Novel Electro-optical Properties of a Semiconductor Superlattice under a Magnetic Field

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We show that the absorption spectrum of a superlattice in the presence of electric and magnetic fields parallel to the growth axis has striking properties. The spectrum contains resonances only and its structure can be simple or very complex depending on the ratio between the fields. As a function of inverse magnetic field the absorption per unit field is periodic and the integrated absorption forms a Devil's staircase within each period.

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Physical systems subject to competing frequencies have received much attention recently. When the motions characterized by two such frequencies are coupled, phase locking may occur if the ratio of the frequencies is a rational number. The width of the locked region is larger for rational numbers that are the ratio of small integers and for strong coupling, thus establishing a hierarchy that distinguishes between various rational and irrational numbers. A variety of systems exhibit this behavior.[1] Also, the energy spectrum in a quantum system may be sensitive to such ratio. An example is an electron in two dimensions in the presence of a perpendicular magnetic field and a periodic potential. The Landau levels are broadened and split in subbands by the potential, the number of subbands in each level being determined by the ratio of the cyclotron frequency, and the frequency for scattering in the crystal planes.[2,3] Another example is the motion of an electron in a sum of periodic potentials whose periods may be commensurate (rational case) or incommensurate (irrational case). In the former case the wave functions are extended while in the latter they may be extended or localized depending on the strength of the coupling.[4]

In this Letter we discuss a case of competing frequencies that exhibits new phenomenology. Consider a superlattice of period a and length $L=\nu a$ in both a magnetic field B and an electric field E along the superlattice axis, z. Motion in the perpendicular x-y plane is quantized by the magnetic field with a characteristic cyclotron frequency $\omega_B=eB/m^*c\hbar$, where m^* is the effective mass. A characteristic Stark frequency $\omega_E=eEa/\hbar$ also arises in the motion along the z axis due to the electric field and the superlattice potential.[5] The spectrum has resonances and, as we will show, exhibits a remarkable structure that depends on the ratio between the two frequencies ω_B/ω_E.

We are interested in the absorption coefficient for interband transitions. For simplicity, we treat a two-band model in the tight-binding approximation and we ignore spin-flip transitions. This will exhibit the basic physical features we want to display. Also, in a tight-binding model the Stark spectrum is rigorously discrete, so the overall spectrum will be a pure point spectrum and the density of states a sequence of δ functions. Using the Landau gauge $\mathbf{A}=(0,Bx,0)$ the envelope function is an eigenstate of the operator,

$$H=\frac{\hbar^2}{2m^*(z)}\left(-\frac{d^2}{dz^2}+\frac{1+2n}{l_B^2}\right)+V(z)+eEz\,, \qquad (1)$$

where $V(z)=V(z+a)$ is the superlattice potential, n a non-negative integer, $l_B=(\hbar c/eB)^{1/2}$, and $m^*(z)$ is the effective mass, which may be different in the alternating layers. Note that the magnetic term is z dependent since the kinetic energy of the Landau orbits changes in regions of different effective mass. This causes the eigenfunctions of (1) to depend on the magnetic quantum number n. The dependence is small at moderate magnetic energies, however, and will be ignored in what follows. This approximation is justified in type-I heterostructures of GaAs/AlGaAs, for example, where the effective superlattice potential is at least several hundred times the usual magnetic energies. The coefficients C_l of the cell eigenfunctions in the tight-binding expansion of the envelope function obey the simple recursion formula $C_{l+1}+(\Delta/\lambda)(\zeta-l)C_l+C_{l-1}=0$, where l denotes a superlattice site, $\Delta=eEa$ is the Stark cell energy, λ is the transfer integral between nearest neighbors, and $\zeta=[\epsilon-\hbar\omega_B(n+\frac{1}{2})]/\Delta$. To minimize edge effects we assume the electric-field-induced localization condition $eEL\gg 2\lambda$.[6] The recursion relation is obeyed by the Bessel functions $J_{l-\zeta}(2\lambda/\Delta)$. Setting $\zeta=m$ to discard divergence for large index yields the energy quantization

$$\epsilon_{n,m}=\hbar\omega_B(n+\tfrac{1}{2})+meEa\,, \qquad (2)$$

with m any integer. Note that the spectrum is entirely discrete, with two superposed ladders of levels whose spacing may be commensurate or incommensurate depending on the ratio of the natural frequencies involved. For each value of n there is a degeneracy per unit area $g=(2\pi l_B^2)^{-1}$. Applying these solutions to both the conduction- and valence-band wave functions and using the properties of Bessel functions one obtains the absorption coefficient for a photon of frequency ω,

$$\alpha = A \sum_{m,m',n} J^2_{m-m'}(\eta)\,\delta(\epsilon^{(c)}_{n,m'} + \epsilon_g - \epsilon^{(v)}_{n,m} - \hbar\omega)\,,$$

where $\eta = 2(\lambda_v + \lambda_c)/\Delta$, ϵ_g is the energy gap, and the indices c and v refer to the conduction and valence bands, respectively. The constant A equals $8\pi^2 e^3 B|\mu|^2 K^2/n_0 m_0^2 c^2 \hbar\omega a$, where μ is the electron-hole cell overlap integral, K is the Kane matrix element, m_0 is the electron rest mass, and n_0 is the index of refraction. Since each term in the sum depends on the unrestricted integers m,m' only through their difference $m-m'$, one may use the constraints imposed by the δ function to recast this equation into the more convenient form,

$$\alpha = \bar{A} \sum_{n=0}^{(\nu-1)/2} J^2_{[\epsilon_B n - \epsilon_f]}(\eta)\,\delta(\epsilon_B n - \epsilon_f - [\epsilon_B n - \epsilon_f])\,, \qquad (3)$$

where $\bar{A} = \nu A/\Delta$ and $[x]$ is the integral part of x. We have used the dimensionless reduced magnetic and photon energies $\epsilon_B = (1/m_e^* + 1/m_h^*)\hbar e B/c\Delta$ and $\epsilon_f = (\hbar\omega - \epsilon_g - \epsilon_e - \epsilon_h)\Delta - \epsilon_B/2$, respectively, where ϵ_e (ϵ_h) and m_e^* (m_h^*) are the isolated-well energy level and effective mass for electrons (holes).

Equation (3) shows that the spectrum consists of point resonances, each with a strength determined by the properties of Bessel functions. Assume first the reduced magnetic energy to be a rational $\epsilon_B = p/q$, with p,q where p and q are integers having no common divisors. A resonance occurs when $q\epsilon_f = np - Mq$, with M,n integers, the first unrestricted, the second non-negative. This is the Diophantine equation, which has an infinity of solutions n,M provided $\epsilon_f = N/q$, with N an integer.[7] Resonances are thus equally spaced, with spacing $1/q$. Once the photon energy is fixed at one of these solutions the Diophantine equation determines M for each value of n. Using the properties of solutions to this equation one can show that the strength of the resonance is determined by the function

$$S(p,q,N) = \sum_{s \geq -Nq_{r-1}/q} J^2_{Np_{r-1}+sp}(\eta)\,. \qquad (4)$$

Here p_{r-1}/q_{r-1} is the next to the last convergent in an even expansion of the rational p/q.[7] This function has the following general properties. If the reduced photon energy ϵ_f increases by p/q, only the limit in the sum (4) changes, becoming $s \geq -(Nq_{r-1}+1)/q$, and either a term is added or the sum remains unchanged. After a term is added, the next $q-1$ increases in the unit p/q leaving the sum the same. This property becomes a true

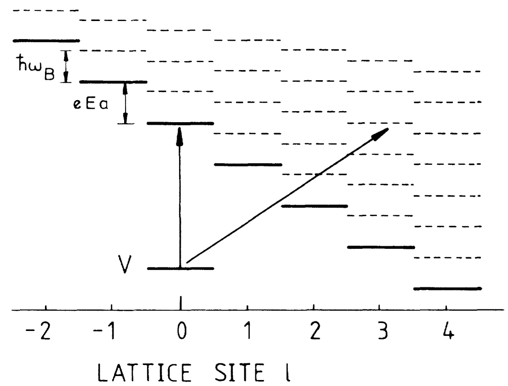

FIG. 1. Schematics of interband transitions from a reference valence-band level (V) to the conduction-band Stark and Landau ladders. Arrows are allowed transitions for a photon energy equal to the effective band gap.

period p/q when the photon energy is large since the terms added are small. At zero reduced photon energy ($n=0$) there is always a resonance since we measured energies from the effective band edge. Decreasing the energy by p units below the edge just causes the lowest-index term in (4) to disappear and the strength is decreased. Absorption in the gap is thus finite but reduced as the energy is made smaller. This analysis, valid for rational ϵ_B, may be extrapolated to irrationals by treating the latter as a limit of rational convergents.

The above properties may be easily visualized with the help of Fig. 1 where a reference level V in the valence band, together with available levels in the conduction band, are depicted. The well label l is also the index $Np_{r-1}+sp$ of the Bessel function that measures the strength of the corresponding transition. For each value of l there is a well level (solid line) and above it a ladder of local Landau levels (dashed lines). The figure shows the case $\epsilon_B = \frac{3}{4}$ and a reduced photon energy $\epsilon_f = 0$. Note that transitions may occur to matching levels that appear with period $p=4$ as prescribed by the terms in the series (4). At the photon energy chosen no transitions appear to levels of negative index so the strength (4) contains Bessel functions of positive index only. One can easily find the structure in the absorption coefficient by changing the photon energy (length of the vertical arrow in the figure), checking which horizontal levels match, and adding them all, each with the proper strength as prescribed by the index of the transition. As an example we show in Fig. 2 the absorption coefficient near the band edge for $\epsilon_B = \frac{1}{5}$ and $\frac{6}{5}$. The cases shown stress the very notorious differences that may occur in the spectra for two values of the reduced magnetic energy.

To check the intuitive idea of continuity as the field is varied we compared spectra for numbers very close to each other, such as $\epsilon_B = \frac{1}{5}$ and $\epsilon_B' = \frac{101}{500}$. Near the band edge the spectra look very much the same, only that

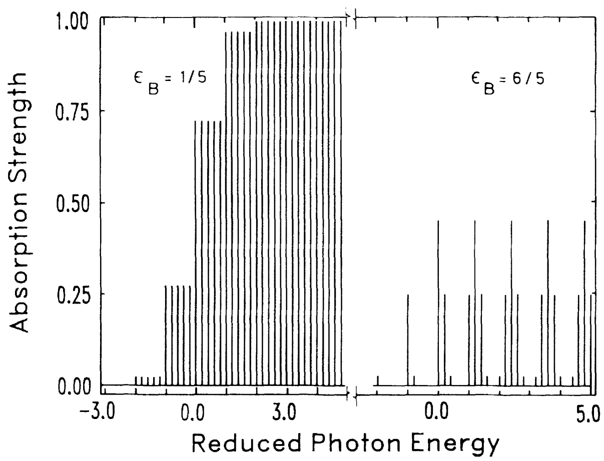

FIG. 2. Absorption strength S for two values of the field parameter ϵ_B and $\eta=1.2$. Energies are in units of the Stark period eEa.

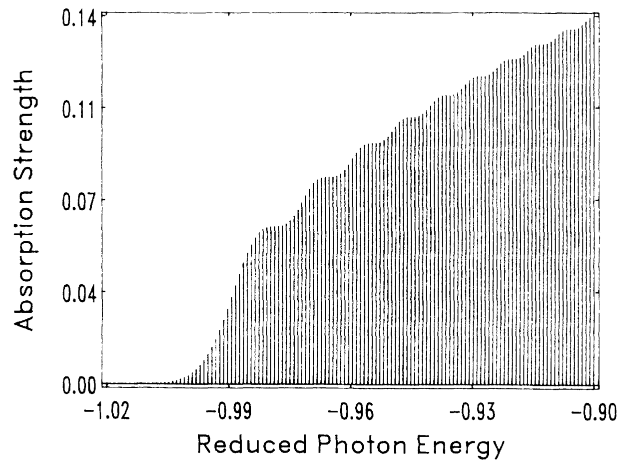

FIG. 3. Absorption in the low-E field limit. Here $\epsilon_B=1$ and $\eta=1000$. Energies are in units of the bandwidth parameter $2(\lambda_c+\lambda_v)$.

about half the weight of the resonances for ϵ_B is distributed in very narrow wings that develop around the same resonances for ϵ_B'. Note that in the latter resonances are $\frac{1}{500}$ apart, yet the strength of them is essentially zero except those common with the former case, in confirmation of the idea of continuity. Also, the position of the main resonances suffers a small displacement that accumulates and becomes 50% at about $\epsilon_B=50$. This noticeable displacement always occurs, however small the difference in the values of ϵ_B; only the smaller that it is, the further away from the band edge the maximum shift occurs for the first time. In the limit of small electric field (large η) the Stark ladders are closely spaced and Franz-Keldysh oscillations may be seen as an envelope to the evenly spaced resonances. This is shown below the band edge in Fig. 3 using $\eta=1000$ and $\epsilon_B=1$. Energies are here measured in units of the bandwidth parameter $2(\lambda_c+\lambda_v)$.

The spectrum also has a period for fixed photon energy and varying magnetic field. To see this we redefine the reduced photon energy as $\bar{\epsilon}_f=\epsilon_f+\epsilon_B/2$ to make it independent of B. Given $\bar{\epsilon}_f=N/Q$, with N,Q prime to each other, the relevant Diophantine equation $p(2n+1)-2Mq=2q\bar{\epsilon}_f$ gives resonances whenever $p/q=2P/Q(2S+1)$, with P,S arbitrary integers. One can show that the absorption strength function is then invariant under the change $q/p \to q/p+Q$ and $q/p \to Q-q/p$. We note that the additional factor $\bar{A}$, proportional to the magnetic field, appears in the absorption coefficient. Thus the latter is not truly periodic in the inverse field, but contains a decaying factor $1/B^{-1}$. Figure 4 shows the integrated absorption strength $\int^{B^{-1}} S\, dB'^{-1}$ for $\eta=4.2$ and a photon energy equal to the gap energy ($N=0$, $Q=1$). The strength has period 1 and a reflection symmetry about ϵ_B^{-1} a half integer. Note that resonances occur at values of ϵ_B^{-1} of the form $(2S+1)/2P$, with S,P integers only. The integrated absorption is a Devil's staircase with a hierarchy that is not strictly self-similar since the Bessel functions are not monotonic in the index but oscillate as they decay.

The case of zero magnetic field has been studied previously by Bleuse, Bastard, and Voisin[8] using a similar approach to ours. The physics is different, however, since a finite magnetic field quantizes the motion in the x-y plane turning steps in the absorption into sharp resonances. This is clearly seen comparing their Fig. 2(b) with our Fig. 3. The presence of this field also has favorable experimental consequences since resonances are more easily observed than plateaus in optical experiments. Assuming a linewidth of about 2 meV due to thickness and barrier composition fluctuations, finite temperature, and other broadening effects, we estimate that the main resonances may be resolved in a GaAs-AlGaAs superlattice of period 50 Å using fields of about $E=10$ kV/cm and $B=2$–3 T. Observing fine structure

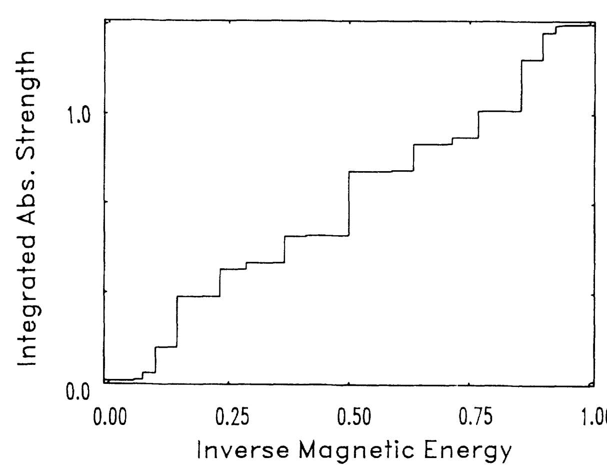

FIG. 4. Integrated absorption strength vs ϵ_B^{-1} for a photon energy equal to the field free band gap and $\eta=4.2$.

in such a system may require the use of larger fields, however. Then, we expect the regularities we predict to be somewhat altered since the hole Landau levels deviate from their linear fanlike behavior at large fields and our simple parabolic two-band model approximation becomes inaccurate.[9] Results in the large-field regime will be reported elsewhere.

In conclusion, we have shown that the absorption of a superlattice subject to electric and magnetic fields parallel to the axis is a sequence of resonances with interesting properties. The spectrum is periodic for large energies and exhibits a fine structure characteristic of the ratio between the fundamental magnetic and electric energies. There is absorption in the gap that acquires a Franz-Keldysh envelope in the limit of low electric fields. The absorption spectrum is proportional to a strength function S that exhibits a characteristic period in the inverse field. The integrated absorption in this variable forms a Devil's staircase in each period.

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