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On the finite size of the bag and the critical deconfining temperature in hybrid models

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We consider a chiral bag model at finite temperature, with a thermal skyrmion of Eskola and Kajantie modelling the exterior sector. Supposing that the "Cheshire Cat" scenario takes care of all zero temperature effects, we show the existence of a critical temperature over which there can only exist a quark–gluon plasma phase, since the bag is no longer stable. This fact is interpreted as the occurrence of a deconfinement phase transition.

It turns out that the critical temperature has an important dependence on the boundary conditions, due to the finite size of the bag, obeyed by the quarks and gluons of the thermal vacuum inside the bag. In fact, the corrections due to these boundary conditions imply a critical temperature bigger by almost a factor two compared to the case where the boundary conditions are ignored.

The analysis of a possible deconfining phase transition of the hadronic world, due to thermal effects, has given rise to an important amount of work during the last years. Different procedures and techniques yield a critical deconfining temperature between 120 MeV and 160 MeV. In particular, in a recent paper [1], this problem has been discussed invoking the hybrid models [2]. The main ingredients of this discussion were a chiral bag and a non-perturbative external pion field configuration, given by a thermal skyrmion of Eskola and Kajantie [3].

It is important to remark here that the pure skyrmion model at zero temperature provides a notable successful description of the nucleon and other baryons and mesons after a suitable modification of the basic model [4]. This represents a strong motivation for discussing the occurrence of thermal deconfinement in this frame.

As it is well known, the consistent construction of chiral bag models requires a careful analysis of the zero point energies at the one loop order, as well as the cancellation of ultraviolet divergences due to the presence of the bag wall [5]. These considerations, together with an external skyrmionic tail, provide a "Cheshire Cat" description of the nucleon, where all low energies physical observables, as, for example, the mass, the rms radius, the isoscalar and vector magnetic moments, etc., turn out to be essentially independent of the bag radius [5,6]. This radius seems to play an irrelevant role in the determination of physical quantities. It corresponds only to a demarcation between two different descriptions of the relevant degrees of freedom: quarks and gluons inside the bag and, outside, free pions and the non-perturbative skyrmion configuration.

The purpose of this letter is to improve the discussion presented in [1], by taking explicitly into account the effect of the boundary conditions, obeyed by the quarks and gluons of the thermal vacuum inside the bag, on the equilibrium condition between both phases. As we will see, the critical temperature emerging from this analysis turns out to be substantially bigger than the value given in [1], where the boundary conditions were neglected.

We will assume here, as in [1], that all zero point effects have been taken into account at zero temperature, giving rise to the "Cheshire Cat" scenario. The value of the equilibrium radius at finite temperature should be determined by the thermal contributions. If for a certain temperature the model is unable to pro-

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ilibrium radius, we will interpret this fact rence of thermal deconfinement.

ısume that the populated thermal vacuum d gluons inside the bag does not have any ı the baryonic number of the hadron, ne- chemical potential. The hadronic quan- rs are provided by the valence quarks, ɔurse, do not play any role in the ther- ies. The description of the external vac- a superposition of the thermal skyrmion free massless pions. Nevertheless, the cru- ıtion for deconfinement comes from to :urbative configuration.

ıs at finite temperature comes from an between the external and internal vari- essure due to thermal effects. Normally, on used for the internal pressure of the ı plasma is just the one of an ideal gas, i.e.

$$= \tfrac{37}{90}\pi^2 T^4 , \tag{1}$$

lavors and three colors are assumed [7]. ʼer, we will calculate the pressure directly tition function, taking into account in this :t of the finite size bag through the bound- ns. These produce, as usual, a set of dis- levels for the thermal partons. The wave he quarks (and antiquarks), according to ındary condition, obeys

$$)|_{r=R} = \psi(\boldsymbol{r})|_{r=R}, \tag{2}$$

$$\begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix} .$$

ʼell known [8], the free solution of the ıon with spherical symmetry is given by

$$\begin{pmatrix} j_{l_\kappa}(pr) X^\mu_\kappa \\ \mathrm{i}\dfrac{\kappa}{|\kappa|}\sqrt{\dfrac{E-m_0}{E+m_0}}\, j_{l_{-\kappa}}(pr) X^\mu_{-\kappa} \end{pmatrix} , \tag{3}$$

$_\kappa(pr)$ are spherical Bessel functions and

$$C(l\tfrac{1}{2}j; \mu - m, m) Y_l^{\mu-m} X^m$$

:tions associated to the spin (X^m) and rees of freedom ($Y_l^{\mu-m}$) through the rdan coefficients (C).

In the expressions above, j is the total angular momentum and l_κ is given by

$$l_\kappa = \begin{cases} \kappa & \kappa > 0 \\ -\kappa - 1 & \kappa < 0 \end{cases} ;$$
$$l_{-\kappa} = \begin{cases} \kappa - 1 & \kappa > 0 \\ -\kappa & \kappa < 0 \end{cases} . \tag{4}$$

From the boundary conditions we get
(a) If $\kappa > 0 \Rightarrow l_{-\kappa} = l_\kappa - 1$,

$$j_{l_\kappa}(pR) = -\sqrt{\frac{E-m_0}{E+m_0}}\, j_{l_\kappa - 1}(pR). \tag{5}$$

(b) If $\kappa < 0 \Rightarrow l_{-\kappa} = l_\kappa + 1$,

$$j_{l_\kappa}(pR) = \sqrt{\frac{E-m_0}{E+m_0}}\, j_{l_\kappa + 1}(pR). \tag{6}$$

Since the mass of the u and d quarks are very small [9] compared to the nucleon mass, we will take here the chiral limit. We will not consider possible contributions due to heavy flavors, since they are strongly suppressed by the Boltzmann factor. In this way, the energy spectrum can be obtained from the equations
(a) $\kappa > 0$ ($l_\kappa = \kappa \equiv l$), $l = 1, 2, \ldots$ ($\kappa = 1, 2, \ldots$),

$$j_l(pR) + j_{l-1}(pR) = 0. \tag{7}$$

(b) $\kappa < 0$ ($l_\kappa = -\kappa - 1 \equiv l$), $l = 0, 1, 2, \ldots$ ($\kappa = -1, -2, \ldots$),

$$j_l(pR) - j_{l+1}(pR) = 0. \tag{8}$$

The free gluons inside are treated here as eight copies of an Abelian gauge field, i.e. as eight photons. The gluon confinement can be modelled through the solution of the Maxwell equations inside a conducting spherical cavity. As it is well known [10], we have two kinds of solutions: the transverse magnetic (TM) and transverse electric (TE) cases which are given by

$$\boldsymbol{B}_{lm} = f_l(kr) \boldsymbol{L} Y_{lm},$$
$$\boldsymbol{E}_{lm} = (\mathrm{i}/k) \boldsymbol{\nabla} \times \boldsymbol{B}_{lm}, \tag{9}$$

and

$$\boldsymbol{E}_{lm} = f_l(kr) \boldsymbol{L} Y_{lm},$$
$$\boldsymbol{B}_{lm} = -(\mathrm{i}/k) \boldsymbol{\nabla} \times \boldsymbol{E}_{lm}, \tag{10}$$

respectively.

The appropriate boundary conditions emerge from imposing

$$\boldsymbol{E}_{\text{tangential}}|_{r=R} = 0\,. \tag{11}$$

Avoiding the singular solutions at the origin, we get for the TE mode

$$j_l(kR) = 0 \tag{12}$$

and for the TM mode

$$(l+1)j_l(kR) - kRj_{l+1}(kR) = 0. \tag{13}$$

In order to compute the partition function, we need the degeneration factor for the different energy levels. For the fermionic case we have a global factor 12 coming from the two flavors and the three colors, and an extra factor two since we have quarks and antiquarks in the thermal vacuum. Additionally, since the degeneration of the levels is $(2j+1)$, we have for the ε_κ level, associated to κ, a total degeneration $\Omega(\varepsilon_\kappa) = 2l \cdot 12$ if $\kappa > 0$ (here $l \equiv l_\kappa$). It is convenient to denote the energy as $\varepsilon_{ln} = a_{ln}/R$ where R is the bag radius. The a_{ln} are determined from $j_l(a_{ln}) + j_{l-1}(a_{ln}) = 0$, i.e. as the nth solution for a given l, which corresponds to radial excited states [11]. For $\kappa < 0$, $\Omega(\varepsilon_\kappa) = 2(l+1)\cdot 12$. In this case if we denote the energy levels by $\varepsilon_{ln} = b_{ln}/R$, the b_{ln} are obtained from $j_l(b_{ln}) - j_{l+1}(b_{ln}) = 0$. For gluons we have a global factor 16 due to the two helicity states and the eight different gluons. We define in an analogous way, the coefficients c_{ln} and d_{ln} such that $j_l(c_{ln}) = 0$ and $(l+1)j_l(d_{ln}) - d_{ln}j_{l+1}(d_{ln}) = 0$, where $\varepsilon_{ln} = c_{ln}/R$ and $\varepsilon_{ln} = d_{ln}/R$ are the respective energy levels.

Since we do not know the number of particles belonging to the thermal vacuum, we will use the grand canonical ensemble. The partition function is given by

$$\Xi = \prod_{\varepsilon_\kappa} (1 \pm \lambda\, \mathrm{e}^{-\beta\varepsilon_\kappa})^{\pm\Omega(\varepsilon_\kappa)}\,. \tag{14}$$

In this expression we have included the degeneration factor for each state. The $+$ $(-)$ sign refers to the Fermi–Dirac (Bose–Einstein) case, $\beta = 1/kT$ and $\lambda = \mathrm{e}^{\mu N/kT}$. Here $\lambda = 1$ since μ, the total chemical potential associated to the baryonic number of the quarks and antiquarks, vanishes. The pressure of this system is given then by

$$PV = \pm kT \sum_{\varepsilon_\kappa} \Omega(\varepsilon_\kappa) \ln(1 \pm \mathrm{e}^{-\beta\varepsilon_\kappa})\,. \tag{15}$$

Now we need only to sum the different contributions to the pressure due to the quarks, including the antiquarks, and the gluons. We will use the unit system where $k = 1$. The bag volume is $\frac{4}{3}\pi R^3$.

(a) Quarks: if $\kappa > 0$, $\Omega(\varepsilon_{ln}) = 12\cdot 2l$, where $\varepsilon_{ln} = a_{ln}/R$,

$$p_1 = \frac{18T}{\pi R^3} \sum_{l=1}^{\infty} l \sum_{n=1}^{\infty} \ln(1 + \mathrm{e}^{-a_{ln}/TR})\,. \tag{16}$$

If $\kappa < 0$, $\Omega(\varepsilon_{ln}) = 12\cdot 2(l+1)$ where $\varepsilon_{ln} = b_{ln}/R$,

$$p_2 = \frac{18T}{\pi R^3} \sum_{l=0}^{\infty} (l+1) \sum_{n=1}^{\infty} \ln(1 + \mathrm{e}^{-b_{ln}/TR}). \tag{17}$$

(b) Gluons: for the TE mode, from $\Omega(\varepsilon_{ln}) = 8\cdot 2\cdot(2l+1)$ and $\varepsilon_{ln} = c_{ln}/R$,

$$p_3 = -\frac{12T}{\pi R^3} \sum_{l=1}^{\infty} (2l+1) \sum_{n=1}^{\infty} \ln(1 - \mathrm{e}^{-c_{ln}/TR})\,. \tag{18}$$

Finally, for the TM mode from $\varepsilon_{ln} = d_{ln}/R$ we have

$$p_4 = -\frac{12T}{\pi R^3} \sum_{l=1}^{\infty} (2l+1) \sum_{n=1}^{\infty} \ln(1 - \mathrm{e}^{-d_{ln}/TR})\,. \tag{19}$$

The total internal pressure is given by

$$P_{\text{in}} = p_1 + p_2 + p_3 + p_4\,. \tag{20}$$

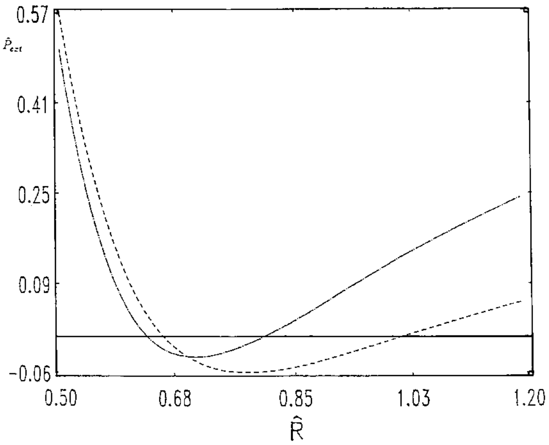
For the external pressure nothing changes with respect to the treatment in [1]. The contribution of the free pion gas is

$$P_{\text{pions}} = \tfrac{3}{90}\pi^2 T^4\,. \tag{21}$$

The variation of the pressure due to the skyrmion,

$$\Delta P_{\text{sk}}(T) = P_{\text{sk}}(T) - P_{\text{sk}}(0), \tag{22}$$

can be calculated from the dependence of the thermal skyrmion energy as a function of the bag volume. The energy (mass) of the skyrmion outside the bag, in units of $F_\pi/4e$ (where e is the constant in front of the



ȷ. 1. In fig. 1 we show the difference between the internal ɗ external pressures ($\widehat{P}_{in} - \widehat{P}_{ext}$) as a function of the bag lius ($\widehat{R}$) for different temperatures: – – – : $\widehat{T} = 0.5$, – · – : $\widehat{T} = 0.6$. Here $\widehat{T} = (2/eF_\pi)T$ corresponds to : dimensionless temperature and T_c to the critical tem-ːature.

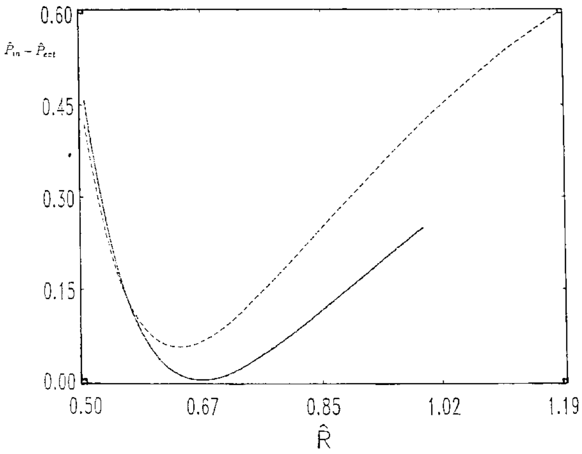


Fig. 2. As fig. 1, with the critical temperature also included: ——— : $\widehat{T_c} = 0.65$, – – –: $\widehat{T} = 0.7$.

ıbilizing term in the Skyrme Lagrangian, and F_π is ɜ pion decay constant [12]) is given by

$$(R,T) = \int_R^\infty I(r,T)\,\mathrm{d}r, \tag{23}$$

th

$$r,T) = 4\pi\left\{\left[r^2\left(\frac{\partial f}{\partial r}\right)^2 + 2\sin^2 f\right] + \frac{\sin^2 f}{r^2}\left[2r^2\left(\frac{\partial f}{\partial r}\right)^2 + \sin^2 f\right]\right\}, \tag{24}$$

ıere R is the radius of the bag measured in units $2/eF_\pi$. In the previous expression, f denotes the ɔfile function of the thermal skyrmion of EK [3]. ıs given by

$$r,T) = \pi\left[1 - \frac{r + \frac{1}{2}\lambda^2(\mu\coth(\mu r) - (1/r))}{\sqrt{r^2 + \frac{1}{2}\mu^2\lambda^4 + \mu r\lambda^2\coth(\mu r)}}\right]. \tag{25}$$

ıe constant μ in this formula is $2\pi T$, where the tem-rature has been expressed in units of $eF_\pi/2$, and s the size (in the same units as R) of the instan-ı which gives rise to the skyrmion. Now, the pressure of the skyrmion at radius R and temperature T, $P_{sk} = \partial M/\partial V_{bag}$, is given by

$$P_{sk}(R,T) = -\frac{I(R,T)}{4\pi R^2}. \tag{26}$$

At a given temperature, a state of equilibrium between the internal and external phases can exist only if there is a radius at which the pressures are equal. The equilibrium condition is $P_{in} = P_{ext}$. In terms of the dimensionless variables defined above, the expression for the internal pressure should be multiplied by $(e^4F_\pi^2/16)$. So we have

$$P_{in} = \frac{(I(R,0) - I(R,T))}{8e^2\pi R^2} + \tfrac{3}{90}\pi^2T^4. \tag{27}$$

For the numerical analysis of this expression we have used the constant values for F_π and e according to ref. [12], fitted from the proton and Δ masses. In ref. [3] it is shown that the size λ varies between 1.45, for $\mu = 0$, and 2.5 in a reasonable range of variation of T. It is important to remark that the value of F_π we have used, 129 MeV, is smaller than the experimental one, 186 MeV, using the normalization of ref. [12].

We find that eq. (27) in general has two solutions (i.e. two possible values of the radius for each temperature), up to a point where both values coincide. Beyond this point there is no solution. This is shown in figs. 1 and 2. Below the critical temperature, only one of the two radii is physical. The real physical radius emerges from imposing the thermodynamical equilibrium criteria, according to which the second derivative of the thermodynamical functions should be negative. In this way, by looking at fig. 1 it is easy to see that the physical radius corresponds to the smaller radius.

We interpret the absence of a real solution of eq. (27) as the occurrence of a deconfining phase transition, since for higher temperatures only the quark–gluon plasma phase does exit. For our estimates we have taken $\lambda = 1.45$. For the parameters written above, we find that the critical temperature is $T_c = 228$ MeV. This temperature is almost a factor two bigger than the value given in [1]. The effect of the finite size of the bag turns out to be extremely important.

The critical temperature grows linearly with F_π. The critical temperature turns out to be much more dependent on F_π than on e. In general, according to EK [3] the value of λ grows with temperature, if we minimize the mass of the thermal skyrmion for each value of T, diminishing the value of the critical temperature for each value of F_π. The dependence on the parameter e turns out to be much less sensitive than in the case with a constant value of λ. From our discussion, it turns out that F_π is the most relevant parameter in order to fix the critical temperature. The detailed discussion about the dependence on the different parameters is presented in [1].

The mechanism responsible for the occurrence of this phase transition can be viewed as a concentration of the skyrmion around the origin as the temperature grows [3]. This has the effect of increasing the pressure on the bag due to the tail of the skyrmion, compared to the zero temperature pressure . In other words, the tail of the skyrmion always "sucks" the bag to the outside, being the pressure proportional to the height of the profile function at R. The temperature diminishes this height, giving rise to an increase of the external pressure. This can compensate the corresponding increase of the internal pressure only up to T_c. Note that this critical temperature is smaller than $1/\lambda \approx 234$ MeV. In fact, $1/\lambda$ is a natural bound for the temperature in order to avoid instanton interactions. We have to remark here that although we have now a higher critical temperature it is still less than this natural bound.

We would like to note that from our analysis the values for the equilibrium radius R are around 0.7 and 1.3 fm. However, this value should not necessarily be identified with the physical radius of the hadron, since the tail of the skyrmion is part of the hadronic structure in this picture.

It is possible to carry out the same analysis for the case of genuine chiral boundary conditions, where a chiral angle θ_5 is introduced. The chiral bag has been thoroughly discussed in the literature [5, 13] and we will not present here the details, but only mention the principal differences. In the chiral bag there are more energy levels than in our case. These levels are classified according to the so-called grand angular momentum $\boldsymbol{K}$, a peculiar mixture between angular momentum and isospin degrees of freedom $\boldsymbol{K} = (\boldsymbol{L}+\boldsymbol{S})+\boldsymbol{I}$.

The higher abundance of energy levels implies that the critical temperature becomes about a 15% smaller than the value we have presented here, for the same values of the parameters and for small chiral angle values (not close to $\theta_5 = \frac{1}{2}\pi$). The critical temperature develops a smooth dependence on the chiral angle, as long as θ_5 remains small. For $\theta_5 = \frac{1}{2}\pi$ a singularity occurs for the ground state because it starts to dive into the Dirac sea.

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