Comment on "Instability and Entanglement of the Ground State of the Dicke Model"

In their recent Letter [1], Buzek, Orszag, and Rosko returned to the fundamental model of quantum optics, the Dicke model. They showed that the well known instability of its ground state has interesting entanglement properties. The Dicke model Hamiltonian

$$H = \frac{\hbar\omega_A}{2}\sum_{j=1}^{N}\sigma_j^z + \hbar\omega_F a^\dagger a + \hbar\kappa\sum_{j=1}^{N}(e^{i\vec{r}_j\cdot\vec{k}}a^\dagger\sigma_j^- + \text{H.c.}), \tag{1}$$

as in Ref. [1], written here in its simplest form (in a rotating wave approximation), is usually used to describe a nearly resonant interaction of N atomic dipoles of a particular transition with a single quantized mode of the electromagnetic field. The Pauli matrices $\sigma^{(z,-,+)}$ are natural to use for the two-dimensional Hilbert space of the two relevant levels of each atom. The electromagnetic field is represented by the oscillatorlike creation and annihilation operators $a^\dagger, a$ of the photons. The authors of Ref. [1] explicitly refer to the electric dipole transitions when they first invoke the Hamiltonian (1).

While what follows in Ref. [1] is certainly correct in a mathematical sense, we wish to point out an obvious physical drawback of this Letter. As we have shown back in 1991 [2], the instability of the ground state of the Dicke model is solely a result of approximations. Most important is the neglect of the A^2 term from the minimal coupling Hamiltonian. It spoils the gauge invariance of the atom-field Hamiltonian (1). In fact, the atomic electron, if found in the anomalous ground state discussed in Ref. [1], would have a negative kinetic energy.

In Ref. [2], we have shown that the full, nonrelativistic atom-field Hamiltonian has a spectrum bounded from below by the energy of the ground state of the bare atom. This rigorous inequality is violated by the anomalous, entangled ground state discussed in Ref. [1].

It is the purpose of this Comment to clarify to the wide audience of Physical Review Letters readers that such an entangled ground state and instability cannot occur in a system involving atom-field interaction.

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