s}^{w^*} w^s\kappa'_\tau f^d\left(\kappa(w^s)\right) dF^s(w^s) - \int_{w^*}^{\hat{w}} w^* F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau\hat{w}'_\tau w^* F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau\int_{w^*}^{\hat{w}} w^*\kappa'_\tau f^d\left(\kappa(w^s)\right) dF^s(w^s) \\
& - \int_{\hat{w}}^{\overline{w}^s} w^* F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau(-\hat{w}'_\tau)w^* F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau\int_{\hat{w}}^{\overline{w}^s} w^*\kappa'_\tau f^d\left(\kappa(w^s)\right) dF^s(w^s)\Bigg\} = 0
\end{aligned}
$$

To simplify the previous expression, use several results and assumptions. First, $g = {}^{(G_{V^i}\times V^i_C)}/_{\delta}$ is defined as the social valuation of agent's consumption. Second, it is necessary to recall that $U^d(\kappa(w^s)) = V^s(w^s,\mathcal{M})$, implying that some terms are canceling among them. Third, by assumption $\kappa(\hat{w}) = \overline{w}^d$, hence, some integrals collapse to zero. Fourth, as a result of the linearity in tax and the audit schedule, $\kappa'_\tau = 0$ for $w^s \leq w^*$ and for $w^s \geq \hat{w}$, resulting in some term to be zero. Finally, the derivatives of consumption respect to marginal tax in each sector are.

$$\frac{dC_d}{d\tau} = -w^d, \qquad \frac{dC_s}{d\tau} = \begin{cases} -w^s & \text{if } w^s < w^* \\ -w^* & \text{if } w^* \leq w^s \end{cases}$$

By using those facts and doing algebra yields

$$\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} g(w^d)(-w^d)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} g(w^d)(-w^d)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} g(w^s)(-w^s)dF^d(w^d)dF^s(w^s)$$
$$+ \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} g(w^s)(-w^*)dF^d(w^d)dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} g(w^s)(-w^*)dF^d(w^d)dF^s(w^s) = -\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s)$$
$$- \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) + \tau\int_{w^*}^{\hat{w}} \kappa'_\tau \kappa(w^s) f^d(\kappa(w^s))dF^s(w^s) - \int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right) dF^s(w^s)$$
$$- \int_{w^*}^{\hat{w}} w^* F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau\int_{w^*}^{\hat{w}} w^*\kappa'_\tau f^d\left(\kappa(w^s)\right) dF^s(w^s) - \int_{\hat{w}}^{\overline{w}^s} w^* F^d\left(\kappa(w^s)\right) dF^s(w^s)$$

Let us note that, $\kappa(w^s) - w^* = \dfrac{w^s - w^*}{1-\tau}$. By replacing it in the above equation and doing algebra, the optimal marginal tax formula is

$$\frac{\tau}{1-\tau}\int_{w^*}^{\hat{w}} (w^s - w^*)\kappa_\tau(w^s) f^d(\kappa(w^s))dF^s(w^s) = \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d)dF^s(w^s)$$
$$+ \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^s dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^* dF^d(w^d)dF^s(w^s)$$
$$+ \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^* dF^d(w^d)dF^s(w^s)$$

2.7.10 Optimal Budget for the IRS

The Lagrangian of the government's problem is

$$\mathcal{L} = \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s, \mathcal{M})\right) dF^d(w^d) dF^s(w^s)$$

$$+ \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s, \mathcal{M})\right) dF^d(w^d) dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s, \mathcal{M})\right) dF^d(w^d) dF^s(w^s) - \delta \left\{ B + R - \tau \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) \right.$$

$$\left. -\tau \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) - \tau \int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau w^* \int_{w^*}^{\hat{w}} F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau w^* \int_{\hat{w}}^{\overline{w}^s} F^d\left(\kappa(w^s)\right) dF^s(w^s) \right\}$$

The first-order condition with respect to B is

$$w^*_B \int_{\kappa(w^*)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) f^s(w^*) + \int_{\underline{w}^s}^{w^*} (\kappa'_B) G\left(U^d(\kappa(w^s))\right) f^d(\kappa(w^s)) dF^s(w^s) + (-w^*_B) \int_{\kappa(w^*)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) f^s(w^*)$$

$$+ \hat{w}'_B \int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) dF^s(w^s) + \int_{w^*}^{\hat{w}} (-\kappa'_B) G\left(U^d(\kappa(w^s))\right) f^d(\kappa(w^s)) dF^s(w^s) + w^*_B \int_{\underline{w}^d}^{\kappa(w^*)} G\left(V^s(w^*, \mathcal{M})\right) dF^d(w^d) f^s(w^*)$$

$$+ \int_{\underline{w}^s}^{w^*} \kappa'_B G\left(V^s(w^s, \mathcal{M})\right) f^d(\kappa(w^s)) dF^s(w^s) + (-w^*_B) \int_{\underline{w}^d}^{\kappa(w^*)} G\left(V^s(w^*, \mathcal{M})\right) dF^d(w^d) f^s(w^*) + \hat{w}'_B \int_{\underline{w}^d}^{\kappa(\hat{w})} G\left(V^s(\hat{w}, \mathcal{M})\right) dF^d(w^d) f^s(\hat{w})$$

$$+ \int_{w^*}^{\hat{w}} \kappa'_B G\left(V^s(w^s, \mathcal{M})\right) f^d(\kappa(w^s)) dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s} V^s_C \frac{dC_s}{dB} dF^d(w^d) dF^s(w^s) + (-\hat{w}'_B) \int_{\underline{w}^d}^{\kappa(\hat{w})} G\left(V^s(\hat{w}, \mathcal{M})\right) dF^d(w^d) f^s(\hat{w})$$

$$+ \int_{\hat{w}}^{\overline{w}^s} \kappa'_B G\left(V^s(w^s, \mathcal{M})\right) f^d(\kappa(w^s)) dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s} V^s_C \frac{dC_s}{dB} dF^d(w^d) dF^s(w^s) - \delta \left\{ 1 - \tau w^*_B \int_{\kappa(w^*)}^{\overline{w}^d} w^d dF^d(w^d) f^s(w^*) \right.$$

$$- \tau \int_{\underline{w}^s}^{w^*} (-\kappa'_B) \kappa(w^s) f^d(\kappa(w^s)) dF^s(w^s) - \tau(-w^*_B) \int_{\kappa(w^*)}^{\overline{w}^d} w^d dF^d(w^d) f^s(w^*) - \tau \hat{w}'_B \int_{\kappa(\hat{w})}^{\overline{w}^d} w^d dF^d(w^d) f^s(\hat{w})$$

$$- \tau \int_{w^*}^{\hat{w}} (-\kappa'_B) \kappa(w^s) f^d(\kappa(w^s)) dF^s(w^s) - \tau w^*_B w^* F^d\left(\kappa(w^*)\right) f^s(w^s) - \tau \int_{\underline{w}^s}^{w^*} w^s \kappa'_B f^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau(-w^*_B) w^* F^d\left(\kappa(w^*)\right) f^s(w^*)$$

$$- \tau \hat{w}'_B w^* F^d\left(\kappa(\hat{w})\right) f^s(\hat{w}) - \tau \kappa'_B w^* f^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau \int_{w^*}^{\hat{w}} \frac{dw^*}{db} F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau(-\hat{w}'_B) w^* F^d\left(\kappa(\hat{w})\right) f^s(\hat{w})$$

$$\left. -\tau \int_{\hat{w}}^{\overline{w}^s} w^* \kappa'_B f^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau \int_{\hat{w}}^{\overline{w}^s} \frac{dw^*}{dB} F^d\left(\kappa(w^s)\right) dF^s(w^s) \right\}$$

Use some results and assumptions to simplify the above expression. First, the social valuation of the agent's consumption can be defined as $g = {(G'_V \times V_C)}/{\delta}$. Second, by definition $U^d(\kappa(w^s)) = V^s(w^s, \mathcal{M})$, makes some terms cancel among them. Third, from the result in the IRS's problem, $\kappa'_B = 0$ where $w^s \leq w^*$ and $w^s > \hat{w}$, result in some terms to be equal to zero. Fourth, by definition $\kappa(\hat{w}) = w^d$ collapsing some terms to zero. Finally, the derivative of consumption in the self-employed sector with respect to the budget is

$$\frac{dC_s}{dB} = \begin{cases} 0 & \text{if } w^s < w^* \\ -\tau \dfrac{dw^*}{dB} & \text{if } w^* \leq w^s \end{cases}$$

Let us recall that, in the dependent sector, an increase in the budget for the IRS does not affect taxes. By using those facts and doing algebra yields

$$\tau \int_{w^*}^{\hat{w}} (w^* - \kappa(w^s)) \frac{dw^*}{dB} \kappa'_B f^d(\kappa(w^s)) dF^s(w^s) + \tau \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} \frac{dw^*}{dB} dF^d(w^d) dF^s(w^s) + \tau \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} \frac{dw^*}{dB} dF^d(w^d) dF^s(w^s)$$

$$= 1 + \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} g(w^s) \tau \frac{dw^*}{dB} dF^d(w^d) dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} g(w^s) \tau \frac{dw^*}{dB} dF^d(w^d) dF^s(w^s)$$

2.7.11 Differential Taxation

Let us define t_s and t_d as the tax rate in the self-employed and the dependent sector, respectively. As it has been explained before, the solution of the optimal audit maintains because differential taxation does not alter the problem. Now, the Lagrangian of the government's problem is

$$\begin{aligned}
\mathcal{L} = & \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s, \mathcal{M})\right) dF^d(w^d) dF^s(w^s) \\
& + \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s, \mathcal{M})\right) dF^d(w^d) dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s, \mathcal{M})\right) dF^d(w^d) dF^s(w^s) - \delta \left\{ B + R - t_d \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) \right. \\
& \left. - t_d \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) - t_s \int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right) dF^s(w^s) - t_s w^* \int_{w^*}^{\hat{w}} F^d\left(\kappa(w^s)\right) dF^s(w^s) - t_s w^* \int_{\hat{w}}^{\overline{w}^s} F^d\left(\kappa(w^s)\right) dF^s(w^s) \right\}
\end{aligned}$$

For simplicity, the same notation in the Lagrange multiplier for the budget constraint is maintained. At this point, it is of great importance to recall that, the threshold function changes, and takes the following form

$$\kappa(w^s) = \begin{cases} \dfrac{w^s(1-t_s)}{1-t_d} & \text{if } w^s < w^* \\ \dfrac{w^s - t_s w^*}{1-t_d} & \text{if } w^* \leq w^s < \hat{w} \\ \overline{w}^d & \text{if } \hat{w} \leq w^s \end{cases}$$

As it has been explained before, the public good provision is the same as in the first-best, because the differential taxation does not alter the marginal rate of substitution between the public goods and consumption. For this reason, the government only chooses both tax rates and the budget for the IRS. First, the optimal tax rate in the dependent sector is obtained. The first-order condition regarding t_d is

$$
w^{*\prime}_{t_d}\int_{\kappa(w^*)}^{\overline{w}^d} G\left(U^d(w^d)\right)dF^d(w^d)f^s(w^*)+\int_{\underline{w}^s}^{w^*}(-\kappa'_{t_d})G\left(U^d(\kappa(w^s))\right)f^d(\kappa(w^s))dF^s(w^s)+\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d}G'_{U^d}U^d_C\frac{dC_d}{dt_d}dF^d(w^d)dF^s(w^s)
$$

$$
+(-w^{*\prime}_{t_d})\int_{\kappa(w^*)}^{\overline{w}^d}G\left(U^d(w^d)\right)dF^d(w^d)f^s(w^*)+\hat{w}'_{t_d}\int_{\kappa(\hat{w})}^{\overline{w}^d}G\left(U^d(w^d)\right)dF^d(w^d)f^s(\hat{w})+\int_{w^*}^{\hat{w}}(-\kappa'_{t_d})G\left(U^d(\kappa(w^s))\right)f^d(\kappa(w^s))dF^s(w^s)
$$

$$
+\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d}G'_{U^d}U^d_C\frac{dC_d}{dt_d}dF^d(w^d)dF^s(w^s)+w^{*\prime}_{t_d}\int_{\underline{w}^d}^{\kappa(w^*)}G\left(V^s(w^*,\mathcal{M})\right)dF^d(w^d)f^s(w^*)+\int_{\underline{w}^s}^{w^*}\kappa'_{t_d}G\left(V^s(w^s,\mathcal{M})\right)f^d(\kappa(w^s))dF^s(w^s)
$$

$$
+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)}G'_{V^s}V^s_C\frac{dC_s}{dt_d}dF^d(w^d)dF^s(w^s)+(-w^{*\prime}_{t_d})\int_{\underline{w}^d}^{\kappa(w^*)}G\left(V^s(w^*,\mathcal{M})\right)dF^d(w^d)f^s(w^*)+\hat{w}'_{t_d}\int_{\underline{w}^d}^{\kappa(\hat{w})}G\left(V^s(\hat{w},\mathcal{M})\right)dF^d(w^d)f^s(\hat{w})
$$

$$
+\int_{w^*}^{\hat{w}}\kappa'_{t_d}G\left(V^s(w^s,\mathcal{M})\right)f^d(\kappa(w^s))dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}G'_{V^s}V^s_C\frac{dC_s}{dt_d}dF^d(w^d)dF^s(w^s)+(-\hat{w}'_{t_d})\int_{\underline{w}^d}^{\kappa(\hat{w})}G\left(V^s(\hat{w},\mathcal{M})\right)dF^d(w^d)f^s(\hat{w})
$$

$$
+\int_{\hat{w}}^{\overline{w}^s}\kappa'_{t_d}G\left(V^s(w^s,\mathcal{M})\right)f^d(\kappa(w^s))dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}G'_{V^s}V^s_C\frac{dC_s}{dt_d}dF^d(w^d)dF^s(w^s)-\delta\left\{-\int_{\hat{w}}^{\overline{w}^s}\int_{\kappa(w^s)}^{\overline{w}^d}w^d dF^d(w^d)dF^s(w^s)\right.
$$

$$
-t_d w^{*\prime}_{t_d}\int_{\kappa(w^*)}^{\overline{w}^d}w^d dF^d(w^d)f^s(w^*)-t_d\int_{\underline{w}^s}^{w^*}(-\kappa'_{t_d})\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s)-\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d}w^d dF^d(w^d)dF^s(w^s)
$$

$$
-t_d(-w^{*\prime}_{t_d})\int_{\kappa(w^*)}^{\overline{w}^d}w^d dF^d(w^d)f^s(w^*)-t_d(-\hat{w}'_{t_d})\int_{\kappa(\hat{w})}^{\overline{w}^d}w^d dF^d(w^d)f^s(\hat{w})-t_d\int_{w^*}^{\hat{w}}(-\kappa'_{t_d})\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s)
$$

$$
-t_s(w^{*\prime}_{t_d})w^*F^d(\kappa(w^*))f^s(w^*)-t_s\int_{\underline{w}^s}^{w^*}w^s\kappa'_{t_d}f^d\left(\kappa(w^s)\right)dF^s(w^s)-t_s w^{*\prime}_{t_s}\int_{w^*}^{\hat{w}}F^d(\kappa(w^s))dF^s(w^s)-t_s w^*(-w^{*\prime}_{t_d})F^d(\kappa(w^*))f^s(w^*)
$$

$$
-t_s w^*\hat{w}'_{t_d}F^d\left(\kappa(\hat{w})\right)dF^s(w^s)-t_s w^*\int_{w^*}^{\hat{w}}\kappa'_{t_d}f^d\left(\kappa(w^s)\right)dF^s(w^s)-t_s(w^{*\prime}_{t_d})\int_{\hat{w}}^{\overline{w}^s}F^d(\kappa(w^s))dF^s(w^s)-t_s w^*(-\hat{w}'_{t_d})F^d\left(\kappa(\hat{w})\right)dF^s(w^s)
$$

$$
\left.-t_s w^*\int_{\hat{w}}^{\overline{w}^s}\kappa'_{t_d}f^d\left(\kappa(w^s)\right)dF^s(w^s)\right\}=0
$$

To simplify the above expression, use some assumptions and results. First, the social value of the agent's consumption is defined as $g = {(G' \times U^d_C)}/{\delta}$. Second, by definition $U^d(\kappa(w^s)) = V^s(w^s, \mathcal{M})$ makes some terms cancel among them. Third, by definition $\kappa(\hat{w}) = \overline{w}^d$, collapsing some terms to zero. Fourth, from the result in the IRS's problem, $\kappa'_{t_d} = {\kappa(w^s)}/{(1-t_d)}$ for $w^s \leq w^*$, $\kappa'_{t_d} = -\dfrac{t_s w^{*\prime}_{t_d}}{1-t_d} + {\kappa(w^s)}/{(1-t_d)}$ for $w^* < w^s \leq \hat{w}$, and $\kappa'_{t_d} = 0$ for $w^s > \hat{w}$. Fifth, ${dC_d}/{dt_d} = -w^d$. Sixth, ${dC_s}/{dt_d} = 0$ for $w^s \leq w^*$, and ${dC_s}/{dt_d} = -\dfrac{dw^*}{dt_d}t_s$ for $w^* < w^s \leq \hat{w}$. By using those facts and doing algebra yields

$$t_d = \frac{\overbrace{\left(\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d}(1-g(w^d))w^d dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d}(1-g(w^d))w^d dF^d(w^d)dF^s(w^s)\right)(1-t_d)}^{A}}{\underbrace{\int_{\underline{w}^s}^{w^*}\kappa(w^s)^2 f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}}\left(\kappa(w^s) - t_s w^{*\prime}_{t_d}\right)\kappa(w^s) f^d(\kappa(w^s))dF^s(w^s)}_{B}}$$

$$+ \frac{\overbrace{w^{*\prime}_{t_d}\left(\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}(1-g(w^s))dF^d(w^d)dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}(1-g(w^s))dF^d(w^d)dF^s(w^s)\right)(1-t_d)}^{D}}{\underbrace{\int_{\underline{w}^s}^{w^*}\kappa(w^s)^2 f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}}\left(\kappa(w^s) - t_s w^{*\prime}_{t_d}\right)\kappa(w^s) f^d(\kappa(w^s))dF^s(w^s)}_{B}}$$

$$+ \frac{t_s \overbrace{\left(\int_{\underline{w}^s}^{w^*} w^s\kappa(w^s) f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}} w^*\left(\kappa(w^s) - t_s w^{*\prime}_{t_d}\right) f^d(\kappa(w^s))dF^s(w^s)\right)}^{C}}{\underbrace{\int_{\underline{w}^s}^{w^*}\kappa(w^s)^2 f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}}\left(\kappa(w^s) - t_s w^{*\prime}_{t_d}\right)\kappa(w^s) f^d(\kappa(w^s))dF^s(w^s)}_{B}}$$

To simplify the exposure of the result, some parts with letters can be denoted. The optimal tax rate is

$$t_d = \frac{A + t_s D}{A + B + t_s D} + t_s \frac{C}{A + B + t_s D}$$

In order to fulfill the limited liability constraint the marginal tax rate in the dependent sector cannot be higher than one, thus, it is necessary that $t_s \leq B/C$. In in other words, to obtain a marginal tax rate in the dependent sector not bigger than one, the marginal tax rate in the self-employed sector cannot be higher than the ratio B/C.

Now, the tax rate in the self-employed sector will be obtained. The first-order condition with respect to

t_s is

$$
\begin{aligned}
& w^{*\prime}_{t_s} \int_{\kappa(w^*)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) f^s(w^*) + \int_{\underline{w}^s}^{w^*} (-\kappa'_{t_s}) G\left(U^d(\kappa(w^s))\right) f^d(\kappa(w^s)) dF^s(w^s) + (-w^{*\prime}_{t_s}) \int_{\kappa(w^*)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) f^s(w^*) \\
& + \hat{w}'_{t_s} \int_{\kappa(\hat{w})}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) f^s(\hat{w}) + \int_{w^*}^{\hat{w}} (-\kappa'_{t_s}) G\left(U^d(\kappa(w^s))\right) f^d(\kappa(w^s)) dF^s(w^s) + w^{*\prime}_{t_s} \int_{\underline{w}^d}^{\kappa(w^*)} G\left(V^s(w^*, \mathcal{M})\right) dF^d(w^d) f^s(w^s) \\
& + \int_{\underline{w}^s}^{w^*} \kappa'_{t_s} G\left(V^s(w^s, \mathcal{M})\right) f^d(\kappa(w^s)) dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s} V^s_C \frac{dC_s}{dt_s} dF^d(w^d) dF^s(w^s) + (-w^{*\prime}_{t_s}) \int_{\underline{w}^d}^{\kappa(w^*)} G\left(V^s(w^*, \mathcal{M})\right) dF^d(w^d) f^s(w^s) \\
& + \hat{w}'_{t_s} \int_{\underline{w}^d}^{\kappa(\hat{w})} G\left(V^s(\hat{w}, \mathcal{M})\right) dF^d(w^d) f^s(\hat{w}) + \int_{w^*}^{\hat{w}} \kappa'_{t_s} G\left(V^s(w^s, \mathcal{M})\right) f^d(\kappa(w^s)) dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s} V^s_C \frac{dC_s}{dt_s} dF^d(w^d) dF^s(w^s) \\
& + (-\hat{w}'_{t_s}) \int_{\underline{w}^d}^{\kappa(\hat{w})} G\left(V^s(\hat{w}, \mathcal{M})\right) dF^d(w^d) f^s(\hat{w}) + \int_{\hat{w}}^{\overline{w}^s} \kappa'_{t_s} G\left(V^s(w^s, \mathcal{M})\right) f^d(\kappa(w^s)) dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s} V^s_C \frac{dC_s}{dt_s} dF^d(w^d) dF^s(w^s) \\
& - \delta \Bigg\{ -t_d w^{*\prime}_{t_s} \int_{\kappa(w^*)}^{\overline{w}^d} w^d dF^d f^s(w^*) - t_d \int_{\underline{w}^s}^{w^*} (-\kappa'_{t_s}) \kappa(w^s) f^d(\kappa(w^s)) dF^s(w^s) - t_d(-w^{*\prime}_{t_s}) \int_{\kappa(w^*)}^{\overline{w}^d} w^d dF^d f^s(w^*) \\
& - t_d \hat{w}'_{t_s} \int_{\kappa(\hat{w})}^{\overline{w}^d} w^d dF^d(w^d) f^s(\hat{w}) - t_d \int_{w^*}^{\hat{w}} \kappa'_{t_s} \kappa(w^s) f^d(\kappa(w^s)) dF^s(w^s) - \int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right) dF^s(w^s) - t_s w^{*\prime}_{t_s} w^* F^d(\kappa(w^*)) f^s(w^*) \\
& - t_s \int_{\underline{w}^s}^{w^*} w^s \kappa'_{t_s} f^d\left(\kappa(w^s)\right) dF^s(w^s) - w^* \int_{w^*}^{\hat{w}} F^d\left(\kappa(w^s)\right) dF^s(w^s) - t_s w^{*\prime}_{t_s} \int_{w^*}^{\hat{w}} F^d(\kappa(w^s)) dF^s(w^s) - t_s w^* (-w^{*\prime}_{t_s}) F^d(\kappa(w^*)) f^s(w^*) \\
& - t_s w^* \hat{w}'_{t_s} F^d\left(\kappa(\hat{w})\right) dF^s(w^s) - t_s w^* \int_{w^*}^{\hat{w}} \kappa'_{t_s} f^d\left(\kappa(w^s)\right) dF^s(w^s) - w^* \int_{\hat{w}}^{\overline{w}^s} F^d\left(\kappa(w^s)\right) dF^s(w^s) - t_s w^{*\prime}_{t_s} \int_{\hat{w}}^{\overline{w}^s} F^d(\kappa(w^s)) dF^s(w^s) \\
& - t_s w^* (-\hat{w}'_{t_s}) F^d\left(\kappa(\hat{w})\right) dF^s(w^s) - t_s w^* \int_{\hat{w}}^{\overline{w}^s} \kappa'_{t_s} f^d\left(\kappa(w^s)\right) dF^s(w^s) \Bigg\} = 0
\end{aligned}
$$

To simplify the expression, use some assumptions and results. First, one can be defined the social value of the agent's consumption as$g = {}^{(G' \times V^s_C)}/_{\delta}$. Second, by definition $U^d(\kappa(w^s)) = V^s(w^s, \mathcal{M})$ makes some terms cancel among them. Third, by definition $\kappa(\hat{w}) = \overline{w}^d$, collapsing some terms to zero. Fourth, from the result in the IRS's problem, $\kappa'_{t_s} = {}^{(-w^s)}/_{(1-t_d)}$ for $w^s \leq w^*$, $\kappa'_{t_s} = {}^{(-w^* - t_s w^{*\prime}_{t_s})}/_{(1-t_d)}$ for $w^s \in (w^*, \hat{w})$, and $\kappa'_{t_s} = 0$ for $w^s > \hat{w}$. Fifth, ${}^{dC_s}/_{dt_s} = -w^s$ for $w^s < w^*$ and ${}^{dC_s}/_{dt_s} = -w^* - t_s w^{*\prime}_{t_s}$ for $w^s \geq w^*$. By using those facts and doing algebra yields

$$t_s = \frac{\overbrace{\left[\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^s dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))(w^* + t_s w^{*\prime}_{t_s})dF^d(w^d)dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))(w^* + t_s w^{*\prime}_{t_s})dF^d(w^d)dF^s(w^s)\right](1-t_d)}^{E}}{\underbrace{\int_{\underline{w}^s}^{w^*} (w^s)^2 f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}} w^*(w^* + t_s w^{*\prime}_{t_s})f^d(\kappa(w^s))dF^s(w^s)}_{F}}$$

$$+ \frac{t_d \overbrace{\left(\int_{\underline{w}^s}^{w^*} \kappa(w^s)w^s f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}} \kappa(w^s)(w^* + t_s w^{*\prime}_{t_s})f^d(\kappa(w^s))dF^s(w^s)\right)}^{C}}{\underbrace{\int_{\underline{w}^s}^{w^*} (w^s)^2 f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}} w^*(w^* + t_s w^{*\prime}_{t_s})f^d(\kappa(w^s))dF^s(w^s)}_{F}}$$

Again, some parts with letters can be denoted to simplify the exposure of the result. Note that the term C is equal to one in the derivation of t_d. Rearranging terms

$$t_s = \frac{E}{F} - t_d\frac{(E-C)}{F}$$

In this case, $t_s \leq 1$ is needed to fulfill the requirement for a direct incentive-compatible mechanism. Applying this condition to the above equation yields

$$\frac{E-F}{E-C} \leq t_d \leq 1 \Rightarrow F \geq C$$

Let us analyze if each possible combination of the marginal tax rates hold the condition for an optimum marginal tax rate obtained above. First, if $t_s < t_d$, $\kappa(w^s) > w^s$ for $w^s \leq \hat{w}$, hence $C > F$ producing a contradiction with the condition to obtain a solution, therefore, it is not a solution. Second, if $t_s = t_d$, $\kappa(w^s) = w^s$ for $w^s \leq w^*$ and $\kappa(w^s) > w^s$ for $w^* \leq w^s \leq \hat{w}$, resulting in $C > F$. Therefore, $t_s = t_d$ cannot be a solution for differential taxation either. Third, if $t_s > t_d$, $\kappa(w^s) < w^s$ for $w^s \leq w^*$ and $\kappa(w^s) \lesseqgtr w^s$ for $w^* \leq w^s \leq \hat{w}$, producing that for some cases $F \geq C$. Hence, the only possible solution for differential taxation is $t_s > t_d$.

2.7.12 Comparative Statics

Taxes

For simplicity, the tax rate formula can be rewritten as $t = {}^{A}/_{(A+X)}$, where A takes the value of

$$
\begin{aligned}
A =& \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d)dF^s(w^s) \\
&+ \int_{\underline{w}^s}^{w^*} \int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^s dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^* dF^d(w^d)dF^s(w^s) \\
&+ \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^* dF^d(w^d)dF^s(w^s)
\end{aligned}
$$

and X the value of

$$X = \int_{w^*}^{\hat{w}} (w^s - w^*)\kappa_\tau(w^s) f^d(\kappa(w^s))dF^s(w^s)$$

With this notation, the partial derivative of the tax rate with respect to the audit cost, c, which is

$$\frac{d\tau}{dc} = \frac{\frac{dA}{dc}X - \frac{dX}{dc}A}{(A+X)^2}$$

This formula simplifies how the overall effect is obtained. First, one can obtain the characterization of the partial derivative of the term A with respect to c

$$
\begin{aligned}
\frac{dA}{dc} =& \frac{dw^*}{dc} \int_{\kappa(w^*)}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d)f^s(w^*) + \int_{\underline{w}^s}^{w^*} (-\kappa_c')(1-g(\kappa(w^s)))\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} (-g'U_c^d)w^d dF^d(w^d)dF^s(w^s) \\
&+ (-\frac{dw^*}{dc}) \int_{\kappa(w^*)}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d)f^s(w^*) + \hat{w}_c' \int_{\kappa(\hat{w})}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d)f^s(\hat{w}) + \int_{w^*}^{\hat{w}} (-\kappa_c')(1-g(\kappa(w^s)))\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s) \\
&+ \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} (-g_c'U_c^d)w^d dF^d(w^d)dF^s(w^s) + \frac{dw^*}{dc} \int_{\underline{w}^d}^{\kappa(w^*)} (1-g(w^*))w^* dF^d(w^d)f^s(w^*) + \int_{\underline{w}^s}^{w^*} \kappa_c'(1-g(w^s))w^s f^d(\kappa(w^s))dF^s(w^s) \\
&+ \int_{\underline{w}^s}^{w^*} \int_{\underline{w}^d}^{\kappa(w^s)} (-g'V_c^s)w^s dF^d(w^d)dF^s(w^s) + (-\frac{dw^*}{dc}) \int_{\underline{w}^d}^{\kappa(w^*)} (1-g(w^*))w^* dF^d(w^d)f^s(w^*) + \hat{w}_c' \int_{\underline{w}^d}^{\kappa(\hat{w})} (1-g(\hat{w}))w^* dF^d(w^d)f^s(\hat{w}) \\
&+ \int_{w^*}^{\hat{w}} \kappa_c'(1-g(w^s))w^* f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} \left[-g'V_c^s w^* + (1-g(w^s))\frac{dw^*}{dc}\right] dF^d(w^d)dF^s(w^s) + (-\hat{w}_c') \int_{\underline{w}^d}^{\kappa(\hat{w})} (1-g(\hat{w}))w^* dF^d(w^d)f^s(\hat{w}) \\
&+ \int_{\hat{w}}^{\overline{w}^s} \kappa_c'(1-g(w^s))w^* f^d(\kappa(w^s))dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} \left[-g'V_c^s w^* + (1-g(w^s))\frac{dw^*}{dc}\right] dF^d(w^d)dF^s(w^s)
\end{aligned}
$$

It is possible to simplify the above expression using the following facts. First, by definition $\kappa(\hat{w}) = \overline{w}^d$, this produces that some terms are equal to zero. Second, by definition $\kappa_c' = 0$ for $w^s > \hat{w}$, and for $w^s < w^*$, producing that some terms are equal to zero. Finally, some terms are canceled among them because they are the same but have the opposite sign. The simplified expression is

$$\begin{aligned}\frac{dA}{dc} =& \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} (-g'U_c^d)w^d dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} (-g_c'U_c^d)w^d dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} (-g'V_c^s)w^s dF^d(w^d)dF^s(w^s)\\ &+ \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} \left[-g'V_c^s w^* + (1-g(w^s))\frac{dw^*}{dc}\right] dF^d(w^d)dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} \left[-g'V_c^s w^* + (1-g(w^s))\frac{dw^*}{dc}\right] dF^d(w^d)dF^s(w^s)\\ &+ \int_{w^*}^{\hat{w}} (-\kappa_c')(1-g(\kappa(w^s)))\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}} \kappa_c'(1-g(w^s))w^* f^d(\kappa(w^s))dF^s(w^s)\end{aligned}$$

Let us use the following results to simplify the solution. First, from the threshold function

$$\kappa_c = \frac{(\frac{d\tau}{dc}(w^s - w^*) - \frac{dw^*}{dc}(t - t^2))}{(1-t)^s}$$

for $w^s \in (w^*, \hat{w})$. Second, from the agent's utility, $U_c^d = -w^d(d\tau/dc)$, and $V_c^s = -w^s(d\tau/dc)$ for $w^s < w^*$ and $V_c^s = -w^*(d\tau/dc) - \tau(d\tau/dc)$ for $w^* \leq w^s$. By using those facts, grouping terms by $d\tau/dc$ and dw^*/dc, and joining together the integrals with $\hat{w}$, which reflect the same effect. Those fact yields

$$\begin{aligned}\frac{dA}{dc} =& \frac{d\tau}{dc}\left\{\int_{\underline{w}^s}^{\overline{w}^s}\int_{\kappa(w^s)}^{\overline{w}^d} (w^d)^2 g_{U^d}^d dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} (w^s)^2 g_{V^s}^s dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} (w^*)^2 g_{V^s}^s dF^d(w^d)dF^s(w^s)\right.\\ &\left.+ \int_{w^*}^{\hat{w}} \frac{(w^s - w^*)}{(1-\tau)^2}(w^* - \kappa(w^s))(1-g^s(w^s))f^d(w^d)dF^s(w^s)\right\} + \frac{dw^*}{dc}\left\{\int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} ((1-g^s(w^s)) + tw^* g_{V^s}^s)dF^d(w^s)dF^s(w^s)\right.\\ &\left.- \int_{w^*}^{\hat{w}} \frac{\tau}{1-\tau}(w^* - \kappa(w^s))(1-g^s(w^s))f^d(w^d)dF^s(w^s)\right\}\end{aligned}$$

For simplicity, the above equation can be denoted as $dA/dc = (d\tau/dc)a_1 + (dw^*/dc)a_2$. Let us recall that, $g_V \leq 0$, hence $a_1 \leq 0$ and $a_2 \lesseqgtr 0$. Now, obtain the derivative of the term X with respect to c

$$\begin{aligned}\frac{dX}{dc} =& (-w^*)(w^* - w^*)\kappa_\tau'(w^*)f^d(\kappa(w^*))f^s(w^*) + \hat{w}_c'(\hat{w} - w^*)\kappa_\tau'(\hat{w})f^d(\kappa(\hat{w}))f^s(\hat{w})\\ &+ \int_{w^*}^{\hat{w}} \left[-\frac{dw^*}{dc}\kappa_\tau' f^d(\kappa(w^s)) + (w^s - w^*)\kappa_{\tau c}' f^d(\kappa(w^s)) + (w^s - w^*)\kappa_\tau' f^{d\prime}(\kappa(w^s))\kappa_c'\right] dF^s(w^s)\end{aligned}$$

Use the following result to simplify the expression. First

$$\kappa_{\tau c} = -\frac{dw^*}{dc}\frac{1}{(1-\tau)^2} + \frac{d\tau}{dc}\frac{2}{1-\tau}\kappa_t$$

for $w^s \in (w^*, \hat{w})$. Second, $\hat{w} = \overline{w}^d(1-\tau) + \tau w^*$, hence $\hat{w}'_c = -\frac{d\tau}{dc}(\overline{w}^d - w^*) + \frac{dw^*}{dc}\tau$. By using this fact, rearranging and grouping by $d\tau/dc$ and dw^*/dc. Those fact yields

$$\begin{aligned}\frac{dX}{dc} =& \frac{d\tau}{dc}\left\{\int_{w^*}^{\hat{w}}\left((w^s - w^*)\frac{2}{1-\tau}\kappa'_\tau f^d(\kappa(w^s)) + (w^s - w^*)(\kappa'_\tau)^2 f^{d\prime}(\kappa(w^s))\right)dF^s(w^s) - (\overline{w}^d - w^*)(\hat{w} - w^*)\kappa'_\tau(\hat{w})f^d(\kappa(\hat{w}))f^s(\hat{w})\right\}\\ &- \frac{dw^*}{dc}\left\{\int_{w^*}^{\hat{w}}\left(\kappa_\tau f^d(\kappa(w^s)) + \frac{(w^s - w^*)}{(1-\tau)^2}f^d(\kappa(w^s)) + (w^s - w^*)\kappa_\tau\frac{\tau}{1-\tau}f^{d\prime}(\kappa(w^s))\right)dF^s(w^s) + \tau(\hat{w} - w^*)\kappa'_\tau(\hat{w})f^d(\kappa(\hat{w}))f^s(\hat{w})\right\}\end{aligned}$$

For simplicity, the above equation is denoted as ${}^{dX}/_{dc} = ({}^{d\tau}/_{dc})x_1 - ({}^{dw^*}/_{dc})x_2$, where $x_1 \lesseqgtr 0$ and $x_2 > 0$.

By using the characterization of ${}^{dA}/_{dc}$ and${}^{dX}/_{dc}$, is possible to obtain the overall effect on the audit cost in the tax rate. Replace the first formula using the simplified version of each characterization and resolve for the effect of the audit cost in the tax rate, obtaining

$$\frac{d\tau}{dc} = \frac{dw^*}{dc}\frac{a_2X + x_2A}{(A+X)^2 - (a_1X - x_1A)}$$

Note that, the denominator of the previous equation is positive; thus, the effect on audit cost in tax rate depends on the sign of the numerator, which is the effect for changes in audit cost on revenue and social welfare for productivities $w^s \in (w^*, \hat{w})$.

Budget for the IRS

Let us derive both sides of equation 2.7 with regard to the marginal audit cost and simplify it to obtain an expression for this effect. First, the derivative with respect to the audit cost is

$$
\begin{aligned}
&\frac{d\tau}{dc}\int_{w^*}^{\hat{w}}(w^*-\kappa(w^s))\frac{dw^*}{dB}\kappa'_B f^d(\kappa(w^s))dF^s(w^s)+\tau(-\frac{dw^*}{dc})(w^*-\kappa(w^*))\frac{dw^*}{dB}\kappa'_B f^d(\kappa(w^*))f^s(w^*)+\tau\hat{w}'_c(w^*-\kappa(\hat{w}))\frac{dw^*}{dB}\kappa'_B f^d(\kappa(\hat{w}))f^s(\hat{w})\\
&\tau\int_{w^*}^{\hat{w}}\Big[(\frac{dw^*}{dc}-\kappa'_c)\frac{dw^*}{db}\kappa'_B f^d(\kappa(w^s))+(w^*-\kappa(w^s))\frac{dw^*}{dB}\frac{dB}{dc}\kappa'_B f^d(\kappa(w^s))+(w^*-\kappa(w^s))\frac{dw^*}{dB}\kappa''_B\frac{dB}{dc}f^d(\kappa(w^s))\\
&+(w^*-\kappa(w^s))\frac{dw^*}{dB}\kappa'_B\kappa'_c f^d(\kappa(w^s))\Big]dF^s(w^s)+\frac{d\tau}{dc}\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)+\tau(-\frac{dw^*}{dc})\int_{\underline{w}^d}^{\kappa(w^*)}\frac{dw^*}{dB}dF^d(w^d)f^s(w^*)\\
&+\tau\hat{w}'_c\int_{\underline{w}^d}^{\kappa(\hat{w})}\frac{dw^*}{dB}dF^d(w^d)f^s(\hat{w})+\tau\int_{w^*}^{\hat{w}}\kappa'_c\frac{dw^*}{dB}f^d(\kappa(w^s))dF^s(w^s)+\frac{d\tau}{dc}\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}\frac{dB}{dc}dF^d(w^d)dF^s(w^s)\\
&+\frac{d\tau}{dc}\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)+\tau(-\hat{w}'_c)\int_{\underline{w}^d}^{\kappa(\hat{w})}\frac{dw^*}{dB}dF^d(w^d)f^s(\hat{w})+\tau\int_{\hat{w}}^{\overline{w}^s}\kappa'_c\frac{dw^*}{dB}f^d(\kappa(w^s))dF^s(w^s)\\
&+\tau\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}\frac{dB}{dc}dF^d(w^d)dF^s(w^s)=(-\frac{dw^*}{dc})\int_{\underline{w}^d}^{\kappa(w^*)}g(w^*)\tau\frac{dw^*}{dB}dF^d(w^d)f^s(w^*)+(\hat{w}'_c)\int_{\underline{w}^d}^{\kappa(\hat{w})}g(\hat{w})\tau\frac{dw^*}{dB}dF^d(w^d)f^s(\hat{w})\\
&+\int_{w^*}^{\hat{w}}\kappa'_c g(w^s)f^d(\kappa(w^s))dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}\Big[g'V^s_c\tau\frac{dw^*}{dB}+g(w^s)\frac{d\tau}{dc}\frac{dw^*}{dB}+g(w^s)\tau\frac{dw^*}{dB}\frac{dB}{dc}\Big]dF^s(w^s)+(-\hat{w}'_c)\int_{\underline{w}^d}^{\kappa(\hat{w})}g(\hat{w})\tau\frac{dw^*}{dB}dF^d(w^d)f^s(\hat{w})\\
&+\int_{\hat{w}}^{\overline{w}^s}\kappa'_c g(w^s)f^d(\kappa(w^s))dF^s(w^s)+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}\Big[g'V^s_c\tau\frac{dw^*}{dB}+g(w^s)\frac{d\tau}{dc}\frac{dw^*}{dB}+g(w^s)\tau\frac{dw^*}{dB}\frac{dB}{dc}\Big]dF^s(w^s)
\end{aligned}
$$

To simplify this expression, use some assumptions and results. First, $\kappa(w^*)=w^*$. Second, $\kappa'_c=0$ for $w^s>\hat{w}$. Third, $\hat{w}'_c=\frac{dw^*}{dc}\tau+\frac{d\tau}{dc}(\overline{w}^d-w^*)$. Fourth, $V^s_c=-\frac{dw^*}{dc}\tau-\frac{d\tau}{dc}w^*$. Fifth, $\kappa'_c=\frac{d\tau}{dc}\frac{w^s-w^*}{(1-\tau)^2}-\frac{\tau}{1-\tau}\frac{dw^*}{dc}$. Finally, the terms can be grouped by the effect on taxes and threshold level. By using those facts, the total impact on audit cost in the budget for the IRS is

$$
\begin{aligned}
&\frac{dB}{dc}\left\{\int\limits_{w^*}^{\hat{w}}\int\limits_{\underline{w}^d}^{\kappa(w^s)}\tau\frac{dw^*}{dB}(1-g(w^s))dF^d(w^d)dF^s(w^s)+\int\limits_{\hat{w}}^{\overline{w}^s}\int\limits_{\underline{w}^d}^{\kappa(w^s)}\tau\frac{dw^*}{dB}(1-g(w^s))dF^d(w^d)dF^s(w^s)+\tau\int\limits_{w^*}^{\hat{w}}(w^*-\kappa(w^s))\frac{dw^*}{dB}(\kappa'_B+\kappa''_B)f^d(\kappa(w^s))dF^s(w^s)\right\}\\
&=\frac{d\tau}{dc}\left\{\tau(\overline{w}^d-w^*)(w^*-\kappa(w^s))\frac{dw^*}{dB}\kappa'_Bf^d(\kappa(w^s))dF^s(w^s)+\int\limits_{w^*}^{\hat{w}}\left[\tau\frac{w^s-w^*}{(1-\tau)^2}(1-w^*+\kappa(w^s))\frac{dw^*}{dB}\kappa'_Bf^d(\kappa(w^s))-(w^*-\kappa(w^s))\frac{dw^*}{dB}\kappa'_Bf^d(\kappa(w^s))\right.\right.\\
&\left.\left.\tau\frac{w^s-w^*}{(1-\tau)^2}\frac{dw^*}{dB}(1-g(w^s))f^d(\kappa(w^s))\right]dF^s(w^s)-\int\limits_{w^*}^{\hat{w}}\int\limits_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}(1-g'w^*\tau+g(w^s))dF^d(w^d)dF^s(w^s)-\int\limits_{\hat{w}}^{\overline{w}^s}\int\limits_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}(1-g'w^*\tau+g(w^s))dF^d(w^d)dF^s(w^s)\right\}\\
&+\frac{dw^*}{dc}\left\{\int\limits_{\underline{w}^d}^{\kappa(w^*)}(1-g(w^*))\tau\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)-\tau^2(w^*-\kappa(\hat{w}))\frac{dw^*}{dB}\kappa'_Bf^d(\kappa(\hat{w}))f^s(\hat{w})+\int\limits_{w^*}^{\hat{w}}\left[\frac{\tau^2}{1-\tau}\frac{dw^*}{dB}(1-g(w^s))f^d(\kappa(w^s))-\frac{\tau}{1-\tau}\frac{dw^*}{dB}\kappa'_Bf^d(\kappa(w^s))\right.\right.\\
&\left.\left.\frac{\tau^2}{1-\tau}(w^*-\kappa(w^s))\frac{dw^*}{dB}\kappa'_Bf^d(\kappa(w^s))\right]dF^s(w^s)-\int\limits_{w^*}^{\hat{w}}\int\limits_{\underline{w}^d}^{\kappa(w^s)}g'\tau^2\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)-\int\limits_{\hat{w}}^{\overline{w}^s}\int\limits_{\underline{w}^d}^{\kappa(w^s)}g'\tau^2\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s\right\}
\end{aligned}
$$

To give a better interpretation, the above expression is conveyed, with those respective signs, as follows

$$
\frac{dB}{dc}\overbrace{b}^{+}=\frac{d\tau}{dc}\underbrace{\left\{\overbrace{t_1}^{+}+\overbrace{t_2}^{-}-\overbrace{t_3}^{+}-\overbrace{t_4}^{+}\right\}}_{+/-}+\frac{dw^*}{dc}\underbrace{\left\{\overbrace{w_1}^{+}-\overbrace{w_2}^{+}+\overbrace{w_3}^{+}-\overbrace{w_4}^{-}-\overbrace{w_5}^{-}\right\}}_{+/-}
$$

2.7.13 Alternative Derivation of the Optimal Government Policies

In this appendix, the solution for the case where $\overline{n}^d > \overline{n}^s$ is shown. In each case, it is demonstrated that it is possible to extrapolate the conclusion given in the main section. For this explanation, let us define the level of productivity in the dependent sector that is equal to the threshold function evaluated in the upper bound of the self-employed productivity as $\hat{w} = \kappa(\overline{w}^s)$.

Optimal BLS Rule

The optimal BLS rule is obtained from the government's problem deriving the Lagrangian with respect to the public goods. The Lagrange of the government problem is

$$
\begin{aligned}
\mathcal{L} = & \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} G\left(U^d(w^d)\right) dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} G\left(U^d(w^d)\right) dF^d(w^d)dF^s(w^s) \\
& + \int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d)dF^s(w^s) \\
& - \delta\left\{ B + R - \tau\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d)dF^s(w^s) \right. \\
& \left. -\tau\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau w^*\int_{w^*}^{\overline{w}^s} F^d\left(\kappa(w^s)\right) dF^s(w^s) \right\}
\end{aligned}
$$

The first-order condition (FOC) with respect to R is

$$
\begin{aligned}
& \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} G'\left(U^d(w^d)\right)\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} G'\left(U^d(w^d)\right)\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} G'\left(U^d(w^d)\right)\phi'(R)dF^d(w^d)dF^s(w^s) \\
& + \int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} G'\left(U^d(w^d)\right)\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G'\left(V^s(w^s,\mathcal{M})\right)\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G'\left(V^s(w^s,\mathcal{M})\right)\phi'(R)dF^d(w^d)dF^s(w^s) - \delta = 0
\end{aligned}
$$

By doing algebra yields

$$
\begin{aligned}
& \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} \frac{G'\left(U^d(w^d)\right)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} \frac{G'\left(U^d(w^d)\right)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} \frac{G'\left(U^d(w^d)\right)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s) \\
& + \int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} \frac{G'\left(U^d(w^d)\right)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} \frac{G'\left(V^s(w^s,\mathcal{M})\right)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} \frac{G'\left(V^s(w^s,\mathcal{M})\right)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s) = 1
\end{aligned}
$$

The derivative of the Lagrangian regarding consumption in each sector are

$$G'(U^d(w^d))f^d(w^d)f^s(w^s) - \delta f^d(w^d)f^s(w^s) = 0$$
$$G'\left(V^s(w^s, \mathcal{M})\right)f^d(w^d)f^s(w^s) - \delta f^d(w^d)f^s(w^s) = 0$$

where the definition of consumption, $C(w) = w - T(w)$, is used to replaces taxes for consumption. By using the above equation to replace the Lagrange multiplier in the FOC of the public good yields

$$\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} \phi'(R)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} \phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} \phi'(R)dF^d(w^d)dF^s(w^s)$$
$$+\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} \phi'(R)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} \phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} \phi'(R)dF^d(w^d)dF^s(w^s) = 1$$

Finally, one can use the fact that the marginal utility of consumption is equal to one. Hence, the marginal rate of substitution only $\phi'(R)$. The final expression is

$$\phi\prime(R) = \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} MRS(RC)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} MRS(RC)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} MRS(RC)dF^d(w^d)dF^s(w^s)$$
$$+\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} MRS(RC)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} MRS(RC)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} MRS(RC)dF^d(w^d)dF^s(w^s) = 1$$

This equation is the same as in the paper, only the limits in the integrals change, but it represents the same result; the solution of the optimal public good provision is the same as in the first-best.

Optimal Marginal Tax Rate

The optimal marginal tax rate solves the Lagrangian in the government's problem. The Lagrangian of the government problem is

$$
\begin{aligned}
\mathcal{L} = & \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\hat{w}} G\left(U^d(w^d)\right) dF^d(w^d) dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \int_{\hat{w}}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) dF^s(w^s) + \int_{w^*}^{\overline{w}^s} \int_{\kappa(w^s)}^{\hat{w}} G\left(U^d(w^d)\right) dF^d(w^d) dF^s(w^s) \\
& + \int_{w^*}^{\overline{w}^s} \int_{\hat{w}}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s, \mathcal{M})\right) dF^d(w^d) dF^s(w^s) + \int_{w^*}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s, \mathcal{M})\right) dF^d(w^d) dF^s(w^s) \\
& - \delta \left\{ B + R - \tau \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d) dF^s(w^s) - \tau \int_{\underline{w}^s}^{w^*} \int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) - \tau \int_{w^*}^{\overline{w}^s} \int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d) dF^s(w^s) \right. \\
& \left. - \tau \int_{w^*}^{\overline{w}^s} \int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) - \tau \int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau w^* \int_{w^*}^{\overline{w}^s} F^d\left(\kappa(w^s)\right) dF^s(w^s) \right\}
\end{aligned}
$$

The first-order condition with respect to τ is

$$\int_{\underline{w}^s}^{w^*}(-\kappa'_\tau)G\left(\kappa(w^s)\right)f^d(\kappa(w^s))dF^s(w^s)+\int_{\underline{w}^s}^{w^*}\hat{w}'_\tau G\left(U^d(\hat{w})\right)f^d(\hat{w})dF^s(w^s)+\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}}G'_{U^d}U^d_C\frac{dC_d}{d\tau}dF^d(w^d)dF^s(w^s)$$
$$+\int_{\underline{w}^s}^{w^*}(-\hat{w}'_\tau)G\left(U^d(w^d)\right)f^d(\hat{w})dF^s(w^s)+\int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d}G'_{U^d}U^d_C\frac{dC_d}{d\tau}dF^d(w^s)dF^s(w^s)+\int_{w^*}^{\overline{w}^s}(-\kappa'_\tau)G\left(U^d(\kappa(w^s))\right)f^d(\kappa(w^s))dF^s(w^s)$$
$$\int_{w^*}^{\overline{w}^s}\hat{w}'_\tau G\left(U^d(\hat{w})\right)f^d(\hat{w})dF^s(w^s)+\int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}}G'_{U^d}U^d_C\frac{dC_d}{d\tau}dF^d(w^d)dF^s(w^s)+\int_{w^*}^{\overline{w}^s}(-\hat{w}'_\tau)G\left(U^d(\hat{w})\right)f^d(\hat{w})dF^s(w^s)$$
$$+\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d}G'_{U^d}U^d_C\frac{dC_d}{d\tau}dF^d(w^d)dF^s(w^s)+\int_{\underline{w}^s}^{w^*}\kappa'^\tau G\left(V^s(w^s,\mathcal{M})\right)f^d(\kappa(w^s))dF^s(w^s)+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)}G'_{V^s}V^s_C\frac{dC_s}{d\tau}dF^d(w^d)dF^s(w^s)$$
$$+\int_{w^*}^{\overline{w}^s}\kappa'_\tau G\left(V^s(w^s,\mathcal{M})\right)f^d(\kappa(w^s))dF^s(w^s)+\int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}G'_{V^s}V^s_C\frac{dC_s}{d\tau}dF^d(w^d)dF^s(w^s)-\delta\left\{-\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}}w^d dF^d(w^d)dF^s(w^s)\right.$$
$$-\tau\int_{\underline{w}^s}^{w^*}(-\kappa'_\tau)\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s)-\tau\int_{\underline{w}^s}^{w^*}\hat{w}'_\tau\hat{w}f^d(\hat{w})dF^s(w^s)-\int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d}w^d dF^d(w^d)dF^s(w^s)-\tau\int_{\underline{w}^s}^{w^*}(-\hat{w}'_\tau)\hat{w}f^d(\hat{w})dF^s(w^s)$$
$$-\int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}}w^d dF^d(w^d)dF^s(w^s)-\tau\int_{w^*}^{\overline{w}^s}(-\kappa'_\tau)\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s)-\tau\int_{w^*}^{\overline{w}^s}\hat{w}'_\tau\hat{w}f^d(\hat{w})dF^s(w^s)-\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d}w^d dF^d(w^d)dF^s(w^s)$$
$$-\tau\int_{w^*}^{\overline{w}^s}(-\hat{w}'_\tau)\hat{w}f^d(\hat{w})dF^s(w^s)-\int_{\underline{w}^s}^{w^*}w^sF^d\left(\kappa(w^s)\right)dF^s(w^s)-\tau\int_{\underline{w}^s}^{w^*}w^s\kappa'_\tau f^d\left(\kappa(w^s)\right)dF^s(w^s)-\int_{w^*}^{\overline{w}^s}w^*F^d\left(\kappa(w^s)\right)dF^s(w^s)$$
$$\left.-\tau\int_{w^*}^{\overline{w}^s}w^*\kappa'_\tau f^d\left(\kappa(w^s)\right)dF^s(w^s)\right\}$$

To simplify the previous expression, one can use several results and assumptions from the model. First $g = {}^{(G_{V^i}\times V^i_C)}/_{\delta}$ can be defined as the social valuation of agent's consumption. Second, it needs to be recalled that $U^d(\kappa(w^s)) = V^s(w^s,\mathcal{M})$, implying that some terms are canceling among them. Third, from the IRS's solution, $\kappa'_\tau = 0$ for $w^s \le w^*$. Finally, the derivative of consumption in regard to marginal tax in each sector are

$$\frac{dC_d}{d\tau} = -w^d, \qquad \frac{dC_s}{d\tau} = \begin{cases} -w^s & \text{if } w^s < w^* \\ -w^* & \text{if } w^* \le w^s \end{cases}$$

By using those facts and doing algebra yields

$$
\begin{aligned}
&-\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} g(w^d)w^d dF^d(w^d)dF^s(w^s) - \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} g(w^d)w^d dF^d(w^d)dF^s(w^s) - \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} g(w^d)w^d dF^d(w^d)dF^s(w^s) \\
&-\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} g(w^d)w^d dF^d(w^d)dF^s(w^s) - \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} g(w^s)w^s dF^d(w^d)dF^s(w^s) - \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} g(w^s)w^* dF^d(w^d)dF^s(w^s) \\
&= -\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d)dF^s(w^s) - \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d)dF^s(w^s) + \tau\int_{w^*}^{\overline{w}^s} \kappa_\tau'\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s) \\
&-\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right)dF^s(w^s) - \int_{w^*}^{\overline{w}^s} w^* F^d\left(\kappa(w^s)\right)dF^s(w^s) - \tau\int_{w^*}^{\overline{w}^s} w^*\kappa_\tau' f^d\left(\kappa(w^s)\right)dF^s(w^s)
\end{aligned}
$$

Note that, $\kappa(w^s) - w^* = \dfrac{w^s - w^*}{1-\tau}$. By using this fact and doing algebra, the optimal marginal tax formula is

$$
\begin{aligned}
\frac{\tau}{1-\tau}\int_{w^*}^{\overline{w}^s}\left[(w^s - w^*)\kappa_\tau(w^s)f^d(\kappa(w^s))\right]dF^s(w^s) =& \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} (1-g(w^d))w^d dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d)dF^s(w^s) \\
&+ \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} (1-g(w^d))w^d dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d)dF^s(w^s) \\
&+ \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^s dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^* dF^d(w^d)dF^s(w^s)
\end{aligned}
$$

This formula is essentially the same as in the main section, only the limits of some integrals change. Moreover, both results represent the same effects and have the same mechanism behind.

Optimal Budget for the IRS

The optimal IRS's budget comes from the maximization of the Lagrangian of the government's problem

$$
\begin{aligned}
\mathcal{L} =& \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} G\left(U^d(w^d)\right) dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} G\left(U^d(w^d)\right) dF^d(w^d)dF^s(w^s) \\
&+ \int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d)dF^s(w^s) \\
&- \delta \left\{ B + R - \tau \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d)dF^s(w^s) - \tau \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d)dF^s(w^s) \right. \\
&\left. - \tau \int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau \int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau w^* \int_{w^*}^{\overline{w}^s} F^d\left(\kappa(w^s)\right) dF^s(w^s) \right\}
\end{aligned}
$$

The first-order condition with respect to B is

$$
\begin{aligned}
& w^*_B \int_{\kappa(w^*)}^{\hat{w}} G\left(U^d(w^d)\right) dF^d(w^d)f^s(w^*) + \int_{\underline{w}^s}^{w^*} \hat{w}'_B G\left(U^d(\hat{w})\right) f^d(\hat{w})dF^s(w^s) + \int_{\underline{w}^s}^{w^*} (-\kappa'_B) G\left(U^d(\kappa(w^s))\right) f^d(\kappa(w^s))dF^s(w^s) \\
& w^*_B \int_{\hat{w}}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d)f^s(w^*) \int_{\underline{w}^s}^{w^*} (-\hat{w}_B) G\left(U^d(\hat{w})\right) f^d(\hat{w})dF^s(w^s) + (-w^*) \int_{\kappa(w^*)}^{\hat{w}} G\left(U^d(w^d)\right) dF^d(w^d)f^s(w^*) \\
& \int_{w^*}^{\overline{w}^s} \hat{w}_B G\left(U^d(\hat{w})\right) f^d(\hat{w})dF^s(w^s) + \int_{w^*}^{\overline{w}^s} (-\kappa'_B) G\left(U^d(\kappa(w^s))\right) f^d(\kappa(w^s))dF^s(w^s) + (-w^*_B) \int_{\hat{w}}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d)f^s(w^*) \\
& + \int_{w^*}^{\overline{w}^s} (-\hat{w}'_B) G\left(U^d(\hat{w})\right) f^d(\hat{w})dF^s(w^s) + w^*_B \int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^*,\mathcal{M})\right) dF^d(w^d)f^s(w^*) + \int_{\underline{w}^s}^{w^*} \kappa'_B G\left(V^s(w^s,\mathcal{M})\right) f^d(\kappa(w^s))dF^s(w^s) \\
& (-w^*_B) \int_{\underline{w}^d}^{\kappa(w^*)} G\left(V^s(w^*,\mathcal{M})\right) dF^d(w^d)f^s(w^*) + \int_{w^*}^{\overline{w}^s} \kappa'_B G\left(V^s(w^s,\mathcal{M})\right) f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s} V^s_C \frac{dC_s}{dB} dF^d(w^d)dF^s(w^s) \\
& - \delta \left\{ 1 - \tau w^*_B \int_{\kappa(w^*)}^{\hat{w}} w^d dF^d(w^d)f^s(w^*) - \tau \int_{\underline{w}^s}^{w^*} \hat{w}_B \hat{w} f^d(\hat{w})dF^s(w^s) - \tau \int_{\underline{w}^s}^{w^*} (-\kappa'_B)\kappa(w^s) f^d\kappa(w^s))dF^s(w^s) - \tau w^*_B \int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)f^s(w^*) \right. \\
& \tau \int_{\underline{w}^s}^{w^*} (-\hat{w}'_B)\hat{w} f^d(\hat{w})dF^s(w^s) - \tau(-w^*_B) \int_{\kappa(w^*)}^{\hat{w}} w^d dF^d(w^d)f^s(w^*) - \tau \int_{w^*}^{\overline{w}^s} \hat{w}'_B \hat{w} f^d(\hat{w})dF^s(w^s) - \tau \int_{w^*}^{\overline{w}^s} \kappa'_B \kappa(w^s) f^d(\kappa(w^s))dF^s(w^s) \\
& - \tau(-w^*_B) \int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)f^s(w^*) - \tau \int_{w^*}^{\overline{w}^s} (-\hat{w}'_B)\hat{w} f^d(\hat{w})dF^s(w^s) - \tau w^*_B w^* F^d\left(\kappa(w^*)\right) f^s(w^*) - \tau \int_{\underline{w}^s}^{w^*} w^s \kappa'_B f^d\left(\kappa(w^s)\right) dF^s(w^s) \\
& \left. -\tau(-w^*_B) w^* F^d\left(\kappa(w^*)\right) f^s(w^*) - \tau \int_{w^*}^{\overline{w}^s} w^* \kappa'_B f^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau \int_{w^*}^{\overline{w}^s} \frac{dw^*}{dB} F^d\left(\kappa(w^s)\right) dF^s(w^s) \right\}
\end{aligned}
$$

To simplify the above expression, use some results and assumptions. First, the social valuation of the agent's consumption is defined as $g = {G'_V \times V_C}/{\delta}$. Second, by definition $U^d(\kappa(w^s)) = V^s(w^s, \mathcal{M})$, make some terms cancel among them. Finally, the derivative of consumption in the self-employed sector respect to the budget is

$$\frac{dC_s}{dB} = \begin{cases} 0 & \text{if } w^s < w^* \\ -\tau \dfrac{dw^*}{dB} & \text{if } w^* \leq w^s \end{cases}$$

At this point, it is important to recall that, in the dependent sector, an increase in the budget for the IRS does not affect taxes. By using those facts and doing algebra yields

$$\tau \int_{w^*}^{\overline{w}^s} (w^* - \kappa(w^s))\kappa'_B f^d(\kappa(w^s)) dF^s(w^s) + \tau \int_{w^*}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} w^*_B dF^d(w^d) dF^s(w^s) = 1 + \int_{w^*}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} g(w^s) \tau w^*_B dF^d(w^d) dF^s(w^s)$$

This equation represents the same as the one shown in the main section.

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