PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE

SCHOOL OF ENGINEERING

HAZARDOUS MATERIALS TRANSPORTATION IN URBAN AREAS

ANDRÉS BRONFMAN CÁCERES

Thesis submitted to the Office of Graduate Studies in partial fulfillment of the requirements for the Degree of Doctor in Engineering Sciences

Advisor:

VLADIMIR MARIANOV KLUGE

Santiago de Chile, Octubre, 2016

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To Susana, Emilia and Gabriel

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Hazardous Materials Transportation in Urban Areas

Tesis enviada a la Dirección de Investigación y Postgrado en cumplimiento parcial de los requisitos para el grado de Doctor en Ciencias de la Ingeniería.

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RESUMEN

Grandes cantidades de materiales peligrosos (MP) son generados como materias primas o como subproducto de la actividad industrial. En la gran mayoría de las zonas urbanas que contienen industrias, estos materiales deben ser transportados transformándose en una fuente de peligro para la población frente a posibles eventos de liberación del material. Se reconoce la existencia de múltiples agentes implicados en la toma de decisiones, quienes se caracterizan por presentar un rango de prioridades y puntos de vista divergentes: los transportistas tratan de minimizar sus costos de transporte; las agencias gubernamentales intentan minimizar el riesgo impuesto a la población distribuyendo en forma equitativa este riesgo sin amenazar la viabilidad económica de la actividad; finalmente el público intentará evitar cualquier actividad peligrosa en su entorno cercano, guiado principalmente por su propia percepción respecto de las actividades que involucran MP. En zonas urbanas este problema se acrecienta producto de la alta población potencialmente expuesta a los peligros de esta actividad, principalmente de aquellos grupos poblacionales que presentan dificultades para ser evacuados. En vista de esto, es razonable prestar especial atención a esta población en situación de riesgo al momento de diseñar las rutas para el transporte de MP.

El principal objetivo de esta tesis es desarrollar una metodología que permita resolver el problema de transporte de MP en zonas urbanas de tal forma de proteger a la población

más vulnerable. Para ello, se presentan dos nuevos enfoques de enrutamiento, en los cuales la distancia es un proxy del peligro al cual están expuestos los centros, de modo que a menor distancia, mayor peligro, y se propone nuevos objetivos que consideran el peligro y el tiempo de exposición.

El primer enfoque se centra en el problema de ruteo de MP desde un origen a un destino, ambos al interior de una zona urbana, minimizando el peligro sobre el centro vulnerable más expuesto. Se propone el *maximin hazmat routing problem* (MmHRP) que, como proxy del peligro, maximiza la distancia entre la ruta y su centro vulnerable más cercano, ponderado por la población del centro. El MmHRP es formulado sobre una red de transporte como un problema de programación entera y, adicionalmente, se presenta un procedimiento de resolución óptima en tiempo polinomial.

El segundo enfoque aborda el problema de transporte de MP entre múltiples pares origen-destino. En primer lugar, se presenta el *maxisum hazmat routing problem* (MsHRP) el cual maximiza la suma de las distancias ponderadas desde todos los centros vulnerables a su punto más cercano en las rutas utilizadas. En segundo lugar, se presenta el *maximin-maxisum hazmat routing problema* (MmMsHRP), el cual combina los criterios *maximin* y *maxisum*. Mediante el criterio *maximin* se buscan soluciones eficientes en términos de proteger al centro vulnerable más afectado, pero el efecto sobre el resto de la población no se considera y en las soluciones, se genera un gran número de centros expuestos. Mediante el criterio *maxisum* se obtienen soluciones que minimizan la exposición promedio de los centros vulnerables, pero sin considerar la magnitud del peligro impuesto sobre cada centro individual. La consideración de ambos objetivos permite soluciones en las cuales ambos criterios son razonablemente tomados en cuenta. Se formula un modelo exacto para cada problema, y se propone un procedimiento heurístico que permite resolver de manera eficiente ambos problemas para instancias de gran tamaño.

Finalmente, en vez de usar la distancia como proxy, se propone un estimador de peligro general para cualquier centro poblado y se incorpora el tiempo de exposición de la

población como otro proxy de peligro. El nivel de peligro al que está expuesto un centro es una función de la distancia entre el centro poblado y cada punto de los arcos al interior de su área de peligro. Se formula y resuelve un problema de diseño de rutas que integra estos nuevos objetivos.

Todos los modelos fueron probados en un caso real para el transporte de HAZMAT sobre la red de transporte de la ciudad de Santiago de Chile.

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PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
ESCUELA DE INGENIERÍA

Hazardous Materials Transportation in Urban Areas

Thesis submitted to the Office of Research and Graduate Studies in partial fulfillment of the requirements for the Degree of Doctor in Engineering Sciences by

ANDRÉS BRONFMAN CÁCERES

ABSTRACT

Large quantities of hazardous materials (HAZMAT) are generated as raw material or by-products of industrial activity. In the great majority of urban areas that contain industries, this type of material must be transported in trucks, which becomes a source of hazard for the population. The HAZMAT transport decision-making process also includes multiple agents, each with different priorities. Freighters seek transportation cost minimization, regulators risk minimization subject to equity and economical concerns, and population hopes avoiding HAZMAT activity, guided by their own perception of hazard. In urban zones, these issues are stressed out, because of the high population density, especially of groups that cannot be easily evacuated. It is reasonable then, to take special care of these groups when designing HAZMAT transportation routes.

The main objective of this thesis is to develop a methodology to solve the HAZMAT transportation problem in urban zones, in order to protect the most vulnerable population. In the first place, we present two different approaches. These use the distance as a proxy of the danger to which vulnerable population centers are exposed, corresponding a lesser distance to a higher danger. Secondly, we propose a new objective that explicitly measures the danger, as well measuring the exposure time as an estimator of danger.

The first approach studies the HAZMAT routing problem for a single origin-destination pair in an urban zone, minimizing the highest danger to which a population center is exposed. We present the maximin hazmat routing problem (MmHRP), which maximizes the distance between the route and its closest vulnerable center, weighted by the population of the center. The MmHRP is formulated as an integer problem, and an optimal polynomial procedure is presented.

The second approach studies the HAZMAT transportation problem between multiple origin-destination pairs. First, we present the maxisum hazmat routing problem (MsHRP), which maximizes the sum of the population-weighted distances from all vulnerable centers to their closest point on the routes. Second, we present the maximin-maxisum hazmat routing problem (MmMsHRP), which combines the maximin and maxisum criteria. The maximin criteria provides solutions that are efficient in protecting the most affected vulnerable center, but it is unable to take into account the effect over the rest of the population. As a consequence, a high proportion of the vulnerable centers is exposed. The maxisum criterion, on the other hand, obtains solutions that minimize the average exposure of all population centers, but ignores the danger to which each individual center is exposed. We formulate an exact model for each problem, and we propose a heuristic procedure to solve efficiently large instances.

Finally, we explicitly consider a general danger estimator for population centers, and we propose considering the exposure time as another danger estimator. The level of hazard to which a center is exposed, is a function of the distance between the center and each point on the link within its hazard circle. We formulate and solve a mathematical programming model that considers these objectives.

All the models were tested in a real case study for HAZMAT transportation, considering Santiago of Chile transportation network.

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1.   INTRODUCTION

This thesis aims to model and solve the HAZMAT (Hazardous Material) Transportation Problem in urban zones. We develop three new techniques to solve the problem, stressing out population protection in case of HAZMAT release events during transport. The first technique focuses on the protection of population groups that are vulnerable or hard to evacuate, through the maximization of the distance between the closest vulnerable center and the designed route. We name it the maximim HAZMAT routing problem. The second technique maximizes the sum of the distances between every vulnerable center and its closest route, and it is denoted the maxisum HAZMAT routing problem. In these approaches, the distance is used as an estimator of the danger to which the vulnerable centers are exposed. In other words, the lesser the distance, the higher the danger. The third technique explicitly considers a general danger estimator for every populated center, incorporating also the population exposure time as another proxy. The resulting models were tested in a real case study of HAZMAT transport in Santiago de Chile.

1.1. The Hazardous Material Transportation Problem

A hazardous material is defined as one which can cause damage to people, property or environment [UN (2015)]. These materials can be classified in nine classes [UN (2015), NCh382 (1998)]: explosive, flammable, carburant, organic peroxides, poisonous (toxic) and infectious, corrosive, radioactive, and other dangerous substances. In most of the large urban areas, HAZMATs must be transported between different generation/attraction points using trucks. This action exposes the population to possible material release events, where both risk and danger are implied.

Despite the efforts of governments, overseers and private agents, accidents during the HAZMAT transport still occur. A HAZMAT liberation event could cause multiple consequences, such as property and environment damage, people injured, deaths,

evacuation and cleaning costs. In Santiago, Chile, data shows that during 2013, 12 HAZMAT release events occurred, affecting 2,279 people with 137 injured of different severity. It also implied the evacuation of thousands of people and high cleaning cost [INE (2015)]. This evidence suggests that both practitioners and researchers must do their best, in order to develop new methodologies for the urban HAZMAT transport. These efforts must be oriented toward protection of the population in case of HAZMAT release events. The development of new methodologies must also consider all difficulties of HAZMAT transport, such as the evaluation of risk estimators, and the existence of multiple stakeholders in the decision making process.

1.2. Multiple stakeholders

There are multiple agents implied in the HAZMAT management and transport decision making activities, with different priorities and viewpoints.

First, the carriers aim to minimize their transportation cost, while following the regulations. Each carrier must solve an independent routing problem for every shipment. This is a minimum cost route between origin and destination (OD), for a single product and type of vehicle. Minimizing transportation costs is their main goal, although they also follow the regulations, which ensures not putting population at a high risk. The literature on this topic is profuse, and it is mainly oriented to the HAZMAT routing problem for one OD pair, using deterministic models [Batta and Chiu (1988)], stochastics models [Miller-Hooks and Mahmassani (1998), Erkut and Ingolfsoon (2000), Hall (1986), Fu and Rilett (1998), and Miller-Hooks (2001)], one objective models [Erkut and Verter (1998), Erkut and Ingolfsson (2005)] and multiobjective models [Sherali et al. (1997), Marianov and ReVelle (1998)].

At the other extreme are the government agencies, who get involved in the system to minimize the public and environmental risk, but taking care not to threaten the economic viability of the activity, and establishing equitable distribution of risk. Their view must

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consider all shipments within their jurisdiction, transforming routing decisions in a multi-product and multi-pair origin-destination problem. The literature about these problems has increased in the last decades. It has been oriented toward the risk distribution [Gopalan et al. (1990b), Lindner-Dutton et al. (1991), Marianov and ReVelle (1998) and Iakovou et al. (1999)], selection of a route set for an OD pair [Akgün et al. (2000) and Dell'Olmo et al. (2005)] and the design of a HAZMAT transportation network [Kara and Verter (2004), Gzara (2013)].

Finally, there is the population, which is guided by their own perception on the activities involving HAZMAT. They aim to avoid any dangerous activity in their closeness. The public perception is usually not fully aligned with the authorities' objective of minimizing risk. A first reason is that overall risk is not the same as individual risk. Another reason is related to the concept of social amplification of risk, which states that public assessment of a risk not only depends on the magnitude, but also the subjective perceptions of the population [Kasperson *et al.* (1988)]. So, HAZMAT release events have a higher perceived risk, as compared to other kinds of events. Few articles in the literature of operations research consider the public perspective on perceived risk in the HAZMAT transport [ReVelle *et al.* (1991), Abkowitz *et al.* (1992)]. Interesting sources of knowledge that can be used in this area, are the literature on risk and danger perceived by the public from the location of undesirable and obnoxious facilities [Hung and Wang (2011), Elliott *et al.* (1999), Brody *et al.* (2004) and Lima (2004)] and the literature on natural disasters [Lindell and Perry (2000), Arlikatti *et al.* (2006), Wachinger *et al.* (2013), Miceli *et al.* (2008), Heitz *et al.* (2009) y Brilly *et al.* (2005)].

1.3.   Evaluation of Risk Estimators

The main difference between HAZMAT transport and other transportation problems is the existence of undesirable health, environmental and property effects, in case of a

HAZMAT release event during transportation. However, there is still no agreement among researchers on how to properly estimate these unwanted effects.

The literature has addressed this problem mainly through vehicle routing models that minimize some risk estimator for the population. Mostly, risk is a probability function of a HAZMAT release event during transportation, or the consequence associated to the event, or a combination of both. Nevertheless, the estimation of these factors is difficult. First, the techniques used to estimate the release event probabilities, based on historic frequency or logic diagrams (failure trees and event trees) assume that the past events can be used to predict the future [Saccomanno and Shortreed (1993)]. Even more, given that the HAZMAT release events have low probabilities, long periods of data gathering are required. During this period, the data obtained become less reliable, because the road characteristics, the transport technology and the population behavior change over the time horizon.

Focusing on consequences, these are a function of the area affected by the release material, the population, properties and environmental conditions inside this area at the time of the event, and the amount of material released. In turn, the shape and size of the impact area not only depend on the substance to be transported, but also on other factors such as weather conditions, topology, speed and wind direction. Although it is appealing, from the theoretical point of view, it is infeasible to estimate precisely the consequences of a particular event/incident, requiring strong assumptions to estimate the consequences [Erkut and Verter (1998) y Kara *et al.* (2003)].

Most of the risk estimation methodologies in HAZMAT transportation consider the computing of the exposed population. These assume that the effects of the event reach a maximum distance λ, which depends on the physical and chemical properties of the transported HAZMAT. A very popular approach, because of its simplicity, consist in summing the population exposed to HAZMAT transportation within a λ-neighborhood of the arc or semicircular exposure zone. The assumption done is that all this population is

equally exposed to the transportation danger. The resulting shape is a rectangle with semicircles in the opposed ends ("stadium"). This technique also assumes that both the event probability and the population density are uniform along every arc of the network (see examples in Figure 1-1).
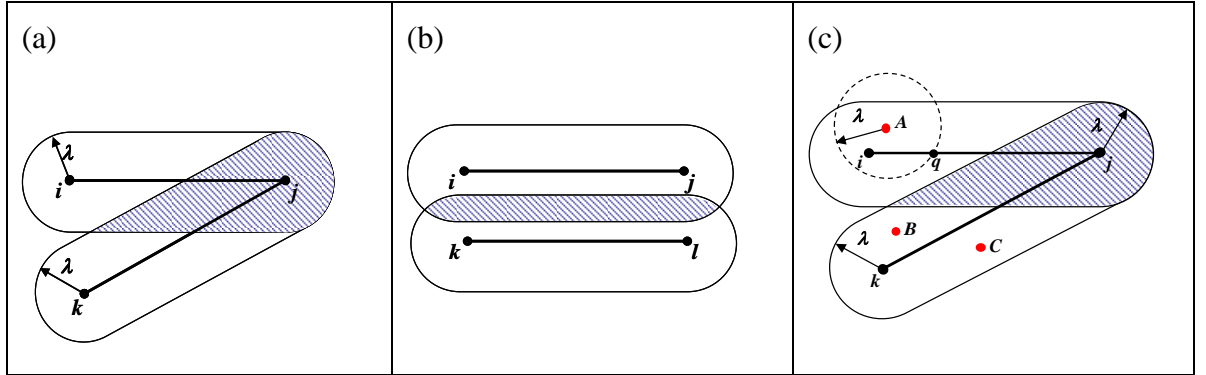
Figure 1-1: (a) Overlap of exposure zones ("stadiums") of two adjacent links in a route, with uniform population density along the link; (b) Overlap with unconnected links; (c) Overlap with non-uniform population density.

This method produces estimation errors of more than one type. Consider Figure 1-1a, which depicts a route including two links (i,j) and (j,k). Assuming uniform density, the population within the λ-neighborhood of each link represents the exposed population. However, as shown in Figure 1-1a, the population within the shaded area will be double-counted due to overlap of the links' exposure zones. Kara *et al.* (2003) compute the resulting error and propose a link-labeling shortest-route algorithm to correctly find the route that minimizes exposed population when density is uniform. A second type of overlap error, which may arise with nearby but disconnected links, is indicated by the shaded area in Figure 1-1b. There are as yet no developed correctives for this source of overestimation. If the density of the population is not uniform or it is concentrated at discrete points, as in Figure 1-1c where it is concentrated at points *A*, *B* and *C*, the methodology of Kara *et al*. cannot be applied, as it could incorrectly eliminate a non-existing population double-count. Again, the consequence can be either underestimated or

overestimated, depending on whether the population is concentrated as shown in Figure 1-1c, or in the shaded area of the same Figure 1-1c.

Erkut and Verter (1998) propose a second method of risk estimation, considering potential effects or consequence, rather than population or environment exposure. They assume that if a spill occurs during HM transport over link (i,j), the material will disperse uniformly and thus create a danger circle of radius λ. Each individual within that circle will be subject to the same undesirable effects, regardless of their distance from the incident. Both the incident probability and the population density are assumed to be uniform over the entire length of each network link. Thus, the affected population (or area) is just a proportion of the population contained in the stadium ($\pi\lambda / [\pi\lambda + 2l]$, where l is the length of the arc.) This method must be applied only when population density is indeed homogeneous, otherwise it can lead to errors. In the heterogeneous density case, it can either overestimate or underestimate the consequence. This is illustrated in Figure 1-2a, where population is concentrated in point A.
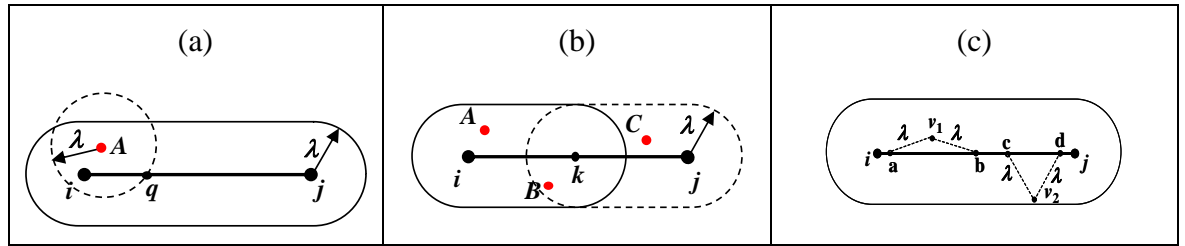
Figure 1-2: (a) Overestimation of the consequence on segment (q,j) and underestimation of consequence at population center A when population is concentrated at that point; (b) division of link (i,j) into two links (i,k) and (k,j) of equal length; (c) population centers v_1 and v_2 within the semi-circular exposure zone of link (i,j).

Because of the uniform density assumption, the danger circle methodology will spread the population of A over the whole link, and will assign a non-existent consequence (overestimation) to segment (q,j) of link (i,j), while for segment (i,q), the consequence will be underestimated. The danger circle method may also generate an error in the case where an intermediate node k is added to the link, as illustrated by the case in Figure 1-2b with

three population concentrations denoted *A*, *B* and *C*. This occurs because part of the population affected by an incident over link (i,j) is now considered again as affected by incidents on two possible links, (i,k) and (k,j), and population center *B* will be counted twice (i.e., once for each link). This error, which can be very significant, has not been solved in the literature on the danger circle method, although an alternative design similar to that of Kara *et al.* (2003) may be possible.

A third methodology [Batta and Chiu (1988)], considering two variations, lacks of some of the errors induced by the previously explained methodologies. The first variation computes the sum of the total length of arc segments that are within the λ-neighborhood of some population center, weighting them according to their amount of population. The second variation add a multiplication of every arc segment by the unitary event probability (per length unit) on this segment. The result is the risk of using the arc.

Even though this methodology produces no errors, the indicators have certain limitations. One of them is that there is no consideration of the time each population center is exposed to a danger. Although in their exposure indicator the link length can be thought of as a proxy for exposure time, since the former was defined as a link attribute rather than a population center attribute, the exposure times of a given center to different links cannot be added together. Another limitation of these indicators is that they do not take account of the distance between each population center and the link affecting it. There are three exceptions that do consider the distance between the population center and the HAZMAT route [Carotenuto *et al.* (2007), List and Mirchandani (1991) and Erkut and Verter (1995)] which are described in section 2.2.1.

To illustrate the effects of distance to the link and exposure time, consider Figure 1-2c in which two units (v_1 and v_2) of equal population are located at different distances from a route. If we assume that a hazmat vehicle travel speed and incident probability are homogeneous along the entire length of the link, a hazard along (i,j) will not only expose population center v_1 to the associated risks for a longer period of time, but also, if we

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suppose that danger is as well a function of nearness to a possible incident, the danger it exposes the center to is greater.

Common to all of the methods described so far is that consequence or risk are always the attributes of a link rather than a population center. In other words, each link is assigned the undesired effect it produces and the amount of population it affects. In transport route designs, two or more links may affect the same population center, a double impact none of these methods can account for. The only way to incorporate the effects of multiple links on the population is to use an approach that assigns the impacts not to links but rather to population centers.

In light of the foregoing observations, in this Thesis, we use a new approach that specifies danger and risk as attributes of the relationship of each population center with each link that affects it. Each population center is represented as a point in a plane around which a circular *danger zone* of radius λ is defined, as shown in Figure 1-3a. The links consist of straight line sections of a path while the population v is represented as concentrated at point k. Since the danger zone is confined to the circle of radius λ, segment (a,b) of link (i,j) is the part that exposes the population to danger and is therefore denoted the *exposure segment*.
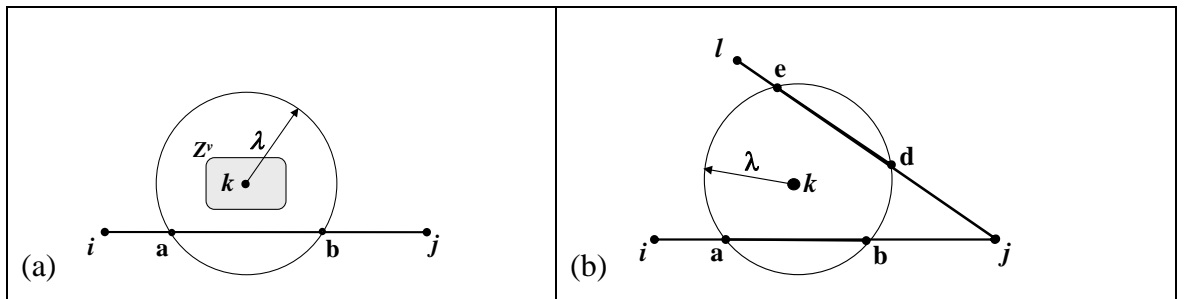
Figure 1-3: (a) Population v represented by its geometric center k, together with its circular danger zone and exposure segment (a,b); (b) Exposure segments (a,b) and (e,d) within danger zone of population center k.

Figure 1-3b depicts how a population center can be affected by more than one link, especially in urban areas. By expressing adverse effects as attributes of a center rather than

a link, we can account for the aggregate effect of all links on a given center. In cases such as the one shown in Figure 1-3b, the consequence is always the population concentrated at k no matter how many links cross its danger zone. The risk and the danger imposed on k are the respective sums of the individual risk and danger values imposed by each of the two link segments (a,b) and (e,d).

In some cases, the population v in an area Z^v is represented as concentrated at its central point (Figure 1-3a). Strictly speaking, this does not introduce significant errors since population data are usually presented as discrete figures (e.g., in census publications) and therefore already contain aggregation errors that cannot be corrected by any model. Furthermore, it is impossible to include into any model the actual location of each person. Of course, if the model uses a higher level of aggregation than the source data, errors will indeed be introduced. A review of the literature on aggregation errors may be found in Sadigh and Fallah (2009) and Francis *et al.* (2004). Methods for reducing them have been proposed by Current and Schilling (1990) and Emir-Farinas and Francis (2005). Although most of these studies focus on the problem as it arises in location modeling, the principles involved here are the same.

1.4. Minimization of the individual danger of population centers in urban zones

In an urban zone, the population is distributed across all the urban surface of a more or less continuously. It implies that it is inevitable to expose part of the population to the HAZMAT transport, independently of the selected route. The population must also be evacuated in a short enough period of time, in order to avoid the consequences of most of the HAZMAT release events. Some population groups are particularly hard to evacuate, by the high concentration of people in small areas (for example schools, large buildings and commercial centers), or because the people has difficulties to be evacuated (patients in hospitals, elders, etc.). In summary, it is reasonable to give special attention to this component of the population when designing HAZMAT transportation routes.

One approach to solve this problem is to represent every vulnerable center as a point in the plane, where all its population is concentrated. Then, it is possible to compute a circular danger zone (Figure 1-3), which is where, if an event occurs, the center would be affected. From this point of view, every route segment within this danger zone around a center would generate danger to its population. It is assumed that danger is the potential to generate an undesirable consequence, independent of the occurrence probability [Rasmussen (1991)]. It is considered that within the danger zone, the danger is a function of the distance between the vulnerable center and the route segment used for the HAZMAT transportation (*danger function*). This danger function can have multiple functional forms. We propose the inverse of the squared distance between the vulnerable center and the danger source; the inverse of an exponential function of the square Euclidian distance between the population and the danger source; and the inverse of the distance between the vulnerable center and the danger source. The function to be used depends on the material and context considered. To estimate the total danger imposed to a population center, the danger function is multiplied by the population of the center.

Any of these functions can be used to minimize the danger imposed to every vulnerable center, in order to assure that the HAZMAT activities are done as far as possible from them. In order to achieve this goal, the danger indicator can be integrated into one of the routing approaches studied in this thesis: maximin and maxisum. The maximin approach maximizes the minimum weighted distance between the HAZMAT transportation route and its closest vulnerable center. It is efficient in protecting the most affected vulnerable center, but it is not able to measure the effect on the rest of the population. As a consequence, in the obtained solution, a great number of centers could be exposed. The maxisum approach, on the other hand, maximizes the weighted sum of the distances between every vulnerable center and its closest arc of the route within its danger zone. Minimizing the total impact over the population, the maxisum approach offers routes that minimize the average exposure, but it does not take into account the maximum danger that impacts individual centers. A third approach is to combine the maximin and maxisum

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approaches. Under the maximin criteria, we can obtain routes that minimize the negative impact to the most affect center, and with the maxisum criteria, we can obtain routes that minimize the negative effect over all the vulnerable centers considered. Multiobjective modeling allows generating a set of efficient solutions that can be proposed to the decision makers to choose among.

1.5. Thesis contributions

The main contributions of this thesis are three:

First, we formulate and solve up to optimality the *maximin HAZMAT routing problem* in urban zones, over a transportation network, considering both vulnerable and hard-to-evacuate population protection. The resulting model maximizes the minimum weighted distance between the route and the closest vulnerable center. It minimizes the consequences to the most exposed part of the population.

Second, we formulate and solve efficiently the *maxisum HAZMAT routing problem* and the *maximin-maxisum HAZMAT routing problem*. The first method maximizes the weighted sum of distance between vulnerable zones and arcs used for HAZMAT transportation. The second method combines the maximin HAZMAT routing problem and the maxisum HAZMAT routing problem using a bi-criteria approach. We generate a set of non-dominated solutions that are options for the decision makers.

Third and final, we propose a methodology that incorporates the population exposure time and danger level as new objectives to be considered in the HAZMAT routing problem in urban zones.

The rest of the thesis is organized as follows. Chapter 2 contains the article "*The maximin hazmat routing problem*"[1]. Chapter 3 presents the article "*The maxisum and maximin-maxisum hazmat routing problems*", submitted to Transportation Research Part E. Chapter 4 is the paper "*Incorporation of hazard and period of exposure as objectives in HM transportation*", to be submitted. Instead of a separate conclusion chapter, every chapter ends with specific concluding remarks and future lines of research.

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[1] Bronfman, A., V. Marianov, G. Paredes-Belmar and A. Lüer-Villagra (2015). "The maximin HAZMAT routing problem." European Journal of Operational Research 241(1): 15-27.

2. THE MAXIMIN HAZMAT ROUTING PROBLEM

The hazardous material routing problem from an origin to a destination in an urban area is addressed. We maximize the distance between the route and its closest vulnerable center, weighted by the center's population. A vulnerable center is a school, hospital, senior citizens' residence or the like, concentrating a high population or one that is particularly vulnerable or difficult to evacuate in a short time. The potential consequences on the most exposed center are thus minimized. Though previously studied in a continuous space, the problem is formulated here over a transport (road) network. We present an exact model for the problem, in which we manage to significantly reduce the required variables, as well as an optimal polynomial time algorithm. The integer programming formulation and the algorithm are tested in a real-world case study set in the transport network in the city of Santiago, Chile.

2.1. Introduction

The hazardous material (HAZMAT) routing problem has been extensively studied in recent decades. For the most part it has been treated as a least cost routing problem between an origin and a destination, in which cost is a combination of transport costs and a risk function.

Although there is no consensus on the best way to model risk, it is generally agreed that any formulation will include two elements: the probability of an accidental HAZMAT release, and its associated consequences. Very few of the risk measures take into account the distance between a population center and the HAZMAT route except in order to define distance thresholds within which the consequence or risk is total and beyond which it is non-existent. In reality, however, the closer a HAZMAT vehicle passes to a population center within such a threshold, the greater is the center's exposure to hazard (where hazard is understood as the potential for producing an undesired consequence without regard to the probability of its occurrence). This observation also fits with public perceptions,

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strongly suggesting that the distance between routes and populated centers warrants greater attention in HAZMAT route modelling, since it is a good proxy for hazard.

Our contribution is oriented to propose an approach oriented to the protection of vulnerable centers, together with a new model and an optimal algorithm for HAZMAT transportation in urban areas. The approach assumes that population in residential or low-rise commercial areas is easier to evacuate, but there are vulnerable centers concentrating high populations of children, senior citizens or ill people, for which is difficult to evacuate or can slowly do so. These vulnerable centers are represented as points on a plane. We incorporate in a new model, the distance between a vulnerable center and a HAZMAT transport route. A maximin objective is used here that, to the best of our knowledge, has not previously been used in the HAZMAT or routing literature in a network context. This objective maximizes the minimum Euclidean distance between the route and the nearest vulnerable center, the distance being weighted by the center's population. We remark that any other distance and hazard measure could be trivially used, as long as it is non-increasing with distance. Since we explicitly assume that hazard is an attribute of each vulnerable center and depends on the center's distance from a HAZMAT route, by using this maximin approach we minimize the hazard facing the center closest to the route (the most exposed center). By using this approach, we obviate the need to set risk or risk difference thresholds, or to compute probabilities. Moreover, the formulation we develop designs a route instead of choosing one from a set.

Maximal values of risk or hazard have seldom being minimized in the HAZMAT literature. In general, the average or total risk or hazard has been the objective to be minimized. Exceptions are some works that locate either straight or broken lines in a plane. However, HAZMAT transport in practice takes place over a network. Modelling the routing problem in a network context, with an integer programming formulation, requires the application of constraints that will relate each population center to its closest link in the route (i.e., the link imposing the greatest hazard). This type of constraints is used in

discrete location problems, e.g., assigning customers to the closest plant of a multi-plant firm.

The new exact model we formulate here (our first solution method) potentially requires a large number of closest assignment constraints. Normally, this would also mean a large number of decision variables, significantly complicating the solution of the problem. Since the route is not known a priori, a variable associating each vulnerable center with each network link has to be added, implying a total of $O(mq)$ variables where m is the number of network links and q the number of vulnerable centers. In our exact model, however, each vulnerable center needs only a subset of these variables identified by the sections of route within the center's danger area. The result is a major reduction in the required number of variables and constraints. Although the problem can be solved using an optimal algorithm also presented here, we offer both procedures, as the variable reducing technique could be applied to harder problems.

Our second approach corresponds to an optimal algorithm, which solves the problem in polynomial time and can be used easily for large real instances.

The remainder of this article is organized into four sections. In Section 2.2, we offer a Literature review. Section 2.3 formulates the maximin problem for hazardous materials routing as an exact formulation and includes an optimal algorithm; Section 2.4 describes a practical application of the proposed methodology and analyses the results; and finally, Section 2.5 presents our conclusions and some possibilities for future research.

2.2.   Literature review

2.2.1. Risk and distance dependent danger

Erkut *et al.* (2007) have identified nine different risk estimators: the exposed population ReVelle *et al.* (1991); the probability of an accident [Saccomanno and Chan (1985), Abkowitz *et al.* (1992) and Marianov and ReVelle (1998)]; the expected consequence,

defined as the product of the probability of an accident and its associated consequences [Pijawka *et al.* (1985), Batta and Chiu (1988), Alp (1995) and Erkut and Verter (1995)]; the expected consequence given that an accident has occurred along the route [Sivakumar *et al.* (1993), Sivakumar *et al.* (1995) and Sherali *et al.* (1997)]; risk aversion, the perceived risk along a link being measured as pC^q, where p is the probability of an accident on the link, C is the consequence of an accident and q is a risk preference parameter [Abkowitz *et al.* (1992)]; a demand satisfaction model proposed by Erkut and Ingolfsson (2005) in which an accident terminates a trip, necessitating a new shipment to fulfil demand; the maximum exposed population [Erkut and Ingolfsson (2000)]; simultaneous consideration of the expected value and variance of the number of people affected by an accident [Erkut and Ingolfsson (2000)]; and expected disutility, using a disutility function of the form $u(X) = \exp(\alpha X)$ where X is the affected population and $\alpha > 0$ a constant measuring catastrophe aversion [Erkut and Ingolfsson (2000)].

In addition to these nine estimators, Jin and Batta (1997) propose six ways of modelling risk based on expected consequence in terms of the number of HAZMAT shipments or trips S to be made and the threshold number of accidents Q. In these formulations, shipments cease once a number Q of accidents have occurred or when S trips have been made, whichever comes first. The shipments are considered as a sequence of independent Bernoulli trials and a trip terminates if either an accident occurs or the destination is reached.

Some authors incorporate equity into the spatial distribution of risk in HAZMAT routing. For example, Zografos and Davis (1989) include the concept indirectly by placing flow capacity constraints on the various links in the transport network. Marianov and ReVelle (1998) propose stipulating an upper bound on the total risk associated with each link. Gopalan *et al.* (1990a) consider a route defined by an origin-destination pair to be equitable if the difference between the risk levels imposed on any pair of zones in the neighborhood of the route stays below a present threshold. They calculate the risk

associated with a link in the route as the sum of the risks imposed on the various zones in the link's neighborhood, an approach that could double-count part of the population. The same authors extend their model in Gopalan *et al.* (1990b) to identify a set of routes for making T trips between a single origin-destination pair. They minimize the total risk over the T trips while maintaining the difference in total risk between every zone pair under a certain equity threshold, the latter given for any pair by the differences in risk summed over the T trips. Lindner-Dutton *et al.* (1991) take this model further, focusing on the search for an equitable sequence for the T trips. They minimize the sum of the maximum differences in risk between any zone pair accumulated over t trips ($t = 1,...,T$). Other approaches to the equitable risk distribution for a set of trips between a given origin-destination pair may be found in Dell'Olmo *et al.* (2005) and Caramia *et al.* (2010).

All of the above-mentioned works use subjective risk thresholds without defining any standard. Moreover, they all consider risk as an attribute of the route links rather than the population centers along it. This approach, if not used carefully, could lead to under-or overestimation of both risk itself and the differences in risk between population centers.

Risk measures do not incorporate distance. However, hazard, defined by Rasmussen (1981) as the potential for producing an undesired consequence without regard to the probability of its occurrence does depend on distance. The phenomenon is acknowledged by Saccomanno and Shortreed (1993), Jonkman *et al.* (2003), Fernández. *et al.* (2000) and Karkazis and Boffey (1995), who note that distance should be a factor to incorporate in models dealing with HAZMAT transportation.

Three studies which do incorporate distance into their formulations are Erkut and Verter (1995), Carotenuto *et al.* (2007) and List and Mirchandani (1991). In the first one, two models are proposed. The first model assumes population concentrated at points on a plane, while the second treats population centers as two-dimensional objects. Both models use probabilities (of an accident, of an incident given an accident, and probability of a material release) that are difficult to estimate. In the paper, the models are used to choose

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among several routes. In Carotenuto *et al.* (2007), assuming the population is located on the transport network links (populated links), the authors calculate, for each unit-length segment x of a link, the risk imposed by its use for HAZMAT transport on each populated segment y in the network. The calculation is made only within a threshold distance measured from the center of segment x. For each populated segment y within that distance, the authors multiply the population along that segment by a function that decreases exponentially with the distance between the two segments and the probability of an incident on segment x. The sum of the risks imposed by the use of each segment x of a link gives the total risk associated with that link, and by the same token, the total risk imposed by a route is the sum of the risks imposed by each of its constituent links. Heuristic procedures are applied to generate a set of alternative routes between an origin–destination pair and the total risk is then minimized, with a preset upper limit on the total risk over the populated links. Note, however, that this methodology could generate a risk overestimation whose magnitude increases with the density of the network.

Finally, List and Mirchandani (1991) also take into account the distance from a HAZMAT route to a population. They calculate, for each population center, the integral over the entire route of a function of the distance between a point representing that center and each point on the route, said distance being weighted by the size of the population center and the probability of an incident at the route point. The authors define the risk associated with each route and each population point as a function of the integral, but do not propose any specific functional form. The total risk imposed by a route is the sum of the individual risks it imposes on each population centers. However, in their case study List and Mirchandani (1991) recognize the complexities of their method and instead of the function as just described they employ the expected fatalities. Furthermore, as with Carotenuto *et al.* (2007) their formulation requires that the candidate routes be explicitly enumerated.

Following List and Mirchandani (1991) and Bronfman and Marianov (2013) propose two new objectives for HAZMAT transport: the hazard or danger a population center located near a route is exposed to (danger exposure) and the length of time it is exposed (time exposure). The level of danger is a function of the distance between a population center and each point on the route segment in its danger area. But neither pair of authors consider possible differences in the spatial distribution of the danger or total imposed risk even though the perception of risk inequity often leads to public opposition to HAZMAT transport along nearby routes Erkut *et al.* (2007).

We remark that there are at least two approaches to risk mitigation. In the first one, which most of the literature follows, routes are prescribed by authorities, while in the second approach, rules are defined that the vehicles carrying HAZMAT must respect. Using this indirect approach, Kara and Verter (2004), Erkut and Gzara (2008) and Amaldi *et al.* (2011) formulate the hazmat network design problem, as a bi-level mathematical programming problem that explicitly captures the leader–follower nature of the relationship between authorities and transportation operators. In the first level, authorities restrict the routes to a subset of the network arcs, which is found by minimizing a risk function. On a second level, operators find cost-minimizing routes over the restricted network. Erkut and Alp (2007) solve the hazmat network design problem for a large urban center. The problem is formulated as a selection of minimum risk Steiner trees. Verter and Kara (2008) approach the same problem by generating a set of feasible routes for HAZMAT transportation. Each route represents a point on the risk-cost trade-off boundary. Marcotte *et al.* (2009) propose the use of tolls to redirect the routes through less populated areas. Again, a bi-level problem is formulated, which can be reduced to a mixed-integer single-level problem. Bianco *et al.* (2009) minimize both risk and risk equity, assuming that authorities limit the amount of HAZMAT traffic over network arcs. A different approach was adopted by Bruglieri *et al.* (2014), who propose to reroute HAZMAT through compulsory control points or gateways. The problem consists of

locating a fixed number of gateways and their assignment to trucks, so that the total risk of the minimum cost route of each truck through gateways is minimized.

2.2.2. Maximin in the plane

Maximin objectives have been considered in previous works, although mostly over a continuous space (in a plane). For example, Drezner and Wesolowsky (1989), determine an obnoxious route through a restraining polygonal region containing a set of points such that the minimum weighted distance between the route and the points is maximized. They also locate a straight-line route that maximizes the minimum weighted distance to the points in a region circumscribed by the points' convex hull. Díaz-Báñez *et al.* (2005) consider both the route cost and the minimization of risk in a continuous space. Their approach finds a route between an origin–destination pair whose length does not exceed a preset value (the route cost objective) while maximizing the minimum distance to a set of population centers or points (the minimum risk objective). Barcia *et al.* (2003) solve the fixed-length obnoxious anchored segment location problem by maximizing the minimum Euclidean distance between *n* points and a route segment with one fixed endpoint. Díaz-Báñez and Hurtado (2006) locate an obnoxious route made up of two links joined at a corner with fixed endpoints at the route origin and destination. Also, for a given set of points and a positive value l_0, the authors calculate an obnoxious 1-corner polygonal chain of maximum length l_0 that maximizes the minimum distance to the points. Díaz-Báñez *et al.* (2007) assume demand is concentrated in polygonal regions instead of at points and then solve the maximin line problem, that is, the problem of locating the straight line that maximizes the minimum weighted distance between the set of polygons and the line. They also address the extension of the problem to three dimensions (Díaz-Báñez *et al.* (2006), Díaz-Báñez *et al.* (2004), Díaz-Báñez *et al.* (2002)), as do Follert (1995) and Follert *et al.* (1997).

2.2.3. Closest assignment constraints

Closest assignment constraints have been used in many location problems (see Gerrard and Church (1996) and Espejo *et al.* (2012)), including the ordered capacitated facility location problem [Kalcsics *et al.* (2010)], *r*-interdiction median problems [Church *et al.* (2004) and Liberatore *et al.* (2011)], the budget constrained median problem [Rojeski and ReVelle (1970)], the competitive location problem [Dobson and Karmarkar (1987)], the plant location problem with order [Cánovas *et al.* (2007)], the obnoxious *p*-median problem [Belotti *et al.* (2007)], the location with equitable load problem [Berman *et al.* (2009)], models for locating regional energy facilities [Church and Cohon (1976)] and location models for maximizing social welfare or consumer surplus [Wagner and Falkson (1975)]. Further applications of closest assignment constraints may be found in Plastria (2002), Hanjoul and Peeters (1987) and Marín (2011), and finally in Lei and Church (2011), who discuss their use in the context of multi-level assignments. We have not found literature on the use of these constraints in routing.

2.3. Formulation of the problem

2.3.1. The base model

Let a transport network be defined by a directed graph $G(N, A)$, where $N = \{1, \ldots, n\}$ is the set of nodes and $A = \{1, \ldots, m\}$ the set of links. There is a cost c_{ij} associated with the use of link $(i,j) \in A$ and a set of vulnerable zones or centers, each of which is represented by a point $p \in P = \{1, \ldots, q\}$ in the plane at which the population D^p is assumed to be concentrated. A HAZMAT load is to be transported between a known origin-destination pair *O-D* along a route that maximizes its minimum distance to the vulnerable centers, weighted by the reciprocal of each center's population. Note that, depending on what type of HAZMAT is being considered, any estimator of danger or risk could be used instead, as long as it is non-decreasing with distance, without changing the structure of the model. In this case, by using the reciprocal of the population, the most populated centers become

more important to protect. Other estimators are the distance itself; functions of the distance and the risk, and so on. Let d_{ij}^p be the Euclidean distance between vulnerable center p and its closest point on link $(i, j) \in A$.

The decision variables are:

$$x_{ij} = \begin{cases} 1 & \text{if link } (i,j) \in A \text{ is used for HAZMAT transport} \\ 0 & \text{if not} \end{cases}$$

$$z_{ij}^p = \begin{cases} 1 & \text{if link } (i,j) \in A, \text{ used for HAZMAT transport, is the closest link to } p \in P \\ 0 & \text{if not} \end{cases}$$

w: minimum weighted distance from the route to a vulnerable center at $p \in P$.

The basic model of the problem, M_1, is formulated as follows:

$M_1)$ Max w $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (2.1)

$s.t.$

$$w \leq \sum_{(i,j) \in A} \left(\frac{1}{D^p} \right) d_{ij}^p z_{ij}^p \qquad\qquad\qquad \forall p \in P \qquad\qquad (2.2)$$

$$\sum_{(i,j) \in A} z_{ij}^p \leq 1 \qquad\qquad\qquad \forall p \in P \qquad\qquad (2.3)$$

$$z_{ij}^p \geq x_{ij} - \sum_{\left\{ (k,l) \in A / d_{kl}^p \leq d_{ij}^p \right\}} z_{kl}^p \qquad\qquad \forall p \in P, \forall (i, j) \in A \qquad (2.4)$$

$$z_{ij}^p \leq x_{ij} \qquad\qquad\qquad \forall p \in P, \forall (i, j) \in A \qquad (2.5)$$

$$\sum_{\{j/(i,j) \in A\}} x_{ij} - \sum_{\{j/(j,i) \in A\}} x_{ji} = \begin{cases} 1 & \text{if} & i = O \\ -1 & \text{if} & i = D \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in N \qquad (2.6)$$

$$w \geq 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2.7)$$

$$x_{ij} \in \{0,1\} \qquad\qquad\qquad \forall (i,j) \in A \qquad\qquad (2.8)$$

$$z_{ij}^{p} \in \{0,1\} \qquad\qquad\qquad \forall p \in P, \forall (i,j) \in A \qquad (2.9)$$

The objective (2.1) maximizes the minimum weighted distance w between a vulnerable center and its assigned link. Constraint set (2.2) equates w to the shortest weighted distance between the route and any vulnerable center $p \in P$. Constraint set (2.3) ensures that only one link on the route is assigned to any vulnerable center, constraint set (2.4) imposes that this single assigned link is the closest one and constraint set (2.5) prevents any vulnerable center from being assigned to a link that is not part of the route. Finally, the network flow conservation conditions are given by (2.6) and the nature of the variables is defined by (2.7), (2.8) and (2.9).

The above formulation contains ($mq + n + 1$) decision variables and ($2q + 2mq + n + 1$) constraints, requiring large amounts of computational memory and execution time to solve even small instances of the problem. We therefore propose the following reformulation, which significantly reduces the number of required variables and constraints.

2.3.2. Reduction of required variables and constraints

Let η be the threshold (maximum) distance from the HAZMAT route to a vulnerable center $p \in P$ beyond which the effects on the center of a HAZMAT release event occurring on the route are insignificant. Since p is a point, η is also the radius of a circle bounding the area within which the center is exposed to danger. This implies that variables and constraints relating a center to links at distances greater than η can be eliminated. Decision variables z_{ij}^{p} are therefore defined only for combinations center p-link (i,j) such that $d_{ij}^{p} \leq \eta$. In other words, for a center located at p, $z_{ij}^{p} = 0$ for all links $(i,j) \in A / d_{ij}^{p} > \eta$. Also, for each center p a variable z_{η}^{p} is defined that is equal to 1 if the distance to the closest link is greater than η, and 0 otherwise. This variable replaces all of the variables

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assigning p to links $(i, j) \in A / d_{ij}^p > \eta$. If, in the solution of the problem, z_η^p is equal to 1, the corresponding center p suffers no consequences of an event along the route.

The model with the number of variables thus reduced, which we name M_2, is formulated as follows:

$$M_2) \quad Max \quad w \tag{2.10}$$

s.t.

$$w \le \sum_{\{(i,j) \in A | d_{ij}^p \le \eta\}} \left(\frac{1}{D^p} \right) d_{ij}^p z_{ij}^p + \frac{\eta}{D^\eta} z_\eta^p \qquad \forall p \in P \tag{2.11}$$

$$\sum_{\{(i,j) \in A | d_{ij}^p \le \eta\}} z_{ij}^p + z_\eta^p \le 1 \qquad \forall p \in P \tag{2.12}$$

$$z_{ij}^p \ge x_{ij} - \sum_{\{(k,l) \in A | d_{kl}^p \le d_{ij}^p\}} z_{kl}^p \qquad \forall p \in P, \forall (i, j) \in A \mid d_{ij}^p \le \eta \tag{2.13}$$

$$z_{ij}^p \le x_{ij} \qquad \forall p \in P, \forall (i, j) \in A \mid d_{ij}^p \le \eta \tag{2.14}$$

$$z_\eta^p \le \sum_{\{(i,j) \in A | d_{ij}^p > \eta\}} x_{ij} \qquad \forall p \in P \tag{2.15}$$

$$z_\eta^p \ge 1 - \sum_{\{(i,j) \in A | d_{ij}^p \le \eta\}} z_{ij}^p \qquad \forall p \in P \tag{2.16}$$

$$\sum_{\{j/(i,j) \in A\}} x_{ij} - \sum_{\{j/(j,i) \in A\}} x_{ji} = \begin{cases} 1 & \text{if} \quad i = O \\ -1 & \text{if} \quad i = D \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in N \tag{2.17}$$

$$w \ge 0 \tag{2.18}$$

$$x_{ij}, z_{ij}^p, z_\eta^p \in \{0,1\} \qquad \forall p \in P, \forall (i, j) \in A \tag{2.19}$$

Constraint set (2.11) is equivalent to (2.2) in M_1 but contains an additional term for the case where $\mathfrak{R}$, the route used, has no link sections within the danger area of a given center. This term consists of the variable z_η^p weighted by the distance η and divided by a factor

D^η such that $w = \eta/D^\eta$ only if no center has a route link within a distance η. For this to be so, it must be true that $\eta/D^\eta \geq \max\left\{d_{ij}^p\right\}\Big/\min_{p \in P}\{D^p\} = \eta\Big/\min_{p \in P}\{D^p\}$. If these conditions are satisfied and $w = \eta/D^\eta$ in the solution of the problem, then no center is located within distance η of the route and none will suffer any consequences of a HAZMAT event along it.

Constraint set (2.12) imposes that each vulnerable center p is assigned no more than one link, which will be either the closest one if $\exists (i, j) \in \Re \mid d_{ij}^p \leq \eta$, or a link representing routes beyond distance η. Constraint set (2.13) ensures that if there is more than one link $(i, j) \in \Re \mid d_{ij}^p \leq \eta$, the link assigned to p will be the closest one.

Constraints sets (2.14) and (2.15) prevent an inactive link from being assigned to a center within or beyond the threshold distance, respectively. Constraint set (2.16) stipulates that if no $(i, j) \in \Re \mid d_{ij}^p \leq \eta$ exists, then the weighted distance η/D^η must be assigned to center p. (Note here that although constraint sets (2.12) and (2.16) could be formulated as a single set with an equal sign, they have been expressed separately for the sake of clarity.) Finally, constraint set (2.17) states the network flow conservation conditions, and constraints (2.18) and (2.19) define the nature of the decision variables.

Since the goal of the model is to find routes that stay clear of vulnerable areas whenever it is possible, it is reasonable to think that the better the value of the objective, the longer are the route and the distance travelled by a vehicle. This is more so as the threshold distance η (and therefore the center danger area) increases. Consequently, the maximin objective is attractive to the resident population and the regulatory authorities, but not for the carrier attempting to minimize operating costs and therefore route length.

The exact model can be easily modified to include the minimization of the carrier's operating costs, in a bi-objective model. Using the same notation and definitions presented above, the bi-criteria HAZMAT routing problem is set out below as model M_3:

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$$M_3) \quad Min \quad \left\{ f_1 = w, \ f_2 = \sum_{(i,j) \in A} c_{ij} x_{ij} \right\} \tag{2.20}$$

$s.t. \qquad (2.11)\text{-}(2.19)$

To generate the set of efficient solutions, the two objectives i are each multiplied by a non-negative weighting factor θ_i and then summed in a single objective function. By parametrically varying the weighting factor vector the efficient set can be generated, or at least approximated [Cohon (1978)]. A possible formulation that normalizes the values of the objectives is the following parametric linear program, denoted M_4:

$$M_4) \quad Max \quad \sum_{i=1}^{2} \theta_i \left[\frac{I_i - f_i}{I_i - AI_i} \right] \tag{2.21}$$

$s.t. \ (2.11)\text{-}(2.19)$

$$f_1 = w \tag{2.22}$$

$$f_2 = -\sum_{(i,j) \in A} c_{ij} x_{ij} \tag{2.23}$$

The normalization is performed by subtracting the objective i from its best achievable value I_i and then dividing the result by $(I_i - AI_i)$, where AI_i is the worst value the objective can attain. The values are thus transformed into percentage deviations, thereby sidestepping problems of dimensional homogeneity and solutions biased towards the objective with the highest absolute value.

This bi-criteria approach would be particularly useful in practical HAZMAT routing decisions. Instead of selecting just one of the objectives, a decision-maker could apply the bi-criteria approach to generate a set of efficient alternatives and then choose a solution based on the interrelationship between the two criteria and their relationships with other attributes such as the value of the distance threshold η.

2.3.3. An optimal algorithm

Let $\delta_{ij}^p = d_{ij}^p / D^p$ be the weighted distance between an arc $(i, j) \in A$ and a node $p \in P$. This weighted distance is defined only for arcs $(i, j) \in A \mid d_{ij}^p \leq \eta$, since beyond η the consequences of an accident for the population at p are negligible. The minimum weighted distance δ_{ij} between any arc $(i, j) \in A$ and any center $p \in P$ is $\delta_{ij} = \underset{\forall p \in P}{Min} \{\delta_{ij}^p\}$. Given a route R_k between an *O-D* pair over the network $G(N,A)$, the minimum weighted distance δ_{R_k} between the route R_k and any point $p \in P$ is $\delta_{R_k} = \underset{\{\forall (i,j) \in R_k\}}{Min} \{\delta_{ij}\}$.

Maximin Obnoxious Route algorithm

Step 0: *Initialisation*

a. $k = 0$. η given.
b. $\forall p \in P, \forall (i, j) \in A \mid d_{ij}^p \leq \eta$, compute δ_{ij}^p. $\forall p \in P, \forall (i, j) \in A \mid d_{ij}^p > \eta$, set $\delta_{ij}^p = \infty$.
c. For all $(i, j) \in A$, compute δ_{ij}.

Step 1: *First route*

a. Find a route R_0 connecting the pair *O-D*, e.g., using a shortest path heuristic.
  i.   If there is no possible route, the graph is disconnected and the problem is unfeasible. **Stop**.
  ii. If there is a route R_0, compute δ_{R_0}.
  - If $\delta_{R_0} = \infty$, R_0 is an optimal solution for the given η, and $w = \infty$. **Stop**.
  - If $\delta_{R_0} < \infty$, continue to Step 2.

Step 2: *Improving the route*

a. Compute $A_k = A - \{(i, j) \in A \mid \delta_{ij} \leq \delta_{R_k}\}$. If $k = 0$, $A_{k-1} = A$. $G_k(N, A_k)$ is the graph containing all nodes, and arcs in A_k.
b. $k = k + 1$.
c. Find a route R_k over $G_{k-1}(N, A_{k-1})$ connecting the pair *O-D*.
  i.   If there is no possible route R_k, the graph is disconnected. R_{k-1} is an optimal solution for the given η, and $w = \delta_{R_{k-1}}$. **Stop**.
  ii. If there is a route R_k, compute δ_{R_k}. Repeat Step 2.

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The algorithm is optimal by construction. It suffices to prove that it converges, and that it converges to the solution in a finite number of steps. Convergence is proved noting that at each iteration, all arcs (i,j) are eliminated such that $\delta_{ij} \leq \delta_{R_k}$, which makes δ_{R_k} increasing with k. Since δ_{R_k} is bounded by the maximum distance between an arc and a center on the network, convergence follows. The algorithm is finite, because the number of arcs to be eliminated is finite, and arcs must be eliminated at each step. It converges to the optimum, because it either finds a route with $w = \infty$, or there is no best possible solution than the last route found right before the problem becomes unfeasible. It is polynomial, because at every iteration a shortest path algorithm is used, which is polynomial, and at most, the number of iterations is equal to the number of arcs, m.

Note that, if a shortest path algorithm is used within the algorithm to find the routes, not only the optimal solution to the minimax problem is found: the algorithm minimizes, as a secondary objective, the length of the route. Furthermore, the heuristic finds the nondominated solutions to the bi-objective problem, for a given η.

2.4. A practical application

We applied the proposed formulations to two real-world instances, both considering the road network of the city of Santiago, Chile. This city has an area of 641 sq km (247.6 sq mi). Model M_2 and the algorithm were applied first to an instance involving the entire urban area of the city. A smaller instance, covering a relatively large section of the city, was used for showing how the bi-objective model M_4 can be used to obtain compromise solutions, between public and government objectives, and those of carriers.

2.4.1. Model M_2 and algorithm

We solved the single-objective M_2 model, and compared run times with the algorithm. The model was slightly modified by adding the term $\varphi \sum_{(i,j) \in A} l_{ij} x_{ij}$ to the objective, where l_{ij} is the length of link (i,j), and φ is a constant sufficiently small so as to guarantee that the added term is a secondary objective, i.e., its minimization will not worsen the optimal value of w in the solution. This secondary objective plays two roles: it helps the model choosing the shortest route among all the available routes with the same value of w and, at the same time, avoids arc variables x_{ij} not corresponding to links of the route, unnecessarily taking the value 1 in the solution.

Both solution methods were applied to the city of Santiago, shown in Figure 2-1, which is crisscrossed by a road system we call Transport Network 1 containing 6,681 directed links and 2,212 nodes. In this area there are 244 vulnerable centers (schools with over one thousand students) with a total of 386,254 people (students). Each school has from 1,070 to nearly 4,500 students. HAZMAT shipments are to be transported between three O-D pairs located in industrial zones and shown in the Figure as O1-D1, O2-D2 and O3-D3.
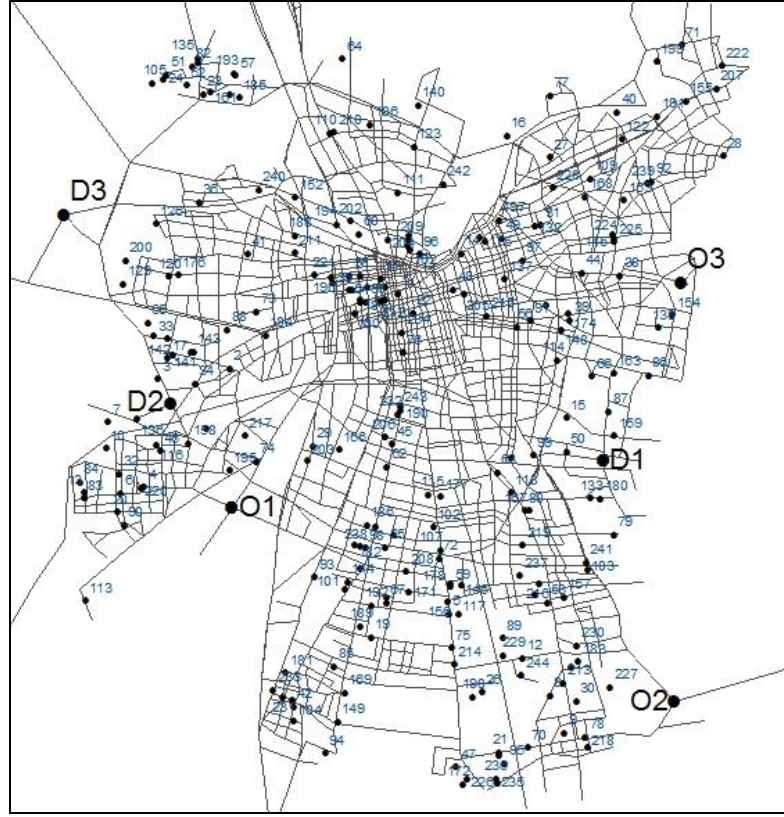
Figure 2-1: Transport network 1 and 244 schools with over a thousand students (vulnerable centers).

Tables 2-1 and 2-2 illustrate the reduction in the number of constraints and variables, respectively, for Transport Network 1, by using the formulation M_2 instead of M_1. The figures are shown for three different values of η. As the Tables show, formulation M_1 requires 1,636,846 such variables and 3,262,289 constraints while M_2 with $\eta = 1,000$ needs only 19,698 and 28,533 constraints respectively. This significant reduction is maintained for all values of η.

The instance was solved on a personal computer running Ubuntu 12.04 LTS with a 3.40 GHz Intel ® Core™ i7-2600 processor and 16 GB of RAM. The implementation was developed in AMPL Cplex 12.5.

Model M_1		Model M_2			
Constraint set	**Number of constraints**	**Constraint set**	**Number of constraints**		
			$\eta=100$	$\eta=500$	$\eta=1000$
2	244	11	244	244	244
3	244	12	244	244	244
4	1,630,164	13	505	3,823	12,772
5	1,630,164	14	505	3,823	12,772
-		15	244	244	244
-		16	244	244	244
6	2,012	17	2,012	2,012	2,012
7	1	18	1	1	1
Total	**3,262,829**	**Total**	**3,999**	**10,635**	**28,533**

Table 2-1: Number of constraints for models M_1 and M_2

Model M_1		Model M_2			
Variable	**Number of variables**	**Variable**	**Number of variables**		
			$\eta=100$	$\eta=500$	$\eta=1000$
x_{ij}	6,681	x_{ij}	6,681	6,681	6,681
z_{ij}^p	1,630,164	z_{ij}^p	505	3,823	12,772
z_η^p	-	z_η^p	244	244	244
w	1	w	1	1	1
Total	**1,636,846**	**Total**	**7,431**	**10,749**	**19,698**

Table 2-2: Number of variables in M_1 and M_2.

The results obtained by M_2 and the algorithm, for different values of η and different O-D pairs are identical, except for the CPU times, and they are set forth in Tables 2-3 to 2-5. The first column shows the value of η. The second and third columns display the value of w and the route length, respectively, obtained by both methods. An entry "-" in the second column indicates that the route does not expose any vulnerable center, i.e., stays out of the danger areas of all points. The fourth and fifth columns show the CPU times for each method. The algorithm never exceeded 1.1 seconds, while the exact model required 374.92 seconds (6.2 minutes) in the worst case. Note that for small values of η, as this parameter increases, the route becomes longer, so that it stays outside of all danger areas (this happens for $\eta = 100$ to 400 for pair O1-D1, $\eta = 100$ to 300 for pair O2-D2 and $\eta = 100$ to 400 for pair O3-D3), until it is not possible to stay clear of the centers.

η (m)	w (m/person)	Route length (km)	CPU time, M_2	CPU time, algorithm
100	-	16.9	9.38	1.09
200	-	17.0	11.37	1.09
300	-	17.2	12.68	1.07
400	-	18.3	14.11	1.08
500	0.39	27.7	26.30	1.06
600	0.35	26.2	42.88	1.04
700	0.35	26.2	138.78	1.01
800	0.35	26.2	127.88	0.99
900	0.35	26.2	218.17	0.97
1,000	0.35	26.2	231.23	0.96

Table 2-3: Values obtained for pair O1-D1 and different values of η. Transport Network 1, model M_2 and algorithm.

η (m)	w (m/ person)	Route length (km)	CPU time, M_2	CPU time, algorithm
100	-	27.3	10.64	1.10
200	-	27.8	12.24	1.10
300	-	31.0	13.74	1.09
400	0.25	31.1	20.67	1.08
500	0.25	31.1	33.70	1.07
600	0.25	31.1	63.34	1.04
700	0.25	31.1	94.92	1.01
800	0.25	31.1	227.06	1.00
900	0.25	31.1	166.55	0.97
1,000	0.25	31.1	374.92	0.96

Table 2-4: Values obtained for pair O2-D2 and different values of η. Transport Network 1, model M_2 and algorithm.

η (m)	w (m/ person)	Route length (km)	CPU time, M_2	CPU time, algorithm
100	-	27.44	8.37	1.09
200	-	28.29	10.32	1.08
300	-	28.86	10.91	1.07
400	-	30.69	12.42	1.08
500	-	33.05	12.94	1.07
600	0.52	40.99	18.90	1.07
700	0.45	43.47	36.95	1.06
800	0.44	44.68	63.12	1.04
900	0.44	78.17	95.47	1.02
1,000	0.44	78.17	83.05	1.00

Table 2-5: Values obtained for pair O3-D3 and different values of η. Transport Network 1, model M_2 and algorithm.

Figure 2-2 to 2-4 show the paths for each one of the O-D pairs for selected values of η. In the figure, the circles represent the danger area of each vulnerable center, and the grey circles represent the danger areas exposed by the selected path.

(a) $\eta = 400$
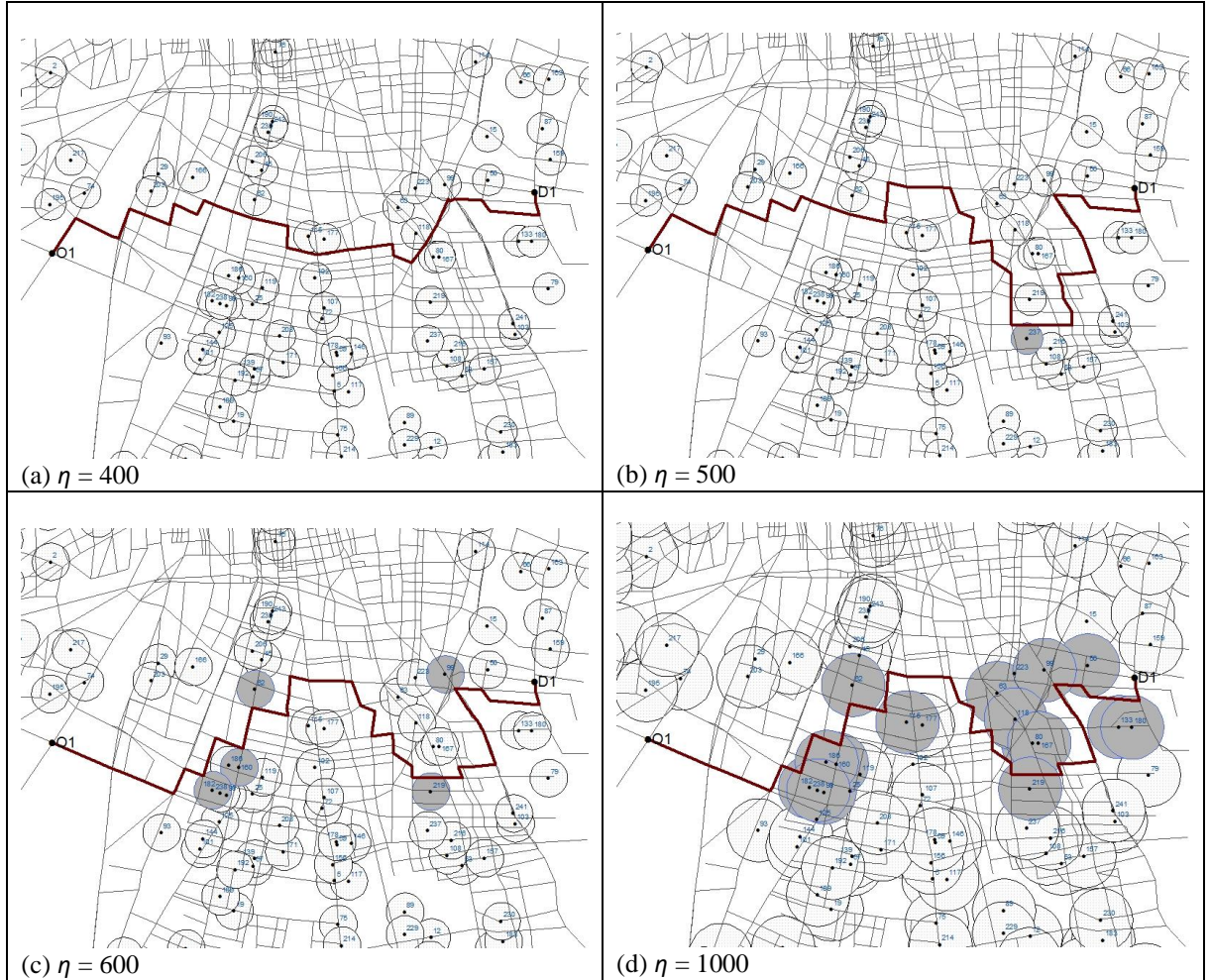
(b) $\eta = 500$

(c) $\eta = 600$

(d) $\eta = 1000$

Figure 2-2: Routes for pair O1-D1 (a), $\eta = 400$ m, (b) $\eta = 500$ m, (c) $\eta = 600$ m, (d) $\eta = 1,000$ m.

Figure 2-3: Routes for pair O2-D2 (a) $\eta = 100$ m, (b) $\eta = 300$ m, (c) $\eta = 400$ m, (d) $\eta = 500$ m, (e) $\eta = 700$ and (f) $\eta = 1,000$.
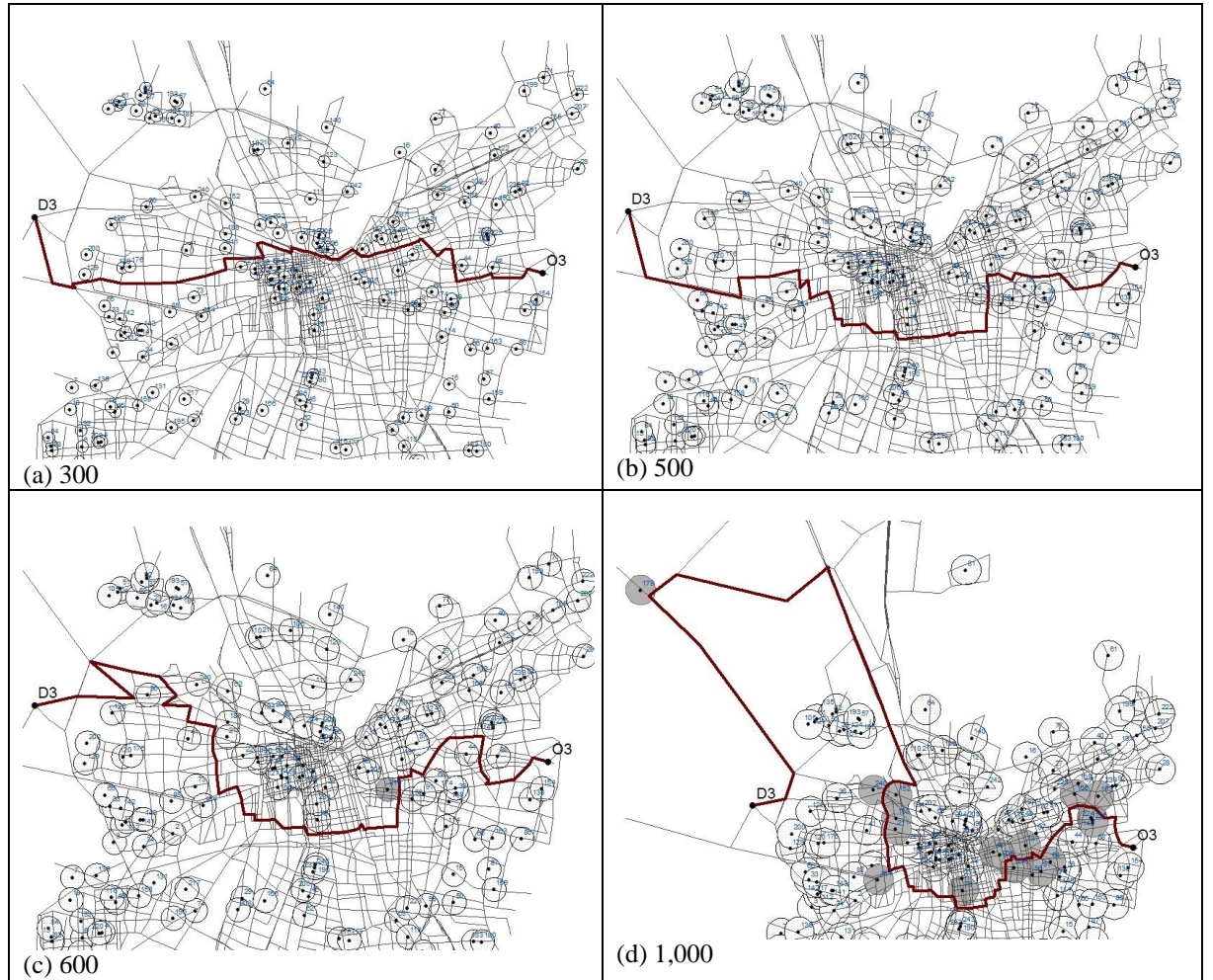
Figure 2-4: Routes for pair O3-D3 (a) $\eta = 300$ m, (b) $\eta = 500$ m, (c) $\eta = 600$ m, (d) $\eta = 1000$ m.

Note in Figures 2-2 to 2-4, how once it is not possible for the route to stay out of the danger areas, it starts crossing one danger area and then, it reaches a point at which it stays the same no matter how large is η. However, as η increases beyond that point, the number of affected centers also increases.

In Figure 2-4, the route for $\eta = 1,000$ needs to be extremely long to maintain the value of w.

When $\eta = 400$ to 1,000 m, the corresponding values of d_{ij}^p and D^p for each vulnerable center p and each route obtained are as shown in Table 2-6 to Table 2-8, corresponding to each one of the O-D pairs. The entries "-" in the tables indicate that $d_{ij}^p > \eta \ \forall (i,j) \in \mathfrak{R}$, i.e., the route stays clear of the centers.

Vulnerable center	D^p (students)	$\eta = 400$		$\eta = 500$		$\eta = 600$		$\eta = 700$		$\eta = 800$		$\eta = 900$		$\eta = 1000$	
		d_{ij}^p (m)	d_{ij}^p/D^p	d_{ij}^p (m)	d_{ij}^p/D^p	d_{ij}^p (m)	d_{ij}^p/D^p	d_{ij}^p (m)	d_{ij}^p/D^p	d_{ij}^p (m)	d_{ij}^p/D^p	d_{ij}^p (m)	d_{ij}^p/D^p	d_{ij}^p (m)	d_{ij}^p/D^p
50	1,820	-	-	-	-	-	-	671	0.369	671	0.369	671	0.369	671	0.369
62	1,695	-	-	-	-	593	0.350	593	0.350	593	0.350	593	0.350	593	0.350
63	1,683	-	-	-	-	-	-	-	-	-	-	877	0.521	877	0.521
80	1,618	-	-	-	-	-	-	-	-	-	-	-	-	942	0.582
98	1,551	-	-	-	-	-	-	-	-	-	-	-	-	970	0.625
99	1,547	-	-	-	-	541	**0.349**	541	**0.349**	541	**0.349**	541	**0.349**	541	**0.349**
115	1,469	-	-	-	-	-	-	-	-	757	0.516	757	0.516	757	0.516
118	1,440	-	-	-	-	-	-	-	-	-	-	861	0.598	861	0.598
133	1,375	-	-	-	-	-	-	-	-	-	-	859	0.624	859	0.624
160	1,286	-	-	-	-	596	0.464	596	0.464	596	0.464	596	0.464	596	0.464
167	1,263	-	-	-	-	-	-	-	-	-	-	892	0.706	892	0.706
180	1,219	-	-	-	-	-	-	-	-	-	-	839	0.688	839	0.688
182	1,217	-	-	-	-	553	0.454	553	0.454	553	0.454	553	0.454	553	0.454
186	1,207	-	-	-	-	-	-	643	0.533	643	0.533	643	0.533	643	0.533
203	1,178	-	-	-	-	-	-	-	-	-	-	-	-	784	0.666
219	1,132	-	-	-	-	439	0.388	439	0.388	439	0.388	439	0.388	439	0.388
237	1,089	-	-	424	**0.389**	-	-	-	-	-	-	-	-	-	-
238	1,088	-	-	-	-	-	-	-	-	787	0.723	787	0.723	787	0.723

Table 2-6: Values of *w* in objective function for $\eta = 400$ to 1,000. Transport Network 1, par O1-D1.

Vulnerable center	D^p (students)	$\eta = 400$		$\eta = 500$		$\eta = 600$		$\eta = 700$		$\eta = 800$		$\eta = 900$		$\eta = 1000$	
		d_{ij}^p (m)	d_{ij}^p / D^p	d_{ij}^p (m)	d_{ij}^p / D^p	d_{ij}^p (m)	d_{ij}^p / D^p	d_{ij}^p (m)	d_{ij}^p / D^p	d_{ij}^p (m)	d_{ij}^p / D^p	d_{ij}^p (m)	d_{ij}^p / D^p	d_{ij}^p (m)	d_{ij}^p / D^p
46	1,900	0	-	473	0.249	473	0.249	473	0.249	473	0.249	473	0.249	473	0.249
80	1,618	0	-	0	-	0	-	0	-	0	-	0	-	942	0.582
102	1,518	0	-	0	-	0	-	0	-	745	0.491	745	0.491	745	0.491
103	1,518	373	**0.246**	373	**0.246**	373	**0.246**	373	**0.246**	373	**0.246**	373	**0.246**	373	**0.246**
115	1,469	0	-	0	-	511	0.348	511	0.348	511	0.348	511	0.348	511	0.348
116	1,454	0	-	466	0.321	466	0.321	466	0.321	466	0.321	466	0.321	466	0.321
118	1,440	0	-	0	-	0	-	0	-	0	-	861	0.598	861	0.598
119	1,429	0	-	0	-	0	-	0	-	787	0.551	787	0.551	787	0.551
157	1,295	0	-	0	-	0	-	689	0.532	689	0.532	689	0.532	689	0.532
158	1,290	0	-	0	-	0	-	636	0.493	636	0.493	636	0.493	636	0.493
160	1,286	0	-	0	-	596	0.464	596	0.464	596	0.464	596	0.464	596	0.464
177	1,225	0	-	450	0.368	450	0.368	450	0.368	450	0.368	450	0.368	450	0.368
186	1,207	0	-	0	-	0	-	643	0.533	643	0.533	643	0.533	643	0.533
216	1,137	0	-	483	0.424	483	0.424	483	0.424	483	0.424	483	0.424	759	0.667
219	1,132	0	-	439	0.388	439	0.388	439	0.388	439	0.388	439	0.388	439	0.388
241	1,085	0	-	0	-	0	-	0	-	703	0.647	703	0.647	703	0.647

Table 2-7: Values of *w* in objective function for η = 400 to 1,000. Transport Network 1, par O2-D2.

Vulnerable center	D^p (students)	$\eta = 400$		$\eta = 500$		$\eta = 600$		$\eta = 700$		$\eta = 800$		$\eta = 900$		$\eta = 1000$	
		d_{ij}^p (m)	d_{ij}^p / D^p	d_{ij}^p (m)	d_{ij}^p / D^p	d_{ij}^p (m)	d_{ij}^p / D^p	d_{ij}^p (m)	d_{ij}^p / D^p	d_{ij}^p (m)	d_{ij}^p / D^p	d_{ij}^p (m)	d_{ij}^p / D^p	d_{ij}^p (m)	d_{ij}^p / D^p
48	1,831	0	-	0	-	0	-	0	-	0	-	0	-	961	0.525
68	1,664	0	-	0	-	0	-	0	-	734	**0.441**	734	**0.441**	734	**0.441**
76	1,635	0	-	0	-	0	-	0	-	743	0.455	743	0.455	743	0.455
91	1,580	0	-	0	-	0	-	0	-	0	-	0	-	915	0.579
106	1,495	0	-	0	-	0	-	689	0.461	0	-	0	-	0	-
111	1,482	0	-	0	-	0	-	0	-	0	-	0	-	865	0.584
137	1,365	0	-	0	-	0	-	631	0.463	631	0.463	631	0.463	631	0.463
145	1,326	0	-	0	-	0	-	681	0.514	600	0.453	600	0.453	600	0.453
152	1,310	0	-	0	-	0	-	0	-	0	-	858	0.655	858	0.655
153	1,310	0	-	0	-	0	-	0	-	0	-	857	0.654	857	0.654
168	1,255	0	-	0	-	0	-	0	-	783	0.624	783	0.624	783	0.624
179	1,221	0	-	0	-	0	-	0	-	0	-	661	0.541	661	0.541
184	1,209	0	-	0	-	0	-	0	-	0	-	0	-	958	0.792
188	1,200	0	-	0	-	0	-	0	-	673	0.561	576	0.480	576	0.480
203	1,178	0	-	0	-	0	-	0	-	784	0.666	0	-	0	-
205	1,173	0	-	0	-	0	-	532	**0.454**	532	0.454	532	0.454	532	0.454
211	1,149	0	-	0	-	0	-	0	-	702	0.611	702	0.611	702	0.611
215	1,137	0	-	0	-	592	**0.520**	568	0.500	568	0.500	561	0.494	568	0.500
224	1,128	0	-	0	-	0	-	0	-	670	0.594	670	0.594	670	0.594
225	1,123	0	-	0	-	0	-	0	-	733	0.653	733	0.653	733	0.653
240	1,085	0	-	0	-	0	-	0	-	0	-	691	0.637	691	0.637

Table 2-8: Values of *w* in objective function for η = 400 to 1000. Transport Network 1, par O3-D3.

The effect of η on the value of w and the route length can be seen, for example, for the pair O1-D1 on Table 2-6. When $\eta = 600$ m, centers 62, 99, 160, 182 and 219 are exposed, with center 99 being the most affected, as d_{ij}^{99} / D^{99} is the lowest weighted distance. In this case, $w = d_{ij}^{99} / D^{99} = 0.349$. Note that, even though center 219 is closer to the route ($d_{ij}^{219} = 439$ m) than center 99 ($d_{ij}^{99} = 542$ m), its population is smaller, which results in a value of $d_{ij}^{219} / D^{219} = 0.388$, above the minimum figure. The route obtained is shown in Figure 2-2c. When $\eta = 700, 800, 900$ and 1000 m, the number of exposed centers increases. Center 99 with $d_{ij}^{99} / D^{99} = 0.349$ is the one that remain setting the value of w.

2.4.2. Bi-objective model

In order to show the effects of different policies, assigning higher weights on either the public concerns (routes farther away from most populated centers) or carrier's issues (cost of the routes), model M_4 was applied to a network representing part of the Santiago network, shown in Figure 2-5.

Figure 2-5: Transport Network 2 and vulnerable centers.

The road system serving this area, denoted Transport Network 2, contains 1,521 links, 504 nodes and 30 vulnerable centers with 81,434 inhabitants. Centers have between 308 and 6,074 people each.

We assume without loss of generality that the cost c_{ij} of using link $(i,j) \in A$ is proportional to its length. The threshold distance η was set to 700 m. To determine the best and worst values (I_i and AI_i, respectively) for the two objectives, each one was optimized separately, using M_2 and a shortest path algorithm. The results, shown in Table 2-9, reveal the essential conflict between the two objectives: When optimizing the value of w, the route length was 22.61 Km, a 21.7% longer than the shortest route, while the value for w was about 5 times the figure of 55.65 m/person obtained when minimizing the length of the route.

Attribute	(maximin)	(minimum distance)
w (m/person)	$0.279 = I_1$	$0.055 = AI_1$
Length (m)	$22.61 = AI_2$	$18.58 = I_2$

Table 2-9: Model M_2 versus shortest path Model ($\eta = 700$)

The efficient frontier between the two objectives was generated by varying the values of parameter θ_i (with $\theta_1 = 1- \theta_2$) in M_4. The results are displayed in Table 2-10, the frontier is shown in Figure 2-6, and the routes obtained depicted in Figure 2-7. Each point in Figure 2-6 represents a different route. The lowest cost solution is equivalent to the one obtained when the model is solved with $\theta_1 = 0.1$ (Figure 2-7a). With values of θ_1 between 0.2 and 0.6, the route is as shown in Figure 2-7b (point B in Figure 2-6) while values 0.7 and 0.8 yield the route in Figure 2-7c (point C in Figure 2-6). Finally, the solution attained with $\theta_1 = 0.9$ (see Table 2-10) dominates the route produced by M_2 (see Table 2-10). For both formulations $w = 0.279$ but the route generated by M_2 is longer, measuring 22.61 Km versus 21.67 Km for M_4, the latter with $\theta_1 = 0.9$ (Figure 2-7d).

θ_1	w (m/person)	c_{ij} (km)	CPU time, M_4
0.1	0.055	18.58	1,41
0.2	0.237	19.04	4,34
0.3	0.237	19.04	2,16
0.4	0.237	19.04	4,07
0.5	0.237	19.04	4,36
0.6	0.237	19.04	3,32
0.7	0.254	19.71	3,52
0.8	0.254	19.71	5,00
0.9	0.279	21.67	1,18

Table 2-10: Approximation of efficient frontier for M_4 obtained by varying parameter θ_1. Transport Network 2, $\eta = 700$.
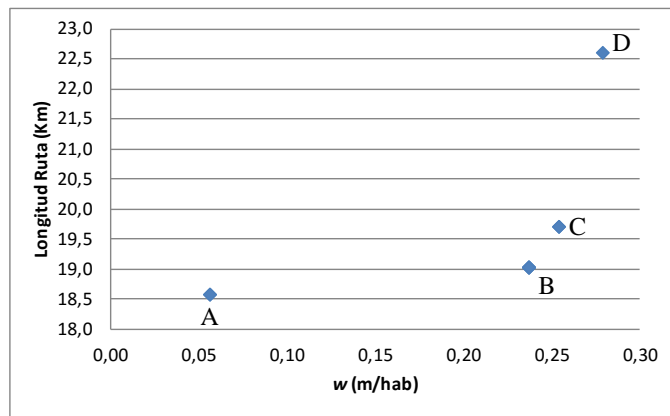
Figure 2-6: Efficient frontier for M_4.Transport Network 2, $\eta = 700$.
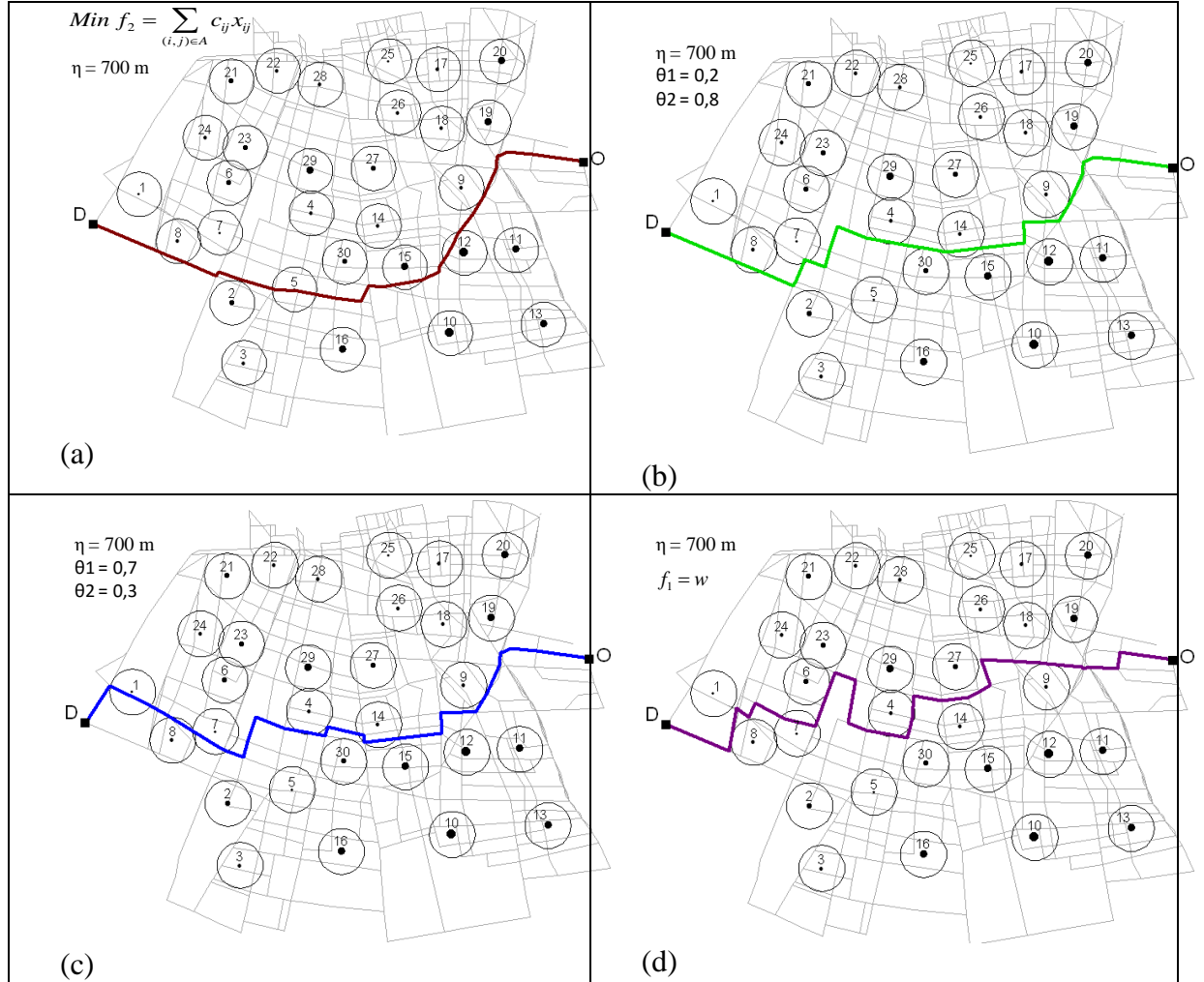
Figure 2-7: Routes obtained for $\eta = 700$ m: (a) Min f_2; (b) $\theta_1 = 0,2$; (c) $\theta_1 = 0,7$; (d) Min f_1. Transport Network 2, model M_4.

2.5. Conclusions and extensions

An approach was proposed for addressing the hazardous materials routing problem on a transport network. The proposed approach maximizes the weighted distance between the route and its closest vulnerable center in order to minimize the potential consequences for the most exposed population. This weighted distance can be replaced by any measure of risk or danger, as long as it is a non-decreasing function of the distance.

Though this maximin objective has been treated in a continuous space setting, it has not been previously studied for an actual road network. A basic formulation of the problem requires a large number of closest assignment constraints and decision variables, making it very difficult to solve. A practical first alternative was therefore developed that required a significantly reduced subset of the variables and constraints for each vulnerable center. The reduction is achieved by replacing a large number of variables by a single variable for each vulnerable center.

A second alternative solution method, also presented here, is an algorithm that is capable of solving large real sized instances to optimality, in a polynomial time. The algorithm is easily implemented and it can be extended to multiple variants of the problem.

Both the exact method and the algorithm were tested by applying them to the transport network in the city of Santiago, Chile. The results illustrated how the solution varies with danger areas of different radii. This problem is easily solved by using the proposed algorithm.

Though the objective of avoiding such areas is attractive for the resident population and the regulatory authorities, it is not necessarily so for the carrier seeking to minimize operating costs. A bi-criteria problem was therefore formulated that captures the relationship between the maximin and minimum cost objectives. The model was applied to a smaller area of the Santiago road network and the efficient frontier between the two objectives was estimated, as a way to show the effects of different policies. The approach proved capable of generating a set of efficient alternatives for different threshold values, in a real-sized instance, suggesting it could be applied also in other real-world situations by decision-makers to find an appropriate HAZMAT route based on assigned priorities.

This study could be extended in various directions. The population's hazard aversion could be represented in the proposed formulation as a function of the distance to the route travelled by means of danger areas and danger function variables that could be modelled to

depend on the vulnerable center, with multiple HAZMAT shipments between multiple origin-destination pairs.

An additional objective could be added to the problem, as a way to take into consideration the fact that the maximin objective does not allow controlling how many vulnerable centers are being put in danger.

The approach developed could also be expanded to address a multi-criteria HAZMAT routing problem incorporating criteria such as expected consequences, time exposure, population exposure or total danger, which could be satisfied jointly with the maximization of the minimum weighted distance and minimum operating cost for both single and multiple shipments between multiple origin-destination pairs.

3. THE MAXISUM AND MAXIMIN-MAXISUM HAZMAT ROUTING PROBLEMS

We design routes for transportation of hazardous materials (HAZMAT) in urban areas, with multiple origin-destination pairs. First, we introduce the maxisum HAZMAT routing problem, which maximizes the sum of the population-weighted distances from vulnerable centers to their closest point on the routes. Secondly, the maximin-maxisum HAZMAT routing problem trades-off maxisum versus the population-weighted distance from the route to its closest center. We propose efficient IP formulations for both NP-Hard problems, as well as a polynomial heuristic that reaches gaps below 0.54% in a few seconds on the real case in the city of Santiago, Chile.

3.1. Introduction

Large quantities of hazardous materials (HAZMAT) are generated as raw material or by-products of industrial activity. These materials may be explosive, flammable, oxidizing, toxic, poisonous, infectious, corrosive and radioactive. In the great majority of urban areas that contain industries, this type of material must be transported in trucks, which becomes a source of hazard for the population. In spite of the efforts made by governmental agencies and transportation industry, accidents happen. Statistics for the city of Santiago, Chile, indicate that during 2013 there were 12 events of liberation of hazardous materials, affecting 2,279 people, 137 of whom suffered some type of injuries. Thousands of people had to be evacuated and the costs of removing the spills were very high [INE (2015)]. The U.S. Department of Transportation *et al.* (2016) reported that, during 2014, there were 3,599 incidents related to HAZMAT transportation, not including loading and unloading, with a total cost of US$ 50,081,168. These facts suggest a need for stronger efforts on part of researchers and practitioners, to develop better methodologies for the protection of both population and environment against the consequences of HAZMAT transportation accidents, especially those occurring in urban areas.

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The literature has mainly addressed this problem through vehicular routing models that minimize some risk function for the population. In practically all of the cases, the risk is a function of the likelihood of an event of release of the material in the process of transport or the consequence associated with such an incident, or a combination of the two factors. A common practice is to trade off the risk indicators against a measure that represents the operational costs of the vehicle such as travel time or distance traveled, representing in this way both the public interests (risk) and the private interests (transport cost).

In an urban environment, the population is distributed in a more or less continuous manner. As such, it is inevitable that the routes used to transport these materials could potentially affect part of the population, regardless of the route chosen. On the other hand, the general population can be evacuated in a short enough period of time to ensure that they do not suffer consequences from most accidents (with the exception of explosive ones). This is not the case with certain at-risk populations or those that are difficult to evacuate, such as large concentrations of people in small surface areas (schools and large buildings, for example), or those that have difficulty evacuating (patients in hospitals, residents of long-term care facilities, etc.). In view of this, it is reasonable to pay extra attention to this at-risk population when designing HAZMAT transport routes.

We present a new method for solving the problem of HAZMAT transport in urban areas. As opposed to dealing with risk, it is designed to decrease the hazard or danger posed by this activity to a set of vulnerable centers located in a large urban area. It can be used as complementary, or instead of risk models, especially if public opposition or evacuations are concerns. Each vulnerable center is represented as a point on the map in which its population is concentrated. With a center at that point, we calculate a circular zone of danger, which is the zone such that, if there were an accident within it, the center would be affected. Thus, the placement of any segment of the route inside of this circular zone would pose a hazard for the population. Hazard is defined as the potential for an undesirable consequence regardless of the likelihood of its occurrence [Rasmussen

(1981)]. The majority of the literature assumes an equal risk for the center, no matter where in the circle an accident occurs. In contrast, we consider that within the circular zone, the hazard is a function of the distance between the vulnerable center and the segment of a route used for HAZMAT transport. So we are able to minimize the hazard that vulnerable centers face, by ensuring that hazardous activities are developed as far away as possible from them.

Our first contribution is a model with a single objective, the maxisum HAZMAT routing problem (MsHRP) for multiple origin-destination (OD) pairs, which consists of maximizing the sum of the population-weighted distances between all vulnerable centers and their closest links belonging to the set of routes used for HAZMAT transportation, whenever these links enter the danger zones of the vulnerable points. This method addresses a shortcoming of the model proposed in Chapter 2 (*the maximin HAZMAT routing problem*). The maximin approach is efficient protecting the most affected vulnerable center, but it does not allow to measure the effect on the rest of the population and, in the solutions, a large number of centers are exposed. By minimizing the total impact on affected population, our maxisum approach provides routes that expose fewer centers. The MsHRP is NP-Hard (Hakimi *et al*. 1993), and as such, besides using a fast exact model, our second contribution is proposing a polynomial heuristic procedure that allows MsHRP to be solved efficiently for large instances.

By focusing on protection of the entire population, MsHRP leaves aside the closest vulnerable center. Our third contribution is combining the maxisum and maximin criteria in the maximin-maxisum HAZMAT routing problem (MmMsHRP). Through the maximin criterion, we seek routes between different pairs OD to minimize the negative impact on the most affected center. Using the maxisum criterion, we seek routes between different pairs OD to minimize the negative effect on all vulnerable centers that could be affected. This bi-objective problem inherits from the maxisum the characteristic of being NP-Hard.

Finally, we conduct a comparative analysis of the results obtained with transportation costs.

Note that in these models the theoretical contribution comes from their novelty in transportation, as well as the proposed solution methods, while their practical significance is the protection of all the vulnerable points, not only the most affected one.

The rest of the chapter is organized as follows: In Section 3.2, we present a review of relevant literature. In Section 3.3, we formulate the Maxisum HAZMAT Routing Problem MsHRP and describe the heuristic. In Section 3.4 we formulate the Maximin-Maxisum version of the problem, MmMsHRP. Section 3.5 describes the application of the proposed methodology to an instance with real data and a detailed analysis of the results. Finally, Section 3.6 presents the conclusions and possible extensions of the research.

3.2.   Literature Review

The HAZMAT transport problem has been widely addressed over the past few decades. Authors have focused mainly on solving a problem of routing vehicles carrying HAZMAT between two points –an origin-destination pair, minimizing an estimator of risk. This estimator depends on the likelihood of an accident in which the transported HAZMAT is released or the consequences of such an incident, or a combination of those factors. Erkut and Verter (1998) and Erkut *et al.* (2007) discuss different forms of modeling risk in HAZMAT transport. One of the most frequently used risk estimators is the expected consequence, which consists of multiplying the likelihood of an incident by the associated consequence [Batta and Chiu (1988), Pijawka *et al.* (1985), Alp (1995) y Erkut and Verter (1995)]. Erkut and Ingolfsson (2005) consider the full expected consequence of all of the trips necessary to meet the demand. The authors assume that an incident ends a trip and that a new shipment must be sent to meet the demand. Sivakumar *et al.* (1993) and Sherali *et al.* (1997) minimize the expected consequence by supposing that there is certainty that an accident will occur on the route. This approach is similar to that of considering risk.

Sivakumar *et al.* (1995) minimize the expected risk of the first accident, also considering equity in the spatial distribution of risk over the region studied. Jin *et al.* (1996) and Jin and Batta (1997) explore diverse objective functions based on the minimization of the expected result given that a specific number of trips will be made and given a maximum number of accidents accepted before the shipments are stopped.

Erkut and Ingolfsson (2000) propose three alternatives for evaluating risk in HAZMAT transport: minimizing the maximum population exposed; simultaneously minimizing the expected value and the variance of the number of people affected by an incident; and minimizing the expected disutility, expressed as the exponential of the size of the affected population, multiplied by a constant that measures aversion to catastrophe. Abkowitz *et al.* (1992) minimize risk aversion where the perceived risk on a link is defined as the product of the likelihood of an incident over the link and the consequence of that incident raised to a risk preference parameter. Other techniques for estimating risk are based on the minimization of the likelihood of an incident [Saccomanno and Chan (1985)] or the exposed population as an estimator of the consequences [ReVelle *et al.* (1991)]. Using risk estimators, some authors address the problem of equity in spatial distribution of risk among the population [Current and Ratick (1995), Gopalan *et al.* (1990), Gopalan *et al.* (1990), Lindner-Dutton *et al.* (1991), Carotenuto *et al.* (2007) and Caramia *et al.* (2010)].

Another line of research recognizes the multi-objective nature and participation of multiple agents in HAZMAT transport decision-making. Zografos and Davis (1989) offer a solution that considers the population at risk, the risk of special categories of the population, property damage, and travel time as proxies of the vehicle operating costs. ReVelle *et al.* (1991) combine the exposure of the population and the cost of transportation of spent nuclear fuel. Marianov and ReVelle (1998) propose a model of linear optimization for the routing of vehicles through hazardous environments or for the routing of the vehicles that transport HAZMATs. The authors minimize the cost and likelihood of an accident. Li and Leung (2011) consider six criteria for HAZMAT routing: travel time,

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likelihood of an incident, users of the highway at risk, population at risk, people with special needs at risk, and possible damages caused to property around the incident. Other studies with multiple objectives include those authored by Wijeratne *et al.* (1993), Zografos and Androutsopoulos (2008), Dell'Olmo *et al.* (2005) y Caramia *et al.* (2010).

More recently, researchers have directly considered the relationship between material carriers and regulatory agents. Officials determine certain rules of use of the transport network which must be respected by HAZMAT materials carriers in their selection of routes [Kara and Verter (2004), Erkut and Gzara (2008), Verter and Kara (2008), Marcotte *et al.* (2009), Erkut and Alp (2007), Gzara (2013), Wang *et al.* (2012) Bruglieri *et al.* (2014) and Xin *et al.* (2013)].

In the literature described, there are various ways to consider distance. The majority of the authors define a threshold distance within which the risk (or, in general, the effects) has a constant value for the entire population and outside of which the risk does not exist. However, researchers have recognized that the danger posed by hazardous activities is a function of the proximity of the population to the source of the hazard [Hung and Wang (2011), Elliott *et al.* (1999), Brody *et al.* (2004), Lima (2004), Lindell and Perry (2000), Arlikatti *et al.* (2006), Wachinger *et al.* (2013), Miceli *et al.* (2008), Heitz *et al.* (2009) and Brilly *et al.* (2005)]. Saccomanno and Shortreed (1993), Jonkman *et al.* (2003) and Fernández. *et al.* (2000) also point to this fact, arguing that possible consequences for the population in the case of HAZMAT release events vary in function of the distance from the event.

There are some exceptions that do consider the distance between the population center and the HAZMAT route. Carotenuto *et al.* (2007) assume that the population is found only over the links of the transportation network (populated links). The risk that a route segment imposes on a populated segment corresponds to the product of the population of the populated segment within the threshold distance of the route segment, the likelihood of an incident in the route segment, and a function that decreases exponentially with the square

of the Euclidean distance between the two segments. List and Mirchandani (1991) propose calculating the integral, on the complete route, of a theoretical function of the distance between the populated point and each point of the route, weighted by the population of the populated point and by the likelihood of occurrence of incidents on the point of the route. However, to solve a practical problem they simplify the method considerably. Carotenuto *et al.* (2007) and List and Mirchandani (1991) require the explicit enumeration of candidate routes and use probabilities to estimate risk. Erkut and Verter (1995) evaluate risk for a populated area within a given distance from the route, assuming uniform population density. The risk to any individual within this area of impact is estimated as the product of the likelihood of an incident over a segment of the road and the length of the segment. The methodology is used to assess routes.

In this paper, we add an objective that integrates the average exposure of the set of centers (maxisum objective), which results in an NP-hard problem. The consideration of this objective as well as its combination with the maximin criterion are new in transportation network routing problems even though both criteria have been used when undesirable or hazardous facilities are located. Note that the modeling in that case is very different [see Church and Garfinkel (1978), Tamir (1991), Erkut and Neuman (1989), Melachrinoudis (1999), Zhang and Melachrinoudis (2001), Moreno-Pérez and Rodríguez-Martín (1999), Saameño Rodríguez *et al.* (2006)]. For a more detailed review of models of location of undesirable facilities and the use of different objectives, see Melachrinoudis (2011), Hosseini and Esfahani (2009), Farahani *et al.* (2010) and Colebrook and Sicilia (2013).

3.3. The Maxisum HAZMAT Routing Problem (MsHRP)

3.3.1. Integer programming formulation

Let a transport network be represented by a graph $G(N,A)$ where $N = \{1,\dots, n\}$ is the set of nodes and $A = \{1,\dots, m\}$ the set of links. Consider that a HAZMATs shipment requires

being transported between different OD (origin-destination) pairs of nodes such that the average distance (or the sum of the distances) between the population centers and their closest route links, weighted by the inverse of the population of each center, is maximized.

The main parameters of the model are as follows:

Q : set of population centers

D^q : Population concentrated at point or population center $q \in Q$.

d_{ij}^q : Euclidian distance between the population center $q \in Q$ y and its closest point on the link $(i, j) \in A$.

G : Set of origin-destination pairs. $G = \{1, \ldots, g\}$.

η : The distance from a center at point q, beyond which the effects on q of an event of release of a HAZMAT are negligible.

Let the following decision variables be defined:

$$x_{ij}^{od} = \begin{cases} 1 & \text{if link } (i, j) \in A \text{ is used for HAZMAT transport between pair } od \in G \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ij}^{q} = \begin{cases} 1 & \text{if link } (i,j) \in A, \text{ used for HAZMAT transport, is the closest link to } q \in Q \\ 0 & \text{otherwise} \end{cases}$$

$$z_{\eta}^{q} = \begin{cases} 1 & \text{if the arc closest to center } q \in Q \text{ is beyond a distance } \eta \\ 0 & \text{Otherwise} \end{cases}$$

The formulation for the maxisum HAZMAT routing problem is as follows:

$$MsHRP) \qquad Max \sum_{q \in Q} \left(\sum_{\{(i,j) \in A | d_{ij}^q \leq \eta\}} \frac{d_{ij}^q}{D^q} z_{ij}^q + \xi z_\eta^q \right) \qquad (3.1)$$

s.t.

$$\sum_{\{(i,j) \in A | d_{ij}^q \leq \eta\}} z_{ij}^q + z_\eta^q \leq 1 \qquad \forall q \in Q \qquad (3.2)$$

$$z_\eta^q \leq \sum_{\{(i,j) \in A | d_{ij}^q > \eta\}} \sum_{od \in G} x_{ij}^{od} \qquad \forall q \in Q \qquad (3.3)$$

$$z_{ij}^q \leq \sum_{od \in G} x_{ij}^{od} \qquad \forall q \in Q, \forall (i,j) \in A | d_{ij}^q \leq \eta \qquad (3.4)$$

$$z_{ij}^q \geq x_{ij}^{od} - \sum_{\{(k,l) \in A | d_{kl}^q \leq d_{ij}^q\}} z_{kl}^q \qquad \forall q \in Q, \forall (i,j) \in A | d_{ij}^q \leq \eta, \forall od \in G \qquad (3.5)$$

$$z_\eta^q \geq 1 - \sum_{\{(i,j) \in A | d_{ij}^q \leq \eta\}} z_{ij}^q \qquad \forall q \in Q \qquad (3.6)$$

$$\sum_{\{j/(i,j) \in A\}} x_{ij}^{od} - \sum_{\{j/(j,i) \in A\}} x_{ji}^{od} = \begin{cases} 1 & \text{if } i = o \\ -1 & \text{if } i = d \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in N, \forall od \in G \qquad (3.7)$$

$$x_{ij}^{od}, z_{ij}^q, z_\eta^q \in \{0,1\} \qquad \forall q \in Q, \forall (i,j) \in A, \forall od \in G \qquad (3.8)$$

In this formulation, maximizing d_{ij}^q / D^q is a proxy for minimizing an estimate of danger. Any estimator or proxy of risk or danger can be used without changing the structure of the model. The objective (3.1) maximizes the sum of the weighted distance between each vulnerable center q and its closest link if it falls within its danger zone (sum within the parentheses), or the number of vulnerable centers with all route segments outside their danger zone (second term inside the parentheses). ξ is a small pre-defined parameter that ensures dimensional homogeneity of the two terms in the objective function. The set of constraints (3.2) establishes that only one link can be assigned as closest link to the center

q, and it may or may not be inside of the danger zone. The constraints (3.3) and (3.4) ensure that only a link belonging to an active route can be considered as the closest to *q,* whether inside or outside of its danger zone. The set (3.5) ensures that, among all links inside the danger zone of the center *q,* the closest one is the one considered for danger effects. Set (3.6) establishes that if there is no active link within the danger zone of *q,* it has still a link assigned, but it is not in danger. The constraints (3.7) are network flow conservation constraints, while the set (3.8) defines the nature of the decision variables. Hakimi *et al.* (1993) show that this problem is NP-Hard.

This model preserves to some extent the risk aversion of a maximin model, as it maximizes the sum of the weighted distances between *each center and its closest* link. However, if required, the model can be easily extended to a more risk-neutral formulation (The RNMsHRP, Risk Neutral MsHRP), by considering not only the effect over each center of the closest link, but the effect of *all the links that fall within its danger zone*. In this case the decision variables z_{ij}^q and z_η^q are not required, resulting in the following problem:

$$RNMsHRP) \quad Max \quad \sum_{q \in Q} \sum_{\{(i,j) \in A | d_{ij}^q \leq \eta\}} \sum_{od \in G} \frac{d_{ij}^q}{D^q} x_{ij}^{od} \tag{3.9}$$

s.t.

$$\sum_{\{j/(i,j) \in A\}} x_{ij}^{od} - \sum_{\{j/(j,i) \in A\}} x_{ji}^{od} = \begin{cases} 1 & \text{if} \quad i = o \\ -1 & \text{if} \quad i = d \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in N, \forall od \in G \tag{3.10}$$

$$x_{ij}^{od} \in \{0,1\} \qquad\qquad \forall (i,j) \in A, \forall od \in G \tag{3.11}$$

If this formulation is to be used, a term $\sum_{od \in G} \sum_{\{(i,j) \in A | d_{ij}^q > \eta\}} x_{ij}^{od}$ weighted by a very small factor can be added to avoid the appearance of disconnected arcs in the solution.

3.3.2. A Heuristic

Note that the RNMsHRP (3.9) – (3.11) can be optimally solved in polynomial time, as it is equivalent to a shortest path problem with non-negative link attributes, when the objective is rewritten as:

$$Min \quad \sum_{q \in Q} \sum_{\{(i,j) \in A | d_{ij}^q \leq \eta\}} \sum_{od \in G} \frac{D^q}{d_{ij}^q + \varepsilon} x_{ij}^{od} = \sum_{q \in Q} \sum_{\{(i,j) \in A\}} \sum_{od \in G} \theta_{ij}^q x_{ij}^{od} \qquad (3.12)$$

where ε is a small parameter that avoids indetermination of the estimate when the center is on the arc, and

$$\theta_{ij}^q = \begin{cases} \dfrac{D^q}{d_{ij}^q + \varepsilon} & d_{ij}^q \leq \eta \\[2mm] 0 & d_{ij}^q > \eta \end{cases} \qquad (3.13)$$

is a proxy of the danger exerted by the arc (i,j) over the vulnerable center q,

Using the *RNHsHRP* with objective (3.12) as an approximation, we developed an efficient polynomial heuristic for the MsHRP.

Maxisum Route Heuristic

Step 0: *Initialization*

$k = 0$; Π, a set of feasible routes $= \phi$; use equation (3.13) to compute $\theta_{ij}^q \quad \forall q \in Q, \forall (i,j) \in A / d_{ij}^q \leq \eta$.

Step 1: *Search for routes*

Solve RNMsHRP using a shortest path algorithm, for a set R^k of routes, $R^k = \{r_1^k, ..., r_g^k\}$, i.e., one route for each origin-destination pair.

i.   (for $k = 0$) If there is no route for some OD pair, the network is not connected and the problem is unfeasible. STOP.

(for $k > 0$) If there is no route for some OD pair, go to Step 2.

ii.   If routes can be found for all OD pairs, compute $\theta_{(a,b)^k} = \underset{q \in Q}{Max}\,\underset{r \in R^0}{Max}\,\underset{(i,j) \in r}{Max}\left\{\theta_{ij}^q\right\}$. Note that $(a,b)^k = \arg\left\{\theta_{(a,b)^k}\right\}$ is the arc that exposes a populated center to the maximum danger.

    a.   If $\theta_{(a,b)^k} = 0$ then the routes constitute no danger and the solution is optimal. STOP.

    b.   If $\theta_{(a,b)^k} > 0$, $\Pi = \Pi \cup \left\{R^k\right\}$; $k = k + 1$; $x_{(a,b)^s} = 0 \quad \forall s < k$; repeat Step 1.

Step 2: *Selection of the solution*

Define $a_q^k = 1$ if, for center q, there are no route arcs within its danger zone, and 0 otherwise. Compute $\theta_{R^k} = \sum_{q \in Q}\left[\underset{r \in R^k}{Max}\,\underset{(i,j) \in r}{Max}\left\{\theta_{ij}^q\right\}\right]$ $\forall k$, i.e., the danger to which the set R^k of routes exposes the population, and $\delta_{R^k} = 1/\theta_{R^k} + \xi \sum_{q \in Q} a_q \quad \forall k$ i.e., the corresponding objective value if this set were chosen. Choose the set R^k of routes for which δ_{R^k} is the maximum (that is, minimum danger).

End

We remark that both RNMsHRP and the heuristic are separable by OD pairs. Note also that the heuristic requires a polynomial number of operations.

3.4.   The maximin-maxisum HAZMAT routing problem (MmMsHRP)

Note that by minimizing the average effect over all of the population centers (maxisum objective), the magnitude of hazard posed to each individual population center is not being

considered, which allows some centers to be highly exposed. On the other hand, a maximin criterion minimizes the impact on only the most affected center. Optimized separately, the maximin and maxisum objectives can generate low quality solutions, assessed by the complementary criterion. Optimizing both concurrently appears then, as a valuable action.

We formulate the maximin-maxisum HAZMAT routing problem (MmMsHRP) combining the maxisum and maximin objectives.

We have chosen to generate a subset of the set of efficient solutions for the problem normalizing the values of each objective and using the weighting method [Cohon (2013)], which consists of multiplying each objective i (normalized in this case) by a non-negative weight α_i such that $\sum_i \alpha_i = 1$, and then adding the weighted objectives. Parametrically varying the values of α_i, a representative subset of the set of efficient or Pareto-optimal solutions is generated. The normalization is achieved by subtracting the objective function i from its best possible value, f_i^+, and then dividing it by ($f_i^+ - f_i^-$), where f_i^- is its worst value.

Let the decision variable w be the weighted distance from the set of routes to its closest vulnerable center $q \in Q$. Note that w is a proxy for hazard that can be replaced by other indicators of risk or hazard as long as they are non-decreasing with the distance.

Using the same notation and definitions presented above, the MmMsHRP for multiple pairs OD can be formulated as follows:

$$MmMsHRP) \quad Max \quad \alpha_1 \left[\frac{f_1 - f_1^+}{f_1^+ - f_1^-} \right] + \alpha_2 \left[\frac{f_2 - f_2^+}{f_2^+ - f_2^-} \right] \tag{3.14}$$

$$s.t. \qquad (3.2)-(3.8)$$

$$f_1 = w \tag{3.15}$$

$$f_2 = \sum_{q \in Q} \sum_{\{(i,j) \in A \mid d_{ij}^q \leq \eta\}} \left(\frac{1}{D^q}\right) d_{ij}^q z_{ij}^q + \xi z_\eta^q \tag{3.16}$$

$$w \leq \sum_{\{(i,j) \in A \mid d_{ij}^q \leq \eta\}} \left(\frac{1}{D^q}\right) d_{ij}^q z_{ij}^q + \frac{\eta}{D^\eta} z_\eta^q \qquad \forall q \in Q \tag{3.17}$$

$$w \geq 0 \tag{3.18}$$

Expression (3.14) maximizes both objectives (3.15) and (3.16). Note how, although they look alike, both objectives are different. Objective f_1 is defined by (3.17). For each $q \in Q$, there is at most one term in the sum on the right-hand side that is nonzero: the term corresponding to the arc (i,j) in the HAZMAT route that is closest to q. If none of the arcs of the route is within distance η, then all the terms in the sum are zero and $z_\eta^q = 1$. In other words, for each q, w is

$$w \leq \begin{cases} \left(\dfrac{d_{ij}^q}{D^p}\right)_{\substack{closest \\ (i,j)}} & \text{if } (i,j) \text{ closer to } q \text{ than } \eta \\[2em] \dfrac{\eta}{D^\eta} & \text{otherwise} \end{cases}$$

As this constraint is formulated for each $q \in Q$, the variable w will take the smallest value among all the q's, i.e. the shortest population-weighted distance between a vulnerable point and an arc of the HAZMAT route. In order for w to take the value η/D^η only when there are no links inside of a danger zone, $\eta/D^\eta \geq \max\{d_{ij}^q\} / \min_{q \in Q}\{D^q\}$ must hold or, equivalently, $D^\eta \leq \min_{q \in Q}\{D^q\}$.

To the contrary, in objective f_2, expression (3.16), again there is only one nonzero term in each inner sum (the sum over arcs (i,j).) However, now the outer sum *adds all the terms* corresponding to all q's, i.e., it adds *all weighted shortest distances between a vulnerable point q and its closest arc.*

The set of constraints (3.18) determines the non-negativity of the w variables.

Given that MmMsHRP is a generalization of MsHRP, it is NP-Hard, as long as $\alpha_2 > 0$.

3.5. Application

The proposed models were applied to a real HAZMAT transport case between different origin-destination pairs in the city of Santiago de Chile. This case covers 244 vulnerable centers (schools with over 1,000 students) in a large urban area and a transport network composed of 6,681 links and 2,212 nodes that establish the connectivity of the area under study (Figure 3-1).

The instances were solved on a personal computer running Ubuntu 12.04 LTS with a 3.40 GHz Intel® Core™ i7-2600 processor and 16 GB of RAM. The implementation was developed in AMPL Cplex 12.5.

3.5.1. The maxisum HAZMAT routing problem (MsHRP) applied to the Santiago case

We solve the MsHRP using the integer programming formulation and the heuristic procedure for various OD pairs and for different values of η. In the IP formulation, we consider $\xi = 1$ m/person and incorporate the term $\psi \sum_{od \in G} \sum_{(i,j) \in A} x_{ij}^{od}$ as a secondary objective where ψ is a constant small enough to guarantee that its minimization does not affect the value of the main objective, i.e., the maxisum. This second objective pushes link variables that do not belong to the route, to take a value 0.

Table 3-1 shows the results of MsHRP for different values of η and the four OD pairs. We do not show the rows with small values of η in which the routes do not expose any vulnerable center. In the table, the left block of columns shows the results of the IP formulation, while the right side block, those of the heuristic. The first column indicates the OD pairs considered. The second column, the value of the threshold distance η beyond which the effects of release of HAZMATs are negligible. The first column of the IP results, shows the value of the maximum danger to which a vulnerable point is exposed, in m/person (distance to the closest link in meters, divided by the population at the center).

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The remaining columns of the IP block display the value of the objective, number of exposed centers, exposed students, the route length in kilometers and the CPU time in seconds. The same results are shown in the heuristic block, on the right side of the Table.

The IP formulation finds the optima in less than 35 seconds for all instances in which the routes are taken one by one. In these cases, the heuristic found very good approximations to the solutions, never exceeding differences of 0.532%, within 2.5 seconds. When multiple OD pairs are considered (see the lowermost block of solutions), the IP formulation requires a much longer time, reaching the 3,000 seconds, while the heuristic still runs in a time that is barely longer than that required for a single pair.

Note that the number of exposed students is almost always higher in the heuristic solutions for large values of η. This is due to the fact that, although both the IP and the heuristic aim at maximizing the sum of the d_{ij}/D^q ratios, both reach their targets at different values of numerator and denominator: the IP formulation exposes less students at shorter distances, while the heuristic exposes more students, but the average distances are longer. It could even happen that a decision maker could prefer the heuristic approach.

For larger instances, although the IP formulation can still be fast for a single OD pair case, as the number of OD pairs increases, it could become intractable, particularly if the problem has to be solved repeatedly. In this case, the heuristic provides an efficient and fast method.
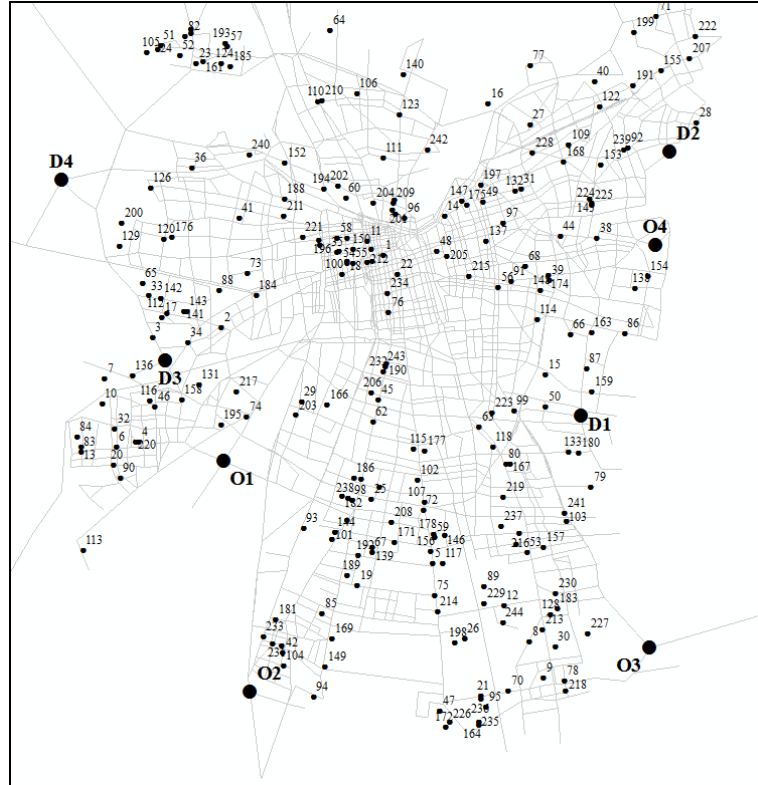
Figure 3-1: Transportation network of the city of Santiago, Chile, and 244 vulnerable centers.

	η (m)	MsHRP results, IP formulation						MsHRP results, heuristic procedure				
		Max Danger Value	Obj	Vulnerable centers exposed	Students exposed	Route Length	CPU Time	Obj	Vulnerable centers exposed	Students exposed	Route Length	CPU Time
OD Pair 1	500	0.389	243.39	1	1,089	27.67	15.37	243.39	1	1,089	29.40	2.40
	600	0.153	243.15	1	1,820	19.37	17.69	243.15	1	1,820	21.74	1.05
	700	0.153	243.15	1	1,820	20.34	18.48	243.15	1	1,820	22.79	1.12
	800	0.153	242.28	3	4,502	21.98	21.04	242.24	3	4,631	22.05	1.09
	900	0.044	242.10	3	4,539	21.86	23.16	242.02	3	5,188	24.26	0.95
	1000	0.044	241.85	4	5,803	21.91	26.5	241.75	4	6,452	25.00	1.14
OD Pair 2	600	0.496	243.01	2	2,236	31.51	16.29	243.02	2	2,236	32.10	0.89
	700	0.196	242.69	2	2,577	33.97	15.79	242.69	2	2,577	36.15	4.13
	800	0.196	241.80	4	5,380	43.41	19.45	241.77	4	5,509	39.74	3.93
	900	0.041	241.04	6	7,716	38.93	26.33	241.03	6	7,879	40.94	4.26
	1000	0.041	240.84	8	10,054	39.16	23.03	240.74	6	8,511	39.50	5.76
OD Pair 3	400	0.246	243.24	1	1,518	31.07	13.37	243.25	1	1,518	34.60	0.95
	500	0.191	242.47	2	3,236	36.96	13.57	242.48	2	3,236	40.05	0.98
	600	0.222	241.85	3	3,897	33.18	16.97	241.86	3	3,897	34.07	1.25
	700	0.023	241.64	3	3,674	34.23	18.77	241.24	4	5,940	33.83	1.21
	800	0.023	241.30	4	4,852	33.66	20.92	240.25	6	8,335	41.17	1.17
	900	0.006	240.98	5	6,788	32.80	21.19	240.34	7	8,730	41.34	1.35
	1000	0.006	240.17	8	11,225	34.17	34.98	238.89	12	14,556	33.62	1.34
OD Pair 4	600	0.520	243.52	1	1,137	41.00	11.91	243.52	1	1,137	45.66	3.82
	700	0.213	243.21	1	1,478	39.74	12.78	243.21	1	1,478	43.85	4.53
	800	0.213	242.66	2	3,146	41.75	13.43	242.66	2	3,146	42.25	3.70
	900	0.213	242.09	3	5,159	45.93	16.39	241.56	4	6,215	36.40	2.43
	1000	0.213	241.64	4	6,804	46.07	19.22	241.12	5	7,860	36.67	5.71
OD pairs 1, 2, 3	400	0.246	243.23	1	1,518	95.30	44.27	243.25	1	1,518	77.94	1.06
	500	0.246	241.91	3	4,061	101.92	58.04	241.87	3	4,325	93.52	3.44
	600	0.112	240.58	5	6,591	91.88	61.35	240.03	6	7,953	87.92	3.19
	700	0.023	239.47	6	8,071	93.10	66.94	239.08	7	10,337	92.76	6.46
	800	0.023	238.54	8	11,332	107.78	142.08	236.01	12	18,475	102.96	6.19
	900	0.041	237.66	10	13,909	106.29	3,059.95	236.63	12	21,978	106.54	6.56
	1000	0.041	236.83	15	19,824	106.54	1,674.99	233.28	22	33,334	98.13	8.25

Table 3-1: MsHRP results for different values of η.

Figure 3-2 depicts a graphic representation of some of the results in Table 3-1. We show only O_4-D_4 and pairs 1, 2 and 3 together. The circles represent the danger zones for each vulnerable center and the grey circles identify the danger zones exposed by the routes. We observe how the number of vulnerable centers and exposed students increases as the value of η increases. In order to minimize the average effect on vulnerable centers, MsHRP avoids, wherever possible, the use of links within any danger zone along the route. As the value of η increases, the number of feasible routes not entering any danger zones decreases, exposing more population centers and students to danger.

Now, as an example of the effects of considering both objectives taken separately, we compare the results of MsHRP with the Maximin HAZMAT Routing Problem (MmHRP) using the O_1-D_1 and O_3-D_3 pairs and different values of η. Table 3-2 shows the results of the MmHRP.

(a) Pair O_4-D_4, $\eta = 400$

(b) Pair O_4-D_4, $\eta = 600$

(c) Pair O_4-D_4, $\eta = 800$

(d) Pair O_4-D_4, $\eta = 1{,}000$

(e) Pairs O_1-D_1, O_2-D_2 and O_3-D_3, $\eta = 700$

(f) Pairs O_1-D_1, O_2-D_2 and O_3-D_3, $\eta = 1{,}000$
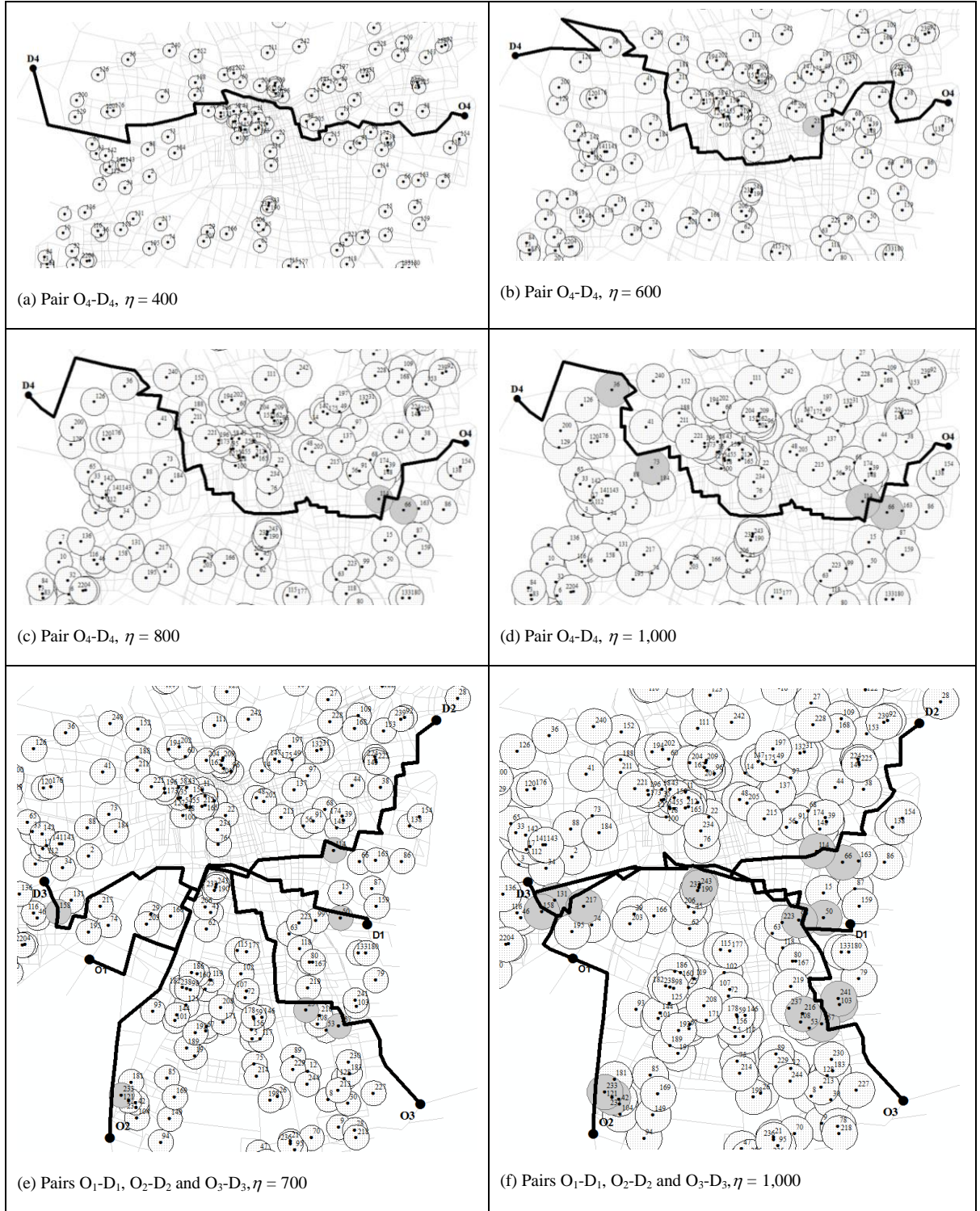
Figure 3-2: Routes obtained by the IP formulation for pair O_4-D_4 and pairs 1, 2 and 3 together, for different values of η.

In Table 3-2, a value "-" indicates that the route does not expose any vulnerable centers. As Tables 3-1 and 3-2 show when compared, for the same values of η, the optimal solution of MsHRP always exposes a lower or at most equal number of students and vulnerable centers than MmHRP. Consider the pair O_1-D_1 and $\eta = 600$ m. In this case, MmHRP exposes 6,877 students and five vulnerable centers. This is 3.8 times the number of students as the route designed using MsHRP (1,820 students and just one school exposed). The results are similar for the others pairs OD. These results suggest that there could be intermediate, non-dominated solutions that could be more appealing to the decision maker.

	η (meter)	MmHRP Value (w)	MsHRP Value	Vulnerable centers exposed	Students exposed	CPU Time MmHRP
OD Pair 1	100	-	244.00	0	0	1.09
	200	-	244.00	0	0	1.09
	300	-	244.00	0	0	1.07
	400	-	244.00	0	0	1.08
	500	0.389	243.39	1	1,089	1.06
	600	0.349	241.01	5	6,877	1.04
	700	0.349	239.91	7	9,904	1.01
	800	0.349	239.15	9	12,461	0.99
	900	0.349	237.28	14	19,441	0.97
	1000	0.349	236.16	17	23,788	0.96
OD Pair 3	100	-	244.00	0	0	1.10
	200	-	244.00	0	0	1.10
	300	-	244.00	0	0	1.09
	400	0.245	243.25	1	1,518	1.08
	500	0.245	240.00	6	8,366	1.07
	600	0.245	238.81	8	11,121	1.04
	700	0.245	237.37	11	14,913	1.01
	800	0.245	236.05	14	18,945	1.00
	900	0.245	235.65	15	20,385	0.97
	1000	0.245	235.48	16	22,003	0.96

Table 3-2: MmHRP results for different values of η.

Figure 3-3 shows the routes associated with the results listed in Table 3-1 and Table 3-2 for $\eta = 800$ and 1,000 m and pairs O_1-D_1 y O_3-D_3. In the figure, the solid line represents the route selected by MsHRP while the dotted line shows the route obtained with MmHRP. The grey circles identify the danger zones exposed by the route of the MsHRP, and the circles with a mesh filling, the zones that are exposed by the MmHRP route. The circles that have both fillings represent danger zones affected by both routes.

(a) Pair O_1-D_1, $\eta = 800$

(b) Pair O_1-D_1 $\eta = 1,000$

(c) Pair O_3-D_3 $\eta = 800$

(d) Pair O_3-D_3 $\eta = 1,000$
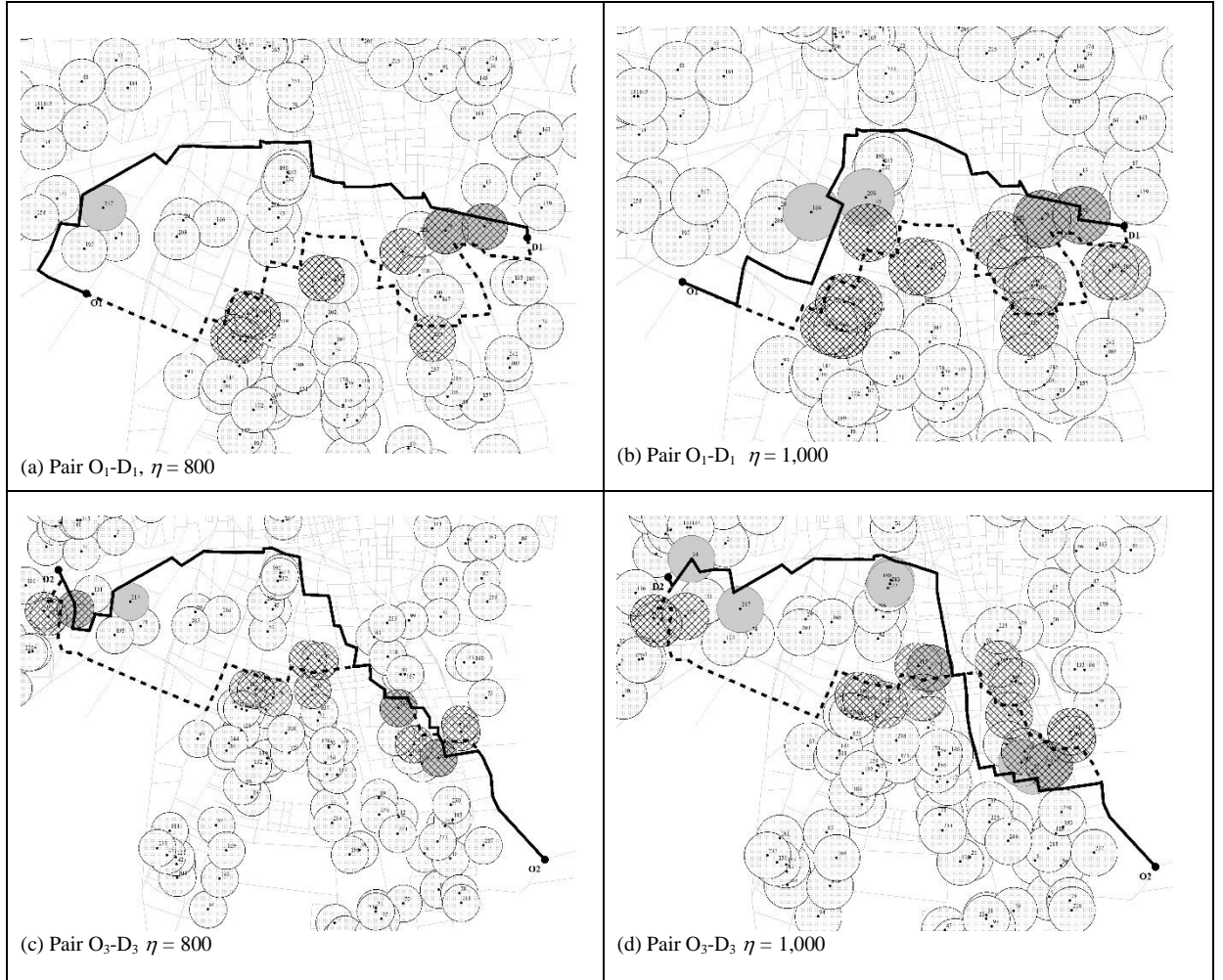
Figure 3-3: Routes for pairs O_1-D_1 and O_3-D_3, MsHRP and MmHRP with $\eta = 800$ and 1,000 m.

On the other hand, the danger proxy w of the vulnerable center closest to the selected route is considerably greater with the MsHRP for each OD pair. For O_1-D_1 and values of η between 600 and 800 m, the weighted distance from the route to the closest vulnerable center is 2.3 times smaller (the hazard is higher) in the MsHRP, with values of $w = 0.153$ in MsHRP and $w = 0.349$ in MmHRP. Intuitively, the first of these values could mean 1,000 people having a potential HAZMAT spill at 153 meters, while the second would represent the same 1,000 people exposed to an incident occurring at 349 meters, more than double the distance. For $\eta = 900$ and 1,000 m, w is eight times lower in MsHRP, with

values of 0.044 in MsHRP and 0.349 in MmHRP. For O_3-D_3, w remains a constant value equal to 0.245 for $\eta \geq 400$ m, and this value decreases in MsHRP as the value of η decreases until it reaches $w = 0.006$. In other words, in this case the weighted distance from the route to the closest vulnerable center is 40.8 times lower (more hazardous).

The results also show that as η increases and when it is no longer possible for the route selected by MmHRP to remain outside the danger zones of the vulnerable centers, a point is reached at which its objective value remains the same regardless of how large η is. Naturally, as η increases beyond that point, the number of centers and people affected increases, but the MmHRP does not recognize this. The contrary effect happens with MsHRP, in which as η increases the risk posed to the closest vulnerable center to the route increases but fewer people and vulnerable centers are exposed compared to MmHRP.

In order to conduct a more detailed analysis, we compiled Tables 3-3 and 3-4, which show the values of D^q and d_{ij}^q for each vulnerable center and for each OD pair obtained after solving MsHRP and MmHRP for $\eta = 800$ and $1,000$ m. As before, the values "-" in the table indicate that the vulnerable center is not exposed.

		Maximin HAZMAT routing problem (MmHRP)				Maxisum HAZMAT routing problem (MsHRP)			
		$\eta = 800$		$\eta = 1000$		$\eta = 800$		$\eta = 1000$	
Vulnerable Center	D^q (students)	d^q_{ij} (m)	d^q_{ij}/D^q	d^q_{ij} (m)	d^q_{ij}/D^q	d^q_{ij} (m)	d^q_{ij}/D^q	d^q_{ij} (m)	d^q_{ij}/D^q
50	1,820	671	0.369	671	0.369	279	0.153	80	0.044
62	1,695	593	0.350	593	0.350	-	-	-	-
63	1,683	-	-	877	0.521	-	-	-	-
80	1,618	-	-	942	0.582	-	-	-	-
98	1,551	-	-	970	0.625	-	-	-	-
99	1,547	541	0.349	541	0.349	713	0.461	713	0.461
115	1,469	757	0.516	757	0.516	-	-	-	-
118	1,440	-	-	861	0.598	-	-	-	-
133	1,375	-	-	859	0.624	-	-	-	-
160	1,286	596	0.464	596	0.464	-	-	-	-
166	1,264	-	-	-	-	-	-	795	0.629
167	1,263	-	-	892	0.706	-	-	-	-
180	1,219	-	-	839	0.688	-	-	-	-
182	1,217	553	0.454	553	0.454	-	-	-	-
186	1,207	643	0.533	643	0.533	-	-	-	-
203	1,178	-	-	784	0.666	-	-	-	-
206	1,172	-	-	-	-	-	-	842	0.718
217	1,135	-	-	-	-	756	0.666	-	-
219	1,132	439	0.388	439	0.388	-	-	-	-
237	1,089	-	-	-	-	-	-	-	-
238	1,088	787	0.723	787	0.723	-	-	-	-

Table 3-3: Values of d^q_{ij}, D^q and d^q_{ij}/D^q to MsHRP and MmHRP, $\eta = 800$ and $1{,}000$ m, pair O$_1$-D$_1$.

From Table 3-3, we observe that for the pair O$_1$-D$_1$ and $\eta = 800$ m, the route obtained through MsHRP exposes vulnerable centers 50, 99 and 217 with a total of 4,502 students. Vulnerable center 50, with a total of 1,820 students, is the most populated and also faces the greatest hazard as it is situated a weighted distance from the route of $w = d^{50}_{ij}/D^{50} = 0.153$. Of the vulnerable centers exposed, the least populated (vulnerable center 217, with 1,135 students) is the one that is exposed to the least amount of risk, with a value of $w = d^{217}_{ij}/D^{217} = 0.666$. For its part, MmHRP exposes 9 centers with a total of 12,461 students. Vulnerable center 99, with 1,547 students, is the most exposed with a value of $w = d^{99}_{ij}/D^{99} = 0.349$. We observe that considering the distribution of the hazard posed to the population, the MsHRP allows the most population centers to face the greatest risk, when that contributes to the objective.

This result is repeated for the other OD pairs analyzed and for the different values of η. Table 3-4 shows the same phenomenon for pair O$_3$-D$_3$.

Vulnerable Center	D^q (students)	Maximin HAZMAT routing problem (MmHRP)				Maxisum HAZMAT routing problem (MsHRP)			
		$\eta = 800$		$\eta = 1000$		$\eta = 800$		$\eta = 1000$	
		d_{ij}^q (m)	d_{ij}^q / D^q	d_{ij}^q (m)	d_{ij}^q / D^q	d_{ij}^q (m)	d_{ij}^q / D^q	d_{ij}^q (m)	d_{ij}^q / D^q
34	2,028	-	-	-	-	-	-	11	0.006
46	1,900	473	0.249	473	0.249	-	-	-	-
53	1,782	-	-	-	-	-	-	341	0.191
80	1,618	-	-	942	0.582	-	-	-	-
102	1,518	745	0.491	745	0.491	-	-	-	-
103	1,518	373	0.245	373	0.245	-	-	-	-
108	1,489	-	-	-	-	-	-	546	0.366
115	1,469	511	0.348	511	0.348	-	-	-	-
116	1,454	466	0.321	466	0.321	-	-	-	-
118	1,440	-	-	861	0.598	-	-	-	-
119	1,429	787	0.551	787	0.551	-	-	-	-
157	1,295	689	0.532	689	0.532	293	0.227	760	0.587
158	1,290	636	0.493	636	0.493	29	0.023	-	-
160	1,286	596	0.464	596	0.464	-	-	-	-
177	1,225	450	0.368	450	0.368	-	-	902	0.736
186	1,207	643	0.533	643	0.533	-	-	-	-
190	1,198	-	-	-	-	-	-	834	0.696
216	1,137	483	0.424	759	0.667	-	-	-	-
217	1,135	-	-	-	-	756	0.666	759	0.669
219	1,132	439	0.388	439	0.388	439	0.388	-	-
237	1,089	-	-	-	-	-	-	-	-
241	1,085	703	0.647	703	0.647	-	-	-	-
243	1,073	-	-	-	-	-	-	993	0.925

Table 3-4: Values of d_{ij}^q, D^q and d_{ij}^q / D^q to MsHRP and MmHRP, $\eta = 800$ and 1,000 m, pair O_3-D_3.

Finally, we analyze a different objective, which is of a great importance for the party in charge of the transportation of HAZMAT: cost. We solve the shortest path problem for all OD pairs and, for the obtained routes, compute the values of the maxisum and maximin criteria. The figures are shown in Table 3-5, for $\eta = 1,000$. We take the link length as its travel cost. Table 3-5, together with Table 1, confirms the conflict between the route cost (or length) and the danger criteria. It is interesting to remark that rather than significant changes in the danger objectives, the differences in route length have a strong effect over the number of centers and students exposed.

Pair	MsHRP Value	MmHRP Value	Vulnerable centers exposed	Students exposed	Route Length (kilometre)
O1-D1	234.97	0.135	16	21,740	17.13
O2-D2	229.40	0.035	23	33,921	29.66
O3-D3	232.84	0.005	16	21,694	27.03
O4-D4	228.62	0.122	28	43,720	28.73

Table 3-5: Shortest path problem results, $\eta = 1,000$.

3.5.2. Compromise maximin-maxisum solutions for Santiago

We now apply the two-objective MmMsHRP to the network of Figure 3-1 and for different values of η under two scenarios: only one OD pair and three OD pairs. The best and worst value (f_i^+ and f_i^-, respectively) of both objectives were obtained separately by solving MsHRP and MmHRP. For simplicity, we will only analyze the results for $\eta = 800$ to 1,000 meters.

Table 3-6 shows the solutions of the MmMsHRP for three OD pairs and different values of η. In all cases, the method found four non-dominated routes. The MmHRP objective ranges between 0.022 and 0.246.

Value η	Weight α over Maximin	MsHRP Value	MmHRP Value (w)	Vulnerable centers exposed	Students exposed	CPU Time, MmMsHRP
$\eta = 800$	≈ 0.0-0.2	239.08	0.022	7	10,233	635.3
	0.3-0.6	238.74	0.153	8	11,612	2,384.7
	0.7-0.9	237.72	0.213	10	14,719	5,754.4
	≈ 1	223.30	0.246	36	50,295	671.6
$\eta = 900$	≈ 0.0-0.5	237.78	0.112	10	14,823	4,055.2
	0.6-0.7	237.40	0.174	11	15,955	5,893.0
	0.8-0.9	236.78	0.213	12	17,983	7,649.3
	≈ 1	221.56	0.246	41	56,967	482.2
$\eta = 1000$	≈ 0.0-0.5	236.62	0.112	15	21,281	6,125.9
	0.6-0.7	236.31	0.153	16	22,568	20,802.1
	0.8-0.9	235.09	0.213	19	26,755	112,194.0
	≈ 1	219.99	0.246	45	63,927	2,653.3

Table 3-6: MmMsHRP results, pairs O_1-D_1, O_3-D_3 and O_4-D_4 together ($\eta = 800$, 900 and 1,000).

Table 3-6 shows, as expected, that there are some compromise solutions that could be of interest to the decision maker. Take $\eta = 800$ for example. The MmHRP objective ranges between 0.022 and 0.246, values obtained with both objectives independently. However, slightly decreasing the MsHRP value from 239.08 to 238.74, allows increasing the MmHRP value from 0.022 to 0.153. This last solution will most probably be very appealing for some decision maker. If we observe what happens with $\eta = 1,000$, going from the maximin solution in which $w = 0.246$ and the maxisum value is 219.99, to the next non-dominated solution, in which the respective values are pretty close (0.213 and 235.09) means reducing the number of exposed centers from 45 to 19, and students from

63,927 to 26,755. Again, an appealing change, due merely to the joint consideration of both objectives.

Figure 3-4 shows the trade-off graphs for Table 3-6. Each point represents a different route. The routes associated with each point of the efficient frontier are shown in Figure 3-5.
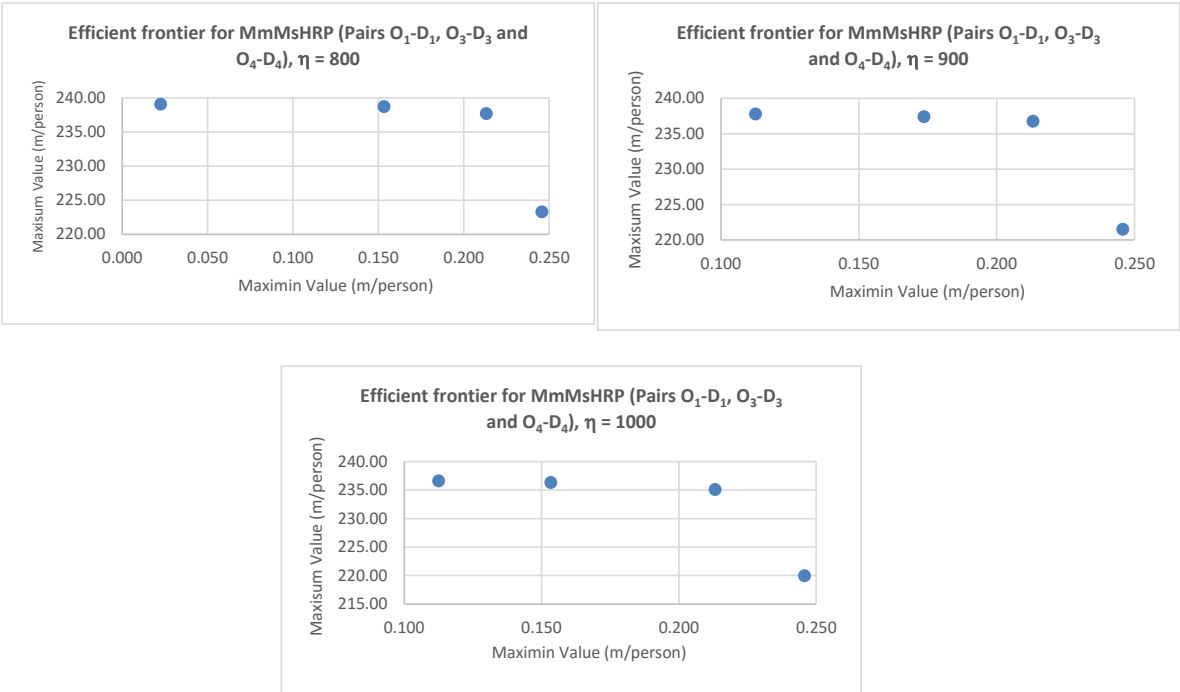
Figure 3-4: MmMsHRP efficient frontier for different values of η, pairs O_1-D_1, O_3-D_3 and O_4-D_4 together .

(a) $\eta = 800$, $\alpha = 0.2$

(b) $\eta = 800$, $\alpha = 0.4$

(c) $\eta = 800$, $\alpha = 0.8$
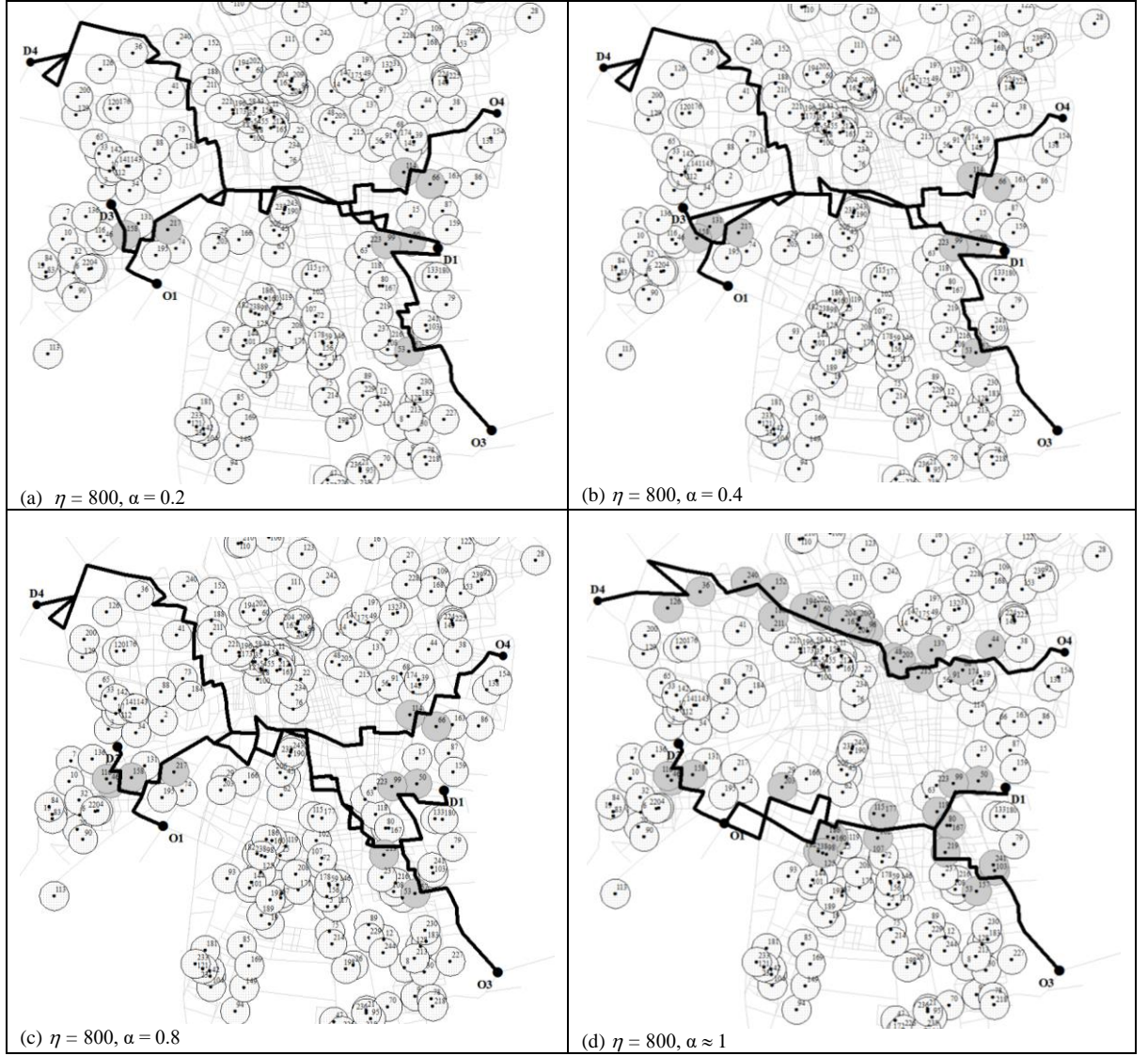
(d) $\eta = 800$, $\alpha \approx 1$

Figure 3-5: Routes for MmMsHRP with $\eta = 800$, pairs O_1-D_1, O_3-D_3 and O_4-D_4 together.

3.6. Conclusions

We present and solve a new approach to the transport of HAZMAT between multiple pairs OD in urban areas, which focuses on the protection of population that is vulnerable or difficult to evacuate, and is aligned with the public perception of hazard or danger. The method, called maxisum HAZMAT routing problem (MsHRP), maximizes the sum of the weighted distances between vulnerable centers and their closest routes links. Furthermore, we merge this method with the previously published Maximin HAZMAT Routing Problem, to propose the maximin-maxisum HAZMAT routing problem (MmMsHRP), a bi-criterion approach that considers the two objectives. In both cases, maximizing weighted distance is a proxy for minimizing hazard. Both the maxisum and the maximin-maxisum methods are adequate for finding a route over a transport network that is "as far as possible" from existing vulnerable centers. Our contribution is the consideration of these objectives, which are new in transport network routing problems.

Despite the fact that the maximin objective in and of itself can be solved in polynomial time, the new MsHRP as well as the MmMsHRP problems are NP-Hard, and we propose (as a further contribution) a tractable and fast IP formulation and an efficient polynomial heuristic procedure that allows us to solve large instances of the problems, especially for cases in which there are multiple OD pairs.

Finally, we apply these methods to a real network with multiple pairs OD in the city of Santiago de Chile, and analyze for managerial insight.

The results show that the MsHRP minimizes the hazard posed to the entire population, reducing the number of vulnerable centers and students exposed, but allows highly populated vulnerable centers to be exposed to a higher degree of hazard. The bi-criterion formulation (MmMsHRP) allows this shortcoming to be addressed. We use the weight method to generate the efficient frontier between the two objectives, presenting different

alternatives for the decision-maker, some of them seemingly more appealing for decision makers.

A last word about different objectives: travel time and probability are important indicators when dealing with HAZMAT transportation. However, not all authors use these parameters as proxies for risk or danger, and we have chosen the same approach. A large number of publications consider a combination of consequence (i.e., number of deaths or injured people, areas of contaminated zones) and probability of an incident, without taking into account travel time or travel speed (see Erkut *et al.* (2007) and Erkut and Verter (1995), for a review of the most common objectives in the literature.)

The approach we use, rather than considering overall risk, is focused on evacuation of individual vulnerable centers, something that usual risk indicators are not able to do. Secondly, we align the objective with what population perceives: danger. We use consequence (population) and distance between a possible incident and the vulnerable point. This indicator considers that people in general fears an accident happening, no matter how low the probabilities are. Thus, should it happen, they want it happening as far as possible, so the damage decreases, or at least, there is opportunity of evacuating. A clear example is the usual perception about nuclear plants. Our danger indicator increases with the population (consequence) and decreases with distance. There are authors that take the consequence as a proxy of risk, as ReVelle *et al.* (1991), but we have not seen a combination between consequence and distance.

Note, however, that the indicator we use can be easily modified to accommodate probability as well as other parameters. In fact, concerning probabilities, they could be added in the denominator of the weighted distance in our model. In other words, the population-weighted distances could be easily become a population-and-probability-weighted distances:

$$\frac{d_{ij}^q}{f(p_{ij})D^q}$$

Where $f(p_{ij})$ is a function of the probability of an incident on arc (i, j), chosen to neither cause scaling problems nor indetermination of the weighted distances. Similarly, the indicator could be multiplied by the travel time, to consider the fact that, the longer the time the vehicles spend close to the point, the higher the danger.

A different alternative is to use, in addition to the maxisum and maximin objectives, independent objectives that consider both probability and travel time.

4. INCORPORATION OF HAZARD AND PERIOD OF EXPOSURE AS OBJECTIVES IN HM TRANSPORTATION

A model for hazardous materials (HM) route design between multiple origin-destination pairs is proposed in which public concerns are taken into account, measured for the first time as a level of perceived or real hazard. Population is assumed to be distributed in discrete points in a plane, denoted population centers, each of which is surrounded by a circular hazard area. The hazard area is defined by the public perception about, or the actual reach of an accident involving HMs. The use of a route segment that falls within a hazard area for HM transportation exposes the corresponding population center to the associated hazard. The proposed methodology incorporates a population center's exposure in terms of time and level of hazard as new objectives. The level of hazard to which a center is exposed, is a function of the distance between the center and each point on the link within its hazard circle. A mathematical programming problem integrating these new objectives is formulated and solved. Finally, the methodology is applied in a real case to define an optimal HM transport route for the city of Santiago, Chile.

4.1. Introduction

The general public typically opposes any dangerous activity in its neighborhood, seeking to minimize hazard –no matter what the probability of an accident is– and individual or community period of exposure. These (hazard and period of exposure) are two measures of danger that the general public understands. This is not the only possible point of view on HM transport, though: government bodies intervene to reduce the risk to the public and the environment, thus attempting to minimize some risk indicator representing the possible consequence of an incident weighted by the probability of its occurrence, and must do so without threatening the economic viability of the dangerous activity. Truck companies or other operators involved in transporting these materials are concerned with minimizing transport costs.

Thus, the various agents that are involved in HM transport have different outlooks and divergent objectives, and a proper balance between transport cost, risk, hazard and period of exposure would be the key to designing HM transport routes that take into account the interests of all stakeholders.

Cost and risk have been dealt with in the literature and practice: the definition of transportation cost is standard, and estimators of risk have been calculated in a variety of ways and used as an estimate of the disutility imposed on the population. Erkut *et al.* (2007) identify nine different models of risk for HM routing, each using its own method of combining the likelihood or probability of an incident on route segments with the associated potential consequences, or one of these factors alone.

However, very little research has been dedicated to hazard and period of exposure, which are the concerns of the population, and they are the subject of the present study.

Hazard, according to Rasmussen (1981), is the potential of an undesirable consequence without regard of how likely it is. On the other hand, any estimator of risk takes into

consideration the probability of an incident. The announcement of the opening of a nuclear plant, for example, will generate strong public opposition, because such a plant is perceived as very hazardous by the public (high hazard), in spite of the fact that the risk to which this population will be exposed is very low, because the probability of an accident is also very low. In other words, when dealing with hazardous activities, people do not care about probabilities, just about hazard. In their minds, people implicitly consider that sooner or later an accident could happen, and the likelihood of such success does not matter much. Another characteristic of hazard is that it is perceived by the public as decreasing with distance. People prefer hazardous activities being carried as far as possible from their places. In spite of this, not all estimators of risk in the literature are explicit functions of distance. Rather, the risk, with a few exceptions, is computed considering that all people within certain distance of a segment of the route are equally affected. By combining this population with a probability, a risk estimator is computed and assigned to the corresponding route segment. Because of this, risk estimators not necessarily capture population's concerns, and we argue that these concerns seem to be better represented by an indicator of hazard.

The period of exposure is another indicator of adverse effects that is easily understood by the public, as opposed to what happens with probability. In people's minds, the longer they are exposed to a dangerous activity, the higher the likelihood of something bad happening to them. In synthesis, we propose to represent people's concern by indicators of hazard and period of exposure. Both indicators can also be adopted by government agencies in an attempt to represent the danger perceived by the population. In turn, the measure of hazard is attractive from the standpoint of damage control, given that an accident release with HM can occur in the transport process.

When applied to urban areas, as it is not possible to keep all population far from hazard, we minimize hazard and period of exposure of those places that are more vulnerable, as hospitals and clinics, schools, and senior homes, all places difficult to evacuate.

We represent the vulnerable places as points in the plane. In our formulation, instead of hazard being an attribute of the route segments, it is an attribute of the populated or vulnerable points. This allows to upper-bound the hazard to which each vulnerable point is exposed, which we do in one of the proposed models. The formulation is intended for designing the routes for HM, as opposed to choosing a route from a set of predetermined routes. It can be used in practice and several objectives can be easily combined, if desired, including risk, consequence and even probability, if there is an adequate estimator for it.

After developing our approach, we apply it to a real case of an HM routing problem with multiple origin-destination pairs in an urban area, evaluating the different objectives and alternative models that combine the estimators. The urban area in our case study is the city of Santiago, Chile. The implementation of study is supported by a geographic information system (GIS).

The remainder of the paper is organized as follows: Section 2 reviews the related literature. Section 3 introduces the estimators of hazard and period of exposure. The formulation of models using these new objectives is contained in Section 4, while Section 5 is devoted to a real case in Santiago. We conclude in Section 6.

4.2.   Literature Review

A range of objectives have been used to minimize adverse effects on the transport of HM. ReVelle *et al.* (1991) minimize exposed population; Saccomanno and Chan (1985) and Abkowitz *et al.* (1992) minimize incident probability; Pijawka *et al.* (1985), Batta and Chiu (1988), Alp (1995) and Erkut and Verter (1995) minimize the product of incident probability and incident consequence; Sivakumar *et al.* (1993), Sivakumar *et al.* (1995), and Sherali *et al.* (1997) minimize the expected consequence given that an accident occurs on the route; Erkut and Ingolfsson (2000) propose diverse objectives: minimization of the maximum population exposure; simultaneous minimization of expected value and the variance of the number of people affected by an incident, with both factors represented as

attributes of each route link; and minimization of the expected disutility, defined as $u(X) = \exp(\alpha X)$ where X is the population affected and $\alpha > 0$ a constant that measures catastrophe aversion. Abkowitz *et al.* (1992) minimize perceived risk imposed by a link, measured as pC^q where p is the probability of an incident on a link, C the incident consequence and q a risk preference parameter; and finally, Erkut and Ingolfsson (2005) assume that the occurrence of an incident terminates a trip so that a new shipment must be sent to satisfy the original demand, and thus use the total expected consequence of all the necessary trips.

The above objectives are used in various approaches for modeling HM transportation. For example, some works recognize the multiple actors involved in decision making and the multi-objective nature of the HM routing problem, such as Zografos and Davis (1989), Marianov and ReVelle (1998) and Li and Leung (2011). Considering the relationship between the carrier and the regulatory agency is a concern in Kara and Verter (2004), Erkut and Gzara (2008), Verter and Kara (2008), Bianco *et al.* (2009) and Bruglieri *et al.* (2014). Another group of works also addresses the issue of population risk equity. For example, Gopalan *et al.* (1990), Lindner-Dutton *et al.* (1991), Carotenuto *et al.* (2007) and Caramia *et al.* (2010) develop models that consider equity in the spatial distribution of risk along the generated routes. Finally, Abkowitz *et al.* (1990), Lepofsky *et al.* (1993), Lovett *et al.* (1997), Chang *et al.* (1997), Brainard *et al.* (1996), Frank *et al.* (2000), Chen *et al.* (2008) and Kim *et al.* (2011) use GIS tools to support the calculation, comparison and visualization of the attributes of alternative routes, as well as to compare different risk modeling techniques and serve as a decision support system for HM transport.

In all of the above approaches, the consequence or the risk are always attributes of a link of the route followed by the HM, rather than measured at the population centers. If two or more links affect a single center, however, the magnitude of the effect over that particular center will not be captured when a route is designed following these approaches. This was recognized by List and Mirchandani (1991), Erkut and Verter (1995) and Bronfman *et al.* (2015). List and Mirchandani (1991) begin by calculating, for each

population center, the integral over the whole route of a hypothetic function of the distance between a point representing the population center location and each point on the route. The risk associated with each route and population point is defined as a function of that integral, although they do not propose any specific functional form. The total risk posed by a given route is thus the sum of the risks each point on it poses to the various population centers. However, in their case study the authors use not this estimator but the expected fatalities as a substitute for it. Furthermore, their formulation requires that the candidate routes be explicitly enumerated and the risk posed by the use of each one of them be calculated. As it stands, it can be used only for choosing routes, not designing them. In a very complete work by Erkut and Verter (1995), a first model assumes population distributed in points (populated points) in the plane, surrounded by a danger area. The risk to which a populated point is exposed is computed as the product of the length of the route segment that falls within its danger area, times the population of the center and the probability of an incident (liberation of HM). Their second model assumes population distributed continuously and uniformly over the plane. A route segment has a rectangular hazard area around it, with a width of twice the reach of an incident (which, in turn, depends on the material being transported). For this representation to be valid, the whole area is decomposed so that all route segments are straight, and the population density is uniform around each route segment. An individual within the rectangle is exposed to a risk that is computed as in the first model, and the risk is integrated over the rectangle and assigned to the segment as an attribute of it. Both models are applied to the selection of one of a set of existing routes, rather than the design of a route composed by segments.

Hazard has not been used as an objective for HM transportation, although it has been studied in relation to dangerous activities or natural events. In those cases, it has been recognized that the hazard a population is exposed to is a function of the distance, an observation that accords with the perception of the general public for hazardous facilities (Hung and Wang (2011), Elliott *et al.* (1999), Brody *et al.* (2004) and Lima (2004), earthquakes [Lindell and Perry (2000)], hurricanes [Arlikatti *et al.* (2006)] and flooding

[Wachinger *et al.* (2013), Miceli *et al.* (2008), Heitz *et al.* (2009) and Brilly *et al.* (2005)]. Saccomanno and Shortreed (1993), Jonkman *et al.* (2003) and Fernández. *et al.* (2000) also point to this fact in their argument that the possible consequences for the population in the case of an HM spill incident vary as a function of the distance from the event.

We next propose an explicit hazard function. Our approach is similar to that of List and Mirchandani; however, we provide explicit functions of hazard, and we formulate a model that allows designing a route, as opposed to choosing one.

4.3. Proposed estimators of adverse effects

Each population or vulnerable center is represented as a point k in a plane around which a circular *hazard zone* of radius λ is defined, as shown in Figure 4-1. The links of the route consist of straight-line sections of it. Segments (a,b) of link (i,j) and (d,e) of link (j,l) are the parts of a hypothetical route that expose the population to hazard and are therefore denoted the *exposure segments*.
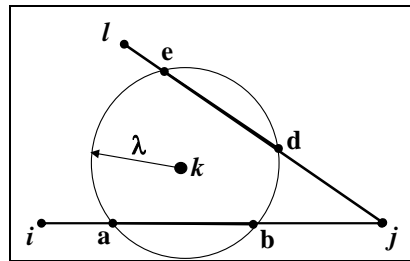
Figure 4-1: Population or vulnerable center k, with its circular hazard zone and exposure segments (a,b) and (d,e)

Figure 4-1 depicts how a population center can be affected by more than one link, especially in urban areas. By expressing adverse effects as attributes of the affected center rather than of a link, we can account for the aggregate effect of all links on a given center. The hazard imposed on k is the sum of the individual hazard values imposed by each of the two link segments (a,b) and (e,d). The period of exposure of the population of center k is

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the sum of the times during which it is exposed due to the use for HM transport of either link.

Note that, in most cases, a vulnerable center can be represented by a point on the plane. If however, population is continuously distributed over the region, strictly speaking, representation of a populated area by its central point does not introduce significant errors since population data are usually presented as discrete figures (e.g., in census publications) and therefore already contain aggregation errors that cannot be corrected by any model. Of course, if the model uses a higher level of aggregation than the source data, errors will indeed be introduced. Aggregation errors are out of the scope of this paper, and reviews of the literature on aggregation errors may be found in Sadigh and Fallah (2009) and Francis *et al.* (2004). Methods for reducing aggregation errors have been proposed by Current and Schilling (1990) and Emir-Farinas and Francis (2005). Although most of these studies focus on the problem as it arises in location modeling, the principles involved here are the same.

4.3.1. Hazard exposure of a population center

Let a transport network be represented by a directed graph $G(N,A)$, where N is the set of nodes and A the set of links. To derive a formal expression for the concept of hazard exposure, let $f^k(x)$ be the hazard to each individual in population center k emanating from a point x on link (i,j). $f^k(x)$ is a non-increasing function of the distance $r^k(x)$ between x on link (i,j) and k, and the form of the function depends on the type of material being transported. Then let f_{ij}^k be the hazard each individual in k is exposed to by the use of exposure segment (a,b) of link (i,j). To determine the value of f_{ij}^k, we divide the exposure segment (a,b) into a finite number n of intervals of equal length Δx (see Figure 4-2). Each interval represents a separate hazard to k that depends on the distance between them. Thus, the contribution to the hazard to k of an HM vehicle travelling each interval in (a,b) is

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given by $f^k(x_q)\Delta x$, where $q = 1, 2, \ldots n = \dfrac{|b-a|}{\Delta x}$. Summing the hazard represented by each interval in segment (a,b), we obtain the following approximation:

$$f^k(x_1)\Delta x + f^k(x_2)\Delta x + \ldots + f^k(x_n)\Delta x = \sum_{q=1}^{n} f^k(x_q)\Delta x$$

The value of f_{ij}^k is the limit of this sum as Δx tends to 0. Thus,

$$f_{ij}^k = \lim_{\Delta x \to 0} \sum_{q=1}^{n} f^k(x_q)\Delta x = \int_{a}^{b} f^k(x)dx \tag{4.1}$$

The hazard function $f^k(x)$ can take various forms. Some of such forms used in modeling real situations of hazardous materials dispersion involve quadratic and exponential functions, as follows.

Example 1: Hazard is inversely proportional to the square of the Euclidean distance between the population unit and the location of the HM vehicle:

$$f^k(x) = \frac{1}{\left[r^k(x)^2 + \varepsilon^2 \right]}, \tag{4.2}$$

where $\varepsilon \geq 0$ is a constant that ensures $f^k(x)$ is not undefined when $r^k(x) = 0$.

Substituting (4.2) into (4.1) and solving the integral, we obtain

$$f_{ij}^k = \frac{1}{\sqrt{h^2 + \varepsilon^2}} \left[ArcTg\left(\frac{b}{\sqrt{h^2 + \varepsilon^2}} \right) - ArcTg\left(\frac{a}{\sqrt{h^2 + \varepsilon^2}} \right) \right] \tag{4.3}$$

where h is the distance between population unit k and link (i,j), measured along a line that is perpendicular to the link.
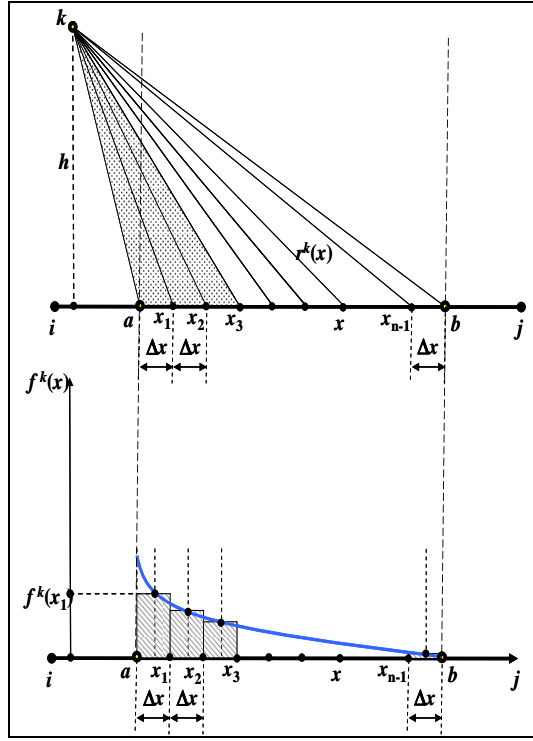
Figure 4-2: Calculation of the hazard k is exposed to by use of segment (a,b) of link (i,j)

Example 2: Hazard is an exponential function of the square of the Euclidean distance between the population unit and the location of the HM vehicle:

$$f^k(x) = e^{-\theta\left[r^k(x)\right]^2}$$

Substituting this expression into (4.1) and solving the integral, we obtain

$$f_{ij}^k = \frac{\sqrt{\pi}}{2\sqrt{\theta}} erf\left[\sqrt{h^2 + x^2}\sqrt{\theta}\right] \qquad \Rightarrow f_{ij}^k = \frac{\sqrt{\pi}}{2\sqrt{\theta}}\left[erf\left[\sqrt{h^2 + b^2}\sqrt{\theta}\right] - erf\left[\sqrt{h^2 + a^2}\sqrt{\theta}\right]\right]$$

where $erf(z)$ is the integral of the normal distribution:

$$erf(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}\,dt$$

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Example 3: Hazard is a logistic function, that is, the exponential of the Euclidean distance between the population unit and the location of the HM vehicle:

$$f^k(x) = \frac{1}{1 + e^{(\alpha + \beta r^k(x))}},$$

where α and $\beta > 0$ are parameters to be estimated.

Substituting this expression into (4.1) and solving the integral, we obtain

$$f_{ij}^k = \left(\sqrt{h^2 + b^2} - \frac{\log\left(e^{\left(\alpha + \sqrt{h^2 + b^2}\right)} + 1 \right)}{\beta} \right) - \left(\sqrt{h^2 + a^2} - \frac{\log\left(e^{\left(\alpha + \sqrt{h^2 + a^2}\right)} + 1 \right)}{\beta} \right)$$

This function can model different intensities of hazard to represent the transport of different types of HM. The smaller is the value of α, the greater is the hazard the population is exposed to, and the larger is the value of β, the greater is the decrease in the hazard as the distance to the link increases.

The total population-weighted hazard F_W^k facing a population center k due to the use of a route W for HM transport is given by the following formula, where G^k is the population of center k:

$$F_W^k = \sum_{(i,j) \in W} f_{ij}^k G^k \tag{4.4}$$

If the travel speed of HM transport over the link segment (a,b) in Figure 4-2, assumed to be constant over the segment's entire length, is doubled, the vehicle's travel time on (a,b) will be reduced by half. Although the hazard to which k is exposed to, does not change, the period of exposure of the population does. Including the additional indicator we propose next can capture this effect.

4.3.2. Period of exposure of the population

The period of exposure of the population (hereafter simply "period of exposure") depends on the length of the route segments that intercept the hazard zone of the population center k, and on the speed s_{ij} of the HM vehicles over each link (i,j). Thus, the period of exposure t_{ij}^k for k due to the use of link segment $(i,j) \in A$ is given by

$$t_{ij}^k = l_{ij}^k / s_{ij} \tag{4.5}$$

Where l_{ij}^k is the length of the segment of link (i,j) which exposes population center k. Assuming s_{ij} is uniform over each link, the period of exposure T_W^k for k due to the use of route W to transport a load of HM is given by the following formula:

$$T_W^k = \sum_{(i,j) \in W} t_{ij}^k \tag{4.6}$$

4.4. HM Routing Models with Multiple OD Pairs

In what follows we formulate two models for using and comparing two different objectives consisting of the indicators proposed in the previous section. These objectives can be easily combined with cost and risk objectives.

The first model, M_1, is bi-objective, and minimizes both total period of exposure and total hazard. The second model, M_2, minimizes total hazard, while constraining both period of exposure and individual hazard to which each vulnerable point k is exposed.

Let N^q be the set of HM shipments between the origin-destination pair $q \in Q$. We define the following binary variables:

$$x_{ij}^{tq} = \begin{cases} 1 & \text{if arc } (i,j) \text{ is used for shipment } t \in N^q \text{ between the origin-destination pair } q \in Q \\ 0 & \text{otherwise} \end{cases}$$

The first model is formulated as follows:

$$M_1) \quad Min \quad \sum_{i=1}^{2} \delta_i \left[\frac{f_i - I_i}{AI_i - I_i} \right] \tag{4.7}$$

Subject to:

$$f_1 = \sum_{k \in K} \sum_{(i,j) \in A} \sum_{q \in Q} \left[\sum_{t \in N^q} \left(f_{ij}^k x_{ij}^{tq} \right) G^k \right] \tag{4.8}$$

$$f_2 = \sum_{k \in K} \sum_{(i,j) \in A} \sum_{q \in Q} \left[\sum_{t \in N^q} \left(t_{ij}^k x_{ij}^{tq} \right) G^k \right] \tag{4.9}$$

$$\sum_{\{j/(i,j) \in A\}} x_{ij}^{tq} - \sum_{\{j/(j,i) \in A\}} x_{ji}^{tq} = \begin{cases} 1 & \text{if} & i = O^q \\ -1 & \text{if} & i = D^q \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in N, \forall q \in Q, \forall t \in N^q \tag{4.10}$$

$$x_{ij}^{tq} \in \{0,1\} \qquad \forall (i,j) \in A, \forall q \in Q, \forall t \in N^q \tag{4.11}$$

Expression (4.7) corresponds to a normalized linear combination of expressions (4.8) and (4.9), which are the population-weighted hazard and period of exposure objectives, respectively. In (4.7), I_i is the best (lowest) possible value of objective f_i, and AI_i is its worst (highest) value. By normalizing the objectives, we avoid scaling problems. Each objective is multiplied by a weight factor δ_i between 0 and 1, with $\delta_1 + \delta_2 = 1$ which is changed in successive runs of the problem, to find an approximation of the efficient frontier [Cohon (1978)]. Constraint set (4.10) represents flow conservation while (4.11) defines the nature of the variables.

Note that M_1 is separable by origin-destination pairs. As in this model the adverse effects (or perceptions) are aggregated over the whole network, this model does not take into account the fact that for a particular center, the hazard or the period of exposure can be

very high. Our second model M_2, addresses the issue. In M_2, one of the objectives is minimized. Alternatively, it is possible to minimize a weighted sum of both objectives. Or only one of them. Without loss of generality, we have chosen to minimize the total hazard. M_2 is as follows:

$$M_2) \quad Min \quad \sum_{k \in K} \sum_{(i,j) \in A} \sum_{q \in Q} \left[\sum_{t \in N^q} \left(f_{ij}^k x_{ij}^{tq} \right) G^k \right]$$

Subject to: (4.10)-(4.11)

$$\left[\sum_{(i,j) \in A} \sum_{q \in Q} \sum_{t \in N^q} \left(f_{ij}^k x_{ij}^{tq} \right) \right] G^k \leq \beta^k \qquad \forall k \in K \qquad (4.12)$$

$$\left[\sum_{(i,j) \in A} \sum_{q \in Q} \sum_{t \in N^q} \left(t_{ij}^k x_{ij}^{tq} \right) \right] G^k \leq \alpha^k \qquad \forall k \in K \qquad (4.13)$$

Where β^k and α^k can be set by the decision maker to represent different "protection levels," e.g., for centers k of different vulnerability.

4.5. Application

The models were applied to the real case of the transport of hazardous industrial solid waste (HW) between five origin-destination pairs in the city of Santiago, Chile (see Figure 4-3 and Table 4-1). The road network and vulnerable centers data are the same as those used in chapter 2 and 3, consisting of 6,681 links, 2,212 nodes and 244 vulnerable centers (schools with over a one thousand seventy students) populated by 386,254 people (students,) distributed as shown in Figure 4-3. The hazard zone radius of an HM incident is $\lambda = 800$ m. For each network link, the data include its length, travel speed for different times of day (morning peak, evening peak and off-peak period) and geographic coordinates. The transport of HM is evaluated during the morning peak period, because students are at schools at these times.
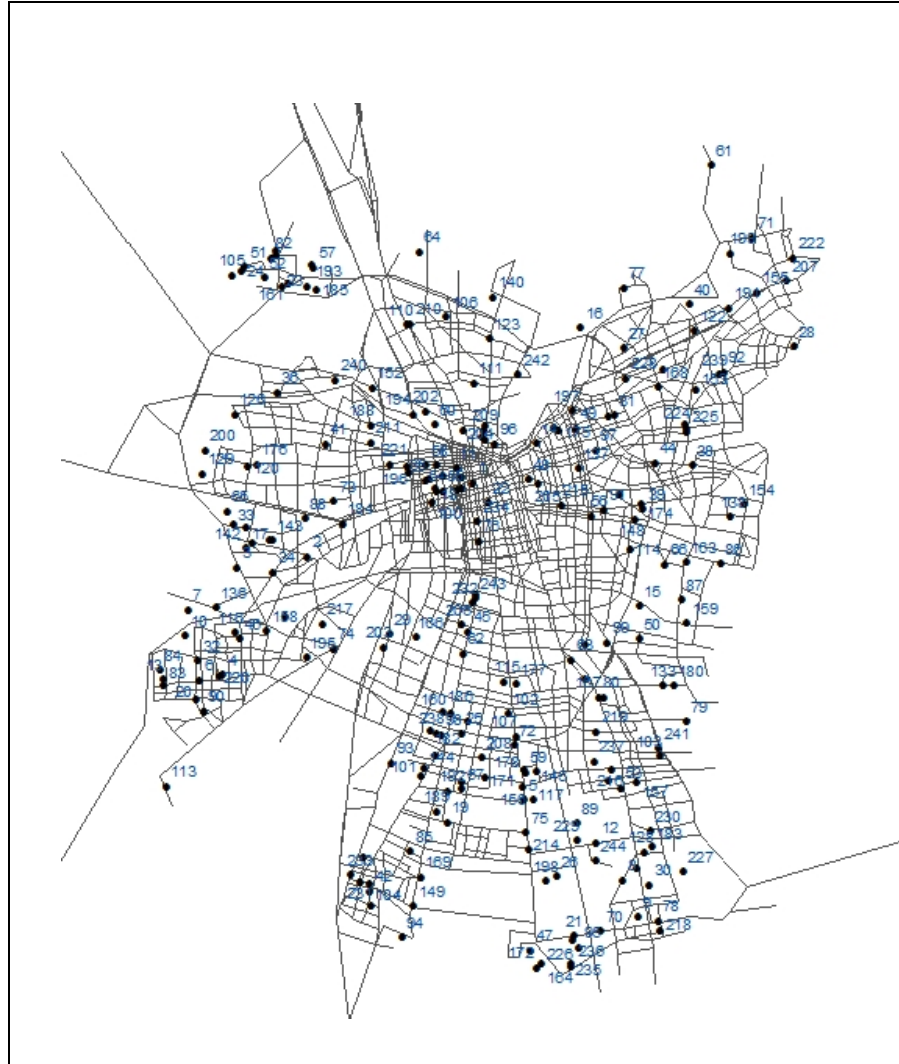
Figure 4-3: Transport network and 244 schools with over a one thousand seventy students (vulnerable centers).

Origin-destination pair	Shipments
O1-D1	2
O2-D2	1
O3-D3	1
O4-D4	3
O5-D5	1

Table 4-1: HM shipments by origin-destination pair, at morning peak period

The students in each school are assumed to be concentrated at its center. We identified the intersections of links with the hazard circles (exposure segments) of each school center

k using simple geometry and a GIS. We then applied equations (4.3) and (4.5) to evaluate the hazard and period of exposure for each population center due to HM transport on each network link. The hazard function was assumed to be the inverse of the square of the distance as in equation (4.2) above, with $\varepsilon = 10^{-10}$. To calculate the hazard exposure for each k, f_{ij}^k as given by equation (4.3) was divided by $\theta = \mathrm{Max}\{\theta^k \mid k \in K\}$, where $\theta^k = \max_{(i,j)\in U^k}\{f_{ij}^k G^k\}$ and U^k is the set of links $(i,j) \in A$ with segments within the hazard circle of k. The resulting hazard values are dimensionless. The instance was solved on a personal computer running Ubuntu 12.04 LTS with a 3.40 GHz Intel ® Core™ i7-2600 processor and 16 GB of RAM. The models were coded and solved using AMPL Cplex 12.5.

4.5.1. Results for M_1, including the effects of the new objectives on transportation costs

We solve M_1 for different values of the weight δ_1 and approximate the efficient frontier. As this version of the problem takes into account only the public point of view, we then analyze the effect of considering each one of the new objectives on the transportation costs —the transportation company concern— represented by the total distance traveled $\sum_{(i,j)\in A}\sum_{q\in Q}\left[\sum_{t\in N^q} l_{ij} x_{ij}^{tq}\right]$, where l_{ij} is the length of arc (i,j). Bi-objective model M_{1*} uses as objectives the total exposure time and the transportation cost, while Bi-objective model M_{1**} trades off the total hazard imposed on the population against the total transportation cost.

The values of I_i and AI_i shown in Table 4-2 to Table 4-4 were obtained by solving each bi-objective model with extreme values of the weights δ_1. Table 4-2 to Table 4-4 and Figure 4-4 to Figure 4-6 show the efficient frontier approximations for the three versions of M_1. Also shown are the corresponding values of δ_1.

δ_1	Hazard	Period of Exposure (hours-person)
≈ 0.0	$52.9 = AI_1$	$1,221.5 = I_2$
0.1	36.7	1,231.7
0.2	34.4	1,248.0
0.3	32.4	1,286.3
0.4	32.4	1,286.3
0.5	32.4	1,286.3
0.6	23.2	1,819.2
0.7	22.5	1,900.9
0.8	22.5	1,900.9
0.9	22.3	1,943.3
≈ 1.0	$21.8 = I_1$	$2,784.4 = AI_2$

Table 4-2: Approximation of the efficient frontier for M_1.
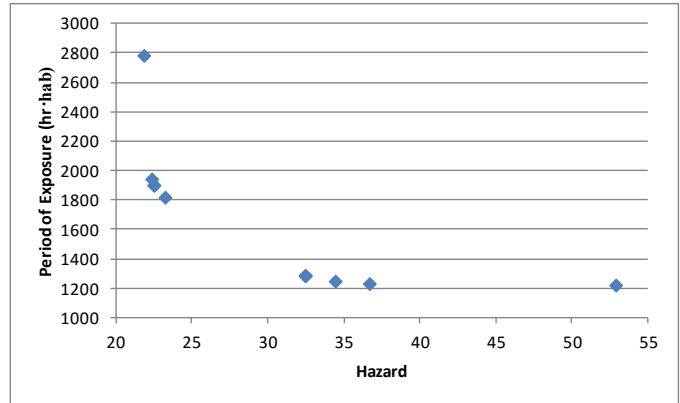
Figure 4-4: Approximation of the efficient frontier for M_1.

δ_1	Period of Exposure (hours-person)	Length (km)
≈ 0.0	$12,340.3 = AI_2$	$201.94 = I_3$
0.1	8690.7	203.1
0.2	6575.1	205.6
0.3	4577.3	211.0
0.4	4113.8	213.1
0.5	2043.2	224.6
0.6	2008.3	224.9
0.7	1833.7	227.1
0.8	1479.9	233.8
0.9	1444.7	234.9
≈ 1.0	$1,221.5 = I_2$	$278.60 = AI_3$

Table 4-3: Approximation of the efficient frontier for M_{1*}.
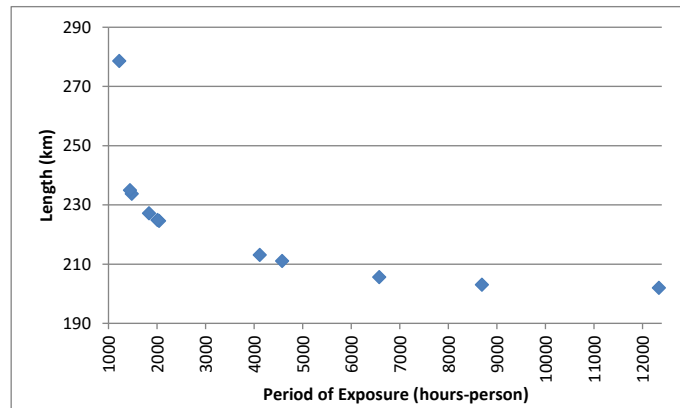
Figure 4-5: Approximation of the efficient frontier for M_{1*}.

δ_1	Hazard	Length (km)
≈ 0.0	$1502.6 = AI_2$	$201.94 = I_3$
0.1	270.0	202.5
0.2	256.3	202.7
0.3	223.2	203.4
0.4	167.5	205.6
0.5	143.4	207.2
0.6	143.4	207.2
0.7	78.2	217.4
0.8	65.1	220.3
0.9	43.2	228.6
≈ 1.0	$21.8 = I_2$	$311.0 = AI_3$

Table 4-4: Approximation of the efficient frontier for M_{1**}.
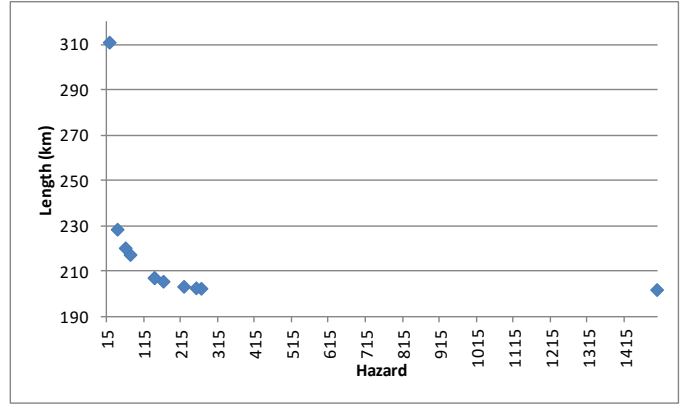

Figure 4-6: Approximation of the efficient frontier for M_{1**}.

Table 4-2 shows how, going from $\delta_1 \approx 1$ to $\delta_1 \approx 0$ in M_1, the total hazard goes from 21.82 to 52.87, an increase of 2.4 times, while the period of exposure decreases from 2,874 to 1,222 hours-person, a reduction of 57%. Good compromise solutions can be found in the efficient frontier, e.g., hazard can be reduced from its maximum at 52.9 to only 32.4, by a small increase in time of exposure (from 1,222 to 1,286 hrs-person).

Table 4-3 shows that a reduction of a 38% in the total transportation cost corresponds to an increase in the period of exposure from 1,221.5 a 12,340.3 hours-person, more than 10 times. Again, if transportation cost is increased from its minimum value by only a 4.5%, the period of exposure decreases to a 37% of its initial value ($\delta_1 = 0.3$). Finally, Table 4-4 shows how an increase of a 54% of the transportation cost corresponds to an increase in hazard from 21.8 to 1,502.6, equivalent to 68.9 times. A small increase of the transportation cost of a 7.1%, reduces hazard in 19 times.

The Figure 4-7 show the transportation paths for the extreme values of δ_1 for each bi-objective model. Origins and destinations are marked in the Figures, except for O3, which is out of the limits of the drawings.
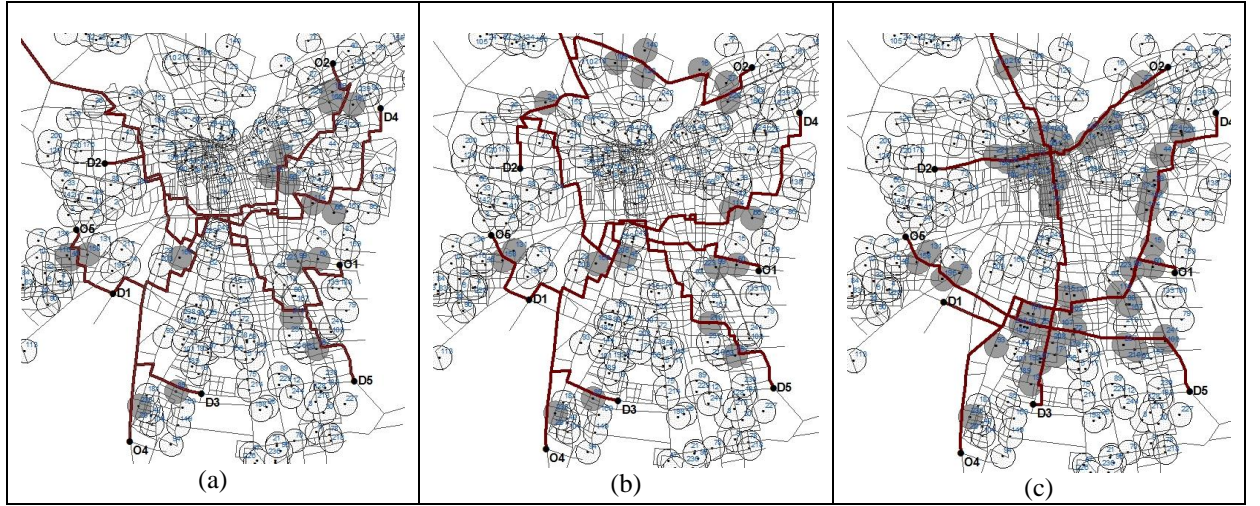
Figure 4-7: HM flows (a) Model M_1 with $\delta_1 \approx 1$, (b) Model M_1 with $\delta_1 \approx 0$, (c) Model M_{1*} and M_{1**} with $\delta_1 \approx 1$ (shortest paths)

The hazard areas marked in gray are those intersected by the route, for one or more HM shipments.

Finally, these three Tables allow choosing a strategy of good compromise between hazard, period of exposure and transportation cost.

4.5.2. Model M_2

Analysis of M_2 for different values of α^k and β^k

We solved M_2 for different values of α^k y β^k, setting the value of β^k at a value high enough ($\beta^k = 30 \; \forall \; k \in K$), to leave constraint (4.12) inactive. The parameter α^k was given values in the range (132.23; 414,99), in steps representing increases of 10% over the previous value. For $\alpha^k < 132.23$ hours-person $\forall \; k \in K$, there is no feasible solution and for $\alpha^k > 414.99$ hours-person $\forall \; k \in K$, we obtain the unconstrained solution for minimum hazard. Table 4-5a and Figure 4-8a show the results. In this instance, an increase of a 33% in the maximum period of exposure of each populated or vulnerable center (132.23 to 176.00 hours-person), the total hazard is reduced in 25.45%. Table 4-5b and Figure 4-8b

show the results of a similar exercise, when leaving α^k fixed at 460 hours-person and changing now the value of β^k by steps of a 10% starting from its minimum feasible value. In this case, a tighter constraint on the individual hazard does not increase total hazard in a significant amount (just 12.09%).

α^k	Hazard	Period of Exposure (hours-person)	CPU Time
132.23	33.41	2,621.61	441.52
145.45	26.85	2,091.55	13.62
160.00	26.21	2,250.79	44.38
176.00	24.91	2,191.14	9.59
193.60	23.77	2,366.81	5.09
212.96	23.00	2,215.55	4.60
234.26	23.00	2,258.52	7.24
257.68	22.39	2,297.96	2.83
283.45	22.18	2,602.25	6.09
311.80	22.18	2,602.25	3.38
342.97	21.87	2,691.68	1.58
377.27	21.87	2,691.68	1.77
414.99	21.82	2,784.42	1.26

β^k	Hazard	Period of Exposure (hours-person)	CPU Time
2.20	24.83	3,807.32	8.25
2.42	23.65	3,382.72	2.58
2.66	23.60	3,475.46	2.20
2.93	23.60	3,475.46	2.47
3.22	23.60	3,475.46	3.12
3.54	23.60	3,475.46	2.80
3.90	23.18	3,096.51	2.66
4.29	22.71	3,129.94	2.11
4.72	22.71	3,129.94	2.17
5.19	22.71	3,129.94	2.80
5.71	22.52	2,734.28	2.49
6.28	21.82	2,784.42	1.08
6.90	21.82	2,784.42	1.06

(a) M_2 with $\beta^k = 30 \ \forall \ k \in K$ and different values of α^k

(b) M_2 with $\alpha^k = 460$ hours-person $\forall \ k \in K$ and different values of β^k

Table 4-5: Hazard and total period of exposure for different values of α^k and β^k.

(a)

(b)
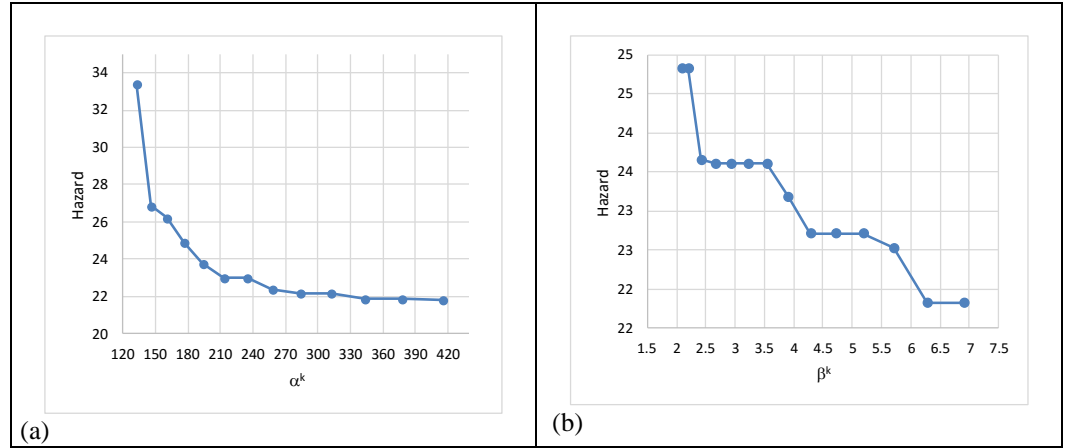
Figure 4-8: Hazard and total period of exposure for different values of α^k and β^k. a) varying α^k and $\beta^k = 30 \ \forall \ k \in K$; b) varying β^k and $\alpha^k = 460$ hours-person $\forall \ k \in K$.

Effects of constraining hazard and period of exposure at individual points

The effects of incorporating the constraints on individual hazard and period of exposure are shown in Table 4-6, Table 4-7 and Figure 4-9. These Tables and Figure show the results of M_2, compared with the results of M_1 for $\delta_1 \approx 1$ and $\delta_1 \approx 0$. The first column of Table 4-6 displays the identification of each vulnerable center exposed to one or more arc segments of the HM routes. The second column shows the number of students in each School. The third, fourth and fifth columns show the hazard and, in parenthesis, the period of exposure of each School for model M_1 with $\delta_1 \approx 1$, M_1 with $\delta_1 \approx 0$, M_2 without constraints (4.13) and M_2, respectively. The chosen values of β^k and α^k are indicated in the top of each column.

Vulnerable center	No. Of students	M_1 with $\delta_1 \approx 1$ (Minimum hazard)	M_1 with $\delta_1 \approx 0$ (Min period of exposure)	M_2 without (4.13) and β^k = 2.2	M_2 with β^k = 2.2 and α^k = 160 (hours-person)
16	2,612	0 (0)	4.14 (136.5)	0 (0)	0 (0)
27	2,094	0 (0)	0.73 (18.8)	0 (0)	0 (0)
39	1,940	0 (0)	0 (0)	0 (0)	2.02 (153.6)
44	1,922	0 (0)	0 (0)	0 (0)	0.57 (94.0)
46	1,900	1.01 (274.7)	0 (0)	1.01 (274.7)	0.22 (120.8)
48	1,831	0 (0)	0 (0)	1.58 (264.6)	1.66 (101.0)
50	1,820	0.88 (210.5)	5.49 (128.2)	0.88 (210.5)	1.61 (128.8)
56	1,750	0.69 (71.5)	0 (0)	1.38 (259.2)	0.80 (67.1)
63	1,683	0 (0)	0 (0)	0 (0)	0.44 (47.4)
66	1,668	0.39 (31.0)	0.39 (31.0)	0.13 (10.3)	0.13 (154.7)
68	1,664	0 (0)	0 (0)	1.30 (113.9)	1.30 (113.9)
85	1,602	2.01 (94.0)	2.01 (94.0)	2.01 (94.0)	2.01 (94.0)
91	1,580	0 (0)	0 (0)	0 (0)	0.03 (4.6)
99	1,547	0.32 (150.0)	0.87 (88.0)	1.26 (197.4)	0.32 (150.0)
106	1,495	0 (0)	0.38 (16.4)	0 (0)	0 (0)
109	1,487	0.92 (70.2)	0 (0)	0.92 (70.2)	0.92 (70.2)
114	1,478	6.17 (318.1)	17.1 (263.9)	2.06 (106.0)	2.06 (87.6)
116	1,454	0.72 (153.2)	0 (0)	0.72 (153.2)	0.19 (100.0)
118	1,440	0 (0)	0 (0)	0 (0)	0.55 (65.7)
122	1,414	0 (0)	0 (0)	0 (0)	0.41 (34.4)
123	1,412	0 (0)	0.45 (13.3)	0 (0)	0 (0)
131	1,379	0 (0)	0.51 (13.0)	0 (0)	0.51 (13.0)
140	1,359	0 (0)	0.15 (7.1)	0 (0)	0 (0)
137	1,365	0.25 (129.6)	0 (0)	0.64 (300.2)	1.82 (138.1)
145	1,326	0 (0)	0 (0)	0.86 (71.1)	0.43 (35.5)
148	1,320	0 (0)	0 (0)	0 (0)	2.00 (144.9)
153	1,310	0.58 (80.0)	0 (0)	1.37 (146.1)	0.39 (33.0)
157	1,295	1.36 (77.5)	1.36 (77.5)	1.36 (77.5)	1.36 (77.5)
158	1,290	0.66 (79.2)	6.71 (22.7)	0.66 (79.2)	1.63 (61.7)
166	1,264	0.07 (329.7)	0.27 (22.1)	0.07 (329.7)	0.31 (111.5)
168	1,255	0.02 (31.9)	0 (0)	0.34 (335.7)	0.16 (151.9)
174	1,237	0 (0)	0 (0)	0 (0)	0.83 (74.9)
179	1,221	0.13 (45.2)	0 (0)	0.13 (45.2)	0.13 (45.2)
205	1,173	0 (0)	0 (0)	0.73 (95.7)	0.45 (37.1)
206	1,172	0 (0)	2.02 (51.1)	0 (0)	0 (0)
215	1,137	0.89 (180.2)	0 (0)	1.55 (255.1)	2.06 (145.9)
216	1,137	0 (0)	0 (0)	0 (0)	0.35 (92.0)
219	1,132	2.66 (411.2)	4.25 (86.3)	1.77 (271.1)	0.89 (131.0)
223	1,128	0 (0)	0 (0)	0 (0)	0.69 (78.6)
224	1,128	0 (0)	0 (0)	0 (0)	0.30 (71.0)
225	1,123	0 (0)	0 (0)	0 (0)	0.63 (66.3)
228	1,110	0 (0)	0.93 (18.3)	0 (0)	0 (0)
233	1,099	2.09 (46.7)	2.09 (46.7)	2.09 (46.7)	2.09 (46.7)
237	1,089	0 (0)	0 (0)	0 (0)	0.98 (147.4)
240	1,085	0 (0)	3.06 (86.6)	0 (0)	0 (0)
Total	64,927	21.,82 (2,784.4)	52.87 (1,221.5)	24.82 (3,807.3)	33.26 (3,291.1)

Table 4-6: Values obtained for M_1 with $\delta_1 \approx 1$, M_1 with $\delta_1 \approx 0$, and M_2, broken down by vulnerable center exposed

Attribute	M_1 with $\delta_1 \approx 1$ (Minimum hazard)	M_1 with $\delta_1 \approx 0$ (Min period of exposure)	M_2 without (4.13) (β^k = 2,2)	M_2 (β^k = 2,2 and α^k = 160 hours-person)
Exposed students	27,074	27,913	33,068	52,588
% Exposed students	7.0%	7.2%	8.6%	13.6%
Total affected vulnerable centers	19	19	23	37
CPU Time (seconds)	1.13	1.08	9.89	121.30

Table 4-7: Values obtained for M_1 with $\delta_1 \approx 1$, M_1 with $\delta_1 \approx 0$, and M_2.

Table 4-7 shows an apparent dominance of model M_1 over M_2 in terms of exposed students and affected schools. However, when the results are analyzed by vulnerable

center, as in Table 4-6, hazard and period of exposure are concentrated in a few vulnerable centers. For example, when $\delta_1 \approx 1$, schools 114 (1,478 students) and 219 (1,132 students) concentrate the 40.5% of the total hazard, and are exposed during long periods (318.1 and 411.2 hours-person, respectively). In this case, the average period of exposure per person is 2.57 minutes.

When M_1 is solved with $\delta_1 \approx 0$, schools 16, 50 and 114 (representing a 9.1% of the students) concentrate the 43.3% of the total period of exposure. However, the average period of exposure per person drops to 1.13 minutes, a decrease of 56.1%. At the same time, school 158 is exposed to the 12.7% of the total hazard during a 1.9% of the total period of exposure.
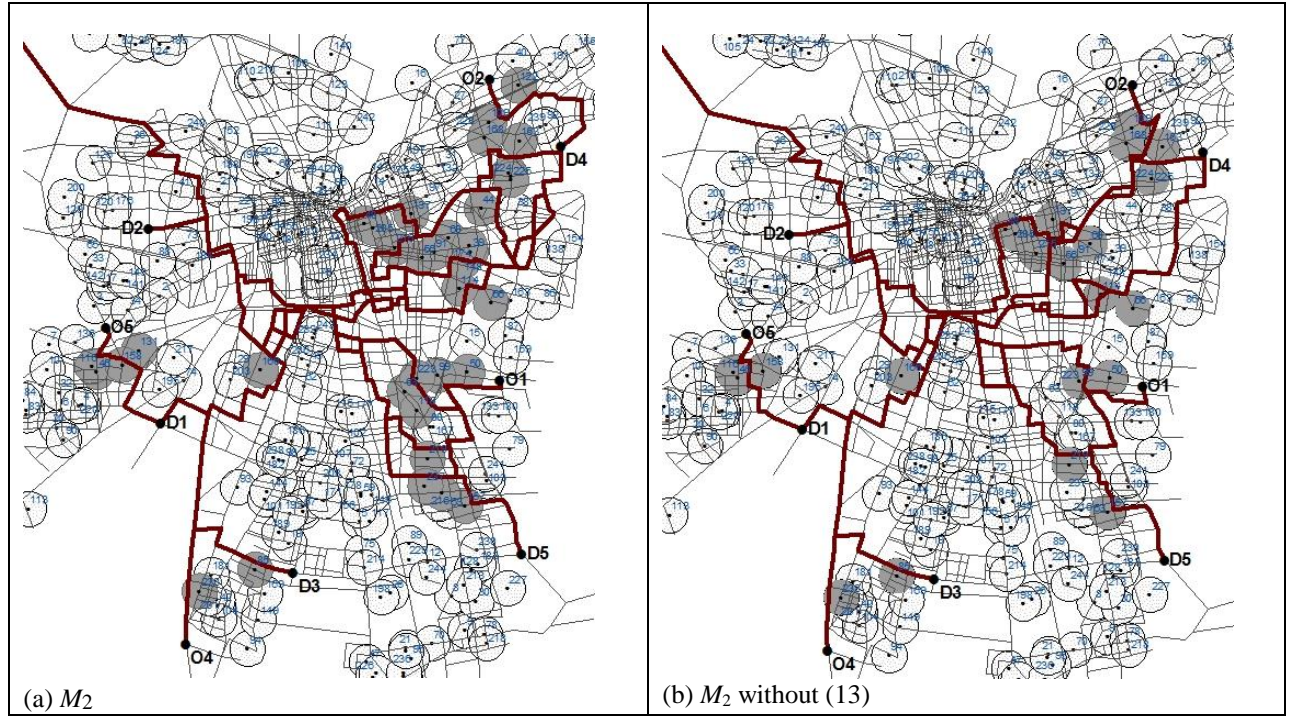

(a) M_2 (b) M_2 without (13)

Figure 4-9: HM flows (a) Model M_2, $\beta^k = 2.2$ and $\alpha^k = 160$ (hr·Hab) $\forall$ $k \in K$; (b) Model M_2, $\beta^k = 2.2$ $\forall$ $k \in K$;

When considering M_2 without constraints (4.13), hazard is shared among more schools, and none of them is overexposed. However, the period of exposure can increase

significantly for some centers, e.g., 56, 137 and 168, which together increase from a 8.4% to a 23.5% of the total period of exposure, when compared with M_1 with $\delta_1 \approx 1$. The average period of exposure per student increases also, to 3.52 minutes. When constraints (4.13) are incorporated, this increase in period of exposure is controlled.

Naturally, there is no free improvement of the individual indicators: the imposed limit on the individual hazard and period of exposure, in this case, is obtained at the expense of an increase of a 52.4% in the aggregated hazard and an increase of a 169.4% in the total period of exposure, as well as an increase of 62.8% in the average period of exposure per student. Also, the number of exposed schools and total number of exposed students increase.

These results are mainly due to the selected very tight values for $\alpha^k = 160$ hours-person and $\beta^k = 2.2$. Recall that the smallest value that α^k and β^k can take are 130 hours-person and 2.2, respectively. For higher values of these parameters, the observed increases in hazard and total exposure period will be naturally lower.

The point here is, however, that the decision maker can find an adequate compromise between total hazard and period of exposure and individual values of both, i.e., equity of exposure, while keeping transportation costs within reasonable values. The models we propose are a useful tool for evaluating each strategy.

4.6. Conclusions and future research

We present an approach to the HM transport route design problem that can be applied to real-world situations. Population in our approach is distributed in discrete points or centers in a plane, with a circle of radius λ around each one determining the associated hazard zone. The use by HM transport of any link segment within that zone constitutes a hazard to the population center.

A general hazard function is defined and a period of exposure function is added, both independent of incident consequence and probability. Both the hazard and the period of exposure of the population, represent the interests of the general public. The indicators are formulated as attributes of the population centers rather than the network links, thus allowing the hazard and period of exposure imposed by several route links to be adequately added for a single populated point, and upper-bounded.

The proposed methodology was applied to a real instance of HW transport in Santiago, Chile. The results demonstrate that hazard exposure is satisfactory as an objective when it is minimized together with the period of exposure. Both objectives can be traded off against transportation costs.

Trading off total hazard and total period of exposure in a two-objective model exposes some of the vulnerable centers to high levels of both hazard and period of exposure. Consequently, we propose a second model that minimizes the total hazard subject to limits on the hazard and exposure period on each population center. We conclude that the incorporation of such thresholds can control the maximum hazard and period of exposure for each population center, naturally at the expense of increased total hazard and total exposure period of the population. The adequate compromise can be easily explored by the decision maker.

The proposed objectives can be combined with other objectives, as risk. Probabilities of events can be included in the models if desired. If speed statistics are known over the network, they can be used to improve the estimations of periods of exposure. The models can be solved for different times of the day, to consider the different distributions of the population along the day.

Yet other possibilities opened up by the proposed approach of representing the undesirable effects of HM transport as attributes of population centers rather than network

links, would be to include emergency response center locations and HM routing as a combined factor in HM transport network design.

5.   CONCLUSIONS

We address the Hazardous Material (HAZMAT) Transportation Problem in urban areas with high population density through three novel techniques, focusing on population protection.

We consider that the relevant population, i.e. most vulnerable or hard to evacuate, is concentrated in points or centers of the plane. We determine a circular danger zone around each of these centers. If an accident occurs inside the danger zone, the respective(s) center(s) would be affected. Then, using an arc that is (partially or completely) within a danger zone must imply danger to the center. Using this representation, we are able to determine both risk and danger that affects individual population centers, calculation that previous risk indicators in literature are unable to do.

In order to represent the interests of the population, we focus our attention to the mainly perceived objective: danger. We use as danger indicator the consequence (population) and distance between the incident and the population center. This indicator considers that the population is insensitive to probabilities, and is only concerned with the fear of an accident. Then, we aim to maximize the distance between HAZMAT shipments and the population centers, minimizing the perceived danger, together with the potential consequences of a HAZMAT liberation event.

From a different point of view, our focus is different from the usual risk minimization, in what it cares about consequences once an accident has occurred and evacuation must be performed of the population in danger. That is the reason why we take into account vulnerable and difficult to evacuate people, as opposed to what is regularly done, which is minimizing population risk, without making any distinction between different classes of population. Both approaches are valid, and complementary.

Based on these considerations, we first aim to maximize the weighted distance between HAZMAT route and the closest population center. The goal is to reduce the possible consequences for the most exposed population center (maximin objective). Although the maximin objective has been addressed on continuous problems, it has not been studied before in real transportation networks. We propose an exact model and a heuristic procedure, able to solve large size instances up to optimality. Both methods were tested in a real case study for HAZMAT transportation in the transportation network of Santiago, Chile. The results of the exact model show how the solutions change depending on the radio of the danger zones. This problem is easily solved using the proposed heuristic.

Note that avoiding danger zones is attractive both for population and regulators. From an opposite side of the road, freighters aim for operating cost minimization. Because of that, we then formulate a bi-criteria problem that captures the relationship between the maximin and minimum cost objectives. The approach is able to generate a set of efficient solutions in a real-size instance, to show the effects of different policies.

The second proposed approach considers HAZMAT transportation between multiple OD pairs in urban zones. It also focuses on the protection of vulnerable and hard-to-evacuate population. The method, denoted maxisum HAZMAT routing problem (MsHRP), maximizes the weighted sum of the distances between vulnerable centers and the arcs of the closest routes (maxisum objective). Then, we combine the maximin and maxisum objectives in the maximin-maxisum HAZMAT routing problem (MmMsHRP), as a bi-criteria approach that considers both objectives. In both cases, maximizing the weighted distances is a proxy of minimization of the danger. The maxisum and maximin-maxisum methods are appropriate to find a route over the transportation network that is 'as far as possible' from the existing vulnerable centers.

As both the MsHRP and the MmMsHRP belong to the class of NP-Hard problems, we propose MIP formulations and heuristic procedures. These procedures allow us to solve large instances of these problems, which is particularly relevant when multiple OD pairs

are considered. Results show that the MsHRP minimizes the danger to which all the population is exposed, reducing the number of vulnerable centers and potentially exposed individuals. On the other hand, there are vulnerable centers that are exposed to a large danger. This issue is addressed by the bi-criteria formulation (MmMsHRP).

In our third approach, we define a general danger function, adding population exposure as a new estimator. The results show that adding both the total danger and exposure time in a bi-objective model allow the appearance of solutions where some vulnerable centers experience high danger levels and exposure periods. Then, we propose a second model that minimizes the total danger to which population is exposed, subject to that both the exposure period and danger for every population center are lower than some predefined upper bounds. We conclude that adding these upper bounds allow us to control the maximum danger and exposure time of every population center. As a trade-off, the total danger and exposure period increased for all the population.

We highlight that the objectives proposed in this thesis can be combined with other risk and danger estimators from the literature. Finally, the modeling approaches proposed on this thesis allow us to support the HAZMAT transportation decision making. We are able to protect the most vulnerable or hard-to-evacuate population, if a HAZMAT incident occurs. Using our approaches, the stakeholders involved in decision making could have a set of efficient solutions. These represent their preferences and interests, but also the consequences (danger) to the population. Then, our approaches are attractive to support decision making from a danger-control perspective.

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