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Integrating Mining Loading and Hauling Equipment Selection and Replacement Decisions Using Stochastic Linear Programming

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Abstract

Equipment selection is a key strategic decision in the design of a material handling system, because an improper one will lead to operational problems and unnecessary investment costs. It involves determining the number and combination of loaders and trucks which will move the material, fulfilling a specified production schedule. Previous works have addressed this problem with deterministic approaches, without considering the inter-dependent availability of trucks and loaders. In order to fill this gap, we developed a stochastic model that combines the selection and equipment replacement problems, subject to a stochastic production rate constraint. This is a new idea that will help decision makers to decide faster and more reliable. The proposed model optimizes the fleet by minimizing the total life-cycle costs. To solve it we used a linearization approach that reduces the computational effort. We tested our approach with a benchmark model, using a mining case study. Results indicate that the solutions ensure with a high probability a determined production target, producing good robust solutions compared to the deterministic counterpart.

Keywords: Equipment Selection, Equipment Replacement, Production Assurance, Linear Stochastic Programming, Mining Industry.

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1. Introduction

The selection and sizing of loading and hauling equipment fleets in capital-intensive industries, like mining, is one of the most important design stages and decisions [1]. It involves a number of technical, geometrical and geographical variables [2], like geotechnical conditions, geological composition, mining and processing recoveries, pit size, stripping ratio, dumping destinations, among others. This decision directly affects mine planning as the pit optimization depends on the type and sized of equipment used, so improving the selection can lower mining costs and change the optimized pit limits. It also affects production strategy as the selectivity of material loaded, the ground conditions and material destination all depend on the type, size and quantity of equipment selected, which directly affects the mine planning and its production strategy [3].

Furthermore, the equipment have considerable purchasing costs and its operations account for approximately two thirds of the total operating costs [4]. To better understand the level of investment involved in this type of decisions, an open pit mining fleet will be in the order of 5-50 trucks for typical iron ore mines [5] or even 50-150 trucks for coal or copper large mine sites. Also a haul truck may cost 2 million US dollars [6], while an electric shovel will have an investment cost of up to 30 million US dollars [7]. The choice of equipment is dependent upon the mining method that will be used, which is based on the economic conditions, geographical variables, human and equipment safety, environmental protection and desired production rates. Before selecting the equipment the mining method is defined, taking it as a constraint for the equipment selection [8]. Finally, a poor choice in the equipment selection can lead to unnecessary expenses and the impossibility to meet capacity constraints [9].

In general, the process of equipment selection and fleet sizing is divided into three decisions steps. First, the type of fleet; second, the equipment capacity; and third, the size of the fleet. Type of the fleet and the equipment capacity are conditioned by the available equipment market supply. The proper combination of trucks and loaders is determined according to the characteristics of the equipment and the operational conditions of the project. While the number of equipment is calculated based on production forecasts [4] and costs. For tractability reasons and to avoid excessive difficulty, the problem is commonly formulated as a deterministic problem [9]. This kind of decision-making approach is useful when the process is considered to be in a strategic time scale. However, the equipment selection problem is located on a tactical and operational time scale, because the selection must be made in order to fulfill the production requirements [9]. Another aspect that has been neglected in the selection process is the fact that each type of equipment is subjected to different maintenance and repair actions. Consequently, the availability of loading and hauling equipment may vary and usually does not exceed 85% [8]. This can lead to larger fleets and maintenance resources [10] mainly because it is almost impossible to achieve a global minimum cost by optimizing sub-problems [11]. Therefore, it would be desirable to integrate both the equipment selection and replacement problems in an integrated robust optimization model, which can give answer to two of the three equipment selection and fleet sizing questions. The third one refers to equipment geometry [12], which is not considered in this work.

Due to the complexity and large dimension of the problem, heuristic methods are commonly used in the industry [13]. The main advantage of these methods is that they can find feasible solutions very quickly. Huang and Kumar develop a selection and sizing model that minimizes the cost of idle machinery and address the variability of some operation parameters [14]. Ta et al.

develop an optimization model that incorporates real-time data to deal with the uncertainty of key operational parameters [15]. Burt relaxes the homogeneity constraint to the match factor, a traditional heuristic that evaluates the efficiency of a fleet through the ratio of loader productivity to truck productivity, and presents a heterogeneous model for equipment selection [11]. Although heuristics rely heavily on the decision maker's experience and knowledge, authors try to capture it into expert systems [16, 17, 18], falling short in the real applicability of such models.

Other solution approaches include queuing theory and mixed integer programming (MIP) algorithms. Najor et al. use queuing theory to model stochastic behaviour of truck and shovel systems. Their model analyses equipment idle time and predicts material throughput by considering the plant capacity [19]. Raman et al. propose some work with queueing theory to deal with the cycle time estimation [20]. Michiotis et al. propose a MIP model for selecting the main excavating equipment in open pit lignite mines [21]. Baxter et al. describe several MIP models for decision-making regarding equipment replacement in the forestry harvesting industry [22]. Topal and Ramazan present a MIP technique to schedule a fleet of mining trucks and minimize maintenance costs in a given operation over a multi-period horizon [23]. Pascual et al. develop an asset-management oriented multi-criteria methodology for the joint estimation of a mobile equipment fleet size, and the maintenance capacity. They evaluate their analytical model using global cost rate, availability and throughput as performance indicators [10]. In a more recent work, Burt et al. develop a MIP model for heterogeneous equipment selection in a surface mine with multiple locations and a multiple period schedule [9].

Most of the previously mentioned analytical models consider as their objective function the average production rate, instead of determining a probability of achieving a determined scheduled mining program. So by using average production rate objective, we do not take into account the risk that the selected fleet may be insufficient or overestimated to meet the future production goals. Moreover, a poor selection may affect the economic performance of the mining operation by raising its operational costs or delaying production [8]. The concept of production assurance and the capacity of systems to meet future requirements have been previously researched. Barabady suggests a methodology for implementation of production assurance programs in production plants that improves performance and helps on decision-making [24]. However, most of the examples have only been developed for the oil & gas industry. There is also some literature in underground mining which addresses the problem of production uncertainty by generating production reliability curves based on geotechnical events and system redundancy [25, 26].

According to the aforementioned review, there is a lack of models in the literature that formulate the equipment selection and replacement problem as a stochastic model and include the concept of production assurance. The main contribution of this work is to present a holistic model that solves the equipment selection problem and its inter-temporal sub problems, simultaneously, subject to stochastic production rate constraints that consider the inter-dependent availability of trucks and loaders.

The paper is structured as follows. Section 2 presents the deterministic and stochastic model formulations. Section 3 describes the case study, with the respective results, discussion and sensitivity analysis. Finally, Section 4 provides the conclusions and potential future developments of this work.

2. Model Formulation

Consider a material transportation fleet composed of hauling and loading equipment. In each period $t \in 1, \ldots, T$, the fleet has to achieve the production rate Q_t. A generic material transportation fleet operation is assumed, where the loaders stay in one site and the transportation equipment moves the load. There are $i = 1, \ldots, I$ different loaders and $j = 1, \ldots, J$ different trucks, giving a total of $I \times J$ different combinations. It is important to assess the combination (loader-truck pair) because the characteristics of single equipment may have an impact on the performance of the whole system. The production rates, the operating costs and availabilities are given for each combination of equipment. Accordingly, the fleet is modeled as a group of independent pieces of equipment subject to unavailability due to failures.

2.1. Deterministic Model

First, we develop a multi-period deterministic model, allowing purchasing and selling equipment between periods, and leaving equipment idle when they are not required. It is denoted as P_D. The assumptions, as defined in [11], parameters and decision variables of the model are listed below.

Assumptions

- *Known schedule*: The production plan is known for each period.
- *Single location*: All loaders work at the same location and loaders can work with any truck.
- *Salvage*: All equipment can be sold at the end of each period with a decreasing price depending on its age.
- *Constant productivity*: Since there is a single location, productivity is constant for equipment of the same type.
- *Heterogeneous fleet compatibility*: Different combinations of equipment can be selected.

- *Idle time*: If equipment are not required to satisfy the production plan, they will be left idle.
- *Full utilization*: A full utilization is considered for all equipment, as long as they are not left idle.
- *Static model*: The production rate defined for each period must be satisfied.

Parameters

$C_{i,j}^{x,t}$ = Cost per time unit of operating type $i = 1, \dots, I$ truck with type $j = 1, \dots, J$ loader in period $t = 1, \dots, T$.

$C_{i,j}^{y,t}$ = Cost per time unit of operating type $j = 1, \dots, J$ loader with type $i = 1, \dots, I$ truck in period $t = 1, \dots, T$.

$D_{i,j}^{x,t}$ = Cost per time unit of not operating type $i = 1, \dots, I$ truck in period $t = 1, \dots, T$.

$D_{i,j}^{y,t}$ = Cost per time unit of not operating type $j = 1, \dots, J$ loader in period $t = 1, \dots, T$.

Q_t = Required tonnage per time unit for period $t = 1, \dots, T$.

$P_{i,j}^{x}$ = Productivity of type $i = 1, \dots, I$ truck working with type $j = 1, \dots, J$ loader.

$P_{i,j}^{y}$ = Productivity of type $j = 1, \dots, J$ loader working with type $i = 1, \dots, I$ truck.

V_i^x = Purchase cost of type $i = 1, \dots, I$ truck.

V_j^y = Purchase cost of type $j = 1, \dots, J$ loader.

$S_{i,t}^{x}$ = Salvage value of type $i = 1, \dots, I$ truck after $t = 1, \dots, T$ periods of use.

$S_{j,t}^{y}$ = Salvage value of type $j = 1, \dots, J$ loader after $t = 1, \dots, T$ periods of use.

$A_{i,j}^{x}$ = Availability of type $i = 1, \dots, I$ truck working with type $j = 1, \dots, J$ loader.

$A_{i,j}^{y}$ = Availability of type $j = 1, \dots, J$ loader working with type $i = 1, \dots, I$ truck.

$r =$ Discount rate.

Decision Variables

$x_{i,j,t} =$ Number of type $i = 1, \dots, I$ trucks working with type $j = 1, \dots, J$ loader during period $t = 1, \dots, T$.

$y_{i,j,t} =$ Number of type $j = 1, \dots, J$ loaders working with type $i = 1, \dots, I$ truck during period $t = 1, \dots, T$.

$d^x_{i,t} =$ Number of type $i = 1, \dots, I$ idle trucks during period $t = 1, \dots, T$.

$d^y_{j,t} =$ Number of type $j = 1, \dots, J$ idle loaders during period $t = 1, \dots, T$.

$b^x_{i,t,u} =$ Number of type $i = 1, \dots, I$ purchased trucks at the beginning of period $t = 1, \dots, T$, and sold at the end of period $u = 1, \dots, T$.

$b^y_{j,t,u} =$ Number of type $j = 1, \dots, J$ purchased loaders at the beginning of period $t = 1, \dots, T$, and sold at the end of period $u = 1, \dots, T$.

The objective function (1) is defined to minimize the operational and maintaining costs, plus the equipment purchase cost and minus the selling incomes, subject to the fulfillment of the production rate for each period.

$$\text{Minimize} \sum_t \frac{1}{(1+r)^t}\Bigg[\sum_{i,j}\left(C^{x,t}_{i,j}\, x_{i,j,t} + C^{y,t}_{i,j}\, y_{i,j,t}\right) + \sum_i \left(V^x_i\left(\sum_{u=t}^{T} b^x_{i,t,u}\right) + D^{x,t}_{i,j}\, d^x_{i,t} - \sum_{u=t}^{T} S^x_{i,u-t+1}\, b^x_{i,t,u}\right)$$
$$+ \sum_j \left(V^y_j\left(\sum_{u=t}^{T} b^y_{j,t,u}\right) + D^{y,t}_{i,j}\, d^y_{j,t} - \sum_{u=t}^{T} S^y_{j,u-t+1}\, b^y_{j,t,u}\right)\Bigg] \tag{1}$$

In the following, if a set of values for an index is not given, the full set is assumed. For the deterministic case, the fleet availability is known, and is the same as a single equipment availability. We added auxiliary decision variables ($R_{i,j,t}$) to assess the productivity of each combination. The productivity constraints are (2), (3) and (4).

$$A_{i,t}^{x}\, x_{i,j,t} P_{i,j}^{x} \geq R_{i,j,t} \,, \forall i,j,t \tag{2}$$

$$A_{i,t}^{y}\, y_{i,j,t} P_{i,j}^{y} \geq R_{i,j,t} \,, \forall i,j,t \tag{3}$$

$$\sum_{i,j} R_{i,j,t} \geq Q_t \,, \forall t \tag{4}$$

Since P_D is a multiperiod model, we need trucks and loaders conservation constraints for every period, i.e. (5)-(8), plus initial conditions (9) and the domain of decision variables (10).

$$\sum_{j} x_{i,j,t} + d_{i,t}^{x} - \sum_{s=1}^{t} b_{i,s,t}^{x} + \sum_{u=t+1}^{T} b_{i,t+1,u}^{x} = \sum_{j} x_{i,j,t+1} + d_{i,t+1}^{x} \,, \forall i, t \in 1, \dots, T-1 \tag{5}$$

$$\sum_{i} y_{i,j,t} + d_{j,t}^{y} - \sum_{s=1}^{t} b_{j,s,t}^{y} + \sum_{u=t+1}^{T} b_{j,t+1,u}^{y} = \sum_{i} y_{i,j,t+1} + d_{j,t+1}^{y} \,, \forall j, t \in 1, \dots, T-1 \tag{6}$$

$$\sum_{j} x_{i,j,t} + d_{i,t}^{x} \geq \sum_{s=1}^{t} b_{i,s,t}^{x} \,, \forall i,t \tag{7}$$

$$\sum_{i} y_{i,j,t} + d_{j,t}^{y} \geq \sum_{s=1}^{t} b_{j,s,t}^{y} \,, \forall i,t \tag{8}$$

$$x_{i,j,0} = y_{i,j,0} = b_{i,0,t}^{x} = b_{j,0,t}^{y} = b_{i,t,0}^{x} = b_{j,t,0}^{y} = d_{i,0}^{x} = d_{j,0}^{y} = 0 \,, \forall i,j,t \tag{9}$$

$$x_{i,j,t} \,, y_{i,j,t} \,, b_{i,t,u}^{x} \,, b_{j,t,u}^{y} \,, d_{i,t}^{x} \,, d_{j,t}^{y} \in \mathbb{Z}^{+} \cup \{0\} \,, \forall i,j,t,u \tag{10}$$

Then P_D is composed by (1)-(10).

2.2. Stochastic Model

An alternative to address the equipment selection problem with a more realistic approach is to consider the availability interaction between fleets. Since the loader production is dependent on the available trucks, the truck downtime distribution must be considered. The same applies to the trucks in relation with the loaders. Relaxing the assumption of independent availabilities between trucks and loaders helps to avoid production shortfalls that occur when short of trucks or loading units [8]. This is also very important if we consider that the opportunity costs in the mining industry increase with non-renewable resources depletion [27].

To determine the most likely number of trucks and loaders needed to fulfill the production schedule for given availabilities, the binomial distribution can be utilized. Since it is a probability distribution, the idea is to minimize the probability of having less trucks or loaders available than the required production. Similarly, it is also desirable to avoid having more trucks or loaders available than needed to reduce capital expenses. The described method is implemented in P_D, by rewriting the productivity constraints as (11)-(13). Thus, we obtain a stochastic model denoted as P_S.

$$\sum_{k=0}^{x_{i,j,t}} \binom{x_{i,j,t}}{k} \left(A_{i,j}^{x}\right)^{k} \left(1 - A_{i,j}^{x}\right)^{x_{i,j,t}-k} k\, P_{i,j}^{x} \geq R_{i,j,t}\ , \forall i,j,t \tag{11}$$

$$\sum_{l=0}^{y_{i,j,t}} \binom{y_{i,j,t}}{l} \left(A_{i,j}^{y}\right)^{l} \left(1 - A_{i,j}^{y}\right)^{y_{i,j,t}-l} l\, P_{i,j}^{y} \geq R_{i,j,t}\ , \forall i,j,t \tag{12}$$

$$\sum_{i,j} R_{i,j,t} \geq Q_t\ , \forall t \tag{13}$$

Given that both fleets should account for the uncertainty, (11)-(13) are equivalent to (14).

$$\sum_{i,j} \sum_{k=0}^{x_{i,j,t}} \sum_{l=0}^{y_{i,j,t}} \left(\binom{x_{i,j,t}}{k} \left(A_{i,j}^{x}\right)^{k} \left(1 - A_{i,j}^{x}\right)^{x_{i,j,t}-k} \binom{y_{i,j,t}}{l} \left(A_{i,j}^{y}\right)^{l} \left(1 - A_{i,j}^{y}\right)^{y_{i,j,t}-l} min\{k\, P_{i,j}^{x}, l\, P_{i,j}^{y}\} \right) \geq Q_t\ , \forall t \tag{14}$$

It should be noted that the minimum function accounts for this inter-dependency between fleets, meaning that if there is no loader available trucks cannot haul the material. The same consideration is applied to the trucks in relation with the loaders.

The rest of the model is analogous to P_D. In other words, P_S is given by (1), (5)-(10) and (14).

Recall that the objective function (1) minimizes the total life-cycle costs, constraints (5)-(8) capture the balance of equipment over time, constraints (9) account for the initial conditions of the problem, and constraints (10) state the domain of the decision variables. Finally constraints (14) ensure that the trucks are capable of matching loader productivity. It should be noted that they are not linear, so they cannot be implemented directly in a mathematical programming software.

2.3. Linearization of constraints (14)

Note the decision variables $x_{i,j,t}$ and $y_{i,j,t}$, i.e. the number of operative trucks and loaders, are present in the limits of the sum in (14). These types of constraints cannot be directly handled by a typical mathematical programming software. Therefore, we need to rewrite them, ideally in a linear form. The proposed transformation is as follows.

We denote by $\bar{x}_{i,j,t}$ the upper bound in variables $x_{i,j,t}$, and analogously $\bar{y}_{i,j,t}$ for $y_{i,j,t}$. Let $O^x_{i,j,t} = \{0, \dots, \bar{x}_{i,j,t}\}$ and $O^y_{i,j,t} = \{0, \dots, \bar{y}_{i,j,t}\}$. We defined the following variables,

$$W^{m,n}_{i,j,t} = \begin{cases} 1, & \text{if } x_{i,j,t} = m \text{ and } y_{i,j,t} = n \\ 0, & \text{otherwise} \end{cases}, \qquad \forall i,j,t,m \in O^x_{i,j,t}, n \in O^y_{i,j,t}$$

Also, the following parameters are required,

$$\beta^{m,n}_{i,j,t} = \sum_{k=0}^{m} \sum_{l=0}^{n} \left(\binom{m}{k} \left(A^x_{i,j}\right)^k \left(1 - A^x_{i,j}\right)^{m-k} \binom{n}{l} \left(A^y_{i,j}\right)^l \left(1 - A^y_{i,j}\right)^{n-l} min\{k\, P^x_{i,j}, l\, P^y_{i,j}\} \right), \forall i,j,t,m \in O^x_{i,j,t}, n \in O^y_{i,j,t}$$

Then, constraints (14) can be rewritten using (15)-(19),

$$\sum_i \sum_j \sum_m \sum_n \beta_{i,j,t}^{m,n} W_{i,j,t}^{m,n} \geq Q_t, \forall t \tag{15}$$

$$\sum_m \sum_n W_{i,j,t}^{m,n} = 1, \forall i, j, t \tag{16}$$

$$x_{i,j,t} = \sum_m \sum_n m\, W_{i,j,t}^{m,n}, \forall i, j, t \tag{17}$$

$$y_{i,j,t} = \sum_m \sum_n n\, W_{i,j,t}^{m,n}, \forall i, j, t \tag{18}$$

$$W_{i,j,t}^{m,n} \in \{0,1\}, \forall i, j, t, m, n \tag{19}$$

This linearized formulation, denoted P_{SL} is given by (1), (5)-(10), and (15)-(19).

3. Case Study

To study the behaviour of the model under real productive information, we use data provided by an industry partner. The case study considers a large open-pit mining operation in the north of Chile. In this mining operation there are three different types of loaders and three different types of trucks, sold by different vendors, giving a total of nine different combinations. The fleets need to achieve an average production rate of $130\, pu/tu$ (production units per time units). The production rate per period is given by Figure 1. The trucks and loaders production rates (pu/tu) are given on Tables 1 and 2. Operating and maintaining costs (mu/tu, monetary units per time units) are given on Tables 3 and 4. The purchase costs (mu) and idle costs (mu/tu) are given on Tables 5 and 6. Salvage value profile, in terms of the purchase cost, is given on Figure 2. Finally, the availability of the different equipment is given on Tables 7 and 8. The discount factor is 10%.

In the first stage we solved the deterministic model P_D to determine the feasibility of the fleet to achieve the required production rate. Then we solved P_{SL} to effectively estimate the fleet size and combination.

[Figure 1 near here]

Figure 1: Production budget.

Table 1: Productivity of trucks working with loaders (pu/tu).

Truck/Loader	**1**	**2**	**3**
1	6	4.7	6
2	5.5	5	5.5
3	5	5.4	6.4

Table 2: Productivity of loaders working with trucks (pu/tu).

Loader/Truck	**1**	**2**	**3**
1	22	21	21
2	30	32	28
3	36	38	37

Table 3: Costs of operating and maintaining trucks with loaders (mu/tu).

Truck/Loader	**1**	**2**	**3**
1	3	3.5	4.2
2	3.2	4.1	3.8
3	2.9	3.1	3.5

Table 4: Costs of operating and maintaining loaders with trucks (mu/tu).

Loader/Truck	**1**	**2**	**3**
1	8	8.5	8.9
2	13	12	12.5
3	16	15	17

Table 5: Purchase costs (mu) and idle costs (mu/tu) of trucks.

Truck	**Purchase Cost**	**Idle Cost**
1	42	0.6
2	37.8	0.7
3	44.8	0.75

Table 6: Purchase costs (mu) and idle costs (mu/tu) of loaders.

Loader	Purchase Cost	Idle Cost
1	154	1.7
2	224	2.5
3	266	3.2

[Figure 2 near here]

Figure 2: Salvage Value Profile.

Table 7: Availability of trucks operating with loaders.

Truck/Loader	1	2	3
1	0.70	0.69	0.68
2	0.72	0.65	0.66
3	0.74	0.72	0.73

Table 8: Availability of loaders operating with trucks.

Loader/Truck	1	2	3
1	0.80	0.79	0.78
2	0.82	0.85	0.86
3	0.79	0.82	0.83

3.1. Results

Both the benchmark and the proposed models were implemented using AMPL, and solved using IBM ILOG CPLEX 12.6 [28]. A comparison of the number of variables, constraints and computing time required for P_D and P_{SL} models is presented in Table 9.

The optimal solution for P_D, the benchmark model, gives a combined fleet of twenty-five type 1 trucks, sixteen type 2 trucks, six type 1 loaders and two type 3 loaders, with a total life-cycle cost of 3,719.7 mu. The total purchase cost is 1,446 *mu* for trucks and 1,410 *mu* for loaders. The total operational and idle costs are 577 *mu* for trucks and 423 *mu* for loaders. To assure the production goal, a fleet of 41 trucks and 8 loaders are purchased within the first three periods, as shown in the summarized Tables 14 and 15. The detailed results of this model are presented on Tables 10 and 11. As shown in Tables 10 and 11, during period 4 due to the lower production target and to reduce operational costs, a type 2 truck and a type 3 loader are left idle because they have higher costs compared to type 1 equipment. During period 7, to achieve the production increase, four type 1 trucks and a type 1 loader are purchased. Type 1 loader is preferred for its lower purchase cost and type 1 trucks are preferred for the best performance combined with type 1 loader. All these additional equipment are sold at the end of the period. Finally, the rest of the equipment are sold within the last two periods.

Similarly, using the proposed stochastic model P_{SL} the optimal solution renders a fleet of forty-four type 1 trucks and ten type 1 loaders, with a total life-cycle cost of 3,803.3 mu. The total purchase cost is 1,620 *mu* for trucks and 1,351 *mu* for loaders. The total operational and idle costs are 616 *mu* for trucks and 383 *mu* for loaders. The detailed results of this model are presented on Tables 12 and 13. As shown in summarized Tables 14 and 15, more trucks and loaders are needed to fulfill the production target than the benchmark model. This is because the fleets' interdependency constraints demand more equipment to increase the probability of achieving the target production. Same as in the benckmark model, the equipment are purchased within the first three periods and sold in the last two periods. However, in this case forty trucks and nine loaders are purchased. Also, two trucks are left idle during period 4, four additional trucks and a loader are purchased for period 7 and they are sold at the end of the period.

Table 9: Resolution Comparison.

	$\mathbf{P_D}$	$\mathbf{P_{SL}}$
Continuous Variables	570	570
Integer Variables	0	298,890
Constraints	310	400
MIP simplex iterations	52,045	1,973
Number of B&B Nodes	7,034	0
Computing time (seconds)	3.63	97.17

Table 10: Results of type 1 and 2 trucks with the benchmark model, P_D.

Period	**Operative Trucks**		**Idle Trucks**		**Purchased Trucks**		**Sold Trucks**	
	Type 1	Type 2	Type 1	Type 2	Type 1	Type 2	Type 1	Type 2
1	11	-	-	-	11	-	-	-
2	16	8	-	-	5	8	-	-
3	21	16	-	-	5	8	-	-
4	21	15	-	1	-	-	-	-
5	21	16	-	-	-	-	-	-
6	21	16	-	-	-	-	-	-
7	25	16	-	-	4	-	4	-
8	21	16	-	-	-	-	-	-
9	21	16	-	-	-	-	4	8
10	17	8	-	-	-	-	17	8

Table 11: Results of type 1 and 3 loaders with the benchmark model, P_D.

Period	**Operative Loaders**		**Idle Loaders**		**Purchased Loaders**		**Sold Loaders**	
	Type 1	Type 3	Type 1	Type 3	Type 1	Type 3	Type 1	Type 3
1	3	-	-	-	3	-	-	-
2	4	1	-	-	1	1	-	-
3	5	2	-	-	1	1	-	-
4	5	2	-	1	-	-	-	-
5	5	2	-	-	-	-	-	-
6	5	2	-	-	-	-	-	-
7	6	2	-	-	1	-	1	-
8	5	2	-	-	-	-	-	-
9	5	2	-	-	-	-	1	1
10	4	1	-	-	-	-	4	1

Table 12: Results of trucks with the proposed linear model, P_{SL}.

Period	**Operative Trucks**	**Idle Trucks**	**Purchased Trucks**	**Sold Trucks**
1	13	-	13	-
2	25	-	12	-
3	40	-	15	-
4	38	2	-	-
5	40	-	-	-
6	40	-	-	-
7	44	-	4	4
8	40	-	-	-
9	40	-	-	12
10	28	-	-	28

Table 13: Results of loaders with the proposed linear model, P_{SL}.

Period	Operative Loaders	Idle Loaders	Purchased Loaders	Sold Loaders
1	3	-	3	-
2	6	-	3	-
3	9	-	3	-
4	9	-	-	-
5	9	-	-	-
6	9	-	-	-
7	10	-	1	1
8	9	-	-	-
9	9	-	-	3
10	9	-	-	6

Table 14: Summary of trucks for both models.

Period	Operative Trucks		Idle Trucks		Purchased Trucks		Sold Trucks	
	P_D	P_{SL}	P_D	P_{SL}	P_D	P_{SL}	P_D	P_{SL}
1	11	13	-	-	11	13	-	-
2	24	25	-	-	13	12	-	-
3	37	40	-	-	23	15	-	-
4	36	38	1	1	-	-	-	-
5	37	40	-	-	-	-	-	-
6	37	40	-	-	-	-	-	-
7	41	44	-	-	4	4	4	4
8	37	40	-	-	-	-	-	-
9	37	40	-	-	-	-	12	12
10	25	28	-	-	-	-	25	28

Table 15: Summary of loaders for both models.

Period	Operative Loaders		Idle Loaders		Purchased Loaders		Sold Loaders	
	P_D	P_{SL}	P_D	P_{SL}	P_D	P_{SL}	P_D	P_{SL}
1	3	3	-	-	3	3	-	-
2	5	6	-	-	2	3	-	-
3	7	9	-	-	2	3	-	-
4	7	9	1	-	-	-	-	-
5	7	9	-	-	-	-	-	-
6	7	9	-	-	-	-	-	-
7	8	10	-	-	1	1	1	1
8	7	9	-	-	-	-	-	-
9	7	9	-	-	-	-	2	3
10	5	9	-	-	-	-	5	6

[Figure 3 near here]

Figure 3: Probability to achieve different production rates in period 4 using benchmark and proposed models.

3.2. Discussion

As mentioned, both models assure that the average the production goal is met in each period. However, it is also important to analyze the probability of achieving a determined production rate. Figure 3 presents a graphic comparison of that for both models in a regular period. The probability for each period is obtained using the binomial function in Equation 14, that accounts for fleet's interdependency, and the optimal operative equipment of each model. We can observe that the solutions given by our model, for the same productivity level, render higher probabilities of achieving that given production rate. Even though the total cost of the proposed model is 2.2% higher than the benchmark model, the proposed model has a much better overall performance regarding the assurance of the production rate, as shown by the dashed lines. Also, it is important to note that although the proposed model requires to purchase more equipment, its total life-cycle costs are only slightly higher than the benchmark model's costs.

3.3. Sensitivity Analysis

We performed a sensitivity analysis on the effect of the production rate on the optimal number of operative trucks and loaders. We considered variations of -20% to 20% in the production rates. Results are detailed on Figures 4 and 5. Each line represents a specific period.

[Figure 4 near here]

Figure 4: Sensitivity analysis results per period of type 1 trucks with the proposed linear model, P_{SL}.

[Figure 5 near here]

Figure 5: Sensitivity analysis results per period of type 1 loaders with the proposed linear model, P_{SL}.

For the given scenarios, the optimal selection consists exclusively of type 1 trucks and loaders. We can observe from Figures 4 and 5 that for a given number of loaders or trucks respectively, when the production rates are varied the required number of trucks and loaders remains relatively unchanged. This is an indication that the proposed model solution P_{SL} is robust against changes in the production rate, as well as the combination of selected equipment. An interesting fact that can be noticed in Figure 4 is that operative trucks in period 1 decrease from 10% to 15%. The same applies to the period 10 from 0% to 5% and from 15% to 20%. These declines are explained by an increase in the number of loaders, which increases the probability of achieving higher production rates according to Equation 14.

4. Conclusions

This paper presents a practical mining equipment stochastic equipment selection model that minimizes the total life-cycle costs, subject to production rate constraints that consider the inter-dependent availability of trucks and loaders. The proposed approach integrates simultaneously the equipment selection and replacement decision, in a multi-period production schedule.

The stochastic model requires a linearization process, producing optimal solutions in reasonable times for practical instance sizes. The introduction of stochastic availability of trucks and loader produces robust solutions that remain feasible under changes in required production rates.

Future lines of work include: (i) Time-depending operating and maintenance costs, which are known to vary with the age of the equipment; (ii) discount factors and depreciations of the material; (iii) diversification under multiple equipment vendors to prevent monopolies in the equipment market; and finally (iv) reductions in the availability as equipment gains usage.

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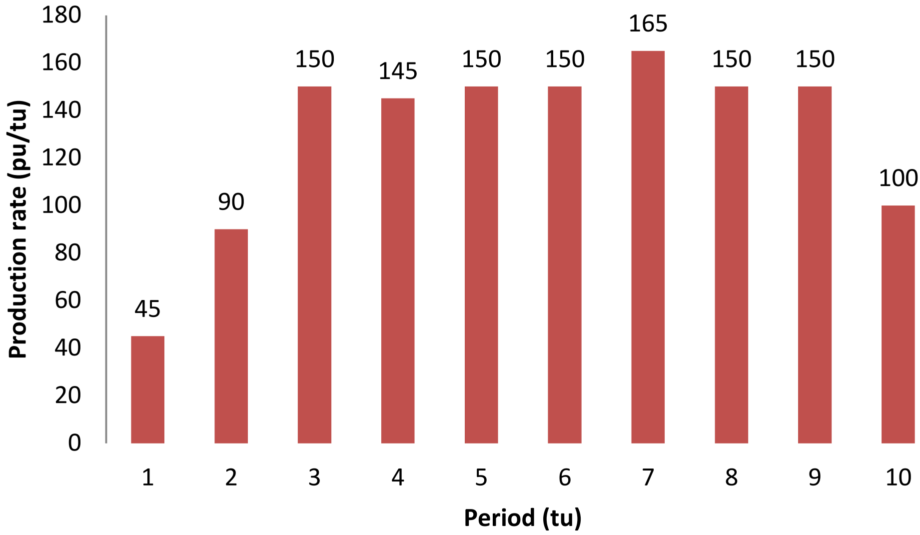
Production rate (pu/tu)
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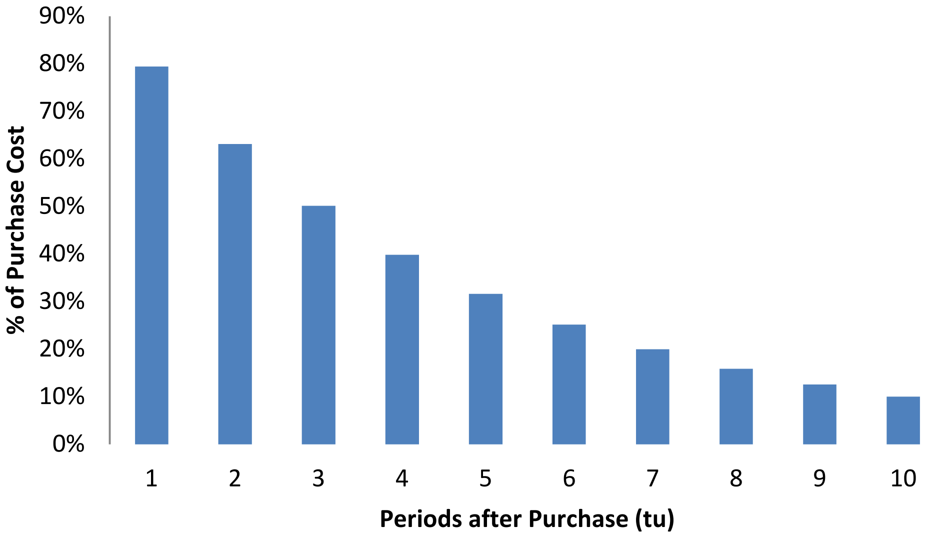
Period (tu)



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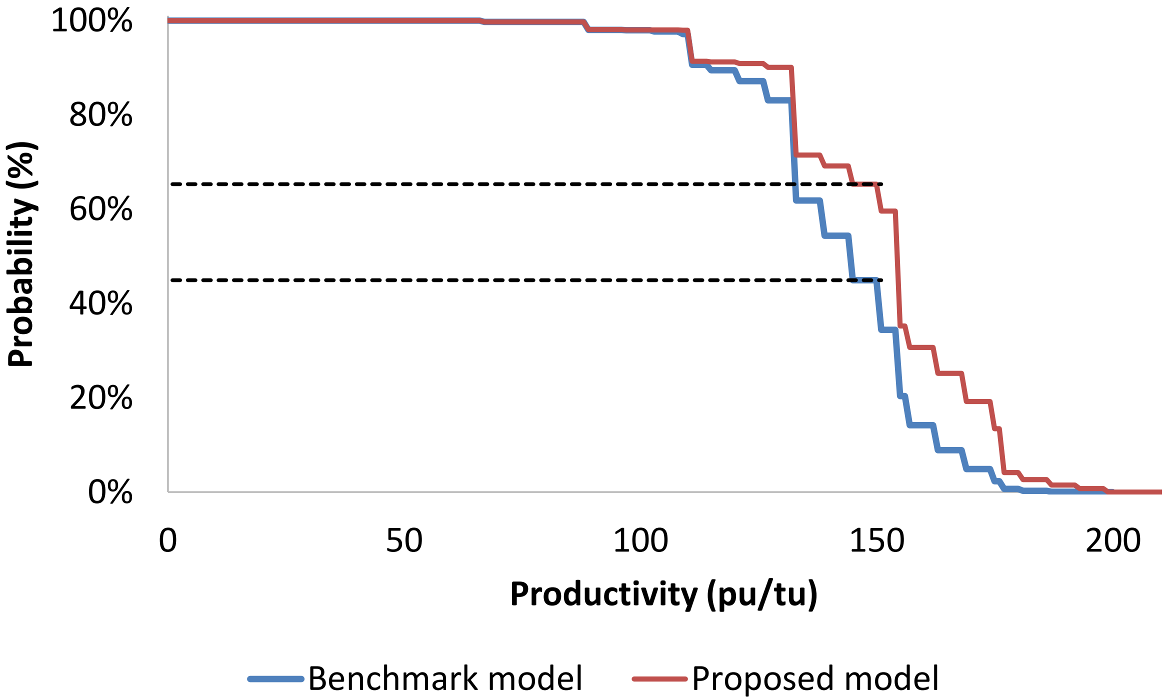
Periods after Purchase (tu)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Probability (%)
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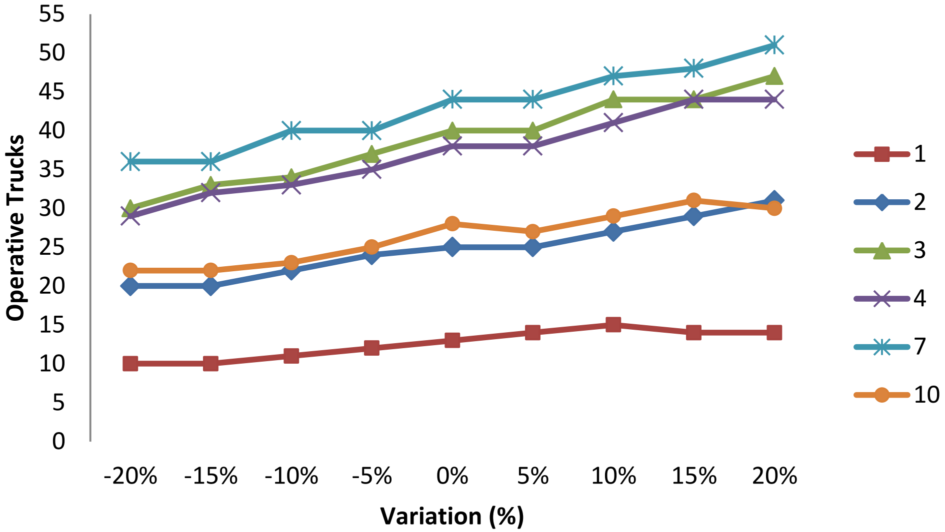
Productivity (pu/tu)
Benchmark model
Proposed model



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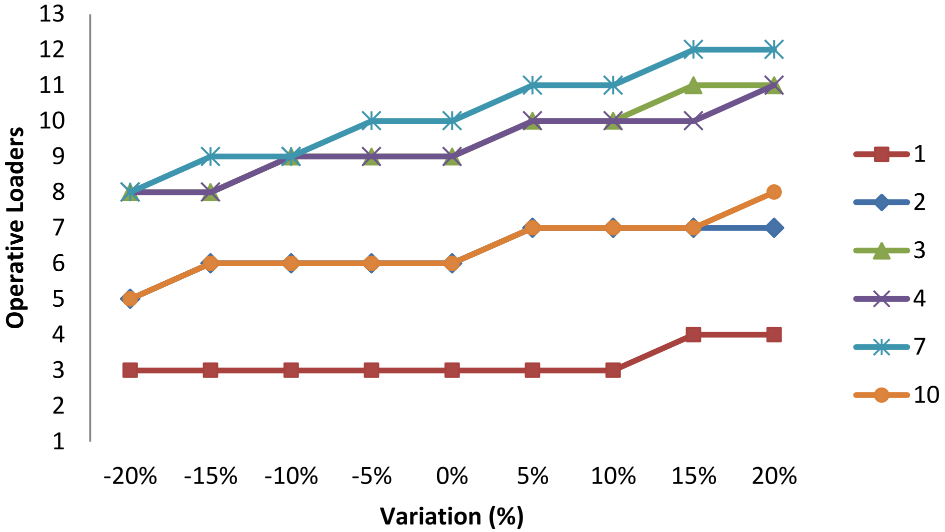
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