

PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE SCHOOL OF ENGINEERING

# A ROBUST LANE GEOMETRY ESTIMATION AND TRACKING APPROACH FOR DRIVER ALERT USING COLOR AND TEXTURE SEGMENTATION

# **RODOLFO ANTONIO TAPIA ESPINOZA**

Thesis submitted to the Office of Research and Graduate Studies in partial fulfillment of the requirements for the degree of Master of Science in Engineering

Advisor: MIGUEL TORRES TORRITI

Santiago de Chile, January 2012

C MMXII, Rodolfo Antonio Tapia Espinoza



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Gratefully to Javiera.

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## ABSTRACT

Real time and accurate lane detection under a wide range of conditions is a critical task in autonomous vehicle guidance and warning driver assistance systems. Most vision-based approaches rely on the analysis of the spacial gradient of the road image. A disadvantage of the edge-based approaches is that if the road structure is not regular and well delimited edges may not be easy to extract and other features must be taken into account. In this work, we evaluate the use of color and textural features to improve the standard gradient-based lane detection and its application as a lane departure detection system. Textural features are obtained using a bank of Gabor Filters and Gauss Markov Random Fields (GMRF), color-based detection use the mean-shift algorithm to cluster large uniform areas. The results from testing the approaches on city roads show that the color and texture analysis can improve the accuracy of road segmentation and lane departure detection.

**Keywords:** computer vision, image processing, lane detection, lane tracking, lane departure warning, color segmentation, texture segmentation, Gabor Filters, Gauss Markov Random Fields, mean-shift, RANSAC

#### RESUMEN

Una detección precisa de la pista en tiempo real y bajo un amplio rango de condiciones es una tarea crítica en el control de vehículos autónomos y en sistemas de alerta al conductor. La mayoría de los métodos basados en visión dependen principalmente de algún tipo de análisis del gradiente espacial de la imagen. Sin embargo, una de las desventajas del método basado en gradiente es que si la estructura del camino no es regular y bien delimitada, los bordes pueden no ser fáciles de extraer y otro tipo de características deben ser empleadas. Este trabajo evalúa el uso de características de color y textura como una manera de mejorar la detección de la pista basada en el método estándar de gradiente. Las características de textura son generadas usando un banco de filtros de Gabor y Campos Gaussianos Aleatorios de Markov, mientras que la detección basada en color usa el algoritmo mean-shift para agrupar áreas uniformes. Los resultados obtenidos al probar los métodos propuestos en rutas urbanas muestran que el análisis usando texturas y color puede mejorar la segmentación del camino y la detección de salidas del carril.

Palabras Claves: visión por computador, procesamiento de imógenes, detección de carril, seguimiento de carril, alerta ante salida del carril, segmentación por color, segmentación por textura, Filtros de Gabor, Campos Gaussianos Aleatorios de Markov, mean-shift, RANSAC

# 1. INTRODUCTION

According to the World Health Organization (WHO), 1.2 million people are estimated to be killed in road traffic accidents each year worldwide. while the number of injured is estimated in 50 million. On the other hand, the economic cost of road crash injuries is estimated at roughly 1% of gross national product in low-income countries, 1.5% in middle-income countries and 2% in high-income countries. The direct cost of global road crashes have been estimated at US\$ 518 billion, with the costs in low-income countries estimated at US\$ 65 billion (Peden et al., 2004).

Despite these figures, public health administrations traditionally view these figures of road injuries and accidents as random events and an inevitable outcome of road transport, leading to few investments to change this situation. However, as stated by WHO, "road crash injury is largely preventable and predictable; it is a human-made problem amenable to rational analysis and countermeasure". In this context, researching and developing systems that address the main causes of collisions, such as unintended maneuvers produced by errors, distraction or drowsiness is essential in the reduction of road accidents.

Some of these systems employ biometric measures of driver's performance parameters like alert state and fatigue level (Jimenez-Pinto & Torres-Torriti, 2011), (Lin et al., 2005) to infer drivers awareness level, other systems propose traffic control schemes (Estrin, Govindan, & Heidemann, 1999) as a safety improvement. While, other approaches rely on vision-based detection of the road and the surrounding conditions to assess possible danger situations, such as proximity of road intersections (Veeraraghavan, Masoud, & Papanikolopoulos, 2003), detection of traffic signs (Escalera, Moreno, Salichs, & Armingol, 1997), pedestrians presence (Dalal & Triggs, 2005) and unintended lane departure, among others.

Thus, the relevance of developing a robust vision-based lane detection and tracking system lies not only in the autonomous vehicle navigation field, but also for warning driver assistance systems, whose purpose is enhance the safety of drivers and pedestrians.

# 1.1. Objectives

The main objective of this work is to develop a robust vision-based road detection, tracking and lane departure warning system capable of preventing unintended lane changes due to distracted or drowsy drivers.

In order to achieve the main objective, the following specific objectives also must be fulfilled:

- extract the area corresponding to the road employing color and textural characteristics,
- detect lane marks and road edges in order to obtain the lane position with respect to a reference system,
- filter lane position measurements to improve estimations under noisy conditions,
- estimate lane departure conditions using the vehicle position relative to the lane,
- the system must be robust under a wide range of conditions like quality of lane marks (if they exist), lighting condition changes, road occlusion by other vehicles and environmental factors, as shown in fig. 1.1,
- minimize the overall cost of the system using off-the-shelf component (regular computers and cameras).

## 1.2. Hypotheses

The main research hypotheses are:

- the use of color and textural characteristic of the road can improve the standard gradient-based road detection,
- the use of texture and color features combined with gradient and RANSAC for lane curve fitting can lead to a reliable detection of the lane position,
- an Extended Kalman Filter can improve the system performance under noisy conditions,



(a) Good quality lane markings.

(b) Poor quality lane markings.



(c) Shadows and no lane marks.

(d) Multiple shadows.

FIGURE 1.1. Variable road conditions.

• metrics can be obtained to infer the vehicle position relative to the lane and possible lane departure situations.

## **1.3.** Existing Approaches

A prerequisite of any lane departure warning system is a robust lane recognition system. Lane recognition in turns requires road color, texture, position and distance features: among other to be extracted in order to be able to infer the lane position relative to a reference frame. The sensing devices employed for lane recognition include:

- radars, which uses radio waves to determine the range or speed of moving and fixed objects (Ma, Lakshmanan, & Hero, 2000),
- ladars (Laser Detection And Ranging, that can measure the distance to the laser illuminating the target (Wijesoma, Kodagoda, & Balasuriya, 2004),

- active infrared sensors, which measure variations in reflection of the infrared beam emitted by a LED on the road,
- standard perspective cameras, used to acquire the scene in front of the vehicle or in omnidirectional configuration.

Although the reported performance of radar, ladar and infrared sensors make them suitable for lane detection tasks, their main drawback is the elevated cost of these solutions. On other hand, one of the main advantages of standard visible spectrum cameras is their low cost, and widespread availability. Thus several image processing methods to extract road features and estimate the lane position have been proposed during the last decade.

Most of the vision-based approaches for lane detection extract lane edge information based on gradient thresholding (Lee, 2002; Otsuka, Muramatsu, Takenaga, Kobayashi, & Monj, 2002). Other approaches are based on template matching scheme (Kosecka, Blasi, Taylor, & Malik, 1998), and steerable filters for detecting solid-line and segmented-line marks (McCall & Trivedi, 2006). Other works have proposed likelihood models of the road (Kluge & Lakshmanan, 1995) in order to improve the performance of lane detection

Approaches for lane extraction considering color, gray intensity and texture segmentation of the pavement have also been suggested. Some use neural networks for classification (Fernandez-Maloigne & Bonnet, 1995), othres combine the features vector for each pixel (Jeong & Nedevschi, 2005), and some employ the covariance matrix of intensity changes in the image (J. Zhang & Nagel, 1994; Thorpe, Hebert, Kanade, & Shafer, 1988). However, to the best of our knowledge there are no approaches that consider the use of Gabor filters, Gauss Markov Random Fields and mean-shift clustering for road recognition. Evaluating and adopting these approaches is one of the contributions of this work.

In order to cope with road variability and make lane estimation more robust to external disturbances (e.g. illumination, visibility, lack of road structure), lane position tracking can be implemented using Kalman filters and sensor fusion techniques based on the vehicle's state monitoring (Ma et al., 2000) and GPS measures (Wang et al., 2005).

# **1.4. Summary of Contributions**

The main contributions of this work can be summarized in:

- (i) The introduction and evaluation of a robust lane detection and tracking system capable of dealing with occlusion, shadows and different lighting conditions.
- (ii) The performance evaluation of a lane departure warning system, whose objective is to warn drivers before an unintended lane changing occurs.
- (iii) The evaluation and comparison of texture and color based road segmentation approaches as a way to improve the standard gradient-based methods.

This research has been partially reported in (Tapia-Espinoza & Torres-Torriti, 2009a) and (Tapia-Espinoza & Torres-Torriti, 2009b).

# 2. PROPOSED APPROACH

The proposed approach for the lane recognition problem and its applications as a lane departure detection system is based on the following sub-processes.

First, the pavement area is segmented from the input video sequence using color and texture features combined or applying a mean-shift clustering approach and subsequent morphological operations. Then, the inverse perspective projection of edge features corresponding to the pavement area are computed. Curves representing the lane boundaries are subsequently computed using a the RANSAC algorithm (Fischler & Bolles, 1981) applied to the edges in the the inverse perspective projection. The output of this process is the position of the lane boundaries. An Extended Kalman filter is employed to track the lane position between frames. Finally, a rule-based approach is used to estimate lane departure situations and the result of the lane detection and tracking is projected to the coordinate frame of the camera's optical plane. The block diagram of this scheme is shown in fig. 2.1. The details of theses process are presented in the next sections.

## 2.1. Road segmentation

Three different methods were implemented to segment the road: a color clustering approach employing the mean-shift algorithm and two approaches using texture features (Gabor filters and Markov Random Fields).

#### 2.1.1. Mean-shift

Mean-shift is an iterative non-parametric feature-space analysis technique (Cheng, 1995). Among its main applications in computer vision and image processing are finding modes and clustering.

Mean-shift considers the feature space as an empirical probability density function. If the input is a set of points then mean-shift considers them as sampled from the underlying



probability density function. Dense regions (or clusters) present in the feature space correspond to the mode (or local maxima) of the probability density function. The goal of the procedure is to find local maxima of the underlying probability density from the samples.

Without loss of generality consider an initial estimate x of the modes and let a kernel function  $K(x_i - x)$  be given. This function determines the weight of nearby points of x in a neighborhood N(x) for re-estimation of the mean.

The weighted mean of the density in the window determined by K is given by

$$m(x) = \frac{\sum_{x_i \in N(x)} K(x_i - x) x_i}{\sum_{x_i \in N(x)} K(x_i - x)}.$$
(2.1)

The mean-shift algorithm sets  $x \leftarrow m(x)$ , and iterates until m(x) converges.

In this work, a mean-shift segmentation procedure is implemented as discussed in (Comaniciu & Meer, 2002) to extract the road area of the image.

The feature space considered is a joint domain consisting of spatial domain features  $x^s$  in image coordinate system and intensity values  $x^r$  in range domain, L\*u\*v\* color space is used because of its property of approximate perceptually uniform color spaces.

The kernel employed in this work is a multivariate function defined as the product of two radially symmetric kernels

$$K_{h_s h_r}(x) = \frac{C}{{h_s}^2 {h_r}^p} k_s \left( \left\| \frac{x^s}{h_s} \right\|^2 \right) k_r \left( \left\| \frac{x^r}{h_r} \right\|^2 \right)$$
(2.2)

where  $h_s$  and  $h_r$  are the bandwidth parameter of space and range domains respectivel, C is a normalization constant and k an Epanechnikov kernel.

Finally, the segmentation is performed by grouping all the filtered pixel values, which are closer than  $h_s$  in the spatial domain and  $h_r$  in the range domain.

An example of the result of the application of the described algorithm is shown in fig. 2.2 using the bandwidth parameters  $h_s = 2$ ,  $h_r = 4$ .





(a) Input image 1.

(b) Application of algorithm to to Input image 1.



(c) Input image 2.

(d) Application of algorithm to Input image 2.

FIGURE 2.2. Mean-shift segmentation algorithm result examples using bandwidth parameters  $h_s = 2$ ,  $h_r = 4$ .

# 2.1.2. Gabor Filters

A second method implemented for road area extraction is the segmentation based on the classification of texture and color features using a standard multivariate Gaussian classifier (MVGC) (Duda, Hart, & Stork, 2000). The texture features associated to each pixel are generated from the response to a bank of Gabor filters (Torres-Torriti & Jouan, 2001). To improve the identification of pavement and other elements, RGB color components of the Gaussian filtered image are also included into the features vector. The impulse response of the Gabor Filter,  $h(s|\sigma, \lambda, \theta, \phi, s_0)$ , in image space domain coordinates  $s = \begin{bmatrix} u & v \end{bmatrix}^T$  is expressed as follows:

$$h(s|\sigma,\lambda,\theta,\phi,s_0) = \exp\left\{\frac{\|s-s_0\|}{2\sigma^2}\right\} \cdot \sin\left(\frac{2\pi}{\lambda}\left(\left[s-s_0\right]^T \begin{bmatrix} \cos\theta\\ \sin\theta \end{bmatrix}\right) + \phi\right), \quad (2.3)$$

where  $\lambda$  corresponds to the wavelength of the sinusoid,  $\sigma$  is the spread of the Gaussian envelope,  $\theta$  and  $\phi$  are the orientation and phase of the Gabor filter respectively and  $s_0$  is a pixel in a neighborhood N(s) around s of the same size as that of the filter mask.

In order to discriminate textures correctly regardless of the phase of the texture, the energy of the response to a filter in pair phase quadrature with  $\phi = \{0, \pi/2\}$  is employed and computed as:

$$E^{2}(\sigma,\lambda,\theta|s) = \sum_{\phi=\{0,\pi/2\}} \left\{ \sum_{s_{0}\in N(s)} h(s|\sigma,\lambda,\theta,\phi,s_{0})I(s_{0}) \right\}^{2}.$$
 (2.4)

With this definition of the response's energy it is now possible to state the proposed feature vector as

$$X_G(s) = \left[ E^2(\lambda_1, \theta_1 | s) \ E^2(\lambda_i, \theta_j | s) \ \cdots \ E^2(\lambda_n, \theta_m | s) \ s_r \ s_g \ s_b \right], \quad (2.5)$$

where  $E^2(\lambda_i, \theta_j | s)$  represents a measure of the response's energy at pixel s to a Gabor filter with wavelength  $\lambda_i$ , orientation  $\theta_j$  and Gaussian filter color responses  $s_r, s_g, s_b$ .

In an initial stage, statistics from representative regions of the road in training images set are computed using this data. The segmentation of the image pixels in road or non road pixels is performed with a MVGC classifier whose parameters are the statistics of the training samples.

An example of the result of the application of Gabor Filters to the road scene of fig. 2.2(a) is shown in fig. 2.3.



(a) Response to parameters  $\lambda = 8$ , (b) Response to parameters  $\lambda = 8$ ,  $\theta = 0^{\circ}$ .  $\theta = 45^{\circ}$ .



(c) Response to parameters  $\lambda = 8$ , (d) Resp  $\theta = 90^{\circ}$ .  $\theta = 135$ 

(d) Response to parameters  $\lambda = 8$ ,  $\theta = 135^{\circ}$ .

FIGURE 2.3. Image response to various Gabor filters parameters.

#### 2.1.3. Gauss Markov Random Fields

Another texture-based segmentation method implemented is Gauss Markov Random Fields (GMRF) (Torres-Torriti & Jouan, 2001). As Gabor filters, texture features are classified using a standard multivariate Gaussian classifier (MVGC).

In the GMRF approach the statistical dependency between a pixel gray level intensity I(s) at site s and its neighbors is represented as a linear combination of gray levels in a neighborhood set N(s). The model assuming zero mean Gaussian observations is expressed as follows:

$$I(s) = \sum_{r \in \Delta N} \theta_r \left( I(s+r) + I(s-r) \right) + e(s),$$
(2.6)

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where  $\Delta N = \{r : s \pm r \in N(s)\}.$ 

The unknown parameters  $\theta$  can be obtained using the least squares method as:

$$\theta^* = \left[\sum_{s \in \Sigma_I} q(s)q(s)^{\mathsf{T}}\right]^{-1} \left[\sum_{s \in \Sigma_I} q(s)I(s)\right],\tag{2.7}$$

where  $\Sigma_I$  is an interior region of the image and q(s) is defined as:

$$q(s) = \begin{bmatrix} I(s+r_1) + I(s-r_1) \\ I(s+r_2) + I(s-r_2) \\ \vdots \\ I(s+r_n) + I(s-r_n) \end{bmatrix}, r_i \in \Delta N.$$
(2.8)  
(2.9)

The estimate  $v^*$  of the noise variance is calculated from:

$$v^{*} = \frac{1}{|\Omega_{I}|} \left[ \sum_{s \in \Sigma_{I}} I(s) - \theta^{*\intercal} q(s) \right]^{2} \right].$$
(2.10)

Finally, textures can be properly characterized by feature vectors,  $X_M$ , defined by the estimated GMRF model parameters as:

$$X_M = \begin{bmatrix} \theta^* & v^* / \rho^2 \end{bmatrix}^{\mathsf{T}},\tag{2.11}$$

where  $\rho$  is the sample variance of the texture as a feature vector  $\begin{bmatrix} \theta^* & v^* \end{bmatrix}^{\mathsf{T}}$ .

#### 2.1.4. Morphological operations

Since the purpose of color or texture recognition is mainly to identify the pavement area, small misclassified regions are removed employing standard morphological operations, which are applied to a binary image whose pixels labeled as one indicate image regions that are likely to contain pavement texture. The morphological operations involve removing isolated regions and filling holes. If  $I_t$  denotes the binary image for the road area and  $I_c$  denotes the image with connected road texture regions, then the image mask  $I_m$  with the area of interest containing the pavement is generated according to the procedure as summarized by the following pseudocode:

Algorithm 1: Morphological operations
<b>Input</b> : $I_t$ , binary image for the road area
<b>Output</b> : $I_m$ , region of interest
// Label connected regions
$I_c(s) \leftarrow \texttt{LabelConnReg}(I_t(s));$
// Compute area of each connected region
$A(label) \leftarrow \texttt{AreaConnReg}(I_c(s));$
// Remove areas below $Area_{min}$
foreach label in A do
if $A(label) \ge Area_{min}$ then
forall s in label do $  I_m(s) \leftarrow I(s);$
end
end
end
// Close holes in open areas
$I_m(s) \leftarrow \texttt{CloseOpenAreas}(I_m(s))$

The result of the pavement identification and posterior morphological operations yielding the region of interest  $I_m$  are shown in fig. 2.4.

# 2.2. Lane detection

The method for estimation of the lane position, relative to the world reference system, employs the RANSAC algorithm to obtain the parameters of the road, which is modeled as polynomial curve that approximates the clothoidal curve of the standard model. By doing



(a) Input image. (b) Road area segmentation result.



(c) Image (b) after morphological filering.

(d) Superimposed road area segmentation.

FIGURE 2.4. Road area morphological operations.

so it is quite possible to quickly and robustly obtain a road model which is sufficiently accurate for lane departure warning purposes. The lane detection procedure and the approach to determine the vehicle's position relative to the road are described in the next subsections.

## 2.2.1. Lane boundaries pixels extraction

The extraction of lane boundaries pixels relies on the application of steerable filters to find edges corresponding to lane marks or road boundaries on the pavement that were extracted in the previous road segmentation step. Steerable filters have proved to be reliable in the detection of solid and segmented line markings under different road conditions (McCall & Trivedi, 2006).

Steerable filters correspond to oriented LoG filters calculated using three pre-computed filters, one in the horizontal direction,  $G_{uu}$ , one in the vertical direction,  $G_{vv}$ , and one in the diagonal direction,  $G_{uv}$ , allowing an efficient calculation of the oriented filter  $G_{\theta}$  according to the following equations (see (Freeman & Adelson, 1991) for details):

$$G_{uu}(u,v) = \frac{\partial^2}{du^2} e^{-(u^2+v^2)/\sigma^2}$$
  
=  $(\frac{4u^4 - 2\sigma^2}{\sigma^4}) e^{-(u^2+v^2)}$  (2.12)

$$G_{vv}(u,v) = \frac{\partial^2}{\partial v^2} e^{-(u^2 + v^2)/\sigma^2}$$
  
=  $(\frac{4v^4 - 2\sigma^2}{\sigma^4}) e^{-(u^2 + v^2)}$  (2.13)

$$G_{uv}(u,v) = \frac{\partial^2}{dudv} e^{-(u^2 + v^2)/\sigma^2} = \frac{4uv}{\sigma^4} e^{-(u^2 + v^2)}$$
(2.14)

$$G_{\theta}(u,v) = G_{uu}\cos^2(\theta) + G_{vv}\sin^2(\theta) + G_{uv}\cos(\theta)\sin(\theta).$$
(2.15)

In order to maximize the response of the lane marks and road boundaries to the steerable filters, a bank composed by two filters is applied to the road area extracted in the previous step. One filter is tuned to be oriented along the normal of the left boundary of the lane and the other along the normal of the right boundary. The thresholded responses to both filters are then combined applying the logical OR operator into one unique response. The orientation of both filters is updated in each iteration according to the orientation of the lane's boundaries that were obtained in the previous iteration.

The application of the steerable filter to the extraction of lane edges is shown in fig. 2.5. As shown in the example fig. 2.5 some elements that do not belong to the lane markings of interest are also detected. In order to filter these undesired elements a geometric criterion is applied to each region of connected pixels in the image.



(a) Segmented road area.

(b) Response to filter tuned on  $\theta = -\frac{\Pi}{6}$ .



(c) Response to filter tuned on  $\theta = +\frac{\Pi}{6}$ .

(d) Response to filter tuned on  $\theta = -\frac{\Pi}{6}$  after thresholding.



(e) Response to filter tuned on  $\theta = +\frac{\Pi}{6}$  after thresholding.



(f) Logical OR operation between thresholded filter responses.

FIGURE 2.5. Applying the steerable filters to a road image.

The geometric criterion is based on the fact that a straight line segment represents the degenerate case of an ellipse whose eccentricity, the ratio of the distance between the foci of the ellipse and its major axis length, is equal to 1. Thus, discarding the regions of connected pixels whose fitted ellipse eccentricity is lower than  $1 - \delta$ , where  $\delta$  is a parameter experimentally adjusted to accept also lines segments that are not strictly straight such as curved marks. The result of the described process is presented in fig. 2.6.

After removing the regions of connected pixels that do not correspond to line segments, the remaining selected pixels become candidates to be part of the left or right lane making



FIGURE 2.6. Result of connected pixels removal based on the geometric eccentricity criterion.

lines or road boundaries, which must be labeled as one of the two edges for subsequent lane position extraction stages.

In order to label the remaining regions of connected pixels, the Hough Transform (Shapiro, Stockman, Shapiro, & Stockman, 2001) of the image containing these elements is computed. To avoid false positives from isolated peaks in the Hough map,  $H(\rho, \theta)$ , we compute mass centers for regions above a predefined threshold  $\lambda_H$ . The mass centers provide the  $(\rho, \theta)$  parameters.

The left and right boundaries label reference parameters are selected as those whose intersect with the horizontal bottom line  $\ell_b$  in the image is closest to the midpoint of  $\ell_b$  from the left and right of the midpoint.

Finally, each region of connected pixels is labeled as part of the left or right lane boundary or none of them according to their  $\rho$  and  $\theta$  peak values obtained from computing the Hough transform and the comparison to the left and right boundaries label parameters.

The lane boundaries pixels extraction procedure is summarized by the pseudocode detailed in Algorithm 2.

#### 2.2.2. Inverse Perspective Mapping

Once the lane boundaries pixels have been obtained from the segmented road area, an Inverse Perspective Mapping (IPM) transformation is applied to the these pixels. The reason behind this procedure is that the road boundaries correspond to a clothoid curve as looked from a top view (Nedevschi et al., 2004). Thus, for proper lane geometry estimation

#### Algorithm 2: Lane boundaries pixels extraction

```
Input: I_m (region of interest); \theta_{left}, \theta_{right} (filter orientation)
Output: LeftBoundPixels, RightBoundPixels (lane boundaries pixels)
// Compute the responses of the road segmented image, I_m,
     to the bank of steerable filters
I_l(s) \equiv G_{\theta_{left}} * I(s) \ \forall s \in \{p \mid I_m(p) = 1\};
I_r(s) \equiv G_{\theta_{right}} * I(s) \ \forall s \in \{p \mid I_m(p) = 1\};
I_e(s) \equiv I_l(s) \lor I_r(s);
// Apply threshold
\lambda_e \leftarrow 0.1 \cdot max(I_e(s));
I_{\lambda}(s) \leftarrow I_{e}(s) \geq \lambda_{e};
// Label regions in I_{\lambda}(s)
L_I(label) \leftarrow \text{AreaConnReg}(I_\lambda(s));
// Calculate eccentricity for every region in L_I(label) and
     discard line unalike regions
forall labelinL_I do
    Eccentricity(label) \leftarrow GetEccentricity(L_I(label));
    if Eccentricity(label) < 1 - \delta then
    | I_e(s)(label) \leftarrow 0;
   end
end
// Compute the Hough transform
H(\rho, \theta) \leftarrow \text{HoughTransform}(I_e(s));
// Apply threshold
\lambda_H \leftarrow 0.5 \cdot \text{Max} (H(\rho, \theta));
H_{\lambda}(\rho, \theta) \leftarrow H(\rho, \theta) \ge \lambda_H;
// Label regions
L(label) \leftarrow \text{AreaConnReg}(H_{\lambda}(\rho, \theta));
// Calculate mass centers for every region in L(label)
forall labelinL do
   LanesCand(label) \leftarrow GetMassCenter(L(label));
end
// Select left and right lines of the lane
forall label in LanesCand do
    Intersec \leftarrow GetIntersection (LanesCand(label), \ell_b);
    if Intersec is the nearest from right to ImageWidth/2 then
        RighBoundPixels \leftarrow LanesCand(label);
    else if Intersec is the nearest from left to ImageWidth/2 then
       LeftBoundPixels \leftarrow LanesCand(label);
    end
end
```

and accurate position computation it is very useful to compute the IPM and top view of the road.

The projection is performed using the standard pinhole camera model as shown in fig. 2.7, under the assumption that:

- the world coordinate system  $S_W$  fixed to camera, and thus to the vehicle, is inertial, while the scene points move relative to a stationary vehicle in a direction opposite to that resulting from the driver's maneuvers,
- the camera is mounted at some given constant height h with respect to the ground,
- the road ahead near the vehicle (first 5-10 m) is flat (has zero slope with respect to the ground tangent plane at the vehicle's current position).

Consider a camera mounted on a car as depicted in fig. 2.7. The world coordinate frame is denoted by  $S_W \stackrel{def}{=} {\mathbf{x}^W, \mathbf{y}^W, \mathbf{z}^W}$  and a point in the world lying on the road plane  $\mathbf{P} = [x, y, z]^T$ . Similarly, consider a coordinate frame attached to the camera's optical plane  $S_I \stackrel{def}{=} {\mathbf{u}, \mathbf{v}}$  and the projection of  $\mathbf{P} \in S_W$  onto  $S_I$  denoted by  $\mathbf{p} = [r, c]^T$ . On the other hand,  $S_A \stackrel{def}{=} {\mathbf{r}, \mathbf{c}}$  corresponds to the coordinates system fixed on the optical plane, where the  $\mathbf{r}, \mathbf{c}$  axes represent the indices of the pixels.

Let f be the focal length of the camera and  $\theta_0$  the tilt angle of the u axis. In order to relate r, v and c, u, also m and n must be defined as the number of pixels rows and columns in the image respectively and  $\rho[pixels/m]$  as the number of pixels per meter on the physical image plane array.

It is possible to derive the following transformation between  $S_A$  and  $S_I$  given by:

$$\mathbf{v}(\mathbf{r}) = \frac{1}{\rho} \left[ \frac{m+1}{2} - \mathbf{r} \right] \Rightarrow \mathbf{r}(\mathbf{v}) = \frac{m+1}{2} - \rho \mathbf{v}, \qquad (2.16)$$



FIGURE 2.7. Geometric projection model (perspective view).

$$\mathbf{u}(\mathbf{c}) = \frac{1}{\rho} \left[ \mathbf{c} - \frac{n+1}{2} \right] \Rightarrow \mathbf{c}(\mathbf{u}) = \frac{n+1}{2} + \rho \mathbf{u}.$$
 (2.17)

From examination of fig. 2.8, the relation between focal length f and the camera's vertical aperture  $\alpha_v$  is given by:

$$tan(\alpha_v) = \frac{v(1)}{f}$$
  

$$\Rightarrow f = v(1)\cot(\alpha_v)$$
  

$$f = \frac{m-1}{2\rho}\cot(\alpha_v).$$
(2.18)



FIGURE 2.8. Geometric projection model (lateral view).

Using the properties of equivalent triangles properties, we can obtain the point  $\mathbf{P}$  coordinates in terms of the image coordinates of point  $\mathbf{p}$  as follows.

$$\frac{x\sin(\theta_0) - h\cos(\theta_0)}{x\cos(\theta_0) + h\sin(\theta_0)} = \frac{x\tan(\theta_0) - h}{x + h\tan(\theta_0)} = \frac{v}{f},$$
(2.19)

Substituting equations (2.16) and (2.18) in (2.19) and solving for X leads to:

$$\frac{x\tan(\theta_0) - h}{x + h\tan(\theta_0)} = \left[1 - 2\left(\frac{r-1}{m-1}\right)\right]\tan(\alpha_v)$$
(2.20)

$$\Rightarrow x(r) = h\left(\frac{1 + \left[1 - 2\left(\frac{r-1}{m-1}\right)\right] \tan(\alpha_v) \tan(\theta_0)}{\tan(\theta_0) - \left[1 - 2\left(\frac{r-1}{m-1}\right)\right] \tan(\alpha_v)}\right)$$
(2.21)

Once again applying equivalent triangles properties of fig. 2.9 it is possible to establish that

$$\frac{y}{h\sin(\theta_0) + x\cos(\theta_0)} = -\frac{u}{f}, \qquad (2.22)$$

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FIGURE 2.9. Geometric projection model (top view).

and substituting equations (2.17) and (2.18) in (2.22):

$$\frac{y}{h\sin(\theta_0) + x\cos(\theta_0)} = \left[1 - 2\left(\frac{c-1}{n-1}\right)\right]\tan(\alpha_u).$$
 (2.23)

Finally, substituting x(r) from equation (2.21) in (2.23) and solving for y leads to:

$$y(r,c) = h\left(\frac{\left[1-2\left(\frac{c-1}{n-1}\right)\right]\tan(\alpha_u)}{\sin(\theta_0) - \left[1-2\left(\frac{r-1}{m-1}\right)\right]\tan(\alpha_v)\cos(\theta_0)}\right), \quad (2.24)$$

that completes with (2.21) the equations system necessary to calculate the coordinates of a point **P** lying on the road plane from the coordinates of a point **p** in the image plane.

It is to be noted that in both expression for x(r) and y(r, c), the camera height h acts as a scale factor, suitable for system calibration if some physical magnitude is known, such as the lane width.

From equation (2.20) and (2.23) it is possible to obtain the coordinates of  $\mathbf{p} = [r, c]^T$ on  $S_A$  of a point  $\mathbf{P} = [x, y, z]^T$  into the scene projected onto the optical plane as follows:

$$\frac{x tan(\theta_0) - h}{x + h tan(\theta_0)} = \left[1 - 2\left(\frac{r - 1}{m - 1}\right)\right] tan(\alpha_v)$$
  

$$\Rightarrow r(x) = \frac{m - 1}{2} \left[1 + \frac{h - x tan(\theta_0)}{h tan(\theta_0) + x} cot(\alpha_v)\right] + 1, \quad (2.25)$$

$$\frac{y}{hsin(\theta_0) + xcos(\theta_0)} = \left[1 - 2\left(\frac{c-1}{n-1}\right)\right] tan(\alpha_u)$$
  

$$\Rightarrow c(x,y) = \frac{n-1}{2} \left[\frac{y}{hsin(\theta_0) + xcos(\theta_0)} cot(\alpha_u)\right] + 1. \quad (2.26)$$

As shown in fig. 2.10(b) the application of equations (2.21) and (2.24) to the lane boundaries pixels of 2.10(a) leads to unevenly spaced points because pixel close to the horizon represents larger distance of the road than pixels of the pavement close to the car. In order to correct this situation, evenly spaced interpolation is performed to ensure a *constant density* of points per unit of length. The result of this interpolation procedure is shown in fig. 2.10(c).

#### 2.2.3. Lane boundaries position estimation

In order to estimate the position of the lane relative to the world coordinate system, the coordinates of the lane boundaries pixels after the IPM transformation are fitted to a polynomial function using the RANSAC algorithm (Fischler & Bolles, 1981).

The RANSAC algorithm (RANdom Sample And Consensus) is a method to estimate the parameters of a certain model from a set of data contaminated by large amounts of outliers. An outlier may be defined as a datum that does not fit to the model instantiated by the correct parameters within some error threshold for the deviation produced by the effects of noise, assuming that there exists a correct set of parameters that can exactly generate the observed measurements if they were observed in absence of noise.

The RANSAC algorithm essentially involves the following steps which are iteratively repeated until certain termination criterion is fulfilled (e.g. an error threshold or a maximum number of iterations):

• Step 1: Hypothesize. A minimal sample set (MSS) is randomly selected from the input data set and the models parameters are computed using the elements of the MSS. The cardinality of MSS is such that its elements are the minimum to



(a) Lane boundaries pixels in image reference system.



(b) Lane boundaries pixels mapped mapped onto world reference system (without interpolation).



(c) Lane boundaries pixels mapped onto the world reference system after evenly spaced interpolation.

FIGURE 2.10. IPM application to lane points.

determine the model parameters. For instance if a parabola must be determined the cardinality of the MSS is 3, since at least 3 different points are required to define such a curve.

• Step 2: Test. The objective of the second step of the algorithm is to verify which elements of the entire data set are consistent with the model estimated from the MSS. This set of elements is called the consensus set (CS).

Without loss of generality, consider the problem of fitting a polynomial function of order k with coefficients  $\theta_0, \theta_1, \theta_2, \dots, \theta_k \in \mathbb{R}$  to a set of N points  $D = \{\mathbf{p_1}, \dots, \mathbf{p_N}\} \subset \mathbb{R}^2$  i.e. each  $p_i = (x_i, y_i)$  must satisfy  $\theta_0 + \theta_1 x_i + \theta_2 x_i^2 + \dots + \theta_k x_i^k - y_i = 0$ . To estimate which elements belong to the CS, the absolute value of the error defined by

$$e\left(\mathbf{p_i};\theta\right) = \frac{\theta_0 + \theta_1 x_i + \theta_2 x_i^2 + \ldots + \theta_k x_i^k - y_i}{\sqrt{\theta_0^2 + \ldots + \theta_k^2}}$$
(2.27)

is computed for each point. The points whose error is below a certain threshold  $\delta$  are those belonging to the CS.

A ranking method must be employed in order to assess if the CS determined in the current iteration is better than a previous one. In the originally proposed RANSAC algorithm (Fischler & Bolles, 1981), the ranking of a CS is determined by its cardinality, thus a larger CS is ranked better.

In mathematical terms, the original RANSAC algorithm can be formulated as an optimization algorithm whose goal is to minimize the cost function:

$$C(D;\theta) = \sum_{i=1}^{N} \rho(\mathbf{p_i},\theta), \qquad (2.28)$$

where:

$$\rho(\mathbf{p_i}, \theta) = \begin{cases} 0 : e(\mathbf{p_i}; \theta) \le \delta \\ 1 : otherwise \end{cases} .$$
(2.29)

However, in this work an alternative approach based on M-Estimators (Z. Zhang, 1997) is employed, where the function  $\rho$  is substituted by

$$\rho(\mathbf{p_i}, \theta) = \begin{cases}
e\left(\mathbf{p_i}; \theta\right) & : e\left(\mathbf{p_i}; \theta\right) \le \delta \\
\delta & : otherwise
\end{cases}.$$
(2.30)

This approach, also called MSAC (M-estimator Sample And Consensus), weights the inliers according to how well they fit to the model, while the outliers are given a constant weight, improving the robustness of the estimation with no additional computational cost (Torr & Zisserman, 2000).

Using the RANSAC procedure, the parameters of the road modeled as a polynomial curve are obtained. This polynomial road model approximates the clothoidal standard model (Nedevschi et al., 2004) used for road construction, whose main characteristic is that it prevents sudden changes in centripetal force while driving, leading to smooth transitions between straight and curved sections of the road and allowing application of steering actions in a gradual manner by the drivers. The parametric clothoidal model  $L \rightarrow (x (L), y (L))$  is given by:

$$x(L) = \sqrt{2R_0 s_0} \int_0^L \cos(s^2) \, ds, \qquad (2.31)$$

$$y(L) = \sqrt{2R_0 s_0} \int_0^L \sin(s^2) ds,$$
 (2.32)

where  $R_0$  corresponds to the radius of the circular curve at the end of the spiral and L the length of the spiral curve.

Considering the normalized clothoid curve ( $\sqrt{2R_0s_0} = 1$ ) and replacing  $\cos(s^2)$  and  $\sin(s^2)$  in (2.31) and (2.32) by their power series expansion allows to find an approximate parametric representation in polynomial form, which is amenable for simpler and faster

computation of the road geometry:

$$\begin{aligned} x(L) &= \int_0^L \left( 1 - \frac{s^4}{2!} + \frac{s^8}{4!} - \frac{s^{12}}{6!} + \dots \right) ds \\ &= L - \frac{L^5}{5 \cdot 2!} + \frac{L^9}{9 \cdot 4!} - \frac{L^{13}}{13 \cdot 6!} + \dots, \end{aligned} \tag{2.33} \\ y(L) &= \int_0^L \left( s^2 - \frac{s^6}{3!} + \frac{s^{10}}{5!} - \frac{s^{14}}{7!} + \dots \right) ds \\ &= \frac{L^3}{3} - \frac{L^7}{7 \cdot 3!} + \frac{L^{11}}{11 \cdot 5!} - \frac{L^{15}}{15 \cdot 7!} + \dots. \end{aligned} \tag{2.34}$$

In this work, in order to estimate the position of the boundaries of the lane, cubic curves

$$y = \theta_3 x^3 + \theta_2 x^2 + \theta_1 x + \theta_0, \qquad (2.35)$$

are fitted onto the the data obtained from the lane boundaries pixels extraction step in world coordinates after IPM projection as a third order approximation of the truncated power series expansion (2.33) and (2.34) of the clothoid curve parametric coordinates.

An example of the RANSAC fitting algorithm applied to the lane detection problem is shown in fig. 2.11, where the threshold  $\delta$  was experimentally set to 0.2 m in order to reject outliers points caused by disturbances such as shadows on the road.

However, sometimes it may be almost impossible to correctly fit a curve onto one of the lane's boundaries due occlusions of the road's edges or lines caused by other vehicles, as well as shadows casted on the road by trees and buildings. In order to make the lane detection algorithm more robust under occlusions, an additional step is performed after the RANSAC curve fitting is applied.

First, the range of the consensus sets obtained from fitting cubic curves onto the left and right lane boundaries are checked. It was experimentally found that the range of the points conforming the consensus sets must be such that its minimum and maximum values are at least between 10 m and 20 m ahead of the vehicle to consider the fitting as reliable, in the sense that the points in the CS represent an important section of the road. Whenever this condition is not fulfilled and the cardinality of the CS is not empty, i. e. the lane boundary







(b) Curve fitting superimposed on lane segmentation (top-view).

FIGURE 2.11. RANSAC cubic fitting (green) application under strong shadow presence.

is not completely occluded, a vertically shifted version of the opposite lane boundary curve:

$$y_{\Delta} = \theta_3 x^3 + \theta_2 x^2 + \theta_1 x + \Delta, \qquad (2.36)$$

where  $\Delta$  corresponds to the distance that the curve must be translated is fitted applying the RANSAC algorithm.

Finally, if any of the CS is empty (the lane in completely occluded) an estimation of the lane position obtained from the Extended Kalman Filter is employed. Fig. 2.12 (b) shows the application of this procedure to the case depicted in fig. 2.12 (a), where a correct curve fitting is achieved through fitting a similar curve vertically shifted.



(a) RANSAC erroneous fitting due occlusion of right boundary of the lane (red).



(b) Correction using the curve fitted to the left boundary of the lane (blue) to fit a curve onto the right boundary segment points (red).

FIGURE 2.12. Example of RANSAC fitting correction procedure.

The lane boundaries position estimation procedure is summarized by the pseudocode detailed in Algorithm 3.

Algorithm 3: Lane boundaries position estimation

```
Input: LeftBoundPixels, RightBoundPixels (lane boundaries pixels); h, \theta_0, \alpha_u, \alpha_v
        (camera parameters)
Output: \Theta_L, \Theta_R (left and right fitted cubic curves parameters)
// Compute IPM transformation to lane boundaries pixels
LeftBoundPixels_{IPM} \leftarrow IPM (LeftBoundPixels, h, \alpha_v, \alpha_u, \theta_0);
RighBoundPixels_{IPM} \leftarrow IPM (RighBoundPixels, h, \alpha_v, \alpha_u, \theta_0);
// Perform RANSAC algorithm to IPM transformation of lane
     boundaries pixels
[CS_L, \Theta_L] \leftarrow \text{RANSAC} (LeftBoundPixels_{IPM}, iterations_{max}, \delta);
[CS_R, \Theta_R] \leftarrow \text{RANSAC} (RighBoundPixels_{IPM}, iterations_{max}, \delta);
// Perform fitting corrections if necessary
if (10 < \text{Min}(CS_L) \lor \text{Max}(CS_L) < 15) \land (10 > \text{Min}(CS_R) \land \text{Max}(CS_R) > 15)
then
    \Theta_L \leftarrow \text{RANSACCorrection} (LeftBoundPixels_{IPM}, \Theta_R, iterations_{max}, \delta);
else
    \Theta_L \leftarrow \text{EKF}();
end
if (10 \leq \text{Min}(CS_R) \vee \text{Max}(CS_R) \leq 15) \land (10 \geq \text{Min}(CS_L) \land \text{Max}(CS_L) \geq 15)
then
    \Theta_R \leftarrow \text{RANSACCorrection} (RightBoundPixels_{IPM}, \Theta_L, iterations_{max}, \delta);
else
    \Theta_R \leftarrow \text{EKF}();
end
```

### 2.3. Lane position tracking method

In order to improve the robustness of the lane detection scheme to erroneous measurements, an Extended Kalman Filter (EKF) is employed to track the lane boundary lines between frames. Since the parameters of a cubic curve can be inferred from the coordinates of 4 points, 8 points are tracked independently to filter the position of the left and right boundaries of the lane, as shown in fig. 2.13.

Consider  $\hat{\mathbf{P}}_{S_W} = [P_{S_W} \mathbf{1}]^T$  as one of the tracked points corresponding to a 3D point in homogeneous coordinates, projected using the Inverse Perspective Mapping procedure previously discussed.

Tracking of point  $\mathbf{P}_{S_W}$  requires some knowledge of the motion of the vehicle. To this end, a simplified motion model of the vehicle is employed under the same assumptions



FIGURE 2.13. In order to filter two cubic curves, 8 points are tracked.

made on the Inverse Perspective Mapping section, namely, the world coordinate system  $S_W$  fixed to the camera at a constant height (and thus to the vehicle) is inertial, while the scene points move relative to the vehicle in a direction opposite to that resulting from the driver's maneuvers. Also the road ahead is assumed to be flat at least in a vicinity of the vehicle.

Under these assumptions, the motion model for the tracked points  $\hat{\mathbf{P}}_{S_W}$  can be constructed as follows. First, the translation and rotation of  $\hat{\mathbf{P}}_{S_W}$  according to the driver's maneuvers are considered. The translation defined along the X is represented by the transformation matrix:

$$\mathbf{T}_{X}(\delta_{D}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\delta_{D} \\ 0 & 0 & 0 & 1 \end{bmatrix},$$
(2.37)

where  $\delta_D$  is the vehicle's longitudinal displacement between two image frames. On the other hand, the rotation around the Z axis is given by the homogeneous transformation

matrix:

$$\mathbf{R}_{Z}(\theta_{D}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta_{D}) & \sin(\theta_{D}) & 0 \\ 0 & -\sin(\theta_{D}) & \cos(\theta_{D}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$
(2.38)

where  $\theta_D$  is the amount of rotation between the acquired frames. The variables  $\delta_D$  and  $\theta_D$  can be estimated from the vehicle's motion model as the result of the vehicle's speed v and amount of steering  $\phi$ :

$$\delta_D = v \cdot \Delta T \tag{2.39}$$

$$\theta_D = k \cdot \phi. \tag{2.40}$$

This static model considers the vehicle's translation  $\delta_D$  and yaw angle  $\theta_D$  as decoupled from the respective inputs  $u_1 = v$  and  $u_2 = \phi$ , respectively.

The point  $\hat{\mathbf{P}}_{S_W}$  is then transformed to the point  $\hat{\mathbf{P}'}_{S_W}$ , in homogeneous coordinates, according to:

$$\hat{\mathbf{P}}'_{S_W} = \mathbf{R}_Z(\theta_D) \cdot \mathbf{T}_X(\delta_D) \cdot \hat{\mathbf{P}}_{S_W}$$
(2.41)

Which completes the formulation of the motion model:

$$f: (\hat{\mathbf{P}}_{S_W}, u_1, u_2) \to \hat{\mathbf{P}}'_{S_W}$$

$$(2.42)$$

In terms of the common notation, the EKF states are the vectors  $\mathbf{x}_{\mathbf{k}} = \hat{\mathbf{P}}_{\mathbf{S}_{\mathbf{W}}}(\mathbf{k})$ , the motion model  $\mathbf{x}_{\mathbf{k}+1} = \mathbf{f}(\mathbf{x}_{\mathbf{k}}, \mathbf{u}_{\mathbf{k}}, \mathbf{v}_{\mathbf{k}})$ , with  $\mathbf{u}_{\mathbf{k}} = (\delta_{\mathbf{k}}, \theta_{\mathbf{k}})$ , and a measurement model  $\mathbf{z}_{\mathbf{k}} = \mathbf{x}_{\mathbf{k}} + \mathbf{w}_{\mathbf{k}}$ . To complete the filter the covariance of the process disturbance  $\mathbf{v}_{\mathbf{k}}$  is

assumed constant:

$$\Sigma_{v}(k) = \begin{bmatrix} \sigma_{v_{u}}^{2} & \sigma_{v_{\delta}}\sigma_{v_{\theta}} \\ \sigma_{v_{\delta}}\sigma_{v_{\theta}} & \sigma_{v_{v}}^{2} \end{bmatrix}, \qquad (2.43)$$

while the covariance of the measurement noise  $w_k$  is defined as:

$$\Sigma_{z}(k) = \begin{bmatrix} \sigma_{z_{u}}^{2} & \sigma_{z_{\delta}}\sigma_{z_{\theta}} \\ \sigma_{z_{\delta}}\sigma_{z_{\theta}} & \sigma_{z_{v}}^{2} \end{bmatrix}.$$
(2.44)

#### 2.4. Lane departure estimation

Among the objectives of this work is to develop a lane departure warning system (LDW), based on the detection and tracking of the lane position relative to the car. The LDW system must be capable of preventing unintended lane changes.

One of the simplest methods employed is to measure the position of the lane boundaries relative to the vehicle, triggering an alarm when the lateral distance falls below a fixed threshold. However, the main drawback of this approach is the high false alarm rate due to the arbitrary low distance to any of the lane boundaries that a driver may keep without changing lane.

A better approach is to consider the time-to-lane-crossing (TLC) as a measure of lane departure danger (Mammar, Glaser, & Netto, 2006). The TLC corresponds to the time remaining before the vehicle crosses one of the lane boundaries. Thus, TLC in iteration k is computed as:

$$TLC_k = \frac{d_k}{v_k} \tag{2.45}$$

where  $d_k$  corresponds to the distance between the vehicle and the lane boundary and  $v_k$  to the lateral speed of the vehicle relative to the lane boundaries position at sampling instant k.

In this work, the distance  $d_k$  can be immediately obtained from the equation of the detected and tracked curves as the constant term of the cubic polynomial model. To estimate the lateral speed, the values of recently computed lateral distance into a fixed time window are employed according to:

$$v_k = \frac{1}{N} \cdot \sum_{i=k-N}^k \frac{d_i - d_{i-1}}{t_i - t_{i-1}},$$
(2.46)

where  $t_i$  represents the time at sampling instant k and N the fixed time windows duration.

In the constant offset case, the lateral speed will be low, thus leading to a large TLC value and no alarm triggering. If fast correction maneuvers are performed by the driver, the TLC value will also be large as the drivers keeps the car centered to the lane and far from its boundaries. A TLC alarm is triggered whenever the lateral distance is low and the lateral speed is high, since this condition results in a small TLC value.

# 3. TESTING METHODOLOGY

## 3.1. Data Acquisition

A sequence of images captured during real driving conditions was used to evaluate the proposed lane extraction method on highway and urban roads. The evaluation considers different light conditions such as daytime, sunset and dusk and various road conditions, presence or absence of lane marks, shadows, occlusions due to other vehicles and varying road geometry including straight and curved streets.

The camera employed was a Point Gray<sup>®</sup> Firewire camera with a  $640 \times 480$  pixels 1/2" CCD and a Tamron varifocal lens with focal distances in the range 6 - 12 mm, corresponding to a field view in the range  $30.4^{\circ} \times 23.1^{\circ}$  (telephoto)  $-58.7^{\circ} \times 44.4^{\circ}$  (wide). The camera was mounted on a structure on the roof of a stock Toyota Yaris, with an ahead viewing distance of about 40 m. The camera setup in shown in fig. 3.1.



FIGURE 3.1. Vehicle for data acquisition.

## 3.2. Results Analysis and Validation

In order to assess the performance of the proposed approach, the algorithm is evaluated under different road conditions and using the three road segmentation procedures discussed in the previous sections: Gabor filters, Gauss Markov Random Fields (GMRF) and meanshift clustering. The performance of the lane detection relying only on the computation of gradients applying steerable filters without any other form of road segmentation is also computed for comparison purposes. In addiction to the cubic curves employed to describe the lane boundaries also, quadratic curves were considered for, quadratic curves were also fitted for comparison.

Approximately 2.5 hrs of city road video footage recorded at 15 fps was captured to evaluate the proposed method. Sequences of images containing different road geometry, lighting conditions and disturbances, such as occlusions and shadows, were taken into account. Table 3.1 summarizes the different scenarios and the number of samples considered for the performance evaluation of the lane detection algorithm.

Scenario	Road geometry	Lane marks quality	Lighting condition	Number of samples
1	Straight	Good	Daytime, no shadows	340
2	Straight and curved	Good	Daytime, no shadows	350
3	Straight	Medium	Sunset, shadows presence	410
4	Straight and curved	Medium	Dusk, shadows presence	380

TABLE 3.1. Scenarios for the performance evaluation of the lane detection.

The metrics computed to evaluate the performance of the lane detection and tracking algorithm are the Mean Absolute Error (MAE), the Root Mean Squere Error (RMSE) and the Standard Deviation of the Error ( $\sigma_E$ ) between the detected and the manually identified lane boundaries in the analyzed sample sequences considering the error in position for both lane boundaries simultaneously. Fig. 3.2 illustrates the procedure to calculate the error between the manually identified lane boundaries (dashed lines), where vertical lines depict the absolute error in lateral position estimation.



FIGURE 3.2. Manually identified lane boundaries (solid line) are compared to the detected lane boundaries (dashed lines) to calculate the error metrics.

In order to assess the proposed lane departure warning approach, a sequence of images of lane departure and lane changing events were considered. Since the estimation of the lane departure condition depends on the position of the lane, the boundaries of the lane in the test images were manually identified to generate the ground truth. The reliability and accuracy of the lane departure warning is measured in terms of False Positive (FP) and False Negative (FN) rates. The FP rate quantifies the amount of false alarms. The driver will cannot rely on a system with a high false alarm rate and will finally loose confidence on the system. On the other hand, the FN rate measures the amount of occasions in which dangerous situations go undetected. A system with a high misdetection rate can be considered inaccurate, and render it unreliable to the user as it may not help to prevent collisions due to unintended lane changes. Table 3.2 summarizes the cases considered for evaluation of the lane departure warning.

Scenario	Lane marks quality	Lighting condition	Number of samples
1	Good	Daytime, no shadows	340
2	Good	Sunset, shadows	280
3	Medium	Dusk, shadows	300

TABLE 3.2. Scenarios for lane departure warning system performance evaluation

#### 4. EXPERIMENTAL RESULTS

#### 4.1. Lane detection and tracking results

The bank of Gabor filters used for the road segmentation procedure was composed of eight filters with the combination of wavelengths  $\lambda = \{8, 4\}$  pixels and orientations  $\theta = \{0^{\circ}, 45^{\circ}, 90^{\circ}, 135^{\circ}\}$  (for optimum Gabor filters parameters selection see Li and Staunton (2008)). In case of GMRF segmentation, the neighborhood N(s) used was the star-like mask as recommended in Torres-Torriti and Jouan (2001). Mean-shift clustering was performed using the bandwidth parameters  $h_s = 2$ ,  $h_r = 4$ , which allow good clustering results under various lighting conditions.

In order to filter and track the position of the lane's boundaries, the EKF was adjusted considering that the vehicle was driven at a constant speed of 45 km/h with almost null steering except at a finite number of lane changes. The noise covariance matrices  $\Sigma_z$  and  $\Sigma_v$  of the measurement process and the driver's maneuvers, respectively, were estimated using collected data and defined for the sampling instant k as:

$$\Sigma_{v}(k) = \begin{bmatrix} 7.6 \cdot 10^{-4} & 1 \cdot 10^{-4} \\ 1 \cdot 10^{-4} & 4 \cdot 10^{-2} \end{bmatrix} \forall k \in \mathbb{Z}_{+},$$
(4.1)

$$\Sigma_{z}(k) = \begin{bmatrix} 500 & 50 \\ 50 & 500 \end{bmatrix} \forall k \in \mathbb{Z}_{+}.$$

$$(4.2)$$

The last three rows of Table 4.1 summarize the results of the three metrics considered (MAE, RMSE,  $\sigma_E$ ) in the performance evaluation of the lane detection and tracking algorithm under the four scenarios described in Table 3.1. The results show an 18.4% reduction in RMSE and MAE on average when Gabor filters are employed. An additional 7.9% reduction on average is possible thanks to the EKF. GMRF segmentation also improves the lane detection reducing the error 18.3% on average for RMSE and MAE metrics compared

to the gradient method. An additional 3.5% reduction is obtained using the EFK. Meanshift segmentation yields the best results with a reduction of 38.7% on average in the same metrics with an additional 6.1% of reduction in the error when EKF is used.

		Method													
		G Gabor filter segme				segmenta	gmentation GMRF segmentation			Mean-shift clustering					
			,	(	3	G +	EKF	(	3	G + 2	EKF	(	3	G +	EKF
		Q	С	Q	С	Q	С	Q	С	Q	С	Q	С	Q	С
	$M_1$	12.12	13.21	10.11	9.81	9.21	9.31	11.13	12.04	11.01	10.98	9.11	9.51	8.51	8.62
$S_1$	$M_2$	13.33	14.81	11.15	10.15	9.51	10.11	13.01	13.05	12.05	12.33	9.47	10.03	9.02	9.11
	$M_3$	3.21	3.92	2.91	3.01	2.51	3.13	4.01	3.98	3.68	3.56	2.19	2.52	2.21	2.41
	$M_1$	13.62	14.11	9.81	9.71	9.11	9.12	11.51	10.65	11.21	10.45	9.31	9.61	8.05	8.42
$S_2$	$M_2$	14.77	14.96	10.52	10.33	9.51	10.11	12.01	11.31	10.56	10.87	9.79	10.03	9.12	9.21
	$M_3$	3.41	3.99	2.16	2.81	2.01	2.38	5.01	4.67	4.51	4.01	2.11	2.42	2.11	2.19
	$M_1$	15.62	19.11	13.81	13.93	12.19	12.82	14.21	13.56	14.11	13.52	9.41	10.11	8.91	9.43
$S_3$	$M_2$	16.75	20.96	14.51	15.11	13.16	13.21	15.32	13.89	14.54	15.02	10.09	11.31	9.52	9.81
	$M_3$	6.11	7.97	4.16	3.81	3.14	3.28	4.71	6.11	4.65	3.98	2.51	2.28	2.19	2.25
	$M_1$	18.21	18.41	17.13	18.95	16.09	14.81	14.65	14.67	13.67	14.01	9.11	10.41	8.21	8.72
$S_4$	$M_2$	19.58	19.96	18.21	19.15	17.66	15.11	15.55	15.51	14.32	14.33	10.01	11.81	9.33	9.29
	$M_3$	7.91	8.12	5.61	5.18	5.04	4.33	4.56	5.21	4.19	4.97	2.11	2.18	2.10	2.17
	$\overline{M_1}$	14.89	16.21	12.71	13.10	11.65	11.51	12.87	12.73	12.50	12.24	9.23	9.91	8.42	8.79
	$\overline{M_2}$	16.10	17.67	13.61	13.68	12.46	12.13	13.97	13.44	12.86	13.13	9.84	10.79	9.25	9.36
	$\overline{M_3}$	5.16	6.00	3.71	3.70	3.17	3.28	4.57	4.99	4.25	4.13	2.23	2.35	2.15	2.25

TABLE 4.1. Lane detection and tracking results

 $S_i$ : Scenario  $i = \{1, 2, 3, 4\}$ 

 $M_1$ : MAE [cm];  $M_2$ : RMSE [cm];  $M_3$ :  $\sigma_E$  [cm] EKF: Extended Kalman Filter

G: Gradient features extraction Q: RANSAC quadratic curve fitting

ve fitting C: RANSAC cubic curve fitting

A difference of 1.1% on average between the results was obtained using a quadratic model and those computed with a cubic polynomial road model applying the RANSAC fitting procedure to the Gabor filters segmentation. A similar difference of 1.3% can be appreciated using GMRF segmentation, while a 3.6% of difference was obtained in case of mean-shift clustering. Thus, there is not a significative difference in the error measured using the quadratic or cubic curves for the road model. However, the cubic fitting procedure exhibits on average a 5% higher standard deviation of the error. This is explained by the difficulties in fitting the extra parameter.

From a practical implementation standpoint, an important comparison is the relative computational time of each approach. As shown in Table 4.2, the Gabor filter road segmentation done with a feature vector of size  $N_F = 19$ , on currently available CPUs, such as an Intel® Core<sup>TM</sup> 2 Duo processor at 2GHz, can be done at 3 frames per second on average. The road segmentation procedure can be performed at 5.5 fps on average using the mean-shift clustering algorithm. On the other hand, the GMRF approach is the most computationally intensive method, requiring 26.7 seconds on average for processing each frame. However, using dedicated hardware or a GPU it should be possible to decrease this time five to ten times. For comparison, the purely gradient-based method can process 11.1 fps on average, although is has a lower accuracy. Hence, the extra computational effort is worth in exchange for a more robust and accurate lane identification.

Method	Average processing time per frame [s]
Gradient	0.09
Mean-shift	0.18
Gabor filters	0.34
GMRF	26.7

TABLE 4.2. Comparison of lane detection and tracking computational time.

In order to represent the lane detection and tracking results in the image coordinate frame, a projection from the world coordinate system must be applied. Although this procedure is not required for the lane departure system, the projection of the results may be useful for the enhancement of the visualization of the road under adverse conditions such as bad weather or poor quality lane marks and providing visual support to drivers.

The coordinates of the points belonging to the curves fitted and then tracked are projected into the image plane using equations (2.25) and (2.26), derived in the Inverse Perspective Mapping section. Examples of the projection of the estimated lane boundaries onto the image plane for the four considered scenarios are shown in fig. 4.1, where challenging conditions are depicted, such as variable lighting conditions, poor lane marks and strong shadow presence.









(c) Scenario 3.

(d) Scenario 4.

FIGURE 4.1. Examples of lane detection and tracking results for the considered scenarios.

#### 4.2. Lane departure warning results

Since the lane departure estimation directly relies on the lane position measurement, the same road segmentation methods analyzed in the last section are considered for evaluation. A cubic polynomial is used for the road model because of the insignificance difference in the lane detection results when using the quadratic model as mentioned in the previous section.

In fig. 4.2 (a) it is possible to see that when the vehicle moves from the left lane to the right lane, the distance to the right boundary of the left lane decreases to zero. Once the car

has crossed over the left lane's boundary (see sampling instant in fig. 4.2 (a)), the system starts to measure the distance to the new right boundary, which now corresponds to that of the right lane. As the lane change progress the distance to the right boundary of the right lane also decreases. A similar observation can be done for fig. 4.2 (b).



(a) Right lateral distance.



(b) Left lateral distance.

FIGURE 4.2. Lane boundaries detected lateral distance (crosses) versus ground-truth (circles) under lane departure situation (sampling instant 90).

A threshold of  $\delta = 1.0 s$  was selected for the minimum allowable time to lane crossing (TLC) before triggering an alarm of unintended lane departure. In real driving scenarios, this alarm must consider the lane change indication performed by the driver under an intended lane changing to avoid false positive warnings.

Results presented on Table 4.3 show that the use of the combined gradient and road segmentation approaches leads to a lower FP and FN rate than the purely gradient-based method.

In comparison to the gradient-based method, a decrease on average of 49.5% and 52.3% of the FP and FN rate respectively is achieved using Gabor filters, with an additional 4.7% and 4.6% of decreasing respectively on both metrics when the EKF is used.

Relative to the GMRF-based segmentation, a decrease on average of 34.8% and 57.1% in the rate of FP and FN is achieved. An additional 11.1% and 7.2% decrease can be achieved if an EKF is also employed.

Comparing the mean-shift clustering approach with the gradient-based method, a decrease on average of 55.3% and 65.5% of the FP and FN rates respectively is possible. An additional 2.9% and 2.1% of decrease of FP and FN rates can be respectively achieved with the EKF.

The lane departure warning estimation has no significative impact on the processing times presented in Table 4.2 since it relies on the results of the lane detection and tracking step to compute simple mathematical operations involved in the TLC measure.

Method	Metric [%]	Scenario 1	Scenario 2	Scenario 3	Mean
Gradiant	FP	8.12	11.81	12.41	10.78
Grautent	FN	12.43	13.44	9.92	11.93
Gabor Filter Segmentation	FP	5.76	4.44	6.11	5.44
and Gradient	FN	5.11	6.31	5.65	5.69
Gabor Filter Segmentation,	FP	5.16	4.09	5.53	4.93
Gradient and EKF	FN	4.45	5.71	5.27	5.14
GMRE and Gradient	FP	6.44	7.94	6.67	7.02
Owner and Oracient	FN	7.12	4.12	4.11	5.12
GMRE Gradient and EKE	FP	5.23	6.23	6.01	5.82
Owner, Oracient and ERI	FN	6.51	3.25	3.01	4.26
Mean-shift Clustering and	FP	4.01	5.32	5.13	4.82
Gradient	FN	4.21	3.31	4.84	4.12
Mean-shift Clustering,	FP	3.71	4.88	4.91	4.50
Gradient and EKF	FN	4.01	3.21	4.39	3.87

TABLE 4.3. Lane Departure Warning results

# 5. CONCLUSION AND FUTURE RESEARCH

This work assess the effectiveness of using texture based segmentation or color-clustering methods for robust lane detection and tracking. The results showed that the road can be extracted robustly using the mean-shift method for clustering regions of similar color despite shadows and other variability in road materials, structure and illumination (results highlighted in Table 4.1). Texture features also improved the effectiveness of the standard methods based on edge-detection. However, compared to the mean-shift color clustering, obtaining textures is computationally demanding, and therefore less adequate for applications that require a high rate of frames per second.

This work also proposes an approach to robustly determine the road geometry despite the presence of occlusions and shadows, which introduce discontinuities in the extracted lane boundaries. To achieve this improvement the standard clothoidal road model is approximated by its power series expansion of order three. Obtaining the parameters of the cubic polynomials corresponding to the correct lane geometry can be carried out successfully despite outliers generated by disturbances such as shadows and occlusions, by means of the MSAC variant of the RANSAC robust parameter estimation approach. The experiments showed that a quadratic model is enough for practical purposes, and that the cubic polynomial model does not yield significally better results.

Further improvements to lane detection and departure warning are possible using an Extended Kalman filter to predict the lane location on subsequent frames whenever there are severe occlusions. It is to be noted that the tracking is performed in the 3D space of motion and not in the image plane as most of the existing lane tracking approaches. This allows us to segment the road and determine its geometry in a more accurate and robust way. Tracking in 3D space requires projection features from the image space back to the 3D space. In general this requires a multiview approach, but in this application the knowledge of a simple parameters as the height and angle at which the camera is mounted on the roof of the car is sufficient to back-project image objects onto the space of the road.

The proposed approach was also employed to calculate the time to lane crossing (TLC), which is a measure of how soon the vehicle will leave the current lane and thus provides a reliable indication of unintended lane departures that should be alerted to the driver.

Results regarding the lane departure warnings show that the use of textural and color features provide an improvement over the standard gradient approach as a reduction of false positive and false negative rates. As the performance of the proposed LDW system relies on the lane detection and tracking stage, the mean-shift clustering approach performs better than the the gradient-based method, Gabor filters and GMRF, with lower false positive and negative rates are achieved.

Future research involves the use of inertial measurements units (IMU) to improve the vehicle's motion estimation. It is also being considered the integration of other sensors, besides standard perspective cameras, such as omnidirectional cameras, stereo cameras, RADAR and LADAR to obtain an accurate 3D model of the road and overcome some limitations in the back projection of image points to the 3D space for points far in the horizon imposed by the locally flat road model.

Ongoing research is also concerned with the application of the proposed lane segmentation and tracking approach to the challenging problem of intersection recognition.

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