RGE Effects on Neutrino Masses in Partial Split Supersymmetry

Fabian Cadiz*

Physique de la matière condensée, Ecole Polytechnique, CNRS, 91128 Palaiseau, France

Marco Aurelio Díaz

Departamento de Física, Pontificia Universidad Católica de Chile,

Avenida Vicuña Mackenna 4860, Santiago, Chile

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We show that the running of the Higgs-gaugino-higgsino couplings present in Partial Split Supersymmetry can severely affect the neutrino masses generated through Bilinear R-parity Violation. We find a working scenario where the predicted neutrino observables satisfy the experimental constraints when the running is neglected. After including the running, we show that already with a split supersymmetric scale of 10^4 GeV the atmospheric mass leaves the allowed experimental window, and that the solar mass leaves it even earlier, with a split supersymmetric scale of 10^3 GeV. This shows that the correct prediction of neutrino observables in these models necessitates the inclusion of the running of these couplings.

I. INTRODUCTION

With the Large Hadron Collider (LHC) successfully running since 2009, the two mayor experiments, ATLAS and CMS, discovered a new particle compatible with the Higgs boson of the Standard Model, with a mass of 126 GeV [1, 2]. In addition, they have been collecting data and looking for signals beyond the Standard Model (SM). In the case of supersymmetry, mainly R-Parity conserving models have been considered. Among the last ones, in ref. [3] a search for supersymmetric events with two leptons (electrons and/or muons) was made. These events are produced from decays of heavy neutralinos or charginos into the lightest neutralino (lightest supersymmetric particle, LSP) via a slepton. In a simplified model where the slepton has a mass half way between the heavy neutralino/chargino and the LSP, a 200 GeV bound on the chargino mass is obtained. In ref. [4] a search for neutralino decaying into a gravitino and a photon was performed in an R-Parity conserving gauge mediated supersymmetry breaking model. Looking for

*Electronic address: fabian.cadiz@polytechnique.edu

di-photon events with missing energy, a limit of 805 GeV for the gluino mass when the neutralino is heavier than 50 GeV was set. In ref. [5] a sbottom pair was searched, with each sbottom decaying into a bottom quark and a neutralino. No signal was observed setting a bound of 60 GeV for the neutralino mass and 390 GeV for the sbottom mass. Other R-Parity conserving searches for supersymmetry can be found in ref. [6] for ATLAS and ref. [7] for CMS. These results mainly point to upper limits for gluino and squark masses which are starting to reach the 1 TeV level.

In the R-Parity violating scenario, in ref. [8] a displaced vertex is looked for in association with a muon, found in the decay of a neutralino. The non-observation of an excess placed limits on the production cross section as a function of the neutralino lifetime. In ref. [9] a massive particle decaying into an electron and a muon is searched, with no excess found. This places limits on a stau that decays via trilinear R-Parity violating couplings, setting a model dependent lower bound on the stau mass of 1.32 TeV. In ref. [10] supersymmetry was searched in the channel with jets and an isolated lepton without the observation of an excess. This was interpreted in the Bilinear R-Parity Violation (BRpV) model in the tree-level dominance scenario, and exclusion limits where set in the gaugino-scalar mass plane ($M_{1/2} - m_0$). For example, the gaugino mass has to be larger than 340 GeV for low scalar masses, and for low gaugino masses the scalar mass has a lower bound that reach around 900 GeV.

Although the LHC experiments have not found evidence for supersymmetry, the available experimental information is ruling out important sectors of the theory, specially where colored particles are light. At least two mayor scenarios are not challenged yet: Split Supersymmetry (SS) and R-Parity violation.

First, in Split supersymmetry [11, 12] all scalar particles are very heavy, with the exception of a SM-like Higgs boson. The idea behind this model is to keep two of the best phenomenological features of supersymmetric theories, namely the unification of gauge couplings and the existence of a viable dark matter candidate. The price to pay is the abandon of the supersymmetric solution to the naturalness problem. Since squark are not seen yet at the LHC, this scenario acquires strength. Second, although signals for R-Parity have been searched for, the constraints are strongly model dependent. In BRpV for example, the search reported in [10] is interpreted in BRpV within the tree-level dominance scenario. In this case, tree level contributions to neutrino mass matrix dominates over the one-loop corrections. But this needs not to be the case, and in other scenarios the interpretation of the search results would have to be re-done. In fact, in a different scenario the decay of the neutralino into a muon and two jets could be very suppressed.

In the original version of Split Supersymmetry, where only one Higgs doublet remains light, it is not sufficient to add BRpV to generate acceptable neutrino masses and mixing angles. One way to fix this problem would be the inclusion of a gravity motivated term to the neutrino mass matrix [13]. A different approach is to keep two Higgs doublets light with all the sfermions masses at a high scale, scenario known as Partial Split Supersymmetry (PSS) [14–16]. In this scenario, despite the fact that the lightest neutralino is unstable due to the presence of BRpV, the gravitino is a viable dark matter candidate [17, 18].

In PSS, after integrating out the sfermions, new couplings are generated between the gauginos, higgsinos, and light Higgs bosons. These couplings are called $\tilde{g}_u$, $\tilde{g}_d$, $\tilde{g}'_u$, and $\tilde{g}'_d$, and have boundary conditions at the split supersymmetric scale $\widetilde{m}$ that relate them to the gauge couplings. They run independently to the weak scale acquiring values that differs from coupling to coupling, running that has been neglected up to now. In this article we find the RGE for these couplings in PSS, and estimate the effect they produce in the neutrino observables.

II. ELECTROWEAK SYMMETRY BREAKING

In PSS the Higgs sector is the same as in the MSSM, with two Higgs doublets H_u and H_d, each acquiring vacuum expectation values v_u and v_d respectively. The Higgs potential is also equal to the one in the MSSM. The presence of BRpV implies that the sneutrinos acquire a vev, v_i, $i = 1, 2, 3$, one for each generation of sneutrinos. If we do not decouple yet the sfermions, the minimization conditions for the scalar potential are the ones in the MSSM-BRpV model [19]. The tadpole condition for the two Higgs boson vevs are given by,

$$\begin{aligned} \left(m_{H_d}^2 + \mu^2\right) v_d + v_d D - B_\mu v_u + \mu \vec{v} \cdot \vec{\epsilon} \;&=\; 0 \\ -B_\mu v_d + \left(m_{H_u}^2 + \mu^2\right) v_u - v_u D + \vec{v} \cdot \vec{B}_\epsilon + v_u \vec{\epsilon}^{\,2} \;&=\; 0 \end{aligned} \tag{1}$$

while the three tadpole conditions associated to the sneutrino vevs are condensed into the following equation,

$$v_i D + \epsilon_i \left(-\mu v_d + \vec{v} \cdot \vec{\epsilon}\right) + v_u B_\epsilon^i + v_i M_{L_i}^2 = 0 \tag{2}$$

Most of the notation in these equations is the usual one, although we note that ϵ_i are the supersymmetric BRpV mass terms in the superpotential, and that B_ϵ^i are the bilinear soft terms associated to the former, analogous to the term B_μ which is related to the CP-odd Higgs mass [14].

In PSS sfermions masses are pushed up to a scale $\widetilde{m} \gg m_Z$, and eq. (2) implies that $B_\epsilon^i \sim (v_i/v_u) M_{L_i}^2$, assuming that the other terms are smaller. This can be achieved in theoretical models as noted in ref. [18]. On the other hand, the two conditions in eq. (1) can be met for a given set of vacuum expectation values if appropriate values of $m^2_{H_d}$ and $m^2_{H_u}$ are chosen, solving for these masses in the tadpole conditions. Even if the CP-odd Higgs mass is larger than m_Z (but much smaller than $\widetilde{m}$), the first condition in eq. (1) is fulfilled if $m_{H_d} \sim m_A$. In addition, even in the case where the split supersymmetric scale is very large, eq. (1) can be satisfied if $m_{H_u} \sim (v_i/v_u)M_{L_i}$. All these considerations imply that the scalar potential can have a minimum with a given set of vevs for the case of very large sfermion masses, provided the soft mass parameters satisfy the tadpole equations. Under these conditions, the mass eigenstate sfermions are integrated out. In particular, the sneutrino mass eigenstates that are integrated out have, by construction, a higgs component. Conversely, the remaining Higgs bosons fields at the low scale, have a sneutrino component. This is crucial for the neutrino mass generation mechanism to work.

III. NEUTRAL FERMIONS IN PARTIAL SPLIT SUSY

Now we work in the low energy effective model where the sfermions are integrated out. As in any supersymmetric model with BRpV, in PSS gauginos and higgsinos mix with neutrinos forming a 7×7 mass matrix. In the base $\psi_0^T = (-i\widetilde{B}, i\widetilde{W}^0, \widetilde{H}^0_d, \widetilde{H}^0_u, \nu_e, \nu_\mu, \nu_\tau)$ we group the mass terms in the lagrangian

$$\mathcal{L}_N = -\frac{1}{2}\psi_0^T \mathcal{M}_N^{PSS} \psi_0 \tag{3}$$

where we divide the mass matrix into four blocks,

$$\mathcal{M}_N^{PSS} = \begin{bmatrix} \mathrm{M}_{\chi^0}^{PSS} & (m^{PSS})^T \\ m^{PSS} & 0 \end{bmatrix}, \tag{4}$$

The upper-left block corresponds to the neutralino sector,

$$\mathbf{M}_{\chi^0}^{PSS} = \begin{bmatrix} M_1 & 0 & -\frac{1}{2}\tilde{g}'_d v_d & \frac{1}{2}\tilde{g}'_u v_u \\ 0 & M_2 & \frac{1}{2}\tilde{g}_d v_d & -\frac{1}{2}\tilde{g}_u v_u \\ -\frac{1}{2}\tilde{g}'_d v_d & \frac{1}{2}\tilde{g}_d v_d & 0 & -\mu \\ \frac{1}{2}\tilde{g}'_u v_u & -\frac{1}{2}\tilde{g}_u v_u & -\mu & 0 \end{bmatrix}. \tag{5}$$

The form of this mass matrix is analogous to the case in the MSSM, the difference being in the Higgs-gaugino-higgsino couplings $\tilde{g}$, whose form in the lagrangian are,

$$\mathcal{L}_{PSS}^{RpC} \ni -\tfrac{1}{\sqrt{2}} H_u^\dagger (\tilde{g}_u \sigma \widetilde{W} + \tilde{g}'_u \widetilde{B}) \widetilde{H}_u - \tfrac{1}{\sqrt{2}} H_d^\dagger (\tilde{g}_d \sigma \widetilde{W} - \tilde{g}'_d \widetilde{B}) \widetilde{H}_d + \mathrm{h.c.} \tag{6}$$

In the MSSM these couplings are equal to the corresponding gauge couplings, thus the boundary conditions they satisfy at the scale $\widetilde{m}$ are

$$\tilde{g}_u(\widetilde{m}) = \tilde{g}_d(\widetilde{m}) = g(\widetilde{m})$$
$$\tilde{g}'_u(\widetilde{m}) = \tilde{g}'_d(\widetilde{m}) = g'(\widetilde{m}) \tag{7}$$

Below the scale $\widetilde{m}$ these couplings are governed by their own RGE, which are developedoped for PSS in the Appendix. The mixing between neutralinos and neutrinos is given by the block

$$m^{PSS} = \left[\begin{array}{cccc} -\frac{1}{2}\tilde{g}'_d b_1 v_u & \frac{1}{2}\tilde{g}_d b_1 v_u & 0 & \epsilon_1 \\ -\frac{1}{2}\tilde{g}'_d b_2 v_u & \frac{1}{2}\tilde{g}_d b_2 v_u & 0 & \epsilon_2 \\ -\frac{1}{2}\tilde{g}'_d b_3 v_u & \frac{1}{2}\tilde{g}_d b_3 v_u & 0 & \epsilon_3 \end{array}\right]. \tag{8}$$

The relevant terms in the lagrangian that account for this mixing matrix are,

$$\mathcal{L}^{RpV}_{PSS} = -\epsilon_i \widetilde{H}_u^T \epsilon L_i \; - \; \tfrac{1}{\sqrt{2}} b_i H_u^T \epsilon (\tilde{g}_d \sigma \widetilde{W} - \tilde{g}'_d \widetilde{B}) L_i \; + \; h.c., \tag{9}$$

where we see the supersymmetric mass parameters ϵ_i, which mix higgsinos with neutrinos. The Higgs-gaugino-lepton term proportional to b_i are induced in the low energy theory after the sleptons are integrated out, contributing to the gaugino-neutrino mixing when the Higgs acquire a vacuum expectation value.

The nature of the b_i terms can be understood as follows. Above the scale $\tilde{m}$ the Higgs scalars gauge eigenstates mix with the sneutrinos gauge eigenstates, both the real parts (CP-even) and the imaginary parts (CP-odd). If we call s_s^i and t_s^i the component of the i-th real-part-sneutrino inside the CP-even Higgs bosons mass eigenstates h and H respectively, it has been proved that they satisfy $s_s^i \sim -b_i c_\alpha \sim -c_\alpha v_i/v_u$ and $t_s^i \sim -b_i s_\alpha \sim -s_\alpha v_i/v_u$, with analogous relations for the imaginary-part-sneutrinos [14]. Therefore, the presence of a non-zero b_i term in eq. (9) indicates that the low energy fields we call Higgs bosons, in fact have a small sneutrino component. The sneutrinos are not completely decoupled from the low energy theory below $\widetilde{m}$, but continue living inside our Higgs bosons. In addition, the fact that b_i is proportional to the sneutrino vev, implies that if the $SU(2)$ breaking is switch off, the b_i disappears.

If eq. (4) is block diagonalized, a neutrino effective mass matrix is generated,

$$\mathbf{M}_\nu^{eff} = -m^{PSS}\,(\mathrm{M}_{\chi^0}^{PSS})^{-1}\,(m^{PSS})^T = \frac{M_1 \tilde{g}_d^2 + M_2 \tilde{g}_d'^2}{4 \det M_{\chi^0}^{PSS}} \left[\begin{array}{ccc} \Lambda_1^2 & \Lambda_1\Lambda_2 & \Lambda_1\Lambda_3 \\ \Lambda_2\Lambda_1 & \Lambda_2^2 & \Lambda_2\Lambda_3 \\ \Lambda_3\Lambda_1 & \Lambda_3\Lambda_2 & \Lambda_3^2 \end{array}\right], \tag{10}$$

with,

$$\det M^{SS}_{\chi^0} = -\mu^2 M_1 M_2 + \frac{1}{2} v_u v_d \mu \left(M_1 \tilde{g}_u \tilde{g}_d + M_2 \tilde{g}'_u \tilde{g}'_d \right) + \frac{1}{16} v_u^2 v_d^2 \left(\tilde{g}'_u \tilde{g}_d - \tilde{g}_u \tilde{g}'_d \right)^2 . \quad (11)$$

This matrix, whose matrix elements we denote $M_\nu^{ij} = A^{(0)} \Lambda_i \Lambda_j$ with $A^{(0)}$ being the tree-level contribution in eq. (10), has only one non-vanishing eigenvalue. Quantum corrections must be included in order to generate a solar as well an atmospheric mass difference. The quantum corrected neutrino effective mass matrix becomes,

$$M_\nu^{ij} = A \Lambda_i \Lambda_j + C \epsilon_i \epsilon_j \quad (12)$$

The C coefficient is generated at one-loop and in PSS, with $m_Z \ll m_A \ll \widetilde{m}$, the important contributions are from loops with neutral Higgs bosons, resulting in [15]

$$C \approx \frac{m_Z^2 \sin^2 2\beta}{64\pi^2 \mu^2 s_\beta^2 m_A^2} \sum_{k=1}^{4} m_{\chi^0_k} \left(\tilde{g}_d N_{k2} - \tilde{g}'_d N_{k1} \right)^2 \quad (13)$$

In addition, the A coefficient receives small one-loop corrections we do not display, and there is a coefficient B that mixes Λ and ϵ, but can be tunned to zero by choosing an appropriate substraction scale.

Notice that the neutrino mass terms satisfy the two requisites a Majorana neutrino mass should. First, lepton number is violated, as seen in eq. (9) by both terms ϵ_i and b_i. Second, this neutrino mass vanishes in the limit where $SU(2)$ symmetry is restored, as we see from Λ_i definition that $\Lambda_i \to 0$ as $v \to 0$, and from eq. (13) that $C \to 0$ in the same limit. Notice also that the general theorem shown in ref. [20] is also satisfied in our model. The theorem states that for a Majorana neutrino mass to be non-zero in a supersymmetric model, a ”Majorana” sneutrino mass should be present also. This means a mass splitting between the real and imaginary sneutrino masses. Remembering that our sneutrinos live in the low energy Higgs fields, this means that the contribution from the Higgs loops should vanish if $m_H \to m_A$. This is satisfied in our model because the above limit is equivalent to $m_A \gg m_Z$, and from eq. (13) we see that the C term goes zero in this limit.

One of the neutrinos remains massless, and the experimental result $\Delta m^2_{sol}/\Delta m^2_{atm} \approx 0.035$ implies $m_{\nu_3} \gg m_{\nu_2}$ implies,

$$\begin{aligned} \Delta m^2_{atm} &\approx \left(A|\vec{\Lambda}|^2 + C|\vec{\epsilon}|^2 \right)^2 - 2AC|\vec{\Lambda} \times \vec{\epsilon}|^2, \\ \Delta m^2_{sol} &\approx \frac{A^2 C^2 |\vec{\Lambda} \times \vec{\epsilon}|^4}{\left(A|\vec{\Lambda}|^2 + C|\vec{\epsilon}|^2 \right)^2}. \end{aligned} \quad (14)$$

Further approximated results can be obtained if one of the terms $A|\vec{\Lambda}|^2$ and $C|\vec{\epsilon}|^2$ dominates over the other. The tree-level dominance scenario is the most commonly assumed [10], and corresponds to the case $A|\vec{\Lambda}|^2 \gg C|\vec{\epsilon}|^2$.

IV. NUMERICAL RESULTS

A. Running Couplings in PSS

One of the guiding principles of Split Supersymmetry is gauge unification. It is expected that above the scale M_{GUT} particles interactions are governed by a gauge theory based on a single gauge group. Therefore we impose the unification of the three gauge couplings at M_{GUT}. The starting point is at the weak scale,

$$g_1^2 = \frac{5}{3}\frac{4\pi\alpha_e}{c_W^2}\,, \qquad g_2^2 = \frac{4\pi\alpha_e}{s_W^2} \tag{15}$$

with $\alpha_e^{-1}(m_Z) = 128.962 \pm 0.014$ [21] and $s_W^2(m_Z) = 0.23116 \pm 0.00013$ [22]. We run the couplings g_1 and g_2 until they meet at a scale we define as M_{GUT}. At that point we impose $g_3 = g_2 = g_1$ and run back g_3 to the weak scale, where we impose that the strong coupling constant satisfy the experimental constraint $\alpha_s(m_Z) = 0.1184 \pm 0.0007$ [22].

As a working example we take $\widetilde{m} = 10^{14}$ GeV and $\tan\beta = 10$, and find $M_{GUT} = 3.6 \times 10^{16}$ GeV with $\alpha_s(m_Z) = 0.1189$, in agreement with the experimental data. The running of the three gauge coupling constans can be seen in Fig. 1. The RGE for g_i above the scale $\widetilde{m}$ are the ones for the MSSM, while below $\widetilde{m}$ the RGE to be used are the ones for PSS and they are given in the Appendix. In the same working scenario we plot in Fig. 2 the Higgs-higgsino-gaugino couplings $\tilde{g}$. The top three curves correspond to $\tilde{g}_d$, $\tilde{g}_u$, and g $(= g_2)$, the third one shown for comparison. Above $\widetilde{m}$ the three coincide as it should be in the MSSM, and below $\widetilde{m}$ they separate for as much as 20% in the case of $\tilde{g}_d$. The lower three curves correspond to $\tilde{g}'_d$, $\tilde{g}'_u$, and g' $(= g_1\sqrt{3/5})$, and they can differ for as much as 10% in this scenario. Considering the high precission for the measurements of neutrino mass differences, these RGE effects will have an impact as we will show next.

B. Neutrino Mass Differences in PSS

It is clear that the larger $\widetilde{m}$ the larger the RGE effects on the gaugino couplings $\tilde{g}$. This is obvious from the definition, but it is also apparent in Fig. 2. This affects the neutrino mass matrix in eq. (12) through the

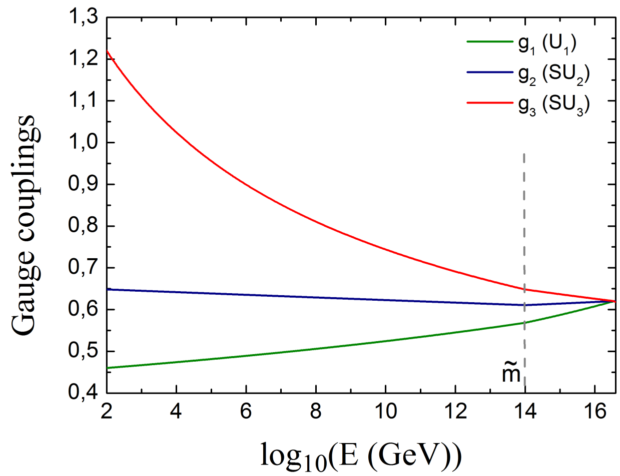

FIG. 1: *Unification of gauge couplings in PSS.*

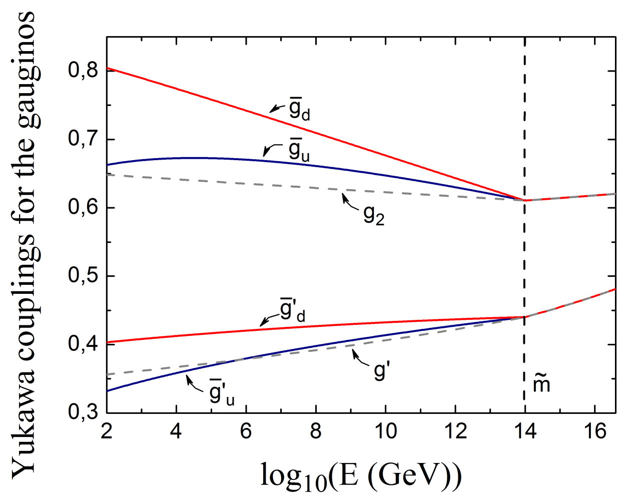

FIG. 2: *Running of Higgs-higgsino-gaugino couplings.*

coefficients A and C, which depend on the gaugino couplings as can be seen in eqs. (10) and (13). The size and shape of the effect depends also on the particular point in parameter space we are working with.

First of all we look for a working scenario with neutrino masses and mixing angles in agreement with experimental results, when neglecting the RGE effect. This means we work in the approximation $\tilde{g}_d = \tilde{g}_u = g$ and $\tilde{g}'_d = \tilde{g}'_u = g'$ at the weak scale. For the PSS parameters we choose the values indicated in Table I.

For the RpV parameters we perform a scan over parameter space according to the intervals indicated in

TABLE I: PSS parameters for the working scenario S

PSS parameter	S	Units
$\tan\beta$	10	...
μ	450	GeV
M_2	300	GeV
M_1	150	GeV
m_h	120	GeV
m_A	1000	GeV
Q	527	GeV

the third column in Table II. For each scanned point we calculate (See ref. [23] for the status of best fits to

TABLE II: RpV parameters for the working scenario S

RpV parameter	S	Scanned range	Units
ϵ_1	0.0346	$[-1, 1]$	GeV
ϵ_2	0.2516	$[-1, 1]$	GeV
ϵ_3	0.3504	$[-1, 1]$	GeV
Λ_1	0.0348	$[-1, 1]$	GeV^2
Λ_2	-0.0021	$[-1, 1]$	GeV^2
Λ_3	0.0709	$[-1, 1]$	GeV^2

neutrino parameters)

$$\chi^2 = \left(\frac{10^3\Delta m^2_{atm} - 2.4}{0.4}\right)^2 + \left(\frac{10^5\Delta m^2_{sol} - 7.7}{0.6}\right)^2 + \left(\frac{\sin^2\vartheta_{atm} - 0.505}{0.165}\right)^2 + \left(\frac{\sin^2\vartheta_{sol} - 0.33}{0.07}\right)^2 \tag{16}$$

and demand $\chi^2 < 1$. Among all the solutions we choose the one given by the second column in Table II with $\chi^2 = 0.072$, and refer to it as scenario S. This is an example of what we call a one-loop dominated solution, since $|A\vec{\Lambda}^2/(C\vec{\epsilon}^2)| = 0.29$. Without approximations in the neutrino mass matrix in eq. (12) we obtain the following observables,

$$\begin{aligned}
\Delta m^2_{\text{atm}} &= 2.4 \times 10^{-3}\ \text{eV}^2 \\
\Delta m^2_{\text{sol}} &= 7.73 \times 10^{-5}\ \text{eV}^2 \\
\sin^2 \vartheta_{\text{atm}} &= 0.4577 \\
\sin^2 \vartheta_{\text{sol}} &= 0.3337 \\
\sin^2 \vartheta_{\text{reac}} &= 2.30 \times 10^{-5} \\
m_{\beta\beta} &= 0.0029\ \text{eV}
\end{aligned} \tag{17}$$

which are well within experimental constraints. The last parameter corresponds to the effective Majorana neutrino mass, which must satisfy $m_{\beta\beta} < 0.7$ eV according to neutrinoless double beta decay experiments. It is useful to confront these results with approximated neutrino mass differences and angles in the one-loop dominated scenario. In this situation we have,

$$\begin{aligned}
\Delta m^2_{\text{sol}} &\approx A^2 \frac{|\Lambda \times \epsilon|^4}{|\epsilon|^4} \\
\Delta m^2_{\text{atm}} &\approx C^2 |\epsilon|^4 \\
\tan^2 \vartheta_{\text{sol}} &\approx \frac{\Lambda_1^2(\epsilon_2^2 + \epsilon_3^2)}{(\Lambda_2\epsilon_3 - \Lambda_3\epsilon_2)^2} \\
\tan^2 \vartheta_{\text{atm}} &\approx \frac{\epsilon_2^2}{\epsilon_3^2} \\
\tan^2 \vartheta_{\text{reac}} &\approx \frac{\epsilon_1^2}{\epsilon_2^2 + \epsilon_3^2}
\end{aligned} \tag{18}$$

Clearly the mixing angles in first approximation are not affected by the RGE effects, while the mass differences are. This is confirmed in the following plots.

In Fig. 3 we have the atmospheric mass squared difference in the y-axis and the sine squared of the atmospheric angle in the x-axis. The lowest point corresponds to $\widetilde{m} = m_Z$, which means we have the MSSM all the way to the weak scale. This is also equivalent to neglect the RGE effects. The Δm^2_{atm} and $\sin^2 \theta_{atm}$ values for this case are the ones given in eq. (17). In the following points we study the effect of the $\tilde{g}$ running, and each of them are defined by an increasing value of $\widetilde{m}$. Two things are immediately apparent: (i) the effect on the atmospheric angle is negligible, and (ii) the effect on the atmospheric mass is very important. Indeed, already at $\widetilde{m} = 10^4$ GeV the value of Δm^2_{atm} leaves the 3σ allowed interval.

In Fig. 4 we have a similar plot, with the solar mass squared difference in the y-axis and the sine squared of the solar angle in the x-axis. The situation is similar, with a very small effect on the solar angle and a

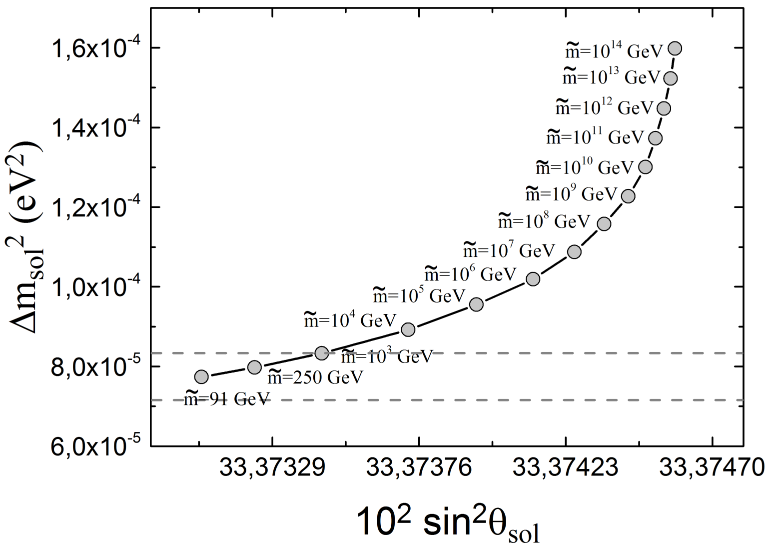

FIG. 3: *Running effects on solar mass and solar angle.*

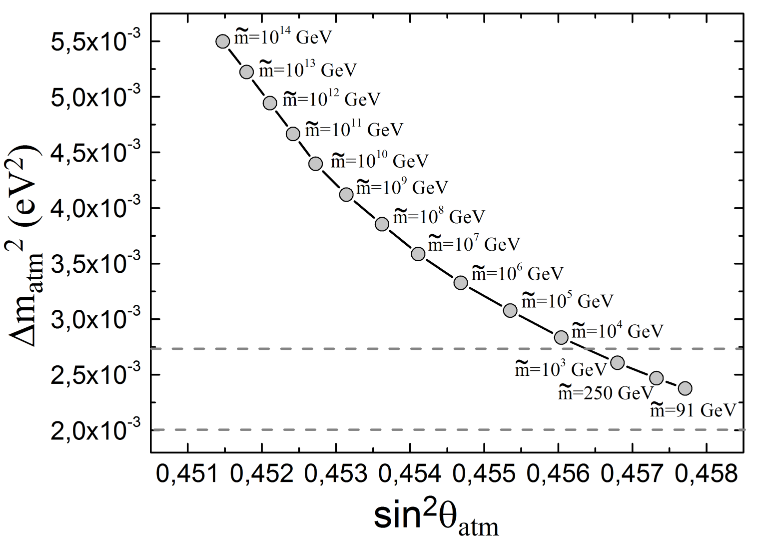

FIG. 4: *Running effects on atmospheric mass.*

very large effect on the solar mass. Indeed, already for $\widetilde{m} = 10^3$ GeV the solar mass calculated including the RGE effect leaves the 3σ allowed interval.

V. SUMMARY

We have seen that neutrino masses can be generated in PSS via a low energy see-saw mechanism, where neutrinos mix with neutralinos through BRpV couplings. The neutrino masses and mixing angles generated depend, through both the tree-level and the one-loop contributions, on the Higgs-gaugino-higgsino couplings. These couplings have boundary conditions at the split supersymmetric scale $\widetilde{m}$ that relate them to the gauge couplings. But the RGEs that govern them are different from the RGEs for the gauge couplings, which implies they have different values at the weak scale. We have found the RGE for the relevant couplings in PSS, and showed that the effect of their running is large enough to affect the neutrino observables. Although mixing angles are not affected, we have shown that the atmospheric and solar mass squared differences are very sensitive to this running, such that with a moderate split supersymmetric scale, it already can change the viability of a given scenario.

Acknowledgments

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VI. APPENDIX: RENORMALIZATION GROUP EQUATIONS

Here we study the Renormalization Group Equations for gauge, Yukawa, and gaugino-Higgs-higgsino couplings in Partial Split Supersymmetry. As usual, we define $t = \ln Q^2$ with Q the arbitrary scale introduced by dimensional regularization. We use the results, and follow as close as possible the notation, given by [24].

A. General Two-loop RGE for Gauge Couplings

In a $SU(3) \times SU(2) \times U(1)$ gauge theory, the three gauge couplings have the following RGE,

$$\beta_{g_i} = \frac{dg_i}{dt} = \frac{1}{16\pi^2} g_i^3 b_i + \frac{1}{(16\pi^2)^2} g_i^3 \Big\{ B_{ij} g_j^2 - 2Y_i(F) \Big\} \tag{19}$$

where there is a sum over j but not over i. Here $i = 1, 2, 3$ refers to the $U(1)$, $SU(2)$, and $SU(3)$ groups respectively, denoted in general as G_i, with g_i being the corresponding gauge coupling. The term proportional to b_i is the one-loop contribution, with

$$b_i = -\frac{11}{3} C_2(G_i) + \frac{2}{3} \sum_f T(R_i^f) d(R_j^f) d(R_k^f) + \frac{1}{3} \sum_s T(R_i^s) d(R_j^s) d(R_k^s) \tag{20}$$

and $i \neq j \neq k$. The first term is the contribution from the gauge bosons, with $C_2(G_i)$ the normalization factor of the quadratic Casimir operator C_2 for the adjoint representation of the group G_i. This Casimir normalization is defined as,

$$C_2 \; = \; A_i^a A_i^a = C_2(G_i) \mathbf{1} \tag{21}$$

also satisfying,

$$\mathrm{Tr}(A_i^a A_i^b) \; = \; C_2(G_i) \delta^{ab} \tag{22}$$

where A_i^a are the N generators in the adjoint representation of the group G_i, labeled by the indices $a, b = 1, ...N$. For $G_N = SU(N)$, $N > 1$ we have $C_2(G_N) = N$, while for $G_1 = U(1)$ we have $C_2(G_1) = 0$. In particular, for the case of $SU(2)$ and $SU(3)$ we have the two well known results,

$$\begin{aligned} \epsilon_{acd}\epsilon_{bcd} = 2\delta_{ab} \, , \qquad & \text{with} \qquad (A_2^a)_{bc} = \epsilon_{abc} \\ f_{acd} f_{bcd} = 3\delta_{ab} \, , \qquad & \text{with} \qquad (A_3^a)_{bc} = f_{abc} \end{aligned} \tag{23}$$

with ϵ_{abc} and f_{abc} the structure constants of the $SU(2)$ and $SU(3)$ Lie groups respectively.

The second term in eq. (20) corresponds to the contribution from fermions living in a general representation R_i^f, where f runs over all different fermions and $i = 1, 2, 3$ refers to the gauge group G_i as before. In our model, fermions live in the fundamental representation, whose generators we call F_i^a. They are equal to half the Pauli matrices in the case of $SU(2)$, $F_2^a = \sigma^a/2$, $a = 1, ...3$, and half the Gelmann matrices in the case of $SU(3)$, $F_3^a = \lambda^a/2$, $a = 1, ...8$. They satisfy,

$$\mathrm{Tr}(F_i^a F_i^b) = T(R_i^f)\delta^{ab} = \frac{1}{2}\delta^{ab} \tag{24}$$

where $T(R_i^f)$ is known as the Dynkin index of the representation R_i^f. In the case of the fundamental representation the conventional normalization is $T(R_i^f) = 1/2$. This last equation is analogous to eq. (22). Finally, factors $d(R_j^f)$ and $d(R_k^f)$ are the dimensions of the fermion multiplet in the other two groups $j, k \neq i$. In the case of $U(1)$ the Dynkin index of the fundamental representation is $T(R_1^f) = y^2$, with y the appropriately normalized hypercharge.

The third term in eq. (20) is the contribution from scalars when they live in the general representation R_i^s, where s runs over all the scalars. If this representation is the fundamental, of course we have $T(R_i^s) = 1/2$.

We turn now to the two-loop contributions to the RGE for the gauge couplings, given in the second term of eq. (19). The term B_{ij} inside the bracket is equal to,

$$\begin{aligned} B_{ii} &= -\frac{34}{3}\left[C_2(G_i)\right]^2 + \sum_f \left[\frac{10}{3}C_2(G_i) + 2C_2(R_i^f)\right] T(R_i^f)d(R_j^f)d(R_k^f) \\ &\quad + \sum_s \left[\frac{2}{3}C_2(G_i) + 4C_2(R_i^s)\right] T(R_i^s)d(R_j^s)d(R_k^s) \\ B_{ij} &= \sum_f 2T(R_i^f)C_2(R_j^f)d(R_j^f)d(R_k^f) + \sum_s 4T(R_i^s)C_2(R_j^s)d(R_j^s)d(R_k^s) \end{aligned} \tag{25}$$

with $i \neq j \neq k$ and there is no sum over repeated i indices. Here we see the quadratic Casimir normalization factor evaluated in the fundamental representation. The version of eq. (21) for the fundamental representation is,

$$C_2 = F_i^a F_i^a = C_2(R_i)\mathbf{1} \tag{26}$$

with an upper index f or s on R_i for the fermion and scalar cases. It can be easily shown the following relation with the Dynkin index,

$$C_2(R_i)d(R_i) = T(R_i)d(G_i) \tag{27}$$

This implies for the fundamental representation of $SU(N)$ that $C_2(R_N) = (N^2 - 1)/(2N)$. For the case of $U(1)$ we have $C_2(R_1) = T(R_1) = y^2$, with y defined before as the normalized hypercharge.

The $Y_i(F)$ term inside the bracket in eq. (19) is given by,

$$Y_i(F) = \frac{1}{d(G_i)} \sum_f \mathrm{tr}\left[C_2(R_i^f) N_c Y_f^a Y_f^{a\dagger}\right] \tag{28}$$

where Y_f^a are Yukawa-type terms (see Appendix C), f are all the fermions that couple through that coupling, and a labels the real scalars coupled to those fermions.

B. Explicit Two-loop RGE for Gauge Couplings in PSS

Here we explicitly apply the above formulae to our model. We find first the one loop parameters b_i, which for $SU(3)$ in PSS is,

$$b_3 = \left\{ -\frac{11}{3} \times 3 + \frac{2}{3} \times \frac{1}{2} \times n_g\,(2+1+1) \right\}_{SM} + \frac{2}{3} \times 3 = -5 \tag{29}$$

which includes contributions from gluons, Q, u_R, d_R, and $\tilde{g}$. Note that n_g is the number of generations, that the gluinos live in the adjoint representation thus we use $T(R_3^f) = 3$, and that PSS does not include squarks. For convenience we include in brackets the SM contribution to the RGE.

Second, for $SU(2)$ we have,

$$b_2 = \left\{ -\frac{11}{3} \times 2 + \frac{2}{3} \times \frac{1}{2} \times n_g\,(3+1) + \frac{1}{3} \times \frac{1}{2} \right\}_{SM} + \frac{2}{3} \times \left[2 + \frac{1}{2}(1+1)\right] + \frac{1}{3} \times \frac{1}{2} = -1 \tag{30}$$

which includes the contributions from W, Q, L, H_d, $\widetilde{W}$, $\widetilde{H}_u$, $\widetilde{H}_d$, and H_u. Note that the winos live in the adjoint representation, thus we take $T(R_2^f) = 2$.

Third, we obtain for $U(1)$,

$$\begin{aligned} b_1 \;=\; & \left\{ -\frac{11}{3} \times 0 + \frac{2}{3} \times \frac{3}{20} \times n_g \Big[(1/3)^2 \times 2 \times 3 + (2/3)^2 \times 3 + (-4/3)^2 \times 3 + \right. \\ & \qquad \left. (-1)^2 \times 2 + (2)^2 \Big] + \frac{1}{3} \times \frac{3}{20} \times (-1)^2 \times 2 \right\}_{SM} \\ & + \frac{2}{3} \times \frac{3}{20} \times \left[(1)^2 \times 2 + (-1)^2 \times 2\right] + \frac{1}{3} \times \frac{3}{20} \times (1)^2 \times 2 = \frac{23}{5} \end{aligned} \tag{31}$$

which includes contributions from B, Q, d_R, u_R, L, e_R, H_d, $\widetilde{H}_u$, $\widetilde{H}_d$, and H_u. Note that the quadratic Casimir for the adjoint representation in $U(1)$ is null, and that the factor $3/20$ is the hypercharge normalization. We summarize the values of b_i in Table III for both the SM and PSS. The corresponding values for Split Supersymmetry can be found in ref. [12].

TABLE III: b_i parameters in the one-loop RGE for gauge couplings.

Model	b_1	b_2	b_3
SM	$\frac{41}{10}$	$-\frac{19}{6}$	-7
PSS	$\frac{23}{5}$	-1	-5

Next we calculate the two-loop parameters B_{ij} in eq. (19). The diagonal terms B_{ii} are calculated from eq. (25),

$$\begin{aligned} B_{11} \;=\; & \Bigg\{ -\frac{34}{3}\times 0 + 2n_g\Big(\frac{3}{20}\Big)^2\Big[(1/3)^4\times 3\times 2 + (2/3)^4\times 3 + (-4/3)^4\times 3 + (-1)^4\times 2 + (2)^4\Big] \\ & +4\Big(\frac{3}{20}\Big)^2(-1)^4\times 2\Bigg\}_{SM} + 2\Big(\frac{3}{20}\Big)^2\Big[(1)^4\times 2 + (-1)^4\times 2\Big] + 4\Big(\frac{3}{20}\Big)^2(-1)^4\times 2 \\ =\; & \frac{217}{50} \end{aligned} \tag{32}$$

where we are including B, Q, d_R, u_R, L, e_R, H_d, $\widehat{H}_u$, $\widehat{H}_d$, and H_u;

$$\begin{aligned} B_{22} \;=\; & \Bigg\{ -\frac{34}{3}(2)^2 + n_g\Big[\frac{10}{3}\times 2 + 2\times\frac{3}{4}\Big]\times\Big(\frac{1}{2}\times 3+\frac{1}{2}\Big) + \Big[\frac{2}{3}\times 2 + 4\times\frac{3}{4}\Big]\times\frac{1}{2}\Bigg\}_{SM} \\ & + \Big[\frac{10}{3}\times 2 + 2\times\frac{3}{4}\Big]\times\Big(\frac{1}{2}+\frac{1}{2}\Big) + \Big[\frac{10}{3}\times 2 + 2\times 2\Big]\times 2 + \Big[\frac{2}{3}\times 2 + 4\times\frac{3}{4}\Big]\times\frac{1}{2} = \frac{225}{6} \end{aligned} \tag{33}$$

where we have included W, Q, L, H_d, $\widetilde{H}_u$, $\widetilde{H}_d$, $\widetilde{W}$, and H_u; and

$$\begin{aligned} B_{33} \;=\; & \Bigg\{ -\frac{34}{3}(3)^2 + n_g\Big[\frac{10}{3}\times 3 + 2\times\frac{4}{3}\Big]\times\Big[\frac{1}{2}\times 2 + \frac{1}{2} + \frac{1}{2}\Big]\Bigg\}_{SM} \\ & + \Big[\frac{10}{3}\times 3 + 2\times 3\Big]\times 3 = 22 \end{aligned} \tag{34}$$

where we have included g, Q, u_R, d_R, and $\tilde{g}$.

The off-diagonal terms B_{ij} are also calculated from eq. (25),

$$\begin{aligned} B_{12} \;=\; & \Bigg\{ 2n_g\times\frac{3}{4}\times\frac{3}{20}\Big[(1/3)^2\times 2\times 3 + (-1)^2\times 2\Big] + 4\times\frac{3}{4}\times\frac{3}{20}(-1)^2\times 2\Bigg\}_{SM} \\ & +2\times\frac{3}{4}\times\frac{3}{20}\Big[(1)^2\times 2 + (-1)^2\times 2\Big] + 4\times\frac{3}{4}\times\frac{3}{20}(1)^2\times 2 = \frac{9}{2} \end{aligned} \tag{35}$$

$$B_{21} = \left\{2n_g \times \frac{1}{2} \times \frac{3}{20}\left[(1/3)^2 \times 3 + (-1)^2\right] + 4 \times \frac{1}{2} \times \frac{3}{20}(-1)^2\right\}_{SM}$$
$$+2 \times \frac{1}{2} \times \frac{3}{20}\left[(1)^2 + (-1)^2\right] + 4 \times \frac{1}{2} \times \frac{3}{20}(1)^2 = \frac{3}{2} \tag{36}$$

where in both B_{12} and B_{21} we included Q, L, H_d, $\widetilde{H}_u$, $\widetilde{H}_d$, and H_u;

$$B_{13} = \left\{2n_g \times \frac{4}{3} \times \frac{3}{20}\left[(1/3)^2 \times 2 \times 3 + (2/3)^2 \times 3 + (-4/3)^2 \times 3\right]\right\}_{SM} = \frac{44}{5} \tag{37}$$

$$B_{31} = \left\{2n_g \times \frac{1}{2} \times \frac{3}{20}\left[(1/3)^2 \times 2 + (2/3)^2 + (-4/3)^2\right]\right\}_{SM} = \frac{11}{10} \tag{38}$$

where in both B_{13} and B_{31} we included Q, d_R, and u_R; and

$$B_{23} = \left\{2n_g \times \frac{4}{3} \times \frac{1}{2} \times 3\right\}_{SM} = 12 \tag{39}$$

$$B_{32} = \left\{2n_g \times \frac{3}{4} \times \frac{1}{2} \times 2\right\}_{SM} = \frac{9}{2} \tag{40}$$

where in both B_{23} and B_{32} we included only Q. We summarize the two-loop parameters B_{ij} in Table IV.

TABLE IV: B_{ij} parameters in the two-loop RGE for gauge couplings.

Model	B_{11}	B_{12}	B_{13}	B_{21}	B_{22}	B_{23}	B_{31}	B_{32}	B_{33}
SM	$\frac{199}{50}$	$\frac{27}{10}$	$\frac{44}{5}$	$\frac{9}{10}$	$\frac{35}{6}$	12	$\frac{11}{10}$	$\frac{9}{2}$	-26
PSS	$\frac{217}{50}$	$\frac{9}{2}$	$\frac{44}{5}$	$\frac{3}{2}$	$\frac{225}{6}$	12	$\frac{11}{10}$	$\frac{9}{2}$	22

Finally we calculate the two-loop terms $Y_i(F)$ given in eq. (28). For the $U(1)$ gauge group we have,

$$Y_1(F) = \left\{3 \times \frac{3}{20}\left[(1/3)^2\,\mathrm{Tr}\left(Y_u Y_u^\dagger + Y_d Y_d^\dagger\right) + (2/3)^2\,\mathrm{Tr}\left(Y_d Y_d^\dagger\right) + (-4/3)^2\,\mathrm{Tr}\left(Y_u Y_u^\dagger\right)\right]\right.$$
$$\left. + \frac{3}{20}\left[(-1)^2\,\mathrm{Tr}\left(Y_e Y_e^\dagger\right) + (2)^2\,\mathrm{Tr}\left(Y_e Y_e^\dagger\right)\right]\right\}_{SM}$$
$$+\frac{3}{20}(1)^2\left[\left(\frac{\tilde{g}_u}{\sqrt{2}}\right)^2 \times 3 + \left(\frac{\tilde{g}'_u}{\sqrt{2}}\right)^2\right] + \frac{3}{20}(-1)^2\left[\left(\frac{\tilde{g}_d}{\sqrt{2}}\right)^2 \times 3 + \left(\frac{\tilde{g}'_d}{\sqrt{2}}\right)^2\right] \tag{41}$$
$$= \left\{\frac{17}{20}\mathrm{Tr}\left(Y_u Y_u^\dagger\right) + \frac{1}{4}\mathrm{Tr}\left(Y_d Y_d^\dagger\right) + \frac{3}{4}\mathrm{Tr}\left(Y_e Y_e^\dagger\right)\right\}_{SM} + \frac{9}{40}\left(\tilde{g}_u^2 + \tilde{g}_d^2\right) + \frac{3}{40}\left(\tilde{g}_u'^2 + \tilde{g}_d'^2\right)$$

where we have included Q, d_R, u_R, L, e_R, $\widetilde{H}_u$, and $\widetilde{H}_d$. Notice that the Yukawa terms that contribute due to the higgsinos are written in eq. (6); the term associated to $SU(2)$ is,

$$\begin{aligned} Y_2(F) &= \left\{\frac{1}{3}\times\frac{3}{4}\left[3\times \mathrm{Tr}\left(Y_uY_u^\dagger + Y_dY_d^\dagger\right) + \mathrm{Tr}\left(Y_eY_e^\dagger\right)\right]\right\}_{SM} + \frac{1}{3}\times 2\left[\left(\frac{\tilde{g}_u}{\sqrt{2}}\right)^2 + \left(\frac{\tilde{g}_d}{\sqrt{2}}\right)^2\right]\times 3 \\ &\quad + \frac{1}{3}\times\frac{3}{4}\left[\left(\frac{\tilde{g}_u}{\sqrt{2}}\right)^2\times 3 + \left(\frac{\tilde{g}'_u}{\sqrt{2}}\right)^2\right] + \frac{1}{3}\times\frac{3}{4}\left[\left(\frac{\tilde{g}_d}{\sqrt{2}}\right)^2\times 3 + \left(\frac{\tilde{g}'_d}{\sqrt{2}}\right)^2\right] \\ &= \left\{\frac{3}{4}\mathrm{Tr}\left(Y_uY_u^\dagger\right) + \frac{3}{4}\mathrm{Tr}\left(Y_dY_d^\dagger\right) + \frac{1}{4}\mathrm{Tr}\left(Y_eY_e^\dagger\right)\right\}_{SM} + \frac{11}{8}\left(\tilde{g}_u^2 + \tilde{g}_d^2\right) + \frac{1}{8}\left(\tilde{g}_u'^2 + \tilde{g}_d'^2\right) \end{aligned} \tag{42}$$

where we have included Q, L, $\widetilde{W}$, $\widetilde{H}_u$, and $\widetilde{H}_d$; the term associated to $SU(3)$ is,

$$\begin{aligned} Y_3(F) &= \left\{\frac{1}{8}\times\frac{4}{3}\times 3\left[\mathrm{Tr}\left(Y_uY_u^\dagger + Y_dY_d^\dagger\right) + \mathrm{Tr}\left(Y_dY_d^\dagger\right) + \mathrm{Tr}\left(Y_uY_u^\dagger\right)\right]\right\}_{SM} \\ &= \left\{\mathrm{Tr}\left(Y_uY_u^\dagger\right) + \mathrm{Tr}\left(Y_dY_d^\dagger\right)\right\}_{SM} \end{aligned} \tag{43}$$

where we have included Q, d_R, and u_R.

To summarize, the two-loop RGE for the gauge couplings in PPS are, for $U(1)$

$$\begin{aligned} \frac{dg_1}{dt} &= \frac{g_1^3}{16\pi^2}\frac{23}{5} + \frac{g_1^3}{(16\pi^2)^2}\left[\frac{217}{50}g_1^2 + \frac{9}{2}g_2^2 + \frac{44}{5}g_3^2 - \frac{9}{20}\left(\tilde{g}_u^2 + \tilde{g}_d^2\right) - \frac{3}{20}\left(\tilde{g}_u'^2 + \tilde{g}_d'^2\right)\right. \\ &\quad \left. - \frac{17}{10}\mathrm{Tr}\left(Y_uY_u^\dagger\right) - \frac{1}{2}\mathrm{Tr}\left(Y_dY_d^\dagger\right) - \frac{3}{2}\mathrm{Tr}\left(Y_eY_e^\dagger\right)\right] \end{aligned} \tag{44}$$

for $SU(2)$,

$$\begin{aligned} \frac{dg_2}{dt} &= -\frac{g_2^3}{16\pi^2} + \frac{g_2^3}{(16\pi^2)^2}\left[\frac{3}{2}g_1^2 + \frac{225}{6}g_2^2 + 12g_3^2 - \frac{11}{4}\left(\tilde{g}_u^2 + \tilde{g}_d^2\right) - \frac{1}{4}\left(\tilde{g}_u'^2 + \tilde{g}_d'^2\right)\right. \\ &\quad \left. - \frac{3}{2}\mathrm{Tr}\left(Y_uY_u^\dagger\right) - \frac{3}{2}\mathrm{Tr}\left(Y_dY_d^\dagger\right) - \frac{1}{2}\mathrm{Tr}\left(Y_eY_e^\dagger\right)\right] \end{aligned} \tag{45}$$

and for $SU(3)$,

$$\frac{dg_3}{dt} = -5\frac{g_3^3}{16\pi^2} + \frac{g_3^3}{(16\pi^2)^2}\left[\frac{11}{10}g_1^2 + \frac{9}{2}g_2^2 + 22g_3^2 - 2\mathrm{Tr}\left(Y_uY_u^\dagger\right) - 2\mathrm{Tr}\left(Y_dY_d^\dagger\right)\right] \tag{46}$$

C. General One-loop RGE for Yukawa Couplings

In the context of Renormalization Group Equations, the usual notation for the Yukawa terms in the lagrangian is,

$$\mathcal{L}_Y = -\psi_i^\dagger Y_{ij}^a \psi_j \phi_a - \psi_j^\dagger Y_{ij}^{a*}\psi_i\phi_a \tag{47}$$

where Y^a is a hermitic matrix, ψ_i $i = 1...n$ are n two-component fermion fields in chiral representation, and ϕ_a $i = 1...m$ are m real scalar fields. The m RGEs for the $n \times n$ Yukawa matrices are

$$\begin{aligned}(16\pi^2)\frac{d}{dt}\mathbf{Y}^a &= \frac{1}{2}\mathbf{Y}^{b\dagger}\mathbf{Y}^b\mathbf{Y}^a + \frac{1}{2}\mathbf{Y}^a\mathbf{Y}^{b\dagger}\mathbf{Y}^b + 2\mathbf{Y}^b\mathbf{Y}^{a\dagger}\mathbf{Y}^b \\ &+ \frac{1}{2}\mathbf{Y}^b\,\mathrm{Tr}\left(\mathbf{Y}^{b\dagger}\mathbf{Y}^a + \mathbf{Y}^{a\dagger}\mathbf{Y}^b\right) - C^a(Y^a_{ij})\mathbf{Y}^a \end{aligned} \tag{48}$$

with

$$C^a(Y^a_{ij}) = 3\sum_{f,k} g_k^2 C_2(R_k) \tag{49}$$

In this last equation the sum over f runs over all fermions that couple by $\mathbf{Y}^a_{ij}$, $C_2(R_k)$ is the normalization of the Casimir operator in the representation R_k of the gauge groups with coupling constant g_k, $k = 1, 2, 3$.

D. Explicit One-loop RGE for Yukawa Couplings in PSS

In our model the terms in the lagrangian usually known as Yukawa couplings are

$$\mathcal{L}_Y = \overline{u}_R\mathbf{h}_u H_u^T \varepsilon\, Q_L - \overline{d}_R\mathbf{h}_d H_d^T \varepsilon\, Q_L - \overline{e}_R\mathbf{h}_e H_d^T \varepsilon L_L \tag{50}$$

The couplings themselves, $\mathbf{h}_u$, $\mathbf{h}_d$, and $\mathbf{h}_e$, are 3×3 matrices in flavour space. The first step is to decompose the Higgs doublets into their real scalar components

$$H_u = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1^u + i\phi_2^u \\ \phi_3^u + i\phi_4^u\end{pmatrix}, \qquad H_d = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1^d + i\phi_2^d \\ \phi_3^d + i\phi_4^d\end{pmatrix} \tag{51}$$

such that the terms in eq. (50) become

$$\begin{aligned}\mathcal{L}_Y &= \frac{1}{\sqrt{2}}\overline{u}_R\mathbf{h}_u\left(\phi_1^u + i\phi_2^u\right)d_L - \frac{1}{\sqrt{2}}\overline{u}_R\mathbf{h}_u\left(\phi_3^u + i\phi_4^u\right)u_L - \frac{1}{\sqrt{2}}\overline{d}_R\mathbf{h}_d\left(\phi_1^d + i\phi_2^d\right)d_L \\ &+\frac{1}{\sqrt{2}}\overline{d}_R\mathbf{h}_d\left(\phi_3^d + i\phi_4^d\right)u_L - \frac{1}{\sqrt{2}}\overline{e}_R\mathbf{h}_e\left(\phi_1^d + i\phi_2^d\right)e_L + \frac{1}{\sqrt{2}}\overline{e}_R\mathbf{h}_e\left(\phi_3^d + i\phi_4^d\right)\nu_L \end{aligned} \tag{52}$$

To these terms we have to add the Higgs-higgsino-gaugino terms in eq. (6), which also are Yukawa type terms. After the decomposition we write them as,

$$\begin{aligned}\mathcal{L}_{Hhg} &= -\frac{1}{2}\left(\phi_1^u - i\phi_2^u\right)\left[\left(\tilde{g}_u\widetilde{W}^3 + \tilde{g}'_u\widetilde{B}\right)\widetilde{H}_u^+ + \tilde{g}_u\left(\widetilde{W}^1 - i\widetilde{W}^2\right)\widetilde{H}_u^0\right] \\ &-\frac{1}{2}\left(\phi_3^u - i\phi_4^u\right)\left[\left(-\tilde{g}_u\widetilde{W}^3 + \tilde{g}'_u\widetilde{B}\right)\widetilde{H}_u^0 + \tilde{g}_u\left(\widetilde{W}^1 + i\widetilde{W}^2\right)\widetilde{H}_u^+\right] \\ &-\frac{1}{2}\left(\phi_1^d - i\phi_2^d\right)\left[\left(\tilde{g}_d\widetilde{W}^3 - \tilde{g}'_d\widetilde{B}\right)\widetilde{H}_d^0 + \tilde{g}_d\left(\widetilde{W}^1 - i\widetilde{W}^2\right)\widetilde{H}_d^-\right] \\ &-\frac{1}{2}\left(\phi_3^d - i\phi_4^d\right)\left[-\left(\tilde{g}_d\widetilde{W}^3 + \tilde{g}'_d\widetilde{B}\right)\widetilde{H}_d^- + \tilde{g}_d\left(\widetilde{W}^1 + i\widetilde{W}^2\right)\widetilde{H}_d^0\right]\end{aligned} \tag{53}$$

The terms in eqs. (52) and (53) are grouped into eight matrices $\mathbf{Y}^a$, whose index a runs over the eight real scalar fields we have, namely, $a = \phi_1^u, \phi_2^u, \phi_3^u, \phi_4^u, \phi_1^d, \phi_2^d, \phi_3^d, \phi_4^d$. Each of these matrices is 15×15, and it is expanded in the base formed by the 15 fermions ν_L, e_L, e_R, u_L, d_L, u_R, d_R, $\widetilde{W}^1$, $\widetilde{W}^2$, $\widetilde{W}^3$, $\widetilde{B}$, $\widetilde{H}_u^+$, $\widetilde{H}_u^0$, $\widetilde{H}_d^0$, $\widetilde{H}_d^-$. These are the matrices whose RGE are given in eq. (48).

Within these RGE, we see the coefficients $C^a(Y^a_{ij})$, which we calculate now. The four scalars belonging to H_u have the same coefficient, which associated to the Yukawa coupling $\mathbf{h}_u$ is,

$$C^{\phi_i^u}(\mathbf{h}_u) = \left\{3g_1^2 \times \frac{3}{20}\Big[(1/3)^2 + (-4/3)^2\Big] + 3g_2^2 \times \frac{3}{4} + 3g_3^2\Big[\frac{4}{3} + \frac{4}{3}\Big]\right\}_{SM} = \frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2 \quad (54)$$

where we have included Q and u_R for $U(1)$ and $SU(3)$, and only Q for $SU(2)$. The same coefficient associated to the Yukawa coupling $\tilde{g}_u$ is,

$$C^{\phi_i^u}(\tilde{g}_u) = \left\{3g_1^2 \times \frac{3}{20}(1)^2 + 3g_2^2\Big[2 + \frac{3}{4}\Big]\right\}_{SM} = \frac{9}{20}g_1^2 + \frac{33}{4}g_2^2 \quad (55)$$

where we included $\widetilde{H}_u$ for $U(1)$, $\widetilde{W}$ and $\widetilde{H}_u$ for $SU(2)$, and none for $SU(3)$. Lastly, the same coefficient but this time associated to the Yukawa coupling $\tilde{g}'_u$ is,

$$C^{\phi_i^u}(\tilde{g}'_u) = \left\{3g_1^2 \times \frac{3}{20}(1)^2 + 3g_2^2 \times \frac{3}{4}\right\}_{SM} = \frac{9}{20}g_1^2 + \frac{9}{4}g_2^2 \quad (56)$$

where we have included $\widetilde{H}_u$ for $U(1)$ and $SU(2)$, and no fields for $SU(3)$.

This allow us to calculate the RGEs for couplings associated to the entries of the matrix $\mathbf{Y}^{\phi_1^u}$,

$$\begin{aligned}
(16\pi^2)\frac{d}{dt}\mathbf{h}_u &= \mathbf{h}_u\left\{\frac{3}{2}\tilde{g}_u^2 + \frac{1}{2}\tilde{g}_u'^2 + \frac{3}{2}\mathbf{h}_u^\dagger\mathbf{h}_u + \frac{1}{2}\mathbf{h}_d^\dagger\mathbf{h}_d + 3\mathrm{Tr}\left(\mathbf{h}_u^\dagger\mathbf{h}_u\right) - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right\} \\
(16\pi^2)\frac{d}{dt}\tilde{g}_u &= \tilde{g}_u\left\{\frac{11}{4}\tilde{g}_u^2 + \frac{3}{4}\tilde{g}_u'^2 + \frac{1}{2}\tilde{g}_d^2 + 3\mathrm{Tr}\left(\mathbf{h}_u^\dagger\mathbf{h}_u\right) - \frac{9}{20}g_1^2 - \frac{33}{4}g_2^2\right\} \\
(16\pi^2)\frac{d}{dt}\tilde{g}'_u &= \tilde{g}'_u\left\{\frac{9}{4}\tilde{g}_u^2 + \frac{5}{4}\tilde{g}_u'^2 + \frac{1}{2}\tilde{g}_d'^2 + 3\mathrm{Tr}\left(\mathbf{h}_u^\dagger\mathbf{h}_u\right) - \frac{9}{20}g_1^2 - \frac{9}{4}g_2^2\right\}
\end{aligned} \quad (57)$$

Now we continue with the coefficients $C^a(Y^a_{ij})$. As before, the four scalars belonging to H_d have the same coefficient. Associated to the Yukawa coupling $\mathbf{h}_e$ we have,

$$C^{\phi_i^d}(\mathbf{h}_e) = \left\{3g_1^2 \times \frac{3}{20}\Big[(-1)^2 + (2)^2\Big] + 3g_2^2 \times \frac{3}{4}\right\}_{SM} = \frac{9}{4}g_1^2 + \frac{9}{4}g_2^2 \quad (58)$$

where we have included L and e_R for $U(1)$, L for $SU(2)$, and none for $SU(3)$. Similarly, the coefficient associated to $\mathbf{h}_d$ is,

$$C^{\phi_i^d}(\mathbf{h}_d) = \left\{3g_1^2 \times \frac{3}{20}\Big[(1/3)^2 + (2/3)^2\Big] + 3g_2^2 \times \frac{3}{4} + 3g_3^2\Big[\frac{4}{3} + \frac{4}{3}\Big]\right\}_{SM} = \frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2 \quad (59)$$

where we have included Q and d_R for $U(1)$ and $SU(3)$, and only Q for $SU(2)$. Finally, note that

$$C^{\phi_i^u}(\tilde{g}_u) = C^{\phi_i^d}(\tilde{g}_d)\,, \qquad C^{\phi_i^u}(\tilde{g}'_u) = C^{\phi_i^d}(\tilde{g}'_d) \tag{60}$$

With these results the four missing RGE from the entries of matrix $\mathbf{Y}^{\phi_1^d}$ are,

$$\begin{aligned}
(16\pi^2)\frac{d}{dt}\mathbf{h}_e &= \mathbf{h}_e\left\{\frac{3}{2}\tilde{g}_d^2 + \frac{1}{2}\tilde{g}_d'^2 + \frac{3}{2}\mathbf{h}_e^\dagger\mathbf{h}_e + 3\mathrm{Tr}\left(\mathbf{h}_d^\dagger\mathbf{h}_d\right) + \mathrm{Tr}\left(\mathbf{h}_e^\dagger\mathbf{h}_e\right) - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2\right\} \\
(16\pi^2)\frac{d}{dt}\mathbf{h}_d &= \mathbf{h}_d\left\{\frac{3}{2}\tilde{g}_d^2 + \frac{1}{2}\tilde{g}_d'^2 + \frac{1}{2}\mathbf{h}_u^\dagger\mathbf{h}_u + \frac{3}{2}\mathbf{h}_d^\dagger\mathbf{h}_d + 3\mathrm{Tr}\left(\mathbf{h}_d^\dagger\mathbf{h}_d\right) + \mathrm{Tr}\left(\mathbf{h}_e^\dagger\mathbf{h}_e\right) - \frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right\} \\
(16\pi^2)\frac{d}{dt}\tilde{g}_d &= \tilde{g}_d\left\{\frac{1}{2}\tilde{g}_u^2 + \frac{11}{4}\tilde{g}_d^2 + \frac{3}{4}\tilde{g}_d'^2 + 3\mathrm{Tr}\left(\mathbf{h}_d^\dagger\mathbf{h}_d\right) + \mathrm{Tr}\left(\mathbf{h}_e^\dagger\mathbf{h}_e\right) - \frac{9}{20}g_1^2 - \frac{33}{4}g_2^2\right\} \\
(16\pi^2)\frac{d}{dt}\tilde{g}'_d &= \tilde{g}'_d\left\{\frac{1}{2}\tilde{g}_u'^2 + \frac{9}{4}\tilde{g}_d^2 + \frac{5}{4}\tilde{g}_d'^2 + 3\mathrm{Tr}\left(\mathbf{h}_d^\dagger\mathbf{h}_d\right) + \mathrm{Tr}\left(\mathbf{h}_e^\dagger\mathbf{h}_e\right) - \frac{9}{20}g_1^2 - \frac{9}{4}g_2^2\right\}
\end{aligned} \tag{61}$$

Thus, eqs. (57) and (61) are the RGE for the Yukawa couplings for the Partial Split Supersymmetry Model.

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