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# An Alternative Model for Describing Even-Aged Stand-Level Sawtimber Growth in Pennsylvania

**Phillip J. Manning, Marc E. McDill, and Horacio Gilabert**

A nonlinear stand-level model was formulated to estimate net and gross sawtimber volume for Pennsylvania forests based on measured plot-level observations from the Pennsylvania Department of Conservation and Natural Resources Bureau of Forestry's continuous forest inventory database. The resulting model, although derived using specific data from a unique inventory system, provides an application of fitting a dimensionally compatible function to model sawtimber yield while maintaining flexibility to differentiate stand-level growth characteristics such as mean annual increment (MAI) and the age of culmination of MAI among seven forest types, three site classes, two stocking levels, and four ecoregion clusters. Results show later culmination and decreased productivity as site quality and stocking level decrease, varying among forest types and ecoregion clusters. Evaluation of the Akaike information criteria showed the strength of the subsequent model in projecting growth and yield compared to the state's current yield model from Gilabert et al. (*North. J. Appl. For.* 27[4]:140–150, 2010).

**Keywords:** growth and yield, management planning, whole stand models, mean annual increment

The ability to reliably estimate future growth and yield is an essential part of landscape-level strategic forest management planning. The Pennsylvania Department of Conservation and Natural Resources (DCNR) Bureau of Forestry (the Bureau) currently uses a sawtimber yield model developed by Gilabert et al. (2010) to project future stand-level yields for planning purposes. The Gilabert et al. (2010) model, however, lacks the flexibility to describe key variations in stand growth among different site classes, stocking levels, forest types, and ecoregion clusters. In addition, growth characteristics such as mean annual increment (MAI) and the age at which it culminates for eastern hardwood forest types are absent from much of the literature surrounding sawtimber yield modeling.

Yield is defined as the net volume of wood per unit area present at a given age, whereas growth refers to the change in yield over a given time period (Vanclay 1994). Of particular interest are two measures of growth: current annual increment (CAI), which is the annual increase in volume per unit area at a given age, and MAI, which is the average annual increase in volume per unit area over the life of the stand at a given age. According to Sharma et al. (2003), evolution of MAI as a stand ages is a critical characteristic of even-aged forests. Rightly or wrongly, the age where MAI reaches a maximum, commonly referred to as the culmination of MAI (CMAI), is often cited as the optimal age at which an even-aged stand should be harvested (Newman 1988, Bettinger et al. 2009) if the objective is to maximize yield harvested in the long-term, or at least as a key guideline for determining when to harvest an even-aged timber stand (US Department of Agriculture [USDA] Forest Service 2006). Maximum MAI is emphasized here because it is one method for determining optimal rotation age and is one way to evaluate how biologically plausible the model is.

Even though MAI provides insight into stand development and productivity and is frequently cited as playing a crucial role in forest management decision making, little information is available regarding the age at which CMAI occurs for eastern hardwood forest types. In addition, few stand-level growth and yield models are flexible enough to allow variation in the timing of the CMAI across different site and stocking classes, forest types, and ecological regions. Although a number of USDA Forest Service growth and yield simulators are available for the Northeast, such as SILVAH (Marquis and Ernst 1992) and NE-TWIGS (Teck 1990) along with spatial growth models like the Forest Vegetation Simulator (FVS) (Dixon 2002), the required inputs for these models (tree lists and site index) are not readily available for planning management units (Gilabert et al. 2010) specifically because the Bureau uses site class definitions in place of site index for measuring forest site quality. Furthermore, predictions from stand-level models are often superior to those of individual tree growth models when stand-level attributes, such as MAI and CMAI, are the primary concern (Cao 2006).

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**Affiliations:** *Phillip J. Manning (pjm294@psu.edu), Pennsylvania State University, State College, PA. Marc E. McDill (mem14@psu.edu), Pennsylvania State University. Horacio Gilabert (hgilab@uc.cl), Pontificia Universidad Catolica de Chile.*

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Table 1. Summary statistics by forest type for the fitting data set.

| Forest type | Dominant tree species | *n* | Age (yr) | Gross sawtimber volume (bd ft/ac) | Net sawtimber volume (bd ft/ac) |
|---|---|---|---|---|---|
| Northern hardwoods | Sugar maple (*Acer saccharum*), basswood (*Tilia americana*), etc. | 595 | 80 (13) | 14,556 (8,099) | 10,861 (6,333) |
| Allegheny hardwoods | Black cherry (*Prunus serotina*) | 112 | 80 (15) | 21,736 (8,368) | 16,187 (6,660) |
| Red maple | Red maple (*Acer rubrum*) | 112 | 71 (15) | 10,147 (6,465) | 7,111 (4,806) |
| Red oak | Northern red oak (*Quercus rubra*), black oak (*Quercus velutina*), scarlet oak (*Quercus coccinea*) | 557 | 82 (17) | 15,175 (7,627) | 11,573 (6,143) |
| Other oaks | White oak (*Quercus alba*), chestnut oak (*Quercus prinus*) | 1,261 | 81 (19) | 7,784 (5,313) | 5,758 (4,147) |
| Other hardwoods | Yellow-poplar (*Liriodendron tulipifera*), ash (*Fraxinus* spp.), hickory (*Carya* spp.), birch (*Betula* spp.), etc. | 92 | 75 (19) | 11,455 (11,384) | 8,670 (8,508) |
| Conifers | All softwoods (*Pinus* spp., *Tsuga canadensis, Picea* spp.) | 118 | 90 (19) | 18,097 (10,343) | 14,371 (8,926) |

Numbers in parentheses are SD.

Table 2. Site class definitions relative to Pennsylvania.

| Site class | Description | *n* |
|---|---|---|
| 1 | Moist (throughout the year), well-drained, fairly deep soils that usually occur along streams or in bottomlands. Dominant and codominant trees have a projected main stem merchantable height of >50 ft at maturity. Total mature tree heights average >85 ft. | 881 |
| 2 | Intermediate soil moisture, depth, drainage, and fertility that may dry out for short periods throughout the year located on slopes between ridge tops and bottomlands. Dominant and codominant trees have a projected main stem merchantable height of 30–40 ft at maturity. Total mature tree heights average >65 ft but <85 ft. | 1,475 |
| 3 | Shallow, mostly dry, stony and compact soils that usually occur on ridges or broad flat plateaus. Dominant and codominant trees have a projected main stem merchantable height of <30 ft at maturity. Total mature tree heights average <65 ft. | 491 |

Data from the DCNR Bureau of Forestry (1999).

This article describes the development of a new model to predict stand-level net and gross sawtimber volume per acre for Pennsylvania forests. It allows for a quantitative analysis of growth characteristics that differentiate yields for three site classes, two stocking levels, seven forest types, and four ecoregion clusters. As in Gilabert et al. (2010), the yield model described here was fit using the Bureau's continuous forest inventory (CFI) data and should effectively represent the growth of the forests found in the agency's planning units. Unlike the model presented in Gilabert et al. (2010), however, the structure of the model presented here permits the shape of the CAI and MAI growth curves to differ across site classes, stocking levels, forest types, and ecoregion clusters. This makes it possible to assess how the MAI and CMAI differ across different types of forests. Stand-level models with this degree of flexibility are not currently available for the region.

## Data

The data set used in this research was compiled from measured plots from the Bureau's CFI system. The CFI system is based on a stratified sampling design, with periodic partial replacement of a small number of plots to account for changes in stand typing, zoning, land acquisitions, and large-scale natural disturbances. In the CFI data set, stands are classified by forest type, site class, and size class (Sterner 2007). Only stands assigned to multiple resource, buffer, or limited resource management zones were included in this study. These management zones represent stands where active forest management activities occur, whereas zones such as special resource or wild areas (not included in this study) limit or restrict harvesting activities and have different management needs outside the scope of this analysis. The available data represent up to four measurement periods from primarily unmanaged stands. The information in these inventories was collected according to the procedures detailed in Bureau inventory manuals (DCNR 1968, 1985, 1997, 1999, 2003).

Variables describing net and gross sawtimber volume for each plot were calculated from the tree-level information, where tree volumes (International ¼ in. rule) were calculated using form class volume tables (Mesavage and Girard 1946) and form classes by species derived by Sterner (2001). Per-acre estimates of volume were expanded from the tree-level data using the appropriate expansion factors. Summary statistics by forest type for key stand-level variables are shown in Table 1.

Site class and stocking levels are uniquely defined by the Bureau. Site class is defined by the aggregation of specific features such as soil moisture and depth, terrain position, slope, and classes of average merchantable height and average total height for dominant and codominant trees. The three site classes represented in the data are described in Table 2. Stocking is evaluated for oak-dominated stands based on Sander (1977) and on Stout and Nyland (1986) for all other stand types. Stocked and understocked stands are separated at the 50% stocking level, corresponding to basal areas of 60 $ft^2$/ac or more for oak forest types and 80 $ft^2$/ac or more for all other forest types.

Plots classified as uneven-aged or with unusable data were excluded from the final data set. Procedures from Gilabert et al. (2010) were used to classify plots into site and stocking classes, forest types, and ecoregion clusters (Figure 1). The final data set for fitting the yield model consisted of 2,847 plots, with the majority in stands between 60 and 100 years old (Table 1). This concentration of stand ages is precisely the range of interest for planning purposes and should provide reliable estimations of growth and yield for those ages. Some caution should be used in applying the yield curves to stands outside these ages, as discussed below. The distribution of plots by site class (Table 2) and the numbers of plots by stocking level (stocked = 2,742; understocked = 105) are representative of the types of stands under management by the Bureau.

## Model Development

The model structure was driven by the available data, planning needs, and limitations of the Bureau with its current yield prediction model. Brooks and Wiant (2006) showed that relatively simple whole-stand models can accurately project yield in similar forest

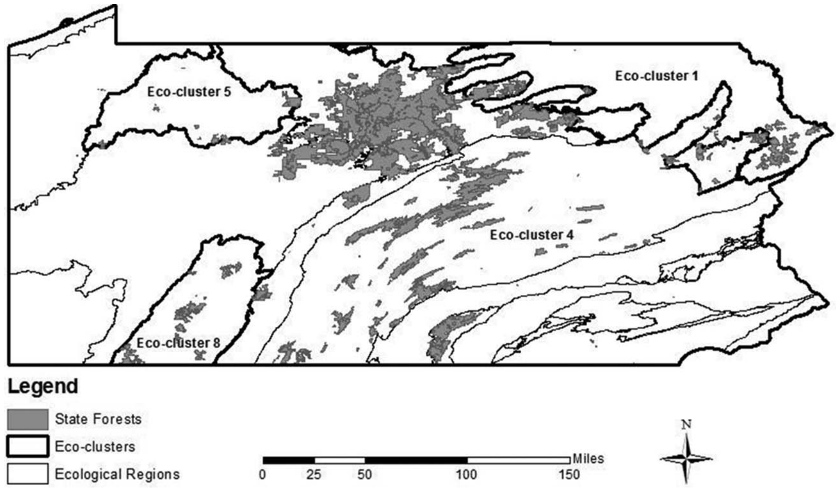


**Figure 1. Ecoregion clusters and underlying Pennsylvania ecological regions.**

types. However, their model and most other stand-level yield models include variables such as basal area and site index (Vanclay 1994). These variables are not available for most stands in the Bureau's planning data set based on historic procedures for collecting inventory information at the plot level and consequently would not be available in projecting future yields for forest planning models. As a result, stand age was the main predictor. Binary variables were used to account for variations in volume across three site classes, seven forest types, two stocking levels, and four ecoregion clusters. Bureau-defined ecological regions were combined into ecoregion clusters 1, 4, 5, and 8 (Figure 1) based on the procedures and criteria described in Gilabert et al. (2010). The model is expected to have a typical sigmoid shape representing biological growth differentiated between site classes, stocking levels, forest types, and ecoregion clusters while capturing variation in the culmination of CAI and MAI.

Several model forms, such as the Richards (1959) and Weibull (1951) models, were initially considered. However, these models were not subsequently fit because of convergence problems and undesirable properties in the shapes of the growth curves. The final model chosen was a dimensionally compatible growth model developed by McDill and Amateis (1992). This equation has been widely used to model dominant height and basal area growth (Wang et al. 2011, Zhang et al. 2011, Aspinwall et al. 2012, Kitikidou et al. 2012, Alcorn et al. 2013), but, to our knowledge, this is the first time it has been used to model sawtimber volume yield. A modified general form of the model is

$$V = \frac{M}{1 - \left(1 - \frac{M}{V_0}\right)\left(\frac{A_0}{A}\right)^{a}} \quad (1)$$

where $V$ is volume (board feet [bd ft]/ac), $V_0$ is initial volume (bd ft/ac), $A_0$ is initial age (years), $A$ is age (years), $M$ is the maximum volume parameter; and $a$ is the rate parameter. The "pinned" form of the model (McDill and Amateis 1992) was used due to the available data and our objectives. This model form provided the flexibility we desired to describe stand-level growth characteristics. The initial volume variable ($V_0$) was estimated as a parameter at a predetermined age of 20 ($A_0$) years following McDill and Amateis (1992). This value was chosen because it represents the minimum age of stands in the fitting data set. The final model that was subsequently fit was

$$\ln(\text{Yield}) = \left[\frac{M + \alpha_i \cdot \text{FT}_i + \beta_j \cdot \text{EcR}_j}{1 - \left(1 - \frac{M + \alpha_i \cdot \text{FT}_i + \beta_j \cdot \text{EcR}_j}{V}\right)\left(\frac{20}{A}\right)^{(a + \varphi_k \cdot \text{Site}_k + \gamma \cdot \text{Stock})}}\right] \quad (2)$$

where ln(Yield) is the natural logarithm of net (gross) sawtimber volume (bd ft/ac); $A$ is age (years); $\text{Site}_k = 1$ for site class $k$ ($k = 1, 3$), 0 otherwise; $\text{FT}_i = 1$ for forest type $i$ ($i = 1, 2, 3, 4, 6, 7$), 0 otherwise; $\text{EcR}_j = 1$ for ecoregion cluster $j$ ($j = 1, 5, 8$), 0 otherwise; Stock = 1 for understocked stands, 0 for stocked stands; and $M$, $V$, $a$, $\alpha_i$, $\beta_j$, $\varphi_k$, and $\gamma$ are model parameters estimated from the data.

The response (yield) was transformed by the natural logarithm to obtain desirable properties (homoscedasticity) of the error variance and to aid in parameter convergence (Wykoff 1990). The transformed responses were transformed back to the original scale before the error statistics were calculated. Transformation bias was accounted for using the methods described by Sprugel (1983). The transformation bias correction factor was 1.236 for the net sawtimber model and 1.201 for the gross sawtimber model. In addition, understocked site class 3 plots were combined with understocked site class 2 plots because of the small number of understocked observations and undesirable properties of the growth curves obtained for these types of stands. The final model (2) was fit using nonlinear least-squares with the nls( ) function in R (R Development Core Team 2013), and separate models were fit for net and gross sawtimber volume (bd ft/ac). The base model (all binary variables equal to zero) is for site class 2, forest type 5 (other oaks), ecoregion cluster 4 (ecoregions 4, 6, 9, 11, and 13) and fully stocked stands. This is the category of land with the largest number of observations.

## Results

### Parameter Estimates

Parameter estimates and their standard errors for the net and gross sawtimber volume models are provided in Table 3. The signs and magnitude of the parameters in each model associated with site class indicate higher yields for better sites (site class 1) and lower yields for poorer sites (site class 3), which is consistent with our expectations and the results in Gilabert et al. (2010). The negative sign for the parameter associated with understocked stands indicates

**Table 3. Parameter estimates for the yield model.**

| Parameter | Corresponding variable | Total gross saw timber volume (bd ft/ac) | Total net sawtimber volume (bd ft/ac) |
|---|---|---|---|
| $M$ | Maximum volume | 9.85218 (0.09110) | 9.54461 (0.09296) |
| $V$ | Volume | 5.77894 (0.18523) | 5.49612 (0.19000) |
| $a$ | Rate | 1.41966 (0.10174) | 1.43872 (0.10508) |
| $\phi_1$ | Site 1 | 0.44000 (0.06679) | 0.49011 (0.07813) |
| $\phi_2$ | Site 3 | −0.54833 (0.03996) | −0.62598 (0.04427) |
| $\alpha_1$ | Northern hardwoods | 0.38545 (0.05079) | 0.34390 (0.05351) |
| $\alpha_2$ | Allegheny hardwoods | 0.79383 (0.08959) | 0.74813 (0.09437) |
| $\alpha_3$ | Red maple | 0.26950 (0.09139) | 0.15676 (0.09708)* |
| $\alpha_4$ | Red oak | 0.41482 (0.04854) | 0.43121 (0.05200) |
| $\alpha_6$ | Other hardwoods | −0.06893 (0.10103)* | 0.01731 (0.10959)* |
| $\alpha_7$ | Coniferous | 0.57393 (0.07885) | 0.61110 (0.08457) |
| $\beta_1$ | Ecoregion Cluster 1 | −0.21396 (0.06544) | −0.26667 (0.07036) |
| $\beta_5$ | Ecoregion Cluster 5 | 0.42808 (0.10627) | 0.47903 (0.11380) |
| $\beta_8$ | Ecoregion Cluster 8 | 0.78791 (0.09764) | 0.59451 (0.10000) |
| $\gamma$ | Understocked | −0.51934 (0.04885) | −0.55541 (0.05257) |

Numbers in parentheses are SE.
* Indicates estimate not significantly different from zero ($\alpha = 0.05$).

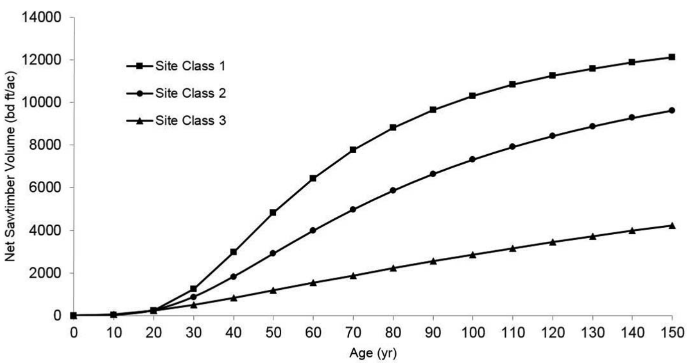


**Figure 2. Net sawtimber yield by site class for stocked other oaks stands in ecoregion cluster 4.**

lower yields for these stands. Even though some parameter estimates for forest types were not significantly different from the base forest type (other oaks), the binary variables for these forest types were left in the model because these values represent our best estimate for projecting yields in these forest types (Table 3). The forest type parameters also indicate that yields are generally lowest on the base forest type. The ecoregion cluster parameters were all significantly different from zero, mainly as a result of merging the original ecoregions represented in the data set into larger ecoregion clusters (Gilabert et al. 2010).

## Model Evaluation

The model was evaluated both qualitatively and quantitatively based on suggestions from Vanclay and Skovsgaard (1997). Qualitatively, we examined the model for logical and biological consistency. For example, Figure 2 compares the projected net sawtimber yield for stocked other oaks stands in ecoregion cluster 4 by site classes, whereas Figure 3 compares the projected net sawtimber yield for a site class 2 other oaks stand in ecoregion cluster 4 by stocking level. The sigmoid shapes of the yield curves are consistent with expectations for stand growth, and the variation in yields across site classes and stocking levels are consistent with our experience. The yield curves for any combination of site classes, forest types, stocking levels, and ecoregion clusters were similar to those in Gilabert et al. (2010). Yield curves and model fits were examined for all combinations of binary variables for both the net and gross sawtimber model, and similar results were obtained.

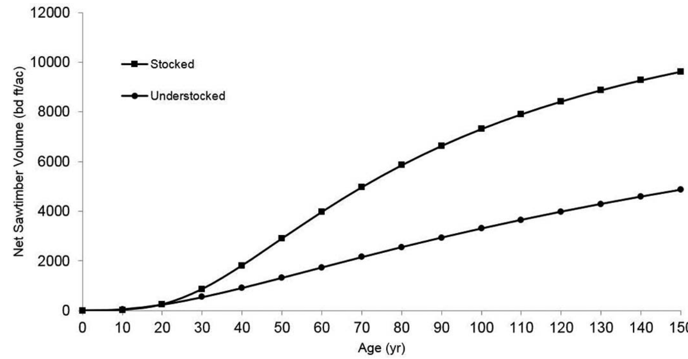


**Figure 3. Net sawtimber yield by stocking level for site class 2 other oaks stands in ecoregion cluster 4.**

**Table 4. Fit statistics for the yield model.**

| | Volume | |
|---|---|---|
| Fit statistic* | Gross sawtimber (bd ft/ac) | Net sawtimber (bd ft/ac) |
| RMSE | 5,898 | 4,704 |
| MD | 1,289 | 1,092 |
| MAD | 4,204 | 3,344 |
| FI | 0.484 | 0.471 |

* The statistics are as follows: root mean squared error (RMSE) = $\sqrt{\Sigma(y_i - \hat{y}_i)^2/(n - p)}$, Mean difference (MD) = $\Sigma(y_i - \hat{y}_i)/n$, mean absolute difference (MAD) = $\Sigma|(y_i - \hat{y}_i)|/n$, fit index (FI) = $1 - \Sigma(y_i - \hat{y}_i)^2/\Sigma(y_i - \bar{y}_i)^2$, where $y_i$ and $\hat{y}_i$ are the observed and fitted values, respectively, and $n$ and $p$ are number of plots and number of parameters, respectively.

Residual plots and fit statistics were used to quantitatively evaluate the model. Fit statistics for both the net and gross sawtimber model are presented in Table 4. Overall, the fit statistics show improved prediction capability compared with that in Gilabert et al. (2010) but indicate opportunities for improvement based on similar models developed by Brooks and Wiant (2006) for Appalachian hardwoods. Diagnostic plots for the net sawtimber model are shown in Figure 4. Similar plots were observed for the gross sawtimber model. The plot of residuals ($y_i - \hat{y}_i$) against the fitted values shows an acceptable random scattering of the errors around zero (Figure 4A). The other residual plots revealed no apparent trends or evidence to suggest that the basic assumptions of nonlinear regression were violated. Even though multiple measurements of the same plot are not independent, each individual plot measurement was assumed to be an independent observation for our purposes. Figure 4D indicates that the effect of autocorrelation in the residuals was probably negligible.

The Akaike information criterion [AIC = $-2\ln(L) \pm 2k$, where $L$ is the likelihood and $k$ is the number of parameters] (Akaike 1981) was used to evaluate the model against the current yield model for the Bureau developed by Gilabert et al. (2010). The difference in AIC (ΔAIC) measures the distance of a candidate model relative to true model ($\Delta_i = \text{AIC}_i - \text{AIC}_{\min}$, where $\text{AIC}_{\min}$ is the minimum of $\text{AIC}_i$ values) while taking into account the number of parameters in the model (Ferreira de Lima et al. 2015). Burnham and Anderson (2002) proposed that ΔAIC values between 4 and 7 (and greater) represent a 95% confidence level for evidence to accept the model with the lower AIC value ($\text{AIC}_{\min}$). The ΔAIC values between the net sawtimber models and the gross sawtimber models were 9.548

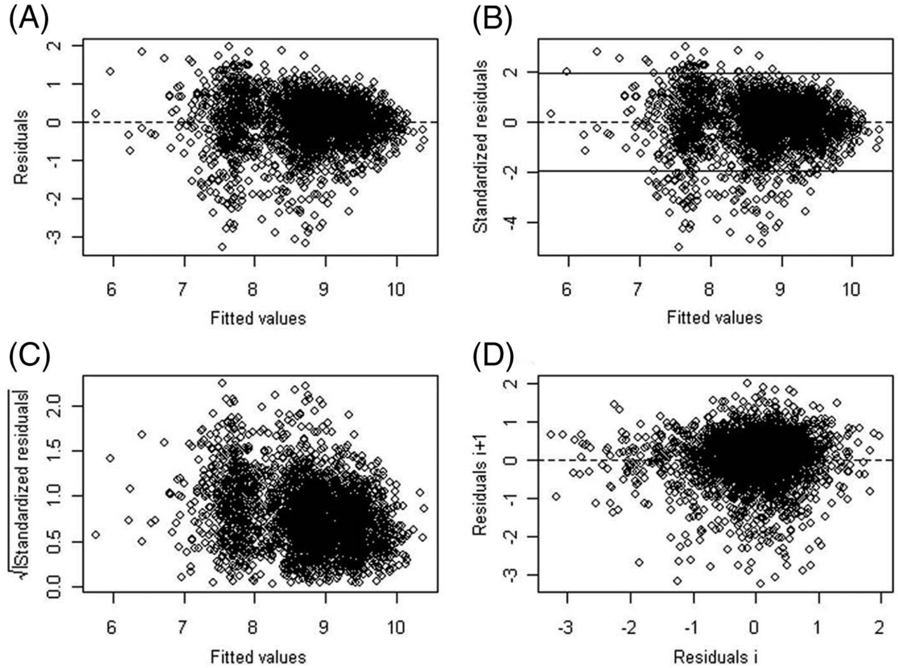


**Figure 4. Plot of residuals (A), standardized residuals (B), square root of standardized residuals (*y*-axis) versus fitted values (*x*-axis) (C), and lagged residuals (*y*-axis) versus residuals for the net sawtimber model (D).**

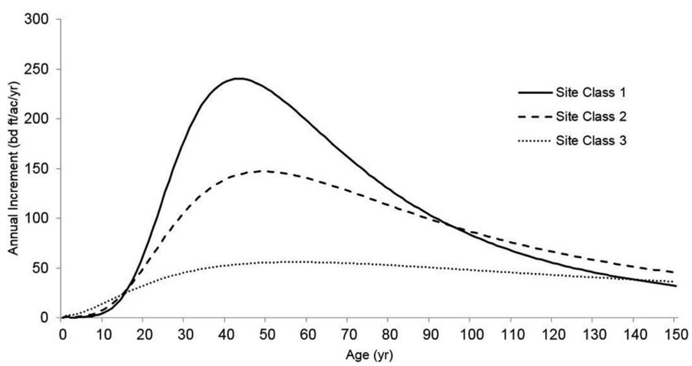


**Figure 5. Comparison of CAI of gross volume by site class, for stocked other oaks stands in ecoregion cluster 4.**

and 33.222, respectively, compared with those for the models fit in Gilabert et al. (2010), indicating evidence to support the nonlinear model described here as the more plausible model for describing stand-level yield in Pennsylvania.

## Growth Analysis

The shapes of the CAI and MAI curves were different for each combination of site, stocking, forest type, and ecoregion cluster based on the available data used to fit the model. For example, Figure 5 shows the gross volume CAI curves for each site class for stocked other oaks stands in ecoregion cluster 4. In the figure, the modeled growth rates are the reverse of what one might expect up to age 16; that is, growth is higher on the poor sites and least on the best sites. However, this trend reverses and is consistent with expectations from age 16 until age 95. After age 95, the growth rate for site 1 stands again drops below the growth rate on site 2 stands. The unexpected results for the younger ages is a result of the fact that the model uses a single initial volume parameter (at age 20) for all forest types, site classes, and ecoregion clusters. Because of this and the lack of data for young stands, predicted yields for younger ages ($<$40 years) from the model probably are not reliable. This is not a problem for planning purposes, as stands are not generally harvested at such young ages. The somewhat anomalous results for older stands are a result of the fact that the maximum volume parameter in the model is the same for all sites. Because of the lack of data for stands more than 100 years old, we were not able to model both the rate parameter and the maximum yield parameter as functions of site index, and we obtained better results by including the site variables only in the expression for the rate parameter. In the future, as additional data are collected on aging plots, we hope to be able to fit models where both parameters are functions of site index. Consequently, the model is most applicable in stands between 40 and 100 years old.

Tables 5 and 6 show the estimated CMAI and maximum MAI for each forest type, site class, and ecoregion cluster for gross and net sawtimber growth. The tables clearly show a trend of increasing CMAI ages for poorer sites. CMAI for gross sawtimber yields occurs between 70 and 97 years, depending on the forest type and ecoregion cluster, on site 1, between 83 and 127 years on site 2, and between 100 and 207 years on site 3. Although CMAI estimates above 100 years are outside the range of the majority of the data and therefore less reliable, the trend of reaching CMAI at higher ages on poorer sites is quite robust. On average, CMAI comes slightly earlier for net sawtimber growth than for gross sawtimber growth. However, there are several exceptions to this trend and, whether positive or negative, the differences tend to be small. CMAI occurs earlier on other hardwood and other oak stands and later on Allegheny hardwood and coniferous stands. CMAI and MAI for understocked stands are not shown. On average, maximum MAI was about 40% lower, and CMAI occurred 16–36 years later on understocked stands than on stocked stands.

The highest maximum MAIs occurred in the Allegheny hardwoods forest type. This is of particular interest because the Allegheny hardwoods forest type is on average 60–70% black cherry

Table 5. Estimated age of CMAI and MAI at CMAI by forest type, ecoregion cluster, and site class for the gross sawtimber yield function.

| | Ecoregion Cluster 1 | | | Ecoregion Cluster 4 | | | Ecoregion Cluster 5 | | | Ecoregion Cluster 8 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Site 1 | Site 2 | Site 3 | Site 1 | Site 2 | Site 3 | Site 1 | Site 2 | Site 3 | Site 1 | Site 2 | Site 3 |
| Other oaks | | | | | | | | | | | | |
| CMAI (yr) | 71 | 85 | 103 | 74 | 89 | 113 | 80 | 99 | 136 | 86 | 108 | 156 |
| MAI (bd ft/ac/yr) | 121.7 | 84.4 | 39.4 | 144.6 | 99.0 | 44.5 | 205.2 | 137.0 | 57.6 | 276.8 | 181.2 | 72.2 |
| Northern hardwoods | | | | | | | | | | | | |
| CMAI (yr) | 77 | 93 | 122 | 80 | 98 | 133 | 86 | 109 | 150 | 91 | 117 | 180 |
| MAI (bd ft/ac/yr) | 166.3 | 112.6 | 49.3 | 198.2 | 132.6 | 56.1 | 282.8 | 184.8 | 73.4 | 382.8 | 245.7 | 92.9 |
| Allegheny hardwoods | | | | | | | | | | | | |
| CMAI (yr) | 83 | 103 | 144 | 86 | 108 | 150 | 92 | 119 | 150 | 97 | 127 | 207 |
| MAI (bd ft/ac/yr) | 232.7 | 154.0 | 63.3 | 278.1 | 182.0 | 72.4 | 398.8 | 255.4 | 94.6 | 541.8 | 341.1 | 122.3 |
| Red maple | | | | | | | | | | | | |
| CMAI (yr) | 75 | 91 | 116 | 78 | 96 | 127 | 84 | 106 | 150 | 89 | 115 | 173 |
| MAI (bd ft/ac/yr) | 151.3 | 103.2 | 46.0 | 180.2 | 121.3 | 52.3 | 256.7 | 168.8 | 68.2 | 347.0 | 224.1 | 86.0 |
| Red oak | | | | | | | | | | | | |
| CMAI (yr) | 77 | 94 | 124 | 80 | 99 | 135 | 86 | 109 | 150 | 91 | 118 | 182 |
| MAI (bd ft/ac/yr) | 170.3 | 115.1 | 50.1 | 203.0 | 135.6 | 57.1 | 289.8 | 189.2 | 74.7 | 392.4 | 251.5 | 94.7 |
| Other hardwoods | | | | | | | | | | | | |
| CMAI (yr) | 70 | 83 | 100 | 73 | 88 | 110 | 79 | 98 | 132 | 85 | 106 | 152 |
| MAI (bd ft/ac/yr) | 115.2 | 80.2 | 37.9 | 136.8 | 94.0 | 42.8 | 193.9 | 129.9 | 55.2 | 261.3 | 171.7 | 69.1 |
| Coniferous | | | | | | | | | | | | |
| CMAI (yr) | 79 | 98 | 132 | 82 | 103 | 144 | 89 | 113 | 150 | 94 | 122 | 192 |
| MAI (bd ft/ac/yr) | 194.0 | 130.0 | 55.2 | 231.6 | 153.3 | 63.1 | 331.2 | 214.5 | 82.6 | 449.1 | 285.7 | 105.3 |

Table 6. Estimated age of CMAI and MAI at CMAI by forest type, ecoregion cluster, and site class for the net sawtimber yield function.

| | Ecoregion Cluster 1 | | | Ecoregion Cluster 4 | | | Ecoregion Cluster 5 | | | Ecoregion Cluster 8 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Site 1 | Site 2 | Site 3 | Site 1 | Site 2 | Site 3 | Site 1 | Site 2 | Site 3 | Site 1 | Site 2 | Site 3 |
| Other oaks | | | | | | | | | | | | |
| CMAI (yr) | 69 | 83 | 99 | 73 | 89 | 114 | 79 | 100 | 142 | 81 | 103 | 150 |
| MAI (bd ft/ac/yr) | 89.4 | 60.5 | 25.0 | 110.9 | 73.7 | 28.8 | 164.5 | 106.1 | 37.7 | 181.1 | 116.1 | 40.4 |
| Northern hardwoods | | | | | | | | | | | | |
| CMAI (yr) | 74 | 91 | 118 | 77 | 97 | 134 | 84 | 108 | 150 | 86 | 111 | 173 |
| MAI (bd ft/ac/yr) | 118.1 | 78.1 | 30.0 | 147.0 | 95.7 | 34.9 | 219.2 | 138.7 | 46.2 | 241.6 | 151.9 | 49.7 |
| Allegheny hardwoods | | | | | | | | | | | | |
| CMAI (yr) | 79 | 100 | 142 | 83 | 107 | 150 | 89 | 118 | 150 | 91 | 121 | 203 |
| MAI (bd ft/ac/yr) | 164.8 | 106.3 | 37.8 | 205.9 | 130.8 | 44.2 | 308.6 | 191.1 | 58.4 | 340.6 | 209.6 | 64.1 |
| Red maple | | | | | | | | | | | | |
| CMAI (yr) | 71 | 87 | 108 | 75 | 93 | 123 | 81 | 104 | 150 | 83 | 107 | 160 |
| MAI (bd ft/ac/yr) | 101.4 | 67.9 | 27.1 | 126.1 | 83.0 | 31.4 | 187.4 | 119.8 | 41.4 | 206.4 | 131.1 | 44.3 |
| Red oak | | | | | | | | | | | | |
| CMAI (yr) | 75 | 93 | 123 | 79 | 99 | 139 | 85 | 111 | 150 | 87 | 113 | 179 |
| MAI (bd ft/ac/yr) | 126.9 | 83.4 | 31.5 | 158.1 | 102.3 | 36.7 | 236.0 | 148.6 | 48.6 | 260.1 | 162.8 | 52.5 |
| Other hardwoods | | | | | | | | | | | | |
| CMAI (yr) | 69 | 83 | 100 | 73 | 89 | 115 | 79 | 101 | 143 | 81 | 103 | 151 |
| MAI (bd ft/ac/yr) | 90.6 | 61.3 | 25.2 | 112.5 | 74.7 | 29.0 | 166.8 | 107.6 | 38.1 | 183.7 | 117.6 | 40.8 |
| Coniferous | | | | | | | | | | | | |
| CMAI (yr) | 77 | 97 | 134 | 81 | 103 | 150 | 88 | 115 | 150 | 89 | 118 | 192 |
| MAI (bd ft/ac/yr) | 147.1 | 95.7 | 34.9 | 183.6 | 117.6 | 40.8 | 274.7 | 171.3 | 54.0 | 303.0 | 187.8 | 58.8 |

(*Prunus serotina*), which is the highest value species on a per-unit basis in the state (Gilabert et al. 2010). In areas where this forest type is most common, i.e., ecoregion clusters 1 and 5, the maximum MAI for net sawtimber volume on site class 1 was 164.8 (bd ft/ac/yr) at age 79 and 308.6 (bd ft/ac/yr) at age 89, respectively.

## Discussion

The growth and yield relationships obtained in this study were consistent with our prior expectations for Pennsylvania forests. Furthermore, the level of errors associated with the models presented here were similar to those reported by Gilabert et al. (2010). However, because of the flexibility of the yield equation used, the shapes of the growth curves were different for different site classes, stocking levels, forest types, and ecoregion clusters, allowing the model to predict different ages for CMAI for different types of forests. The use of a more flexible, dimensionally compatible function provided the ability to estimate MAI and CMAI with no deterioration in predictive accuracy for eastern hardwood forest types. The AIC model selection criteria along with the flexibility of the model described here indicate an improvement over the model described by Gilabert et al. (2010).

There are two practical concerns with the model discussed here, however. The first is that the level of yields projected for younger ages, particularly those less than 40 years, were higher for poorer sites and lower for better sites and in general higher than is typically observed. This can be attributed to the model form and lack of data within these ages. However, Pennsylvania stands at these ages are not typically harvested, and their yields are not of great interest for planning purposes. Similarly, only a small number of plots from older stands were available. As the inventory cycles continue, more data will become available to better model the yields in these ages and ultimately provide an understanding of early and late stand growth. The other concern is that growth and yield cannot be assessed for understocked site class 3 stands because these plots were merged with understocked site class 2 plots. Understocked site class 3 stands represent a small proportion of the total acreage within planning units. These stands typically do not fit into any of the other defined categories. Future inventory cycles should better classify plots in this category and target more low site, understocked stands to better model the growth of these stands.

The model presented here provides an empirical approach for projecting future stand conditions requiring only stand-level inputs. Their flexibility allows for the analysis of stand-level growth and yield across a broad distribution of forest characteristics suitable for long-term forest planning. The intended use and limitations of available data constrained us from incorporating other stand-level variables to describe growth such as basal area and further comparing our results with more widely accepted models such as that of Clutter (1963). This, along with the unconventional protocols used by the Bureau in defining site classes and stocking levels, limits the usefulness of the models outside of Pennsylvania. However, the model is based on a large and well-defined continuous inventory data set with a wide geographical distribution and for that reason should provide useful general growth and yield information for eastern hardwood forests in the region. In addition, the model presented provides an application of fitting a dimensionally compatible function flexible enough to examine forest growth characteristics for eastern hardwood species previously lacking in the region and much of the literature. The future ability to incorporate more conventional stand descriptors and additional data may lead to a less complex model that overcomes the limitations discussed and that can be used by more forest managers across the Northeast for planning. Practitioners interested in using the model presented here are referred to Gilabert et al. (2010) for detailed instructions on how to use the model. Gilabert et al. (2010) also provide species proportions that allow users of the model to estimate the species composition of the total yields predicted by the model.

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