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Computer simulations of modern cosmic microwave background experiments, and an application to the Cosmology Large Angular Scale Surveyor

BY

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- *Why would anyone reinvent the wheel?*
- *Because a Porsche does not roll on stones.*

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¹I keep a screenshot of that first email around

Abstract

The study of the Cosmic Microwave Background is one of the corner-stones in the understanding of our universe. In recent years, ground and space born telescope have evidence that the observable universe is well described by a relatively simple model, the Λ CDM model. Ongoing and future experiments aim at going even further than Λ CDM by probing the very first moments of our universe using CMB polarization. Within this polarization field lies a possible signature of inflation, the leading theory that explains why do we observe a flat, isotropic and homogeneous universe. This signal corresponds to primordial B-modes, and its faintness makes its detection a major technical challenge. In this work, we present computer simulations of the CLASS Q-band telescope, one of the four telescopes of the CLASS experiment aiming at characterizing, among other things, primordial B-modes and thus inflation.

This work is divided in several chapters. The first two briefly introduce the reader to the general concepts of cosmology and the Cosmic Microwave Background (CMB). The third chapter describes the method used to model the polarizing properties of antennas. Chapter four presents the algorithm and prototype implementation of a new computer simulation code for CMB, which was used to build simulations of the CLASS experiment. Chapter five generally describes the CLASS telescope, and presents the methodology and results from electromagnetic simulations. This work finalizes by presenting the results of an application of the simulation code to CLASS using realistic parameters for its scanning strategy, beams and sky models.

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Chapter 1

Physics of the early universe

1.1 The current model for the universe

Humanity has always been in the search for understanding the origins of life, the universe and everything it contains¹. This need to understand Nature can be found in many different cultures across the globe, spanning thousands of years in the past. While primitive “models” of the universe included deities and magic, there is evidence of past civilizations having a remarkable understanding of the physical world around us. As humanity evolved, so did the dialectic with the outside world. Back to the present day, and having experienced an exponential growth in almost every area of knowledge, Humanity is still asking the same question: when, why and how did the universe “happen”.

To answer those questions, modern cosmologists have devised physical models and mathematical tools to describe the evolution of the universe. Observational evidence has made the Λ Cold Dark Matter model and the Big Bang theory (Λ CDM for short) to be the most widely accepted cosmological model of the universe. Λ CDM provides a framework for describing how the universe began by expanding from a very high-density, high-temperature state and offers a comprehensive explanation for a broad range of phenomena. This includes, but it is not limited to, the abundance of light elements, Hubble’s law and the presence of the Cosmic Microwave Background.

1.1.1 Evidence supporting Λ CDM

In the mid 20’s, the scientific community was divided. Part of it supported a model where the universe was static and eternal, called the steady-state model. Others were in favor of a dynamic scenario, where the universe was allowed to evolve. Both models were compatible with Einstein’s field equations; Einstein himself was more of a supporter of the steady-state model. He was so convinced of this that, when the expansion of the universe was discovered, he deliberately added a “cosmological

¹The answer is “42”

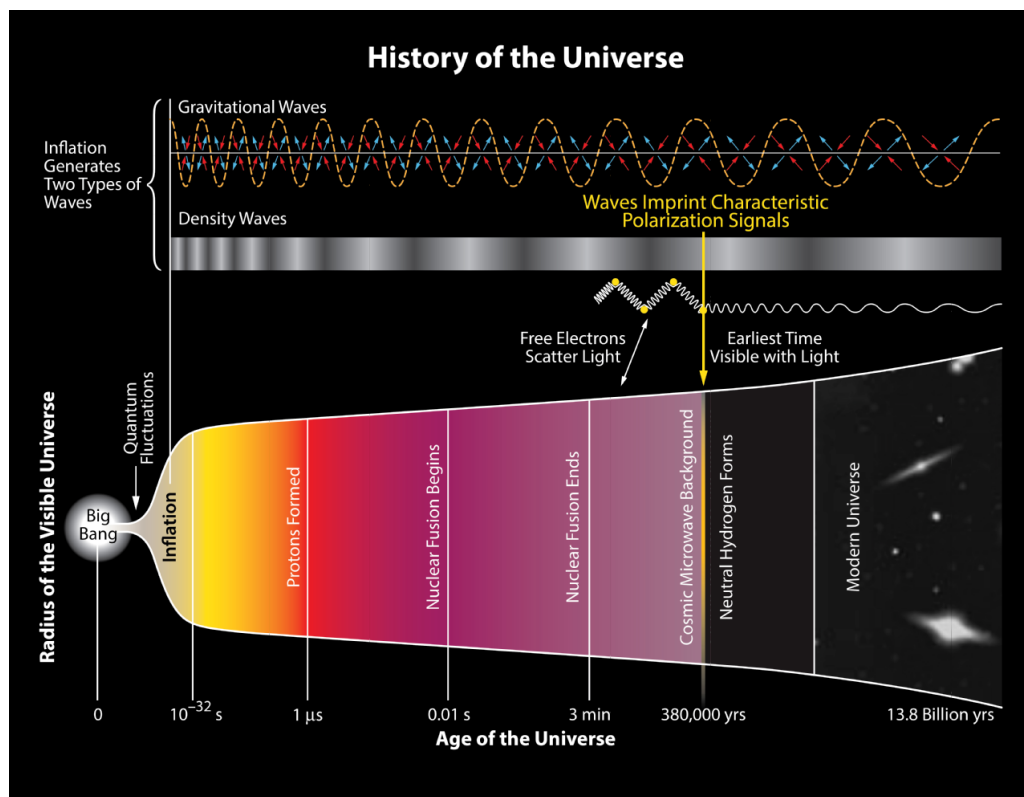


Figure 1.1: History of the Universe. The bottom part of this illustration shows the scale of the universe versus time. Specific events are shown such as the formation of neutral Hydrogen at 380 000 years after the big bang. Prior to this time, the constant interaction between matter (electrons) and light (photons) made the universe opaque. After this time, the photons we now call the CMB started streaming freely. The fluctuations (differences from place to place) in the matter distribution left their imprint on the CMB photons. The density waves appear as temperature and "E-mode" polarization. The gravitational waves leave a characteristic signature in the CMB polarization: the "B-modes". Both density and gravitational waves come from quantum fluctuations which have been magnified by inflation to be present at the time when the CMB photons were emitted. National Science Foundation (NASA, JPL, Keck Foundation, Moore Foundation, related) - Funded BICEP2 Program (<http://bicepkeck.org/faq.html>. Figure source: <http://bicepkeck.org/visuals.html>)

constant” to the equations so that the universe remained stationary.

The first piece of evidence in favor of the Big Bang, precursor of the Λ CDM model, came from Mount Wilson Observatory, the world’s most powerful observatory as of 1929. Using this facility, Edwin Hubble found a striking correlation between redshift from distant galaxies and their distance to Earth. Particularly, he found that the more distant the galaxy was, the larger the redshift. Solutions to Einstein’s field equations that were compatible with this discovery were derived in 1927, before Hubble’s measurement. This became the first piece of evidence supporting an expanding universe.

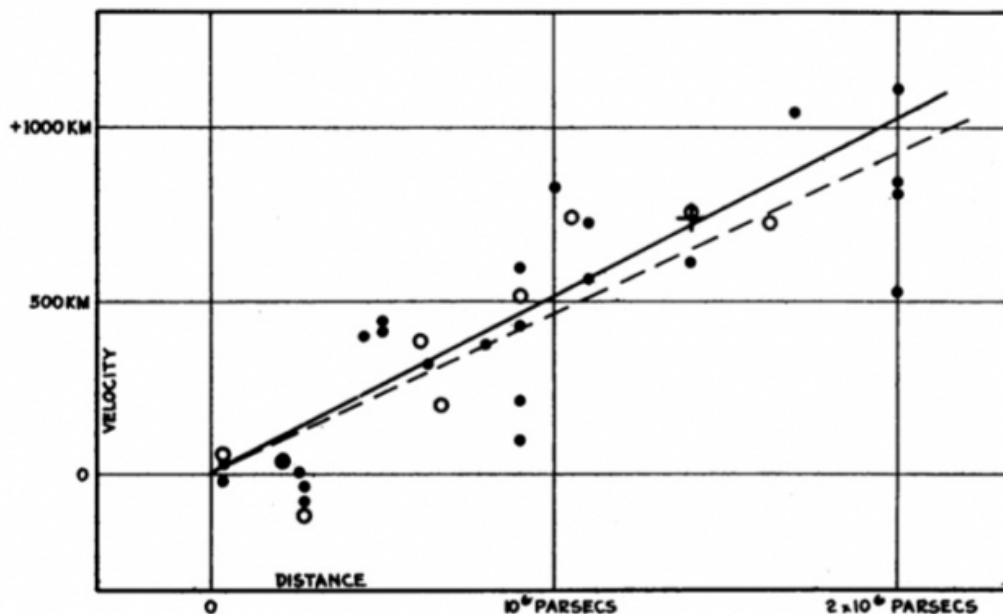


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

Figure 1.2: Original plot from Hubble (1929) showing the velocity-distance relation between galaxies.

Hubble’s discovery was not enough to convince the complete scientific community, as reasonable modifications to the steady-state model were made to accommodate this new observational data (see Bondi & Gold (1948)). The second piece of evidence came from a relatively simple prediction: if the universe is expanding now, it means that it should have been smaller in the distant past. In 1948, the work of Alpher,

Bethe and Gamow (see Alpher et al. (1948)) showed that, during the phase when the universe was very hot and dense, synthesis of elements like helium, hydrogen and lithium took place. This process is known as Big Bang Nucleosynthesis (BBN). Subsequent observations and experiments conducted in particle accelerators proved this theory to be correct, which was another big push in favor of the Big Bang model.

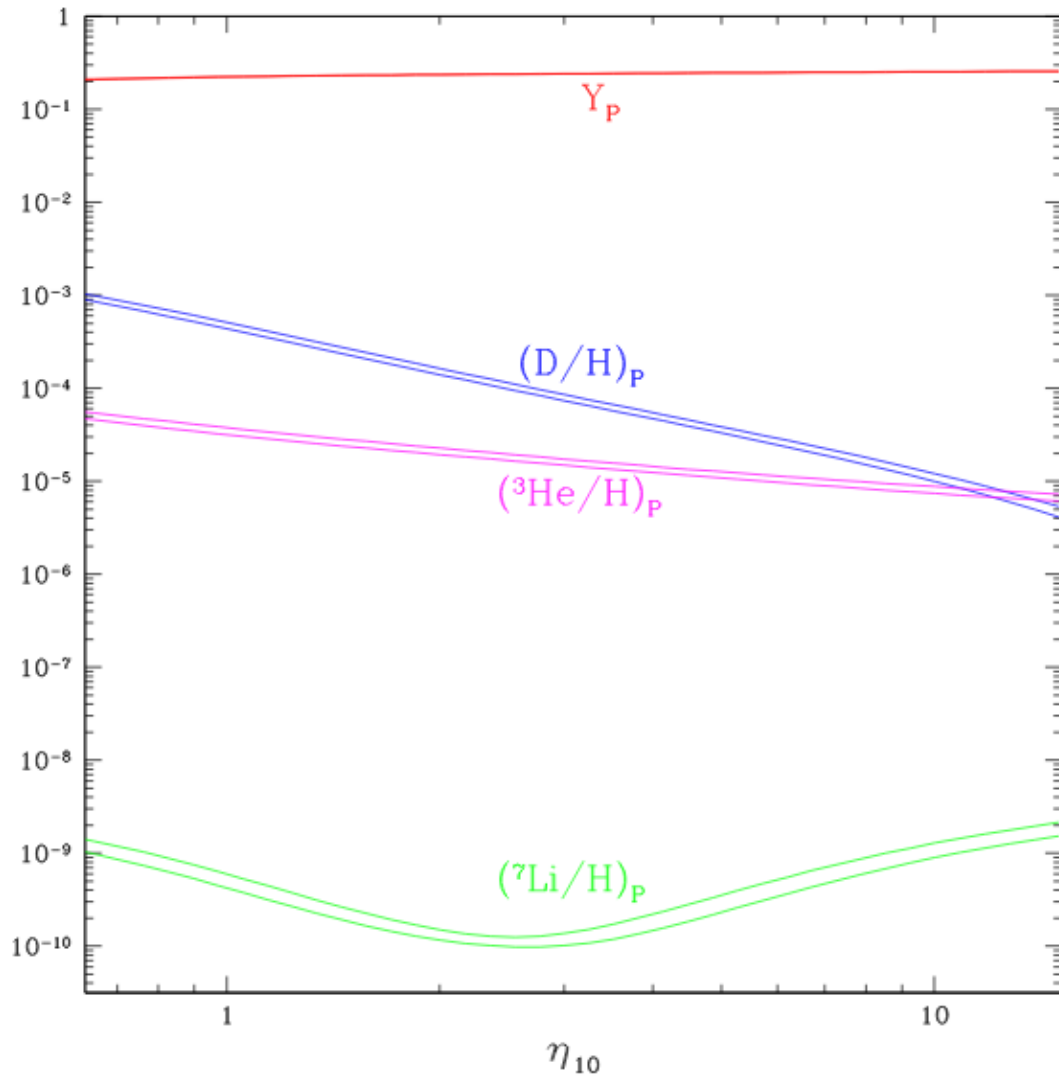


Figure 1.3: The predicted primordial abundances of D, ${}^3\text{He}$ and ${}^7\text{Li}$ (by number density n , with respect to hydrogen), and the ${}^4\text{He}$ mass fraction Y as a function of the nucleon abundance. The widths of the bands reflect the theoretical uncertainties. Figure source: Big Bang Nucleosynthesis: Probing the First 20 Minutes, by Gary Steigman

While predictions of a background radiation are as old as 1886², the first arguments pointing to the existence of a cosmic microwave background were developed around 1950 (see Assis et al. (2001) for a more detailed description) The arguments in favor of a cosmic microwave background pointed out that, after the synthesis of light elements, the universe continued to expand and cooling until atomic nuclei and electrons were not energetic enough to overcome their attraction so that hydrogen was formed. This would have “set free” the photons, which would have then free streamed across the universe. Photons from this distant event would permeate the universe until the current epoch, but would be much less energetic due to the expansion of the universe.

In 1964, Arno Penzias and Robert Woodrow Wilson at the Crawford Hill location of Bell Telephone Laboratories were conducting experiments using a high sensitivity horn antenna. They soon discovered a constant background of random noise that was present in the data, day and night, during the entire year. Penzias and Wilson found themselves with a puzzle: no matter where the antenna was pointing and independently of the time of the day, there was always an “excess” noise being recorded by the system. This noise was roughly 100 times more powerful than expected. They performed an exhaustive set of tests, looking for any source of interference of terrestrial origin. They even shot pigeons that would not desists from roosting in the antenna³ Having discarded all plausible explanations for the noise being a systematic error, Penzias and Wilson concluded that the noise they were measuring was in fact coming from outer space, and was likely of extra galactic origin.

In parallel, Robert H. Dicke, Jim Peebles and David Wilkinson were designing an experiment to look for the ancient CMB light. When Bernard F. Burke, professor

²It is important to remark that this background radiation has a different origin than the CMB radiation

³According to Penzias, this was done with a close range shotgun, so birds were killed instantly.



Figure 1.4: Picture of Robert Woodrow Wilson (in the back) and Arno Penzias. In the back, the “sugarspoon” antenna that was used to produce the first measurement of the CMB. Date and source of the picture are uncertain, but it is from a time when astronomers wore ties.

of Physics at MIT, told Penzias about a preprint paper he had seen by Jim Peebles on the possibility of finding radiation left over from an explosion that filled the universe at the beginning of its existence, he got in touch with Dicke. After a short period of time, he and Wilson decided to publish their results. The paper was called “A measurement of excess antenna temperature at 4080 Mc/s” Penzias & Wilson (1965). Penzias and Wilson were awarded with the Nobel Prize for their discovery, the Cosmic Microwave Background (CMB). Five decades have passed since Penzias and Wilson’s discovery and, regardless of the exponential growth in CMB related research, the scientific community keeps finding ways of using this electromagnetic fossil to extract information about the universe.

1.1.2 Problems with the Big Bang paradigm

The discovery of the CMB by Penzias and Wilson, Hubble’s measurement’s of an expanding universe and the corroboration of nucleo-synthesis provided the necessary evidence to support the Λ CDM model. However, there were a few issues yet to be explained. First, the CMB has a high degree isotropy. While this is indeed a good sign as it supports the assumption of an isotropic universe, the observed isotropy implied that causally disconnected regions in the early universe had to be at the same temperature when the CMB was formed. How could those regions “know” which temperature to be?

On the other hand, general relativity predicts that an “inward curved” space-time produces an additional pull that would have made easier for the large scale structure of the universe to form. In contrast, an open curvature would have made the formation of this structure more difficult. Modern measurements of the CMB imply that the curvature of space-time in the early universe was very close to zero, also referred as a “flat geometry” (see Bennett et al. (2013)). In fact, it was just *flat enough* to allow formation of large scale structure, but not curved enough to make the universe shortly re-collapse after the Big Bang. Again, how could space-time “know” what curvature to have? Finally, in the extreme conditions of the very early universe, quantum chromodynamics predict exotic particles such as magnetic monopoles and other species must have been created. If present today those particles should dominate the matter content of the universe. Detection of such particles has been largely unsuccessful (see Acharya et al. (2017)) These problems are known as the horizon problem, the flatness problem and the non-relics problem, which the Big Bang model alone could not explain without recurring to very fine tuned initial conditions.

1.2 Inflation

The problems presented above are solved naturally by the introduction of inflation. Though some of the key ideas were developed earlier and independently by Starobinsky and others (see Starobinskiĭ (1979)), it is widely accepted that inflation was proposed by Guth in 1981 (see Guth (1981)). Inflation is a period in the very early universe, around 10^{-34} seconds after the Big Bang, in which the dominant component was the inflaton field. The key property of the inflaton field is that it produced an accelerated expansion, very much like what we observe today in the form of a cosmological constant. Inflation would have then “erased” any trace of the initial conditions of the universe. Inflation also provides an explanation for the large-scale structure of the universe by magnifying quantum fluctuations in the inflaton field to macroscopic size, generating the seeds of regions where galaxies, and clusters of galaxies, would form. Inflation also solves the horizon problem by providing a mechanism in which small, causally connected regions of the early universe, can grow to roughly the size of the universe today. Finally, the rapid expansion of the universe produced by the inflation would have greatly diluted the number density of exotic particles to less than one exotic particle per causally connected region. While the exact mechanism that drove inflation is unknown, the theory can be tested by measuring properties of the observable universe. A particularly rich source of information is provided by the large scale features of the CMB. Inflation is an active field of research in cosmology as there is a lack of physical constraints on the models.

1.2.1 Base concepts

In an expanding universe, photons emitted from a distant object lose energy as they travel through the ever stretching space-time. Note that this energy loss is associated with the expansion of space-time itself: this is *interpreted* by the receiver of the photon as if the source is moving with respect to it. For convenience, we associate redshift z with the Doppler reddening of light as

$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emitted}}} = \frac{1}{a} \quad (1.1)$$

where a is the scale factor of the universe, a unit-less parameter that sets the physical scale of the universe. High redshift sources are far away in the past, meaning the universe was smaller at that time.

Hubble noted that distant galaxies are moving away from us. He also noticed a trend implying that the receding velocity increases with distance. This is exactly what we expect from an expanding universe, for the physical distance between two galaxies is $d = ax$, where x is the comoving distance between the galaxies and a is the scale factor of the Universe (set to be 1 at the current epoch)⁴ In the absence of peculiar motion, that is, $\dot{x} = 0$, the relative velocity $v = \dot{d}$ is equal to $\dot{a}x = Hd$. Therefore, we expect the velocity to increase linearly with distance, with a slope given by H , the so called Hubble constant. The above means that it is possible to determine H by measuring the distances and redshifts to distant objects. H is defined in terms of the scale factor a , as

$$H(t) = \frac{\dot{a}(t)}{a(t)} \quad (1.2)$$

Thus $H(t)$ measures how the scale factor changes over time.

On the other hand, the evolution of the scale factor is determined by the Friedmann equation

$$H(t)^2 = \frac{8\pi G}{3}\rho(t) - \frac{\kappa c^2}{a(t)^2} \quad (1.3)$$

where $\rho(t)$ is the energy density of the universe as a function of time, κ is the curvature of the universe, and c is the speed of light. Current measurements using WMAP data placed the Hubble constant to be around $69.4 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ (see

⁴Note that the physical distance between two objects at cosmologically relevant distances is ill-defined from the perspective of General Relativity. Corrections must be applied when dealing with distances in this regime.

Bennett et al. (2013))

1.2.2 Slow roll inflation

During the time inflation took place, the dominant component of the energy density of the universe was the inflaton field, ϕ , its equation of motion given by

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi} \quad (1.4)$$

where H is the Hubble parameter defined in 1.2, and V is the potential energy associated with the inflation field ϕ . For a universe dominated solely by the inflaton field, the equation for H becomes

$$H^2 = \frac{8\pi}{3m_{\text{pl}}^2} \left[V(\phi) + \frac{1}{2}\dot{\phi}^2 \right] \quad (1.5)$$

where m_{pl} is the Planck mass. To have an accelerated expansion of the universe, we must have $\ddot{a} > 0$, which translates into $1/2\dot{\phi}^2 < V(\phi)$. In slow-roll inflation, it is assumed that $1/2\dot{\phi}^2 \ll V(\phi)$. In this approximation and using the continuity equation, Friedmann equations can be solved yielding

$$H^2 \approx \frac{8\pi}{3m_{\text{pl}}^2} V(\phi) \quad (1.6)$$

$$3H\dot{\phi} \approx -V'' \quad (1.7)$$

where $'$ means derivation with respect to ϕ . From here, the slow-roll parameters are defined

$$\epsilon(\phi) = \frac{m_{\text{pl}}^2}{16\pi} \left(\frac{V'}{V} \right)^2 \quad (1.8)$$

$$\eta(\phi) = \frac{m_{\text{pl}}^2}{8\pi} \left(\frac{V''}{V} \right) \quad (1.9)$$

ϵ measures the slope of the potential, while η the curvature of it. The conditions necessary for inflation require both ϵ and η to be much less than 1 for a sufficiently long amount of time.

During inflation, everything except the inflaton field is redshifted to very low densities. As mentioned before, this explains the lack of topological defects in the observable universe. Once the slow-roll conditions break down, $\phi(t)$ switches from being over-damped to being under-damped and begins to oscillate. As it does, the inflaton field decays into conventional particles. The details of this process, called reheating, are an important area of research in inflationary cosmology, but are not important for the generation and evolution of perturbations that arise from this process.

1.2.3 Primordial perturbations

During inflation, the evolution of small perturbations to a free-field ϕ (a field that with potential energy $V(\phi) = 0$) can be modeled, in Fourier space, as

$$\delta\ddot{\phi}_k + 3H\dot{\phi}_k + \left(\frac{k}{a}\right)^2 \delta\phi_k = 0 \quad (1.10)$$

where the subscript k denotes the k -th of both the field ϕ and associated perturbation $\delta\phi$. k is the *comoving* wavenumber, which stays constant during inflation. The momentum of the particle associated with the field θ is given by $\mathbf{p} = \mathbf{k}/a$. During inflation, the wavelength of the quantum mode is stretched by the rapid expansion, while the particle horizon, the largest distance a particle can travel in a Hubble time, remains roughly constant. Short-wavelength fluctuations are then quickly redshifted by the expansion until their wavelength becomes larger than the particle horizon of the universe. When this happens, the fluctuation starts evolving as classical perturbations.

An heuristic approach to the generation of primordial fluctuations comes from considering arbitrary perturbations to the metric $g_{\mu\nu}$ during inflation. This pertur-

bation can be expressed as the composition of a scalar (ρ), vector ($\vec{v}$) and tensor ($h_{\mu\nu}$) components. It is worth noting that no free-field known to date can produce vector perturbations, meaning that primordial vector perturbations produced during inflation are expected to be zero.

In the case of perturbations to the metric, each mode in the perturbation field follows the equation of motion of a free-field. This implies that classical fluctuations are generated during inflation and, because these perturbations arise in the metric of space time, it is expected their macroscopical counterpart to become gravitational waves. It is worth noting that this process will arise only if gravity can be quantized. Hence, detection of this primordial gravitational wave background would provide strong evidence supporting the existence of a quantum theory of gravity.

It can be shown that the energy scale of primordial gravity waves at horizon crossing, that is, when the wavelength of the perturbation equaled the size of the universe, is given by

$$h_{+ \times} = \frac{H}{2\pi m_{\text{pl}}} \quad (1.11)$$

while for scalar (density) perturbations, it can be shown that it follows

$$\left(\frac{\delta\rho}{\rho}\right) = \frac{H^2}{4\pi^{3/2}\dot{\phi}} \quad (1.12)$$

In the slow-roll approximation, both H and ϕ are slowly varying, so that the amplitude of the modes at horizon crossing changes very little during inflation. This can be described formally by considering the power spectrum of a perturbation field as its variance per logarithmic interval

$$\left(\frac{\delta X}{X}\right)^2 = \int P_s(k) d\log k \quad (1.13)$$

where X is the amplitude of some the perturbation.

Many inflation models predict a power spectrum in the form of a power law,

$P_s(k) \propto k^{n-1}$, where n is referred as the spectral index. Measurements of the spectral index are consistent with values of $n = 0.91^{+0.15}_{-0.07}$, as measured by the Planck satellite (see Planck Collaboration et al. (2018)) This value of n implies that the primordial power spectrum for scalar perturbations is indeed nearly scale invariant, which is consistent with the slow roll inflation paradigm. Similarly, metric perturbation modes are also expected to produce a power spectra in the form of $P_T(k) \propto k^{n_T}$.

The case of the tensor spectral index is very interesting, as the slow-roll inflation paradigm relates ϵ , the slow roll parameter, to n_T as

$$n_T = -2\epsilon \quad (1.14)$$

This allows using the equations of motion of a free-field in the slow roll approximation and find the ratio between amplitudes of tensor and scalar mode fluctuations,

$$\frac{P_T}{P_S} = 4\pi \left(\frac{\dot{\phi}}{m_{\text{pl}} H} \right)^2 = -n_T/2 \quad (1.15)$$

With these relations, it is usual to define r , the tensor-to-scalar ratio, as

$$r \approx 16\epsilon \quad (1.16)$$

Note that r is a direct proxy to the energy scale of the inflation.

The bottom line of section is that a rather simple model for inflation not only solves fundamental problems with of the Λ CDM/Big Bang model, but also provides three independent, *observable* parameters: the amplitude of scalar fluctuations $\left(\frac{\delta\rho}{\rho} \right)$ the tensor/scalar ratio r and both scalar and tensor spectral indices. Measuring these parameters will provide new and exciting insight on the physics that governed at the beginning of the universe.

Chapter 2

The cosmic microwave background

In this section, we shall introduce the reader to the Cosmic Microwave Background in the context of the Λ CDM model and inflation. We will first give some insight on the mathematical machinery that is necessary to analyze the CMB fluctuations across the sky. We then describe the physical processes that give rise to the CMB. The final section describes how does inflation can be studied using the CMB anisotropy field.

2.1 Basics for the analysis of cosmic microwave background

2.1.1 Polarization of light

Consider an electromagnetic plane wave with angular frequency ω , propagating on free space with wave-number k along $-\hat{r}'$, towards an observer. This plane wave can be written, in phasor form, as

$$\vec{E}(z, t) = \left(\epsilon_x e^{i\phi_x} \hat{x}' + \epsilon_y e^{i\phi_y} \hat{y}' \right) e^{kz - \omega t} \quad (2.1)$$

where ϵ and ϕ are real numbers denoting the amplitude and phase of the two transverse oscillatory modes in the x' and y' directions. The angular frequency ω is related to the wave number by $\omega = kc$, with c the speed of light.

Taking the real part of $\vec{E}(z, t)$ yields

$$\text{Re}\{\vec{E}\} = \epsilon_x \cos(kz - \omega t + \phi_x) \hat{x}' + \epsilon_y \cos(kz - \omega t + \phi_y) \hat{y}' \quad (2.2)$$

Looking in a perpendicular direction to the $x - y$ plane, the tip of $\vec{E}$ will trace an ellipse as a function of time. A convenient way of modeling this effect is by taking the projection of the electric field quantities, ϵ_x and ϵ_y , on the coordinate system which defining the ellipse major and minor axes, $\hat{e}_a$ and $\hat{e}_b$. In this new coordinate system, we have

$$\vec{E} = E_1 \cos(\omega t) \hat{e}_a + E_2 \sin(\omega t) \hat{e}_b \quad (2.3)$$

where

$$E_1 = \epsilon_0 \cos(\beta) \quad (2.4)$$

$$E_2 = \epsilon_0 \sin(\beta) \quad (2.5)$$

β , the ellipticity angle, belongs to the interval $[\pi/2, \pi/2]$. The angle measures how “in phase” are the x and y components of the plane wave: if $\beta = \pi/4$, they are perfectly in phase and the resulting plane-wave is circularly polarized, while if $\beta = \pm\pi/2$, or $\beta = 0$, the plane wave is linearly polarized. Note that, by conservation of energy,

$$E_1^2 + E_2^2 = \epsilon_x^2 + \epsilon_y^2 = \epsilon_0^2 \quad (2.6)$$

This way of expressing plane waves allows the introduction of the Stokes parameters I, Q, U and V

$$I = \epsilon_x^2 + \epsilon_y^2 = \epsilon_0^2 \quad (2.7)$$

$$Q = \epsilon_x^2 - \epsilon_y^2 = \epsilon_0^2 \cos(2\beta) \cos(2\chi) \quad (2.8)$$

$$U = 2\epsilon_x \epsilon_y \cos(\phi_x - \phi_y) = \epsilon_0^2 \cos(2\beta) \sin(2\chi) \quad (2.9)$$

$$V = 2\epsilon_x \epsilon_y \sin(\phi_x - \phi_y) = \epsilon_0^2 \sin(2\beta) \quad (2.10)$$

The Stokes parameters defined in this way are all real quantities. The I parameter measures the radiation intensity and is always positive. The other parameters describe the polarization state and can take either positive or negative sign.

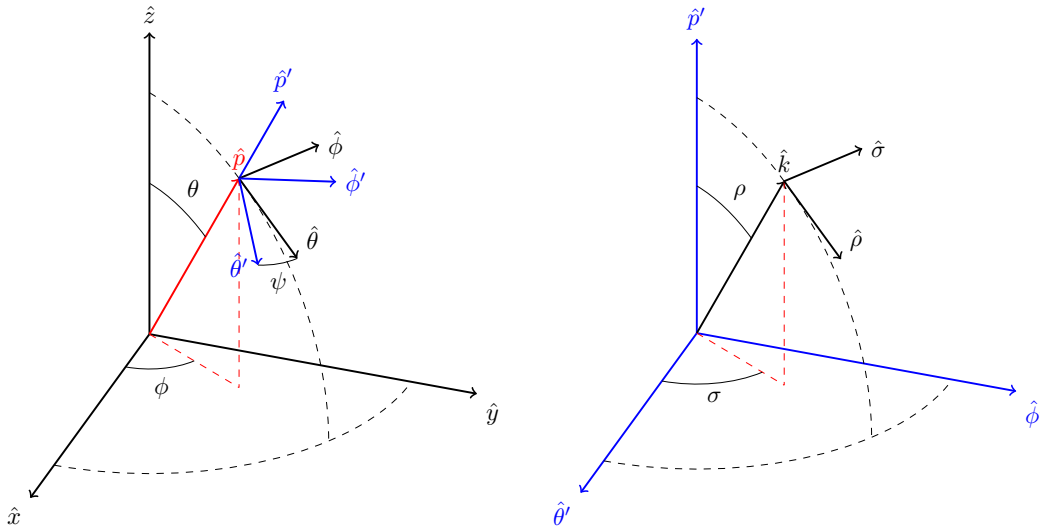


Figure 2.1: Left panel: sky basis. The sky basis is a generic spherical coordinate system. $\hat{x}$, $\hat{y}$ and $\hat{z}$ form an orthonormal basis. Unit vector $\hat{p}$ is defined by its spherical coordinates, co-latitude θ and longitude ϕ . Co-latitude increases from the north pole towards the south pole. Longitude increases from west to east. Tangent vectors at $\hat{p}$, $\hat{\theta}$ and $\hat{\phi}$, can be rotated around $\hat{p}$ by angle ψ to generate vectors $\hat{\theta}'$ and $\hat{\phi}'$. Note an observer looking towards the sky along $\hat{p}$ will measure angle ψ as increasing clockwise from South. Right panel: antenna basis. The antenna basis is the orthonormal system resulting from rotating $\hat{\theta}$ and $\hat{\phi}$ by an angle ψ . Very much like the sky basis, a unit vector $\hat{k}$ is described by its antenna basis co-latitude ρ and longitude σ . We can also build tangent vectors to a unit sphere centered at the origin of the antenna basis. These vectors are $\hat{\rho}$ and $\hat{\sigma}$.

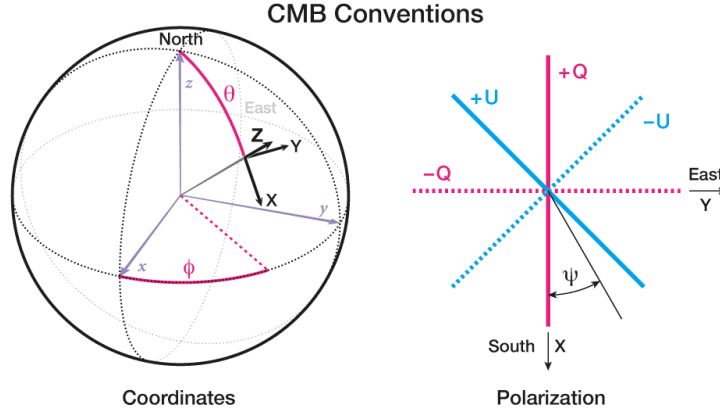


Figure 2.2: Figure showing the convention used for polarization among the CMB community. In this figure, z is parallel to $\hat{p}_0$, x is parallel to $\hat{\theta}$ and y points along $\hat{\phi}$. ψ is the angle between the antenna “up” direction, and x ($\hat{\theta}$). The sign conventions of Stokes parameters Q and U used by the CMB community are: positive Q if the polarization vector is aligned with $\hat{\theta}$ (North-South direction), negative Q if the polarization vector is aligned with $\hat{\phi}$ (East-West direction), positive U is aligned with $(\pm(\hat{\phi} + \hat{\theta})/\sqrt{2})$ (North/West-South/East direction), and negative U is aligned with $(\pm(\hat{\phi} - \hat{\theta})/\sqrt{2})$ (North/East-South/West direction). Note that the right-most panel of this figure corresponds to an observer looking towards *Earth*. Figure source: LAMBDA website

2.1.2 Coordinate systems

Because the CMB is an observable defined on the celestial sphere, it is natural to use a spherical coordinate system. It is also convenient to define the relations between the celestial coordinate system and a local coordinate system “attached” to an observer, as showed in Figure 2.1. The coordinate system used to describe the CMB (left panel in Figure 2.1) is the *sky basis*. Unit base vectors of these coordinate system are $\hat{x}$, $\hat{y}$ and $\hat{z}$, with $\hat{z}$ pointing towards the north pole. For convenience, we will refer as *pointing* as a 3-tuple $\bar{q}$, so that an observer aiming at co-latitude θ_0 and longitude ϕ_0 with position angle ψ_0 has pointing

$$\bar{q}_0 = (\theta_0, \phi_0, \psi_0) \quad (2.11)$$

Then the *pointing direction*, denoted by vector $\hat{p}_0$, can be expressed as a linear combination of the sky basis unit vectors and spherical coordinates (θ_0, ϕ_0) via

$$\hat{p}_0 = \sin(\theta_0) \cos(\phi_0) \hat{x} + \sin(\theta_0) \sin(\phi_0) \hat{y} + \cos(\theta_0) \hat{z} \quad (2.12)$$

Conversely, the vectors $\hat{\theta}_0$ and $\hat{\phi}_0$ are computed using

$$\begin{aligned} \hat{\theta}_0 &= \cos(\theta_0) \cos(\phi_0) \hat{x} + \cos(\theta_0) \sin(\phi_0) \hat{y} - \sin(\theta_0) \hat{z} \\ \hat{\phi}_0 &= -\sin(\phi_0) \hat{x} + \cos(\phi_0) \hat{y} \end{aligned} \quad (2.13)$$

These vectors can be used to build a second coordinate system which we will refer to as the *antenna basis* (see Figure 2.1, right panel). Given a pointing $\bar{q}_0$, the antenna basis base vectors can be written in term of sky basis coordinates as

$$\hat{p}'_0 = \hat{p}_0 \quad (2.14)$$

$$\hat{\theta}'_0 = \cos(\psi_0) \hat{\theta}_0 + \sin(\psi_0) \hat{\phi}_0 \quad (2.15)$$

$$\hat{\phi}'_0 = -\sin(\psi_0) \hat{\theta}_0 + \cos(\psi_0) \hat{\phi}_0 \quad (2.16)$$

Note that, in the antenna basis, coordinates analog to sky basis co-latitude and longitude are ρ and σ , respectively. As in equation 2.12, a vector $\hat{k}$ can be similarly written in terms of antenna basis coordinates as

$$\hat{k} = \sin(\rho) \cos(\sigma) \hat{\theta}'_0 + \sin(\rho) \sin(\sigma) \hat{\phi}'_0 + \cos(\rho) \hat{p}'_0 \quad (2.17)$$

while vectors analog to the ones described by equation 2.13 are

$$\begin{aligned} \hat{\rho} &= \cos(\rho) \cos(\sigma) \hat{\theta}'_0 + \sin(\rho) \sin(\sigma) \hat{\phi}'_0 - \sin(\rho) \hat{p}'_0 \\ \hat{\sigma} &= -\sin(\sigma) \hat{\theta}'_0 + \cos(\sigma) \hat{\phi}'_0 \end{aligned} \quad (2.18)$$

Finally, consider an observer in the antenna basis, equipped with a device that is

most sensitive to electromagnetic radiation oscillating along some direction, and define this direction as a unit vector $\hat{e}_{\parallel}$. By construction, this device will have minimum sensitivity to the perpendicular direction, which will denote by $\hat{e}_{\times}$. Electromagnetic radiation coming from the sky has its polarization basis defined with respect to the sky (see Figure 2.2), so any polarization sensitive device must compensate for the apparent rotation of its own polarization basis with respect to the sky. This can be accomplished by rotating the incoming Stokes vector by the angle between $\hat{e}_{\parallel}$ and $\hat{\theta}$, namely

$$\chi(\rho, \sigma) = \arctan \left(\frac{|\hat{e}_{\parallel} \times \hat{\theta}|}{\hat{e}_{\parallel} \cdot \hat{\theta}} \right) \quad (2.19)$$

Note that $\chi(\rho, \sigma)$ is defined in the antenna basis. Reader is referred to Appendix A for details on the computation of $\psi(\rho, \sigma)$ using spherical trigonometry.

2.1.3 Statistical analysis of a Stokes field on the sphere

Scalar spherical harmonic transform

Any scalar (rotationally invariant) field $T = T(\theta, \phi) = T(\hat{r})$ can be decomposed into spherical harmonics, very much like any scalar function f on the $x - y$ plane can be decomposed into Fourier modes. The spherical harmonic decomposition of T is an infinite sum of properly weighted natural resonance modes of the sphere. To obtain the spherical transform of T , one must solve

$$T(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi). \quad (2.20)$$

where the spherical harmonic of order ℓ and quantum number m , Y_{ℓ}^m is

$$Y_{\ell m}(\theta, \phi) = \sqrt{\frac{(2\ell+1)(\ell-m)!}{4\pi(\ell+m)!}} P_{\ell m}(\cos \theta) e^{im\phi} \quad (2.21)$$

and $P_{\ell m}(\cos \theta)$ is the associated Legendre polynomial of degree ℓ and order m .

The coefficients $a_{\ell m}$ are the spherical transform of T . Since spherical harmonics form a complete and orthonormal basis in Hilbert space, they are computed as

$$a_{\ell m} = \int_{4\pi} T(\theta, \phi) Y_{\ell m}(\theta, \phi)^* d\Omega \quad (2.22)$$

Spin-2 spherical harmonic transform

Consider an observer equipped with a detection device capable of measuring Stokes vector coming from the sky. Since the Stokes vector field S will follow the angle conventions on the definition of polarization, it is usually assumed that the “ Q -basis” of the detection device is aligned with $\hat{\theta} - \hat{\phi}$. However, if the detection device has its basis rotated by angle ψ with respect to the $\hat{\theta} - \hat{\phi}$ basis, the measured Stokes parameters will be related to S by

$$I_m = I \quad (2.23)$$

$$Q_m = \cos(2\psi)Q + \sin(2\psi)U \quad (2.24)$$

$$U_m = -\sin(2\psi)Q + \cos(2\psi)U \quad (2.25)$$

$$V_m = V \quad (2.26)$$

$$(2.27)$$

The above can also be written in terms of Q and U fields only, as

$$Q' \pm iU' = e^{\mp 2i\psi} (Q \pm iU) \quad (2.28)$$

where primes are for the Stoke vector elements on the rotated basis.

This extra symmetry breaks the assumption of the field being rotationally invariant, to that a more general spherical harmonic decomposition must be used. The work of Goldberg et al. (1967) describes the spin-weighted spherical harmonic decomposition, which is the generalization of the scalar spherical harmonic transform

for spin- s fields. In the case of cosmic microwave background analysis, however, it is sufficient to define the relevant operators for the case where s , the spin number, is equal to 2. One convenient way of computing the SHT of a spin-2 field is described in the work Zaldarriaga et al. (see Zaldarriaga & Seljak (1997)) The key behind this is to define two auxiliary scalar fields, with opposite parities under transformations, from a single spin-2 field. In the context of CMB, these are called the E-modes and B-modes, whose relations to Q and U fields are given by

$$E(\hat{r}) = -\frac{1}{2} [(\delta^-)^2(Q(\hat{r}) + iU(\hat{r})) + (\delta^+)^2(Q(\hat{r}) - iU(\hat{r}))] \quad (2.29)$$

and

$$B(\hat{r}) = \frac{i}{2} [(\delta^-)^2(Q(\hat{r}) + iU(\hat{r})) - (\delta^+)^2(Q(\hat{r}) - iU(\hat{r}))] \quad (2.30)$$

where $\delta^\pm$ are the spin raising and lowering operators, defined in the appendix of Zaldarriaga & Seljak (1997).

2.1.4 Connection with statistics

Statistics of the CMB are conveniently defined in terms of the power spectrum C_ℓ , which can be computed using the spherical harmonic transform by

$$\langle a_{\ell m} a_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell \quad (2.31)$$

where the δ -functions arise from the fact the CMB is isotropic, and the average is taken over a sufficiently large amount of CMB realizations. This leads to the following expression for the power spectrum coefficients

$$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} [a_{\ell m} a_{\ell m}^*] \quad (2.32)$$

Similarly, power spectrum relating temperature to polarization anisotropy are given

by

$$C_{\ell}^{XY} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} [a_{\ell m, X} a_{\ell m, Y}^*] \quad (2.33)$$

where $X, Y = T, E, B$.

In real space, the power spectrum is related to the expectation value of the correlation of the temperature and polarization anisotropies between two points in the sky. This implies that, if the CMB anisotropy field “seeded” by a random Gaussian process, like inflation, the resulting anisotropy field would be characterized by only six power spectra. The reader is referred to the work of Kamionkowski et al. (see Kamionkowski et al. (1997)) for a more complete description.

2.2 Physics of the CMB

2.2.1 Recombination

After nucleosynthesis took place, the Universe was still a hot and dense place. Photons continuously transferred momentum to electrons via Thomson scattering, preventing hydrogen atoms to form. An important fact about this process is that it follows Gaussian random statistics, because photons followed a random walk while interacting with electrons. As the universe kept expanding and cooling, the interactions between photons and electrons became less frequent and less energetic. At some point, the mean temperature of the photon field was low enough so hydrogen atoms could form. This process is called recombination.

It is possible to calculate when recombination took place. The reaction we are interested in is the formation of neutral hydrogen from the interaction of an electron and a proton



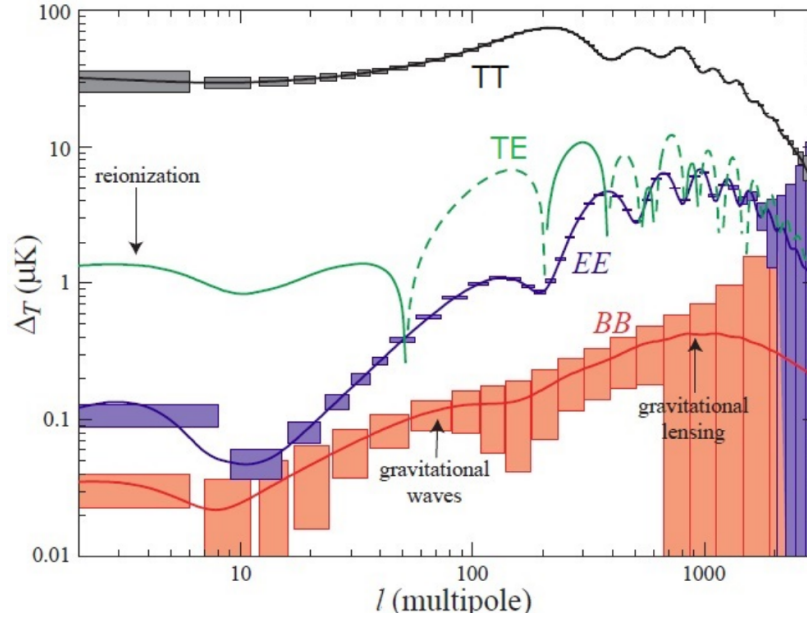


Figure 2.3: Figure showing the power spectrum of the CMB temperature anisotropy. Transparent polygons are representative of the uncertainties in the measured C_ℓ . First peak position of TT provides information about the curvature of the universe. Ratio between second and first peak heights of TT is related to the baryon density, while the third peak height in TT is a proxy for the density of dark matter. TE and EE provides extra constraints on the above cosmological parameters, and the redshift of reionisation. BB (B-modes) would probe generation of gravity waves during inflation.

where the $(1s)$ means the hydrogen in its ground state. Because the universe was in equilibrium after inflation, photons and electrons were, thermodynamically speaking, in chemical balance. Thus, at recombination, the rate at which electrons recombined with protons to form neutral hydrogen atoms was the same. This means that

$$\mu(p^+) + \mu(e^-) = \mu(H(1s)) \quad (2.35)$$

On the other hand, the equation for the chemical potential of a species with number density n_x and degeneracy number s_x is

$$\mu_x = m_x + T \log \left[\frac{n_x}{s_x} \left(\frac{2\pi}{m_x T} \right)^{3/2} \right] \quad (2.36)$$

If we plug this into the above equation and collect the terms, we arrive to the Saha

equation

$$\frac{n_p n_e}{n_{H(1s)}} = \left(\frac{m_p m_e T}{2\pi m_H} \right)^{3/2} e^{-\epsilon_0/T} \quad (2.37)$$

where ϵ_0 is the binding energy of the hydrogen atom, approximately 13.6 eV.

It is generally assumed that, after inflation, the universe was neutral, so that $n_e = n_p$, hence

$$n_{H(1s)} = n_{\text{tot}} - n_e \quad (2.38)$$

which can be used to solve equation 2.37, and obtaining the fraction of ionized hydrogen as a function of temperature. In particular, it can be shown that when the temperature dropped below 3800 Kelvin, more than half of the Hydrogen nuclei in the universe was recombined into neutral hydrogen. Because the universe expands adiabatically, temperature and redshift are the inverse of one another, so we can compute the redshift at which recombination took place, $z_{\text{rec}} \approx 1400$. At a redshift of $z = 1100$, more than 99% of the hydrogen in the universe was already recombined into neutral hydrogen, making the universe effectively transparent.

2.2.2 Polarization by Thomson scattering

During recombination, the dominant process was Thomson scattering. The scattering cross-section, defined as the radiated intensity per unit solid angle divided by the incoming intensity per unit area, is given by

$$\frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{8\pi} |\hat{\epsilon}' \cdot \hat{\epsilon}|^2 \quad (2.39)$$

where σ_T is the total Thomson scattering cross-section. $\hat{\epsilon}'$ and $\hat{\epsilon}$ are unit vectors which are parallel to the polarization vectors of incoming and outgoing light, respectively.

Following Kosowsky (see Kosowsky (1996)), it is useful to define the quantities

$I_x = I + Q$ and $I_y = I - Q$. Note that incoming light is unpolarized, so $Q' = U' = 0$, which in turn means that $I'_x = I'_y = I'/2$. The scattered intensities can be calculated as

$$I_x = \frac{3\sigma_T}{8\pi} \left(I'_x |\hat{\epsilon}'_x \cdot \hat{\epsilon}_x|^2 + I'_y |\hat{\epsilon}'_y \cdot \hat{\epsilon}_x|^2 \right) = \frac{3\sigma_T}{16\pi} I' \quad (2.40)$$

$$I_y = \frac{3\sigma_T}{8\pi} \left(I'_x |\hat{\epsilon}'_x \cdot \hat{\epsilon}_y|^2 + I'_y |\hat{\epsilon}'_y \cdot \hat{\epsilon}_y|^2 \right) = \frac{3\sigma_T}{16\pi} I' \cos^2(\theta) \quad (2.41)$$

from which the scattered Stokes parameters can be obtained

$$I = I_x + I_y = \frac{3\sigma_T}{16\pi} I' (1 + \cos^2(\theta)) \quad (2.42)$$

and

$$Q = I_x - I_y = \frac{3\sigma_T}{16\pi} I' \sin^2(\theta) \quad (2.43)$$

The above result applies for a single beam of light that propagates along the z -axis. In the case of an incoming, unpolarized radiation field of intensity $I(\theta, \phi)$, the above expressions must be integrated over all incoming directions. Care must be taken to ensure outgoing U and Q fluxes from a given incoming direction are rotated to a common coordinate system. This can be done via its transformation properties under rotations, yielding

$$I(\hat{z}) = \frac{3\sigma_T}{16\pi} \int_{4\pi} (1 + \cos^2(\theta)) I'(\theta, \phi) \quad (2.44)$$

$$Q(\hat{z}) - iU(\hat{z}) = \frac{3\sigma_T}{16\pi} \int_{4\pi} e^{2i\phi} I' \sin^2(\theta)(\theta, \phi) \quad (2.45)$$

where $\hat{z}$ is the outgoing direction of the scattered light. Expanding $I'(\theta, \phi)$ into spherical harmonics using equation 2.20, and using their orthogonality properties, the above result can be expressed as

$$I(\hat{z}) = \frac{3\sigma_T}{16\pi} \left[\frac{8}{3} \sqrt{\pi} a_{00} + \frac{4}{3} \sqrt{\frac{\pi}{5}} a_{20} \right] \quad (2.46)$$

$$Q(\hat{z}) - iU(\hat{z}) = \frac{3\sigma_T}{4\pi} \sqrt{\frac{2\pi}{15}} a_{22} \quad (2.47)$$

Thus, polarization is generated along the outgoing scattering direction provided that a_{22} , the quadrupole moment of the incoming radiation, is different from zero.

2.2.3 Origin of primordial CMB polarization

Quadrupole moments in the primordial photon field can have two physical sources. One of such sources are primordial, scalar perturbations to the matter density field. This produces a modulation of the linear polarization, as seen by an observer “inside” the CMB. The polarization field for a single scalar perturbation mode is shown in figure 2.4. It can be shown (see Kosowsky (1996)) that the polarization field generated by scalar perturbations corresponds to a pure E-mode field, and that is correlated with the temperature anisotropy field.

Another source for quadrupole moments in the photon field are gravitational waves produced during inflation. The polarization field produced by a single mode of the gravitational wave background is shown in figure 2.4. Because the evolution of perturbations to the space-time metric evolve independently, the polarization field produced by this primordial gravitational wave background is expected to be uncorrelated to the primordial density perturbation field. It has been shown that a primordial gravitational wave background would have produced a pure B-mode pattern in the CMB. The amplitude of this anisotropy is small, making its detection a challenging task. Confirming the existence of this polarization pattern would provide strong evidence supporting the inflationary paradigm.

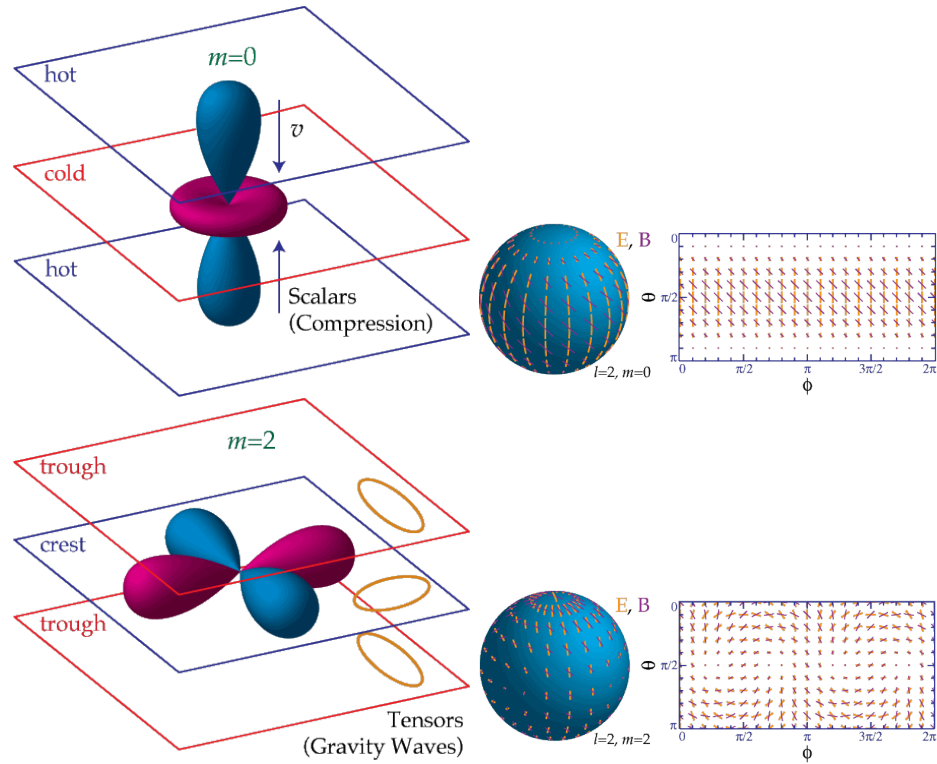


Figure 2.4: Upper figure: representation of the generation of polarization by a scalar (density) perturbation mode. The projection of the single mode on the sky generates the polarization field shown on the right. Bottom figure: representation of the generation of polarization by a single tensor perturbation mode, and its projection on the sphere. Figure source: A CMB Polarization Primer (Hu, 2011)

Chapter 3

Antennas

3.1 Antennas

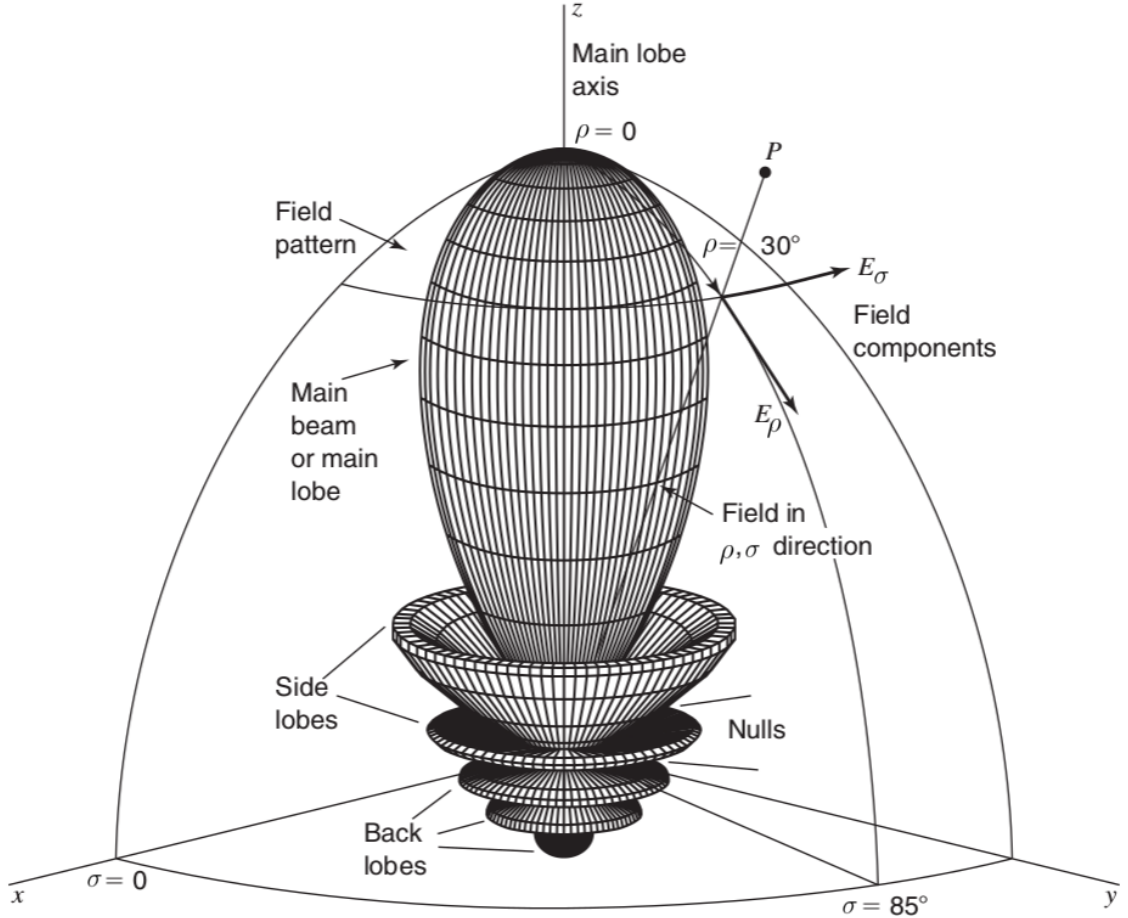


Figure 3.1: Three dimensional representation of an antenna beam, showing the main characteristics of a beam. Note the coordinates used for representing points defined in the antenna basis are labeled as ρ and σ . Figure source: Antennas for All Applications.

For the purposes of this thesis we will restrict ourselves to antennas used in CMB applications, that is, millimeter and sub-millimeter telescopes. The main goal of a telescope is to capture the energy carried by photons and convert it to another form of energy which can be recorded by either analog or digital systems. To do so, such an antenna must be able to focus electromagnetic radiation from the sky into a small area where the detection device is located. The reciprocity theorem allows to treat this system in two different, but mathematically equivalent ways. (see Jackson (1999)) We can either consider light to be coming from the sky, focused

by the antenna and then captured by the detection device (time-forward), or place an hypothetical source at the detection device and propagate electromagnetic fields through the antenna onto the sky (time-reversed). In what follows, we will treat an antenna in the time-reversed sense.

3.1.1 Beam, directivity and effective collecting area

Consider a monochromatic source of electromagnetic radiation oscillating at frequency ν (wavelength λ), so that all of its radiated power gets “captured” by an antenna. Call the radiated power ϵ . In an ideal situation, the optical elements of the antenna produce no energy loss, so the amount of power radiated by the antenna is ϵ as well. Usually, antennas for astrophysical applications are directional. We will call the direction where the radiated power density is maximal to be the origin of the *antenna basis*, its z axis pointing from the telescope to the sky.

Evaluating the electromagnetic fields that propagated through the antenna in the far-field or Fraunhofer zone (see Baars & Swenson (2008), chapter 3), allows us to define a distribution of radiation intensity (watts per steradian) around the antenna $U(\rho, \sigma)$, such that

$$\epsilon = \int_{4\pi} U(\rho, \sigma) d\Omega \quad (3.1)$$

This allows to define the antenna beam, $b(\rho, \sigma)$, as

$$b(\rho, \sigma) = \frac{U(\rho, \sigma)}{\text{MAX}[U(\rho, \sigma)]} = \frac{U(\rho, \sigma)}{U(0, 0)} \quad (3.2)$$

And the beam solid angle, Ω

$$\Omega = \int_{4\pi} b(\rho, \sigma) d\Omega \quad (3.3)$$

The solid angle is a quantity that measures how directional an antenna is: the smaller the solid angle, the more directional the antenna.

Another important concept in antenna theory is the directivity $D(\rho, \sigma)$. Directivity, the ability of an antenna to direct radio waves in one direction or receive from a single direction, is the ratio of the power transmitted by the antenna in direction (ρ, σ) to the power P_{iso} that would be transmitted by a hypothetical isotropic antenna. Because an isotropic antenna transmits power isotropically in all directions, we can write

$$D(\rho, \sigma) = \frac{U(\rho, \sigma)}{\int_{4\pi} U(\rho, \sigma) d\Omega / 4\pi} = 4\pi \frac{U(\rho, \sigma)}{\int_{4\pi} U(\rho, \sigma) d\Omega} \quad (3.4)$$

where we can use equation 3.2 to express D in terms of the beam

$$D(\rho, \sigma) = 4\pi \frac{U(0, 0)b(\rho, \sigma)}{U(0, 0) \int_{4\pi} b(\rho, \sigma) d\Omega} = 4\pi \frac{b(\rho, \sigma)}{\Omega} \quad (3.5)$$

In a time-forward sense, an antenna can be modeled as a “photon collector” with some effective collecting area $A_{\text{eff}}(\rho, \sigma)$, where the effective collecting area depends on the direction of the incoming light. In this case, the power per unit solid angle that gets *captured* by the antenna will be some radiant intensity times the effective collecting area

$$U'(\rho, \sigma) = A_{\text{eff}}(\rho, \sigma)F_0 \quad (3.6)$$

Similarly to the time reverse case, we can apply the definition of directivity to obtain

$$D'(\rho, \sigma) = \frac{A_{\text{eff}}(\rho, \sigma)F_0}{\int_{4\pi} A_{\text{eff}}^{\text{iso}}(\rho, \sigma)F_0 d\Omega} \quad (3.7)$$

where $A_{\text{eff}}^{\text{iso}}(\rho, \sigma)$ is the effective collecting area of an isotropic antenna, which can be shown to be

$$A_{\text{eff}}^{\text{iso}}(\rho, \sigma) = A_{\text{eff}}^{\text{iso}} = \frac{\lambda^2}{4\pi} \quad (3.8)$$

so that directivity D' becomes

$$D'(\rho, \sigma) = (4\pi) \frac{A_{\text{eff}}(\rho, \sigma)}{\lambda^2} \quad (3.9)$$

However, because of the reciprocity theorem (see Jackson (1999)), $D'(\rho, \sigma) = D(\rho, \sigma)$

$$D'(\rho, \sigma) = D(\rho, \sigma) \quad (3.10)$$

$$(4\pi) \frac{A_{\text{eff}}(\rho, \sigma)}{\lambda^2} = 4\pi \frac{b(\rho, \sigma)}{\Omega} \quad (3.11)$$

from which we arrive to the well known relation

$$A_{\text{eff}}(\rho, \sigma) = \frac{\lambda^2}{\Omega} b(\rho, \sigma) \quad (3.12)$$

3.1.2 Electrical properties of antennas

The formulation showed above is more of a microscopical description of an antenna. A much more detailed physical model of an antenna can be obtained by solving Maxwell equations. Unfortunately, analytical solutions can only be found in a handful of cases and, in general, numerical solutions must be used instead. Solving Maxwell equations numerically for systems where the wavelength λ is much smaller than the characteristic physical scale l is computationally demanding, and even intractable for the largest computing facilities on Earth, so for the sake of the argument we will consider an accurate solution to the antenna being excited by a source has already been computed. Since Maxwell equations are linear, solutions are often obtained using a monochromatic source of electric and magnetic fields,

$$\vec{E}_{\text{source}} = a(\hat{x})e^{\omega t} \quad (3.13)$$

$$\vec{H}_{\text{source}} = b(\hat{x})e^{\omega t} \quad (3.14)$$

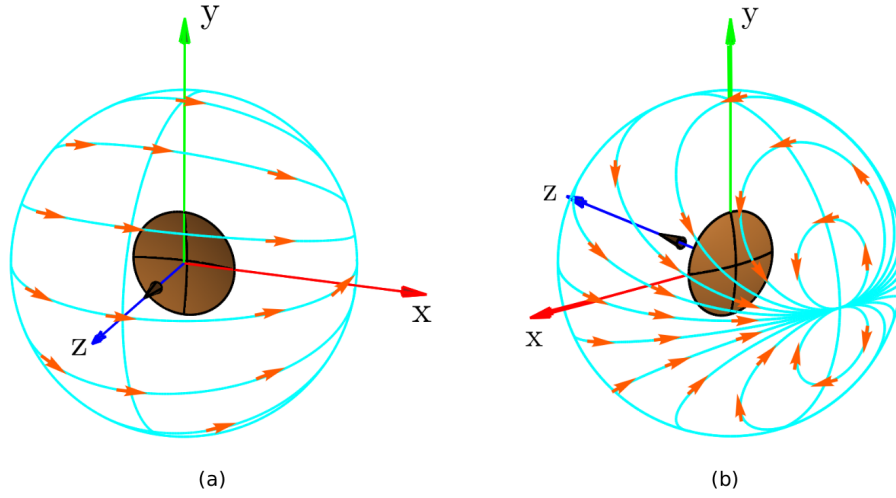


Figure 3.2: Graphical representation of the co-polarization field $\hat{e}_{\parallel}(\rho, \sigma)$ as defined by Ludwig's 3rd definition. The co-polar direction at beam center is parallel to the x axis. Note the co-polarization field becomes singular behind the antenna. Figure courtesy of TICRA.

Solution to Maxwell equations provide the distribution of electric and magnetic *currents* at the surface of the conducting elements of an antenna. It is possible to transform these currents into magnetic and electric fields around the antenna, $\vec{E}(\vec{x}, t)$ and $\vec{H}(\vec{x}, t)$. Note that $\vec{x} = \vec{x}(R, \rho, \sigma)$ is defined in the antenna basis. It can be shown that, for sufficiently large values of R , the distribution of electric and magnetic fields around the antenna can be represented as an infinite set of planar waves modes (see Baars & Swenson (2008), chapter 3), such that at any point in space the electromagnetic field is well characterized by a plane wave propagating along $\vec{x}$, that is, a plane wave with propagation vector

$$\hat{k} = \frac{1}{R} \vec{x} \quad (3.15)$$

This regime is called the far field, or Fraunhofer zone, of the antenna. In this regime, the electric and magnetic fields are interchangeable in CGS units.

It is convenient to separate the electromagnetic field distribution as the sum of components $\vec{E}_{\parallel}$ and $\vec{E}_{\times}$ (see figure 3.2), such that

$$\vec{E}_{\parallel}(\rho, \sigma) = (\vec{E}(\rho, \sigma) \cdot \hat{e}_{\parallel}(\rho, \sigma))\hat{e}_{\parallel}(\rho, \sigma) \quad (3.16)$$

$$\vec{E}_{\times}(\rho, \sigma) = (\vec{E}(\rho, \sigma) \cdot \hat{e}_{\times}(\rho, \sigma))\hat{e}_{\times}(\rho, \sigma) \quad (3.17)$$

where $\hat{e}_{\parallel}$ and $\hat{e}_{\times}$ are known as the co and cross polar basis vectors, which are defined according to Ludwig's 3rd-III definition (see Ludwig (1973)). The coordinate transformation described in Ludwig (1973) is necessary to guarantee that polarization is expressed with respect to a consistent basis across the sphere. In terms of the antenna basis unit vectors $\hat{\rho}$ and $\hat{\sigma}$

$$\hat{e}_{\parallel}(\rho, \sigma) = \sin(\sigma)\hat{\rho}(\rho, \sigma) + \cos(\sigma)\hat{\sigma}(\rho, \sigma) \quad (3.18)$$

$$\hat{e}_{\times}(\rho, \sigma) = \cos(\sigma)\hat{\rho}(\rho, \sigma) - \sin(\sigma)\hat{\sigma}(\rho, \sigma) \quad (3.19)$$

Expressing the co and cross polar components in phasor form yields

$$\vec{E}_{\parallel} = \epsilon_{\parallel} e^{i(kR + \phi_{\parallel} - \omega t)} \hat{e}_{\parallel} \quad (3.20)$$

and

$$\vec{E}_{\times} = \epsilon_{\times} e^{i(kR + \phi_{\times} - \omega t)} \hat{e}_{\times} \quad (3.21)$$

We can use these components to define the time averaged co and cross polar Poynting vectors, namely

$$\langle \vec{S}_{\parallel} \rangle = \text{Re} \left\{ \frac{1}{2} \vec{E}_{\parallel} \vec{E}_{\parallel}^* \right\} \quad (3.22)$$

$$\langle \vec{S}_{\parallel} \rangle = \frac{1}{2} \epsilon_{\parallel}^2 \quad (3.23)$$

and

$$\langle \vec{S}_\times \rangle = \text{Re} \left\{ \frac{1}{2} \vec{E}_\times \vec{E}_\times^* \right\} \quad (3.24)$$

$$\langle \vec{S}_\times \rangle = \frac{1}{2} \epsilon_\times^2 \quad (3.25)$$

Because the Poynting vector represents a distribution of power per unit area per unit frequency, it is natural to consider

$$U(\rho, \sigma)_{\parallel, \times} \propto \epsilon(\rho, \sigma)_{\parallel, \times}^2 \quad (3.26)$$

from where macroscopic quantities like the directivity and effective collecting area can be derived for both the co and cross polarized components.

3.1.3 Mueller matrix analysis for antennas: beam tensors

The Mueller matrix formalism is a vector-matrix framework that allows to express the polarized transfer function of an optical system. Mueller matrices deal with the polarization properties of incoherent light and optical elements by expressing incoherent electromagnetic radiation as Stokes vectors. Electromagnetic radiation with Stokes vector $\vec{S}_{\text{in}}$ that passes through an optical system with Mueller matrix $\mathbf{M}$, a 4×4 matrix, will experience a change of its polarization state. The output Stokes vector, $\vec{S}_{\text{out}}$, can then be expressed as

$$\vec{S}_{\text{out}} = \mathbf{M} \vec{S}_{\text{in}} \quad (3.27)$$

An antenna can also be modeled using Mueller matrices, as described in the work of Piepmeier et al. (2008) and O'Dea et al. (2007). The later describes a formalism that is more suitable for application to CMB experiments, while the former better fits general applications of antennas.

The polarization transfer function of the antenna can be characterized by a tensor field which we will call across this work as a beam tensor, or beamsor for short. A beamsor can be interpreted as a field of Mueller matrices, such that for each direction there is an associated Mueller matrix that quantifies how the antenna couples to Stokes vector. We will denote beamsors by letter $B = B(\rho, \sigma)$. Single elements of B are denoted as B^i_j . The value of $B(\rho, \sigma)$ is

$$B^i_j = \frac{1}{\tilde{\Omega}} \begin{bmatrix} B_{TT} & B_{TQ} & B_{TU} & B_{TV} \\ B_{QT} & B_{QQ} & B_{QU} & B_{QV} \\ B_{UT} & B_{UQ} & B_{UU} & B_{UV} \\ B_{VT} & B_{VQ} & B_{VU} & B_{VV} \end{bmatrix} \quad (3.28)$$

where $\tilde{\Omega}$ is a normalization factor

$$\tilde{\Omega} = \int_{4\pi} B_{TT}(\rho, \sigma) d\Omega \quad (3.29)$$

The work of O’Dea et al. (2007) describes a way of computing a beamsor¹. In this work, we are using a transposed version of this formalism. This was done intentionally for convenience: for instance, consider the product of B with a Stokes vector representing an unpolarized source. This yields

$$\begin{bmatrix} B_{TT} \\ B_{TQ} \\ B_{TU} \\ B_{TV} \end{bmatrix} = \begin{bmatrix} B_{TT} & B_{QT} & B_{UT} & B_{VT} \\ B_{TQ} & B_{QQ} & B_{UQ} & B_{VQ} \\ B_{TU} & B_{QU} & B_{UU} & B_{VU} \\ B_{TV} & B_{QV} & B_{UV} & B_{VV} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.30)$$

where it is more evident that elements B_{Ti} , with $i = Q, U, V$, correspond to the temperature to polarization leakage beams. In the work of O’Dea et al. (2007), this would have yielded the B_{iT} components instead.

A practical way of calculating beam tensor elements can be carried out by using

¹The term used in the original paper is “beam Mueller fields”

the “beam Jones matrices” described in the work of Rosset et al. (2010) (Michael K. Brewer, private communication). This formalism assumes that, for a given antenna, a model or solution for the distribution of electric fields in the far-field is already known. For the purposes of this derivation, we consider such solutions as two distribution of electric fields on the sphere: $\vec{E}_x = E_{x,\parallel}\hat{e}_\parallel + E_{x,\times}\hat{e}_\times$, and the distribution of fields for another detector that was placed perpendicular to x at the focal plane, $\vec{E}_y = E_{y,\parallel}\hat{e}_\parallel + E_{y,\times}\hat{e}_\times$. Using the formalism described in the work of Rosset et al. (2010), we can build a Jones matrix for the experiment

$$\mathbf{J} = \begin{bmatrix} E_{x,\parallel} & E_{x,\times} \\ -E_{y,\times} & E_{y,\parallel} \end{bmatrix} \quad (3.31)$$

where the minus sign in $E_{y,\times}$ comes from the definition of beam Mueller fields (see O’Dea et al. (2007)). The associated beam tensor elements can then be obtained as

$$B_j^i = \sigma_i \mathbf{J} \sigma_j \mathbf{J}^\dagger \quad (3.32)$$

where $i, j = T, Q, U, V$ and σ_i are the Pauli matrices

$$\sigma_T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (3.33)$$

$$\sigma_Q = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (3.34)$$

$$\sigma_U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (3.35)$$

$$\sigma_V = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad (3.36)$$

For completeness, we also show the expanded the beam tensor elements, which match the definition given in O’Dea et al. (2007)

$$\begin{aligned}
 B_{TT} &= \frac{1}{2} \left(\left| \vec{E}_x \right|^2 + \left| \vec{E}_y \right|^2 \right) \\
 B_{QT} &= \frac{1}{2} \left(\left| \vec{E}_{x,\parallel} \right|^2 - \left| \vec{E}_{x,\times} \right|^2 + \left| E_{y,\times} \right|^2 - \left| E_{y,\parallel} \right|^2 \right) \\
 B_{UT} &= \frac{1}{2} \left(\vec{E}_{x,\parallel} E_{x,\times}^* - E_{y,\parallel} E_{y,\times}^* \right) + \text{c.c.} \\
 B_{VT} &= \frac{1}{2} i \left(\vec{E}_{x,\parallel} \vec{E}_{x,\times}^* + \vec{E}_{y,\parallel} \vec{E}_{y,\times}^* \right) + \text{c.c.} \\
 B_{TQ} &= \frac{1}{2} \left(\left| \vec{E}_x \right|^2 - \left| \vec{E}_y \right|^2 \right) \\
 B_{QQ} &= \frac{1}{2} \left(\left| \vec{E}_{x,\parallel} \right|^2 - \left| \vec{E}_{x,\times} \right|^2 + \left| \vec{E}_{y,\parallel} \right|^2 - \left| \vec{E}_{y,\times} \right|^2 \right) \\
 B_{UQ} &= \frac{1}{2} \left(\vec{E}_{x,\parallel} \vec{E}_{x,\times}^* + \vec{E}_{y,\parallel} \vec{E}_{y,\times}^* \right) + \text{c.c.} \\
 B_{VQ} &= \frac{1}{2} i \left(\vec{E}_{x,\parallel} E_{x,\times}^* - E_{y,\parallel} E_{y,\times}^* \right) + \text{c.c.} \\
 B_{TU} &= \frac{1}{2} \left(-\vec{E}_{x,\parallel} \vec{E}_{x,\times}^* + \vec{E}_{y,\times} \vec{E}_{y,\parallel}^* \right) + \text{c.c.} \\
 B_{QU} &= \frac{1}{2} \left(-\vec{E}_{x,\parallel} \vec{E}_{y,\times}^* - \vec{E}_{x,\times} \vec{E}_{y,\parallel}^* \right) + \text{c.c.} \\
 B_{UU} &= \frac{1}{2} \left(\vec{E}_{x,\parallel} \vec{E}_{y,\parallel}^* - \vec{E}_{x,\times} \vec{E}_{y,\times}^* \right) + \text{c.c.} \\
 B_{VU} &= \frac{1}{2} i \left(\vec{E}_{x,\parallel} \vec{E}_{y,\parallel}^* + \vec{E}_{x,\times} \vec{E}_{y,\times}^* \right) + \text{c.c.} \\
 B_{TV} &= \frac{1}{2} i \left(\vec{E}_{x,\parallel} \vec{E}_{y,\times}^* - \vec{E}_{x,\times} \vec{E}_{y,\parallel}^* \right) + \text{c.c.} \\
 B_{QV} &= \frac{1}{2} i \left(\vec{E}_{x,\parallel} \vec{E}_{y,\times}^* + \vec{E}_{x,\times} \vec{E}_{y,\parallel}^* \right) + \text{c.c.} \\
 B_{UV} &= \frac{1}{2} i \left(-\vec{E}_{x,\parallel} \vec{E}_{y,\parallel}^* + \vec{E}_{x,\times} \vec{E}_{y,\times}^* \right) + \text{c.c.} \\
 B_{VV} &= \frac{1}{2} \left(\vec{E}_{x,\parallel} \vec{E}_{y,\parallel}^* + \vec{E}_{x,\times} \vec{E}_{y,\times}^* \right) + \text{c.c.}
 \end{aligned} \tag{3.37}$$

where c.c. stands for complex conjugate.

3.2 Data model for observations using polarized antenna

3.2.1 Detected Stokes parameters

CMB measurements are carried out by antennas equipped with polarization sensitive devices. Usually, this is actually a partially polarized, total power detector. We can model the process by which one of these detectors transforms incoming radiation in a Stokes vector to either voltage or current by calculating the dot product of the with vector (see Jones et al. (2007))

$$D_j(\zeta, \epsilon, s) = \frac{s}{2} [(1 + \epsilon), (1 - \epsilon) \cos(2\zeta), (1 - \epsilon) \sin(2\zeta), 0] \quad (3.38)$$

where we have defined the polarization leakage term, ϵ , such that $1 - \epsilon$ is the polarization efficiency, and s is the voltage (or current) responsivity of the detector. Finally, the angle ζ is the orientation of the axis of sensitivity of the linear polarizer with respect to $+Q$ on the sky. The action of taking a total power measurement on some Stokes vector S^i is then given by

$$d = D_i S^i \quad (3.39)$$

Placing a partially polarized, total power detector after an antenna requires to take into account the polarizing properties of the antenna. This can be carried out using beam tensors. For simplicity, consider the case where antenna pointing is $\bar{q} = (0, 0, 0)$, such that the sky basis coincides with the antenna basis. Note that we are making this fact explicit by writing θ, ϕ instead of ρ, σ since, in this particular case, they are equivalent

$$S^i \propto \int_{4\pi} B_j^i(\theta, \phi) S^j(\theta, \phi) \mathrm{d}\Omega \quad (3.40)$$

For an arbitrary pointing $\bar{q}$, we need to account for the apparent rotation of the

beam tensor and the antenna polarization basis with respect to the sky. We thus need to de-rotate the Stokes vector field by angle χ as defined in Equation 2.19 using a new operator

$$\Lambda_k^j(\rho, \sigma; \bar{q}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\chi) & \sin(2\chi) & 0 \\ 0 & -\sin(2\chi) & \cos(2\chi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.41)$$

so that the convolution between a beam tensor and the sky becomes

$$S^i(\bar{q}) \propto \int_{4\pi} B_j^i [\Lambda_k^j S^k] \, \mathbf{d}\Omega \quad (3.42)$$

where we note that the beamsor has been implicitly aligned with the sky basis according to pointing $\bar{q}$.

Finally, using Equation 3.42 and 3.39, we arrive to an expression that describes every sample $d(\bar{q})$ in the data stream,

$$d(\bar{q}) = D_i(\zeta, \epsilon, s) \int_{4\pi} B_j^i [\Lambda_k^j S^k] \, \mathbf{d}\Omega \quad (3.43)$$

Equation 3.43 allows to model the complex interaction between the beamsor and the sky, for arbitrary scan strategies of the antenna and sky models. For many applications, it is not necessary to compute 3.43 to estimate the effect of the beam in the power spectra, as the most relevant effect of the beam is to act as a low pass filter in harmonic space (see Page et al. (2003)). However, when sidelobes are included, analytic approximations become unpractical. Moreover, far sidelobes can produce unwanted pickup of bright sources, like the Sun, the Moon, planets or the Galaxy into the CMB maps. The net effect is then highly specific to the experiment, and must be estimated numerically using a formalism similar to the one described above.

Chapter 4

Computer simulations of CMB experiments

Since the discovery of the CMB, sensitivity and complexity of experiments aiming at characterizing it has scaled exponentially; from the single horned “Sugarspoon” antenna, to spaceborn missions like Planck or WMAP. Because of the boost in sensitivity, systematic effects that were neglected before might now play a major role. This is of particular importance for measuring faint signals like primordial B-modes. For this reason, modeling, control and mitigation on these effects is a crucial area of development among the CMB community.

Because building a CMB telescope is expensive and time-consuming, the only way of estimating the performance of the mission is to perform computer simulations of possible scenarios. This allows to explore the impact that systematic effects have on the science outcomes. A remarkable example of such a pipeline are the Full Focal Plane simulations (FFP) performed the Planck mission (Planck Collaboration et al. (2016a)) FFP simulations were very compute intensive, taking millions of CPU hours on a world-class supercomputer at the time they were performed. The results, though, were outstanding, and allowed the Planck mission to obtain the best constrain on the Λ CDM model parameters to date. This points to the importance of having fast and accurate computer codes for ongoing and future missions aiming at characterizing the CMB anisotropy field.

4.1 Convolution of discrete fields on the sphere

4.1.1 Pixelization

It is convenient to define a few conventions that will be used in what follows. For instance, continuous fields on the sphere will be named using capital letters, like F . Pixelated counterparts will be denoted as $\mathbb{F}$. Finally, the k -th pixel will be denoted as ${}_k\mathbb{F}$. Pixelization of a function defined on the sphere can thought as a discretization process, where the continuous function $F = F(\hat{u})$ becomes $\mathbb{F}$. If F represents a distribution of Stokes vectors on the sphere, then $\mathbb{F}$ would become a matrix of N_p

(number of pixels) rows and four columns.

A pixelization scheme that minimizes aliasing can be constructed by using a pixel weighting function, $w(\hat{u})$. The goal of a pixel weighting function is to smooth rapid variations of F inside a pixel, so that ${}_k\mathbb{F}$ becomes

$${}_k\mathbb{F} = \int {}_k w(\hat{u}) F(\hat{u}) d\hat{u} \quad (4.1)$$

One of such pixelization schemes is HEALPix (Gorski et al. (1999)). HEALPix is used extensively among the CMB community because the pixelization scheme guarantees that all pixels have the same solid angle Ω . The size of every pixel is determined by a single parameter NSIDE, so that

$$\Omega(\text{NSIDE}) = \frac{4\pi}{12 \times \text{NSIDE}^2} \quad (4.2)$$

Note that, from the above, we infer that the amount of pixels N_p for a given NSIDE parameter is given by

$$N_p = 12 \times \text{NSIDE}^2 \quad (4.3)$$

4.1.2 Convolution

As described in Chapter 3, measuring the sky signal using a detector can be modeled as the convolution of a beamsor and the sky. Key to this process is the fact that the beam tensor needs to be properly aligned with respect to the sky basis for any given pointing $\bar{q}$. For the sake of the argument, we will start by describing a simple scenario where the beam tensor and sky are scalar and the pointing tuple $\bar{q}$ is $\bar{q} = (0, 0, 0)$. According to the definitions described in 2.1.2, this means the beamsor is pointing at the north pole with position angle $\psi_0 = 0$. The discrete, or pixel-space convolution then becomes a dot product, namely

$$s(\bar{q}_0) = \Omega \sum_p^{N_p} {}_p\mathbb{B}_p \mathbb{S} \quad (4.4)$$

where we implicitly assumed both distributions were pixelated using the same **NSIDE** parameter.

In general, $\bar{q}_0$ will not point at the North pole, meaning the beam tensor must be re-pixelated accordingly. This is because applying a rotation to a sky basis vector can result in an antenna basis vector that does not coincide with the center of any beamsor pixel. In practice, this is done not by rotating the beam tensor, but by interpolating it at the corresponding antenna basis coordinates. Denoting $\mathbb{B}'$ to the re-pixelated beam, the more general case becomes

$$s(\bar{q}) = \Omega \sum_p^{N_p} {}_p\mathbb{B}'_p \mathbb{S} \quad (4.5)$$

The above definitions work in the case of a scalar beamsor and sky, while the more realistic case requires considering their dependence on the beam tensor orientation. This can be accomplished by including discrete version of operator $\Lambda_j^i, {}_p\lambda_j^i$. The pixelated counterpart of 3.43 then becomes

$$S^i(\bar{q}) = \Omega \sum_p^{N_p} {}_p\mathbb{B}_j^{i'} \lambda_k^j {}_p\mathbb{S}^k \quad (4.6)$$

Finally, equation 4.6 allows us to also write Equation 3.43 for the pixelated case as

$$d(\bar{p}) = \Omega D_i(\zeta, \epsilon, s) \sum_p^{N_p} {}_p\mathbb{B}_j^{i'} \lambda_k^j {}_p\mathbb{S}^k \quad (4.7)$$

4.2 The PIXel SPAce CONvolver: PISCO

Pixel space convolution codes for CMB have been used in the past. A good example is FEBeCoP (see Mitra et al. (2011)), which as used to simulate the Planck mission (see Mitra et al. (2011)) Unfortunately, FEBeCoP is not public. No other

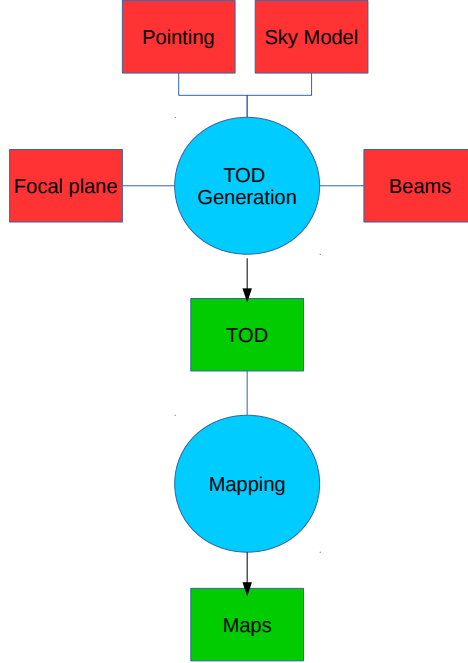


Figure 4.1: Basic flow of a typical PISCO simulation pipeline. Red polygons show the required user input. PISCO uses this input and produces TOD (green polygon). This TOD stream is calculated using equation 4.7 for all pointing directions. TOD can then be sent into a mapper and, finally, to a power spectra estimator tool. PISCO does not compute pointing nor produces maps from TOD by itself; these tasks are left to external programs.

code that performs the convolution in pixel domain was found in the literature, so we developed our own implementation of pixel space convolution, the **PI**xel **S**pace **CO**nvolver (PISCO). PISCO is a tool with the capability of generating TOD provided a beamsor, the experiment’s scanning strategy and a sky model. In this section, we present a pathfinder implementation of PISCO that makes use of the massively parallel architecture of modern Graphics Processing Units (GPU) and is designed for scalability, portability and ease of use.

PISCO is the software tool in charge of generating mock TOD given a beamsor,

a sky model and a scanning strategy. A diagram showing the general workings of PISCO is shown in Figure 4.1. PISCO receives as input a sky model in the form of 4 maps representing Stokes parameters I, Q, U and V , a beamsor, pointing and focal plane information. PISCO stores the beamsor elements and sky model as HEALPix maps. HEALPix was chosen because it is widely used among the CMB community, and because it naturally handles the closed surface topology of the sphere. HEALPix also provides equal area pixels, which is a desirable feature when computing convolution in pixel space. The focal plane specifications are only needed if multiple detectors are being included in the pointing stream, as PISCO needs the angle ζ of each detector to compute equation 4.7. All the inputs are sent to the TOD generation function, which returns the data streams. At this point, the data can either be saved to disk or sent to a map-making code. This last step is preferred as, usually, input-output operations are time consuming. Finally, maps can be analyzed using external tools to calculate the power spectra.

4.2.1 Design

The impact of any scientific tool depends not only on efficiently producing accurate results, but also on its capability to be used by as many users as possible. PISCO was designed from scratch with these directives in mind, so effort was made to provide an simple user-level layer while still retaining performance and scalability on HPC environments. This was achieved by combining the scriptability of `Python` and the performance of `C`. The user-level interface is based on `Python`, providing the user with a familiar language and the ability to design and control the overall execution flow using high level abstractions. `Python` also provides interfaces to application programming interfaces (API) like the Message Passing Interface (MPI) via `mpi4py`, which enables PISCO to take advantage of distributed computing systems. Performance, on the other hand, required all algorithms making computationally expensive calculations to be implemented in a “machine friendly” language. Compiled

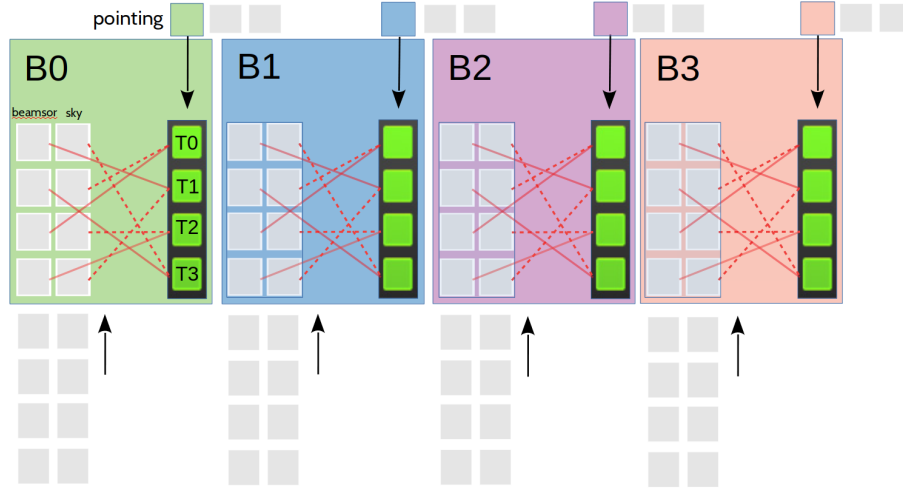


Figure 4.2: Parallelization scheme. This figure shows the case of PISCO executing in 4 blocks ($B = 4$), with four threads per block ($T = 4$). Arrays with beamsor and sky elements are at the bottom. Each block has access to four beamsor and sky pixels (gray boxes inside colored boxes) and one pointing entry (colored small boxes) at a time. Green boxes represent the multiplication process of a single beamsor pixel with a single sky pixel. This includes rotating the sky pixel to the antenna polarization basis, and computing the re-pixelization of B at the corresponding coordinates. Solid and dotted red lines represent the complex memory access pattern generated by this process. Every thread within a block writes its result to shared memory space. When a thread finishes its computation, it waits until all threads have finished and a reduction on the shared memory space is performed across all blocks. This process is repeated for every pointing. At the end of the procedure, each block has computed the convolution of a beamsor with the sky for a particular pointing, and every shared memory space of the block has the corresponding result. These results are collected into the GPU global memory, which is then transferred back to CPU (host) memory.

languages, like FORTRAN or C/C++ are known to deliver the best performance, at the expense of increased developing time, code complexity and some loss in portability. We chose C because Python has built-in capability to interface with it. Also, C is the officially supported language by one of the most prominent API to implement algorithms using Graphics Processing Units (GPUs), CUDA, which was used to accelerate the TOD generation procedure described in figure 4.2.

4.2.2 Implementation using CUDA

GPUs allow for substantial acceleration of algorithms that perform a large amount of independent operations. TOD generation using equation 4.7 presents an optimal application case because all operations are independent of each other. In this work, we used the Compute Unified Device Architecture framework from NVIDIA to implement the TOD generation routine. The reader is referred to Sanders & Kandrot (2010) for an excellent description of CUDA and associated capabilities.

To better understand how the parallelism in 4.7 can be exploited, consider the process of synthesizing N_T measurements using a CUDA grid of B blocks and T threads. Consider each measurement to have an associated pointing $\bar{q}_t$ with $t = 0..N_T$. PISCO performs a double parallelization scheme: the “slow” loop ($L1$) scans the pointing stream and associates every block to a pointing $\bar{q}_t$. A second, “faster” loop ($L2$), iterates over a list of pixels, which correspond to sky pixels that are “inside” the beams extension. This list of pixels is constructed in advance and then transferred to the GPU. $L2$ executes T operations in parallel. Great care was taken to ensure no race conditions arise when multiple threads try to read (write) from (to) the same memory address. As every block executes T convolution operations in parallel, and the CUDA grid runs B simultaneous blocks, the parallelism is $B \times T$. Furthermore, if G GPUs are available, the computation can be distributed among them, increasing the parallelism to $G \times B \times T$. A graphical description of this process is shown in figure 4.2.

4.2.3 Performance

A simulation of a realistic CMB experiment took approximately 45 minutes using a node equipped with two Intel Xeon E5-2610 processors (10 physical cores and 20 threads per processor), 256 GB of RAM and one NVIDIA GTX 1080. This simulation generated 1 week of TOD, meaning that PISCO executed around 224 times faster than “real time operation” of the experiment. Measuring performance of the current

implementation in FLOPS is a highly nontrivial task, as the TOD generation routine performs both integer and floating point mathematical operations. FLOPS are also not a representative metric given that there is a major impact of the memory access pattern inside the main CUDA routine. This memory access pattern is caused by having to fetch beamsor pixels in a pseudo-random manner. In an effort to provide a comparison basis with other implementations, we report that this simulation can achieve 322560 convolutions per second.

We note that, for this particular test, it is quite possible for programs like `beamconv` (see Duivenvoorden et al. (2018)) to achieve much higher performance than *PISCO*. It is worth noting, however, that *PISCO* should have a smaller cost associated to increasing the complexity of the beam, i.e., by adding ghosting, high frequency features (in angular space) and time-dependent beam parameters, or transient events on the sky model, like varying temperature of the ground surrounding the receiver. We believe adding this extra level of realism would make *PISCO* comparable to `beamconv` in terms of performance. This also makes *PISCO* a good complement to other software tools that perform similar tasks.

4.2.4 Future improvements

The current implementation must calculate the list of sky pixels involved in each convolution, for all pointing directions, before the CUDA routine is launched. Having this list of pixels in memory decreases the available parallelism, as fewer pointing directions can be used at a given time. While the wall-time associated with this operation is modest, the result must be kept in memory and transferred to the GPU, so that the associated buffer quickly becomes too large to be held in the VRAM. Currently, *PISCO* handles this situation by performing the generation of TOD in blocks to avoid memory overflow. In the test machine, computing and transferring the lists of pixels can take up to 13% of the *overall* simulation wall-time. A solution to this problem has already been devised and will be implemented in future releases.

Another drawback of the current implementation is the use of global memory to hold the beam tensor elements. Future releases will exploit data locality by making use of the CUDA texture memory pipeline (see Sanders & Kandrot (2010)). Finally, while the current implementation of PISCO was designed to execute in multiple GPU nodes, significant coding effort is required to provide the user with an easy to use interface. Experiments were performed emulating a multi-node system by making PISCO use all 3 GPUs of the machine. These tests showed an almost linear increase in performance, but more work is required in order to find the knee of the curve between performance and available GPUs.

4.3 Code validation

4.3.1 Point sources

The first test involved comparing the capability of PISCO to correctly reproduce observations of a point source at an arbitrary location on the sphere. The beamsor used in this test corresponded to the equivalent of a circularly symmetric Gaussian beam. The simulation setup can be summarized as

- Build a sky map with a single pixel with coordinates (θ_k, ϕ_k) having a Stokes vector $S^i = (1, Q, U, 0)$, such that $Q^2 + U^2 = 1$.
- Set up a raster scan around (θ_k, ϕ_k) for a detector with $\zeta = 0$. In order to have full polarization coverage, the scan is performed 3 times with angles $\psi = 0^\circ, 45^\circ, 90^\circ$.
- Make maps of TOD generated by PISCO.
- Compare the maps with a harmonic space convolution of the single pixel map with the Gaussian beam.

Convolution in harmonic space was performed using the routines available in the HEALPix software package Górski et al. (2005), particularly the `smoothing` routine

provided by `healpy` Python wrapper. `smoothing` performs the harmonic space tensor convolution of a circularly symmetric Gaussian with a given sky model. In order to ease the comparison, TOD produced by PISCO were also projected to a HEALPix map. Since synthetic TOD are free of noise, an unweighted co-adding algorithm was used for map-making. We note that at least three observations at different position angles are needed in order to recover the Stokes parameters I, Q and U from each pixel.

In order to find the optimal HEALPix resolution parameters for the beamsor and the sky, we used the prescription described in the FeBeCOP as a starting point (see Mitra et al. (2011)). This yielded a “rule of thumb” for the ratio $\text{NSIDE}_{\text{beamsor}}/\text{NSIDE}_{\text{sky}} = 4$ so that the convolution preserved the input map flux to better than 0.1%. Subsequent experiments revealed that error in the flux error independent of the location of the point source, so its net effect on whole sky maps should be a constant amplitude bias on the power spectra.

It is also crucial to note that, given the non-zero extension of beamsor, intra-beam variations of χ can occur. This can become problematic near the poles and, if not taken into account properly, can yield to leakage from E-mode to B-modes. In order to correctly take this into account, the computation of χ used by PISCO was derived from first principles using spherical trigonometry (see Appendix A)

4.3.2 Ideal CMB experiment

Description

The simulation of an ideal CMB experiment was accomplished by the following:

- Build a beamsor without cross-polarization Each B_{ii} component is a circular Gaussian beam with FWHM of 1.5° .
- Build mock CMB whole sky maps with a tensor-to-scalar ratio $r = 0.0$.

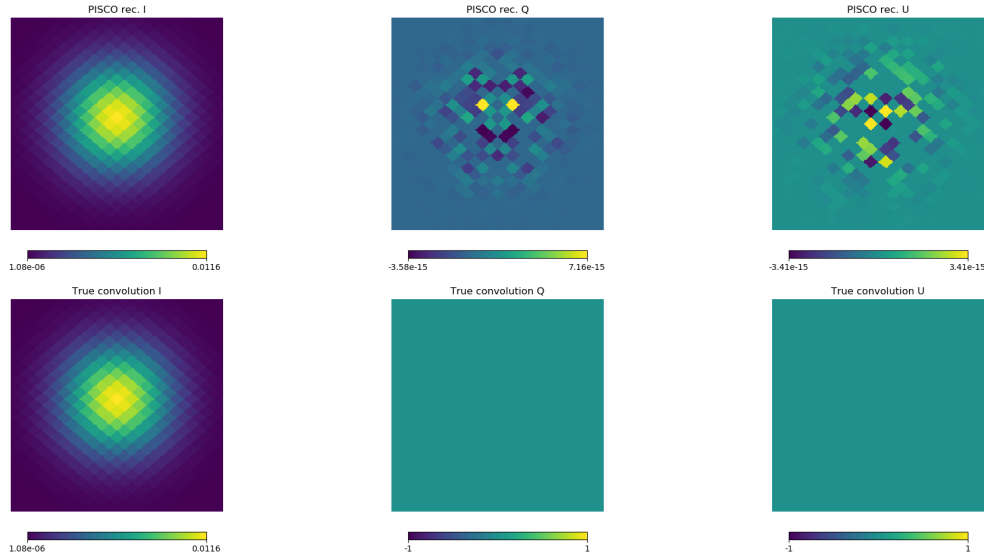


Figure 4.3: First row: PISCO convolution of a point source located at declination zero with Stokes vector $(1, 0, 0, 0)$. Amplitudes of Q and U maps are consistent with zero when the noise generated by rounding errors, inherent to floating point arithmetic, are considered. Bottom row: $\ell - m$ space convolution of the same source. Amplitudes of the PISCO and exact convolution agree to better than 0.1%. Input map NSIDE is 256. Beam has an NSIDE parameter of 1024.

- Set up a scanning strategy to visit each pixel center at 3 different position angles $\psi_0 = 0^\circ, 45^\circ, 90^\circ$.
- Make maps of TOD generated by PISCO.
- Compare spectra generated from the maps with spectra of the input maps.

The input sky maps were generated using a combination of CAMB (see Lewis & Bridle (2002)) to generate C_ℓ , and **synfast** to generate maps from the C_ℓ . The cosmological parameters are consistent with those reported by the Planck satellite collaboration (see Planck Collaboration et al. (2016b)). This procedure returns 3 CMB anisotropy maps, one for each Stokes parameter¹ CAMB was configured to return a CMB with no primordial B-modes ($r = 0$) and no lensing, as this last effect is expected to transform E-modes to B-modes. The resulting B-mode power

¹It is usually assumed the CMB has no circular polarization, so the V map was set to zero.

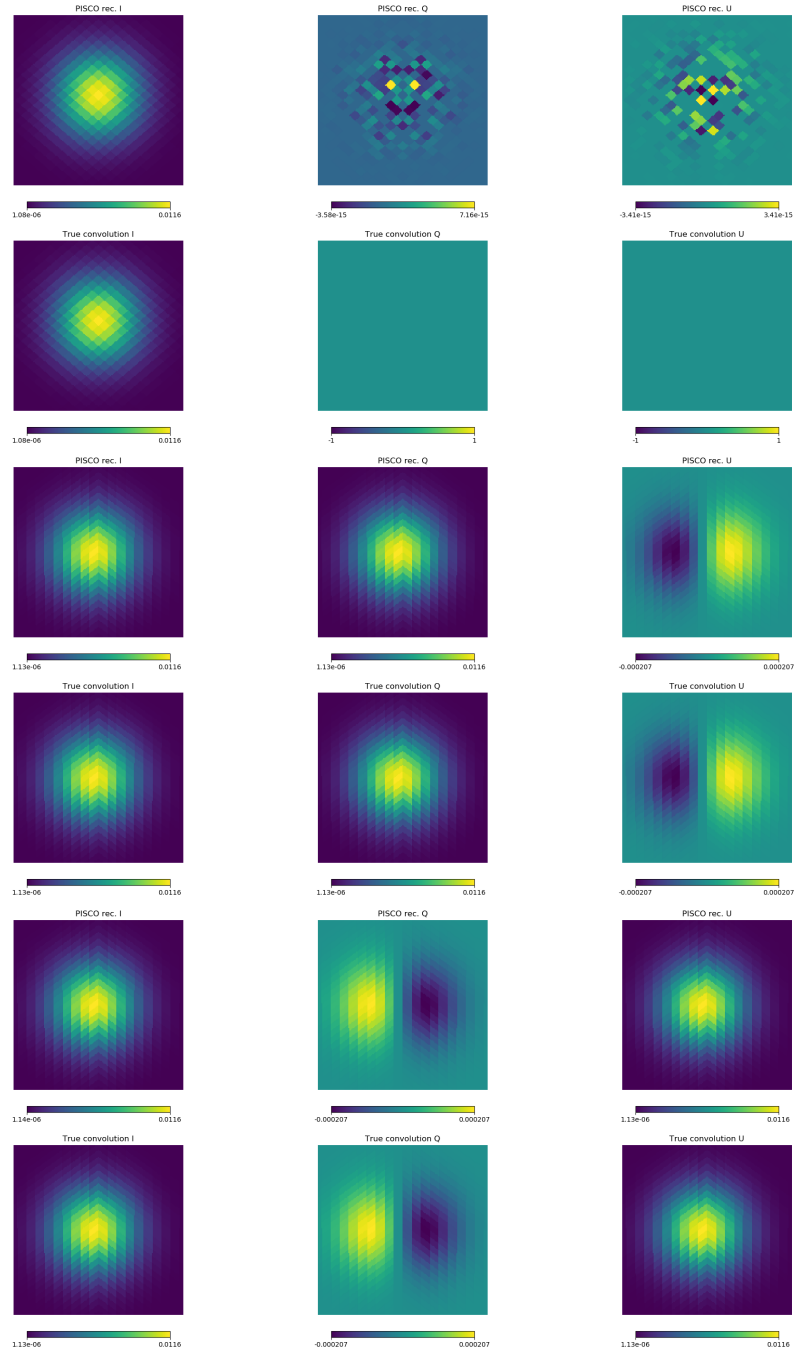


Figure 4.4: First plot, first row: PISCO convolution of a point source located at 45 degrees of declination, with Stokes vector $(1, 1, 0, 0)$. Amplitudes of I , Q and U maps are consistent with the result of `healpy.smoothing` (first plot, bottom row). Note the U map different from zero, and shows a dipole feature. This behavior is expected and it is due to the finite size of the beam and the variation of position angle within it. The coordinate transformation engine of PISCO is able to correctly take this into account when computing the convolution between the beamsor and the sky. Amplitudes of the PISCO and exact convolution agree to better than 0.1%. Input map NSIDE is 256. Beam has an NSIDE parameter of 1024. Bottom plot is analogous, but using a source with Stokes vector $(1, 0, 1, 0)$

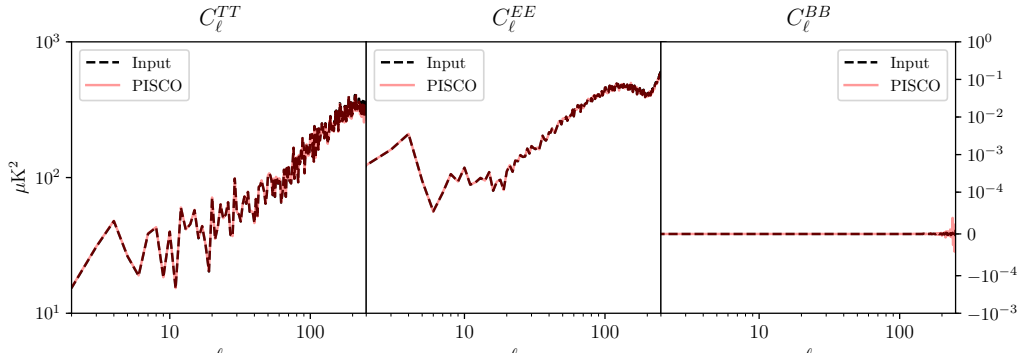


Figure 4.5: Plot showing spectra from PISCO generated TOD (red, solid curve) for the whole sky CMB test (black, dashed curve). Excellent agreement is observed, with no leakage from E-modes to B-modes or from temperature to polarization.

spectrum is effectively zero at all angular scales. No foreground or other sources were added on top of the simulated CMB. All maps use the `HEALPix` pixelization and were generated at a resolution of `NSIDE`= 128. This restricts the analysis in harmonic to $\ell < 384$.

The scanning strategy was built so that every pixel on the sky gets visited exactly three times, each time at a different beam orientation angle. In addition, every pixel was observed at its center, which is an important requirement that ensures the intra-pixel coverage does not affect the estimation of the power spectra at high ℓ . Since only three hits per pixel at different values of ψ_0 are required to recover the polarization field of the CMB, the scanning was generated for a single detector with a polarization sensitive angle $\zeta = 0$.

Finally, power spectra were calculated using `anafast`. No further post-processing of the power spectra was needed given that this simulated observation covers the whole sky, and hence no masking effects arise. The power spectra corresponding to maps that were generated using PISCO TOD were corrected by the equivalent beam transfer function of a circular Gaussian beam of FWHM 1.5° . We note that, while the pixelization allows for harmonic analysis to reach $\ell = 384$, in practice the beam transfer function smears out all information at $\ell \approx 250$, which is the limit in ℓ used in Figure 4.5.

Chapter 5

Electromagnetic characterization of the CLASS Q-band telescope

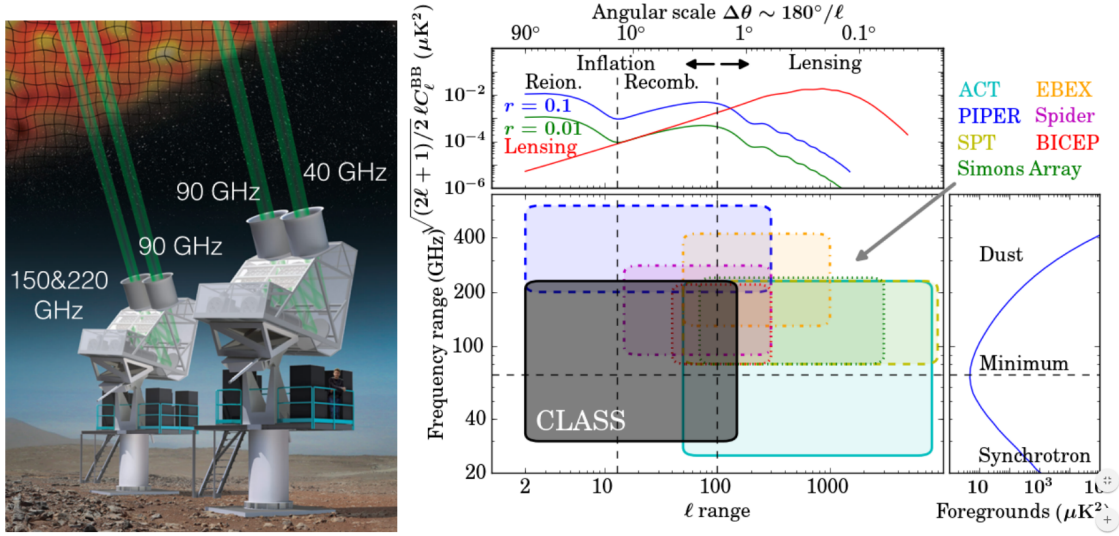


Figure 5.1: Figure showing the CLASS telescope array and the survey’s targets. Left: computer generated image of the four CLASS telescopes on two 3-axis mounts, with the 5200 meter site in the Atacama desert in the background. The telescopes operate across four frequencies which correspond to minimums in atmospheric emission. The right figure shows how the CLASS survey is designed to measure the primordial B-mode signal from both reionization and recombination. The figure gives the multipole (ℓ) and frequency range of current surveys with forecasted constraints at the $r \approx 0.01$ level, similar to the aim of CLASS. Top and side plots show the B-mode angular power spectrum and the frequency spectrum of polarized dust emission and synchrotron radiation. Figure source: Harrington et al. (2016)

5.1 The Cosmology Large Angular Scale Surveyor: CLASS

5.1.1 Science outcomes

Inflation

In the Λ CDM model, inflation is a key process that provides a mechanism explaining many of the observed characteristics of the cosmos. As seen in Chapter 1, inflation yields both scalar (density) and tensor (gravitational wave) perturbations. In order to distinguish between different inflationary models, measurements of the CMB temperature and polarization anisotropy field are required. These measurements must target the large angular scales, so as to sample the primordial perturbations.

Currently available data has only been able to set limits on the amplitude of the scalar perturbation by using the temperature and E-mode components of the CMB anisotropy field. On the other hand, constraints on the tensor perturbations predicted by inflation can only be provided by measurements of the B-mode component at large angular scales. Because of the above, characterization of the primordial B-mode signal has plays an crucial role in modern CMB polarization experiments. CLASS aims at the detection of the B-mode signal at large angular scales, with an expected sensitivity to r of $r = 0.01$.

Reionization

Reionization corresponds to a second “phase transition” of the gas (mainly neutral Hydrogen) in the universe. Long after recombination, most of the neutral hydrogen was formed and large clouds of gas had enough time to collapse, forming the first generation of stars. Some of these stars had enough mass to produce large amounts of UV light, which subsequently ionized the surrounding hydrogen. This ionized medium provided conditions that were similar to when recombination occurred. The main difference between reionization and recombination is that, for recombination, the background radiation field was colder and the scattering rate much lower. Reionization left an imprint on the CMB power spectra, which can be detected at large angular scales. This is known as the “reionization bump”, an excess power in both E-mode and B-mode power spectra produced by the extra scattering.

The details of the process of reionization remain largely unconstrained. CLASS is designed to provide a cosmic variance limited measurement of the E-mode spectrum below $\ell \approx 100$. This is required to better constraint the epoch of reionization. This measurement will be an important complement and crosscheck to the current, and next generation, 21-cm measurements aiming at the characterization of reionization.

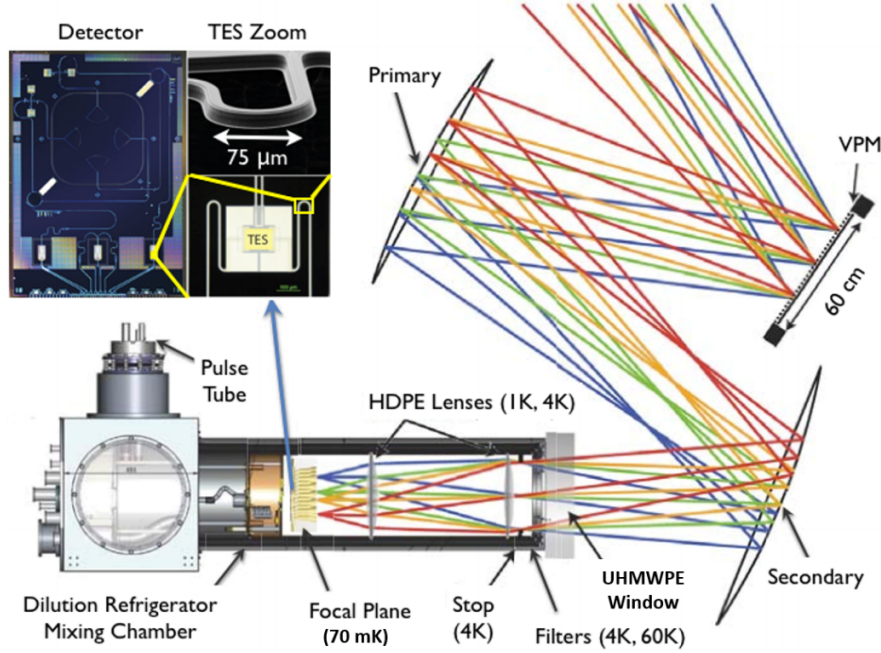


Figure 5.2: Figure showing the optical design of the CLASS Q-band telescope. Light enters through the closeout and the first element that encounters is the VPM, which reflects the light to the primary and secondary mirror. CLASS follows an a-focal design, and so the image at the VPM is re-imaged by the lenses, one of them at the 4 Kelvin stage of the cryostat (the 4 Kelvin lens) and another inside the 1 Kelvin stage. The image is then formed at the focal plane, where the 70 mK cooled TES perform the detection. Figure source: Harrington et al. (2016)

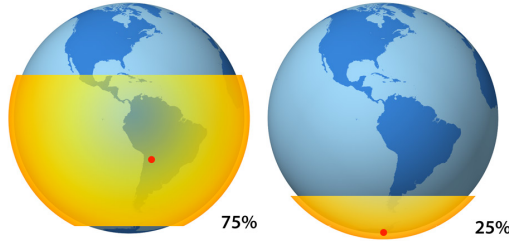


Figure 5.3: Graphical representation of the sky coverage for the CLASS experiment (leftmost globe), compared to the coverage provided by an experiment located at the South Pole (rightmost pole). Covering a larger fraction of the sky is key to allow the recovery of the large angular scales fluctuations of the CMB. Credits to the CLASS collaboration (2018)

5.1.2 The CLASS experiment

The CLASS experiment is located near Llano de Chajnantor, at 5180 meters above sea level in northern Chile’s Atacama Desert. The site offers a unique combination of accessibility and low atmospheric water vapor and oxygen lines. Being at $\approx 23^\circ$ Southern latitude is also desirable, as it allows observations over approximately 75% of the sky. This is a very important requirement for CLASS, and other experiments aiming at recovering large angular scale features of the CMB anisotropy field.

The primary observing mode of CLASS consists in 720° “sweeps” at constant elevation. Due to mechanical constraints, the telescope can only perform one turn before having to rotate in azimuth in the other direction. Constant elevation scans keep the atmospheric optical depth held constant while the observation is performed, and hence are the preferred observing mode.

5.1.3 The CLASS Q-band telescope

The CLASS Q-band telescope is designed to measure the polarization anisotropy field of the sky. The Q-band telescope is intended to act as a cleaning channel to help removing polarized emission between Earth and the CMB, particularly synchrotron. For this reason, the observing band spans from 33 GHz to 43 GHz. While the CLASS Q-band is intended to measure large angular scale features, a resolution of 1.5 degrees was chosen to guarantee to still be able to characterize foregrounds on smaller scales. To minimize hot spill-over from the ground, the CLASS Q-band telescope has all of its primary optical elements shielded from the environment by a cage.

The CLASS Q-band telescope is based in a novel optical design (see Eimer et al. (2012)). The optics provide a large Field Of View (FOV) with minimal optical distortions, while keeping the mechanical design as compact as possible. The sensitivity and stability needed to recover primordial B-modes requires a system capable of mapping the sky as fast as possible. CLASS also utilizes a “lock-in” technique to separate polarized signals from other systematic effects, which is a requirement to

recover such a small amplitude signal over large angular scales.

The Variable-delay Polarization Modulator

A key component of the CLASS experiment is the Variable delay Polarization Modulator (VPM). The purpose of the VPM is to amplitude-modulate the polarization signal entering the optical chain. If the modulation transfer function is known, the data can be demodulated to recover the polarization signal. Because the VPM is the first element in the optics, any polarization systematics that arise *after* the VPM, like the cross polarization response of the feedhorn-coupled TES, will not be modulated and thus will not affect polarization measurements.

A VPM consists on two elements: a conducting mirror and a grid made of conducting wires parallel to each other. The basic operation principle is that electric fields that are polarized parallel to the wires are totally reflected by them, while orthogonal ones will get reflected by the mirror instead. Because of the grid and the mirror are separated, one of the polarization states will get a phase delay with respect to the other one. A more detailed description of the VPM inner workings is given in Kathleen Harrington (2018).

In the case of CLASS, detectors are oriented at $\pm 45^\circ$ with respect to the wires of the grid, so that a measurement with $\psi = 0$ produces, to first order,

$$d(t) = T + Q \cos(\phi(t)) - V \sin(\phi(t)) + n(t) \quad (5.1)$$

where $n(t)$ is the noise in the measurement, while $\phi(t)$ is the phase delay. A first order model of the VPM yields

$$\phi(t) = \frac{4\pi z(t)}{\lambda} \cos(\theta) \quad (5.2)$$

where $z(t)$ is the grid-mirror distance, λ is the observing wavelength, and θ is the incidence angle of the ray of light hitting the VPM measured with respect to a vector normal to the plane defined by the VPM wire grid. The reason of CLASS being a

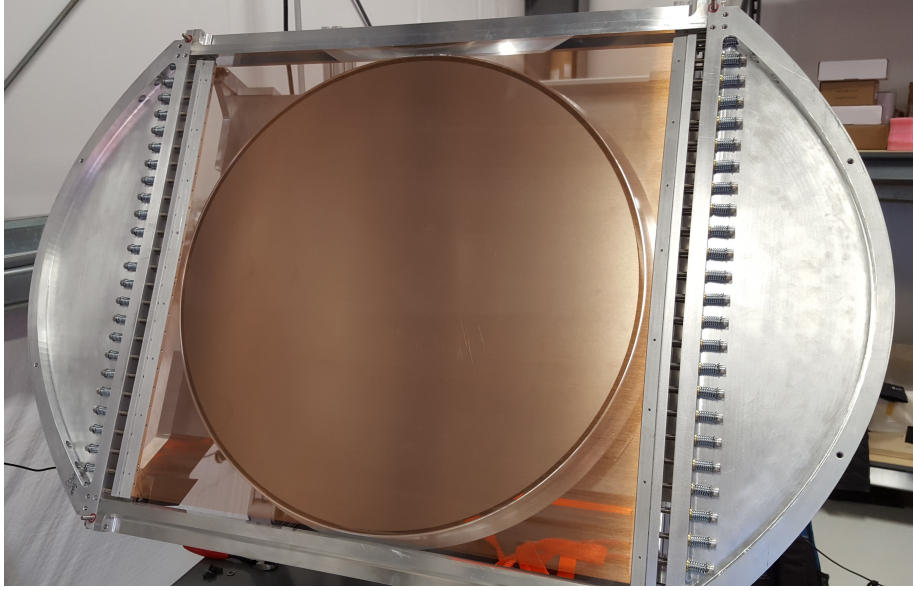


Figure 5.4: Picture of the Variable delay Polarization Modulator (VPM) at the facilities in John Hopkins University. The circular, flat mirror can be seen behind the dense wire grid, as well as the frame providing mechanical support for the structure. Picture credit: Kathleen Harrington, CLASS Collaboration 2018.

3-axis telescope is that rotating the VPM with respect to the telescope pointing modifies the modulation from Q/V to U/V . The VPM also explicitly modulates the V Stokes parameter, which is expected to be zero in the case of the CMB. This way, V maps will become a powerful way of checking VPM related systematics.

Cryogenic camera

The optics of the cryogenic camera were designed to map spherical wavefronts, produced by the primary optical elements, onto the focal plane. Of similar importance is the fact that the cryogenic camera provides the necessary rejection of out of band light that could reach the detectors, particularly infrared light.

Spherical to flat wavefront conversion is performed by a dual lens system; while possible to achieve using just one, the thermal mass of the optimal lens is too large to be practically cooled. The lenses are made of High Density PolyEthylene (HDPE), a material that was chosen due to its low price, relative ease of machining and refraction index in the band of interest. To minimize in-band transmission losses,

each lens surface was provided an Anti Reflection coating. Optimization of this system yielded convex-convex lenses with on-axis, ellipsoidal surfaces. Parameters and surface equations for the lenses are given in Eimer et al. (2012).

The focal plane consists of an array of feedhorn coupled, polarization-sensitive detectors continuously cooled to 70 mK by a dilution refrigerator. The coupling of the feedhorns with the on-chip detector circuitry define polarized beams that propagate through the telescope. Two transition edge sensors (TES) detect the power corresponding to orthogonal linear polarizations. The TES are read out with time-domain Superconducting Quantum Interference Devices (SQUID) multiplexing electronics.

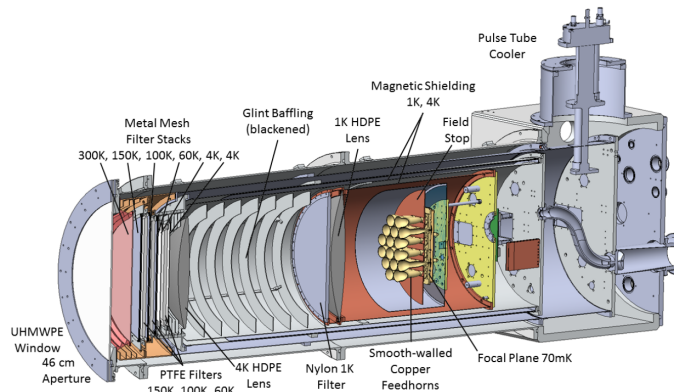


Figure 5.5: The 38 GHz receiver as it was fielded in early 2016. The vacuum is held over a 46 cm aperture by a 4.8 mm thick ultra-high molecular weight polyethylene window. Six multi-layer stacks of reflective metal-mesh filters interspersed between three polytetrafluoroethylene (PTFE) filters drastically reduce the loading from infrared radiation before the 4 K cold stop. Blackened glint baffling and a blackened field stop absorb stray light while two high-density polyethylene (HDPE) lenses image incident light onto the focal plane. The focal plane sits at 70 mK and is surrounded by two layers of magnetic shielding. Figure source: Harrington et al. (2016)

Primary optical elements

The primary optical elements of the Q-band telescope comprise the VPM, primary and secondary mirrors. The shapes and relative positions of these elements was optimized to map plane waves coming from the sky into aberration-free spherical

waves at the focal plane. The VPM and mirrors were also designed to be sub-illuminated, so as to prevent unwanted diffraction effects at the edges of the mirrors, but particularly by the VPM mounting structure. An optimization process lead to the primary optical elements to form an off-axis, dual mirror, a-focal system. Details of the design process, as well as relative positions and tolerances can be found in Eimer et al. (2012).

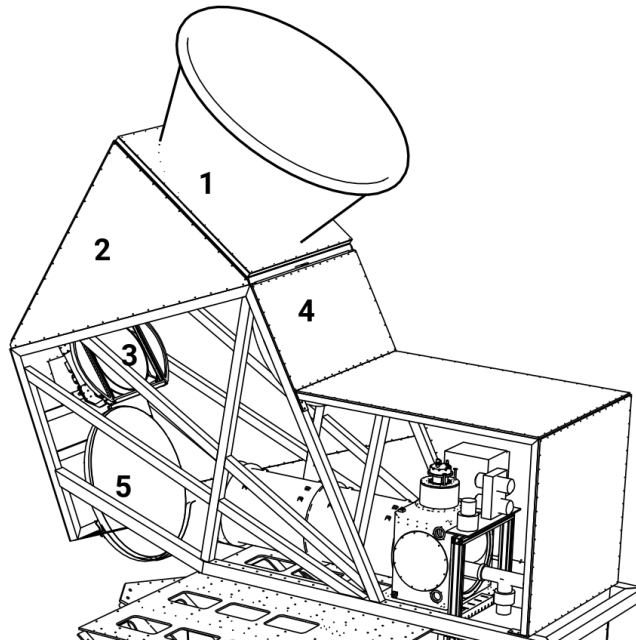


Figure 5.6: Rendering of the CLASS Q-band telescope. Numbered parts correspond to: 1. Baffle, 2. Upper panel of the co-moving shield, 3. VPM, 4. (behind the panel) Primary mirror, 5. secondary mirror. The interface between the baffle and the enclosing structure, the co-moving shield, is called the clouseout. Credits to the CLASS collaboration (2018).

5.2 Electromagnetic simulations of the CLASS Q-band telescope

Electromagnetic simulations are a valuable tool to gain insight on any systematic effects caused by the optical design. Effects such as beam eccentricity, distortion to the polarization signal and sidelobes are among these. Given the sensitivity con-

straints to detect primordial B-modes, understanding how these artifacts affect the science outcomes of the experiment becomes a priority. This section describes the methodology that was used in order to produce a model of the CLASS Q-band telescope.

5.2.1 Simulation tools

To simulate the Q-band telescope, we used General Reflector Antenna Software Package, GRASP¹. GRASP is composed of a Computing Assisted Drawing (CAD) interface and an analysis module that performs an electromagnetic simulation of the CAD model. GRASP uses several approximations and acceleration techniques to solve Maxwell equations for a given model. These approximations are Geometrical Optics (GO), Geometrical Theory of Diffraction (GTD), Physical Optics (PO) and Physical Theory of Diffraction (PTD). GRASP also has an implementation of the Method Of Moments to solve Maxwell Equations using the Fast Multiple Method.

All simulations were carried out in **hades**, a compute node at Centro de Astroingeniería UC. The **hades** node is an Supermicro system, based in a quad-socket architecture powered by 4 AMD Opteron 6380 SE processors, each processor capable of executing 16 threads simultaneously. **hades** is equipped with 128 GB of DDR3 RAM, which is connected to the CPU using AMD's proprietary HyperTransport technology, so that each processor can access a quarter of the memory at twice the DDR3 speeds. Input-output operations are accelerated using a RAID level 0 array of 5, 4 Terabyte Toshiba X300 hard drives. Sequential input-output operations on the array exceed 450 Megabytes per second, consistent with a saturation of the SATA II protocol the RAID-0 array relies on.

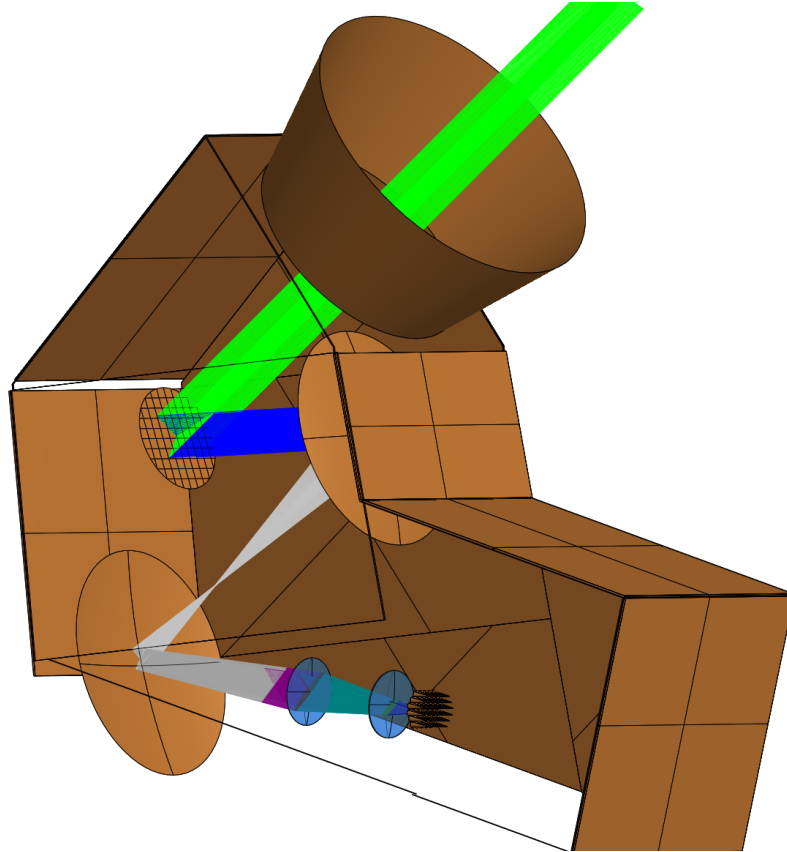


Figure 5.7: Render of the CLASS Q-band telescope in GRASP. In this figure, the primary mirror and the insides of the cryogenic camera are visible. Color lines are representative of a ray-trace emanating from a central feedhorn. Reader is referred to figure 5.6 to compare the GRASP model with the actual telescope design.

5.2.2 Elements of the simulation

Feedhorn and focal plane

CLASS uses a novel feedhorn design described in Zeng (2012). When high directivity and low cross polarization are required over a wide bandwidth, corrugated feedhorns are used. This type of feedhorn has been proved to be effective but expensive and difficult to machine (see McCarthy et al. (2016)) To overcome this issue, the feedhorn of the CLASS Q-band telescope have a smooth, rotationally symmetric profile. The profile increases in radius monotonically along the feedhorn symmetry axis. This simplifies the fabrication process. The beam of prototype feedhorns were measured using the Anechoic Chamber at the Goddard Space Flight Center and validated the predictions made by Zeng (2012). It was also validated that optical performance of smooth-walled feedhorn closely matches equivalent corrugated feedhorns.

It is usually assumed that the beam of a feedhorn can be well represented using Gaussian optics. We performed Gaussian fits to the beams presented in Zeng (2012). Fits only accounted for the main lobe. While the real beam from a feedhorn has non negligible sidelobes, these features will be suppressed as they illuminate the insides of the cryogenic camera and not the lenses. Including these secondary reflections would also dramatically increase the computational cost of simulating the camera.

The Q-band focal plane consists of 36 feedhorns distributed on a flat surface. The positions of individual feedhorns were obtained from the mechanical design of the focal plane. Each feedhorn is electromagnetically coupled to a pair of TES bolometers, which are oriented 45 (V detectors) and -45 degrees (H detectors) with respect to the feedhorn's axis of symmetry. This behavior was taken into account by rotating the polarization of the beam radiated by the feedhorn by ± 45 degrees, depending on the type of detector being simulated.

¹www.ticra.com/grasp

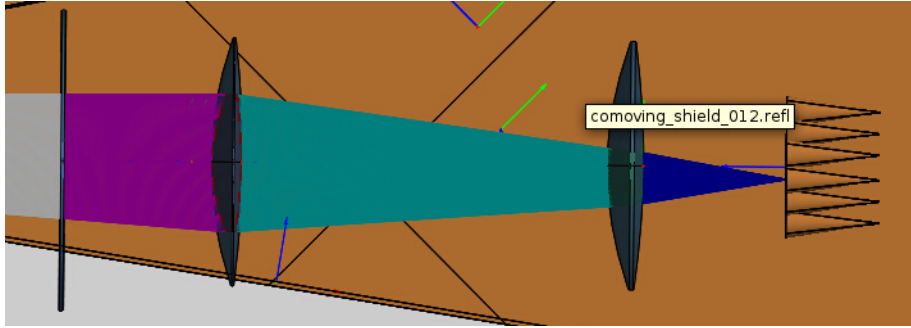


Figure 5.8: GRASP rendering of a side view of the elements of the re-imaging optics. From right to left: array of feedhorns, 1 Kelvin lens, 4 Kelvin lens, and window. The cold-stop is modeled as an infinite sheet of thin, perfectly conducting material with a circular hole, so it is not visible in the side render. The cold-stop is located between the the window and th 4 Kelvin lens.

Re-imaging optics

Re-imaging optics comprise 2 lenses and the cold stop. The lenses were drawn following the shapes and relative distances given in Eimer et al. 2012, Table 3. The refractive index of the lenses was set to 1.564, as expected from HDPE at cryogenic temperatures. The cold stop was modeled as a circular aperture on a an infinite, perfectly conducting plane.

The simulation method to propagate the fields from the feedhorns through the re-imaging optics was Physical Optics. Physical Optics is orders of magnitude faster than MoM, at the expense of producing less accurate results. Despite this relative loss in accuracy, PO produces reliable, accurate results, and is widely used among the scientific community. The speed of PO also comes from a the fact that interactions between optical elements such as shadowing are not taken into account. Finally, while Physical Optics is can handle dielectric sheets on certain kind of surfaces, it doesn't support Anti-Reflection coating on lenses. This last limitation makes GRASP simulations of the re-imaging optics not reliable for studying effects like optical band-passes.

Primary optical elements

The cryogenic camera, where the re-imaging optics operate, illuminates the secondary mirror, which then reflects the fields onto the primary mirror, which in turn redirects the fields to the VPM mirror. Primary and secondary mirrors are sections of ellipsoids, with the main parameters and relative positions of these described in Eimer et al 2012, tables 1 and 2. All mirrors were taken to behave as a Perfectly Electrical Conductor (PEC), including the VPM mirror, was modeled as a flat, circular surface reflector.

The interaction between the VPM grid and the mirror was not included in the simulations. The reason is that the VPM is a complex electromagnetic system that requires specialized treatment and, possibly, different simulation techniques. This is due to the fact that a proper simulation needs to capture the micro interactions between the wires that form the grid, and how the grid interacts with the mirror. This required an exceedingly expensive computational effort. Simulations of a scaled model of the VPM were performed to estimate the computation time of simulating the CLASS VPM. The results showed that more than 400 hours of computation were needed for a single detector, and would have taken more RAM than the available.

Baffle

The baffle is a conic section made of aluminum. It was designed to avoid stray light from the ground, as well as to protect the interface between the primary optical elements (the closeout) from weather inclemencies. Note that this makes the baffle be the first optical element, not the VPM. The baffle was modeled as a perfectly conducting conical section. Its shape and position, relative to the rest of the optical elements, was obtained from the mechanical design of the telescope. The interaction of the baffle with the rest of the optical elements was a challenging task, as the closeout and the inner walls of the baffle are too close to each other, making the Physical Optics produce unreliable results.

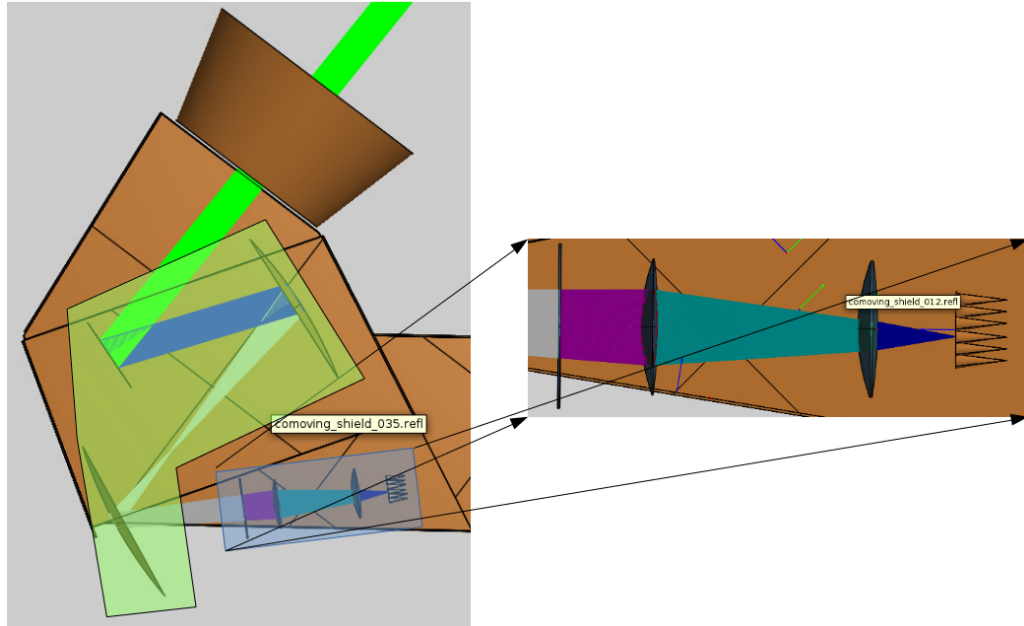


Figure 5.9: GRASP render of a side view of the Q-band telescope. The transparent green polygon shades the primary optical elements: secondary mirror (bottom-left) primary mirror (upper-right) and VPM mirror (upper-left). Blue transparent polygon shades the re-imaging optics region, which is shown as part of the complete model for reference. After the VPM mirror, rays (shown in green) pass through the telescope closeout and propagate inside the conical geometric shape on top of it, which represented the baffle.

To overcome this problem, we used a combination of Plane Wave Expansion (PWE) and the MoM solver. The problem of illuminating *only* the inner walls of the baffle was solved by performing a PWE of the fields at the closeout. Plane Wave Expansion requires careful tuning of the coordinate system used for the propagation of the (expanded) fields, particularly, the orientation of its z axis. As all Q-band detectors point to different directions on the sky, the coordinate system used for the PWE was setup based on a previous simulation used to obtain the pointing solution of the telescope. The fields from the PWE were used as the electromagnetic source of the MoM simulation of the baffle. Finally, the currents inside baffle were converted to electric fields at the telescope’s far field, and added to the electric fields forming the main beam.

5.3 Simulations

5.3.1 Methodology

To obtain the properties of the “temperature” beam (the B_{TT} component of the beamsor) for the CLASS Q-band telescope, we performed simulation for all 72 detectors in the focal plane across a frequency band spanning 30 GHz to 46 GHz, in 1 GHz steps.

We performed a single full focal plane simulation of the Q-band telescope at 38 GHz. The output of this simulation was a data-cube, each slice of the cube being a grid with the values of the co and cross polar electric fields radiated by a single detector. The grid extends 20×20 degrees in azimuth and elevation, and its origin being coincident with boresight pointing. After the simulation finished, the data-cube was scanned to find the locations of the beam maximums. The sky coordinates of the beam maxima were cross referenced with the mechanical positions of each detector/feedhorn in the focal plane, so as to build a pointing solution. The process was repeated for a simulation at 30 GHz and 43 GHz to check for chromatic

aberration, which was found to be negligible when compared to the fit error.

As part of the simulation processes, we developed the Q-band Electro-Magnetic Simulator, QEMS. QEMS is a collection of Python scripts that interfaces with the GRASP native scripting language, in a way that allows automating the process of simulating the whole telescope. QEMS takes a template of the GRASP model, with key parameters flagged for further replacing by the software. QEMS then uses the pointing model, mechanical positions of the detectors and feedhorn models to build a project suitable to be simulated by GRASP. Simulations are launched asynchronously, so that parallel execution is possible. The software is also designed to be simple to deploy in a High Performance Computing environment.

Computational considerations prevented us from running QEMS on the GRASP model that includes the baffle, which must be simulated using different techniques. QEMS uses Physical Optics to propagate the fields from the feedhorn through the Q-band optics. The simulation follows a sequential execution, that is, the fields emanating from the feedhorn pass through the lenses, get reflected by the mirrors and get projected to the sky. No interactions between elements were included in this analysis. The output of this run corresponds to 72 datasets, where each set stores the electric fields radiated by a particular detector in the focal plane at a particular frequency.

5.3.2 Main beam parameters

While a mathematically consistent description of a Gaussian distribution on the sphere must be formulated using Kent distributions (see Kasarapu (2015)), it is much simpler and convenient to parametrize the main beam by the planar projection of a two dimensional Gaussian. This planar projection follows equation

$$b(x, y) = ae^{\frac{1}{2}\left(\frac{x^2}{\sigma_x} + \frac{y^2}{\sigma_y}\right)} \quad (5.3)$$

where x and y are offsets with respect to beam maximum. To avoid ambiguity in

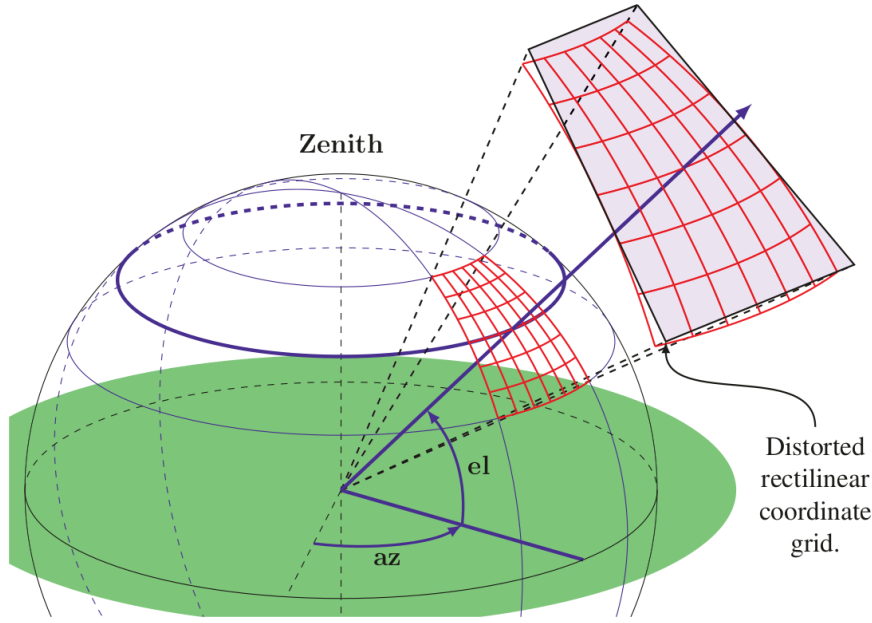


Figure 5.10: Graphical representation of the projection effect arising from considering a planar projection of the beam. The x -axis of the planar projection increases in the $+\hat{\sigma}$ direction, while the y axis does so in the $-\hat{\rho}$ direction. Figure source. Dominik Gothe thesis dissertation (Gothe (2015))

the results, the offsets are defined in a coordinate system that has its x axis aligned with the major axis of the ellipse, so that $\sigma_x > \sigma_y$. This coordinate system defines a plane parallel projection of the spherical cap that encloses the Gaussian part of the beam. As the Q-band beam is contained within a small region, we can approximate $\sin(\alpha) \approx \alpha$ inside this zone, so that the plane-parallel approximation is valid. Note that, in this coordinate system, x and y are projections of arcs on the sphere, and do not correspond spherical angles like azimuth or elevation, as shown in Figure 5.10.

In the planar projection regime, the solid angle of a Gaussian beam is given by

$$\Omega_g \approx 6.278\sigma_x\sigma_y \text{ strad} \quad (5.4)$$

The CLASS Q-band telescope was designed to have its main beam characterized by a circularly symmetric, 1.5 degrees Full Width at Half Maximum (FWHM) Gaussian at 38 GHz. The relation between $\sigma_{x,y}$ and the FWHM is given by

$$\text{FWHM}_{x,y} \approx 2.354\sigma_{x,y} \quad (5.5)$$

Applying this relation to the “nominal” Q-band beam yields

$$\Omega_0 \approx 776 \mu\text{strad} \quad (5.6)$$

The results of the 38 GHz QEMS run show that the average Q-band temperature beam is characterized by a 2-dimensional Gaussian of $\langle \text{FWHM}_x \rangle = 1.58 \pm 0.21$ degrees and $\langle \text{FWHM}_y \rangle = 1.44 \pm 0.23$ degrees. Averaging both $\langle \text{FWHM}_x \rangle$ and $\langle \text{FWHM}_y \rangle$ gives an effective FWHM, $\text{FWHM}_{\text{eff}} = 1.51$ degrees, which matches the optical specifications to better than 1%.

5.3.3 Broadband beams

While the Q-band telescope was designed to operate at 38 GHz, there is an inherent broad-band frequency response of the whole optical system. This response, in principle, can vary across detectors. While there might be several reasons behind this behavior, the most evident comes from the fact that the beam produced by the feedhorn changes with frequency. This means that all optical elements are illuminated differently, and thus the telescope beam will not be the same at all frequencies. As described in the work of Page et al. (2003), proper characterization of this phenomenon is relevant for the absolute calibration of the experiment.

To obtain the broad-band beams of CLASS Q-band, we followed a similar method to the one described in the above section, but repeated 16 times: each detector beam was simulated for 16 different frequencies between 30 and 46 GHz. Then, we performed a per-pixel weighted average of each beam using an optical band-pass function (sometimes called optical transmission function), $W(\nu)$, as the weighting function. The frequency averaged beam $\bar{B}$ for detector i is defined as

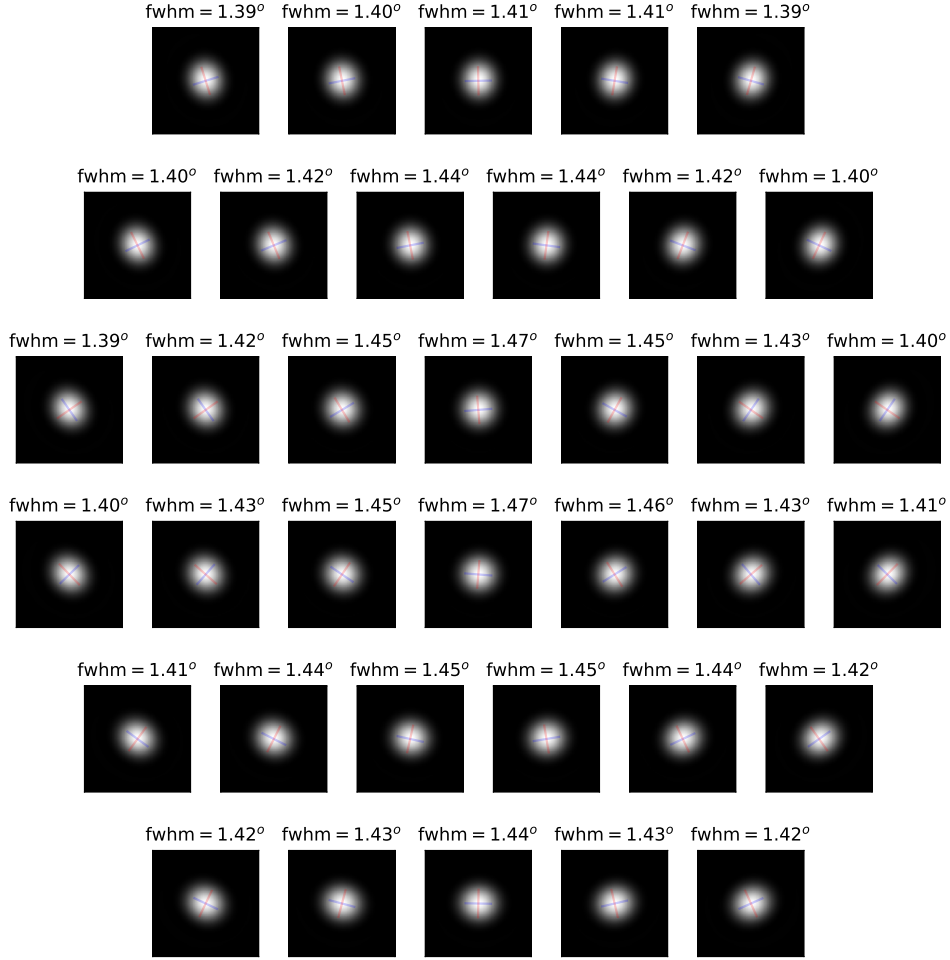


Figure 5.11: Visual representation of CLASS Q-band beams from every feedhorn in the focal plane. Relative positions of sub-plots are representative of the positions of feedhorns at the focal plane, with feedhorn 1 being the bottom-left sub-plot. The title over each sub-plot corresponds to the major axis of the -3dB elliptical contour of the beam (FWHM_x). Non negligible amounts of eccentricity can be seen in the temperature beams, and a visible correlation between feedhorn location, eccentricity and orientation of the beam major (minor) axis can be seen. The cause of this correlation is unclear, but is believed to be caused by the novel optical design, and as a consequence of the large FOV required for the CLASS experiment.

$$\bar{B}_i(\rho, \sigma) = \frac{\sum_k b_i^k(\rho, \sigma) W(f_k)}{\sum_f W(f_k)} \quad (5.7)$$

where k spans the range from $[1, 16]$. f_k , in GHz, is given by

$$f_k = k \times 1 \text{ GHz} + 30 \text{ GHz} \quad (5.8)$$

The band-pass function comes from Fourier Transform Spectrometry (FTS) measurements made in laboratory conditions. The measurements were made for a single detection device that was placed behind a Q-band feedhorn. This measurement neglects effects in the band-pass that might arise from the re-imaging optics as, at the time of writing, measurements of $W(\nu)$ including these effects are not available. While it is possible to perform a simulation to estimate $W(\nu)$, it was found that they would exceed the available computational capability. Future work should address this matter when more realistic models or measurements of the band-pass become available.

Simulations imply that the average *FWHM* of the CLASS Q-band broad-band beams for Rayleigh-Jeans sources to be $FWHM_{\text{eff,bb}} = 1.50$. Compared to $FWHM_{\text{eff}} = 1.51$, which was computed using 38 GHz simulations only, the difference is negligible. The impact of the broad-band beam response becomes more evident in the case of calibration. As an example, CLASS Q-band uses the Moon to calibrate the detector response to absolute units (watts). For this purpose, the Moon can be considered as an unresolved Rayleigh-Jeans source, so that the measured solid angle *considering the broad-band response of the beam*, becomes a convolution, namely

$$\Omega_{\text{RJ}} = \frac{\int \Omega(\nu) W(\nu) \nu^2 d\nu}{\int W(\nu) \nu^2 d\nu} \quad (5.9)$$

For a lossless telescope, the detected power P' is related to the power P reaching the telescope aperture via

$$P' \propto \eta_{\text{RJ}} P \quad (5.10)$$

where

$$\eta_{\text{RJ}} = \frac{\Omega_{\text{source}}}{\Omega_{\text{RJ}}} \quad (5.11)$$

5.3.4 Polarization angle rotation

The CLASS Q-band telescope is designed to produce low distortion to the polarization properties of the beam (see Eimer et al. (2012)). Particularly, the optics guarantee the cross-polar gain to be -40 dB below the co-polar beam gain. To perform polarization analysis, it is crucial to keep track of what is “up” and “right”, as this sets the basis of co and cross polar unit vectors.

A convenient “fiducial” direction to refer each antenna basis to, is the boresight pointing direction. The boresight pointing direction alone, however, does not completely define a coordinate basis, so it becomes unavoidable to introduce an external spherical basis (see Figure 2.1) Without loss of generality, we take the boresight pointing direction to point at $(\theta_b = \pi/2, \phi_b = 0, \psi_b = 0)$ in auxiliary basis coordinates. Using this convention, the “up” direction is given by $-\hat{\theta}_b$ (recall that $\hat{\theta}$ points towards the South pole, see Figure 2.2), while the “right” direction is parallel to $\hat{\phi}_b$. This is the boresight basis, denoted by $\mathbf{C}_{\text{bor}}$.

The pointing of a particular beam is specified as beam offsets δ_x and δ_y . δ_x is the translation along the x -axis described in figure 5.10, while δ_y is along the y -axis. Note that these translations are valid in the plane-parallel projection. For a beam centroid located at (δ_x, δ_y) with respect to the boresight basis, $\mathbf{C}_{\text{beam}}$ can be found by parallel transporting $\mathbf{C}_{\text{bor}}$ across the sphere to the centroid of the beam using three rotations, namely

$$\mathbf{C}_{\text{beam}} = \mathbf{R}_z(-\gamma)\mathbf{R}_y(r)\mathbf{R}_z(\alpha)\mathbf{C}_{\text{bor}} \quad (5.12)$$

where $\mathbf{R}_i(\delta)$, $i = x, y, z$ is the 3D rotation matrix that rotates a vector around axis i by an angle δ . Angles α , r and γ are related to the beam centroid offsets (δ_x, δ_y)

$$\alpha = \arctan\left(\frac{\sin(\delta_y)}{\sin(\delta_x)\cos(\delta_y)}\right) \quad (5.13)$$

$$r = \arccos(\cos(\delta_x)\cos(\delta_y)) \quad (5.14)$$

$$\gamma = \arctan\left(\frac{\cos(\delta_x)\sin(\delta_y)}{\sin(\delta_x)}\right) \quad (5.15)$$

These rotations imply that, for a detector with polarization sensitive angle ζ , the position angle q is given by

$$q = \psi_0 + \zeta_{eff} \quad (5.16)$$

where $\zeta_{eff} = \zeta + \gamma$.

In the case of CLASS Q-band, the above expression for q is accurate for detectors close to the center of the FOV. However, it was found that detectors near the edge will experience a rotation of their polarization basis. Indications of this phenomena were first seen as edge beams having cross-polar components 2 orders of magnitude larger than specified by the optical design. Later, evidence that this phenomena is real was found in the literature (see Koopman et al. (2016)). This polarization angle distortion changes the definition of ζ_{eff} by adding an extra term,

$$\zeta_{eff} = \zeta + \gamma + \Upsilon \quad (5.17)$$

where $\Upsilon = \Upsilon(\delta_x, \delta_y)$ varies across positions on the FOV. The amplitude of this correction depends on the optical design, and affects every detector in a different way.

The original method for calculating Υ was suggested by Stig Busk Sørensen.² As part of the output options, GRASP can provide

²sbs@ticra.com

$$j = \sqrt{\frac{E_{\text{RHC}}}{E_{\text{LHC}}}} \quad (5.18)$$

where j is a complex number. Using the definitions of Stokes parameters (see 2.10), it can be shown that *phase of j* is “the rotation angle of the major axis of the polarization ellipse”.

The procedure to find Υ then becomes straightforward. First, values of γ for all feedhorns are computed using the GRASP pointing solution. Then, individual simulations return j for every feedhorn. Then, we calculate $\Upsilon = \text{phaseof}(j)$ for all the beams. The results are shown in figure 5.12.

5.3.5 Far sidelobes

Sidelobes and far sidelobes are a major concern for high sensitivity experiments like CLASS. In particular, it has been shown that far sidelobes can produce significant leakage from temperature to polarization at the map-making stage (see Fluxá et al. (2016)). Far sidelobes might also capture thermal emission from the ground, reducing the experiment’s sensitivity and producing scanning dependent features on the maps. Finally, there exists the possibility of the telescope having *polarized* far sidelobes. A careful search in the literature did not yield results regarding analytical models for the impact of far sidelobes in CMB maps. Alternatively, electromagnetic simulations can be used to estimate the order of magnitude of the effect. In addition, simulations might reveal the cause of the far sidelobes, enabling mitigation measures to be taken.

CLASS Q-band telescope was designed with sidelobe suppression in mind. The optical design exhibits low levels of “spill”, the amount of light hitting regions outside the primary optical elements. In addition to this, CLASS Q-band implements a strict tapering to avoid diffraction at the edge of the mirrors or support structure of the VPM. Finally, the whole optical system is enclosed inside a thermally isolated, metallic cage. Not only the cage prevents thermal fluctuations from warping the optics, but it also makes the closeout be the only possible entry for light. This design

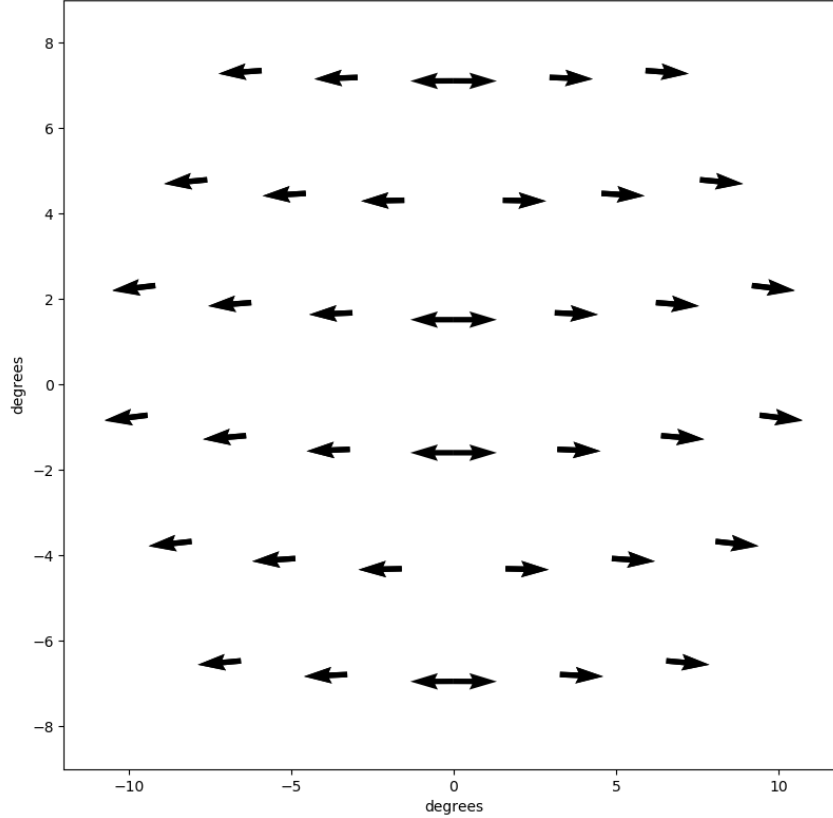


Figure 5.12: Figure showing a visualization of Υ for every feedhorn of the CLASS Q-band telescope. The plot corresponds to a plane-parallel projection with respect to boresight centered coordinates. Every arrow starts at the on-sky position of its corresponding feedhorn and shares the same length with all other arrows. The tilt of the arrow with respect to the x -axis of the figure corresponds to the value of Υ . Feedhorns that are close to the symmetry axis of the optics are not expected to experience significant deflection in their effective polarization sensitive angle, which is consistent with the simulations. The maximum deflection is approximately 7.5 degrees, and corresponds to feedhorns located at the very edge of the focal plane. The consequence of this deflection is a matter of active study in the CLASS collaboration.

ensures far sidelobes to be minimal, which is desirable because it is very difficult to predict how far sidelobes affect the overall performance of the experiment.

There is another level of sidelobe suppression that, and it is provided by the VPM. Because the polarization signal is continuously modulated at 10 Hz, sidelobes produced by light that does not interact with the VPM will not get modulated. Demodulation will then suppress this signal, hence preventing these sidelobes from affecting the data quality significantly. However, sidelobes that are produced by, for example, diffraction of the VPM itself will be modulated and will have a non trivial impact on the data. The goal of this section is to study these type of sidelobes.

GRASP was used to compute the far sidelobes at 38 GHz. The simulation was setup to include all interactions between elements that get direct illumination by the VPM, that is, the closeout, baffle and projecting panels around the VPM. Notably, the region where sidelobes are generated corresponds to the cage panels around the VPM as shown in figure 5.13. GRASP propagates the beam from the feedhorn through the cryogenic camera using Physical Optics up to the VPM mirror. The VPM mirror was then set to illuminate the panels surrounding the VPM (component 2 of figure 5.6). Since panels are close to each other, it was necessary to use MoM to capture all crossed interactions. The electric fields reflected by the panels was then passed through the closeout, and then used to illuminate the inner walls of the baffle via a PWE. Finally, the fields from inside the baffle were projected to the sky.

Figure 5.14 shows the far sidelobes computed this way, for a detector at the center of the focal plane. While the main beam peaks at 42.3 dB, the most prominent feature in the map does at -25 dB. This means that far sidelobes produced by spill or diffraction at the VPM mirror are attenuated by 3×10^{-6} . This is a first order estimation of the effect, as the real VPM as a much more complex mounting structure that might affect this result.

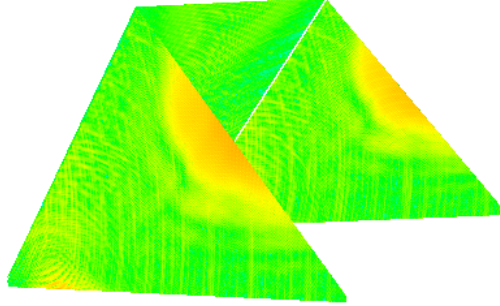


Figure 5.13: Figure showing the amplitude of the electric fields at the co-moving shield panels that get illuminated by the VPM (see figure 5.6 for reference)

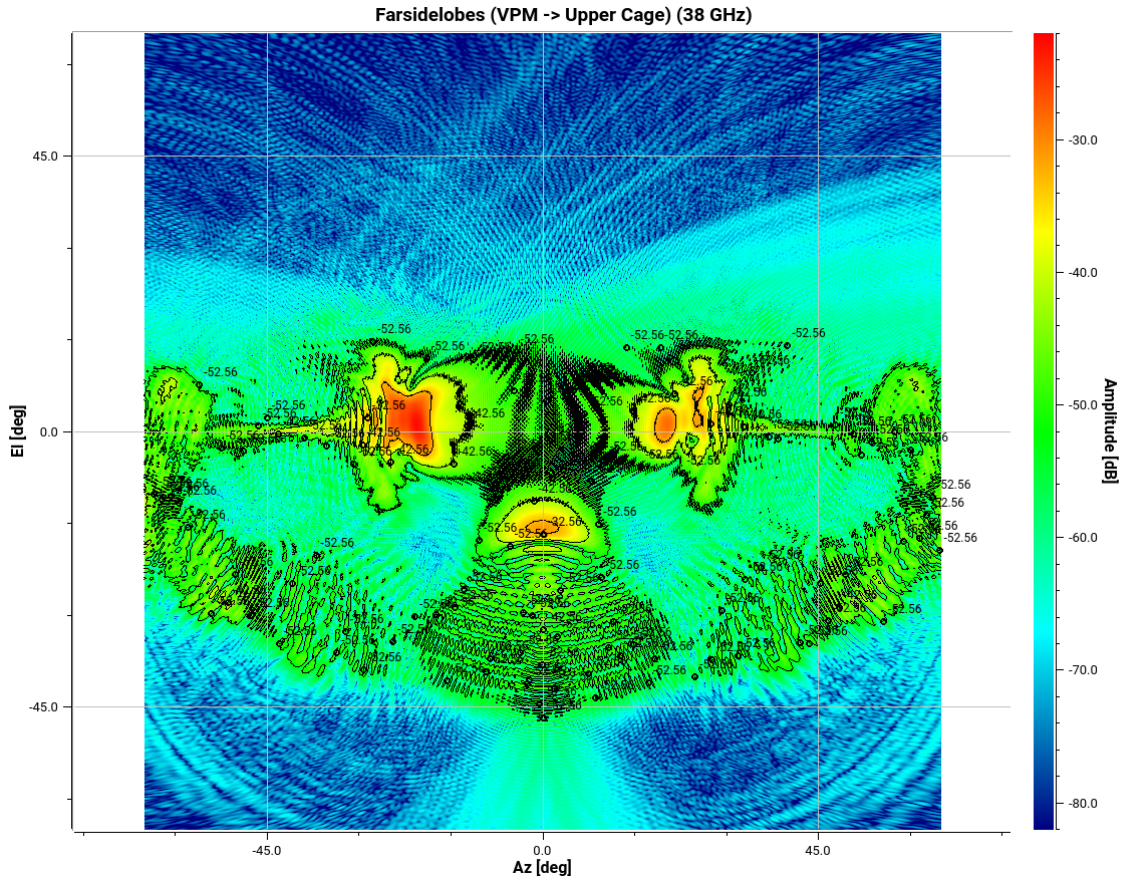


Figure 5.14: Far sidelobes calculated using GRASP, for a central detector. The asymmetry in the pattern is due poor convergence of the calculation performed by MoM. This was necessary to speed-up the computation, which would have taken several days to complete otherwise. The most prominent feature in the map is a “blob”, 30 degrees to the East. The amplitude of this sidelobe is -25 dBi. for comparison the amplitude of the main beam is 42 dBi.

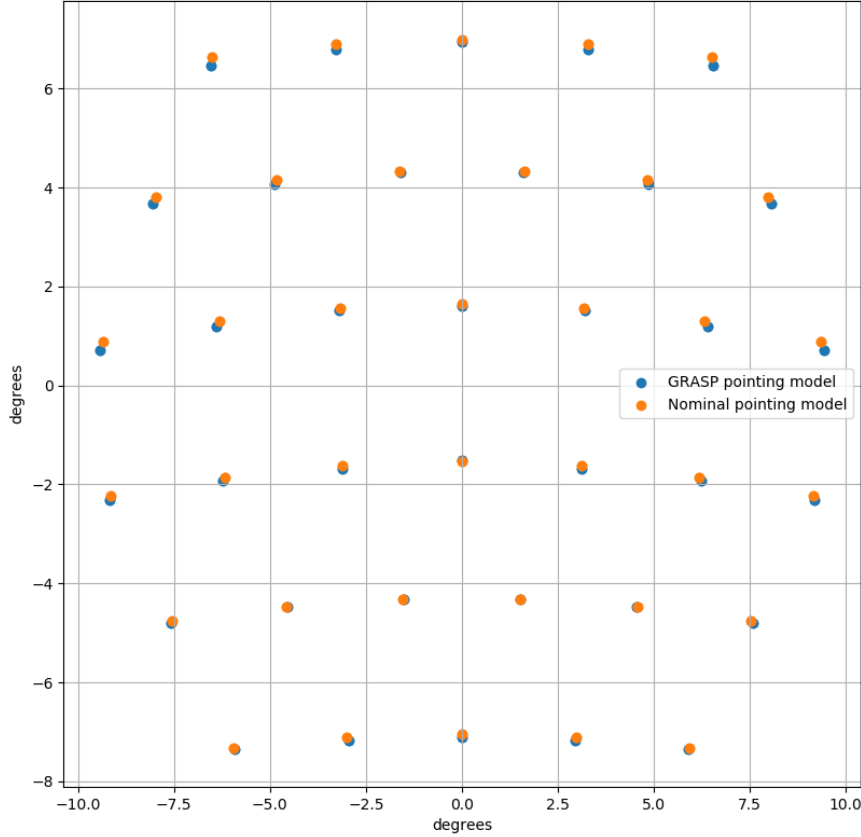


Figure 5.15: Comparison between the nominal pointing solution (orange circles) with the one obtained from GRASP simulations at 38 GHz (blue circles).

5.4 Validation

5.4.1 Pointing solution.

As part of setting up the full focal plane simulation, we obtained the pointing solution of the Q-band telescope. The pointing solution is a mapping between the position of the feedhorn in focal plane coordinates and the pointing of the respective beam on the sky, in boresight centered coordinates. GRASP simulations match the pointing solution obtained from Moon measurements during commissioning up to a

magnification factor. The source of this magnification is believed to be caused by a combination of misalignment of the optical elements and the bowing of the vacuum window in the cryogenic camera. Efforts to study the impact of the bowing at the vacuum window were unsuccessful given the added complexity to the model. Future work should address this by properly including the bowed window in the model.

5.4.2 Broadband beams and calibration.

As described in Appel et al. (2018), CLASS Q-band uses the Moon as a calibration source. Careful analysis yielded the conversion factor κ between measured power and brightness temperature of a Rayleigh-Jeans source. The work presented in Appel et al. (2018), however, does not take into account the frequency dependence of the beam and its coupling to the bandpass $W(\nu)$ of the optical system, as the simulations presented in this thesis were not available at the moment.

The conversion factor κ relates detected power by detectors and the thermodynamic temperature of a Rayleigh-Jeans source. This means that it includes an broad-band average of all sources of optical losses in the system. One of the sources is the broad-band beam filling factor, which as not accounted for as frequency dependent. The value of κ reported in Appel et al. (2018) is an average across the focal plane, assuming the Moon has a thermodynamic temperature of 210 Kelvin. Using the same Moon temperature model and GRASP simulations, we computed κ_s , where “s” stands for “simulation”, taking into account the broad-band beam dilution factor and obtained

$$\kappa_s = 13.9 \frac{\text{Kelvin}}{\text{pW}} \quad (5.19)$$

while the reported κ is $13.1 \frac{\text{Kelvin}}{\text{pW}}$.

This result shows that power to brightness temperature conversion factor computed from simulations is roughly 7% larger than the measured one. This implies that a lossless telescope, affected only by beam dilution, would receive around 7%

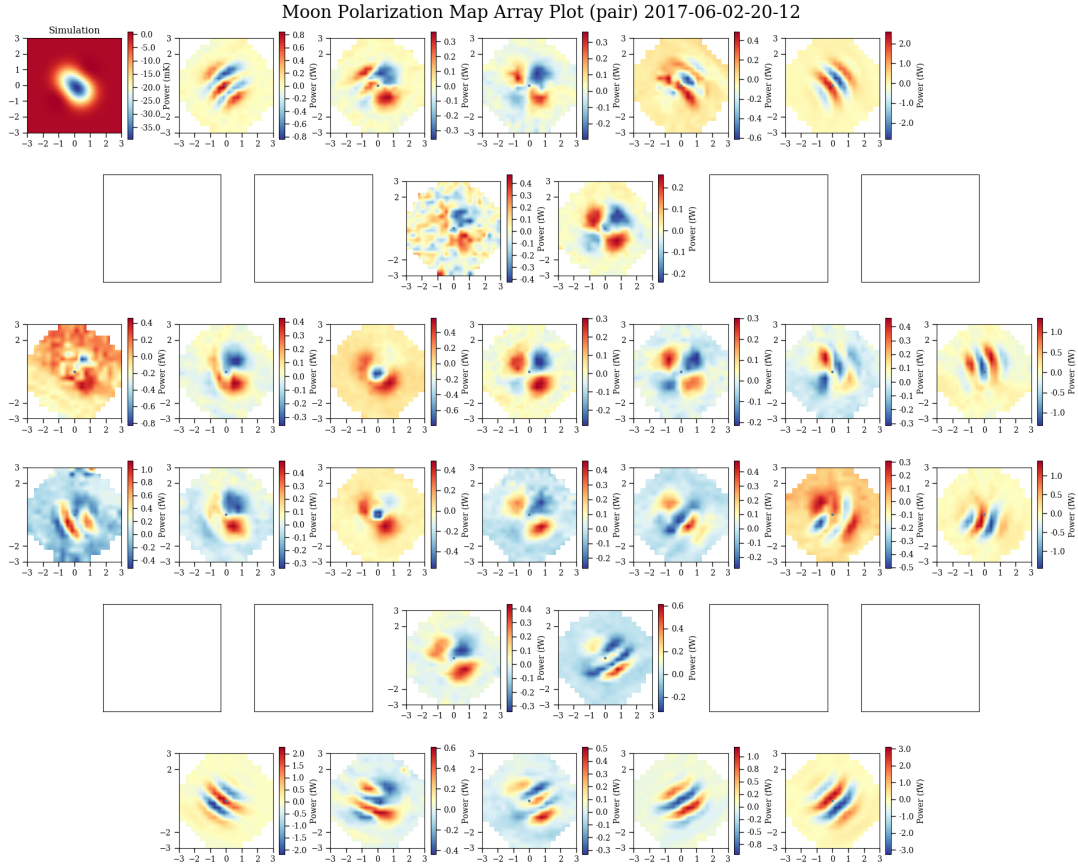


Figure 5.16: Local Q (sky U) maps of the polarization signal of the Moon using pair differentiation and polarization demodulation. Credits to Zhilei Xu (CLASS Collaboration, 2018)

more energy of a Rayleigh-Jeans source than the real one. For reference, the difference between beam efficiency and “global” efficient for the APEX telescope is around 12%. The origin of this discrepancy is related to optical imperfections.

5.4.3 Polarization leakage

CLASS uses VPM technology to enhance its sensitivity to polarization. Because the VPM is the first “active” element, leakage caused by inherent cross-polarized response of the optical setup is not of major concern. However, CLASS Q-band uses a baffle to protect the optics from weather inclemencies as well as to prevent thermal

radiation from the ground to reach the camera. Indications that the baffle causes polarization leakage was observed in polarization maps of the Moon. While central detectors show the expected quadruple pattern from a radially polarized Moon, the polarization maps for edge detectors are dominated by a stripy pattern. The data also shows that the further away from boresight pointing the detector is, the larger the amplitude of the stripes.

CLASS Q-band measures polarization by translating the scan synchronous polarized signal to 10 Hz, using the VPM. Time streams are then AM demodulated and the “clean” data projecting the result to the sky. In the case of Moon measurements though, a 10 Hz temperature common of unknown origin forced the analysis to be carried out by differentiating detector streams on the same feedhorn in addition to VPM demodulation. The consequence of this using pair differentiating and VPM demodulation is that the resulting maps are a combination of B_{QQ} and B_{UU} components of the beamsor, which couple to the polarization signal, and off-diagonal terms B_{QT} and B_{UT} , responsible for leakage from temperature to polarization.

To model the effect the baffle has on the system, we computed

$$B_{js}^i = B_{jb}^i - B_{jc}^i \quad (5.20)$$

where c corresponds to the beamsor obtained from a simulation carried out without baffle (“c” from clouseout) and b for a simulation with baffle. Since the beamsor is, by definition, unit-less, we scaled the simulations by a factor f such that the simulations could be compared to Moon measurements. f is given by

$$f = 10^3 \times 210 \text{ K} \times \kappa^{-1} \text{ pW K}^{-1} \quad (5.21)$$

where 210 Kelvin corresponds to the brightness temperature of the Moon at 38 GHz (Appel et al. (2018)), and κ to the conversion factor.

Figure 5.17 we show $f \times B_{js}^i$ for feedhorn 1, which is at the lower edge of the focal plane. We note that B_{TQ} shows an amplitude of $\approx \pm 3 \text{ fW}$, which is in good

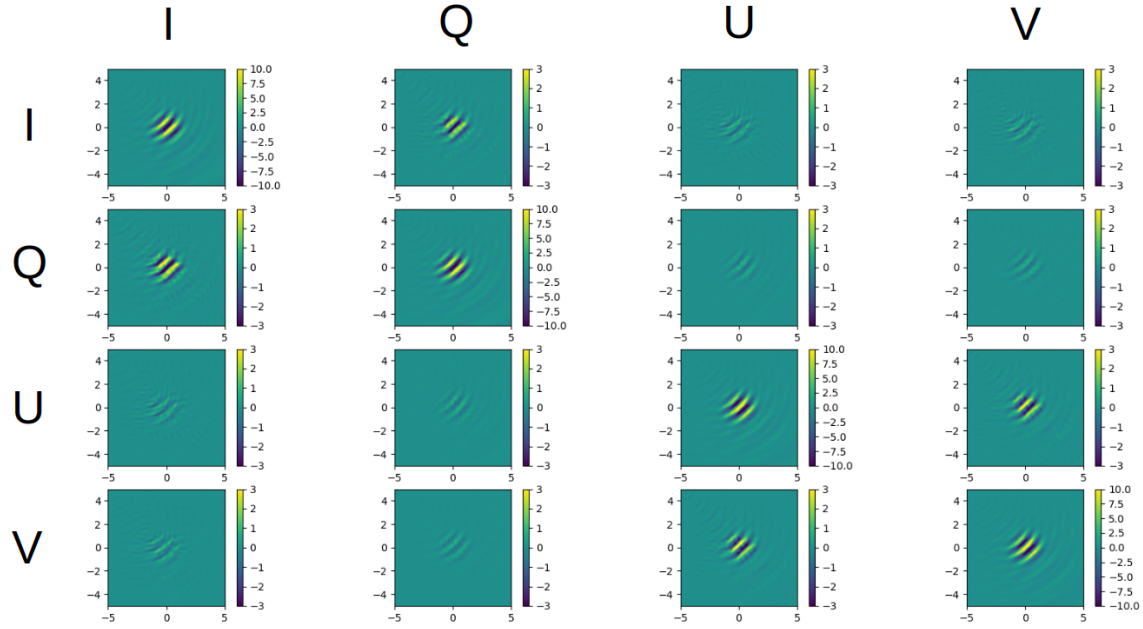


Figure 5.17: Difference between the beams of feedhorn 1, computed with the baffle and without the baffle. The second plot in the first row corresponds to the leakage beam from temperature to polarization, particularly Q . Note the Q beam is in the feedhorn's coordinate system. Moon polarization measurements were performed using pair differencing, so that unpolarized light would be measured by a combination of the QQ beam (second row, second column), the UU beam and leakage terms. Since the unpolarized brightness of the Moon is much larger than the polarized signal, the leakage beam dominates the measurements. Plot units are degrees from beam centroid. Color bar is in femtoWatts (fW).

agreement to the measurements. A final confirmation of the stripes being produced by the baffle comes from measurements of the Moon polarization signal carried out with a blackened baffle. The inner walls and outer edge of the baffle were covered with Eccosorb (r), which dramatically improved the quality of the Moon polarization maps.

5.4.4 Far sidelobes

An estimation of how this sidelobes would affect observations can be made by considering their interaction with the most powerful source on the sky, the Sun. The Sun has a brightness temperature at 38 GHz of 9000 Kelvin, reaching 10000 Kelvin when the solar cycle peaks in activity. This means that the measured brightness at the blob will be

$$T_{\text{sidelobe}} = \frac{\Omega_{\text{sun}}}{\Omega_{\text{sidelobe}}} \times 10^4 \text{ Kelvin} \approx 250 \mu\text{K} \quad (5.22)$$

CLASS uses a Sun Avoidance algorithm to keep the array center at a minimum distance of 20 degrees from the Sun. However, the large FOV makes edge detectors get as close at 12 degrees. This means that contamination caused by the Sun being captured by the far sidelobes is possible. From GRASP simulations, we see that the expected level of contamination does not exceed $250 \mu\text{K}$. This is contamination is, however, of the same order of magnitude than the CMB temperature anisotropy field.

To put an upper limit to far sidelobes, we used 2016 data to build Sun centered maps. Developing a map making code was out of the scope of this thesis, and efforts to create interfaces with existing codes, like MADAM, were unsuccessful. To overcome this issue, we developed `navmap`, an implementation of the co-adding map-making algorithm (also called “naive” map-making). `navmap` also uses a custom implementation of the HEALPix C routines that allows to efficiently build maps of regions of the sky. Together with `navmap`, we also developed an interface to

efficiently read CLASS data from disk. This interface is currently being used in the official CLASS pipeline. Finally, a fast pointing library that takes into account boresight rotations was developed as well: `mkpoint`. `mkpoint` is a joint effort between Michael K. Brewer from the CLASS collaboration and the author of this work. It is a pointing library that handles coordinate transformations between the horizontal and equatorial system. It also has the capability to re-center the coordinates of any focal plane into a second coordinate system. `mkpoint` also has a GPU accelerated version, which allows for a remarkable speed-up of repetitive operations. This is particularly useful for large arrays of detectors.

In order to maximize the signal to noise ratio, several data selection techniques were applied. The first one regards choosing a date range where the regions around the Sun have the best possible coverage (recall that CLASS has a Sun avoidance algorithm) Given CLASS location and scanning strategy, we selected data between 11/2016 and 01/2017. A second data cut revealed all data packages, in that date range, with valid detector data, the Sun above 30° in elevation, the telescope scanning in its standard scanning mode. Having identified the first set of useful data, we applied several cleaning techniques to it. This included fixing or discarding regions with “jumps” (DC offsets) in the data, glitches, detectors that would randomly turn off, sections of the data where the read-out system misbehaved and where detectors were unstable.

A second post processing layer addressed the presence of $1/f$ noise in the data. $1/f$ noise is shows up in the data as correlated random fluctuations in the streams. Since it is correlated, it cannot be removed by increasing the amount of data, i.e., integration time in the maps. Removing $1/f$ noise is a major challenge in CMB data reduction, requiring a detailed and deep understanding of the systematic effects caused by, for example, thermal fluctuations at the focal plane, sidelobes, optical deformations and others. $1/f$ noise, though, is expected to dominate in the low frequency regime of the streams, and hence it can be suppressed by applying frequency domain filtering to the signals. Unfortunately, this translates into a loss of sensitivity

at large angular scales. $1/f$ noise caused by thermal fluctuations can be removed by a Common Mode Subtraction (CMS), because the thermal drift in the focal plane is expected to affect all detectors simultaneously. Similar to frequency filtering, CMS might also reduce sensitivity to large angular scales features. Both band-pass filtering and CMS were applied to generate Sun centered maps, in an effort to reduce the $1/f$ noise as much as possible. Finally, every clean detector stream was processed by cutting regions of the data with an amplitude larger than 1 Kelvin.

Maps were generated applying `navmap` to the “clean” data. Each month of data was projected into 1296 maps (5 maps per detector), each map being generated using approximately the same amount of data for each detector. We calculated the RMS noise of each map and selected the ones with RMS noise below a certain threshold in millikelvin. Finally, maps that matched the RMS noise criteria were stacked. This process was embedded in a brute force minimization procedure in order to find an optimal threshold that minimized the RMS noise of the final map. The threshold that giving the best results was 100 milliKelvin, and resulted on a final maps composed of 582 (out of 1296) maps. The RMS noise in the stacked map was 7 milliKelvin (see Figure 5.18).

Unfortunately, this RMS level is insufficient to detect signals at 250μ Kelvin. Nevertheless, this process is it still a valuable test as it puts an upper limit to the amplitude of far sidelobes. The antenna temperature of the Sun is given by

$$T_{\text{sun}} = \frac{\Omega_{\text{sun}}}{\Omega_{\text{beam}}} \times 10^4 \text{ Kelvin} \approx 740 \text{ K} \quad (5.23)$$

which implies that CLASS Q-band appears to have no indication of significant temperature sidelobes down to $0.007/740 \approx -50$ dB with respect to the main beam.

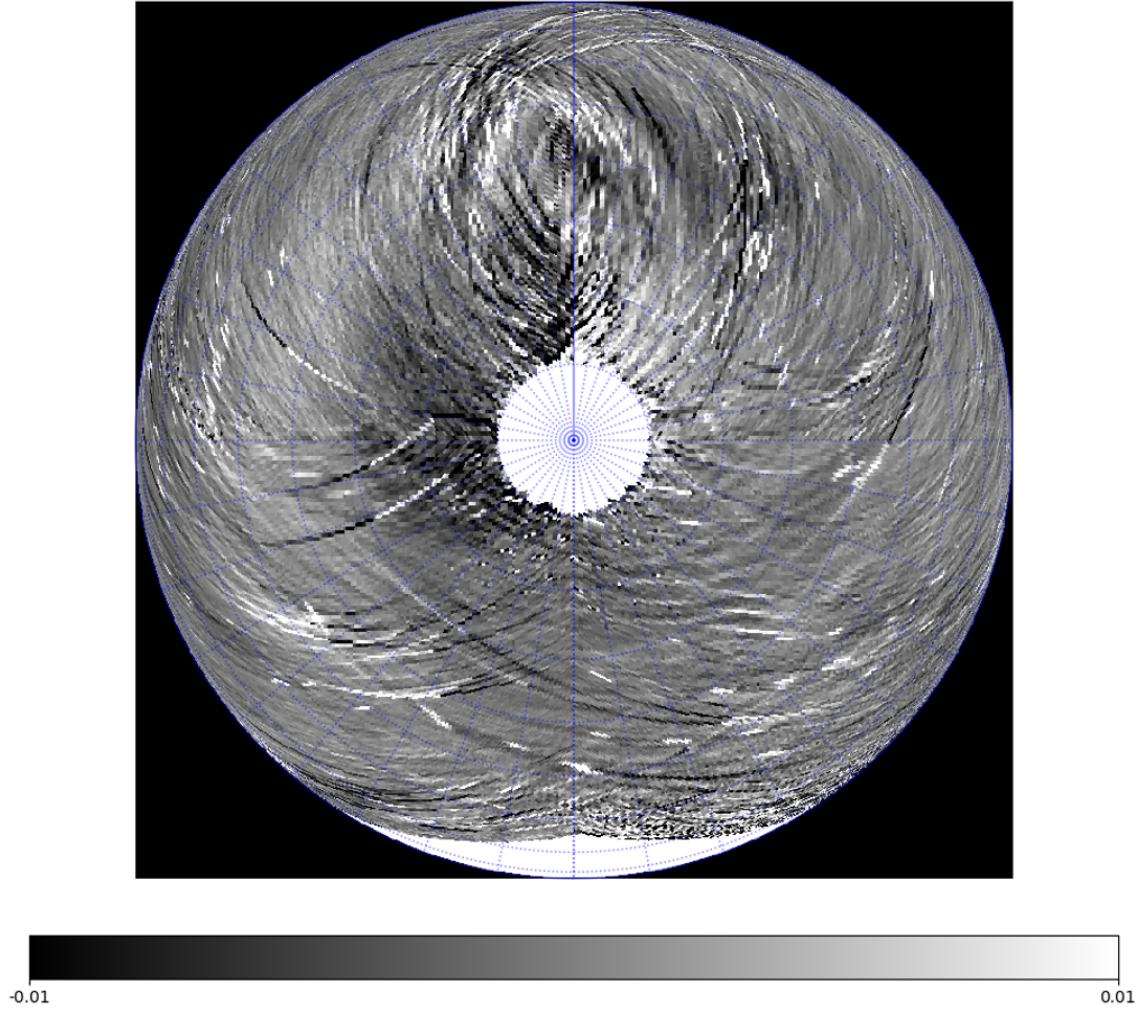


Figure 5.18: Orthographic projection of a temperature Sun centered map in boresight centered coordinates, in brightness temperature units. Color bar is in Kelvin, ranging from -0.01 to 0.01 Kelvin. North points up, while East does to the left. This map was computed using more than 10000 individual data packages. RMS noise in the map is 7 milliKelvin. The “cool” spot near the East is not consistent with sidelobes coming from the GRASP simulations. Signal to noise is insufficient to rule-out the presence of the sidelobes predicted by GRASP simulations, but still enough to rule out presence of sidelobes at -50 dB down the main beam.

Chapter 6

Simulating the CLASS Q-band experiment using PISCO

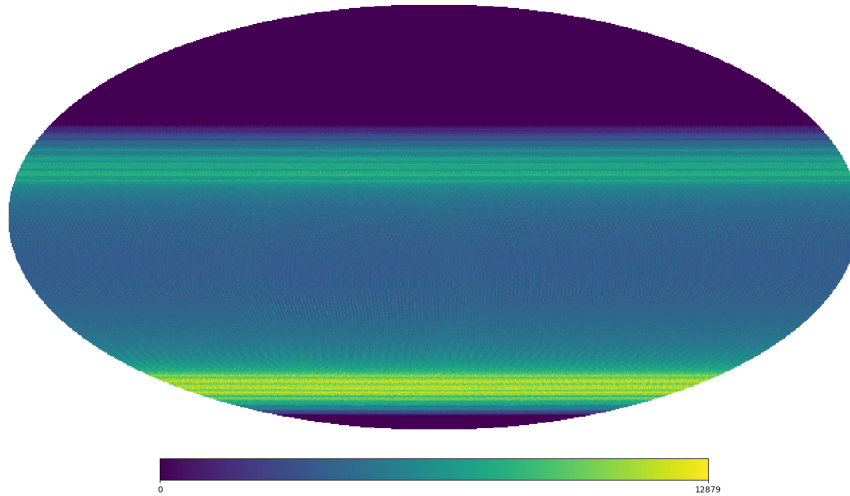


Figure 6.1: Simulation showing the amount of observations on each sky pixel for one week of a “nominal” scanning strategy of the CLASS experiment. More observations are seen near the poles, which is due to the asymmetry in the projection of the scanning strategy, defined in horizontal coordinates, into equatorial coordinates.

6.1 Simulation setup

This section describes the conditions on which we performed a computer simulation of CLASS Q-band using PISCO. This section briefly describes the scanning strategy, beamsors and sky models, as well as the tool used to compute power spectra out of maps.

6.1.1 Pointing

The scanning strategy of CLASS consists of constant elevation scans (CES). Elevation is kept at 45° while the telescope rotates 720° in azimuth at 1 degree per second. This process is repeated for around 18 hours per day. The boresight is rotated from -45° to $+45^\circ$ by 15° per day on a weekly schedule. This scanning strategy, in combination with the large CLASS field of view results in the telescope covering more than 70 percent of the sky every 24 hours. In addition, because of the boresight rotation, only seven days are needed to provide excellent position angle coverage. Boresight

rotation is also key to allowing modulation of both Q and U signals (see Harrington et al. (2016)) While CLASS records data 200 times per second, the pointing streams were generated at 20 Hz. This down-sampling factor was selected so as not to produce pixel misses, with the median number of hits per pixel being on the order of thousands. Down-sampling allows for a ten-fold decrease in computation time. Even with this significant reduction, the pointing stream resulted in more than 870 million individual directions.

The equatorial coordinates of every detector were computed from the scanning strategy in horizontal coordinates and the beam center offsets of every detector. Representative beam center offsets of the Q-band receiver were provided by the CLASS collaboration. Two streams were generated by considering detector pairs to have matched or mismatched offsets. The case of matched offsets was simulated by forcing each pair to share the same beam center offsets, which in turn was calculated as the average of the individual pair offsets. Treating the deviation of every offset with respect to the pair average as a Gaussian random error yields a root mean squared mismatch of approximately 5 arc-seconds ($\approx 0.001^\circ$).

6.1.2 Beams

Chapter 5 describes the procedure that was followed to obtain CLASS Q-band beams. PISCO is, in principle, capable of performing simulations of the CLASS experiment using the full 4π beamsor of each detector. This would have required accurate electromagnetic simulations of each beamsor, possibly including second order effects such as sidelobes caused by spill inside the cage and polarization distortions caused by the baffle. Since these electromagnetic simulations are computationally expensive, we limited our analysis to the main beams only.

Main beams of the CLASS Q-band telescope show non negligible amounts of eccentricity and correlation of the ellipse orientation with the position on the sky. There is extensive literature on how beam related systematics, like mismatch, can

produce leakage from temperature to polarization (T to P leakage) (see Shimon et al. (2008), O’Dea et al. (2007) and Das et al. (2015)) Most of the results are obtained using semi-analytical methods that rely on some harmonic space representation of the beams. It is of particular importance to note that CLASS Q-band uses *paired* detectors. While the pipeline does not perform pair-difference in time domain, individual detector maps are effectively combined by the map making algorithm, so pair-difference happens in map domain. This makes CLASS sensitive to leakage caused by beam mismatch, which is one of the scenarios presented below. Simulations of the CLASS Q-band receiver provided us with representative main beam parameters. We did not include cross-polarized response of detectors in the beamsors. The main beam parameters correspond to FWHM in the East-West direction ($FWHM_x$), FWHM in the North-South direction ($FWHM_y$) and the rotation angle of the major axis of the corresponding elliptical profile. We calculated the degree of beam mismatch from beam parameters following a similar procedure as for the beam center offsets. The root mean square mismatch for $FWHM_x$ and $FWHM_y$ is 0.01° . Beam tensors were generated using main beam parameters (FWHMs and rotation angle) as obtained from GRASP simulations. The beam tensors were directly generated into HEALPix grids, which used an NSIDE parameter of 512.

6.1.3 Sky

To generate the maps for this simulation, we followed a similar procedure to the one described in section 4.3.2, with the main difference being the addition of an unpolarized CMB. This unpolarized caseh was used to check for T to P leakage, which could arise by beam mismatch, for example. All maps use the HEALPix pixelization with an NSIDE parameter of 128.

Power spectra and beam transfer function

Given that, at present, CLASS only covers $\approx 75\%$ of the sky, computing the power spectra of simulated maps requires the use of a tool than can handle a sky mask. For this reason, we changed the estimator from `anafast` to `Spice` though the `PolSpice` implementation (see Chon et al. (2004)). In addition, the CLASS collaboration provided a realistic sky mask that includes the galactic plane as well as the natural incomplete coverage.

We also need to account for the effect of an elliptical beam on the CMB power spectra. We do this by symmetrizing the beam. This symmetrized beam can be thought of as a low-pass filter in harmonic space (see Page et al. (2003)). We estimated the equivalent beam transfer function of an arbitrary number of elliptical beams by averaging the radial profiles obtained from an analytical integration over ϕ of all of the beams in “real” space, and calculating the harmonic transform of the average profile (Michael K. Brewer, private communication).

6.2 Results and discussion

Pointing mismatch

CMB experiments that rely on detector pairs are subject to leakage caused by pointing mismatch. Leakage arises at the map-making stage and is caused by an incomplete cancellation of the Stokes I term when solving for the individual pixel-covariance matrices. This residual term from the Stokes I is interpreted by the map-making algorithm as a polarized signal, hence causing the leakage shown in figure 6.2. We stress the fact that the information regarding pointing mismatch between pairs of detectors in CLASS Q-band was *not obtained from GRASP simulations*, but rather from preliminary estimations of beam center offsets carried out by the CLASS collaboration team.

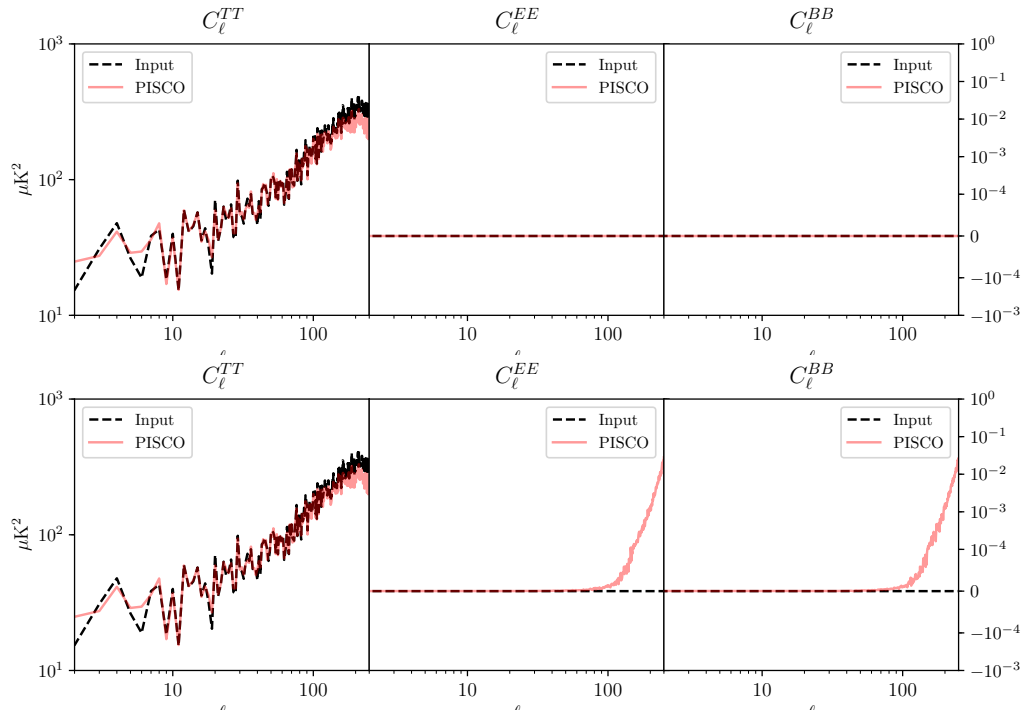


Figure 6.2: Resulting power spectra for a realistic simulation using matched pointing (upper figure), and mismatched pointing (bottom figure). The input CMB was unpolarized. The amplitude of the mismatch was provided by the CLASS collaboration team. Significant leakage from TT to EE and BB power spectra is present for the mismatched pointing case. The spurious signal reaches in the order of 30 nK^2 at $\ell = 250$ for both EE and BB .

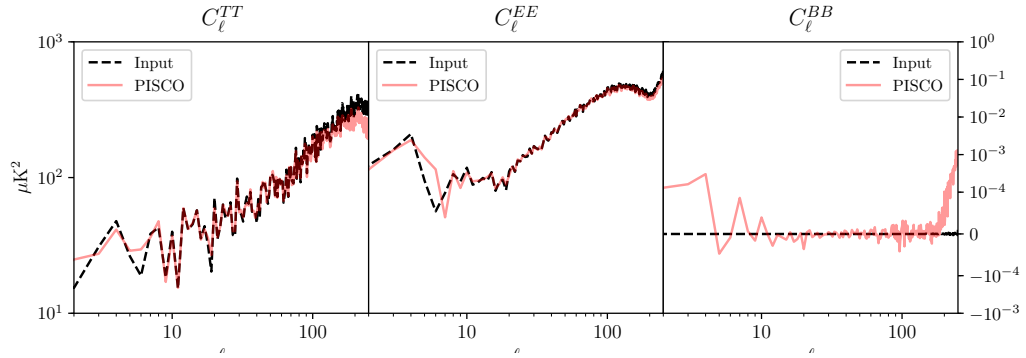


Figure 6.3: Resulting power spectra for the case of matched pointing with uneven intra-pixel coverage using as input a polarized CMB without B-modes. The B-mode power spectrum shows non-negligible amounts of a spurious signal not present in the input power B-mode power spectrum. The spurious B-mode spectrum reaches in the order of 1 nK^2 at $\ell = 250$.

Uneven intra-pixel coverage

Simulating a CMB experiment using a more realistic scanning strategy can produce another systematic effect at the power spectra level. This source of noise is related to the intra-pixel coverage of the sky. In section 4.3.2, all pixels were observed exactly at their centers while, in a real experiment, every sample of the TOD “hits” a given pixel at an arbitrary location within it. If the distribution of hits inside a pixel is symmetric with respect to pixel center coordinates, map-making will average all observations and the power spectra from the resulting map will not be affected. However, if this distribution is asymmetric and gradients between pixels are present (recall that pixel space convolution is affected by neighbor pixels) the resulting power spectra may suffer from P to P leakage. This was discussed in more detail in Poutanen et al. (2005).

Figure 6.3 shows the result of running the realistic simulation using a polarized CMB with matched pointing and it is clear that a spurious signal is present in the B-mode power spectrum. In the upper plot of Figure 6.2, the same simulation was performed but with an unpolarized CMB as input, the resulting polarized power spectra being consistent with zero. This indicates that the effect of uneven intra-pixel coverage is P to P leakage. We note that this systematic effect is subdominant

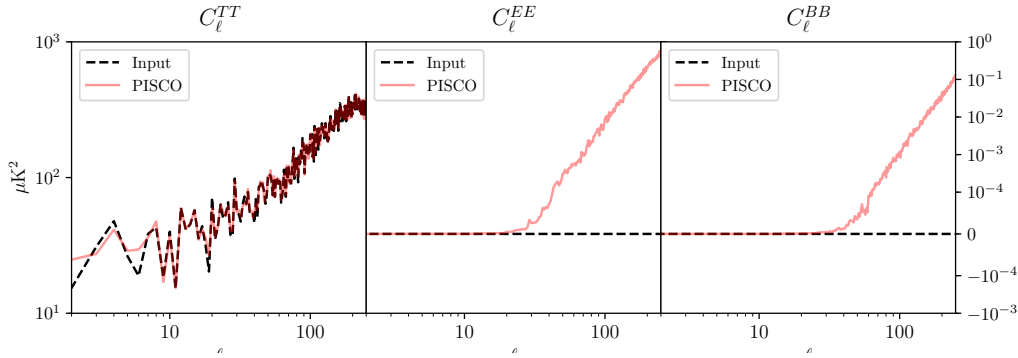


Figure 6.4: Resulting power spectra of a realistic simulation from which the effects of uneven intra-pixel and pointing mismatch have been suppressed. The input CMB was unpolarized, and so the resulting E-mode and B-mode power spectra can only be a result of T to P leakage. The amplitude of the T to P leakage reaches roughly $0.1 \mu\text{K}^2$ at $\ell = 250$.

with respect to the T to P leakage caused by pointing mismatch by roughly a factor of 30.

Beam mismatch

The effect of beam mismatch can be thought of as similar to pointing mismatch in that the net effect is to create a spurious polarization signal from the temperature signal. Figure 6.4 shows the resulting power spectra of a simulation with matched pointing, without the effects of uneven intra-pixel coverage. Suppressing the effect of intra-pixel coverage systematics was achieved by forcing the pointing of every detector to aim at its closest sky pixel, for an NSIDE resolution of 128. Results show that PISCO is capable of reproducing the expected systematic effect caused by beam mismatch, which is T to P leakage. We also note that, of all three systematic effects investigated, this produces the largest amount of leakage which, for beam mismatch, reaches approximately $0.1 \mu\text{K}^2$ at $\ell = 250$.

Chapter 7

Conclusions

7.1 Main summary

This thesis describes the procedure followed to develop a computer simulation of a CMB experiment, particularly, to the Q-band receiver of the CLASS experiment. In this work,

- Development of a mathematical framework that describes the polarizing properties of a CMB telescope.
- Implementation of a computer code to simulate the impact of a CMB experiment, in particular, the CLASS Q-band telescope.
- Development and validation of electromagnetic simulations of the CLASS Q-band telescope.
- Application of the computer simulation code to the CLASS Q-band telescope.

The first item corresponds to the discussion presented in Chapter 3 (antennas). This chapter describes a way of applying the formalism presented in the work of O’Dea et al. (2007), originally envisioned for harmonic space analysis, to the real domain. This resulted in a mathematical framework that allows to express the TOD as a function of the telescope beams, scanning strategy and beam tensor in a general scenario. The advantages of performing these simulations in the real domain become relevant when including time-dependent effects, complex scanning strategies or transient events on the sky. All of the above systematic effects cannot be trivially included in current formalism, like the one described in Duivenvoorden et al. (2018).

Limitations of harmonic space convolution yielded to the creation of a new computer simulation code, the Pixel Space CONvolver (PISCO), as well as an associated CMB simulation pipeline that includes a pointing library, `mkpoint` and a map-making code, `navmap`. Parts of this library are currently used by the core of the official CLASS experiment analysis pipeline. PISCO, on the other hand, is a code with the capability to synthesize TOD following the framework developed in Chapter

3. PISCO is unique in its field by the use of GPUs to accelerate the procedure of beamsor-sky multiplication. The current implementation of PISCO is also flexible enough to allow future improvements. Validation tests indicate that PISCO produces correct results, and so it is very likely to become a valuable tool for forecasting beam related systematics for ongoing and future experiments, like CMB Stage-4 or the Simmons Observatory.

As part of the scope of this thesis, we also developed an electromagnetic model of the CLASS Q-band telescope using GRASP. This pipeline showed to closely reproduce main beams, band-averaged beams, polarizing properties of the optics (by applying the formalism developed in Chapter 3) as well estimations of far sidelobes caused by spill-over on off-axis optical elements. Results from electromagnetic simulations were compared and validated using observational data, like polarized beam maps obtained from Moon observations, calibration sources and Sun centered maps. All this reinforces the idea that electromagnetic simulations not only are a valuable tool to gain insight on the electromagnetic properties of a telescope, but might also serve as a way of forecasting how these properties affect the scientific results.

Finally, we combined chapters 3, 4 and 5 to produce a computer simulation of the CLASS Q-band telescope. Simulations to estimate the impact of polarizing properties of Q-band, as well as its far sidelobes, require significant computational effort. For this reason, we only explored the impact of a three systematic effects: pointing mismatch, beam mismatch and uneven intra-pixel sky coverage. It was found that both pointing and beam mismatch produces T to P leakage at angular scales comparable to the beam size. Quantifying the effect of this leakage on the estimation of cosmological parameters was not addressed, as this is a challenging task on its own which requires a large amount of man and computational power.

7.2 Discussion

The main driver of this thesis was to prove that it was possible to realistically simulate a CMB experiment using a computer. The results presented in this work indicate that this was a success, as we were able to characterize a CMB experiment using two computer codes, GRASP and PISCO. PISCO, in turn, was an “in-house” solution, developed with the sole purpose of simulating a CMB mission. We believe this is a remarkable result in the sense that a great amount of insight was obtained without the need of physically building the experiment. This is probably the main product of this work, that proves not only that simulating a CMB experiment with a computer is possible and accurate, but also that it can become a crucial part of the analysis pipeline producing scientific results.

The work that was carried out to build an electromagnetic model of Q-band was shown to be valid by comparing simulations to real data. This validation process also showed that the formalism developed in Chapter 3 is correct, and that it is possible to apply it for real experiments. EM simulations showed that Q-band main beam parameters are compatible with design specifications. We did not find any signs of optical aberrations caused by the broad-band response of the telescope. For polarization, we did find two systematic effects. The first one is related to the baffle, which causes temperature to polarization leakage. While the exact mechanism by which this occurs is not yet fully understood, mitigation measures were taken and the leakage was greatly reduced. The second polarization systematic is related to the orientation of the co-polar direction for beams originating from feeds that are far away from the center of the focal plane. The origin of this offset in the polarization sensitive angle is not yet fully validated, but indications of its impact had recently been devised. More work needs to be carried out in this front to fully understand, and possibly mitigate, this systematic error. Finally, analysis of far sidelobes using GRASP revealed that, to first order, they are of no major concern to Q-band, being at suppressed at least 50 dB with respect to the main beam. There are signs that

there might be far sidelobes not being predicted by the simulations, which is a strong driver to keep working on this field.

While the challenge of simulating a CMB experiment had already been addressed, this work presents a different way of accomplishing the same goal. This was done not only by providing a new¹, accurate formalism that works in pixel domain, but also a code (PISCO) with built-in capability to scale well in HPC environments as well as making use of modern accelerators like GPUs. We believe PISCO will become a key complement to existing simulation pipelines, in the sense that PISCO can probe systematics that were difficult to test for. Worked is already being carried out by the author of this work and the Cosmology Computational Center at Lawrence Berkeley Labs to port PISCO to upcoming a pre-exascale supercomputer, Perlmutter, at NERSC.

Finally, simulations of CLASS Q-band using PISCO indicate that pointing mismatch and beam mismatch can cause T to P leakage. More work needs to be carried out in order to quantify this leakage properly, particularly by performing a larger number of simulations with more realistic parameters, like pointing, beams with cross-polarization and far sidelobes, the VPM and, possibly, time-varying atmosphere. We note that the design of the current pipeline allows for such a dramatic increase in complexity with relatively low effort, so it is quite possible that future work in this field gets carried out in the middle term.

¹Though based in the work of O’Dea et al. (2007)

Appendices

Appendix A

Computation of antenna coordinates from sky coordinates and antenna pointing

A.1 Derivation using spherical trigonometry

This derivation was kindly provided by Michael K. Brewer from the CLASS collaboration

A.2 Computation of antenna basis coordinates and the angle *chi* from sky coordinates

For a beam center pointing (θ_0, ϕ_0) , rotation angle ψ_0 and off beam center pointing (θ, ϕ) in the sky basis, we can derive the antenna basis coordinates (ρ, σ) and the angle χ between $\hat{e}_\parallel$ and $+Q$ ($\hat{\theta}$) by using spherical trigonometry (see Figure A.1). The identities used in this derivation are: the law of cosines, the law of sines and the analogue (or five part) formula.

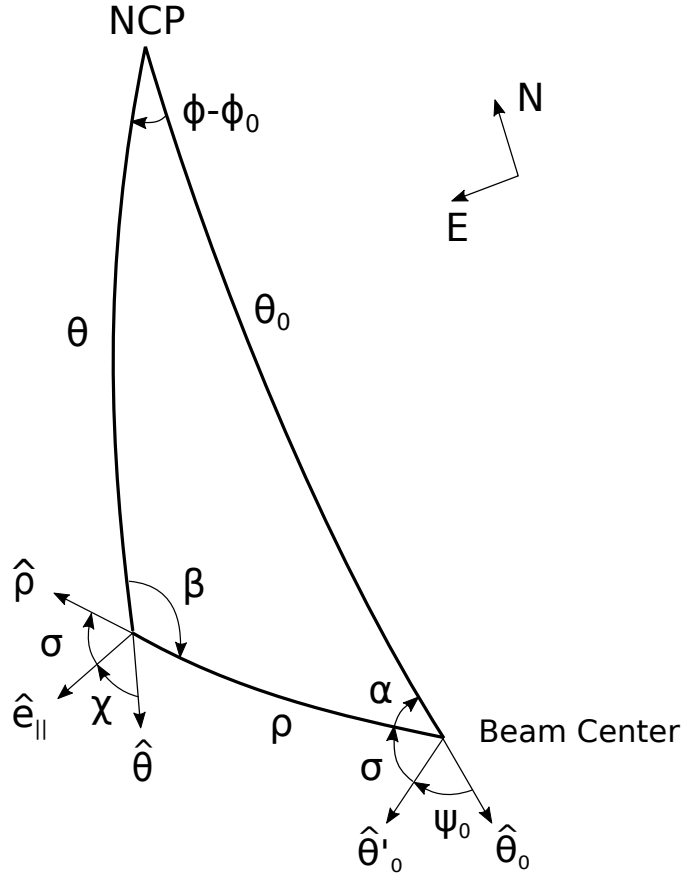


Figure A.1: Sky and antenna basis coordinates for beam center pointing and off beam center pointing from the view point of an observer looking at the sky. Here NCP is the North Celestial Pole and χ is the angle between $\hat{e}_{\parallel}$ and $\hat{\theta}$ according to Ludwig's 3rd definition. Credit to Michael K. Brewer, CLASS collaboration (2018)

A.2. COMPUTATION OF ANTENNA BASIS COORDINATES AND THE ANGLE χ FROM SKY COORDINATES

Defining $\Delta\phi \equiv \phi - \phi_0$, the antenna basis coordinate ρ is given by

$$\rho = \arccos(\cos(\theta)\cos(\theta_0) + \sin(\theta)\sin(\theta_0)\cos(\Delta\phi)) \quad (\text{A.2.1})$$

Then defining α as the angle between ρ and θ_0 , the antenna basis coordinate σ is given by

$$\sin(\alpha) = \frac{\sin(\theta)\sin(\Delta\phi)}{\sin(\rho)} \quad (\text{A.2.2})$$

$$\cos(\alpha) = \frac{\cos(\theta)\sin(\theta_0) - \sin(\theta)\cos(\theta_0)\cos(\Delta\phi)}{\sin(\rho)} \quad (\text{A.2.3})$$

$$\alpha = \arctan\left(\frac{\sin(\theta)\sin(\Delta\phi)}{\cos(\theta)\sin(\theta_0) - \sin(\theta)\cos(\theta_0)\cos(\Delta\phi)}\right) \quad (\text{A.2.4})$$

$$\sigma = 180^\circ - \psi_0 - \alpha \quad (\text{A.2.5})$$

Following Ludwig's third definition of cross polarization (see Ludwig (1973)), the unit vectors of co and cross polarization in the antenna basis are

$$\hat{e}_{\parallel} = \cos(\sigma)\hat{\rho} - \sin(\sigma)\hat{\sigma} \quad (\text{A.2.6})$$

$$\hat{e}_{\times} = -\sin(\sigma)\hat{\rho} - \cos(\sigma)\hat{\sigma} \quad (\text{A.2.7})$$

Thus $\hat{e}_{\parallel}$ is offset from $\hat{\rho}$ by the angle $-\sigma$. Finally, defining β as the angle between θ and ρ yields

$$\sin(\beta) = \frac{\sin(\theta_0)\sin(\Delta\phi)}{\sin(\rho)} \quad (\text{A.2.8})$$

$$\cos(\beta) = \frac{\cos(\theta_0)\sin(\theta) - \sin(\theta_0)\cos(\theta)\cos(\Delta\phi)}{\sin(\rho)} \quad (\text{A.2.9})$$

$$\beta = \arctan\left(\frac{\sin(\theta_0)\sin(\Delta\phi)}{\cos(\theta_0)\sin(\theta) - \sin(\theta_0)\cos(\theta)\cos(\Delta\phi)}\right) \quad (\text{A.2.10})$$

$$\chi = \beta - \sigma \quad (\text{A.2.11})$$

A.3 Code listing

```
void rho_sigma_chi_pix(double *rho, double *sigma, double *chi,
double ra_bc, double dec_bc, double psi_bc,
double ra_pix, double dec_pix )

/*
 *   Calculates radial, polar offsets and position angle of the
 *   co-polarization
 *   unit vector using Ludwig's 3rd definition at a location
 *   offset from beam center. All outputs use the HEALPix coordinate system
 *   and the CMB polarization angle convention.
 *
 *   Outputs: rho is the radial offset
 *            sigma is the polar offset clockwise positive on the sky from
 *            South
 *            chi is the position angle clockwise positive from South
 *
 *   Inputs:  ra_bc  Right Ascension of beam center
 *            dec_bc Declination of beam center
 *            psi_bc Position angle of beam center clockwise positive
 *            from North
 *            ra_pix Right Ascension of offset position
 *            dec_pix Declination of offset position
 */

{
double cdc = cos(dec_bc);
double sdc = sin(dec_bc);
double cdp = cos(dec_pix);
double sdp = sin(dec_pix);
```

```
double dra = ra_pix - ra_bc;
double sd = sin(dra);
double cd = cos(dra);

double gamma = atan2(sd * cdp, sdp * cdc - cdp * sdc * cd);
double delta = atan2(sd * cdc, sdc * cdp - cdc * sdp * cd);
double crho = sdc * sdp + cdc * cdp * cd;

if(crho >= 1.0) {
    *rho = 0.0;
    *sigma = 0.0;
    *chi = psi_bc;
}
else {
    *rho = acos(crho);
    *sigma = M_PI - gamma - psi_bc;
    if(*sigma < 0.0)*sigma += 2.0 * M_PI;
    else if(*sigma > 2.0 * M_PI)*sigma -= 2.0 * M_PI;
    *chi = delta - *sigma;
    if(*chi < -M_PI)*chi += 2.0 * M_PI;
}
```

Appendix B

Main beam parameters for the CLASS Q-band telescope

Table B.1: Main beam parameters for 38 GHz simulation.

detector number	FWHM _{<i>x</i>} (degrees)	FWHM _{<i>y</i>} (degrees)	Ω (μ sterad)
0	1.568	1.435	776.3
1	1.568	1.435	776.3
2	1.550	1.456	779.1
4	1.548	1.457	778.7
5	1.597	1.393	767.9
7	1.591	1.396	766.6
8	1.562	1.417	764.0
9	1.560	1.418	763.5
11	1.594	1.403	772.0
12	1.587	1.407	770.5
13	1.530	1.497	790.5
14	1.531	1.496	790.6
15	1.552	1.472	788.3
16	1.546	1.476	787.6

Table B.1: Main beam parameters for 38 GHz simulation.

detector number	FWHM _{<i>x</i>} (degrees)	FWHM _{<i>y</i>} (degrees)	Ω (μ sterad)
17	1.577	1.437	782.2
18	1.569	1.442	780.7
19	1.615	1.405	782.9
21	1.605	1.409	780.4
22	1.618	1.423	794.4
23	1.608	1.427	791.8
24	1.561	1.478	796.2
25	1.554	1.483	795.1
26	1.589	1.442	790.9
27	1.580	1.447	789.2
30	1.621	1.415	791.4
32	1.611	1.419	788.8
34	1.598	1.450	799.6
35	1.590	1.454	798.1
36	1.579	1.471	801.7
37	1.575	1.474	801.3
38	1.631	1.433	806.9
39	1.623	1.437	804.8
40	1.619	1.432	800.4
41	1.614	1.435	799.4
42	1.613	1.445	804.3
43	1.611	1.446	804.1
45	1.591	1.454	798.2
46	1.598	1.450	799.5
47	1.576	1.473	801.3
48	1.578	1.472	801.7

*APPENDIX B. MAIN BEAM PARAMETERS FOR THE CLASS Q-BAND
TELESCOPE*

Table B.1: Main beam parameters for 38 GHz simulation.

detector number	FWHM _{<i>x</i>} (degrees)	FWHM _{<i>y</i>} (degrees)	Ω (μ sterad)
49	1.624	1.436	804.8
51	1.630	1.434	806.7
52	1.615	1.435	799.7
53	1.618	1.433	800.2
55	1.608	1.427	791.8
56	1.617	1.423	794.3
57	1.540	1.503	798.6
58	1.539	1.504	798.6
59	1.554	1.482	795.0
60	1.561	1.478	796.2
61	1.580	1.447	789.1
62	1.589	1.442	790.9
63	1.611	1.419	788.9
65	1.621	1.415	791.2
66	1.586	1.407	770.4
67	1.595	1.403	772.0
68	1.545	1.477	787.5
69	1.552	1.471	788.3
70	1.569	1.442	780.6
71	1.578	1.437	782.3
72	1.605	1.409	780.3
76	1.615	1.405	782.7
77	1.562	1.438	775.3
78	1.569	1.434	776.4
79	1.547	1.458	778.5
80	1.551	1.455	779.2

Table B.1: Main beam parameters for 38 GHz simulation.

detector number	FWHM _{<i>x</i>} (degrees)	FWHM _{<i>y</i>} (degrees)	Ω (μ sterad)
82	1.598	1.393	768.0
83	1.559	1.419	763.2
84	1.563	1.417	764.2
85	1.559	1.426	767.2
86	1.560	1.426	767.3

Table B.2: Polarization angle rotation Υ for CLASS Q-band, at 38 GHz.

detector number	Υ (degrees)
38	-5.161
39	-5.088
40	-2.758
41	-2.391
42	-0.227
43	0.226
52	2.390
53	2.757
49	5.091
51	5.160
22	-5.949
23	-6.405
34	-3.721
35	-3.775
36	-1.298
37	-1.166
47	1.166
48	1.298

*APPENDIX B. MAIN BEAM PARAMETERS FOR THE CLASS Q-BAND
TELESCOPE*

45	3.775
46	3.721
55	6.404
56	5.949
30	-6.608
32	-7.484
26	-4.618
27	-4.982
24	-2.365
25	-2.444
57	-0.006
58	0.006
59	2.444
60	2.365
61	4.982
62	4.618
63	7.485
65	6.608
19	-6.309
21	-7.169
17	-4.372
18	-4.729
15	-2.245
16	-2.309
13	-0.016
14	0.016
68	2.309
69	2.245

70	4.729
71	4.372
72	7.169
76	6.308
11	-5.169
12	-5.591
00	-3.230
01	-3.230
02	-1.162
04	-1.002
79	1.002
80	1.162
77	3.253
78	3.230
66	5.592
67	5.169
05	-4.208
07	-4.083
08	-2.319
09	-1.898
85	-0.258
86	0.258
83	1.898
84	2.319
81	4.084
82	4.207

7.1 On the process of getting this PhD

The original idea that drives this thesis came from a fruitful discussion I had with Dominik Gothe, back in 2014. We were talking about how easily things could go wrong with CLASS, so as to introduce spurious B-modes into the results. I started digging into how to address this using very simple methods, until I found a code from Nathan Miller. His pipeline was able to produce synthetic TOD with the VPM systematics, as well as maps using the CLASS scanning strategy. There was a comment in the source code referring to the need of including the beam into this pipeline, so I thought “this sounds like a good thing to work on”. In parallel, I was working with Michael “Mike” Brewer to build a code that would interface to the mount computer, so as to control and monitor the whole telescope remotely. We started exchanging emails and messages about this around March 2015, and as I kept asking stuff to him, he kept answering. I did not know it back then, but the seed of this work was planted, and the right person to help me make it grow was already there.

Then, around December 2015, my supervisor Rolando “Rolo” Dünner suggests me to write my thesis about electromagnetic simulations of CLASS. This made a lot of sense, since I had already been doing something similar for ACT since 2013. I started working on this, but I could not stop wondering how to propagate the results from electromagnetic simulations to maps and power spectra. The culprit of this came from Thomas Puzia, a professor at the AIUC who asked me how would cross-polarized beams impact the results of CMB experiments, to which I answered “I don’t know”. He might have been joking, but he shouted “well, you should simulate it!”. I sort of took that as a personal challenge, and started working on it.

I started looking on some methods, and built a sketchy, horrible code that would take a fairly complex beam and get a map out of it (see Fluxá et al. (2016)) The code was so slow I *had* to make it run on the Graphics Processing Unit. It is important to remark that, to get that running, I had to hack the ACT pointing library that

had no documentation whatsoever, used quaternions and did not allow for boresight rotation like CLASS does. This made me start developing a new pointing library which Mike and I finally got running. Part of this library is now used by the CLASS mapping code. I kept working on this all across 2016. As a side note, I was supposed to present this work at the SPIE of 2016, but the tremendous effort of getting the code running made me sick. I suffer from atopic dermatitis, and a few days before traveling I ended up at the emergency room with 1500 times more IgE (an enzyme that control the inflammatory response) than normal. Bad times.

Then, one good (or bad!) day around March 2017, Rolo showed me the work of O’Dea et al. (2007) and I immediately thought “That’s it, everything is solved now”. I was *so wrong* about it. Until only a few days ago (this text was written in July 24th, 2019) I was still finding subtleties and doing math with a pen and paper (actually, I was using `sympy`) to check if everything was fine. One of the most prominent contributions to my work came from Mike, who derived formulae to keep track of the intra-beam variations of position angle. Note that Mike was in charge of the mount computer, which had absolutely nothing to do with simulations, except maybe for the pointing. 2017 was a rough year, with a lot going on in both the GRASP and PISCO fronts. Funding was cut in August 2017, but fortunately Rolo had budget to keep me around and continue my job. Be aware, I entered PhD program in August 2013, so almost 4 years had passed already. No real progress (at least, if you asked me about it) had been made. I was jumping all over the place between GRASP simulations for CLASS and ACT, the CLASS analysis pipeline, PISCO and taking a course on cosmology. I was not enjoying things.

The year 2018 was key. I was able to make crucial improvements to the CLASS Q-band simulations. Data from Moon observations did finally converge to publication quality results, and I was able to validate all of the GRASP simulations. During that year, I made significant improvements to the GRASP model, which was completed around August 2018 (5 years now!). Around November 2018, Mike noticed I had not applied a change to PISCO that he suggested me to apply in *around April*. With

that in place, the code finally worked! I was like being hit by a bottle of pisco, and realized I had everything I needed to assemble the document you are reading. By the way, this thesis was horribly written in the beginning. Rolo and Toby made a great job at correcting it. I submitted a first, crappy draft, December the 14th 2018.

There were still a lot of issues with PISCO: not only it was slow and complicated to run, but also simulations were not yielding the results I expected. This meant the paper that I had to pull out to graduate was wrong, and so did the results of the thesis. In addition to this, after two years without funding and living at my parents expenses, I was sort of forced to start working on the private sector. I also won a small fund to keep me around campus one day every week, which turned out to be a nice compromise that I might keep doing. It was February 2019 already, and I was just about to graduate, got a decent job to pay the bills, and everything seemed fine. I kept working on my thesis, with great support from the company I now work at, my family, friends, Rolo, Toby, Mike and, last but not least, Camila. After quite a bit of coding and mental gymnastics that encompassed April, May and most of June, Mike, Rolo and I were finally convinced the simulations were correct, and I was ready to go. For the record, today, July the 24th 2019 (almost 6 years now), Mike and I finally confirmed that the polarization formalism used by PISCO is correct. Now that took long enough!

During the last 6 years, I had become an expert on GRASP simulations and pixel space convolution. I now know a lot more of spherical trigonometry, celestial mechanics, electromagnetism, cosmology, quantum and nuclear physics. I learned a lot of C, C++, FORTRAN and Python, databases and GNU/Linux. I found out how to port algorithms to graphic processing units, and learned how to develop code for general HPC environments. I got a really good grasp on how a CMB experiment works, literally by getting involved in every single step of the process; cryogenics, detectors, electronics, some robotics, mechanical designs, data analysis, software development and, finally, scientific results. It is true that I am exhausted but, as shown in a comic I once saw, I know own a small bubble in the knowledge-sphere.

A bubble that I created and kept from popping until it got stable by itself. A small, insignificant piece of space-time where I put my hopes, effort, tears and every single neuron I had for the sole purpose of answering a question I had back then in 2014, when I was younger and foolish. And I would do it again, just *quicker*.

Rolo, Mike, Toby, and many others, it was an honor to work along your side. I very much hope that my work will become useful one day, hopefully soon enough to see it either succeed, or fail. And to whoever that might read this, if you are doing a PhD or MsC, just hang in there: if you look for help, you will find it, sometimes from unsuspected people.

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