On the endogenous sustainability of the non-funded Social Security System.

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Abstract

The paper describes the effects of the non-funded social security systems over fertility rate and labor supply(typically family choice variables). We show that changes on fiscal policy may induce subsequent changes on family choices which produce an endogenous problem of sustainability over the social security system.

INTRODUCTION

Governments that manage non-funded social security programs generally face fiscal problems caused by demographic transition as fewer individuals pay taxes but more individuals receive the social security benefits over time. This process finally requires the replacement of the non-funded social security system by a fully-funded system. This paper addresses this phenomena by stressing a feedback effect from social security to demographic transition. It also shows a negative effect of the non-funded system over labor supply.

In this scenario, the non-funded system becomes unsustainable because tax rates raise continuously over time.

THE ENVIRONMENT

In this economy, there is a representative household which has three overlapping generations: newborns, middle age and elderly individuals. Each middle age individual will

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solve a dynastic problem that can be written in a recursive setup.

She is endowed with H_t units of human capital and b_t units of financial wealth obtained as bequest from her parents. She maximizes a welfare function composed by her current utility level, her discounted utility level at old age and their children's discounted utility. Let β be the individual's discount factor and, as in Becker, Murphy and Tamura(1990) -BMT-, let $\alpha n_t^{-\epsilon}$ be a constant elasticity function of altruism per children, where n_t is the number of children the individual decides to bear. The instantaneous utility function will be $u(c_t)=(c_t)^\sigma/\sigma$, where σ is the constant elasticity of substitution of consumption. The individual chooses the number of children to bear, the fraction of time invested on human capital on each of them (y_t), the savings to carry over to next period (s_t) and the level of bequest left to each child (b_{t+1}). No constraint is assumed on bequest, meaning that they could be negative, which is the case of children supporting their parents.

Total income depends on labor income, financial return from investments on capital market and social security payment received during retirement age. Labor income during adulthood depends on (1) the amount of time supplied to the labor market and (2) the human capital of the individual. We assume that each individual is endowed with 1 unit of time which might be used either to work or as an input in childbearing. If she only works, she provides H_t units of human capital. However, bringing up children takes y_t units of time, and since we have n_t children, we are left just with $(1-n_t y_t)$ units of time to work. In this case the after tax labor income will be $w_t H_t(1-n_t y_t)(1-\tau_t)$, where w_t is the wage rate per unit of human capital and τ_t is a social security tax rate levied by the government. Additionally income is obtained as return from the capital market of the investment of bequests, b_t. The capital market pays a rate of return equal to r_t. Finally, during old age each individual receives social security benefits composed by a lump sum benefit, G_{t+1}, and a return, ϕ, over the individual's contribution.

The evolution of human capital stock over time is described by $H_{t+1} = Ay_t^\theta H_t$, where A is a technology parameter, H_t is parents' human capital and H_{t+1} is child's human capital. The parameters of the model have the following properties:

Assumption 1: $0 < \beta, \alpha, \theta, \epsilon, \sigma < 1; 0 < A, \theta < \epsilon$.

Assumption 2: $0 < \phi < (1 + r_t)$.

Assumption 3: $w_t = w$ and $r_t = r > 0, \forall\ t$

Assumption 1 states that parents are selfish ($\alpha < 1$) as they care more on their own welfare than on their children's welfare. The fact that $0 < \theta < 1$ implies that the rate of return of investment on human capital is decreasing on y_t. The assumptions about σ and ϵ assure that the individual's utility function is concave on consumption and the altruism function per child is also concave. The assumption $\theta < \epsilon$ assure the existence and uniqueness of an equilibrium (see below).

The assumption $\phi < (1 + r_t)$ basically indicates that the system is not fully funded. Assumption 3 allows us to focus on a small open economy that faces prices. Finally even when we did not state any assumption on tax rates, it should be noticed that an upper and a lower bound for taxes exist. In fact, as the government requires positive revenues we should have $\tau_t > 0$. Also if individuals face $\tau_t > 1$, they would obtain negative labor income and they would not supply labor. Thus we would require $\tau_t < 1$ to obtain revenues. This last condition will be used later when defining the sustainability of the system.

Summing up, this setup is similar to BMT, but allowing for three period of time during lifecycle, a decreasing rate of return on human capital, individuals facing prices -as in any lifecycle context- and the existence of a social security system. The individual's problem, given initial human capital and assets carried over from childhood will be:

$$V_t(H_t, b_t) = \max_{c_t^a, c_{t+1}^o, s_t, n_t, y_t} \frac{(c_t^a)^\sigma}{\sigma} + \beta \frac{(c_{t+1}^o)^\sigma}{\sigma} + \beta \alpha n_t^{1-\epsilon} V_{t+1}(H_{t+1}, b_{t+1}) \tag{1}$$

$$c_t^a = (1 + r_t)b_t + w_t H_t (1 - n_t y_t)(1 - \tau_t) - s_t \tag{2}$$

$$c_{t+1}^o = (1 + r_{t+1})s_t + \phi w_t H_t \tau_t (1 - n_t y_t) + G_{t+1} \tag{3}$$

$$H_{t+1} = A y_t^\theta H_t \tag{4}$$

Where $V_t(H_t, b_t)$ is the value function for the individual in her adulthood, given the human capital stock and assets she carries over while c_t^a, c_{t+1}^o are consumption during adulthood and retirement age respectively. Additionally to assumption 1, to satisfy second order conditions we require $1 - \sigma - \epsilon > 0$ -as in Becker and Barro (1988)- and $0 < \theta < 1$.

The first order conditions and the envelope conditions determine a set of three equations. The first condition is the following:

$$R_H = [A\theta y^{\theta-1}](1 - n_{t+1}y_{t+1}) \frac{w_{t+1}}{w_t} \left[\frac{1 - \tau_{t+1} + \frac{\phi \tau_{t+1}}{1 + r_{t+2}}}{1 - \tau_t + \frac{\phi \tau_t}{1 + r_{t+1}}} \right] = 1 + r_{t+1} = R_k \tag{5}$$

The condition equates marginal return from human capital with marginal return from bequests (financial wealth). The first term of the left hand side, $A\theta y^{\theta-1}$, is the physical marginal return on human capital if the individual supply a unit of time to the labor market. However, the effective return on human capital depends on the amount of time that each child works, $(1-n_{t+1}y_{t+1})$. Finally we multiply by the ratio of relative wages corrected by taxes. Hence this equation determines y_t such that at the margin, the rate of return on human capital is equal to the rate of return on bequests.

The next two equations are the followings:

$$(\frac{c_{t+1}^a}{c_t^a})^\sigma = (Ay^\theta)^\sigma = \beta\alpha n_t^{-\epsilon}(1+r_{t+1}) \tag{6}$$

$$\frac{b_{t+1}}{H_{t+1}} = \frac{w_t}{A\theta y^{\theta-1}}(1-\tau_t+\frac{\phi\tau_t}{1+r_t})(\frac{1-\epsilon-\sigma\theta}{\sigma}) \tag{7}$$

Equation (6) is just an Euler equation. The first equality assumes a stable growth path. The right hand side of the equation is the usual discount factor corrected by interest rate and fertility rate. Equation (7) states that the ratio of bequest versus next period human capital is function of relative prices. The bigger is the wage rate, the more the individual is willing to supply effective human capital to the market. In that case, less time is available for investment on children's human capital and parents substitute away from investment on human capital to bequests. The bigger is the return on human capital, the more human capital is accumulated and thus the less work is supplied. In this case, the ratio of bequest to human capital decreases because at the margin, parents prefer to spent on children's human capital rather than on bequests.

We have a system of three equations that determines the family decision variables. Equations (5) and (6) fully interact and jointly determine the number of children and the time spent on human capital accumulation on each child. Given the value of y_t, the ratio of bequest to human capital is determined.

Lemma 1. Under assumptions 1-3, there exists a unique and stationary equilibrium $(n*, y*, \frac{b*}{H*})$.

Proof: See mathematical appendix.

Notice that given those values we can calculate the stationary level of per capita income growth on the dynastic family, $g*=Ay*^\theta$, and the level of stationary aggregate growth of the family's income, $(1+g*)n*$.

The set of equation that determine optimal allocations depends on the parameters of the model including the tax rate. We analyze next how fiscal policy might affect the household's allocations. The following proposition states the effect of a change on τ_t over y^*, n^* and $(1+g^*)n^*$ when $\phi < (1+g^*)n^*$. This case is stressed here because the rate of return provided by governments is generally smaller than the aggregate growth rate of the economy -See Song (2000).

Proposition 1. When $\phi < (1+g^*)n^*$ and assumptions 1-3 hold, an increase in the social security tax rate at time t, τ_t, impacts negatively fertility rate -n^*- and labor supply.

Proof: See Mathematical appendix

The intuition of those results is the following. Consider an increase on τ_t holding constant the level of benefits. This case is associated with a current superavit on the fiscal budget which will be returned to future generations. Thus there is not a direct income effect over the family budget constraint. However, there might exist a substitution effect. In fact, the family has two options two allocate its time: (1) working or (2) childbearing. As government is taxing labor income there is an effect over the current return of labor supply while future return of labor supply (the social security component, ϕ) is held constant though.

Will the household allocate more time to childbearing? It depends on childbearing return. This return is determined by the increase on future family income. In fact as we spent more time on children, the family will have larger aggregated future income due to larger human capital accumulation (holding constant fertility rate) or to larger fertility rate (large number of individuals, holding constant human capital). Thus this return is given by the aggregate growth rate of the economy, $n^*(1+g^*)$.

As the tax rate increases, the family compares the old-age return of working versus childbearing. If $\phi < n^*(1 + g^*)$, there are incentives to allocate more time to childbearing and current labor supply decrease. Also as there will be more resources available to future resources, the current generation is able to leave a smaller level of bequests (which might be even negative) which is accomplish by choosing a smaller level of children and fewer bequest per children. Thus when $\phi < n^*(1 + g^*)$, we obtain lower labor supply and lower fertility rate but larger human capital accumulation.

THE PATH OF TAXES OVER TIME ON THE NON-FUNDED SOCIAL SECURITY SYSTEM

In this section, we focus on determining if the fiscal system is sustainable. A sustainable system will be understood as a system where the government is able of collecting revenues to pay the social security benefits. The next definition illustrates the idea.

Definition 1. A non-funded social security system is sustainable if the fiscal budget balances and $\tau_t < 1, \forall t$.

The above definition rules out cases where $\tau_t > 1$ simply because in that case individuals do not obtain labor income when supplying labor and thus they prefer not to work. Whenever $\tau_t < 1$, the government is able to collect positive revenues.

The government promises to pay some benefits during retirement age. We assume that those promises are not broken and the government adjusts its level of taxes if required to keep in balance the fiscal budget.

Thus the government has the following budget constraint:

$$n_t \tau_{t+1} w_{t+1} (1 - n_{t+1} y_{t+1}) H_{t+1} = \phi \tau_t w_t (1 - n_t y_t) H_t + G_{t+1} \tag{8}$$

Assuming a stable growth path and the lump sum level of benefit being equal to a fraction $\gamma > 0$ of total income at t+1, we obtain:

$$\tau_{t+1} = [\frac{\phi}{n^*(1 + g^*)}] \tau_t + \gamma \tag{9}$$

This equation indicates the determinant of tax rates over time. The rate of return of social security system plays a main role. In fact if ϕ is bigger than the aggregate growth rate of the economy, $n^*(1 + g^*)$, tax rate increases continuously over time. Consider next the case $\phi < n^*(1 + g^*)$. In this case, if $n^*(1 + g^*)$ was constant, the tax rate would converge to $\frac{\gamma n^*(1+g^*)}{n^*(1+g^*) - \phi}$. However $n^*(1+g^*)$ is a function of tax rate -as shown above- and thus the tax rate might not converge to a stable value and further it might raise over time as in the former case. This continuous increase of tax rates would produce the system to become unsustainable.

The next proposition states the main result of this section.

Assumption 4: $0 < \phi < n^*(1 + g^*)$

Proposition 2. When assumption 1-4 hold, the social security tax raises continuously and the system becomes unsustainable in a finite time horizon.

Proof: See mathematical appendix.

The main intuition for the result is the following. The government might keep the level of taxes stable if it is able to collect enough revenues to pay the social security benefits. This tax collection depends on the aggregate growth rate of the economy. However $n^*(1 + g^*)$ is negatively affected by an increase on social security tax due to the negative effect on fertility rate and on labor supply. Thus an exogenous increase on tax rate require larger tax rates in the future to collect enough revenues to pay the benefits.

CONCLUSION

The paper shows that the non-funded social security system might become unsustainable under fairly weak assumptions. The main force driving the result is an endogenous demographic transition produced by the system. Thus the fiscal problems caused by demographic transition which finally requires the implementation of a fully funded system are -at least partly- caused by the non-funded system itself.

REFERENCES

[1] Barro, R., (1974), "Are government bonds net wealth?" *Journal of Political Economy*,vol 82, No 6.

[2] Becker, G. Murphy, K. and Tamura, R., (1993) "Human capital, Fertility and Economic Growth", *In Human Capital, A Theoretical and Empirical analysis with special reference to education*, Third Edition.

[3] Ehrlich, I. and Zhong J. (1998), "Social security and the real economy: An inquiry into some neglected issues", *American Economic Review*, May.

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[4] Kotlikoff, L. (1996) "Simulating the Privatization of social security in General equilibrium", *NBER working paper No 5776*, September.

[5] Song, Chen (2000)" The nature of social security and its impact on family", *PhD dissertation*, University of Chicago.

MATHEMATICAL APPENDIX

Proof of lemma 1

Equations (5) and (6) determine the following set of implicit functions:

$$[A\theta y^{\theta-1}](1-n_{t+1}y_{t+1})\frac{w_{t+1}}{w_t}\left[\frac{1-\tau_{t+1}+\frac{\phi\tau_{t+1}}{1+r_{t+2}}}{1-\tau_t+\frac{\phi\tau_t}{1+r_{t+1}}}\right]=1+r_{t+1}\Rightarrow y^I=y^I(n)$$

$$(Ay^\theta)^\sigma=\beta\alpha n_t^{-\epsilon}(1+r_{t+1})\Rightarrow y^{II}=y^{II}(n)$$

As technology and tastes are smooth continuous functions, so are the above functions. Also those equations imply that as n→0, $y^I\to\infty$ while $y^{II}\to[(1+r)/A\theta]^{\frac{1}{\theta-1}}$.

Further notice that the slopes of the two functions are:

$$\frac{\partial y^I}{\partial n}\frac{n}{y^I}=-\frac{\epsilon}{(1-\sigma)\theta}\tag{10}$$

$$\frac{\partial y^{II}}{\partial n}\frac{n}{y^{II}}=-\frac{1}{\theta+\frac{1-\theta}{ny}}\tag{11}$$

Both elasticies are negatives and furthermore the elasticity (I) is always larger in absolute value than the one of (II). The proof is by contradiction. Suppose the elasticity of (II) is larger. Then:

$$\frac{\epsilon}{(1-\sigma)\theta}<\frac{1}{\theta+\frac{1-\theta}{ny}}\Rightarrow\frac{1-\sigma-\epsilon}{1-\sigma}>\frac{\epsilon(1-\theta)}{(1-\sigma)\theta ny}\Rightarrow 1>ny>\frac{\epsilon}{1-\sigma-\epsilon}\frac{1\theta}{\theta}$$

Since 1> ny by time constraint, $(1-\sigma-\epsilon)>0$ by SOC. Thus the inequality requires $\epsilon<\theta(1-\sigma)$ which is a contradiction by assumption 1. Thus the slope of (I) is always larger than the one of (II) in absolute values.

It follows that both lines cross once on the plane (y,n) and thus a unique stationary equilibrium exists. Q.E.D.

Proof of proposition 1

Simply comparative statics on equations (5) and (6) yields:

$$\begin{bmatrix} \sigma\theta & \epsilon \\ c & d \end{bmatrix}\begin{bmatrix} \frac{\partial y}{\partial \tau}\frac{1}{y} \\ \frac{\partial n}{\partial \tau}\frac{1}{n} \end{bmatrix} = \begin{bmatrix} 0 \\ f \end{bmatrix} \Leftrightarrow A\begin{bmatrix} \frac{\partial y}{\partial \tau}\frac{1}{y} \\ \frac{\partial n}{\partial \tau}\frac{1}{n} \end{bmatrix} = \begin{bmatrix} 0 \\ f \end{bmatrix}$$

where

c$=\frac{(\theta-1)-ny\theta}{1-ny}$

d$=\frac{-ny}{1-ny}$

f$=\frac{(\frac{\phi}{n(1+g)}-1)(1-\frac{\phi}{1+r})}{[1-\frac{\phi\tau}{n(1+g)}(1-\frac{\phi}{1+r})][1-\tau(1-\frac{\phi}{1+r})]}$

Notice that assumptions $\phi < (1+r)$ and $\tau < 1$ imply f< 0 when $\phi < n(1+g)$ and by second order conditions $det(A) > 0$.

Using Cramer's rule, we obtain:

$\frac{\partial y}{\partial \tau}\frac{1}{y} = -\frac{f\epsilon}{det(A)}$

$\frac{\partial n}{\partial \tau}\frac{1}{n} = \frac{\sigma f\theta}{det(A)}$

$\frac{\partial ny}{\partial \tau}\frac{1}{ny} = \frac{\partial y}{\partial \tau}\frac{1}{y} + \frac{\partial n}{\partial \tau}\frac{1}{n} = \frac{f}{det(A)}(\sigma\theta - \epsilon)$

Further, since $\sigma\theta > 0, \epsilon > 0$ and $\sigma\theta - \epsilon < 0$ we obtain:

$\frac{\partial y}{\partial \tau}\frac{1}{y} > 0, \frac{\partial n}{\partial \tau}\frac{1}{n} < 0, \frac{\partial ny}{\partial \tau}\frac{1}{ny} > 0$. Q.E.D.

Proof of proposition 2

The evolution of taxes follows equation (5). However we should notice that the aggregate growth rate of the economy is function of tax rate: $n^*(1 + g^*) = n^*(1 + g^*)(\tau)$. Thus it follows that:

$$\tau_{t+1} = [\frac{\phi}{n^*(1 + g^*)}]\tau_t + \gamma \Rightarrow \tau_{t+1} = \tau_{t+1}(\tau_t) \tag{12}$$

In fact, the effect of taxes over the aggregate growth rate is given by:

$$\frac{\partial n(1+g)}{\partial \tau}\frac{1}{n(1+g)} = \frac{\partial n}{\partial \tau}\frac{1}{n} + \theta\frac{\partial y}{\partial \tau}\frac{1}{y} = \frac{f(1-\sigma-\epsilon)\theta}{det(A)} < 0$$

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Where the last inequality follows from SOC and $f < 0$.

To study the evolution of taxes over time, we linearize equation (12) by using the Euler method to obtain:

$$\tau_{t+1} = \tau_t + \frac{\partial \tau_{t+1}}{\partial \tau_t} = \frac{\phi}{n^*(1+g^*)} + (1 - \frac{\phi}{n^*(1+g^*)} \frac{(1-\sigma-\epsilon)\theta f}{det(A)})\tau_t \tag{13}$$

Thus it follows that the change of taxes over time is:

$$\tau_{t+1} - \tau_t = \frac{\phi}{n^*(1+g^*)}(1 - \frac{(1-\sigma-\epsilon)\theta f \tau_t}{det(A)}) \tag{14}$$

Fix any initial positive level of taxes, $\tau_t > 0$ and assume that tax rate decreases over time. Thus we should have:

$$1 < \frac{(1-\sigma-\epsilon)\theta f \tau_t}{det(A)} \tag{15}$$

But this is a contradiction because $\tau_t > 0$ and $f < 0$. Thus tax rate increase unambiguously over time. Q.E.D.

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