PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
SCHOOL OF ENGINEERING

ENERGY EFFICIENT WIRELESS COMMUNICATIONS

FERNANDO ERNESTO ROSAS DE ANDRACA

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Doctor in Engineering Sciences

Advisor:
CHRISTIAN OBERLI GRAF

Santiago de Chile, December 2012

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To all the wonder that surround us,
which for being so overwhelming,
takes the form of our own glasses.

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ESCUELA DE INGENIERÍA

COMUNICACIONES INALÁMBRICAS ENERGÉTICAMENTE EFICIENTES

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FERNANDO ERNESTO ROSAS DE ANDRACA

RESUMEN

En los últimos años, la reducción de la energía necesaria para transferir información entre transmisor a receptor se ha constituido en un objetivo importante para las técnicas de comunicación inalámbricas. La búsqueda de nuevos métodos para aumentar la eficiencia energética de las comunicaciones inalámbricas se ha transformado en un activo campo de investigación. Algunos investigadores han sugerido que el uso de sistemas de múltiples antenas podría generar nuevas reducciones en el consumo energético de las comunicaciones inalámbricas. A pesar de un reciente interés en este tema, ningún análisis completo ha sido reportado sobre como el tamaño del arreglo de antenas, el tamaño de la modulación y la potencia irradiada han de elegirse para lograr comunicaciones energéticamente eficientes sobre canales con desvanecimiento.

En esta tésis, enfrentamos este problema presentando un modelo que determina la energía consumida por cada bit de información transferido sin error a través de canales con diversas estadísticas. Usando este modelo de consumo energético, primero presentamos las reglas para elegir el tamaño de la modulación y la potencia de irradiación de un sistema de una sola antena que logra la eficiencia energética máxima en función de la distancia de enlace. Luego, extendemos el análisis a sistemas de multiples antenas que usan la técnica de descomposición de valor singular, y presentamos el número óptimo de canales paralelos usados para reducir el consumo energético. Usando estos resultados, comparamos el consumo mínimo alcanzable por sistemas de comunicaciones equipados con distinto número de antenas. Nuestro análisis muestra que los sistemas de comunicaciones equipados con un gran arreglo de antenas son óptimos para realizar comunicaciones a través de largas distancias. Por otra parte, sistemas equipados con una sola antena son la mejor opción para comunicaciones sobre distancias cortas.

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PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
ESCUELA DE INGENIERÍA

ENERGY EFFICIENT WIRELESS COMMUNICATIONS

Thesis submitted to the Office of Research and Graduate Studies in partial fulfillment of the requirements for the Degree of Doctor in Engineering Sciences by

FERNANDO ERNESTO ROSAS DE ANDRACA

ABSTRACT

In the last years, the reduction of the total energy necessary for transferring information from sender to receiver have emerged as an important goal of wireless communication techniques. The search of new methods for increasing the energy efficiency of wireless communications have become an active field of research. Some researchers have suggest that multiple-input multiple-output systems could be used for achieving further reductions of the energy consumption of wireless communications. Despite of a recent interest in this topic, no complete analysis have been reported so far on how the antenna array size, the modulation size and transmission power must be chosen in order to achieve energy-efficient communications over fading channels.

In this thesis, we adress this problem by presenting a model that determines the energy consumed per payload bit transferred without error over fading channels of various statistics. Using this energy-consumption model, we first derive rules for choosing the modulation size and irradiation power of single antenna systems which achieves highest energy efficiency as a function of link distance. Then, we extend this analysis to MIMO SVD systems, and present the optimal number of eigenchannels to be used for reducing the energy consumption. Using these results, we compared the minimal consumption achievable by communication systems of different antenna array sizes. Our analysis show that communication systems equipped with large antenna arrays are optimal for long range

communications. On the other hand, single antenna systems equipped with large constellation sizes are the best choice for communications over short link distances.

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LIST OF PAPERS

This dissertation is based on the following papers, which are included at the end of this document.

I Rosas, F. and Oberli, C.
Modulation optimization for achieving energy efficient communications over fading channels, *Proceedings of IEEE VTC2012-Spring*.

II Rosas, F. and Oberli, C.
Nakagami-m approximations for MIMO SVD transmissions, *Proceedings of WSA 2012*.

III Rosas, F. and Oberli, C.
Energy-efficient SVD MIMO communications, *Proceedings of IEEE PIMRC'12*.

IV Rosas, F. and Oberli, C.
Modulation and SNR optimization for achieving energy-efficient communications over short-range fading channels, *IEEE Transactions on Wireless Communications* (in press).

V Rosas, F. and Oberli, C.
Nakagami-m approximations for MIMO SVD transmissions, *IET Transactions on Communications* (under review).

VI Rosas, F. and Oberli, C.
Energy efficient SVD MIMO communications, *IEEE Transactions on Wireless Communications* (under review).

1. INTRODUCTION

Through history, one of the goals of communication engineers has been to increase the efficiency of the process of transmitting information. In the days of the Roman or Inca empires messages were carried by runners, who were aided by an extensive network of roads and relay posts (Hagen, 2011). A significant improvement was achieved in Africa, where people developed a system to code messages into drumming. This allowed to share news among villages by means of sound waves (Carrington, 1949). Further improvements were achieved in the 19th century with the invention of the telegraph, which enabled news to travel at light speed (Gleick, 2011). In the 20th century, the telephone replaced the cold beeps of the telegraphed Morse code by the warm sound of the human voice (Sterling, Bernt, & Weiss, 2005). Today, advances in digital communication technologies allows millions of people to share high definition images and sounds worldwide in real time.

Since the advent of digital communications, the quest for efficiency has been mainly taken on as the maximization of throughput. This begun with the seminal work of Claude Shannon (Shannon, 1948), who proved that the maximum theoretically achievable throughput is given by

$$R \leq W \log_2(1 + P/N) \ , \tag{1.1}$$

where W is the bandwidth and P/N is the signal-to-noise (SNR) ratio. The Shannon bound establishes the region of allowable performances for real communication systems. Much of the work done since then has been a search for achieving Shannon's upper bound with practical systems (Forney & Costello, 2007).

As the Shannon curve gives the optimal relationship between bandwidth, power and throughput, it was thought that the optimal power level could be found just by inverting the Shannon equation (Sklar, 2001). Therefore, power efficiency would be achieved by the same technical means as throughput efficiency (see Figure 1.1).

Nevertheless, during the first years of this millenium it was shown that energy efficiency of single antenna systems is achieved by solutions that do not spring from Shannon

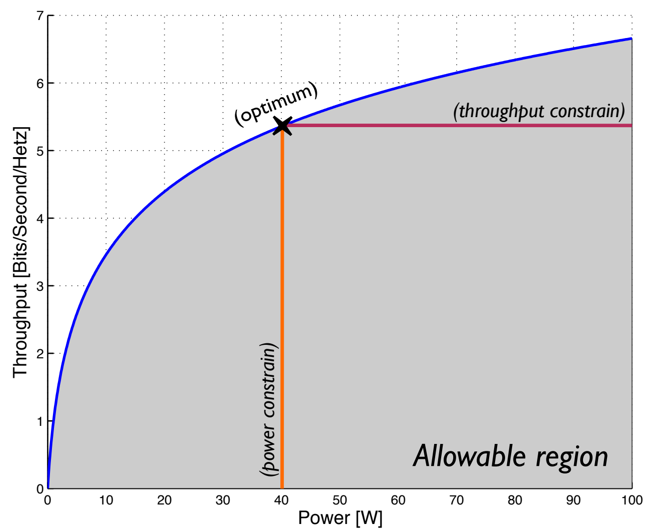


FIGURE 1.1. Allowable region given by (1.1). To maximize the throughput of a real system given a power constrain is equivalent to minimize the signal power given a throughput constraint.

equations (Cui, Goldsmith, & Bahai, 2005; Raghunathan, Schurgers, Park, & Srivastava, 2002). Since then, the idea of energy efficient communications has gathered a lot of attention among researchers and became a prominent area of investigation.

In this dissertation we consider energy efficiency to be an independent criterion, whose development leads to new principles and strategies for designing communication systems. We address questions such as: what is the relationship between throughput and energy efficiency? In particular, which are the rules by which the modulation and the antenna array size of a communication system shall be chosen to attain energy-efficient communications? Are multiple antenna systems capable of reducing the overall energy consumption of wireless communication devices?

Although these questions are purely technical in principle, we think that they are related with a desire for new solutions and paradigms which is blooming nowadays in different areas of our society. In fact, we believe that the throughput maximization criterion and standard economics theory are strongly correlated, as both are based on the principle of "infinite needs" (or bits to transmit) and scarcity of resources (power or bandwidth).

On the contrary, energy-efficient communications are more related to sustainable initiatives, as they share the idea of reducing the consumption of resources (energy) necessary to fullfil a fixed need (transfer of a message). We hope that this work may stimulate further developments on this latter line of thought, which is so needed in a world overwhelmed by the consequences of the abuse of the “more is better” principle.

2. STATE OF THE ART AND CONTRIBUTIONS OF THIS THESIS

In the following, Section 2.1 formally introduces the general concept of energy-efficient communication techniques. Sections 2.2 and 2.3 outline the state of the art of energy-efficient communication techniques for optimizing the physical layer parameters of single and multiple antenna systems, respectively. Finally, Section 2.4 presents the goals of this dissertation and Section 2.5 summarizes its main contributions.

2.1 Energy-efficient communications techniques

Energy-efficient communication techniques are methods which seek to minimize the total energy necessary for transferring one bit of data successfully from transmitter to receiver. Such a bit is henceforth called a *goodbit*. These methods are not constrained by bit-rates: they simply seek to reduce the overall energy consumption. For this reason, energy efficiency optimization is independent of the throughput maximization criterion.

	Optimization	Constraints
Energy-efficient communications	Minimize energy consumption	Bandwidth Amount of data
Standard communications	Maximize throughput	Bandwidth Irradiated power

2.2 Energy-efficient SISO communications

The communication energy budget of single-input single-output systems (SISO), which use a single antenna for transmitting and receiving signals, depends on choices such as the modulation scheme, packet structure and transmission power. When attaining high data rates is not a requirement and when the communication system is power-limited, the common notion is to choose low-order modulations such as BFSK or BPSK, whose bandwidth efficiency is lowest in favor of a lower SNR requirement for achieving a desired bit error rate (Sklar, 2001). These modulations are, in fact, the only ones used in commercially available low power transceivers like TI CC1000 (Chipcon, 2002) or CC2420 (Chipcon, 2006). Nevertheless, it has been shown that the above notion leads to suboptimal operation for communication over short distances through deterministic channels (Ammer & Rabaey, 2006; Cui et al., 2005; Raghunathan et al., 2002; T. Wang, Heinzelman, & Seyedi, 2008).

The rules by which the modulation size and transmission power shall be chosen to attain energy-efficient communications through fading channels have not yet been studied thoroughly. Most of the reported work about how to achieve energy efficient communications focuses on the additive white Gaussian noise channel (AWGN) (Ammer & Rabaey, 2006; Cui et al., 2005; Holland, 2007; Hou, Hamamura, & Zhang, 2005; Kan, Cai, Zhao, & Xu, 2007; Raghunathan et al., 2002; Schurgers, Aberthorne, & Srivastava, 2001; A. Wang, Cho, Sodini, & Chandrakasan, 2001; Q. Wang, Hempstead, & Yang, 2006; T. Wang et al., 2008), and has no straightforward generalization to the analysis of random channels. Therefore, their results are not directly applicable to wireless communications, which are characterized by fading channels with important outage probabilities (Rappaport, 2002).

Attempts of considering other channels than AWGN have been found in (Holland & Wang, 2011; T. Wang & Heinzelman, 2010). In (Holland & Wang, 2011), energy consumption of block fading Rayleigh channels is studied. We do not like their approach, as they consider the effect of the random channel fading via its outage probability, rather than by taking into account the actual symbol error rate (SER) degradation. In (T. Wang & Heinzelman, 2010), physical layer parameters of ultra-wide-band communications over fading channels are optimized by numerical evaluations. However, the model used is only valid for fast-fading channels and cannot be extended for fading channels with correlation over time.

Many existing energy consumption models such as the ones reported in (Cui et al., 2005; Kan et al., 2007; Raghunathan et al., 2002; Schurgers et al., 2001; A. Wang et al., 2001; Q. Wang et al., 2006) share the assumption that the bit error rate is a given constant, which is determined by upper layer requirements. The idea that the bit error rate should not be a constant but a parameter to be optimized is analyzed in (T. Wang et al., 2008) and (Hartwell, Messier, & Davies, 2007), but those results are only valid for AWGN channels. Their approach cannot be extended to fading channels because expressing the SNR as function of the bit error rate leads to intractable mathematics.

2.3 Energy efficient MIMO communications

Multiple-input multiple-output (MIMO) communication systems, which use multiple antenna arrays in both transmitter and receiver, were originally introduced as a way for achieving higher data rates or for improving the reliability of wireless links (Foschini & Gans, 1998). More recently, researchers have started to realize that the MIMO techniques can also be used for reducing the energy consumption of wireless communications (Belmega & Lasaulce, 2009; Bravos & Kanatas, 2008; Cui, Goldsmith, & Bahai, 2004; Glazunov, 2012; Heliot, Imran, & Tafazolli, 2011; Jiang & Cimini, 2011; Kim, Chae, de Veciana, & Heath, 2009; Prabhu & Daneshrad, 2010; Siam, Krunz, Cui, & Muqattash, 2010; Xu & Qiu, 2012).

The *multiple-input multiple-output singular value decomposition* (MIMO SVD) modulation is widely known as an capacity achieving scheme for sending data through a multi-antenna communications link in which the transmitter has knowledge of the channel state (Tse & Viswanath, 2005). Consider a MIMO channel in which the received signal vector $\boldsymbol{v} = (v_1, \ldots, v_{N_r})^t$ can be expressed in terms of the transmitted symbol vector $\boldsymbol{u} = (u_1, \ldots, u_{N_t})^t$ as

$$\boldsymbol{v} = \boldsymbol{H}\boldsymbol{u} + \boldsymbol{w} \ , \tag{2.1}$$

where u_j is the complex symbol transmitted through the j-th antenna, v_i is the complex symbol received by the i-th transmission branch, $\boldsymbol{H}$ is a random matrix with coefficients

$h_{i,j}$ which are i.i.d. standard complex normal random variables and $\boldsymbol{w} = (w_1, \ldots, w_{N_r})^t$ is the vector of additive white Gaussian noise terms experimented in each branch of the receiver (Goldsmith, 2005). Using the singular value decomposition, the channel matrix $\boldsymbol{H}$ is diagonalized creating $N = \min\{N_{\mathrm{t}}, N_{\mathrm{r}}\}$ non-interfering channels (*eigenchannels* in the following). This can be described as

$$y_k = \sqrt{\lambda_k} x_k + n_k \qquad k = 1 \ldots N \quad , \tag{2.2}$$

where k indexes the eigenchannels, x_k are the transmitted symbols, n_k are additive white Gaussian noise terms (AWGN) and $\sqrt{\lambda_k}$ are the singular values of the channel matrix H (Eckardt & Young, 1939). It is worth to notice that although the MIMO SVD modulation provides N eigenchannels, there is no need to use them all. Using all the eigenchannels maximizes the data rate, but sacrifices symbol error rate (SER). Conversely, using only the $n < N$ eigenchannels with most favorable fading statistics yields a better SER but at the cost of decreasing the data-rate (Tse & Viswanath, 2005).

Most of the existent models for analyzing the energy consumption of MIMO SVD communications reported so far in the literature (Belmega & Lasaulce, 2009; Glazunov, 2012; Heliot et al., 2011; Jiang & Cimini, 2011; Kim et al., 2009; Prabhu & Daneshrad, 2010; Xu & Qiu, 2012) are based on the abstract definition of the capacity of a MIMO random fading channel. These models are not adequate for determining attainable performances of concrete modulations with a specific number of eigenchannels used. In (Cui et al., 2004) a model is presented that provide a more concrete framework. Nevertheless, using the bit error rate instead of the SNR as a variable, their formulation finds big mathematical difficulties.

More important, none of these models consider the effect of retransmissions required to guaranteeing error-free transmissions. To consider this, it is mandatory to know the symbol error rate of the MIMO system. Despite the importance and popularity of the MIMO SVD modulation, no simple formula for the symbol error rate (SER) of the eigenchannels has been reported yet, even though the topic has seen much recent activity (e.g.

(Au et al., 2008; Edelman & Persson, 2005; Jin, McKay, Gao, & Collings, 2008; Kang & Alouini, 2003; Kwan, Leung, & Ho, 2007; Onatski, 2008; Zanella, Chiani, & Win, 2009; Zhang, Guan, & Zhou, 2011)). The common approach for studying the statistics of the eigenchannels is to consider λ_k as eigenvalues of the complex Wishart matrix $\boldsymbol{W} = \boldsymbol{H}\boldsymbol{H}^\dagger$ (Muirhead, 1982). The eigenvalues of $\boldsymbol{W}$ have a joint probability distribution (p.d.f.) given by (Edelman, 1989)

$$p = K_N \exp\left(-\sum_{k=1}^{N} \lambda_k\right) \prod_{i=1}^{N} \lambda_i^{|N_\mathrm{t}-N_\mathrm{r}|} \prod_{i>j}^{N} (\lambda_i - \lambda_j)^2 \ , \tag{2.3}$$

with $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$ and K_N a constant. Deriving the statistics of each eigenchannel requires to determine the exact marginal p.d.f. $p_{\lambda_k}(\lambda_k)$ of each eigenvalue from (2.3). Following this approach, it is shown in (Edelman, 1989) that the SNR of the smallest eigenchannel of a $N \times N$ MIMO channel have the same statistics as a Rayleigh channel with power gain $1/N$ (i.e. $p_{\lambda_1}(\lambda_1) = Ne^{-N\lambda_1}$). Although expressions for the marginal p.d.f. of the other eigenvalues have been found (Edelman & Persson, 2005; Kang & Alouini, 2003; Kwan et al., 2007; Onatski, 2008; Zanella et al., 2009), they are mathematically complex and do not provide much insight about the performance of the corresponding eigenchannels. In (Ordoez, Palomar, Pages-Zamora, & Rodriguez Fonollosa, 2007), it was shown that in the high signal-to-noise ratio (SNR) regime the SER of each eigenchannel can be expressed as

$$\bar{P}_s(\bar{\gamma}) = (G_c\bar{\gamma})^{-G_d} + o(\bar{\gamma}^{-G_d}) \ , \tag{2.4}$$

where $\bar{\gamma}$ is the SNR, G_c is the power gain of the channel and G_d is the diversity degree (Zheng & Tse, 2003). The limitation of this result is that the high-SNR restriction leads to insights of little practical interest. In (Taniguchi, Sha, Karasawa, & Tsuruta, 2007), the idea of approximating the statistics of the largest eigenchannel by a Nakagami-m fading is presented. The value of m is chosen in order to approximate the outage statistics of this eigenchannel. Although the approximation thus obtained is accurate, it is not obvious if the proposed method can be extended to model the statistics of other eigenchannels.

2.4 Research goals

The main goal aimed at by this dissertation is to answer the question: can MIMO systems be used for reducing the energy consumption of wireless communications? Although this is plausible, the available literature has not yet given a satisfactory answer.

In order to perform a proper comparison between SISO and MIMO systems, we will seek the rules by which the main physical-layer parameters of each kind of system shall be chosen for attaining energy-efficient communications. Using them, we will be able to compare the minimal energy consumption of SISO and MIMO systems of arbitrary size.

2.5 Summary of contributions of this thesis

In the following, we present the a summary of the main contributions of this thesis.

I We have developed an energy consumption model that provides a general framework that can be applied to a variety of situations, which may include:

- time correlated or uncorrelated random channels with arbitrary statistics (Rayleigh, Nakagami-m, etc) (Rosas & Oberli, in press),
- MIMO systems with different antenna array size (Rosas & Oberli, submitted).

To the best of our knowledge, our model is the first in considering the energy consumption of wireless communications to be a random variable, one that depends on both the channel and the thermal noise statistics (Rosas & Oberli, 2012b).

II We have found an explicit formula for the mean number of transmission trials (τ) necessary for conveying a goodbit (i.e. until a frame is decoded without error) (Rosas & Oberli, in press):

$$\bar{\tau} = 1 + \sum_{n=1}^{\infty} \mathbb{E}\left\{ \prod_{j=1}^{n} P_j \right\} \quad . \tag{2.5}$$

Above, $\mathbb{E}\{\cdot\}$ denotes the expectation operator and P_j is the probability of decoding the frame with error during the j-th transmission trial. The generality of

(2.5) allows to estimate the energy consumption of retransmissions in both SISO or MIMO communications over channels with arbitrary fading statistics.

III We developed a method for approximating the statistics of the eigenchannels of MIMO SVD communications using the Nakagami-m fading model (Rosas & Oberli, 2012c). This method was used for deriving an approximation of the mean symbol error rate (SER) of MIMO SVD transmissions that use only the n eigenchannels with better fading statistics. The approximation found is

$$\bar{P}_{\mathrm{s}}^{(n)}(\bar{\gamma}_{N-n+1}, \ldots, \bar{\gamma}_n) \approx \frac{1}{n} \sum_{k=N-n+1}^{N} \bar{P}_{\mathrm{s}}(\bar{\gamma}_k, \mu_k, m_k) \ , \tag{2.6}$$

where $\bar{P}(\bar{\gamma}, \mu, m)$ is the SER of a Nakagami-m channel with mean SNR $\bar{\gamma}$ and mean power gain μ. This expression is simple, accurate, easily computable and provides intuition about the quality of the channel for data transmission (Rosas & Oberli, 2012c). All these qualities are absent in alternative expressions found in the literature (see Section 2.3). Using (2.6) we were able to a number of important insights about the diversity degree and the power gain of the eigenchannels of a $N \times N$ MIMO system (Rosas & Oberli, under review).

IV Using our energy consumption model, we derive rules for choosing the modulation size that achieves highest energy efficiency as a function of link distance on fast fading channels. For SISO systems, we found that for long transmission distances low bandwidth efficiency modulations (with small M-ary number like BPSK) are optimal in the energy consumption sense (see Section 3.5) (Rosas & Oberli, 2012b). As the transmission distance shortens, the optimal modulation size grows. We found that this principle can be generalized for MIMO SVD communications. In effect, the strategy that minimizes the energy consumption of short range communications is to maximize the throughput using a large constellation size over all the available eigenchannels (Rosas & Oberli, 2012a). As the transmission distance increases, the optimal number of used eigenchannels and the optimal modulation size decreases. For long link distances, the optimal strategy is to use beamforming with a binary modulation.

V We found that small antenna array systems are more energy efficient than single antenna systems for short range communications (see Section 3.6) (Rosas & Oberli, submitted). On the contrary, we found that large MIMO systems are optimal for long transmission distances.

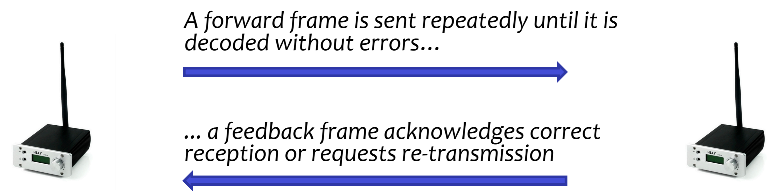
3. METHODOLOGY AND RESULTS

In order to find the rules by which the main physical-layer parameters may be optimized, we have developed a model for the total energy necessary for transferring one goodbit from transmitter to receiver. In this chapter we present our model and use it for comparing the energy consumption of SISO and MIMO systems of different antenna array size. For SISO systems, we use the model for optimizing the SNR and the M-QAM constellation size. For MIMO system, we focus on the MIMO SVD modulation and use the model for also optimizing the number of used eigenchannels. Finally, we compared the energy consumption of the optimized SISO and MIMO systems.

The chapter is structured as follows: Section 3.1 presents the energy consumption model. Section 3.2 presents results regarding to mean number of retransmissions of SISO and MIMO systems. Section 3.3 presents our work on characterizing the statistics of the MIMO SVD channel, which was used later for evaluating the energy consumption of MIMO SVD transmissions. Section 3.4 present our results about how to optimize the SNR for further also minimizing the energy consumption of SISO or MIMO SVD systems. Section 3.5 shows the relationship between the throughput and energy efficiency of a given antenna array size. Finally, Section 3.6 compares the energy consumption between optimized SISO and MIMO systems.

3.1 Energy consumption model

We assume that every frame transmitted in the *forward* direction is matched by a feedback frame in the *reverse* direction, to acknowledge correct reception or requests a re-transmission. We also assume that the irradiated power is determined by the transmitter based upon knowledge of the statistics of the signal-to-noise ratio (SNR) at the intended receiver. We further assume that all frames in both directions are always detected and that all feedback frames are decoded without error.



Transmissions in both directions cause energy expenses at respective transmitters and receivers. In short range communications, the energy consumption for receiving a frame is known to be on the same order as the consumption for transmitting it (Raghunathan et al., 2002) and must hence be accounted for.

In our model, the energy consumed by the transmitter of forward frames per error-free transferred bit, and for also decoding the corresponding feedback frames, is given by (Rosas & Oberli, in press)

$$\mathcal{E}_{\mathrm{T}} = \mathcal{E}_{\mathrm{st}} + \left[B_{\mathrm{tx}} + (P_{\mathrm{el,tx}} + P_{\mathrm{PA}}) T_{\mathrm{b}} + P_{\mathrm{el,rx}} \frac{T_{\mathrm{fb}}}{L} \right] \tau \ . \tag{3.1}$$

Here $\mathcal{E}_{\mathrm{st}}$ is the energy needed to wake up the transmitter from a low power consumption (sleep) mode, divided by the number of payload bits that are going to be transmitted before the transceiver goes again into low power consumption mode. B_{tx} stands for the energy consumption of the baseband processing per bit. $P_{\mathrm{PA}} = \sum_{j=1}^{N} P_{\mathrm{PA}}^{(j)}$ is the total power consumption of the power amplifiers ($N = 1$ in the SISO case). $P_{\mathrm{el,tx}}$ (respectively $P_{\mathrm{el,rx}}$) is the power consumed by the remaining baseband and radio-frequency electronic components that perform the forward transmission (respectively the feedback frame reception). T_{b} is the average air time per payload bit on a forward frame, which includes acquisition, synchronization and frame overhead. $T_{\mathrm{b}} = R^{-1}(1 + O/L)$, where R is the physical layer bit-rate, L is the number of payload bits per frame and O is a measurement of the overhead in bits. $T_{\mathrm{fb}} = F/R$ is the air time of the feedback frame, where F is the feedback frame length. Finally τ is the number of trials until the frame that contains the considered bit is decoded without errors in the receiver.

By analogy, the total energy used by the receiver of forward frames for demodulating τ forward transmissions, and for transmitting the corresponding τ feedback frames, is

$$\mathcal{E}_{\mathrm{R}} = \mathcal{E}_{\mathrm{st}} + \left[B_{\mathrm{rx}} + P_{\mathrm{el,rx}} T_{\mathrm{b}} + \left(P_{\mathrm{el,tx}} + P_{\mathrm{PA}} \right) \frac{T_{\mathrm{fb}}}{L} \right] \tau \ \ . \tag{3.2}$$

The total energy consumption per bit transmitted without error is the sum of (3.1) and (3.2):

$$\mathcal{E}_{\mathrm{b}} = 2\mathcal{E}_{\mathrm{st}} + \left[B_{\mathrm{tx}} + B_{\mathrm{rx}} + \left(P_{\mathrm{el,tx}} + P_{\mathrm{PA}} + P_{\mathrm{el,rx}} \right) \left(T_{\mathrm{b}} + \frac{T_{\mathrm{fb}}}{L} \right) \right] \tau \tag{3.3}$$

$$= S + \left[B + (P_{\mathrm{el}} + P_{\mathrm{PA}}) T \right] \tau \ \ , \tag{3.4}$$

where we have defined $S = 2\mathcal{E}_{\mathrm{st}}$, $B = B_{\mathrm{tx}} + B_{\mathrm{rx}}$, $P_{\mathrm{el}} = P_{\mathrm{el,tx}} + P_{\mathrm{el,rx}}$ and $T = T_{\mathrm{b}} + T_{\mathrm{fb}}/L$.

Because of τ, $\mathcal{E}_{\mathrm{b}}$ is a random variable that depends on the realizations of the channel and of the thermal noise. Its mean value is

$$\bar{\mathcal{E}}_{\mathrm{b}} = \mathbb{E}\left\{ \mathcal{E}_{\mathrm{b}} \right\} = S + \left[B + \left(P_{\mathrm{el}} + P_{\mathrm{PA}} \right) T \right] \bar{\tau} \ \ . \tag{3.5}$$

Expressions for $\bar{\tau}$ are discussed in the sequel.

3.2 Mean number of retransmissions

A key contributor to the energy consumption is the need for re-transmissions due to forward frames that get decoded with errors at the receiver. The probability of frame error (and hence the probability of re-transmission) depends on the mean received SNR, $\bar{\gamma}$, and on the statistics of the wireless channel. Therefore, the number of trials (τ) until a frame is decoded without error is a random variable.

To calculate the mean number of trials, $\bar{\tau}$, we have found a explicit formula (Rosas & Oberli, in press):

$$\bar{\tau} = 1 + \sum_{n=1}^{\infty} \mathbb{E} \left\{ \prod_{j=1}^{n} P_j \right\} \ \ , \tag{3.6}$$

where $\mathbb{E}\{\cdot\}$ denotes the expectation operator and P_j is the probability of decoding the frame with error during the j-th transmission trial. Each of the terms of the infinite sum in (3.6) correspond to the expected value of the probability of having n successive frames decoded with error. The generality of (3.6), which can be applied for SISO or MIMO systems with different channel fading statistics, is a key element of the flexibility of our energy consumption model.

The value of $\bar{\tau}$ depends on the joint distribution of the probabilities P_j for $j = 1, \ldots, \infty$. In effect, consider first a static channel, where the frame error probabilities P_j are fully correlated which each other, and hence $P_j = P_1 \; \forall j \in \mathbb{N}$. In this case, (3.6) becomes

$$\bar{\tau}_{\text{static}} = 1 + \sum_{n=1}^{\infty} \mathbb{E}\left\{P_1^n\right\} = \mathbb{E}\left\{\frac{1}{1-P_1}\right\} \quad . \tag{3.7}$$

Consider now a fading channel in which the SNR levels of any two frame transmission trials are statistically independent. Then (3.6) can be re-written as

$$\bar{\tau}_{\text{f}} = 1 + \sum_{n=1}^{\infty} \prod_{j=1}^{n} \mathbb{E}\left\{P_j\right\} = \frac{1}{1-\mathbb{E}\left\{P_1\right\}} \quad , \tag{3.8}$$

where $\{P_j\}_{k=1}^{\infty}$ is now a collection of i.i.d. random variables.

Using the Jensen inequality for the convex function $\Phi(x) = (1-x)^{-1}$ with $x \in [0, 1)$, it can be shown that

$$\bar{\tau}_{\text{f}} \leq \bar{\tau}_{\text{static}} \quad , \tag{3.9}$$

where the equality is attained by the AGWN channel. This result shows that transferring successfully one entire frame of data across uncorrelated channels takes, on average, fewer transmission attempts than doing it over fully correlated channels. An intuitive explanation for this is that unfavorable (initial) realizations of static channels have a permanent low SNR level, and require therefore a large number of trials until a frame is received without error. However unlikely, the poor performance of these unfavorable cases raise the mean number of trials enough to spoil the average performance beyond the case of uncorrelated channels.

3.3 Characterization of MIMO SVD channels

In order to use (3.7) or (3.8) for calculating the corresponding expression for MIMO SVD transmissions, we need to find expressions for the fading statistics of the MIMO channel. For this, we have developed a method for approximating the statistics of the eigenchannels of MIMO SVD communications using the Nakagami-m fading model (Rosas & Oberli, 2012c). Maximum Likelihood Estimation of Nakagami-m channel parameters is performed for each of the eigenchannels of the MIMO system. The accuracy of the results can be shown by minimizing the mean square error for the 2×2 case, and using the Kolmogorov-Smirnov test (Gregory & Corder, 2009) for larger system sizes.

The proposed method was used as a starting point for deriving an approximation of the mean SER of MIMO SVD transmissions where only the n channels with better fading statistics are used. Our approximation is given by

$$\bar{P}_{\mathrm{s}}^{(n)}(\bar{\gamma}_{N-n+1}, \ldots, \bar{\gamma}_n) \approx \frac{1}{n} \sum_{k=N-n+1}^{N} \bar{P}_{\mathrm{s}}(\bar{\gamma}_k, \mu_k, m_k) \ , \tag{3.10}$$

where $\bar{P}_{\mathrm{s}}(\bar{\gamma}, \mu, m)$ is the SER of a Nakagami-m channel with mean SNR $\bar{\gamma}$ and mean power gain μ. This expression is simple, accurate, easily computable and provides intuition about the quality of the channel for data transmission (see Figure 3.1). Using this approximation, we have presented an upper and lower bound for the SER of MIMO SVD based just on the SER of the weakest eigenchannel used for the transmission.

We also show that, for $N > 15$, the eigenchannels of a $N \times N$ MIMO channel fit the following general characterization: there are $N - 5$ eigenchannels which error statistics are similar to an AWGN channel (within 1 dB SNR), while the five weakest eigenchannels perform like a Rayleigh and Nakagami-m channels with $m = 4, 9, 16$ and 25, respectively (Rosas & Oberli, under review). We also provide a number of insights about the mean power gain distribution among the eigenchannels for large antenna arrays, and show that 75% of the total mean power gain of the MIMO channel goes to the top third of all the eigenchannels.

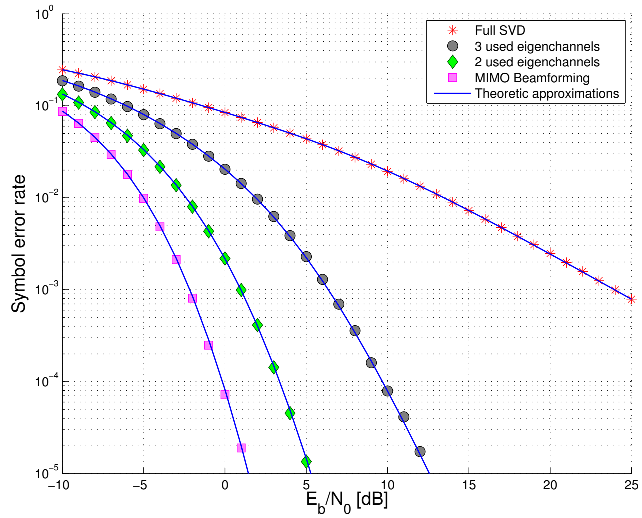


FIGURE 3.1. Average SER of SVD transmissions over a 4×4 MIMO channel using uncoded BPSK and 1, 2, 3 or 4 eigenchannels of the SVD modulation. Markers show the average SER obtained by computer simulation of 10^6 symbols. Solid lines show the corresponding proposed approximation (3.14). An increasing diversity gain (i.e. the slope of the SER in the high SNR regime) can be observed as the number of used eigenchannels is reduced.

3.4 Optimization of the SNR

Consider rewriting (3.5) so that the terms that depend on the mean SNR, $\bar{\gamma}$, observed at the decision stage of the receiver, become explicit:

$$\bar{\mathcal{E}}_{\mathrm{b}}\left(\bar{\gamma}\right) = S + \left(B + \left[P_{\mathrm{el}} + P_{\mathrm{PA}}(\bar{\gamma})\right]T\right)\bar{\tau}(\bar{\gamma}) \quad . \tag{3.11}$$

Above, $P_{\mathrm{PA}}(\bar{\gamma})$ represents the dependency between the PA consumption and the irradiated electromagnetic energy. We have shown that $P_{\mathrm{PA}}(\bar{\gamma}) = Ad^{\alpha}\bar{\gamma}$, where A is a constant, d is the link distance in meters and α is the path loss exponent (Rappaport, 2002). The mean number of transmission trials, $\bar{\tau}(\bar{\gamma})$, is a strictly decreasing function of $\bar{\gamma}$ which satisfies $\lim_{\bar{\gamma}\to\infty}\bar{\tau}(\bar{\gamma}) = 1$ (Rosas & Oberli, 2012a, in press).

By construction, (3.11) is the product of the decreasing function $\bar{\tau}(\bar{\gamma})$ and the increasing linear function $P_{\mathrm{PA}}(\bar{\gamma})$. Such a product attains a unique minimum at the SNR level

$\bar{\gamma}^*$. Lower SNR levels are suboptimal because they force the system to do too many retransmissions, and higher SNR levels are also suboptimal because the overall irradiated power is excessive. This behavior is common to both SISO and MIMO SVD systems (see Figure 3.2 and 3.3).

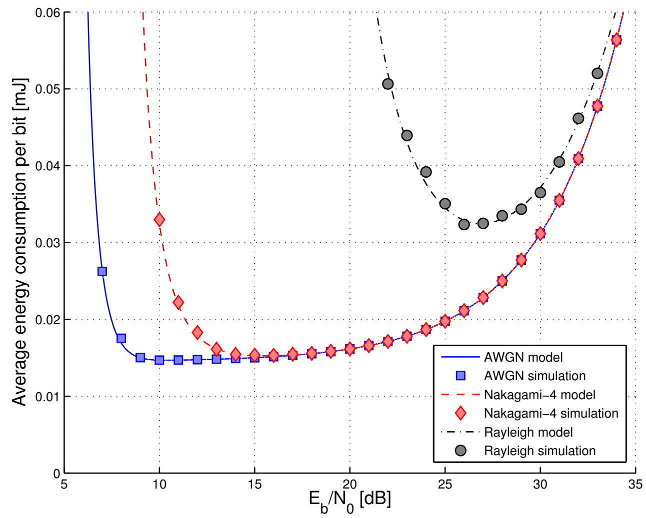


FIGURE 3.2. Simulated average energy consumption per effective transmitted bit of a SISO system for various fading channels (markers) vs. numerical evaluations of the model presented in (3.11) (solid lines) for a low-power transceiver with typical parameters. Modulation is QPSK and link distance is $d = 35$ meters. A unique minimum is observed, which corresponds to the optimal SNR for maximum energy efficiency.

We further found that the optimal energy consumption per bit and the optimal SNR at which this occurs take larger values for channels with less favorable error statistics. In effect, for a generic SISO low-power device, the minimal energy consumption in Rayleigh fading can double the optimal consumption in AWGN, and the corresponding SNR for Rayleigh fading can be 15 dB higher than the one for AWGN (see Figure 3.2).

We found explicit expressions for the optimal SNR value of SISO systems for AWGN, Rayleigh and Nakagami-m fading statistics (Rosas & Oberli, in press). For example, we

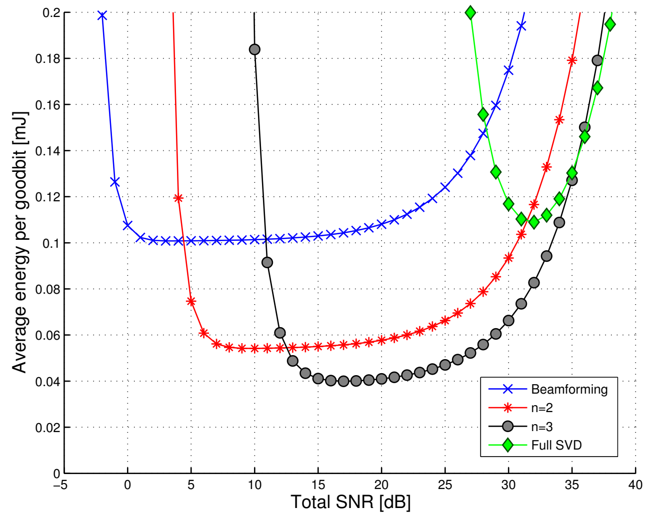


FIGURE 3.3. Mean energy consumption of a low power 4×4 MIMO SVD system with equal power allocation and all possibilities of used eigenchannels. Modulation is uncoded BPSK and link distance is d = 50 meters.

found that the optimal SNR for a Rayleigh fading channel is given by

$$\bar{\gamma}_0 \approx \frac{\bar{\gamma}_{\text{long}}}{2}\left(1+\sqrt{1+\frac{4P_{\text{el}}}{\bar{\gamma}_{\text{long}}Ad^{\alpha}}}\right) \quad , \tag{3.12}$$

where P_{el} is as defined in (3.4). $\bar{\gamma}_{\text{long}}$ is the optimal SNR as $d\to\infty$, which is given by

$$\bar{\gamma}_{\text{long}} = 0.81\frac{c}{a}\left[1+\frac{L+H}{\log_2 M}\right] \quad , \tag{3.13}$$

where L is the number of payload bits, H is the number of bits of header, M the order of the M-QAM modulation, $c = 2\left(1-1/\sqrt{M}\right)$ and $a = 3/(M-1)$. $\bar{\gamma}_{\text{long}}$ approximates the optimal SNR in the traditional sense of wireless communications, where the link budget neglects the energy consumption of the electronics and only considers link gains and losses. Using the first order Taylor approximation $\sqrt{1+x}\approx 1+x/2$ in (3.12), we obtain

$$\bar{\gamma}_0 \approx \bar{\gamma}_{\text{long}} + \frac{P_{\text{el}}}{Ad^{\alpha}} \quad . \tag{3.14}$$

This expression reflects the counter-intuitive fact that the optimal SNR *increases* as distance decreases (see Figure 3.4).

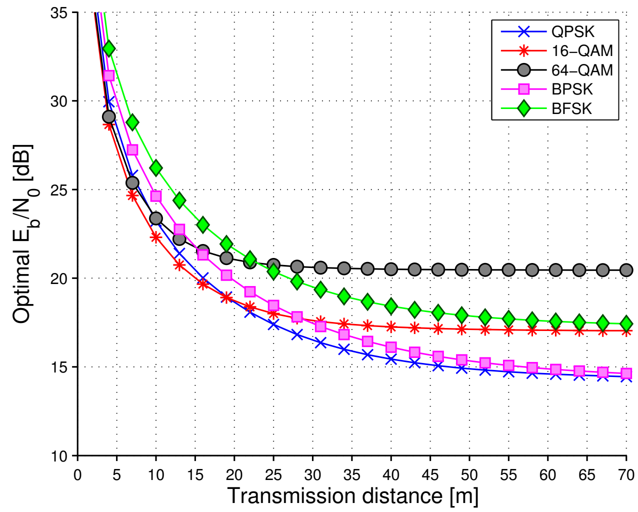


FIGURE 3.4. Optimal SNR for achieving energy efficiency as function of link distance for a fast fading Rayleigh channel.

3.5 Optimization of the throughput

We want to study the minimal average energy consumption achievable for a given transmission distance, namely

$$\bar{\mathcal{E}}_{\mathrm{b}}^{*}(d) = \min_{\bar{\gamma}\in[0,\infty]} \bar{\mathcal{E}}_{\mathrm{b}}(\bar{\gamma}, d) = \bar{\mathcal{E}}_{\mathrm{b}}(\bar{\gamma}^{*}, d) \ , \tag{3.15}$$

with $\bar{\mathcal{E}}_{\mathrm{b}}(\bar{\gamma}, d)$ given by (3.11) explicating its dependance on the link distance d.

We compared the value of (3.11) achieved by different modulation. For SISO systems, we found that for long transmission distances low bandwidth efficiency modulations (with small M-ary number like BPSK) achieve a smaller energy consumption (see Figure 3.5) (Rosas & Oberli, 2012b). As the transmission distance shortens, the optimal modulation size grows. In short range communications, the power consumed by electronic components dominates over the irradiated power, and dominates also over the energy consumption of the power amplifier. Under these conditions, the average air time spent per data bit becomes a relevant parameter in the total energy budget. This makes optimal to pack more bits into each symbol and hence to choose a larger M-ary number. Our results show that lifetime extensions up to 500% can be gained in short range networks by selecting modulations with larger constellations than BPSK.

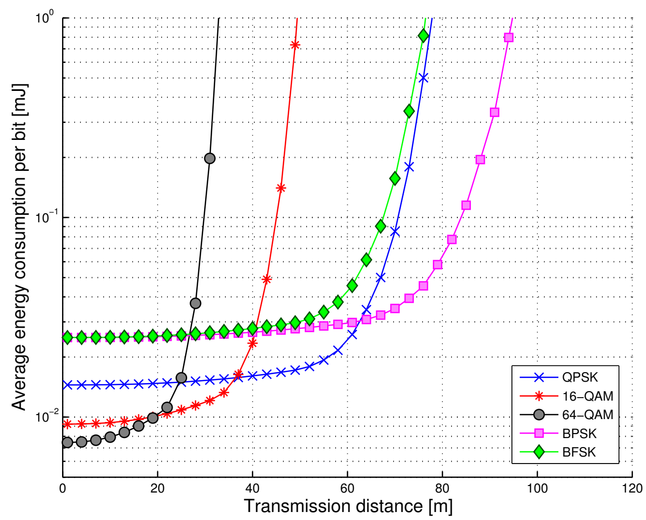


FIGURE 3.5. Energy consumption of a SISO system per successfully transmitted bit of various modulations over a Nakagami-m channel with $m = 4$ as a function of the link distance. Each modulation is operated at its own optimal SNR for the given distance. As distance decreases, modulations with higher spectral efficiency become energy optimal.

We also found that the above principle can be generalized for MIMO systems (Rosas & Oberli, 2012a). In effect, the strategy that minimizes the energy consumption of short range communications is to maximize the throughput using a large constellation size over all the available eigenchannels (see Figures 3.6 and 3.7). As the transmission distance increases, the optimal number of used eigenchannels and the optimal modulation size decreases. For long link distances it is optimal to reduce the irradiated power by reducing the throughput and sending all the power only through the more favorable direction, which is given by the beamforming pattern.

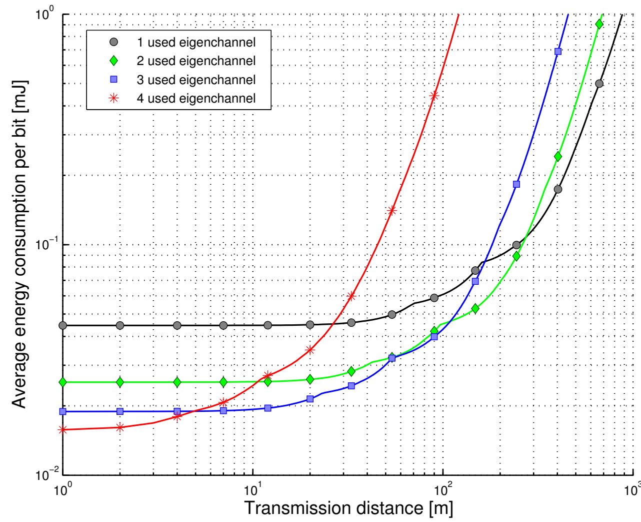


FIGURE 3.6. Minimal energy consumption of 4x4 MIMO SVD communications using various numbers of used eigenchannels. Full SVD, where all the available eigenchannels are being used, is the optimal strategy for short link distances. On the contrary, beamforming (only using one eigenchannel) is optimal for long range communications.

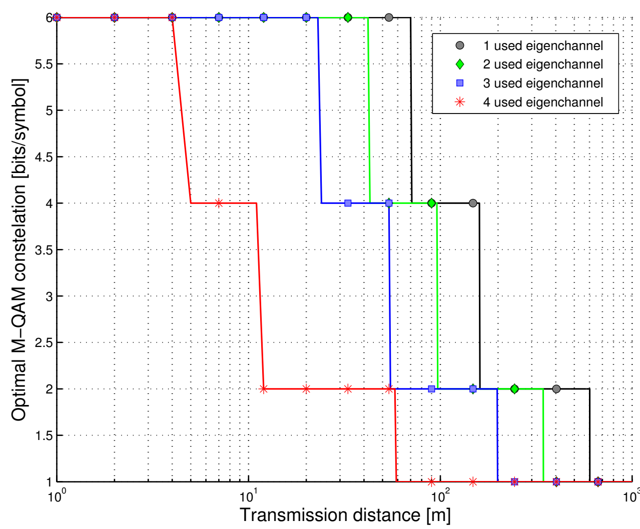


FIGURE 3.7. Size of the M-QAM modulation which minimizes the energy consumption of a 4×4 MIMO system over various link distances. Small modulations are optimal for long link distances, while the optimal modulation size grows as the link distance shortens.

3.6 Comparison between SISO and MIMO systems

We compared the minimal average energy consumption achievable by an $N \times N$ MIMO SVD system for different antenna array sizes (N). This minimal consumption is found optimizing the modulation, the SNR used in each eigenchannel and the number of used eigenchannels (Rosas & Oberli, submitted). The SISO case is recovered using $N = 1$.

We found that small antenna arrays systems are more energy efficient than single antenna system in short range communications (see Figure 3.8) (Rosas & Oberli, submitted). In this regime, large antenna arrays are suboptimal, as the reduction in the time per bit achieved by a higher throughput is not enough to compensate the increase in the electric power, which is linear on the antenna array size. The role played by the MIMO overhead is critical, as it is the only component of the time per bit that do not decreases when the throughput grows.

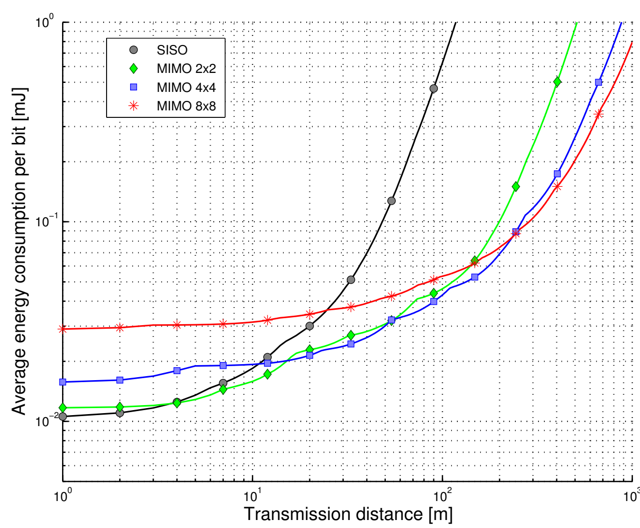


FIGURE 3.8. Minimal energy consumption of different antenna array sizes. Single antenna systems consume less energy in short range communications, while large antenna arrays achieve better performances when the link distance is large.

On the contrary, we found that large MIMO systems are optimal for long transmission distances (see Figure 3.8). The use of beamforming with a large antenna array generates important reductions in the frame error rate. These more favorable error statistics allow to reduce the irradiated energy needed to reach the receiver, which is the main source of energy consumption in long transmission distances.

4. CONCLUSIONS

In this thesis, we studied the optimization of physical-layer parameters in order to reduce the energy consumed by a transceiver for delivering one bit of data without error. In our study, we considered different transmission distances and various channel statistics, as well as the energy cost of retransmissions, feedback frames and the consumption of electronic components.

Most of the available literature about how to achieve energy efficient communications focuses on the AWGN channel and has no straightforward generalization to the analysis of random channels. We developed an energy consumption model which is, to the best of our knowledge, the first in considering the energy consumption of wireless communications to be a random variable, one that depends on both the channel and the thermal noise statistics. The flexibility of our model allows to compare the energy consumption of systems with different antenna array sizes and to evaluate the effect of channels with various statistics.

Our analysis shows that, for a given antenna array size, the optimal choice for reducing the energy consumption of short range communications is to maximize the throughput by using large constellations over all the available eigenchannels. As the transmission distance increases, the optimal modulation size and the optimal number of used eigenchannels decreases. For long-range communications, the optimal choice is to reduce the throughput by using a binary modulation and sending all the power through the largest eigenchannel, which is given by the beamforming patter.

For long transmission distances, our results support the intuitive fact that important energy savings can be achieved by using small constellations and performing beamforming with large antenna arrays. On the contrary, our results show that single antenna systems equipped with large modulation sizes are more efficient in the energy sense for performing short range communications than MIMO systems.

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Modulation optimization for achieving energy efficient communications over fading channels

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Modulation optimization for achieving energy efficient communications over fading channels

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***Abstract*—It is commonly assumed that the energy consumption of wireless communications is minimized when low-order modulations such as BPSK are used. Nevertheless, the literature provides some evidence that low-order modulations are suboptimal for short transmission distances. A thorough analysis on how the modulation scheme and transmission power must be chosen as a function of distance in order to achieve energy-efficient communications over fading channels has not been reported yet. In this paper we provide this analysis by presenting a model that determines the energy consumed per payload bit transferred without error over correlated or uncorrelated random channels.**

We find that each modulation scheme has a single optimal signal-to-noise ratio (SNR) at which the energy consumption is minimized. We also find that if all modulations are operated at their optimal SNR, BPSK and QPSK are the optimal choices for long transmission distances, but as the transmission distance shortens the optimal modulation size grows to 16-QAM and even to 64-QAM. This result leads to showing that for short-range communications the lifetime of a typical low-power transceiver can be increased by up to 600% by selecting the optimal constellation rather than BPSK.

I. Introduction

Attaining high energy efficiency is a key condition that wireless communications technologies like wireless sensor networks (WSN) must satisfy in order for the technology to prosper into large-scale, autonomous networks. Requirements on size and cost of the nodes pose vital constraints to the problem. In fact, battery depletion has been identified as one of the primary causes of lifetime limitation of these networks, and replacing them regularly is impractical in large networks or may even be impossible in hostile environments [1].

The communications energy budget depends on choices such as the modulation scheme, packet structure and transmission power. When the communication system is power-limited (as in WSN), the common notion is to choose low-order modulations such as BFSK or BPSK, which has a low SNR requirement for achieving a desired bit error rate [2]. These modulations are, in fact, the ones used in commercially available low-power transceivers like the TI CC1000 [3] or CC2420 [4], often used for WSN applications. Nevertheless it has been shown that the above notion leads to suboptimal operation for short link distances [5]–[7].

The rules by which the modulation scheme and transmission power shall be chosen to attain energy-efficient communications through random channels have not yet been clearly established. Most of the work reported so far focuses on the additive white Gaussian noise channel (AWGN) [5]–[12]. In [13], energy consumption of block fading Rayleigh channels is studied, but channel fading is included by its outage probability rather than by taking into account the actual symbol error rate (SER) degradation due to the fading process.

In this paper, we present an energy consumption model which allows for optimizing the modulation scheme and transmission power for communications over correlated or uncorrelated random channels.

Furthermore, we establish rules for choosing the modulation size that achieves highest energy efficiency as a function of link distance on fast fading channels. For large transmission distances, our model confirms the common notion discussed above for power-limited systems, while for short transmission distances it coincides with the results reported in [14] and [7] for AWGN, extending them to fast-fading random channels.

Our model also shows that a single optimal SNR level exists at which the least amount of energy is consumed per data bit transferred without error, and reveals how it depends on the packet frame length, modulation size and channel statistics. Many existing energy consumption models such as the ones reported in [5], [6], [8]–[11] share the assumption that the bit error rate is a given constant, which is determined by upper-layer requirements independently of physical layer parameters such as the modulation type or the power consumption of electronic components. The idea that the bit error rate should be a parameter to be optimized can be found in [7] and [15], but those results are only valid for AWGN channels.

The paper is organized as follows: Section II presents the energy consumption model, Section III uses this model for optimizing the SNR to achieve energy efficiency, and Section IV presents the principles for selecting modulation scheme and SNR. Finally, Section V summarizes our conclusions.

II. Energy Consumption Model

Our goal is to determine the total energy that is necessary for transferring one bit of data successfully, without error, in a point-to-point packet-switched wireless communication link (e.g. between two sensor nodes). We assume that every frame transmitted in the *forward* direction is matched by a feedback

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frame in the *reverse* direction, which acknowledges correct reception or requests a re-transmission. We also assume that the irradiated power is determined by the transmitter based upon knowledge of the statistics of the signal-to-noise ratio (SNR) at the decision stage of the intended receiver. We further assume that all frames in both directions are always detected and that all feedback frames are decoded without error.

Transmissions in both directions cause energy expenses at respective transmitters and receivers. In short range communications, the energy consumption for receiving a frame is known to be on the same order as the consumption for transmitting it [5] and must hence be accounted for.

In the sequel, we first analyze the components of energy consumption of a transceiver from the standpoint of a node that transmits one payload frame and receives the corresponding feedback frame (the reverse case —a transceiver that receives one payload frame and transmits the corresponding feedback frame— follows by analogy). We continue by analyzing the statistics of re-transmissions and finally we present of our total energy consumption model.

A. Total Energy Consumption per Successfully Transferred Bit

The energy consumed by the transmitter of forward frames per error-free transferred bit, and for also decoding the corresponding feedback frames, is given by

$$\mathcal{E}_{\mathrm{T}} = \mathcal{E}_{\mathrm{st}} + \left[(P_{\mathrm{el,tx}} + P_{\mathrm{PA}})T_{\mathrm{b}} + P_{\mathrm{el,rx}}\frac{T_{\mathrm{fb}}}{L}\right]\tau \ . \quad (1)$$

Here $\mathcal{E}_{\mathrm{st}}$ is the energy needed to wake up the transmitter from a low power consumption (sleep) mode, divided by the number of payload bits that are going to be transmitted before the transceiver goes again into low power consumption mode. P_{PA} is the power consumed by the power amplifier (PA), and $P_{\mathrm{el,tx}}$ (respectively $P_{\mathrm{el,rx}}$) is the power consumed by the remaining baseband and radio-frequency electronic components that perform the forward transmission (respectively the feedback frame reception). T_{b} is the average air time per payload bit on a forward frame, which includes acquisition, synchronization and frame overhead. $T_{\mathrm{b}} = R^{-1}(1 + O/L)$, where R is the physical layer bit-rate, L is the number of payload bits per frame and O is a measurement of the overhead in bits. $T_{\mathrm{fb}} = F/R$ is the air time of the feedback frame, where F is the feedback frame length. Finally τ is the number of trials until the frame that contains the considered bit is decoded without errors in the receiver.

By analogy, the total energy used by the receiver of forward frames for demodulating τ forward transmissions, and for transmitting the corresponding τ feedback frames, is

$$\mathcal{E}_{\mathrm{R}} = \mathcal{E}_{\mathrm{st}} + \left[P_{\mathrm{el,rx}}T_{\mathrm{b}} + (P_{\mathrm{el,tx}} + P_{\mathrm{PA}})\frac{T_{\mathrm{fb}}}{L}\right]\tau \ . \quad (2)$$

The total energy consumption per bit transmitted without error is the sum of (1) and (2):

$$\begin{aligned}\mathcal{E}_{\mathrm{b}} &= 2\mathcal{E}_{\mathrm{st}} + (P_{\mathrm{el,tx}} + P_{\mathrm{PA}} + P_{\mathrm{el,rx}})\left(T_{\mathrm{b}} + \frac{T_{\mathrm{fb}}}{L}\right)\tau \quad (3)\\ &= S + (P_{\mathrm{el}} + P_{\mathrm{PA}})T\tau \ , \quad (4)\end{aligned}$$

where we have defined $S = 2\mathcal{E}_{\mathrm{st}}$, $P_{\mathrm{el}} = P_{\mathrm{el,tx}} + P_{\mathrm{el,rx}}$ and $T = T_{\mathrm{b}} + T_{\mathrm{fb}}/L$.

It is to be noted that because of τ, $\mathcal{E}_{\mathrm{b}}$ is a random variable that depends on the realizations of the channel and of the thermal noise. Its mean value is

$$\bar{\mathcal{E}}_{\mathrm{b}} = \mathbb{E}\{\mathcal{E}_{\mathrm{b}}\} = S + (P_{\mathrm{el}} + P_{\mathrm{PA}})T\bar{\tau} \ . \quad (5)$$

Expressions for $\bar{\tau}$ are discussed in the sequel.

B. Re-transmission Statistics

A key contributor to the energy consumption is the need for re-transmissions due to forward frames that get decoded with errors at the receiver. The probability of frame error (and hence the probability of re-transmission) depend on the mean received SNR, $\bar{\gamma}$, and on the statistics of the wireless channel. Therefore, the number of trials (τ) until a frame is decoded without error is a random variable. It can be shown that its mean value, $\bar{\tau} = \mathbb{E}\{\tau\}$, where $\mathbb{E}\{\cdot\}$ denotes the expectation operator, can be expressed as

$$\bar{\tau} = 1 + \sum_{n=1}^{\infty}\mathbb{E}\left\{\prod_{j=1}^{n}P_j\right\} \ , \quad (6)$$

where P_j is the probability of decoding the frame with error during the j-th transmission trial. For lack of space, the complete derivation will appear in a future journal article.

The value of $\bar{\tau}$ depends on the joint distribution of the probabilities $\{P_j\}_{j=1}^{\infty}$. In effect, consider first a static channel where $P_j = P_1 \ \forall j \in \mathbb{N}$. In this case, (6) becomes

$$\bar{\tau} = 1 + \sum_{n=1}^{\infty}\mathbb{E}\{P_1^n\} = \mathbb{E}\left\{\frac{1}{1-P_1}\right\} \triangleq \bar{\tau}_{\mathrm{static}} \ . \quad (7)$$

Consider now a fading channel in which the SNR levels of any two frame transmission trials are statistically independent. Then (6) can be re-written as

$$\bar{\tau} = 1 + \sum_{n=1}^{\infty}\prod_{j=1}^{n}\mathbb{E}\{P_j\} = \frac{1}{1-\mathbb{E}\{P_1\}} \triangleq \bar{\tau}_{\mathrm{f}} \ , \quad (8)$$

where P_j and P_k are i.i.d. random variables whenever $j \neq k$.

Using the Jensen inequality for the convex function $\Phi(x) = (1-x)^{-1}$ with $x \in [0, 1)$, it can be shown that

$$\bar{\tau}_{\mathrm{f}} \leq \bar{\tau}_{\mathrm{static}} \ , \quad (9)$$

where the equality is attained by the AGWN channel. This result shows that transferring successfully one frame across uncorrelated channels takes, on average, fewer transmission attempts than doing it over fully correlated channels. An intuitive explanation for this is that unfavorable (initial) realizations of static channels have a permanent low SNR level, and require therefore a large number of trials until a frame is received without error. However unlikely, the poor performance of these unfavorable cases raise the mean number of trials enough to spoil the average performance beyond the case of uncorrelated channels.

III. Optimal mean SNR Level for Energy Efficiency

In this section, we seek to determine the mean SNR for which a communication with M-QAM modulation uses, on the average, the least amount of energy per bit transferred without error.

Consider rewriting (5) so that the terms that depend on the mean SNR, $\bar{\gamma}$, observed at the decision stage of the receiver, become explicit:

$$\bar{\mathcal{E}}_{\mathrm{b}}(\bar{\gamma}) = S + [P_{\mathrm{el}} + P_{\mathrm{PA}}(\bar{\gamma})]\, T\bar{\tau}(\bar{\gamma}) \ . \tag{10}$$

Above, $P_{\mathrm{PA}}(\bar{\gamma})$ is a linear function of $\bar{\gamma}$. In effect, the PA's power consumption is related with the transmission power as $P_{\mathrm{tx}} = (\eta/\xi)P_{\mathrm{PA}}$ where ξ is the peak-to-average power ratio of the transmitted signal and η is the drain efficiency of the PA [6]. The transmitted power attenuates over the air with path loss and arrives at the receiver with a mean power given by $P_{\mathrm{rx}} = P_{\mathrm{tx}}/(Ad^{\alpha})$, where A is a parameter that depends on the transmitter and receiver antenna gains and the transmission wavelength, d is the distance between transmitter and receiver and α is the path loss exponent [16]. At the input of the decision device of the receiver, $\bar{\gamma}$ is related to P_{rx} as $\bar{\gamma} = P_{\mathrm{rx}}/(N_0 W N_{\mathrm{f}} M_{\mathrm{l}})$, where N_0 is the power spectral density of the baseband-equivalent additive white Gaussian noise (AWGN), W is the transmission bandwidth, N_{f} is the noise figure of the receiver's front end and M_{l} is a link margin term which represents any other additive noise or interference [6]. Putting all these relationships together, we find that

$$P_{\mathrm{PA}}(\bar{\gamma}) = \frac{\xi A d^{\alpha} N_0 W N_{\mathrm{f}} M_{\mathrm{l}}}{\eta}\bar{\gamma} = A_{\mathrm{total}}\bar{\gamma} \ , \tag{11}$$

with A_{total} a constant.

Assume that an uncoded M-ary modulation is used for these transmissions. Then, there are $\lambda = (L+O)/\log_2(M)$ symbols per frame, where L is the number of payload bits per frame and O is the overhead (c.f. Section II-A). The frame error rate P_f can be written in terms of the symbol error rate $P_{\mathrm{s}}(\gamma)$ as $P_f = 1 - \prod_{k=1}^{\lambda}[1 - P_{\mathrm{s}}(\gamma_k)]$, where γ_k is the SNR in effect during the k-th symbol. Using this in (8) and assuming that that all γ_k are i.i.d. random variables with mean $\bar{\gamma}$ (i.e. fast fading) we obtain:

$$\bar{\tau}_{\mathrm{f}}(\bar{\gamma}) = \frac{1}{\prod_{k=1}^{\lambda}\left[1 - \mathbb{E}\left\{P_{\mathrm{s}}(\gamma_k)\right\}\right]} = \frac{1}{\left[1 - \mathbb{E}\left\{P_{\mathrm{s}}(\gamma_1)\right\}\right]^{\lambda}} \ , \tag{12}$$

where the first equality follows from the assumption that each symbol is decoded independently from all others.

Under fast fading conditions, replacing (11) and (12) into (10), we find

$$\mathcal{E}_{\mathrm{b}}(\bar{\gamma}) = S + \frac{(P_{\mathrm{el}} + A_{\mathrm{total}}\bar{\gamma})\, T}{[1 - \bar{P}_{\mathrm{s}}(\bar{\gamma})]^{\lambda}} \ , \tag{13}$$

where we are using the shorthand notation $\bar{P}_{\mathrm{s}}(\bar{\gamma})$ for $\mathbb{E}\{P_{\mathrm{s}}(\gamma)\}$. In general, $\bar{P}_{\mathrm{s}}(\bar{\gamma})$ is a strictly decreasing function of $\bar{\gamma}$ that satisfies $\lim_{\bar{\gamma}\to\infty}\bar{P}_{\mathrm{s}}(\bar{\gamma}) = 0$. Therefore, the average number of transmissions needed to successfully transfer one frame under fast fading conditions, given by (12), is also a strictly decreasing function of $\bar{\gamma}$ and satisfies $\lim_{\bar{\gamma}\to\infty}\bar{\tau}(\bar{\gamma}) = 1$ (this reflects the intuitive fact that the average number of retransmissions drops as the SNR grows). By construction, (13) is the product of the decreasing function $\bar{\tau}(\bar{\gamma})$ and the increasing linear function $P_{\mathrm{PA}}(\bar{\gamma})$. Such a product attains a unique minimum at the optimal SNR level $\bar{\gamma}_0$. Lower SNR levels are suboptimal because they force the system into too many retransmissions, and higher SNR levels are also suboptimal because the overall irradiated power is excessive.

The minimization of (13) over $\bar{\gamma}$ is straightforward by taking derivative and equating the result to zero. This leads to the following implicit expression for $\bar{\gamma}_0$:

$$\lambda\left(\frac{P_{\mathrm{el}}}{A_{\mathrm{total}}} + \bar{\gamma}_0\right)\frac{d\bar{P}_{\mathrm{s}}}{d\bar{\gamma}}(\bar{\gamma}_0) - \bar{P}_{\mathrm{s}}(\bar{\gamma}_0) + 1 = 0 \ . \tag{14}$$

It is to be noted that the only parameters that influence $\bar{\gamma}_0$ are the mean symbol error rate, $\bar{P}_s(\bar{\gamma})$, the number of payload symbols per frame, λ, and the ratio between the power consumption of electronic components, P_{el}, and the coefficient A_{total}, which is proportional to the irradiated power.

Equation (14) can be used to find the optimal SNR for different random channel models. The corresponding analysis for AWGN, Rayleigh and Nakagami- m channels is currently work in progress.

IV. Optimal modulation as a function of distance

We wish to understand how the energy consumption of a given modulation varies with the transmission distance. Our investigation will be focused on the case of fast fading channels as defined in Section II-B. We will study the energy consumption of M-QAM modulations compared against BPSK and BFSK, motivated by the popularity of these modulations among available of-the-shelf low-power transceiver components [3], [4].

A. Energy consumption analysis

Numerical evaluations of (13) using the parameters presented in Table I show that BPSK, BFSK and various M-QAM modulations attains it minimum energy consumption at a different SNR. As can be seen in Figure 1, the SNR at which these minima occur varies with transmission distance (curves are plotted against E_{b}/N_0 to compare the results against an equal amount of energy per bit.).

The mean energy consumption (13) evaluated at the optimal mean SNR as function of link distance ($\bar{\gamma}\{d\}$) gives the minimal consumption for that modulation at a given distance, which we denote as $\bar{\mathcal{E}}_{\mathrm{b}}(d)$ (Figure 2). Analytically, $\bar{\mathcal{E}}_{\mathrm{b}}(d)$ can be expressed from (10) as

$$\bar{\mathcal{E}}_{\mathrm{b}}(d) = S + [P_{\mathrm{el}} + P_{\mathrm{PA}}\left\{d, \bar{\gamma}(d)\right\}]\, T\bar{\tau}\left\{\bar{\gamma}(d)\right\} \ . \tag{15}$$

In long range communications, the power consumed by the power amplifier (P_{PA}) dominates over the power consumed by the electronic components (P_{el}). Under these conditions, the

‡Source: [6]

TABLE I
GENERIC LOW-POWER DEVICE PARAMETERS

Parameter	Description	Value
R_s	Symbol rate	10 kBaud ‡
L	Frame Payload	98 bits
O	Overhead	30 bits
F	Feedback frame length	11 bits
$\mathcal{E}_{st}$	Start-up energy	0.125 nJ ‡
α	Path-loss coefficient	3.5 ‡
A	Channel loss	30 dB ‡
η	PA efficiency	0.35% ‡
$P_{el,tx}$	Tx electric power consumption	98.2 mW ‡
$P_{el,rx}$	Rx electric power consumption	112.5 mW ‡
N_f	Receiver noise figure	10 dB ‡
M_l	Link margin	40 dB ‡

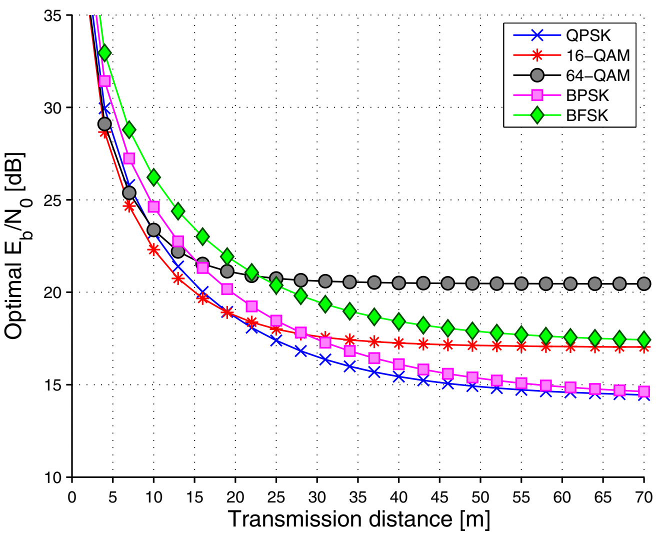

Fig. 1. Optimal SNR for achieving energy efficiency as function of link distance for a fast fading Rayleigh channel.

curves in Figure 2 confirm that the energy-optimal modulations are the ones with lowest spectral efficiency (low M-ary number).

However, for short transmissions distances (less than 15 meters in Figure 2) the power consumed by electronic components ($P_{\rm el}$) dominates over the irradiated power and therefore also over the consumption of the power amplifier ($P_{\rm PA}$). The energy consumption for this case can therefore be approximated as $\bar{\mathcal{E}}_{\rm b}(d) \approx S + P_{\rm el} T \bar{\tau}\{\bar{\gamma}(d)\}$. This shows that for these conditions the average time per bit, T, becomes a relevant parameter in the total energy budget. This compels to pack more bits into each symbol in order to reduce the transmission time T of each bit.

It can further be seen in Figure 2 that for long range transmissions BPSK and QPSK are optimal among the studied modulations and nearly equally energy-efficient. At short distances, on the other hand, when $P_{\rm el}$ is relevant, BPSK almost doubles the energy consumption per bit of QPSK because of the doubly long time per bit.

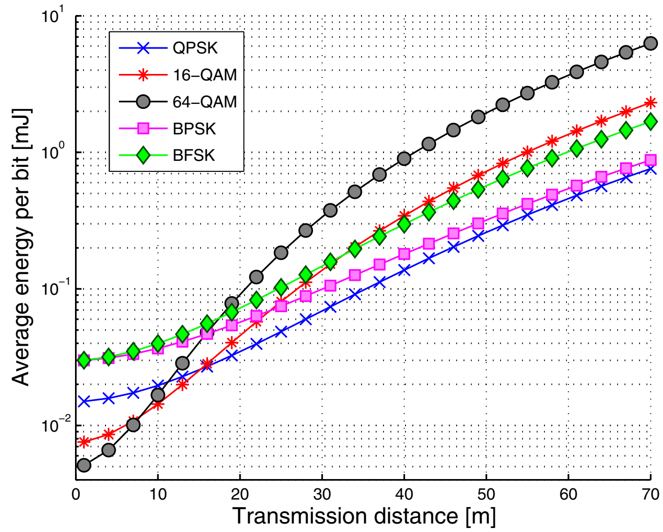

Fig. 2. Energy consumption per successfully transmitted bit of various modulations over a fast fading Rayleigh channel as a function of link distance. Each modulation is operated at its own optimal SNR. As distances decreases, modulations with higher spectral efficiency become optimal.

BFSK is never an optimal modulation (Figure 2). For long range transmissions it suffers the classic 3dB SNR gap with respect to BPSK and is hence suboptimal in the energy sense. For short range, it performs similarly to BPSK because of their equal spectral efficiency, but worse than larger size modulations.

B. Transceiver lifetime analysis

The results presented so far allow for studying the lifetime of networks with finite energy supply. For illustration, consider a simple network composed by two wireless sensor nodes with parameters as given in Table I. The nodes exchange 10 kbits of data every 5 minutes. Each node is powered by an ideal 1.2 Volt AA battery with a 2000 mAh initial energy charge. This charge is used exclusively for the communications tasks described in Section II.

Using this model, the average lifetime of the batteries of these two nodes was calculated for BPSK, BFSK and M-QAM transmissions over different channel models as a function of link distance, with each modulation operated at its optimal SNR. It was found that as distance decreases, the longest network lifetime is achieved by more spectrally efficient modulations (Figure 3 for the AWGN channel and Figure 4 for fast fading Rayleigh channel). It is apparent that, regardless of the channel type, lifetime extensions up to 600% can be gained in short range networks by selecting modulations with larger constellations than BPSK.

V. CONCLUSIONS

We studied the optimization of the SNR and modulation size in order to minimize the energy consumed by a transceiver for delivering one bit of data without error. In our study we considered different transmission distances and various

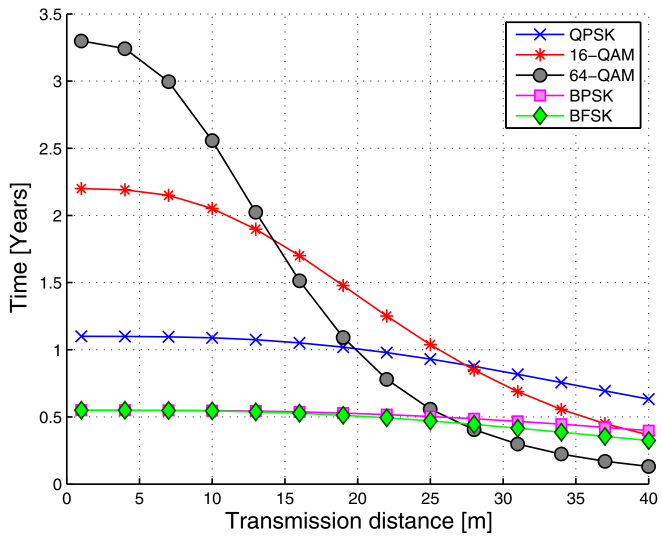

Fig. 3. Lifetime of two wireless sensor nodes that exchange 10 kbits of payload data every five minutes over an AWGN channel. At short transmission distances, BPSK yields a shorter lifetime than higher order modulations.

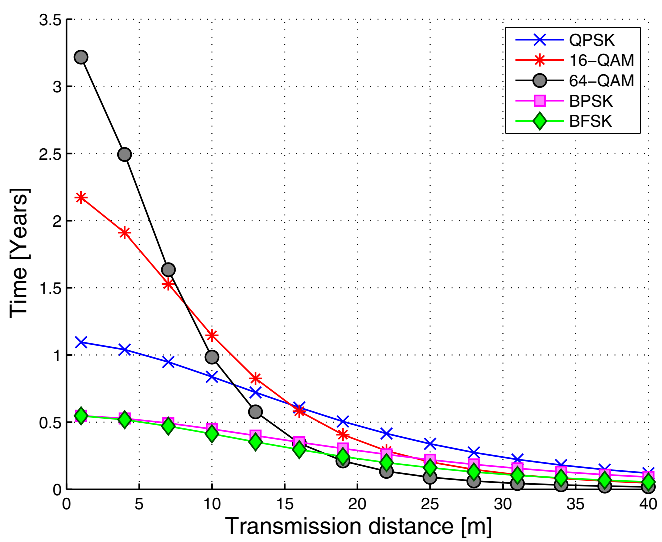

Fig. 4. Same as in Figure 3 but for the fast fading Rayleigh channel case. It is to be noted that the optimality of more spectrally efficient modulations at short distances holds.

channel statistics, as well as the energy cost of retransmissions, feedback frames and the consumption of electronic components.

We found that for a given modulation scheme the average energy consumed per bit by transmissions over a fast fading channel as function of the SNR has a unique minimum value, which is obtained at an SNR which is optimal in the energy consumption sense. The parameters that influence this optimal SNR are the mean symbol error rate, the number of payload symbols per transmission frame and the ratio between the power consumption of electronic components versus the irradiated power.

We prove that transferring successfully one frame of data across a fading channel in which the SNR levels of any two frame transmission trials are statistically independent takes, on the average, fewer transmission attempts than doing it over static channels.

We also found that for long transmission distances, low bandwidth efficiency modulations (small M-ary number, like BPSK) are optimal in the energy consumption sense. As the transmission distance shortens the optimal modulation size grows. In short range communications the power consumed by electronic components dominates over the irradiated power, and hence also does so over the energy consumption of the power amplifier. Under these conditions the average air time spent per data bit becomes a relevant parameter in the total energy budget. This makes optimal to pack more bits into each symbol and thereby to chose a larger modulation size. Finally, our results show that lifetime extensions up to 600% can be gained in short range networks by selecting modulations with larger constellations than BPSK.

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II

Nakagami-m approximations for MIMO SVD transmissions

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Nakagami-m approximations for MIMO SVD transmissions

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***Abstract*—MIMO SVD modulation is an efficient way of sending data through a multi-antenna communication link in which the transmitter has knowledge of the channel state. Despite its importance, there is no simple formula available for its symbol error rate (SER) and therefore no intuitive characterization of the quality of this technique for data transmission. In this paper, we show that the statistics of each eigenchannel of MIMO SVD transmissions can be effectively approximated using the Nakagami-m fading model. We show that the SER of the entire MIMO SVD link can be approximated by the average of the SER of Nakagami-m channels. The expression found is simple and provides intuition about the quality of the channel for data transmission. We also establish upper and lower bounds for the SER of MIMO SVD, which allows us to estimate the diversity degree of the MIMO SVD transmission. Finally, we also present an approximation for the SER of MIMO SVD communications with channel coding.**

I. Introduction

Multiple-input multiple-output (MIMO) wireless channels are created by the use of multiple antennas in both transmitter and receiver. The most popular MIMO channel model used to study narrow-band communications through fast fading scenarios is the *I.i.d. Rayleigh MIMO Fading Model* [1]. This model assumes that the received signal vector $\vec{v} = (v_1, \ldots, v_{N_r})^t$ can be expressed in terms of the transmitted symbol vector $\vec{u} = (u_1, \ldots, u_{N_t})^t$ as

$$\vec{v} = H\vec{u} + \vec{w} \tag{1}$$

where u_j is the complex symbol transmitted through the j-th antenna, v_i is the complex symbol received by the i-th antenna, H is a random matrix with coefficients $h_{i,j}$ which are i.i.d. standard complex normal random variables, and w_j are the thermal noise experimented in the j-th receiver antenna which distribute as i.i.d. zero mean complex normal variables with variance $\sigma_{\mathrm{n}}^2 = N_0$.

The MIMO SVD modulation is widely known as an efficient way of sending data through a MIMO channel like (1) when the transmitter has knowledge of the channel state [1]. Using this procedure the matrix channel is diagonalized creating $N = \min\{N_{\mathrm{t}}, N_{\mathrm{r}}\}$ non interfering channels (*eigenchannels* in the following), which can be described as

$$y_k = \sqrt{\lambda_k} x_k + n_k \qquad k = 1 \ldots N \tag{2}$$

where x_k are the transmitted symbols, n_k is the thermal noise and $\sqrt{\lambda_k}$ are the singular values of the matrix H [2].

Despite the importance and popularity of the MIMO SVD modulation, no simple formula for the symbol error rate (SER) of the eigenchannels has been reported yet, even though the topic has seen much recent activity [3]–[6]. The expressions for the SER reported in these articles are quite complex and do not give an insight about the quality of each eigenchannel for transmitting information. The common approach for studying the statistics of the eigenchannels is to consider λ_k as eigenvalues of the matrix $W = HH^\dagger$. W is a complex Wishart matrix [7], whose eigenvalues have a joint probability distribution (p.d.f.) given by [8]

$$p = K_N \exp\left(-\sum_{k=1}^{N} \lambda_k\right) \prod_{i=1}^{N} \lambda_i^{|N_{\mathrm{t}} - N_{\mathrm{r}}|} \prod_{i>j}^{N} (\lambda_i - \lambda_j)^2 \tag{3}$$

with $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$ and K_N a constant. Deriving the statistics of each eigenchannel requires then to determinate the exact marginal p.d.f. $p_{\lambda_k}(\lambda_k)$ of each eigenvalue from (3).

In [8], it was shown that the SNR of the smallest eigenchannel of a square MIMO channel have the same statistics as a Rayleigh channel with power gain $1/N$ (i.e. $p_{\lambda_1}(\lambda_1) = Ne^{-N\lambda_1}$). Although expressions for the marginals of the other eigenvalues have been found [6], [9]–[12], they are mathematically complex and do not provide much insight about the performance of the corresponding eigenchannels. In [13], it was shown that in the high SNR regime the SER of each eigenchannel can be expressed as

$$\bar{P}_s(\bar{\gamma}) = (G_c \bar{\gamma})^{-G_d} + o(\bar{\gamma}^{-G_d}) \ , \tag{4}$$

where G_c is the power gain of the channel and G_d represents its diversity degree [14]. The limitation of this result is that the high-SNR restriction leads to insights of little practical interest. For example, (4) only gives accurate information of the SER of the largest eigenchannel of a 4×4 MIMO channel for $\bar{P}_s(\bar{\gamma}) < 10^{-8}$. In [15], the idea of approximating the eigenchannel of the largest eigenvalue of a MIMO channel by a Nakagami-m channel is presented. The value of m is choose in order to approximate the outage statistics of this eigenchannel. Although the approximation thus obtained is

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accurate, it is not obvious if the proposed method could be extended to other eigenchannels.

In this paper we show that the statistics of all the eigenchannels of MIMO SVD transmissions through a MIMO channel with i.i.d. Rayleigh statistics can be approximated using the Nakagami-m fading model. These approximations allow us to derive an accurate approximation for the SER of the entire MIMO SVD link, one that provides a strong insight on the performance of each eigenchannel.

The rest of the paper is organized as follows: Section II explains how Nakagami-m approximations can be calculated for each eigenchannel of a MIMO system. In Section III, a simple expression for the SER of MIMO SVD transmissions is derived using the Nakagami-m approximations. Section IV presents our conclusions.

II. Statistical analysis of the MIMO eigenchannels

For clarity of the exposition we focus on a MIMO communication system with equal number of transmit and receive antennas, denoted by N. However, extending the ideas to MIMO systems in which $N_{\mathrm{r}} \neq N_{\mathrm{t}}$ is straightforward. Lets denote by λ_k the k-th smallest eigenvalue of a $N \times N$ Wishart matrix $HH^{\dagger}$, which correspond to the channel coefficients of the eigenchannel of a MIMO SVD transmission through a MIMO channel H with i.i.d. Rayleigh statistics.

We seek to approximate the fading statistics of each eigenchannel using the well known Nagakami-m channel model [16], [17]. The power gain of the Nakagami-m channel, denoted here by λ, is a Gamma random variable with p.d.f. given by [16]

$$p_{\lambda}(\lambda) = \left(\frac{m}{\mu}\right)^{m} \frac{\lambda^{m-1}}{\Gamma(m)} \exp\left(-\frac{m\lambda}{\mu}\right) \quad . \tag{5}$$

The Gamma distribution is characterized by the *mean power gain* of the channel μ (as $\mathbb{E}\{\lambda\} = \mu$), and the *diversity degree* of the channel m (m is equal to the absolute value of the asymptotic logarithmic slope of the SER, i.e. $p_s \propto \mathrm{SNR}^{-m}$ for the high SNR regime) [17]. The diversity degree m is related to outage statistics [14]. In effect, when $m = 1$ (5) turns into the p.d.f. of the exponential distribution, which corresponds to a Rayleigh channel, well known for having the most adverse outage statistics. When $m \to \infty$ the channel tends to the AWGN channel, which has no outage at all.

For a given MIMO size N we seek to find parameters μ_k and m_k, with $k \in \{1 \dots N\}$, for which a Gamma random variable with these parameters best fits the p.d.f. of the k-th eigenvalue λ_k. For a 2×2 MIMO system these parameters can be determined by minimizing the mean square error between the marginal distributions found from (3) and a Gamma distribution (the actual calculation is presented in the Appendix). When $N > 2$ that analytic minimization lead into intractable mathematics (c.f. [6], [12]), and hence other kind of analysis is needed.

In our approach, we propose to find values for m_k and μ_k by performing a maximum likelihood estimation (MLE) using computer generated samples of the eigenvalues. The MLE procedure finds the parameters m_k and μ_k of the Gamma distribution that has the larger probability of generating random numbers with the statistics observed in the given sample [18]. It has been shown that the Maximum Likelihood principle for the Gamma distribution is equivalent to selecting μ_k and m_k as [19]

$$\mu_k = \frac{1}{n_0} \sum_{j=1}^{n_0} x_j \tag{6}$$

$$\ln(m_k) - \psi(m_k) = \ln\left(\frac{1}{n_0} \sum_{j=1}^{n_0} x_j\right) - \frac{1}{n_0} \sum_{j=1}^{n_0} \ln(x_j) \tag{7}$$

where $\{x_j\}_{j=1}^{n_0}$ is a sample of n_0 realizations of the k-th eigenvalue and $\psi(x) = \Gamma'(x)/\Gamma(x)$ is the Psi (digamma) function [20]. Although (6) is the classic estimator of the mean value of a population, (7) gives a nontrivial estimator to its diversity degree. For a 2×2 MIMO system, the MLE method over samples of size $n_0 = 10^6$ of both eigenvalues λ_1 and λ_2 gives values for m_1, m_2, μ_1 and μ_2 with less than 0.1% of error compared to the values found by analytic optimization in the Appendix (Table I).

For larger MIMO systems we have calculated the values for μ_k and m_k using samples of size $n_0 = 10^6$ (Table I). It is worth noting that these numbers provide the complete characterization of the Nakagami-m approximation of any MIMO systems of the same sizes and Rayleigh fading statistics.

TABLE I
Values of the Nakagami-m distribution parameters for MIMO SVD eigenvalue statistics approximation

2×2 MIMO System		
Eigenvalue	m	μ
λ_2	3.82	3.5
λ_1	1	.5
4×4 MIMO System		
Eigenvalue	m	μ
λ_4	12.72	9.77
λ_3	8.66	4.41
λ_2	4.09	1.57
λ_1	1	.25
8×8 MIMO System		
Eigenvalue	m	μ
λ_8	38	23.73
λ_7	37.66	15.85
λ_6	31.32	10.63
λ_5	23.52	6.82
λ_4	15.89	4.02
λ_3	9.26	2.05
λ_2	4.15	.76
λ_1	1	.125

III. Symbol error rate approximations

In the sequel we derive an approximation for the average SER of uncoded MIMO SVD communications using the method of Nakagami-m approximations presented in Section II. The approximation holds for a large family of commonly used modulations. We also present upper and lower bounds for the SER, with which the diversity degree of the MIMO SVD transmission can be estimated. Finally, we show an application of these approximations for coded communications.

A. SER approximation

Lets assume that the eigenchannels described by (2) are used to send data using a modulation whose SER for a given SNR γ can be express as

$$P_{\mathrm{s}} = cQ(\sqrt{a\gamma}) \ . \tag{8}$$

Above, $Q(x)$ is the tail probability of the standard normal distribution and c and a are constants that depend on the choice of modulation scheme and constellation size. The SER of such a modulation over a Nakagami-m channel with power gain λ is calculated as

$$\begin{aligned}\bar{P}_{\mathrm{s}}(\bar{\gamma}, \mu, m) &= \mathbb{E}\{cQ(\sqrt{a\lambda\bar{\gamma}})\} &(9)\\ &= \int_0^\infty cQ(\sqrt{a\lambda\bar{\gamma}}) p_\lambda(\lambda, \mu, m)\mathrm{d}\lambda \ , &(10)\end{aligned}$$

where λ is a Gamma random variable with p.d.f. $p_\lambda(\lambda, \mu, m)$ as given in (5).

Consider one realization of the eigenvalues $\lambda_1, \ldots, \lambda_N$. Then, the SNR of the eigenchannels given by (2) are given by

$$\mathrm{SNR}_k = \lambda_k \bar{\gamma}_k \ . \tag{11}$$

Lets assume for simplicity of notation that all the eigenchannels share the same $\bar{\gamma}_k = \bar{\gamma}$. Then lets define as A_k the event in which the transmitted symbol was send through the k-th eigenchannel, and E the event that the symbol were decoded with error. Assuming that the N eigenchannels are equally used then the probability of A_k is given by $\mathbb{P}(A_k) = 1/N \quad \forall k = 1 \ldots N$. As the A_k are jointly exhaustive, we can decompose the error event E as $E = \cup_{k=1}^N (E \cap A_k)$, where each event $E \cap A_k$ denotes the possibility that the error has occurred in the k-th eigenchannel. Therefore the symbol error rate (P_{s}) can be calculated as:

$$\begin{aligned}P_{\mathrm{s}}(\bar{\gamma}, \lambda_1 \ldots \lambda_N) &= \mathbb{P}(E|\bar{\gamma}, \lambda_1 \ldots \lambda_N) &(12)\\ &= \mathbb{P}\left(\cup_{k=1}^N (E \cap A_k)|\bar{\gamma}, \lambda_1 \ldots \lambda_N\right) &(13)\\ &= \sum_{k=1}^N \mathbb{P}\left(E \cap A_k|\bar{\gamma}, \lambda_k\right) &(14)\\ &= \sum_{k=1}^N \mathbb{P}\left(A_k\right)\mathbb{P}\left(E|A_k, \bar{\gamma}, \lambda_k\right) &(15)\\ &= \frac{1}{N}\sum_{k=1}^N cQ\left(\sqrt{a\lambda_k\bar{\gamma}}\right) &(16)\end{aligned}$$

The equality from (13) to (14) follows because if $k \neq j$ the events $E \cup A_k$ and $E \cup A_j$ are disjoint, and $E \cup A_k$ is independent of λ_j; the equality from (14) to (15) follows from the definition of conditional probability and from the fact that A_k is independent of $\bar{\gamma}$ and of λ_k. The step from (15) into (16) considers the following rationale: as the values of $\lambda_1 \ldots \lambda_N$ are given, the randomness that influences on the errors is only the thermal noise. Because data symbols and noise in each eigenchannel are uncorrelated, the demodulation of each symbol is an independent event.

Now assume that the eigenvalues are random with their joint p.d.f. given by (3). Using the Nakagami-m approximations described in Section II, the mean SER can be approximated, using by (16), as:

$$\begin{aligned}\bar{P}_{\mathrm{s}}(\bar{\gamma}) &= \mathbb{E}\left\{P_{\mathrm{s}}(\bar{\gamma}, \lambda_1 \ldots \lambda_N)\right\} &(17)\\ &= \frac{1}{N}\sum_{k=1}^N \mathbb{E}\left\{cQ\left(\sqrt{a\lambda_k\bar{\gamma}}\right)\right\} &(18)\\ &\approx \frac{1}{N}\sum_{k=1}^N \bar{P}_{\mathrm{s}}(\bar{\gamma}, \mu_k, m_k) \ , &(19)\end{aligned}$$

where $\bar{P}_{\mathrm{s}}(\bar{\gamma}, \mu_k, m_k)$ denotes the mean SER of a Nagakami-m channel as given by (10) and m_k and μ_k are parameters determined by the method explained in Section II.

It is to be noticed that the above result can be generalized in the following way. Although a full range $N \times N$ MIMO system provides N parallel channels there is no need to use them all. As eigenchannels with smaller eigenvalues have worse statistics, a relevant reduction of the mean SER can be achieved by transmitting data only through the best $n < N$ eigenchannels, at the expense of lowering the data rate. Furthermore, if unequal power allocation among the n eigenchannels is used, then an analogous deduction allows to express the mean SER as

$$\bar{P}_{\mathrm{s}}(\bar{\gamma}_1, \ldots, \bar{\gamma}_n) \approx \frac{1}{n}\sum_{k=N-n}^N \bar{P}_{\mathrm{s}}(\bar{\gamma}_k, \mu_k, m_k) \ . \tag{20}$$

The accuracy of this approximation has been tested by computer simulation of 10^6 BPSK symbols through a $N \times N$ MIMO system, for various values of N. For each transmitted symbol vector, an independent narrow-band MIMO channel matrix H was generated following the MIMO channel i.i.d. Rayleigh fading model. After SVD decomposition of each realization H, statistics were calculated for transmissions that use $n = 1$ (MIMO beamforming), $n = 2$, $n = 3$ and $n = 4$ (full SVD). We allocated equal power to the used eigenchannels. Each run of 10^6 symbols was evaluated at various SNR levels. The results show that the approximation proposed in (20) is accurate for any choice of n (Figure 1).

B. SER bounds

For equal power allocation, the weakest eigenchannel mean SER will be the worst among all the used eigenchannels, so it SER will be an upper bound to the average SER. For finding a lower bound, it is enough to realize that (20) is a sum of positive terms. Therefore, any of them is smaller than the sum. Thus, the right side of (20) can be upper and lower bounded as follows

$$\bar{P}_{\mathrm{s}}(\bar{\gamma}, \mu_n, m_n) \geq \bar{P}_{\mathrm{s}}(\bar{\gamma}, n) \geq \frac{1}{n}\bar{P}_{\mathrm{s}}(\bar{\gamma}, \mu_n, m_n) \ , \tag{21}$$

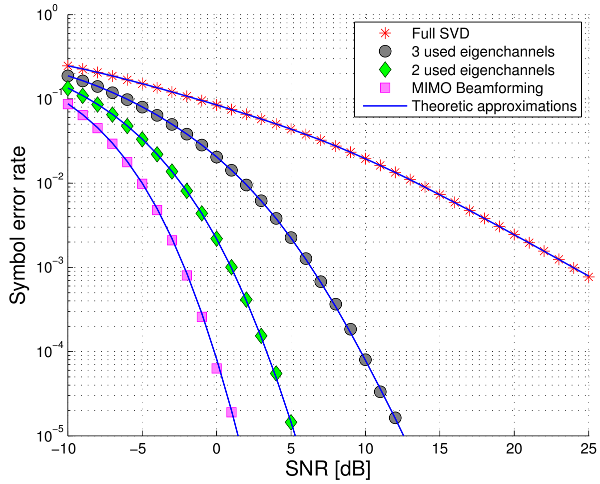

Fig. 1. Average SER of MIMO SVD transmissions using uncoded BPSK using 1, 2, 3 or 4 eigenchannels of the SVD modulation. Markers show the average SER obtained by computer simulation of 10^6 symbols, and solid lines show the corresponding proposed approximation (20). An increasing diversity gain (i.e. the slope of the SER in the high SNR regime) can be observed as the number of used eigenchannels is reduced.

where $\bar{P}_s(\bar{\gamma}, n)$ equals (20) with $\bar{\gamma}_j = \bar{\gamma} \quad \forall j = 1 \dots n$ and $n < N$.

Simulations show that these bounds are not only valid for the approximated SER $\bar{P}_s(\bar{\gamma}, n)$, but also for the exact MIMO SVD SER (Figure 2). The lower bound becomes exact as SNR$\rightarrow \infty$, because the error rate of better eigenchannels tend to zero faster than the error rate of the worst eigenchannel.

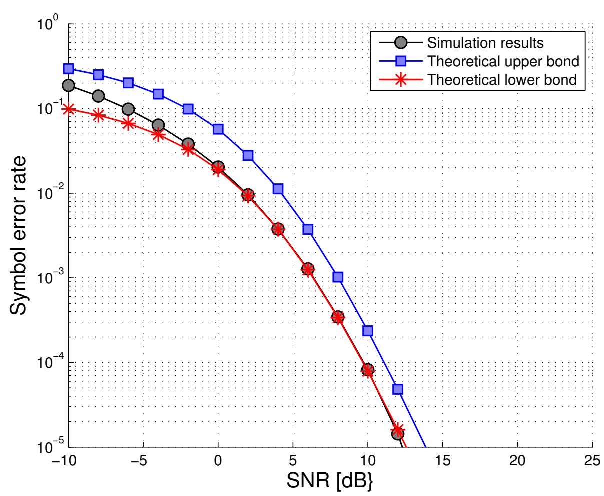

Fig. 2. The theoretical bounds presented in (21) effectively enclose the statistics obtained by computer simulations of MIMO SVD transmissions using BPSK through the three best eigenchannels. It can be seen that in the high-SNR regime the lower bound become exact.

The lower bound presented in (21) can be used to estimate the diversity degree [14] of an uncoded MIMO SVD transmissions with $n \leq N$ eigenchannels in use. In the high SNR regime $\bar{P}_s \approx n^{-1}\bar{P}_s(\mu_n\bar{\gamma}, m_n)$, as can be seen in Figure 2. It is thus apparent that the diversity degree of the n-th eigenchannel, m_n, approximates the diversity gain of this MIMO SVD link. As a consequence, full SVD transmissions ($n = N$) always have diversity degree $m_1 = 1$, similar to the Rayleigh channel. On the other hand, beamforming MIMO (with $n = 1$, also known as MIMO MRC [21]) has diversity degree m_N, which grows with N. This implies that as the array size (N) grows, the SER statistics of beamforming MIMO tend to those of an AWGN channel.

C. Application to coded transmissions

In the sequel we illustrate how the work presented so far in this chapter can be used to find a closed form formula for the SER of channel-coded MIMO communications, which uses a $(\hat{n}, k)$ maximum distance separable (MDS) linear block code to perform forward error correction (FEC) [22]. Using a transceiver architecture as the one shown in Figure 3, the $(\hat{n}, k)$ MDS linear FEC code is able to correct $t = \lfloor (\hat{n} - k)/2 \rfloor$ bit errors per block, where $\lfloor x \rfloor$ denotes the largest integer smaller than x. In [23], a SER formula for linear block codes is found in terms of the uncoded SER of the underlying modulation. By plugging in (21) for the uncoded SER, the SER of the coded MIMO SVD system is given by

$$\bar{P}_s^{(\hat{n},k)}(\bar{\gamma}, n) \approx \frac{1}{\hat{n}} \sum_{j=t+1}^{\hat{n}} j \binom{\hat{n}}{j} [\bar{P}_s(\bar{\gamma}, n)]^j [1 - \bar{P}_s(\bar{\gamma}, n)]^{\hat{n}-j} \ . \tag{22}$$

(22) can be computed numerically, and can be used to study analytically the dependance of the SER of the coded system on the number of eigenchannels used (n) and on the code parameters $\hat{n}$ and k.

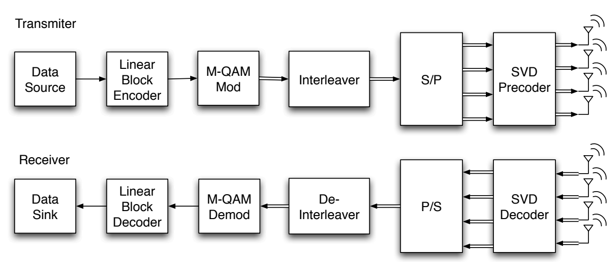

Fig. 3. Architecture of a coded MIMO SVD transceiver.

IV. Conclusions

We have developed a method for approximating the statistics of the eigenchannels of uncoded MIMO SVD communications using the Nakagami-m fading model. Maximum Likelihood Estimation of Nakagami-m channel parameters is performed for each of the eigenchannels of the MIMO system. The accuracy of the results was shown analytically for the 2×2 case and by simulations for larger MIMO system sizes.

The method was used as an starting point for deriving an approximation of the mean SER of MIMO SVD transmissions. The approximation found is simple, accurate, easily computable and provides intuition about the quality of the channel for data transmission.

Furthermore we have presented an upper and lower bound for the SER of MIMO SVD based on the SER of the weakest eigenchannel used for the transmission. These bounds allow quick calculations, and shows that the diversity degree of the MIMO channel can be approximated by the diversity degree of the weakest eigenchannel in use.

Finally, use of our results in coded MIMO SVD communications was illustrated for linear FEC block codes.

Appendix

The parameters of a Nakagami-m approximation of a 2×2 MIMO system can be found as follows. From [8] we know that the smallest eigenvalue of such a system has a p.d.f. given by $\lambda_1 \sim \exp(1/2)$, which implies that $m_1 = 1$ and $\mu_1 = 1/2$. On the other hand, the probability distribution of the largest eigenvalue of a $N \times N$ MIMO system can be expressed as [24]

$$p_{\lambda_N}(\lambda_N) = \left(\prod_{j=1}^{N} \frac{1}{(N-j)!} \right)^2 \frac{d}{d\lambda_2} \det\{S(\lambda_2)\} \qquad (23)$$

where $S(\lambda_2)$ is a matrix with coefficients given by $[S(\lambda_2)]_{i,j} = \int_0^{\lambda_2} x^{i+j-2} e^{-x} \mathrm{d}x$. Following [25] we find that for $N = 2$, (23) can be simplified to obtain

$$p_{\lambda_2}(\lambda_2) = \left[(\lambda_2)^2 - 2\lambda_2 + 2 - 2e^{-\lambda_2}\right] e^{-\lambda_2} \qquad (24)$$

By defining $g_{m,\theta}(x)$ to be a p.d.f. of a Gamma random variable as defined in (5), parametrized on m and $\theta = \mu/m$, the mean square error between p_{λ_2} and $g_{m,\theta}$ can be expressed in terms of m and θ as

$$\int_0^{\infty} |p_{\lambda_2}(\lambda_2) - g_{m,\theta}(\lambda_2)|^2 \mathrm{d}\lambda_2 = 2m\theta \frac{2 + \theta - m\theta}{(\theta+1)^{m+2}} \cdots + \frac{4}{(2\theta+1)^m} - \frac{4}{(\theta+1)^m} + \frac{\Gamma(2m-1)}{2^{2m-1}\theta[\Gamma(m)]^2} + \frac{19}{108} \ . \qquad (25)$$

This expression can be minimized on m and θ using numerical methods, finding a minimum in $m = 3.82$ and $\theta = 0.911$, which gives $\mu = k\theta = 3.48$. The mean square error at the minimum is $\approx 6 \times 10^{-6}$, which shows that the two distributions p_{λ_2} and $g_{m,\theta}$ are identical for any practical purpose.

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III

Energy-efficient MIMO SVD communications

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Energy-efficient MIMO SVD communications

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***Abstract*—Multiple-input multiple-output (MIMO) techniques can be used for reducing the energy consumption of wireless communications. Although some research has been reported on this topic, the rules by which the MIMO physical layer parameters should be chosen in order to achieve energy efficiency have not yet been formally established. In this paper, we analyze the case of MIMO singular value decomposition (SVD) technique. We present a model for the mean energy consumption of a MIMO SVD system per data bit transferred without error.**

We find that, for a given number of eigenchannels used with equal power allocation, exists a single optimal radiation power level at which the mean energy consumption is minimized. We also find that beamforming (only the best eigenchannel is used) is optimal in the energy consumption sense for long transmission distances, while the optimal number of eigenchannels to be used grows as transmission distance shortens. Using all the eigenchannels is optimal only for very short transmission distances.

I. Introduction

Multiple-input multiple-output (MIMO) communication systems were originally introduced as a way for achieving higher data rates or for improving the reliability of wireless links [1]. More recently, researchers have started to realize that the MIMO techniques can also be used for reducing the energy consumption of wireless communications [2]–[11]. Despite this recent interest, the rules by which the main physical-layer parameters shall be chosen for attaining energy efficiency in the MIMO system have not yet been formally established [12].

The MIMO SVD technique is a well known method for sending data through a MIMO communication link in which the transmitter has knowledge of the channel state [13]. The core concept considers the diagonalization of the channel H, which we will assume to be a $N \times N$ matrix of full rank. The diagonalization establishes N non interfering channels (henceforth *eigenchannels*), whose input-output relationships can be described as

$$y_k = \sqrt{\lambda_k} x_k + w_k \qquad k = 1 \dots N \ . \tag{1}$$

Above, k indexes the eigenchannels, x_k are the transmitted symbols, w_k are additive white Gaussian noise terms (AWGN) and $\sqrt{\lambda_k}$ are the singular values of the channel matrix H [14]. It is worth mentioning that although the MIMO SVD modulation provides N eigenchannels, there is no need to use them all. Using all the eigenchannels maximizes the data rate, but sacrifices symbol error rate (SER). Conversely, using only the $n < N$ eigenchannels with most favorable fading statistics yields a better SER but at the cost of decreasing the data-rate [15].

In this paper we develop a formal model for analyzing the energy consumption of MIMO SVD communications. Most of the existent models reported so far in the literature [5]–[11] are based on the abstract definition of the capacity of a MIMO random fading channel. These models are not adequate for determining attainable performances of concrete modulations with a specific number of eigenchannels used. Our model incorporate these elements. It shares some features with the one reported in [2], but our model finds a more straightforward mathematical formulation. Our approach to analize the energy consumption of MIMO systems is also novel for considering the effect of retransmissions required to guaranteing error free transmissions.

Our model allows for optimizing the radiated power and the number of eigenchannels ($n \leq N$) used for the transmission as a function of link distance. In effect, we show that beamforming ($n = 1$) is the energy-optimal transmission strategy for large transmission distances, while for short transmission distances a larger number of eigenchannels is optimal in the energy sense. Full SVD ($n = N$) is optimal only for very short link distances.

The paper is organized as follows: Section II presents the energy consumption model, Section III specifies the dependence of the energy consumption on the signal-to-noise ration (SNR), and Section IV presents an analysis of the energy consumption of a MIMO SVD system using various numbers of used eigenchannels and link distances. Finally, Section V presents our conclusions.

II. Energy consumption model

Our goal is to determine the total energy that is necessary for transferring one bit of data successfully, henceforth called a *goodbit* as in [7], in a point-to-point packet-switched MIMO SVD communication. We assume that every frame transmitted in the *forward* direction is matched by a feedback frame in the *reverse* direction that acknowledges correct reception or requests a re-transmission. We also assume that the irradiated power is determined based upon knowledge of the statistics of the signal-to-noise ratio (SNR) at the decision stage of the receiver. We further assume that all frames in both directions are always detected and that all feedback frames are decoded without error.

The energy consumption analysis has been made for a specific MIMO transceiver architecture, popular among academic [16], [17] and commercial [18] products (Figure 1).

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In the sequel, we present the analysis of the components of energy consumption of the MIMO SVD transceiver from the standpoint of a node that transmits one payload frame and receives the corresponding feedback frame (the reverse case —a transceiver that receives one payload frame and transmits the corresponding feedback frame— follows by analogy), followed by the analysis of the statistics of re-transmissions and finally by a synthesis of our total energy consumption model.

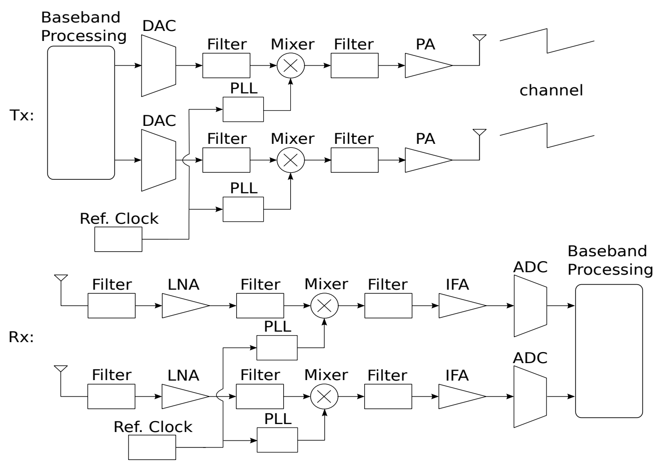

Fig. 1. Common architecture of a MIMO SVD transceiver.

A. Components of Energy Consumption of the Forward Transceiver

The energy consumption of the MIMO SVD transceiver that transmits forward frames and receives feedback frames is composed by three terms, each one described below.

1) RF electronic consumption: The total air time per forward frame is composed by T_L seconds used for the transmission of the L bits that compose the frame, and T_O seconds used for the transmission of overhead signals for tasks such as acquisition, channel estimation, synchronization, frame parameters signaling, etc. The air time per bit is therefore

$$\hat{T}_\mathrm{b} = \frac{T_L + T_\mathrm{O}}{L} \ . \tag{2}$$

T_O is composed by overhead for acquisition, which depends linearly on the number of transmitter antennas, and tasks like synchronization, which are approximately independent of N. By noting that $L/T_\mathrm{L} = nbR_s$ is the total bit-rate of the MIMO system, where n is the number of used eigenchannels (cf. Section I), R_s is the symbol-rate of each eigenchannel and b is the number of bits modulated in each symbol, we may express $\hat{T}_\mathrm{b}$ as

$$\hat{T}_b = \frac{1}{R_\mathrm{s}}\left(\frac{1}{nb} + \frac{NO_\mathrm{a} + O_\mathrm{b}}{L}\right) \ , \tag{3}$$

where O_a is the acquisition overhead per branch and O_b is the remaining overhead, each one measured in bits.

During the $\hat{T}_b$ seconds, the MIMO transceiver consumes $\hat{P}_\mathrm{el,tx}$ Watts, which is largely dominated by the consumption of passband processing components such as filters, mixers and frequency synthesizers engaged in the forward transmission [19]. It can be inferred from Figure 1 that $\hat{P}_\mathrm{el,tx}$ grows linearly with the number of antennas (N). Therefore, the energy per goodbit consumed in the transmission processing may be expressed as

$$\mathcal{E}_\mathrm{el,tx} = \hat{P}_\mathrm{el,tx}\hat{T}_\mathrm{b} = NP_\mathrm{el,tx}\frac{1}{R_\mathrm{s}}\left(\frac{1}{nb} + \frac{NO_\mathrm{a} + O_\mathrm{b}}{L}\right) \ , \tag{4}$$

where $P_\mathrm{el,tx}$ stands for the electric power consumed by each branch of the transceiver.

2) Energy Consumption due to Electromagnetic Radiation: Each frame is aired through the N branches with a transmission power $P_\mathrm{A}^{(j)}$ irradiated by the j-th antenna, supplied by corresponding power amplifier (PA) (Figure 1). The power consumption of the j-th PA, $P_\mathrm{PA}^{(j)}$, is modeled by

$$P_\mathrm{PA}^{(j)} = \frac{\xi}{\eta}P_\mathrm{A}^{(j)} \ , \tag{5}$$

where ξ is the peak-to-average ratio of the transmitted signal and η is the drain efficiency of the PA [19]. Thus, the energy per bit used for electromagnetic radiation is given by

$$\mathcal{E}_\mathrm{RF} = \left(\sum_{j=1}^{N} P_\mathrm{PA}^{(j)}\right)\hat{T}_\mathrm{b} = \hat{P}_\mathrm{PA}\hat{T}_\mathrm{b} \ , \tag{6}$$

where $\hat{T}_\mathrm{b}$ is given by (3), and we have defined $\hat{P}_\mathrm{PA}$ as the total power consumption of all the PA's.

3) Energy Consumption of Electronic Components due to the Processing of Feedback Frames: Feedback frames are assumed to last $\hat{T}_\mathrm{fb} = F/(nbR_\mathrm{s})$ seconds, where F is the number of bits that compose the feedback frame. During that time, the MIMO receiver consumes $\hat{P}_\mathrm{el,rx}$ Watts, which mainly includes the power needed to energize the passband receiver elements (low-noise amplifiers, mixers, filters, frequency synthesizers, etc.) of all the branches [19]. Hence, it grows linearly with the number of antennas (N). Therefore, the energy per forward payload bit spent by the transmitter for decoding the corresponding feedback frame is

$$\mathcal{E}_\mathrm{fb,rx} = \hat{P}_\mathrm{el,rx}\frac{\hat{T}_\mathrm{fb}}{L} = NP_\mathrm{el,rx}\frac{F}{nbR_\mathrm{s}L} \ , \tag{7}$$

where $P_\mathrm{el,rx}$ is the electronic power consumption of one branch of the transceiver.

B. Re-transmission Statistics

A key contributor to the energy consumption is the need for re-transmissions due to forward frames that get decoded with errors at the receiver. The number of trials, τ, until a frame is decoded without error is a random variable, whose mean value has been shown to be [20]

$$\bar{\tau} = 1 + \sum_{r=1}^{\infty}\mathbb{E}\left\{\prod_{i=1}^{r} P_\mathrm{f}(i)\right\} \ , \tag{8}$$

where $\mathbb{E}\{\cdot\}$ denotes the expectation operator and $P_\mathrm{f}(i)$ is the probability of decoding the frame with error during the i-th transmission trial. In general, the $P_\mathrm{f}(i)$ are random variables that depend on the number of antennas, the frame size, modulation type and received SNR during the i-th trial. It is to be noted that (8) is valid for correlated or uncorrelated channel fading statistics [20].

C. Total Energy per Goodbit

The discussion in Sections II-A and II-B allows for stating our model of the total energy consumption. Concretely, the energy consumed by the transmitter of forward frames per goodbit, which also decodes feedback frames, is given by

$$\mathcal{E}_{\mathrm{T}} = (\mathcal{E}_{\mathrm{el,tx}} + \mathcal{E}_{\mathrm{RF}} + \mathcal{E}_{\mathrm{fb,rx}})\tau \tag{9}$$

$$= \left[(\hat{P}_{\mathrm{el,tx}} + \hat{P}_{\mathrm{PA}})\hat{T}_{\mathrm{b}} + \frac{\hat{P}_{\mathrm{el,rx}}\hat{T}_{\mathrm{fb}}}{L}\right]\tau \ . \tag{10}$$

By analogy, the total energy used by the receiver for demodulating τ forward transmissions and for transmitting the corresponding τ feedback frames, is

$$\mathcal{E}_{\mathrm{R}} = \left[\hat{P}_{\mathrm{el,rx}}\hat{T}_{\mathrm{b}} + \left(\hat{P}_{\mathrm{el,tx}} + \hat{P}_{\mathrm{PA}}\right)\frac{\hat{T}_{\mathrm{fb}}}{L}\right]\tau \ . \tag{11}$$

The total energy consumption per goodbit is the sum of (10) and (11):

$$\mathcal{E}_{\mathrm{b}} = \ (\hat{P}_{\mathrm{el,tx}} + \hat{P}_{\mathrm{PA}} + \hat{P}_{\mathrm{el,rx}})\left(\hat{T}_{\mathrm{b}} + \frac{\hat{T}_{\mathrm{fb}}}{L}\right)\tau \tag{12}$$

$$= \ (\hat{P}_{\mathrm{el}} + \hat{P}_{\mathrm{PA}})\hat{T}\tau \ , \tag{13}$$

where we have defined $\hat{P}_{\mathrm{el}} = \hat{P}_{\mathrm{el,tx}} + \hat{P}_{\mathrm{el,rx}}$ as the total power consumed by electronic components and $\hat{T} = \hat{T}_{\mathrm{b}} + \hat{T}_{\mathrm{fb}}/L$ the total time per bit per transmission trial.

Because of τ, $\mathcal{E}_{\mathrm{b}}$ is a random variable that depends on the realizations of the channel and the thermal noise. Its mean value is

$$\bar{\mathcal{E}}_{\mathrm{b}} = \mathbb{E}\left\{\mathcal{E}_{\mathrm{b}}\right\} = \left(\hat{P}_{\mathrm{el}} + \hat{P}_{\mathrm{PA}}\right)\hat{T}\bar{\tau} \ , \tag{14}$$

with $\bar{\tau}$ as given by (8).

III. Energy consumption as function of the SNR

We seek an explicit expression for the dependence on the SNR of the energy consumption per goodbit of a MIMO SVD system which only uses the n eigenchannels with the most favorable fading statistics. To achieve this, we first analyze the PA's total power consumption ($\hat{P}_{\mathrm{PA}}$) and the mean number of transmission trials ($\bar{\tau}$), followed by the actual formulation of the relationship between $\bar{\mathcal{E}}_b$ an the SNR.

A. PA's total power consumption as function of the SNR

The transmission power that has been allocated to the k-th eigenchannel, $\bar{P}_{\mathrm{tx}}^{(k)}$ ($k \in \{1, \ldots, n\}$), attenuates over the air with path loss and arrives at the receiver with a mean power given by

$$\bar{P}_{\mathrm{rx}}^{(k)} = \frac{\bar{P}_{\mathrm{tx}}^{(k)}}{A_0 d^{\alpha}} \ , \tag{15}$$

where A_0 is a parameter that depends on the transmitter and receiver antenna gains and the transmission wavelength, d is the distance between transmitter and receiver and α is the path loss exponent. The SNR of the k-th eigenchannel is given by

$$\mathrm{SNR}_k = \frac{\lambda_k \bar{P}_{\mathrm{rx}}^{(k)}}{\sigma_{\mathrm{n}}^2} = \lambda_k \phi_k \bar{\gamma} \ , \tag{16}$$

where λ_k is the square of the k-th singular value of the channel matrix H (cf. (1)), σ_n^2 is the noise power, $\bar{\gamma} = (\sum_{k=1}^{n} \bar{P}_{\mathrm{rx}}^{(k)})/\sigma_{\mathrm{n}}^2$ is the total SNR (which may be produced if all the radiated power was allocated to only one eigenchannel) and $\phi_k = \bar{P}_{\mathrm{rx}}^{(k)}/(\sum_{k=1}^{n} \bar{P}_{\mathrm{rx}}^{(k)})$ is the percentage of the total SNR that goes to the k-th eigenchannel. Furthermore, we can express the noise power as $\sigma_{\mathrm{n}}^2 = N_0 W N_{\mathrm{f}} M_{\mathrm{L}}$, where N_0 is the power spectral density of the baseband-equivalent additive white Gaussian noise (AWGN), W is the transmission bandwidth, N_{f} is the noise figure of the receiver's front end and M_{L} is a link margin term which represents any other additive noise or interference [2].

Finally, using the result presented in Appendix A in addition to (15) and (16), the following relation can be found:

$$\hat{P}_{\mathrm{PA}} = \frac{\xi}{\eta}\sum_{k=1}^{n} P_{\mathrm{tx}}^{(k)} = \frac{\xi A_0 d^{\alpha}}{\eta}\sum_{k=1}^{n} P_{\mathrm{rx}}^{(k)} \tag{17}$$

$$= \frac{\xi A_0 d^{\alpha}\sigma_{\mathrm{n}}^2}{\eta}\bar{\gamma} = A d^{\alpha}\bar{\gamma} \ , \tag{18}$$

with A a constant.

B. $\bar{\tau}$ as function of the SNR

Consider the assumption that the probabilities of frame error of each transmission trial, $\{P_{\mathrm{f}}(i)\}_{i=1}^{\infty}$ (cf. (8)), are a set of i.i.d. random variables. Define their mean value as $\mathbb{E}\{P_{\mathrm{f}}\} := \mathbb{E}\{P_{\mathrm{f}}(i)\}$, where we have dropped the index i for simplicity of notation. Using these conditions on (8), it can be shown [20] that

$$\bar{\tau} = \frac{1}{1 - \mathbb{E}\{P_{\mathrm{f}}\}} \ . \tag{19}$$

An alternative derivation of (19) uses the proof presented in Appendix B, which shows that τ is a Geometric random variable with parameter $1 - \mathbb{E}\{P_{\mathrm{f}}\}$.

The value of the mean frame error rate, $\mathbb{E}\{P_{\mathrm{f}}\}$, and hence the value of $\bar{\tau}$, depends on how the data symbols are fed into the SVD engine. To illustrate this, we present the analysis of two cases that use uncoded M-ary modulation for transmitting forward frames composed of L bits, hence by $l = L/\log_2(M)$ M-ary symbols. (The possibility of using different modulations on the different channels of the SVD decomposition is a topic for future research). We will assume that the transmitter is equipped with a randomizer device (e.g. an interleaver), which decorrelates the eigenchannel coefficients -and therefore the SNR- that occur during the transmission of each symbol. For simplicity of the analysis, we will also assume that l is an integer multiple of n.

1) Case I: Suppose that the data symbols are fed into the SVD pre-coder using the same fixed order, in which the i-th symbol of each frame is always sent through the $k \equiv i \bmod n$ eigenchannel. For a given realization of the channel matrix H, the frame error rate $P_{\rm f}$ can be written in terms of the symbol error rate (SER), $P_{\rm s}(\text{SNR}_k)$, as

$$P_{\rm f} = 1 - \prod_{i=1}^{l} [1 - P_{\rm s}(\text{SNR}_{i \bmod n})] \ , \tag{20}$$

where $\text{SNR}_{i \bmod n}$ is the signal to noise ratio of the $i \bmod n$ -th eigenchannel, as given by (16). Using (20) in (19), the following expression is obtained:

$$\bar{\tau}_{\rm ordered} = \prod_{k=1}^{n} [1 - \bar{P}_{\rm s}(\lambda_k \phi_k \bar{\gamma})]^{-l/n} \ , \tag{21}$$

where $\bar{P}_{\rm s}(\lambda_k \phi_k \bar{\gamma}) = \mathbb{E}\{P_{\rm s}(\lambda_k \phi_k \bar{\gamma})\}$ is a shorthand notation for the mean SER of the k-th eigenchannel.

2) Case II: Suppose now that the SVD engine is fed using a different order for each transmission trial in a pseudo-random fashion. Define A_k as the event in which a symbol is assigned to the k-th eigenchannel, and E the event that a symbol is decoded with error. As all events A_k are jointly exhaustive and mutually exclusive, we can decompose the error event as $E = \cup_{k=1}^{N} (E \cap A_k)$. Hence, the average SER for a given realization of the channel matrix H can be calculated as

$$\mathbb{P}\{E\} = \mathbb{P}\left\{\cup_{k=1}^{n} (E \cap A_k)\right\} = \sum_{k=1}^{n} \mathbb{P}\left(E \cap A_k\right) \tag{22}$$

$$= \sum_{k=1}^{n} \mathbb{P}\left(A_k\right) \mathbb{P}\left(E|A_k\right) = \frac{1}{n} \sum_{k=1}^{n} P_{\rm s}\left(\lambda_k \phi_k \bar{\gamma}\right) \ . \tag{23}$$

where we are using that $\mathbb{P}(A_k) = 1/n \quad \forall k = 1 \ldots n$. It is to be noted that this average SER is valid for all the l symbols of the frame, and therefore the frame error rate for a given channel realization can be expressed as $P_{\rm f} = 1 - (1 - \mathbb{P}\{E|H\})^l$. Using this result in (19), the following is obtained:

$$\bar{\tau}_{\rm mixed} = \left[1 - \frac{1}{n} \sum_{k=1}^{n} \bar{P}_{\rm s}(\lambda_k \phi_k \bar{\gamma})\right]^{-l} \ . \tag{24}$$

It can be shown that $\bar{\tau}_{\rm mixed} \leq \bar{\tau}_{\rm ordered}$. In effect, define $\alpha_k = 1 - \bar{P}_{\rm s}(\lambda_k \phi_k \bar{\gamma})$. Their arithmetic mean is $A(\alpha_k) = 1/n \sum_{k=1}^{n} \alpha_k$, and their geometric mean is $G(\alpha_k) = \prod_{k=1}^{n} \alpha_k^{1/n}$. Then, $\bar{\tau}_{\rm ordered} = G(\alpha_k)^{-l}$ and $\bar{\tau}_{\rm random} = A(\alpha_k)^{-l}$. The inequality follows from the well known fact that the arithmetic mean is always larger than the geometric mean.

C. Final equation

Under the conditions stated in III-B2, the mean total energy consumption per goodbit (14) can be re-written using (18) and (24) as

$$\bar{\mathcal{E}}_{\rm b} = \frac{(N P_{\rm el} + A d^{\alpha} \bar{\gamma}) \hat{T}}{\left[1 - \bar{P}_{\rm s}^{\rm (n)}(\{\phi_k\}, \bar{\gamma})\right]^{l}} \ , \tag{25}$$

where we have defined the mean MIMO SVD SER as $\bar{P}_{\rm s}^{\rm (n)}(\{\phi_k\}, \bar{\gamma}) = \frac{1}{n} \sum_{k=1}^{n} \bar{P}_{\rm s}(\lambda_k \phi_k \bar{\gamma})$, and $P_{\rm el} = P_{\rm el,tx} + P_{\rm el,rx}$ is the total electronic power consumption per branch of the transceiver (see Section II-A1).

IV. Optimal number of used eigenchannels as function of distance

We seek to determine the radiation power and number of eigenchannels for minimizing the average energy consumption per goodbit in a MIMO SVD system. Throughout this section we will assume that the MIMO channel matrix H has i.i.d. complex gaussian fading coefficients [13]. We will consider the case of equal power allocation among the n used eigenchannels, i.e. $\phi_k = n^{-1} \quad \forall k = 1, \ldots, n$ (the optimization of the ϕ_k's for further energy minimization is ongoing work).

Previous work of the authors provides three key insights for analyzing the case at hand:

1.- The mean SER of MIMO SVD (denoted above as $\bar{P}_{\rm s}^{\rm (n)}(\{\phi_k\}, \bar{\gamma})$) can be approximated by the mean SER of a single-antenna Nakagami-m fading channel [15].
2.- The mean energy consumption per goodbit of a single-antenna system, which operates over a Nakagami-m fading channel using a setup similar to the one presented in this work, has the same algebraic structure than the mean consumption (25) of a MIMO SVD system [20].
3.- The mean energy consumption per goodbit of transmissions over a single-antenna Nakagami-m fading channel as function of $\bar{\gamma}$ has a unique minimum [20].

Therefore, we conclude that the energy consumption of a MIMO SVD system as function of $\bar{\gamma}$ must also have a unique minimum. This insight has been confirmed by numerical evaluations of (25) using the parameters presented in Table I (Figure 2).

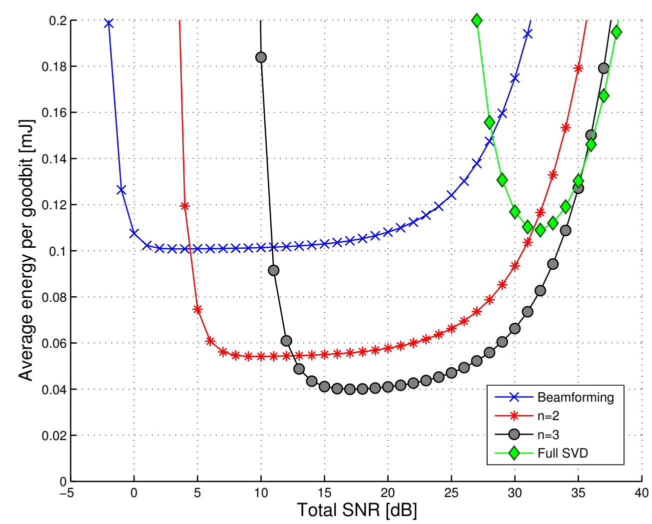

Fig. 2. Mean energy consumption of a 4×4 MIMO SVD system with equal power allocation and all possibilities of used eigenchannels. Modulation is uncoded BPSK and link distance is $d = 50$ meters. System parameters where taken from Table I.

By denoting $\bar{\gamma}^*$ the SNR value at which the MIMO SVD energy consumption achieves its minimum, the minimal energy consumption achievable for a given link distance can be expressed as

$$\bar{\mathcal{E}}_{\rm b}^*(d) = \frac{(N P_{\rm el} + A d^{\alpha} \bar{\gamma}^*) \hat{T}}{\left[1 - \bar{P}_{\rm s}^{\rm (n)}(\{n^{-1}\}, \bar{\gamma}^*)\right]^{l}} \ . \tag{26}$$

TABLE I
GENERIC LOW-POWER DEVICE PARAMETERS

Parameter	Description	Value
R_{s}	Symbol rate	10 kBaud †
L	Frame length	108 bytes
O_{a}	Acquisition overhead	1 byte
O_{b}	Estimation and synchronization overhead	4 bytes
F	Feedback frame length	11 bytes
α	Path-loss coefficient	3.2
A_0	Channel loss	30 dB †
η	PA efficiency	35% †
$P_{\text{el,tx}}$	Tx electric power consumption	98.2 mW †
$P_{\text{el,rx}}$	Rx electric power consumption	112.5 mW †
N_0	Noise power density	-174 dBm/Hz
N_{f}	Receiver noise figure	10 dB †
M_{l}	Link margin	30 dB

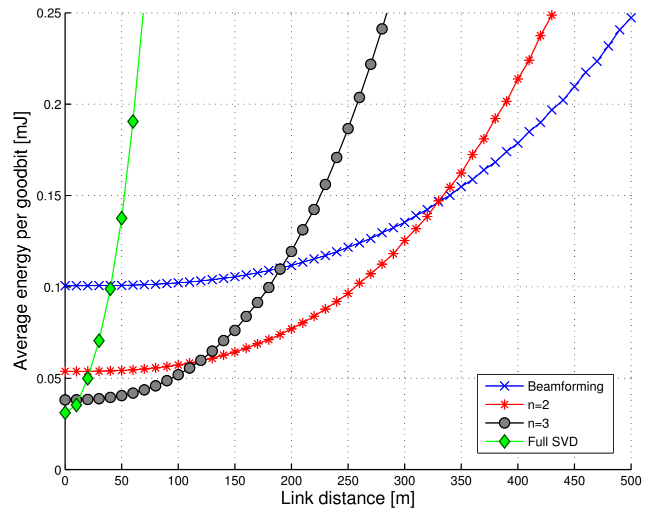

Fig. 3. Mean energy consumption per goodbit of a 4×4 MIMO SVD system that uses uncoded BPSK with equal power allocation, as function of link distance. For each amount of used eigenchannels, n, the system is operated at the optimal total SNR for the given distance. As distance decreases, larger values of n become optimal in the energy sense.

From studying (26) we have found that in long range communications, the power consumed by the power amplifiers (equal to $Ad^{\alpha}\bar{\gamma}^*$ Watts) dominates over the power consumed by the electronic components (NP_{el} Watts). Under these conditions, beamforming is the energy-optimal transmission scheme (Figure 3), because it invests the radiated electromagnetic energy entirely on the best eigenchannel. This reduces the mean SER and thereby limits the average number of retransmissions.

For medium link distances (below 350 meters in Figure 3) the power consumed by electronic components begins to dominate over the radiated power and therefore also over the consumption of the power amplifiers. In this case, the optimal energy consumption can be approximated as $\bar{\mathcal{E}}_{\text{b}}^*(d) \approx NP_{\text{el}}\hat{T}\bar{\tau}$, which shows that under these conditions the total time per bit, $\hat{T}$, becomes a relevant parameter in the energy budget. Therefore, it is attractive to use more eigenchannels simultaneously, because it increases the baud rate and thereby reduces the transmission time per bit. At very short link distances ($d \leq 10$ meters in Figure 3), this notion leads to using full SVD ($n = N$) as the most energy-efficient scheme. We point out that this assumes that the channel matrix has a full rank at all link distances.

V. Conclusions

We studied the optimization of the radiated power and the number of eigenchannels to be used in order to minimize the energy consumption of a MIMO SVD communication link for delivering one bit of data without error.

We found that a MIMO SVD system that uses a given number of eigenchannels with equal power allocation among them, achieves a minimal mean energy consumption per bit transferred without error at one unique optimal radiation power level.

We also showed that for large link distances the more energy-efficient transmission strategy is using only the eigenchannel with best fading statistics (beamforming). As the link distance shortens, the power consumed by electronic components starts to dominate over the radiated power. This compels to reduce the average air time spent per data bit by increasing the bit-rate using a larger number of eigenchannels. Using all the eigenchannels (full SVD) is only optimal for very short transmission distances.

† Source: [2]

Appendix A
Alternative Expression for $\hat{P}_{\text{PA}}$

Lets deduce an alternative expression for the total power consumption of all the power amplifiers $\hat{P}_{\text{PA}}$ (cf. Section II-A2).

If only the best n eigenchannels are being used, the transmitted vector $\vec{x}$ is related to the transmitted symbol vector $\vec{s}$ as $\vec{x} = V_n\vec{s}$, where V_n is the $N \times n$ precoding matrix which is composed by the first n columns of the unitary matrix V which is obtained from the singular value decomposition $H = U^*\Lambda V$ [14]. Therefore

$$|x_j|^2 = \sum_{k=1}^{n} |v_{j,k}|^2 \bar{P}_{\text{tx}}^{(k)} \tag{27}$$

where $v_{j,k}$ is the coefficient of the j-th row and k-th column of V_n, and $\bar{P}_{\text{tx}}^{(k)}$ is the power allocated into the k-th eigenchannel for $j \in \{1, \ldots, n\}$ (cf. Section III-A).

Assuming that $|x_j|^2 = P_{\text{A}}^{(j)}$, where $P_{\text{A}}^{(j)}$ denotes the power aired by the j-th antenna, for $j \in \{1, \ldots, N\}$ (cf. II-A2), and using the definition of $\hat{P}_{\text{PA}}$ (cf. Section 5), then the total power consumption of the PA's can be written as

$$\hat{P}_{\text{PA}} = \sum_{j=1}^{N} P_{\text{PA}}^{(j)} = \frac{\eta}{\xi} \sum_{j=1}^{N} |x_j|^2 \tag{28}$$

$$= \frac{\eta}{\xi} \sum_{k=1}^{n} \sum_{j=1}^{N} |v_{j,k}|^2 \bar{P}_{\text{tx}}^{(k)} = \frac{\eta}{\xi} \sum_{k=1}^{n} \bar{P}_{\text{tx}}^{(k)} \tag{29}$$

where we are using the fact that $\sum_{j=1}^{N} |v_{j,k}|^2 = 1$ as the columns of the unitary matrix V are orthonormal.

Appendix B
Comment about a hierarchical model

In statistics, a hierarchical model is a random variable which possesses one or more parameters that are themselves random variables [21].

Lemma: Lets $\{p_i\}_{i=1}^{\infty}$ be a collection of i.i.d. random variables, each one of which take positive values in $[0, 1]$ and have a finite mean $\mathbb{E}\{p\} = \bar{p}$. Lets $\{X_i\}_{i=1}^{\infty}$ be another collection of i.i.d. random variables, where each X_j can be either 0 or 1, and its distribution is determined by p_i as $\mathbb{P}\{X_i = 0\} = p_i$ and $\mathbb{P}\{X_i = 1\} = 1 - p_i$. Finally lets be Y a random variable which takes the value n if $X_n = 1$ and $X_i = 0 \quad \forall\, 1 \leq i < n$. Then Y is a Geometric random variable with parameter $\bar{p}$.

Proof: Lets start finding the distribution of X_i, which is calculated as follows

$$\mathbb{P}\{X_i = 0\} = \int_0^1 \mathbb{P}\{X_i = 0|p_j\} f_{p_i}(p_i) \mathrm{d}p_i \qquad (30)$$

$$= \int_0^1 p_i f_{p_i}(p_i) \mathrm{d}p_i = \bar{p} \qquad (31)$$

where $\mathbb{P}\{A|B\}$ denotes the conditional probability of A given B, and $f_{p_i}(p_i)$ is the p.d.f. of the random variable p_i. As X_i can only show two outcomes, it is clear that $\mathbb{P}\{X_i = 1\} = 1-\bar{p}$. Therefore, X_i is a Bernoulli random variable with parameter $\bar{p}$.

Now lets find the distribution of Y. As it only takes integer values, its distribution can be found by direct calculation as

$$\mathbb{P}\{Y = n\} = \mathbb{P}\{X_1 = 0, \dots, X_{n-1} = 0, X_n = 1\} \qquad (32)$$

$$= \mathbb{P}\{X_n = 1\} \prod_{i=1}^{n-1} \mathbb{P}\{X_i = 0\} \qquad (33)$$

$$= (1 - \bar{p})\bar{p}^{n-1} \ . \qquad (34)$$

Therefore, Y is a geometric random variable with parameter $1 - \bar{p}$. It is to be noted that this result only depends on the mean value of the random variable p_i, being independent of its actual distribution $f_{p_i}(p_i)$.

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IV

Modulation and SNR optimization for achieving energy-efficient communications over short-range fading channels

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Modulation and SNR Optimization for Achieving Energy-Efficient Communications over Short-Range Fading Channels

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***Abstract*—It is commonly assumed that the energy consumption of wireless communications is minimized when low-order modulations such as BPSK are used. Nevertheless, the literature provides some evidence that low-order modulations are suboptimal for short transmission distances. No complete analysis on how the modulation size and transmission power must be chosen in order to achieve energy-efficient communications over fading channels has been reported so far. In this paper we provide this analysis by presenting a model that determines the energy consumed per payload bit transferred without error over fading channels of various statistics.**

We find that each modulation scheme has a single optimal signal-to-noise ratio (SNR) at which the energy consumption is minimized. The optimal SNR and the minimal energy consumption are larger for channels with less favorable error statistics. We also find that, if each modulations is operated at its optimal SNR, BPSK and QPSK are the optimal choices for long transmission distances, but as the transmission distance shortens the optimal modulation size grows to 16-QAM and even to 64-QAM. This result leads to showing that for short-range communications the lifetime of a typical low-power transceiver can be up to 500% longer by selecting the optimal constellation instead of BPSK.

***Index Terms*—Energy-efficiency, cross-layer design, fading channels, modulation optimization, energy consumption modeling, wireless sensors networks, low-power communications.**

I. INTRODUCTION

ATTAINING high energy efficiency is a key condition that wireless communications technologies like wireless sensor networks (WSN) must satisfy in order for the technology to prosper into large-scale autonomous networks. Requirements on size and cost of the nodes pose vital constraints on the problem. In fact, battery depletion has been identified as one of the primary causes of lifetime limitation of these networks [1]. Replacing them regularly is impractical in large networks or may even be impossible in hostile environments [2].

The main tasks that WSN nodes perform are sensing the environment, processing the data and communicating it wirelessly across the network. The latter task dominates the overall energy budget [3] and, therefore, optimizing it has a direct impact on a network's lifetime [4].

The communication energy budget depends on choices such as the modulation scheme, packet structure and transmission power, whose choices have a direct impact on the link's frame error probability. The frame error probability, in turn, affects the number of re-transmissions that are necessary and thereby also affects the overall energy needed to convey successfully each bit of information from one node to the next.

When attaining high data rates is not a requirement and when the communication system is power-limited (as in WSN), the common notion is to choose low-order modulations such as BFSK or BPSK, whose bandwidth efficiency is lowest in favor of a lower SNR requirement for achieving a desired bit error rate [5]. These modulations are, in fact, the only ones used in commercially available low power transceivers like TI CC1000 [6] or CC2420 [7], often used in WSN nodes. Nevertheless, it has been shown that the above notion leads to suboptimal operation for communication over short distances through deterministic channels [8]–[11].

The rules by which the modulation size and transmission power shall be chosen to attain energy-efficient communications through fading channels have not yet been studied thoroughly. Most of the reported work focuses on the additive white Gaussian noise channel (AWGN) [8]–[17]. In [18], energy consumption of block fading Rayleigh channels is studied by considering the channel fading via its outage probability, rather than by taking into account the actual symbol error rate (SER) degradation. In [19], physical layer parameters of ultra-wide-band communications are optimized by numerical evaluations. However, the model used is only valid for fast-fading channels and cannot be extended for fading channels with correlation over time.

In this paper we present an energy consumption model which allows for optimizing the modulation scheme and transmission power for communication over correlated and uncorrelated fading channels. Furthermore, we derive rules for choosing the modulation size that achieves highest energy efficiency as a function of link distance on fast fading channels. For large transmission distances, our model confirms the common notion discussed above for power-limited systems, while for short transmission distances it coincides with the results reported in [11] and [14] for AWGN channels, extending them to fading channels.

Our model also shows that a single optimal SNR exists for each type of wireless channel and that it depends on the frame length, modulation size and channel statistics. Many existing energy consumption models such as the ones reported in [8], [9], [12], [13], [15], [16] share the assumption that the bit error

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rate is a given constant, which is determined by upper layer requirements. The idea that the bit error rate should not be a constant but a parameter to be optimized is analyzed in [11] and [20], but those results are only valid for AWGN channels. Their approach cannot be extended to fading channels because expressing the SNR as function of the bit error rate leads to intractable mathematics. Using the SNR as a variable, our model allows to derive analytic expressions for the optimal SNR for AWGN, Rayleigh and Nakagani-m channels.

The rest of this article is organized as follows: Section II presents the energy consumption model, Section III uses it to optimize the SNR, Section IV studies the optimization of the modulation scheme and finally Section V summarizes our conclusions.

II. Energy Consumption Model

Our goal is to determine the total energy that is necessary for transferring one bit of data successfully, without error, in a point-to-point packet-switched wireless communication link (e.g. between two sensor nodes). We assume that every frame transmitted in the *forward* direction is matched by a feedback frame in the *reverse* direction, to acknowledge correct reception or requests a re-transmission. We also assume that the irradiated power is determined by the transmitter based upon knowledge of the statistics of the signal-to-noise ratio (SNR) at the intended receiver. We further assume that all frames in both directions are always detected and that all feedback frames are decoded without error.

Transmissions in both directions cause energy expenses at respective transmitters and receivers. In short range communications, the energy consumption for receiving a frame is known to be on the same order as the consumption for transmitting it [8] and must hence be accounted for.

In the sequel, we first analyze the components of energy consumption of a transceiver from the standpoint of a node that transmits one payload frame and receives the corresponding feedback frame (the reverse case —a transceiver that receives one payload frame and transmits the corresponding feedback frame— follows by analogy). We continue then by analyzing the statistics of re-transmissions and finally we present our total energy consumption model.

A. Components of the Energy Consumption of the Forward Transceiver

The energy consumption of the transceiver that transmits forward frames and receives feedback frames is composed of four terms, each one described next.

1) Start-up Energy Consumption: We assume that the transmitter is by default in a low power consumption (sleep) mode. Hence, it must be brought online before it can make a transmission. The energy spent in the activation process can be significant [4]. We will denote this energy, divided by the number of payload bits that are going to be transmitted before the transceiver goes into low power consumption mode again, by $\mathcal{E}_{\rm st}$. The value of $\mathcal{E}_{\rm st}$ depends on the device architecture and electronic components.

2) Energy Consumption of Electronic Components due to Pre-transmission Processing: We assume that each physical-layer forward frame carries H bits of header with essential transmission parameters and L bits of payload data. The total duration of a forward frame is shared by $T_{\rm L}$ seconds for transmitting the L bits of payload (with a suitable modulation), $T_{\rm H}$ seconds for the transmission of the header and $T_{\rm O}$ seconds for the transmission of overhead signals for acquisition and tracking (channel estimation, synchronization, etc.). The average air time per payload bit in a forward frame is

$$T_{\rm b} = \frac{T_{\rm L} + T_{\rm H} + T_{\rm O}}{L} \ . \tag{1}$$

Lets assume that an M-ary modulation is being used. Each payload symbol therefore carries $\log_2(M)$ bits. If $R_{\rm s}$ denotes the physical layer symbol-rate, then (1) can be formulated alternatively as

$$T_{\rm b} = \frac{1}{R_{\rm s}} \left(\frac{1}{\log_2(M)} + \frac{H+O}{L} \right) \ , \tag{2}$$

where O is a measure of the total overhead per forward frame, measured in bits [8].

Following (2), we may write the energy per bit per forward frame used for transmit processing as

$$\mathcal{E}_{\rm el,tx} = P_{\rm el,tx} T_{\rm b} \ , \tag{3}$$

where $P_{\rm el,tx}$ is the power consumption of the baseband and radio-frequency electronic components that perform the forward transmission. It is to be noted that $\mathcal{E}_{\rm el,tx}$ is largely dominated by passband processing components such as filters, mixer and frequency synthesizer [21], whose consumption is typically orders of magnitude larger than the one of the digital baseband processing modules [4].

3) Energy Consumption due to Electromagnetic Radiation: Each frame is aired with a transmission power $P_{\rm tx}$ provided by the power amplifier (PA). The PA's power consumption is modeled by

$$P_{\rm tx} = \frac{\eta}{\xi} P_{\rm PA} \ , \tag{4}$$

where ξ is the peak-to-average ratio of the transmitted signal and η is the drain efficiency of the PA [9]. Thus, the energy per bit per forward frame used for electromagnetic radiation is

$$\mathcal{E}_{\rm PA} = P_{\rm PA} T_{\rm b} \ . \tag{5}$$

4) Energy Consumption of Electronic Components due to the Processing of Feedback Frames: Feedback frames are assumed to be $T_{\rm fb}$ seconds long and consume $P_{\rm el,rx}$ Watts during that time at the receiver for decoding. $T_{\rm fb} = F/R_{\rm s}$, where F is the number of bits that compose the feedback frame and $R_{\rm s}$ is the physical layer symbol-rate. $P_{\rm el,rx}$ includes mainly the power needed to energize the passband receiver elements (low-noise amplifier, mixer, filters, frequency synthesizer, etc.) [21]. The energy per forward payload bit spent by the transmitter of a forward frame for decoding the corresponding feedback frame is

$$\mathcal{E}_{\rm fb,rx} = \frac{P_{\rm el,rx} T_{\rm fb}}{L} \ . \tag{6}$$

B. Re-transmission Statistics

A key contributor to the energy consumption is the need for re-transmissions due to forward frames that get decoded with errors at the receiver. The probability of frame error (and hence the probability of re-transmission) depends on the mean received SNR, $\bar{\gamma}$, and on the statistics of the wireless channel. Therefore, the number of trials (τ) until a frame is decoded without error is a random variable, whose mean value can be calculated to be (see Appendix A)

$$\bar{\tau} = 1 + \sum_{n=1}^{\infty} \mathbb{E}\left\{ \prod_{j=1}^{n} P_j \right\} \ , \tag{7}$$

where $\mathbb{E}\{\cdot\}$ denotes the expectation operator and P_j is the probability of decoding the frame with error during the j-th transmission trial. In general, the P_j are random variables that depend on the frame size, modulation type and received SNR during the j-th trial.

The value of $\bar{\tau}$ depends on the joint distribution of the probabilities P_j for $j = 1, \ldots, \infty$. In effect, consider first a static channel, where the frame error probabilities P_j are fully correlated which each other, and hence $P_j = P_1 \, \forall j \in \mathbb{N}$. In this case, (7) becomes

$$\bar{\tau} = 1 + \sum_{n=1}^{\infty} \mathbb{E}\left\{P_1^n\right\} = \mathbb{E}\left\{ \frac{1}{1-P_1} \right\} \triangleq \bar{\tau}_{\text{static}} \ . \tag{8}$$

Consider now a fading channel in which the SNR levels of any two frame transmission trials are statistically independent. Then (7) can be re-written as

$$\bar{\tau} = 1 + \sum_{n=1}^{\infty} \prod_{j=1}^{n} \mathbb{E}\left\{P_j\right\} = \frac{1}{1-\mathbb{E}\left\{P_1\right\}} \triangleq \bar{\tau}_{\text{f}} \ , \tag{9}$$

where $\{P_j\}_{k=1}^{\infty}$ is now a collection of i.i.d. random variables.

Using the Jensen inequality for the convex function $\Phi(x) = (1-x)^{-1}$ with $x \in [0, 1)$, it can be shown that

$$\bar{\tau}_{\text{f}} \leq \bar{\tau}_{\text{static}} \ , \tag{10}$$

where the equality is attained by the AGWN channel. This result shows that transferring successfully one entire frame of data across uncorrelated channels takes, on average, fewer transmission attempts than doing it over fully correlated channels. An intuitive explanation for this is that unfavorable (initial) realizations of static channels have a permanent low SNR level, and require therefore a large number of trials until a frame is received without error. However unlikely, the poor performance of these unfavorable cases raise the mean number of trials enough to spoil the average performance beyond the case of uncorrelated channels.

C. Total Energy per Successfully Transferred Bit

The discussion in Sections II-A and II-B leads to the following model for the total energy consumption. The energy consumed by the transmitter of forward frames per bit transferred to the receiver without error is given by

$$\begin{aligned} \mathcal{E}_{\text{T}} &= \mathcal{E}_{\text{st}} + (\mathcal{E}_{\text{el,tx}} + \mathcal{E}_{\text{PA}} + \mathcal{E}_{\text{fb,rx}})\tau & (11) \\ &= \mathcal{E}_{\text{st}} + \left[(P_{\text{el,tx}} + P_{\text{PA}})T_{\text{b}} + P_{\text{el,rx}} \frac{T_{\text{fb}}}{L} \right] \tau \ , & (12) \end{aligned}$$

where τ is the number of trials until the frame is decoded without errors at the receiver.

By analogy, the total energy used by the receiver of forward frames for demodulating τ forward transmissions, and for transmitting the corresponding τ feedback frames, is

$$\mathcal{E}_{\text{R}} = \mathcal{E}_{\text{st}} + \left[P_{\text{el,rx}} T_{\text{b}} + (P_{\text{el,tx}} + P_{\text{PA}}) \frac{T_{\text{fb}}}{L} \right] \tau \ . \tag{13}$$

The total energy consumption per bit transmitted without error is the sum of (12) and (13):

$$\begin{aligned} \mathcal{E}_{\text{b}} &= 2\mathcal{E}_{\text{st}} + (P_{\text{el,tx}} + P_{\text{PA}} + P_{\text{el,rx}}) \left(T_{\text{b}} + \frac{T_{\text{fb}}}{L} \right) \tau & (14) \\ &= S + (P_{\text{el}} + P_{\text{PA}}) T \tau \ , & (15) \end{aligned}$$

where we have defined $S = 2\mathcal{E}_{\text{st}}$ as the total start-up energy per bit, $P_{\text{el}} = P_{\text{el,tx}} + P_{\text{el,rx}}$ as the total power consumed by electronic components and $T = T_{\text{b}} + T_{\text{fb}}/L$ the total time per bit per transmission trial.

It is to be noted that because of τ, $\mathcal{E}_{\text{b}}$ is a random variable that depends on the realizations of the channel and of the thermal noise. Its mean value is

$$\bar{\mathcal{E}}_{\text{b}} = \mathbb{E}\left\{\mathcal{E}_{\text{b}}\right\} = S + (P_{\text{el}} + P_{\text{PA}}) T \bar{\tau} \ , \tag{16}$$

with $\bar{\tau}$ as discussed in Section II-B.

III. Optimal mean SNR Level for M-QAM Transmissions

The received SNR varies over time according to the statistical properties of the wireless channel. In this section we seek to determine the mean SNR for which a communication with M-QAM modulation uses, on the average, the least amount of energy per bit transferred without error.

In the sequel, a general framework for studying this problem based on the results of Section II is presented (Section III-A). Then, the accuracy of the results is tested by simulations in Section III-B. Finally, a method for finding approximate expressions for the optimal SNR is presented in Section III-C.

Although we analyze the performance of M-QAM transmissions, the results can be applied to any other modulation scheme whose symbol error rate over an AWGN channel can be written in the form $P_{\text{e}}(\gamma) = cQ(\sqrt{a\gamma})$, where $Q(x)$ is the tail probability of the standard normal distribution, a and c are appropriate constants and γ is the SNR. This will be used in Section IV for the cases of BPSK and BFSK modulations.

A. General Case

Consider rewriting (16) so that the terms that depend on the mean SNR, $\bar{\gamma}$, observed at the decision stage of the receiver, become explicit:

$$\bar{\mathcal{E}}_{\text{b}}(\bar{\gamma}) = S + \left[P_{\text{el}} + P_{\text{PA}}(\bar{\gamma})\right] T \bar{\tau}(\bar{\gamma}) \ . \tag{17}$$

Above, $P_{\text{PA}}(\bar{\gamma})$ is a linear function of $\bar{\gamma}$. In effect, the transmission power given in (4) attenuates over the air with path loss and arrives at the receiver with a mean power given by

$$P_{\text{rx}} = \frac{P_{\text{tx}}}{A_0 d^{\alpha}} \ , \tag{18}$$

where A_0 is a parameter that depends on the transmitter and receiver antenna gains and the transmission wavelength, d is the distance between transmitter and receiver and α is the path loss exponent [22]. At the input of the decision stage of the receiver, $\bar{\gamma}$ is related to $P_{\rm rx}$ as

$$\bar{\gamma} = \frac{P_{\rm rx}}{N_0 W N_{\rm f} M_{\rm l}} \ , \tag{19}$$

where N_0 is the power spectral density of the baseband-equivalent additive white Gaussian noise, W is the transmission bandwidth, $N_{\rm f}$ is the noise figure of the receiver's front end and $M_{\rm l}$ is a link margin term that represents any other additive noise or interference [9]. From (4), (18) and (19) we find that

$$P_{\rm PA}(\bar{\gamma}) = \frac{\xi A_0 d^\alpha N_0 W N_{\rm f} M_{\rm l}}{\eta} \bar{\gamma} = A d^\alpha \bar{\gamma} \ , \tag{20}$$

with A a constant.

Assume that the L bits of payload are transmitted using an uncoded M-ary modulation, and that the H bits of header are modulated using a binary modulation for minimizing their probability of error. Hence, the frame is composed by L/b M-ary symbols and H binary symbols, where $b = \log_2(M)$. Then, the frame error rate P_f can be written in terms of the M-ary modulation symbol error rate $P_{\rm s}(\gamma)$ and the binary modulation symbol error rate $P_{\rm bin}(\gamma)$ as

$$P_f = 1 - \prod_{k=1}^{H} [1 - P_{\rm bin}(\gamma_k)] \prod_{i=H+1}^{H+L/b} [1 - P_{\rm s}(\gamma_i)] \ , \tag{21}$$

where γ_k is the SNR during the k-th symbol. Using this in (9) and assuming that γ_k are i.i.d. random variables with mean $\bar{\gamma}$ (fast fading scenario) we obtain:

$$\bar{\tau}_{\rm f}(\bar{\gamma}) = \frac{1}{[1 - \bar{P}_{\rm bin}(\bar{\gamma})]^H [1 - \bar{P}_{\rm s}(\bar{\gamma})]^{L/b}} \ , \tag{22}$$

where we are using the shorthand notation $\bar{P}_{\rm bin}(\bar{\gamma}) = \mathbb{E}\{P_{\rm bin}(\gamma)\}$ and $\bar{P}_{\rm s}(\bar{\gamma}) = \mathbb{E}\{P_{\rm s}(\gamma)\}$.

Under fast fading conditions, replacing (20) and (22) into (17) yields

$$\bar{\mathcal{E}}_{\rm b}(\bar{\gamma}, d) = S + \frac{(P_{\rm el} + A d^\alpha \bar{\gamma})\, T}{[1 - \bar{P}_{\rm bin}(\bar{\gamma})]^H [1 - \bar{P}_{\rm s}(\bar{\gamma})]^{L/b}} \ . \tag{23}$$

It is to be noted that $\bar{P}_{\rm s}(\bar{\gamma})$ and $\bar{P}_{\rm bin}(\bar{\gamma})$ are strictly decreasing functions of $\bar{\gamma}$ that satisfy $\lim_{\bar{\gamma}\to\infty} \bar{P}_{\rm s}(\bar{\gamma}) = \lim_{\bar{\gamma}\to\infty} \bar{P}_{\rm bin}(\bar{\gamma}) = 0$. Therefore, the average number of transmission trials needed to successfully transfer one frame under fast fading conditions, given by (22), is also a strictly decreasing function of $\bar{\gamma}$ and satisfies $\lim_{\bar{\gamma}\to\infty} \bar{\tau}(\bar{\gamma}) = 1$. This reflects the intuitive fact that the average number of retransmissions drops as the SNR grows. By construction, (23) is the product of the decreasing function $\bar{\tau}(\bar{\gamma})$ and the increasing linear function $P_{\rm PA}(\bar{\gamma})$. Such a product attains a unique minimum at the SNR level $\bar{\gamma}^*$. Lower SNR levels are suboptimal because they force the system to do too many retransmissions, and higher SNR levels are also suboptimal because the overall irradiated power is excessive.

TABLE I
GENERIC LOW-POWER DEVICE PARAMETERS

Parameter	Description	Value
W	Bandwidth	10 kHz †
$R_{\rm s}$	Symbol rate	10 kBaud †
L	Frame Payload	98 bytes §
H	Frame Header	2 bytes §
O	Overhead	5 bytes §
F	Feedback frame length	11 bytes §
$\mathcal{E}_{\rm st}$	Start-up energy	0.125 nJ †
α	Path-loss coefficient	3.2
A_0	Free space path loss	30 dB †
$P_{\rm el,tx}$	Tx electric power consumption	98.2 mW †
η	PA efficiency	35% †
$P_{\rm el,rx}$	Rx electric power consumption	112.5 mW †
N_0	Noise power density	-174 dBm/Hz
$N_{\rm f}$	Receiver noise figure	10 dB †
$M_{\rm l}$	Link margin	30 dB

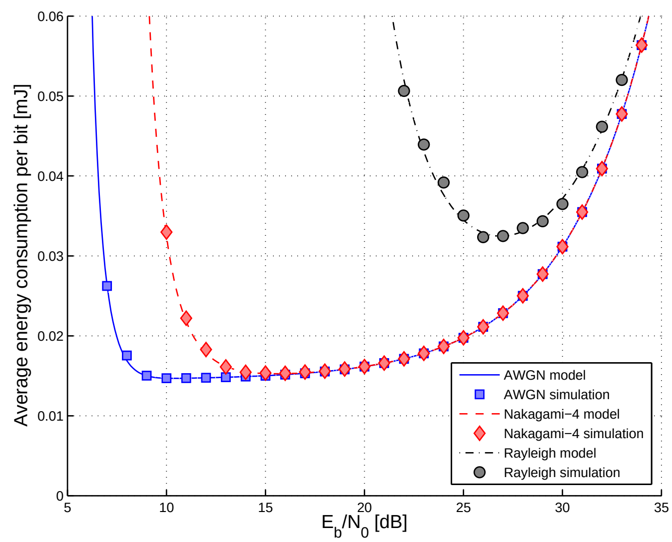

Fig. 1. Simulated average energy consumption per effective transmitted bit for various fading channels (markers) vs. numerical evaluations of the model presented in (23) (solid lines) for a low-power transceiver with typical parameters. A unique minimum is observed, which corresponds to the optimal SNR for maximum energy efficiency.

B. Simulations

We have simulated a communication between two low-power transceivers, such as wireless sensor node radios, separated by 35 meters, for various wireless channel models. System parameters were taken from Table I. One simulation consists of transmitting 10^3 frames, each one composed of 98 bytes of payload and a header composed by 2 bytes (cf. Section II-A2). Each frame was re-transmitted until received without error. For each symbol of each transmission trial, an independent narrowband complex baseband equivalent channel coefficient was randomly generated according to the desired channel model. Every simulation of 10^3 frames was repeated for various mean SNR levels. Statistics were calculated afterwards.

†Source: [9]
§Source: IEEE 802.15.4 standard [23]

The results show that (23) is an accurate model for the energy consumption per bit transferred without error, and confirms the existence and uniqueness of the optimal SNR for energy efficiency (Figure 1). Results also validate the intuition that the minimal mean energy consumption per bit (24), $\bar{\mathcal{E}}_{\text{b}}^*$, and the optimal SNR $\bar{\gamma}^*$ at which this occurs, are larger for channels with less favorable error statistics (Figure 1). In effect, the optimal E_{b}/N_0 for Rayleigh fading is about 15 dB higher than the one for AWGN, and the minimal energy consumption in Rayleigh fading is twice the minimal consumption in AWGN, when each channel is operated at its optimal mean SNR.

C. Minimization of the energy consumption

We want to study the minimal average energy consumption achievable for a given transmission distance, namely

$$\bar{\mathcal{E}}_{\text{b}}^*(d) = \min_{\bar{\gamma}\in[0,\infty]} \bar{\mathcal{E}}_{\text{b}}(\bar{\gamma}, d) = \bar{\mathcal{E}}_{\text{b}}(\bar{\gamma}^*, d) \ , \tag{24}$$

with $\bar{\mathcal{E}}_{\text{b}}(\bar{\gamma}, d)$ given by (23).

Solving for $\bar{\gamma}^*$ in closed-form is mathematically intractable. Therefore, we seek to find a suitable approximation. For this, consider the following inequality, valid for any M-QAM constellation size:

$$[1 - \bar{P}_{\text{bin}}(\bar{\gamma})]^b \geq [1 - \bar{P}_{\text{s}}(\bar{\gamma})] \ . \tag{25}$$

The above inequality follows from the fact that is more probable, at the same SNR, to decode without error b bits transmitted with a binary modulation than one M-ary symbol that carries the same amount of bits. Using (25), the following upper bound for (23) can be constructed:

$$\bar{\mathcal{E}}_{\text{b}}(\bar{\gamma}, d) \leq S + \frac{(P_{\text{el}} + Ad^\alpha \bar{\gamma})\, T}{[1 - \bar{P}_{\text{s}}(\bar{\gamma})]^{(L+H)/b}} \ . \tag{26}$$

Equality is attained for $M = 2$. For the high SNR regime, (26) approaches the equality, as the denominators of (23) and (26) both tend to 1.

An upper bound for the minimum stated in (24) can be found by minimizing the right side of (26) over $\bar{\gamma}$. This can be done by taking derivative and equating the result to zero, which leads to the following implicit expression for the SNR $\bar{\gamma}_0$ that minimizes the upper bound:

$$\lambda \left(\frac{P_{\text{el}}}{Ad^\alpha} + \bar{\gamma}_0 \right) \frac{d\bar{P}_{\text{s}}}{d\bar{\gamma}}(\bar{\gamma}_0) - \bar{P}_{\text{s}}(\bar{\gamma}_0) + 1 = 0 \ , \tag{27}$$

where $\lambda = (L + H)/b$ is a shorthand notation. It is to be noted that the only parameters that influence $\bar{\gamma}_0$ are the mean symbol error rate, $\bar{P}_s(\bar{\gamma})$, λ and the ratio between the power consumption of electronic components, P_{el}, and a coefficient proportional to the irradiated power, Ad^α.

The value of $\bar{\gamma}_0$ depends on the channel statistics. Formulations of (27) for the specific cases of the AWGN, Rayleigh and Nakagami-m channel models, along with approximated solutions for $\bar{\gamma}_0$ in each case, are presented in Appendix B. The Appendix also shows, by means of a few numerical examples, the accuracy of (27) and of the found solutions for the three channel models.

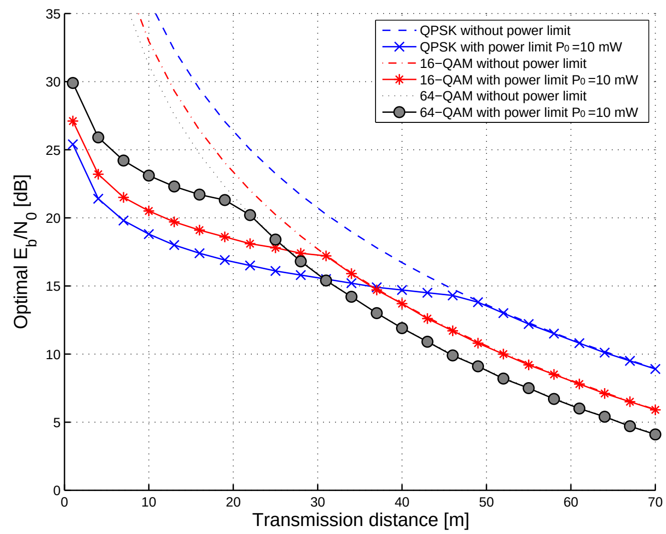

Fig. 2. Optimal SNR that solves (28) for minimum the energy consumption of a low-power transceiver with irradiation power limit of $P_0 = 10$ mW, as a function of link distance for a Nakagami-m channel with $m = 4$. Dotted lines show the unconstrained optimum SNR that minimizes (24).

IV. Optimal Modulation as a Function of Distance

We wish to understand how the energy consumption of a given modulation varies according to the transmission distance. Our investigation will be focused on the case of the fast fading channels introduced in Section II-B. We will restrict our study to the energy consumption of M-QAM modulations in comparison to BPSK and BFSK, motivated by the popularity of these latter two modulations among commercially available low-power transceiver components [6], [7]. We have included in our considerations the effect of transmission power limitations and variable payload size on the energy consumption.

A. Energy consumption with irradiation power limit

Real transmitters have irradiation power limits due to hardware and regulatory constraints. If the power limit is P_0, then, using (18), (19) and (20) it can be shown that the condition $P_{\text{tx}} \leq P_0$ is equivalent to $\bar{\gamma} \leq Kd^{-\alpha}$, where $K = \xi P_0/(\eta A)$ is independent of the transmission distance. Therefore, the minimization of the energy consumption (24), with an added irradiation power constraint, can be stated as

$$\bar{\mathcal{E}}_{\text{b}}^{P_0}(d) = \min_{\bar{\gamma}\in[0,Kd^{-\alpha}]} \bar{\mathcal{E}}_{\text{b}}(\bar{\gamma}, d) \ . \tag{28}$$

The SNR at which the minimum (28) is attained, which we denote $\bar{\gamma}_{P_0}^*$, relates to the SNR at which the unconstrained minimum (24) is attained, $\bar{\gamma}^*$, as follows:

$$\bar{\gamma}_{P_0}^*(d) = \begin{cases} \bar{\gamma}^*(d) & \text{if} \quad \bar{\gamma}^*(d) \leq Kd^{-\alpha} \\ Kd^{-\alpha} & \text{otherwise} \end{cases} \ . \tag{29}$$

The upper bound $Kd^{-\alpha}$ occurs for large transmission distances, when the allowable transmit power is not enough to achieve the unconstrained optimum (Figure 2).

The minimum energy consumption (28) thus attained is

$$\bar{\mathcal{E}}_{\text{b}}^{P_0}(d) = S + \left[P_{\text{el}} + P_{\text{PA}}\left\{d, \bar{\gamma}_{P_0}^*(d)\right\}\right] T\bar{\tau}\left\{\bar{\gamma}_{P_0}^*(d)\right\} \ , \tag{30}$$

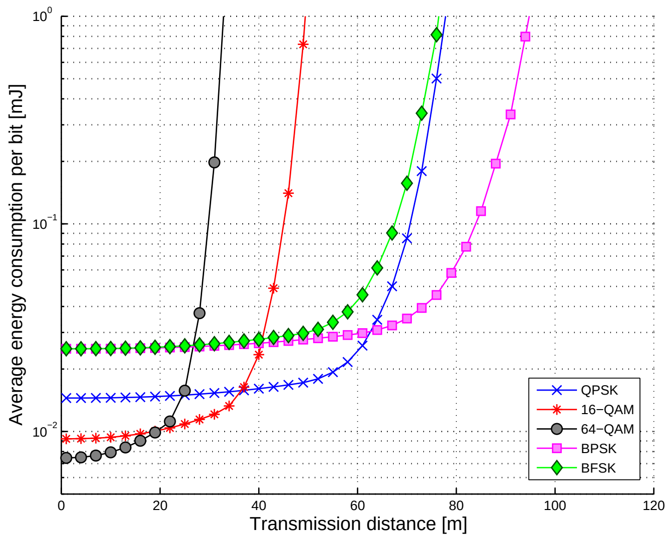

Fig. 3. Energy consumption per successfully transmitted bit of various modulations over a Nakagami-m channel with $m = 4$ as a function of the link distance. Power limit is $P_0 = 10$ mW. Each modulation is operated at its own optimal SNR for the given distance. As distance decreases, modulations with higher spectral efficiency become energy optimal.

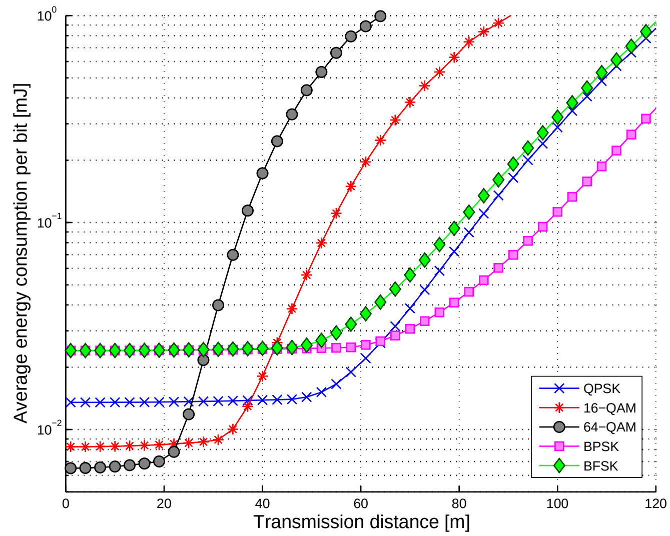

Fig. 4. Average energy consumption of various modulations for transmissions with SNR and payload optimization according to (31). Power limit P_0 is 10 mW, maximum payload size L_0 is 127 bytes and channel is Nakagami-m with $m = 4$.

which is shown in Figure 3 for different modulations schemes using the parameters of Table I.

In long range communications, the power consumed by the power amplifier (P_{PA}) dominates over the power consumed by the electronic components (P_{el}). Figure 3 confirms that the energy-optimal modulations for long range links are the ones with lowest spectral efficiency (small M-ary number).

For short transmissions distances (less than approx. 40 meters in Figure 3) the power consumed by electronic components (P_{el}) dominates over the irradiated power and hence also over the consumption of the power amplifier (P_{PA}). The energy consumption (30) can therefore be approximated for this case as $\bar{\mathcal{E}}_{\mathrm{b}}(d) \approx S + \; P_{\mathrm{el}} T \bar{\tau}\{\bar{\gamma}(d)\}$. This shows that for these conditions the total time per bit, T, becomes a relevant parameter in the total energy budget. This makes attractive to pack more bits into each symbol in order to reduce the transmission time of each bit. It can be seen in Figure 3 that BPSK almost doubles the energy consumption per bit of QPSK at short distances, because of the longer time per bit.

BFSK is never an optimal modulation (Figure 3). For long range transmissions it suffers from the classic 3 dB SNR gap with respect to BPSK. For short range, it performs similarly to BPSK because of their equal spectral efficiency, but worse than larger size modulations.

Figure 3 also shows that when the irradiation power limit is reached, the slope of the energy consumption curves becomes sharply steeper. This illustrates that a transmission range threshold ensues from the irradiation power restriction.

B. Energy consumption with variable payload size

Further minimization of the energy consumption can be achieved by optimizing the number of payload bits per frame. MAC protocols often give freedom to choose the length of the payload, L, up to an upper bound L_0 (for instance $L_0 = 127$ bytes in the IEEE 802.15.4 standard [23]). As the current analysis considers fast fading channels, experience from previous transmission trials does not provide useful information for optimizing the length of the current trial. Therefore, frame length optimization is attained by finding the fixed number of payload bits per frame that minimizes the average energy consumption. The optimization problem can be formally stated as

$$\bar{\mathcal{E}}_{\mathrm{b}}^{P_0,L_0}(d) = \min_{\substack{\bar{\gamma}\in[0,Kd^{-\alpha}] \\ L\in[0,L_0]}} \bar{\mathcal{E}}_{\mathrm{b}}(\bar{\gamma}, d) \; . \tag{31}$$

The frame length optimization softens the range threshold caused by the power limit discussed above and allows for a more graceful loss of coverage at large distances (Figure 4). The optimal payload size grows as the transmission distance shortens, reaching eventually the upper bound L_0 (Figure 5).

It is to be noted that the optimal modulation as function of distance does not vary significantly when frame size optimization is introduced.

C. Transceiver lifetime analysis

The results presented so far allow for studying the lifetime of networks with finite energy supply. For illustration, consider a simple network composed by two wireless sensor nodes with parameters as given in Table I. The nodes exchange 10 kbits of data every 5 minutes. Each node is powered by an ideal battery with a 2000 mAh initial energy charge. This charge is used exclusively for the communications tasks described in Section II.

Using our energy consumption model, we calculate the average lifetime of the batteries of these two nodes for lifelong operation using BPSK, BFSK and M-QAM over different channel models. Each modulation operated at its optimal SNR and frame size. The maximum irradiation power is

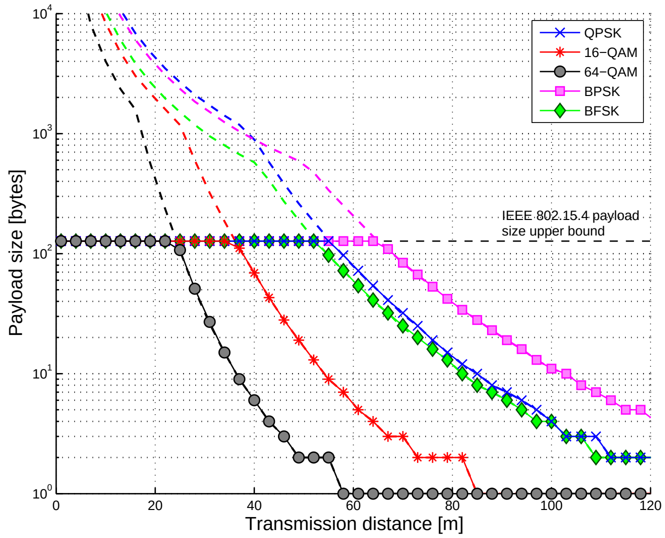

Fig. 5. Optimal payload size that minimizes the energy consumption of transmissions over a Nakagami-m channel with $m = 4$. The markers show the optimal payload size obtained by numerical minimization of (31) with irradiation power limit P_0 at 10 mW and payload size upper limit L_0 of 127 bytes. The doted lines show the corresponding optimal size without payload size limit.

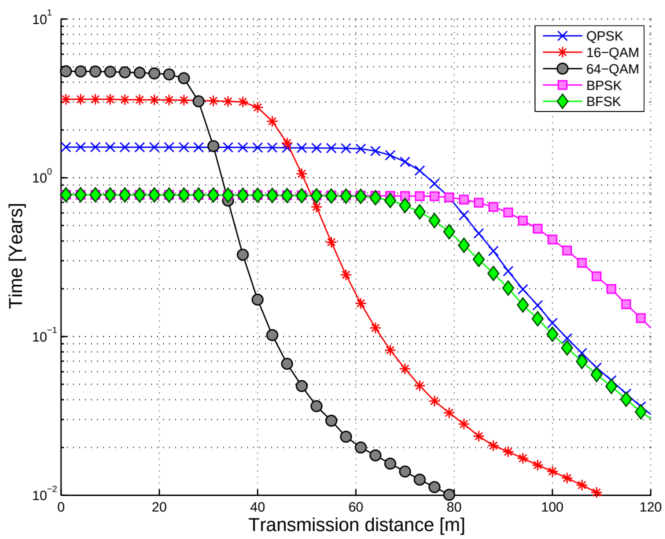

Fig. 6. Lifetime of two wireless sensor nodes that exchange 10 kbits of payload data every five minutes over an AWGN channel. At short transmission distances, BPSK yields a shorter lifetime than more spectrally efficient modulations.

$P_0 = 10$ mW and the frame size is limited to $L_0 = 127$ bytes. We find that as distance decreases, the longer network lifetime is achieved by more spectral efficient modulations (Figure 6 for the AWGN channel and Figure 7 for the Rayleigh channel). In fact, Figures 6 and 7 show that lifetime extensions up to 500% can be gained in short range networks by selecting modulations with larger constellations than BPSK.

Three regions can be identified in both figures: long range, where BPSK is the optimal choice, short range, where 64-QAM is the optimal modulation, and a transition region between them. The three regions exist for any Nakagami-m channel (which includes the AWGN and Rayleigh channels as extreme cases). The sizes of these regions vary according to the randomness of the channel: the lower limit of the long range changes from 80 m to 40 m between AWGN and Rayleigh channels respectively, and the high limit of the short range moves from 28 m to 7 m, respectively.

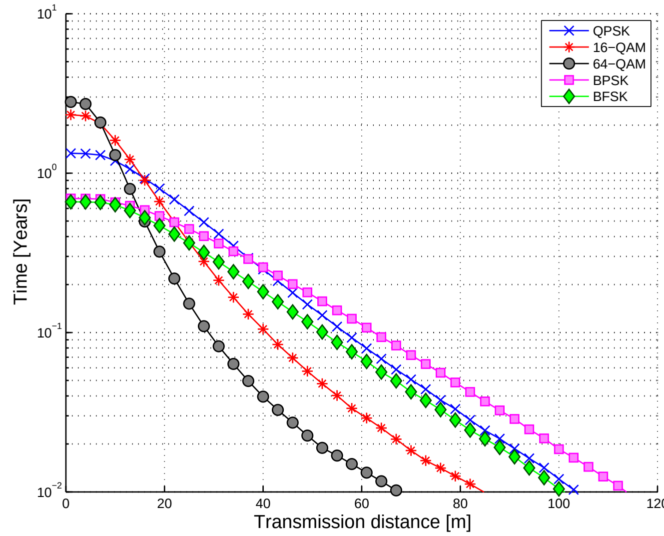

Fig. 7. Same as in Figure 6 but for the Rayleigh fading channel case. The optimality of larger modulations as distance shortens holds.

The observations presented above are valid for typical parameters of commercially available IEEE 802.15.4 Zigbee transceivers [24]. Although the three mentioned regions are found in all cases, the energy consumption can vary by more than a 500% between different transceiver modules and parameter choices.

V. Conclusions

We studied the optimization of the SNR and modulation size in order to minimize the energy consumed by a transceiver for delivering one bit of data without error. In our study, we considered different transmission distances and various channel statistics, as well as the energy cost of retransmissions, feedback frames and the consumption of electronic components.

We found that, for a given modulation scheme, the average energy consumed per bit by transmissions over a fast fading channel as function of the SNR has a unique minimum value. We further found that the optimal energy consumption per bit and the optimal SNR at which this occurs take larger values for channels with less favorable error statistics. In effect, for a generic low-power device, the minimal energy consumption in Rayleigh fading can double the optimal consumption in AWGN, and the corresponding SNR for Rayleigh fading can be 15 dB higher than the one for AWGN.

We proved that transferring successfully one entire frame of data across a fading channel in which the SNR levels of any two frame transmission trials are statistically independent takes, on the average, fewer transmission attempts than doing it over static channels.

We found that the optimal payload size is inversely proportional to the transmission distance. For short-range transmissions, capping the payload size limits the potential energy savings.

We also found that for long transmission distances low bandwidth efficiency modulations (with small M-ary number like BPSK) are optimal in the energy consumption sense. As the transmission distance shortens, the optimal modulation size grows. In short range communications, the power consumed by electronic components dominates over the irradiated power, and dominates also over the energy consumption of the power amplifier. Under these conditions, the average air time spent per data bit becomes a relevant parameter in the total energy budget. This makes optimal to pack more bits into each symbol and hence to chose a larger M-ary number. Our results show that lifetime extensions up to 500% can be gained in short range networks by selecting modulations with larger constellations than BPSK.

Appendix A
Proof of (7)

Lemma: Suppose that a frame is transmitted repeatedly until it is decoded by the receiver without errors. Consider the probability of success of the j-th trial $(1 - P_j)$. If τ denotes the number of trials until the transmission is successful, then its mean value is given by

$$\bar{\tau} = 1 + \sum_{n=1}^{\infty} \mathbb{E}\left\{\prod_{j=1}^{n} P_j\right\} \ . \tag{32}$$

Proof: Let e_n be an indicator function, whose value is 1 if at least n trials were needed to achieve a successful transmitted frame and 0 otherwise. The number of trials needed for a successful transmission can be characterized as follows:

$$\tau = \sum_{n=1}^{\infty} e_n \ . \tag{33}$$

Lets first find an expression for the conditional probability for the event $\{e_n = 1\}$ for given frame error rates. By definition, the value of e_n is 1 if and only if all the frames sent during the first $n-1$ trials where decoded with errors. In general, errors depend on the channel and the thermal noise realizations. Let P_j be the probability of decoding with error the frame sent during the j-th trial for a given channel realization. If the values of P_j are given, then the channel is fixed and the noise is the only random variable involved in the desicion process. As noise is independent from frame to frame, then each frame decoding is an independent event. Therefore, it can be found that for any $n > 1$

$$\mathbb{P}\left(e_n = 1 \mid \{P_j\}_{j=1}^{\infty}\right) = \mathbb{P}\left(\text{first n-1 trials in error} \mid \{P_j\}_{j=1}^{\infty}\right) \tag{34}$$

$$= \prod_{j=1}^{n-1} P_j \ . \tag{35}$$

The case for $n = 1$ is trivial as $e_1 = 1$ by definition.

Now lets calculate the expected value of e_n. Note that e_n given $\{P_1 \dots P_{n-1}\}$ is a Bernoulli random variable with its parameter given by (35). As the expected value of a Bernoulli random variable is its parameter, using (35) and the definition of conditional expectation, it can be stated that for any $n > 1$

$$\mathbb{E}\{e_n\} = \mathbb{E}\left\{\mathbb{E}\left\{e_n \mid \{P_j\}_{j=1}^{\infty}\right\}\right\} = \mathbb{E}\left\{\prod_{j=1}^{n-1} P_j\right\} \ . \tag{36}$$

This value is a number that depends on the compound probability density function of the random variables $P_1 \dots P_{n-1}$.

Finally, using Equations (33) and (36) the lemma is proved.

Appendix B
Solution of (27) for various channel models

In the following, we study (27) for AWGN, Rayleigh and Nakagami-m channel models, and find closed-form approximations for $\bar{\gamma}_0$ for each case.

A. AWGN Channel

The symbol error rate of M-QAM transmissions over the AWGN channel is given by

$$P_{\mathrm{s}}(\gamma) = 1 - \left(1 - cQ(\sqrt{a\gamma})\right)^2 \ , \tag{37}$$

where $c = 2\left(1 - 1/\sqrt{M}\right)$ and $a = 3/(M-1)$. An approximation of (37) is

$$P_{\mathrm{s}}(\gamma) \approx \frac{2c}{5} e^{-0.54a\gamma} \ , \tag{38}$$

which results from fitting the curve $a_1 e^{-a_2\gamma}$, with a_1 and a_2 constants, to (37) for $10^{-8} < P_{\mathrm{s}} < 10^{-1}$ and $M = \{4, 16, 64\}$ and minimizing the mean square error (MMSE). It should be noted that (38) is not a bound but simply a MMSE best fit curve.

Even though strictly speaking the AWGN channel is not a fast fading channel, (27) remains valid for this case. By replacing (38) in (27) we find:

$$\frac{5^3}{3^3 ac\lambda} \exp\left(0.54a\bar{\gamma}_0\right) = \bar{\gamma}_0 + \frac{P_{\mathrm{el}}}{Ad^{\alpha}} + \frac{1}{0.54a\lambda} \ . \tag{39}$$

Despite being an approximation, the virtue of (39) is that it provides insight into the fact that the optimal SNR is unique. In effect, (39) is the intersection of an exponential function with a straight line of unit slope. There can be only one intersection in the first quadrant, which marks $\bar{\gamma}_0$.

B. Fast Fading Rayleigh Channel

The symbol error rate for uncoded M-QAM over a Rayleigh channel is [25]

$$\bar{P}_s(\bar{\gamma}) = c(1-s) - c^2\left[\frac{1}{4} - \frac{s}{\pi}\arctan\left(\frac{1}{s}\right)\right] \ , \tag{40}$$

with c and a defined as in (37) and $s = \sqrt{a\bar{\gamma}/(2 + a\bar{\gamma})}$. Equation (40) can be approximated by

$$\bar{P}_s(\bar{\gamma}) \approx \frac{B}{\bar{\gamma}} \ , \tag{41}$$

with $B = 0.81c/a$. This constant has been found by fitting (41), with B constant, to (40) for $10^{-6} < P_{\mathrm{s}} < 10^{-1}$ and $M \in \{4, 16, 64\}$ by means of MMSE.

Replacing (41) into (27) yields a parabola with only one positive root given by

$$\bar{\gamma}_0 = \frac{(1+\lambda)B}{2}\left(1+\sqrt{1+\frac{4\lambda P_{\mathrm{el}}}{(1+\lambda)^2 BAd^{\alpha}}}\right) \quad . \tag{42}$$

For large distances (e.g. $d > 100$ m for low power devices) $\bar{\gamma}_0$ converges to

$$\lim_{d\to\infty} \bar{\gamma}_0 = (1+\lambda)B \triangleq \bar{\gamma}_{\mathrm{long}} \quad . \tag{43}$$

$\bar{\gamma}_{\mathrm{long}}$ approximates the optimal SNR in the traditional sense of wireless communications, where the link budget neglects the energy consumption of the electronics and only considers link gains and losses.

Assuming that a frame contains enough data symbols so that $\hat{\lambda}/(\lambda+1) \approx 1$, we rewrite (42) using (43) as

$$\bar{\gamma}_0 \approx \frac{\bar{\gamma}_{\mathrm{long}}}{2}\left(1+\sqrt{1+\frac{4P_{\mathrm{el}}}{\bar{\gamma}_{\mathrm{long}} Ad^{\alpha}}}\right) \quad . \tag{44}$$

Finally, using the first order Taylor approximation $\sqrt{1+x} \approx 1 + x/2$ in (44), we obtain

$$\bar{\gamma}_1 \triangleq \bar{\gamma}_{\mathrm{long}} + \frac{P_{\mathrm{el}}}{Ad^{\alpha}} \quad . \tag{45}$$

Clearly, $\bar{\gamma}_1$ is only a valid approximation for $\bar{\gamma}_0$ (and hence for the optimal SNR) for large link distances, but it is a useful rule of thumb. It also reflects the counter-intuitive fact that the optimal SNR *increases* as distance decreases. This aspect is analized in Section IV-A.

C. Channels with Nakagami-m fading statistics

The Nakagami-m distribution happens naturally when modeling maximum ratio combining receivers without power gain under independent Rayleigh fading per receiver branch [26]. It is often used as an approximation for channels with Ricean statistics, as we do in the sequel, because the probability of symbol error for Ricean channels does not lend itself well for analytical treatment of (27).

The SNR of a Nakagami-m channel is a Gamma-distributed random variable [25], parametrized by its mean SNR $\bar{\gamma}$ and its diversity degree m [27]. The symbol error rate of square M-QAM modulations under Nakagami-m fading can be calculated as [25]

$$\bar{P}_{\mathrm{s}}(\bar{\gamma}) = 2cI\left(\frac{\pi}{2}\right) - c^2 I\left(\frac{\pi}{4}\right) \quad , \tag{46}$$

with $I(x)$ defined by the following integral:

$$I(x) = \frac{1}{\pi}\int_0^x \left(1+\frac{a\bar{\gamma}}{2m\sin^2\theta}\right)^{-m} \mathrm{d}\theta \quad . \tag{47}$$

Above, a and c are defined as in (37) and (40). We point out that general SER formulas for arbitrary rectangular M-QAM modulations exist [28], but we restrict our analysis to square modulations for the sake of mathematical tractability.

We propose the following approximation for (46):

$$\bar{P}_{\mathrm{s}}(\bar{\gamma}) \approx hc\left(1.2+\frac{a\bar{\gamma}}{h}\right)^{-h} \quad , \tag{48}$$

where $h = (14m-4)/(m+9)$. This approximation is adecuate for $10^{-5} < \bar{P}_{\mathrm{s}} < 0.5*10^{-2}$, $M = \{4, 16, 64\}$ and $m \in [1, \infty)$.

Defining $u = 1.2 + a\bar{\gamma}/h$, and using (48) in (27), the following polynomial can be found:

$$u^{h+1} - \alpha u - \beta = 0 \quad . \tag{49}$$

Above, $\alpha = hc(1+\lambda h)$ and $\beta = hc\lambda[aP_{\mathrm{el}}/(Ad^{\alpha}) - 1.2h]$. For $h = 1, 2$ and 3, the polynomial can be solved using well-known closed-form formulas. When $h \geq 4$ (this implies $m \geq 4$) there exists no closed-form formula for u (Galois Theorem, [29]). To find an approximate solution for the general case, consider the function $\phi(u) = u^{h+1} - \alpha u$. The largest real root of this polynomial is $\alpha^{1/h}$, and the Taylor series expansion of first order centered on this root is:

$$\phi(u) \approx \alpha h(u - \alpha^{\frac{1}{h}}) \quad . \tag{50}$$

Equating the right side of the above expression to β and solving for u gives the following approximated solution to (49):

$$\hat{u} = \alpha^{\frac{1}{h}} + \frac{\beta}{\alpha h} \quad , \tag{51}$$

Expression (51) allows for determining an approximate optimal SNR $\bar{\gamma}_0$ by a simple change of variables, resulting:

$$\bar{\gamma}_0 = \frac{h}{a}(\hat{u} - 1.2) \tag{52}$$

$$= \frac{1}{a}\left(h\alpha^{1/h} + \frac{\beta}{\alpha} - 1.2h\right) \quad . \tag{53}$$

The estimate $\bar{\gamma}_0$ allows for quick back-of-the-envelope calculations.

The accuracy of (53) decreases for large values of β, which are found in short transmission distances ($d \leq 50$ m for typical low power devices), when $P_{\mathrm{el}}/(Ad^{\alpha}) \gg ah$ and therefore $\beta \approx hc\lambda a P_{\mathrm{el}}/(Ad^{\alpha}) \gg 0$. In these cases, higher order Taylor approximations are needed. For example, the third order approximation also results in (52) but with $\hat{u}$ as the only real root of the following third order polynomial:

$$(\hat{u} - \alpha^{\frac{1}{h}}) + \frac{(h+1)}{\alpha^{\frac{1}{h}}}(\hat{u} - \alpha^{\frac{1}{h}})^2 + \frac{(h+1)(h-1)}{\alpha^{\frac{2}{h}}}(\hat{u} - \alpha^{\frac{1}{h}})^3 = \frac{\beta}{h\alpha}, \tag{54}$$

which is the third order Taylor expansion of $\phi(u) = u^{h+1} - \alpha u$ centered in $\alpha^{\frac{1}{h}}$, equated to β.

It is to be noted that the expression given by equation (39) does not provide a closed-form solution for the optimal SNR in AWGN channels, whereas (53) and (54) does give explicit, albeit approximate, formula for it for $m \to \infty$.

We also point out that the expressions presented in (53) and (54) can be used for Ricean channels of parameter $K \geq 0$ by choosing $m = (1+K)^2/(1+2K)$ [30].

D. Numerical evaluations

Using the parameters of Table I we have calculated the average energy consumed by a communication system operated under the SNR given by the various approximations developed in this appendix (Figure 8). The energy consumption achieved using the approximations presented closely matches the minimal consumption found by solving (24) numerically.

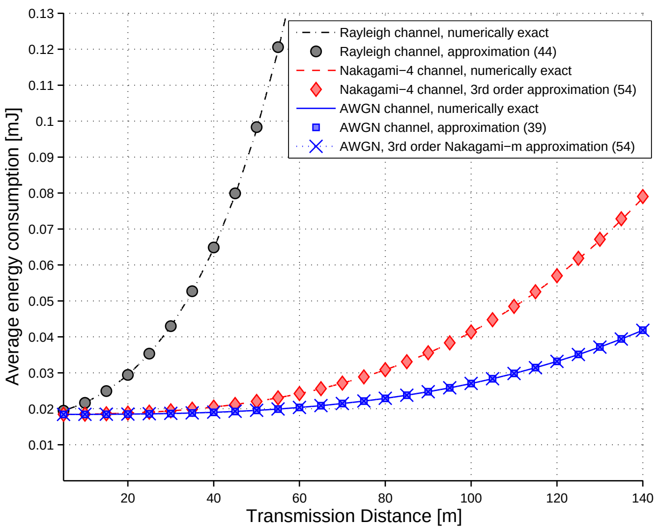

Fig. 8. Average energy consumption using the optimal SNR determined numerically form (24) (lines) and using the various approximations proposed in Appendix B (markers) for QPSK.

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V

Nakagami-m approximations for MIMO SVD transmissions

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Nakagami-m approximations for MIMO SVD transmissions

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***Abstract*—MIMO SVD modulation is an efficient way of sending data through a multi-antenna communications link in which the transmitter has knowledge of the channel state. Despite its importance, no simple formula for its symbol error rate (SER) has been found, and hence no intuitive characterization of the quality of this technique for data transmission is available at the present. In this paper, we present a method for approximating the statistics of each eigenchannel of MIMO SVD using the Nakagami-m fading model. Using our method, we show that the SER of the entire MIMO SVD link can be approximated by the average of the SER of Nakagami-m channels. The expression found is simple and yet accurate. This leads to characterize the eigenchannels of $N \times N$ MIMO channels with N larger than 14, showing that the smallest eigenchannel distributes as a Rayleigh channel, the next four eigenchannels closely distributes as Nakagami-m channels with $m = 4, 9, 25$ and 36, and the $N - 5$ remaining eigenchannels have statistics similar to an AWGN channel within 1 dB SNR. We also show that 75% of the total mean power gain of the MIMO SVD channel goes to the top third of all the eigenchannels.**

I. Introduction

The *multiple-input multiple-output singular value decomposition* (MIMO SVD) modulation is widely known as an efficient way for sending data through a multi-antenna communications link in which the transmitter has knowledge of the channel state [1]. Consider a MIMO channel in which the received signal vector $\boldsymbol{v} = (v_1, \dots, v_{N_r})^t$ can be expressed in terms of the transmitted symbol vector $\boldsymbol{u} = (u_1, \dots, u_{N_t})^t$ as

$$\boldsymbol{v} = \boldsymbol{H}\boldsymbol{u} + \boldsymbol{w} \ , \tag{1}$$

where u_j is the complex symbol transmitted through the j-th antenna, v_i is the complex symbol received by the i-th transmission branch, $\boldsymbol{H}$ is a random matrix with coefficients $h_{i,j}$ which are i.i.d. standard complex normal random variables and $\boldsymbol{w} = (w_1, \dots, w_n)^t$ is the vector of additive white gaussian noise terms experimented in each branch of the receiver [1]. Using the singular value decomposition, the channel matrix $\boldsymbol{H}$ is diagonalized creating $N = \min\{N_\mathrm{t}, N_\mathrm{r}\}$ non-interfering channels (*eigenchannels* in the following). This can be described as

$$y_k = \sqrt{\lambda_k} x_k + n_k \qquad k = 1 \dots N \ , \tag{2}$$

where x_k are the transmitted symbols, n_k is the thermal noise and $\sqrt{\lambda_k}$ are the singular values of the channel matrix $\boldsymbol{H}$ [2].

Despite the importance and popularity of the MIMO SVD modulation, no simple formula for the symbol error rate (SER) of the eigenchannels has been reported yet, even though the topic has seen much recent activity (e.g. [3]–[10]). The common approach for studying the statistics of the eigenchannels is to consider λ_k as eigenvalues of the complex Wishart matrix $\boldsymbol{W} = \boldsymbol{H}\boldsymbol{H}^\dagger$ [11]. The eigenvalues of $\boldsymbol{W}$ have a joint probability distribution (p.d.f.) given by [12]

$$p = K_N \exp\left(-\sum_{k=1}^{N} \lambda_k \right) \prod_{i=1}^{N} \lambda_i^{|N_\mathrm{t} - N_\mathrm{r}|} \prod_{i>j}^{N} (\lambda_i - \lambda_j)^2 \ , \tag{3}$$

with $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_N$ and K_N a constant. Deriving the statistics of each eigenchannel requires to determine the exact marginal p.d.f. $p_{\lambda_k}(\lambda_k)$ of each eigenvalue from (3). Following this approach, it is shown in [12] that the SNR of the smallest eigenchannel of a $N \times N$ MIMO channel have the same statistics as a Rayleigh channel with power gain $1/N$ (i.e. $p_{\lambda_1}(\lambda_1) = N e^{-N\lambda_1}$). Although expressions for the marginal p.d.f. of the other eigenvalues have been found [6]–[10], they are mathematically complex and do not provide much insight about the performance of the corresponding eigenchannels. In [13], it was shown that in the high signal-to-noise ratio (SNR) regime the SER of each eigenchannel can be expressed as

$$\bar{P}_s(\bar{\gamma}) = (G_c \bar{\gamma})^{-G_d} + o(\bar{\gamma}^{-G_d}) \ , \tag{4}$$

where $\bar{\gamma}$ is the SNR, G_c is the power gain of the channel and G_d is the diversity degree [14]. The limitation of this result is that the high-SNR restriction leads to insights of little practical interest. In [15], the idea of approximating the statistics of the largest eigenchannel by a Nakagami-m fading is presented. The value of m is chosen in order to approximate the outage statistics of this eigenchannel. Although the approximation thus obtained is accurate, it is not obvious if the proposed method can be extended to model the statistics of other eigenchannels.

In this paper we show that the statistics of all the eigenchannels of MIMO SVD transmissions over MIMO channels with i.i.d. Rayleigh fading statistics can be approximated using the Nakagami-m fading model. These approximations allow us to derive an accurate approximation for the SER of the entire MIMO SVD link, one that provides a strong insight on the performance of each eigenchannel. Using these approximations we will show that for any $N \times N$ MIMO system with $N > 15$ the following characterization of its eigenchannels can be made: there are $N-5$ eigenchannels which error statistics are similar to an AWGN channel (within 1 dB SNR), while the five weakest eigenchannels perform like a Rayleigh and Nakagami-m channels with $m = 4, 9, 16$ and 25, respectively.

The rest of the paper is organized as follows: Section II explains how Nakagami-m approximations can be calculated for each eigenchannel of a MIMO system. In Section III, a simple expression for the SER of MIMO SVD transmissions is derived using the Nakagami-m approximations. Section IV provides insights about the fading statistics of the eigenchannels, and introduces the characterization of the eigenchannels of $N \times N$ MIMO systems with $N > 15$. Finally, Section V presents our conclusions.

II. Statistical analysis of the MIMO eigenchannels

Denote by λ_k the k-th smallest eigenvalue of a Wishart matrix $\boldsymbol{H}\boldsymbol{H}^\dagger$. λ_k is the channel coefficient of the k-th eigenchannel of a MIMO SVD transmission through a $N_\text{r} \times N_\text{t}$ MIMO channel matrix $\boldsymbol{H}$ with i.i.d. Rayleigh statistics as in (1). Denote $N = \min\{N_\text{r}, N_\text{t}\}$ be the number of eigenchannels available for data transmission.

We seek to approximate the fading statistics of each eigenchannel using the well known Nagakami-m fading channel model [16], [17]. The power gain of the Nakagami-m channel, denoted here by λ, is a Gamma random variable with p.d.f. given by [16]

$$p_{\mu,m}(\lambda) = \left(\frac{m}{\mu}\right)^m \frac{\lambda^{m-1}}{\Gamma(m)} \exp\left(-\frac{m\lambda}{\mu}\right) \quad . \tag{5}$$

The Gamma distribution is characterized by the *mean power gain* μ of the channel, equal to the expected value of λ, and the *diversity degree* m of the channel. It can be shown that m is equal to the absolute value of the asymptotic logarithmic slope of the SER, i.e. $\bar{P}_s \propto \text{SNR}^{-m}$ for the high SNR regime [17]. The diversity degree m is also related to the outage statistics of the channel [14]. In effect, when $m = 1$, (5) turns into the p.d.f. of the exponential distribution, which corresponds to a Rayleigh channel, whose outage statistics are well known to be the most adverse. When $m \to \infty$, the channel statistics tend to those of the AWGN channel, which has no outage at all.

For a given $N_\text{r} \times N_\text{t}$ MIMO system, we seek to find parameters μ_k and m_k for which a Gamma random variable best fits the p.d.f. of the k-th eigenvalue λ_k. For a 2×2 MIMO system, these parameters can be determined by minimizing the mean square error between the marginal distributions found from (3) and a Gamma distribution (the actual derivation is presented in Appendix A). For larger system sizes, the mean square error minimization lead to intractable mathematics.

In our approach, we find values for m_k and μ_k by performing a maximum likelihood estimation (MLE) using computer generated samples of the eigenvalues. The MLE procedure finds the parameters m_k and μ_k of the Gamma distribution that has the largest probability of generating random numbers with the statistics observed in the given sample [18]. It has been shown in [19] that the Maximum Likelihood principle for the Gamma distribution is equivalent to choosing μ_k as

$$\mu_k = \frac{1}{n_0} \sum_{j=1}^{n_0} x_j \ , \tag{6}$$

and solving for m_k from

$$\ln(m_k) - \psi(m_k) = \ln\left(\frac{1}{n_0}\sum_{j=1}^{n_0} x_j\right) - \frac{1}{n_0}\sum_{j=1}^{n_0} \ln(x_j) \ . \tag{7}$$

Above, $\{x_j\}_{j=1}^{n_0}$ is a sample of n_0 realizations of the k-th eigenvalue and $\psi(x) = \Gamma'(x)/\Gamma(x)$ is the Psi (digamma) function [20]. Although (6) is the classic estimator of the mean value of a population, (7) gives a nontrivial estimator to its diversity degree.

For samples of size $n_0 = 10^6$ of a 2×2 MIMO channel, the MLE method gives values for m_1, m_2, μ_1 and μ_2 with less than 0.1% of error compared to the values found by minimizing the mean square error in the Appendix A (Table I). For larger MIMO systems, we have calculated the values for μ_k and m_k for some typical MIMO sizes using samples with $n_0 = 10^6$ (Table I). It is worth noting that these numbers provide the complete characterization of the Nakagami-m approximation of any MIMO system of the given sizes and Rayleigh fading statistics.

We tested the accuracy of the Nakagami-m approximations for 2×2, 4×4 and 8×8 Rayleigh fading MIMO channels using the Kolmogorov-Smirnov (KS) test [21]. The KS test provides an indication of the likelihood that a given sample comes from a population that follows a certain candidate probability distribution (see Appendix B). The tests were performed using channel samples of size $n_0 = 10^4, 10^5$ and 10^6, with Gamma distributions as candidates with parameters found using (6) and (7) and a false negative probability $\alpha = 5\%$.

TABLE I
Parameter values of the Nakagami-m approximations and Kolmogorov-Smirnov test results

2×2 MIMO System					
Eigenvalue	m	μ	$n_0 = 10^4$	$n_0 = 10^5$	$n_0 = 10^6$
λ_2	3.82	3.5	✓	✓	×
λ_1	1	0.5	✓	✓	✓
4×4 MIMO System					
Eigenvalue	m	μ	$n_0 = 10^4$	$n_0 = 10^5$	$n_0 = 10^6$
λ_4	12.72	9.77	✓	×	×
λ_3	8.66	4.41	✓	✓	✓
λ_2	4.09	1.57	✓	✓	×
λ_1	1	0.25	✓	✓	✓
8×8 MIMO System					
Eigenvalue	m	μ	$n_0 = 10^4$	$n_0 = 10^5$	$n_0 = 10^6$
λ_8	38	23.73	✓	×	×
λ_7	37.66	15.85	✓	×	×
λ_6	31.32	10.63	✓	✓	×
λ_5	23.52	6.82	✓	✓	✓
λ_4	15.89	4.02	✓	✓	×
λ_3	9.26	2.05	✓	✓	×
λ_2	4.15	0.76	✓	×	×
λ_1	1	0.125	✓	✓	✓

We found that λ_1 always passes the KS test for all system sizes (Table I). This makes sense because it follows a exponential distribution (i.e. a Gamma distribution with $m = 1$) [12]. The largest eigenvalue of a 2×2 MIMO channel passes the test for $n_0 = 10^4$ and 10^5, but fails it for $n_0 = 10^6$. This happens because the distribution of λ_2 does not belong to the Gamma family, even though there are values of μ and m that provide a very close fit, as shown in Appendix A. As n_0 grows, the KS test eventually fails because a larger sample provides more detailed information, which makes the test more sensitive.

The fact that the Nakagami-m approximations do not fail the test with a sample size of 10^4 (see Table I) is not to be overlooked. It means that 10^4 channel realizations do not contain enough evidence to conclude that the eigenchannels do not fit the Nakagami-m model.

III. Bit error rate approximation

N eigenchannels may be exploited in a MIMO system. Nevertheless, a relevant reduction of the mean SER can be achieved by transmitting data only through the $n < N$ eigenchannels with most favorable statistics, at the expense of lowering the data rate. This is a manifestation of the diversity-multiplexing tradeoff of MIMO channels [14].

In the sequel, we derive an approximation for the average SER of uncoded MIMO SVD communications using the method of Nakagami-m approximations presented in Section II. The approximation holds for any number of eigenchannels used. We also present upper and lower bounds for the SER, which are based only on the Nakagami-m approximation of the weaker eigenchannel in use.

A. SER approximation

Lets assume that the eigenchannels described by (2) are used to send data using a modulation whose SER for a given SNR γ can be expressed as

$$P_\text{s} = cQ(\sqrt{a\gamma}) \ . \tag{8}$$

Above, $Q(x)$ is the tail probability of the standard normal distribution and c and a are constants that depend on the choice of modulation scheme and constellation size. The SER of such a modulation over a Nakagami-m channel with power gain λ is calculated as

$$\begin{aligned}\bar{P}_\text{s}(\bar{\gamma}, \mu, m) &= \mathbb{E}\{cQ(\sqrt{a\lambda\bar{\gamma}})\} &(9)\\ &= \int_0^\infty cQ(\sqrt{a\lambda\bar{\gamma}})p_{\mu,m}(\lambda)\mathrm{d}\lambda \ , &(10)\end{aligned}$$

where λ is a Gamma random variable with p.d.f. $p_{\mu,m}(\lambda)$ as presented in (5).

Consider one realization of the eigenvalues $\lambda_1, \ldots, \lambda_N$. Then, the SNR of the k-th eigenchannel, as presented in (2), is $\text{SNR}_k = \lambda_k\bar{\gamma}_k$, where $\bar{\gamma}_k = P_k/\sigma_\text{n}^2$ is the ratio between the signal power P_k allocated to the k-th eigenchannel and the noise power σ_n^2. Lets define A_k as the event in which the transmitted symbol was sent through the k-th eigenchannel, and E the event that the symbol is decoded with error. Assuming that the $n \leq N$ eigenchannels with more favorable statistics are used equally often, then the probability of A_k is given by

$$\mathbb{P}(A_k) = \begin{cases} 1/n & \text{if } k \in \{\nu, \ldots, N\} \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

where $\nu = N - n + 1$ is just a shorthand notation for the eigenchannel to be used which has the least favorable statistics. As the A_k are jointly exhaustive and mutually exclusive events, we can decompose the error event E as $E = \cup_{k=1}^{N}(E \cap A_k)$, where each event $E \cap A_k$ denotes the event that the error has occurred in the k-th eigenchannel. Therefore, if we define $\boldsymbol{\gamma} = (\bar{\gamma}_1, \ldots, \bar{\gamma}_N)^t$ and $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_N)^t$, then the symbol error rate of a given channel realization can be calculated as:

$$\begin{aligned}P_\text{s}^{(n)}(\boldsymbol{\gamma}, \boldsymbol{\lambda}) &= \mathbb{P}(E|\boldsymbol{\gamma}, \boldsymbol{\lambda}) &(12)\\ &= \mathbb{P}\left(\cup_{k=1}^{N}(E \cap A_k)|\boldsymbol{\gamma}, \boldsymbol{\lambda}\right) &(13)\\ &= \sum_{k=1}^{N}\mathbb{P}\left(E \cap A_k|\boldsymbol{\gamma}, \boldsymbol{\lambda}\right) &(14)\\ &= \sum_{k=1}^{N}\mathbb{P}\left(A_k\right)\mathbb{P}\left(E|A_k, \boldsymbol{\gamma}, \boldsymbol{\lambda}\right) &(15)\\ &= \frac{1}{n}\sum_{k=\nu}^{N} cQ\left(\sqrt{a\lambda_k\bar{\gamma}_k}\right) &(16)\end{aligned}$$

The equality from (13) to (14) follows because for $k \neq j$, the events $E \cup A_k$ and $E \cup A_j$ are disjoint; the equality from (14) to (15) follows from the definition of conditional probability. The step from (15) to (16) considers the following rationale: as the values of $\lambda_1 \ldots \lambda_N$ are given, the error rates depends only on the thermal noise. As data symbols and noise in each eigenchannel are uncorrelated, symbol errors are independent events.

The mean SER is obtained by averaging $P_\text{s}^{(n)}(\boldsymbol{\gamma}, \boldsymbol{\lambda})$ over the possible eigenchannel coefficient realizations. If the channel coefficients have a joint p.d.f. given by (3), then, using (16) and the Nakagami-m approximations described in Section II, the mean SER can be approximated as:

$$\begin{aligned}\bar{P}_\text{s}^{(n)}(\boldsymbol{\gamma}) &= \mathbb{E}\left\{P_\text{s}^{(n)}(\boldsymbol{\gamma}, \boldsymbol{\lambda})\right\} &(17)\\ &= \frac{1}{n}\sum_{k=\nu}^{N}\mathbb{E}\left\{cQ\left(\sqrt{a\lambda_k\bar{\gamma}_k}\right)\right\} &(18)\\ &\approx \frac{1}{n}\sum_{k=\nu}^{N}\bar{P}_\text{s}(\bar{\gamma}_k, \mu_k, m_k) \ , &(19)\end{aligned}$$

where $\bar{P}_\text{s}(\bar{\gamma}_k, \mu_k, m_k)$ is the mean SER of a Nagakami-m channel, as given by (10), with m_k and μ_k determined by the method explained in Section II.

The accuracy of this approximation has been tested by simulating the transmission of 10^6 BPSK symbols over $N \times N$ MIMO systems of various sizes. For each transmitted symbol vector, an independent narrow-band MIMO channel matrix H was generated following the MIMO channel i.i.d. Rayleigh fading model. Statistics were calculated afterwards for each number n of used eigenchannels in the range $1 \leq n \leq N$. Each run of 10^6 symbols was evaluated at various SNR levels.

The results show that the approximation proposed in (19) is accurate for any choice of n (Figure 1).

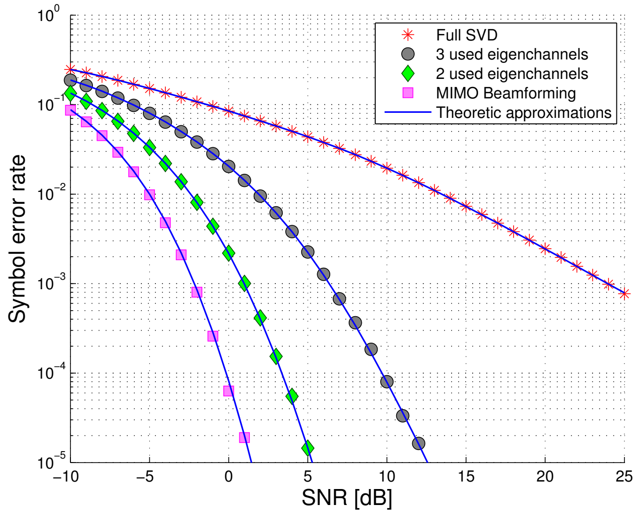

Fig. 1. Average SER of SVD transmissions over a 4×4 MIMO channel using uncoded BPSK and 1, 2, 3 or 4 eigenchannels of the SVD modulation. Markers show the average SER obtained by computer simulation of 10^6 symbols. Solid lines show the corresponding proposed approximation (19). An increasing diversity gain (i.e. the slope of the SER in the high SNR regime) can be observed as the number of used eigenchannels is reduced.

B. SER bounds

Consider equal power allocation among all eigenchannels to be used, i.e. $\bar{\gamma}_k = \bar{\gamma}$ for all $\nu \leq k \leq N$. In this case, the SER of the eigenchannel with less favorable statistics, $\bar{P}_{\mathrm{s}}(\bar{\gamma}, \mu_\nu, m_\nu)$, is larger than the SER of all the other eigenchannels in use. Hence, it gives an upper bound for the average SER of the n eigenchannels. For finding a lower bound, it is enough to realize that (19) is a sum of positive terms. Therefore, any of them is smaller than the sum. Thus, (19) can be upper and lower bounded as follows:

$$\bar{P}_{\mathrm{s}}(\bar{\gamma}, \mu_\nu, m_\nu) \geq \bar{P}_{\mathrm{s}}^{(n)}(\bar{\boldsymbol{\gamma}}) \geq \frac{1}{n} \bar{P}_{\mathrm{s}}(\bar{\gamma}, \mu_\nu, m_\nu) \quad . \tag{20}$$

where $\bar{\boldsymbol{\gamma}} = (\bar{\gamma}, \ldots, \bar{\gamma})$. Simulations show that he lower bound becomes exact as SNR $\rightarrow \infty$, because the error rate of eigenchannels with larger eigenvalues tends to zero faster than the error rate of the smallest used eigenchannel.

IV. Insights about the eigenchannels fading statistics

The idea of using Nakagami-m approximations for modeling the error statistics of MIMO SVD transmissions allows for reaching interesting insights about the eigenchannel fading statistics.

Throughout this section, we will denote the squared singular values of an $N \times N$ MIMO channel as λ_k^N, with $\lambda_1^N \leq \lambda_2^N \leq \cdots \leq \lambda_N^N$. We denote by m_k^N the diversity degree and μ_k^N the mean power gain of the Nakagami-m approximation of the eigenchannel that corresponds to λ_k^N, as developed in Section II. The convenience of the non-conventional ordering of the eigenchannels from smallest to largest will become apparent in Section IV-A2.

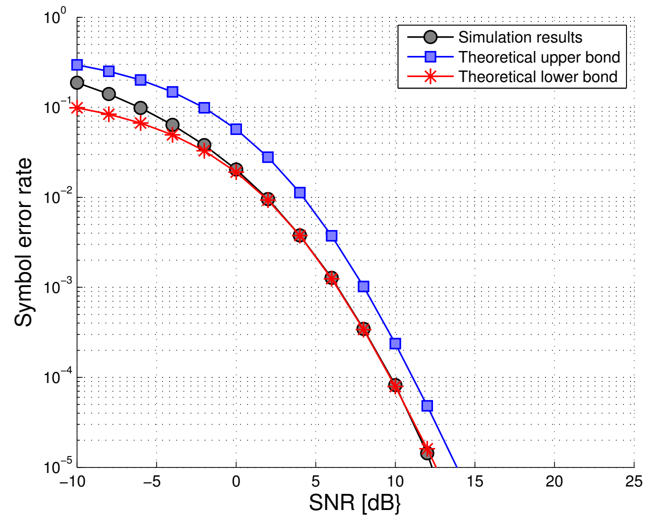

Fig. 2. Upper and lower bounds (20) for the SER of a 4×4 MIMO SVD link in which $n = 3$ out of $N = 4$ eigenchannels are used to transmit BPSK symbols. Simulation parameters as for Figure 1.

A. Insights about the diversity degree

1) Evolution of m_k^N as N grows: It is known that, for any system size, the diversity degree of the smallest eigenchannel is $m_1^N = 1$ [12]. For a given $k \neq 1$, numerical evaluations show that m_k^N tends monotonically to k^2 as N increases (Figure 3). The gap between m_k^N and k^2 is smaller than 10% for $N > 3k$.

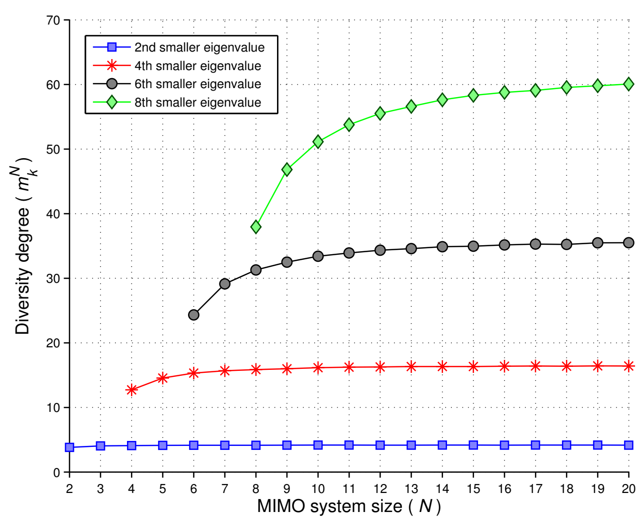

Fig. 3. For $3k \geq N \geq k$ the values of the diversity degree m_k^N grow approaching k^2, stabilizing for $N \geq 3k$.

2) Characterization of $N \times N$ MIMO systems with $N \geq 15$: As m grows, the error statistics of the Nakagami-m channel converge to the statistics of an AWGN channel. For example, if we consider symbol error rates above 10^{-5}, the error curves of any Nakagami-m channels with $m \geq 30$ differ by less than

1 dB from the error curve of the AWGN channel (Figure 4) (more precision can be achieved by increasing the value of m). Based on this fact, and on the observations presented in Section IV-A1, we can formulate the following qualitative characterization of the N eigenchannels of a $N \times N$ MIMO system with $N \geq 15$:

- The smallest eigenchannel behave like a Rayleigh channel (corresponding to λ_1^N).
- The next four channels satisfy $k \leq N/3$. They behave statistically as Nakagami-m eigenchannels with $m = 4, 9, 16$ and 25 (corresponding to $\lambda_2^N \dots \lambda_5^N$).
- The remaining $N - 5$ eigenchannels (corresponding to $\lambda_6^N \dots \lambda_N^N$) behave like AWGN channels, because $m_k^N > 30$ holds for all of them.

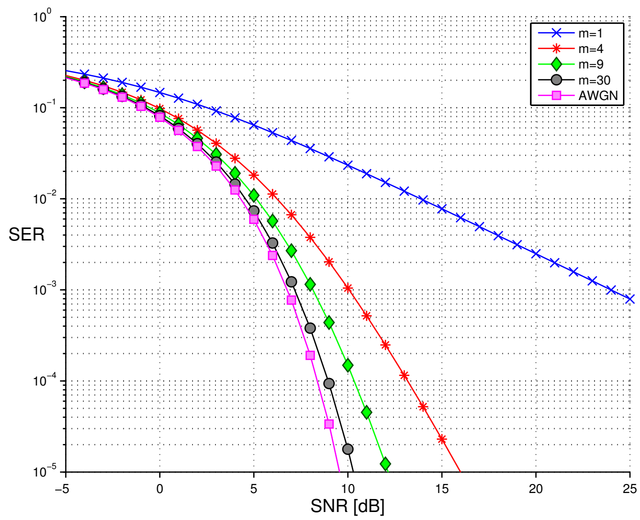

Fig. 4. Symbol error rate of BPSK transmissions over Nakagami-m channels for various values of m. As m increases, the curves approach the curve of the AWGN channel. For $m \geq 30$, the Nakagami-m and the AWGN curves differ by less than 1 dB for $P_s > 10^{-5}$.

B. Insights about the mean power gain μ_k^N

1) Evolution of μ_k^N as N grows: It is known that for any system size the mean value of the power gain of the smallest eigenchannel is $\mu_1^N = 1/N$ [12]. Numerical evaluations of the mean power gains of the remaining eigenchannels suggest that μ_k^N can be approximated by

$$\mu_k^N \approx \frac{\nu(k)}{N} \quad , \tag{21}$$

with $\nu(k)$ a function independent of N (Figure 5).

2) Mean value of the largest eigenvalue μ_N^N: By defining $M_N = \sum_{k=1}^{N} \mu_k^N$ as the total mean power gain of an $N \times N$ MIMO system, μ_N^N can be expressed as:

$$\mu_N^N = M_N - \sum_{n=1}^{N-1} \mu_k^N \quad . \tag{22}$$

Considering (21) for μ_k^N and μ_k^{N-1}, it is immediate that $N\mu_k^N \approx (N-1)\mu_k^{N-1}$. Using this in (22), the following

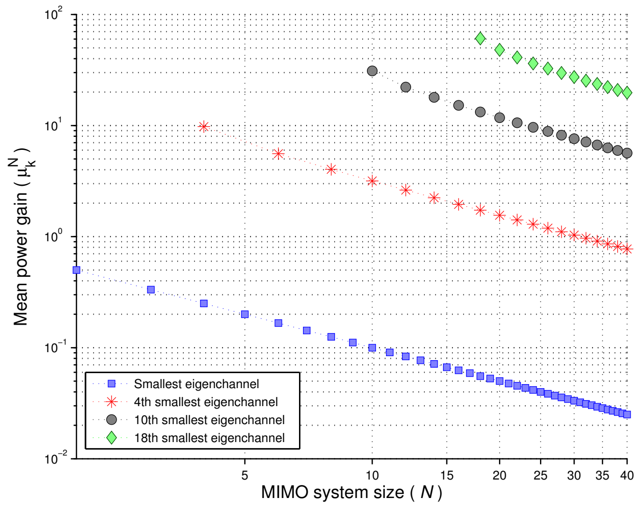

Fig. 5. The log-log plot suggests that, for a given k, the mean power gain has a linear dependence on $1/N$.

approximate expression for μ_N^N can be found:

$$\mu_N^N \approx M_N - \frac{N-1}{N} M_{N-1} \quad . \tag{23}$$

Lets calculate the value of M_N. As the trace of a symmetric matrix is equal to the sum of its eigenvalues, it can be shown that

$$\sum_{k=1}^{N} \lambda_k^N = \mathrm{Tr}(\boldsymbol{W}), \tag{24}$$

where $\boldsymbol{W}$ is a random matrix which distributes following a complex central Wishart distribution $\mathcal{W}_n(n, I)$ [12]. Applying the expected value operator $\mathbb{E}\{.\}$ at both sides of (24), and using the fact that $\mu_k^N = \mathbb{E}\left\{\lambda_k^N\right\}$, it is immediate that $M_N = \mathbb{E}\left\{\mathrm{Tr}(\boldsymbol{W})\right\}$. Furthermore, it can be proved that $\mathbb{E}\left\{\mathrm{Tr}(\boldsymbol{W})\right\} = N^2$ [22], and therefore the total mean power gain of a $N \times N$ MIMO system is

$$M_N = N^2 \quad . \tag{25}$$

Finally, replacing (25) in (23), the following rule of thumb for the mean power gain of the largest eigenchannel results:

$$\mu_N^N \approx 3N - 3 + \frac{1}{N} \quad . \tag{26}$$

Because $3N > 1/N$, the dependence of μ_N^N on the system size is dominantly linear, as confirmed by numerical evaluations. This observation is further supported by results about the asymptotic behavior of the mean value of the largest eigenvalue of Wishart matrices [23].

3) Relative mean power gain: We want to understand how the mean total power gain is distributed among the N eigenchannels. For this, we define the relative mean power gain $\hat{\mu}_k^N = \mu_k^N / \mu_N^N$. Numerical evaluations of $\hat{\mu}_k^N$ show that it is essentially a function of the quotient k/N. This observation has prompted us to consider

$$\hat{\mu}_k^N \approx \left(\frac{k}{N}\right)^2 e^{\frac{k}{N}-1} \tag{27}$$

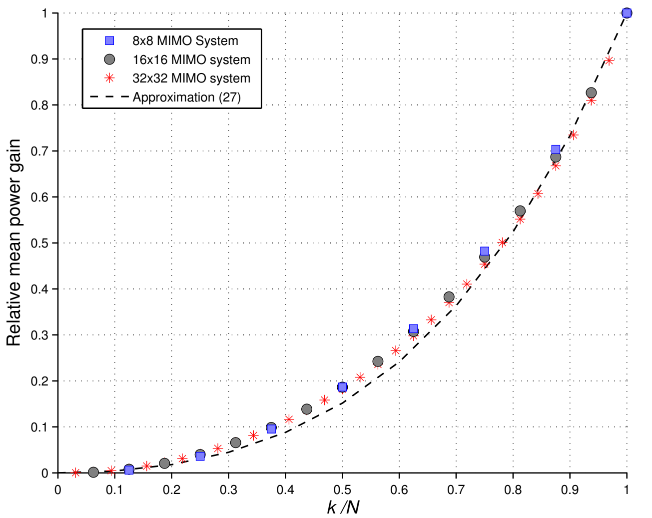

Fig. 6. The relative mean power gains ($\hat{\mu}_k^N$) are essentially a function of k/N. The resulting curve can be approximated by (27).

as an empirical model for the relative power gains (see Figure 6).

From (27), many questions about the mean power gain distribution among the eigenchannels can find an answer by studying the curve x^2e^{x-1}. The convexity of the curve says that the differences $\mu_{k+1}^N - \mu_k^N$ grow with k. Also, by studying the area $\mathcal{A}(y) = \int_0^y x^2 e^{x-1}\mathrm{d}x$, one can conclude that 75% of the mean power gain is shared by the top third of all the eigenchannels (i.e. $k/N > 2/3$).

V. Conclusions

We have developed a method for approximating the statistics of the eigenchannels of MIMO SVD communications using the Nakagami-m fading model. Maximum Likelihood Estimation of Nakagami-m channel parameters is performed for each of the eigenchannels of the MIMO system. The accuracy of the results was shown by minimizing the mean square error for the 2×2 case, and using the Kolmogorov-Smirnov test for larger system sizes.

The proposed method was used as a starting point for deriving an approximation of the mean SER of MIMO SVD transmissions. The approximation found is simple, accurate, easily computable and provides intuition about the quality of the channel for data transmission. Using this approximation, we have presented an upper and lower bound for the SER of MIMO SVD based just on the SER of the weakest eigenchannel used for the transmission.

We also show that, for $N > 15$, the eigenchannels of a $N \times N$ MIMO channel fit the following general characterization: the five eigenchannels with less favorable statistics (i.e. $k = 1, \dots, 5$) behaves like Nakagami-m channels with diversity degree $m_k = k^2$, while the remaining $N-5$ eigenchannels behave approximately like AWGN channels. We also provide a number of insights about the mean power gain of the eigenchannels, and show that 75% of the total mean power gain of the MIMO channel goes to the top third of all the eigenchannels.

Appendix A
Finding the Nakagami-m parameters for a MIMO 2×2

The parameters of a Nakagami-m approximation of a 2×2 MIMO system can be found as follows. From [12], we know that the smallest eigenvalue of such a system has a p.d.f. given by $\lambda_1 \sim \exp(1/2)$, which implies that $m_1 = 1$ and $\mu_1 = 1/2$. On the other hand, the probability distribution of the largest eigenvalue of a $N\times N$ MIMO system can be expressed as [24]

$$p_{\lambda_N}(\lambda_N) = \left(\prod_{j=1}^{N} \frac{1}{(N-j)!}\right)^2 \frac{d}{d\lambda_2}\det\{S(\lambda_2)\} \tag{28}$$

where $S(\lambda_2)$ is a matrix with coefficients given by $[S(\lambda_2)]_{i,j} = \int_0^{\lambda_2} x^{i+j-2}e^{-x}\mathrm{d}x$. Following [25], we find that for $N=2$, (28) can be simplified to

$$p_{\lambda_2}(\lambda_2) = \left[(\lambda_2)^2 - 2\lambda_2 + 2 - 2e^{-\lambda_2}\right]e^{-\lambda_2} \tag{29}$$

By defining $g_{\theta,m}(x)$ to be the p.d.f. of a Gamma random variable as defined in (5), parametrized on m and $\theta = \mu/m$, the mean square error between p_{λ_2} and $g_{m,\theta}$ can be expressed in terms of m and θ as

$$\int_0^\infty |p_{\lambda_2}(\lambda_2) - g_{m,\theta}(\lambda_2)|^2 \mathrm{d}\lambda_2 = 2m\theta\frac{2+\theta-m\theta}{(\theta+1)^{m+2}}\dots + \frac{4}{(2\theta+1)^m} - \frac{4}{(\theta+1)^m} + \frac{\Gamma(2m-1)}{2^{2m-1}\theta[\Gamma(m)]^2} + \frac{19}{108}\ . \tag{30}$$

This expression can be minimized on m and θ using numerical methods, finding a minimum in $m^* = 3.82$ and $\theta^* = 0.911$, which gives $\mu^* = k^*\theta^* = 3.48$. Using these values, the mean square error is $\approx 6\times 10^{-6}$, which shows that the two distributions p_{λ_2} and g_{m^*,θ^*} are identical for any practical purpose.

Appendix B
Introduction to the KS test

The KS test works as follows. Consider a random variable X, whose cumulative distribution function is $F(x)$, and some sample $\{y_j\}_{j=1}^{n_0}$, whose empirical distribution function is

$$F_0(y) = \frac{1}{n_0}\sum_{j=1}^{n_0} 1_{y_j}(y)\ . \tag{31}$$

Above $1_{y_j}(y)$ is the indicator function of y_j, whose value is 1 if $y_j \le y$ and zero otherwise. We wish to determine the likelihood of the hypothesis that *"the sample $\{y_j\}_{j=1}^{n_0}$ follows the distribution F(x)"*.

The KS test [21] is built on the statistic

$$D_{n_0} = \sup_{x\in[0,\infty]} |F_0(x) - F(x)|\ . \tag{32}$$

This statistic is a measure of the largest discrepancy between F and F_0. Using (32), the KS test estimates the probability that the empirical distribution of a sample of size n_0, generated according to the c.d.f. of X, has a discrepancy of magnitude D_{n_0} from F. This probability, which we will denote as

$\mathbb{P}(D_{n_0})$, is used to test the hypothesis, which is is rejected if $\mathbb{P}(D_{n_0})$ is below a certain threshold value α. Common values of α range between 1% and 10%, and represent the probability of rejecting the hypothesis when it is correct (false negative). As the sample size n_0 grows, $\mathbb{P}(D_{n_0})$ converges to the exact probability of observing a discrepancy D_{n_0}, and hence the KS test improves it accuracy.

In the case presented in Section II, the KS test will eventually reject the proposed hypothesis as n_0 increases. This happens because we know that the eigenvalues λ_k are not Gamma ($k = 1$ is an exception). The interesting element to observe in the test is the value of n_0 at which the KS test starts rejecting the hypotesis.

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VI

Energy efficient SVD MIMO communications

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Energy efficient SVD MIMO communications

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***Abstract*—Multiple-input multiple-output (MIMO) techniques can be used for reducing the energy consumption of wireless communications. Although some research has been reported on this topic, the rules by which the MIMO physical layer parameters should be chosen in order to achieve energy efficiency have not yet been formally established. In this paper, we analyze the case of MIMO singular value decomposition (SVD) technique. We present a model for the mean energy consumption of a MIMO SVD system per data bit transferred without error.**

We find that, for a given number of eigenchannels used with equal power allocation, exists a single optimal radiation power level at which the mean energy consumption is minimized. We also find that beamforming (only the best eigenchannel is used) is optimal in the energy consumption sense for long transmission distances, while the optimal number of eigenchannels to be used grows as transmission distance shortens. Using all the eigenchannels is optimal only for very short transmission distances.

I. Introduction

Multiple-input multiple-output (MIMO) communication systems were originally introduced as a way for achieving higher data rates or for improving the reliability of wireless links [1]. More recently, researchers have started to realize that the MIMO techniques can also be used for reducing the energy consumption of wireless communications [2]–[11]. Despite this recent interest, the rules by which the main physical-layer parameters shall be chosen for attaining energy efficiency in MIMO systems have not yet been formally established [12].

The MIMO SVD technique is well known as an efficient method for sending data through a MIMO communications link in which the transmitter has knowledge of the channel state [13]. The core concept considers the diagonalization of the channel $\mathbf{H}$, which we will assume to be a full rank $N \times N$ matrix, for establishing N non interfering channels (henceforth *eigenchannels*), whose input-output relationships can be described as

$$y_k = \sqrt{\lambda_k} x_k + w_k \qquad k = 1 \dots N \ . \tag{1}$$

Above, k indexes the eigenchannels, x_k are the transmitted symbols, n_k are additive white Gaussian noise terms (AWGN) and $\sqrt{\lambda_k}$ are the singular values of the channel matrix $\mathbf{H}$ [14].

In this paper we extend our energy-consumption model for single antenna systems reported in [15] to the analysis of the energy consumption of MIMO SVD communications. Most of the existent models reported so far in the literature are based on the abstract definition of the capacity of a MIMO random fading channel [5]–[11]. These models are not adequate for determining attainable performances of concrete modulations with a specific number of eigenchannels used. Our model incorporates these elements, sharing some features with the one reported in [2] but having a more straightforward mathematical formulation that enables more insightful analysis. Moreover, our MIMO energy consumption model is also novel because it considers the effect of retransmissions required to guarantee error free transmissions.

Furthermore, although the MIMO SVD modulation provides, in general, N eigenchannels, there is no need to use them all. Using all the eigenchannels maximizes the data rate, but sacrifices symbol error rate (SER) [16]. Conversely, using only the $n < N$ largest eigenchannels yields a better SER but at the cost of decreasing the data-rate. Our model allows for optimizing the radiated power and the number of eigenchannels used for the transmission as a function of link distance. In effect, we show that beamforming ($n = 1$) is the energy-optimal transmission strategy for large transmission distances, while a larger number of eigenchannels is optimal for short transmission distances. Full SVD ($n = N$) is optimal only for very short link distances.

By analyzing the energy consumption as a function of the antenna array size (N), we show that single antenna systems are more energy-efficient for performing short range transmissions than multiple antenna systems. Conversely, multiple antenna systems of growing size achieve important savings when the link distance increases. Nevertheless, we show that an antenna array size exists, beyond which larger systems do not yield significant savings anymore.

The paper is organized as follows: Section II presents the energy consumption model, Section III specifies the dependence of the energy consumption on the signal-to-noise ratio (SNR), and Section IV presents an analysis of the energy consumption of a MIMO SVD system using various numbers of used eigenchannels (n), antenna array sizes (N) and link distances. Finally, Section V presents our conclusions.

II. Energy Consumption Model

Our goal is to determine the total energy that is necessary for transferring one bit of data successfully, henceforth called a *goodbit* [7], in a point-to-point packet-switched MIMO SVD communication. As in [15] we assume that every frame transmitted in the *forward* direction is matched by a feedback frame in the *reverse* direction that acknowledges correct reception or requests a re-transmission. We also assume that the irradiated power is determined based upon knowledge of the statistics of the signal-to-noise ratio (SNR) at the decision stage of the receiver. We further assume that all frames in both directions are always detected and that all feedback frames are decoded without error.

The energy consumption analysis has been made for a specific MIMO transceiver architecture, popular among academic [17], [18] and commercial [19] products (Figure 1). In the sequel, we present the analysis of the components of energy consumption of the MIMO SVD transceiver from the standpoint of a node that transmits one payload frame and receives the corresponding feedback frame (the reverse case —a transceiver that receives one payload frame and transmits the corresponding feedback frame— follows by analogy), followed by the analysis of the statistics of re-transmissions and finally by a synthesis of our total energy consumption model.

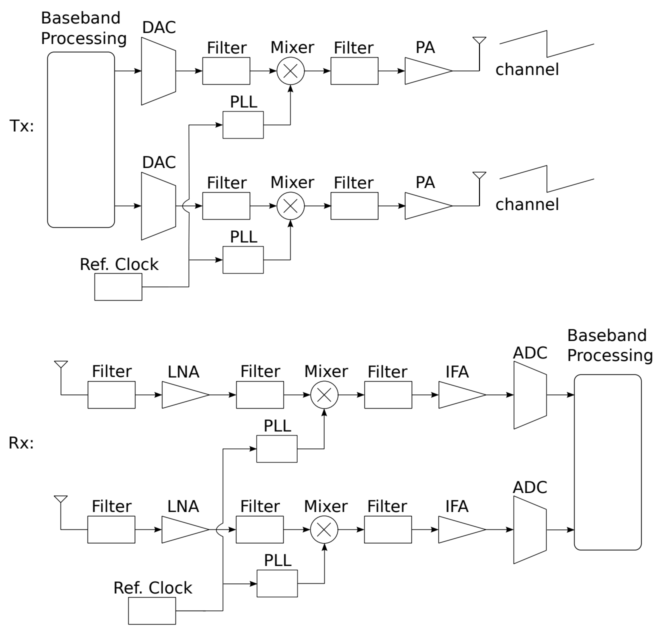

Fig. 1. Common architecture of a MIMO SVD transceiver.

A. Components of energy consumption of the forward transceiver

The energy consumption of the MIMO SVD transceiver that transmits forward frames and receives feedback frames is composed by five terms, each one described below.

1) Startup energy consumption: We assume that the transmitter is by default in a low power consumption (sleep) mode. Hence, it must be brought online before it can make a transmission. We will denote $\hat{\mathcal{E}}_{\text{st}}$ the total startup energy divided by the number of payload bits that are going to be transmitted before the transceiver goes into low power consumption mode again. In a MIMO system, $\hat{\mathcal{E}}_{\text{st}}$ is largely dominated by the energy spent in the stabilization of the N phase-lock-loops (PLL) of the transceiver (see Figure 1), while startup costs of components common to all branches are negligible [20]. Therefore $\hat{\mathcal{E}}_{\text{st}} = N\mathcal{E}_{\text{st}}$, where $\mathcal{E}_{\text{st}}$ is the startup energy consumption per branch.

2) Baseband electronic consumption: Performing the SVD of the MIMO channel matrix is the more demanding baseband operation. Each SVD computation involve K different arithmetic operations, each of which has an energy consumption $\mathcal{E}_k$ and is performed n_k times during the algorithm. Thus, the energy consumption of performing one SVD, $\mathcal{E}_{\text{SVD}}$, is given by

$$\mathcal{E}_{\text{SVD}} = \sum_{k=1}^{K} n_k \mathcal{E}_k \ . \tag{2}$$

If the operations are performed by an arithmetic processing unit (APU), the energy consumption of the k-th operation can be modeled as [21]

$$\mathcal{E}_k = V_{\text{dd}} I_0 \Delta t_k \ , \tag{3}$$

where V_{dd} is the APU operating voltage and I_0 is the average current during the execution time of the arithmetic operations. It is to be noted that I_0 depends on V_{dd} and on the clock frequency, f_{APU}. Δt_k is the time required for executing the k-th operation. It is related to f_{APU} and to the number of clock cycles required by the operation, c_k, as follows:

$$\Delta t_k = \frac{c_k}{f_{\text{APU}}} \ . \tag{4}$$

Replacing these terms in (2), the energy required for estimating the channel is given by

$$\mathcal{E}_{\text{SVD}} = \frac{V_{\text{dd}} I_0}{f_{\text{APU}}} \sum_{k=1}^{K} n_k c_k . \tag{5}$$

The SVD decomposition has to be performed each time the MIMO channel has significatly changed. This can be measured by the coherence time, which can be defined as [22]

$$T_{\text{c}} = \frac{9 c_0}{16 \pi v_{\text{m}} f_{\text{c}}} \ , \tag{6}$$

where c_0 is the speed of light, f_{c} is the carrier frequency and v_{m} is the maximum speed found in the mobile environment. Therefore, the energy consumption of performing one SVD decomposition is shared among the payload bits that are transmitted in one coherence time. That number of bits can be approximated by $\nu = nbR_{\text{s}}T_{\text{c}}$, where n is the number of eigenchannels used for the transmission (cf. Section I), $b = \log_2 M$ is the number of bits modulated in each M-ary data symbol and R_{s} is the symbol-rate per eigenchannel. Therefore, the energy consumption per bit of the baseband processing can be expresed as

$$\mathcal{E}_{\text{baseband}} = \frac{\mathcal{E}_{\text{SVD}}}{\nu} \ . \tag{7}$$

3) RF electronic consumption: The total air time per forward frame is composed by T_L seconds used for the transmission of the L payload bits that compose the frame, T_H seconds for the transmission of the H bits that compose the frame header and T_{O} seconds used for the transmission of overhead signals for tasks such as acquisition, channel estimation, synchronization, frame parameters signaling, etc. The air time per bit is therefore

$$\hat{T}_{\text{b}} = \frac{T_L + T_H + T_{\text{O}}}{L} \ . \tag{8}$$

$T_{\rm O}$ is composed by overhead for acquisition, which depends linearly on the number of transmitter antennas (N), and tasks like synchronization, which are approximately independent of N. By noting that $L/T_{\rm L} = nbR_s$ is the total bit-rate of the MIMO system (where n and b are used as defined in Section II-A2), we may express $\hat{T}_{\rm b}$ as

$$\hat{T}_b = \frac{1}{R_{\rm s}}\left(\frac{1}{nb} + \frac{H}{nL} + \frac{NO_{\rm a} + O_{\rm b}}{L}\right) \ , \tag{9}$$

where $O_{\rm a}$ is the acquisition overhead per branch and $O_{\rm b}$ is the reminding overhead, each one measured in bits. Equation (9) assumes that the header bits are sent throught the n eigenchannels using a binary modution.

During the $\hat{T}_b$ seconds, the MIMO transceiver consumes $\hat{P}_{\rm el,tx}$ Watts, which is largely dominated by the consumption of passband processing components such as filters, mixers and frequency synthesizers engaged in the forward transmission [23]. It can be inferred from Figure 1 that $\hat{P}_{\rm el,tx}$ grows linearly with the number of antennas (N). Therefore, the energy per goodbit consumed in the transmission processing may be expressed as

$$\mathcal{E}_{\rm el,tx} = \hat{P}_{\rm el,tx}\hat{T}_{\rm b} = NP_{\rm el,tx}\hat{T}_{\rm b} \ , \tag{10}$$

where $P_{\rm el,tx}$ stands for the electric power consumed by each branch of the transmiter.

4) Energy consumption due to electromagnetic radiation: Each frame is aired out of all the N branches. We define $P_{\rm A}^{(j)}$ as the power irradiated by the antenna of the j-th branch, which is supplied by a corresponding power amplifier (PA) (Figure 1). The power consumption of the j-th PA, $P_{\rm PA}^{(j)}$, is modeled by

$$P_{\rm PA}^{(j)} = \frac{\xi}{\eta}P_{\rm A}^{(j)} \ , \tag{11}$$

where ξ is the peak-to-average ratio of the transmitted signal and η is the drain efficiency of the PA [23]. Thus, the energy per bit used for electromagnetic radiation is given by

$$\mathcal{E}_{\rm RF} = \left(\sum_{j=1}^{N} P_{\rm PA}^{(j)}\right)\hat{T}_{\rm b} = \hat{P}_{\rm PA}\hat{T}_{\rm b} \ , \tag{12}$$

where $\hat{T}_{\rm b}$ is given by (9), and we have defined $\hat{P}_{\rm PA}$ as a shorthand notation for the total power consumption of all the PA's.

5) Energy Consumption of Electronic Components due to the Processing of Feedback Frames: Feedback frames are assumed to last $\hat{T}_{\rm fb} = F/(nbR_{\rm s})$ seconds, where F is the number of bits that compose the feedback frame and b and $R_{\rm s}$ are as defined in Section II-A3. During that time, the MIMO receiver consumes $\hat{P}_{\rm el,rx}$ Watts, which mainly includes the power needed to energize the passband receiver elements (low-noise amplifiers, mixers, filters, frequency synthesizers, etc.) of all the branches [23]. Hence, it grows linearly with the number of antennas (N). Therefore, the energy per forward payload bit spent by the transmitter for decoding the corresponding feedback frame is

$$\mathcal{E}_{\rm fb,rx} = \hat{P}_{\rm el,rx}\frac{\hat{T}_{\rm fb}}{L} = NP_{\rm el,rx}\frac{F}{nbR_{\rm s}L} \ , \tag{13}$$

where $P_{\rm el,rx}$ is the electronic power consumption of one branch of the receiver.

B. Re-transmission statistics

A key contributor to the energy consumption is the need for re-transmissions due to forward frames that get decoded with errors at the receiver. The number of trials, τ, until a frame is decoded without error is a random variable, whose mean value has been shown to be [15]

$$\bar{\tau} = 1 + \sum_{r=1}^{\infty}\mathbb{E}\left\{\prod_{i=1}^{r}P_{\rm f}(i)\right\} \ , \tag{14}$$

where $\mathbb{E}\{\cdot\}$ denotes the expectation operator and $P_{\rm f}(i)$ is the probability of decoding the frame with error during the i-th transmission trial. In general, the $P_{\rm f}(i)$ are random variables that depend on the number of antennas, the frame size, modulation type, the number of eigenchannels used in the link and the received SNR during the i-th trial. It is to be noted that (14) is valid for any correlated or uncorrelated channel fading statistics [15].

C. Total energy per goodbit

The material presented in Sections II-A and II-B allows for stating our model of the total energy consumption. Concretely, the energy consumed per goodbit by the transmitter of forward frames, which also decodes feedback frames, is given by

$$\mathcal{E}_{\rm T} = \mathcal{E}_{\rm st} + (\mathcal{E}_{\rm baseband} + \mathcal{E}_{\rm el,tx} + \mathcal{E}_{\rm RF} + \mathcal{E}_{\rm fb,tx})\tau \tag{15}$$

$$= \mathcal{E}_{\rm st} + \left[\frac{\mathcal{E}_{\rm SVD}}{\nu} + (\hat{P}_{\rm el,tx} + \hat{P}_{\rm PA})\hat{T}_{\rm b} + \frac{\hat{P}_{\rm el,rx}\hat{T}_{\rm fb}}{L}\right]\tau \ . \tag{16}$$

By analogy, the total energy used by the receiver for demodulating τ forward transmissions and for transmitting the corresponding τ feedback frames (recall that the SVD algorithm must also be performed at the receiver for findind the decoding matrix), is

$$\mathcal{E}_{\rm R} = \mathcal{E}_{\rm st} + \left[\frac{\mathcal{E}_{\rm SVD}}{\nu} + \hat{P}_{\rm el,rx}\hat{T}_{\rm b} + \left(\hat{P}_{\rm el,tx} + \hat{P}_{\rm PA}\right)\frac{\hat{T}_{\rm fb}}{L}\right]\tau \ . \tag{17}$$

The total energy consumption per goodbit is the sum of (15) and (17):

$$\mathcal{E}_{\rm b} = 2\mathcal{E}_{\rm st} + \left[2\frac{\mathcal{E}_{\rm SVD}}{\nu} + (\hat{P}_{\rm el} + \hat{P}_{\rm PA})\left(\hat{T}_{\rm b} + \frac{\hat{T}_{\rm fb}}{L}\right)\right]\tau \tag{18}$$

$$= \hat{S} + \left[\hat{B} + (\hat{P}_{\rm el} + \hat{P}_{\rm PA})\hat{T}\right]\tau \ , \tag{19}$$

where we have defined $\hat{P}_{\rm el} = (\hat{P}_{\rm el,tx} + \hat{P}_{\rm el,rx})$ as the total power consumed by electronic components, $\hat{S} = 2\mathcal{E}_{\rm st}$ as the total startup energy consumption per bit, $\hat{B} = 2\mathcal{E}_{\rm SVD}/\nu$ as the total baseband processing consumption per bit and $\hat{T} = (\hat{T}_{\rm b} + \hat{T}_{\rm fb}/L)$ the total time per bit per transmission trial.

Because of τ, $\mathcal{E}_{\rm b}$ is a random variable that depends on the realizations of the channel and the thermal noise. Its mean value is

$$\bar{\mathcal{E}}_{\rm b} = \mathbb{E}\{\mathcal{E}_{\rm b}\} = \hat{S} + \left[\hat{B} + (\hat{P}_{\rm el} + \hat{P}_{\rm PA})\hat{T}\right]\bar{\tau} \ , \tag{20}$$

with $\bar{\tau}$ as given by (14).

III. Energy consumption as a function of the SNR

We seek an explicit expression for the dependence on the SNR of the energy consumption per goodbit of a MIMO SVD system, in which only the n eigenchannels with the largest singular values are used for data transmission in each frame (see Section I). To achieve this, we first analyze the PA's total power consumption ($\hat{P}_{\text{PA}}$) and the mean number of transmission trials ($\bar{\tau}$) as functions of the SNR. Finally, we present how the mean energy consumption can be minimized by optimizing the SNR.

A. PA's total power consumption as function of the SNR

The transmission power that has been allocated to the k-th eigenchannel, $\bar{P}_{\text{tx}}^{(k)}$ ($k \in \{1, \ldots, n\}$), attenuates over the air with path loss and arrives at the receiver with a mean power given by

$$\bar{P}_{\text{rx}}^{(k)} = \frac{\bar{P}_{\text{tx}}^{(k)}}{A_0 d^\alpha} \ , \tag{21}$$

where A_0 is a parameter that depends on the transmitter and receiver antenna gains and the transmission wavelength, d is the distance between transmitter and receiver and α is the path loss exponent. The SNR of the k-th eigenchannel is given by

$$\text{SNR}_k = \frac{\lambda_k \bar{P}_{\text{rx}}^{(k)}}{\sigma_{\text{w}}^2} = \lambda_k \phi_k \bar{\gamma} \ , \tag{22}$$

where λ_k is the square of the k-th singular value of the channel matrix H (cf. (1)), σ_{w}^2 is the noise power, $\bar{\gamma} = (\sum_{k=1}^n \bar{P}_{\text{rx}}^{(k)})/\sigma_{\text{w}}^2$ is the total SNR (which may be produced if all the radiated power was allocated to only one eigenchannel) and $\phi_k = \bar{P}_{\text{rx}}^{(k)}/(\sum_{k=1}^n \bar{P}_{\text{rx}}^{(k)})$ is the percentage of the total SNR that goes to the k-th eigenchannel. Furthermore, we can express the noise power as $\sigma_{\text{w}}^2 = N_0 W N_{\text{f}} M_{\text{L}}$, where N_0 is the power spectral density of the baseband-equivalent additive white Gaussian noise (AWGN), W is the transmission bandwidth, N_{f} is the noise figure of the receiver's front end and M_{L} is a link margin term which represents any other additive noise or interference [2].

Finally, using the result presented in Appendix A along with (21) and (22), the following relationship can be found:

$$\hat{P}_{\text{PA}} = \frac{\xi}{\eta} \sum_{k=1}^{n} P_{\text{tx}}^{(k)} = \frac{\xi A_0 d^\alpha}{\eta} \sum_{k=1}^{n} P_{\text{rx}}^{(k)} \tag{23}$$

$$= \frac{\xi A_0 d^\alpha \sigma_{\text{w}}^2}{\eta} \bar{\gamma} = A d^\alpha \bar{\gamma} \ , \tag{24}$$

with A a constant.

B. $\bar{\tau}$ as function of the SNR

Assume that the probabilities of frame error of each transmission trial, $\{P_{\text{f}}(i)\}_{i=1}^{\infty}$ (cf. (14)), are a set of i.i.d. random variables. Then, define their mean value as $\bar{P}_{\text{f}} := \mathbb{E}\{P_{\text{f}}(i)\}$. Using these conditions on (14), it can be shown that [15]

$$\bar{\tau} = \left(1 - \bar{P}_{\text{f}}\right)^{-1} \ . \tag{25}$$

Lets assume that H bits of header are transmitted using a binary modulation for minimizing their probability of error, and L bits of payload are modulated using an uncoded M-ary modulation with $b = \log_2 M$ bits per symbol. Hence, the frame is composed by H binary symbols and L/b M-ary symbols. We will assume that the transmitter is equipped with a deep interleaver [24], which completely decorrelates the MIMO channel between any sucessive symbols.

There are different ways in which the symbols that compose the frame can be fed into the SVD engine. Following [25], we will consider the case in which these symbols are assigned to the eigenchannels using a different order for each transmission trial in a pseudo-random fashion. This has been shown to outperform an ordered feeding [25]. Define A_k as the event in which a symbol is assigned to the k-th eigenchannel and E_{p} as the event that a payload symbol were decoded with error. As all events A_k are jointly exhaustive and mutually exclusive, we can decompose the error events as $E_{\text{p}} = \cup_{k=1}^{N} (E_{\text{p}} \cap A_k)$. Hence, for a given realization of the channel matrix $\mathbf{H}$, the average error rate of a payload symbol can be written in terms of the symbol error rate (SER), P_{s} , as

$$\mathbb{P}\{E_{\text{p}}\} = \mathbb{P}\left\{\cup_{k=1}^{n}(E_{\text{p}} \cap A_k)\right\} = \sum_{k=1}^{n} \mathbb{P}\left(E_{\text{p}} \cap A_k\right) \tag{26}$$

$$= \sum_{k=1}^{n} \mathbb{P}\left(A_k\right) \mathbb{P}\left(E_{\text{p}} | A_k\right) = \frac{1}{n} \sum_{k=1}^{n} P_{\text{s}}\left(\text{SNR}_k\right) \ , \tag{27}$$

where SNR_k is as defined in (22). Above, we are using that $\mathbb{P}(A_k) = 1/n \quad \forall k = 1 \ldots n$. In similar fashion, by defining E_{h} as the event that a header symbol is decoded with error, it can be proved that $\mathbb{P}\{E_{\text{h}}\} = \frac{1}{n} \sum_{k=1}^{n} P_{\text{bin}}\left(\text{SNR}_k\right)$, where P_{bin} is the SER of the binary modulation.

It is to be noted that these SER are valid for all the header and payload symbols that compose the frame. It follows that, for a given channel realization, the frame error rate can be expressed as $P_{\text{f}} = 1 - (1 - \mathbb{P}\{E_{\text{h}}\})^H (1 - \mathbb{P}\{E_{\text{p}}\})^{L/b}$. Using this and (27) in (25), the following is obtained:

$$\bar{\tau} = \left[1 - \bar{P}_2^{(n)}(\tilde{\phi}_k, \bar{\gamma})\right]^{-H} \left[1 - \bar{P}_M^{(n)}(\phi_k, \bar{\gamma})\right]^{-L/b} \ . \tag{28}$$

where we have defined the mean SVD SER as

$$\bar{P}_M^{(n)}(\phi_k, \bar{\gamma}) = \frac{1}{n} \sum_{k=1}^{n} \bar{P}_{\text{s}}(\lambda_k \phi_k \bar{\gamma}) \ , \tag{29}$$

which is the average of the mean SER of the used eigenchannels which use a M-ary modulation. Above, $\bar{P}_2^{(n)}(\tilde{\phi}_k, \bar{\gamma})$ is (29) for $M = 2$, and $\tilde{\phi}_k$ is the percentage of the total SNR that goes to the k-th eigenchannel during the transmission of the header.

C. Optimizing the SNR

In the following, we will present how the total SNR $\bar{\gamma}$ and coefficients ϕ_k and $\tilde{\phi}_k$ can be optimized to achieve energy efficiency.

1) Optimization of ϕ_k and $\tilde{\phi}_k$: The mean total energy consumption per goodbit (20) can be re-written using (24) and (28) as

$$\bar{\mathcal{E}}_{\mathrm{b}}(\phi_k,\tilde{\phi}_k,\bar{\gamma}) = \hat{S} + \ldots + \frac{\hat{B} + (\hat{P}_{\mathrm{el}} + Ad^{\alpha}\bar{\gamma})\hat{T}}{\left[1-\bar{P}_2^{(n)}(\tilde{\phi}_k,\bar{\gamma})\right]^H \left[1-\bar{P}_M^{(n)}(\phi_k,\bar{\gamma})\right]^{L/b}} \ . \quad (30)$$

The values of ϕ_k that minimize (30) are the ones that minimize $\bar{P}_M^{(n)}(\phi_k,\bar{\gamma})$. From (29), the optimal values $\phi_1^*,\ldots,\phi_n^*$ for a given realization of the fading coefficients $\lambda_1,\ldots,\lambda_n$, are given by

$$(\phi_1^*,\ldots,\phi_n^*) = \underset{\phi_1,\ldots,\phi_n}{\operatorname{argmin}} \left\{ \frac{1}{n}\sum_{k=1}^{n} P_{\mathrm{s}}(\lambda_k\phi_k\bar{\gamma}) \right\} \ , \quad (31)$$

for which one solution is presented in Appendix B. Using the solution to (31), the minimized mean SVD SER can be expressed as

$$\bar{P}_M^{(n)}(\phi_k^*,\bar{\gamma}) = \mathbb{E}\left\{ \frac{1}{n}\sum_{k=1}^{n} P_{\mathrm{s}}(\lambda_k\phi_k^*\bar{\gamma}) \right\} \ , \quad (32)$$

where the expectation is taken over the fading coefficients.

The values $\tilde{\phi}_1^*,\ldots,\tilde{\phi}_n^*$ that minimize $\bar{P}_2^{(\mathrm{n})}(\tilde{\phi}_k,\bar{\gamma})$ can be found as above using (31) and (32) for the specific case of a binary modulation.

2) Optimization of the total SNR: Using (31) and (32), the minimal energy consumption per goodbit for a given total SNR $\bar{\gamma}$ can be found by evaluating (30) at ϕ^* and $\tilde{\phi}^*$. It can be seen from (30) that the energy consumption is large at extreme values of the total SNR. In effect, if the SNR is low, then the symbol error rate tends to 1, in which case the denominator in (30) is small. This reflects the intuitive fact that at low SNR the energy consumption is high because of the large number of retransmissions needed for a succesful frame reception. On the contrary, at a high SNR, (30) is also large because the numerator is proportional to $\bar{\gamma}$. This reveals that the energy consumption is large because the irradiated power is excessive. We thus infer that an optimal SNR that minimizes the energy consumption must exist in between.

The previous analysis is analogous to the one made for single antenna systems in [15]. Following that work, we define the SNR at which the system attains a minimal average energy consumption as

$$\bar{\gamma}^* = \underset{\bar{\gamma}\in[0,\infty)}{\operatorname{argmin}}\, \bar{\mathcal{E}}_{\mathrm{b}}(\phi_k^*,\tilde{\phi}_k^*,\bar{\gamma}) \ , \quad (33)$$

which represents an optimal tradeoff between irradiation power and retransmission consumption.

IV. Optimization of the antenna array size

The goal of this section is to compare the energy consumption of MIMO systems of different sizes. For this, we first optimize the modulation size and number of eigenchannels to be used in order to minimize the energy consumption of a MIMO system of a given size. Then, we compare the optimal performances thus found amoung MIMO systems with different sizes, and study how the minimal energy consumption scales with the antenna array size.

A. Optimization of the number of eigenchannels used (n)

We begin by evaluating (30) at the optimal quantities ϕ_k^*, $\tilde{\phi}_k^*$ and $\bar{\gamma}^*$ (from (31) and (33)) and rewriting it so that the dependence on the number of bits per symbol (b), on the number of used eigenchannels (n) and on the antenna array size (N) becomes explicit. Concretely:

$$\bar{\mathcal{E}}_{\mathrm{b}}(\phi_k^*,\tilde{\phi}_k^*,\bar{\gamma}^*) = NS + \ldots + \frac{(nb)^{-1}B + [NP_{\mathrm{el}} + Ad^{\alpha}\bar{\gamma}^*]\hat{T}}{\left[1-\bar{P}_2^{(n)}(\tilde{\phi}_k^*,\bar{\gamma}^*)\right]^H \left[1-\bar{P}_M^{(n)}(\phi_k^*,\bar{\gamma}^*)\right]^{L/b}} \ , \quad (34)$$

where $S = 2\mathcal{E}_{\mathrm{st}}$, $B = 2\mathcal{E}_{\mathrm{SVD}}/(R_{\mathrm{s}}T_{\mathrm{c}})$ and $P_{\mathrm{el}} = P_{\mathrm{el,tx}} + P_{\mathrm{el,rx}}$ is the total electronic power consumption per transceiver branch (see Section II). It is to be notice that the quantities ϕ_k^*, $\tilde{\phi}_k^*$ and $\bar{\gamma}^*$ are also functions of b, n and N.

Numerical evaluations of (34) were performed using parameters of a typical low-power device, which are given in Tables I and II. Parameters $n_{\mathrm{add}}, n_{\mathrm{prod}}, n_{\mathrm{div}}$ and n_{root} where found calculating averanges over the iterative algorithms presented in [14] and [28].

TABLE I
Generic low-power device parameters

Parameter	Description	Value
f_{c}	Carrier frequency	2.4 GHz §
$v_{\max}$	Maximal mobility	3 m/s
W	Bandwidth	10 kHz †
R_{s}	Symbol rate	10 kBaud †
L	Frame Payload	98 bytes §
O_{a}	Adqusition overhead	1 byte §
O_{b}	Estimation and sincronization overhead	3 bytes §
H	Frame header	26 bytes §
F	Feedback frame length	11 bytes §
$\mathcal{E}_{\mathrm{st}}$	Start-up energy per branch	0.125 nJ †
α	Path-loss coefficient	3.2 ‖
A_0	Free space path loss	30 dB †
η	PA efficiency	35% †
$P_{\mathrm{el,tx}}$	Tx electric power consumption	98.2 mW †
$P_{\mathrm{el,rx}}$	Rx electric power consumption	112.5 mW †
N_0	Noise power density	-174 dBm/Hz
N_{f}	Receiver noise figure	10 dB †
M_{l}	Link margin	30 dB ‖

Consider now optimizing the modulation size for a given number n out of N eigenchannels in a link distance of d meters. Concretely, using (34), define

$$\bar{\mathcal{E}}_{\mathrm{b}}^*(d,n,N) = \min_{b\in\mathcal{M}} \bar{\mathcal{E}}_{\mathrm{b}}(\phi_k^*,\tilde{\phi}_k^*,\bar{\gamma}^*) \ , \quad (35)$$

where $\mathcal{M}$ is a set of modulations. Computing (35) for various M-QAM modulations, we find that smaller modulations are optimal for long link distances. The optimal modulation size grows as link distance decreases (see Figure 2). This generalizes the results presented in [15] for single antenna systems.

†Source: [20]
§Source: IEEE 802.15.4 standard [29]
‖Source: [15]
*Source: [30]
‡Source: [31]
††Source: [32]

TABLE II
APU PARAMETERS

Parameter	Description	Value
f_{ALU}	ALU frequency	20 MHz ‡
V_{dd}	ALU voltage	3 V ‡
I_0	Average current	6.37 mA ‡
c_{add}	Adding cost	6 cycles *
c_{prod}	Product cost	13 cycles *
c_{div}	Division cost	21 cycles *
c_{root}	Root cost	149 cycles ††
n_{add}	ALU cycles per addition	$\frac{16}{3}N^3 + 10N^2 - \frac{28}{3}N + 10$
n_{prod}	ALU cycles per multiplication	$\frac{16}{3}N^3 + 16N^2 - \frac{70}{3}N + 4$
n_{div}	ALU cycles per division	$4N^2 - 2N - 3$
n_{root}	ALU cycles per square root	$2N^2 - 3$

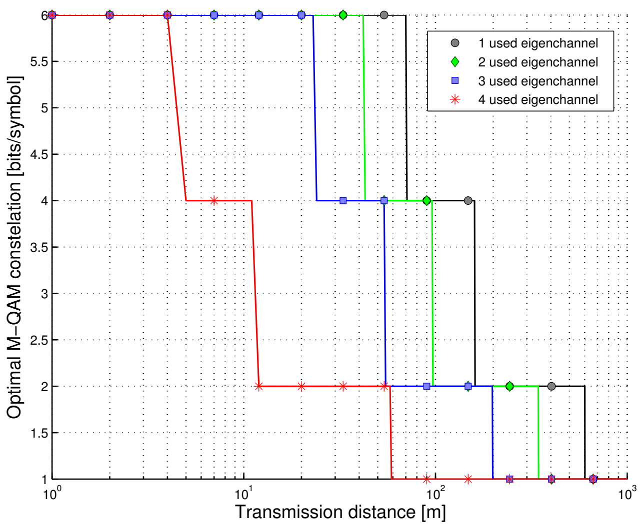

Fig. 2. Size of the M-QAM modulation which miminizes the energy consumption of a 4×4 MIMO system over various link distances. Small modulations are optimal for long link distances, while the optimal modulation size grows as the link distance shortens.

Evaluations of (35) for varying values of n shows that beamforming ($n = 1$), used together with BPSK, is the optimal choice for minimizing the energy consumption in long range communications (see Figure 3). This agrees with intuition, because the power consumed by the power amplifiers ($\hat{P}_{\text{PA}}$) dominates over the power consumed by the electronic components ($\hat{P}_{\text{el}}$) when the link distance is large. It is therefore convenient to reduce the irradiated power by investing it exclusively in the most favorable eigenchannel.

On the other end, full SVD ($n = N$) used along with spectrally efficient modulations is optimal in the energy sense for short transmission distances (see Figure 3). In this scenario, the power consumed by electronic components dominates over the consumption of the power amplifiers. The energy consumption (34) can therefore be approximated for this case as

$$\bar{\mathcal{E}}_{\text{b}} \approx NS + [(nb)^{-1}B + NP_{\text{el}}]\hat{T} \ . \qquad (36)$$

Hence, under these conditions it is attractive to increase the throuput in order to reduce the total transmission time per bit

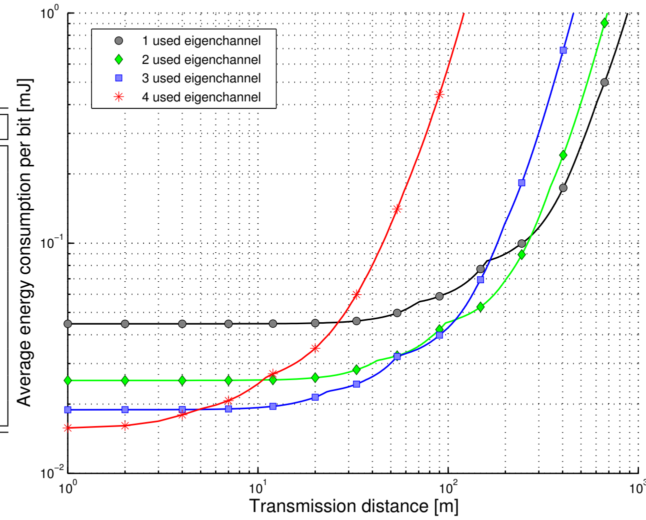

Fig. 3. Minimal energy consumption of 4x4 MIMO SVD communications using various numbers of used eigenchannels. Full SVD, where all the available eigenchannels are being used, is the optimal strategy for short link distances. On the contrary, beamforming (only using one eigenchannel) is optimal for long range communications.

$\hat{T}$.

B. Optimization of array size (N)

We define the minimal energy consumption per goodbit of an $N \times N$ MIMO system as

$$\bar{\mathcal{E}}_{\text{b}}^{\star}(d, N) = \min_{n \in \{1,\ldots,N\}} \bar{\mathcal{E}}_{\text{b}}^{*}(d, n, N) \ , \qquad (37)$$

where $\bar{\mathcal{E}}_{\text{b}}^{*}(d, n, N)$ is given by (35).

Numerical evaluations of (37) show that fpr long link distances large MIMO systems are more energy efficient than single antenna systems (see Figure 4). This is because the use of beamforming with a large antenna array generates important reductions in the frame error rate. These more favorable statistics allow in turn for reducing the irradiated energy needed to reach the receiver with an adequate SNR.

On the contrary, small antenna arrays are optimal for short transmissions distances (see Figure 4). Large antenna arrays are suboptimal, as the reduction in the time per bit achieved by a higher throughput is not enough to compensate the increase in the electric power $\hat{P}_{\text{el}}$, which is linear on N (see Equations (36) and (39)). The rol played by the MIMO overhead $(NO_{\text{a}} + O_{\text{b}})/L$ is critical, as it is the only term in (39) that do not decreases when the throughput grows.

It is important to notice that the above analysis continue to hold if different modulation sizes are used over the eigenchannels. In effect, it was shown in Section IV-A that, for short link distances, the minimal energy consumption is achieved maximizing the throuput, which is achieved using the larger allowable modulation over all the eigenchannels. In fact, using a different modulations over the eigenchannels will always reduce the overall throughput, which will increase the total time per bit increasing the mean energy consumption of short range communications. Similarly, beamforming equiped with

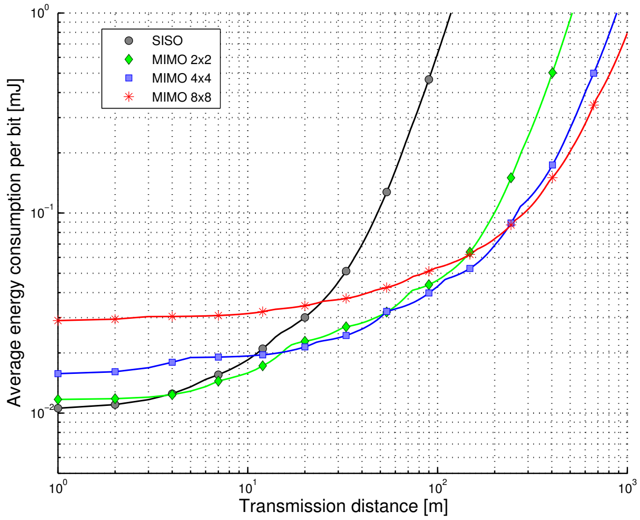

Fig. 4. Minimal energy consumption of different antenna array sizes. Single antenna systems consume less energy in short range communications, while large antenna arrays achieve better performances when the link distance is large.

BPSK will continue to be the optimal transmission strategy for large link distances, as the achieved mean SVD SER cannot be improved by any other combination of modulations and used eigenchannels [16].

C. Performance of very large MIMO systems

Section IV-B presents the idea that the strategy that minimizes the energy consumption of communications through long link distances is to use a binary modulation over beamforming performed on a large antenna array. We want to study how this performance scales when the array size grows.

In [16] is shown that the SER of beamforming in large MIMO systems ($N \geq 15$) is essentialy equal to the SER of an AWGN channel with a array gain μ_N (cite Dresden), i.e. $\bar{P}_2^{(1)}(\tilde{\phi}_k, \bar{\gamma}) \approx Q(\sqrt{2\mu_N\bar{\gamma}})$. Morover, at very long transmission distances the electronic consumption can be neglected. Therefore, the minimal energy consumption given by (37) can be approximated by

$$\bar{\mathcal{E}}_{\mathrm{b}}^{\star}(d,N) \approx \min_{\bar{\gamma}\in(0,\infty)} \frac{Ad^{\alpha}\bar{\gamma}\hat{T}}{\left[1-Q(\sqrt{2\mu_N\bar{\gamma}})\right]^{H+L/b}} \ , \tag{38}$$

where $\hat{T}$ is given by (see Section II)

$$\hat{T} = \frac{1}{R_{\mathrm{s}}}\left(\frac{1}{nb} + \frac{F}{nbL} + \frac{H}{nL} + \frac{NO_{\mathrm{a}}+O_{\mathrm{b}}}{L}\right) \ . \tag{39}$$

For large values of N, (39) can be approximated as $\hat{T} \approx N(O_{\mathrm{a}}/L)$. Also, simulations shows that, when beamforming is used over long transmission distances, the optimal SNR ($\bar{\gamma}^*$) is such that the average number of retransmissions is low, so $\bar{\tau}(\bar{\gamma}^*) = \left[1-Q(\sqrt{2\mu_N\bar{\gamma}^*})\right]^{-H-L/b} \approx 1$. This is achieved at an approximately fixed SNR level, which we will denote as $K = \mu_N\bar{\gamma}^*$. Therefore, (38) can be rewriten as

$$\bar{\mathcal{E}}_{\mathrm{b}}^{\star}(d,N) \approx \frac{N}{\mu_N}\left(\frac{Ad^{\alpha}KO_{\mathrm{a}}}{L}\right) \ . \tag{40}$$

Above, the term in parenthesis is a constant that do not depend on N. Morover, it has been shown that $\mu_N \propto N$ when N is large [16]. Hence, it can be seen from (40) that the benefits of a larger array gain (μ_N) is compensated with the increase in the mean time per bit ($\hat{T}$), which grows with the array size because of the need of estimating a larger MIMO channel matrix. Therefore, the energy consumption of very large MIMO systems is approximately the same, and there is a critical antenna array size, beyond which no significant energy savings can be achieved (see Figure 5).

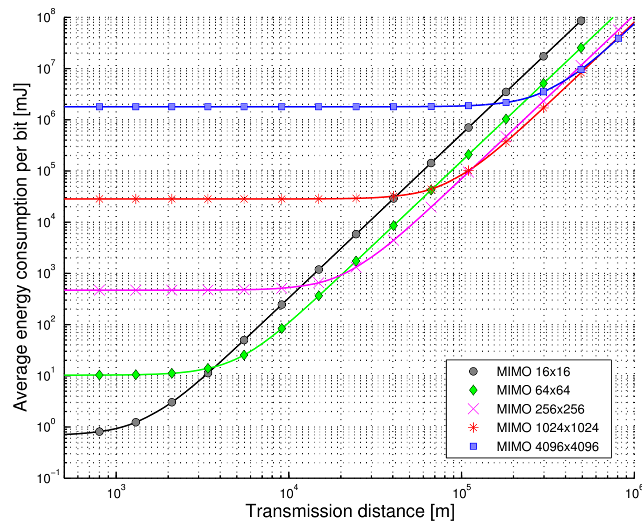

Fig. 5. Minimal energy consumption of large MIMO systems, which use BPSK modulation over beamforming. It can be seen that, for long tranmssion distances, the performance of very large MIMO systems is essentially the same, and therefore there is no reason to go beyond arrays of 256 antennas.

V. Conclusions

We studied the energy consumed in a MIMO SVD communication link for delivering one bit of data without error, and how it could be minimized by optimizing the radiated power, modulation size and number of eigenchannels to be used. In our study, we considered different transmission distances and various channel statistics, as well as the energy cost of retransmissions, feedback frames and the consumption of electronic components.

We found that, for a given antenna array size, the strategy that minimizes the energy consumption of short range communications is to maximize the throughput using a large constelation size over all the available eigenchannels. As the transmission distance increases, the optimal number of used eigenchannels and the optimal modulation size decreases. For long link distances it is optimal to reduce the irradiated power by reducing the throughput and sending all the power only thought the more favorable direction, which is given by the beamforming pattern.

We also found that single antenna systems are more energy efficient for performing short range communications than MIMO systems. On the contrary, large antenna arrays are the optimal choice for long distance links, where beamforming can achieve important array gains. Nevertheless, we found that there is a critical array size, over which larger array sizes do not achieve further energy savings, and therefore their higher cost and complexity cannot be justified.

APPENDIX A
ALTERNATIVE EXPRESSION FOR $\hat{P}_{\text{PA}}$

Lets deduce an alternative expression for the total power consumption of all the power amplifiers $\hat{P}_{\text{PA}}$ (cf. Section II-A4).

If only the best n eigenchannels are being used, the transmitted vector $\vec{x}$ is related to the transmitted symbol vector $\vec{s}$ as $\vec{x} = V_n\vec{s}$, where V_n is the $N \times n$ precoding matrix which is composed by the first n columns of the unitary matrix V which is obtained from the singular value decomposition $H = U^*\Lambda V$ [14]. Therefore

$$|x_j|^2 = \sum_{k=1}^{n} |v_{j,k}|^2 \bar{P}_{\text{tx}}^{(k)} \tag{41}$$

where $v_{j,k}$ is the coefficient of the j-th row and k-th column of V_n, and $\bar{P}_{\text{tx}}^{(k)}$ is the power allocated into the k-th eigenchannel for $j \in \{1, \ldots, n\}$ (cf. Section III-A).

Assuming that $|x_j|^2 = P_{\text{A}}^{(j)}$, where $P_{\text{A}}^{(j)}$ as defined in Section II-A4), and using (11) and the definition of $\hat{P}_{\text{PA}}$ (cf. Equation 12), then the total power consumption of the PA's can be written as

$$\hat{P}_{\text{PA}} = \sum_{j=1}^{N} P_{\text{PA}}^{(j)} = \frac{\eta}{\xi}\sum_{j=1}^{N} |x_j|^2 \tag{42}$$

$$= \frac{\eta}{\xi}\sum_{k=1}^{n}\sum_{j=1}^{N} |v_{j,k}|^2 \bar{P}_{\text{tx}}^{(k)} = \frac{\eta}{\xi}\sum_{k=1}^{n} \bar{P}_{\text{tx}}^{(k)} \tag{43}$$

where we are using the fact that $\sum_{j=1}^{N} |v_{j,k}|^2 = 1$ as the columns of the unitary matrix V are orthonormal.

APPENDIX B
SOLUTION OF (31)

The problem stated in (31) is equivalent to

$$\begin{aligned} \text{minimize} \quad & f(\phi_1, \ldots, \phi_n) = \frac{1}{n}\sum_{k=1}^{n} P_{\text{s}}(\lambda_k\phi_k\bar{\gamma}) \\ \text{subject to} \quad & \sum_{k=1}^{n} \phi_k = 1, \\ & \phi_k \geq 0, \quad k = 1, \ldots, n. \end{aligned}$$

Above, the SER function is given by

$$P_{\text{s}}(\lambda_k\phi_k\bar{\gamma}) = \begin{cases} cQ\left(\sqrt{a\lambda_k\phi_k\bar{\gamma}}\right) & \text{BPSK} \\ 1 - \left[1 - cQ\left(\sqrt{a\lambda_k\phi_k\bar{\gamma}}\right)\right]^2 & \text{M-QAM} \end{cases} \tag{44}$$

where a and c are appropiate constants [24]. We will first show that the previous is a convex optimization problem, and then apply the Karush-Kuhn-Tucker (KKT) conditions.

Lets write the objetive function as $f = (1/n)\sum_{k=1}^{n} f_k$, where $f_k(\phi_1, \ldots, \phi_n) = P_{\text{s}}(\lambda_k\phi_k\bar{\gamma})$ as given in (44). It can be shown that, for any value of k, $f_k : \mathbb{R}^n \to \mathbb{R}$ is a convex function, as it only depends on ϕ_k and $\frac{\partial^2 f_k}{\partial\phi_k^2} > 0$. Therefore, f is also convex because it is the sum of convex functions. The domain of allowable solutions is a *probability simplex*, which is a well known convex set [33].

As the above problem is convex, the KKT conditions are necessary and sufficient to characterize the optimal solution [33]. Therefore, expressions for the optimal values $(\phi_1^*, \ldots, \phi_n^*)$ can be found using the following Lagrangian:

$$\Lambda = \frac{1}{n}\sum_{k=1}^{n} P_{\text{s}}(\lambda_k\phi_k\gamma) + \eta_0\left(\sum_{k=1}^{N}\phi_k - 1\right) - \sum_{k=1}^{n}\eta_k\phi_k \ . \tag{45}$$

Using the KKT conditions on (45), it can be shown that the optimal values ϕ_k^* satisfy the conditions

$$\frac{s\phi_k^* e^{s\phi_k^*}}{[1 - rQ(\sqrt{s\phi_k^*})]^2} = \beta\lambda_k^2 \qquad \forall k \in \{1, \ldots, n\} \ , \tag{46}$$

where $s = a\lambda_k\bar{\gamma}$, β is a constant, and $r = 2(1 - 1/\sqrt{M})$ for M-QAM modulations or $r = 0$ for BPSK.

Solving (46) for BPSK can be done using the Lambert function $W(x)$ [34]. To be able to solve (46) for the general case, we define a *generalized Lambert function* $W_r(x)$ as

$$W_r(x) = y \Longleftrightarrow \frac{ye^y}{(1 - rQ(\sqrt{y}))^2} = x \ . \tag{47}$$

The general $W_r(x)$ exists and is unique for all $x \geq 0$ and $r \geq 0$, because it is the inverse of a strictly increasing function.

Using the generalized Lambert function, the solution can be written as

$$\phi_k^* = \frac{1}{a\lambda_k\bar{\gamma}} W_r(\beta\lambda_k^2) \ . \tag{48}$$

The constant β is the number that satisfies the condition $\sum_{k=1}^{n}\phi_k^* = 1$. Replacing (48) into this condition we find

$$a\bar{\gamma} = \sum_{k=1}^{n}\frac{1}{\lambda_k}W_r(\beta\lambda_k^2) \ , \tag{49}$$

from where β must be found numerically.

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